Permutations and combinations

In order to answer many probability questions, we need to understand permutations and combinations. A **permutation** is the number of ways we can arrange a set of things, and the order matters.

The formula for a permutation is

$$ {}_nP_k = \frac{n!}{(n-k)!} $$

where n is the total number of items we have, and k is the number of items we want to arrange.

Example

I have 4 scoops of ice cream: 1 chocolate, 1 strawberry, 1 vanilla, and 1 mint. I want to eat only 3 of the scoops. How many different ways can I eat 3 of the scoops if I consider both which scoops I eat and the order in which I eat them?

This is a permutation question, since I care about the order in which I eat the scoops. There are 4 total scoops, but I only want to eat 3 of them. Therefore, the number of ways I could eat three of the scoops of ice cream is

$$ {}_nP_k = \frac{n!}{(n-k)!} $$

$$ {}_4P_3 = \frac{4!}{(4-3)!} $$

$$ {}_4P_3 = \frac{4!}{1!} $$

$$ {}_4P_3 = \frac{4 \cdot 3 \cdot 2 \cdot 1}{1} $$

$$ {}_4P_3 = 4 \cdot 3 \cdot 2 $$

$$ {}_4P_3 = 24 $$

There are 24 different ways that I could eat 3 of the 4 scoops. For example, chocolate-strawberry-vanilla would be 1 of the 24 options, but since order matters, chocolate-vanilla-strawberry would be another option.

On the other hand, a **combination** is the number of ways we can arrange a set of things, but the order doesn't matter. The formula for a combination is

$$ {}_nC_k = \frac{n!}{k!(n-k)!} $$

where n is the total number of items we have, and k is the number of items we want to choose. Sometimes people write ${}_nC_k$ as

$$ \binom{n}{k} $$

which is called the binomial coefficient, and read as "n choose k."

So to continue with the example from earlier, chocolate-strawberry-vanilla and chocolate-vanilla-strawberry would not count separately, because we don't care about the order when we're talking about combinations. All we care about is which items we picked, so chocolate-strawberry-vanilla and chocolate-vanilla-strawberry would count as the same thing.

Example

I have the same 4 scoops of ice cream: 1 chocolate, 1 strawberry, 1 vanilla, and 1 mint. I want to eat only 3 of the scoops, and I don't care about the order in which I eat my 3 scoops. How many different combinations of 3 scoops can I create?

This is a combination question, since I don't care about the order in which I eat the scoops. There are 4 total scoops, but I only want to eat 3 of them. Therefore, the number of ways I could eat three of the scoops of ice cream is

$$ {}_nC_k = \frac{n!}{k!(n-k)!} $$

$$ {}_4C_3 = \frac{4!}{3!(4-3)!} $$

$$ {}_4C_3 = \frac{4!}{(3!)(1!)} $$

$$ {}_4C_3 = \frac{4 \cdot 3 \cdot 2 \cdot 1}{(3 \cdot 2 \cdot 1)(1)} $$

$$_4C_3 = \frac{4}{1}$$

$$_4C_3 = 4$$

There are 4 different ways that I could eat 3 of the 4 scoops. For example, chocolate-strawberry-vanilla would be 1 of the 4 options, but since order doesn't matter, chocolate-vanilla-strawberry would be the same combination, and wouldn't count as another one of the 4 combinations.