

B.Sc(V), Odd Semester Examination, 2021-22

Subject: Theory of computation

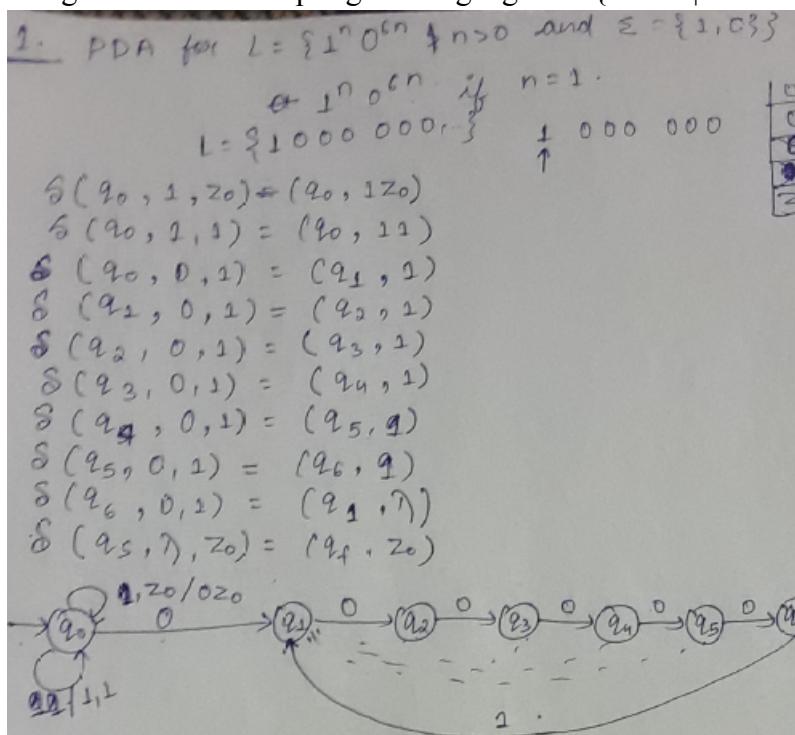
Date: 26/11/2021, Time: 10:00 am – 12:00 pm

Total Marks: 70

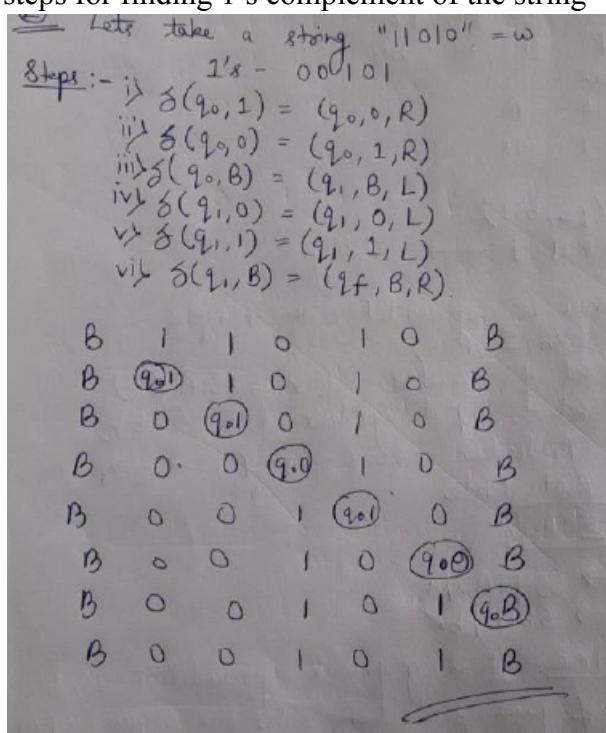
Attempt any 14 questions.

Marks: $14 \times 5 = 70$

- ✓ 1. Design a PDA for accepting the language $L = \{ 1^n 0^{6n} \mid n > 0 \text{ and } \Sigma = \{1, 0\} \}$



2. Design a Turing machine to find 1's complement of binary number. Write the sequence of steps for finding 1's complement of the string "11010"



3. Define Turing machine.

③ Turing Machine is represented by - 7 tuples
 $M = (Q, \Sigma, \Gamma, \delta, q_0, B, F)$
 where Q: a finite set of states
 Σ : a set of Γ not including B is set of Input symbols.
 B: a symbol of Γ is the blank
 Γ : the finite set of allowable tape symbols
 including $(B + \Sigma) \cup \Gamma$
 δ : go to next move function, a mapping from $Q \times \Gamma$ to $Q \times \Gamma \times \{L, R\}$
 F : go to the set of final states.
 q_0 : in Q is the start state.

