CONTROL ENGINEERING LAB

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GROUP 18

AMAN SAINI (2020EEB1155)

ANIRUDH SHARMA (2020EEB1158)

ANSAF AHMAD (2020EEB1160)

OBJECTIVE:

Given a three-variable digital system with parameters susceptible to the arbitrariness of values (adjustable by the operator), we need to check out the scope and limits of possible deadbeat type performance.

INTRODUCTION:

The following three-variable system has arbitrary values or settings possible for parameters a and b:

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \\ x_3(k+1) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ a & b & -(a/b) \end{bmatrix} \cdot \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} [u(k)]$$

We need to test if the following requirements can be fulfilled:

1.) With initial state $\mathrm{x}(0) = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, if $\mathrm{x}(3) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ is achievable.

2.) With initial state $\mathrm{x}(0) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$, if $\mathrm{x}(3) = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ is achievable.

In the case of achievability (*for open-loop operation*), we need to find out the range for values of parameters ***a*** and ***b*** and the required input corresponding to those values. If the above requirements are unachievable (*for the open-loop system*), we would require a feedback control for the system to fulfill the given requirements.

It is evident from the problem statement that the studies on the system are limited to just 3-time steps and how the system behaves throughout that duration; after that, the problem reduces to keeping the states constant at those fixed values.

SIMULINK MODEL USED:

The following Simulink model was used to simulate the given digital system model:

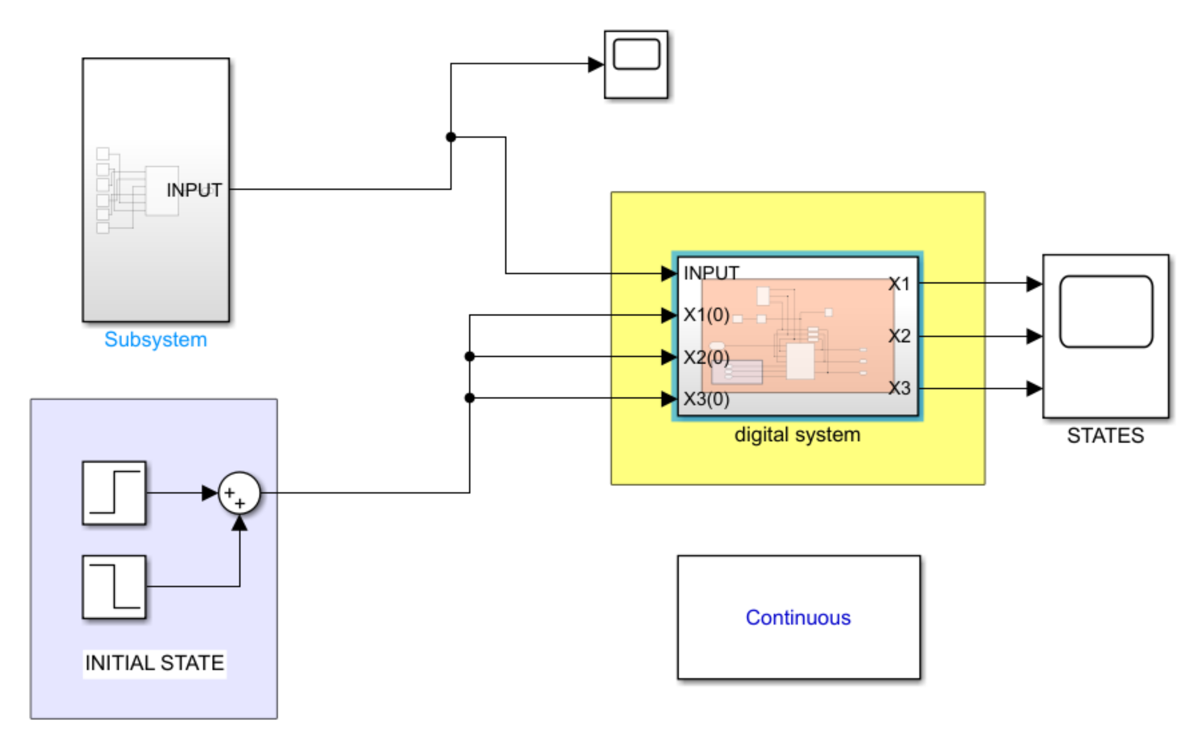

Fig 0a: *the given digital system is a triggered system (which would become evident once we look under the mask), the initial state is loaded at t = 1, and simultaneously input is provided to the system. This delay of one-time step is done because the triggered subsystem cannot detect a rising edge at t = 0 (when the simulation begins).*

