Topic: Sampling distribution of the sample mean

Question: If the original population is normally distributed, then the sampling distribution of the sample mean...

Answer choices:

A will also be normally distributed, if the sample size is large enough.

B will also be normally distributed, regardless of the sample size.

C will not be normally distributed.

D will have an unknown shape.

Solution: B

The Central Limit Theorem tells us that, if the original population is normally distributed, then the SDSM will also be normally distributed, regardless of the sample size n that we use.

If the original population is not normally distributed, or if we don't know the shape of the population distribution, then the SDSM is only guaranteed to be normally distributed when we use a sample size of at least $n = 30$.

Topic: Sampling distribution of the sample mean

Question: A hospital finds that the average birth weight of a newborn is 7.5 lbs with a standard deviation of 0.4 lbs. What is the standard deviation of the sampling distribution, if the hospital randomly selects 45 newborns to test this claim?

Answer choices:

A $\sigma_{\bar{x}} = 0.0596$

B $\sigma_{\bar{x}} = 1.118$

C $\sigma_{\bar{x}} = 0.0533$

D $\sigma_{\bar{x}} = 0.0089$

Solution: A

To find the standard deviation of the sampling distribution, we'll plug population standard deviation $\sigma = 0.4$ and the sample size $n = 45$ into the formula for the standard error.

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

$$\sigma_{\bar{x}} = \frac{0.4}{\sqrt{45}}$$

$$\sigma_{\bar{x}} \approx 0.0596$$

Topic: Sampling distribution of the sample mean

Question: A group of marathon runners have the following finishing times in hours: 3.2, 3.5, 3.8, 4.2, 4.5. Given a sample of size 2 if we're sampling with replacement, find the standard error $\sigma_{\bar{x}}$.

Answer choices:

A $\sigma_{\bar{x}} \approx 0.2104$

B $\sigma_{\bar{x}} \approx 0.2504$

C $\sigma_{\bar{x}} \approx 0.2904$

D $\sigma_{\bar{x}} \approx 0.3304$

Solution: D

Let's first determine the total number of possible samples, using N^n, given $N = 5$ and $n = 2$.

$$N^n = 5^2 = 25$$

The complete sample space, and the mean for each sample, is

Sample (Sample mean)				
3.2, 3.2 (3.20)	3.5, 3.2 (3.35)	3.8, 3.2 (3.50)	4.2, 3.2 (3.70)	4.5, 3.2 (3.85)
3.2, 3.5 (3.35)	3.5, 3.5 (3.50)	3.8, 3.5 (3.65)	4.2, 3.5 (3.85)	4.5, 3.5 (4.00)
3.2, 3.8 (3.50)	3.5, 3.8 (3.65)	3.8, 3.8 (3.80)	4.2, 3.8 (4.00)	4.5, 3.8 (4.15)
3.2, 4.2 (3.70)	3.5, 4.2 (3.85)	3.8, 4.2 (4.00)	4.2, 4.2 (4.20)	4.5, 4.2 (4.35)
3.2, 4.5 (3.85)	3.5, 4.5 (4.00)	3.8, 4.5 (4.15)	4.2, 4.5 (4.35)	4.5, 4.5 (4.50)

Build a table for the probability distribution of the sample mean. Because there are 25 total samples, the probability of each sample mean will be given by the number of times that sample mean occurs, divided by the total number of possible samples, so "count/25."

Sample mean	$P(x_i)$
3.20	1/25
3.35	2/25
3.50	3/25
3.65	2/25
3.70	2/25
3.80	1/25
3.85	4/25
4.00	4/25
4.15	2/25
4.20	1/25
4.35	2/25
4.50	1/25

Now we can calculate the mean of the sampling distribution of the sample mean, $\mu_{\bar{x}}$, where $\bar{x}_i$ is a given sample mean, $P(\bar{x}_i)$ is the probability of that particular sample mean occurring, and N is the number of samples.

$$\mu_{\bar{x}} = \sum_{i=1}^{N} \bar{x}_i P(\bar{x}_i)$$

$$\mu_{\bar{x}} = 3.20\left(\frac{1}{25}\right) + 3.35\left(\frac{2}{25}\right) + 3.50\left(\frac{3}{25}\right) + 3.65\left(\frac{2}{25}\right) + 3.70\left(\frac{2}{25}\right)$$

$$+3.80\left(\frac{1}{25}\right) + 3.85\left(\frac{4}{25}\right) + 4.00\left(\frac{4}{25}\right) + 4.15\left(\frac{2}{25}\right)$$

$$+4.20\left(\frac{1}{25}\right)+4.35\left(\frac{2}{25}\right)+4.50\left(\frac{1}{25}\right)$$

$$\mu_{\bar{x}}=\frac{3.20}{25}+\frac{6.70}{25}+\frac{10.50}{25}+\frac{7.30}{25}+\frac{7.40}{25}+\frac{3.80}{25}+\frac{15.40}{25}$$

$$+\frac{16.00}{25}+\frac{8.30}{25}+\frac{4.20}{25}+\frac{8.70}{25}+\frac{4.50}{25}$$

$$\mu_{\bar{x}}=\frac{96}{25}$$

$$\mu_{\bar{x}}=3.84$$

Because we're sampling with replacement, we would expect this mean of the SDSM to be equivalent to the mean of the population, $\mu_{\bar{x}}=\mu$, and we can see that it is if we calculate the mean of the population.

$$\mu=\frac{3.2+3.5+3.8+4.2+4.5}{5}=\frac{19.2}{5}=3.84$$

Both means are $\mu_{\bar{x}}=\mu=3.84$. The population variance is

$$\sigma^2=\frac{\sum_{i=1}^{N}(x_i-\mu)^2}{N}$$

$$\sigma^2=\frac{(3.2-3.84)^2+(3.5-3.84)^2+(3.8-3.84)^2+(4.2-3.84)^2+(4.5-3.84)^2}{5}$$

$$\sigma^2=\frac{(-0.64)^2+(-0.34)^2+(-0.04)^2+0.36^2+0.66^2}{5}$$

$$\sigma^2=\frac{0.4096+0.1156+0.0016+0.1296+0.4356}{5}$$

$$\sigma^2 = \frac{1.092}{5}$$

$$\sigma^2 = 0.2184$$

which means that the population standard deviation is

$$\sigma = \sqrt{0.2184}$$

$$\sigma \approx 0.4673$$

Because we're sampling with replacement, we can use the simplified formula for standard error (the one *without* the FPC).

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

$$\sigma_{\bar{x}} = \frac{0.4673}{\sqrt{2}}$$

$$\sigma_{\bar{x}} \approx 0.3304$$