

TOC

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→ Initial state :-

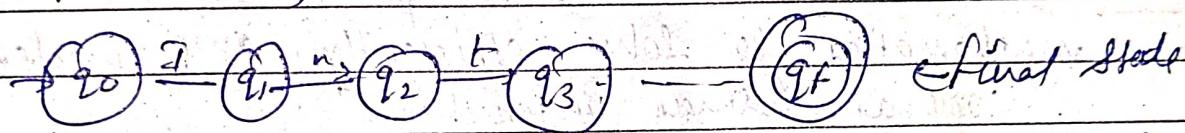
denoted by single circle and arrow

→ q_0

→ Final states - denoted by double circle



put a, b :



get Q contains no. of state

$$Q = \{q_0, q_1, q_2, \dots, q_3\}$$

→ initial state = q_0 . final state = q_3

→ input symbol = $\{0, 1\} \subseteq \Sigma$

transition: in which state which input it applied gives detail about path.

represent with delta

$\delta(q_0, 1) = q_2$

↑
previous state input next state

$$\delta(q_0, 0) = q_1$$

$$\delta(q_2, 0) = q_3$$

$$\delta(q_1, 0) = q_3$$

Table of previous transition:

rows = state

columns = input

	0	1	
1 $\rightarrow q_0$	q_1	q_2	language accepted
2 $\rightarrow q_1$	q_3	-	$L \{ 10, 100 \}$
3 $\rightarrow q_2$	q_3	-	
4 $\rightarrow q_3$	-	-	

cifiot22

Definition → It is a branch of computer science and mathematics to solve efficiently any problem without human interaction.

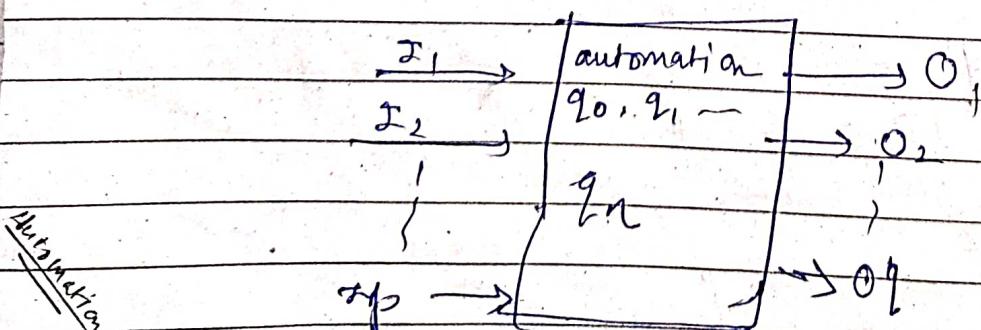
Automation:

An automation is defined as a system where energy, material and information are transferred and used for performing functions without direct participation of man.

or

It is a system where information and knowledge is transferred for solving problem without human interaction.

Model of automation →



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Language

Automata

Grammars

Symbol → Building block of the language
smallest unit of the language.

$$= \{ a, b, c, 0, 1, 2, \dots \}$$

Alphabate $\rightarrow \Sigma$ - sigma

finite set of symbols \rightarrow It can't be infinite

String \rightarrow Collection of Alphabates
Sequence of Alphabates

$$\Sigma \{a, b\} \rightarrow \{a, b, aa, bb, ba\}$$

length of the string \rightarrow How many symbols are present
in string

If length is 2 then

$$\Sigma \{a, b\} = \{aa, ab, ba, bb\}$$

$$\text{if } w = 01101 = |w| = 5$$

$$w = abcdbab = |w| = 6$$

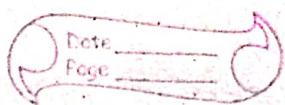
Language \rightarrow Collection of all the possible strings
language can be infinite also

Ex) string starts with a and with a.

Ex) L1 = { strings
of length
3 }

$$\zeta^0 = \underline{\epsilon} - \underline{\text{Epsilon}}$$

2 Length of string can be ^{also} 0 and it is defined by



Automata \rightarrow (model, machine)

→ By this it's a mathematical model which helps us to know whether the string is present in a part of language or not.

Power of Σ (sigma) :-

Minimum power can be 0.

Σ^0 = Set of all strings with length '0' : = ϵ , \emptyset

$$\zeta^1 = \underline{\quad} + \underline{\quad} \quad \zeta^1 = \{a, b\}$$

$$g^2 = \underline{\hspace{1cm}} \text{ " } \underline{\hspace{1cm}} \text{ " } \underline{\hspace{1cm}} \text{ " } \underline{\hspace{1cm}} \text{ " } \underline{\hspace{1cm}}$$

$$\Sigma \cdot \Sigma = \{aa, bb\} \cup \{a, b\} = \{aa, ab, ba, bb\}$$

$$z^3 = \underline{\hspace{2cm}} \quad \alpha = \underline{\hspace{2cm}} \quad '3' \text{ apd!}$$

$$\Sigma \cdot \Sigma \cdot \Sigma = \{a,b\} \{a,b\} \{a,b\}$$

$$= \{aa, ab, ba, bb\}^{\{a,b\}}$$

{aaa, aab, aba, abb, baa, bab, bba, bbb}

$$\Sigma = \{a, b\}$$

Σ^* (kleene closure)

\rightarrow (set of all strings of all length possible) on

→ Infinite language

\rightarrow infinite language
 $\{t \rightarrow \text{positive closure} \mid \text{closure} \rightarrow \text{all string possible except } t\}$

Identifying element
which is null

$$2 \boxed{\Sigma^* = \emptyset = \Sigma^+}$$

$\{S, Q, P, \delta\}$

Grammars: —

~~Constituted DFA in TOL~~ →

Grammar: —

A grammar G is defined as quadruple

$$G = \{V, T, P, S\}$$

- It's a standard way of representing a language
- Rules used to generate string of language L.

