# CONTROL ENGINEERING LAB

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GROUP 18

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**OBJECTIVE:**

Dynamic study of the given crane trolley system on Simulink, with which different Lyapunov control designs are to be incorporated and the systems individually simulated.

**INTRODUCTION:**

The motion of an overhead crane can be represented by the following system of differential equations:

$$[m_L + m_c] \cdot \ddot{x}_1(t) + m_L\, l \cdot [\ddot{x}_3(t) \cdot cos x_3(t) - \dot{x}_3^2\,(t) \cdot sin x_3(t)] = u(t)$$

$$m_L\, [\ddot{x}_1(t) \cdot cos x_3(t) + l \cdot \ddot{x}_3(t)] = -m_L\, g \cdot sin x_3(t)$$

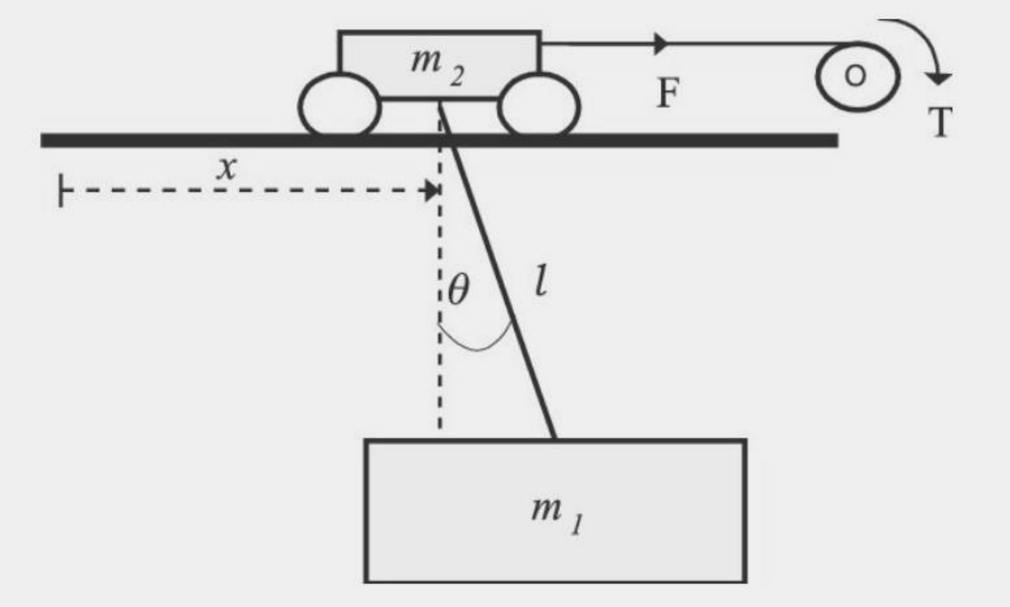


*Fig 1: illustration of system with the help of a diagram*

***Here, mC*** *= mass of trolley; (10 kg)*

***mL*** *= Mass of hook and load; the hook mass is 10 kg; load can be varied from 0 to several hundred kg*

***l =*** *Rope length; 1m or higher, const. for a given operation*

***g =*** *Acceleration due to gravity*

***Variables for the problem include:***

***INPUT:*** *u: Force in Newtons, applied to the trolley*

***OUTPUT:*** *y: Position of load in meters, $y(t) = x1\ (t) + l \cdot sinx3\ (t)$*

***STATES:***

*$x1$: Position of trolley in meters*

*$x2$: Speed of trolley in m/s*

*$x3$: Rope angle in rads*

*$x4$: Angular speed of rope in rad/s*

As already done in experiment 8 we use the value of the $\ddot{x_1}(t)$ and $\ddot{x_3}(t)$ to create the non-linear system simulation on Simulink.

$$\ddot{x}_1 = -\frac{m_L cos(x_3)sin(x_3)g + m_L l\dot{x}_3^2 sin(x_3) + u}{-m_L - m_C + m_L cos^2(x_3)}$$

$$\ddot{x}_3 = -\frac{m_L\dot{x}_3^2 l \cdot cos(x_3)sin(x_3) + m_L g \cdot sin(x_3) + m_C g \cdot cos(x_3) + u \cdot cos(x_3)}{l \cdot (-m_L - m_C + m_L cos^2(x_3))}$$

In addition to above information about the system we are also provided with four different energy-based function components any linear combination of which could be choice for the Lyapunov function for the system. These functions are:

***A: Proportionate to square of linear potential energy:*** $\boldsymbol{K_{PE}^{l} \cdot (x_{1,ref} - x_1)^2}$

***B: Proportional to linear kinetic energy:*** $\boldsymbol{K_{KE}^{l} \cdot {x_2}^2}$

***C: Proportional to square of rotary potential energy:*** $\boldsymbol{K_{PE}^{l} \cdot {x_3}^2}$

***D: Proportional to rotary kinetic energy:*** $\boldsymbol{K_{KE}^{l} \cdot {x_4}^2}$

Having already created the patch diagram in Simulink to simulate the given Non-Linear system, we can proceed to design a control law for the same using Lyapunov control design methods. Now, preparing the control for the system calls for additional Simulink blocks and subsystems to provide feedbacks, estimate the value of Lyapunov functions and simulate the model for different sets of input. The next section describes these blocks and subsystems in detail.

## SIMULINK MODEL:

The following images provide a detailed description of the Simulink model.

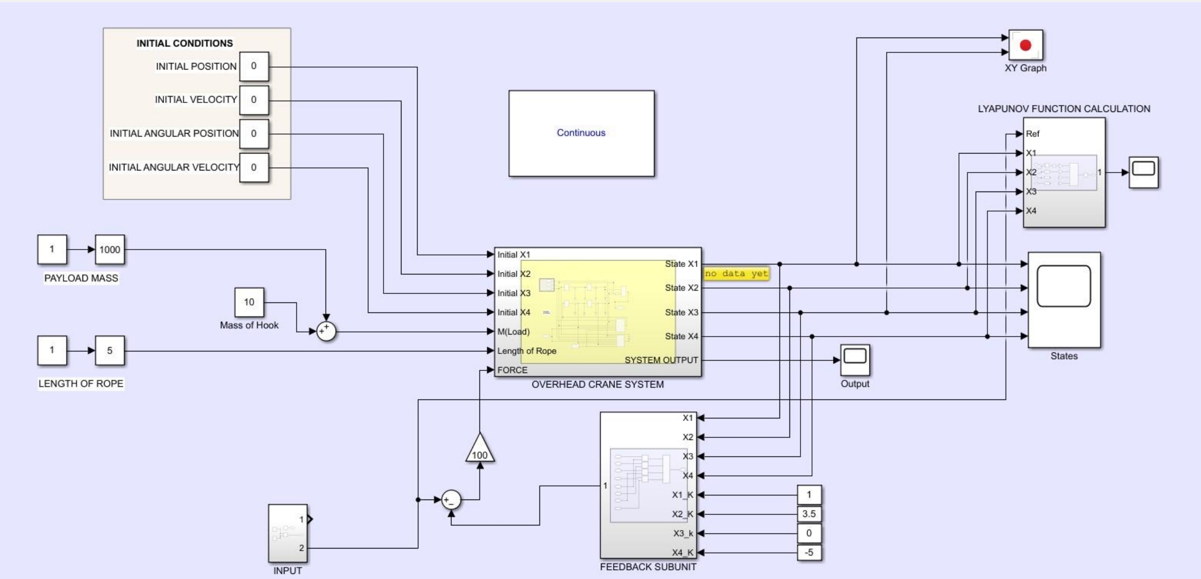


***Fig 2a:*** *The basic structure of the model remains the same as previous experiment, in addition to that we add some additional blocks and* ***subsystems to calculate the value of Lyapunov function and the state feedback****. Input in this case is of the form of reference position for trolley "x1", the force that has to be provided to the system is therefore calculated from reference signal and state feedback. So, every input we provide in the discussions that follow does not represent force but the reference position.*