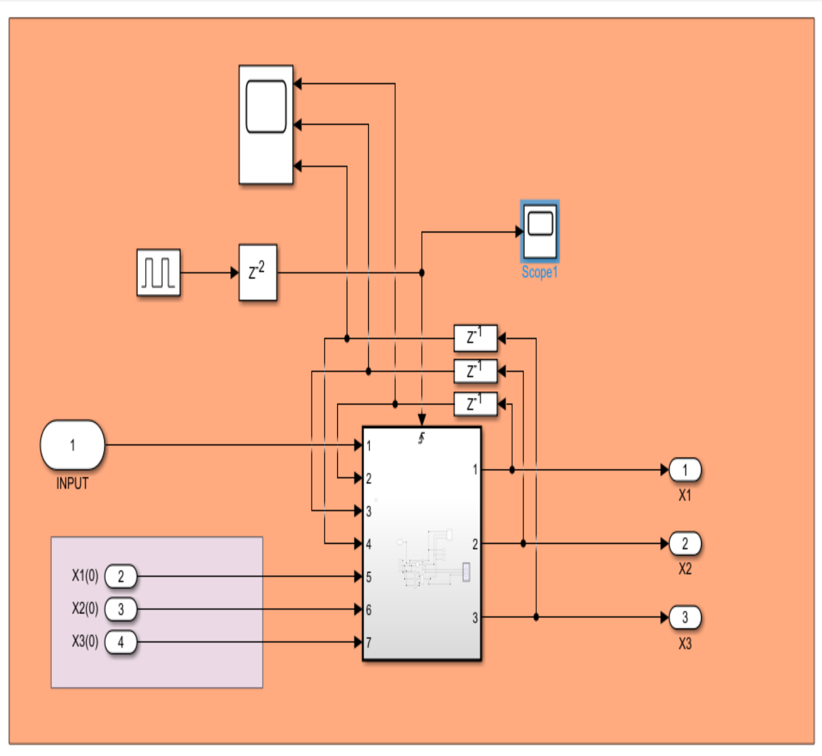

Fig0b: *the digital subsystem, is triggered by a clock signal with time period of 1 sec and the inputs provided to the system includes u(k - 1) the previous states x(k-1) which are then used to compute x(k). The delay blocks are used for delaying the signals. Notice how no delay block is connected to the input block, that is because it is compensated in the input subsystem (in fig 0a) where we provide the input with a delay of one-time step and hence the delay is internally compensated by the input block*

