**Topic**: Conditions for inference with the SDSP

**Question**: A math class wants to know how many of the $2{,}000$ students in their school carry a blue backpack. Which sample meets all conditions of a normal sampling distribution?

**Answer choices**:

A The class randomly surveys $250$ students during lunch

B The class surveys $100$ students from freshmen classes

C The class surveys $500$ students from senior classes

D The class randomly surveys $100$ students during passing period

**Solution**: D

Surveying $100$ students randomly during a passing period is a valid sampling distribution because it’s random and keeps the number of subjects in the sample below $10\%$, which maintains independence.

The other choices either don’t keep the sample below $10\%$, and/or aren’t random because they’re surveying only freshmen or seniors.

**Topic**: Conditions for inference with the SDSP

**Question**: A restaurant wants to know the percentage of their customers who order desert. The restaurant has $1{,}500$ customers in one week and finds by randomly surveying $100$ of them that $35$ order desert. What is the standard error of the SDSP?

**Answer choices**:

A $\sigma_{\hat{p}} \approx 0.047697$

B $\sigma_{\hat{p}} \approx 0.015096$

C $\sigma_{\hat{p}} \approx 0.052303$

D $\sigma_{\hat{p}} \approx 0.084900$

**Solution**: A

To verify normality, our sample space should be random, no more than $10\%$ of the population, and the expected number of successes and failures should each be at least $5$.

Independence: $\frac{100}{1{,}500} = 0.067 = 6.7\% \leq 10\%$

Successes: $100(0.35) = 35 \geq 5$

Failures: $100(0.65) = 65 \geq 5$

The sample space was random, so we've met the conditions of normality. The standard error, or sampling distribution of the sample proportion, is given by

$$\sigma_{\hat{p}} = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$\sigma_{\hat{p}} = \sqrt{\frac{0.35(1-0.35)}{100}}$$

$$\sigma_{\hat{p}} = \sqrt{\frac{0.35(0.65)}{100}}$$

$$\sigma_{\hat{p}} = \sqrt{\frac{0.2275}{100}}$$

$$\sigma_{\hat{p}} \approx 0.047697$$

**Topic**: Conditions for inference with the SDSP

**Question**: A group of scientists is studying $10,000$ manatees and finds that $20\%$ are calves. We want to verify their claim, but can't conduct a study of all $10,000$, so we randomly sample just $500$. What's the probability that our results are within $5\%$ of the scientists' study?

**Answer choices**:

A $13.5\%$

B $73.72\%$

C $99.48\%$

D $99.8\%$

**Solution**: C

To verify normality, our sample space should be random, no more than $10\%$ of the population, and the expected number of successes and failures should each be at least $5$.

$$\text{Independence: } \frac{500}{10{,}000} = 0.05 = 5\% \leq 10\%$$

$$\text{Successes: } 500(0.2) = 100 \geq 5$$

$$\text{Failures: } 500(0.8) = 400 \geq 5$$

The sample space was random, so we've met the conditions of normality. Now we'll find the mean and standard deviation of the sampling distribution of the sample proportion.

$$\mu_{\hat{p}} = p = 0.2$$

$$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{0.2(0.8)}{500}} \approx 0.0179$$

We need to find the probability that our results are within $5\%$ of the population proportion $p = 20\%$. In other words, how likely is it that the sample proportion falls between $15\%$ and $25\%$?

$$\frac{0.05}{0.0179} \approx 2.79$$

We want to know the probability of $P(-2.79 < z < 2.79)$. Using a $z$-table, $-2.79$ gives us $0.0026$

| z | .00 | .01 | .02 | .03 | .04 | .05 | .06 | .07 | .08 | .09 |
|---|---|---|---|---|---|---|---|---|---|---|
| -2.8 | .0026 | .0025 | .0024 | .0023 | .0023 | .0022 | .0021 | .0021 | .0020 | .0019 |
| -2.7 | .0035 | .0034 | .0033 | .0032 | .0031 | .0030 | .0029 | .0028 | .0027 | .0026 |
| -2.6 | .0047 | .0045 | .0044 | .0043 | .0041 | .0040 | .0039 | .0038 | .0037 | .0036 |

and $2.79$ gives us $0.9974$.

| z | .00 | .01 | .02 | .03 | .04 | .05 | .06 | .07 | .08 | .09 |
|---|---|---|---|---|---|---|---|---|---|---|
| 2.6 | .9953 | .9955 | .9956 | .9957 | .9959 | .9960 | .9961 | .9962 | .9963 | .9964 |
| 2.7 | .9965 | .9966 | .9967 | .9968 | .9969 | .9970 | .9971 | .9972 | .9973 | .9974 |
| 2.8 | .9974 | .9975 | .9976 | .9977 | .9977 | .9978 | .9979 | .9979 | .9980 | .9981 |

So the probability we want to find is

$$P(-2.79 < z < 2.79) = 0.9974 - 0.0026$$

$$P(-2.79 < z < 2.79) = 0.9948$$

There's a $99.48\%$ chance that our sample proportion will fall within $5\%$ of the first study's claim.