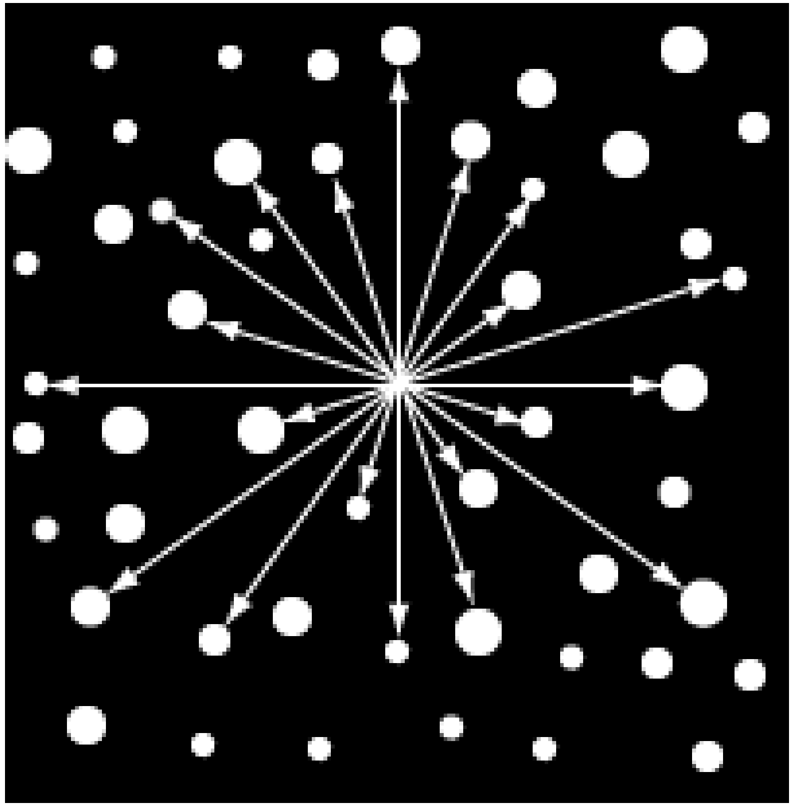
Fundamental Observations

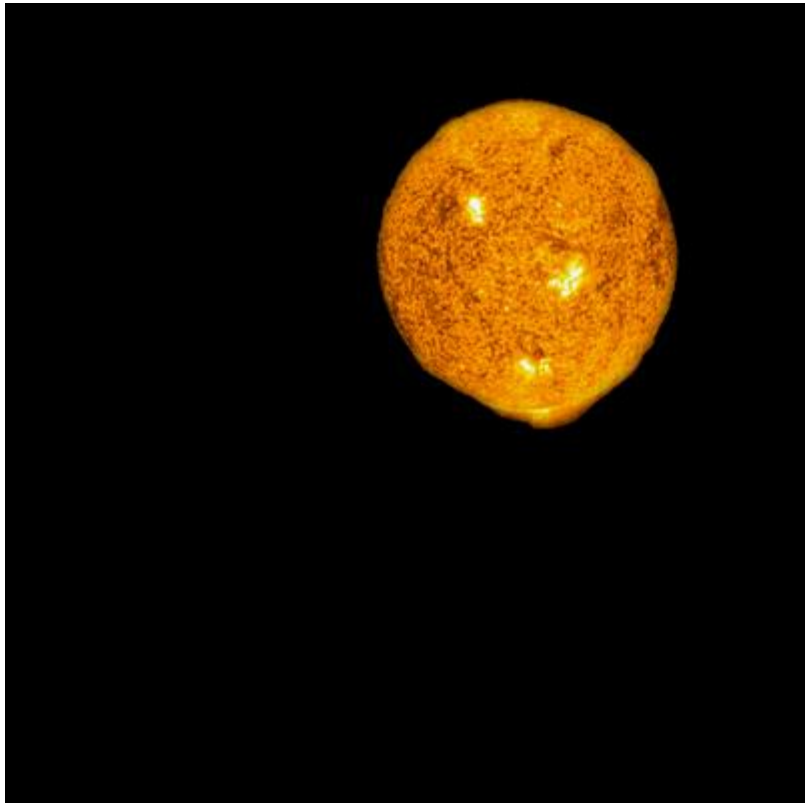
Observations that led to the current standard model of the Universe (Hot Big Bang Model):

- Dark night sky
- Large scale isotropy and homogeneity of universe
- Redshift and distance relation
- Astro-particles
- CMB

1. Olbers’ Paradox- the night sky is dark



In a static, infinitely old and infinitely large universe with an infinite number of stars, space would be bright rather than dark.

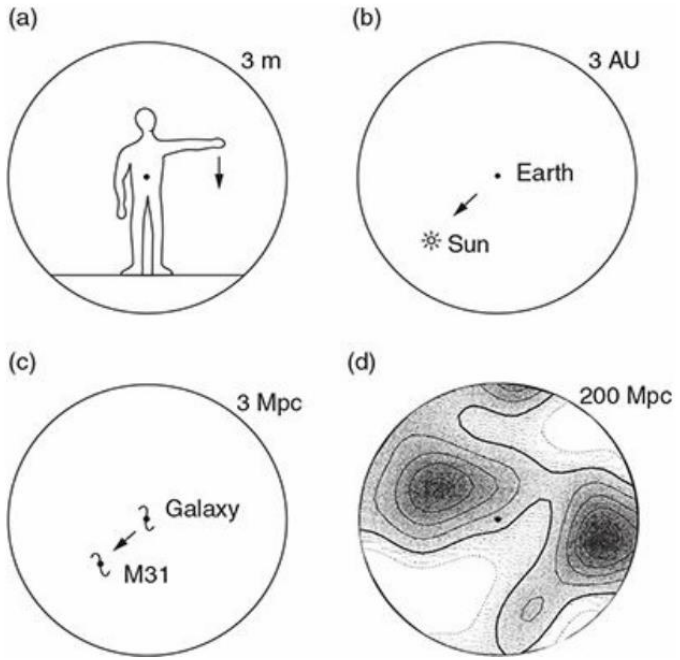


Under certain simplifying assumptions, like all stars being identica
the Sun, the sky should be uniformly bright

Assumptions/Resolutions

- Space is transparent everywhere
- Universe is infinitely large
- Universe is infinitely old (Primary resolution. What about redshift?)
- Surface brightness of stars is independent of distance

2. The Universe is Isotropic and Homogeneous



This is true only on very large scales, usually of the order of 100 Mpc or more

Redshift

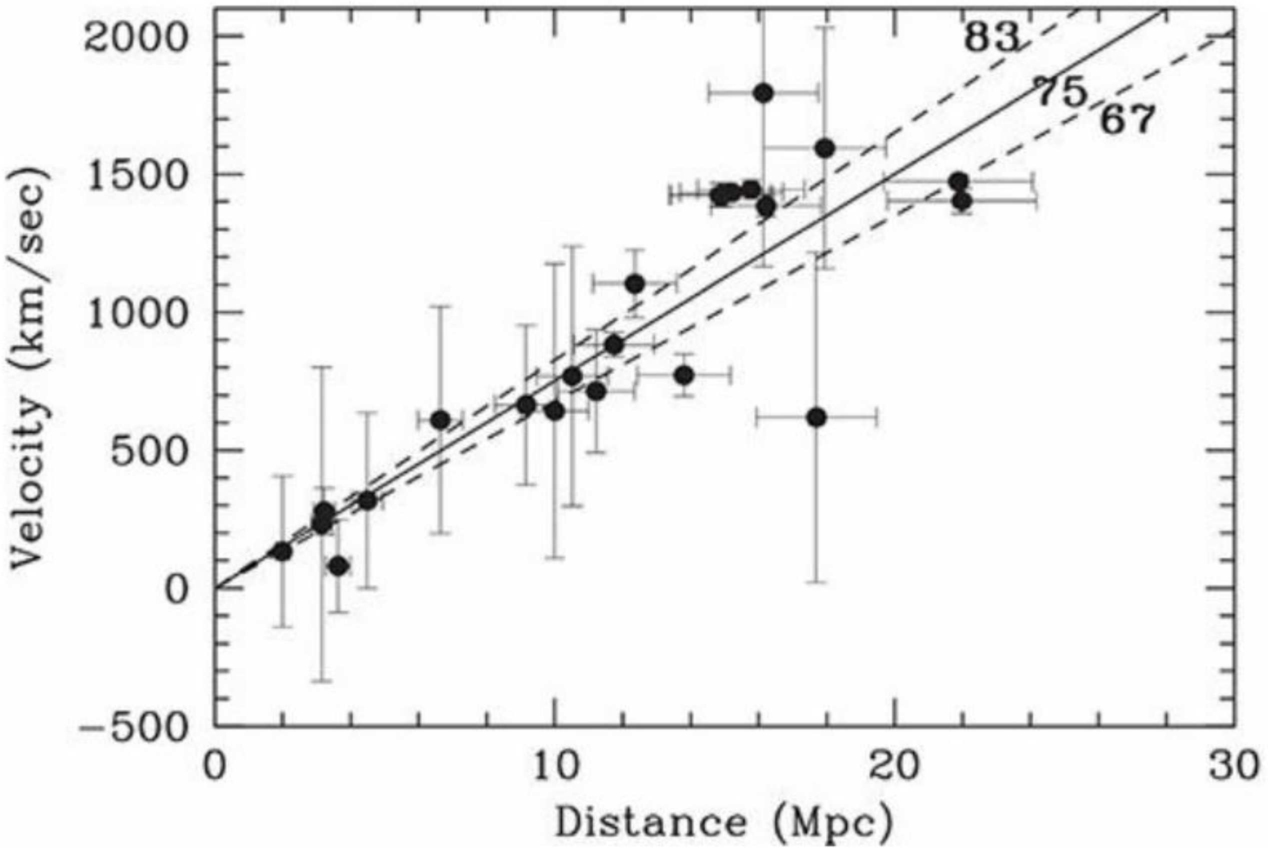
- It was observed that all galaxies, with a few expansions, had a redshift when their absorption and emission lines were measured.
- The redshift z of an object (here galaxy) is defined by:

$$z \equiv \frac{\lambda_{ob} - \lambda_{em}}{\lambda_{em}}$$

where $\lambda_{em} = wavelength\ measured\ in\ stationery\ frame$

$$\lambda_{ob} = wavelength\ emitted\ by\ moving\ object$$

Hubble Expansion



$$v = H_o r$$

$$H_0 = 68 \pm 2\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$$

Hubble Time

- Relative velocity b/w 2 galaxies: $v = H_o r$
- Time at which they were at a single point:

$$t_o = \frac{r}{v} = H_o^{-1} \approx 14.38 \pm 0.42\text{Gyr}$$

Different types of particles

Particle	*Symbol*	*Rest energy (MeV)*	*Charge*
Proton	p	938.27	+1
Neutron	n	939.57	0
Electron	e^-	0.5110	−1
Neutrino	ν_e, ν_μ, ν_τ	$< 3 \times 10^{-7}$	0
Photon	γ	0	0
Dark matter	?	?	0

