AMAN SAHOO, 22164 HE 386 PROBLEM SET 19/10/24

1. We know $F = \frac{mv^2}{R}$ (Centripetal force)

$$\Rightarrow qvB = \frac{mv^2}{R} \Rightarrow p = qBR \quad \text{(in S.I units)}$$

$$p \; kg\,m/s = p' \; \frac{GeV}{c} \Rightarrow p\left(\frac{J}{m/s}\right) = p'\left(\frac{GeV}{c}\right)\left(\frac{1.6\times10^{-19}}{10^{-9}}\frac{J}{GeV}\right)\left(\frac{1}{3\times10^{8}}\frac{c}{m/s}\right)$$

$$\Rightarrow p = p' \frac{1.6\times10^{-10}}{3\times10^{8}}$$

||ly $q(C) = q'(e)\left(\frac{1.6\times10^{-19}C}{e}\right) \Rightarrow q = q'(1.6\times10^{-19})$

$$\therefore p'\left(\frac{1.6}{3}\times10^{-18}\right) = q'(1.6\times10^{-19})BR \Rightarrow p' = 0.3q'BR$$

where p' in $\frac{GeV}{c}$ & q' in e (can be assumed = 1)

2. $\cancel{[(511c^2)+1]^2 = (511c^2)^2 + p^2c^2}$

$\cancel{\Rightarrow (511c^2)^2+1+2(511c^2) = (511c^2)^2 + p^2c^2} \Rightarrow \cancel{p^2 = \left(1022 + \frac{1}{c^2}\right)\left(\frac{KeV}{c}\right)^2}$

$\cancel{\Rightarrow p = \sqrt{1022+\frac{1}{c^2}} \frac{KeV}{c}} =$

2. @1 keV e^- $\Rightarrow (511+1)^2 = (511)^2 + p^2c^2$

$$\Rightarrow 1023 \frac{keV^2}{c^2} = p^2 \Rightarrow p = 31.98 \frac{keV}{c}$$

$$= 31.98\times10^{-6} \frac{GeV}{c}$$

$$\therefore R = \frac{31.98\times10^{-6}}{0.3\times1\times1} = 10.66\times10^{-5} m \Rightarrow 0.1066 \text{ mm}$$

(b) 100 keV.

$(511+100)^2 = (511)^2 + p^2c^2 \Rightarrow \cancel{1132000}\, p = 334.96 \frac{\text{keV}}{c} = 334.96\times10^{-6}\frac{\text{GeV}}{c}$

$\Rightarrow R = \frac{334.96\times10^{-6}}{0.3\times1} = 1116.54\times10^{-6}\,\text{m} \Rightarrow 1.11\,\text{mm}.$

Yes, modern HEP detectors/trackers can detect these electrons due to their high radius of curvature.

③ Using Synchrotron Radiation formula $\rightarrow P_\gamma = \frac{q^2 c}{6\pi\epsilon_0}\frac{\beta^4\gamma^4}{\rho^2}$

where ρ = radius of curvature of particle

We know $E = \gamma m_0c^2 \Rightarrow (m_0c^2 + KE) = \gamma m_0c^2 \Rightarrow \gamma = 1 + \frac{KE}{m_0c^2}$

For a proton at LHC $\rightarrow KE = 6.8\,\text{TeV},\ m_0c^2 = 0.938\,\text{GeV}.$

$\rho = \frac{27\times10^3}{2\pi}\,\text{m}.$

$\therefore \gamma = 1 + \frac{6800}{0.938} \approx 7250 \quad \therefore \beta \approx 1.$

$\therefore P_\gamma = \frac{2}{3}\frac{(1.6\times10^{-19})^2 \times (3\times10^8)(7250)^4 \times 9\times10^9}{\left(\frac{27000}{2\pi}\right)^2} = 6.89\times10^{-12}\,\text{W}$

4) for $\bar{e}$ at LEP $\rightarrow KE = 209\,\text{GeV},\ m_0c^2 = 0.511\,\text{GeV}$

$\rho = \frac{27\times10^3}{2\pi}\,\text{m}.$

$\therefore \gamma = 1 + \frac{209}{0.511} \approx 410 \Rightarrow \beta = 0.99999703$

$\therefore P_\gamma = \frac{2}{3} \times 9\times 10^{9} \times \frac{(1.6\times10^{-19})^2 \times (3\times10^8)\times(410)^4}{\left(\frac{27000}{2\pi}\right)^2} = 7.04\times10^{-17}\,W$

$\therefore \frac{P_{p^+}}{P_{e^-}} = 97869.32$

For e^- at Super KEKB $\rightarrow$ $KE = 7\,GeV$, $m_0c^2 = 0.511\,GeV$

$\rho = \frac{3016}{2\pi}\,m$

$\therefore \gamma = 1 + \frac{7}{0.511} = 14.7$, $\beta = 0.997$ $\therefore P_\gamma \approx 1.15\times10^{-22}\,W$

$\therefore \frac{P_{LEP}}{P_{SKEKB}} = 6.12173\times10^5$

4) $\therefore -\frac{dE}{dx} = \frac{E}{X_0} \Rightarrow \frac{dE}{E} = \frac{dx}{X_0} \Rightarrow \ln E = -\frac{x}{X_0} \Rightarrow E = E_0 e^{-\frac{x}{X_0}}$

$\Rightarrow E(X_0) = \frac{E_0}{e}$ $\therefore$ Loss $= E_0\left(1-\frac{1}{e}\right) = 0.63\,E_0$

Radiation Length of Be = 35.28 cm.

