MTH165

Unit 5

MULTIPLE INTEGRALS

L 29-30- Double integrals and change of order of integrals

REVISION

Find the minimum value of $xy+a^3\left(\frac{1}{x}+\frac{1}{y}\right)$.

a) $3a^2$

b) a^2

c) a

d) 1

Answer: a

Explanation:

Given,f(x,y) = $xy + a^3(\frac{1}{x} + \frac{1}{y})$

Now, $\frac{\partial f}{\partial x} = y - \frac{a^3}{x^2}$ and $\frac{\partial f}{\partial y} = x - \frac{a^3}{y^2}$

Putting, $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$=0,and solving two equations,we get,

(x,y)=(a,a) or (-a,a)

Now, at (a,a) r = $\frac{\partial^2 f}{\partial x^2} = \frac{2a^3}{x^3}$=2>0 and $t = \frac{\partial^2 f}{\partial y^2} = \frac{2a^3}{y^3}$=2>0 and $s = \frac{\partial^2 f}{\partial x \partial y}$=1

hence, rt-s^2=3>0 and r>0,hence it has minimum value at (a,a).

Now, at (-a,a) r=$\frac{\partial^2 f}{\partial x^2} = \frac{2a^3}{x^3}$=-2<0 and $t = \frac{\partial^2 f}{\partial y^2} = \frac{2a^3}{y^3}$=2>0 and s=$\frac{\partial^2 f}{\partial x \partial y}$=1

hence, rt-s^2=-5<0,hence it has no extremum at this point.

Hence maximum value is, f(a,a)=a^2+a^3 $(\frac{1}{a} + \frac{1}{a}) = a^2 + 2a^2 = 3a^2$

REVISION

If $u=x+3y^2-z^3$, $v=4x^2 yz$, $w=2z^2-xy$ then $\frac{\partial(u,v,w)}{\partial(x,y,z)}$ at (1,1,1).

a) -184

b) -90

c) 20

d) 40

Answer: a

Explanation: Given that $u=x+3y^2-z^3$, $v=4x^2yz$, $w=2z^2-xy$

$$\frac{\partial(u,v,w)}{\partial(x,y,z)}=\begin{vmatrix}\frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} & \frac{\partial u}{\partial z}\\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} & \frac{\partial v}{\partial z}\\ \frac{\partial w}{\partial x} & \frac{\partial w}{\partial y} & \frac{\partial w}{\partial z}\end{vmatrix}=\begin{vmatrix}1 & 6y & -3z^2\\ 8xyz & 4x^2z & 4x^2y\\ -y & -x & 4z\end{vmatrix}$$

$$\frac{\partial(u,v,w)}{\partial(x,y,z)}\ at\ (1,1,1)=\begin{vmatrix}1 & 6 & -3\\ 8 & 4 & 4\\ -1 & -1 & 4\end{vmatrix}$$

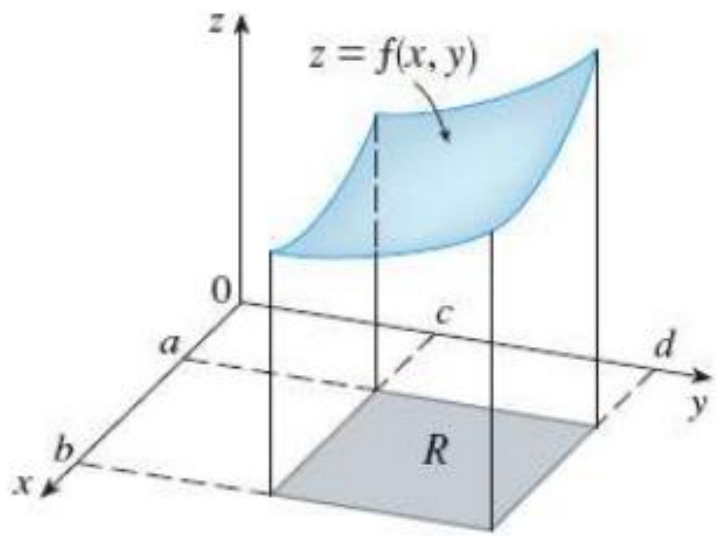
$=1(16+4) - 6(32+4) - 3(-8+4) = -184.$

Double integral

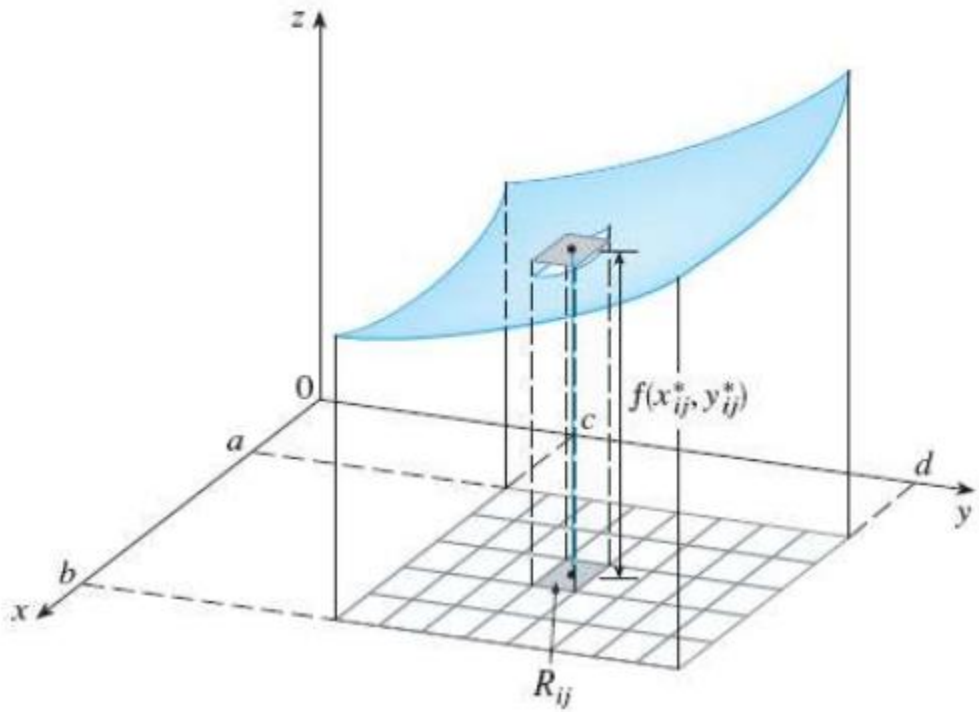
- **DEFINITION** The **double integral** of f over the rectangle R is

$$\iint_R f(x,y)\,dA = \lim_{\max \Delta x_i, \Delta y_i \to 0} \sum_{i=1}^{m} \sum_{j=1}^{n} f(xi\overset{*}{j}, yi\overset{*}{j}) \Delta A_{ij}$$

if this limit exists.