Baryons- Made up of 3 quarks:

a) Proton (p) – 2 up, 1 down
b) Neutron (n) – 1 up, 2 down

- Up quarks have charge of +2/3 e
- Down quarks have charge of -1/3 e

Leptons- not made up of quarks:

a) Electron (e)
b) Neutrino (ν) – Interact only through weak interaction and gravity (very less mass)

Neutrinos

- Flavor states – electron, muon and tau $(\nu_e, , \nu_\mu, \nu_\tau)$
- Mass states - m_1, m_2, m_3 (ν_1, ν_2, ν_3)
- Each flavor state is a quantum superposition of the 3 mass states.
- A neutrino oscillates b/w the 3 flavor states

$$(m_1 + m_2 + m_3)c^2 = [m(\nu_e) + m(\nu_\mu) + m(\nu_\tau)]c^2 \geq 0.057\,\mathrm{eV}$$

$$(m_1 + m_2 + m_3)c^2 = [m(\nu_e) + m(\nu_\mu) + m(\nu_\tau)]c^2 \leq 0.3\,\mathrm{eV}$$

Blackbody radiation

$$\epsilon(f)df = \frac{8\pi h}{c^3}\frac{f^3 df}{\exp\left(\frac{hf}{kT}\right)-1} \Rightarrow n(f)df = \frac{8\pi}{c^3}\frac{f^2 df}{\exp\left(\frac{hf}{kT}\right)-1}$$

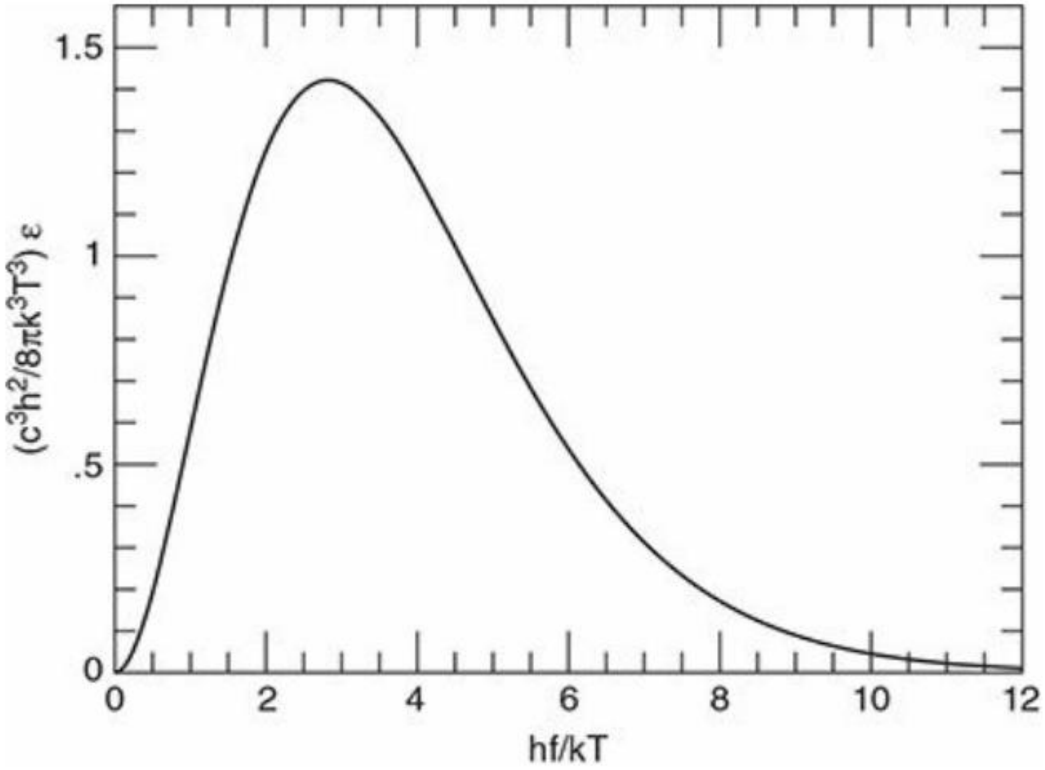
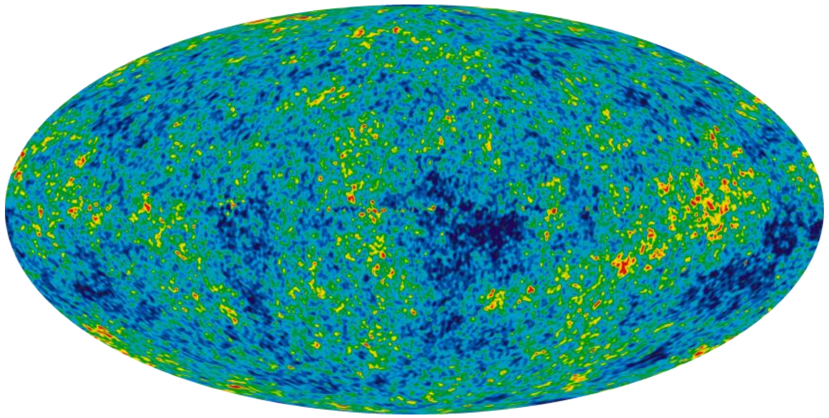
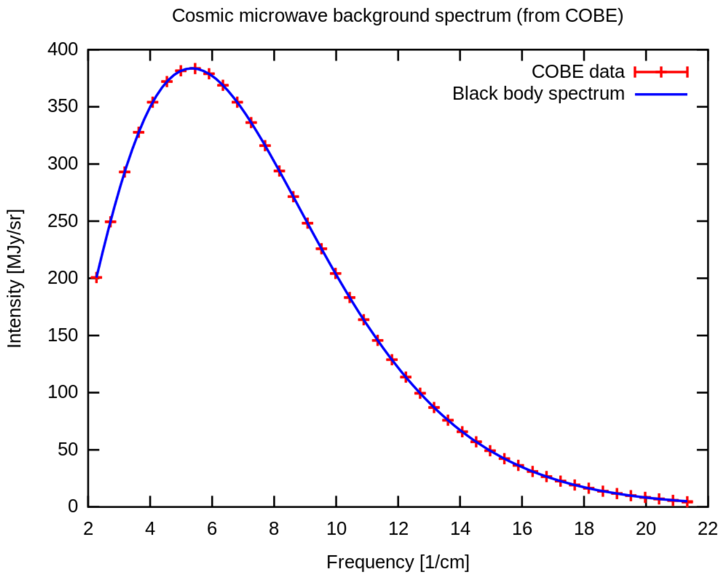


Figure 2.7 The energy density of blackbody radiation, expressed as a function of frequency *f*.

Cosmic Microwave Background (CMB)



The universe is filled with microwave radiation that is (almost isotropic).



Overlap with theoretical blackbody spectrum of T=2.72 K

4 fundamental forces

In increasing order of magnitude:

1. **Gravity**
   - **Dominant force over cosmological scales**
2. Electromagnetism
   - Also acts over huge length scales, but Universe is neutral over large scales
3. Weak Interaction (10^{-18}m)
4. Strong Interaction (10^{-15}m)

Newtonian description of Gravity

- $F = -\frac{GM_g m_g}{r^2}$ & $F = m_i a$
- $m_g = m_i \text{ is just empirical}$
- Poisson's eqn:

$$\nabla^2 \phi = 4\pi G\rho$$

$$\Rightarrow \phi(\vec{r}) = -G \int \frac{\rho(\vec{x})}{|\vec{x} - \vec{r}|} d^3x$$

$$\vec{a} = -\vec{\nabla}\phi$$

Special Relativity and the Lorenz Transformation

Two Postulates of Special Relativity

The Relativity Postulate: The laws of physics are the same for all observers moving at constant velocity, regardless of what their actual velocity is. An observer moving at constant velocity is said to be in an *inertial reference frame*.

