



Problems

Repeats?
Room/time change?

Deletes?

<u>Properties</u>

Class -> Room/time Room -> Lat, Lng

(more compact)

### Example Enrollment table - "v1"

	SID	Class
	4749732	cs 145
	2720942	cs 145
	4823984	cs 145
	4287594	cs 145
75	2984994	cs 145
s145	8472374	cs 145
tudents	4723663	cs 145
	2478239	cs 145
	4763268	cs 145
	2364532	cs 145
	2364573	cs 145
	3476382	cs 145
	2347623	cs 145
00	2364579	cs 245
cs245 students	3476343	cs 245
	2322232	cs 245

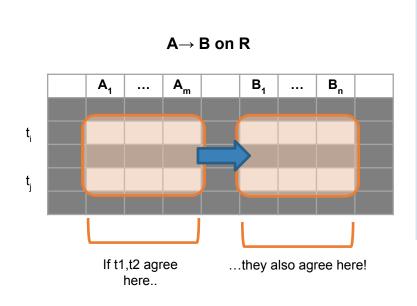


Class	Room	Time
cs 145	Nvidia Aud	T/R 4:30-6
cs 245	Nvidia Aud	T/R 3-4:30
cs 246	Nvidia Aud	M/W 3-4:30

Room	Lat	Lng
Nvidia Aud	37.4277° N	122.1742° W



### A Picture Of FDs [recall]



#### Defn (again):

Given attribute sets  $A=\{A_1,...,A_m\}$  and  $B=\{B_1,...B_n\}$  in R,

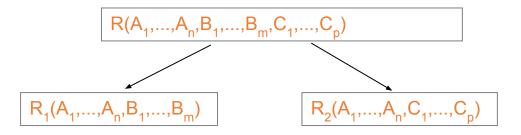
The *functional dependency*  $A \rightarrow B$  on R holds if for *any*  $t_i, t_j$  in R:

$$\begin{split} & \underline{\textbf{if}} \ t_i[A_1] = t_i[A_1] \ \text{AND} \ t_i[A_2] = t_j[A_2] \ \text{AND} \\ & \dots \ \text{AND} \ t_i[A_m] = t_j[A_m] \end{split}$$

 $\begin{array}{l} \underline{\textbf{then}} \; t_i[B_1] = t_j[B_1] \; \text{AND} \; t_i[B_2] = t_j[B_2] \\ \text{AND} \; \dots \; \text{AND} \; t_i[B_n] = t_i[B_n] \end{array}$ 



### **Table Decomposition**



 $R_1$  = the *projection* of R on  $A_1$ , ...,  $A_n$ ,  $B_1$ , ...,  $B_m$ 

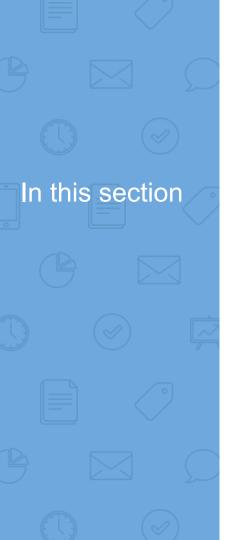
 $R_2$  = the *projection* of R on  $A_1$ , ...,  $A_n$ ,  $C_1$ , ...,  $C_p$ 

## **Conceptual Design**

### For a "mega" table

- Search for "bad" <u>dependencies</u>
- If any, keep <u>decomposing</u> the table into sub-tables until no more bad dependencies
- When done, the database schema is *normalized*

Note: there are several "good" (normal) forms...



### 1. Finding FDs

- Closures: How to compute FDs?
- SuperKeys: One 'good' kind of FDs

### 2. Decomposing mega tables into 'good' tables

Boyce-Codd Normal Form, 3NF

### **Finding Functional Dependencies**

### **Example:**

#### **Products**

Name	Color	Category	Dep	Price
Gizmo	Green	Gadget	Toys	49
Widget	Black	Gadget	Toys	59
Gizmo	Green	Whatsit	Garden	99

#### **Provided FDs:**

- 1.  $\{Name\} \rightarrow \{Color\}$
- 2. {Category} → {Department}
- 3. {Color, Category}  $\rightarrow$  {Price}

Given the provided FDs, we can see that  ${\text{Name, Category}} \rightarrow {\text{Price}}$  must also hold on **any instance**...

