CONTROL ENGINEERING LAB

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GROUP - 18

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OBJECTIVE

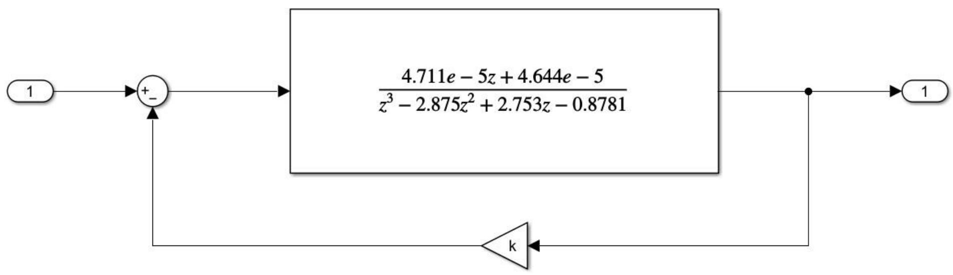
The given problem requires the analysis of Gain and Phase Margins of a Digital System as we change the Sampling Time and feedback gain “K”.

INTRODUCTION

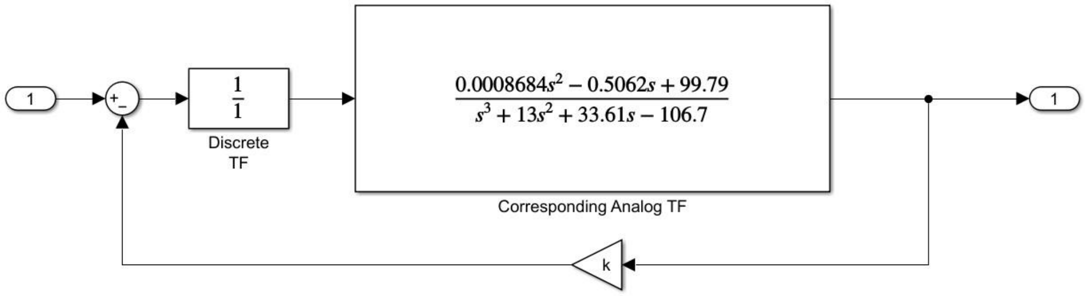
We are given the OLTF of an Electric Furnace Model, inclusive of a first order actuator.

$$G_{OL}(z) = 10^{-5}\frac{4.711z+4.644}{z^3-2.875z^2+2.753z-0.8781}$$

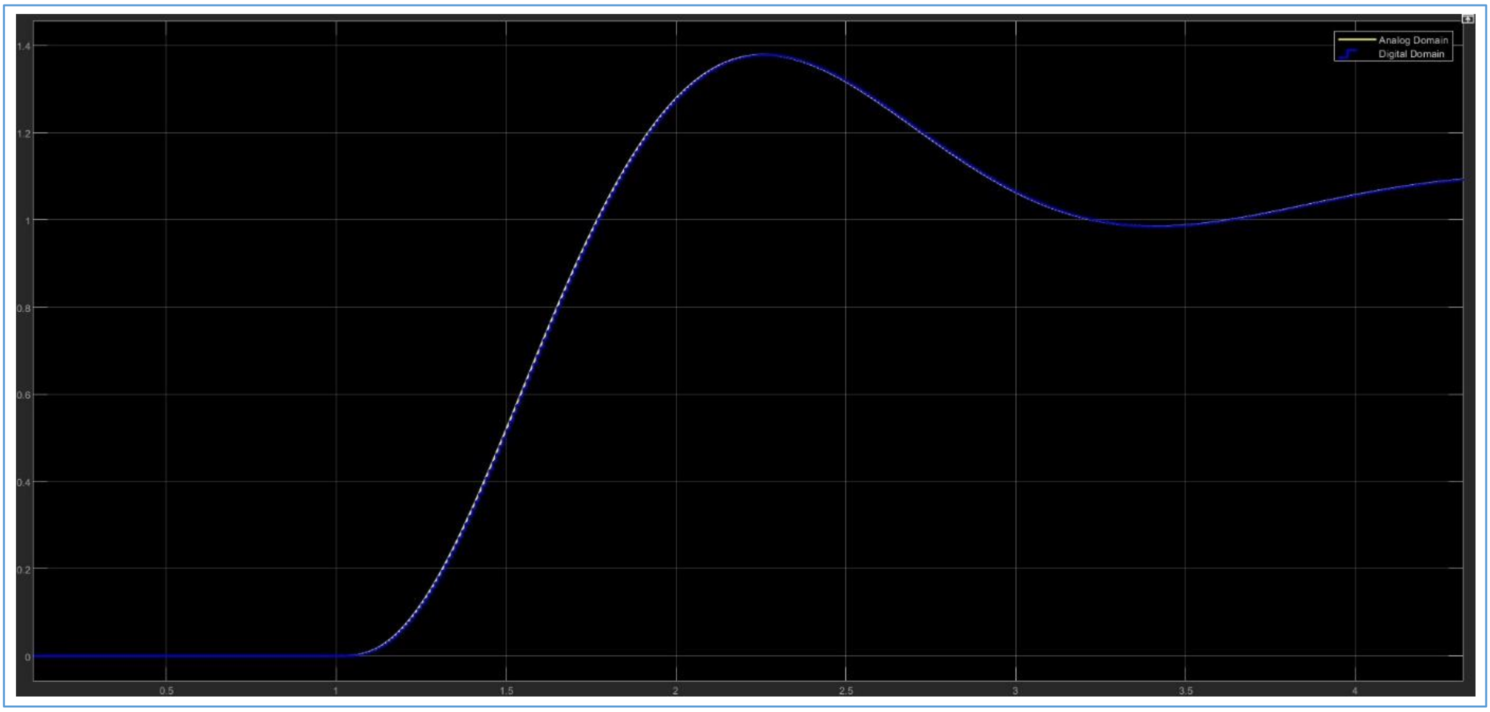
The output of the system can be fed back to the actuator input using negative feedback with different positive integer gains (in the feedback loop). The nominal Sampling time of the system is given to be 0.01 sec. Thus, we can represent the overall system using the figure below:



We know that the actuator and the plant, both are analog systems, which are being controlled using a digital controller, which samples the output (at steps of time T_s) and provides the input to the actuator using a Zero-Order Hold. Therefore, we can replace the given OLTF with a corresponding analog transfer function cascaded to a unity gain digital block and desired sampling time.



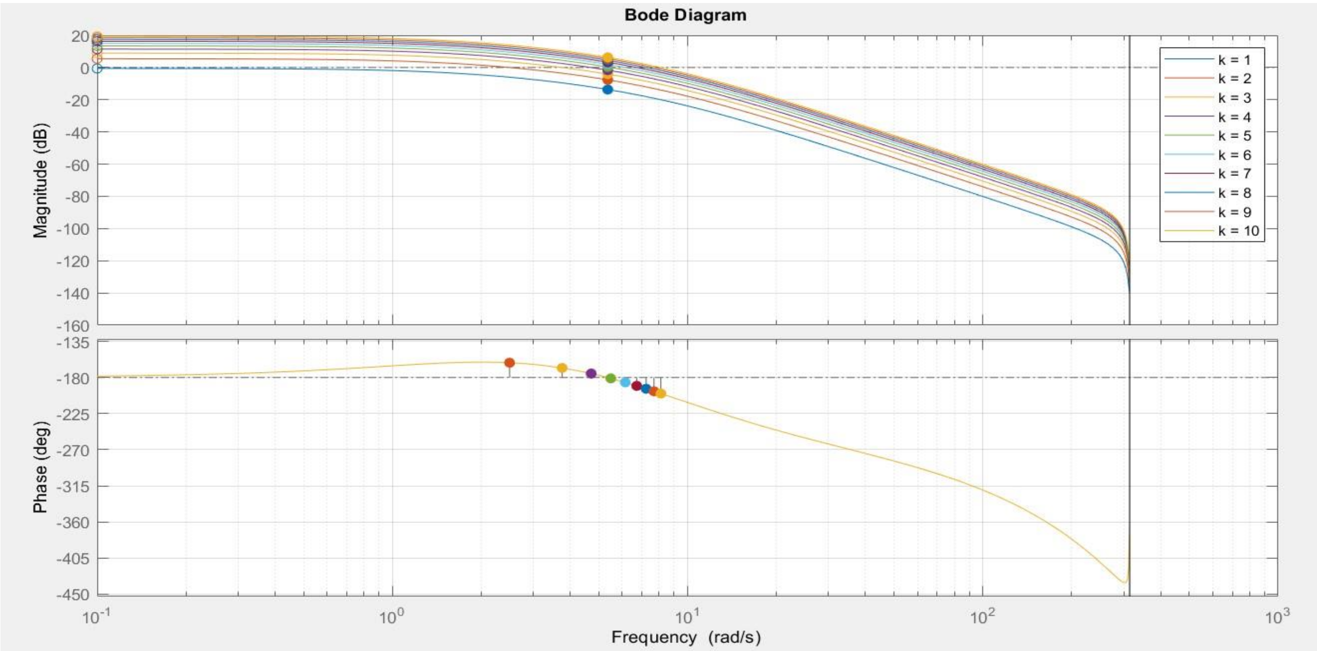
Using the step responses of the above 2 models at the sampling time T_s = 0.01 s (nominal sampling time) and k = 2, we can confirm that the both behaves in a similar manner.