4. Describe the model of a PDA.

Answer ④ Model of PDA's

① Input Tape \Rightarrow The input tape is divided in many cells or symbols. It is read the input head is read only and may only move from left to right each cell contains one symbol at a time.

② Finite Control \Rightarrow The finite control has some pointer which points the current symbol which is to be read.

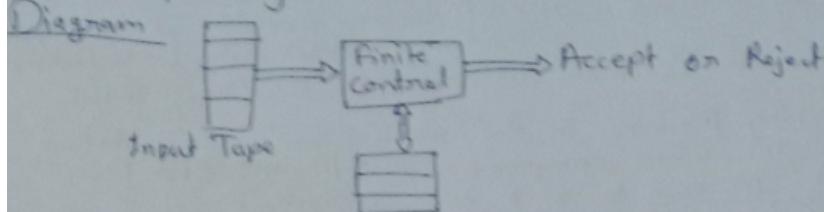
③ Read/write head \Rightarrow The input head is read only and may only move from left to right.

④ Stack \Rightarrow The stack is a structure in which we can push and remove the item from



Answer ⑤ One end only. It has an infinite size. In PDA the stack is used to store the item temporarily.

Diagram



Under Finite control write $\delta \rightarrow (Q \times (\Sigma \cup \lambda))^* \times \Gamma = (Q, \Gamma^*)$

5. Show that the intersection of two regular set is regular with example.

6. Use pumping lemma to show that the language $L = \{a^{2^n} \mid n > 0\}$ and $\Sigma = \{a\}$ is not regular.

Ans-6b $L = \{a^{2^n} \mid \text{where } n > 0\}$ $\Sigma = \{a\}$ is not regular.

Step 1 \rightarrow Let L be a regular language so we can be able to design DFA with n states.

Step 2 \rightarrow $w \in L \mid w \geq n$. $w = xyz \mid |xyz| \leq n$

$$w = a^{2^n} \mid |w| = 2^n \geq n$$

$$w = a^{2^n} = xyz \text{ such that } |xy| \leq n$$

$$\mid y \mid > 0$$

y can be Null. $|y| \leq n$ max length of y can be no and minimum length of y is 1

Step 3 - $|yz|^2 = |yz| + |y| = 2^n + 1 > 2^n$

$$|yz|^2 = |yz| + |y| = 2^n + n < 2^n + 1 \text{ for } n > 1$$

$$\therefore a^{2^n} \notin L$$

OR

[similarly]

$$L = \{a^{2^n} \mid \text{where } n > 0\}$$

Similarly,

Step 1 Initially assume L is regular

Step 2 $L = \{a, a^4, a^8, a^{16}, a^{32}, \dots\}$

$$w = a^4 = \frac{a}{2} \cdot \frac{a}{2} \cdot \frac{a}{2} \cdot \frac{a}{2} \quad \text{total } w=4$$

