Topic: Combinations of random variables

Question: Let A and B be independent, continuous random variables with $\mu_A = 85$, $\sigma_A = 6$, $\mu_B = 92$, and $\sigma_B = 8$. Find μ_{A+B} and σ_{A+B}.

Answer choices:

A $\mu_{A+B} = 177$ and $\sigma_{A+B} = 14$

B $\mu_{A+B} = 177$ and $\sigma_{A+B} = 100$

C $\mu_{A+B} = 177$ and $\sigma_{A+B} = 10$

D $\mu_{A+B} \approx 125.26$ and $\sigma_{A+B} = 10$

Solution: C

The mean for the sum of two or more random variables is the sum of the respective means.

$$\mu_{A+B} = \mu_A + \mu_B$$

$$\mu_{A+B} = 85 + 92$$

$$\mu_{A+B} = 177$$

The variance for the sum of independent random variables is the sum of the respective variances. Finding the square root of that value gives us the standard deviation of the sum.

$$\sigma_{A+B} = \sqrt{{\sigma_A}^2 + {\sigma^2}_B}$$

$$\sigma_{A+B} = \sqrt{6^2 + 8^2}$$

$$\sigma_{A+B} = \sqrt{100}$$

$$\sigma_{A+B} = 10$$

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Question: Let X be the mass of a farm fresh egg. Assume X follows a normal distribution with $\mu_X = 64$ grams and $\sigma_X = 6$ grams. Suppose we package a dozen eggs into an egg carton, and the egg carton itself has a fixed mass of 40 grams. Let C be the mass of a carton full of a dozen farm fresh eggs. Find the mean and standard deviation for C.

Answer choices:

A $\mu_C = 808$ grams and $\sigma_C \approx 20.78$ grams

B $\mu_C = 768$ grams and $\sigma_C \approx 20.78$ grams

C $\mu_C = 808$ grams and $\sigma_C \approx 72$ grams

D $\mu_C = 808$ grams and $\sigma_C \approx 60.78$ grams

Solution: A

Let T be the total mass of the dozen eggs:

$$T = X_1 + X_2 + X_3 + \ldots + X_{12}$$

Since the mass of the eggs follow a normal distribution with $\mu_X = 64$ grams and $\sigma_X = 6$ grams, the mean for the total mass of the dozen eggs can be found using the following formula:

$$\mu_T = \mu_{X_1} + \mu_{X_2} + \ldots + \mu_{X_{12}}$$

$$\mu_T = 64 + 64 + \ldots + 64$$

$$\mu_T = 12(64)$$

$$\mu_T = 768 \text{ grams}$$

The mean mass for the entire carton filled with eggs is

$$C = \mu_T + 40$$

$$C = 768 + 40$$

$$C = 808 \text{ grams}$$

The variance for the total mass of the dozen eggs is

$$\sigma^2{}_T = \sigma^2{}_{X_1} + \sigma^2{}_{X_2} + \ldots + \sigma^2{}_{X_{12}}$$

$$\sigma^2{}_T = 6^2 + 6^2 + \ldots + 6^2$$

$$\sigma^2{}_T = 12(6^2)$$

$$\sigma^2{}_T = 432$$

Which means the standard deviation is

$$\sqrt{\sigma^2{}_T} = \sqrt{432}$$

$$\sigma_T \approx 20.78 \text{ grams}$$

The standard deviation for the entire carton filled with eggs will not change when we add the fixed mass of the 40 grams since this is just a shift and does not change the variability in the overall mass of the carton of eggs.

$$\sigma_C \approx 20.78 \text{ grams}$$

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Question: Sandwiches can be purchased in the school cafeteria. Bread is baked each day and the sandwich is topped with meat and cheese, then sold for $\$3.50$.

The weight of the bread used for each sandwich is normally distributed with mean of 2.3 ounces and standard deviation of 0.4 ounces. The weight of the meat and cheese used for each sandwich is normally distributed with mean of 2.5 ounces and standard deviation of 0.6 ounces. Suppose you purchase a sandwich at random from the school cafeteria. What is the probability that the overall weight of the sandwich exceeds 6 ounces? Assume the two variables are independent.

Answer choices:

A 0.0961

B 0.0485

C 0.1151

D 0.0000

Solution: B

Let B be the weight of the bread and let M be the weight of the meat and cheese. Because we'll be creating a sandwich by combining the weights together, we'll let T be the total weight of the sandwich.

$$T = B + M$$

For bread, $\mu_B = 2.3$ and $\sigma_B = 0.4$, and for meat and cheese, $\mu_M = 2.5$ and $\sigma_M = 0.6$. The mean for the sum of two or more random variables is the sum of the respective means.

$$\mu_T = \mu_{B+M}$$

$$\mu_T = \mu_B + \mu_M$$

$$\mu_T = 2.3 + 2.5$$

$$\mu_T = 4.8 \text{ ounces}$$

The variance for the sum of independent random variables is the sum of the respective variances. Finding the square root of that value gives us the standard deviation of the sum.

$$\sigma_T = \sigma_{B+M}$$

$$\sigma_T = \sqrt{{\sigma_B}^2 + \sigma_M^2}$$

$$\sigma_T = \sqrt{(0.4)^2 + (0.6)^2}$$

$$\sigma_T = \sqrt{0.52}$$

$$\sigma_T \approx 0.7211 \text{ ounces}$$

Because B and M were both normally distributed, the new random variable T will also be normally distributed.

To find the probability that the overall weight of the sandwich exceeds 6 ounces, the value of 6 is converted to a z-score and then the normal model is used to find the corresponding area under the curve.

$$P(T > 6) = P\left(Z > \frac{6 - 4.8}{0.7211}\right)$$

$$P(T > 6) = P(Z > 1.66)$$

$$P(T > 6) \approx 0.0485$$