Property 1 *The union of two regular set is regular.*

Closure property of Regular sets

Proof −

Let us take two regular expressions

$RE_1 = a(aa)^*$ and $RE_2 = (aa)^*$

So, $L_1 = \{a, aaa, aaaaa, ...\}$ (Strings of odd length excluding Null)

and $L_2 = \{ \varepsilon, aa, aaaa, aaaaaa, ...\}$ (Strings of even length including Null)

$L_1 \cup L_2 = \{ \varepsilon, a, aa, aaa, aaaa, aaaaa, aaaaaa, ...\}$

(Strings of all possible lengths including Null)

$RE (L_1 \cup L_2) = a^*$ (which is a regular expression itself)

Hence, proved.

After carrying out some operation

If the result is also regular

Express the result in R.E form

Perform the op, get result, express the result in the form R.E

Property 1. *The union of two regular set is regular.*

Proof −

Let us take two regular expressions

$RE_1 = a(aa)^*$ and $RE_2 = (aa)^*$

$a(aa)^*$ — null

$L_1 = \{a, aaa, aaaaa, aaaaaaa, \ldots\}$

RE1=Regular Set

a(aa)*= Regular Expression

odd no of a's without null

$(aa)^*$

$L_2 = \{null, aa, aaaa, aaaaaa, \ldots\}$

even no of a's with null

$L = L_1 \cup L_2 = \{null, a, aa, aaa, aaaa, a^5, a^6, a^7 \ldots\} = a^*$

Property 2. *The intersection of two regular set is regular.*

Proof –

Let us take two regular expressions

$RE_1 = a(a^*)$ and $RE_2 = (aa)^*$

So, L_1 = { a,aa, aaa, aaaa, ...} (Strings of all possible lengths excluding Null)

L_2 = { ε, aa, aaaa, aaaaaa,.......} (Strings of even length including Null)

$L_1 \cap L_2$ = { aa, aaaa, aaaaaa,.......} (Strings of even length excluding Null)

$RE (L_1 \cap L_2) = aa(aa)^*$ which is a regular expression itself.

Hence, proved.

Property 2. *The intersection of two regular set is regular.*

$a(a^*) \rightarrow$ null

Proof –

$L_1 = \{a, aa, aaa, aaaa, a^5, a^6\}$

Let us take two regular expressions

$L_2 = \{null, aa, aaaa, a^6 \ldots\}$

$RE_1 = a(a^*)$ and $RE_2 = (aa)^*$

$(aa)^*$

$L_1 \cap L_2 = \{aa, aaaa, a^6, a^8, \ldots\}$

common

$RE = aa\boxed{(aa)^*}$

The complement of regular set is regular

RE = (aa)*

So, L = {ε, aa, aaaa, aaaaaa, ...} (Strings of even length including Null)

Complement of **L** is all the strings that is not in **L**.

So, L' = {a, aaa, aaaaa, ...} (Strings of odd length excluding Null)

RE (L') = a(aa)* which is a regular expression itself.

Hence, proved.

The complement of regular set is regular

RE = $(aa)^*$

L = { null, aa, a^4, a^6, a^8, ... }

Complemet= {a, aaa, aaaaa, aaaaaaa,aaaaaaaaa, ...}

RE= $a(aa)^*$

Property 5. *The reversal of a regular set is regular.*

Proof –

We have to prove $\mathbf{L}^R$ is also regular if **L** is a regular set.

Let, L = {01, 10, 11, 10}

RE (L) = 01 + 10 + 11 + 10

L^R = {10, 01, 11, 01}

100

= 001

Property 5. *The reversal of a regular set is regular.*

Proof −

We have to prove L^R is also regular if **L** is a regular set.

Let, L = {01, 10, 11, 10}

$L = \{a, b\}$

$RE = a + b$

RE= 01 +10 + 11 + 10 or

RE= (1+0)(1+0)

Reversal={10,01,11,01}

RE=(1+0)(1+0)

10+01+11+01

Property 6. *The closure of a regular set is regular.*

Proof −

If L = {a, aaa, aaaaa, ...} (Strings of odd length excluding Null)

i.e., RE (L) = a (aa)*

L* = {a, aa, aaa, aaaa , aaaaa,……………} (Strings of all lengths excluding Null)

RE (L*) = a (a)*

Hence, proved.

$aa(aa^*)$

aa

Property 6. *The closure of a regular set is regular.*

Proof –

If L = {a, aaa, aaaaa, ...} (Strings of odd length excluding Null)

i.e., RE (L) = a (aa)*

$\boxed{a(aa)^*}$

$L^* = (a(aa))^*$

aa

$= \text{null}, \; a\,\text{null}, \; a\,aa, \; a\,aaaa, \; a\,a^6, \; a\,a^8 \ldots$

$= \{\text{null}, a, aaa, a^5, a^7, a^9 \ldots\}$

a(aa) RE {null + a(a·a)* }

$$\left(a\,(aa)^*\,x\right)^* \quad 2 \quad 4$$

null

null

$$= aa,\ aaaa,\ a$$

$$= \text{null}, a, aa, aaa, aaaa, \ldots$$

$$= a^*$$

null, a, aa, aa $= a^*$

$L = a^*$

a^{17} a^{32}

$= \{$ null, a, aa, aaa . . . $\}$

$L^* = (a^*)^{*1}$ $(a^2)^3$ $= \{a^2 a^2 a^2$

$= a^6$

a aa

Property 7. *The concatenation of two regular sets is regular.*

Proof –

Let $RE_1 = (0+1)^*0$ and $RE_2 = 01(0+1)^*$

Here, L_1 = {0, 00, 10, 000, 010, ...} (Set of strings ending in 0)

and L_2 = {01, 010, 011,.....} (Set of strings beginning with 01)

Then, $L_1 L_2$ =

{001,0010,0011,0001,00010,00011,1001,10010,.............}

Set of strings containing 001 as a substring which can be represented by an RE – $(0 + 1)^*001(0 + 1)^*$

Hence, proved.

$(0+1)^*0$

{0, 00, 10, 000, 010 ...}

$01(0+1)^*$

$(0+1)^*001(0+1)^*$ = {01, 010, 011, 0100, 0101}