$$|xy| \leq n \quad |y| = 1 \geq 1$$

$3 \leq y$ this is not null

Step 3 $L = \{a^2\}$ $w = \frac{a^4}{2} \cdot \frac{a}{2} \cdot \frac{a}{2}$

$$w' = aaaa = a^5 \notin L$$

so the language $L = \{a^{2^n} \mid n > 0\}$ is not regular

7. Design Turing machine for finding 2's Complement of a binary number.

7) Let's take a binary number

$$01011000$$

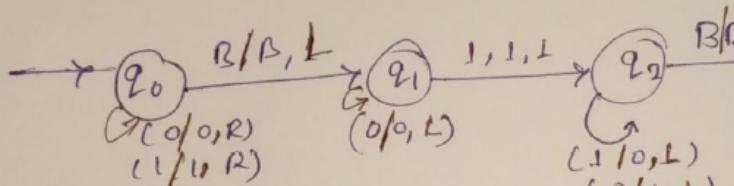
$$1's \rightarrow 10100111$$

$$+ 11$$

$$2's \rightarrow \underline{\underline{10101000}}.$$

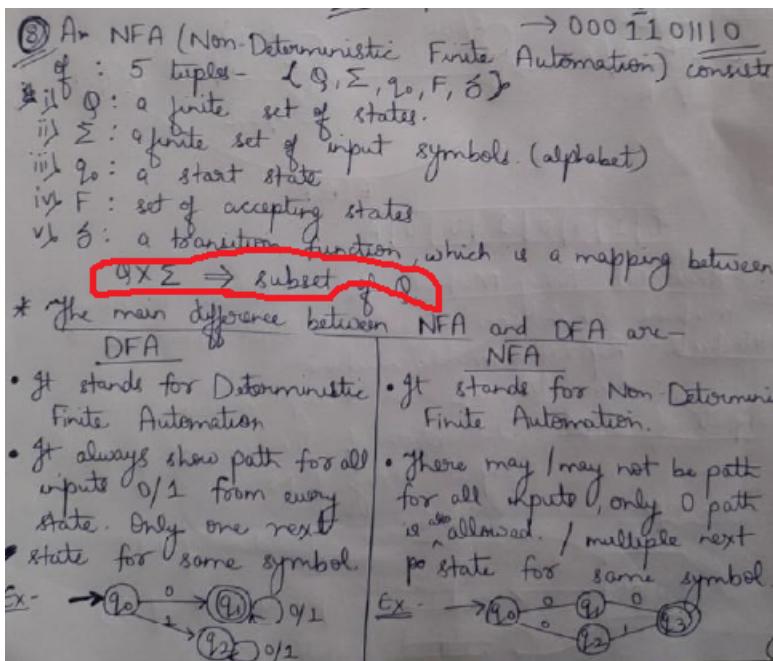
B	B	0	1	0	1	1	0	0	B	B
---	---	---	---	---	---	---	---	---	---	---

B	0	1	0	1	B
B	0	0	1	0	A
B	0	0	0	1	B
B	0	1	0	0	A
B	0	1	0	0	B
B	0	1	0	0	B
B	0	1	0	0	B
B	0	1	0	0	B
B	0	1	0	0	B
B	0	1	0	0	B



B	B	1	0	1	0	1	0	0	B	B
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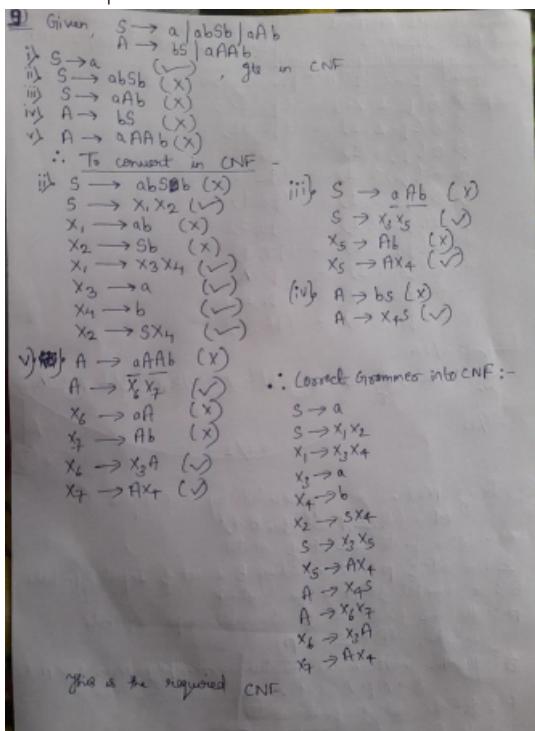
8. Define NFA. Write the main difference between DFA and NFA.