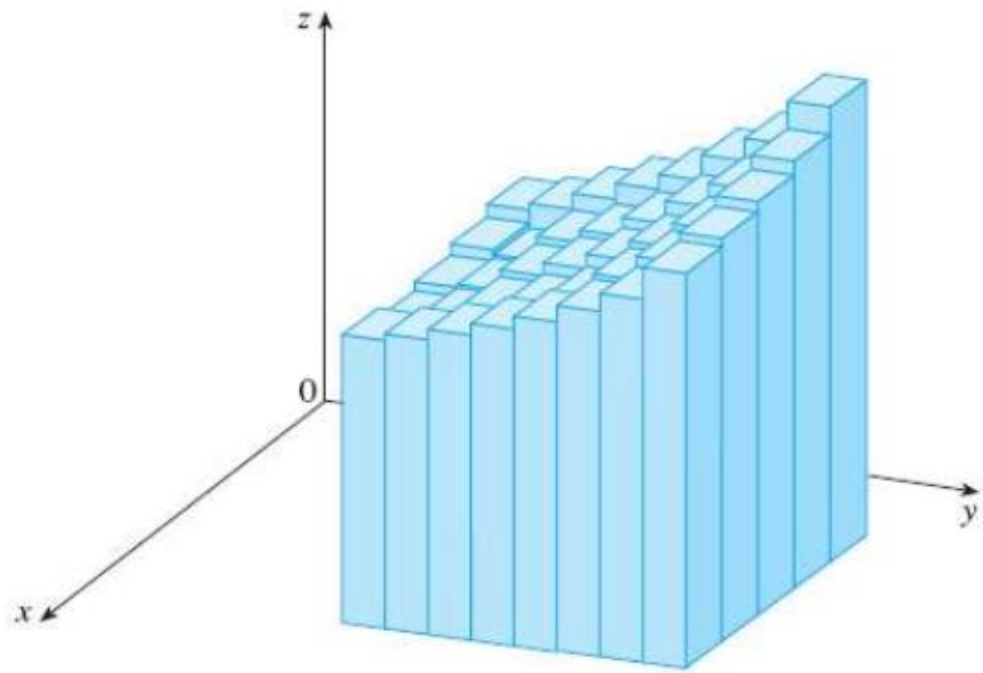


$$\iint_R f(x,y)\,dA = \lim_{m,n\to\infty} \sum_{i=1}^{m} \sum_{j=1}^{n} f(x_i, y_j)\Delta A$$



If f (x, y) ≥ 0, then the volume V of the solid that lies above the rectangle R and below the surface z=f (x, y) is

$$v = \iint_R f(x,y)\,dA$$

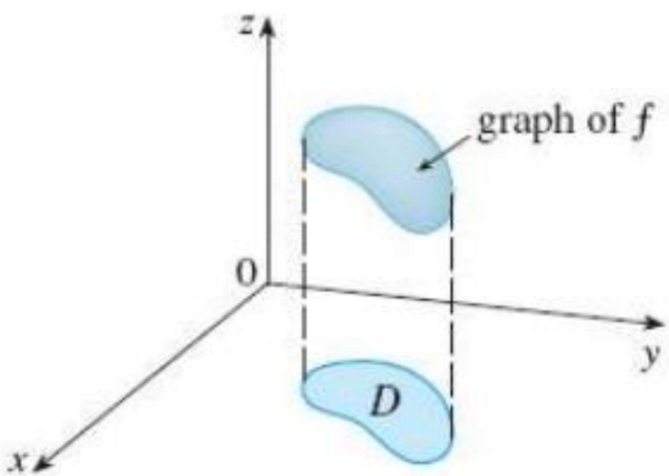


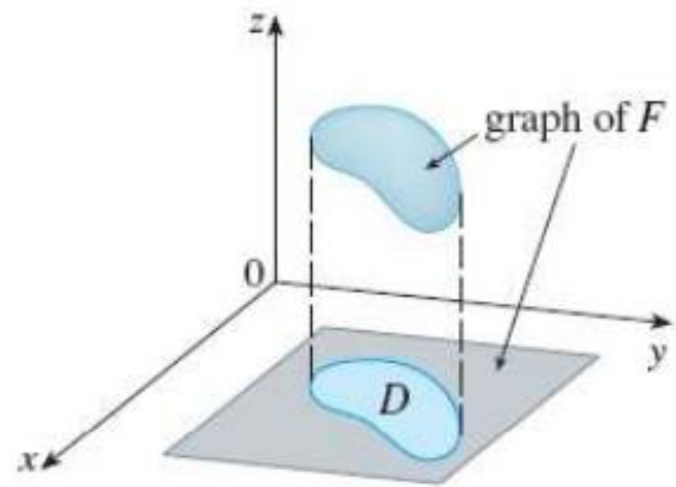
- **FUBINI'S THEOREM** If f is continuous on the rectangle R={(x, y} |a ≤ x≤ b, c ≤ y ≤d} , then

$$\iint_R f(x,y)\,dA = \int_a^b \int_c^d f(x,y)\,dy\,dx = \int_c^d \int_a^b f(x,y)\,dx\,dy$$

More generally, this is true if we assume that f is bounded on R, is discontinuous only on a finite number of smooth curves, and the iterated integrals exist.

$\iint_R g(x)h(y)\,dA = \int_a^b g(x)\,dx \int_c^d h(y)\,dy$ where R $R = [a,b] \times [c,d]$


