GROUP 18

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CONTROLLER DESIGN ON MATLAB PLATFORM USING ANALOG ROOT LOCI

OBJECTIVE

- To design of a parameter for a given analog transfer function according to desired specifications
- To perform sensitivity analysis for variation of key parameters.

ANALOG OPEN LOOP TRANSFER FUNCTION TO BE OPERATED IN CLOSED LOOP

The given Open Loop Transfer Function is

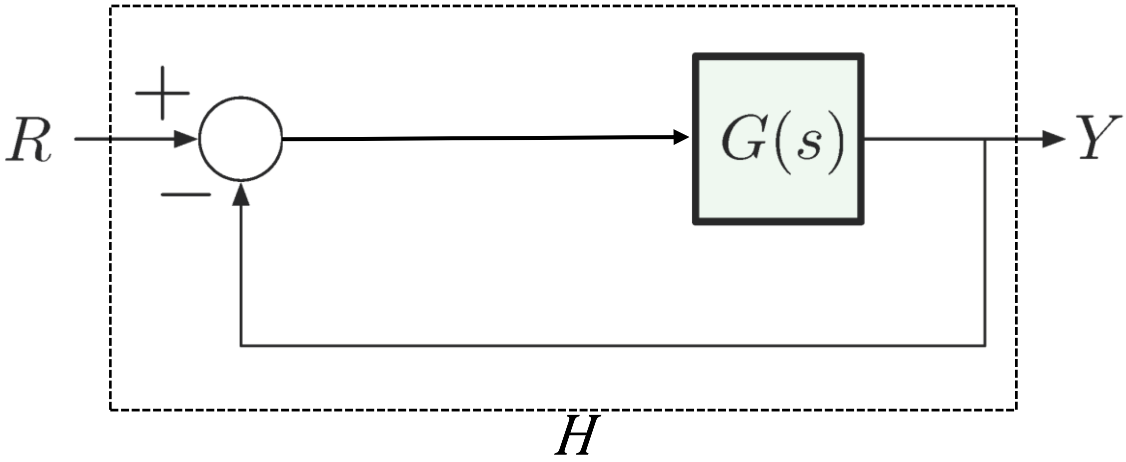
$$G = \frac{30}{s(1+0.1s)(1+0.2s)(1+Ts)}$$

We are required to select T such that the complex poles have a damping ratio between 0.2 to 0.25 taking into account that each parameter in the transfer function is prone to variation of $\pm$20% of the original values.

TRANSFER FUNCTION TAKEN IN CLOSED LOOP

$$G = \frac{30}{0.02T\ s^4 + (0.02+0.3T)s^3 + (0.3+T)s^2 + s}$$

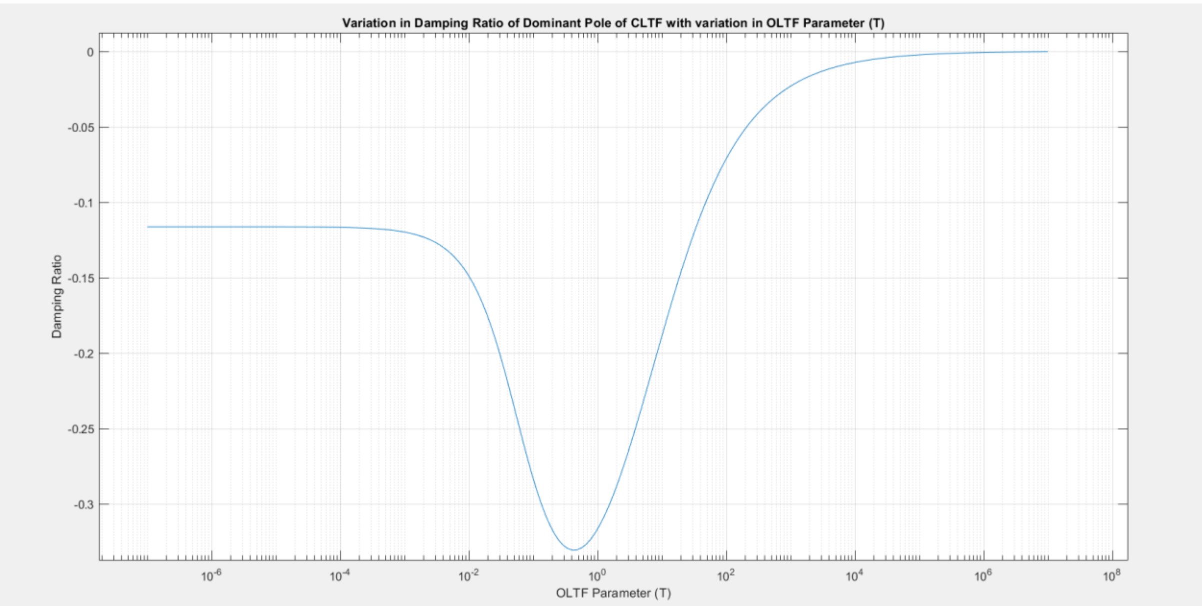
Taking this function in the following configuration-



We get the following Closed Loop Transfer Function-

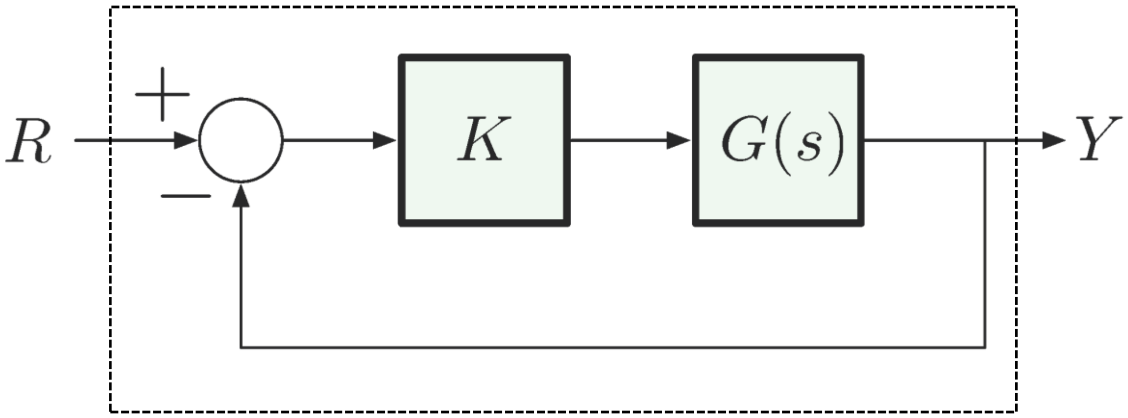
$$H = \frac{30}{0.02T\,s^4 + (0.02 + 0.3T)s^3 + (0.3 + T)s^2 + s + 30}$$

Given the system, we plot the Damping Vs Variation of Parameter ‘T’ as below.



We observe that for **no values** of ‘**T**’ the System Requirement (0.25>Damping Ratio>0.2) is fulfilled. Hence no such system can be made without implementing any changes to the given system. To make our analysis less complicated we make the most simplistic change to the given system. We add a proportional gain ‘K’ in cascade with the OLTF. Hence our analysis revolves around two parameters i.e., ‘T’ & ‘K’.

