Topic: Permutations and combinations

Question: Out of 30 students in a math class, how many study groups of 5 students can be formed from the class members?

Answer choices:

A 6 groups

B 150 groups

C $142{,}506$ groups

D $17{,}100{,}720$ groups

Solution: C

This is a combination question where $n=30$ and $k=5$. The order in which we choose the 5 study group members doesn't matter in this situation.

If Person A, B, C, D, and E end up in a group together, this is equivalent to Person E, D, C, B, and A ending up in a group together.

We'll use the combination formula.

$$_nC_k=\binom{n}{k}=\binom{30}{5}=\frac{n!}{k!(n-k)!}=\frac{30!}{5!25!}=142{,}506 \text{ groups}$$

Topic: Permutations and combinations

Question: Four children are sledding in a toboggan. How many ways can the children arrange themselves on the toboggan?

Answer choices:

A 4 ways

B 16 ways

C 24 ways

D 256 ways

Solution: C

This is a permutation question. We have 4 people we're arranging and we'll arrange those 4 people as many different ways as we can. Set $n = 4$ and $k = 4$ and use the permutations formula.

$$_nP_k = \frac{n!}{(n-k)!} = \frac{4!}{0!} = \frac{4!}{1} = 4! = (4)(3)(2)(1) = 24 \text{ ways}$$

Topic: Permutations and combinations

Question: Sawyer is taking a 5-question biology test, and the test only requires him to answer 3 out of the 5 questions. He gets to choose which 3 he answers. How many different ways could he choose exactly 3 of the 5 questions?

Answer choices:

A 10 ways

B 15 ways

C 60 ways

D 125 ways

Solution: A

To figure out how many different ways could Sawyer could answer exactly 3 of the 5 questions, we need the formula for combinations.

We have 5 questions and want to know how many ways we can pick 3 of the 5 questions. The order won't matter, which is why we need the combination, and not the permutation. For example, answering questions $\#1$, $\#2$, and $\#3$ is the same as answering questions $\#2$, $\#1$, and $\#3$.

Therefore, we find the combination ${}_5C_3$.

$${}_nC_k = \binom{n}{k} = \binom{5}{3} = \frac{n!}{k!(n-k)!} = \frac{5!}{3!2!} = 10 \text{ ways}$$

There are 10 different ways that Sawyer could answer exactly 3 of the 5 questions.