9. Convert the following grammar into Chomsky Normal Form

$$S \rightarrow a \mid abSb \mid aAb$$

$$A \rightarrow b \mid S \mid aAAb$$



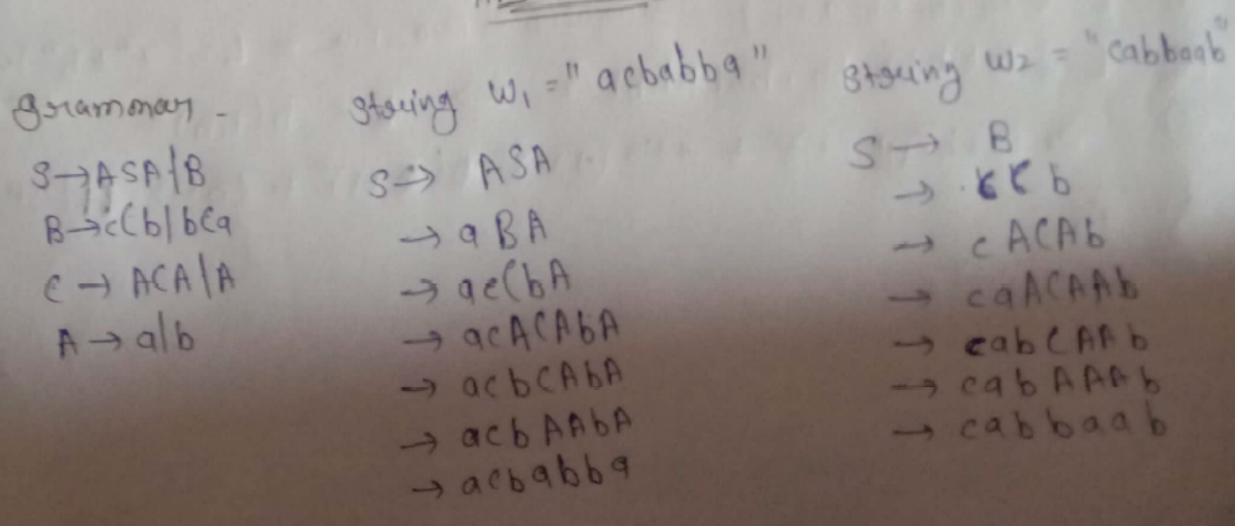
10. Construct any two strings of length 7 from the following grammar

$$S \rightarrow ASA \mid B$$

$$B \rightarrow cCb \mid bCa$$

$$C \rightarrow ACA \mid A$$

$$A \rightarrow a \mid b$$



11. Describe basic operations on language with example.

Operations on Language -

i) Union - If L_1 and L_2 are 2 regular language then union $L_1 \cup L_2$ will also be regular.
 For example: $L_1 = \{a^n \mid n > 0\}$ and $L_2 = \{b^n \mid n > 0\}$, $L_3 = L_1 \cup L_2 = \{a^n \cup b^n \mid n > 0\}$ is also regular.

ii) Intersection - Intersection of 2 $L_1 \cap L_2$ also regular.
 Ex: $L_1 = \{a^m b^n \mid m > 0 \text{ and } n > 0\}$ &
 $L_2 = \{a^m b^n \cup b^m a^n \mid n > 0 \text{ and } m > 0\}$
 $L_3 = L_1 \cap L_2 = \{a^m b^n \mid m > 0 \text{ and } n > 0\}$ is also regular.

iii) Concatenation - Concatenating $L_1 \cdot L_2$ also regular.
 Ex - $L_1 = \{a^m \mid m > 0\}$ and $L_2 = \{b^n \mid n > 0\}$
 $L_3 = L_1 \cdot L_2 = \{a^m b^n \mid m > 0 \text{ and } n > 0\}$ is also regular.

iv) Kleene Star - closure.
 If L is CFL then L^* is context free.
 Ex - $L = \{a^n b^n\}^*$
 Then $L^* = \{a^n b^n\}^*$ is also regular.

v) Complement - Let $L(G)$ is regular language.
 Ex - complement also regular.
 $L = \{a^n \mid n > 3\}$
 $L'(G) = \{a^n \mid n \leq 3\}$ is also regular.