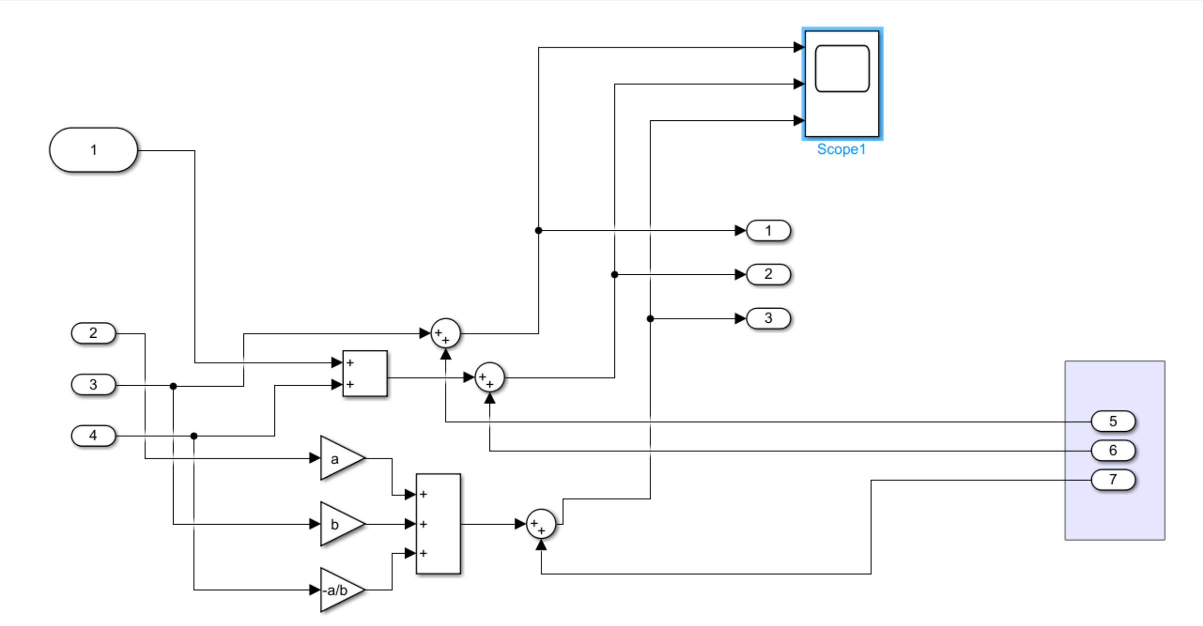

Fig 0c: *the triggered subsystem, it receives the value of previous states and input on the previous step and calculates the current state using the parameters a & b.*

OPEN-LOOP DYNAMICS:

From the given matrices describing the system, we obtain the following equations:

$$\boldsymbol{x_1(k+1) = x_2(k)}$$

$$\boldsymbol{x_2(k+1) = x_3(k) + u(k)}$$

$$\boldsymbol{x_3(k+1) = a \cdot x_1(k) + b \cdot x_2(k) - {}^{a}/_{b} \cdot x_3(k)}$$

The state x_1 purely depends on the previous value of x_2; the state x_2 is the only state that is directly controllable by the input that we provide to the system; and the state x_3 is a linear combination of the states at the previous time step with arbitrariness due to ***a, b & -a/b*** present as parametric gains.

Requirement No. 1

$$x(0) = \begin{bmatrix}1\\1\\1\end{bmatrix};\ x(3) = \begin{bmatrix}0\\0\\0\end{bmatrix}$$

We start out the system from $[1 \quad 1 \quad 1]^T$ and check if achieving $x(3) = [0 \quad 0 \quad 0]^T$ at 3rd time step is possible or not. It is clear equations driving the system that the output (in our case all the states of the systems) would only be influenced by the input at ***k = 0, 1 & 2*** and therefore we can achieve the value of states at 3rd time step in terms of ***a, b, u(0), u(1) and u(2).***

$$x_1(3) = a + b + u(1) - a/b = 0$$

$$x_2(3) = u(2) + b \cdot (u(0) + 1) - {a^2}/{b} + {a^2}/{b^2} = 0$$

$$x_3(3) = {a^3}/{b^2}\left(1 - {1}/{b}\right) = 0$$

From the above equations it is clear to see that either *b = 1 and a is arbitrary* **or** a = 0 and *b is arbitrary*, we solve for the input for both the cases.

<u>Case I: (b = 1 and a is arbitrary)</u>

From $\boldsymbol{x_1(3) = 0}$ *we have:* $\boldsymbol{u(1) = -1}$

From $\boldsymbol{x_2(3) = 0}$ *we have:* $\boldsymbol{u(2) + u(0) = -1}$

This tells us on how to shape the input to the system, we also get the range for the parameters a and b with *a being arbitrary and b = 1*. These conditions result in the system attaining the state $x = [0 \quad 0 \quad 0]^T$ at ***3rd time step*** and after that the system can maintain this value with 0 input (because term reduces to 0) as is clear from the equations given below:

$$x_1(k+1) = x_2(k) = 0 \ (for\ k \geq 3)$$

$$x_2(k+1) = x_3(k) + u(k) = 0 \ (as\ u(k) = 0\ for\ k \geq 3)$$

$$x_3(k+1) = a \cdot x_1(k) + b \cdot x_2(k) - {a}/{b} \cdot x_3(k) = 0 + 0 - 0 = 0 \ (for\ b = 1\ and\ a\ any\ arbitrary\ constant)$$