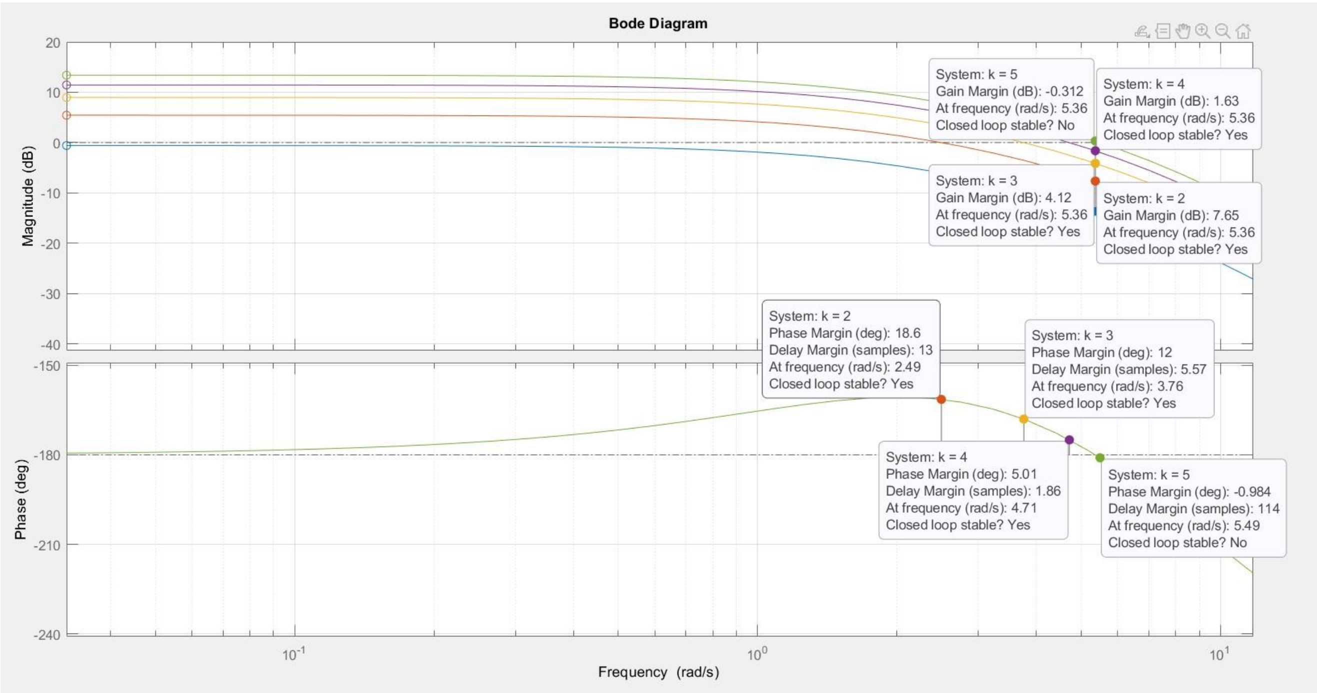


One essential difference is that the step response of the system (completely described digital domain) is not continuous, that is due to the fact that the information of the output is itself being sampled (not because output itself is incrementing or decrementing in steps). The above discussion tells us as to how we can change the Sampling rate of the system. Once we change the Sampling Rate the digital domain representation of the system is also bound to change, hence we can convert the given digital OLTF to analog domain, and obtain the digital transfer function at different sampling rate directly from analog OLTF.

EFFECT OF FEEDBACK GAIN ‘K’ ON GAIN AND PHASE MARGIN OF THE SYSTEM

We vary the feedback gain ‘K’ of the system and study its impact on Gain and Phase Margin for nominal Sampling Time value $T_s = 0.01$ sec. Changing the feedback gain does not affect the phase plot, however it does shift the gain plot, hence altering the stability margins.