The Speed of Light Postulate: The speed of light in a vacuum is the same value (*c=3.00x10⁸ m/sec)* in all directions and in all *inertial reference frames*.

$$x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$y' = y$$

$$z' = z$$

$$t' = \frac{t - \frac{vx}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\begin{aligned}(\Delta s')^2 &= -c^2(\Delta t')^2 + (\Delta \ell')^2 \\ &= -\gamma^2\left[c(t_1 - t_2)^2 - \frac{v}{c}(x_1 - x_2)^2\right]^2 \\ &\quad + \gamma^2[x_1 - x_2 - v(t_1 - t_2)]^2 \\ &\quad + (y_1 - y_2)^2 + (z_1 - z_2)^2 \\ &= -c^2(\Delta t)^2 + (\Delta \ell)^2 \\ &= (\Delta s)^2\end{aligned}$$

GR and Equivalence Principle

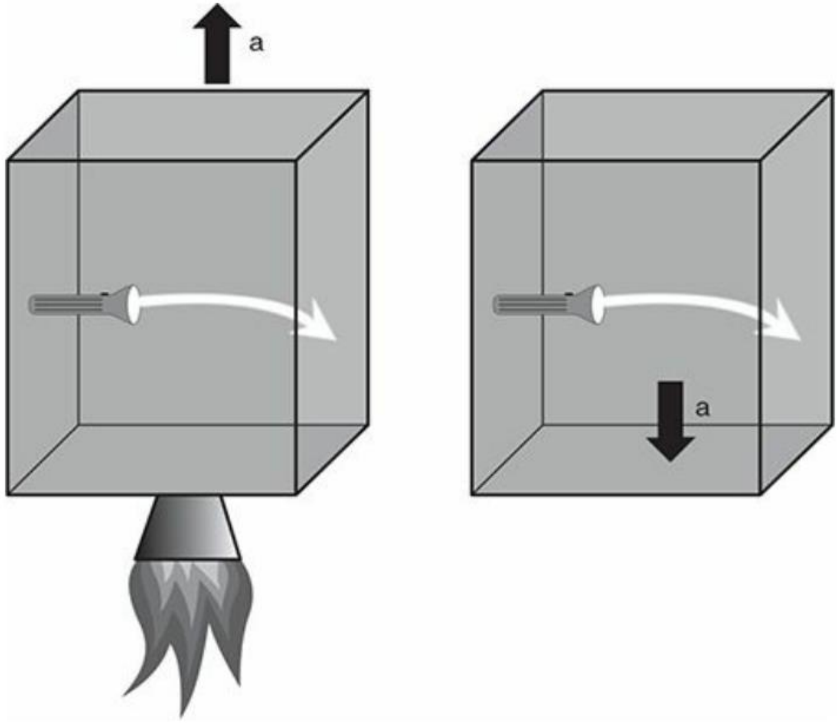
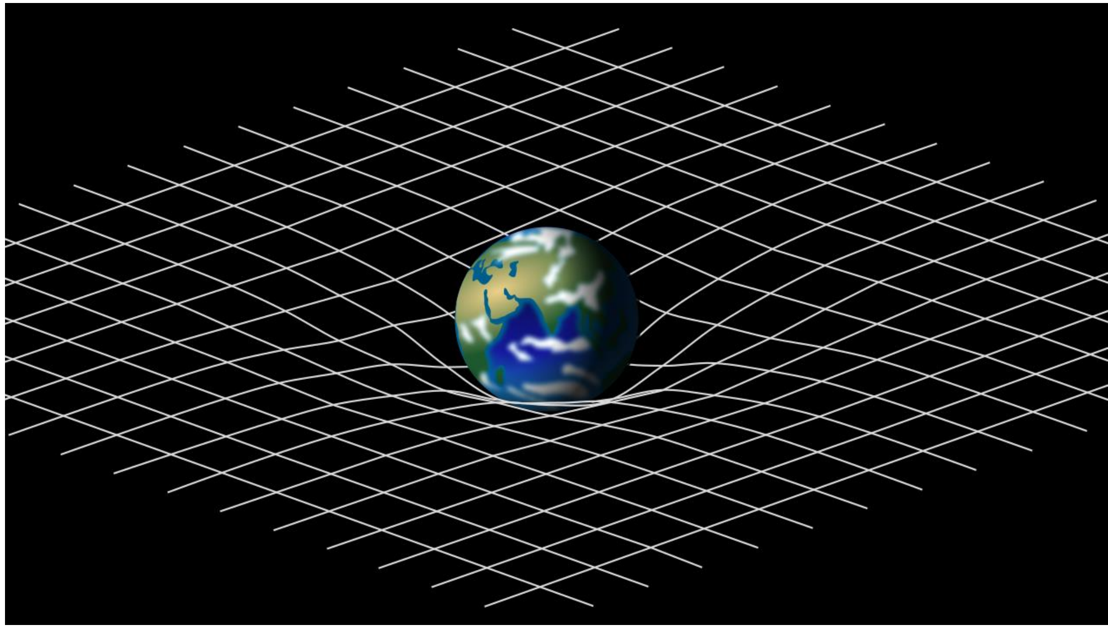


Figure 3.3 Equivalence principle (photon version). The path followed by a light beam in an accelerated box (left) is identical to the path followed by a light beam being accelerated by gravity (right). [The deflection shown is greatly exaggerated for the sake of visualization. The actual deflection will be ~ 2×10^{-14} m if the box is 2 meters across.]



The (General) Way of Einstein:

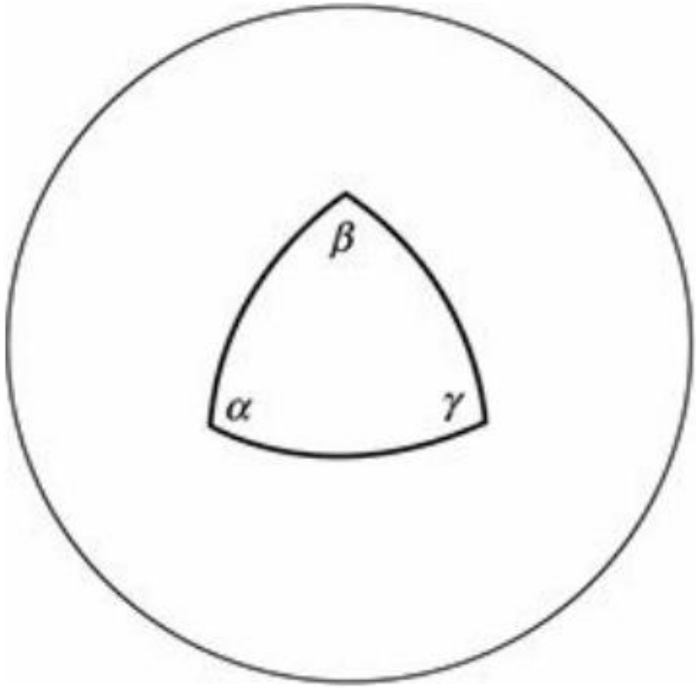
Mass-energy tells spacetime how to curve,

Curved spacetime tells mass-energy how to move.

Curvature

If curvature is positive
$\alpha + \beta + \gamma > \pi$

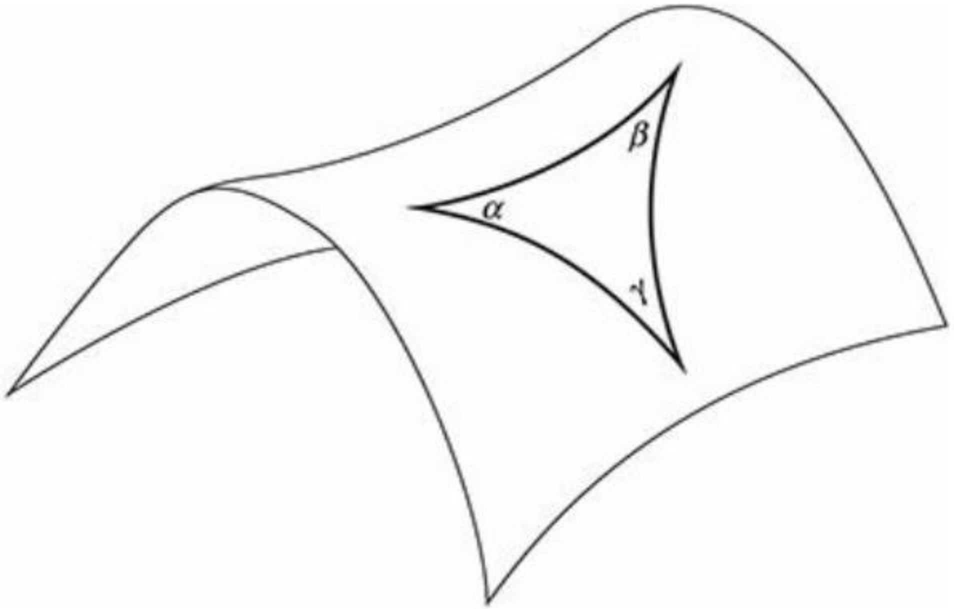
For Uniform Positive Curvature



$$\alpha + \beta + \gamma = \pi + A/R^2$$

If curvature is negative
$\alpha + \beta + \gamma < \pi$

For Uniform Negative Curvature



$$\alpha + \beta + \gamma = \pi - A/R^2$$

Metrics: gives distance between two nearby points

If space is homogeneous and isotropic, 3 possibilities:

1. Flat (k=0):

$$d\ell^2 = dr^2 + r^2[d\theta^2 + \sin^2\theta\, d\phi^2]$$

2. Uniform Positive (k=1):

$$d\ell^2 = dr^2 + R^2\sin^2(r/R)[d\theta^2 + \sin^2\theta\, d\phi^2]$$

3. Uniform Negative (k=-1):

$$d\ell^2 = dr^2 + R^2\sinh^2(r/R)[d\theta^2 + \sin^2\theta\, d\phi^2]$$

$$d\ell^2 = dr^2 + S_\kappa(r)^2 d\Omega^2$$

$$d\Omega^2 \equiv d\theta^2 + \sin^2\theta\, d\phi^2$$

$$S_\kappa(r) = \begin{cases} R\sin(r/R) & (\kappa = +1) \\ r & (\kappa = 0) \\ R\sinh(r/R) & (\kappa = -1) \end{cases}$$

Metrics for 4-D Spacetime

Minkowski Metric

$$ds^2 = -c^2dt^2 + dr^2 + r^2d\Omega^2$$

Applicable only for Minkowski spacetime, which is

- Static
- Euclidian

Robertson-Walker Metric

$$ds^2 = -c^2dt^2 + a(t)^2\left[dr^2 + S_\kappa(r)^2d\Omega^2\right]$$

Applicable when spacetime is

- Expanding/contracting: given by scale factor a(t)
- Has uniform curvature: given by curvature function S_κ

Relation between redshift(z) and the scale factor(a)