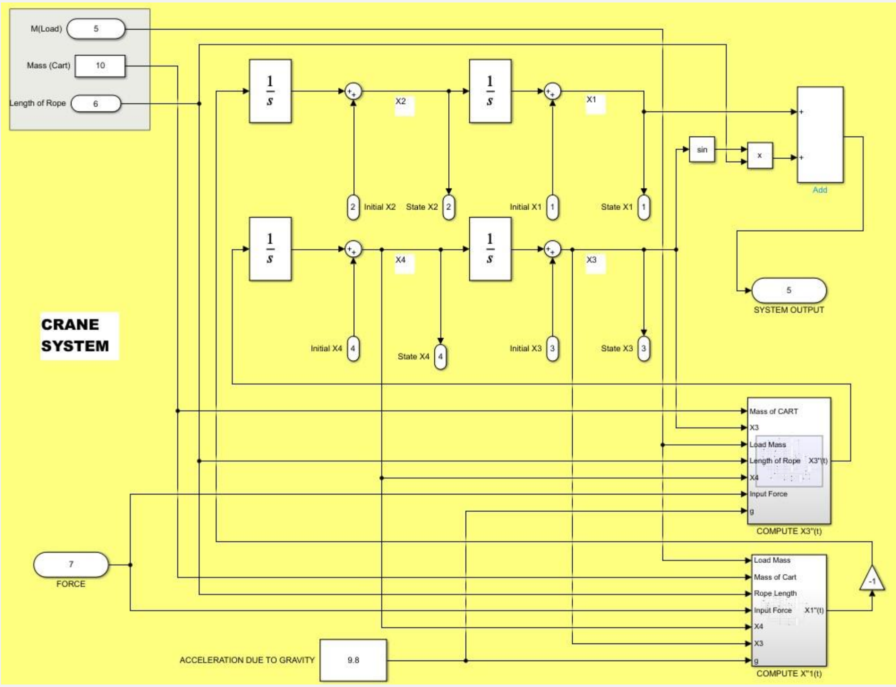


***Fig 2b:*** *The overhead crane system subunit takes in input of* ***initial conditions*** *and the parameters like* ***load mass, length of rope*** *and* ***force applied on the trolley,*** *hence it calculates the states of the system and the output based on states. The compute blocks are used to calculate double derivatives for the state x1, x3 as discussed in experiment 10. This block remains untouched as the we in no way effect the open loop system by establishing a state feedback control.*

The feedback subunit added receives the value of states x1, x2, x3 & x4 and the gains K for each state. The feedback can be easily generated using these signals which is then subtracted from the **INPUT** (which represents the reference position) signal. The force can then be calculated from this difference which is supplied as an input for the **OVERHEAD CRANE SYSTEM** block. The figure below shows the internal structure of the feedback subunit block.

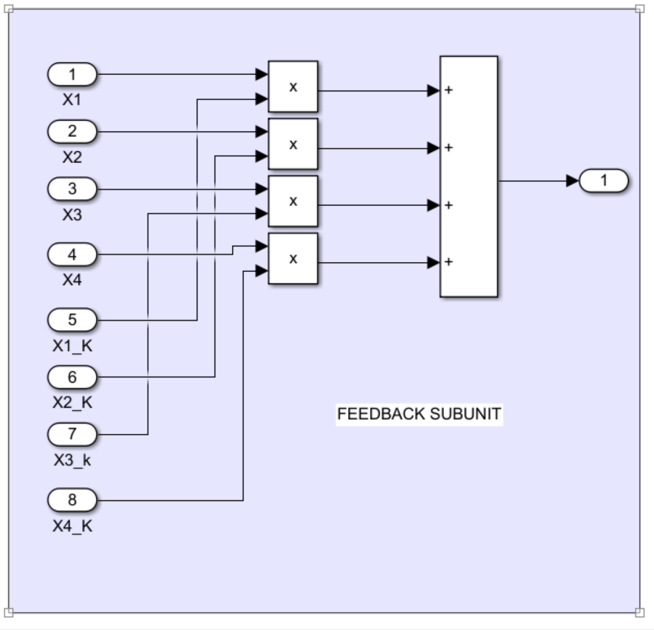


***Fig 3**: The feedback subunit block, it generates the inputs based on the gains and states given as input.*

And lastly, we added a block to compute the value for Lyapunov function. This block can be used to generate and study different linear combinations of energy-based function components (by choosing different values for K-constants) as a candidate for Lyapunov function.

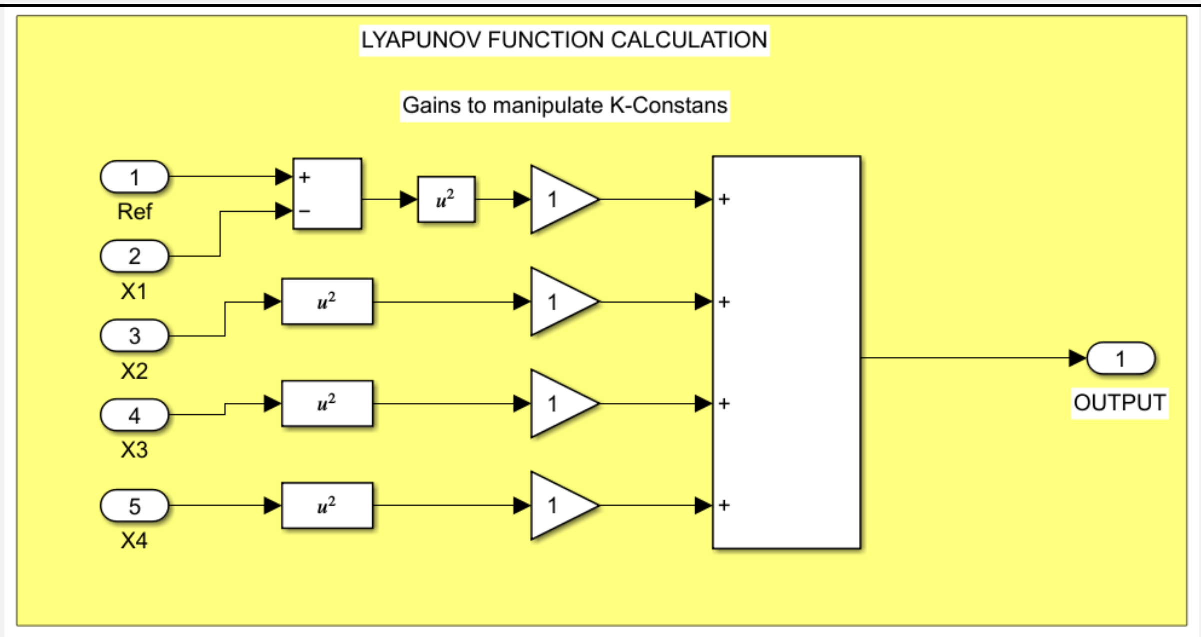


*Fig 4: Subsystem to calculate the value of Lyapunov function, here gains associated with each energy-based component function can be manipulated to generate different combinations. The output of the subsystem is fed into a scope for visualization.*

The next section involves discussions on energy-based component functions and what will be a suitable choice for the Lyapunov Function.