4 tuple:

$$(V, T, P, S)$$

variable terminal production rule starting symbol

SS

$$S \rightarrow SS / aAS / a,$$

$$A \rightarrow bA / ba$$

"aabaa", "aabaa"

$$\Rightarrow S \rightarrow SS$$

$$\Rightarrow a \cdot aAS$$

$$\Rightarrow a \cdot ab \cdot AS$$

$$\Rightarrow aabaa$$

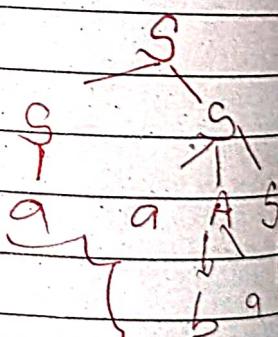
$$S \rightarrow SS$$

$$qS$$

$$a \cdot aAS$$

$$a \cdot abAS$$

$$aabaa$$



→ characteristic of automation are now described as

- (1) Input → at each of the discrete instant of time t_1, t_2, \dots, t_n the input value I_1, I_2, \dots which can take finite number of input to fixed value from input alphabet Σ .
- (2) Output → O_1, O_2, \dots, O_q are the output of model each of which can take a finite number of fixed value from an output Π .
- (3) State → at any instant of time the automation can be one of states q_1, q_2, \dots
- (4) State relation → the next state of an automation at any instant of time is determined by the present state and present input.
- (5) Output relation → the output is related to either state only depends or to both the input and state. Only one state can be taken as once on reading "input" symbol the automation moves to next state → see Aman pdf

★ Alphabets = finite non-empty set of symbols denoted by Σ

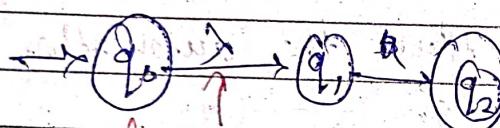
e.g. $\Sigma = \{a, b, c\}$ ← set of alphabet this is
 $\Sigma = \{0, 1\}$ known as ternary
 binary

★ String : A sequence of symbols from the alphabets.

Ex. $w = 01101$, where $w \in \{a, b, c\}$

$$\Sigma = \{0, 1\} \quad \Sigma = \{a, b, c\}$$

★ Empty string : represented by ϵ or λ
 also string with no symbol defined as
 empty string



here we are not consuming any symbol while going
 from q_0 to q_1 .

★ Length of a string - No. of symbols present in the string

$$\text{If } w = 01101, |w| = 5$$

$$w = abcba, |w| = 6$$

★ Substring : X is a substring if it appears consecutively
 in a string w .

X can be at front portion or of
 middle, end.

$$L_1 = \{\epsilon, a, aa, b\}$$

$$L_2 = \{bb, ab, cb\} \quad L_1 \cdot L_2 = \epsilon$$

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Operations on string :-

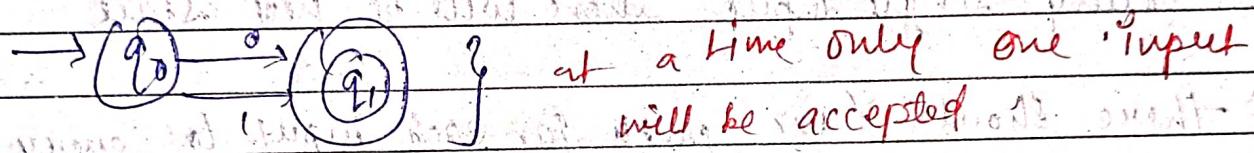
- (1) Concatenation :-

$$X = 00$$

$$Y = 11$$

AFTER concatenation, $X+Y = 0011$ (not sum of numbers)

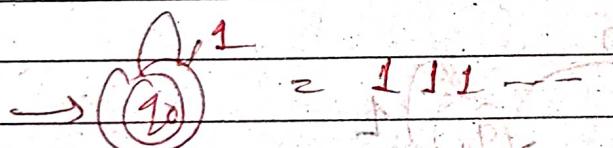
- (2) OR \rightarrow This makes parallel path while transition



- (3) Closure (Repetition) :- It can be repeated many times no limit

→ Closure means repetition of any symbol

→ denoted as ϵ^* or 1^*



set of this = { null, 1, 11, 111, ... }

If we remove null it'll become equivalence and denoted by $1^+ = \{ 1, 11, 111, \dots \}$

DFA to NFA[DFA & NFA]

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1. DFA it refers to Deterministic finite Automaton

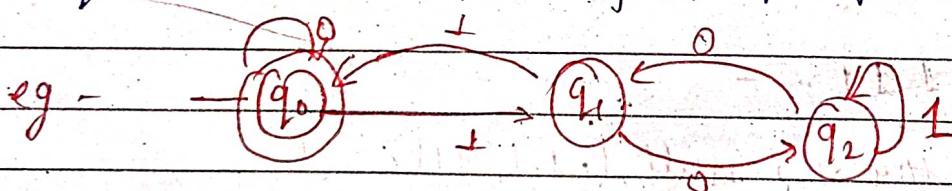
A finite automaton is said to be deterministic if corresponding to an input symbol there is a single resultant state i.e. there is only one transition.

or
for every single input there will be next state.

→ there should have path for each input to every next state

⇒ every input there is 1 next state

→ automation is represented with M , Q is set of state, Σ is set of alphabets. δ shows mapping or transition q_0 is for initial state f is for final state.



$$Q = \{q_0, q_1, q_2\} \quad \Sigma = \{0, 1\} \quad \text{check } w = 0010010$$

Accepted string

$$\delta(q_0, 0010010)$$

$$\delta(q_1, 10)$$

$$\delta(q_0, 010010)$$

$$\delta(q_0, \lambda)$$

$$\delta(q_0, 10010)$$

$\delta(q_0, \lambda)$ — when nothing left in string take λ and if

$$\delta(q_1, 0010)$$

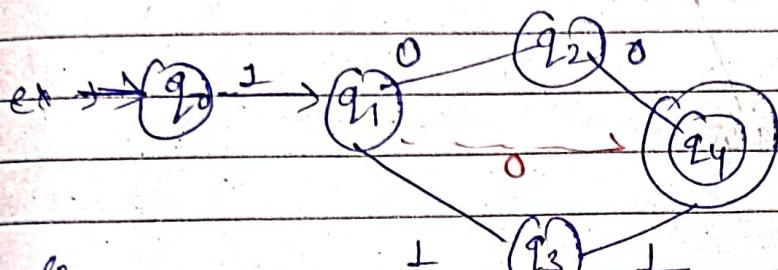
state is == final state

$$\delta(q_2, 010)$$

then we found the string

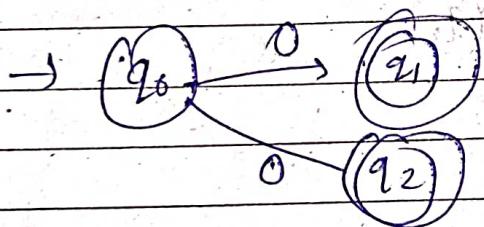
2. NFA (Non-deterministic Finite automaton)

It is said if there is more than one possible transition from one state on same input.