Hence, our new configuration is-

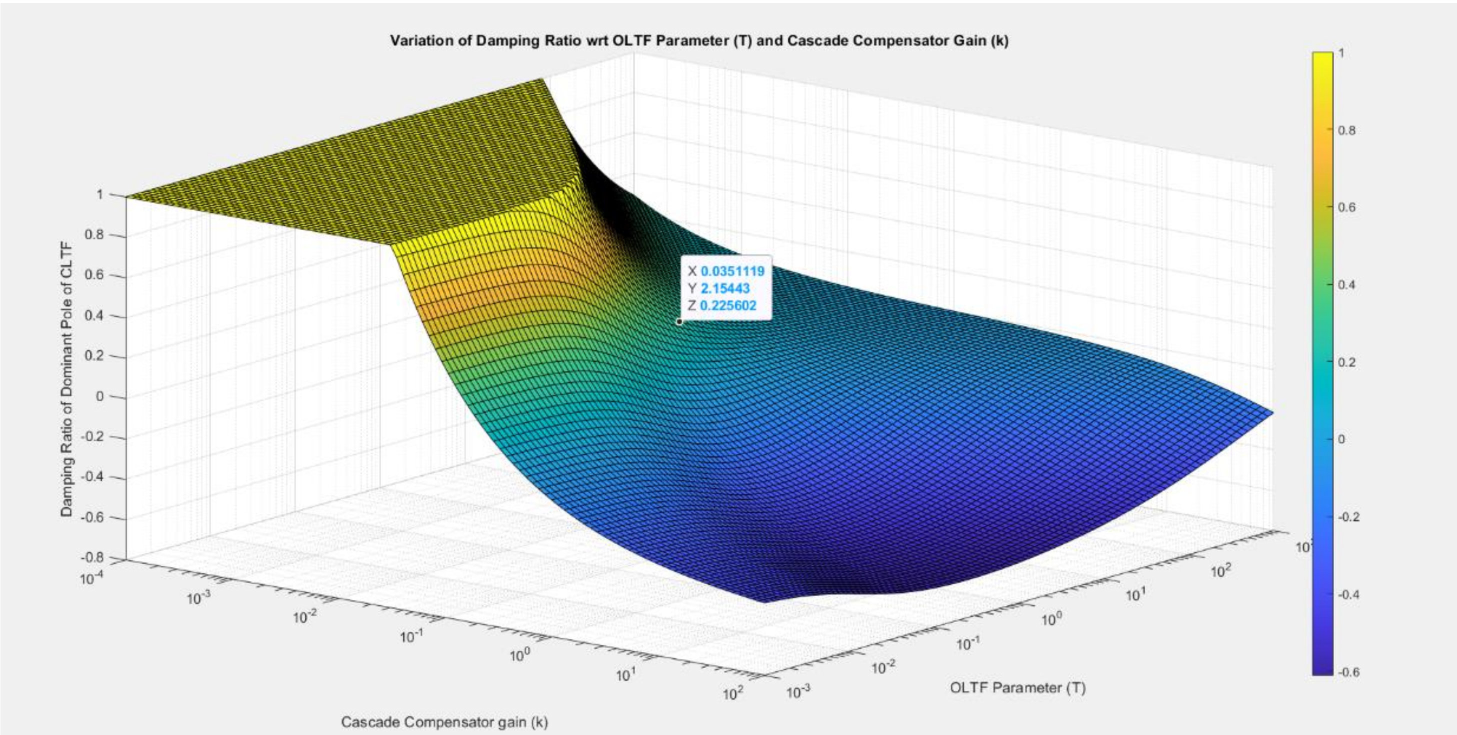


H

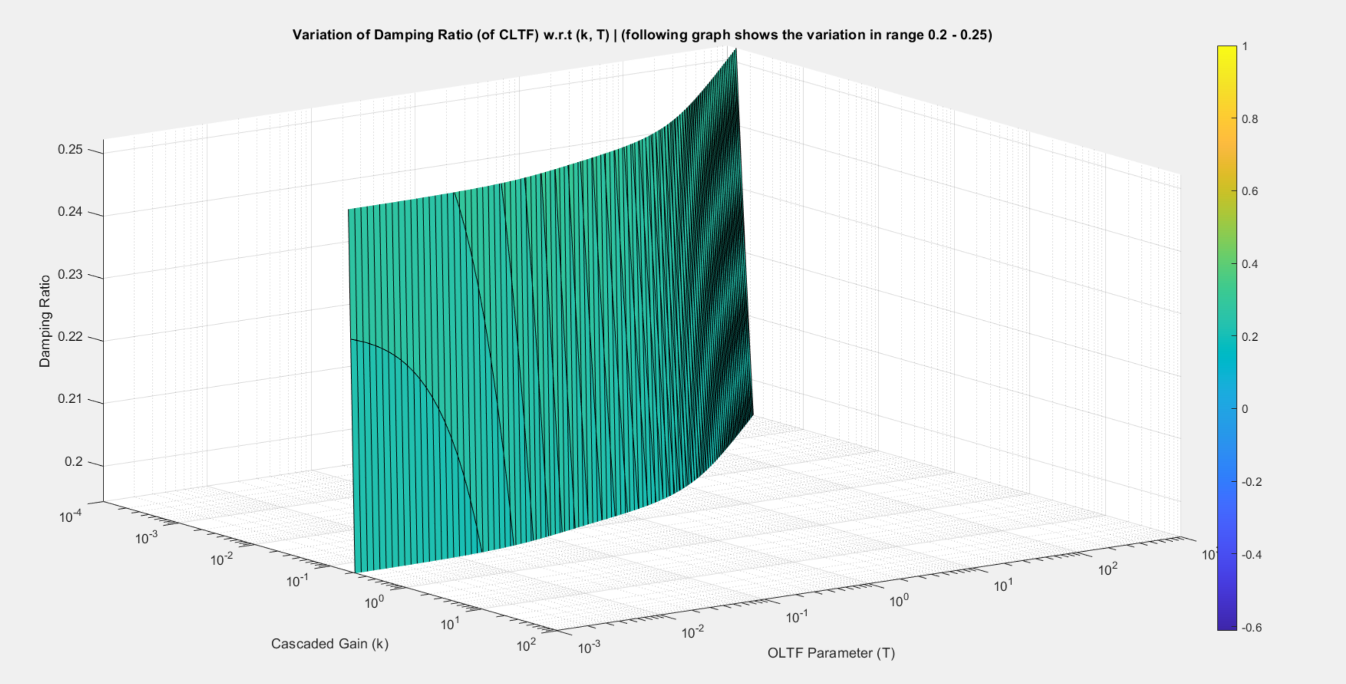
Hence, the CLTF is-

$$H = \frac{30K}{0.02T\ s^4 + (0.02 + 0.3T)s^3 + (0.3 + T)s^2 + s + 30K}$$

To cement upon a value for 'T' & 'K' we plot a 3-D graph of **Damping Ratio (ζ)** (z-axis) vs '**T**' vs '**K**'.

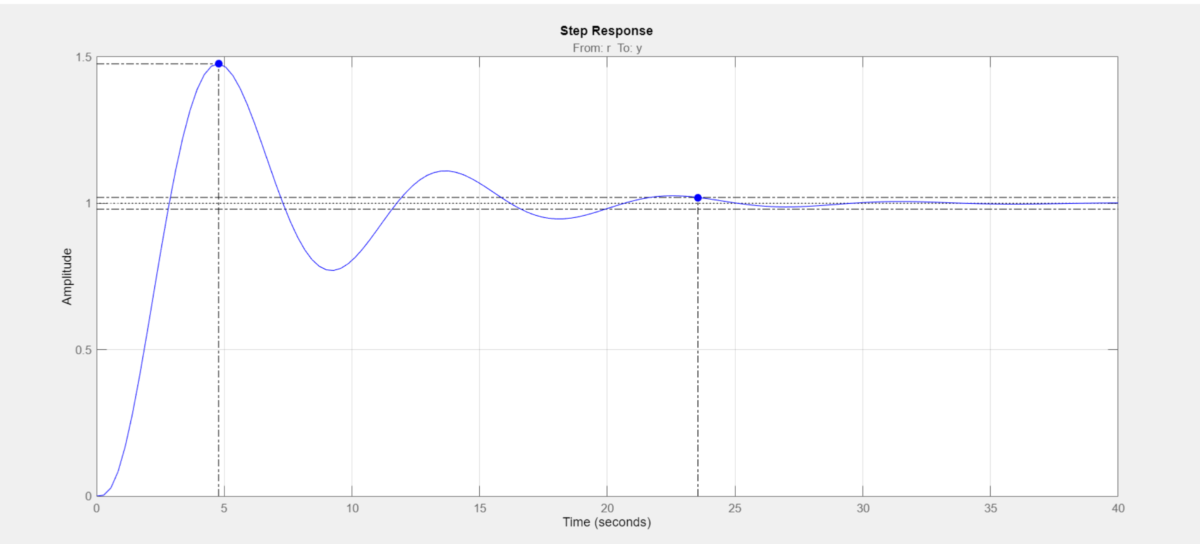


On closer look,

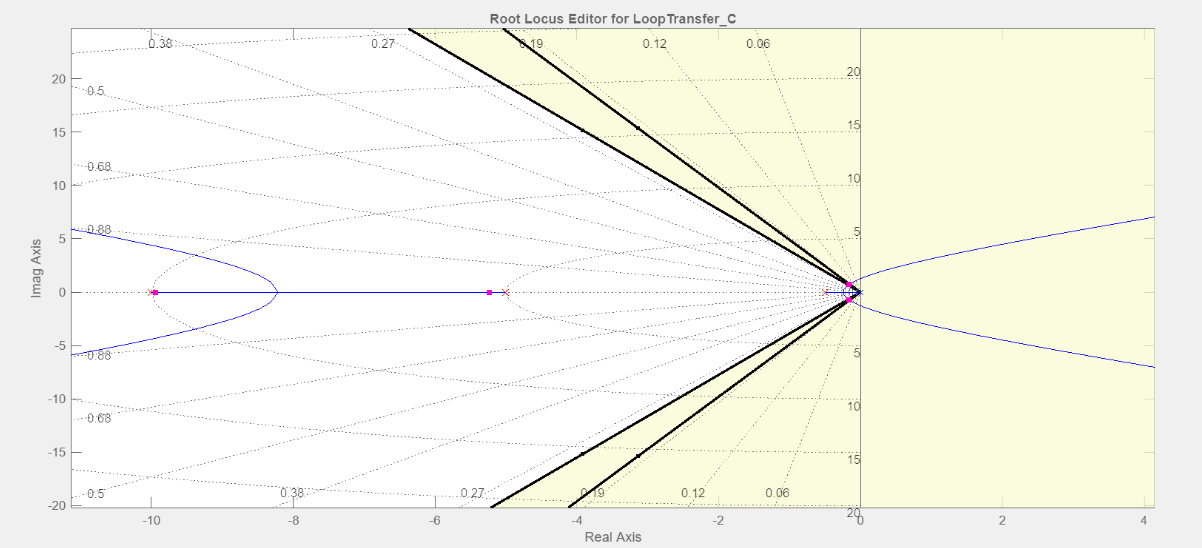


We obtain a value of T~2 and K~0.035 which satisfies our System Requirements. After incorporating variations of ±20% in denominator parameters (0.1 & 0.2) we need to do some tuning by hit and trial to make our system robust. Hence, we experiment with a range of values near about T=2 & K=0.035 and identify T=2 & K=0.0364 as our final value of parameters as they fulfil our system requirements and also enable a stable system.

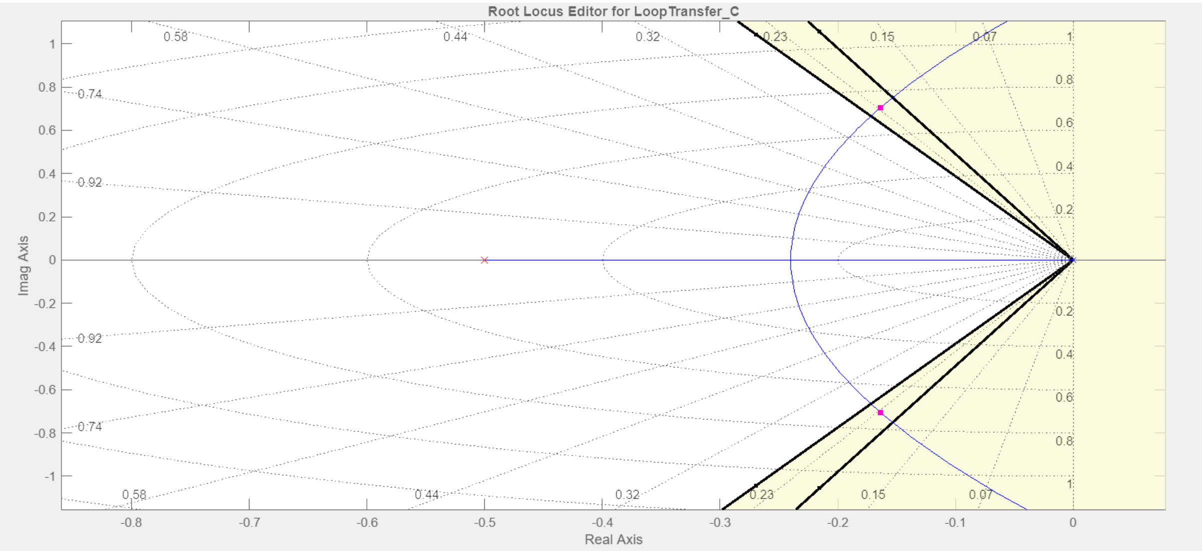
The following step response is obtained when a step input is given to our system with the chosen value of 'T' & 'K'.