12. Define the following with example.

- a) Alphabets b) String c) empty string d) substring e) Initial and final state

- (Q) a) Alphabet - It is defined as a finite non-empty set Σ of symbols.
 $\Sigma = \{a, b, \dots, z\}$
 Binary alphabet $\Sigma = \{0, 1\}$
 Small letters are used for alphabet.
- b) String - It is a finite sequence of symbols from alphabet.
- "0110" is a string over alphabet $\Sigma = \{0, 1\}$
 - "abc" is a string over $\Sigma = \{a, b, c\}$
 - String is represented by u, v, w, x, y, z, \dots in lower case.
- c) Empty String - A string with no symbol
- Denoted by ϵ / λ .
 - Ex - $w = \{\emptyset\}$ or $w = \{\text{null}\}$
- d) Substring - It is a part of a string.
- x is a substring of w if x appears consecutively with w .
 - The substring x can be at end or start or middle of string w .
 - Ex - $x = 111$
- e) Initial state - It is the starting state represented as q_0 .
Final state - It is the ending or final state where string ends.
- Ex - $q_0 \xrightarrow{1} q_1 \xrightarrow{0} q_2 \quad w = 10$

Initial state is shown by \leftarrow
 Final state is shown by $(*) / (0)$.

13. Derive the string "0001101110" using left most and right most derivation from the grammar

$$S \rightarrow 0B \mid 1A$$

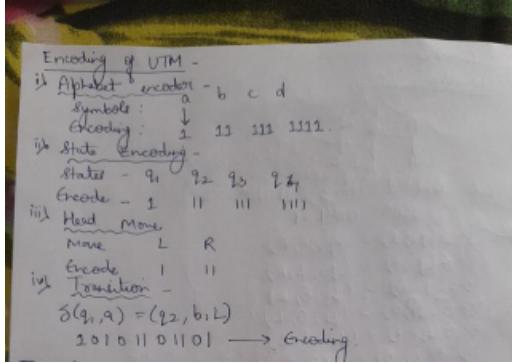
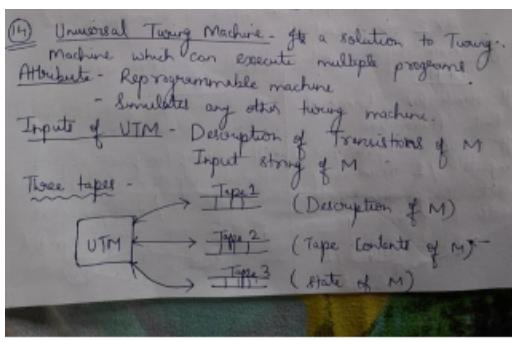
$$A \rightarrow 0S \mid 1AA \mid 0$$

$$B \rightarrow 1S \mid 0BB \mid 1$$

<u>(Q)</u> Given, $w = "0001101110"$	$S \rightarrow 0B \mid 1A$
LMD :-	$A \rightarrow 0/S \mid 1AA$
$S \rightarrow 0B$	$B \rightarrow 1/S \mid 0BB$
$\rightarrow 00BB$	RMD :- $S \rightarrow 0B$
$\rightarrow 000BB$	$\rightarrow 00BB$
$\rightarrow 0001BB$	$\rightarrow 00B1S$
$\rightarrow 00011B$	$\rightarrow 00B11A$
$\rightarrow 000110BB$	$\rightarrow 00B110$
$\rightarrow 0001101B$	$\rightarrow 000B0B110$
$\rightarrow 00011011S$	$\rightarrow 000B110$
$\rightarrow 000110111A$	$\rightarrow 000B10B110$
$\rightarrow 0001101110$	$\rightarrow 000B101110$
	$\rightarrow 0001101110$

(Q) An NFA (Non-Deterministic Finite Automaton)

14. Write a brief note on Universal Turing machine.



$d(q_1, a) = (q_2, b, L)$, $d(q_2, b) = (q_3, c, R), \dots$

10101101101 00 1101101110111011

15. Define and give example of each of the following

- a) Grammar
- b) Derivation Tree

Derivation Tree

A derivation Tree / Parse tree for a Context free Grammar G = (Vn, Sigma, P, S) is a tree satisfying the following conditions

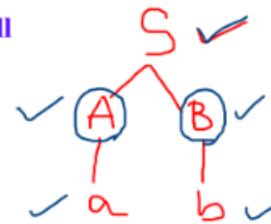
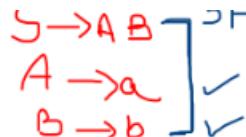
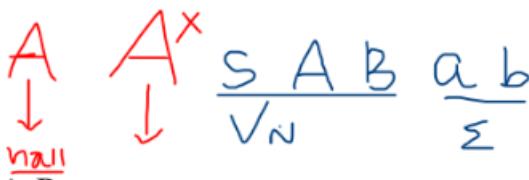
- 1) Every vertex has a label which is a variable or terminal or null
- 2) The root has label S
- 3) The label of an internal vertex is a variable.