Since there is two path for $q_1 \rightarrow q_4$ this NFA
 → even if in Σ there exist more symbol which is not present in automata then it is NFA

→ this also has same 5 tuples as DFA but it does have difference in δ (delta)



$$\delta(q_0, a) \rightarrow \{q_1, q_2\}$$

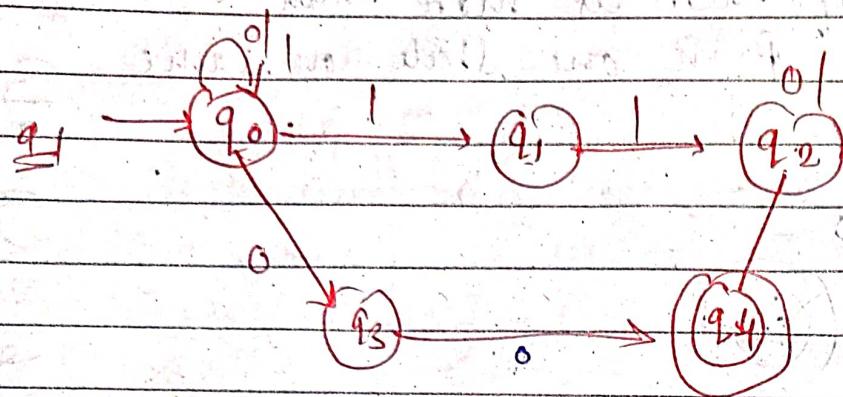
$Q = \{q_0, q_1, q_2\}$ then will get power set for all the state

$$Q \times \Sigma = Q^{\Phi} = \{ \{\emptyset\}, \{q_1\}, \{q_0\}, \{q_2\}, \{q_0, q_1\}, \{q_0, q_2\}, \{q_1, q_2\}, \{q_0, q_1, q_2\} \}$$

$$\{a, b, c\}$$

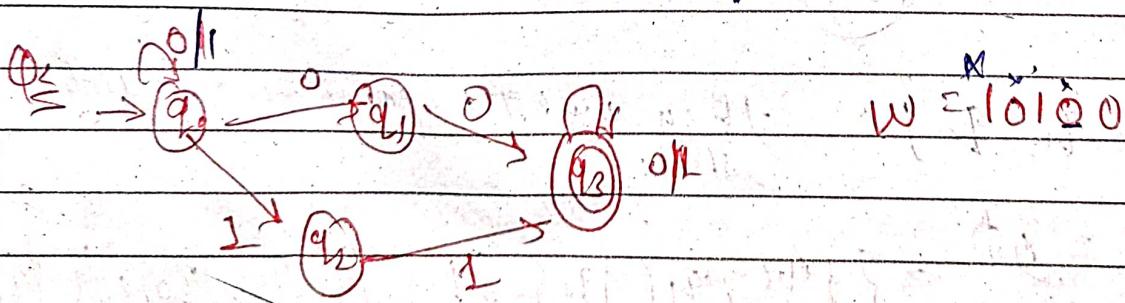
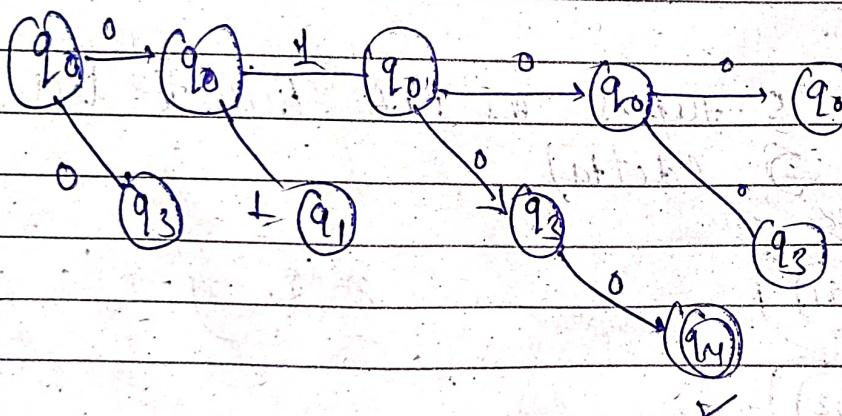
$$\text{powers} = \{ \emptyset, a, b, c, ab, ac, bc, abc, a^2, b^2, c^2 \}$$

Checking whether string is accepted or not

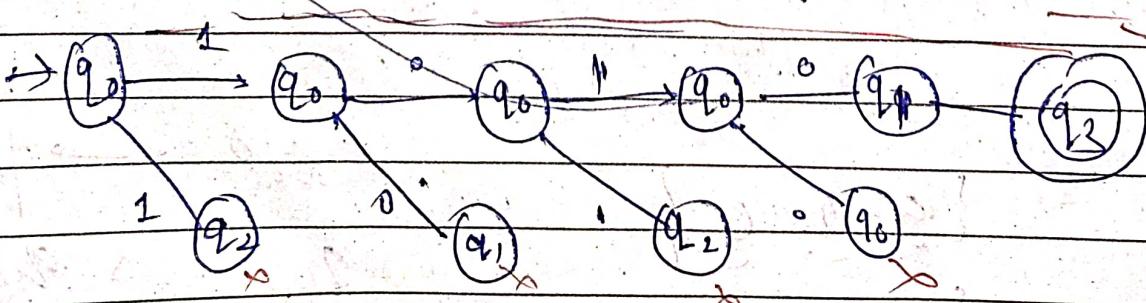


$$w = 0100$$

= transition system for a ~~NDFA~~ non-deterministic aut



$$w = 10100$$



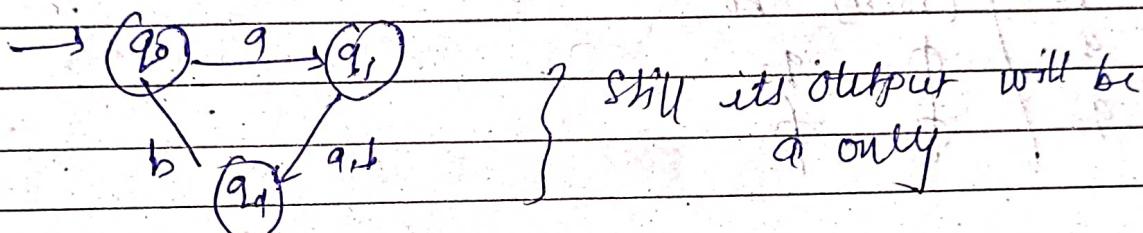
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Conversion of NFA to DFA

