CONTROL ENGINEERING LAB

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GROUP - 18

Aman Saini (2020EEB1155)
Anirudh Sharma (2020EEB1158)
Ansaf Ahmad (2020EEB1160)

OBJECTIVE

To design the state feedback gain matrix for a given analog state-space system to satisfy required performance specifications.

INTRODUCTION

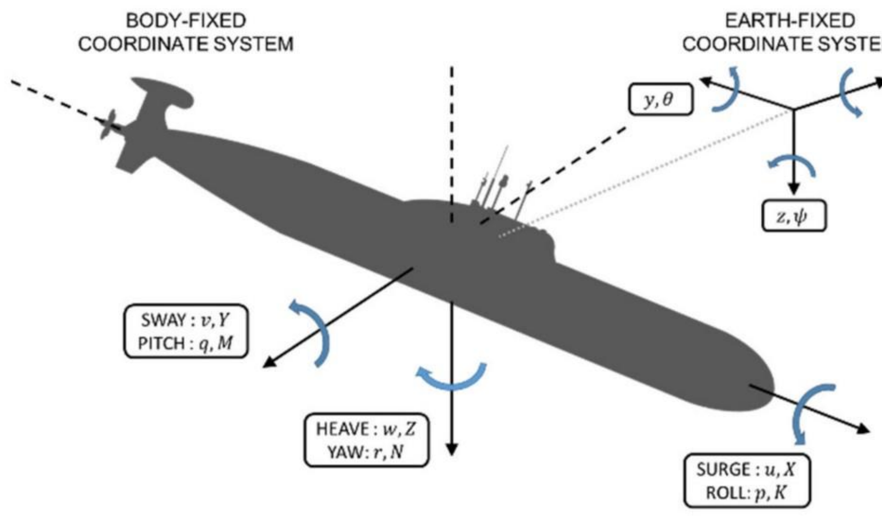
We are given state space matrices for an autonomous underwater vehicle (AUV), which is a robotic submarine that can be used for different underwater studies. The horizontal plane movements have the sway speed (sidewards motion), yaw angle (the angle between line of direction of flight and a plane through the longitudinal and vertical axis of AUV) and yaw rate as state variables x_1, x_2, x_3 respectively. The rudder angle is the single input to control the state variables and the output (**sway speed & yaw angle**). We have been given A, B, and C matrices linearized about a nominal operating point, which characterizes the given analog system.

$$\mathbf{A} = \begin{bmatrix} -0.14 & -0.69 & 0.0 \\ -0.19 & -0.048 & 0.0 \\ 0.0 & 1.0 & 0.0 \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} 0.056 \\ -0.23 \\ 0.0 \end{bmatrix} \quad \boldsymbol{C} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

We need to design state feedback matrices such that:

- Settling times are retained at those of the nominal eigen values.
- Maximum magnitude of all eigen values is as in the nominal set.
- The CL system is always observable and controllable.

The motion of the AUV can be easily understood through the following figure:



Horizontal motion is characterized by yaw and sway, that is exactly what we are required to control (meeting the desired specifications) in the given project.

The eigen values of A matrix which gives us information about the poles of the system are: {0.0; 0.2710; -0.4590}. It is also obvious from the impulse response of the open loop system that the matrices have been obtained by linearizing the system about the point of an unstable equilibrium.

