CONTROL ENGINEERING LAB

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GROUP 18

AMAN SAINI (2020EEB1155)

ANIRUDH SHARMA (2020EEB1158)

ANSAF AHMAD (2020EEB1160)

OBJECTIVE:

Dynamic study of the given crane trolley system on Simulink, using a detailed non-linear state space simulation in the four state variables.

INTRODUCTION:

The motion of an overhead crane can be represented by the following system of differential equations:

$$[m_L + m_c] \cdot \ddot{x}_1(t) + m_L\, l \cdot [\ddot{x}_3(t) \cdot cos x_3(t) - \dot{x}_3^2\,(t) \cdot sin x_3(t)] = u(t)$$

$$m_L\,[\ddot{x}_1(t) \cdot cos x_3(t) + l \cdot \ddot{x}_3(t)] = -m_L\, g \cdot sin x_3(t)$$

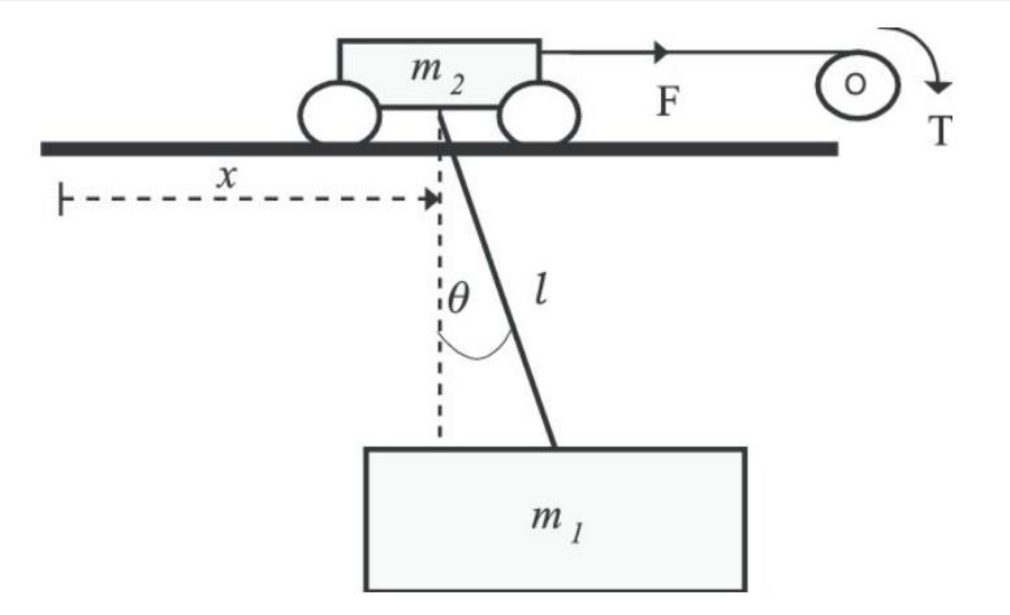


Fig: illustration of system with the help of a diagram

Here, m_C *= mass of trolley; (10 kg)*

m_L *= Mass of hook and load; the hook mass is 10 kg; load can be varied from 0 to several hundred kg*

l = *Rope length; 1m or higher, const. for a given operation*

g = *Acceleration due to gravity*

Variables for the problem include:

INPUT: *u: Force in Newtons, applied to the trolley*

OUTPUT: *y: Position of load in meters,* $y(t) = x1\ (t) + l \cdot sin x3\ (t)$

STATES:

$x1$*: Position of trolley in meters*

$x2$*: Speed of trolley in m/s*

$x3$*: Rope angle in rads*

$x4$*: Angular speed of rope in rad/s*

We can obtain the value of the $\ddot{x}_1(t)$ and $\ddot{x}_3(t)$ from the given differential equations, which can be used to create the non-linear system simulation on Simulink.

$$\ddot{x}_1 = -\frac{m_L cos(x_3) sin(x_3) g + m_L l \dot{x}_3^2 sin(x_3) + u}{-m_L - m_C + m_L cos^2(x_3)}$$

$$\ddot{x}_3 = -\frac{m_L \dot{x}_3^2 l \cdot cos(x_3) sin(x_3) + m_L g \cdot sin(x_3) + m_C g \cdot cos(x_3) + u \cdot cos(x_3)}{l \cdot (-m_L - m_C + m_L cos^2(x_3))}$$

These expressions can be used to create the patch diagram in Simulink, once we have $\ddot{x}_1$ and $\ddot{x}_3$, we can use them to estimate the future values of angular velocity and speed of the crane, which can be further used to calculate angular position of the load and position of the trolley. Once we have the value of these derivatives, we only need initial conditions to obtain the complete response of the system.

SIMULINK MODEL:

The following images provide a detailed description of the Simulink model.

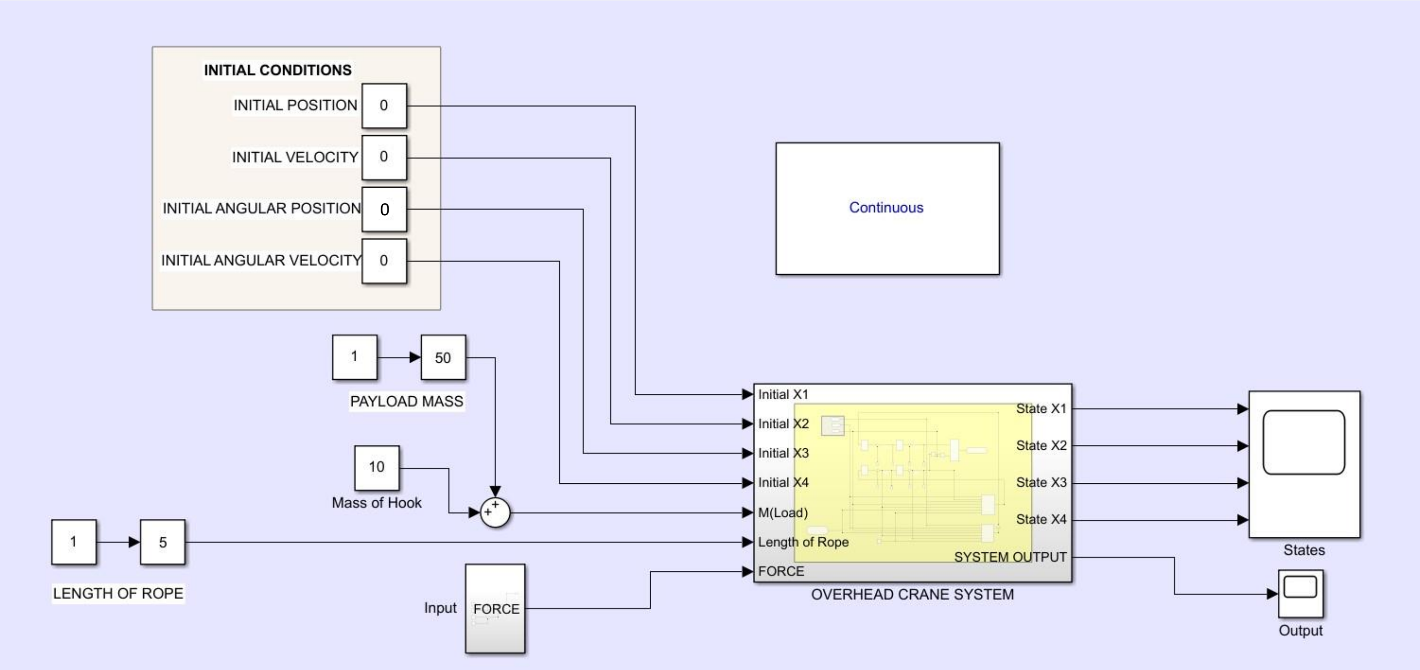


Fig 1: Model of Crane system (inputs: initial condition, length of rope, mass of load; outputs: states of the system and output y(t))

We created a subsystem to represent the overhead crane system, and provide inputs to the system in form of initial conditions and system variables (length of rope and mass of load), the output and states of the system are connected to scope for visualization.