Which / how many other FDs do?!?



# **Finding Functional Dependencies**

Given a set of FDs,  $F = \{f_1, ..., f_n\}$ , does an FD g hold?

Inference problem: How do we decide?

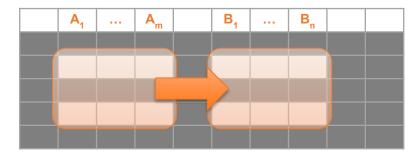
Answer: Three simple rules called **Armstrong's Rules.** 

- 1. Split/Combine
- 2. Reduction
- 3. Transitivity





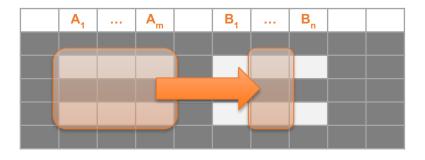
# 1. Split/Combine



$$\boldsymbol{A}_{1},\,...,\boldsymbol{A}_{m} \rightarrow \boldsymbol{B}_{1},\!...,\!\boldsymbol{B}_{n}$$



# 1. Split/Combine



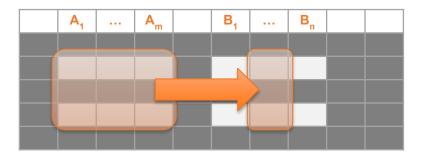
$$A_1, ..., A_m \rightarrow B_1, ..., B_n$$

... is equivalent to the following *n* FDs...

$$A_1,...,A_m \rightarrow B_i$$
 for i=1,...,n



# 1. Split/Combine



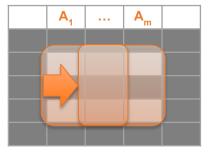
*And vice-versa,*  $A_1,...,A_m \rightarrow B_i$  for i=1,...,n

... is equivalent to ...

$$A_1, ..., A_m \rightarrow B_1, ..., B_n$$



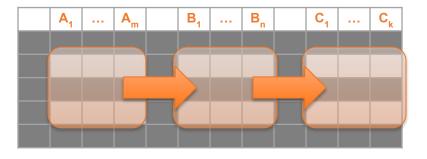
# 2. Reduction/Trivial



If 
$$A_1,...,A_m \rightarrow A_j$$
 for any j=1,...,m
$$A_i \rightarrow A_i \text{ for } i = 1...m$$



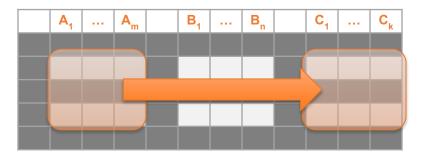
### 3. Transitive Closure



$$A_1, ..., A_m \rightarrow B_1, ..., B_n$$
 and  $B_1, ..., B_n \rightarrow C_1, ..., C_k$ 



### 3. Transitive Closure



$$A_1, ..., A_m \rightarrow B_1, ..., B_n$$
 and  $B_1, ..., B_n \rightarrow C_1, ..., C_k$ 

implies

$$A_1,...,A_m \rightarrow C_1,...,C_k$$

### **Finding Functional Dependencies**

### **Example:**

#### **Products**

Name	Color	Category	Dep	Price
Gizmo	Green	Gadget	Toys	49
Widget	Black	Gadget	Toys	59
Gizmo	Green	Whatsit	Garden	99

#### **Provided FDs:**

- 1.  $\{Name\} \rightarrow \{Color\}$
- 2. {Category} → {Department}
- 3. {Color, Category} → {Price}

Which / how many other FDs hold?

# **Finding Functional Dependencies**

#### **Example:**

#### **Provided FDs:**

Inferred FD	Rule used	1. {Name} → {Color}
4. {Name, Category} -> {Name}	Trivial	2. {Category} → {Dept.} 3. {Color, Category} → {Price}
5. {Name, Category} -> {Color}	Transitive (4 -> 1)	3. (Color, Category) → (Trice)
6. {Name, Category} -> {Category}	Trivial	
7. {Name, Category} -> {Color, Category}	Split/Combine (5 + 6)	
8. {Name, Category} -> {Price}	Transitive (7 -> 3)	

What's an algorithmic way to do this?