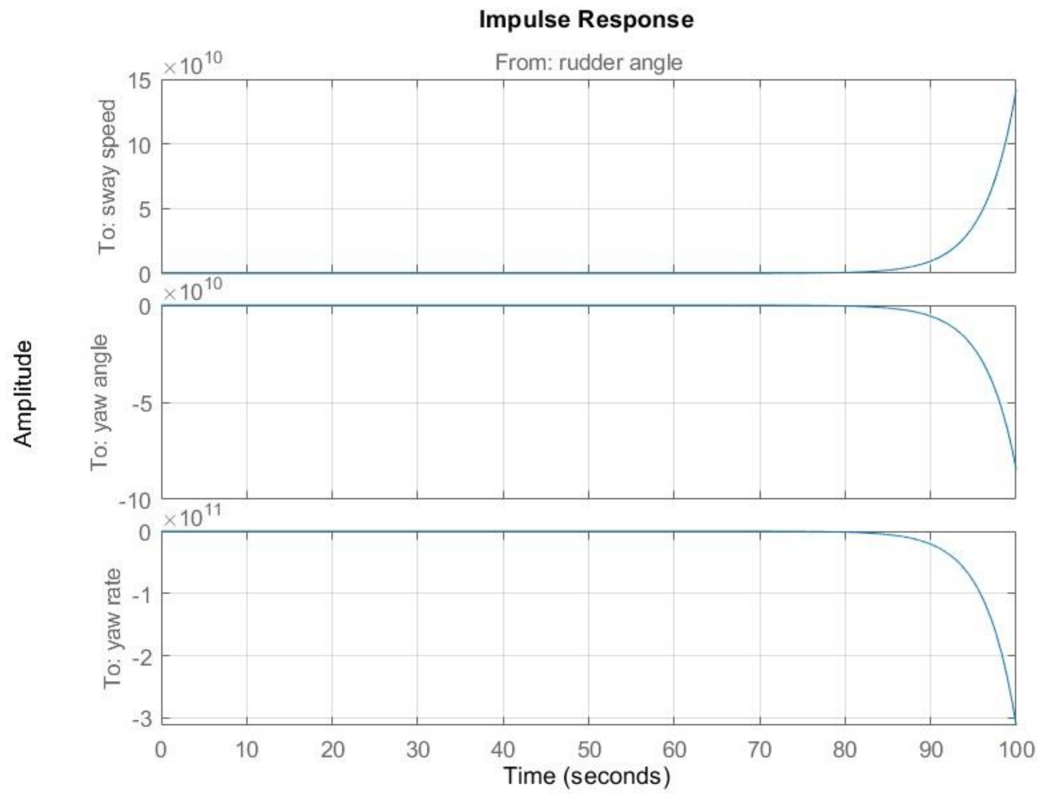


Fig: Impulse response of Open loop system

Having discussed about the problem, we can obtain the state space equations from the given matrices and we get the following system of equations.

