

## Example

Language

Grammar

$$L_1 = \{a^n b^n\}$$

$$S_1 \rightarrow aS_1b \mid \lambda$$

$$L_2 = \{ww^R\}$$

$$S_2 \rightarrow aS_2a \mid bS_2b \mid \lambda$$

Union

$$L = \{a^n b^n\} \cup \{ww^R\}$$

$$S \rightarrow S_1 \mid S_2$$

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$$S_1 \rightarrow aS_1b \mid \lambda$$

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### Concatenation

$$L = \{a^n b^n\} \{ww^R\}$$

$$S \rightarrow S_1 S_2$$



## Example

Language

Grammar

$$L = \{a^n b^n\}$$

$$S \rightarrow aSb \mid \lambda$$

### Star Operation

$$L = \{a^n b^n\}^*$$

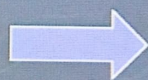
$$S_1 \rightarrow SS_1 \mid \lambda$$

# Intersection

Context-free languages  
are not closed under: **intersection**

$L_1$  is context free

$L_2$  is context free



$L_1 \cap L_2$

**not** necessarily  
context-free



## Example

$$L_1 = \{a^n b^n c^m\}$$

Context-free:

$$S \rightarrow AC$$

$$A \rightarrow aAb \mid \lambda$$

$$C \rightarrow cC \mid \lambda$$

$$L_2 = \{a^n b^m c^m\}$$

Context-free:

$$S \rightarrow AB$$

$$A \rightarrow aA \mid \lambda$$

$$B \rightarrow bBc \mid \lambda$$

**IMP**  
Intersection

$$L_1 \cap L_2 = \{a^n b^n c^n\} \quad \text{NOT context-free}$$

# Complement

Context-free languages  
are not closed under: **complement**

$L$  is context free  $\Rightarrow \bar{L}$  **not** necessarily  
context-free



## Example

$$L_1 = \{a^n b^n c^m\}$$

Context-free:

$$S \rightarrow AC$$

$$A \rightarrow aAb \mid \lambda$$

$$C \rightarrow cC \mid \lambda$$

$$L_2 = \{a^n b^m c^m\}$$

Context-free:

$$S \rightarrow AB$$

$$A \rightarrow aA \mid \lambda$$

$$B \rightarrow bBc \mid \lambda$$

Complement

$$\overline{L_1 \cup L_2} = L_1 \cap L_2 = \{a^n b^n c^n\}$$

NOT context-free