Property 1 The union of two regular set is regular.

# Closure property of Regular sets

Proof -

Let us take two regular expressions

$$RE_1 = a(aa)^*$$
 and  $RE_2 = (aa)^*$ 

So,  $L_1 = \{a, aaa, aaaaa,....\}$  (Strings of odd length excluding Null)

and  $L_2 = \{ \epsilon, aa, aaaaa, aaaaaaa,.....\}$  (Strings of even length including Null)

(Strings of all possible lengths including Null)

RE  $(L_1 \cup L_2) = a^*$  (which is a regular expression itself)

Hence, proved.

After carrying out some operation If the result is also regular

**Express the result in R.E form** 

Perform the op, get result, express the result in the form R.E

**Property 1**. The union of two regular set is regular.

Proof -

Let us take two regular expressions

$$RE_1 = a(aa)^*$$
 and  $RE_2 = (aa)^*$ 

$$L_1 = \left\{ \begin{array}{c} 1 \\ OL \end{array} \right\}$$

odd no of a's without null

Property 2. The intersection of two regular set is regular.

#### Proof -

Let us take two regular expressions

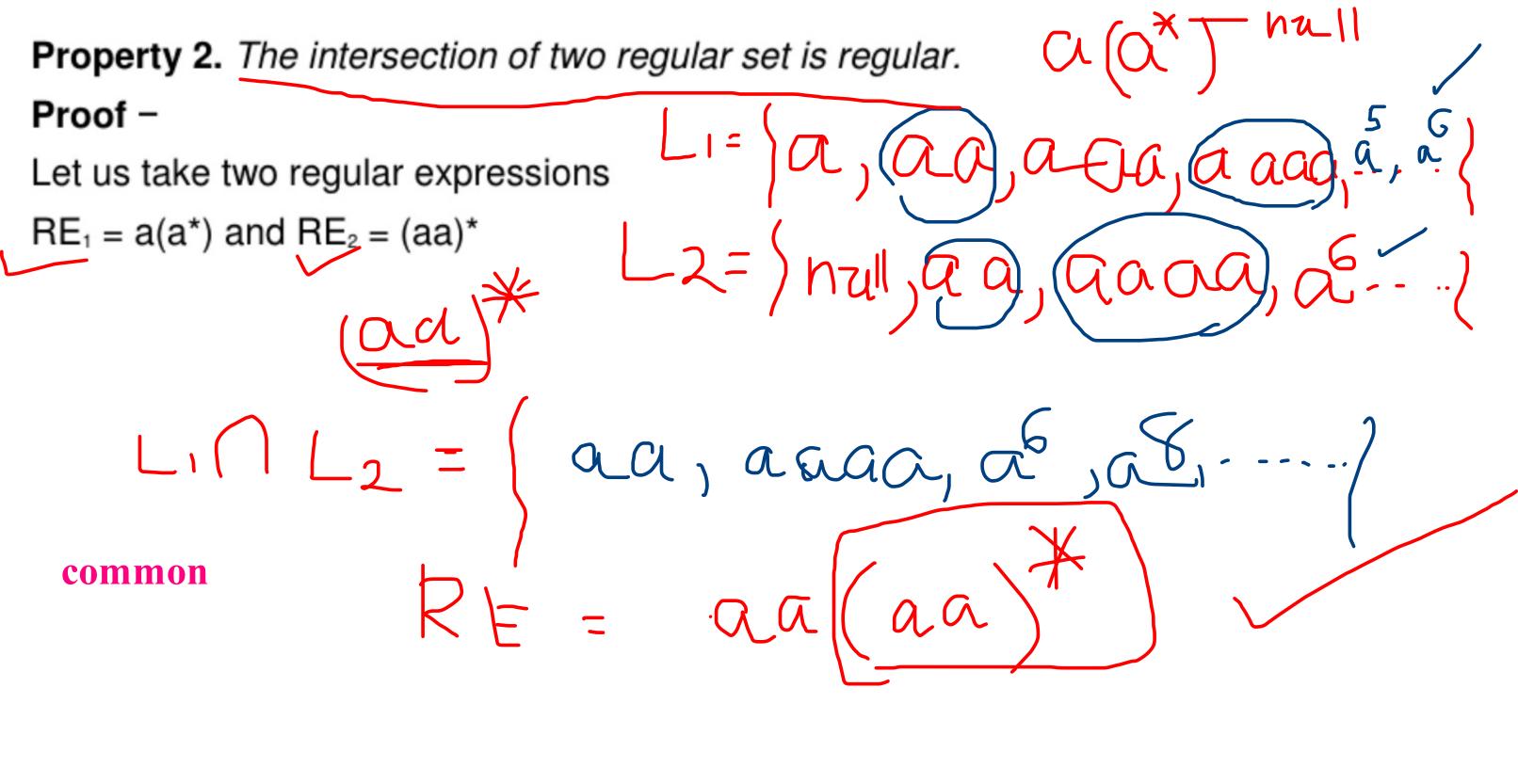
 $RE_1 = a(a^*)$  and  $RE_2 = (aa)^*$ 

So,  $L_1 = \{a,aa, aaa, aaaa, ....\}$  (Strings of all possible lengths excluding Null)

 $L_2 = \{ \epsilon, aa, aaaa, aaaaaa,.....\}$  (Strings of even length including Null)

 $L_1 \cap L_2 = \{ aa, aaaa, aaaaaa,.....\}$  (Strings of even length excluding Null)

RE  $(L_1 \cap L_2)$  = aa(aa)\* which is a regular expression itself.



## The complement of regular set is regular

 $RE = (aa)^*$ 

So, L = {ε, aa, aaaa, aaaaaa, ......} (Strings of even length including Null)

Complement of **L** is all the strings that is not in **L**.

So, L' = {a, aaa, aaaaa, .....} (Strings of odd length excluding Null)