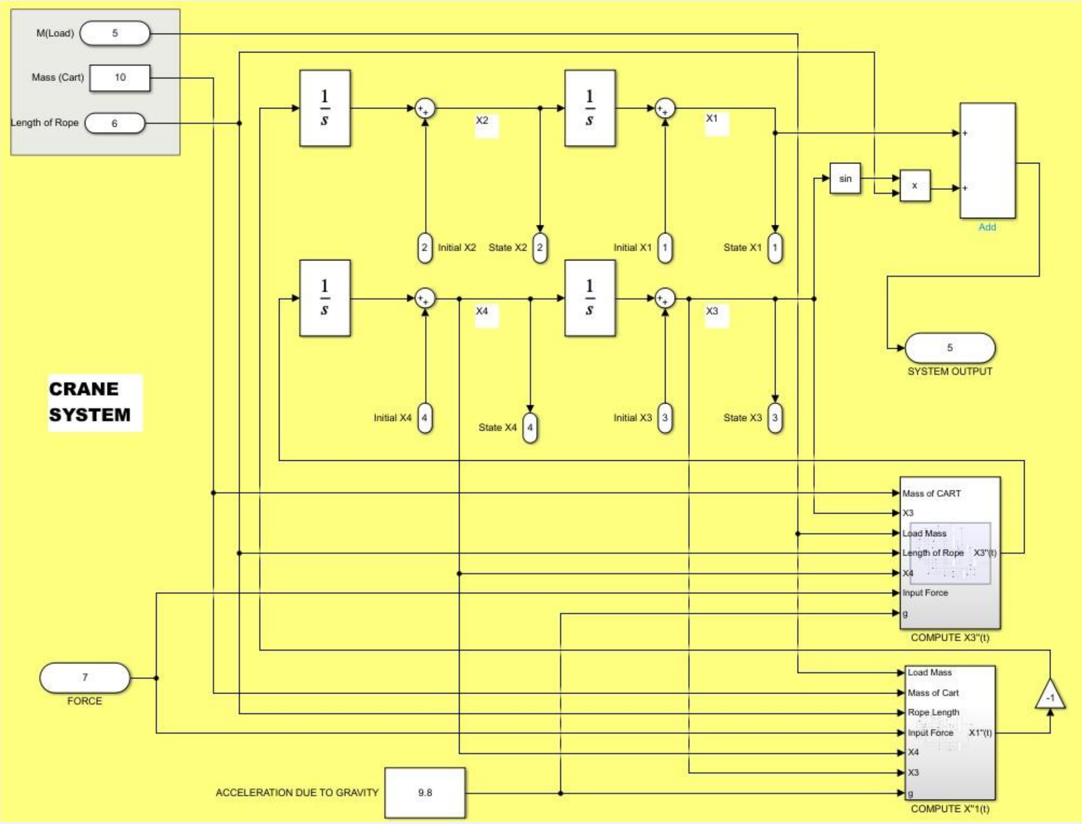


Fig 2: the crane subsystem, inputs and states are used to calculate the value of $\ddot{x}_1$ and $\ddot{x}_3$

The inputs to the crane subsystem include initial conditions, which are directly summed to finalize the value of states, moreover inputs like Load Mass, length of rope is used to calculate $\ddot{x}_1$ and $\ddot{x}_3$ in compute blocks as visible in the above figure. Once we have these values, they can be passed through integrators to calculate the states of the system.

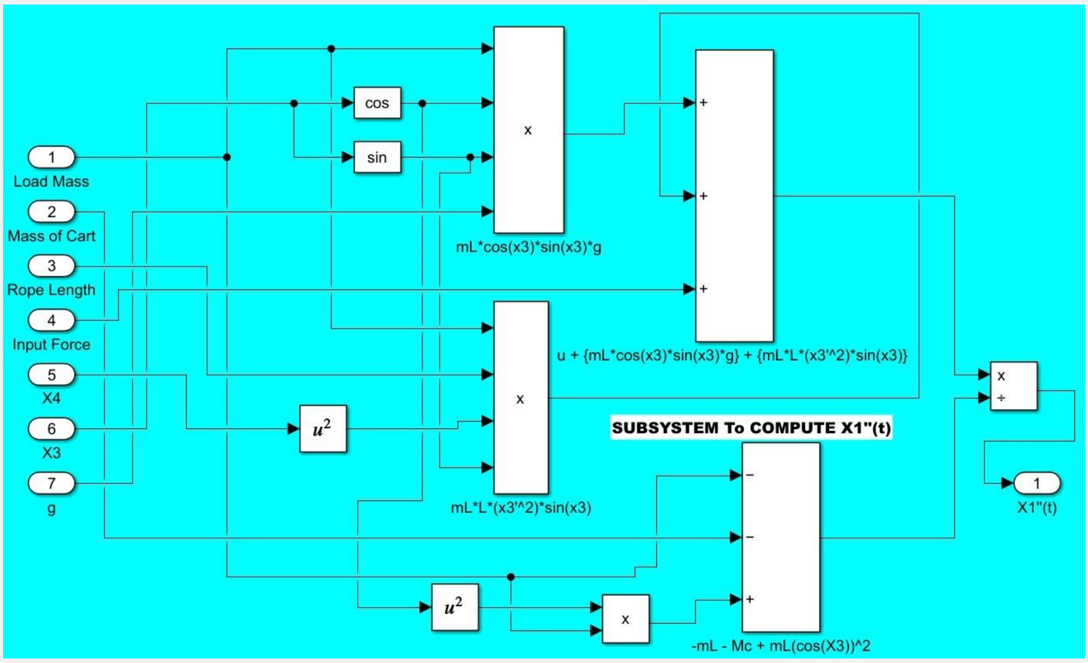


Fig 3; Compute x1''(t) block. {computes x1''(t)} as per the differential equation obtained in previous discussions

