Topic: Matched-pair hypothesis testing

Question: Which samples are used in a matched-pair test?

Answer choices:

A Independent samples

B Dependent samples

C Both independent and dependent samples

D Neither independent nor dependent samples

Solution: B

Matched-pair tests are conducted with dependent samples.

Dependent samples are related to one another in the sense that they contain the same subjects, and each subject produces a value for each sample.

One common type of matched-pair test is "change over time" or "before and after," where each subject in the test produces a "before" value for the first sample and an "after" value for the second sample.

Topic: Matched-pair hypothesis testing

Question: A test prep company believes their new SAT prep program will improve learners SAT scores by at least 150 points. How should they define their populations and write their hypothesis statements?

Answer choices:

A Population 1 will be the SAT scores of students after they complete the prep program, and Population 2 will be the SAT scores of students before they complete the prep program. Then the hypothesis statements are $H_0 : \mu_2 - \mu_1 > 150$ and $H_a : \mu_2 - \mu_1 \leq 150$.

B Population 1 will be the SAT scores of students before they complete the prep program, and Population 2 will be the SAT scores of students after they complete the prep program. Then the hypothesis statements are $H_0 : \mu_2 - \mu_1 > 150$ and $H_a : \mu_2 - \mu_1 \leq 150$.

C Population 1 will be the SAT scores of students after they complete the prep program, and Population 2 will be the SAT scores of students before they complete the prep program. Then the hypothesis statements are $H_0 : \mu_2 - \mu_1 \leq 150$ and $H_a : \mu_2 - \mu_1 > 150$.

D Population 1 will be the SAT scores of students before they complete the prep program, and Population 2 will be the SAT scores of students after they complete the prep program. Then the hypothesis statements are $H_0 : \mu_2 - \mu_1 \leq 150$ and $H_a : \mu_2 - \mu_1 > 150$.

Solution: D

The test prep company will define the SAT scores of students before they've completed the prep program as Population 1, and define the SAT scores of students after they've completed the prep program as Population 2.

Then their null and alternative hypothesis will be

$$H_0 : \mu_2 - \mu_1 \leq 150$$

$$H_a : \mu_2 - \mu_1 > 150$$

where μ_1 is the mean SAT score before the students complete the prep program, and μ_2 is the mean SAT score after the students complete the prep program.

Topic: Matched-pair hypothesis testing

Question: A pharmaceutical company believes their new weight-loss drug produces more weight loss than the current market-leader, which produces a mean monthly weight loss of 10 pounds. They record the before and after weights of 10 people who volunteer to try to new drug.

Participant	1	2	3	4	5	6	7	8	9	10
Before x_1	150	157	206	344	193	188	168	272	245	222
After x_2	142	145	198	330	191	185	150	252	233	206
Difference, d	8	12	8	14	2	3	18	20	12	16
d^2	64	144	64	196	4	9	324	400	144	256

Can the company conclude at 5% significance that their drug works better than the market-leading weight loss drug?

Answer choices:

A Yes, they can conclude that the new drug works better

B No, they can't conclude that the new drug works better

Solution: B

The pharmaceutical company will define the "before" responses as Population 1, and the "after" responses as Population 2. The samples are dependent because it's reasonable to see how a participant's "after" response could be affected by their "before" response.

Then their null and alternative hypotheses will be

$$H_0: \mu_1 - \mu_2 \leq 10$$

$$H_a: \mu_1 - \mu_2 > 10$$

where μ_1 is the mean starting weight before participants begin taking the new weight-loss drug, and μ_2 is the mean ending weight. And because $\mu_1 - \mu_2$ is the difference in weight, the hypothesis statements could also be written as

$$H_0: \mu_d \leq 10$$

$$H_a: \mu_d > 10$$

where μ_d is the mean difference between the two populations.

To find the mean difference, we'll sum the differences and divide by the number of matched-pairs in our sample, $n = 10$.

$$\bar{d} = \frac{\sum_{i=1}^{n} d_i}{n} = \frac{8 + 12 + 8 + 14 + 2 + 3 + 18 + 20 + 12 + 16}{10} = \frac{113}{10} = 11.3$$

So the sample mean tells us that mean weight loss is 11.3. Then the sample standard deviation is

$$s_d = \sqrt{\frac{\sum_{i=1}^{n}(d_i - \bar{d})^2}{n-1}}$$

To calculate this, we'll first find

$$\sum_{i=1}^{n}(d_i - \bar{d})^2$$

$$(8-11.3)^2 + (12-11.3)^2 + (8-11.3)^2 + (14-11.3)^2 + (2-11.3)^2$$

$$+(3-11.3)^2 + (18-11.3)^2 + (20-11.3)^2 + (12-11.3)^2 + (16-11.3)^2$$

$$(-3.3)^2 + 0.7^2 + (-3.3)^2 + 2.7^2 + (-9.3)^2 + (-8.3)^2 + 6.7^2 + 8.7^2 + 0.7^2 + 4.7^2$$

$$10.89 + 0.49 + 10.89 + 7.29 + 86.49 + 68.89 + 44.89 + 75.69 + 0.49 + 22.09$$

$$328.1$$

Then the sample standard deviation is

$$s_d = \sqrt{\frac{328.1}{9}}$$

$$s_d \approx \sqrt{36.456}$$

$$s_d \approx 6.038$$

Because the population standard deviations are unknown, and/or because both sample sizes are small, $n_1, n_2 < 30$, the test statistic will be

$$t = \frac{\bar{d} - \mu_d}{\frac{s_d}{\sqrt{n}}}$$

$$t \approx \frac{11.3 - 10}{\frac{6.038}{\sqrt{10}}}$$

$$t \approx 1.3 \cdot \frac{\sqrt{10}}{6.038}$$

$$t \approx 0.681$$

and the degrees of freedom are

$$\text{df} = n - 1 = 10 - 1 = 9$$

At a significance level of 5% for an upper-tail test, and df $= 9$, the t-table gives 1.833.

	Upper-tail probability p									
df	**0.25**	**0.20**	**0.15**	**0.10**	**0.05**	**0.025**	**0.01**	**0.005**	**0.001**	**0.0005**
8	0.706	0.889	1.108	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	0.703	0.883	1.100	1.383	**1.833**	2.262	2.821	3.250	4.297	4.781
10	0.700	0.879	1.093	1.372	1.812	2.228	2.764	3.169	4.144	4.587
	50%	**60%**	**70%**	**80%**	**90%**	**95%**	**98%**	**99%**	**99.8%**	**99.9%**
	Confidence level C									

The company's t-test statistic $t \approx 0.681$ doesn't meet the threshold $t = 1.833$, so the critical value approach tells them that they can't reject the null hypothesis, and therefore can't conclude that their new weight-loss drug produces more weight loss than the current market-leading drug.