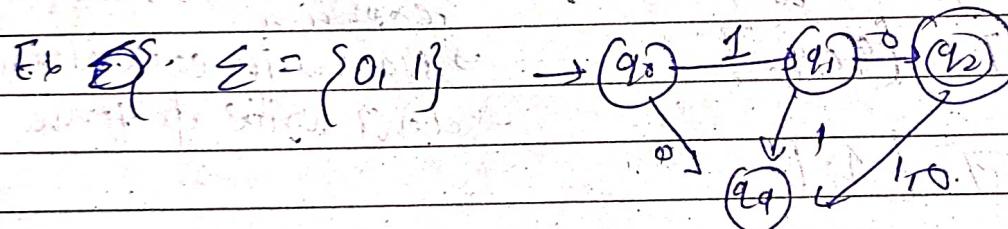
Given $\Sigma = \{a, b\}$

transitional system - $\rightarrow (q_0) a \rightarrow (q_1)$

Since its NFA we need to leave path with all input and send them to dummy state $(q_d) \rightarrow \{\phi\}$



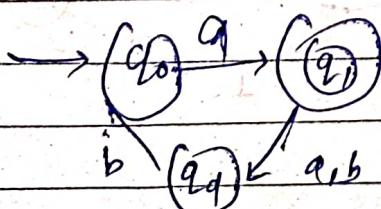
This is easy to implement when no multiple path available in NFA.



Now steps of conversion

① Convert into tabular form

State \rightarrow row symbol \rightarrow column
 Σ



tabular

q_0	a	b
q_1	q_2	ϕ
q_2	q_3	ϕ

2nd step copy first row as it is and then evaluate the unevaluated expressions

$$\begin{array}{c} \text{ex-2} \\ \Rightarrow \\ \begin{array}{c} 0 \\ \rightarrow \\ q_0 \end{array} \end{array} \rightarrow \begin{array}{c} 0 \\ \rightarrow \\ q_0 \end{array} \quad \begin{array}{c} 1 \\ \rightarrow \\ q_1 \end{array} \quad \begin{array}{c} 1 \\ \rightarrow \\ q_2 \end{array}$$

tabular form

$$\begin{array}{c} 0 \quad 1 \\ \rightarrow q_0 \quad | \quad [q_0, q_1] \quad q_0 \\ q_1 \quad | \quad \emptyset \quad q_2 \\ \# q_2 \quad | \quad \emptyset \quad \emptyset \end{array}$$

Now copy first row as it

$$\begin{array}{c} 0 \quad 1 \\ \rightarrow q_0 \quad | \quad [q_0, q_1] \quad q_0 \\ [q_0, q_1] \quad | \quad [q_0, q_1] \quad [q_0, q_2] \\ \# [q_0, q_2] \quad | \quad [q_0, q_1] \quad [q_0] \end{array}$$

→ evaluate the unevaluated expres.
→ start with each element
→ don't write \emptyset term

now changing the variable

$$q_0 \rightarrow A \quad [q_0, q_1] \rightarrow B \quad [q_0, q_2] \rightarrow C$$

$$\begin{array}{c} 0 \quad 1 \\ \rightarrow A \quad | \quad B \quad A \\ B \quad | \quad B \quad C \\ \# C \quad | \quad B \quad A \end{array}$$



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Regular Expression

The regular expressions are useful for representing certain sets of strings in an algebraic fashion.

→ It is a common representation of all strings in a language.

Set $L = \{ \text{all accepted string} \}$

→ Initial state can be also final state

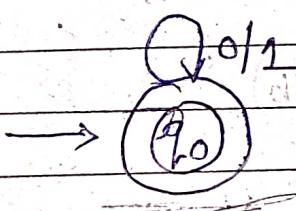
Ex: $a = e^* = \{ \text{null, } a, aa, \dots \}$
collection of all strings - $\Rightarrow (a^*)^*$

① $\Sigma = \{0, 1\}$

write RE for binary no.

→ at a time we 0 or 1

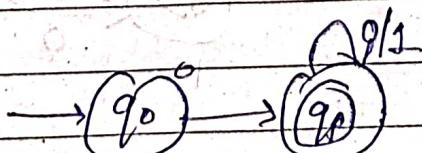
$(0+1)^*$



~~min/wt = 1~~

② Binary numbers starting with string 0.

$0(0+1)^*$

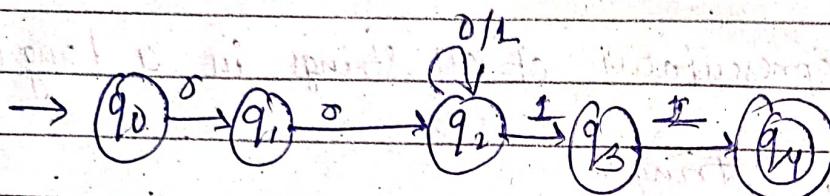


~~min/wt = 1~~

length of the min string

③ Binary nos starting with 00 & ending with 11

$$\Rightarrow 00 (0+1)^* 11$$



Length of min string $\underline{\min L} = 4$

④ Binary string with 111 as a substring

$$\Rightarrow (0+1)^* 111 (0+1)^*$$

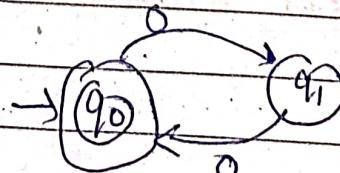
⑤ $\Sigma = \{a, b\}$

String of $a+b$ with ~~at least~~ atleast two a's

$$\Rightarrow (a+b)^* \cdot qa \cdot (a+b)^*$$

⑥ $\Sigma = \{0\}$ even no. of 0's

$$(00)^*$$



doubt ?

Even number of a's followed by odd no' of b's

$$(aa)^* b (bb)^*$$

③ $\Sigma = \{0,1\}$ At least two 0's anywhere not mandatory $x001$

$$(0+1)^* 0 (0+1)^* 0 (0+1)^*$$

④ $\Sigma = \{0,1\}$

At least one 0 anywhere

$$- (0+1)^* 0 (0+1)^*$$

⑤ $\Sigma = \{0,1\}$ All strings of binary no. where 1st & last bit is always 1.

$$1 (0+1)^* 1$$

⑥ $\Sigma = \{0,1\}$ 2nd bit from left is 1 & 3rd bit from right is 0.

Ans
bit 0/1

$$(0+1) \underline{1} (0+1)^* \underline{0} \underline{(0+1)} (0+1)$$

$$\min(n) = 5$$

⑦ 1st bit from left is 0 and 2nd bit right is 1

$$\underline{0} (0+1)^* \underline{1} (0+1) \quad (n)_{\min} = 3$$

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★ Operations on Languages

$$L_1 = \{ E, a, aa \}$$

$$L_2 = \{ a, aa \}$$

↓
set

↓
all possible
strings

→ Regular expression means
common representation



① Concatenation → means with itself.

If L_1 and L_2 are two regular languages their concatenation will also be regular.