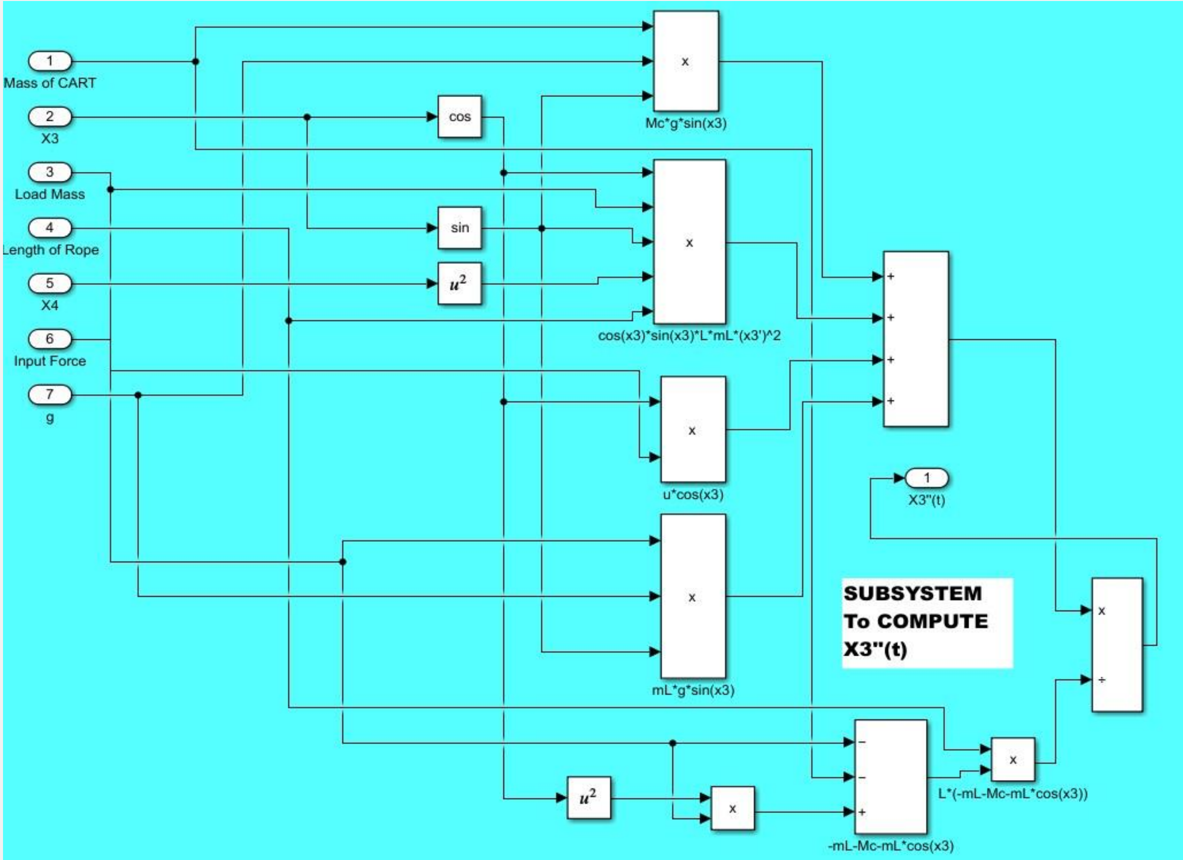


Fig 4: Compute x3''(t) Block, computes the value of x3''(t) as per the differential equation obtained in previous discussions

EQUILIBRIUM POINT ANALYSIS:

Now, as discussed in Expt. 8, the given system of differential equations represents the system in its ideal sense, i.e., no losses of energy in form of friction, tension in rope, elasticity and slip. And the equilibrium points of the given system are $\{x_1, 0, n\pi, 0\}$, also it is obvious that the condition, where n is odd, or $x_3 = 180^o$ is impractical and not desired, since in reality the rope and the system overall won't be ideal, and a condition of $x_3 = 180^o$ would be highly unstable (if reached at all, as it would be requiring a feedback control that too only for the ideal case), would lead to high oscillations or disasters for heavy loads. So, it is clear that the initial conditions for the system (or the point from where the crane would be starting out with the load) would be of the form $\{x_1, 0, 2n\pi, 0\}$. Having established the starting point of the system, we can now study the response of the system for various input signals.

RESPONSE OF THE SYSTEM:

The response of the system is needed to be analyzed for different operating conditions (i.e., different values of rope length and load mass), we start by studying the impulse response of the system for different value of load mass and rope length. Providing input to the system in form of force provides the momentum to the crane system, and further as the system is loss-less, it maintains a constant momentum, so to stop the motion of the crane, we need to provide a negative impulse input (such that it imparts equal momentum to the system as the first one but in opposite direction).

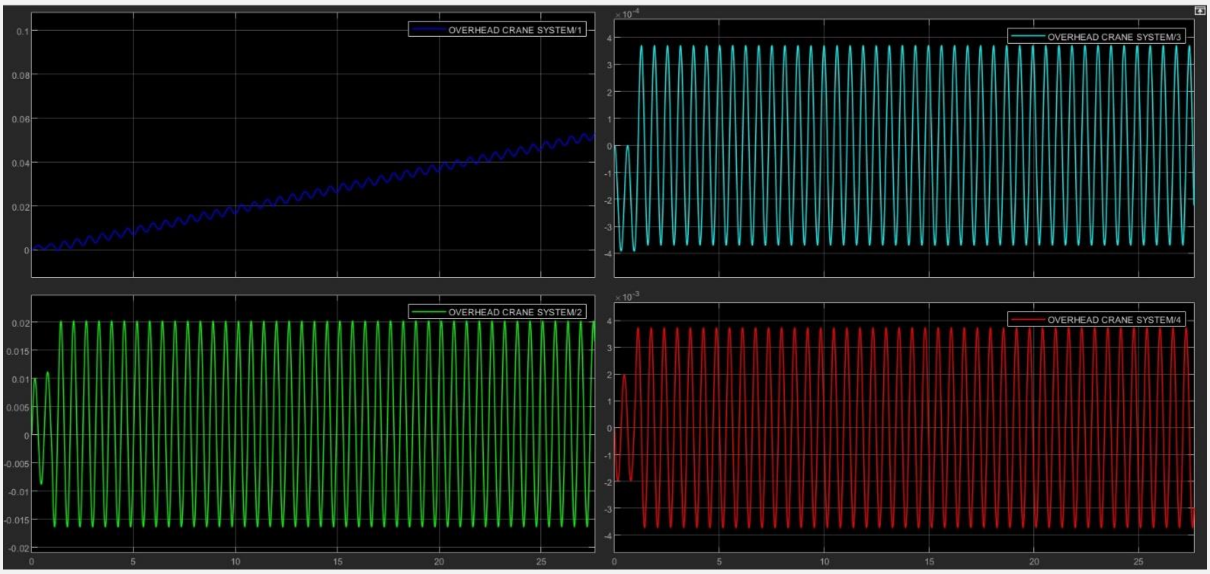


Fig 5: Variation of states with time on providing impulse input-

*for **M = 500 kg and l = 5m (for initial conditions {0,0,0,0})***

As is clear from the above figure, the states x_2, x_3 and x_4 oscillate sinusoidally however the magnitude of oscillations for x_3 and x_4 are very small (which is also desirable since we do not want high sway while carrying some load). The position x_1 of the trolley increases linearly with sinusoids super-imposed on it, which can be understood from the variation of x_2, which shows sinusoid oscillations with equilibrium slightly above 0. Now, regardless of the mass of the load and length of the rope, the output and variation of states remains similar, the only thing which changes is the amplitude, time period, equilibrium points of oscillations, which changes also changes the slope of position-time graph. The output for this case is shown in the figure below:

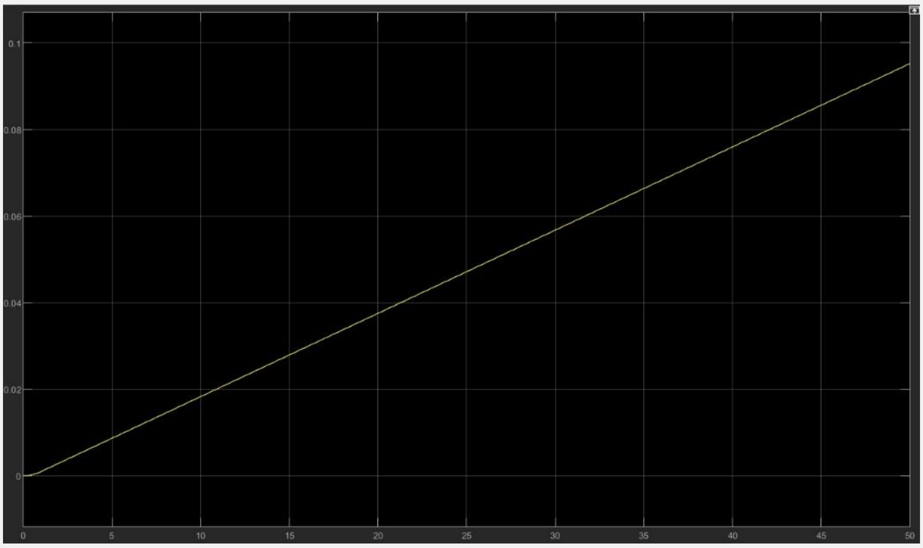


Fig 6: output for the impulse input for M = 500 kg and l = 5m

Unlike the states, the output shows a smooth linear increase with time, as the magnitude of angulate oscillations (of payload) and oscillations in speed (of trolley) is very small. However, these oscillations are still undesirable as they can lead to significant variation in position for long rope lengths and small masses, even when we stop the trolley. The following plot illustrates the problems associated with sway after stopping the trolley for low mass, and long lengths of the rope.

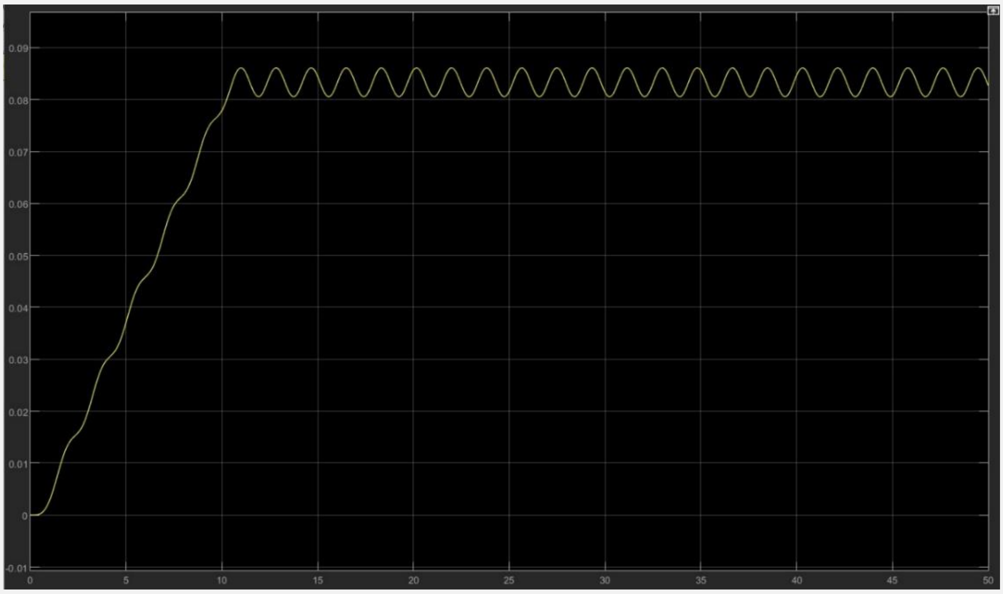


Fig 7: Non-Decaying oscillations, after we stop the system (reduce the momentum to 0) the position of payload keeps on oscillating and does not decay.

The above scenario is undesirable, as for transitions from one equilibrium point to another, we need to stop these oscillations, which is however not possible without feedback control. Although the oscillations do not stop, but still we manage to get a finite final position around which the output oscillates by imparting equal momentum in opposite direction which tells us about the way crane has to be operated.

<u>Step Input</u>: Providing a continuous step input for the ideal case makes little sense, however it would be the most important input while operating the crane in real world, as a continuous constant force is required to overcome friction and constant losses. However, for the ideal case, the step input leads to unstability as shown in the following figure:

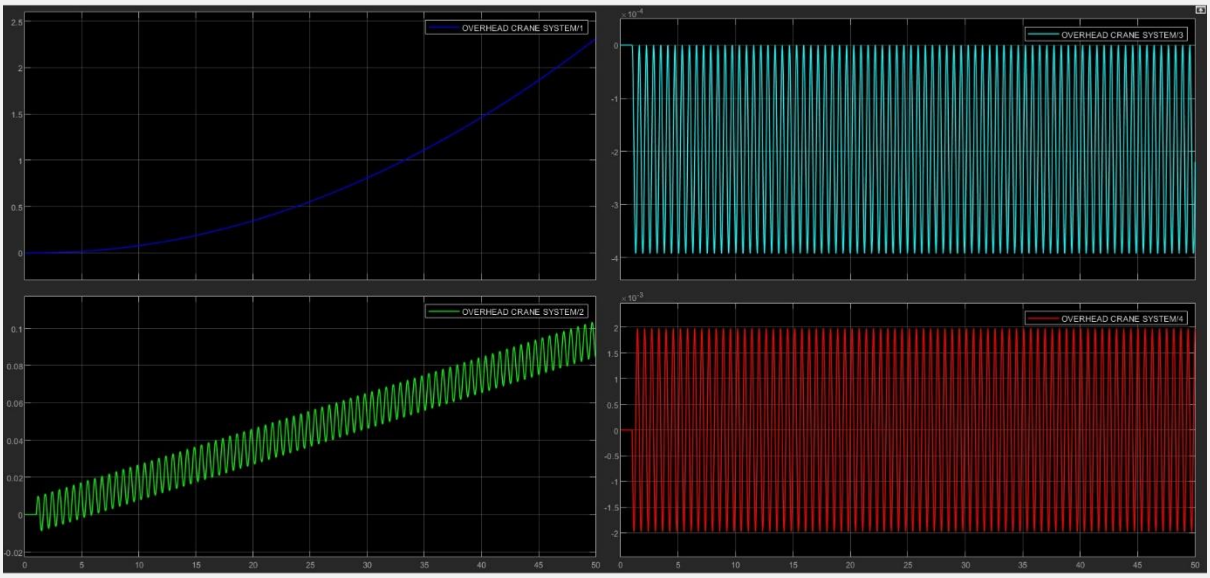


Fig 8: Step input for M = 500 kg, l = 5m

The state x_2 speed of the trolley shows sinusoid variations with equilibrium increasing linearly. The sinusoids are due to oscillations of Load, which also impacts the velocity of trolley because the **given system is isolated and conservation of momentum applies.** The equilibrium position of payload oscillation is also shifted (negative) and constant because of pseudo-force as a constant step input has been applied. State x4 shows sinusoidal variation with equilibrium at 0. The position x1 of the trolley is directly proportional to square of time (with small sinusoidal oscillations superimposed). The output for step input (also directly proportional to square of time) is given in the following figure.

