Topic: Hypothesis testing for the difference of means

Question: Two math teachers, Mr. Johnson and Mr. Adams, want to determine whose students performed better on a recent exam. Mr. Johnson sampled 40 of his students and found a mean score of 84 with the standard deviation of 3.5, while Mr. Adams sampled 38 of his students and found a mean score of 82 with the standard deviation of 3.9. Assuming the exam scores are normally distributed, what can they conclude at 0.01 level of significance?

Answer choices:

A They reject the null; the result is significant at $\alpha = 0.01$.

B They reject the null; the result isn't significant at $\alpha = 0.01$.

C They fail to reject the null; the result is significant at $\alpha = 0.01$.

D They fail to reject the null; the result isn't significant at $\alpha = 0.01$.

Solution: D

The teachers are looking for a difference in exam scores, without any suspicion about the direction of the difference, so they'll use a two-tailed test, and their hypothesis statements will be

$H_0: \mu_J - \mu_A = 0$; the exam scores don't differ significantly

$H_a: \mu_J - \mu_A \neq 0$; the exam scores differ significantly

The samples are large, $n_1 \geq 30$ and $n_2 \geq 30$, and neither population variance is more than twice the other, so we'll assume equal population variances. Therefore, we'll start by calculating pooled variance.

$$s_p = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}}$$

$$s_p = \sqrt{\frac{(40 - 1)3.5^2 + (38 - 1)3.9^2}{40 + 38 - 2}}$$

$$s_p = \sqrt{\frac{39(12.25) + 37(15.21)}{76}}$$

$$s_p \approx 3.70$$

Then the test statistic will be

$$z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

Because $\mu_1 - \mu_2 = 0$, the test statistic formula simplifies.

$$z = \frac{\bar{x}_1 - \bar{x}_2}{s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

$$z = \frac{84 - 82}{3.70\sqrt{\frac{1}{40} + \frac{1}{38}}}$$

$$z = \frac{2}{3.70\sqrt{\frac{1}{40} + \frac{1}{38}}}$$

$$z \approx 2.39$$

We find $z \approx 2.39$ in the z-table, and we get 0.9916.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	**.9916**
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936

The p-value is therefore $p = 2(1 - 0.9916) = 0.0168$, and we have $p = 0.0168 > \alpha = 0.01$, so the teachers will fail to reject the null hypothesis. They can't conclude that the mean exam scores differ.

Topic: Hypothesis testing for the difference of means

Question: An engine manufacturer claims their new engine consumes at least 0.5 gallons less gasoline per 100 miles, compared to older engines. An independent auditing company wants to test this claim and randomly selects 25 cars with new engines and 25 cars with old engines and finds that the new engine consumes 2.95 gallons of gasoline per 100 miles with a standard deviation of 0.14, while the old engine consumes 3.24 gallons of gasoline per 100 miles with a standard deviation of 0.19. What can the auditing company conclude at a 0.05 level of significance?

Answer choices:

A They fail to reject the null; the result is significant at $\alpha = 0.05$.

B They fail to reject the null; the result is not significant at $\alpha = 0.05$.

C They reject the null; the result is significant at $\alpha = 0.05$.

D They reject the null; the result is not significant at $\alpha = 0.05$.

Solution: C

Because the manufacturer suspects that the new engine consumes "at least 0.5 gallons less gasoline," they'll use a one-tailed test (specifically a lower-tailed test), and their hypothesis statements will be

$H_0 : \mu_1 - \mu_2 \geq -0.5$; the new engine doesn't consume at least 0.5 gallons less gasoline

$H_a : \mu_1 - \mu_2 < -0.5$; the new engine consumes at least 0.5 gallons less gasoline

Neither sample variance is more than twice the other, so we can assume that the population variances are equal. Therefore, with small samples $n_1, n_2 < 30$, pooled variance is

$$s_p = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}}$$

$$s_p = \sqrt{\frac{(25 - 1)0.14^2 + (25 - 1)0.19^2}{25 + 25 - 2}}$$

$$s_p = \sqrt{\frac{24(0.0196) + 24(0.0361)}{48}}$$

$$s_p = \sqrt{\frac{0.0196 + 0.0361}{2}}$$

$$s_p = \sqrt{\frac{0.0557}{2}}$$

$$s_p \approx 0.167$$

Then the test statistic is

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

$$t = \frac{(2.95 - 3.24) - (-0.5)}{0.167\sqrt{\frac{1}{25} + \frac{1}{25}}}$$

$$t \approx \frac{0.21}{0.167(0.2828)}$$

$$t \approx 4.45$$

Because we're using a lower-tailed test, we'll make this t-score negative and use $t \approx -4.45$. The degrees of freedom is

$$\text{df} = n_1 + n_2 - 2$$

$$\text{df} = 25 + 25 - 2$$

$$\text{df} = 48$$

From the t-table, $\text{df} = 48$ at a 0.05 level of significance for a lower-tailed test, we get -1.677. The t-value we found is significantly less than -1.677, so the critical-value approach lets the engine manufacturer reject the null hypothesis and conclude that, at 0.05 significance, the new engine consumes at least 0.5 fewer gallons of gasoline per 100 miles than the old engine.

Topic: Hypothesis testing for the difference of means

Question: A pollster wants to determine whether male financial directors earn more than female financial directors. It's known that the standard deviations of mean monthly salaries for men and women are $\$459$ and $\$430$, respectively. The pollster samples 35 male directors and 35 female directors and found that mean monthly salaries were $\$8,504$ and $\$7,845$, respectively. What can the pollster conclude at a 0.10 level of significance?

Answer choices:

A They fail to reject the null; the result is significant at $\alpha = 0.10$.

B They fail to reject the null; the result isn't significant at $\alpha = 0.10$.

C They reject the null; the result is significant at $\alpha = 0.10$.

D They reject the null; the result isn't significant at $\alpha = 0.10$.

Solution: C

The pollster believes that male financial directors earn more than female directors, so they'll use an upper-tailed test and their hypothesis statements will be

$H_0: \mu_1 - \mu_2 \leq 0$; men don't earn more than women

$H_a: \mu_1 - \mu_2 > 0$; men earn more than women

The samples are large $n_1, n_2 \geq 30$, and neither population variance is more than twice the other, so we'll calculate pooled variance as

$$s_p = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}}$$

$$s_p = \sqrt{\frac{(35 - 1)459^2 + (35 - 1)430^2}{35 + 35 - 2}}$$

$$s_p = \sqrt{\frac{34(459^2) + 34(430^2)}{68}}$$

$$s_p \approx 444.74$$

and then the test statistic will be

$$z = \frac{\bar{x}_1 - \bar{x}_2}{s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

$$z = \frac{8,504 - 7,845}{444.74\sqrt{\frac{1}{35} + \frac{1}{35}}}$$

$$z \approx \frac{659}{444.74(0.239)}$$

$$z \approx 6.20$$

This z-value is well above the largest value in the z-table, so the pollster will reject the null hypothesis and conclude that the mean monthly salary for men is greater than the mean monthly salary for women.