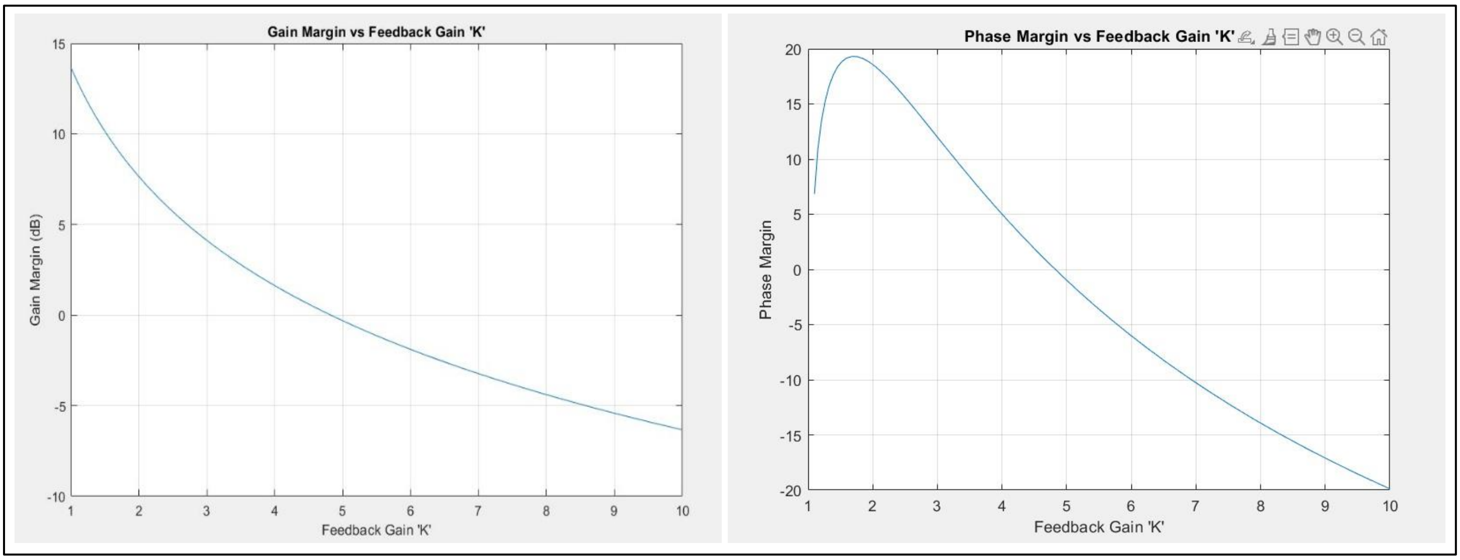




From the bode diagrams, we see that the closed loop system is stable for following Feedback Gains: {K = 2, 3, 4} (at the nominal Sampling Time value). Furthermore, at K = 1, the gain of the system never crosses 0dB, hence the P.M. is inf (or undefined), in addition to this the system is open loop unstable, which requires us to analyse stability at K = 1, using root-locus plot where we see that the system comes out to be unstable. Therefore, we can conclude that the system is stable for K = 2, 3, 4 and drifts towards unstability for K ≥ 5.

Feedback Loop Gain	Gain Margin (dB)	Phase Margin (deg)	Stability
K = 1	**13.67**	**inf**	**Unstable**
K = 2	**7.65**	**18.6**	**Stable**
K = 3	**4.12**	**12**	**Stable**
K = 4	**1.63**	**5.01**	**Stable**
K = 5	**-0.312**	**-0.984**	**Unstable**
K = 6	-2.3	-6.2	Unstable
K = 7	**-3.5**	**-10.15**	**Unstable**
K = 8	**-4.1**	**-13.99**	**Unstable**
K = 9	**-5.23**	**-17.1**	**Unstable**
K = 10	**-6.2**	**-19.98**	**Unstable**

