

Topic: Confidence interval for the proportion

Question: A study shows that 78 % of patients who try a new medication for migraines feel better within 30 minutes of taking the medicine. If the study involved 120 patients, construct a 95 % confidence interval for the proportion of patients who feel better within 30 minutes of taking the medicine.

Answer choices:

- A $(a, b) \approx (0.73, 0.83)$
- B $(a, b) \approx (0.72, 0.84)$
- C $(a, b) \approx (0.71, 0.85)$
- D $(a, b) \approx (0.68, 0.88)$



Solution: C

We know that the sample proportion is $\hat{p} = 0.78$, and that the confidence level is 95 %. The critical value for this confidence level is $z^* = 1.96$. The sample size is $n = 120$. So we can plug these values into the confidence interval formula.

$$(a, b) = \hat{p} \pm z^* \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}}$$

$$(a, b) = 0.78 \pm 1.96 \sqrt{\frac{0.78(1 - 0.78)}{120}}$$

$$(a, b) \approx 0.78 \pm 0.0741$$

$$(a, b) \approx (0.78 - 0.0741, 0.78 + 0.0741)$$

$$(a, b) \approx (0.71, 0.85)$$

This means that we're 95 % confident that the proportion of patients who feel better within 30 minutes of taking the medicine is between 71 % and 85 %.



Topic: Confidence interval for the proportion

Question: A study shows that 243 of 500 randomly selected households were using a family member to care for their young children. Build a 90 % confidence interval for the proportion of households using a family member to care for young children.

Answer choices:

- A $(a, b) \approx (0.45, 0.52)$
- B $(a, b) \approx (0.44, 0.53)$
- C $(a, b) \approx (0.43, 0.54)$
- D $(a, b) \approx (0.42, 0.55)$



Solution: A

The population proportion is

$$\hat{p} = \frac{243}{500} = 0.486$$

With $\hat{p} = 0.486$, a confidence level of 90 % and therefore a critical value of $z^* = 1.65$, and a sample size of $n = 500$, the confidence interval will be

$$(a, b) = \hat{p} \pm z^* \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}}$$

$$(a, b) = 0.486 \pm 1.65 \sqrt{\frac{0.486(1 - 0.486)}{500}}$$

$$(a, b) \approx 0.486 \pm 0.0369$$

$$(a, b) \approx (0.486 - 0.0369, 0.486 + 0.0369)$$

$$(a, b) \approx (0.45, 0.52)$$

This means that we're 90 % confident that the proportion of households using a family member to care for young children is between 45 % and 52 %.



Topic: Confidence interval for the proportion

Question: Bentley is a dog who helps police find drugs, but he has a low success rate. In Bentley's time on the job, it's estimated that he correctly identified drugs 59 % of the time. How many different trials would the police need to put him through to verify that this is his actual success rate, at a 95 % confidence level and with a margin of error of 0.05?

Answer choices:

A $n = 264$

B $n = 372$

C $n = 413$

D $n = 645$



Solution: B

The minimum sample size n that gives a fixed margin of error is given by

$$n = \left(\frac{z^* \sqrt{p(1-p)}}{ME} \right)^2$$

Bentley's success rate is $p = 59\% = 0.59$, the confidence level is 95% , and the critical value for this confidence level is $z^* = 1.96$. The margin of error is $ME = 0.05$. If we plug these values into the formula, we can find the minimum number of trials we need to verify Bentley's success rate.

$$n = \left(\frac{1.96 \sqrt{0.59(1-0.59)}}{0.05} \right)^2$$

$$n = \left(\frac{1.96 \sqrt{0.59(0.41)}}{0.05} \right)^2$$

$$n = \left(\frac{1.96 \sqrt{0.2419}}{0.05} \right)^2$$

$$n \approx 371.71$$

Since we need more than 371 trials to test Bentley, we have to round up to 372 trials, so we can say $n = 372$.

