Topic: Sampling distribution of the sample proportion

Question: In the case of a population proportion, the original population will be modeled by...

Answer choices:

A a binomial distribution.

B a normal distribution.

C a skewed distribution.

D a uniform distribution.

Solution: A

For any proportion, a response is always classified as either a “success” or a “failure.” Because exactly two outcomes are possible, the probability distribution for a proportion is always binomial.

Topic: Sampling distribution of the sample proportion

Question: A population proportion is $p = 0.7$. Find the standard error of the proportion for samples of size $n = 100$.

Answer choices:

A $\sigma_{\hat{p}} \approx 0.0337$

B $\sigma_{\hat{p}} \approx 0.0458$

C $\sigma_{\hat{p}} \approx 0.0548$

D $\sigma_{\hat{p}} \approx 0.0837$

Solution: B

The standard deviation of the sampling distribution of the sample proportion $\sigma_{\hat{p}}$, also called the standard error of the proportion, is given by

$$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$$

$$\sigma_{\hat{p}} = \sqrt{\frac{0.7(1-0.7)}{100}}$$

$$\sigma_{\hat{p}} = \sqrt{\frac{0.7(0.3)}{100}}$$

$$\sigma_{\hat{p}} = \sqrt{\frac{0.21}{100}}$$

$$\sigma_{\hat{p}} \approx \frac{0.4583}{10}$$

$$\sigma_{\hat{p}} \approx 0.0458$$

Topic: Sampling distribution of the sample proportion

Question: A group of 3 siblings have the following eye color: blue, blue, green. Find the mean $\mu_{\hat{p}}$ and standard error $\sigma_{\hat{p}}$ of the sampling distribution of the sample proportion for the proportion of siblings with blue eyes, if we take 2-sibling samples, with replacement.

Answer choices:

A $\mu_{\hat{p}} = 1/3$ and $\sigma_{\hat{p}} = 1/3$

B $\mu_{\hat{p}} = 1/3$ and $\sigma_{\hat{p}} = 2/3$

C $\mu_{\hat{p}} = 2/3$ and $\sigma_{\hat{p}} = 1/3$

D $\mu_{\hat{p}} = 2/3$ and $\sigma_{\hat{p}} = 2/3$

Solution: C

Determine the total number of possible samples, using N^n, given $N = 3$ and $n = 2$.

$$N^n = 3^2 = 9$$

The complete sample space, and the proportion for each sample, is

Sample	Sample proportion
blue, blue	1
blue, blue	1
blue, green	1/2
blue, blue	1
blue, blue	1
blue, green	1/2
green, blue	1/2
green, blue	1/2
green, green	0

Build a table for the probability distribution of the sample proportion. Because there are 9 total samples, the probability of each sample proportion will be given by the number of times that sample proportion occurs, divided by the total number of possible samples, so "count/9."

Sample proportion	P(pᵢ)
0	1/9
1/2	4/9
1	4/9

Now we can calculate the mean of the sampling distribution of the sample proportion, $\mu_{\hat{p}}$, where $\hat{p}_i$ is a given sample proportion, $P(\hat{p}_i)$ is the probability of that particular sample proportion occurring, and N is the number of samples.

$$\mu_{\hat{p}} = \sum_{i=1}^{N} \hat{p}_i P(\hat{p}_i)$$

$$\mu_{\hat{p}} = 0\left(\frac{1}{9}\right) + \frac{1}{2}\left(\frac{4}{9}\right) + 1\left(\frac{4}{9}\right)$$

$$\mu_{\hat{p}} = \frac{2}{9} + \frac{4}{9}$$

$$\mu_{\hat{p}} = \frac{6}{9}$$

$$\mu_{\hat{p}} = \frac{2}{3}$$

Because we're sampling with replacement, we would expect this mean of the SDSP to be equivalent to the population proportion, $\mu_{\hat{p}} = p$, and we can see that it is if we calculate the population proportion.

$$p = \frac{2 \text{ people with brown hair}}{3 \text{ people in the population}} = \frac{2}{3}$$

Both proportions are $\mu_{\hat{p}} = p = 2/3$. The variance of the SDSP would be

$$\sigma_{\hat{p}}^2 = \sum_{i=1}^{N} (\hat{p}_i - p)^2 P(\hat{p}_i)$$

$$\sigma_{\hat{p}}^2 = \left(0 - \frac{2}{3}\right)^2 \left(\frac{1}{9}\right) + \left(\frac{1}{2} - \frac{2}{3}\right)^2 \left(\frac{4}{9}\right) + \left(1 - \frac{2}{3}\right)^2 \left(\frac{4}{9}\right)$$

$$\sigma_{\hat{p}}^2 = \left(-\frac{2}{3}\right)^2 \left(\frac{1}{9}\right) + \left(-\frac{1}{6}\right)^2 \left(\frac{4}{9}\right) + \left(\frac{1}{3}\right)^2 \left(\frac{4}{9}\right)$$

$$\sigma_{\hat{p}}^2 = \left(\frac{4}{9}\right) \left(\frac{1}{9}\right) + \left(\frac{1}{36}\right) \left(\frac{4}{9}\right) + \left(\frac{1}{9}\right) \left(\frac{4}{9}\right)$$

$$\sigma_{\hat{p}}^2 = \frac{4}{81} + \frac{1}{81} + \frac{4}{81}$$

$$\sigma_{\hat{p}}^2 = \frac{9}{81}$$

$$\sigma_{\hat{p}}^2 = \frac{1}{9}$$

and then the standard error would be

$$\sigma_{\hat{p}} = \sqrt{\frac{1}{9}}$$

$$\sigma_{\hat{p}} = \frac{1}{3}$$