## LYAPUNOV FUNCTION FOR CHARACTERISING STABILITY:

After fixing the Lyapunov function "V(***X***) where ***X*** = [*x1; x2; x3; x4*]" to be some linear combination of the given energy-based functions, the equilibrium position must represent the point corresponding to V(***X***) = 0. It is clear that the desired state while operating a crane (and also from the energy functions provided) would be $\{\boldsymbol{x_{1,ref}}, 0, 0, 0\}$. Leaving out the contribution of any energy function (corresponding to a state) in Lyapunov function would ultimately result in V(***X***) = 0 while that particular state is arbitrary. It is also noticeable that K-constant as a multiplication factor only acts as a scaling term, and does not affect the derivative with time of the Lyapunov function, so for now we choose it to be 1 for every energy-based function that we include. When we change or provide a reference input to the system, it changes the equilibrium position (or the position of global stability for the system {***operating with a state-feedback, as the feedback have a tendency to change the position of the system, till it doesn't reach the reference input***}) so even if the energy of the system was initially 0 at some point providing a reference introduces a potential energy which eventually reduces to 0 as the system approaches the reference, hence the function $\boldsymbol{K_{PE}^{l}} \cdot (\boldsymbol{x_{1,ref}} - \boldsymbol{x_1})^{\boldsymbol{2}}$ is important and needs to be included in the Lyapunov function, so for now we choose $\boldsymbol{K_{PE}^{l} = 1}$. Further it is also evident that x2 is an important factor, as high values for this parameter leads to disastrous conditions for heavy loads. So, we need also need the function $\boldsymbol{K_{KE}^{l}} \cdot \boldsymbol{x_2}^{\boldsymbol{2}}$ to be included. The value for x3 and x4 remains negligible for system starting out from a point of equilibrium (of the open loop system), which guarantees their squares to be even less, hence these 2 functions could be excluded from the Lyapunov function as their contribution to the total energy is very less (even for heavy loads). So, the final Lyapunov function we come up with is given as:

$$\boldsymbol{V(X) = K_{PE}^{l} \cdot \left(x_{1,ref} - x_1\right)^2 + K_{KE}^{l} \cdot x_2{}^2}$$

***Here,***

$\boldsymbol{K_{PE}^{l} = 1}$ *and* $\boldsymbol{K_{KE}^{l} = 1}$

## THE STATE FEEDBACK CONTROL:

We establish the control for the system using a state-feedback, it is also obvious that because the system is ideal in the sense that there is no friction, tension, elasticity and other loses, we require simple proportional feedback i.e., no integrator blocks because the system can respond to very small inputs (***we do not need to overcome friction***) and no derivative blocks, as how fast the system responds can be manipulated using only proportional block {***and also because we are not using an integrator block***}.

The value for the feedback gains we come up with are presented in the following picture:

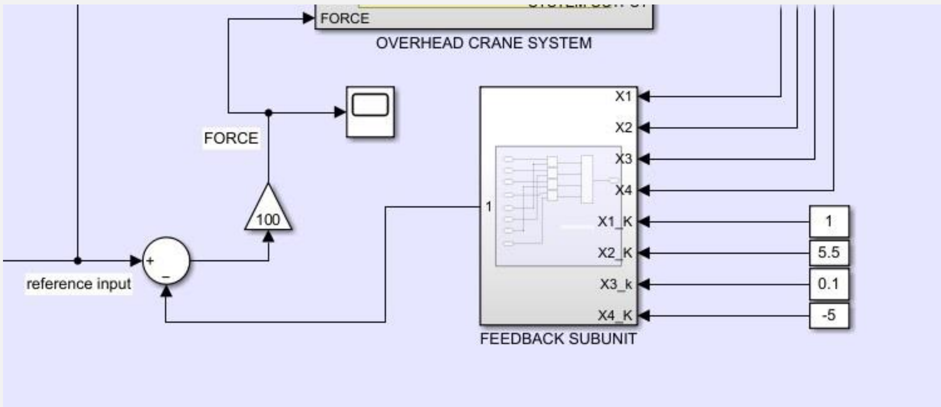


*Fig 5: A full state feedback has been used for our case, with the feedback gains for* ***{x1, x2, x3, x4} = {1; 5.5; 0.1; -5}*** *the given feedback is subtracted from the reference input, then the result is used to calculate the input, (force being just a scaled-up version (by a factor of 100) of the result).*

Having established a control law for the system, we can therefore proceed to study the response of the system to different inputs for the different operating conditions.

## THE RESPONSE OF THE SYSTEM OPERATING UNDER STATE FEEDBACK CONTROL:

**IMPULSE INPUT:**

The impulse input for the reference position is not very practical to be used in real life crane operations, the input shows some overshoot for the given input and eventually settles to zero and the response for various states can be summarized using the following figure:

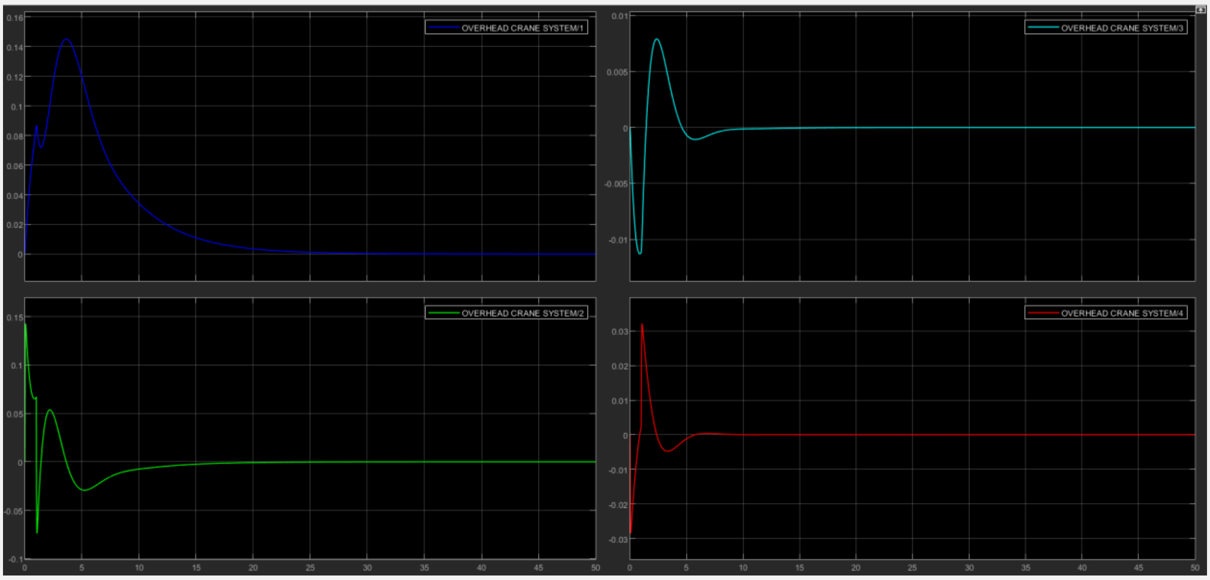


*Fig 6: The impulse response of the system states for M = 500kg and l = 5m, the position of the trolley increases up to a point due to an initial impulse but it eventually settles to zero due to feedback* ***(and does not increase to infinity by acquiring some constant average velocity)****. The output of the system for this case ≈ x1 as the value of x3 is negligible.*