4) Next Page

5) Next Page

Root = S

Internal Vertex = A, B



Derivation Tree
for deriving "ab"

A grammar is represented with 4 tuples

$S \rightarrow AB$

$A \rightarrow a$

$B \rightarrow b$

$$(V_n \cap \Sigma) \\ = \emptyset$$

(V_n , Σ , P , S)

V_n = Capital letters = Set of variables

Σ = Small letters = Set of input symbols /

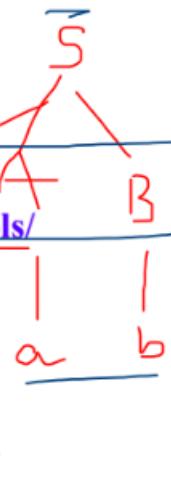
terminal symbols / non variables

P = production rule of the form

alpha \rightarrow Beta

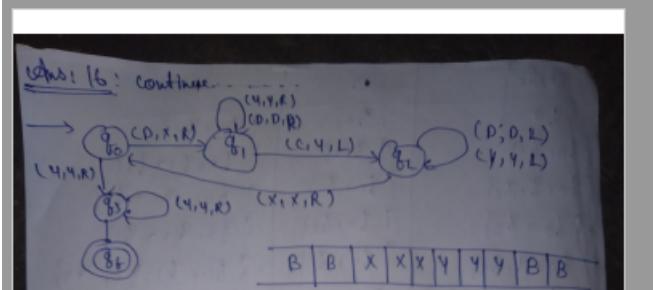
V_n intersection Σ = phi

S = special variable called starting symbol



16. Design Turing machine for accepting the pattern $L = \{ D^n C^n \mid n > 0 \}$

<u>Qns: 16:</u>	$L = \{ D^n C^n \mid n > 0 \}$
for a string DDD...ccc	$(q_0, X) \rightarrow (X, q_0, R)$
$(q_0, D) \rightarrow (X, q_1, R)$	$(q_0, D) \rightarrow (X, q_1, R)$
$(q_1, D) \rightarrow (D, q_1, R)$	$(q_1, Y) \rightarrow (Y, q_1, R)$
$(q_1, C) \rightarrow (Y, q_2, L)$	$(q_1, Y) \rightarrow (Y, q_2, L)$
$(q_2, D) \rightarrow (D, q_2, L)$	$(q_2, Y) \rightarrow (Y, q_2, L)$
$(q_2, X) \rightarrow (X, q_3, L)$	$(q_2, Y) \rightarrow (Y, q_3, L)$
$(q_3, D) \rightarrow (X, q_4, R)$	$(q_3, Y) \rightarrow (Y, q_4, R)$
$(q_4, D) \rightarrow (D, q_4, R)$	$(q_4, Y) \rightarrow (Y, q_4, R)$
$(q_4, Y) \rightarrow (Y, q_5, R)$	$(q_5, Y) \rightarrow (Y, q_5, R)$
$(q_5, C) \rightarrow (Y, q_6, L)$	$(q_6, Y) \rightarrow (Y, q_6, L)$
$(q_6, Y) \rightarrow (Y, q_7, L)$	$(q_7, Y) \rightarrow (Y, q_7, L)$