<u>Case II: (a = 0 and b is arbitrary)</u>

From $\boldsymbol{x_1(3) = 0}$ *we have:* $\boldsymbol{u(1) = -b}$

From $\boldsymbol{x_2(3) = 0}$ *we have:* $\boldsymbol{u(2) + b \cdot u(0) = -b}$

For this case, the input is needed to be provided in accordance to what value we decide for parameter ***b*** unlike the previous case where we dealt with a simple constant. This may place some limitations on the value of b in accordance to how high of an input signal can be given to the system. Also, this means designing and providing a different input every time we change the value for parameter b. And lastly just like the previous case, the input can be reduced to 0 once we achieve the desired state as every term in the equations describing the system also reduces to 0 (*in fact providing an input to the system would disturb the state*).

The following plots illustrate the response of the system to the above conditions (case I & II):

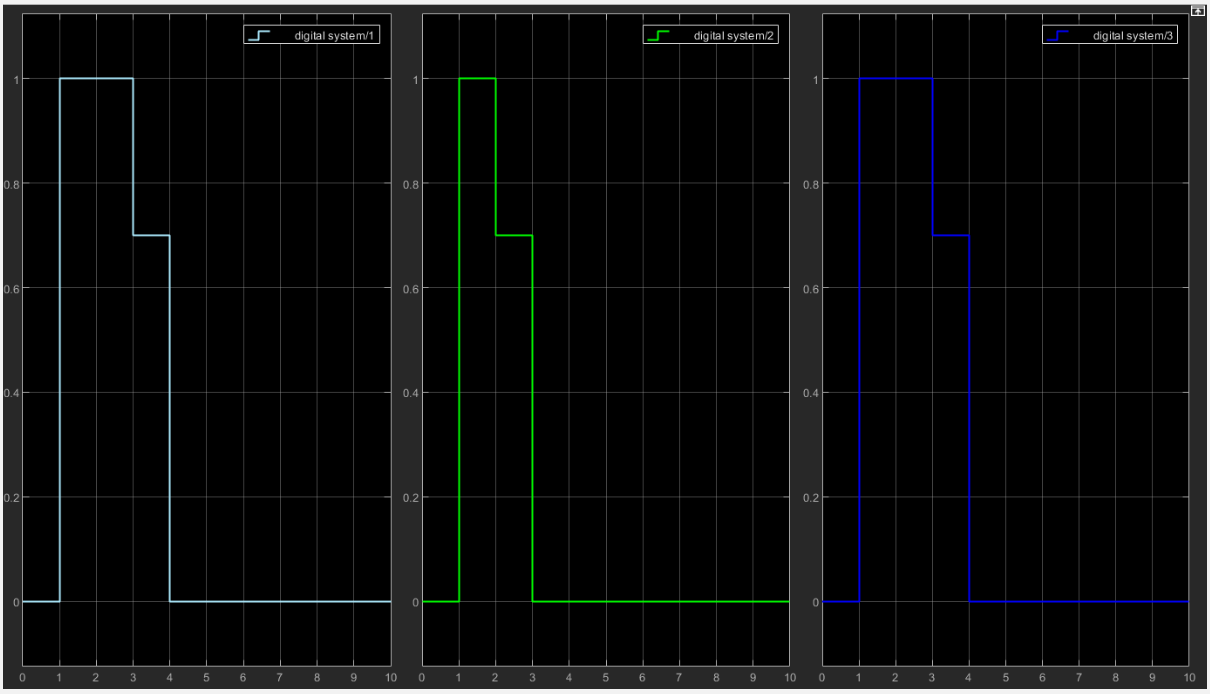

Fig 1a: *[a = 1, b = 1, u = {-0.3, -1, -0.7, 0, 0...}] On providing input corresponding to case-I we see that the system settles at* $[0 \quad 0 \quad 0]^T$. *The simulation begins at t = 1 i.e., system acquires the initial state at t = 1 and we start giving input at the same time which makes it appear that system is reaching desired state at t = 4, which in reality corresponds to* 3^{rd} *time step. (We can set any arbitrary value for parameter a with the same input and b = 1 and we will still satisfy this requirement)*