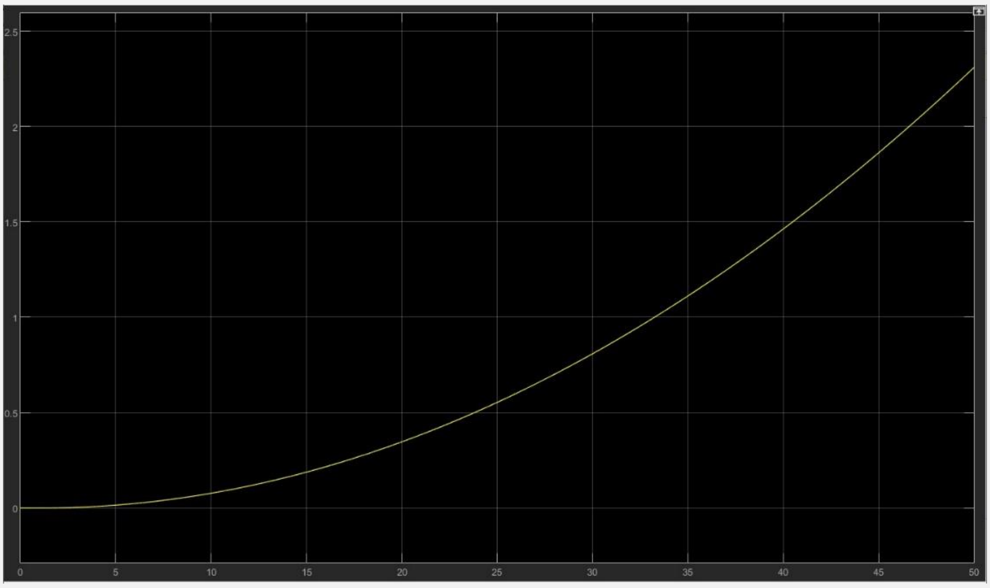
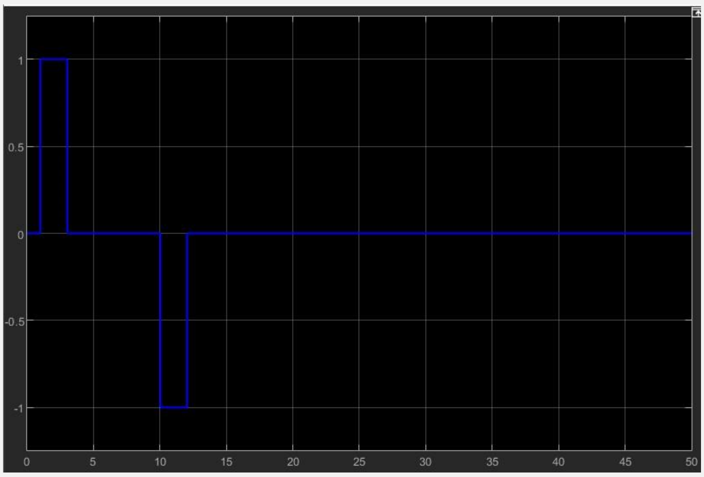


Fig 9: output for the step input for M = 500 kg and l = 5m

The output that we obtain is not desirable for any mode of operation of the machine, therefore a better option would be to provide the step input till we reach an optimal speed of operation for the trolley, let the crane reach the desired location (travelling with this velocity) and finally apply the similar input but in opposite direction when we need to stop the crane. We can definitely scale up the input for heavier loads as it would require more energy for a heavier load to achieve the same velocity (or to overcome friction and other losses (for real world scenario) in case of heavier loads). The input that we provide to the crane system is given below:



<u>Fig 10</u>: *input to the system (step input is turned on for some time, and after the system achieves the desired speed, it is turned off, and we apply the similar input in the opposite direction to stop the crane)*

The output and the states of the system for above input varies as follows:

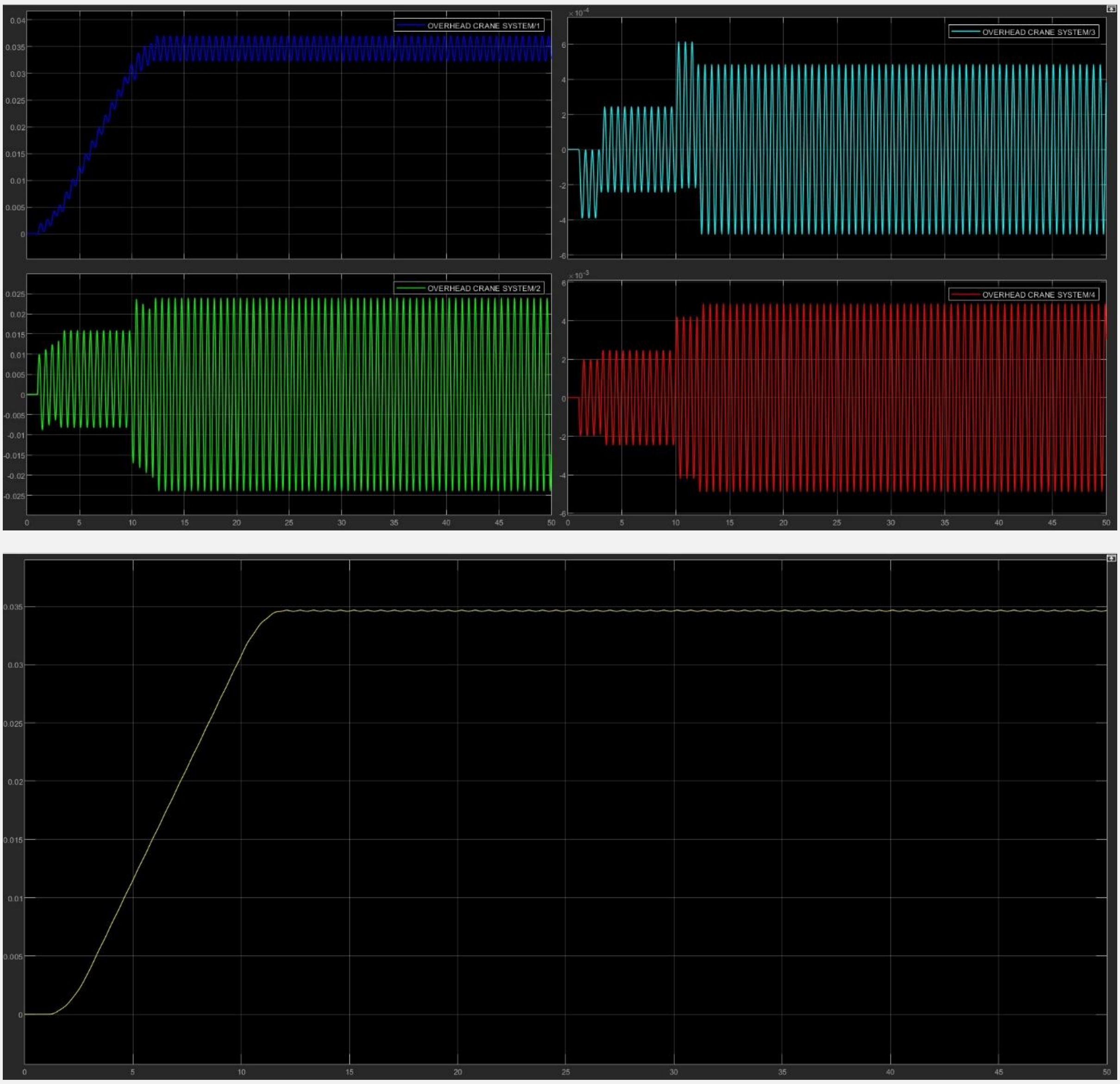


Fig 11: *the output and the states of the system (M = 500 kg, l = 5m) for the above input. (Notice that the even though the momentum of the complete system reduces to zero, the states and output still oscillates in such a way that the sum of momentum of trolley and load is zero, also we see that the oscillations in output are much less than the oscillations in position of trolley, which is due to the fact that the angular velocity of the rope is in opposite direction to that of the trolley and hence cancels its effect on load)*