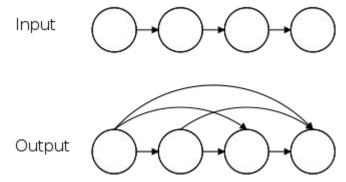


### Algebra (reminder)



$$B \Rightarrow C, C \Rightarrow D, ... X \Rightarrow Y, ... (transitive closures)$$

(Think of Closures as "reachability" in graph)





### Closure of a set of Attributes

Given a set of attributes  $A_1, ..., A_n$  and a set of FDs F: Closure  $\{A_1, ..., A_n\}^+$  is the set of attributes B s.t.  $\{A_1, ..., A_n\} \to B$ 

#### Closure Algorithm

$$\begin{split} & \text{Start with X} = \{A_1, \, ..., \, A_n\}, \, \text{FDs F.} \\ & \text{Repeat until X doesn't change; do:} \\ & \text{if } \{B_1, \, ..., \, B_n\} \rightarrow C \text{ is in F and } \{B_1, \, ..., \, B_n\} \subseteq X: \\ & \text{then } \text{add C to X.} \\ & \text{Return X as X}^+ \end{split}$$

color

category

price

Example: F = {name} → {color} {category} → {department}

Example Closures:

{color, category} → {price}

department

{name}+ = {name, color}

{name, category}+ = {name, category, color, dept, price}

{color}+ = {color}

name

```
Start with X = \{A_1, ..., A_n\}, FDs F.

Repeat until X doesn't change; do:

if \{B_1, ..., B_n\} \rightarrow C is in F and \{B_1, ..., B_n\} \subseteq X:

then add C to X.

Return X as X<sup>+</sup>
```

```
{name, category}<sup>+</sup> = {name, category}
```

```
F = \begin{cases} \text{(name)} \rightarrow \{\text{color}\} \\ \text{(category)} \rightarrow \{\text{dept}\} \\ \text{(color, category)} \rightarrow \{\text{price}\} \end{cases}
```

```
Start with X = \{A_1, ..., A_n\}, FDs F.

Repeat until X doesn't change; do:

if \{B_1, ..., B_n\} \rightarrow C is in F and \{B_1, ..., B_n\} \subseteq X:

then add C to X.

Return X as X<sup>+</sup>
```

```
{name, category}<sup>+</sup> = {name, category}
```

```
{name, category}<sup>+</sup> = {name, category, color}
```

```
F = \begin{cases} \text{(name)} \rightarrow \{\text{color}\} \\ \text{(category)} \rightarrow \{\text{dept}\} \\ \text{(color, category)} \rightarrow \{\text{price}\} \end{cases}
```

```
Start with X = \{A_1, ..., A_n\}, FDs F.

Repeat until X doesn't change; do:

if \{B_1, ..., B_n\} \rightarrow C is in F and \{B_1, ..., B_n\} \subseteq X:

then add C to X.

Return X as X<sup>+</sup>
```

```
F = \{name\} \rightarrow \{color\}
\{category\} \rightarrow \{dept\}
\{color, category\} \rightarrow \{price\}
```

```
{name, category}<sup>+</sup> = {name, category}
```

```
{name, category}<sup>+</sup> = {name, category, color}
```

```
{name, category}<sup>+</sup> = {name, category, color, dept}
```

```
Start with X = \{A_1, ..., A_n\}, FDs F.

Repeat until X doesn't change; do:

if \{B_1, ..., B_n\} \rightarrow C is in F and \{B_1, ..., B_n\} \subseteq X:

then add C to X.

Return X as X^+
```

```
{name, category}<sup>+</sup> = {name, category}
```

```
{name, category}<sup>+</sup> = {name, category, color}
```

```
{name, category}<sup>+</sup> = {name, category, color, dept}
```

```
{name, category}<sup>+</sup> = {name, category, color, dept, price}
```