The variations in Overshoot, Settling Time, Root Loci in our tuned system are explained by the graphs below-

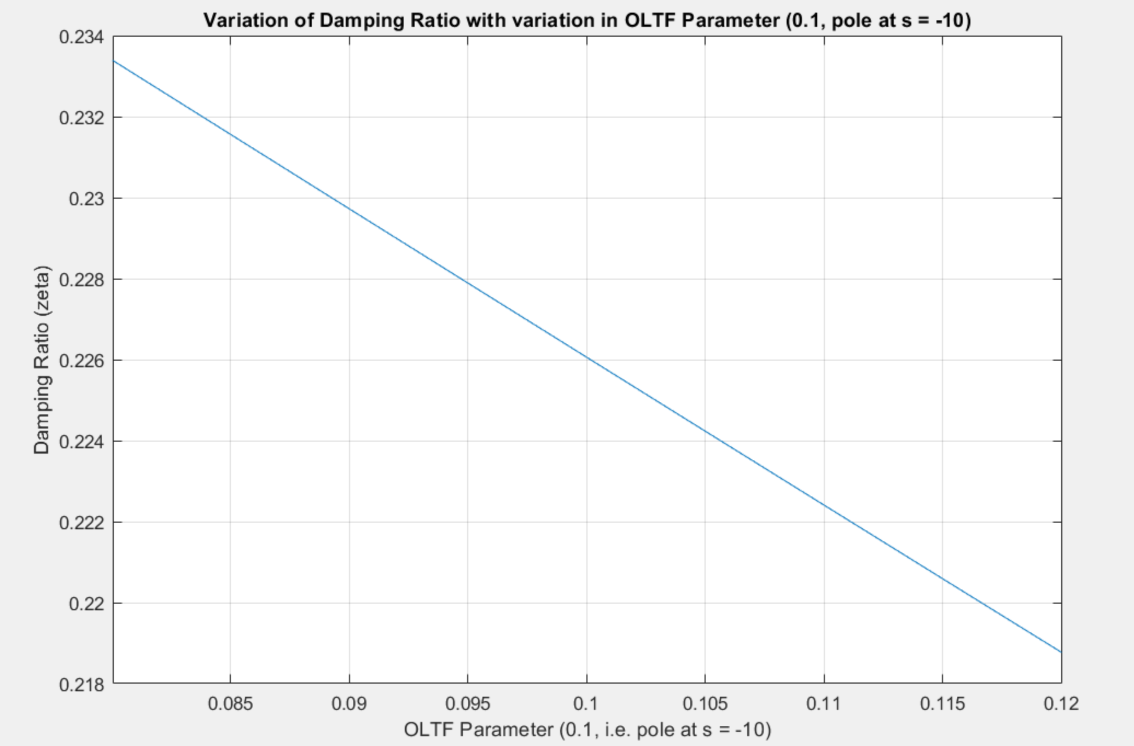


This is the Root Loci obtained for our tuned values of T & K.
A closer look gives us the location of complex poles inside the (0.2,0.25) interval of Damping Ratio.

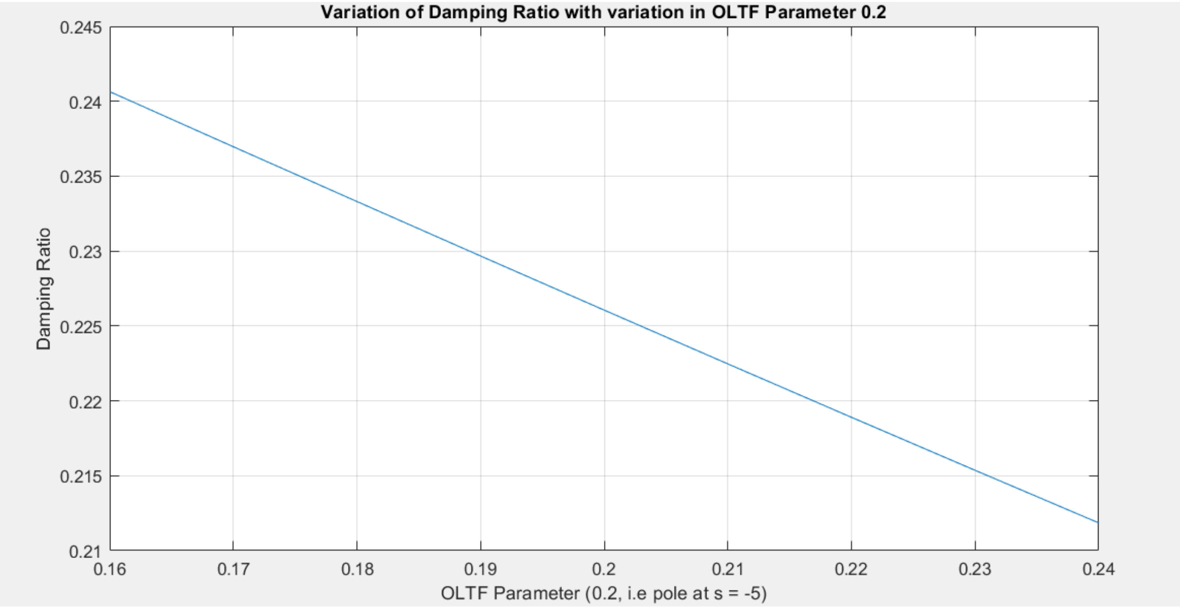


We thus explore the variations in denominator parameters within ±20% from the following graphs-

CHANGE IN DAMPING RATIO (ζ)



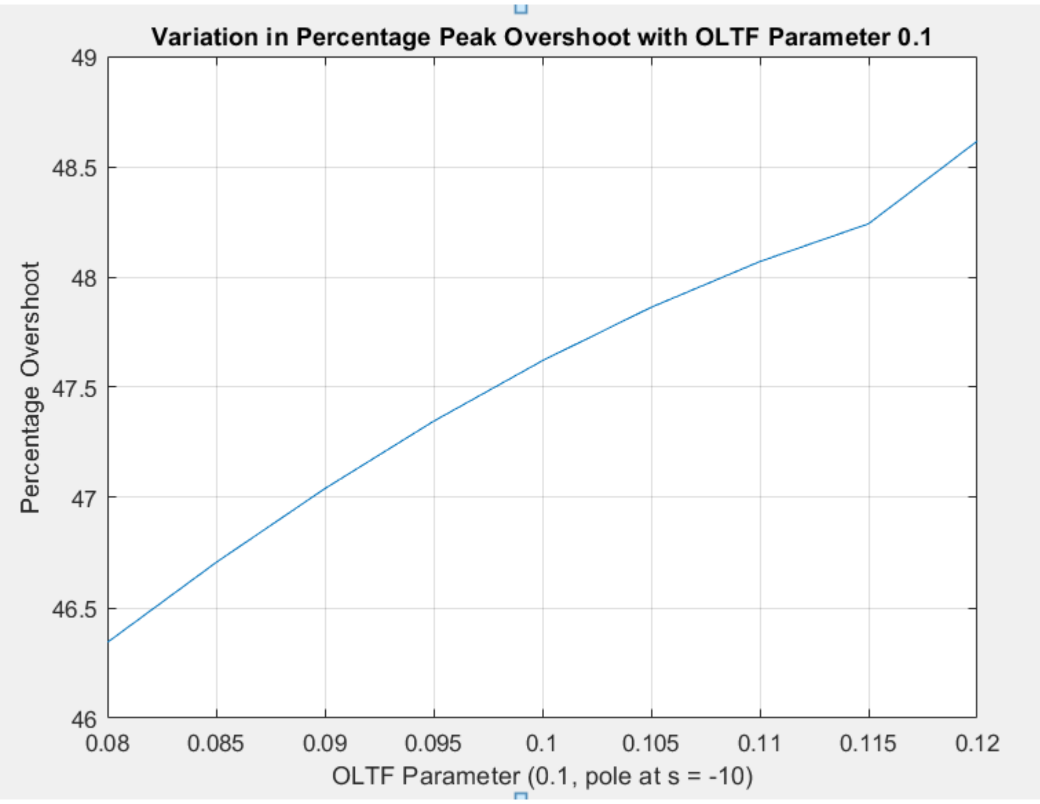
±20% variation in parameter 0.1 (pole at s=-10) yields the following variation in Damping Ratio.



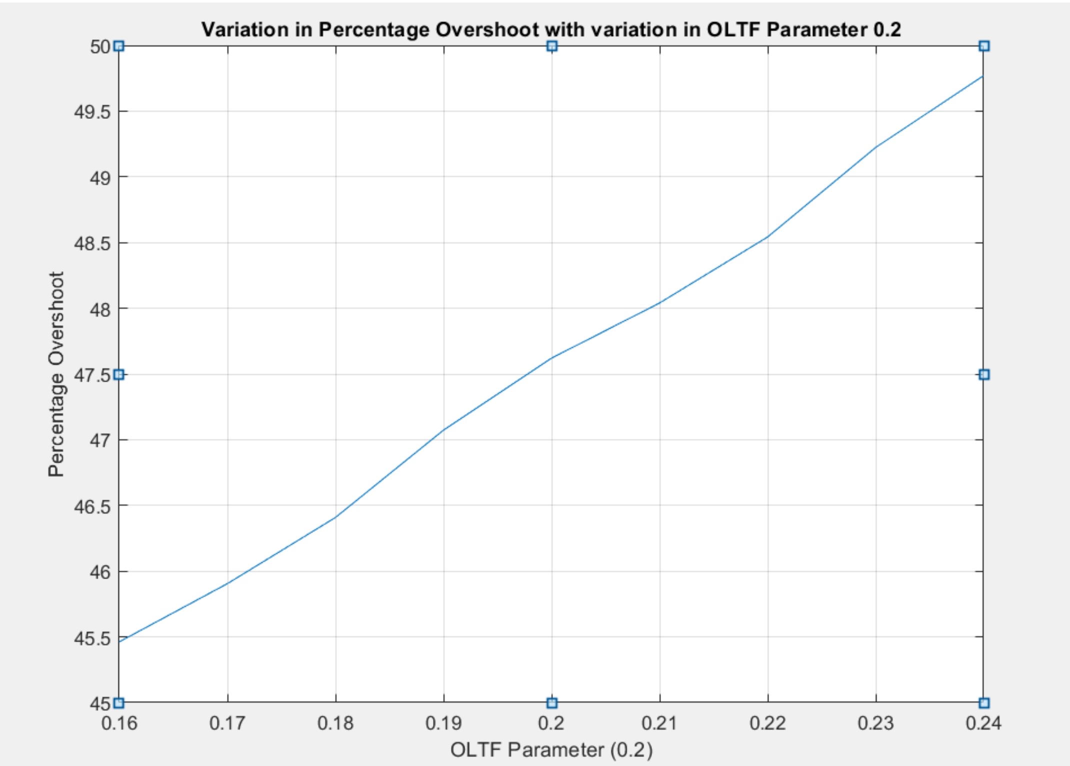
±20% variation in parameter 0.2 (pole at s=-5) yields the following variation in Damping Ratio.