$$\dot{x}_1 = -0.14x_1 - 0.69x_2 + 0.056\,u$$

$$\dot{x}_2 = -0.19x_1 - 0.048x_2 - 0.23\,u$$

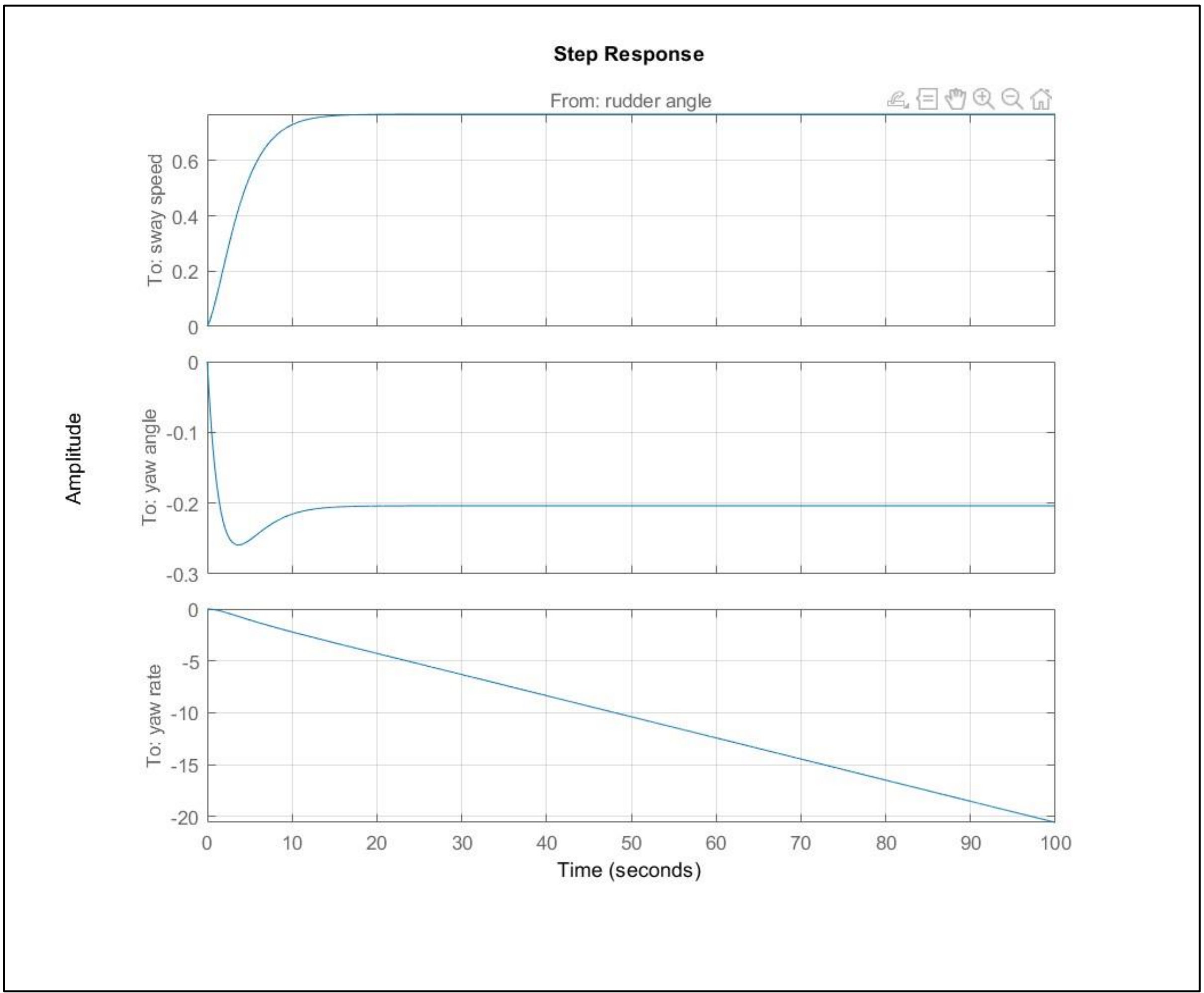
$$\dot{x}_3 = x_2$$

Here, x_1 = Sway Speed; x_2 = Yaw Angle; $\dot{x}_2$ = x_3 = Yaw Rate; $\dot{x}_3$ = Yaw Acceleration & u = Rudder Angle

From the above equations we see that the yaw acceleration is directly proportional to yaw angle (at the point of linearization), thus a little disturbance from the equilibrium position (i.e., x_1 = 0, x_2 = 0, x_3 = 0) drives the system to unstability.