Compute 
$$\{A, F\}^+ = \{A, F, A, F, A,$$

$${A,B} \rightarrow {C}$$
  
 ${A,D} \rightarrow {E}$   
 ${B} \rightarrow {D}$   
 ${A,F} \rightarrow {B}$ 

Compute 
$$\{A,B\}^+ = \{A, B, C, D\}$$

Compute 
$$\{A, F\}^+ = \{A, F, B\}$$

R(A,B,C,D,E,F)

$${A,B} \rightarrow {C}$$
  
 ${A,D} \rightarrow {E}$   
 ${B} \rightarrow {D}$   
 ${A,F} \rightarrow {B}$ 

Compute  $\{A,B\}^+ = \{A, B, C, D, E\}$ 

Compute  $\{A, F\}^+ = \{A, B, C, D, E, F\}$ 

### **Using Closure to Infer ALL FDs**

Compute X<sup>+</sup>, for every set of attributes X:

Example:
Given F =

 $\begin{aligned} \{A,B\} &\to C \\ \{A,D\} &\to B \\ \{B\} &\to D \end{aligned}$ 

```
{A}^{+} = {A}

{B}^{+} = {B,D}

{C}^{+} = {C}

{D}^{+} = {D}

{A,B}^{+} = {A,B,C,D}

{A,C}^{+} = {A,C}

{A,D}^{+} = {A,B,C,D}

{A,B,C}^{+} = {A,B,D}^{+} = {A,C,D}^{+} = {A,B,C,D}, {B,C,D}^{-} = {B,C,D}

{A,B,C,D}^{+} = {A,B,C,D}
```

No need to compute all of these- why?



### **Keys and Superkeys**

A <u>superkey</u> is a set of attributes  $A_1, ..., A_n$  s.t. for *any other* attribute **B** in R, we have  $\{A_1, ..., A_n\} \rightarrow B$ 

I.e. all attributes are functionally determined by a superkey

A key is a minimal superkey

Meaning that no subset of a key is also a superkey

#### Superkey Algorithm:

For each set of attributes X

- Compute X<sup>+</sup>
- If X<sup>+</sup> = set of all attributes then
   X is a superkey
- If X is minimal, then it is a **key**

### **Example of Finding Keys**

Product(name, price, category, color)

```
{name, category} → price
{category} → color
```

What is a key?

### **Example of Keys**

Product(name, price, category, color)

```
{name, category} → price
{category} → color
```

```
{name, category}+ = {name, price, category, color}
= the set of all attributes

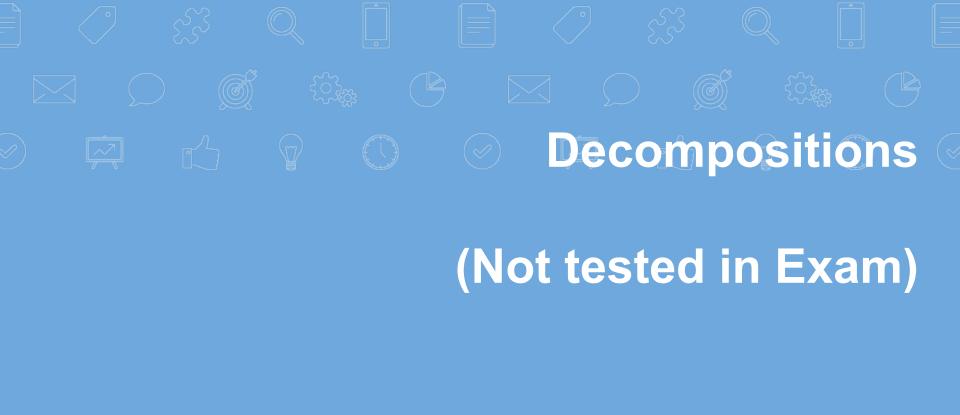
⇒ this is a superkey
⇒ this is a key, since neither name nor category alone is a superkey
```



### 1. Finding FDs

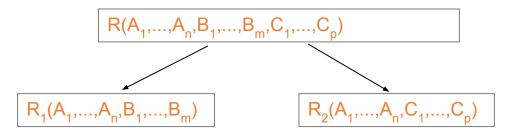
- Closures: How to compute FDs?
- SuperKeys: One 'good' kind of FDs

- 2. Decomposing mega tables into 'good' tables
  - Boyce-Codd Normal Form, 3NF





### **Decompositions**



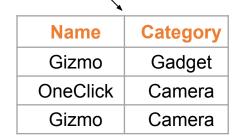
 $R_1$  = the *projection* of R on  $A_1$ , ...,  $A_n$ ,  $B_1$ , ...,  $B_m$ 