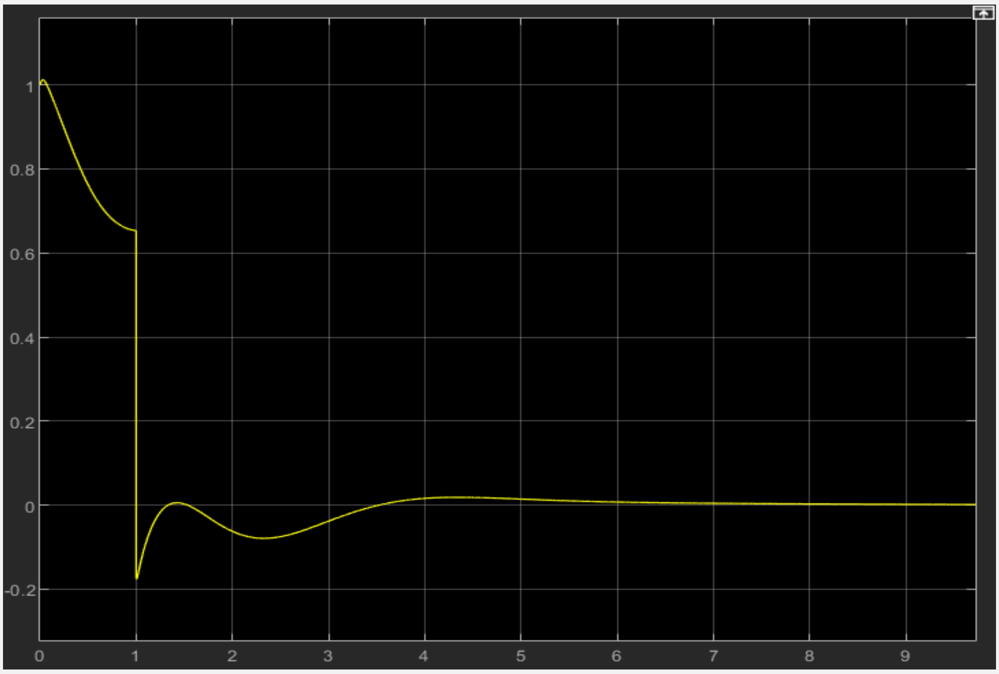


***Fig 7:*** *The variation of the value of Lyapunov function with time after providing impulse as a reference to the system for M = 500 kg, and l = 5m. As soon as the input is provided the value of V(X) jumps from 0 to 1 and eventually decays back to zero (****i.e., to the equilibrium position)****. Some oscillations are also observed because of coupling (between load and trolley) and inherent oscillatory nature of the system.*

The maximum position reached by trolley varies with the load mass. The dependence is depicted in the following plot.

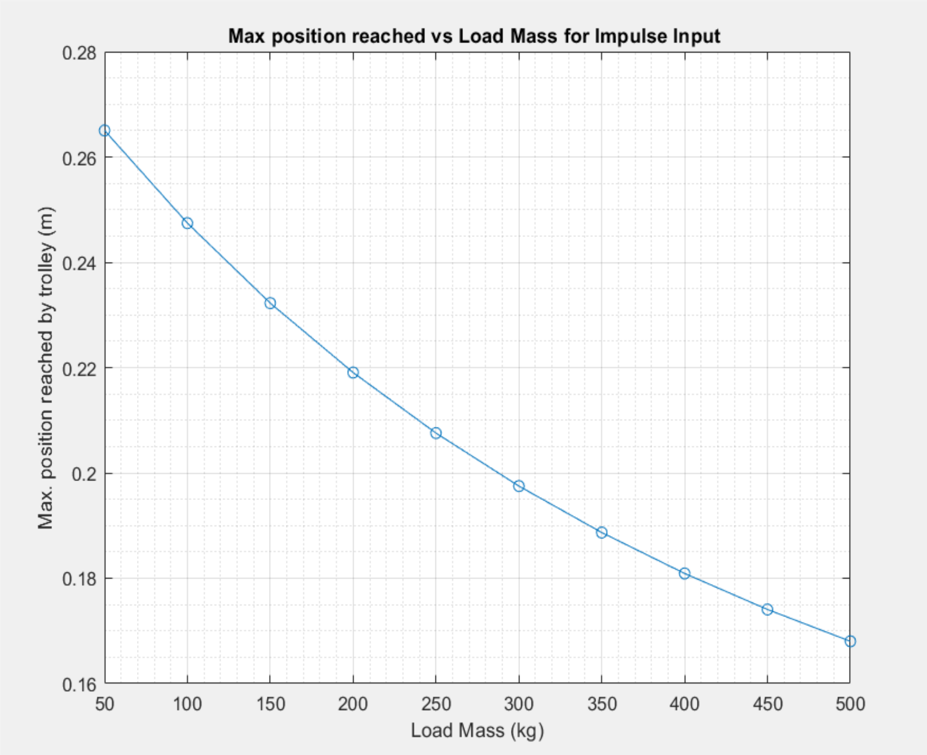


***Fig 8:*** *Max position reached by trolley varies inversely with the load Mass which is depicted in the given plot*

Apart from position the velocity, angular position and angular velocity also remains well within acceptable range for the operation of the crane, for example too high velocities may damage the machine and surroundings and hence are to be avoided while designing the control or while providing the reference input, for example in case of providing a step input to the system reference too far from the current position of trolley leads to high velocities as will be evident in the following section.

**STEP INPUT:**

Providing the step input changes the reference position of the system and hence shifts the equilibrium point to a different point $\{x_{1,\,ref}\ 0\ 0\ 0\}$. The following plots shows the response of the system states and output to the step input.