<u>Ramp Input</u>: In this case, unlike step and discrete input, we increase the force on trolley gradually, which reduces the jerk that the system receives, and hence reduces the amplitude of oscillations as compared to previous 2 cases. However, providing a ramp input to the system is a difficult job as the force increases linearly with time (it needs to be switched off at some point due to limitations of actuating devices in real world) and an unbounded ramp input would lead to unstability in position of trolley, its speed and the angular position of the load. The following responses illustrate the above points.

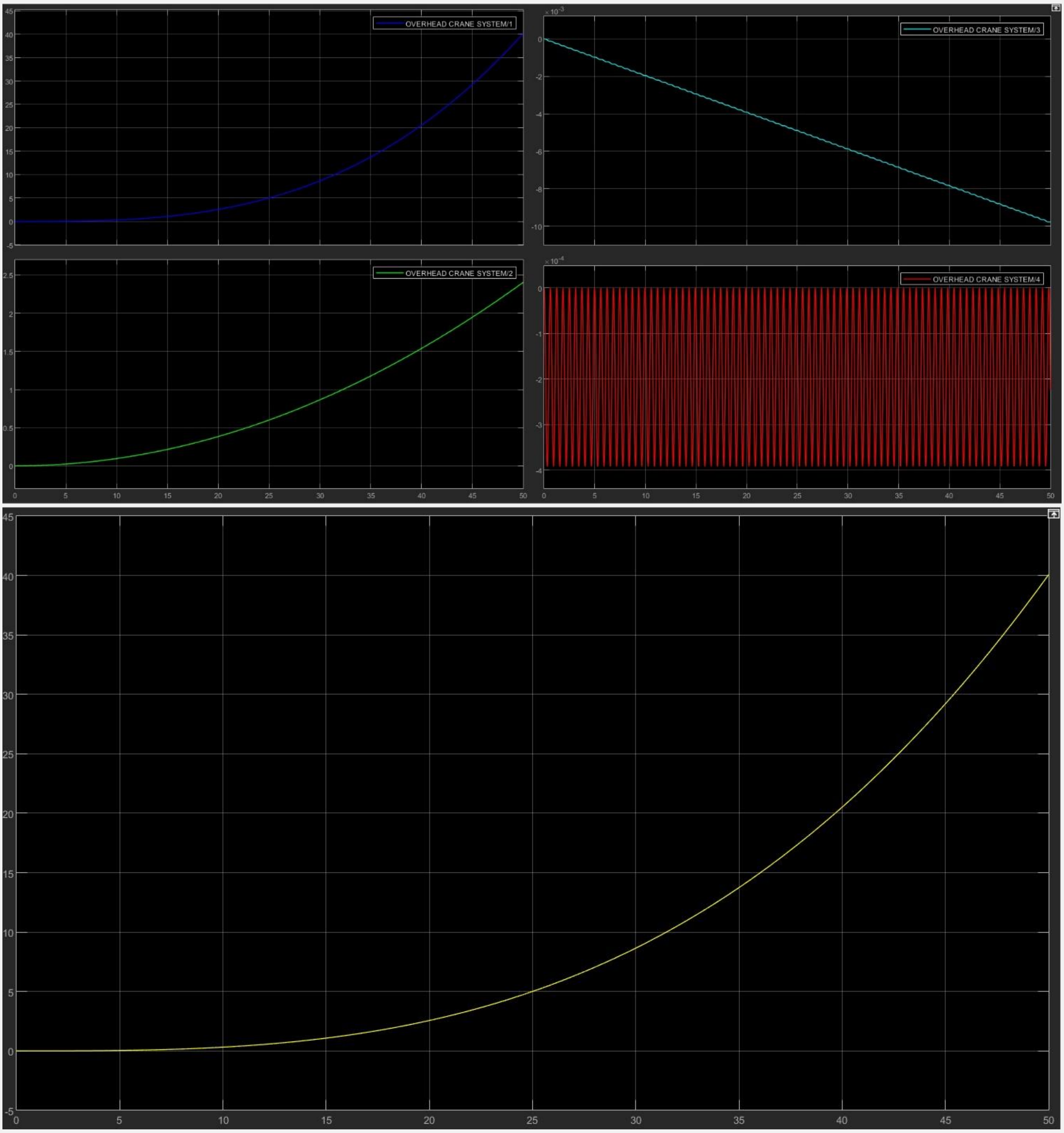


Fig 12: *States and output of the system for Ramp input for M = 500 kg and l = 5 m (Notice that the velocity of the trolley is directly proportional to* t^2 *and position is directly proportional to* t^3*, the equilibrium point of angular oscillations decreases linearly with time & angular velocity shows sinusoid oscillations completely in negative region). Also, the magnitude of amplitude of oscillations for angular velocity in this case is almost* $2e^{-4}$ *which is less that step input by almost a factor of 10, hence ramp type of input can be utilized to increase the force on the trolley in the beginning of operation (because of a finite amount of jerk (unlike step and impulse inputs)).*

DEPENDENCE OF SYSTEM DYNAMICS ON THE MASS OF THE LOAD:

Mass is one of the key parameters in deciding the response of the system to any kind of input. With an increase in load mass, the same kind of input produces less variation in the states and hence the output of the system. The amplitude of oscillation of angular position decreases with increase in Mass loaded and hence the time period reduces (however time period is one of the parameters that we are not particularly interested in since our primary goal is to have small swing in load which is decided by the amplitude of oscillations).

The following plot shows the variation in amplitude of angular position for step and impulse input.

