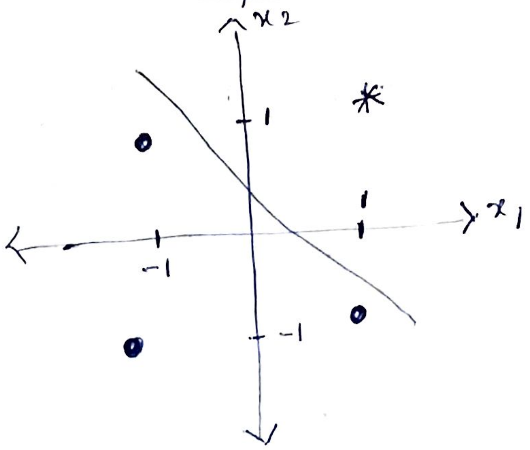
Linear separable problem / Inseparable

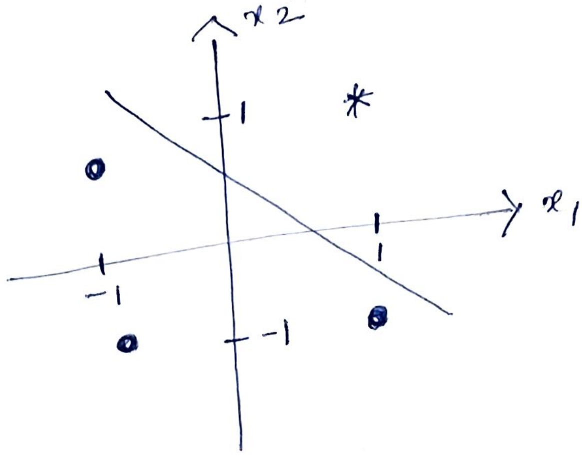
OR gate

x_1	x_2	y	
-1	-1	-1	class 1
-1	1	1	class 2
1	-1	1	
1	1	1	



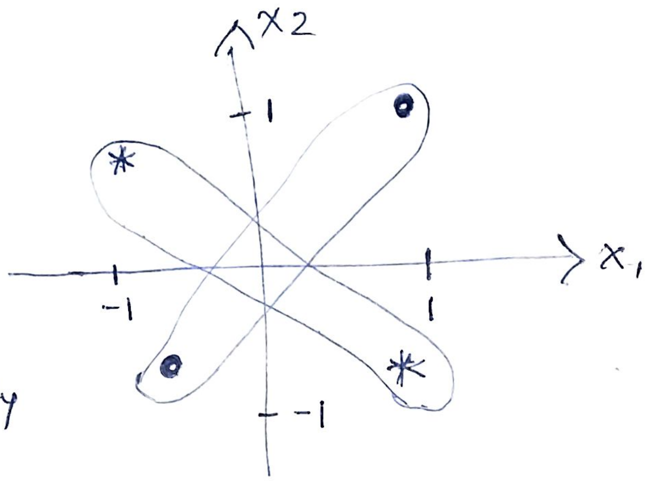
AND gate

x_1	x_2	y	
-1	-1	-1	class 1
-1	1	-1	
1	-1	-1	
1	1	1	class 2



XOR gate

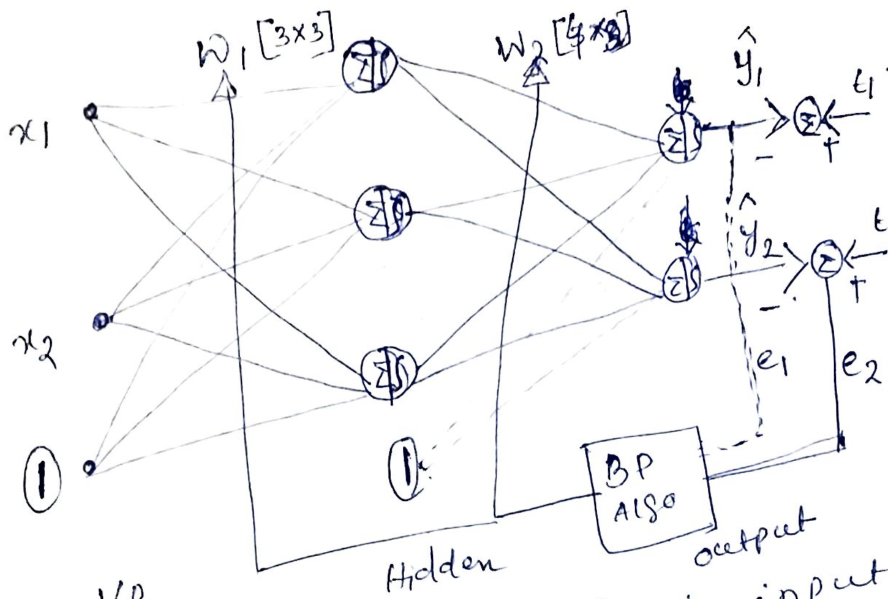
x_1	x_2	y	
-1	-1	-1	even parity
1	1	-1	
-1	1	1	odd parity
1	-1	1	



XOR- is a nonlinear separable problem or Linearly Inseparable problem.

Hence, we can use a MLANN for its classification.

Classification of XOR problem



x_0	x_1	x_2	target y	
1	.1	.1	.1	class 1
1	.9	.9	.1	
1	.1	.9	.9	class 2
1	.9	.1	.9	

$t = [1\ 0;\ 0\ 1]$

$W_1 = rand(3,3) - 0.5$

$W_2 = rand(4,2) - 0.5$

1) First apply the first input pattern, (.1, .1) to the input layer. The inputs are multiplied with the corresponding weights (W_1), summed together and passed through the activation function (tanh) to give output at hidden layer.

2) The output of hidden layer again get multiplied with the weights (W_2), summed and passed through the activation function (tanh) to give $\hat{y}_1$ and $\hat{y}_2$.

(3) The estimated outputs are compared with the target values to produce errors.

$$e_1 = t_1 - \hat{y}_1$$
$$e_2 = t_2 - \hat{y}_2$$

(4) Then the weights are updated using Back propagation rule. First the values of W_2 are updated and then W_1.

(5) Apply all the input patterns in the same way and updates the weights after each pass. This completes one experiment.

6) Repeat the experiment for few iterations until the MSE (mean squared error) is minimized.