 $R_2$  = the *projection* of R on  $A_1$ , ...,  $A_n$ ,  $C_1$ , ...,  $C_p$ 

Name	Price	Category
Gizmo	19.99	Gadget
OneClick	24.99	Camera
Gizmo	19.99	Camera

We need a decomposition to be "correct"

Name	Price
Gizmo	19.99
OneClick	24.99
Gizmo	19.99

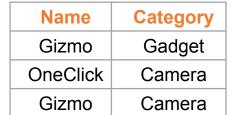


### **Example2: Lossy Decomposition**

Name	Price	Category
Gizmo	19.99	Gadget
OneClick	24.99	Camera
Gizmo	19.99	Camera

Need to avoid "bad" decompositions

What's wrong here?



Price	Category	
19.99	Gadget	
24.99	Camera	
19.99	Camera	

I.e. it is a **Lossy** decomposition

(Lossy ⇒ making up data, "losing' shape of data)

# **Lossy Decomposition**

Name	Price	Category
Gizmo	19.99	Gadget
OneClick	24.99	Camera
Gizmo	19.99	Camera

M



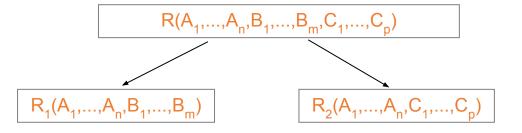
Name	Category
Gizmo	Gadget
OneClick	Camera
Gizmo	Camera



Name	Price	Category
Gizmo	19.99	Gadget
OneClick	24.99	Camera
Gizmo	19.99	Camera
OneClick	19.99	Camera
Gizmo	24.99	Camera



## **Lossless Decompositions**



A decomposition R to (R1, R2) is **lossless** if  $R = R1 \bowtie R2$ 





## **Boyce-Codd Normal Form**

BCNF is a simple condition for removing anomalies from relations:

A relation R is <u>in BCNF</u> if: if  $\{A_1, ..., A_n\} \rightarrow B$  is a FD (*non-trivial*) in R then  $\{A_1, ..., A_n\}$  is a superkey for R

In other words: there are no "bad" FDs

## Example

Name	SSN	PhoneNumber	City
Fred	123-45-6789	206-555-1234	Seattle
Fred	123-45-6789	206-555-6543	Seattle
Joe	987-65-4321	908-555-2121	Westfield
Joe	987-65-4321	908-555-1234	Westfield

 $\{SSN\} \rightarrow \{Name, City\}$ 

This FD is *bad* because it is **not** a superkey

 $\Rightarrow \underline{\text{Not}}$  in BCNF

What is the key? {SSN, PhoneNumber}



## **Example decomposition**

Name	SSN	City
Fred	123-45-6789	Seattle
Joe	987-65-4321	Madison

<u>SSN</u>	<u>PhoneNumber</u>
123-45-6789	206-555-1234
123-45-6789	206-555-6543
987-65-4321	908-555-2121
987-65-4321	908-555-1234

Now in BCNF!

 $\{SSN\} \rightarrow \{Name, City\}$ 

This FD is now good because it is the key

## Let's check anomalies:

- Redundancy?
- Update?
- Delete?



BCNFDecomp(R):



## BCNFDecomp(R):

Find a set of attributes X s.t.:  $X^+ \neq X$  and  $X^+ \neq [$ all attributes]

Find a set of attributes X which has non-trivial "bad" FDs, i.e. is not a superkey, using closures



BCNFDecomp(R):

Find a set of attributes X s.t.:  $X^+ \neq X$  and  $X^+ \neq [$ all attributes]

if (not found) then Return R

If no "bad" FDs found, in BCNF!



BCNFDecomp(R):

Find a set of attributes X s.t.:  $X^+ \neq X$  and  $X^+ \neq [$ all attributes]

if (not found) then Return R

**decompose** R into  $R_1(X^+)$  and  $R_2(X \cup Rest)$ 

R2: Rest of attributes not in X<sup>+</sup>



BCNFDecomp(R):

Find a set of attributes X s.t.:  $X^+ \neq X$  and  $X^+ \neq [$ all attributes]