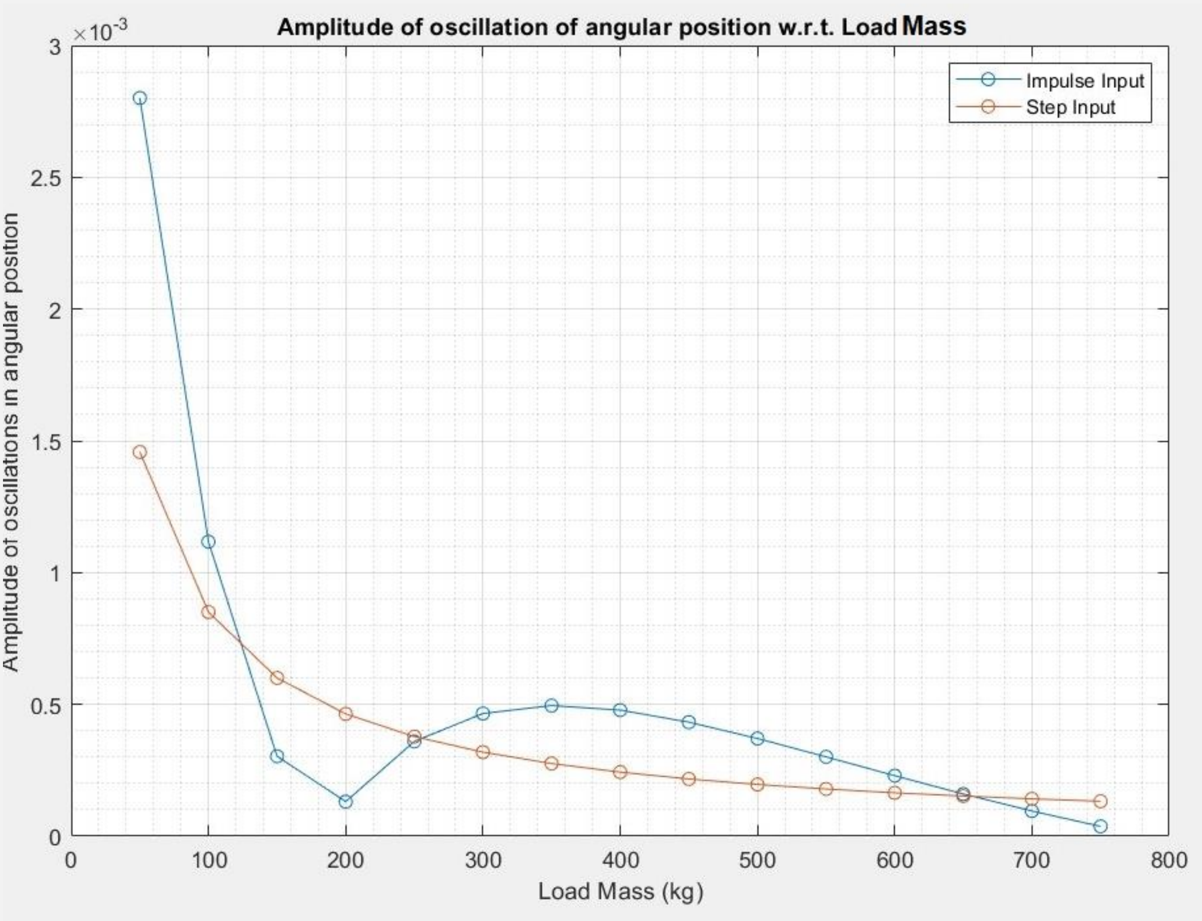


Fig 13: Amplitude of oscillations in angular position (radians) w.r.t load mass (Notice that the amplitude decreases with increase in loading mass, although there are certain irregularities for impulse input), the amplitude generally decays with increasing mass. Also, the amplitude of oscillations observed in step input is lower for a wide range of Load masses.

Apart from reduction in amplitude of oscillations, increasing the mass also reduces the acceleration of the system, therefore we require a scaled-up input for the case of heavy loads to achieve similar acceleration.

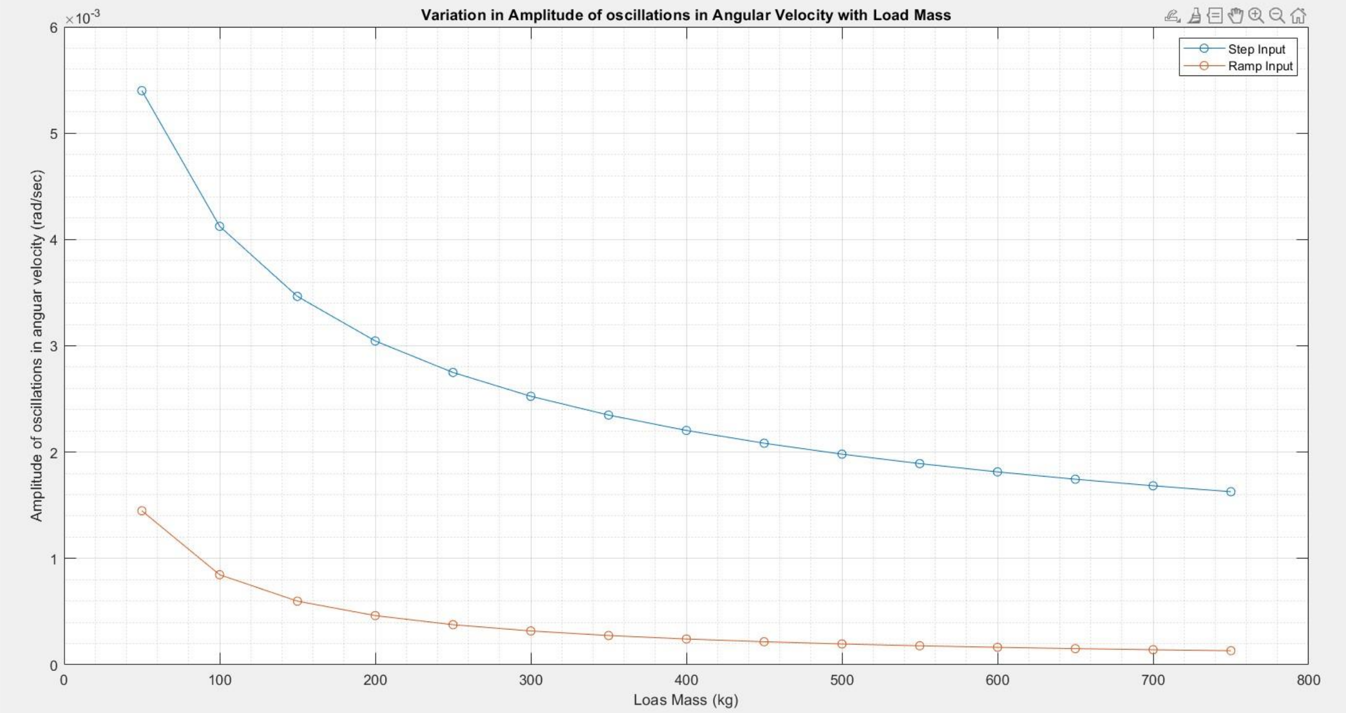


Fig 14: Variation in amplitude of oscillations in angular velocity with load mass, (Notice that the amplitude is of the order of 10^{-3}, decreases with increasing mass, and is much less for ramp input as compared to step input, which is due to gradual increase in force (and hence less jerk) in case of ramp input as compared to the step input case) this property can be utilized to generate an optimal input signal to control the machine.