RE (L') = a(aa)\* which is a regular expression itself.

The complement of regular set is regular

$$RE = a(aa)*$$

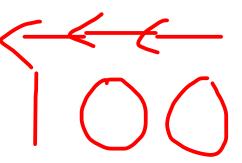
### Property 5. The reversal of a regular set is regular.

### Proof -

We have to prove L<sup>R</sup> is also regular if L is a regular set.

Let, 
$$L = \{01, 10, 11, 10\}$$
  
RE (L) =  $01 + 10 + 11 + 10$ 

$$L^{R} = \{10, 01, 11, 01\}$$



# Property 5. The reversal of a regular set is regular.

### Proof -

We have to prove  $L^R$  is also regular if L is a regular set.  $L = \{01, 10, 11, 10\}$ 

$$RE = 01 + 10 + 11 + 10$$
 or  $RE = (1+0)(1+0)$ 

Property 6. The closure of a regular set is regular.

#### Proof -

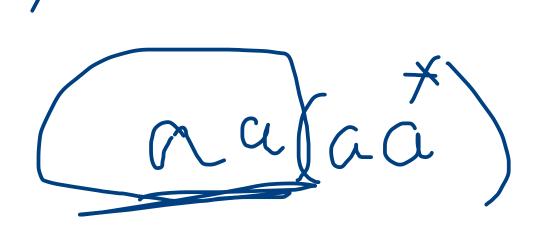
 $\bigvee$ If L = {a, aaa, aaaaa, ......} (Strings of odd length excluding Null)

i.e., RE (L) = 
$$a (aa)^*$$

L\* = {a, aa aaa, aaaa, aaaaa, .....} (Strings of all lengths excluding Null)

$$RE(L^*) = a(a)^*$$





**Property 6.** The closure of a regular set is regular. Proof -If L = {a, aaa, aaaaa, ......} (Strings of odd length excluding Mull) i.e., RE (L) =  $a (aa)^*$ 

null null, a, a a, aca,

Mull, a, aa

Property 7. The concatenation of two regular sets is regular.

#### Proof -

Let  $RE_1 = (0+1)^*0$  and  $RE_2 = 01(0+1)^*$ 

Here,  $L_1 = \{0,00,10,000,010,....\}$  (Set of strings ending in 0)

and  $L_2 = \{01, 010, 011, \dots\}$  (Set of strings beginning with 01)

Then,  $L_1 L_2 =$ 

{001,0010,0011,0001,00010,00011,1001,10010,.....}

Set of strings containing 001 as a substring which can be represented by an RE - (0 + 1)\*001(0 + 1)\*

$$e^{-\frac{1}{2}}$$
  $e^{-\frac{1}{2}}$   $e^{-$ 

