Probability & Statistics Workbook

Discrete random variables

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MATH

DISCRETE PROBABILITY

1. Let X be a discrete random variable with the following probability distribution. Find $P(X \geq 3)$.

X	1	2	3	4	5
P(X)	0.35	0.25	0.20	0.15	?

2. Let B be a discrete random variable with the following probability distribution. Find μ_B and σ_B.

B	0	5	10	15
P(B)	1/5	1/5	2/5	1/5

3. The table shows the distribution of size of households in the U.S. for 2016. Suppose we select a household of size at least 2 at random. What is the probability that this household has a size of at least 4?

Size of household	1	2	3	4	5	6	7+
P(size)	0.281	0.340	?	0.129	0.060	0.023	0.013

■ 4. A standard deck of cards is shuffled, and two cards are selected without replacement. Let R be the number of red cards selected. Construct a probability distribution for R.

■ 5. A local restaurant features a wheel we can spin before paying the bill. The wheel is split into 8 equal size pieces. One of the sections gives us a $\$10$ discount on the bill, two sections give a $\$5$ discount, three sections give a $\$2$ discount, and the rest of the sections give no discount. Find the expected value for the discount given by the wheel.

■ 6. John stops at the local gas station and decides to buy lottery tickets. Each ticket has a $20\,\%$ chance of being a winner. He will buy a lottery ticket and check to see if it's a winner. If it's a winner, he'll collect his money and be done. If it's not a winner, he'll buy another. He'll repeat this until he gets a winning ticket. But if he hasn't won by his fifth ticket, he won't buy any more tickets. Let L be the number of lottery tickets John will buy, then find $E(L)$.

TRANSFORMING RANDOM VARIABLES

1. We use the formula

$$^{\circ}F = \frac{9}{5}{}^{\circ}C + 32$$

to convert from Celsius to Fahrenheit. August is the hottest month in Hawaii with a mean temperature of $27^{\circ}C$. What is the mean temperature in Hawaii in $^{\circ}F$.

2. Let Z be a random variable with $\sigma_Z^2 = 49$. Let $W = (1/2)Z - 10$. Find σ_W.

3. The students in each 8th period classroom were asked to donate money for a school fundraiser. The class who raises the most money is awarded a pizza party. The school secretary records the amount raised by each class and makes a five-number summary for the data.

Min	Q1	Median	Q3	Max
4.50	15.25	22.00	38.75	95.50

Suppose the school has 45 8th period classrooms with 20 students per classroom. What was the median amount donated per student? With what IQR?

4. The number of items sold at a concession stand is normally distributed with $\mu = 323$ and $\sigma = 30$. The average price per item sold is $\$1.25$. Different student clubs volunteer to work the concession stand throughout the year and get to keep half of their sales to go towards their club's activities. What is the probability that a club will get to keep more than $\$220$ in sales?

5. The average length of a full-term new born baby is 20 inches with variance 0.81 inches. What are the mean and standard deviation of the length of a full-term new born, expressed in centimeters? Use 1 in $= 2.54$ cm.

6. The weights of full-term new born babies are normally distributed with $\mu = 120$ ounces and $\sigma = 20$ ounces. Describe the shape, center, and spread for the weights of full-term new born babies as measured in pounds. Use 1 pound $= 16$ ounces.

COMBINATIONS OF RANDOM VARIABLES

1. X and Y are independent random variables with $E(X) = 48$, $E(Y) = 54$, $SD(X) = 3$ and $SD(Y) = 5$. Find $E(X - Y)$ and $SD(X - Y)$.

2. A and B are independent random variables with $E(A) = 6.5$, $E(B) = 4.4$, $SD(A) = 1.6$, and $SD(B) = 2.1$. Find $E(4A + 2B)$ and $SD(4A + 2B)$.

3. The time it takes students to complete multiple choice questions on an AP Statistics Exam has a mean of 55 seconds with a standard deviation of 12 seconds. If the exam consists of 40 multiple choice questions, find the mean total time to finish the exam. Then find the standard deviation in the total time. What assumption must be made?