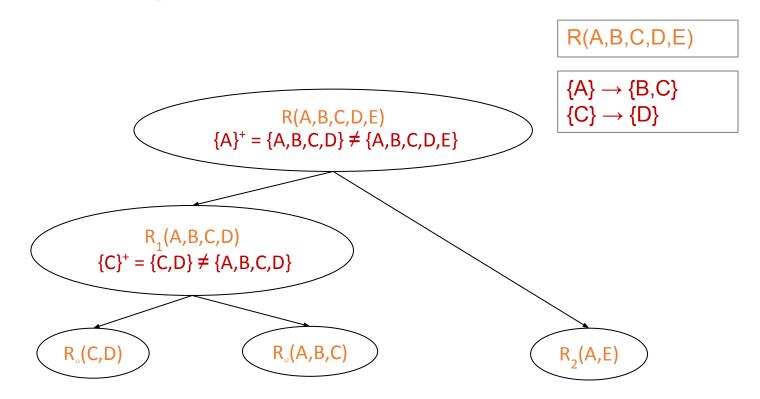
if (not found) then Return R

decompose R into  $R_1(X^+)$  and  $R_2(X \cup Rest)$ 

**Return** BCNFDecomp(R<sub>2</sub>), BCNFDecomp(R<sub>2</sub>)

Proceed recursively until no more "bad" FDs!

## Example

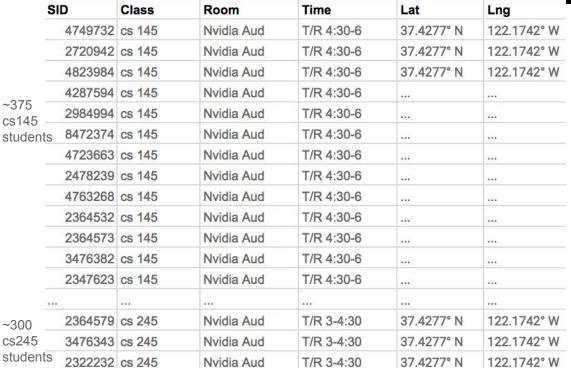


## **Conceptual Design (recap)**

## For a "mega" table

- Search for "bad" <u>dependencies</u>
- If any, *keep <u>decomposing</u>* (lossless) the table into sub-tables until no more bad dependencies
- When done, the database schema is *normalized*

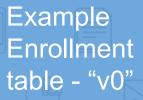






<u>FDs</u> Class -> Room,Time Room -> Lat, Lng

(more compact)



# BCNF decomposition

SID	Class	Room	Time	Lat	Lng
4749732	cs 145	Nvidia Aud	T/R 4:30-6	37.4277° N	122.1742° W
2720942	cs 145	Nvidia Aud	T/R 4:30-6	37.4277° N	122.1742° W
4823984	cs 145	Nvidia Aud	T/R 4:30-6	37.4277° N	122.1742° W
4287594	cs 145	Nvidia Aud	T/R 4:30-6		
2984994	cs 145	Nvidia Aud	T/R 4:30-6		
8472374	cs 145	Nvidia Aud	T/R 4:30-6		
4723663	cs 145	Nvidia Aud	T/R 4:30-6		
2478239	cs 145	Nvidia Aud	T/R 4:30-6		
4763268	cs 145	Nvidia Aud	T/R 4:30-6		
2364532	cs 145	Nvidia Aud	T/R 4:30-6		
2364573	cs 145	Nvidia Aud	T/R 4:30-6		
3476382	cs 145	Nvidia Aud	T/R 4:30-6		
2347623	cs 145	Nvidia Aud	T/R 4:30-6		
2364579	cs 245	Nvidia Aud	T/R 3-4:30	37.4277° N	122.1742° W
3476343	cs 245	Nvidia Aud	T/R 3-4:30	37.4277° N	122.1742° W
2322232	cs 245	Nyidia Aud	T/R 3-4:30	37 4277° N	122 1742° W

Schema: SID, Class, Room, Time, Lat, Lng

#### **FDs**

Class -> Room,Time Room -> Lat, Lng

## **BCNF** decomposition

- Find bad FD #1: Class<sup>+</sup> -> Class, Room, Time, Lat, Lng
   Decomposed: R1(Class, Room, Time, Lat, Lng) and R2(SID, Class)
- Find bad FD #2: Room<sup>+</sup> -> Room, Lat, Lng
   Decompose R1 into R11(Room, Lat, Lng) and R12(Class, Room, Time)
- ⇒ BCNF schema: R2(SID, Class), R12(<u>Class</u>, Room, Time), R11(<u>Room</u>, Lat, Lng)



