Topic: Confidence interval for the mean

Question: The height of students in our school is normally distributed with a standard deviation of $\sigma = 4$ inches. We sample 50 of our classmates (with replacement) and get a sample mean of $\bar{x} = 66$ inches. What is the confidence interval for a confidence level of $95\,\%$?

Answer choices:

A $(a, b) \approx (64.54, 67.46)$

B $(a, b) \approx (64.89, 67.11)$

C $(a, b) \approx (65.07, 66.93)$

D $(a, b) \approx (65.74, 66.26)$

Solution: B

A 95% confidence level is associated with z-scores of $z = \pm 1.96$.

If we plug everything we know into the confidence interval formula for a known population standard deviation, we get

$$(a, b) = \bar{x} \pm z^* \frac{\sigma}{\sqrt{n}}$$

$$(a, b) = 66 \pm 1.96 \cdot \frac{4}{\sqrt{50}}$$

$$(a, b) \approx 66 \pm 1.1087$$

Therefore, we can say that the confidence interval is

$$(a, b) \approx (66 - 1.1087, 66 + 1.1087)$$

$$(a, b) \approx (64.8913, 67.1087)$$

$$(a, b) \approx (64.89, 67.11)$$

We could also express this as the sample mean plus or minus the margin of error, 66 ± 1.1087 inches. We're 95% certain that the actual mean height of students in our school is between 64.89 inches and 67.11 inches.

Topic: Confidence interval for the mean

Question: The weight of chickens on a farm is normally distributed with a standard deviation of $\sigma = 3.5$ ounces. What is the smallest sample we can take if we want a margin of error of ± 2.5 ounces, and we want to be 99% confident?

Answer choices:

A $n = 10$ chickens

B $n = 13$ chickens

C $n = 14$ chickens

D $n = 30$ chickens

Solution: C

Solve the margin of error formula for n.

$$ME = z^* \frac{\sigma}{\sqrt{n}}$$

$$ME\sqrt{n} = z^*\sigma$$

$$\sqrt{n} = \frac{z^*\sigma}{ME}$$

$$n = \left(\frac{z^*\sigma}{ME}\right)^2$$

Now we can plug the values we were given into this equation, remembering that a confidence level of 99% is associated with critical values of $z = \pm 2.58$.

$$n = \left(\frac{2.58 \cdot 3.5}{2.5}\right)^2$$

$$n \approx 13.05$$

Because we can't sample 0.05 of a chicken, we round up to $n = 14$ chickens. Then we can say that, to meet that threshold, and keep a margin of error of ± 2.5 at 99% confidence, we'd need to take a sample size of at least $n = 14$ chickens.

Topic: Confidence interval for the mean

Question: We want to know the mean number of daylight hours (the time between sunrise and sunset) in a day in our city over the course of a year. We take a random sample of 30 days throughout the year and get a sample mean of $\bar{x} = 13.15$ hours and a sample standard deviation of $s = 0.85$ hours. What is the confidence interval for a confidence level of 90%?

Answer choices:

A $(a, b) \approx (12.89, 13.41)$

B $(a, b) \approx (12.85, 13.45)$

C $(a, b) \approx (12.75, 13.55)$

D $(a, b) \approx (12.28, 14.02)$

Solution: A

Because population standard deviation is unknown, we have to use the t-distribution instead of the z-distribution.

A 90% confidence level with $n-1=30-1=29$ degrees of freedom is associated with t-scores of $t=\pm 1.699$.

$$(a,b)=\bar{x}\pm t^*\frac{s}{\sqrt{n}}$$

$$(a,b)=13.15\pm 1.699\cdot\frac{0.85}{\sqrt{30}}$$

$$(a,b)\approx 13.15\pm 0.2637$$

Therefore, we can say that the confidence interval is

$$(a,b)\approx(13.15-0.2637,13.15+0.2637)$$

$$(a,b)\approx(12.8863,13.4137)$$

$$(a,b)\approx(12.89,13.41)$$

We could also express this as the sample mean plus or minus the margin of error, 13.15 ± 0.2637 hours. We're 90% certain that the actual population mean of hours of daylight in a day is between 12.89 hours and 13.41 hours.