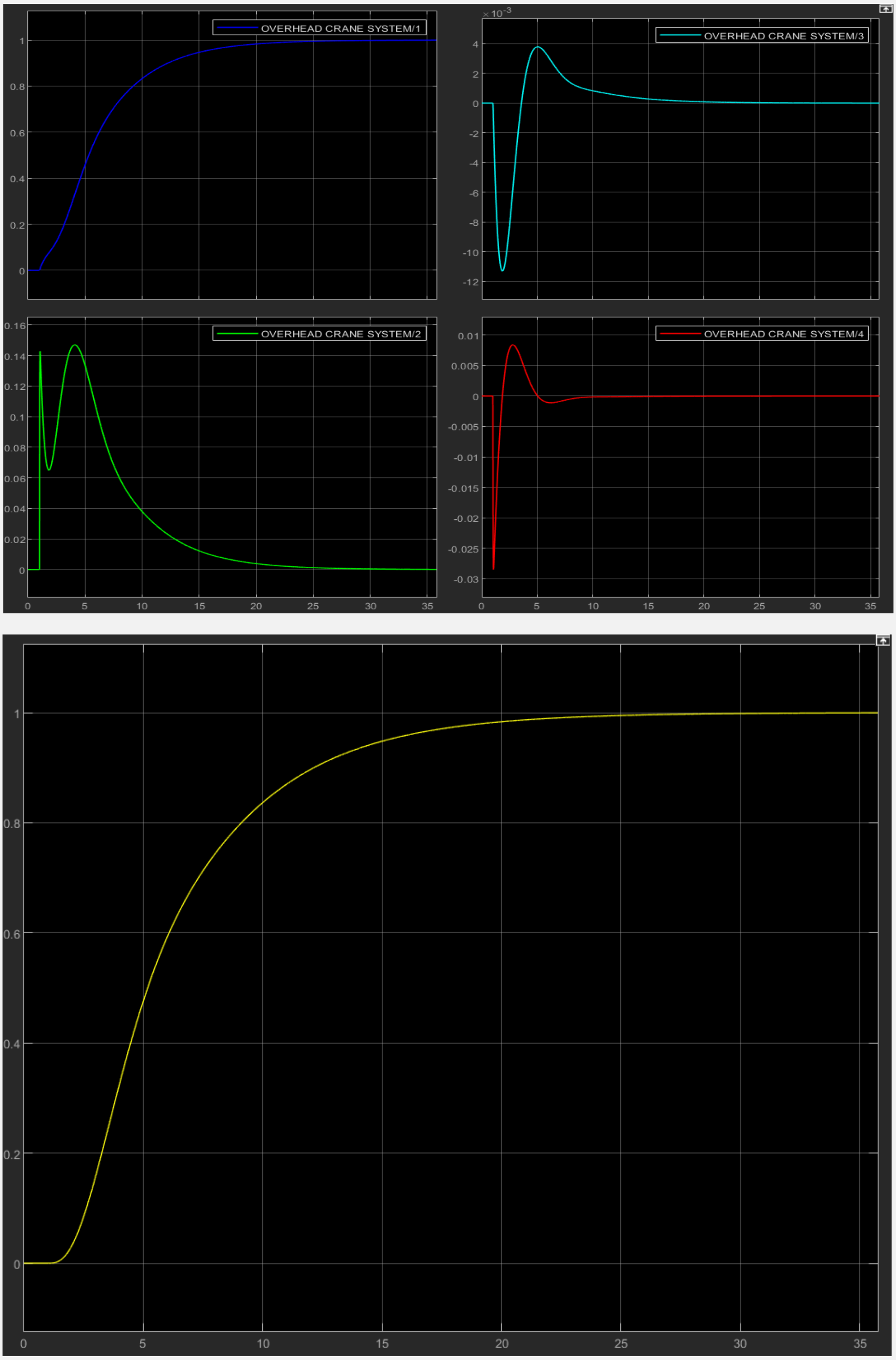


***Fig 9:*** *The unit-step response of the system (states and the output), it is noticeable that the dynamics for the state x1 is lag type which is also desirable as an oscillatory-type dynamics leads to overshoot which would be troublesome if the reference given to the system corresponds to either end of the bridge rail supporting the crane. For example, if a bridge rail spans 30-40m, the overshoot of even 2-3% would be undesired for the system.*

The variation of Lyapunov function with time for the unit step input (at t = 1sec) for above operating conditions is shown in the following plot.

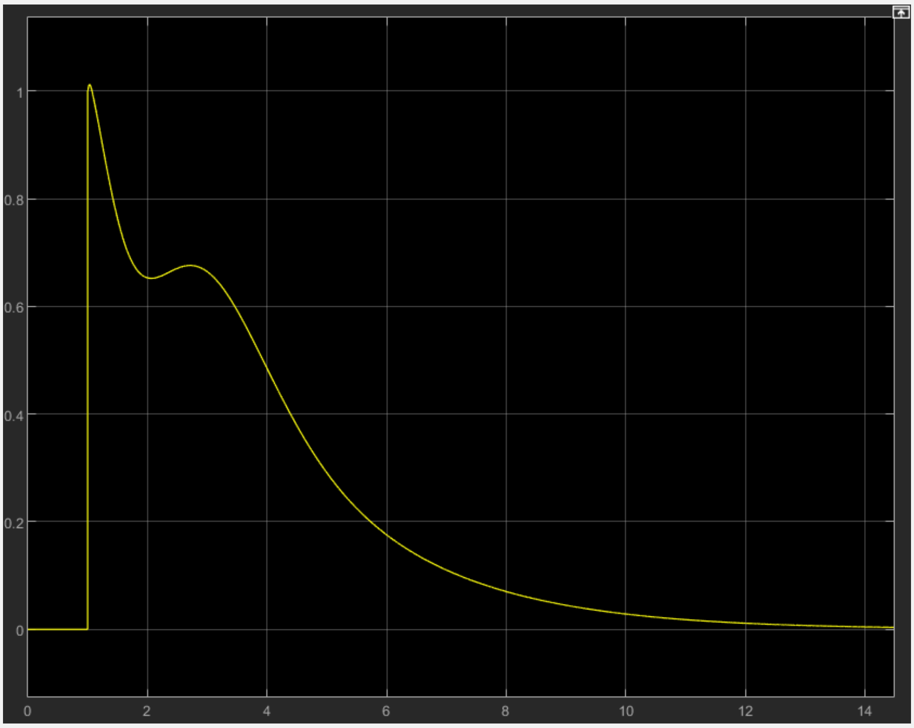


***Fig 10:*** *The variation of Lyapunov function with time for the unit step input and operating conditions i.e., M = 500 kg, l = 5m; As we change the reference at time t = 1 sec, the value of V(x) jumps from 0 to 1, and thereafter it decays to 0 as the system reaches the reference position "1m in this case"*

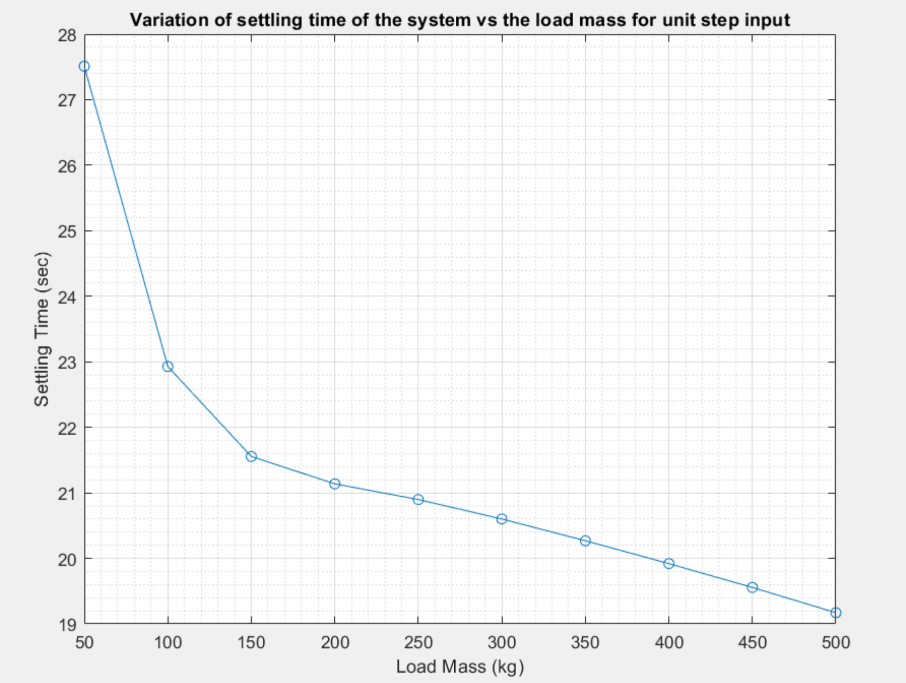


***Fig 11:*** *The given plot illustrates how the settling time varies with the load mass; we can see that with increase in load the settling time of the system reduces.*

One of the features of this type of input for our control design is that scaling the input (let's say by a factor of 10) scales the output such that the settling time remains constant, which obviously means that the velocity of the trolley too gets scaled up by the same factor. Such a scale up in a parameter like velocity which directly correlates to the Kinetic Energy of the system is not desired hence the input needs to be modified to move crane for large distances (for example 40-50 meters). The following plots illustrate the effect on the system of large step change in reference position.