CHANGE IN OVERSHOOT

±20% variation in parameter 0.1 (pole at s=-10) yields the following variation in Overshoot

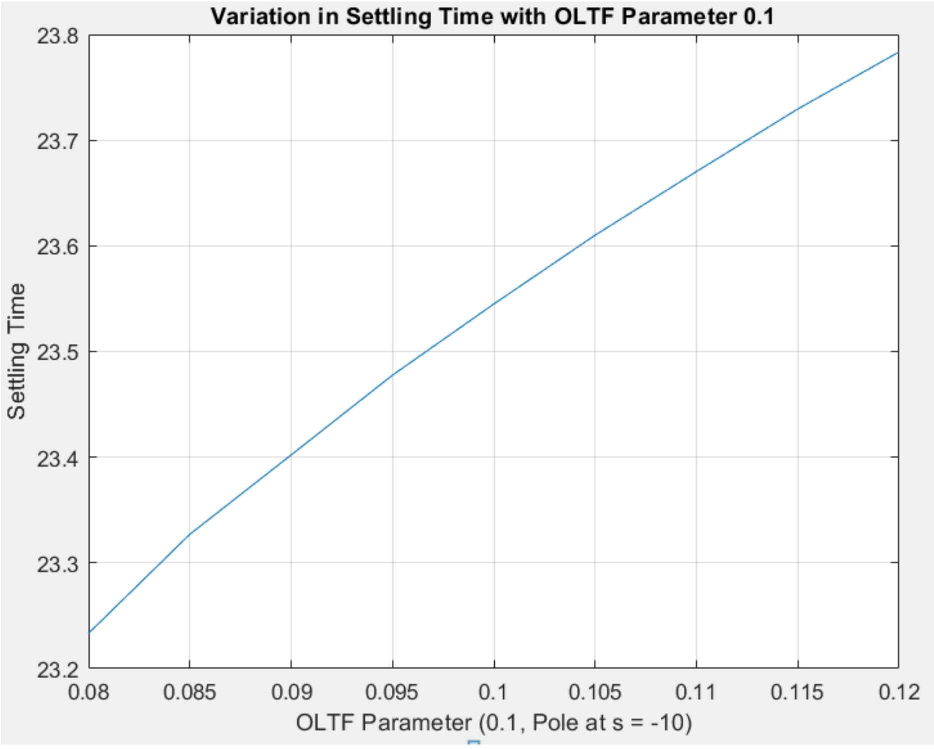


±20% variation in parameter 0.2 (pole at s=-5) yields the following variation in Overshoot

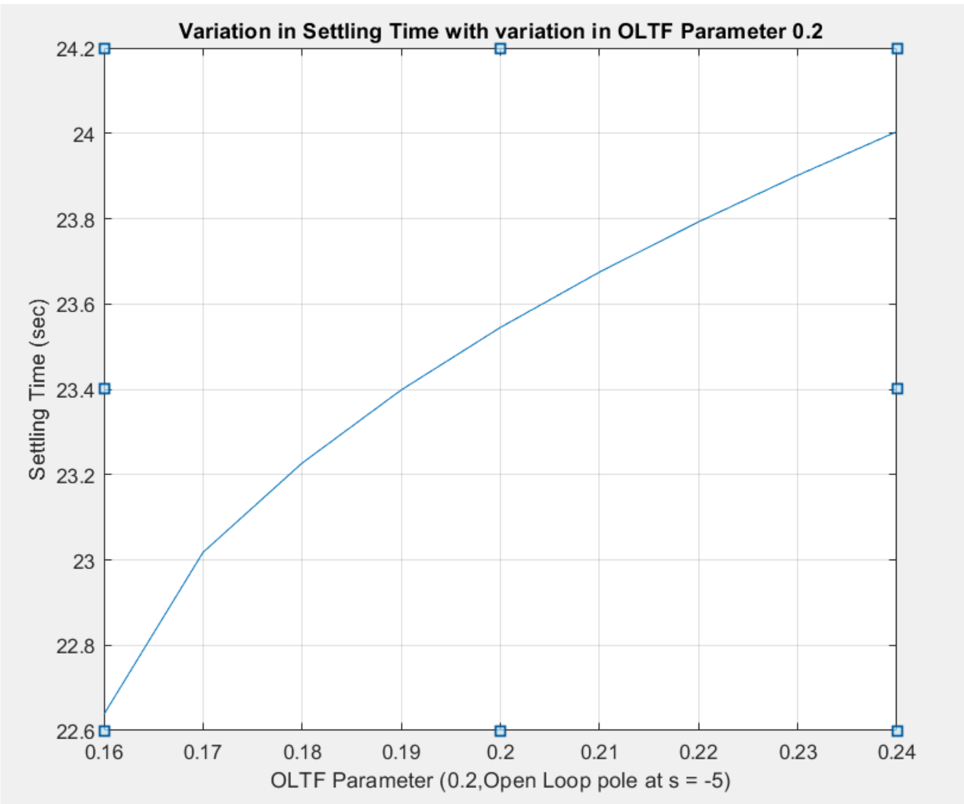


CHANGE IN SETTLING TIME

±20% variation in parameter 0.1 (pole at s=-10) yields the following variation in Settling Time

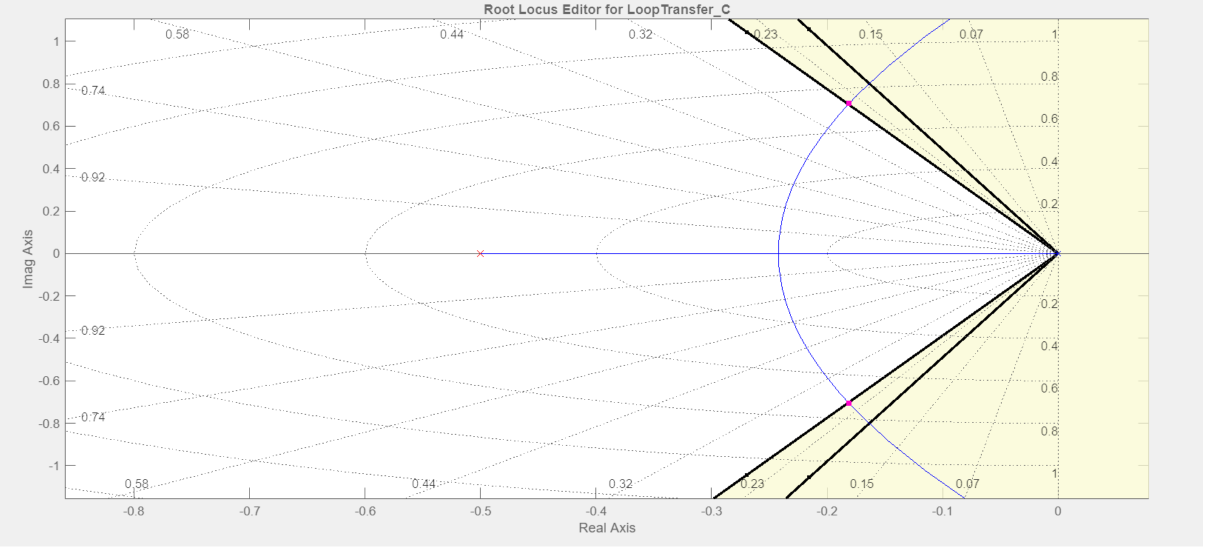


±20% variation in parameter 0.2 (pole at s=-5) yields the following variation in Settling Time

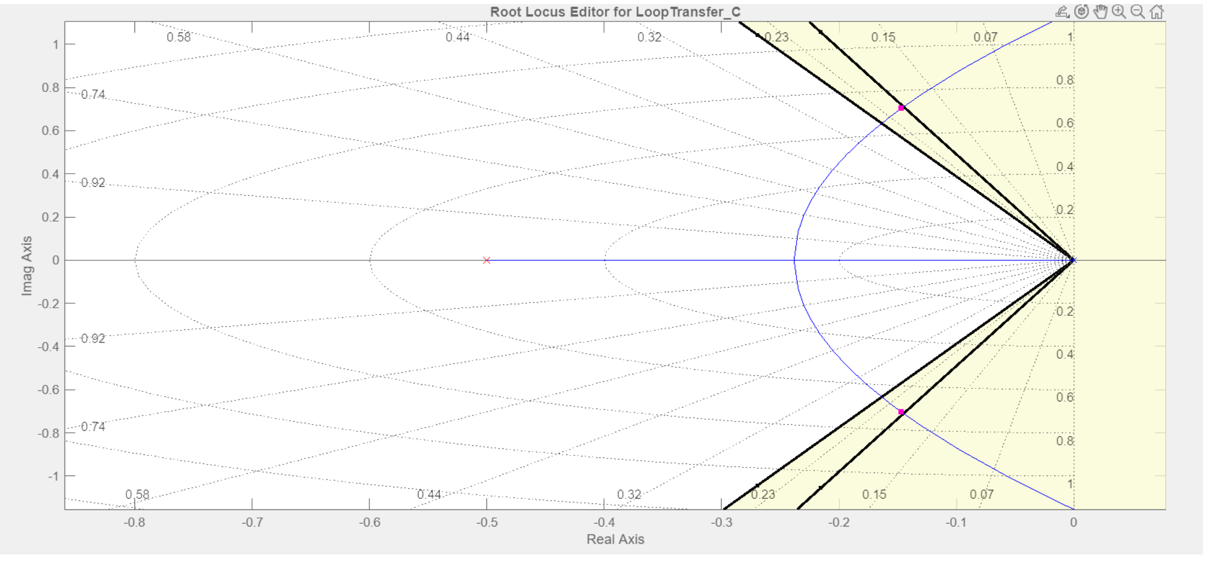


CHANGE IN ROOT LOCI

+20% variation in parameters 0.1 & 0.2 (pole at s=-5 & s=-10) yields the following variation in Location of Root Loci Poles



-20% variation in parameters 0.1 & 0.2 (pole at s=-5 & s=-10) yields the following variation in Location of Root Loci Poles.



We thus tabulate our findings from the above graphs.