Let - $L_1, L_2 \in \Sigma^*$

$$L_1 L_2 = \{ xy \mid x \in L_1 \text{ and } y \in L_2 \}$$

$$\text{Ex} \rightarrow \Sigma = \{ a, b \}$$

$$L = \{ a, b \} = L^1 \rightarrow \text{concatation with itself (original string)}$$

L^0 = null (not taken any symbol or string)

$$L^2 = L \cdot L^1 = \{ a, b \} \{ a, b \} = \{ aa, ab, ba, bb \}$$

$$L^3 = L^2 \cdot L = \{ aa, ab, ba, bb \} \{ a, b \}$$

$$= \{ aaaa, aaab, aabb, abbb, bbbb, bbaa, bbba, bbaa, bbba \}$$

$$bba, bbb \}$$

$L^i = L^{i-1} \cdot L$ For all $i \geq 1$

Stone Closure :-

$$L^* = L^0 U L^1 U L^2 \dots$$

$L\{a\}$

Symbol = a

= {null, a, aa, aaa, ...} - with null value

Positive closure:-

$$(a) L^+ = \{ L^1 U L^2 U \dots \}$$

$$\downarrow = \{a, aa, aaa, \dots\}$$

without null.

(The set Σ^+ is the infinite set of all possible strings of all possible lengths over Σ excluding λ.)

Representation - $\Sigma^+ = \Sigma_1 U \Sigma_2 U \Sigma_3 U \dots$

$$\text{Ex - If } \Sigma = \{a, b\}$$

$$\Sigma^+ = \{a, b, aa, ab, ba, bb, \dots\}$$

Closure properties of regular set :-

Union :-

The union of the regular set is also regular.

$$RE_1 = a(aa)^* \quad L_1 = \{a, aaa, aaaaa, \dots\}$$

$$RE_2 = (aa)^* \quad L_2 = \{\phi, aa, aaaa, aaaaaaaaa, \dots\}$$

$$L_1 U L_2 = \{\phi, a, aa, aaa, aaaa, aaaaa, aaaaaa, \dots\}$$

$$RE_3 = (a^g)^*$$

(2) Intersection: — The intersection of regular set is also regular

$$RE_1 = (aa)^* \quad L_1 = \{ \text{null}, aa, aaaa, \dots \}$$

$$RE_2 = aa(aa)^* \quad L_2 = \{ aa, aaaa, \dots \}$$

$$L_1 \cap L_2 = \{ aa, aaaa, aaaaaa, aaaaaaaa, \dots \}$$

$$RE_3 = aa(aa)^*$$

(3) Closure: —

Closure of regular set is also regular

$$RE = a^* \quad L = \{ \text{null}, a, aa, aaa, \dots \}$$

$$(a^*)^* = \{ \text{null}, a, aa, aaa, \dots \}^*$$

$$= \{ \text{null}, a, aa, aaa, aaaaa, \dots \}$$

\downarrow

a^*

(4) Difference: — $A - B$

Difference of a regular set is also regular.

$$RE_1 = a(a^*) = L_1 \{ a, aa, aaaa, \dots \} = (a, b, c) - (b, c)$$

$$RE_2 = (aa)^* \quad L_2 = \{ \text{null}, aa, aaaa, \dots \} = (a)$$

$$L_1 - L_2 = \{ a, aaa, aaaaa, \dots \}$$

$$RE_2 = a(aa)^*$$

(5) Reverse:-

Reverse of regular expression is also regular.

$$L = \{01, 10, 11, 00\}$$

$$RE = 01 + 10 + 11 + 00$$

$$L^R = \{10, 01, 11, 00\}$$

$$RE = 10 + 01 + 11 + 00$$

(6) Complement:-

Complement of regular expression is also regular.

$$RE(L) = a(aa)^*$$

$$L = \{a, aaa, aaaa, \#a's, \#a's \dots\}$$

Complement of $L = \{\text{null}, aa, aaaa, aaaaa, \dots\}$

$$RE = (aa)^*$$

(7) Concatenation is also regular:-

$$RE_1 = (0+1)^* 0$$

$$RE_2 = 0 L (0+1)^*$$

$$L_1 L_2 = (0+1)^* 0 0 L (0+1)^*$$

$$a(a)^*$$

$$aa,$$

$$a(a), a, aaaa,$$

$$(a, aa, aaaa, aaaa)$$

$$\rightarrow aa(aa)^*$$

$$\underline{aa}(\text{null}, aa, aaaa, aaaaa, \dots)$$

$$\underline{aa}, \underline{aaa}, \underline{aaaa}, \dots$$

$$a \{aa\}^*$$

$$\rightarrow \text{null}, a, aa, \{aa\}, aaaa \}$$

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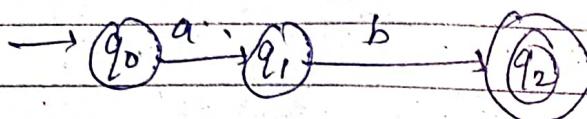
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~~Grammars~~

Rules used to generate strings of language L.

Example $\rightarrow L = \{a, b\}$

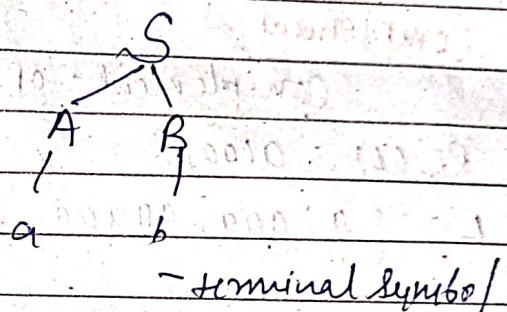
RE = a.b



derivation tree / parse tree

$S \rightarrow A B$
 $A \rightarrow a$
 $B \rightarrow b$

} example of
grammar



- terminal symbols /

Definition :- It is represented with 4 tuple,

(S, V_n, Σ, P)

AU
Capital letters

Small letters

production rule

S = starting symbol

V_n = set of variables (capital L) (S,A,B)

Σ = set of input signal or terminals (small L) (a,b)

P = production rule (no. of arrow that much production rule)

$A \rightarrow d$ where A is variable & $d \in (V_n \cup \Sigma)^*$

Q → Derive the string from the grammar!

