Inferential statistics and hypotheses

One thing we like to do in statistics is make a statement about a population parameter, collect a sample from that population, investigate the sample, and then make a clear statement about whether or not that sample supports our original statement about the population.

We'll cover each part of this process throughout this section. Here's where we're headed: There are five steps for **hypothesis testing**:

1. State the null and alternative hypotheses.

2. Determine the level of significance.

3. Calculate the test statistic.

4. Find critical value(s) and determine the regions of acceptance and rejection.

5. State the conclusion.

This process is also called **inferential statistics**, because we're using information we have about the sample to make *inferences* about the population.

For instance, we might have been told that 40% of cars in our town are blue. We could try to take a random sample of cars in our town, look at the proportion of cars in that sample which are blue (maybe 37%), and then build a confidence interval to state how confident we are that our sample, which produced $\hat{p}=0.37$, supports the claim that $p=0.40$.

Proof vs. support

But it's important to make the distinction right up front between proving a claim and providing support for a claim.

When we do inferential statistics, we're usually not able to prove something with certainty (our $\hat{p}=0.37$ doesn't *prove* that $p=0.40$, even though it might lend some support for that claim). Instead, we use the data to support a theory that we have. Hopefully, if the data is strong enough, we can provide strong, or confident support for our theory, but we still can't necessarily prove it.

Hypotheses for means and proportions

Before we can use inferential statistics, we first need a **hypothesis**, which is a statement of expectation about a population parameter that we develop for the purpose of testing it ($40\,\%$ of cars in our town are blue).

In any hypothesis test, the first thing we always want to do is state what are called the null and alternative hypotheses. Every hypothesis test contains this set of two opposing statements about a population parameter.

The **alternative hypothesis** H_a is the abnormality we're looking for in the data; it's the significance we're hoping to find. Once we have an alternative hypothesis, we always want to state the opposite claim, which we call the **null hypothesis**, H_0.

For example, if our city releases data stating that 40% of the cars in our town are blue, then this 40% figure is the status quo; it's our normal baseline; it should be our null hypothesis. If we then want to test this claim, to see if we can find interesting new data that shows that the city's claim is wrong, then we'd be looking for data that goes against their stated figure. So the alternative hypothesis will be that the proportion of blue cars is contradictory to what the city has stated.

H_a: the proportion of blue cars in our town is not 40%

H_0: 40% of cars in our town are blue

Interestingly enough, we always test the null hypothesis H_0, not the alternative hypothesis H_a. We say that, if our sample gives us good enough evidence, then we can *reject* the null hypothesis, and therefore provide evidence that supports our alternative hypothesis.

In this section we'll focus on hypothesis tests about two population parameters: the population mean μ and the population proportion p.

For population means

Remember that the population mean is the mean value of some characteristic that we're interested in. For example, the mean height of American females might be $\mu = 65$ inches if the average American woman is $5'5''$ tall.

Whether we're investigating a population proportion or a population mean, the null hypothesis states the status quo that the population parameter is $\leq$, $=$, or $\geq$ the claimed value. The null hypothesis always says that the population mean (or parameter) is normal; nothing new or different is happening.

So if we're testing the claim that the mean height of American females is $\mu = 65$ inches, the null hypothesis is $H_0 : \mu = 65$.

If we think the mean height of American females is different than this claim, then we state that in the alternative hypothesis as

- The mean height of American females is different than $\mu = 65$:

$$H_a : \mu \neq 65$$

If, on the other hand, we'd started with a null hypothesis of $H_0 : \mu \leq 65$, then our alternative hypothesis would be

- The mean height of American females is greater than $\mu = 65$:

$$H_a : \mu > 65$$

and if we'd started with a null hypothesis of $H_0 : \mu \geq 65$, then our alternative hypothesis would be

- The mean height of American females is less than $\mu = 65$:

$$H_a : \mu < 65$$

For population proportions

On the other hand, the population proportion is the proportion that meets some sort of criteria we've established. For example, the proportion of American females with blue eyes might be $p = 0.15$ if 15% of American females have blue eyes. The null hypothesis would be $H_0 : p = 0.15$, stating that the population proportion is 15%.

If we think the population proportion is different than this 15% claim, then we state that in the alternative hypothesis as

- The proportion of American females with blue eyes is different than $p = 0.15$:

$$H_a : p \neq 0.15$$

If, on the other hand, we'd started with a null hypothesis of $H_0 : p \leq 0.15$, then our alternative hypothesis would be

- The proportion of American females with blue eyes is greater than $p = 0.15$:

$$H_a : p > 0.15$$

and if we'd started with a null hypothesis of $H_0 : p \geq 0.15$, then our alternative hypothesis would be

- The proportion of American females with blue eyes is less than $p = 0.15$:

$$H_a : p < 0.15$$

As we can see for both population means and population proportions, the alternative hypothesis states the opposite of the null hypothesis. We find support for the alternative hypothesis only because we find a reason to reject the null hypothesis. Because the null hypothesis always includes a $\leq$,

$=$, or $\geq$ sign, the alternative hypothesis always includes a $>$, $\neq$, or $<$ sign, respectively. In summary,

If H_0 is $\mu =$ or $p =$, then H_a is $\mu \neq$ or $p \neq$

If H_0 is $\mu \leq$ or $p \leq$, then H_a is $\mu >$ or $p >$

If H_0 is $\mu \geq$ or $p \geq$, then H_a is $\mu <$ or $p <$

Let's practice writing pairs of hypothesis statements.

Example

Write different sets of hypothesis statements to test the claims that students at Springdale High School perform 1) differently than, 2) better than, and 3) worse than students at Greenville High School on the SAT test.

1) To test the claim that students at SHS perform differently on the SAT than students at GHS, we would write these hypothesis statements:

Null: Students at Springdale High School do not perform differently on the SAT than students at Greenville High School:

$$H_0: \ \ \mu_S = \mu_G$$

Alternative: Students at Springdale High School perform differently on the SAT than students at Greenville High School:

$$H_a: \ \ \mu_S \neq \mu_G$$

2) To test the claim that students at SHS perform better on the SAT than students at GHS, we would write these hypothesis statements:

Null: Students at Springdale High School perform no better on the SAT than students at Greenville High School:

$$H_0: \ \mu_S \leq \mu_G$$

Alternative: Students at Springdale High School perform better on the SAT than students at Greenville High School:

$$H_a: \ \mu_S > \mu_G$$

3) To test the claim that students at SHS perform worse on the SAT than students at GHS, we would write these hypothesis statements:

Null: Students at Springdale High School perform no worse on the SAT than students at Greenville High School:

$$H_0: \ \mu_S \geq \mu_G$$

Alternative: Students at Springdale High School perform worse on the SAT than students at Greenville High School:

$$H_a: \ \mu_S < \mu_G$$

Keep in mind that some statisticians and statistics textbooks will use the convention where the null hypothesis always and only includes an $=$ sign, with the alternative hypothesis stated with $<$, $\neq$, or $>$.

We can use either convention, we just have to stay consistent with whichever one we choose. For our purposes, we'll always stick with the matching pairs we outlined earlier:

If H_0 is $\mu =$ or $p =$, then H_a is $\mu \neq$ or $p \neq$

If H_0 is $\mu \leq$ or $p \leq$, then H_a is $\mu >$ or $p >$

If H_0 is $\mu \geq$ or $p \geq$, then H_a is $\mu <$ or $p <$