OPTIMAL INPUT SIGNAL TO OPERATE THE CRANE:

Having studied the system dynamics for ramp, step and impulse input we can conclude that the best input to work with (for ideal case as in the problem) can be sketched as follows:

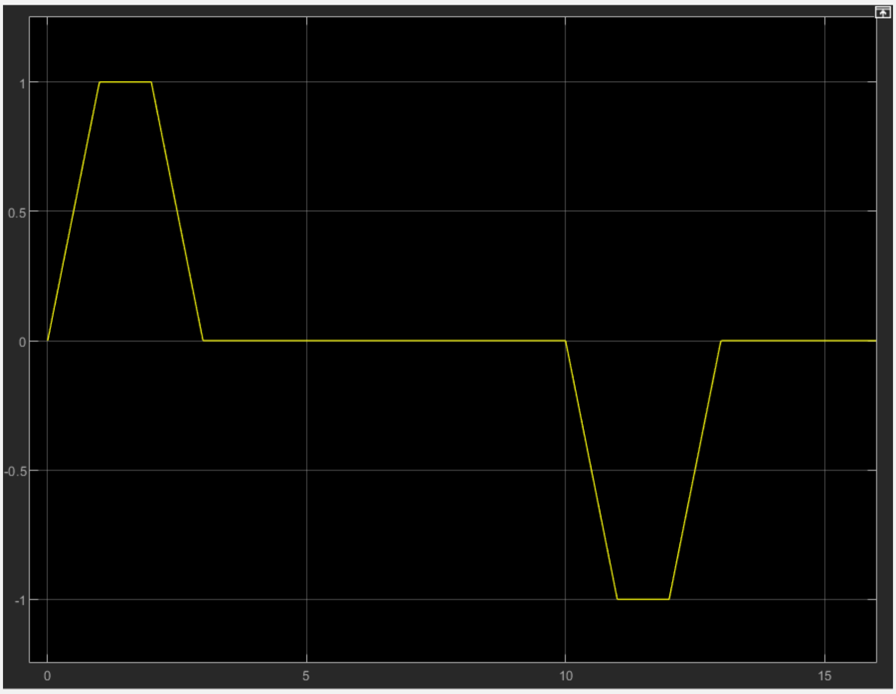


Fig 15: *The ramped part of the signal limits the amount of swing during the operation, then a constant force is maintained (to achieve the desired speed) and the input is then gradually reduced (again, to limit the swing in the load). To stop the crane similar input is applied but in the opposite direction.*

The dynamics of output and the states of the system (for M = 500 kg and l = 5m) for the above input can be studied using the following plots.

Fig 16: the output and the states of the system for the above input (scaled by a factor of 100), the scaling is essential to produce a significant amount of change in position (almost 4m for our case), however scaling also increases the jerk, and hence the amplitude of oscillations, even for an optimal input like we provided for our case. Apart from that, the output shows a smooth variation (for the reason stated in Fig 11) but still the oscillations and the swing we are getting is undesirable.

SWING CONTROL:

Starting out the machine from one steady state and reaching another is not possible for any type of input discussed till now, nor with their combination as discussed in previous section. For the response discussed in previous section, the output of the system was a smooth curve with no visible oscillations, states however are varying with time, and hence the system still hasn't reached a steady state. A feedback control can be established to reduce the swing once the trolley reaches the desired position. The problem provides us with no desired parameters like desired settling time, peak overshoot etc. which we can take into consideration while constructing a feedback control, so to illustrate the swing control mechanism, we take one such feedback gain in consideration which gives reasonable satisfying values of the parameters (like settling time, transient time, peak overshoot etc.)

We simply provide constructive feedback of angular velocity with gain = 100 and obtain the following plots.

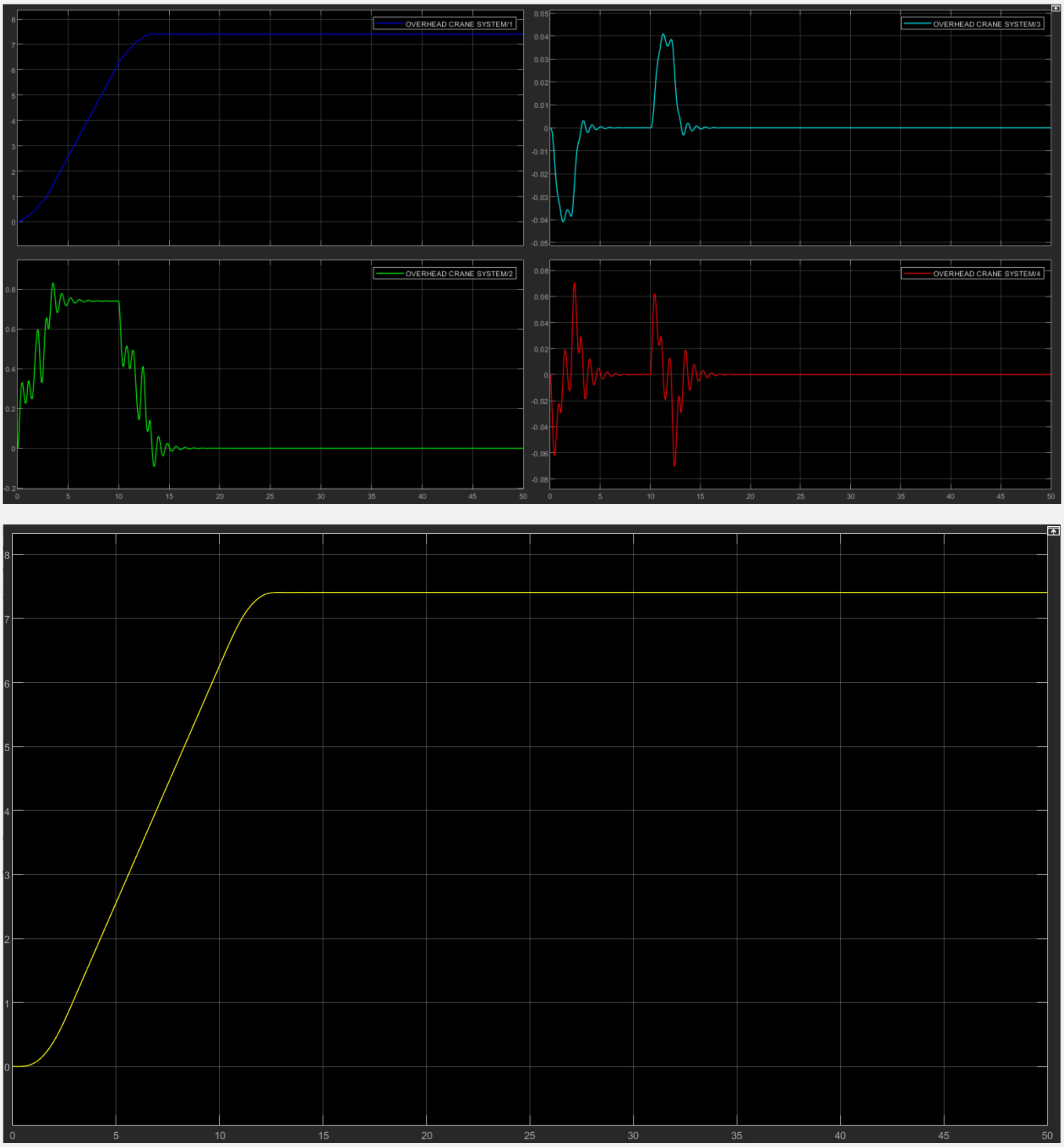


Fig 17: *The performance of the system for M = 250 kg, and l = 5m. With a feedback control, the system is able to reach a steady state and a position of global stability* $\{x_1, 0, 0, 0\}$*, the length of rope and load mass change the settling time and amplitude of oscillations (amplitude and settling time increases with increase in length and decrease in mass) & the given control is valid for a wide range of practical values of length and mass.*

CONCLUSIONS:

We simulated the given non-linear system in Simulink, obtained the responses for a variety of inputs, and hence constructed the best input to operate the Crane. Studied the dependence of system dynamics on the mass of the load with the help of trends in states of interest. We tried simulating the model to reach from one steady state to another, resulting trends showed continued oscillations, even after reducing the momentum to zero, therefore built one such feedback controller which is able to decay the oscillations in states for a wide range of practical Loaded Masses and rope lengths.