$$S \rightarrow SS \mid aAS \mid a$$

$$A \rightarrow SbA \mid ba$$

"aabaa"

$$\Rightarrow S \rightarrow SS$$

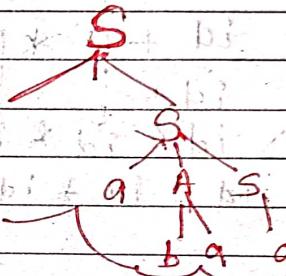
$$\rightarrow aS \quad (S \rightarrow a)$$

$$\rightarrow aAAS \quad (S \rightarrow aAS)$$

$$\rightarrow aa\overset{A}{b}AS \quad (A \rightarrow SbA)$$

$$\rightarrow aabaa \quad (S \rightarrow a)$$

Derivation tree.



$$Q \rightarrow S \rightarrow aC\bar{b}$$

$$C \rightarrow aC\bar{b}$$

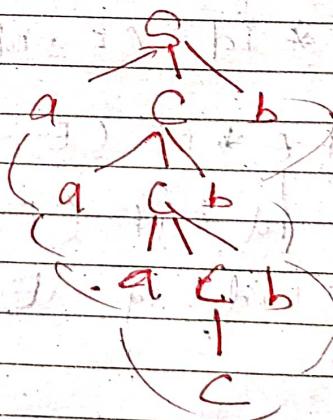
$$C \rightarrow C \quad "a\overset{a}{a}a\overset{C}{\cancel{C}}\overset{\bar{b}}{b}b\overset{b}{b}"$$

$$S \rightarrow aC\bar{b}$$

$$\rightarrow a\overset{a}{a}C\bar{b}\overset{\bar{b}}{b} \quad (C \rightarrow aC\bar{b})$$

$$\rightarrow a\overset{a}{a}a\overset{C}{\cancel{C}}\overset{\bar{b}}{b}b\overset{b}{b} \quad (C \rightarrow aC\bar{b})$$

$$\rightarrow a\overset{a}{a}a\overset{\bar{b}}{b}b\overset{b}{b} \quad (C \rightarrow C)$$



left to right → left most derivation



$$Q \rightarrow E \rightarrow E+E$$

$$E \rightarrow E * E$$

$$E \rightarrow id$$

"id + id * id"

$$\rightarrow E \rightarrow E+E$$

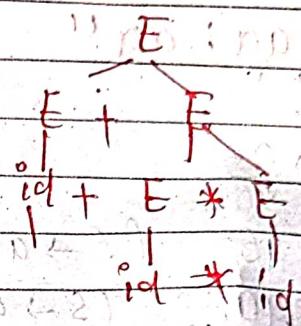
$$\rightarrow id + E \quad (E \rightarrow id)$$

$$\rightarrow id + E * E \quad (E \rightarrow E * E)$$

$$\rightarrow id +$$

$$\rightarrow id + id * id \quad (E \rightarrow id)$$

$$\rightarrow id + id * id \quad (E \rightarrow id)$$



→ Now right to left! →

for same question

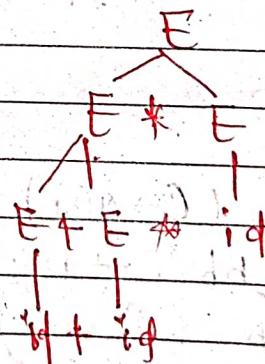
$$E \rightarrow E * E$$

$$\rightarrow E * id \quad (E \rightarrow id)$$

$$\rightarrow E + E * id \quad (E \rightarrow E+E)$$

$$\rightarrow id + id * id \quad (E \rightarrow id)$$

$$\rightarrow id + id * id \quad (E \rightarrow id)$$



Right to left

$\emptyset \rightarrow S \rightarrow S_b S | a$

(abababa)

$\rightarrow S \rightarrow S_b S$

$\rightarrow S_b a \quad (S \rightarrow a)$

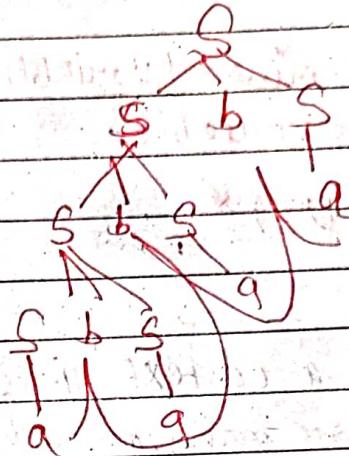
$\rightarrow S_b S b a \quad (S \rightarrow S_b S)$

$\rightarrow S_b a b a \quad (S \rightarrow a)$

$\rightarrow S_b S b a b a \quad (S \rightarrow S_b S)$

$\rightarrow S_b a b a b a \quad (S \rightarrow a)$

$\rightarrow a b a b a b a \quad (S \rightarrow a)$



= Left to right

$\rightarrow S \rightarrow S_b S$

$\rightarrow a^l b^l S \rightarrow (S \rightarrow a)$

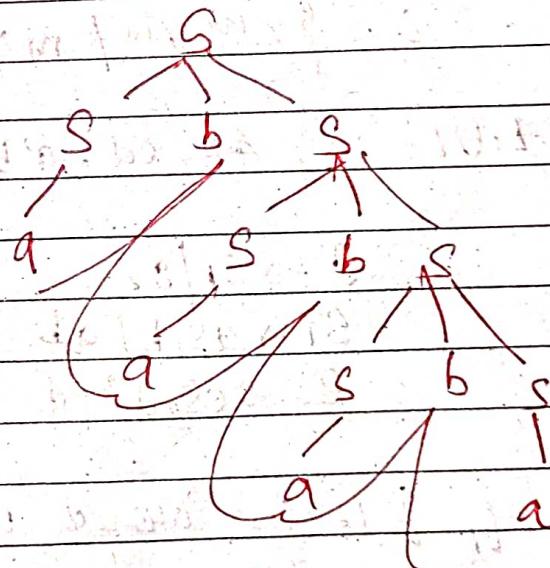
$\rightarrow a b S b S \rightarrow (S \rightarrow S_b S)$

$\rightarrow a b a b S \rightarrow (S \rightarrow a)$

$\rightarrow a b a b a b S \rightarrow (S \rightarrow S_b S)$

$\rightarrow a b a b a b a S \rightarrow (S \rightarrow a)$

$\rightarrow a b a b a b a b a \quad (S \rightarrow a)$



$\Rightarrow G_1$ is ambiguous (Because tree is different)

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Definition of Grammar: A Grammar in which each production is of the form

* Properties of context free grammar:-

$$A \rightarrow \alpha$$

α \rightarrow Terminal / Variable

A is a variable.