Light from a distance galaxy travels along a null geodesic

$\therefore\ c^2dt^2 = a(t)^2dr^2 \Rightarrow c\dfrac{dt}{a(t)} = dr$ Integrating over times $t_e \to t_o$ & $t_e + \frac{\lambda_e}{c} \to t_o + \frac{\lambda_o}{c}$ and distance $0 \to r$, we get:

$$c\int_{t_e}^{t_0} \frac{dt}{a(t)} = \int_0^r dr = r = c\int_{t_e+\lambda_e/c}^{t_0+\lambda_0/c} \frac{dt}{a(t)} \Rightarrow \frac{1}{a(t_e)}\int_{t_e}^{t_e+\lambda_e/c} dt = \frac{1}{a(t_0)}\int_{t_0}^{t_0+\lambda_0/c} dt$$

$$\therefore 1 + z = \frac{a(t_0)}{a(t_e)} = \frac{1}{a(t_e)}$$

Einstein’s (original) Field Equation

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

$G_{\mu\nu} = Einstein\ Tensor$

- 4 x 4 symmetric tensor
- Describes space-time curvature at (t,x,y,z)

$T_{\mu\nu} = Stress - Energy\ Tensor$

- 4 x 4 symmetric tensor
- Describes describes density and flux of energy and momentum at (t,x,y,z)

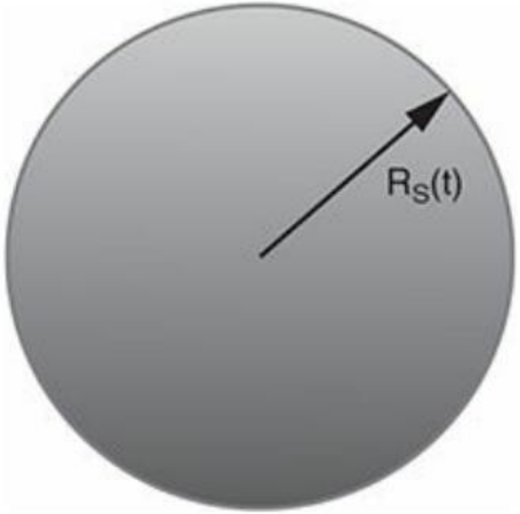
Due to symmetry of both tensors, the equation provides 10 nonlinear 2nd order DEs. The nature of the solutions can be analyzed to predict properties of space-time. Also yields propagating wave solutions.

Friedmann Equation

- Equation that links the Universe's parameters of curvature [a(t), κ, $\mathrm{R_o}$] to its parameters of content [$\varepsilon(t), P(t)$]
- Parameters are functions of only time because Universe contains homogeneous and isotropic perfect gas
- Non-relativistic/Newtonian version:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho(t) + \frac{2U}{r_s^2}\frac{1}{a(t)^2}$$

Where $r_s = comoving\ radius\ of\ sphere$



$$F = -\frac{GM_s m}{R_s(t)^2} \Rightarrow \frac{d^2R_s}{dt^2} = -\frac{GM_s}{R_s(t)^2} \Rightarrow \frac{1}{2}\left(\frac{dR_s}{dt}\right)^2 = \frac{GM_s}{R_s(t)} + U$$

Let $R_s(t) = a(t)r_s$

$$\therefore\ \frac{1}{2}r_s^2\dot{a}^2 = \frac{4\pi}{3}Gr_s^2\rho(t)a(t)^2 + U$$

$$\Rightarrow \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho(t) + \frac{2U}{r_s^2}\frac{1}{a(t)^2}$$

Friedmann Equation

- Relativistic version, derived from Einstein's original field equation: $\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3c^2}\varepsilon(t) - \frac{\kappa c^2}{R_0^2}\frac{1}{a(t)^2}$
- We know, $H(t) \equiv \dot{a}/a$ $\therefore$ $H(t)^2 = \frac{8\pi G}{3c^2}\varepsilon(t) - \frac{\kappa c^2}{R_0^2 a(t)^2}$
- At present, $t = t_o$ $\therefore$ $H_0 = H(t_0) = \left(\frac{\dot{a}}{a}\right)_{t=t_0} = 68 \pm 2\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ & $H_0^2 = \frac{8\pi G}{3c^2}\varepsilon_0 - \frac{\kappa c^2}{R_0^2}$
- For $\kappa = 0$, critical density $\varepsilon_c(t) \equiv \frac{3c^2}{8\pi G}H(t)^2$ & $\varepsilon_{c,0} = \frac{3c^2}{8\pi G}H_0^2 = 4870 \pm 290\,\mathrm{MeV\,m^{-3}}$
- Density Parameter $\Omega(t) \equiv \frac{\varepsilon(t)}{\varepsilon_c(t)}$ In terms of $\Omega(t)$, Friedmann Equation:

$$1 - \Omega(t) = -\frac{\kappa c^2}{R_0^2 a(t)^2 H(t)^2}$$

- At present, $1 - \Omega_0 = -\frac{\kappa c^2}{R_0^2 H_0^2}$ $\Rightarrow$ $\frac{\kappa}{R_0^2} = \frac{H_0^2}{c^2}(\Omega_0 - 1)$

Fluid Equation, Acceleration Equation and Equation of State

- First Law of Thermodynamics and dynamics of an expanding Universe gives: $V\left(\dot{\varepsilon} + 3\frac{\dot{a}}{a}\varepsilon + 3\frac{\dot{a}}{a}P\right) = 0 \Rightarrow \ \dot{\varepsilon} + 3\frac{\dot{a}}{a}(\varepsilon + P) = 0$ (Fluid Equation)

- Taking time derivative of Friedmann Eq and combining with above gives: $\frac{\ddot{a}}{a} = -\frac{4\pi G}{3c^2}(\varepsilon + 3P)$ (Acceleration Equation)

- Relation between Pressure and Density: $P = w\varepsilon$

$$\left\{\begin{array}{l} w \approx \frac{\langle v^2\rangle}{3c^2} \ll 1 \text{ for nonrel particle (matter)} \\ w = \frac{1}{3} \text{ for relativistic particles (radiation)} \\ w = -1 \text{ for dark energy (cosmo const } \Lambda) \end{array}\right.$$

The Cosmological Constant Λ

- Correction term initially added to achieve a static universe conforming to 19th century observations
- Corrected Friedmann eq: $\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3c^2}\varepsilon - \frac{\kappa c^2}{R_0^2 a^2} + \frac{\Lambda}{3}$
- Corrected acceleration eq: $\frac{\ddot{a}}{a} = -\frac{4\pi G}{3c^2}(\varepsilon + 3P) + \frac{\Lambda}{3}$
- Energy density of the additional term: $\varepsilon_\Lambda \equiv \frac{c^2}{8\pi G}\Lambda$
- From fluid eq: $P_\Lambda = -\varepsilon_\Lambda = -\frac{c^2}{8\pi G}\Lambda (\because \dot{\varepsilon} = 0)$

Predictions and Problems of Λ

Predictions about Einstein's static universe:

- $\ddot{a} = 0 \Rightarrow \Lambda = 4\pi G\rho$
- $\dot{a} = 0 \Rightarrow \frac{\kappa c^2}{R_o^2} = 4\pi G\rho$

$$\therefore \kappa = 1$$

$$R_o = \frac{c}{\Lambda^{1/2}}$$

Problems with Einstein's static universe:

- The static model was in **unstable equilibrium**
- The Universe was found to be **expanding**

- The above problems led to $\Lambda = 0$
- It was brought back under the assumption that $\Lambda > 4\pi G\rho \Rightarrow \ddot{a} > 0$
- Λ represents a vacuum energy that remains constant irrespective of the dynamics of the universe

Evolution of Model Universes

Energy density as a function of scale factor (a):

- From fluid equation: $\dot{\varepsilon} + 3\frac{\dot{a}}{a}(\varepsilon + P) = 0$
- $P = \sum_i w_i \varepsilon_i \Rightarrow \dot{\varepsilon}_i + 3\frac{\dot{a}}{a}(1 + w_i)\varepsilon_i = 0$

  $\Rightarrow \varepsilon_i(a) = \varepsilon_{i,0} a^{-3(1+w_i)}$
- Matter (w=0): $\varepsilon_m(a) = \varepsilon_{m,0}/a^3$
- Radiation (w=$^1/_3$): $\varepsilon_r(a) = \varepsilon_{r,0}/a^4$
- Λ (w= -1): $\varepsilon_\Lambda = \varepsilon_{\Lambda,0}$

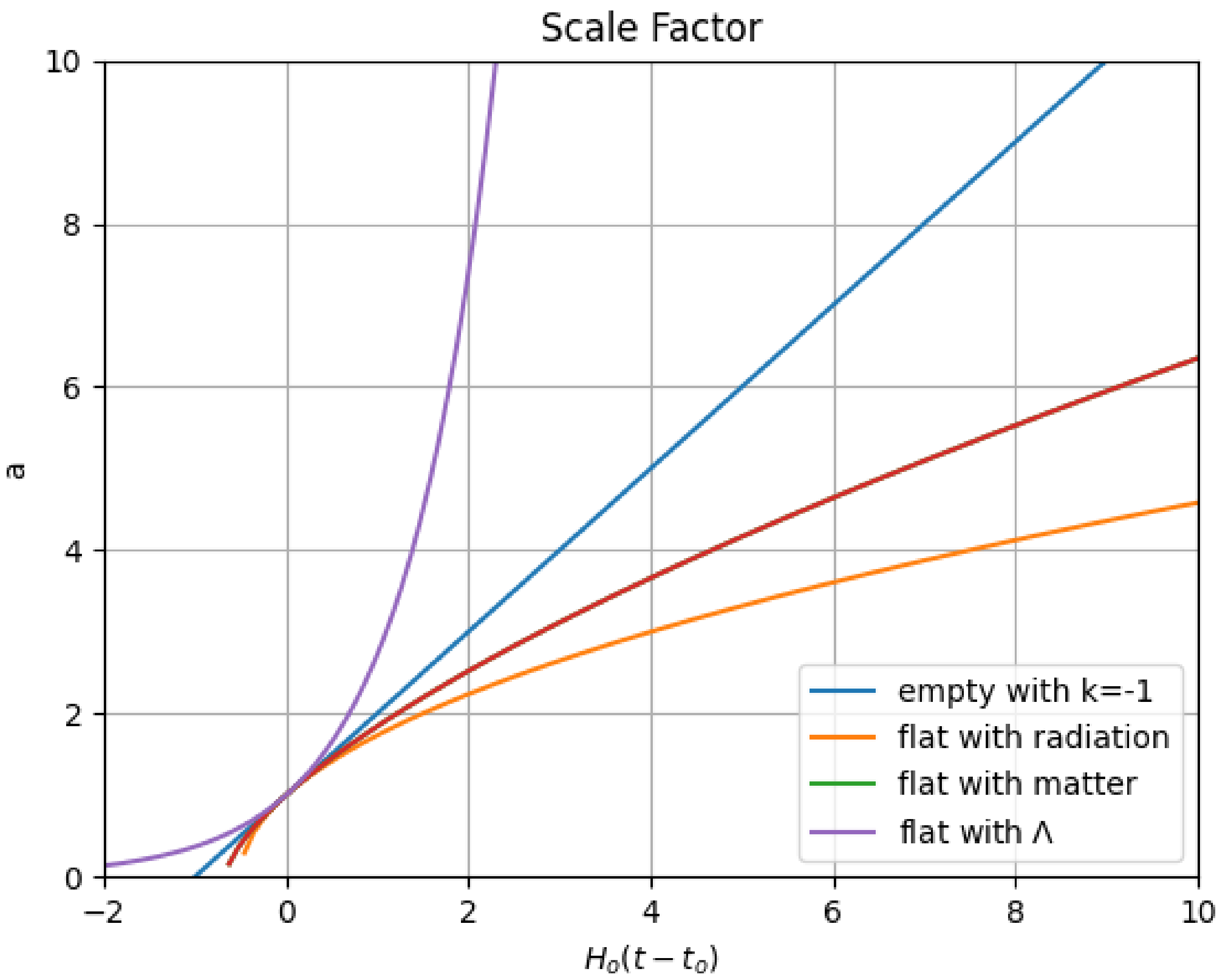
Benchmark Model:

- $\Omega_{r,0} = 9 * 10^{-5}; \Omega_{m,0} = 0.31$

  $\Omega_{r,0} \approx 0.69$
- $\frac{\varepsilon_\Lambda(a)}{\varepsilon_m(a)} = \frac{\varepsilon_{\Lambda,0}}{\varepsilon_{m,0}/a^3} = \frac{\Omega_{\Lambda,0}}{\Omega_{m,0}} a^3 \Rightarrow a_{m\Lambda} = \left(\frac{\Omega_{m,0}}{\Omega_{\Lambda,0}}\right)^{1/3} \approx 0.766$
- $\frac{\varepsilon_m(a)}{\varepsilon_r(a)} = \frac{\varepsilon_{m,0}}{\varepsilon_{r,0}} a \Rightarrow a_{rm} = \frac{\varepsilon_{m,0}}{\varepsilon_{r,0}} \approx 2.9 \times 10^{-4}$
- Friedmann Equation:

$$\dot{a}^2 = \frac{8\pi G}{3c^2} \sum_i \varepsilon_{i,0} a^{-1-3w_i} - \frac{\kappa c^2}{R_0^2}$$

Evolution of Single-Component Universes



Equations governing the evolution of the scale factor:

- Empty Universe, $k = -1$: $a(t) = \dfrac{t}{t_0}$
- Flat Universe, matter/radiation:

$$\dot{a}^2 = \frac{8\pi G\varepsilon_0}{3c^2}a^{-(1+3w)} \Rightarrow a(t) = \left(\frac{t}{t_0}\right)^{2/(3+3w)}$$

where,

$$t_0 = \frac{1}{1+w}\left(\frac{c^2}{6\pi G\varepsilon_0}\right)^{1/2}$$

- Flat Universe, dark matter:

$$\dot{a}^2 = \frac{8\pi G\varepsilon_\Lambda}{3c^2}a^2 \Rightarrow a(t) = e^{H_0(t-t_0)}$$

, where:

$$H_0 = \left(\frac{8\pi G\varepsilon_\Lambda}{3c^2}\right)^{1/2}$$

Evolution of Multi-Component Universes

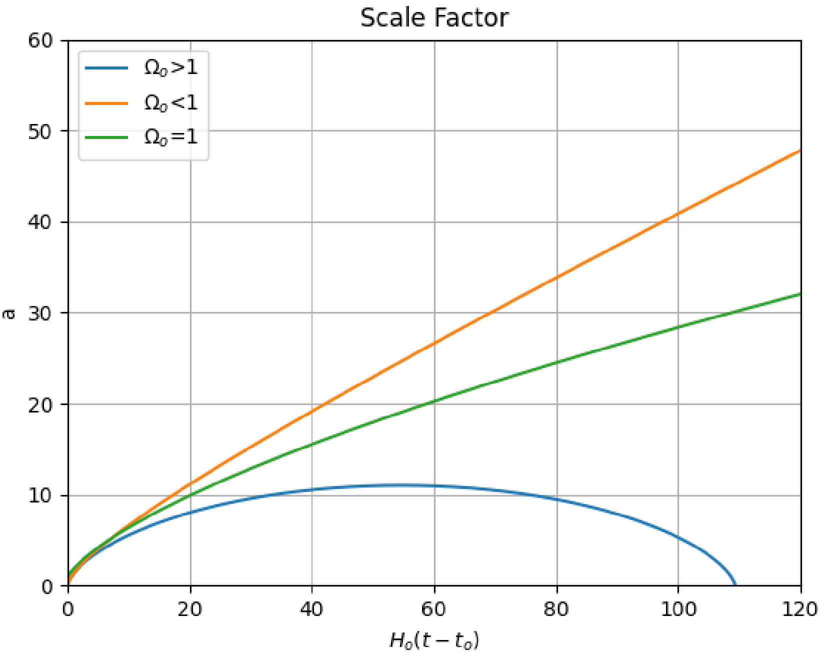
- Substituting $\frac{\kappa}{R_0^2} = \frac{H_0^2}{c^2}(\Omega_0 - 1)$ in Friedmann Equation, we get: $\frac{H(t)^2}{H_0^2} = \frac{\varepsilon(t)}{\varepsilon_{c,0}} + \frac{1 - \Omega_0}{a(t)^2}$

- $\because \varepsilon(t) = \varepsilon_m + \varepsilon_r + \varepsilon_\Lambda = \frac{\varepsilon_{m,0}}{a^3} + \frac{\varepsilon_{r,0}}{a^4} + \varepsilon_{\Lambda,0} \therefore \frac{H^2}{H_0^2} = \frac{\Omega_{r,0}}{a^4} + \frac{\Omega_{m,0}}{a^3} + \Omega_{\Lambda,0} + \frac{1 - \Omega_0}{a^2}$

  where $\Omega_{r,0} = \varepsilon_{r,0}/\varepsilon_{c,0}$, $\Omega_{m,0} = \varepsilon_{m,0}/\varepsilon_{c,0}$, $\Omega_{\Lambda,0} = \varepsilon_{\Lambda,0}/\varepsilon_{c,0}$, and $\Omega_0 = \Omega_{r,0} + \Omega_{m,0} + \Omega_{\Lambda,0}$

- $\because H(t) = \frac{\dot{a}}{a} \Rightarrow H_0^{-1}\dot{a} = \left[\frac{\Omega_{r,0}}{a^2} + \frac{\Omega_{m,0}}{a} + \Omega_{\Lambda,0}a^2 + (1 - \Omega_0)\right]^{1/2} = \int_0^a \frac{da}{[\Omega_{r,0}/a^2 + \Omega_{m,0}/a + \Omega_{\Lambda,0}a^2 + (1 - \Omega_0)]^{1/2}} = H_0 t.$

• Matter + Curvature



$$\frac{H(t)^2}{H_0^2} = \frac{\Omega_0}{a^3} + \frac{1-\Omega_0}{a^2}$$

$$\frac{\dot{a}^2}{H_0^2} = \frac{\Omega_0}{a} + (1-\Omega_0)$$

$$H_0 t = \int_0^a \frac{da}{[\Omega_0/a + (1-\Omega_0)]^{1/2}}$$

When $k = +1$ $(\Omega_o > 1)$:
Universe stops expanding when H(t) = 0

$$a_{\max} = \frac{\Omega_0}{\Omega_0 - 1}$$

$$a(\theta) = \frac{1}{2}\frac{\Omega_0}{\Omega_0 - 1}(1-\cos\theta)$$

$$t(\theta) = \frac{1}{2H_0}\frac{\Omega_0}{(\Omega_0-1)^{3/2}}(\theta - \sin\theta)$$

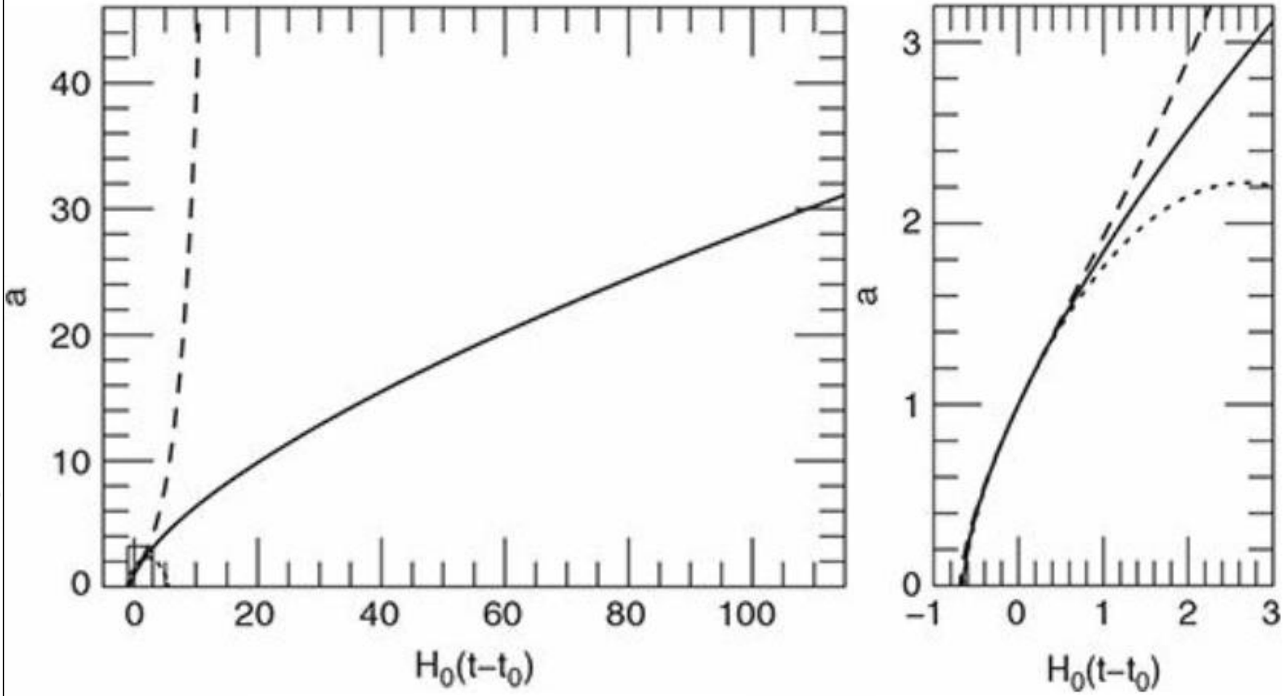
When $k = -1$ $(\Omega_o < 1)$:
Universe keeps expanding as LHS > 0:

$a \propto t^{\frac{2}{3}}$ when $\Omega_m > \Omega_0$
$a \propto t$ when $\Omega_m \ll \Omega_0$

$$a(\eta) = \frac{1}{2}\frac{\Omega_0}{1-\Omega_0}(\cosh\eta - 1)$$

$$t(\eta) = \frac{1}{2H_0}\frac{\Omega_0}{(1-\Omega_0)^{3/2}}(\sinh\eta - \eta)$$

• Matter + Lambda



$$\frac{H^2}{H_0^2} = \frac{\Omega_{m,0}}{a^3} + (1-\Omega_{m,0}) \because \Omega_{\Lambda,0} = 1 - \Omega_{m,0}$$

$\Omega_{\Lambda,0} < 1$:Universe stops expanding when H(t) = 0

$$a_{\max} = \left(\frac{\Omega_{m,0}}{\Omega_{m,0}-1}\right)^{1/3}$$

$$H_0 t = \frac{2}{3\sqrt{\Omega_{m,0}-1}}\sin^{-1}\left[\left(\frac{a}{a_{\max}}\right)^{3/2}\right]$$

$\Omega_{\Lambda,0} > 1$:Universe keeps expanding-

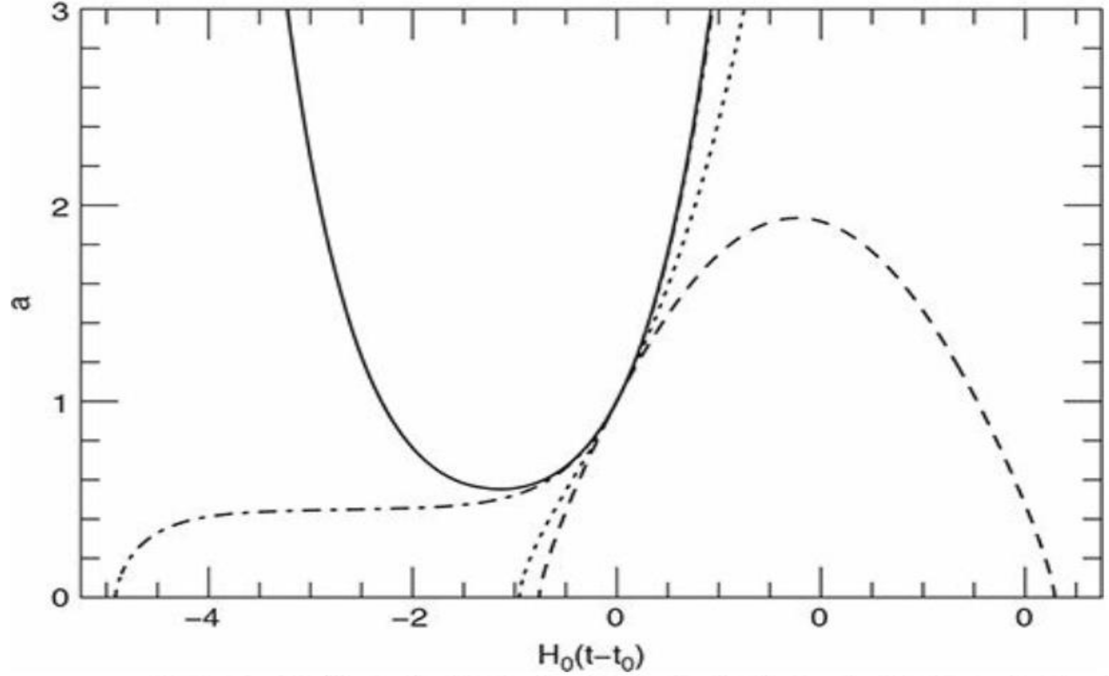
When $a \ll a_{m\lambda}$:

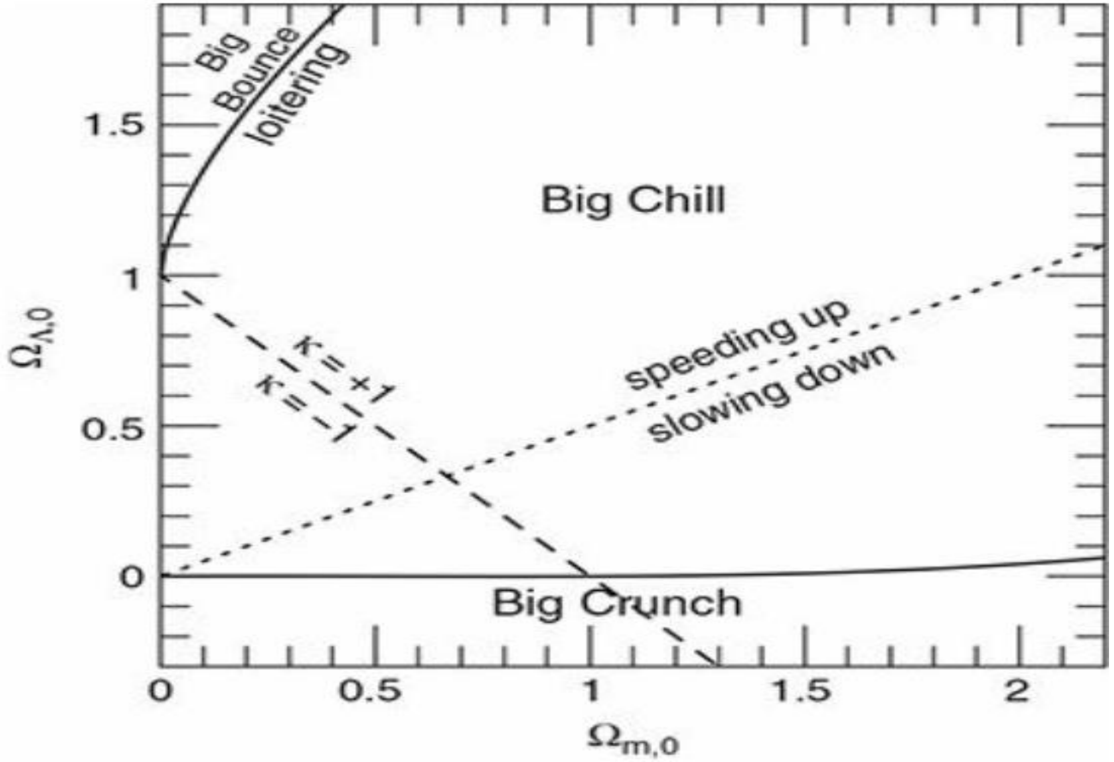
$$a(t) \approx \left(\frac{3}{2}\sqrt{\Omega_{m,0}}H_0 t\right)^{2/3}$$

When $a \gg a_{m\lambda}$:

$$a(t) \approx a_{m\Lambda}\exp(\sqrt{1-\Omega_{m,0}}H_0 t)$$

Matter + Curvature + Lambda

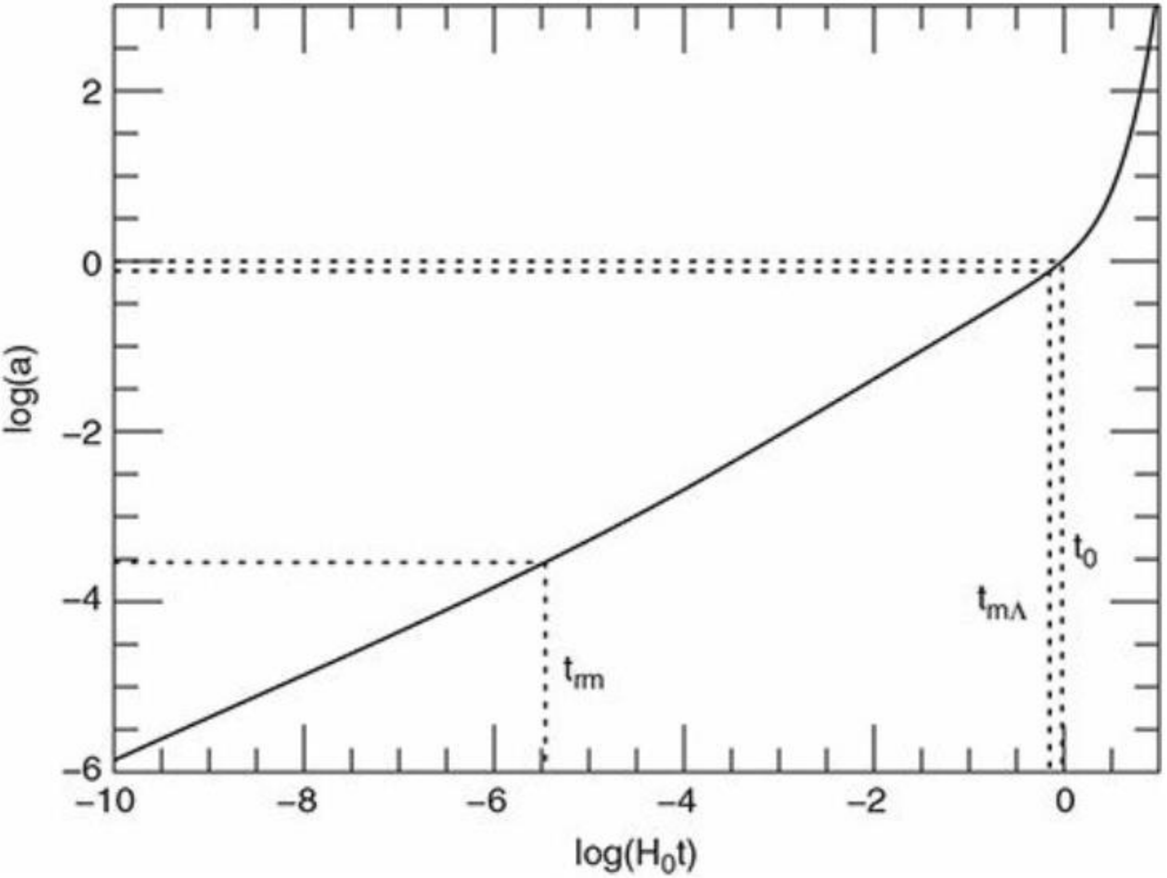




$$\frac{H^2}{H_0^2} = \frac{\Omega_{m,0}}{a^3} + \frac{1 - \Omega_{m,0} - \Omega_{\Lambda,0}}{a^2} + \Omega_{\Lambda,0}$$