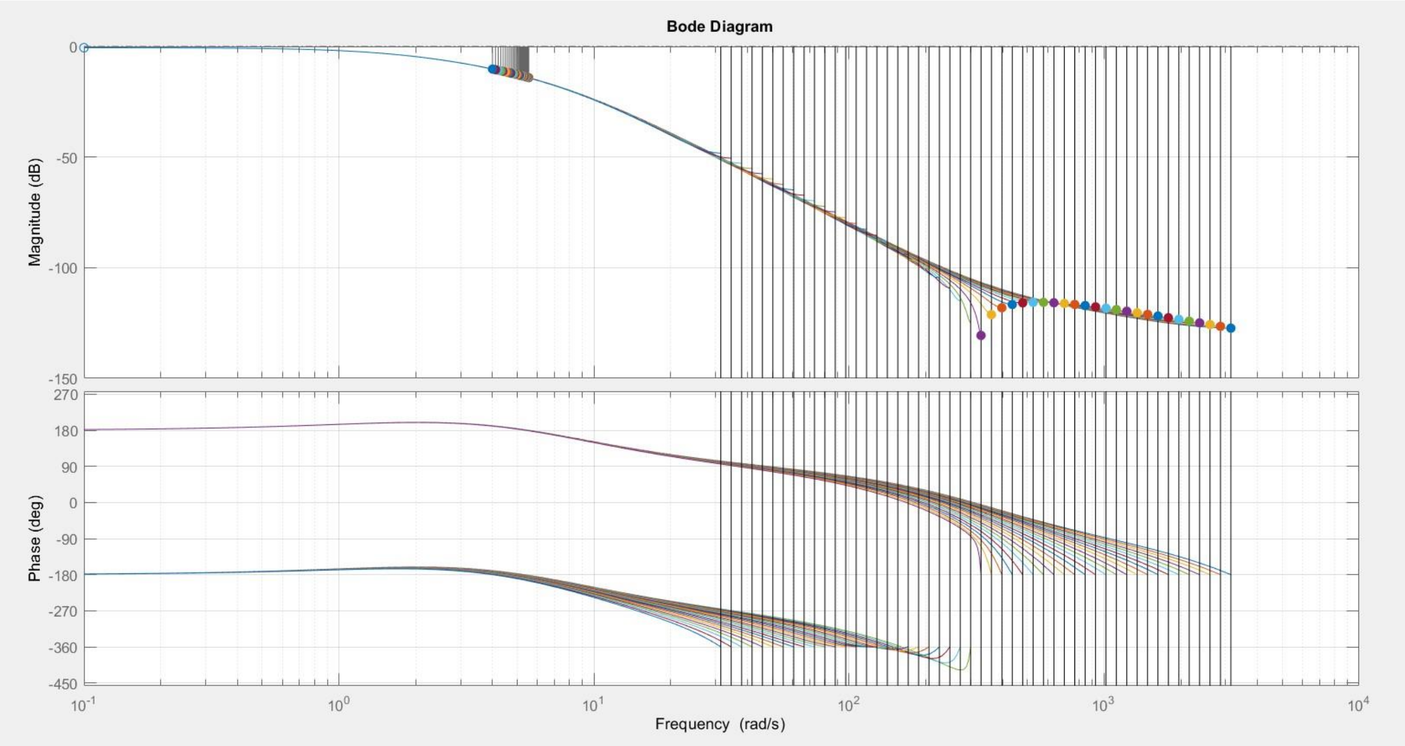
The following plots illustrates the variation of Phase and Gain Margins with the variation of Feedback Gain K.



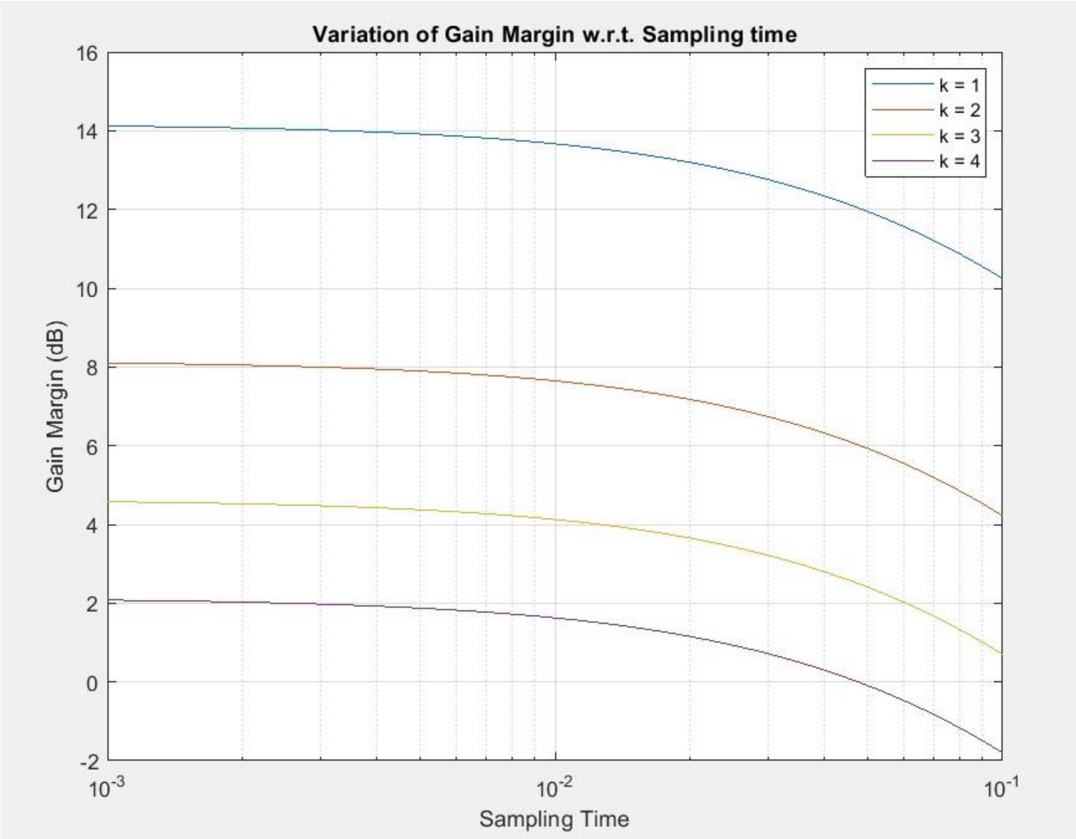
We see that with increasing value of 'K', the Gain cross-over frequency increases and as K becomes very large, it approaches Nyquist Sampling Frequency, and the Corresponding Phase Margin Decreases, shifting to negative side at K = 5. It is also evident that the Phase and Gain margins are largest for K = 2 (among all the positive integer gains) and with increasing K, the stability margins reduce.