$$\alpha \in (V \cup U^*)^*$$

{ If L_1 is a context free grammar and L_2 is also a context free grammar then

$L_1 \cup L_2$ is also a context free grammar (closed)

$$Ex \rightarrow L_1 = \{ a^n b^n \mid n \geq 1 \} \rightarrow S_1 \rightarrow ab a^2 b^2 a^3 b^3$$

$$L_2 = \{ c^m d^m \mid m \geq 1 \} \rightarrow S_2 \rightarrow C d c^2 d^2 c^4 d^4$$

$$L_1 \cup L_2 = \{ ab, cd, a^2 b^2, c^2 d^2 \}$$

$$S \rightarrow S_1 | S_2$$

$$S_1 \rightarrow aS, b | ab$$

$$S_2 \rightarrow cS_2 d | cd$$

2) $L_1 \& L_2$ is closed under concatenation

$$L_1 L_2 = \{ abcde, abc^2d^2, abc^3d^3, \dots, a^4b^4c^4d^4 \}$$

$$S \rightarrow S_1 S_2$$

$$S_1 \rightarrow aS, b | ab$$

$$S_2 \rightarrow cS_2 d | cd$$

3) kleene closure

$$S_1 \rightarrow aSb$$

$$S_1 \rightarrow \text{null}$$

$$L = \{ a^n b^n \mid n \geq 0 \} \quad \text{null included}$$

$$L = \{ \text{null}, ab, a^2b^2, a^3b^3, \dots \}$$

$$\Rightarrow L^* = \{ ab, aba^2b^2, a^4b^4; abababab \dots \}$$

Grammar $\rightarrow S_1 \rightarrow S_1 S_1$

4) Intersection of $L_1 \cap L_2$ may or may not be context free Grammars

$$\text{Let } L_1 = \{ a^n b^n c^m \mid n, m > 0 \}$$

$$L_2 = \{ a^m b^n c^n \mid n, m > 0 \}$$

$$L_1 \cap L_2 = \{ abc, a^2b^2c^2, a^3b^3c^3, \dots \}$$

$$= \{ a^n b^n c^n \mid n > 0 \}$$

It does not belong to a C.F.G.

abc

$a^2b^2c^2$

$a^3b^3c^3$

a^2b^2c

$a^2b^3c^3$

abc

abc

abc

a^2b^2c

$a^2b^2c^2$

a^3b^3c

$a^3b^2c^2$

$a^3b^3c^3$

a^3b^3c

$a^3b^3c^2$

$a^3b^3c^3$

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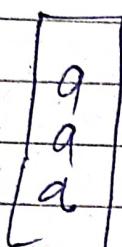
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PUSH Down Automation

$a^n b^n \rightarrow$ should be same, take as same as b).

A regular set fails here and this is solved by using push down automation.

Stack will be used for this



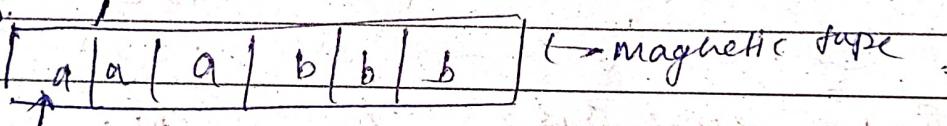
One by one will pop from stack and take
the same no. of b

For memorising (a)

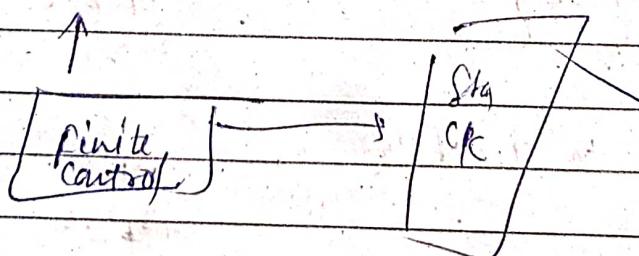
$a^n b^n$

any one from them can be pushed into
stack

model ->



read/write head



④ finite control

$$(, ,) = (,)_j$$

(current state, current input, top of stack) = (next state, top of stack)

$q_0 b^3$

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Initially q_0

1st a $(q_0, a, z_0) \Rightarrow (q_0, a)$

(push)

a
a
a
z_0

Initially null since same process repeated

for remaining a

2: $(q_0, a, a) \Rightarrow (q_0, a)$ push

3: $(q_0, a, a) \Rightarrow (q_0, aa)$

Now from b reading b POP(a)

$(q_0, b, a) \Rightarrow (q_1, \lambda)$

(for pop)

$(q_1, b, a) \Rightarrow (q_1, \lambda)$

for all memory b

$(q_1, \lambda, z_0) \Rightarrow (q_F)$ final state reached

Design of PDA For $L = a^n b^n, n \geq 0$

(A) $a^3 b^3$ pattern

Rule 1: $\delta(q_0, a, z_0) \rightarrow (q_0, az_0)$

Rule 2: $\delta(q_0, a, a) \rightarrow (q_0, aa)$

Rule 3: $\delta(q_0, a, a) \rightarrow (q_0, aa)$

<u>q</u>	/
a	/
a	/
z ₀	/

Rule 4: $\delta(q_0, b, a) \rightarrow (q_1, \lambda)$

Rule 5: $\delta(q_1, b, a) \rightarrow (q_1, \lambda)$

Rule 6: $\delta(q_1, a, a) \rightarrow (q_1, \lambda)$

Rule 7: $\delta(q_1, \lambda, z_0) \rightarrow f$

push Down Automation for $a^n b^n$

$a^n b^{2n} \rightarrow a a b b b b b$ ($n=2$)

single a - two b's

now for first

$\delta(q_0, a, z_0) = (q_0, az_0)$

<u>q</u>	/
z ₀	/

$\delta(q_0, a, a) = (q_0, aa)$

<u>q</u>	/
a	/
z ₀	/

{(a) push}

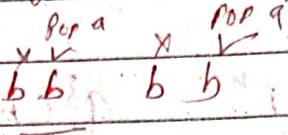
now if we find b the state will change but will skip that

$$\delta(q_0, b, a) = (q_1, a)$$

$$\delta(q_1, b, a) = (q_2, \lambda)$$

$$\delta(q_2, b, a) = (q_1, a)$$

$$\delta(q_1, \lambda, z_0) = (q_f, z_0)$$



03/11/22 Definition of PDA

$S \rightarrow$ set of finite non-empty stack symbols

$$\delta(\emptyset, \Sigma, S) \rightarrow (\emptyset, \Sigma^*, S^*)$$

mean many symbols appears

Transition diagram of PDA

$a^n b^n \mid n > 0$

$$\delta(q_0, a, z_0) \rightarrow (q_0, az_0)$$

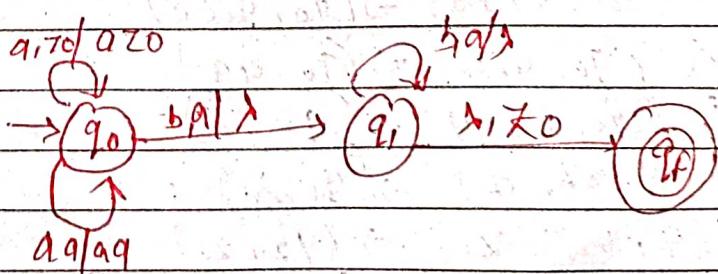
now drawing automata

$$\delta(q_0, a, a) \rightarrow (q_0, aa)$$

$$\delta(q_0, b, a) \rightarrow (q_1, \lambda)$$

$$\delta(q_1, b, a) \rightarrow (q_1, \lambda)$$

$$\delta(q_1, \lambda, z_0) \rightarrow q_f$$



Transition diagram for the pattern!

