

operation on language

two language set let  $A$  &  $B$ .

union -  $A \cup B = \{ x \mid x \in A \text{ or } x \in B \}$   
 $\rightarrow$  taking all the elements in  $A$  &  $B$

concatenation -  $A \circ B = \{ xy \mid x \in A \text{ and } y \in B \}$   
joining one element from  $A$  and another from  $B$ .

Star (Closure) -  $A^* = \{ x_1 x_2 x_3 \dots x_n \mid n \geq 0 \text{ and each } x_i \in A \}$

example  $\rightarrow$

$$A = \{ pq, r \}$$

$$B = \{ t, uv \}$$

union,  $A \cup B = \{ pq, r, t, uv \}$

concatenation,  $A \circ B = \{ pqt, ruv, pq, uv \}$

$$A \circ B = \{ pqt, pqruv, rt, ruv \}$$

closure (star)  $A^* = \{ \epsilon, pq, r, pqr, r pq, pqpq \dots \}$   
 $\uparrow$  epsilon / empty str.

$\rightarrow$  we can take as many symbol as we like and we can join as we like

## Defn of a language in toc

Language  $\rightarrow$  language is a set of string.

when any language does not contain any string then it is said as empty language.

B.

operation on language -  
total 5 operation

1. union
2. intersection
- ~~3. difference~~
4. concatenation
5. Kleen closure

1. union

let two languages  $L_1$  &  $L_2$

~~$$L_1 = \{0, 1\}$$~~

$$L_1 = \{a^n \mid n > 0\}$$

~~$$L_2 = \{1, 0\}$$~~

$$L_2 = \{b^n \mid n > 0\}$$

$n > 2$

~~$$L_1 = \{aa\}$$~~

~~$$L_2 = \{bb\}$$~~

$$L_3 = L_1 \cup L_2$$

$$= \{a^n \cup b^n \mid n > 0\}$$

$\Rightarrow$  that means all the string from both language will be in same language after union

~~$$L_1 \cup L_2 = \{aa, bb\}$$~~

2. Intersection

$$L_1 = \{a^m b^n \mid n > 0 \text{ and } m > 0\}$$

$$L_2 = \{a^m b^n \cup b^n a^m \mid n > 0, m > 0\}$$

$$L_1 \cap L_2 = \{a^m b^n \mid n > 0 \text{ and } m > 0\}$$

$$n=2 \quad m=2$$

$$L_1 = \{a^2, b^2\}$$

1) union.  $\leftarrow$  all the elements from both language.

$$\text{Let } L_1 = \{p, q, r\}$$

$$L_2 = \{q, r, s\}$$

$$L_1 \cup L_2 = \{p, q, r, s\}$$

$\Rightarrow$  elements in  $L_1 \cup L_2 \in \underline{L_1}$  or  $\underline{L_2}$

2) intersection.

getting common strings.

$$L_1 = \{p, q, r\} \quad L_2 = \{q, r, s\}$$

$$L_3 = L_1 \cap L_2 = \{q, r\} \quad \text{where element of } L_3 \in \underline{L_1} \text{ and } \underline{L_2}.$$

3) concatenation.

$$L_1 = \{p, q, r\} \quad L_2 = \{q, r, s\}$$

$$\text{then } L_3 = L_1 \cdot L_2 \\ = \{p, q, r\} \{q, r, s\}$$

$$= \{pq, pr, ps, qq, qr, qs, rq, rs, ss\}$$



4) Kleene closure -

When operation is performed on a single language

$$\text{Let } L_1 = \{a, abc, ba\}$$

$$L_1^2 = \{a, abc, ba\} \cdot \{a, abc, ba\}$$

$$= \{aa, aabc, abaa, abca, abcb, abba, baa, baabc, baba\}$$

$$L_3 = \underline{L_1 \cdot L_1^2}$$

$$\text{but } L_1^0 = \{\epsilon\}$$

↑ epsilon → for null

$$\Rightarrow \text{Kleene closure of } L_1, L_1^* = \{\epsilon, L_1, L_1^2, L_1^3, \dots\}$$