z
graph of f
0
y
D
x


z
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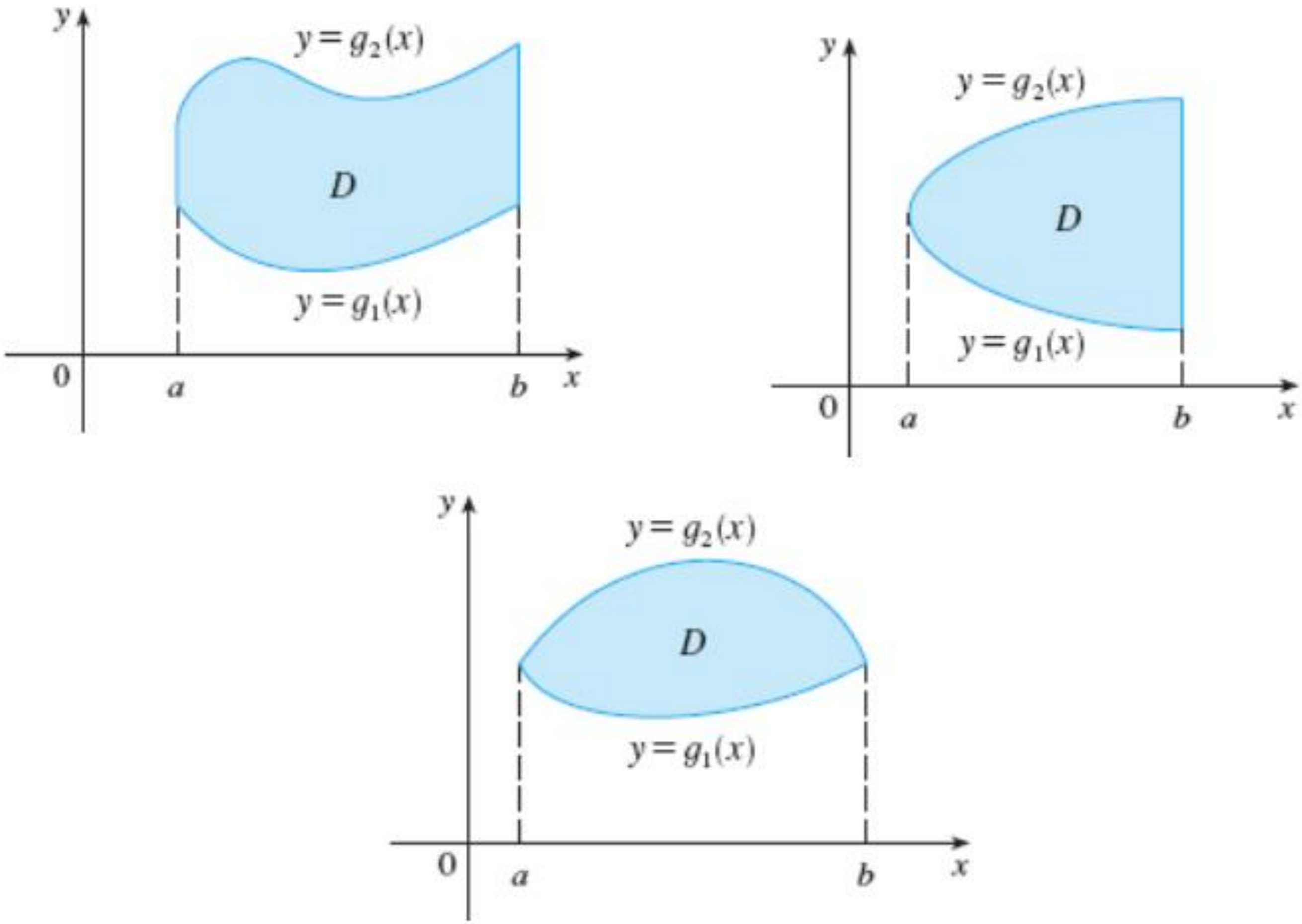


Some type I regions

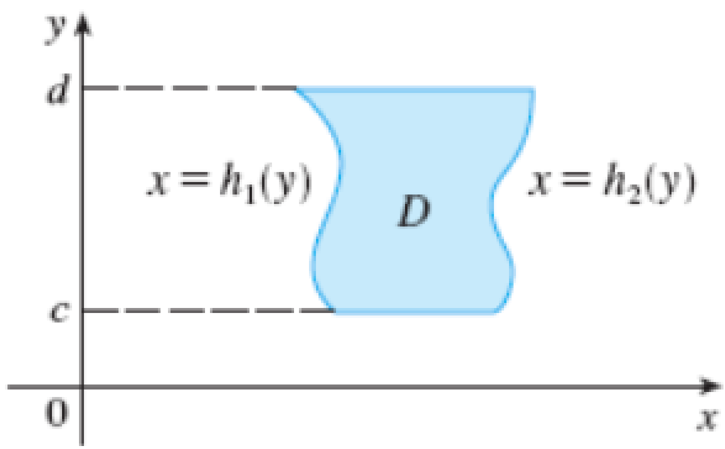
If f is continuous on a type I region *D* such that

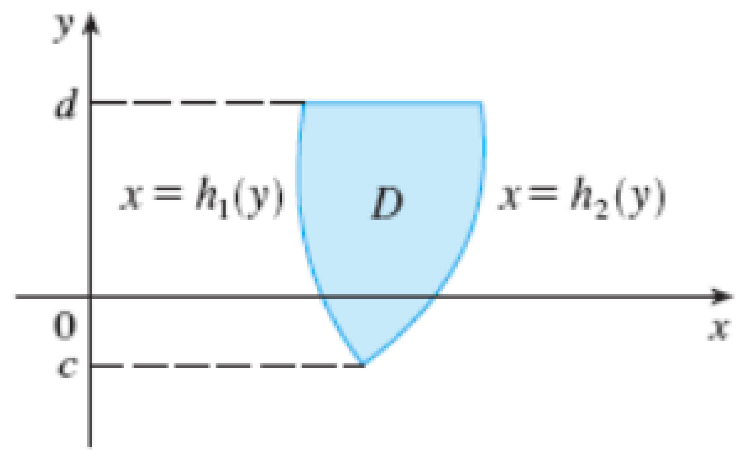
$$D = \{(x, y) | a \le x \le b, g_1(x) \le y \le g_2(x)\}$$

then

$$\iint_D f(x, y)\,dA = \int_a^b \int_{g_1 x}^{g_2 x} f(x, y)\,dy\,dx$$

$$\iint_D f(x,y)\,dA = \int_c^d \int_{h_1(y)}^{h_2(y)} f(x,y)\,dx\,dy$$





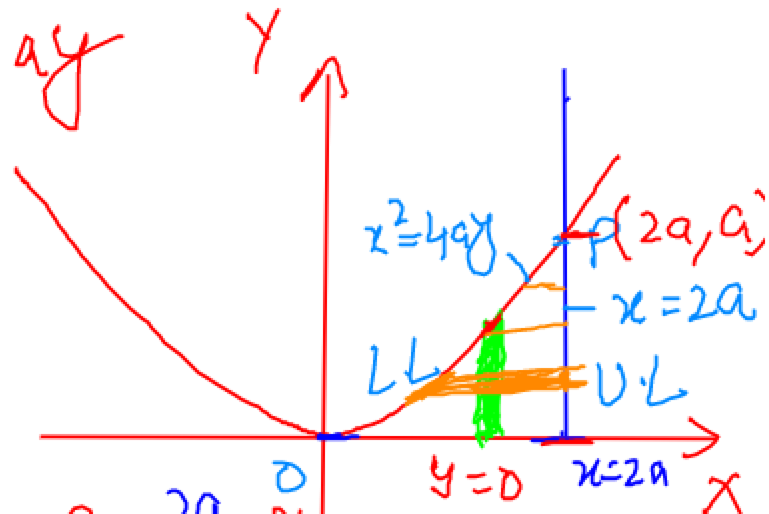
Some type II regions

➢ **CHANGE TO POLAR COORDINATES IN A DOUBLE INTEGRAL** If f is continuous on a polar rectangle R given by 0≤a≤r≤b, a≤θ≤ β , where 0≤ β - α ≤2π , then

$$\iint_R f(x,y)\,dA = \int_\alpha^\beta \int_a^b f(r\cos\theta, r\sin\theta)\, r\,dr\,d\theta$$