EFFECT OF SAMPLING TIME ON GAIN AND PHASE MARGINS

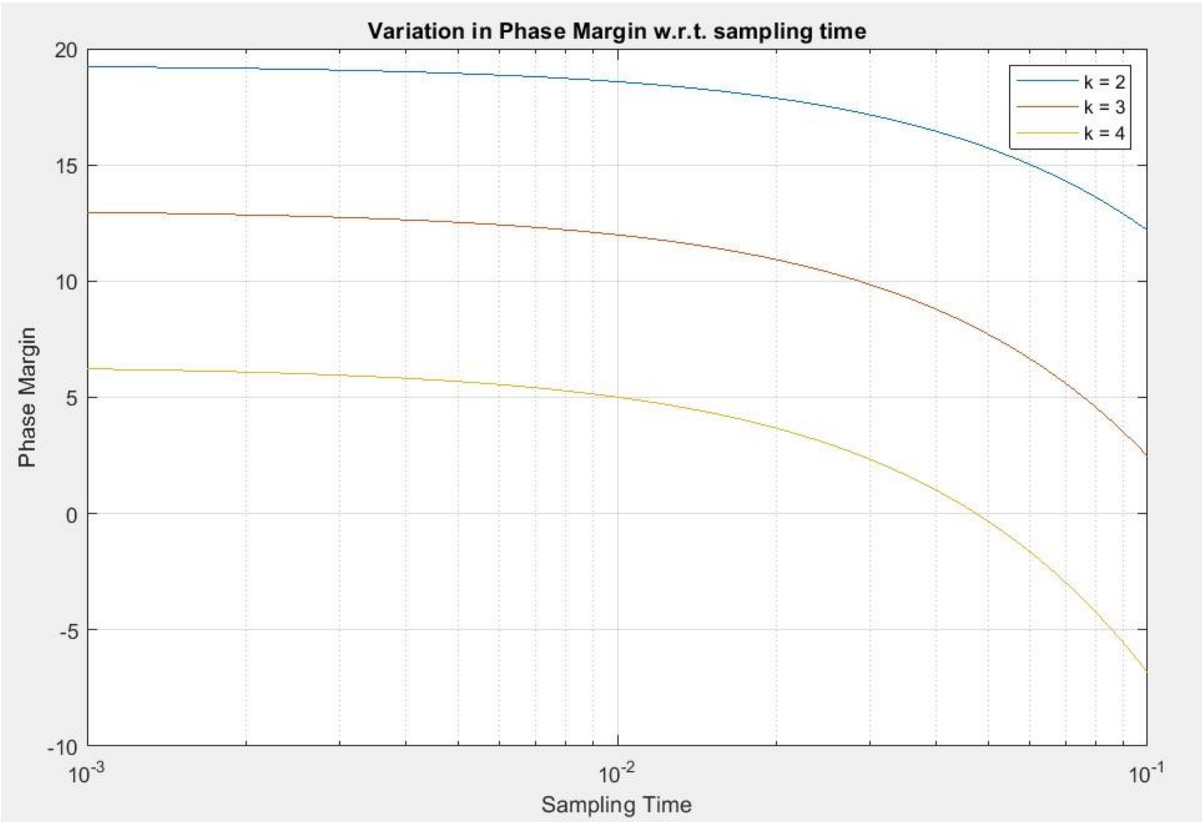
Discretizing a system always leads to loss of information about the output, here our electric furnace and actuator, both are systems in analog domain and their transfer functions are expected to remain fixed, which means when we digitize the plant (in this case Furnace + actuator), by cascading it to a ZOH, and sampling the output at fixed steps, the digital OLTF is bound to change. The following bode plot shows how the system behaves when we vary the sampling time.