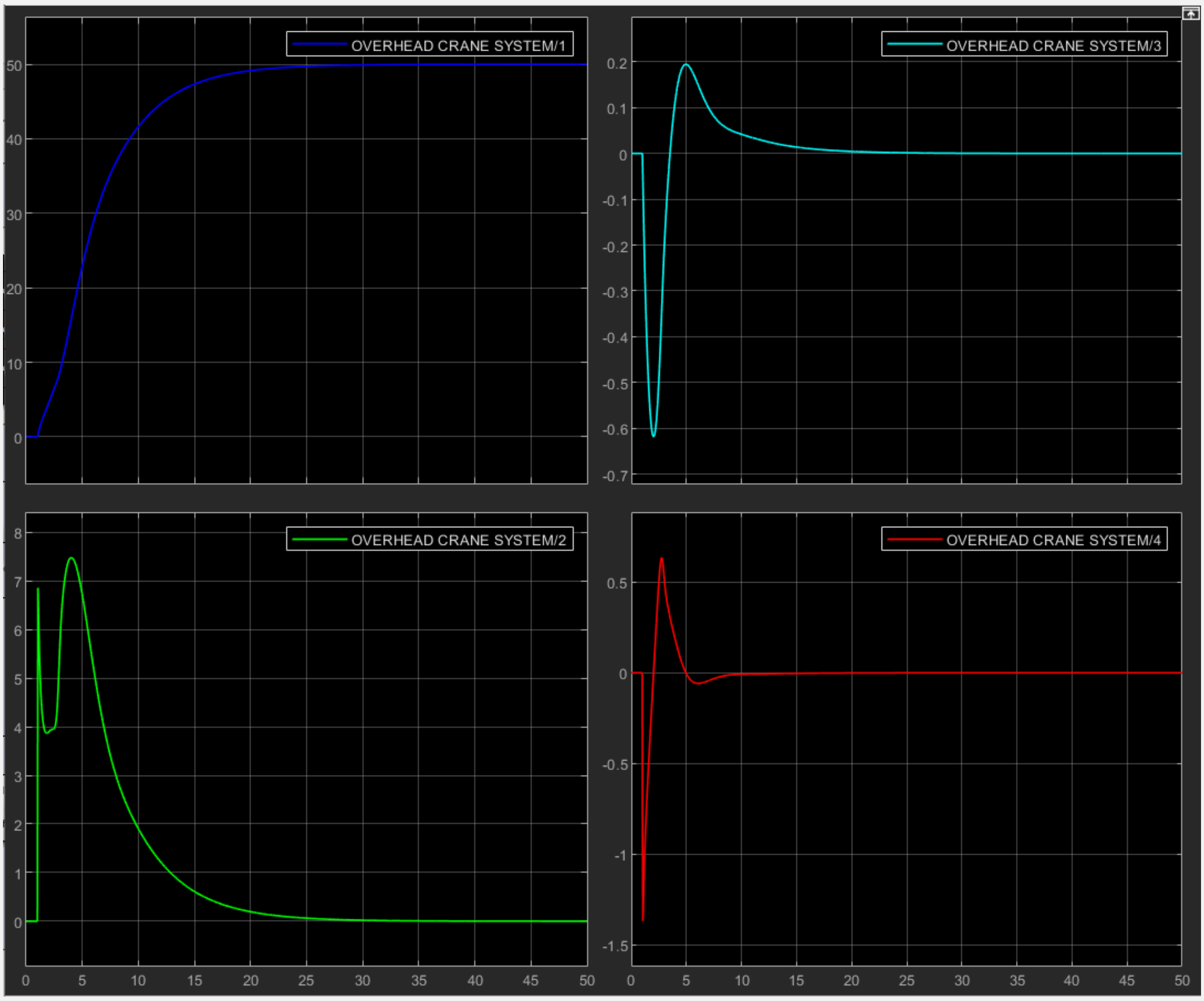


***Fig 12:*** *For a step change in reference to 50 meters the velocity reaches almost 7-8 m/s for load mass M = 500 kg, and l = 5m, which is undesirable as high velocities can be dangerous during the operation of the crane. The problem that the given situation presents can be solved by using a ramp type input whose slope can also be modified to set an optimal speed for the trolley and reach the reference position. This method using ramp input is discussed in the next section.*

**RAMP INPUT:**

Providing a ramp input involves changing the reference continuously either linearly increasing it or decreasing it. It is also clear that from a practical stand point this input cannot be provided continuously as a real-world crane have limitations on how far it can travel due to finite length of bridge rail, therefore at some point in time ***(when the trolley reaches either end of bridge rail)*** the reference input is needed to be made constant at that position. The following plots summarizes the behaviour of the system to ramp input with a slope = 1.

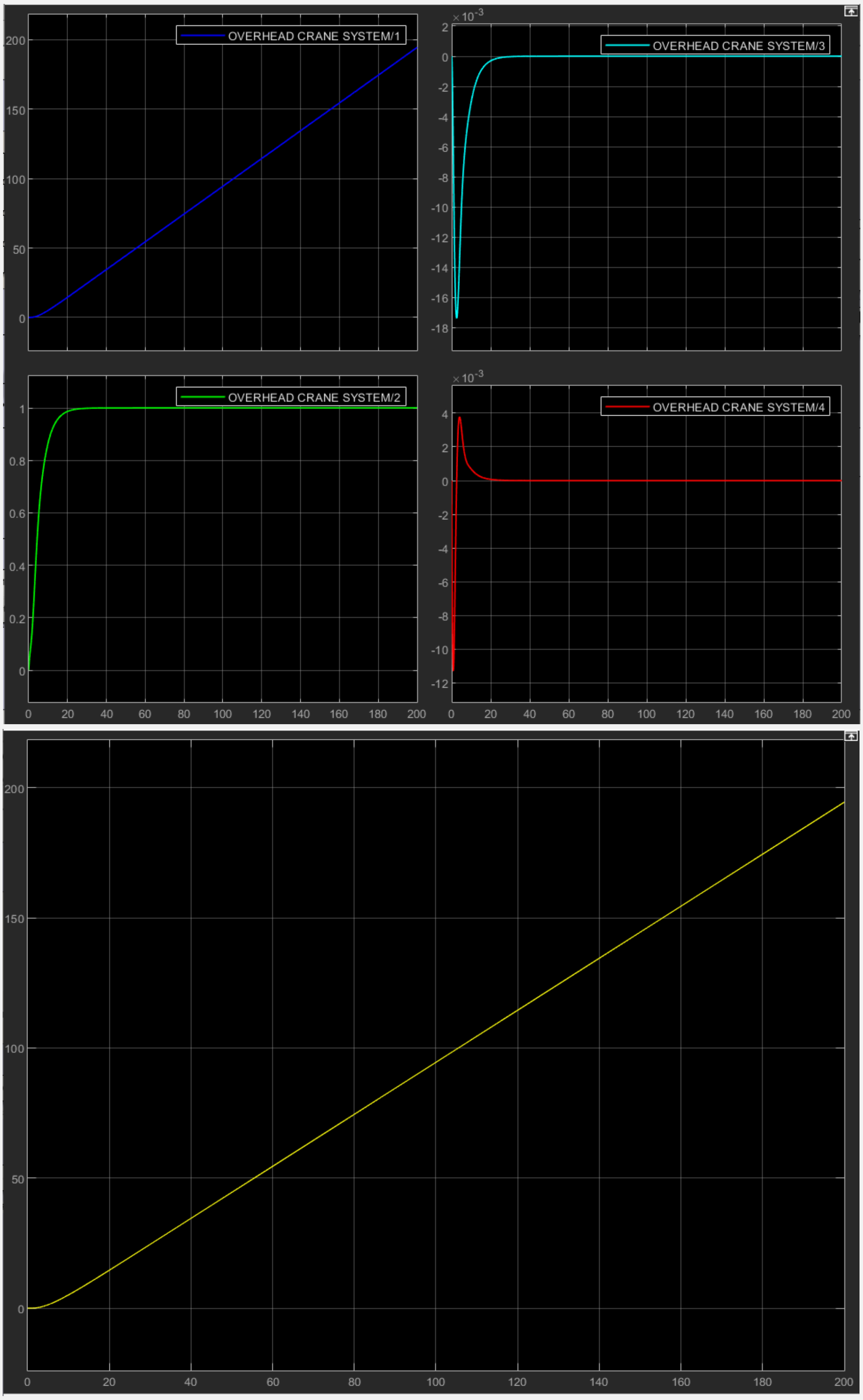


***Fig 13:*** *The states and output of the system for ramp input for M = 500kg and l = 5m, the velocity becomes constant at 1m/s the same speed our reference is increasing, so the state x1 and hence the output (because x3 decays to zero after some initial negative impulse) increases linearly with* ***v = 1m/s****. hence* ***this kind of input lets us work with a constant velocity*** *(which can be controlled through the slope of the ramp input), which is also desirable.*

The Lyapunov function for the ramp input varies with time in the following manner.

