Topic: Confidence interval for the difference of proportions

Question: A video game developer wants to know how the number of male video game players compares to the number of female players. They randomly select 500 males and 500 females and find that 368 of the males play video games, while 230 of the females play video games. Find a 95% confidence interval around the difference between the number of male and female players.

Answer choices:

A $(0.227, 0.325)$

B $(0.218, 0.334)$

C $(0.160, 0.392)$

D $(0.460, 0.736)$

Solution: B

The sample proportions for the males and females, respectively, are

$$\hat{p}_1 = \frac{368}{500} = 0.736$$

$$\hat{p}_2 = \frac{230}{500} = 0.460$$

The value of $z_{\alpha/2}$ for 95% confidence in a two-tailed test is 1.96, so the confidence interval is

$$(a, b) = (\hat{p}_1 - \hat{p}_2) \pm z_{\alpha/2}\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$$

$$(a, b) = (0.736 - 0.460) \pm 1.96\sqrt{\frac{0.736(1-0.736)}{500} + \frac{0.460(1-0.460)}{500}}$$

$$(a, b) = 0.276 \pm 1.96\sqrt{\frac{0.736(0.264)}{500} + \frac{0.460(0.540)}{500}}$$

$$(a, b) = 0.276 \pm 1.96\sqrt{\frac{0.194304}{500} + \frac{0.2484}{500}}$$

$$(a, b) = 0.276 \pm 1.96\sqrt{\frac{0.442704}{500}}$$

$$(a, b) \approx 0.276 \pm 0.058$$

Therefore, the 95% confidence interval is

$$(a, b) \approx (0.276 - 0.058, 0.276 + 0.058)$$

$$(a,b) \approx (0.218, 0.334)$$

We can be 95% confident that the true difference of population proportions of males who play video games and females who play video games is between 0.218 and 0.334. Notice that both ends of the confidence interval are positive, so we can conclude that more males than females play video games.

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Question: A college director wonders about the difference between the number of male and female students who scored higher than 90 on a recent final exam. He randomly selects 25 males and 25 females and finds that 15 males and 12 females scored more than 90. Find a $99\,\%$ confidence interval around the true difference of male students and female students who scored higher than 90 on the recent exam.

Answer choices:

A $(-0.481,0.241)$

B $(-0.152,0.391)$

C $(-0.241,0.481)$

D $(0.480,0.600)$

Solution: C

The sample proportions for males and females, respectively, are

$$\hat{p}_1 = \frac{15}{25} = 0.60$$

$$\hat{p}_2 = \frac{12}{25} = 0.48$$

The value of $z_{\alpha/2}$ for $99\,\%$ confidence in a two-tailed test is 2.58, so the confidence interval is

$$(a, b) = (\hat{p}_1 - \hat{p}_2) \pm z_{\alpha/2}\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$$

$$(a, b) = (0.60 - 0.48) \pm 2.58\sqrt{\frac{0.6(1-0.6)}{25} + \frac{0.48(1-0.48)}{25}}$$

$$(a, b) = 0.12 \pm 2.58\sqrt{\frac{0.6(0.4)}{25} + \frac{0.48(0.52)}{25}}$$

$$(a, b) = 0.12 \pm 2.58\sqrt{\frac{0.24}{25} + \frac{0.2496}{25}}$$

$$(a, b) = 0.12 \pm 2.58\sqrt{\frac{0.4896}{25}}$$

$$(a, b) \approx 0.12 \pm 0.361$$

Therefore, the $99\,\%$ confidence interval is

$$(a, b) \approx (0.12 - 0.361, 0.12 + 0.361)$$

$$(a, b) \approx (-0.241, 0.481)$$

We can be 99% confident that the true difference of population proportions of males and females who scored higher than 90 on the recent exam is between -0.241 and 0.481. But because the confidence interval includes 0, we can't conclude that there's a significant difference between the number of male and female students who scored higher than 90.

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Question: Directors at colleges A and B are interested whether there's a difference in the number of students who work while attending their colleges. They take a random sample of 100 students from each college and find that 37 students at college A and 40 students at college B are currently working. Find a 90% confidence interval around the difference of population proportions.

Answer choices:

A $(-0.14,0.08)$

B $(-0.165,0.105)$

C $(-0.1,0.04)$

D $(0.37,0.40)$

Solution: A

The sample proportions for colleges A and B, respectively, are

$$\hat{p}_1 = \frac{37}{100} = 0.37$$

$$\hat{p}_2 = \frac{40}{100} = 0.40$$

The value of $z_{\alpha/2}$ for 90% confidence in a two-tailed test is 1.65, so the confidence interval is

$$(a, b) = (\hat{p}_1 - \hat{p}_2) \pm z_{\alpha/2}\sqrt{\frac{\hat{p}_1(1 - \hat{p}_1)}{n_1} + \frac{\hat{p}_2(1 - \hat{p}_2)}{n_2}}$$

$$(a, b) = (0.37 - 0.40) \pm 1.65\sqrt{\frac{0.37(1 - 0.37)}{100} + \frac{0.40(1 - 0.40)}{100}}$$

$$(a, b) = -0.03 \pm 1.65\sqrt{\frac{0.37(0.63)}{100} + \frac{0.40(0.60)}{100}}$$

$$(a, b) = -0.03 \pm 1.65\sqrt{\frac{0.2331}{100} + \frac{0.24}{100}}$$

$$(a, b) = -0.03 \pm 1.65\sqrt{\frac{0.4731}{100}}$$

$$(a, b) \approx -0.03 \pm 0.113$$

Therefore, the 90% confidence interval is

$$(a, b) \approx (-0.03 - 0.113, -0.03 + 0.113)$$

$$(a, b) \approx (-0.143, 0.083)$$

We can be 90% confident that the true difference of population proportions of the number of working students between the two colleges is between -0.143 and 0.083. But because the confidence interval includes 0, we can't conclude that there's a significant difference between the number of working students at each college.