Topic: Discrete probability

Question: A red and blue die are rolled. Both are six-sided fair dice. Let X represent the sum of the dice. Which of the following is the correct probability distribution for X?

Answer choices:

A

X	1	2	3	4	5	6
P(X)	1/6	1/6	1/6	1/6	1/6	1/6

B

X	2	3	4	5	6	7	8	9	10	11	12
P(X)	1/12	1/12	1/12	1/12	1/12	1/12	1/12	1/12	1/12	1/12	1/12

C

X	2	3	4	5	6	7	8	9	10	11	12
P(X)	1/36	1/18	1/12	1/9	5/36	1/6	5/36	1/9	1/12	1/18	1/36

D

X	2	3	4	5	6	7	8	9	10	11	12
P(X)	1/36	1/36	1/36	1/36	1/36	1/36	1/36	1/36	1/36	1/36	1/36

Solution: C

Because the smallest value we can roll on each die is 1, the smallest sum we can get is $X = 1 + 1 = 2$. The largest value we can roll on each die is 6, so the largest sum we can get is $X = 6 + 6 = 12$.

Therefore, the sample space for X is $\{2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$. This table shows all possible ways to roll two dice and the sum of each roll.

		Red die					
		1	**2**	**3**	**4**	**5**	**6**
Blue die	**1**	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

There are 11 possible sums (from 2 to 12) and 36 different pairs (from $(1,1)$ all the way to $(6,6)$).

By looking at the table, we can start calculating probabilities for each sum.

$$P(\text{sum of } 2) = \frac{1}{36}$$

$$P(\text{sum of } 3) = \frac{2}{36} = \frac{1}{18}$$

...

Already, the only probability distribution that matches these calculations is the table from answer choice C.

X	2	3	4	5	6	7	8	9	10	11	12
P(X)	1/36	1/18	1/12	1/9	5/36	1/6	5/36	1/9	1/12	1/18	1/36

Topic: Discrete probability

Question: You purchase a raffle ticket for $\$125$. In exchange, you'll be allowed to participate in two drawings. In each drawing, you blindly pick one of three tokens. One token is worth $\$0$, one is worth $\$50$, and one is worth $\$100$. Let Y be the profit made by a raffle ticket. Find the expected value for Y after the two drawings.

Answer choices:

A $-\$50$

B $-\$25$

C $\$50$

D $\$75$

Solution: B

In the first drawing, you can pick $\$0$, $\$50$, or $\$100$. The same is true with the second drawing. So after two drawings, your possible earnings are given in the table.

		Second drawing		
		0	**50**	**100**
First drawing	**0**	0	50	100
	50	50	100	150
	100	100	150	200

The sample space is therefore $\$0$, $\$50$, $\$100$, $\$150$, or $\$200$. Because there are 9 possible combinations, from $(0,0)$ to $(100,100)$, the probability of winning each amount of money is

Y	0	50	100	150	200
P(Y)	1/9	2/9	3/9	2/9	1/9

But your ticket cost you $\$125$, which means we need to adjust the probability distribution by subtracting the cost from each potential profit.

Y	-125	-75	-25	25	75
P(Y)	1/9	2/9	3/9	2/9	1/9

Therefore, the expected value for Y is

$$E(Y) = -125\left(\frac{1}{9}\right) - 75\left(\frac{2}{9}\right) - 25\left(\frac{3}{9}\right) + 25\left(\frac{2}{9}\right) + 75\left(\frac{1}{9}\right)$$

$$E(Y) = -\frac{125}{9} - \frac{150}{9} - \frac{75}{9} + \frac{50}{9} + \frac{75}{9}$$

$$E(Y) = -\frac{225}{9}$$

$$E(Y) = -25$$

Topic: Discrete probability

Question: The following table shows the 2017 AP Statistics Exam score distribution for all students taking the test in the United States. Let Z represent the exam score. Find μ_Z and σ_Z.

Score	1	2	3	4	5
Probability	0.136	0.159	0.248	0.202	0.255

Answer choices:

A $\mu_Z = 3.281$ and $\sigma_Z = 1.846$

B $\mu_Z = 2.719$ and $\sigma_Z = 1.846$

C $\mu_Z = 3.281$ and $\sigma_Z = 1.359$

D $\mu_Z = 2.719$ and $\sigma_Z = 1.359$

Solution: C

Z is a discrete random variable with sample space $\{1, 2, 3, 4, 5\}$. The percentage of students taking the exam who received each of those scores is given in the table as

Score	1	2	3	4	5
Probability	0.136	0.159	0.248	0.202	0.255

We'll find the mean of this discrete random variable as

$$\mu_Z = 1(0.136) + 2(0.159) + 3(0.248) + 4(0.202) + 5(0.255)$$

$$\mu_Z = 0.136 + 0.318 + 0.744 + 0.808 + 1.275$$

$$\mu_Z = 3.281$$

We'll find the variance in order to get to standard deviation. The variance of Z is

$$\sigma_Z^2 = \sum_{i=1}^{5} (Z_i - \mu_Z)^2 P(Z_i)$$

$$\sigma_Z^2 = (1 - 3.281)^2(0.136) + (2 - 3.281)^2(0.159) + (3 - 3.281)^2(0.248)$$

$$+(4 - 3.281)^2(0.202) + (5 - 3.281)^2(0.255)$$

$$\sigma_Z^2 = 1.846$$

So the standard deviation of Z is

$$\sqrt{\sigma_Z^2} = \sqrt{1.846}$$

$\sigma_Z \approx 1.359$