$\therefore E(1\,mm) = 0.5\,e^{-\frac{1}{352.8}}\,GeV \Rightarrow E_{loss} = 0.5\left(1-e^{-\frac{1}{352.8}}\right)$

$= 1.415\,MeV$

5) Scattering angle $\theta_{ms} \approx \dfrac{13.6\ \text{MeV}}{\beta c p} z \sqrt{\dfrac{x}{X_0}} \left[1 + 0.038 \log_{10}\left(\dfrac{x z^2}{X_0 \beta^2}\right)\right]$

For 1 GeV pion(π^-) $\rightarrow z = 1,\ X_0 = 8.897$ cm.
through Aluminium.

$\gamma = \frac{1}{\sqrt{1-\beta^2}}$ $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$
$\Rightarrow 1-\beta^2 = \frac{1}{\gamma^2}$
$\Rightarrow \sqrt{1-\frac{1}{\gamma^2}} = \beta$
$d + vt = ct$

$$E^2 = m^2c^4 + p^2c^2 \Rightarrow (139.57 + 1000)^2 = (139.57)^2 + p^2c^2$$

$$\Rightarrow (1279.14)(1000) = p^2c^2 \Rightarrow p \approx 1131\ \frac{\text{MeV}}{c}$$

$$\gamma = 1 + \frac{KE}{mc^2} = 1 + \frac{1000}{139.57} \approx 8.164 = \frac{1}{\sqrt{1-\beta^2}} \Rightarrow \beta = \sqrt{1 - \frac{1}{(8.164)^2}}$$

$$\approx 0.9925$$

$$\therefore \theta_{ms} = \frac{13.6\ \text{MeV}}{(0.9925)(1131\ \text{MeV})} \sqrt{\frac{1}{8.897}} \left[1 + 0.038 \log_{10}\left(\frac{1}{8.897\ (0.9925)^2}\right)\right]$$

$$= 3.72\ \text{mrad}$$

6) In rest frame $\rightarrow \tau_0 = 8.954 \times 10^{-11}$ s., $m_0 = 497.611\ \frac{\text{MeV}}{c^2}$

In lab frame $\rightarrow \tau_L = \gamma\tau_0 = \left(1 + \frac{10000}{497.611}\right)\tau_0 = 21 \times 8.954 \times 10^{-11}$ s

$= 188.89 \times 10^{-11}$

$= 1.88$ ns

$\therefore$ Flight Distance $= v\tau_L = \beta c \tau_L$

$$= \sqrt{1 - \frac{1}{(21.1)^2}}\ (1.88\ \text{ns})c \approx 0.498\ \text{m}$$

(7) given $\frac{\sigma_E}{E} = \frac{2.3\%}{(E/GeV)^{1/4}} \oplus 1.4\%$

For 5 GeV $\gamma \rightarrow \frac{\sigma_E}{E} = \frac{2.3\%}{5^{1/4}} \oplus 1.4\% = 1.54\% \oplus 1.4\%$

$= \sqrt{(1.54)^2 + (1.4)^2} \approx 2.08\%$ $\therefore \frac{\sigma_E}{E} = 2.08\%$

(8) From PDG, we find $\left|-\frac{dE}{dx}\right|_{min} = 3.876$ MeV/cm for Si

$\therefore n_{eh} = \frac{3.876 \times \cancel{10^6} \times 10^2 \times 100 \times \cancel{10^{-6}}}{3.6}$

$\approx 1.0766 \times 10^4 = 10766$ e-hole pairs

(9) For Photoelectric effect $\sigma_{PE} \propto \frac{Z^5}{E^3}$

Compton Scattering $\sigma_{CS} \propto \frac{Z}{E}$

Pair Production $\sigma_{PP}\uparrow$ as $E, Z \uparrow$

(a) for C, $\sigma_{PE} = \sigma_{CS} = K$ at ~ 100 keV.

$\therefore K = \alpha \frac{Z^5}{E^3} = \alpha \frac{6^5}{(100)^3}$ & $K = \beta \frac{Z}{E} = \beta \frac{(6)}{(100)}$

$\therefore$ for $E = 1000$ keV in Al (13 Z)

$\frac{\sigma_{PE}}{\sigma_{CS}} = \frac{\alpha \left(\frac{13^5}{1000^3}\right)}{\beta \left(\frac{13}{1000}\right)} = \frac{K \left(\frac{13}{6}\right)^5 \left(\frac{100}{1000}\right)^3}{K \left(\frac{13}{1000}\right)\left(\frac{100}{6}\right)} = \left(\frac{13}{6}\right)^4 \left(\frac{100}{1000}\right)^2 = \cancel{0.022}\ 0.22$

$\therefore$ Compton Scattering dominates

(b) As before, for H(Z=2) at E = 100 keV

$$\frac{\sigma_{PE}}{\sigma_{CS}} = \frac{K \frac{2^5}{100^3}\frac{100^2}{6^5}}{K \frac{2}{100}\frac{100}{6}} = \left(\frac{2}{6}\right)^4 = \frac{1}{81}$$

∴ Compton scattering is dominant

(c) For Fe (Z=26) at E = 100 keV

$$\frac{\sigma_{PE}}{\sigma_{CS}} = \left(\frac{26}{6}\right)^4 >> 1$$

∴ Photo electric effect is more dominant

(d) From the figure ~~mentioned~~ 34.15 in referred document, we see that at 10 MeV in C(Z=6), Compton scattering is slightly more dominant than pair production

(e) From the same figure, we note that at 10 MeV in Pb (Z=82) pair production is slightly more dominant than Compton scattering.