## Example Enrollment table - "v1"

	SID	Class
	4749732	cs 145
	2720942	cs 145
	4823984	cs 145
	4287594	cs 145
375	2984994	cs 145
s145	8472374	cs 145
tudents	4723663	cs 145
	2478239	cs 145
	4763268	cs 145
	2364532	cs 145
	2364573	cs 145
	3476382	cs 145
	2347623	cs 145
300	2364579	cs 245
s245	3476343	cs 245
tudents	2322232	cs 245



Class	Room	Time
cs 145	Nvidia Aud	T/R 4:30-6
cs 245	Nvidia Aud	T/R 3-4:30
cs 246	Nvidia Aud	M/W 3-4:30

Room	Lat	Lng
Nvidia Aud	37.4277° N	122.1742° W



## Goals

## Course Summary

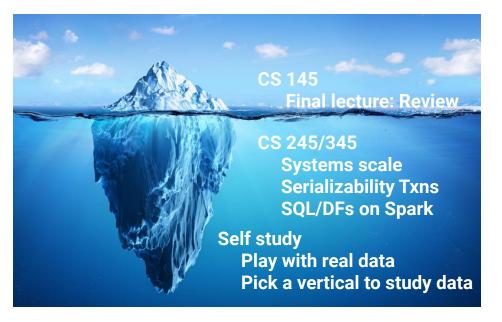
## We'll learn How To...

- Query over small-med-large data sets with SQL? [Weeks 1 and 2]
  - On relational engines, and "big data" engines
- Scale for "big queries"? On Clusters? [Weeks 3, 4, 5]
  - OLAP/Analytics, 1st principles of scale
- $\circ$  **Scale** for "big writes"? [Weeks 6, 7, 8]
  - Writes, Transactions, Logging, ACID properties
- Design "good" databases? [Weeks 9, 10]
  - Big Schemas, design, functional dependencies, query optimizers

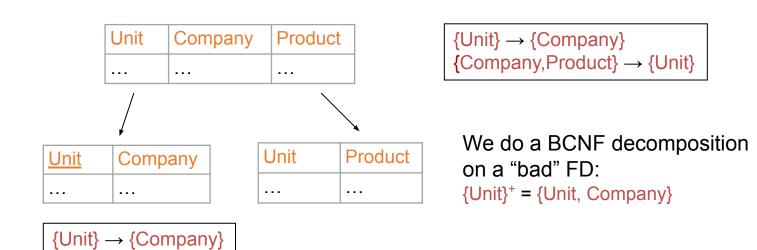
**Project**: Query-Visualize-Learn on GB/TB scale data sets on a Cloud [sql + python]

# Next Steps

## Course Summary



## A Problem with BCNF



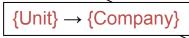
We lose the FD {Company, Product}  $\rightarrow$  {Unit}!!

# So Why is that a Problem?

<u>Unit</u>	Company
Galaga99	UW
Bingo	UW

Unit	Product
Galaga99	Databases
Bingo	Databases

No problem so far. All *local* FD's are satisfied.



Unit	Company	Product
Galaga99	UW	Databases
Bingo	UW	Databases

Let's put all the data back into a single table again:

Violates the FD {Company,Product} → {Unit}!!



## The Problem

- We started with a table R and FDs F
- We decomposed R into BCNF tables R<sub>1</sub>, R<sub>2</sub>, ... with their own FDs F<sub>1</sub>, F<sub>2</sub>, ...
- We insert some tuples into each of the relations—which satisfy their local FDs but when reconstruct it violates some FD across tables!

<u>Practical Problem</u>: To enforce FD, must reconstruct R—on each insert!

## **Possible Solutions**

 Various ways to handle so that decompositions are all lossless / no FDs lost

 Usually a tradeoff between redundancy / data anomalies and FD preservation...



## **Summary**

- Constraints allow one to reason about redundancy in the data
- Normal forms describe how to remove this redundancy by decomposing relations
  - Elegant—by representing data appropriately certain errors are essentially impossible
  - For FDs, BCNF is the normal form.
- A tradeoff for insert performance: 3NF