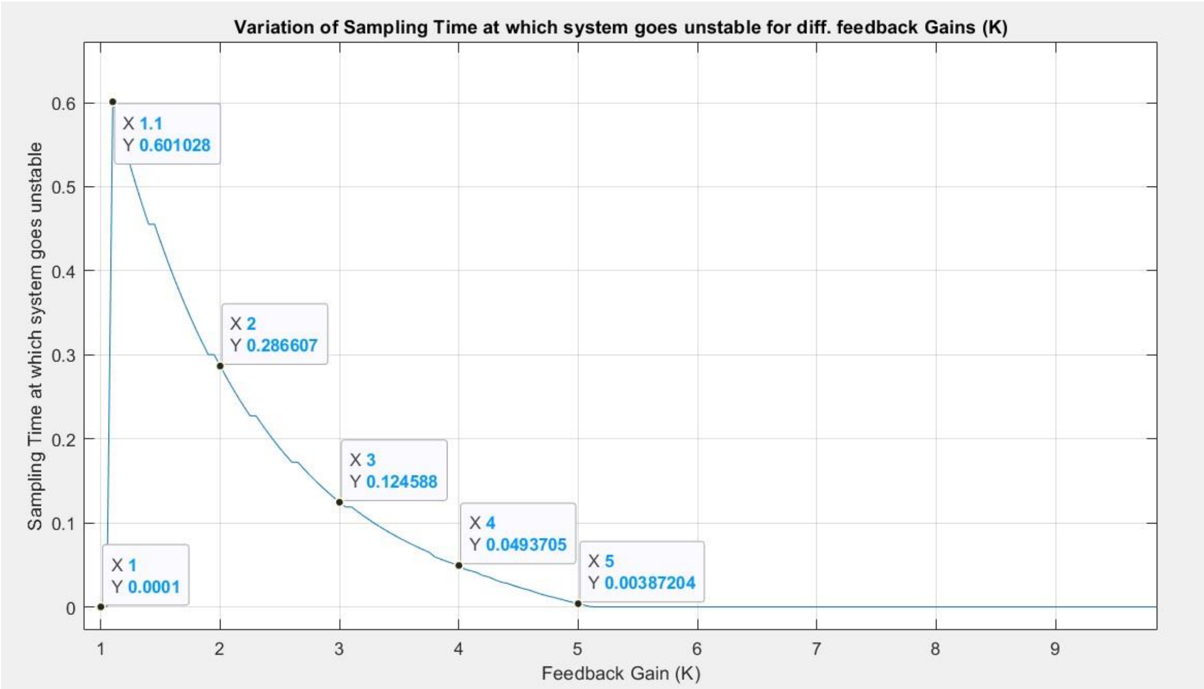
The above plots were obtained for K = 1, hence the phase margin of the system is undefined however it provides the idea of how the change in sampling time affects the system. Increasing the Sampling Time drifts the system closer to unstability. Also, we observe that at higher sampling frequencies the phase plot of the system shifts from -180º to +180º (at $f = 0$). This shift introduces another phase crossover frequency, Gain Margins corresponding to which are higher than the Gain Margin at first Phase crossover frequency. Thus, we study the variations in correspondence to lower gain margins only. The following plot illustrates how



We can similarly study the variation in phase margin with sampling time when K = 2, 3, 4 and we see that the phase margin in all 3 cases, decreases with increasing sampling time.



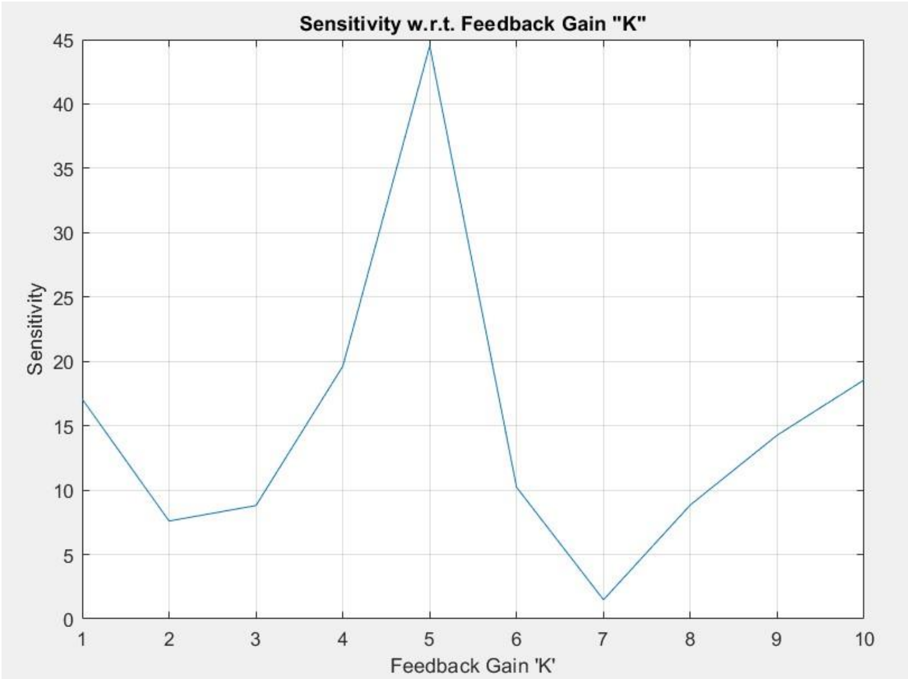
From the above plots, we see that both the phase and gain margins follow similar trends with increase in sampling time and the closed loop system shifts towards unstability region with increasing Sampling Time. We can further see that exactly at what point our system goes unstable by using the function (isstable(sys)) and obtain the following plot.