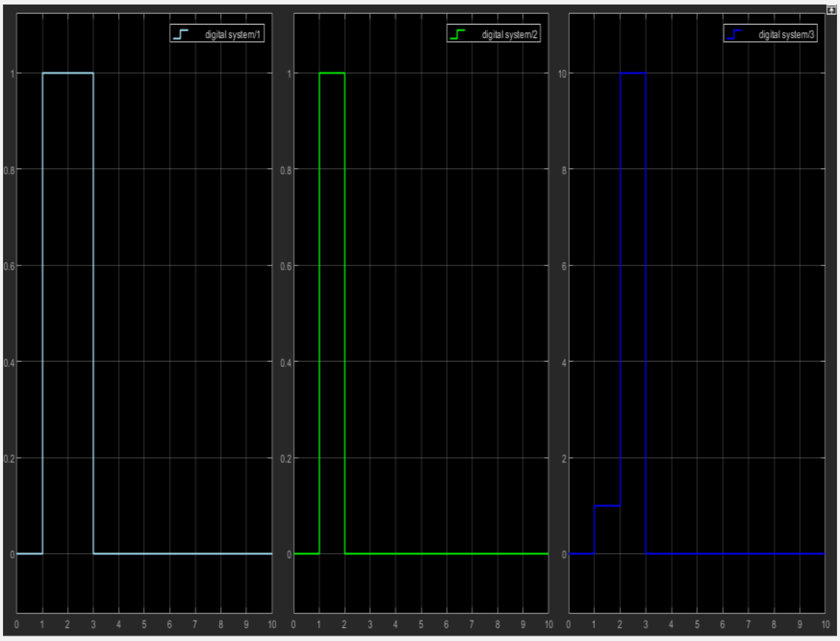

Fig1b: *providing input to the system corresponding to case II i.e., a = 0, b is arbitrary (10 for the present case) and u = {-1,-10,0,0, …}, the desired state* $[0\ 0\ 0]^T$ *is reached within 2-time steps however there is a very high overshoot (equal to b) for* x_3 *state due to high value of parameter b, which places a limit on large the value of parameter b can be corresponding to limitations on state 3.*

Requirement No. 2

$$x(0) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix};\ x(3) = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

As done for the previous case the value of states at k = 3 can be obtained in terms of ***a, b and u(0), u(1) & u(2)*** using **MATLAB**, these results then can be used to obtain the range for parameters ***a & b*** and the constraints on input ***u***. The following equations are obtained:

$$x_1(3) = u(1) = 1$$

$$x_2(3) = u(2) + b \cdot u(0) = 1$$

$$x_3(3) = b \cdot u(1) = 1$$

It is obvious from the equations obtained that $\boldsymbol{u(1) = 1}$, therefore **b = 1** (from $x_3(3) = 1$) and so $u(2) + b \cdot u(0) = \boldsymbol{u(2) + u(0) = 1}$ and ***parameter a is arbitrary.*** Imposing these conditions on input and parameters a and b we obtain the following plots in Simulink:

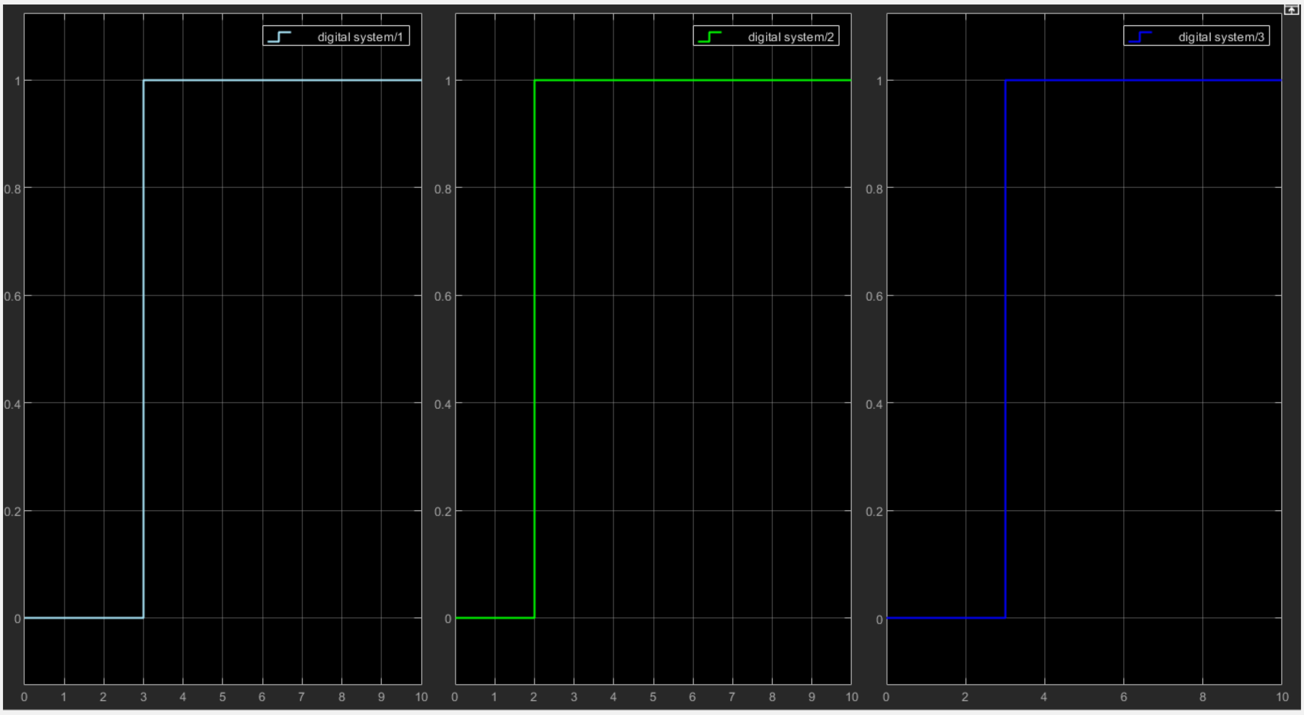

Fig 2: *for this case we have a = 1 and b = 1, and the input is: u = {1, 1, 0, 0…} and as in the previous case the simulation starts from t = 1 i.e., we update the system with initial states at 1st time step and simultaneously start providing input. The system acquires the desired state* ***x = [1 1 1]*** ^{T} *at t = 3 i.e., on 2nd time step after switching on the input. Also, in this case as b = 1, therefore the system maintains the state* ***[1 1 1]*** ^{T} *even after the input is switched off. (a + b - a/b = 1 for b = 1).*

As is evident from the above plots that both the requirements are possible to be met for the open loop system by adjusting parameters and input which is summarized below:

Requirement No. 1:

Case I: (b = 1 and a is arbitrary): $u(1) = -1$ & $u(2) + u(0) = -1$

Case II: (a = 0 and b is arbitrary): $u(1) = -b$ & $u(2) + b \cdot u(0) = -b$

Requirement No. 2:

(b = 1 and a is arbitrary): $u(1) = 1$ & $u(2) + u(0) = 1$

*And for **b = 1** and choosing an arbitrary **a** lead to both the requirements being met by the system provided that input meets the desired conditions.*

DEADBEAT (FEEDBACK) CONTROL:

We find the gains for state feedback in such a way which place all 3 eigen-values of the system at the origin of z-plane. Let the feedback gain matrix be $K = [c \quad d \quad e]$ which alters the A matrix as:

$$A = A - B \cdot K = \begin{bmatrix} 0 & 1 & 0 \\ -c & -d & 1-e \\ a & b & -{}^{a}/_{b} \end{bmatrix}$$

The eigen values for this system would be given as:

$$\boldsymbol{eig(1) = -{}^{d}/_{2} - \sqrt{d^2 + 4 \cdot b - 4 \cdot c - 4 \cdot b \cdot e}/_{2}}$$

$$\boldsymbol{eig(2) = -{}^{d}/_{2} + \sqrt{d^2 + 4 \cdot b - 4 \cdot c - 4 \cdot b \cdot e}/_{2}}$$

$$\boldsymbol{eig(3) = -{}^{a}/_{b}}$$

First two conditions that we obtain from equating the above eigen-values to 0 are: **a = 0 & d = 0**, then we have *b – c – be = 0, i.e., b (1 – e) = c, setting the value of parameters in accordance to these parameters we get the following set:*

a = 0, b = 1, c = 1, d = 0, e = 0 (there is a reason for choosing b = 1 which is explained in fig 4)