17. Represent the following by regular expression

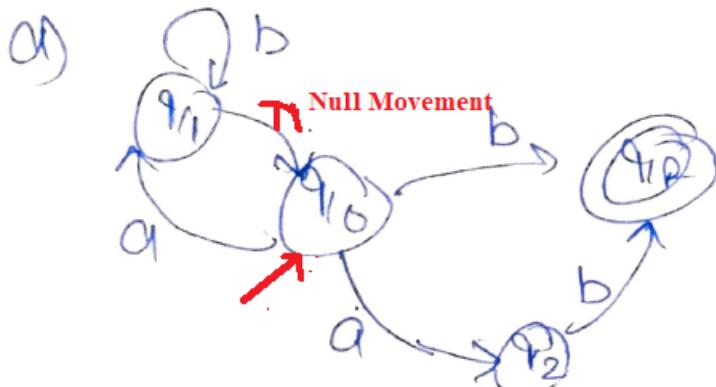
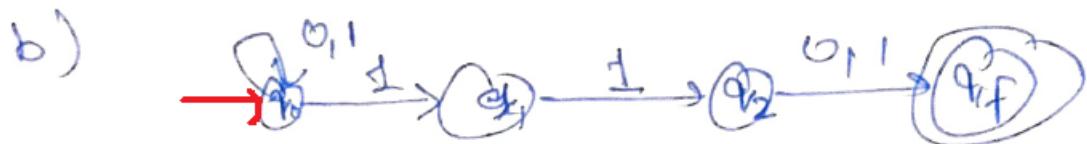
- $L = \{ab, b, bb, aaa\}$ where $\Sigma = \{a, b\}$
- $L = \{b, bbb, bbbbb, bbbbbbb, \dots\}$ where $\Sigma = \{b\}$
- All binary strings with 00 or 11 as substring where $\Sigma = \{0, 1\}$
- All Strings containing exactly one a and one b where $\Sigma = \{a, b\}$
- All strings starting with 1 and ending with 0 where $\Sigma = \{0, 1\}$

Answer No. 17

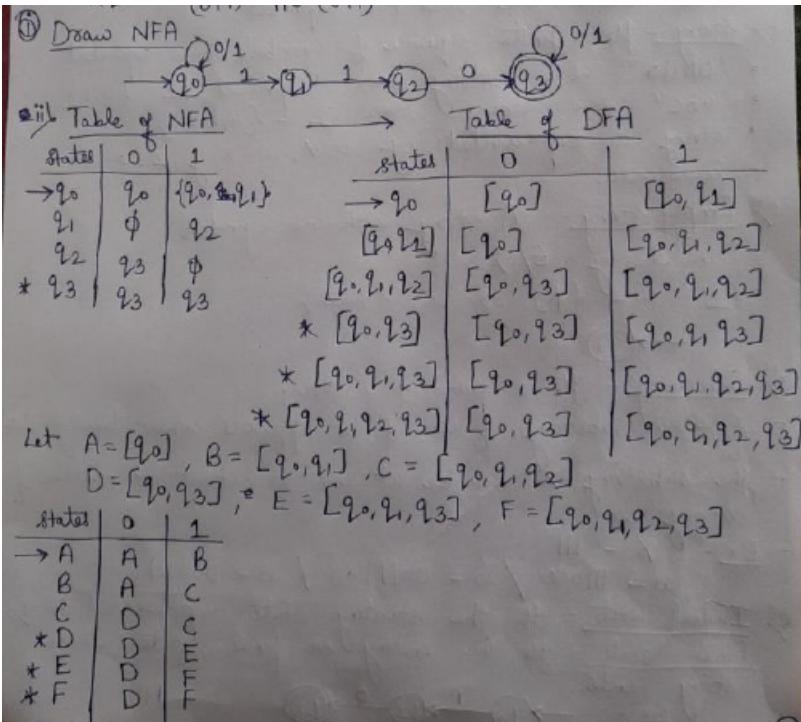
- (a) $ab + b + bb + \varnothing \varnothing \varnothing$
- (b) $b(bb)^*$
- (c) $(0+1)^*(00+11)(0+1)^*$
- (d) $ab + b \alpha$
- (e) $\varnothing (0+1)^*\varnothing$

18. Draw NFA for the following

- a) NFA for $(ab^*)^*b + ab$
- b) All strings of 0's and 1's whose 2nd and 3rd symbol from right is always "1" where $\Sigma = \{0,1\}$



19. Design DFA for accepting the substring "110" where $\Sigma = \{0,1\}$



Extra Calculation: AS last two rows are same E and F are same state.

States	0	1
$\rightarrow A$	A	B
B	A	C
C	D	C
$* D$	D	C
$* E$	D	E
$* F$	D	E

Same so D and E are same state

Final Table

States	0	1
$\rightarrow A$	A	B
B	A	C
C	D	C
$* D$	D	C

with 4 state only