$a^n c b^n$

$a^3 c b^3$

a
a
a
z_0

$$\delta(q_0, a, z_0) \rightarrow (q_0, a z_0)$$

$$\delta(q_0, a, a) \rightarrow (q_0, aa)$$

$$\delta(q_0, c, a) \rightarrow (q_1, ca)$$

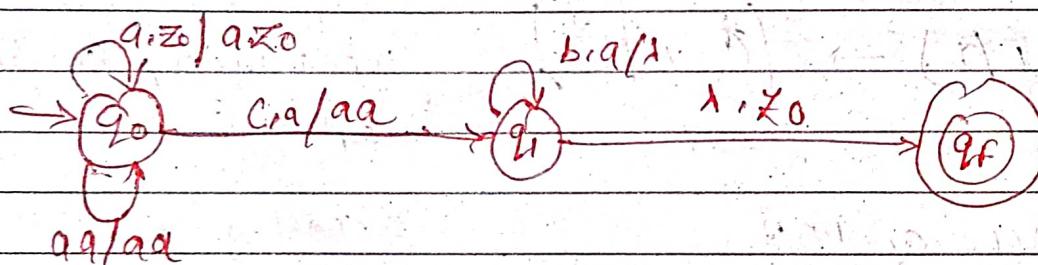
C. is only reading
and moving forward

$$\delta(q_1, b, a) \rightarrow (q_1, \lambda)$$

If it's not associated with
pop or push so it can be used
in b.

$$\delta(q_1, b, a) \rightarrow (q_1, \lambda)$$

$$\delta(q_1, \lambda, z_0) \rightarrow q_f$$



Pattern $a^n b^n c^m$

$$\delta(q_0, a, z_0) \rightarrow (q_0, a z_0)$$

$$\delta(q_0, a, a) \rightarrow (q_0, aa)$$

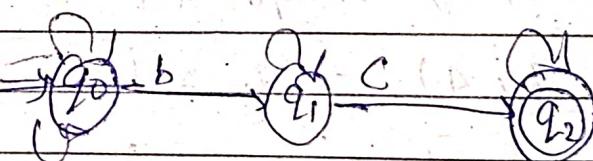
$$\delta(q_0, b, a) \rightarrow (q_1, \lambda)$$

$$\delta(q_1, b, a) \rightarrow (q_1, \lambda)$$

$$\delta(q_1, c, z_0) \rightarrow (q_2, z_0)$$

$$\delta(q_2, c, z_0) \rightarrow (q_2, z_0)$$

$$\delta(q_2, \lambda, z_0) \rightarrow q_f$$



Ratio Pattern:

$a^4 b^3 n \rightarrow$ Here dependency is where ever the last symbol is dependent on previous one will write b/x_0 last

aaa bbb bbb bbb

$a \rightarrow 3$ time

Starting for a and rest of a

$$S(q_0, a, z_0) \rightarrow (q_0, a z_0)$$

$$S(q_0, a, q_1) \rightarrow (q_0, a q_1)$$

$$S(q_0, b, q_1) \rightarrow (q_1, a q_1)$$

$$S(q_1, b, q_2) \rightarrow (q_2, a q_1)$$

$$S(q_2, b, q_3) \rightarrow (q_3, a q_1)$$

$$S(q_3, b, q_4) \rightarrow (q_4, a q_1)$$

$$S(q_4, b, q_5) \rightarrow (q_5, a q_1)$$

$$S(q_5, a, z_0) \rightarrow q_f$$

019 \rightarrow bbbb bbbb

$q_0 \rightarrow q_5$

$$S(q_5, b, q_6) \rightarrow (q_6, a q_1)$$

$(a z_0 / z_0) \rightarrow b, q_1$

$\rightarrow (q_0) \xrightarrow{b} (q_1) \xrightarrow{b} (q_2) \xrightarrow{b} (q_3) \xrightarrow{b} (q_4) \xrightarrow{b} (q_5) \xrightarrow{a z_0 / z_0}$

$(a q_1 / q_1)$

b

If more b are there

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Design a PDA for:-

$$L = \{ w c w^R \mid w \in \{a, b\}^*, c \text{ is a terminal} \}$$

The above can be written as

a a c a a

b b c b b

a b c b a

b a c a b

$$R_1 \quad S(q_0, a, z_0) = (q_0, az_0)$$

$$R_2 \quad S(q_0, b, z_0) = (q_0, bz_0)$$

$$R_3 \quad S(q_0, a, a) = (q_0, aa)$$

$$R_4 \quad S(q_0, b, a) = (q_0, ba)$$

$$R_5 \quad S(q_0, a, b) = (q_0, ab)$$

$$R_6 \quad S(q_0, b, b) = (q_0, bb)$$

for \in

$$S(q_0, c, a) = (q_1, a)$$

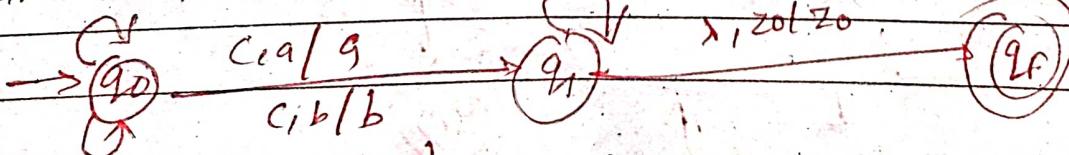
$$S(q_0, c, b) = (q_1, b)$$

$$S(q_1, a, a) = (q_1, \lambda)$$
 } - If a with a

$$S(q_1, b, b) = (q_1, \lambda)$$
 } - If b with b

$$S(q_1, \lambda, z_0) = q_f$$

($a_1, a_1 a_2, b_2 b_1, a_2 a_1 / a_2 z_0$)



($a_1 b_1 / a_2 b_2, b_1 a_1 / b_2 a_2, b_2 b_1 / b_2 b_1$)