① Evaluate $\iint_A xy\,dx\,dy$, where A is the region bounded by the x-axis, $x=2a$ and $x^2=4ay$

Point of intersection of $x^2=4ay$ of $x=2a$ is



$$4a^2=4ay \Rightarrow y=a$$

$$\int_0^{2a}\int_{2\sqrt{ay}}^{2a} xy\,dx\,dy = I=\int_0^{2a}\int_{y=0}^{x^2/2a} xy\,dy\,dx$$

MCQ

$\int_0^1 \int_0^2 x^2 y\,dx\,dy$ is equal to

$a)\ \frac{2}{3} \quad b)\ \frac{1}{3} \quad c)\ \frac{4}{3} \quad d)\ \frac{8}{3}$

MCQ

$\int_0^1 \int_0^1 (x+y)\,dx\,dy$ is equal to

$a)$ 1 $\quad b)$ 2 $\quad c)$ 3 $\quad d)$ 4

MCQ

In polar the integral $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dxdy =$

$a)\ \int_0^{\pi/2} \int_0^\infty e^{-r^2} drd\theta$ $b)\ \int_0^{\pi/4} \int_0^\infty e^{-r} drd\theta$ $c)\ \int_0^{\pi/2} \int_0^\infty e^{-r^2} rdrd\theta$ $d)\ \int_0^{\pi/2} \int_0^\infty e^{-r} drd\theta$

MCQ

Change the order of integration in $\int_0^a \int_0^x dxdy$ is

$a)\ \int_0^a \int_0^x dxdy$ $\qquad b)\ \int_0^a \int_0^x xdydx$ $\qquad c)\ \int_0^a \int_y^a dxdy$ $\qquad d)\ \int_0^a \int_0^y dxdy$

Change of Order

Q1. Change the order of integration in the integral

$$I = \int_{-a}^{a} \int_{0}^{\sqrt{a^2-y^2}} f(x,y)\,dx\,dy$$

Sol:

Limits of x are

Lower Limit $= 0$.

Upper Limit $= \sqrt{a^2-y^2}$

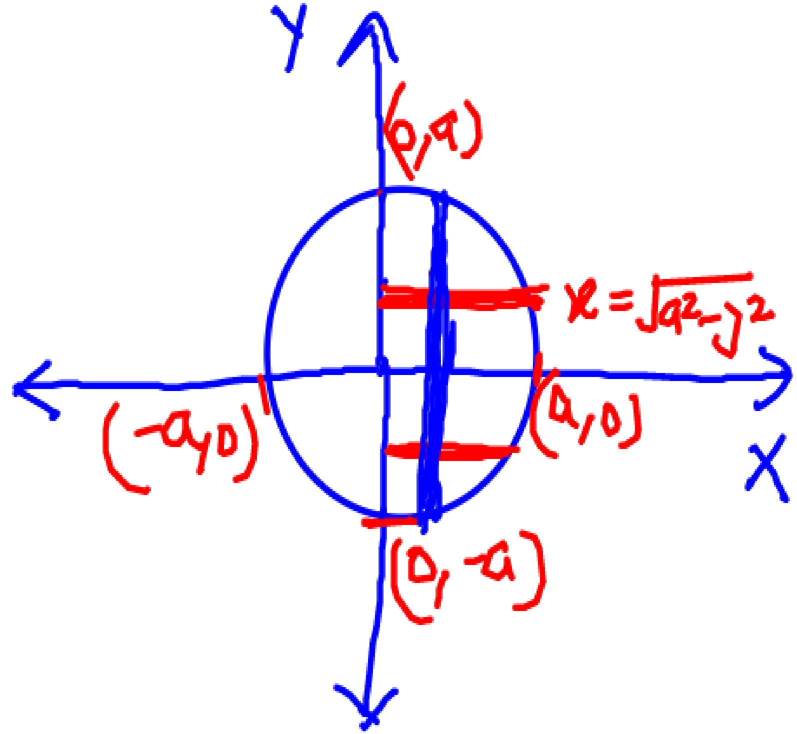
$$x = \sqrt{a^2-y^2}$$

$$x^2 = a^2-y^2 \Rightarrow x^2+y^2=a^2$$

Limits of y

L.L of $y = -a$

U.L of $y = a$



After changing the order

New limits of y are

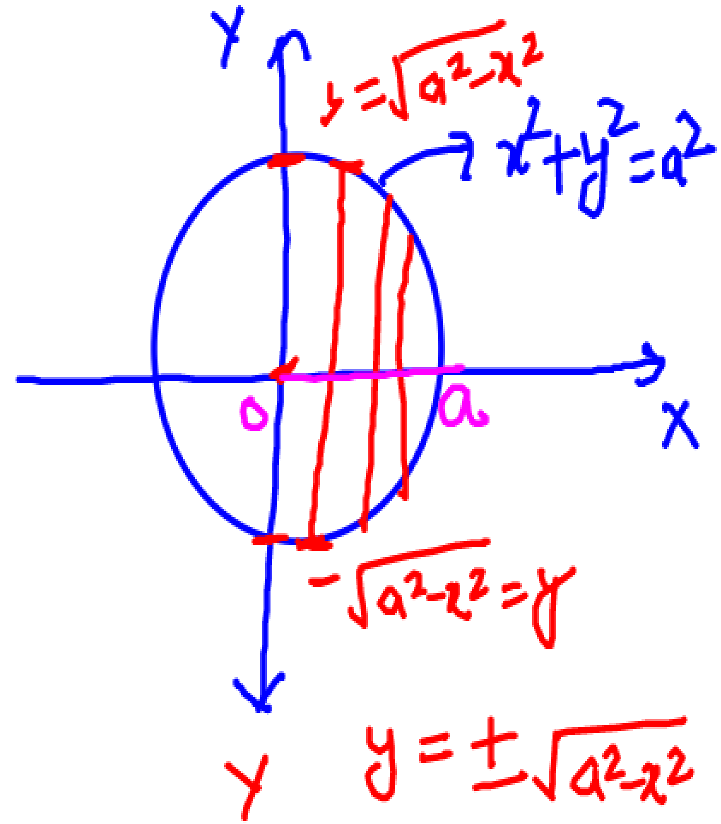
L.L of $y = -\sqrt{a^2-x^2}$

U.L of $y = \sqrt{a^2-x^2}$

New limits of x are

L.L of $x = 0$

U.L of $x = a$



$$\int_{-a}^{a}\int_{0}^{\sqrt{a^2-y^2}} f(x,y)\,dx\,dy = \int_{0}^{a}\int_{-\sqrt{a^2-x^2}}^{\sqrt{a^2-x^2}} f(x,y)\,dy\,dx$$