- Big Bounce: $\Omega_{m,0} > 0; \Omega_{\Lambda,0} > 0;\ \Omega_{m,0} + \Omega_{\Lambda,0} > 1$
  $H < 0, a \gg 1$
- Loitering: initial- $a \propto t^{\frac{2}{3}}$; int- $a = k$; final- $a \propto e^{kt}$
- Big Crunch: can have $k = \pm 1{,}0$
- Big Chill: $k = \pm 1{,}0$; $a \to \infty\ as\ t \to \infty$

Benchmark Model



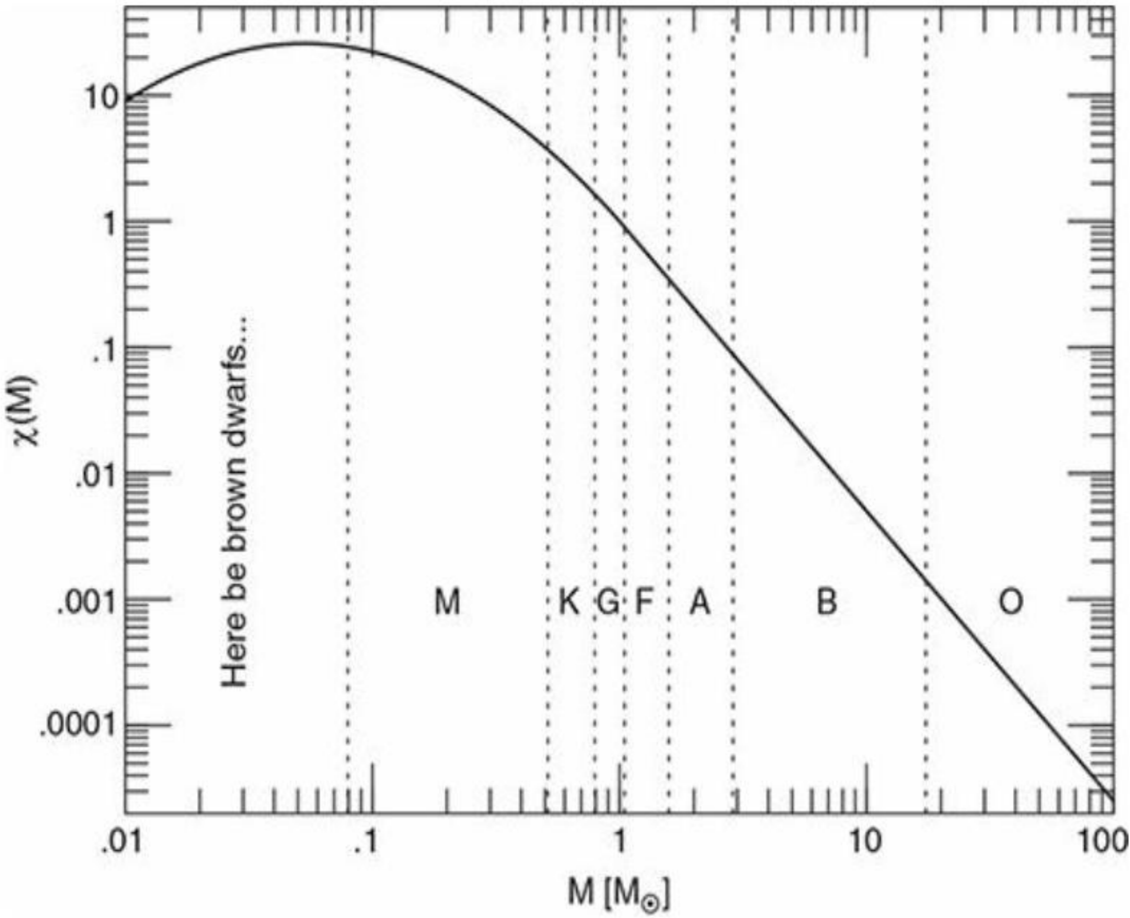
Photons:	$\Omega_{\gamma,0} = 5.35 \times 10^{-5}$
Neutrinos:	$\Omega_{\nu,0} = 3.65 \times 10^{-5}$
Total radiation:	$\Omega_{r,0} = 9.0 \times 10^{-5}$
Baryonic matter:	$\Omega_{\mathrm{bary},0} = 0.048$
Nonbaryonic dark matter:	$\Omega_{\mathrm{dm},0} = 0.262$
Total matter:	$\Omega_{m,0} = 0.31$
Cosmological constant:	$\Omega_{\Lambda,0} \approx 0.69$

Important epochs		
Radiation–matter equality:	$a_{rm} = 2.9 \times 10^{-4}$	$t_{rm} = 0.050$ Myr
Matter–lambda equality:	$a_{m\Lambda} = 0.77$	$t_{m\Lambda} = 10.2$ Gyr
Now:	$a_0 = 1$	$t_0 = 13.7$ Gyr

Detecting Matter

- Info from photodetection:
  - $\Psi_V = 1.1 \times 10^8 L_{\odot,V} Mpc^{-3}$
  - Active Galaxies: $M/L_V \approx 0.3\ M_\odot / L_{\odot,V}$
  - Quiescent Galaxies: $M/L_V \approx 8\ M_\odot / L_{\odot,V}$
  - $\langle M/L_V \rangle = 4.15 \frac{M_\odot}{L_{\odot,V}} \Rightarrow \rho_{*,0} \approx 4 \times 10^8 M_\odot Mpc^{-3}$
  - $\Omega_{*,0} \approx 0.003$ (very low contribution)
  - Gases measured by observing V-band and x-rays
- Info from CMB: temperature fluctuations dependent on photon-baryon ratio of early universe.
- Current element composition of universe is dependent on photon-baryon ratio of nascent universe.

$$\Omega_{bary,0} \approx 0.048 \pm 0.003$$

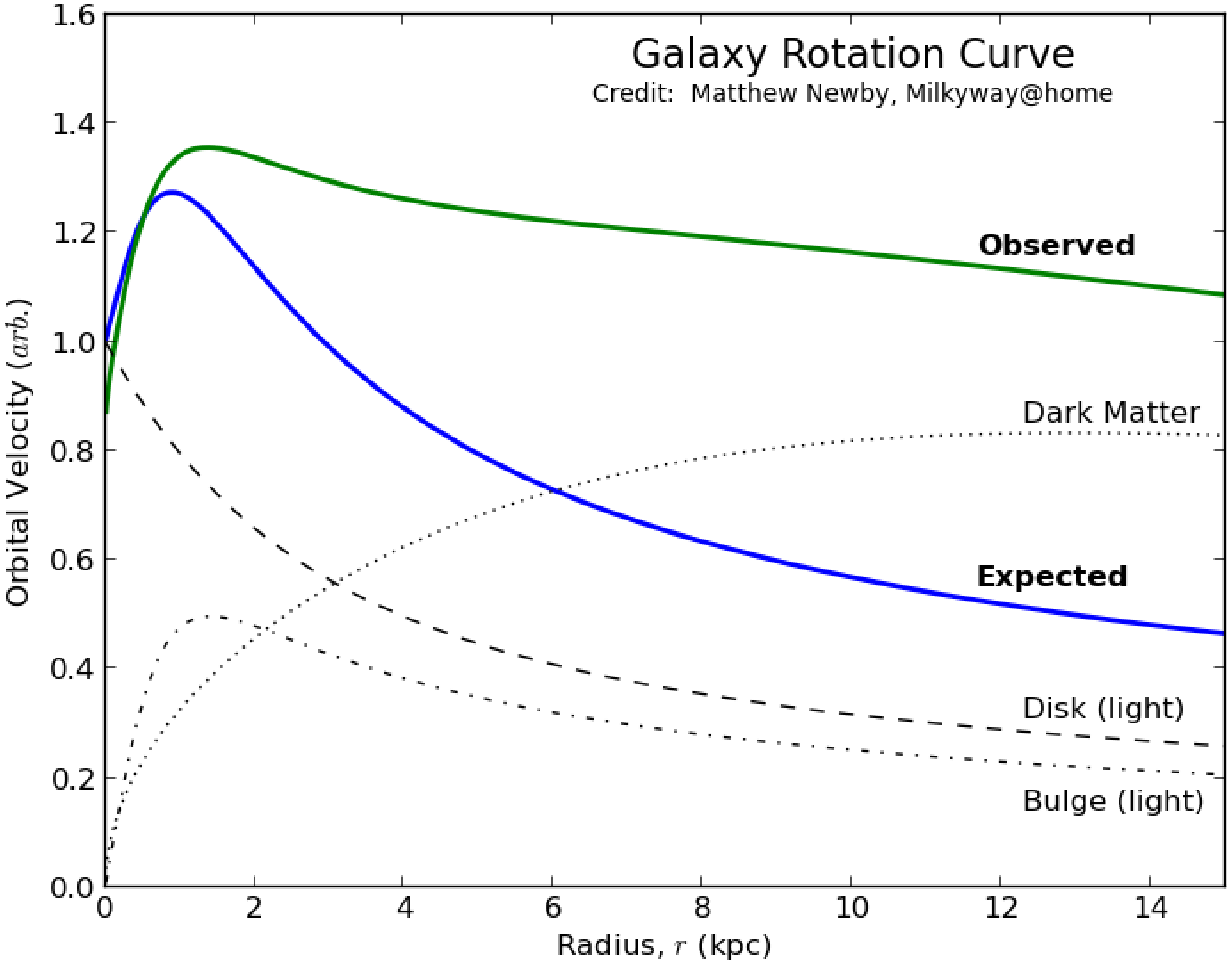


Dark Matter

- Interacts through gravity and not EM
- Existence predicted by observing galaxy rotation.
- $v = \sqrt{\frac{GM(R)}{R}}$ Keplerian Prediction
- $I(R) = I(0)\exp\left(-\frac{R}{R_s}\right)$ Star brightness from centre of galaxy
- So $v \propto R^{-1/2}$ for $R > R_s$