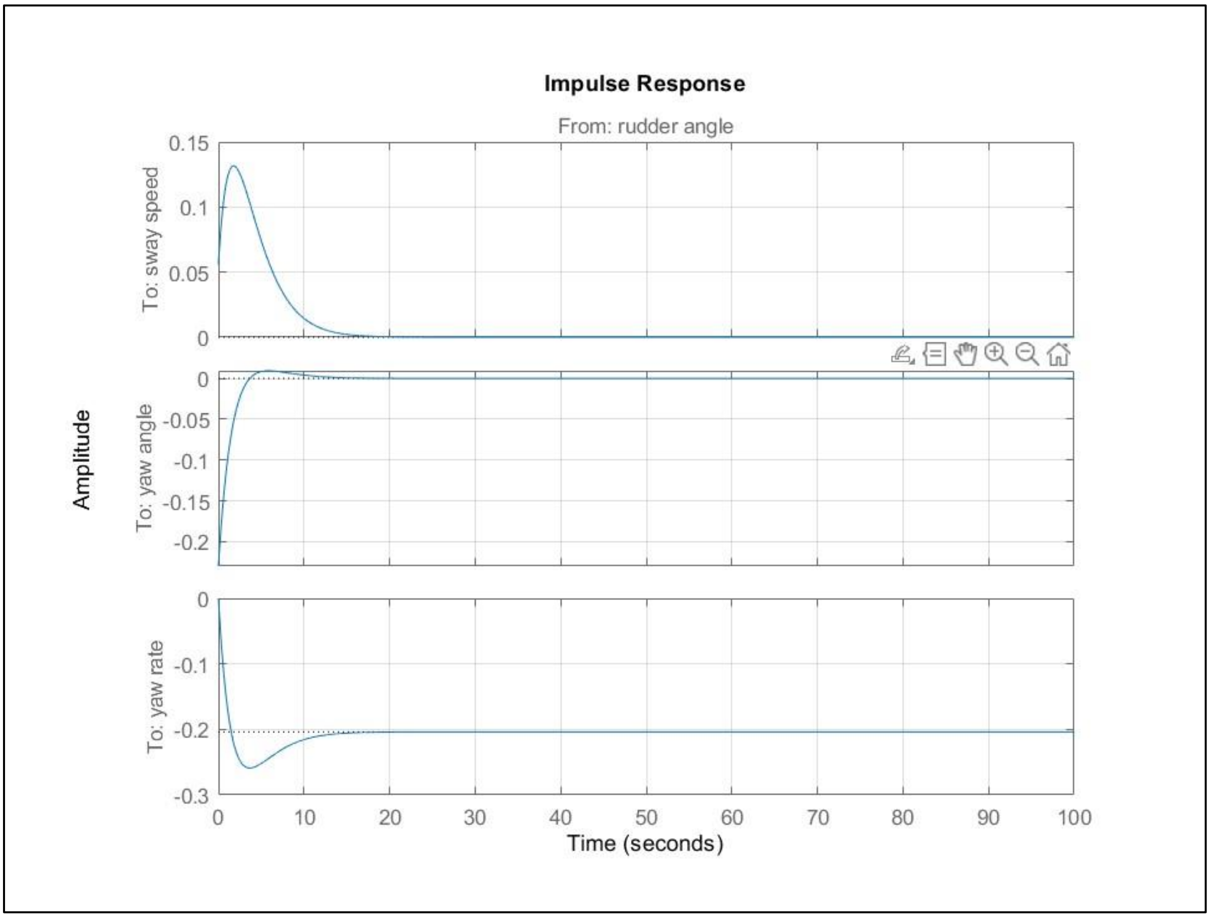
POLE PLACEMENT APPROACH

We can use the pole placement method to make the closed loop system stable. The method requires us to just choose the poles of the closed loop system and it designs the feedback gain matrix accordingly. While placing the poles of the system we observe, that poles with highly negative real part ensures a faster response (lesser sampling time) however that also leads to a reduction in steady state gain for instance if we were to place the poles at **{0, -10, -11}** we get a very small settling time but the sway speed and yaw angle settles at 0.0014 and -0.0082 respectively which directly reduces the responsiveness of the system, hence there is a trade-off. Therefore, we place the close loop poles at {0, -0.458, -0.459} such that first 2 requirements are met i.e., settling time of respective states are maintained at their nominal values, (which is clear by comparing the real part of the respective eigen values), also the maximum magnitude of all the eigen values is as in nominal set. The feedback matrix generated is: **K = [1.3170; -2.8489; 0.0]**. We obtain the following step response:



Since Yaw angle measures the angle between the direction of motion and longitudinal axis of the vehicle, it is clear the if there is no sway motion (i.e., vehicle possess only surge velocity, the yaw angle equals zero), providing a step input to the rudder has a turning effect on the AUV with a constant sway velocity and yaw angle, which is clear from step response graphs. Also, we notice that the yaw angle response lies in the negative region, which is due to the fact that a change in rudder angle in a particular (clockwise and anticlockwise) direction, introduces a yaw angle in opposite direction. It is also noticeable that the yaw rate increases linearly, indefinitely, but since we are dealing with a non-linear system, which has been linearized at some operating point (for which yaw acceleration happens to be proportional to yaw angle), we can safely say that the yaw rate becomes zero as soon as the yaw angle becomes constant.

We also plotted the impulse response of the system, and obtained the following plots.