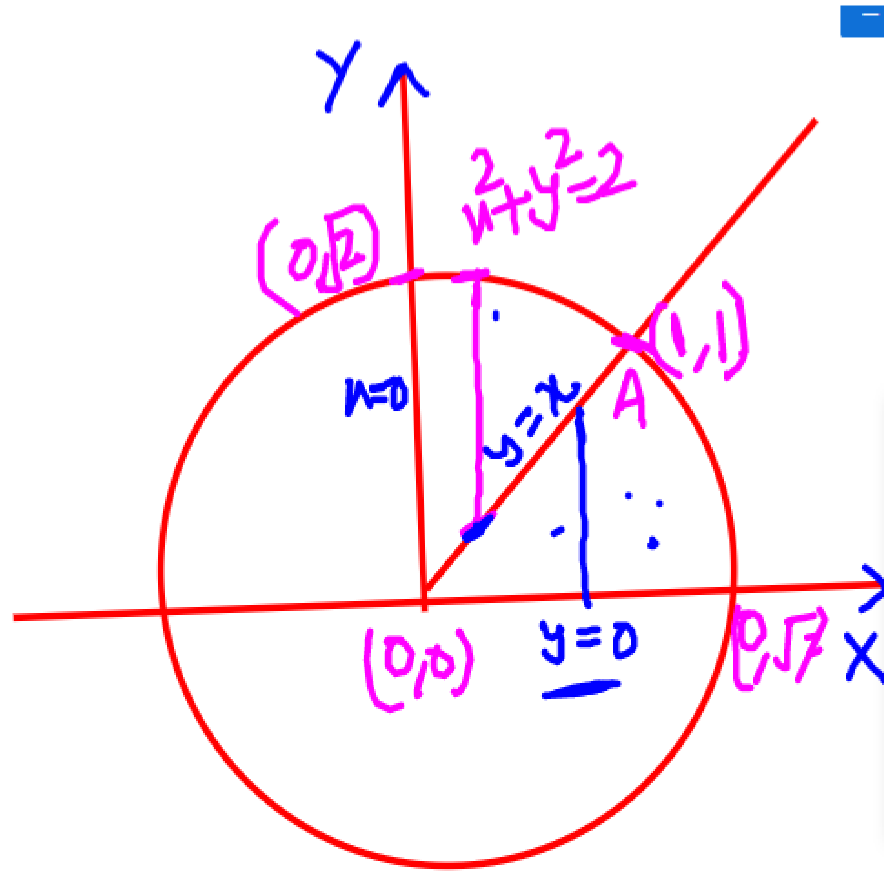
Q-1 Change the order of integration in

$$I = \int_0^1 \int_0^{\sqrt{2-x^2}} \frac{x}{\sqrt{x^2+y^2}}\,dy\,dx$$

$$\int_0^1 \int_x^{\sqrt{2-x^2}} dy\,dx$$

$y = x$

$y = \sqrt{2-x^2} \Rightarrow x^2+y^2=2$



$2x^2=2$

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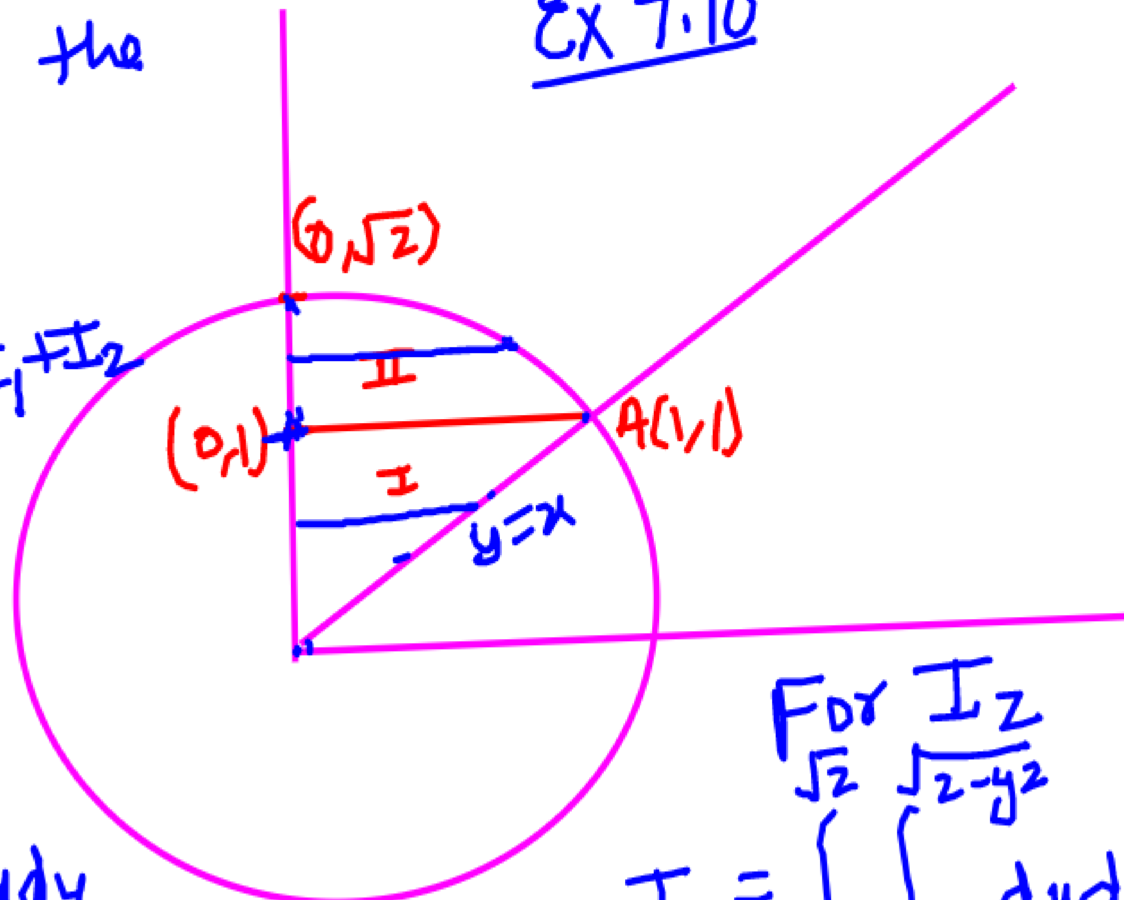
After changing the order

$$I = \underset{\mathrm{I}}{\iint} + \underset{\mathrm{II}}{\iint} = I_1 + I_2$$



For I_1

$$I_1 = \int_0^1 \int_{x=0}^{y} dx\,dy$$

For I_2

$$I_2 = \int_1^{\sqrt{2}} \int_{x=0}^{\sqrt{2-y^2}} dx\,dy$$

MTH165

Unit 5
MULTIPLE INTEGRALS

Lec 29-31 Double and Triple integrals and change of variable

In this :