Simulating the system for this value of feedback provides us the following response:

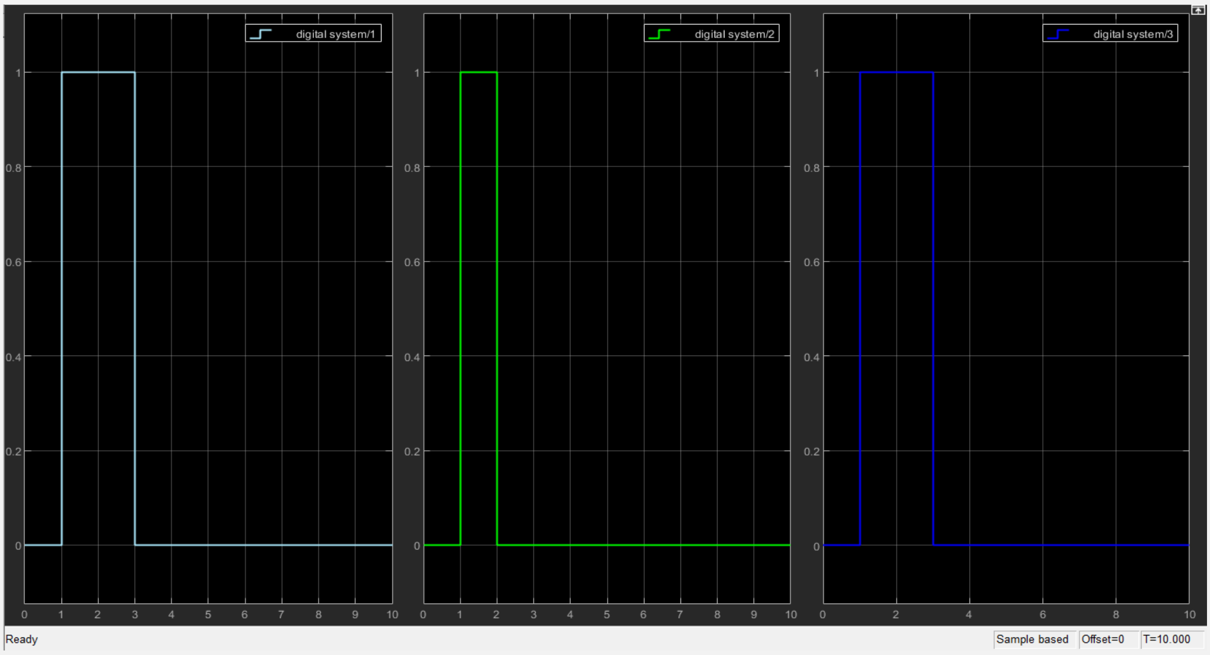

Fig 3: *the response of the closed loop system to the initial state [1 1 1] ^{T} which is loaded into the system at t = 1, and we see that the system achieves the desired state at t = 3 i.e., on the 2nd time step (one-time step earlier) and maintains the value, it should also be noted that no input has been provided for this case, we just provide the initial conditions and feedback loop does the rest, this way system shows deadbeat performance i.e., achieving the steady state in least number of time steps possible.*