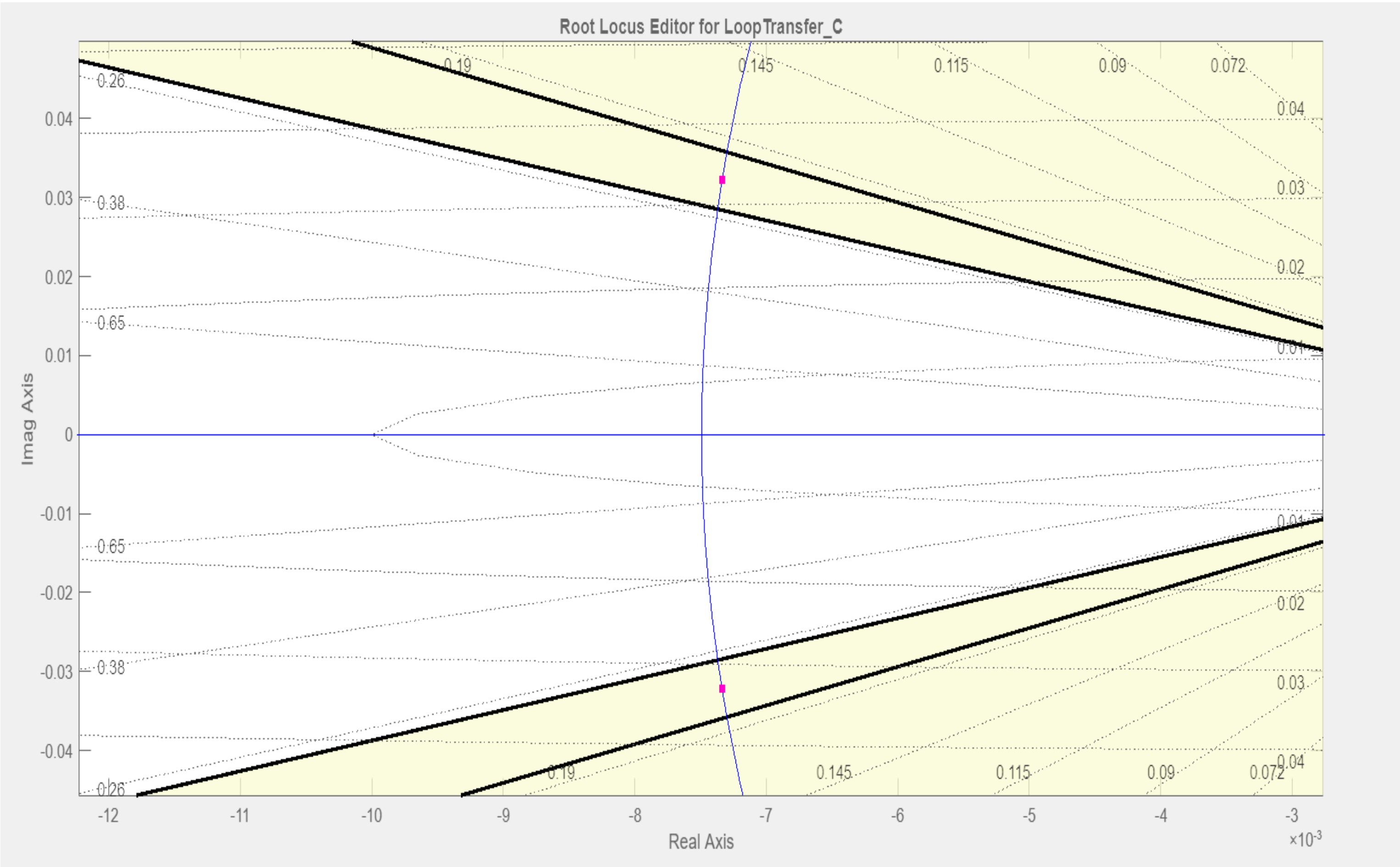
Property	Parameter Varied					
	0.1			**0.2**		
	+20%	**Original**	**-20%**	**+20%**	**Original**	**-20%**
Settling time	*~23.78*	*23.54*	*~23.24*	*~24.00*	*23.54*	*~22.64*
Overshoot	*~48.65*	*47.6205*	*~46.40*	*~49.75*	*47.6205*	*~45.50*
Damping Ratio	*~0.219*	*0.225*	*~0.2335*	*~0.212*	*0.225*	*~0.241*

TABLE-1

We note that we obtain a system fulfilling our system requirements but we haven't incorporated the possibility of variation in Gain Parameter (given to be 30). To make a more robust system that fulfils the system requirements when Gain is also prone to variations, we need new values of 'T' and 'K'. We thus search for another set of 'T' and 'K' from our graph **Damping Ratio (ζ) (z-axis) vs '**T**' vs '**K**'. This gives us our new set of values as T~66, K~0.0025. We tune it from Control System Designer Toolbox and thus revolve our study upon T=66.7 & K=0.002425.

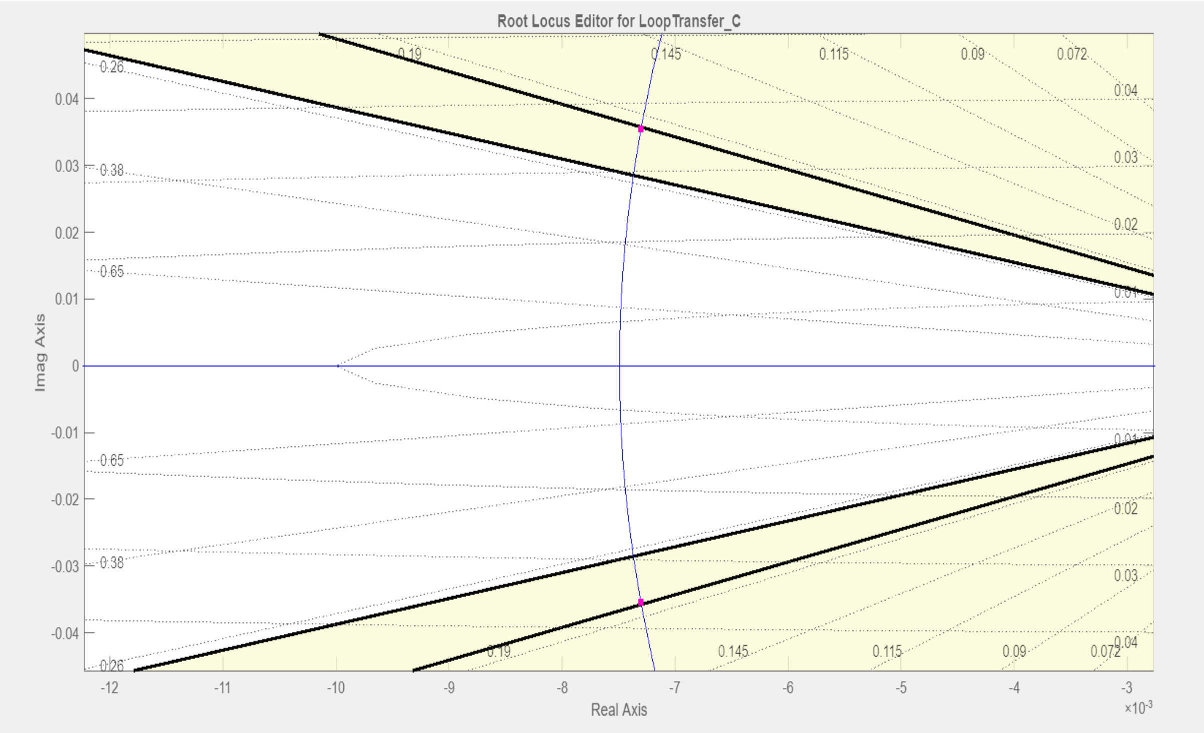
LOCATION OF CLOSED LOOP POLES ON ROOT LOCI

(AT T = 66.7, K = 0.002425)

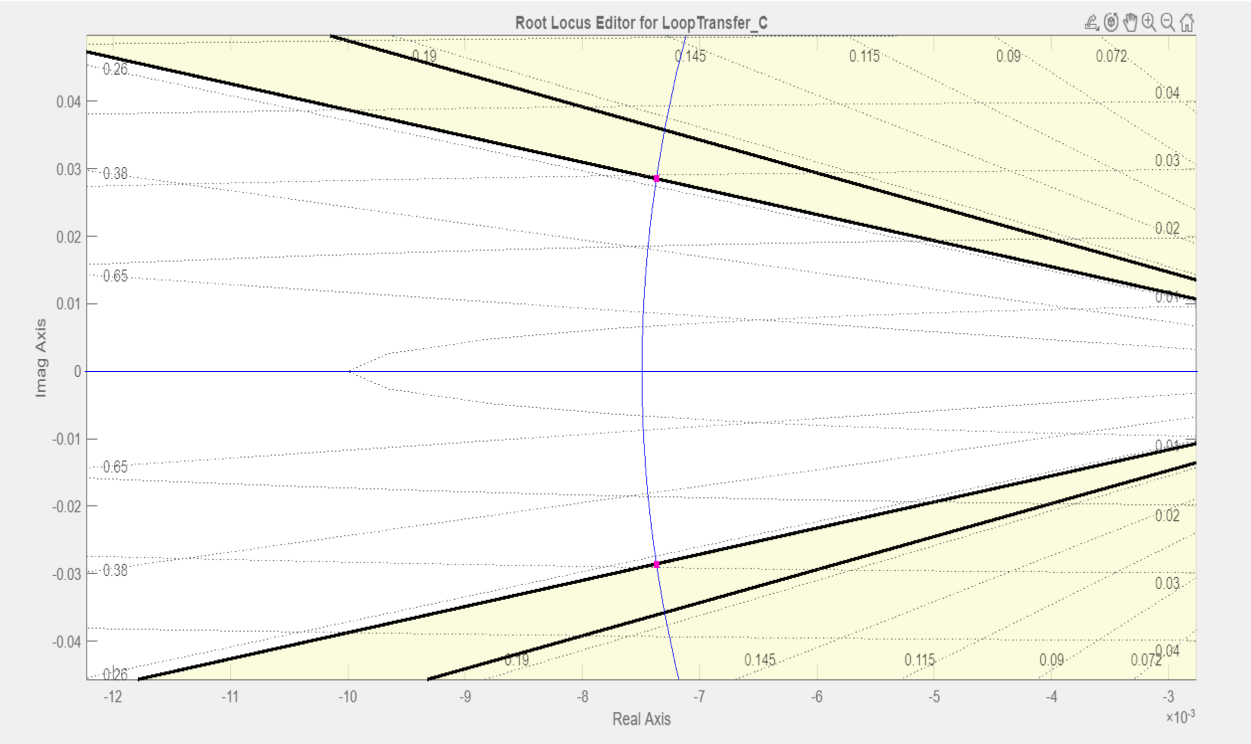


ROOT LOCUS PLOTS

20% variation in Gain Parameter (30+20%) yields the following variation in Location of CLTF Poles on Root Loci