- 1 Double Integrals over Rectangles
- 2 Double Integrals over General Regions
- 3 Double Integrals in Polar Coordinates
- 4 Applications of Double Integrals
- 5 Triple Integrals
- 6 Triple Integrals in Cylindrical Coordinates
- 7 Triple Integrals in Spherical Coordinates
- 8 Change of Variables in Multiple Integrals

Review

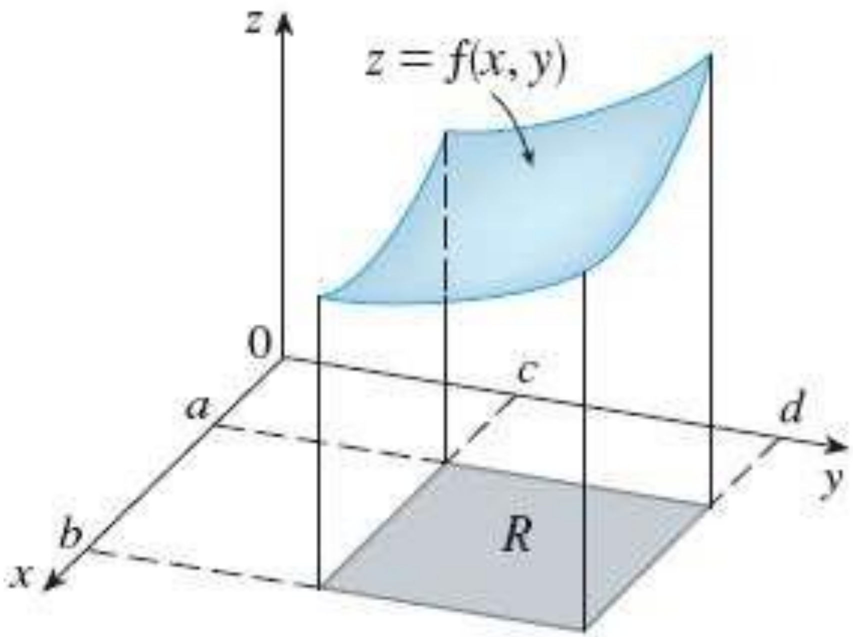


FIGURE 2

$$s = \{(x, y, z) \in R^3 \mid 0 \le z \le f(x, y), (x, y) \in R\}$$

(See Figure 2.) Our goal is to find the volume of *S*.