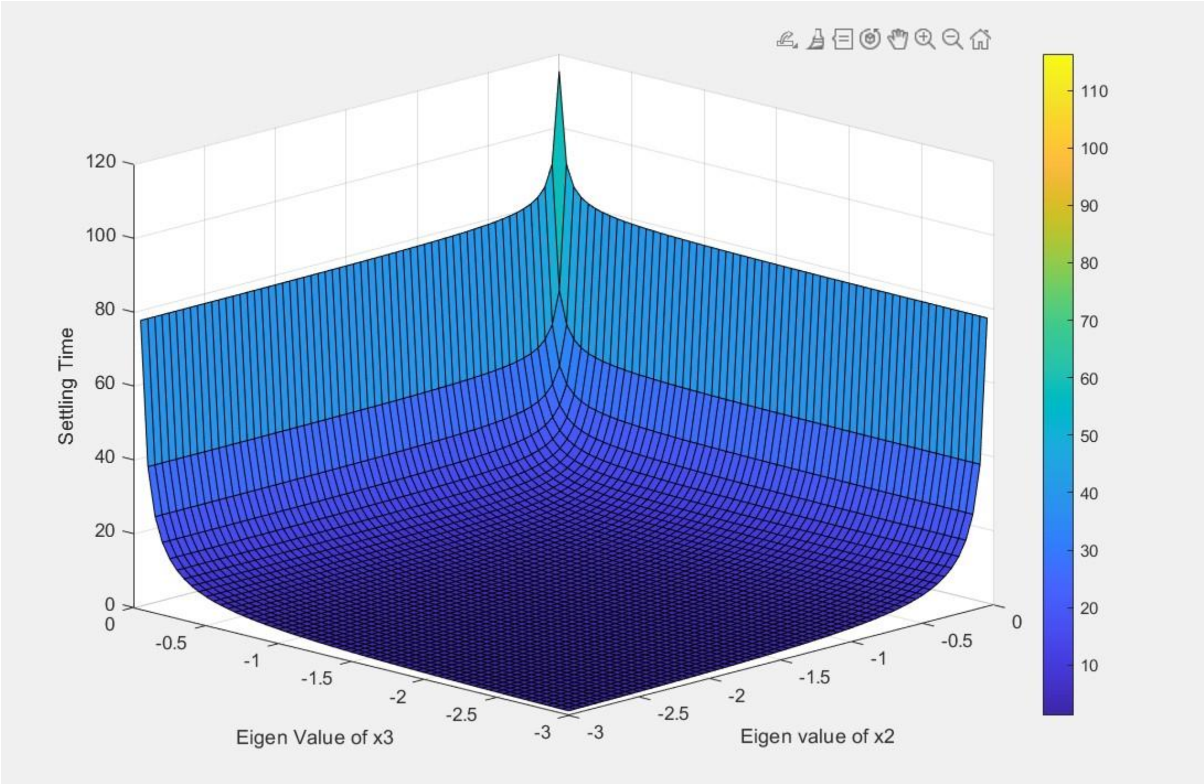


The sway speed and the yaw angle die out with time, when provided with an impulse input, and does not diverge to infinity, it is clear that the system has been stabilized at the given point, and unstable equilibrium at {x1 = 0, x2 = 0, x3 = 0} has been converted to a stable one using pole placement approach.

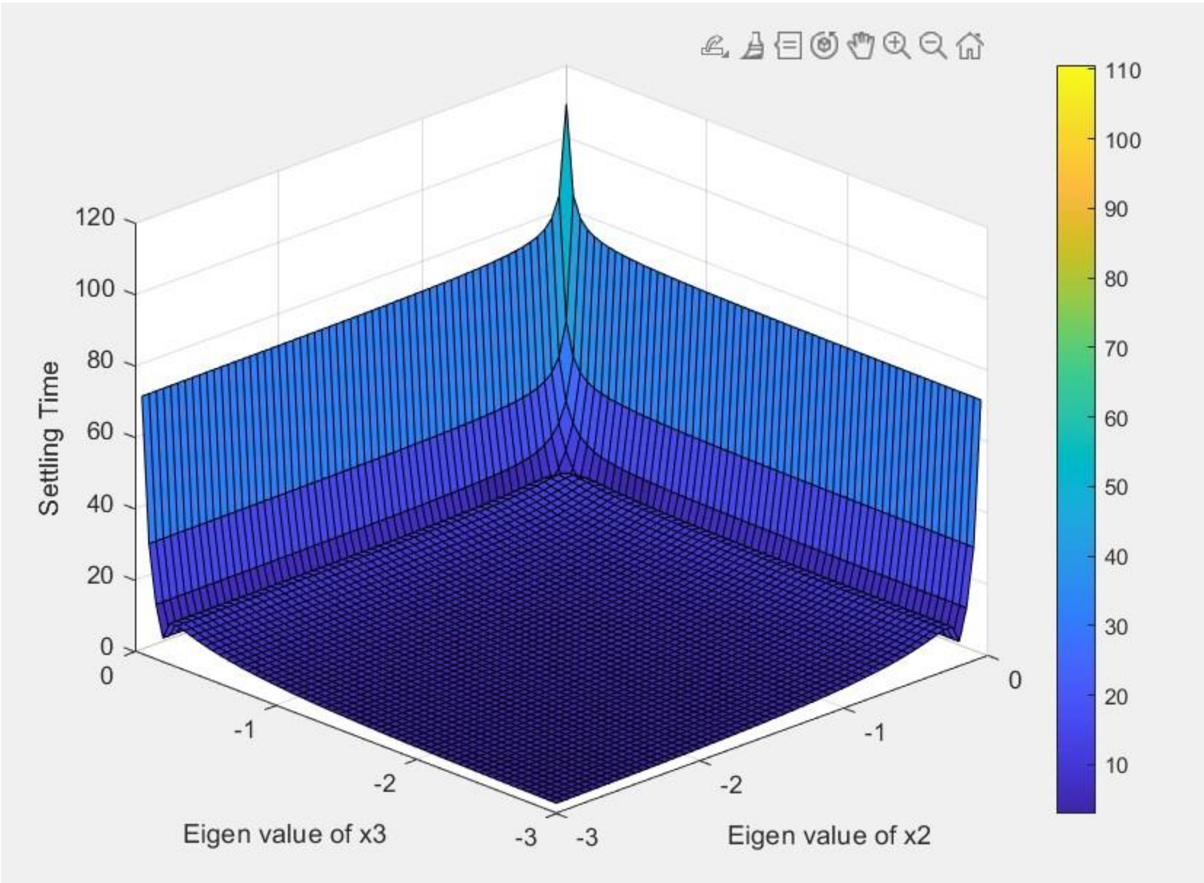
The table below summarizes the step response characteristics.

