Property 1. *The union of two regular set is regular.*

Proof −

Let us take two regular expressions

RE_1 = a(aa)* and RE_2 = (aa)*

So, L_1 = {a, aaa, aaaaa,.....} (Strings of odd length excluding Null)

and L_2 ={ ε, aa, aaaa, aaaaaa,.......} (Strings of even length including Null)

$L_1 \cup L_2$ = { ε, a, aa, aaa, aaaa, aaaaa, aaaaaa,.......}

(Strings of all possible lengths including Null)

RE ($L_1 \cup L_2$) = a* (which is a regular expression itself)

Hence, proved.

Property 2. *The intersection of two regular set is regular.*

Proof −

Let us take two regular expressions

RE_1 = a(a*) and RE_2 = (aa)*

So, L_1 = { a,aa, aaa, aaaa, ...} (Strings of all possible lengths excluding Null)

L_2 = { ε, aa, aaaa, aaaaaa,.......} (Strings of even length including Null)

$L_1 \cap L_2$ = { aa, aaaa, aaaaaa,.......} (Strings of even length excluding Null)

RE ($L_1 \cap L_2$) = aa(aa)* which is a regular expression itself.

Hence, proved.

Property 3. *The complement of a regular set is regular.*

Proof −

Let us take a regular expression −

RE = (aa)*

So, L = {ε, aa, aaaa, aaaaaa, ...} (Strings of even length including Null)

Complement of **L** is all the strings that is not in **L**.

So, L’ = {a, aaa, aaaaa, ...} (Strings of odd length excluding Null)

RE (L’) = a(aa)* which is a regular expression itself.

Hence, proved.

Property 4. *The difference of two regular set is regular.*

Proof −

Let us take two regular expressions −

RE_1 = a (a*) and RE_2 = (aa)*

So, L_1 = {a, aa, aaa, aaaa, ...} (Strings of all possible lengths excluding Null)

L_2 = { ε, aa, aaaa, aaaaaa,.......} (Strings of even length including Null)

$L_1 – L_2$ = {a, aaa, aaaaa, aaaaaaa, ...}

(Strings of all odd lengths excluding Null)

RE ($L_1 – L_2$) = a (aa)* which is a regular expression.

Hence, proved.

Property 5. *The reversal of a regular set is regular.*

Proof −

We have to prove $\mathbf{L^R}$ is also regular if **L** is a regular set.

Let, L = {01, 10, 11, 10}

RE (L) = 01 + 10 + 11 + 10

L^R = {10, 01, 11, 01}

RE (L^R) = 01 + 10 + 11 + 10 which is regular

Hence, proved.

Property 6. *The closure of a regular set is regular.*

Proof −

If L = {a, aaa, aaaaa, ...} (Strings of odd length excluding Null)

i.e., RE (L) = a (aa)*

L* = {a, aa, aaa, aaaa , aaaaa,……………} (Strings of all lengths excluding Null)

RE (L*) = a (a)*

Hence, proved.

Property 7. *The concatenation of two regular sets is regular.*

Proof −

Let RE_1 = (0+1)*0 and RE_2 = 01(0+1)*

Here, L_1 = {0, 00, 10, 000, 010, ...} (Set of strings ending in 0)

and L_2 = {01, 010,011,.....} (Set of strings beginning with 01)

Then, L_1 L_2 = {001,0010,0011,0001,00010,00011,1001,10010,.............}

Set of strings containing 001 as a substring which can be represented by an RE − (0 + 1)*001(0 + 1)*

Hence, proved.