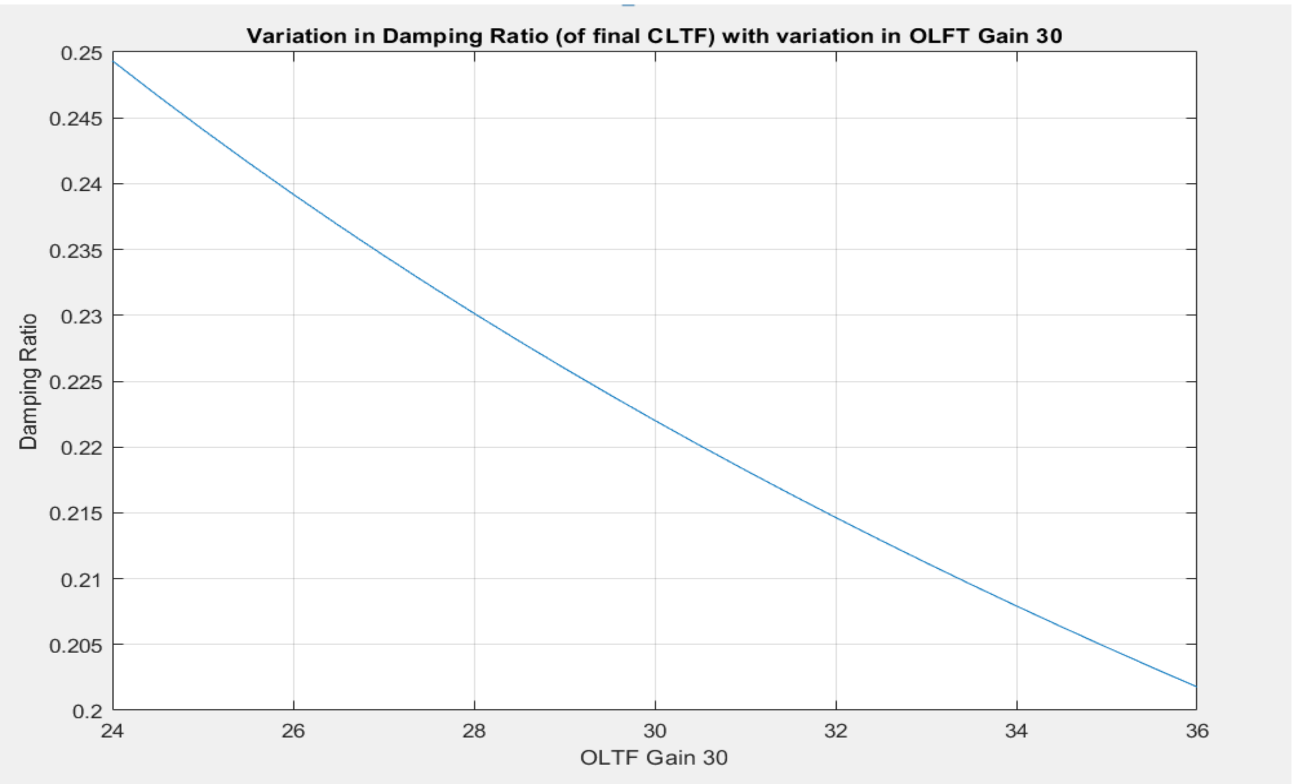


-20% variation in Gain Parameter (30-20%)

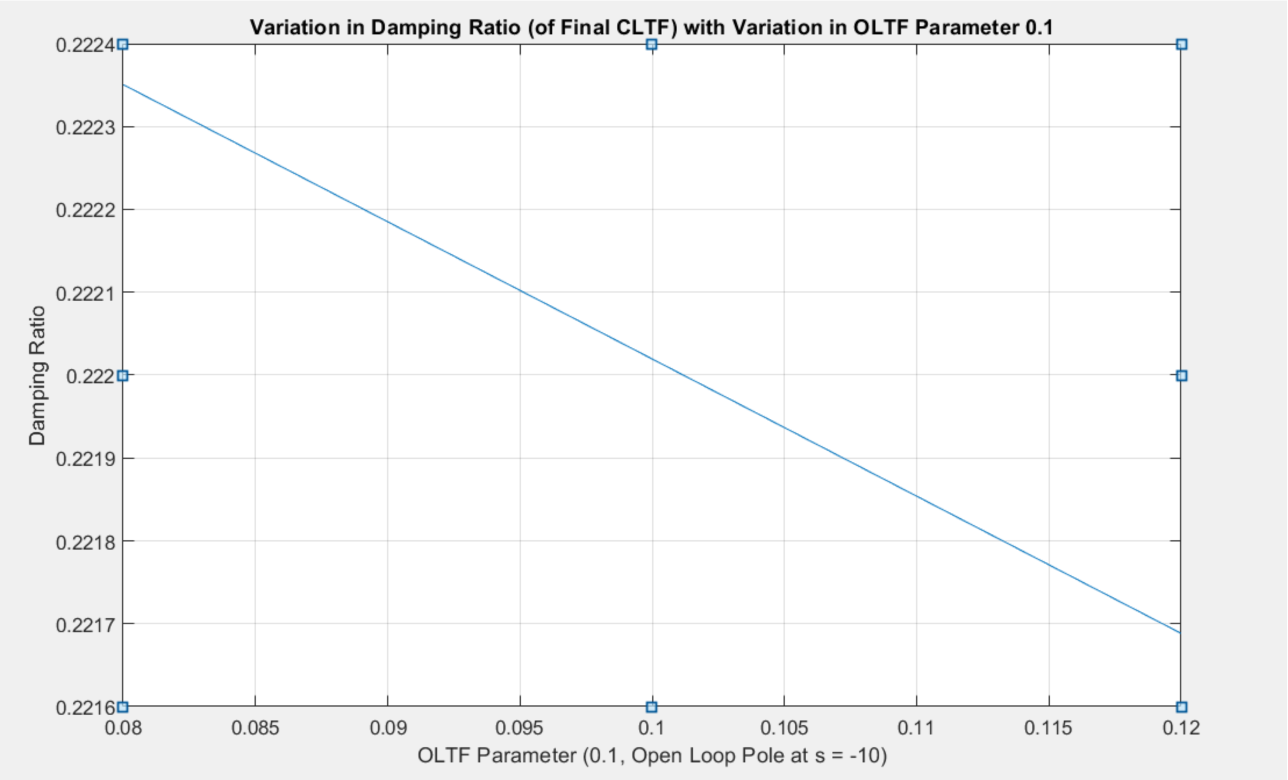


DAMPING RATIO

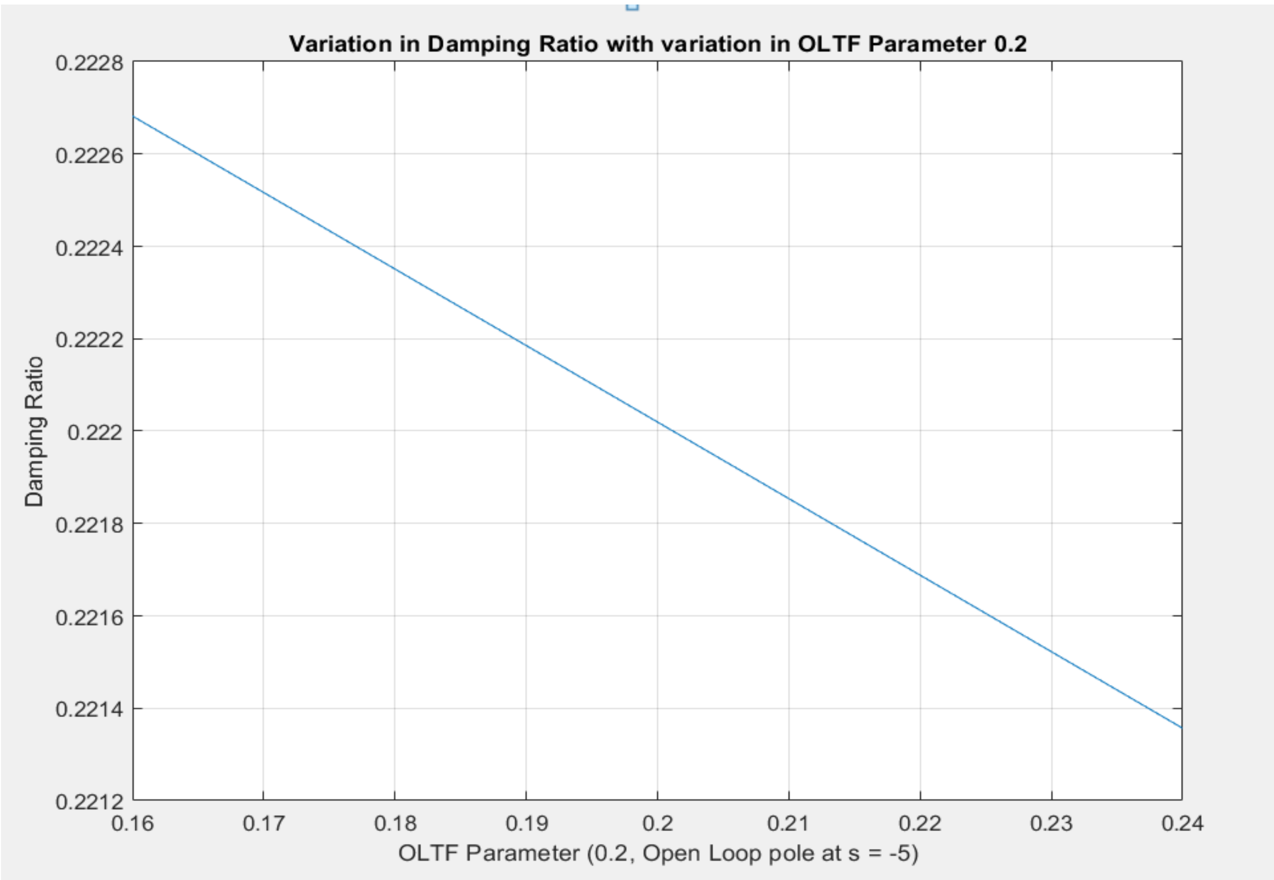
Variation in Damping ratio with variation in Gain Parameter (30±20%):



Variation in Damping ratio with variation in OLTF Parameter 0.1 (0.1±20%):