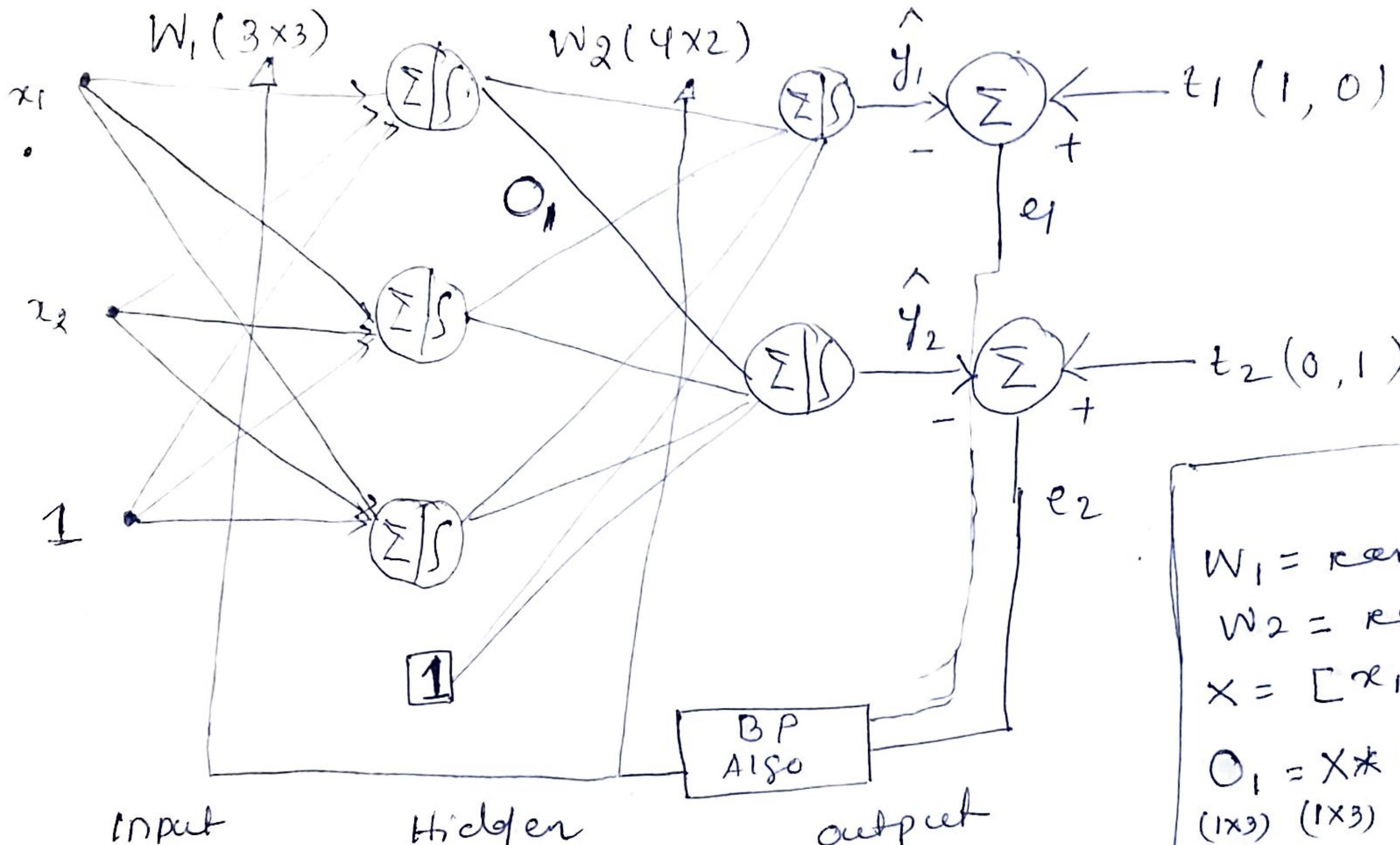
7) Once the MSE is minimized, stop the training process and freeze the final values of w_1 and w_2.

Testing Phase

1) Apply the first input (·1, ·1) to the input layer and calculate the value of $\hat{y}_1$ and $\hat{y}_2$ in the forward pass.

2) Since (·1, ·1) belongs class 1, the $\hat{y}_1$ will give a higher value than $\hat{y}_2$. Means the first neuron will fire.

3) Next time if the input is (·1, ·9) then $\hat{y}_2$ will give a higher value than $\hat{y}_1$. Means the second neuron will fire and indicates the input pattern belongs to class 2.



$W_1 = rand(3,3) - 0.5$

$W_2 = rand(4,2) - 0.5$

$X = [x_1, x_2, 1]$

$\underset{(1\times3)}{O_1} = \underset{(1\times3)}{X} * \underset{(3\times3)}{W_1}$; $O_{11} = [O_1, 1]$

$\underset{(1\times2)}{\hat{y}} = \underset{(1\times4)}{O_{11}} * \underset{(4\times2)}{W_2}$

$e = t - \hat{y}$

$\delta_1 = e * \left(1 - \frac{\hat{y}^2}{2}\right)$

$W_2 = W_2 + \mu * \delta_1 * O_{11}$

$\delta_2 = \delta_1 * W_2$

$W_1 = W_1 + \mu * \delta_2 * X$