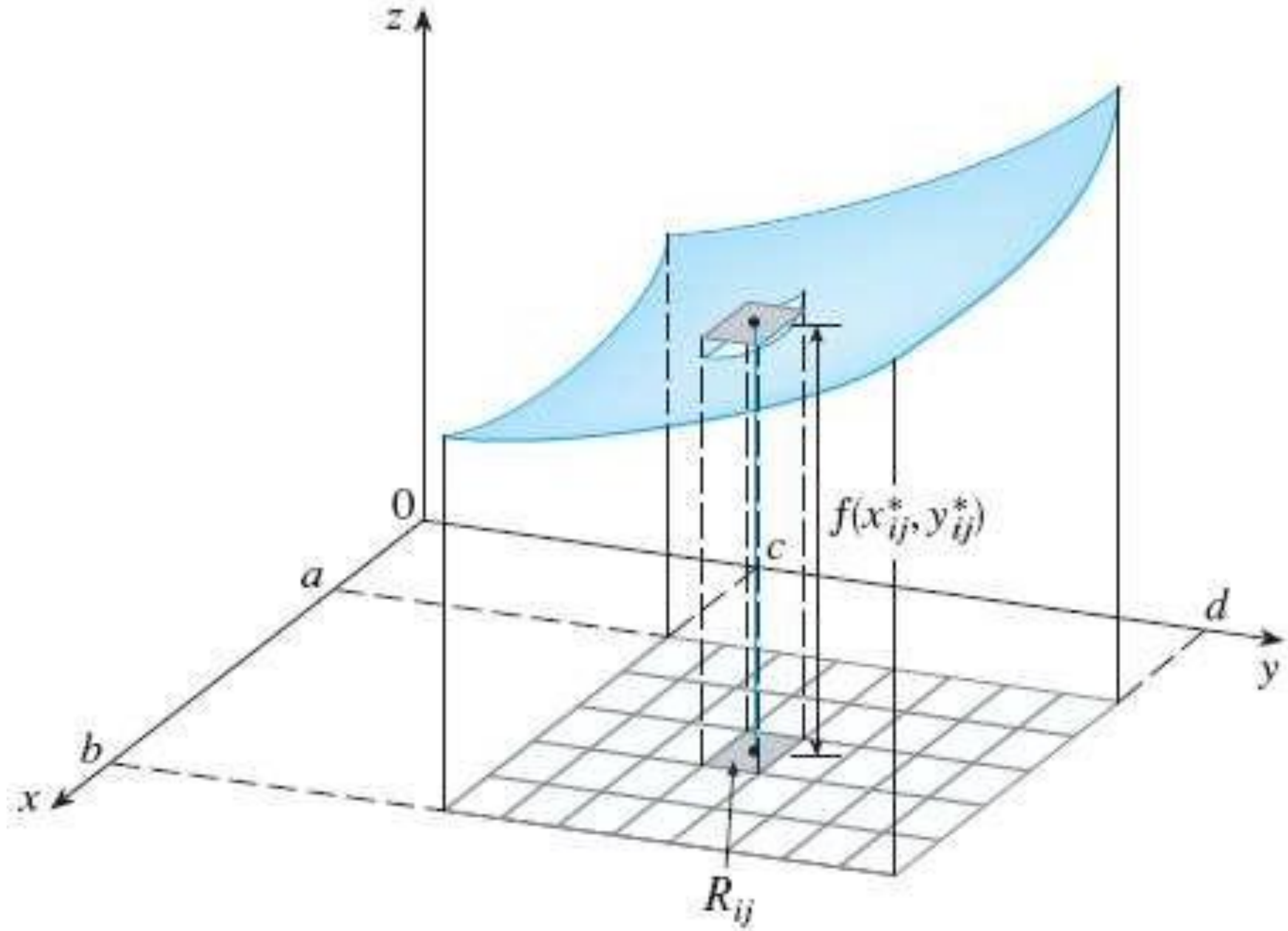


FIGURE 4

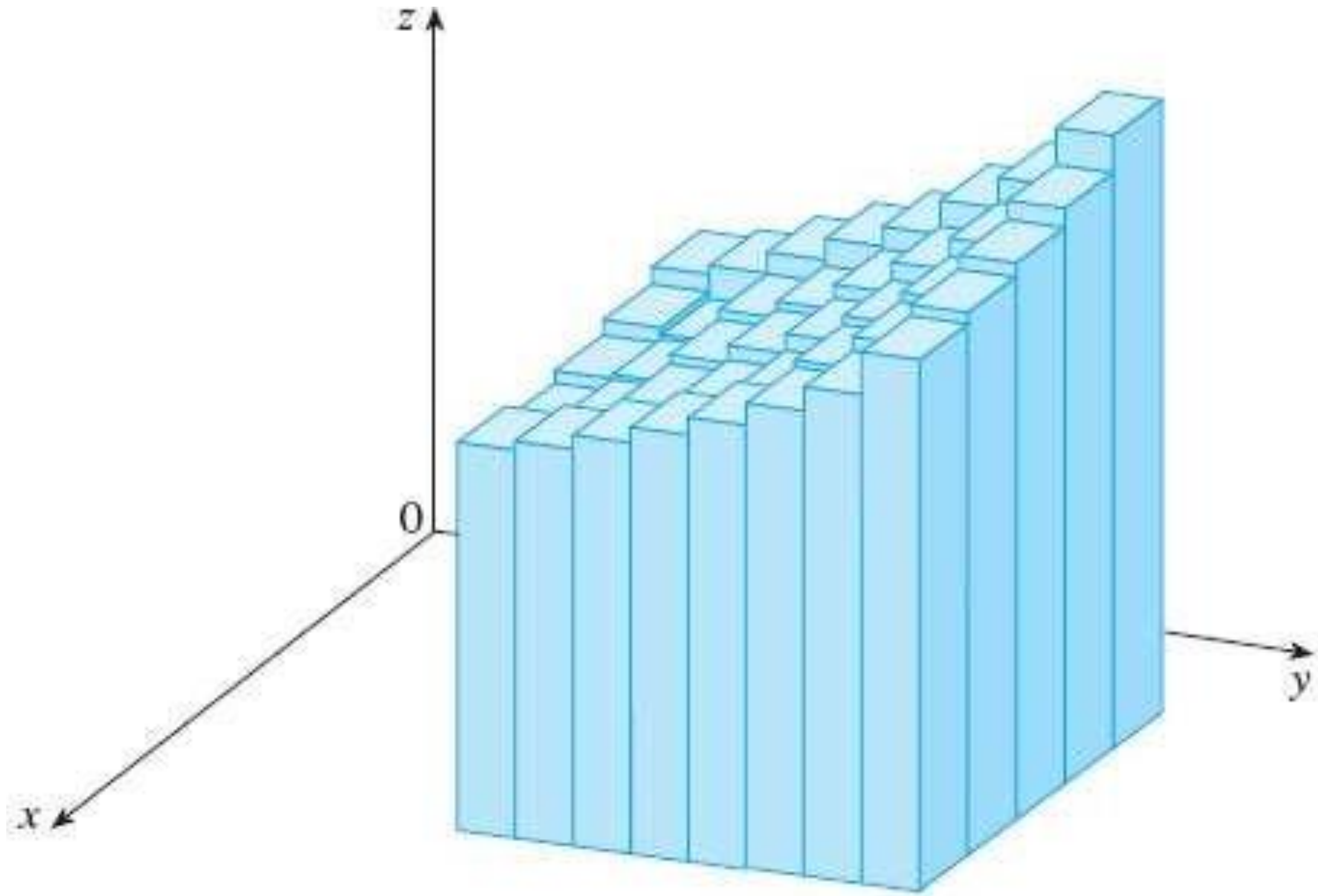


FIGURE 5

➢**DEFINITION** The **double integral** of f over the rectangle R is

$$\iint_R f(x,y)\,dA = \lim_{\max \Delta x_i, \Delta y_i \to 0} \sum_{i=1}^{m} \sum_{j=1}^{n} f(x_{ij}^*, y_{ij}^*)\,\Delta A_{ij}$$

if this limit exists.

$$\iint_R f(x, y)\,dA = \lim_{m,n\to\infty} \sum_{i=1}^{m} \sum_{j=1}^{n} f(x_i, y_j)\,\Delta A$$

If f (x, y) ≥ 0, then the volume V of the solid that lies above the rectangle R and below the surface z=f (x, y) is

$$v = \iint_R f(x, y)\,dA$$

➢MIDPOINT RULE FOR DOUBLE INTEGRALS

$$\iint_R f(x, y)\,dA \approx \sum_{i=1}^{m}\sum_{j=1}^{n} f(\overline{x_i}, \overline{y_j})\,\Delta A$$

where $\bar{x}_i$ is the midpoint of $[x_{i-1}, x_i]$ and $\bar{y}_j$ is the midpoint of $[y_{j-1}, y_j]$.

- **FUBINI'S THEOREM** If f is continuous on the rectangle $R=\{(x, y\} \mid a\leq x\leq b, c \leq y \leq d\}$, then

$$\iint_R f(x, y)dA=\int_a^b\int_c^d f(x, y)dydx=\int_c^d\int_a^b f(x,y)dxdy$$

More generally, this is true if we assume that f is bounded on R, is discontinuous only on a finite number of smooth curves, and the iterated integrals exist.

$$\iint_R g(x)h(y)\,dA = \int_a^b g(x)\,dx \int_c^d h(y)\,dy$$ where R $R = [a,b] \times [c,d]$

