Topic: Hypothesis testing for the difference of proportions

Question: Assuming $p_1 - p_2 = 0$, find the value of the z-test statistic, given $\hat{p}_1 = 0.295$ for $n_1 = 130$, and $\hat{p}_2 = 0.226$ for $n_2 = 110$.

Answer choices:

A $z \approx -1.21$

B $z \approx 1.18$

C $z \approx 1.21$

D $z \approx 1.27$

Solution: C

First, we'll calculate the proportion of the combined sample.

$$\hat{p} = \frac{\hat{p}_1 n_1 + \hat{p}_2 n_2}{n_1 + n_2}$$

$$\hat{p} = \frac{0.295(130) + 0.226(110)}{130 + 110}$$

$$\hat{p} = \frac{38.35 + 24.86}{240}$$

$$\hat{p} = \frac{63.21}{240}$$

$$\hat{p} \approx 0.263$$

Now we can find the value of the z-test statistic.

$$z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1 - \hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$$z = \frac{0.295 - 0.226}{\sqrt{0.263(1 - 0.263)\left(\frac{1}{130} + \frac{1}{110}\right)}}$$

$$z = \frac{0.069}{\sqrt{0.263(0.737)\left(\frac{11}{1{,}430} + \frac{13}{1{,}430}\right)}}$$

$$z = \frac{0.069}{\sqrt{0.193831\left(\frac{24}{1,430}\right)}}$$

$$z \approx \frac{0.069}{\sqrt{0.003}}$$

$$z \approx 1.21$$

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Question: A scientist wants to test how fast two flu drugs help patients recover from flu. He randomly assigns 100 patients each to two groups, and gives group 1 the first drug and group 2 the second drug. In the first group, 57 patients recovered from flu 5 days, while 49 patients in the second group recovered from flu in 5 days. Using a critical value approach at a 99% confidence level, can the scientist conclude that either drug is more effective than the other?

Answer choices:

A Yes, the first drug is more effective than the second drug

B Yes, the second drug is more effective than the first drug

C No, there's no significant difference in their effectiveness

D None of these

Solution: C

We want to determine if either of the flu drugs is significantly more effective than the other, which means we need to perform a two-tailed test, and our hypothesis statements are

$$H_0 : p_1 - p_2 = 0$$

$$H_a : p_1 - p_2 \neq 0$$

The proportion of the combined sample is

$$\hat{p} = \frac{x_1 + x_2}{n_1 + n_2}$$

$$\hat{p} = \frac{57 + 49}{100 + 100}$$

$$\hat{p} = \frac{106}{200}$$

$$\hat{p} = 0.53$$

The sample proportions are

$$\hat{p}_1 = \frac{57}{100} = 0.57$$

$$\hat{p}_2 = \frac{49}{100} = 0.49$$

Then the z-test statistic will be

$$z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$$z = \frac{0.57 - 0.49}{\sqrt{0.53(1-0.53)\left(\frac{1}{100} + \frac{1}{100}\right)}}$$

$$z = \frac{0.08}{\sqrt{0.53(0.47)\left(\frac{1}{50}\right)}}$$

$$z = \frac{0.08}{\sqrt{0.004982}}$$

$$z \approx 1.13$$

For two-tailed test with $\alpha = 0.01$, the critical value of z is 2.576. It means that we can reject the null hypothesis if z-statistic is larger than 2.58 or smaller than -2.58. Since $1.13 < 2.58$, the test statistic falls into the region of acceptance, so we fail to reject the null hypothesis, and we can't conclude that either of the drugs is more effective than the other.

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Question: 50 randomly chosen well-prepared students, and 50 randomly chosen poorly-prepared students, all took a math test. The response to a specific question was examined by the professor, who was interested whether the proportion of well-prepared students who answered the question correctly was at least 18% higher than the proportion of poorly-prepared students who answered the question correctly. The professor found that 39 of the well-prepared and 35 of the poorly-prepared students answered the question correctly. What can he conclude at a 95% confidence level?

Answer choices:

A He fails to reject the null hypothesis, so he concludes that the proportion of well-prepared students who answers the question correctly is at least 18% higher than the proportion of poorly-prepared students.

B He fails to reject the null hypothesis, so he can't conclude that the proportion of well-prepared students who answers the question correctly is at least 18% higher than the proportion of poorly-prepared students.

C He rejects the null hypothesis, so he concludes that the proportion of well-prepared students who answers the question correctly isn't 18% higher than the proportion of poorly-prepared students.

D He rejects the null hypothesis, so he can't conclude that the proportion of well-prepared students who answers the question correctly is 18% higher than the proportion of poorly-prepared students.

Solution: B

The professor is using an upper-tailed test because he's investigating whether the proportion of well-prepared students who answer the question correctly is at least 18% larger than the proportion of poorly-prepared students who answer correctly.

$$H_0: p_1 - p_2 \le 0.18$$

$$H_a: p_1 - p_2 > 0.18$$

The sample proportions are

$$\hat{p}_1 = \frac{39}{50} = 0.78$$

$$\hat{p}_2 = \frac{35}{50} = 0.70$$

so, with $p_1 - p_2 = 0.18$, the z-test statistic will be

$$z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}}$$

$$z = \frac{0.78 - 0.70 - 0.18}{\sqrt{\frac{0.78(1-0.78)}{50} + \frac{0.70(1-0.70)}{50}}}$$

$$z = \frac{-0.10}{\sqrt{\frac{0.78(0.22)}{50} + \frac{0.70(0.30)}{50}}}$$

$$z = \frac{-0.10}{\sqrt{\frac{0.3816}{50}}}$$

$$z = -0.10\sqrt{\frac{50}{0.3816}}$$

$$z \approx -1.14$$

For an upper-tailed test at a 95% confidence level, the critical z-value is 1.65. Since $-1.14 < 1.65$, the professor fails to reject the null hypothesis, and therefore can't conclude that there's an at least 18% difference between the well-prepared and poorly-prepared students.