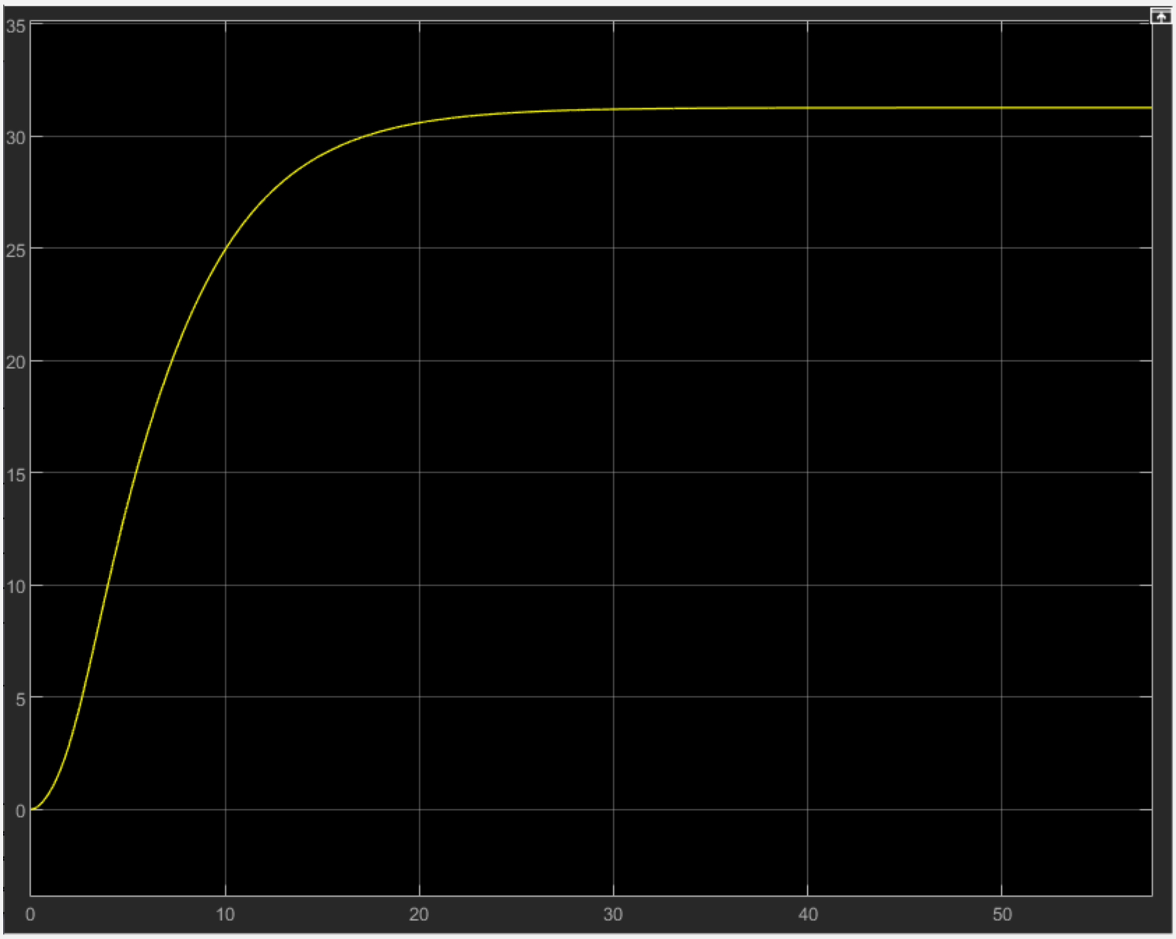


***Fig 14:*** *Lyapunov function increases with time and becomes constant at a value, this is due to the fact that the reference input is constantly increasing so as soon as the velocity of the trolley becomes equal to the rate of change of reference, the Lyapunov function V(x) becomes constant.*

This ramp input is an ideal input to provide the reference to the system, the reference can be increased with a particular rate (which determines the speed of the trolley) and as soon as the input reaches the desired state it can be made constant. For example, for a reference position of 50 meter, with a desired speed of 1m/s the input can be made to look like this.

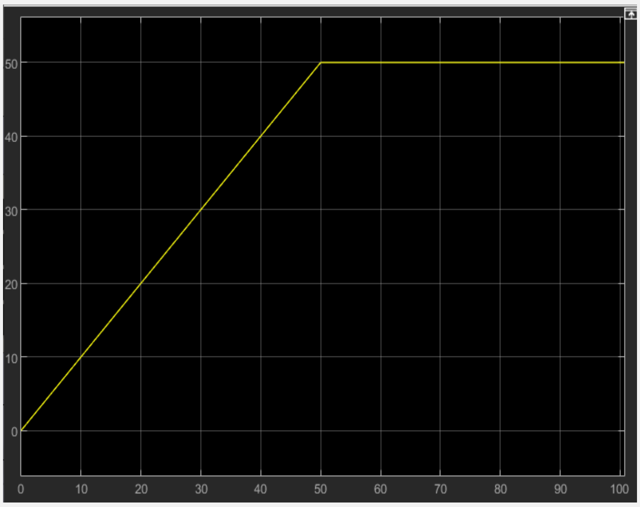


***Fig 15:*** *The ideal input to work with the given system with a full state feedback, the reference is increased linearly with a slope in accordance to the desired speed for trolley, and made constant after the input reaches desired point.*

The states of the system for the above input varies as shown in the following plots:

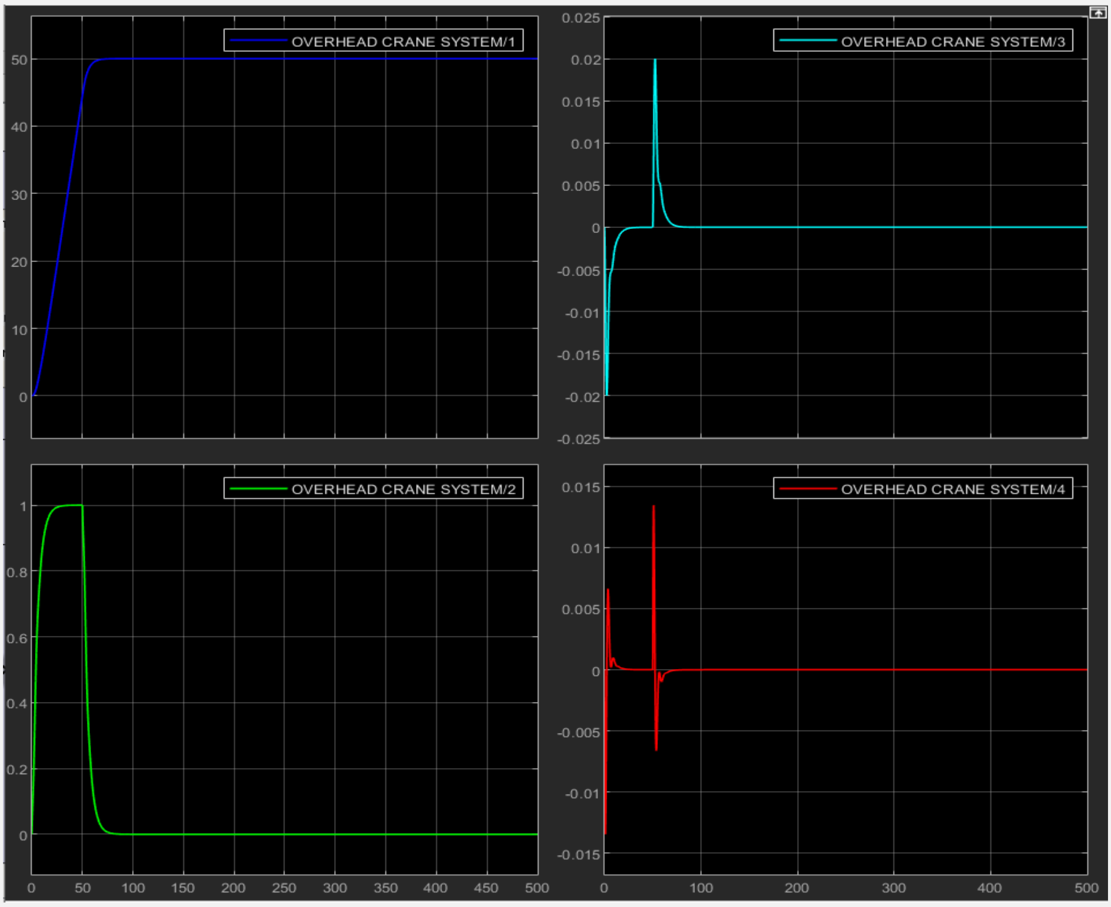


***Fig 16:*** *the state x1 of the system (for M = 500 kg and l = 5m) reaches to desired position (50m) within 100 seconds and does not cross the velocity of 1m/s for the trolley (kinetic energy remains well within acceptable range).*