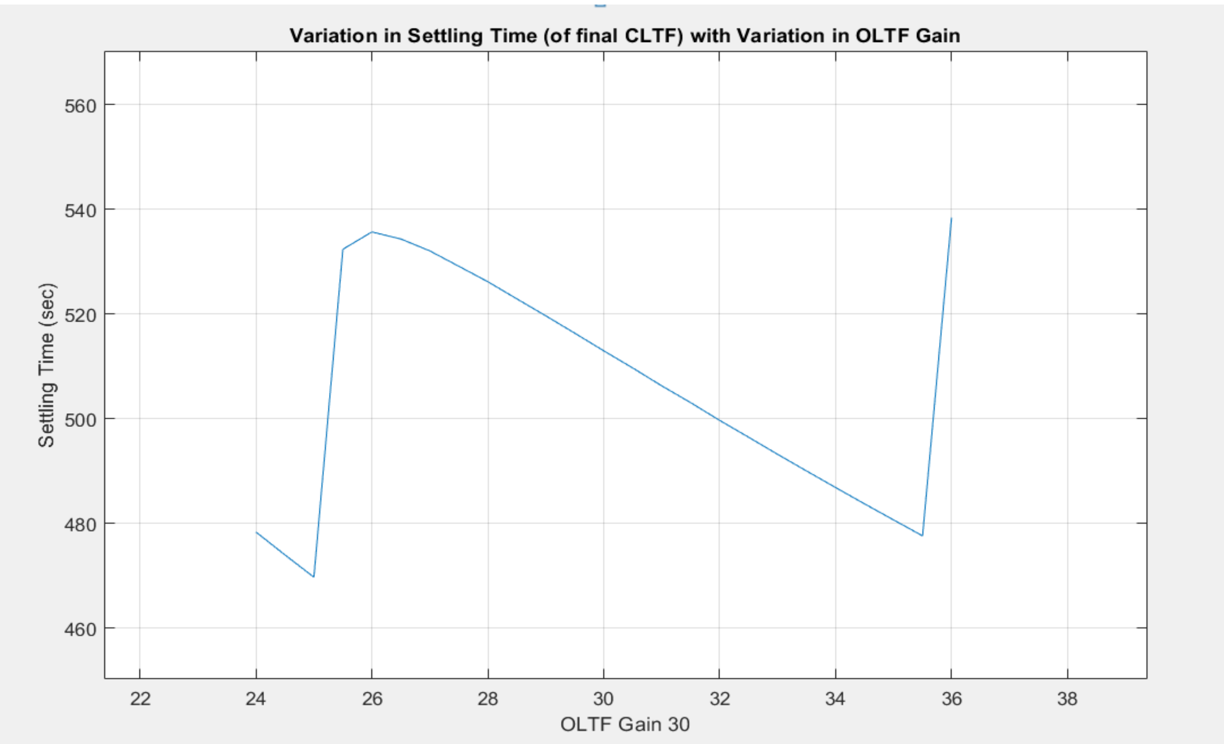
Variation in Damping ratio with variation in OLTF Parameter 0.2 (0.2±20%):



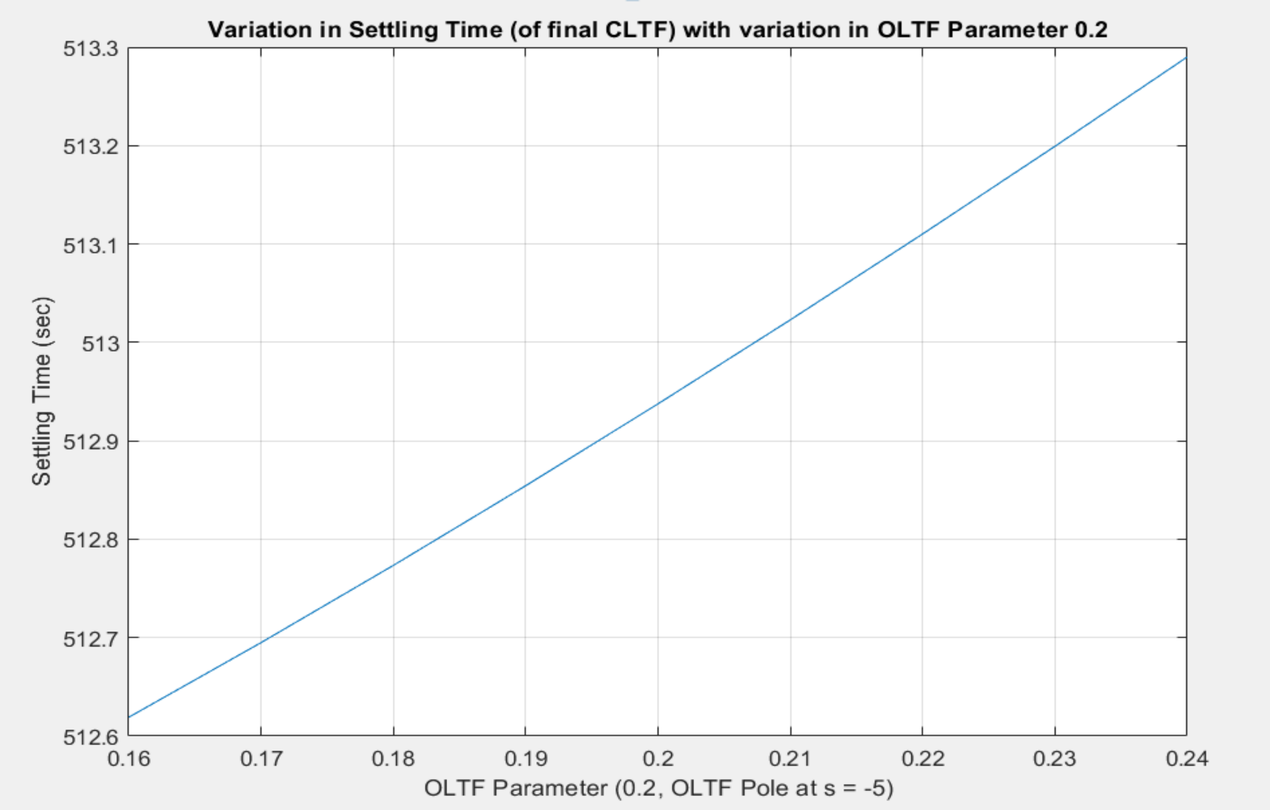
****In the cases of Parameters 0.1 & 0.2 the damping ratio decreases linearly with increase in the Parameters and decreases almost linearly with increase in Gain Parameter. On further observation we can see that the decrease is much more drastic when gain is varied.**

SETTLING TIME

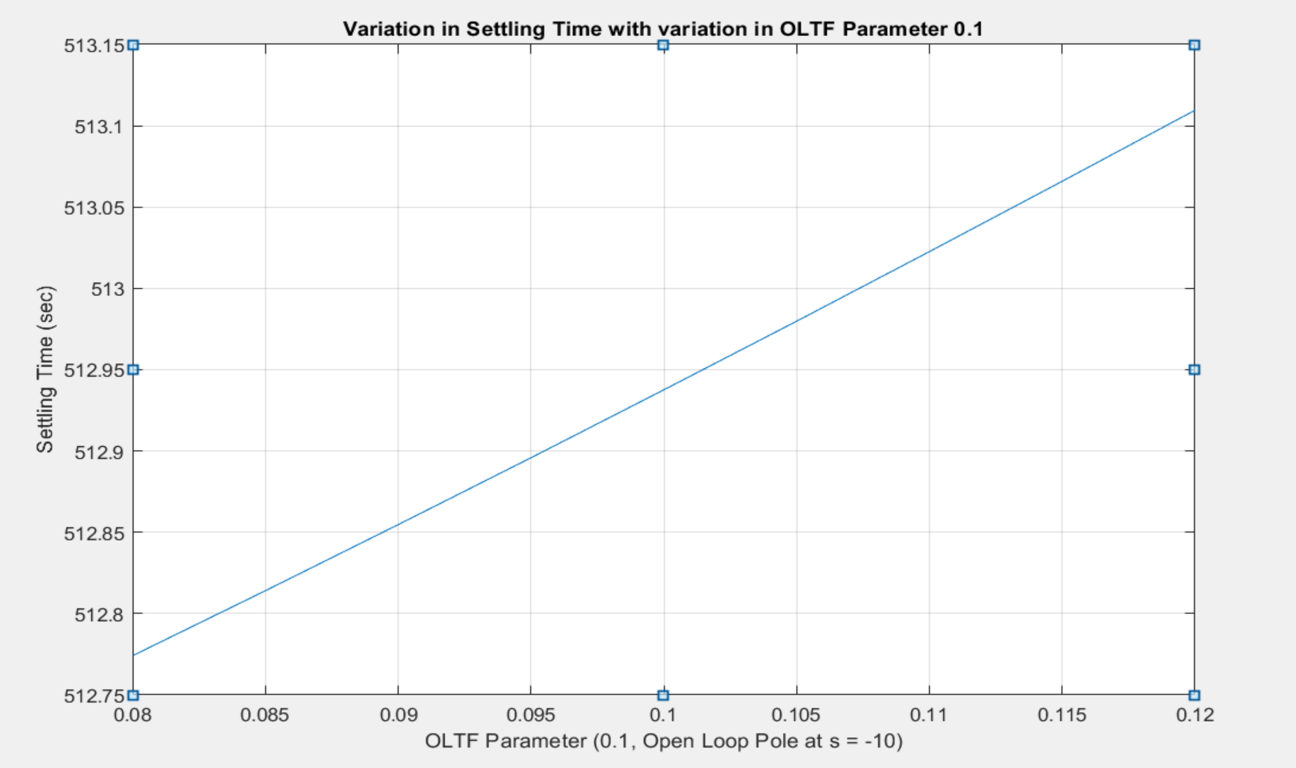
Variation in Settling Time with Variation in OLTF Gain (30±20%):



Variation in Settling time with Variation in OLTF Parameter 0.2 (0.2±20%):



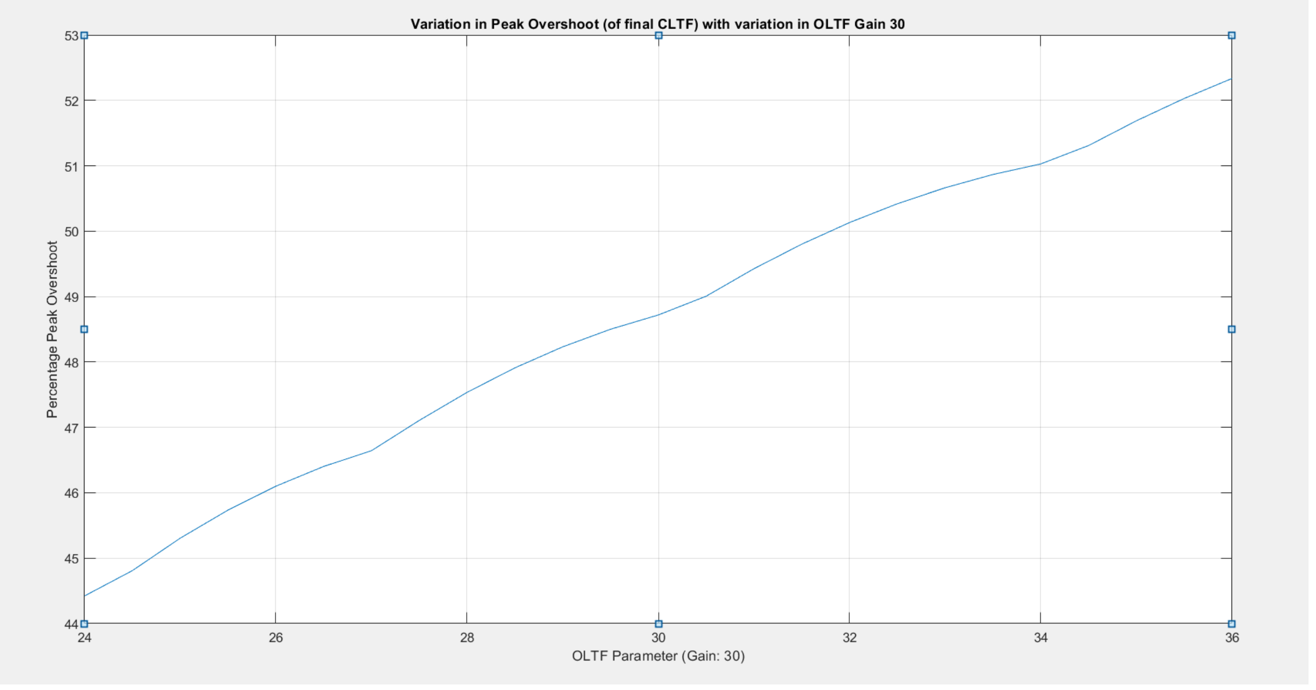
Variation in Settling Time with Variation in OLTF parameter 0.1 (0.1±20%):



