Probability & Statistics Final Exam Solutions

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MATH

Probability & Statistics Final Exam Answer Key

1. (5 pts)	■	B	C	D	E
2. (5 pts)	A	B	■	D	E
3. (5 pts)	A	B	C	D	■
4. (5 pts)	A	B	C	■	E
5. (5 pts)	A	■	C	D	E
6. (5 pts)	A	B	■	D	E
7. (5 pts)	A	B	C	■	E
8. (5 pts)	■	B	C	D	E

9. (15 pts) $x = 89$, Median 84.5, Range 32

10. (15 pts) Median 75.35, IQR 8.4, Range 46.2

11. (15 pts) $\mu_Y = 245$ and $\sigma_Y = 12$

12. (15 pts) $7.13\,\%$

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1. A. Find the mean of the original data set.

$$\mu = \frac{26 + 28 + 32 + 40 + 42 + 44 + 54}{7} = \frac{266}{7} = 38$$

Find the mean of the data set, including the outlier of 100.

$$\mu = \frac{26 + 28 + 32 + 40 + 42 + 44 + 54 + 100}{8} = \frac{366}{8} = 45.75$$

Find the difference between the means.

$$45.75 - 38 = 7.75$$

The mean increased by 7.75 when the outlier was added to the set.

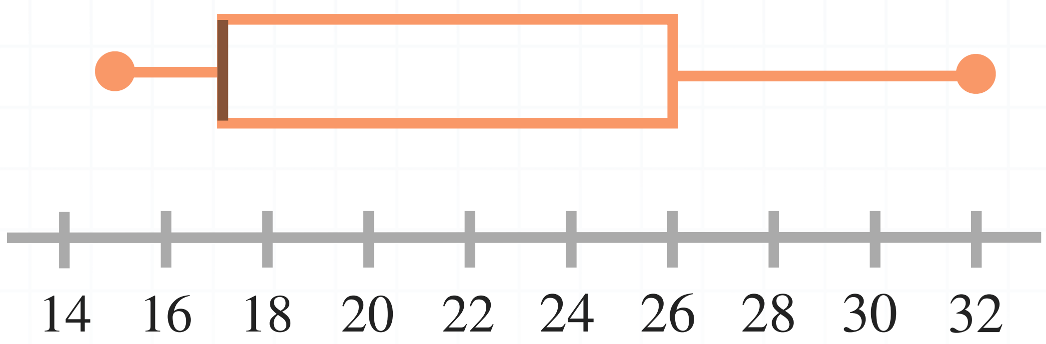
2. C. Arrange the data from smallest to largest.

$$15,\ 17,\ 17,\ 17,\ 26,\ 26,\ 32$$

The minimum is 15 and the maximum is 32. These values will be the ends of the whiskers. The data is already in ascending order, so we can see that the median (the middle number) is 17. This will be the line in the middle of the box.

The lower half of the data is 15, 17, 17, and the median of that lower half is $Q_1 = 17$. The upper half of the data is 26, 26, 32, and the median of that upper half is $Q_3 = 26$.

Answer choice D has the correct box-and-whisker plot with the minimum at 15, Q_1 at 17, the median at 17, Q_3 at 26, and the maximum at 32.



3. E. To find the percentage of architecture students that are Seniors, we'll look at the column for Architecture,

	Architecture
Freshman	25
Sophomore	12
Junior	45
Senior	60
Total	**142**

and turn it into a column-relative frequency.

	Architecture
Freshman	25/142=0.18
Sophomore	12/142=0.08
Junior	45/142=0.32
Senior	60/142=0.42
Total	**142/142=1.00**

We can see from the column-relative frequency that 42% of the architecture students are Seniors.

4. D. Use the addition rule for probability.

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A) = P(\text{face card}) = \frac{12}{52} = \frac{6}{26}$$

$$P(B) = P(\text{black}) = \frac{26}{52} = \frac{13}{26}$$

$$P(A \cap B) = P(\text{face card and black}) = \frac{6}{52} = \frac{3}{26}$$

Therefore, the probability that we get a jack or a red card is

$$P(A \cup B) = \frac{6}{26} + \frac{13}{26} - \frac{3}{26}$$

$$P(A \cup B) = \frac{16}{26}$$

$$P(A \cup B) = \frac{8}{13}$$

$$P(\text{face card or black}) = \frac{8}{13}$$

5. B. To find the median of movies watched by the 26 students, find the middle number of movies watched. This can be done by crossing off 13 dots from each side, working your way to the middle of the graph.