$$M(R) = \frac{v^2 R}{G} = 1.05 \times 10^{11}\,\mathrm{M}_\odot \left(\frac{v}{235\,\mathrm{km\,s^{-1}}}\right)^2 \left(\frac{R}{8.2\,\mathrm{kpc}}\right)$$

$$\langle M/L_V \rangle_{\mathrm{gal}} \approx 64\,\mathrm{M}_\odot/\mathrm{L}_{\odot,V} \left(\frac{R_{\mathrm{halo}}}{100\,\mathrm{kpc}}\right)$$



Dark Matter in Galaxy Clusters

Virial Theorem: For a cluster of mass M

- $K = \frac{1}{2} M \langle v^2 \rangle$ & $W = -\frac{G}{2} \sum_{\substack{i,j \\ j \neq i}} \frac{m_i m_j}{|\vec{x}_j - \vec{x}_i|} \Rightarrow W = -\alpha \frac{GM^2}{r_h}$
- $\ddot{I} = 2 \sum_i m_i (\vec{x}_i \cdot \ddot{\vec{x}}_i + \dot{\vec{x}}_i \cdot \dot{\vec{x}}_i) \Rightarrow \ddot{I} = 2W + 4K$
- In steady state: $\ddot{I} = 0 \Rightarrow K = -\frac{W}{2} \therefore M = \frac{\langle v^2 \rangle r_h}{\alpha G}$
- Using observational data, $\langle v^2 \rangle$ & $\mathrm{r_h}$ can be estimated with certain assumptions
- Eg: $M_{coma} = 2 \times 10^{15} M_\odot$

Hydrostatic equilibrium:

$$\frac{dP_{\mathrm{gas}}}{dr} = -\frac{GM(r)\rho_{\mathrm{gas}}(r)}{r^2}$$

Using $P_{\mathrm{gas}} = \frac{\rho_{\mathrm{gas}} k T_{\mathrm{gas}}}{\mu}$ above, we get:

$$M(r) = \frac{kT_{\mathrm{gas}}(r) r}{G\mu} \left[-\frac{d \ln \rho_{\mathrm{gas}}}{d \ln r} - \frac{d \ln T_{\mathrm{gas}}}{d \ln r} \right]$$

- X-ray data and modeling gives composition, temperature and density distribution of gas
- Eg: $M_{coma} \approx 1.3 \times 10^{15} M_\odot$

$$\boxed{\Omega_{\mathrm{clus},0} \approx 0.2}$$

Gravitational Lensing

- Photon deflection $\alpha = \dfrac{4GM}{c^2 b}$

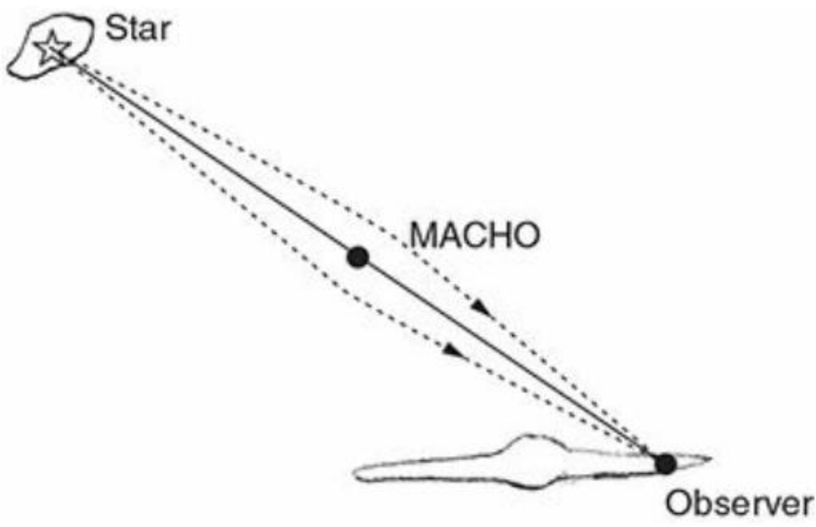
- Einstein angle $\theta_E = \left(\dfrac{4GM}{c^2 d}\dfrac{1-x}{x}\right)^{1/2}$

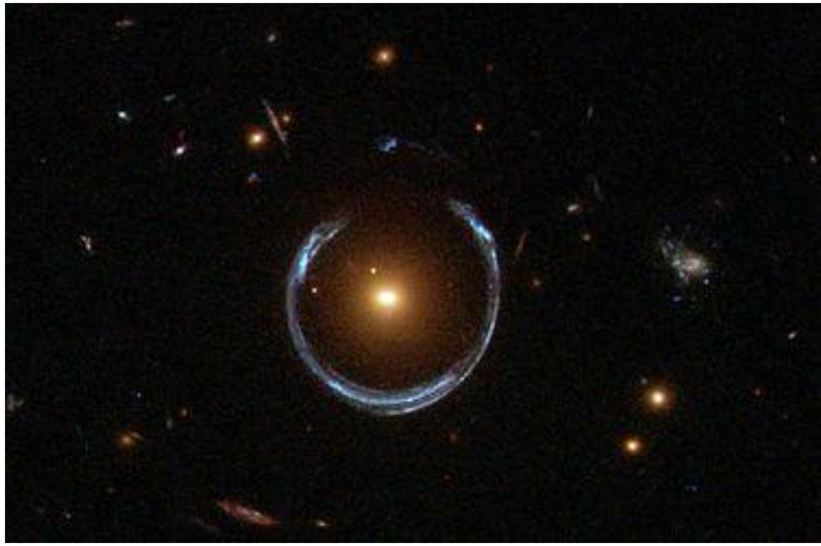
- Lensing leads to increase in flux

$$\Delta t = \frac{d\,\theta_E}{2v} \approx 90\,\text{days}\left(\frac{M}{1\,\mathrm{M}_\odot}\right)^{1/2}\left(\frac{v}{200\,\mathrm{km\,s^{-1}}}\right)^{-1}$$

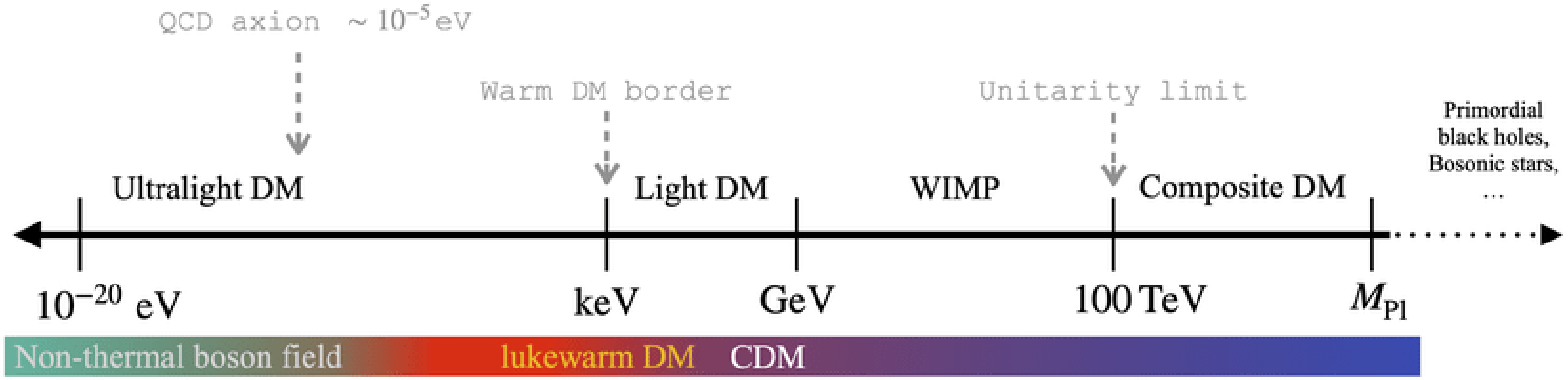
- Mass of galaxy clusters can be estimated by:

$$\theta_E \approx 0.5\,\text{arcmin}\left(\frac{M}{10^{14}\,\mathrm{M}_\odot}\right)^{1/2}\left(\frac{d}{1000\,\mathrm{Mpc}}\right)^{-1/2}$$





What is DM?



- Predictions have mass range of 76 orders of magnitude
- Neutrinos were considered as candidate but contradicting evidence

$$n_\nu = 3\left(\frac{3}{11}\right)n_\gamma = 3.36 \times 10^8\,\mathrm{m}^{-3} \qquad m_\nu c^2 = \frac{\Omega_{\mathrm{dm},0}\varepsilon_{c,0}}{n_\nu} \approx 3.8\,\mathrm{eV}$$

But we know $0.019\,\mathrm{eV} < m_\nu c^2 < 0.1\,\mathrm{eV}$.

- SUSY predicts existence of WIMPs: weak force, gravity and much more massive than neutrinos