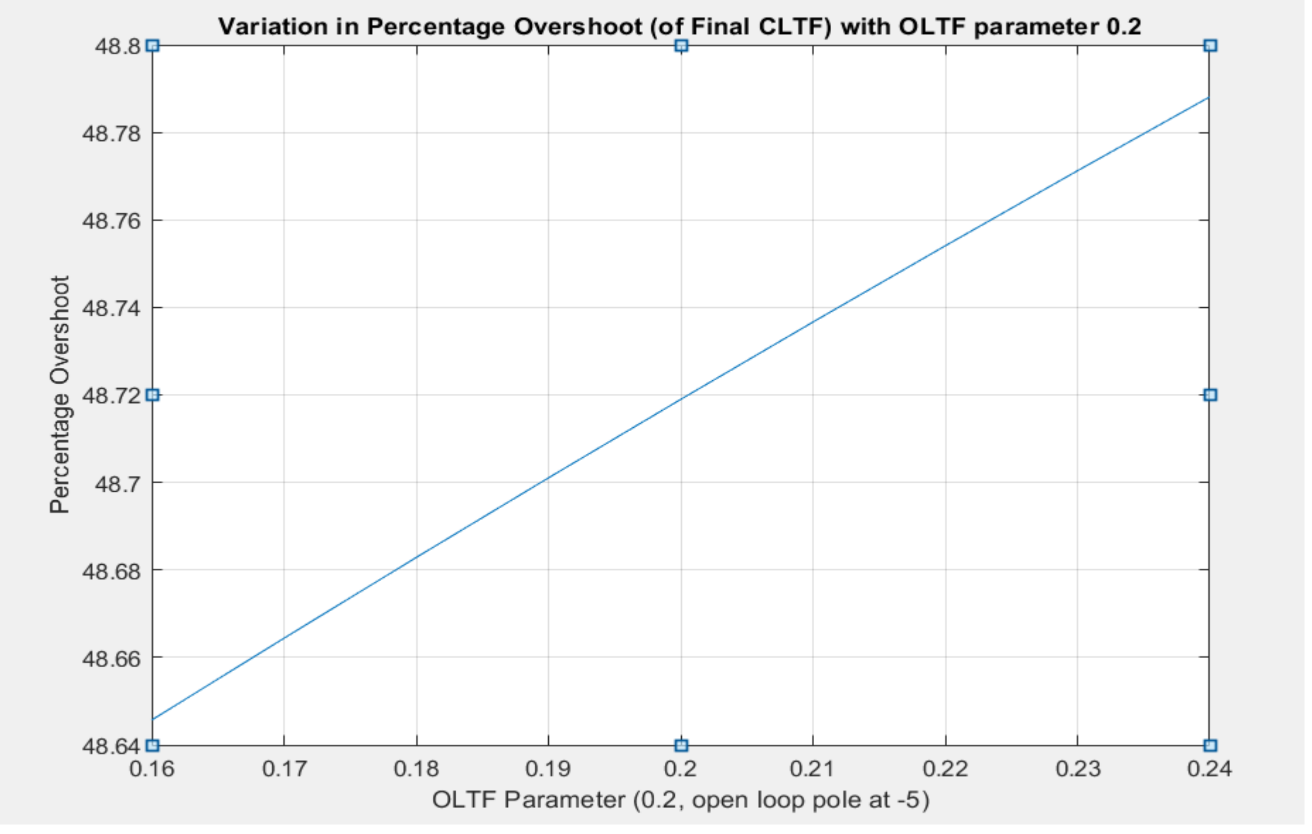
****For the Parameters in the denominator (i.e., 0.1 & 0.2), Settling Time increases. But when Gain (i.e., 30) is varied, the Settling Time graph comes out to be an Oscillating curve with Maxima at Gain=36 and Minima at Gain ≅ 25**

PEAK OVERSHOOT

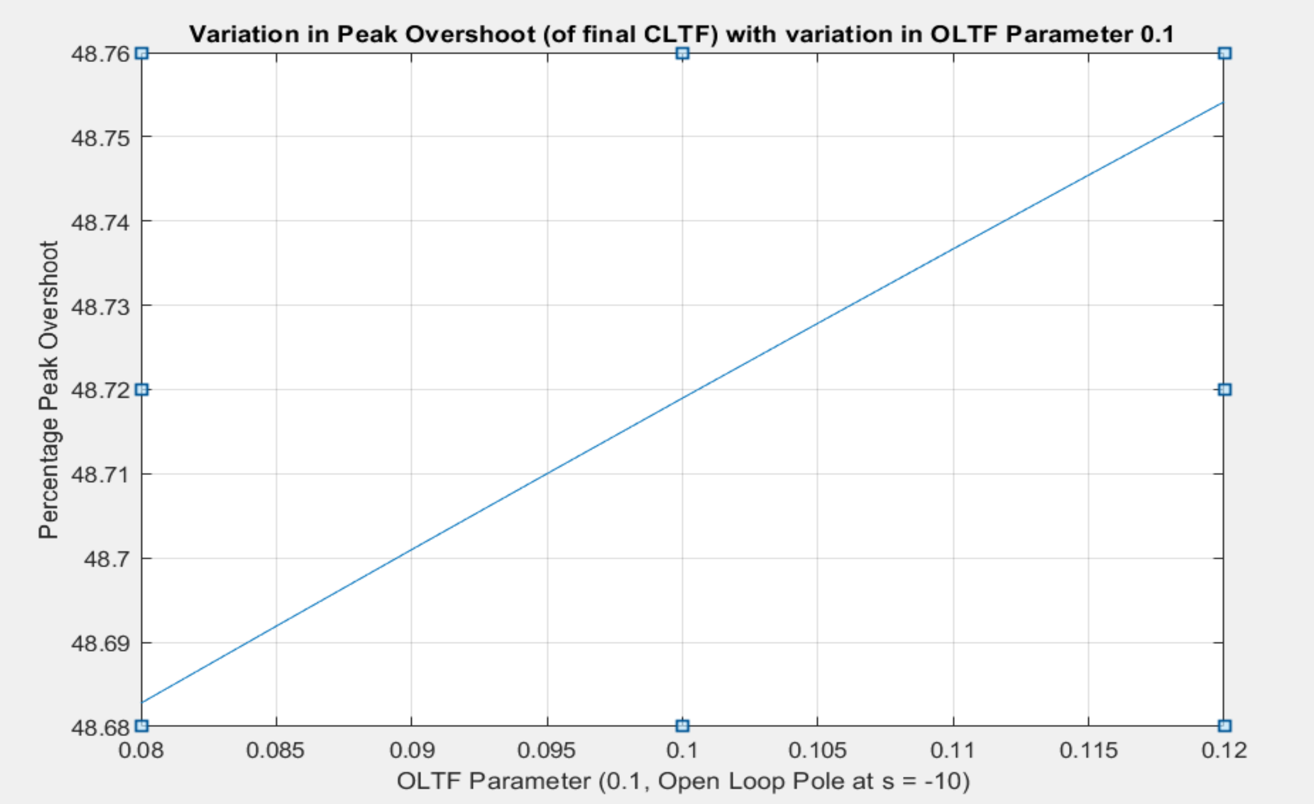
Variation in Peak Overshoot with Variation in OLTF Gain (30±20%):



Variation in Peak Overshoot with Variation in OLTF Parameter 0.2 (0.2±20%):



Variation in Peak Overshoot with Variation in OLTF Parameter 0.1 (0.1±20%):



****In the cases of Parameters 0.1 & 0.2 the Peak Overshoot increases linearly with increase in the Parameters and increases almost linearly with increase in Gain Parameter (30). On further observation we can see that the increase is much more drastic when Gain is varied.**

Property	Parameter Varied							
	0.1			0.2			Gain	
	+20%	Original	-20%	+20%	Original	-20%	+20%	-20%
Settling time	*~513.11*	*512.9376*	*~512.77*	*~513.28*	*512.9376*	*~512.62*	*~528*	*~478*
Overshoot	*~48.754*	*48.7190*	*~48.683*	*~48.79*	*48.7190*	*~48.646*	*~52.4*	*~44.5*
Damping Ratio	*~0.22169*	*0.222*	*~0.22235*	*~0.22136*	*0.222*	*~0.22168*	*~0.202*	*~0.248*

TABLE-2

CONCLUSIONS

We observed that no value of '**T**' fulfilled our System Requirements even without Parameter Variations. Hence, we made a system fulfilling our System Requirement (0.25 > Damping Ratio >0.2) in two sets of Parameters '**T**' & '**K**'. The two sets are-(**T, K) € (2, 0.0364) & (66.7, 0.02425).**

The second set of Parameters is more robust as it allows variations in all Parameters (0.1 ,0.2 and Gain-30) and is almost immune to changes when variations occur in denominator Parameters although Settling Time increases drastically for the second set of Parameters.

We also observe the sensitivity of our system w.r.t changes in Real Poles at s=-10 and s=-5 by studying the Parameters 0.1 & 0.2 respectively.

From *TABLE-1* & *TABLE-2* we study the bandwidth of changes in Settling Time, Overshoot & Dampening Ratio. We find the following

T=2, K=0.0364

Property	Bandwidth	
	Parameter 0.1 (Pole at s=-10)	Parameter 0.2 (Pole at s=-5)
Settling Time	0.54	1.36
Overshoot	2.25	4.25
Damping Ratio	0.0145	0.029

Hence, we see that Sensitivity of our system is more w.r.t Parameter 0.2 (Pole at s=-5) since a change of ±20% induces more variation and bandwidth in the Settling Time, Overshoot and Damping Ratio.

T=66.7, K=0.002425

Property	Bandwidth	
	Parameter 0.1 (Pole at s=-10)	Parameter 0.2 (Pole at s=-5)
Settling Time	0.34	0.66
Overshoot	0.071	0.144
Damping Ratio	0.00066	0.00032

Hence, we see that Sensitivity of our system is more w.r.t Parameter 0.2 (Pole at s=-5) since a change of ±20% induces more variation and bandwidth in the Settling Time, Overshoot with an exception to Damping Ratio where it's comparable.