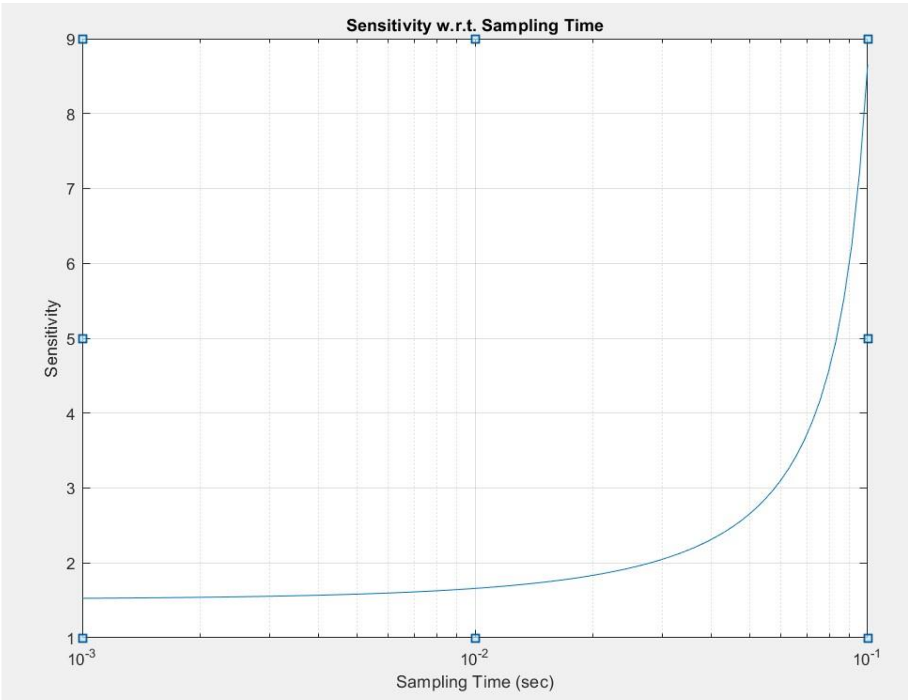
We see from the above plot that for increasing value of ‘K’, the sampling time for which system goes unstable decreases, also any point that lies above the given graph leads to an unstable system. And hence we can conclude that too long Sampling Time or too large feedback gains can destabilise the closed loop system.

SENSITIVITY ANALYSIS OF THE SYSTEM

We measured the sensitivity of the closed loop system w.r.t. gain “K” at nominal sampling time of 0.01 sec and obtained the following plot. We observe that sensitivity peaks at K = 5 and reaches its minima at K = 7.



Also, we plotted the graph of Sensitivity w.r.t. Sampling Time and obtain the following Data.



From the above graph we see that the sensitivity increases exponentially for higher values of Sampling Time, which is evident from how the phase and gain margin starts decreasing rapidly as we approach the sampling time of 0.1 sec.

CONCLUSION

From the above discussion we can conclude that increasing the sampling time and increasing the feedback gain, both result in decrease of Gain and Phase margin, shifting our closed loop system closer to unstability. We saw that we cannot obtain a stable system for ($K \geq 6$) whatever value Sampling Time we may choose. Therefore, it is preferable to operate the system at a comparatively lesser value of feedback gain (i.e., $K = 2$) and a high value of Sampling Rate (which can be handled by our discrete controller (0.01 – 0.001) sec).

MATLAB SCRIPT USED:

```
num = [4.711 4.644];
den = [1 -2.875 2.753 -0.8781];
Gz = 1e-5 * tf(num, den, 0.01); % given digital TF at Ts = 0.01s

G = d2c(Gz); % Converting to analog
k = logspace(-3, -1, 50);
for i = 1:50
    G1 = c2d(G, k(i)); % Transfer Function at a diff. sampling rate.
    bode(G1);
    hold on;
end
k = logspace(-3, -1, 500);
Gm = zeros(1, 500);
Gm2 = zeros(1, 500);
for i = 1:500
    G1 = c2d(G, k(i)); % Transfer Function at a diff. sampling rate.
    S = allmargin(G1);
    Gm(i) = 20*log10(S.GainMargin(2));
    if(size(S.GainMargin) == [1 3])
        Gm2(i) = 20*log10(S.GainMargin(3));
    else
        Gm2(i) = inf;
    end
    hold on;
end
figure; plot(k, Gm), grid;
hold on
%plot(k, Gm2);
x = gca();
set(x, 'xscale', 'log');
```