Step Response Characteristics	Sway speed	Yaw angle
Settling Time (sec)	12.3522	12.9848
Overshoot	0	27.12 %
Peak Time (sec)	30.0643	3.7203
Peak	0.7677	0.2591

The original eigen values of the system were: {0.0; 0.2710; -0.4590}, and the open loop system as a whole is unstable, that is settling time is not really defined, however we can consider the value -0.4590 and ensure that the settling time (represented by pole at s = -0.4590) is retained. We can further plot the dependence of settling time for both sway speed and yaw angle on the eigen values of the closed loop system and we obtain the following plots.



Settling time of sway speed vs eigen values of CL system



Settling time of yaw angle vs eigen values of CL system

At the same time, we required the maximum magnitude to be in the nominal set (0.4590 as per the nominal set), which places an upper limit on the magnitude of the closed loop poles.

Finally, we look at the rank of controllability and observability matrices which comes out to be 3 and 2 respectively for controllability and observability matrix, ensuring that our system is controllable and observable at all points.

Possible Situations for which no design of feedback gain matrix is possible:

- Multiplicity of desired eigenvalues exceeds 1 (number of inputs), for which no feedback gain matrix can be obtained using pole placement method.
- System diverts from the point of linearization (in which case, the performance of designed controller is unknown to us).

CONCLUSION:

Therefore, we designed the feedback gain matrix that meets all the requirements i.e., the settling time of individual states is retained at the nominal values, the maximum magnitude is as in nominal set and the system is always observable and controllable (in the vicinity of the equilibrium point about which the system has been linearized).

MATLAB CODE USED:

```
A = [-0.14 -0.69 0; -0.19 -0.048 0; 0 1 0];
B = [0.056; -0.23; 0];
C = [1 0 0; 0 1 0];
D = [0; 0];
% outputs => sway speed, yaw angle
% state space => sway speed, yaw angle, yaw rate
states = {'sway speed', 'yaw angle', 'yaw rate'};
output = {'sway speed', 'yaw angle'};
input = {'rudder angle'};
sys = ss(A, B, C, D, 'statename', states,...
    'inputname', input,...
    'outputname', output); % state space model
G = tf(sys);
eig(sys)
t = 0:0.01:100;
G(1)
step(G(1), t), grid;
G(2)
step(G(2), t), grid;
P = [0 -0.458 -0.459];
K = place(A, B, P);
syscl = ss(A-B*K, B, C, D, 'statename', states,...
   'inputname', input,...
   'outputname', output);
G = tf(syscl)
rank(ctrb(sys))
rank(obsv(sys))
figure; step(syscl, t), grid;
figure; impulse(syscl, t), grid;
damp(syscl)
stepinfo(G(1))
stepinfo(G(2))
```