

Topic: Chebyshev's theorem

Question: The shape of a probability distribution is unknown. At least what percentage of the data falls within 2.5 standard deviations of the mean?

Answer choices:

- A 78 %
- B 80 %
- C 84 %
- D 86 %



Solution: C

Using Chebyshev's theorem with $k = 2.5$, the percentage of data that falls within 2.5 standard deviations of the mean is

$$1 - \frac{1}{k^2}$$

$$1 - \frac{1}{2.5^2}$$

$$1 - \frac{1}{6.25}$$

$$1 - 0.16$$

$$0.84$$

$$84\%$$



Topic: Chebyshev's theorem

Question: Find the interval, in terms of standard deviations, that contains at least 90 % of the data in a probability distribution, regardless of the shape of the distribution.

Answer choices:

- A ± 3.02 standard deviations
- B ± 3.04 standard deviations
- C ± 3.10 standard deviations
- D ± 3.16 standard deviations



Solution: D

To find the number of standard deviations that contain a specific percentage of the data in a probability distribution, we can set Chebyshev's expression equal to the percentage we're interested in. Since we're looking for the interval for 90 % of the data, we'll set Chebyshev's expression equal to 0.9.

$$0.9 = 1 - \frac{1}{k^2}$$

$$\frac{1}{k^2} = 1 - 0.9$$

$$1 = (1 - 0.9)k^2$$

$$k = \pm \sqrt{\frac{1}{1 - 0.9}}$$

Now that we've solved for k , we can simplify.

$$k = \pm \sqrt{\frac{1}{0.1}}$$

$$k = \pm \sqrt{10}$$

$$k \approx \pm 3.16$$



Topic: Chebyshev's theorem

Question: A particular school of fish has a mean body length of 8 inches, with a standard deviation of 0.75 inches. What's the minimum body length of a fish in the middle 85 % of the school?

Answer choices:

- A 6.07 inches
- B 6.10 inches
- C 6.12 inches
- D 6.15 inches



Solution: A

Using Chebyshev's theorem,

$$0.85 = 1 - \frac{1}{k^2}$$

$$\frac{1}{k^2} = 1 - 0.85$$

$$1 = 0.15k^2$$

$$k^2 = \frac{1}{0.15}$$

$$k \approx 2.58$$

Approximately 2.58 standard deviations above the mean gives us a body length of

$$8 + 2.58(0.75)$$

$$9.94$$

And 2.58 standard deviations below the mean gives us a body length of

$$8 - 2.58(0.75)$$

$$6.07$$

So at least 85 % of the fish fell between 6.07 and 9.94 inches.