Now although it looks like that the system is lag type in nature however there are some oscillations due to the inherent oscillatory nature of the system, these oscillations become significant for low values for load mass. The following plot illustrates the overshoot vs load mass for low value of masses.

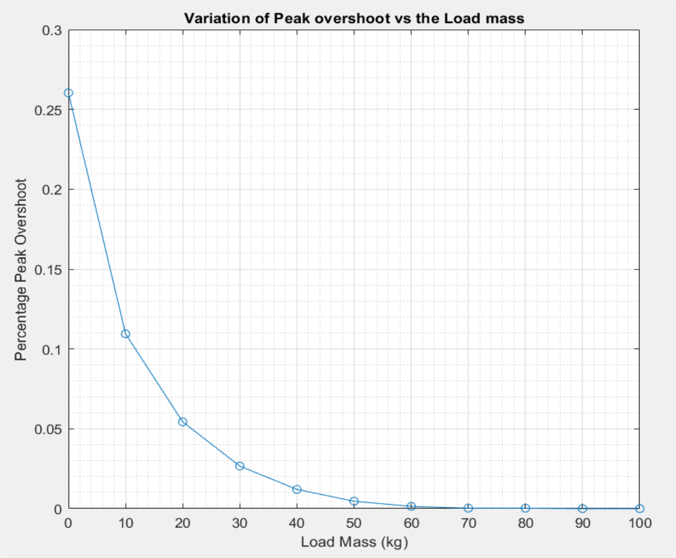


***Fig 17:*** *variation of peak overshoot vs the load mass, it is noticeable that the peak overshoot for the system is very less, and hence negligible, also it decays rapidly for high masses such that the dynamics of the system appears almost lagging type (which is also desirable).*

As discussed above we limited the speed of the crane by carefully shaping the input to a certain shape. This means that for this particular setting of input we can also use a different linear combination of energy-based components for Lyapunov Function ***(which does not include the velocity of trolley term).*** A simple example for such a function would be to only use first energy-based function $\boldsymbol{K^l_{PE}} \cdot (\boldsymbol{x_{1,ref}} - \boldsymbol{x_1})^2$ with $(\boldsymbol{K^l_{PE}} = 1)$**.**

The following plots shows how Lyapunov function evolves with time for different values of inputs.

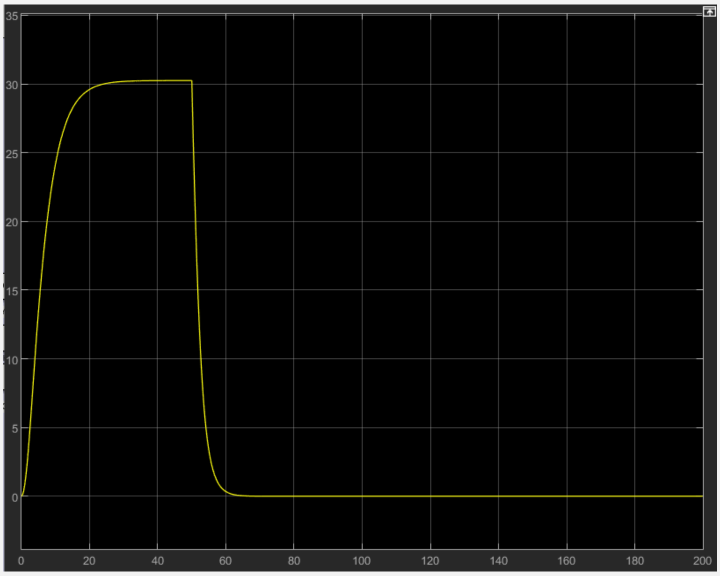


***Fig 18:*** *The value for the Lyapunov function increases for as long as the input is increasing and then it decreases and decays to 0 when the input becomes constant after a certain time.*

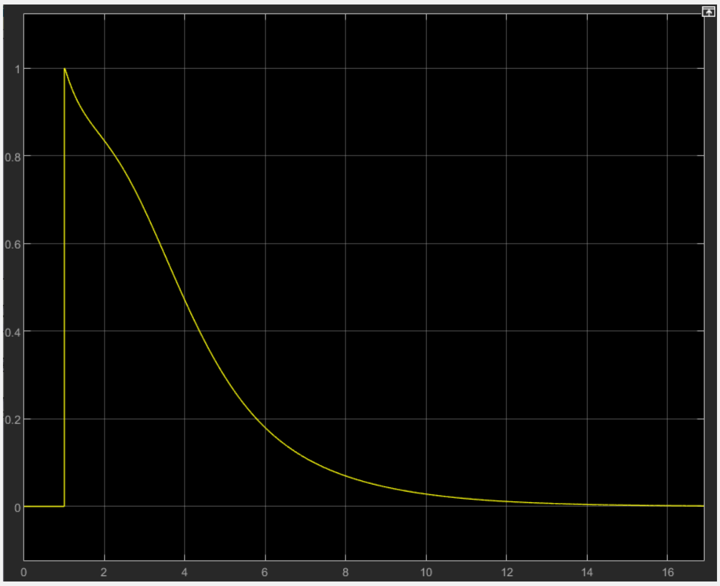


***Fig 19:*** *The value of Lyapunov function with time for a unit step input at t = 1, as we change the reference position by 1 meter, the value of the function increases to 1 and thereafter it decays to zero (time derivative is negative) for every point in time.*

Lyapunov function behave more or less similarly for different values of Load mass and hence the given Lyapunov function ***V(x)*** = $\boldsymbol{K^l_{PE}} \cdot (\boldsymbol{x_{1,ref}} - \boldsymbol{x_1})^2$ where $(\boldsymbol{K^l_{PE}} = 1)$ would also be a suitable choice for the Lyapunov function when we are operating the crane with practical inputs and not impulses or ever-increasing ramp inputs.

## CONCLUSION:

We designed the state feedback control for the give overhead crane system and analyzed the given control logic by studying the evolution of 2 different Lyapunov functions with time for different inputs (step, impulse, and ramp). We also found the best type of input for reference to operate the system such that the Kinetic Energy of the system remains in operable range. Having studied the different Lyapunov function we conclude that the given control law is compatible with the 2$^{nd}$ kind of Lyapunov function which involves only the potential energy due to position of trolley (in the case when speed of the trolley remains within bounds) otherwise the 1$^{st}$ Lyapunov function (involving terms from both speed of trolley and its position) would be better to use. It is also a major observation that the given control law makes the system almost lagging (***which is desirable as a high overshoot in position could be dangerous for high load mass***) in nature (***because there are still oscillations for low value of load mass, however they are negligible the percentage overshoot being of the order of*** $10^{-1}$).