Topic: Hypothesis testing for the population proportion

Question: We've heard that 10% of people are left-handed. We want to verify this claim, so we collect a random sample of 500 people and find that 43 of them are left-handed. What can we conclude at a significance level of $\alpha = 0.10$?

Answer choices:

A We'll reject the null hypothesis; our result is significant at the $\alpha = 0.10$ level

B We'll reject the null hypothesis; our result is significant at the $\alpha = 0.05$ level

C We'll reject the null hypothesis; our result is significant at the $\alpha = 0.01$ level

D We'll fail to reject the null hypothesis

Solution: D

First, build the hypothesis statements.

H_0: 10% of people are left-handed, $p = 0.1$

H_a: The proportion of left-handed people is different than 10%, $p \neq 0.1$

The sample proportion is

$$\hat{p} = \frac{x}{n} = \frac{43}{500} = 0.086$$

Then the standard error of the proportion is

$$\sigma_{\hat{p}} = \sqrt{\frac{p_0(1 - p_0)}{n}} = \sqrt{\frac{0.1(1 - 0.1)}{500}} \approx 0.0134$$

Now we have enough to find the z-test statistic.

$$z = \frac{\hat{p} - p}{\sigma_{\hat{p}}} = \frac{0.086 - 0.10}{0.0134} \approx -1.04$$

The critical z-values for 90% confidence with a two-tailed test are $z = \pm 1.65$. Since the test statistic we found is negative ($z = -1.04$), we'll compare it to $z = -1.65$.

Our z-value of $z = -1.04$ is not less than $z = -1.65$, and therefore falls in the region of acceptance, which means we'll fail to reject the null hypothesis and fail to conclude that the proportion of left-handed people is different than 10%.

Topic: Hypothesis testing for the population proportion

Question: A breakfast company claims at least 80% of Americans eat breakfast. We want to verify this claim, so we collect a random sample of 650 Americans and find that 496 of them eat breakfast. What can we conclude at a significance level of $\alpha = 0.05$?

Answer choices:

A We'll reject the null hypothesis; our result is significant at the $p = 0.0094$ level

B We'll reject the null hypothesis; our result is significant at the $p = 0.9500$ level

C We'll reject the null hypothesis; our result is significant at the $p = 0.9864$ level

D We'll fail to reject the null hypothesis

Solution: A

First, build the hypothesis statements.

H_0: At least 80% of Americans eat breakfast, $p \geq 0.8$

H_a: Fewer than 80% of Americans eat breakfast, $p < 0.8$

The sample proportion is

$$\hat{p} = \frac{x}{n} = \frac{496}{650} \approx 0.7631$$

and the standard error of the proportion is

$$\sigma_{\hat{p}} = \sqrt{\frac{p_0(1-p_0)}{n}} = \sqrt{\frac{0.8(1-0.8)}{650}} \approx 0.0157$$

Now we have enough to find the z-test statistic.

$$z = \frac{\hat{p}-p}{\sigma_{\hat{p}}} = \frac{0.7631-0.8}{0.0157} \approx -2.35$$

The critical z-value for 95% confidence with a lower-tailed test is $z = -1.65$.

Our z-value of $z \approx -2.35$ falls to the left of $z = -1.65$, and therefore falls in the region of rejection, which means we'll reject the null hypothesis and conclude that the proportion of Americans who eat breakfast is less than 80%.

We know our findings are significant at $\alpha = 0.05$, but we can find the p-value to state a higher level of significance that corresponds to $z \approx -2.35$

and not just $z = -1.65$. The test statistic $z \approx -2.35$ gives a value of 0.0094 in the z-table.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
-2.3	.0107	.0104	.0102	.0099	.0096	**.0094**	.0091	.0089	.0087	.0084
-2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110

Which means the conclusion isn't only significant at $\alpha = 0.05$, but it's actually significant at 0.0094. As long as $\alpha \geq 0.0094$, we'll be able to reject H_0.

Topic: Hypothesis testing for the population proportion

Question: The NBA (National Basketball Association) claims that no more than 25% of NBA players started playing basketball before age 5. We want to verify this claim, so we collect a random sample of 117 NBA players and find that 34 of them started playing before age 5. What can we conclude at a significance level of $\alpha = 0.01$?

Answer choices:

A We'll reject the null hypothesis; our result is significant at the $\alpha = 0.01$ level

B We'll reject the null hypothesis; our result is significant at the $\alpha = 0.05$ level

C We'll reject the null hypothesis; our result is significant at the $\alpha = 0.10$ level

D We'll fail to reject the null hypothesis

Solution: D

First, build the hypothesis statements.

H_0: At most 25% of NBA players started playing before 5, $p \leq 0.25$

H_a: More than 25% of NBA players started playing before 5, $p > 0.25$

The sample proportion is

$$\hat{p} = \frac{x}{n} = \frac{34}{117} \approx 0.2906$$

and the standard error of the proportion is

$$\sigma_{\hat{p}} = \sqrt{\frac{p_0(1-p_0)}{n}} = \sqrt{\frac{0.25(1-0.25)}{117}} \approx 0.0400$$

Now we have enough to find the z-test statistic.

$$z = \frac{\hat{p} - p}{\sigma_{\hat{p}}} = \frac{0.2906 - 0.25}{0.0400} \approx 1.0142$$

The critical z-value for 99% confidence with an upper-tailed test is $z = 2.33$.

Our z-value of $z = 1.015$ falls to the left of $z = 2.33$, and therefore falls in the region of acceptance, which means we'll fail to reject the null hypothesis and fail to conclude that the proportion of NBA players who started playing basketball before age 5 is more than 25%.