The median number of movies watched by the students is 4.

6. C. Let X be the number of 8th grade students that take the eye exam out of 90 trials.

X follows a binomial distribution with a trial representing choosing a random student from our nation and recording whether or not they're taking an eye exam. These trials will be independent and the probability of success remains constant at $p = 0.32$. There are a fixed number of trials, $n = 90$.

$$X \sim B(90,0.32)$$

The goal is to find the probability of exactly $k = 40$ successes.

$$P(k \text{ successes in } n \text{ trials}) = \binom{n}{k}(p)^k(1-p)^{n-k}$$

Start by finding the number of ways to have exactly 40 successes in 90 trials using the combination formula.

$$\binom{90}{40} = {}_{90}C_{40} = \frac{90!}{40!(90-40)!}$$

Now we can find the probability.

$$P(x = 40) = \binom{90}{40}(0.32)^{40}(1-0.32)^{50} = 0.0041$$

7. D. There are 5 red marbles out of 12 marbles total.

$$P(\text{red}) = \frac{5}{12} \approx 0.4167 \approx 41.7\,\%$$

8. A. The graph is of a right-skewed distribution, which has a tail on the right. The mean is further to the right than the median. Therefore, the first statement is true.

9. The mean of the data set is $\mu = 82$, and the data set contains the six values $61, 78, 80, x, 91, 93$.

To find the mean, add all of the numbers in the data set together and divide by how many data points are in the set. This time we know the mean is 78, so we can set up an equation.

$$\mu = \frac{\sum_{i=1}^{n} x_i}{n}$$

$$78 = \frac{61 + 78 + 80 + x + 91 + 93}{6}$$

When we solve for x, we get

$$82(6) = 61 + 78 + 80 + x + 91 + 93$$

$$492 = x + 403$$

$$492 - 403 = x + 403 - 403$$

$$x = 89$$

The full data set is $61, 78, 80, 89, 91, 93$. The median is the average of the middle two numbers, 80 and 89.

$$\frac{80 + 89}{2} = \frac{169}{2} = 84.5$$

The range is the difference between the largest and smallest values.

$$93 - 61 = 32$$

10. Each student's old score was substituted into the formula to give a new score for that student. For example, if a student had an original score of 70, his new score would be

$$\text{New score} = 1.05(\text{Old score}) + 5$$

$$\text{New score} = 1.05(70) + 5$$

$$\text{New score} = 73.5 + 5$$

$$\text{New score} = 78.5$$

All original scores would be transformed into new, curved scores in this same way. We’re only given the median, IQR, and range for the original test scores.

The median measures the center for the test scores and the IQR and range both measure the spread in the scores. The median will be transformed in the same way as each individual score.

$$\text{New median} = 1.05(\text{Old median}) + 4$$

$$\text{New median} = 1.05(67) + 5$$

$$\text{New median} = 70.35 + 5$$

$$\text{New median} = 75.35$$

The measures of spread are transformed using only the scale factor of 1.05, but are not affected by adding 5 to each value. Remember that adding a constant k will move every value up by k units, but won't make the data any more or less spread out. Therefore, we'll convert the IQR as

$$\text{New IQR} = 1.05(\text{Old IQR})$$

$$\text{New IQR} = 1.05(8)$$

$$\text{New IQR} = 8.4$$

and the range as

$$\text{New range} = 1.05(\text{Old range})$$

$$\text{New range} = 1.05(44)$$

$$\text{New range} = 46.2$$

11. Since $Y = 5 + 3X$, we'll scale each value in our data set by the constant 3 and then add a constant of 5. The scaling by 3 will affect the mean and standard deviation, but the shifting by 5 will only affect the mean. The mean of Y will therefore be

$$\mu_Y = 5 + 3(\mu_X)$$

$$\mu_Y = 5 + 3(80)$$

$$\mu_Y = 245$$

and the standard deviation of Y will be

$$\sigma_Y = 3(\sigma_X)$$

$$\sigma_Y = 3(4)$$

$$\sigma_Y = 12$$

12. We know this is a Poisson experiment with

$\lambda = 14$, the average number of canvases painted in a month

$x = 17$, the number of canvases required to be painted next month

The Poisson probability is

$$P(x) = \frac{\lambda^x e^{-\lambda}}{x!}$$

$$P(17) = \frac{14^{17} e^{-14}}{17!}$$

$$P(17) \approx 0.0713$$

So the probability the artist will paint 17 canvases next month is approximately 0.0713, or 7.13%.