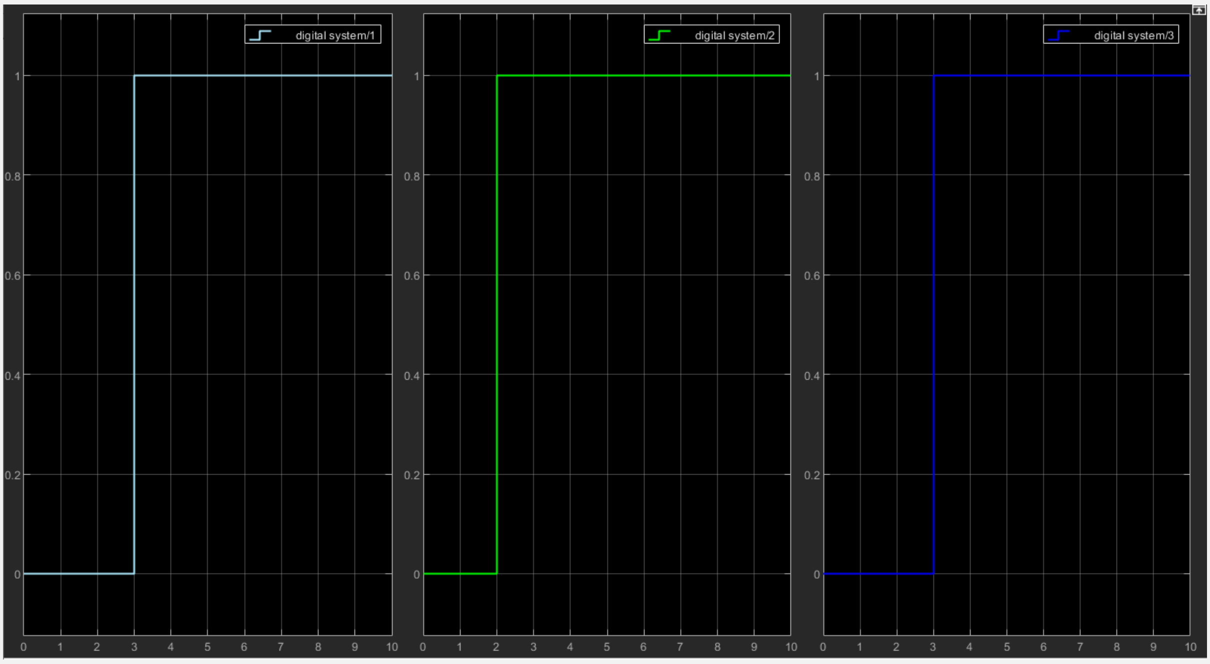

Fig 4: *the step response of the system operating with the designed feedback, the system achieves the desired state [1 1 1]* ^{T} *from initial state [0 0 0]* ^{T} *within 2-time steps after we start providing input (at t = 1, step input for this case). It is also important to know that for achieving the given step response it is essential that b = 1 (and not some arbitrary value) otherwise at steady state* $x_3(k) = b\, x_2(k)$ *which means that system settles at [1 1 b]* ^{T} *which explains why it is important that we choose b = 1 for the proper operating feedback loop which meets both of our conditions.*

It is also important to note that here the input serves as reference for each state of the system and the design of feedback ensures that every state of the system reaches that reference point within 3-time steps (as the order of the system N = 3 therefore it requires a maximum of 3 steps to drive the system from any state to the desired state). Once we reach the steady state, we have $x_1 = x_2$ in every case and x_3 can be adjusted using the parameter: b (as a = 0 for the eigen values to lie at origin for closed loop system) and accordingly we need to adjust the feedback gains c and e which also depends on the value of b, hence we gain control over 2 of the states of the system (*x_2 and x_3 (with the help of adjustable parameter b); $x_1 = x_2$ at steady state for every case*) which is also evident from the controllability matrix whose rank comes out to be 2.

$$C = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & b \\ 0 & b & 0 \end{bmatrix}$$

CONCLUSIONS:

We studied the given digital system for both the open loop and closed loop case, for open loop case we found the range of parameters a and b and corresponding conditions on input for achieving the desired requirements that system needs to fulfill, then we studied the system in closed loop (such that the eigen values of the closed loop system all lie on the z-plane origin), for which case we were able to achieve the desired state within 2 time steps with no complexity of input (input being the reference), we also noted that for closed loop operation the parameter b can be used to alter the steady state value for state x_3 which make it sort of independent from state x_2 however state x_1 is equal to state x_2 when the steady state is achieved.

MATLAB SCRIPT USED:

```
syms a b c d e;

A = [0 1 0; 0 0 1; a b -(a/b)];
B = [0; 1; 0];
C = [1 0 0; 0 1 0; 0 0 1];
D = [0; 0; 0];
K = [c d e];
%sys = ss(A, B, C, D, 1);

x = [1; 1; 1]; %initial state
syms u0 u1 u2; % input provided to the system
for i = 1:3
    if(i == 1)
        x = A*x + B*u0;
    end
    if (i == 2)
        x = A*x + B*u1;
    end
    if (i == 3)
        x = A*x + B*u2;
    end
end
```