4. Let M represent the height of a male over 21 years of age and let W represent the height of a female over 21 years of age. Let D represent the difference between their heights ($D = M - W$). Let $E(M) = 70$ inches, $\sigma_M = 2.8$ inches, $E(W) = 64.5$ inches and $\sigma_W = 2.4$ inches.

What is the mean and standard deviation of the difference between the two heights?

5. The Ironman is a challenge in which a competitor swims 2.4 miles, then bikes 112 miles, and finally runs 26.2 miles. Suppose the times for each of the legs are normally distributed with the given means and standard deviations.

Swim: $\mu_S = 76$ minutes and $\sigma_S = 18$ minutes

Bike: $\mu_B = 385$ minutes and $\sigma_B = 32$ minutes

Run: $\mu_R = 294$ minutes and $\sigma_R = 25$ minutes

What percent of the competitors finish the Ironman in under 710 minutes?

6. We buy a scratch-off lottery ticket for $\$1$ at the local gas station. If we get three hearts in a row on the scratch-off, the state will pay us $\$500$. Let X be the amount the state pays us and let X have the following probability distribution.

X	\$0	\$500
P(X)	0.999	0.001

Suppose we buy one of these scratch-off tickets every day for a week (7 days). Find the expected value and standard deviation of our total winnings.

PERMUTATIONS AND COMBINATIONS

1. Calculate the binomial coefficient.

$$\binom{12}{7}$$

2. Calculate ${}_{10}P_3$.

3. How much greater is ${}_5P_2$ than ${}_5C_2$?

4. The high school girls' basketball team has 8 players, 5 of whom are seniors. They need to figure out which senior will be captain and which senior will be co-captain. To make it fair, they choose two players out of a hat. The first drawn will be captain and the second will be co-captain. How many different captain/co-captain pairs are possible?

5. How many different ways can the letters in the word "SUCCESS" be rearranged?

6. Mrs. B's kindergarten class has 14 students and Mr. G's kindergarten class has 16 students. Three students will be selected at random from each of these classrooms to ride on a float in the school parade coming up next week. How many different groups of 6 can be chosen to ride the float?

BINOMIAL RANDOM VARIABLES

1. We toss a fair coin 15 times and record the number of tails. Is this experiment modeled by a binomial random variable? If it isn't, explain why. If it is, determine its parameters n and p and express the binomial random variable as $X \sim B(n, p)$.

2. We randomly select students from our school until we find a student in the school band. Assume there are 900 students in the school and 80 participate in the school band. Is this experiment modeled by a binomial random variable? If it isn't, explain why. If it is, determine its parameters n and p and express the binomial random variable as $X \sim B(n, p)$.

3. Let $X \sim B(n, p)$ be a binomial random variable with $n = 12$ and $p = 0.08$. Find $P(X = 4)$.

4. Let Y be the number of times we roll a 1 on a fair 6-sided die if we do 10 trials. Fill in the following probability distribution for Y, rounding each probability to 4 decimal places.

Y	0	1	2	3	4	5	6	7	8	9	10
P(Y)											

5. For each binomial random variable, determine whether the shape of the probability distribution will be skewed right, skewed left, or symmetrical.

1. $X \sim B(n, p)$ with $n = 10$ and $p = 0.15$

2. $Y \sim B(n, p)$ with $n = 10$ and $p = 0.75$

3. $Z \sim B(n, p)$ with $n = 10$ and $p = 0.50$

6. Suppose an environmental biologist is studying juvenile sunfish mortality. He finds that only 30% of juvenile sunfish survive in a certain lake. Out of 8 randomly selected juvenile sunfish, what is the probability that exactly 3 will survive?

POISSON DISTRIBUTIONS

1. A student is able to solve 10 practice problems per hour, on average. Find the probability that she can solve 12 in the next hour.

2. A student is able to solve 6 practice problems per hour, on average. Find the probability that she can solve at least 4 in the next hour.

3. A student is able to solve 5 practice problems per hour, on average. Find the probability that she solves at most 3 in the next hour.

4. A baker is able to bake 50 loaves of bread per day, on average. Find the probability that he can bake 60 on Friday.

5. A baker is able to bake 10 cakes per hour, on average. Find the probability that he can bake more than 5 in the next hour.

6. A baker is able to frost 2 cakes per hour, on average. Find the probability that he frosts fewer than 5 cakes in the next hour.

"AT LEAST" AND "AT MOST," AND MEAN, VARIANCE, AND STANDARD DEVIATION

■ 1. Assume X is a binomial random variable. Let $X \sim B(n, p)$ with $n = 15$ and $p = 0.45$. Find $P(X > 7)$.

■ 2. According to a 2017-2018 survey, 68% of U.S. households own a pet. Suppose we select 12 households at random. What is the probability that fewer than 8 of them own a pet?

■ 3. According to a 2017-2018 survey, 68% of U.S. households own a pet. Suppose 200 households are selected at random. Find the expected value and standard deviation for the number of households that own a pet.

■ 4. 3% of runners in the Boston Marathon do not finish. Suppose we select a SRS of 140 Boston Marathon runners. How many do we expect to finish the race?

■ 5. We roll a fair die 6 times. What is the probability we'll observe an even number in at most 3 of the rolls?

6. We roll two fair 6-sided die 10 times and observe the sum. What is the probability of rolling a sum of 7 on at least six of the rolls?

BERNOULLI RANDOM VARIABLES

■ 1. A game at the local county fair involves spinning a circular spinner that's divided into 8 congruent sections, only two of which are "winners." We buy 5 spins for $\$3.00$. If we land on "winner" on any of our 5 spins, we get to choose a stuffed animal. Is this an example of Bernoulli trials?

■ 2. A game at the local county fair involves spinning a circular spinner that's divided into 8 congruent sections, only two of which are "winners." We buy 5 spins for $\$3.00$. If we land on "winner" on any of our 5 spins, we get to choose a stuffed animal. Find the mean and standard deviation for each trial.

■ 3. A game at the local county fair involves spinning a circular spinner that's divided into 8 congruent sections, only two of which are "winners." We buy 5 spins for $\$3.00$. If we land on "winner" on any of our 5 spins, we get to choose a stuffed animal. Find the mean and standard deviation for the number of winners expected in a set of 5 spins.

■ 4. A game at the local county fair involves spinning a circular spinner that's divided into 8 congruent sections, only two of which are "winners." We buy 5 spins for $\$3.00$. If we land on "winner" on any of our 5 spins, we get to choose a stuffed animal. Find the probability of observing no winners in a set of 5 spins.

■ 5. A game at the local county fair involves spinning a circular spinner that's divided into 8 congruent sections, only two of which are "winners." We buy 5 spins for $\$3.00$. If we land on "winner" on any of our 5 spins, we get to choose a stuffed animal. What is the probability of observing at least 1 winner in a set of 5 spins?

■ 6. Our goal is to learn about the percentage of students with high ACT scores. We randomly select high school seniors and record their highest ACT score. Explain why these aren't Bernoulli trials. Then design a way to conduct the experiment differently so that they can be considered Bernoulli trials.

GEOMETRIC RANDOM VARIABLES

■ 1. We toss a coin until we get “tails.” Does this experiment represent a geometric random variable? If it doesn’t, explain why. If it does, determine its parameter p and express the variable as $X \sim \text{Geom}(p)$.

■ 2. We randomly select students from our school until we find a student in the school band. Assume there are 900 students in the school and 80 participate in the school band. Does this experiment represent a geometric random variable? If it doesn’t, explain why. If it does, determine its parameter p and express the variable as $X \sim \text{Geom}(p)$.

■ 3. Let $X \sim \text{Geom}(p)$ with $p = 0.25$. Find $P(X = 5)$.

■ 4. Suppose we roll a 6-sided fair die until we observe a 2. What is the probability that a 2 will be observed within the first 5 trials?

■ 5. Suppose we roll a 6-sided fair die until we observe a 2. What is the probability that a 2 won’t be observed until at least the 6th trial?

6. According to a 2017-2018 survey, 68% of U.S. households own a pet. Suppose we start randomly surveying households and asking whether they are pet owners. How many do we expect we will need to survey to find our first household that owns a pet?

W W W . K R I S T A K I N G M A T H . C O M