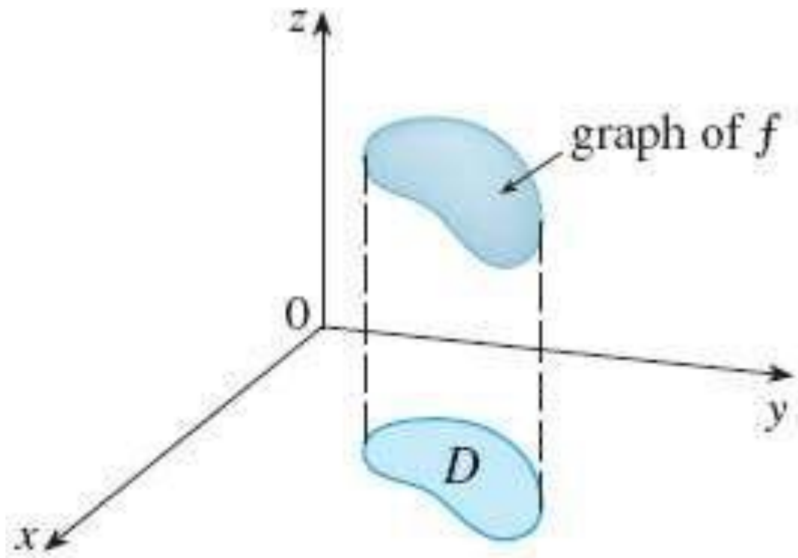


FIGURE 3

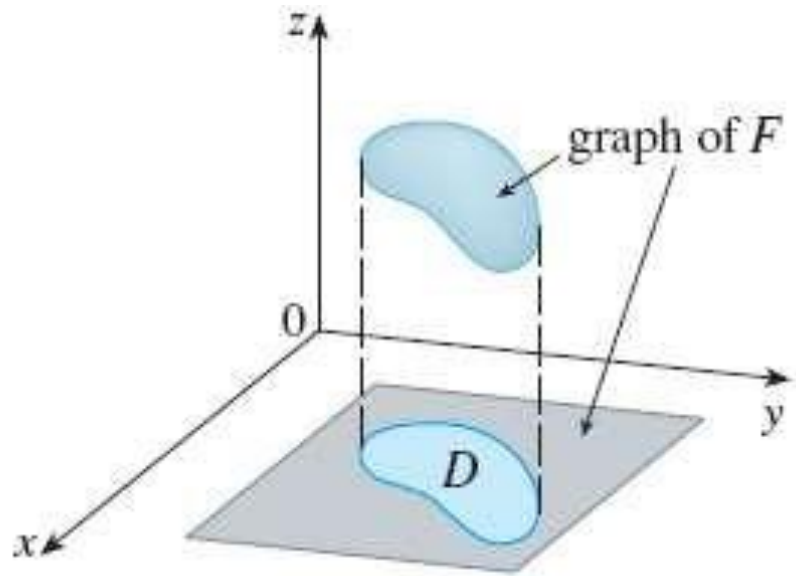
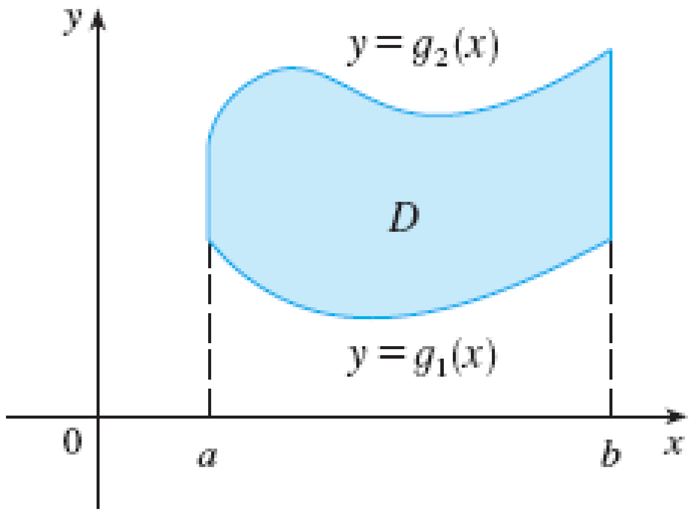
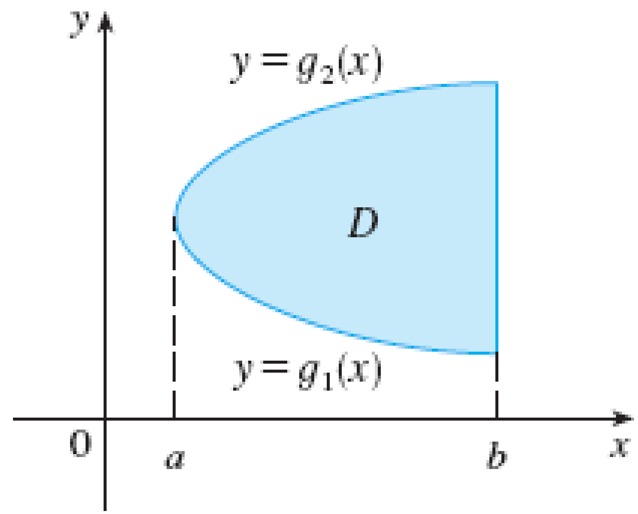


FIGURE 4





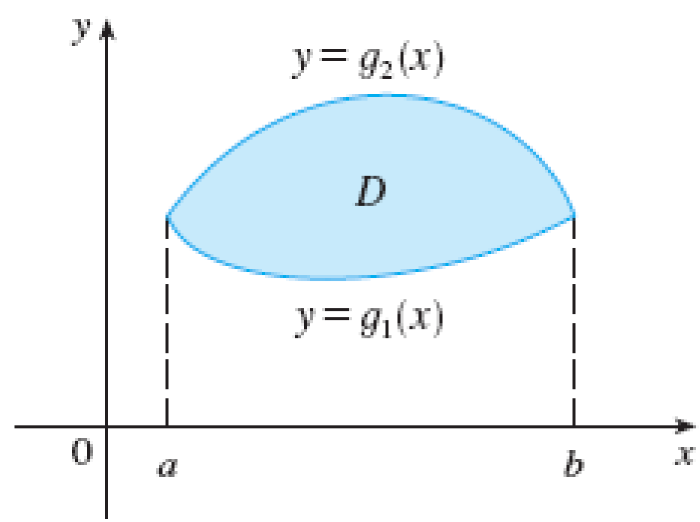


FIGURE 5 Some type I regions

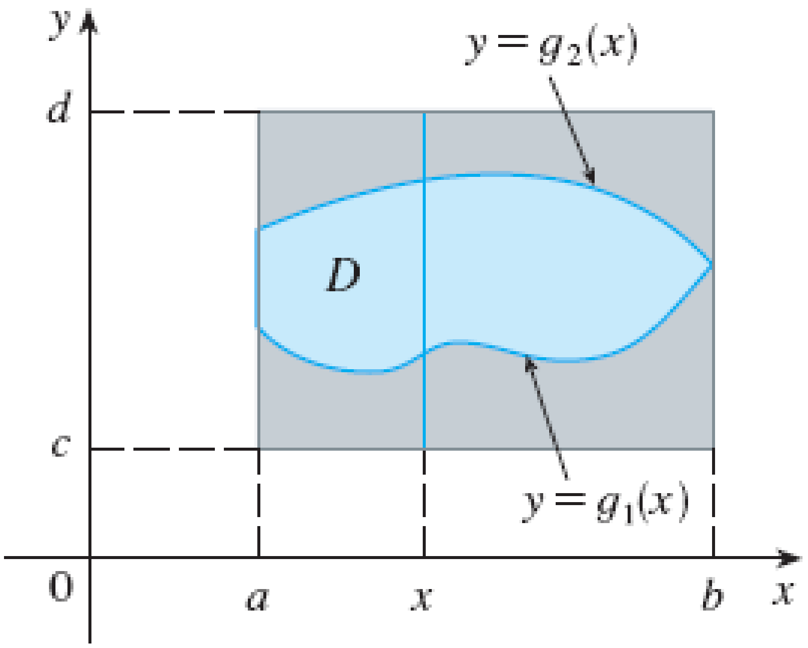


FIGURE 6

3.If f is continuous on a type I region *D* such that

$$D = \{(x, y) \mid a \le x \le b, g_1(x) \le y \le g_2(x)\}$$

then

$$\iint_D f(x, y)\,dA = \int_a^b \int_{g_1 x}^{g_2 x} f(x, y)\,dy\,dx$$

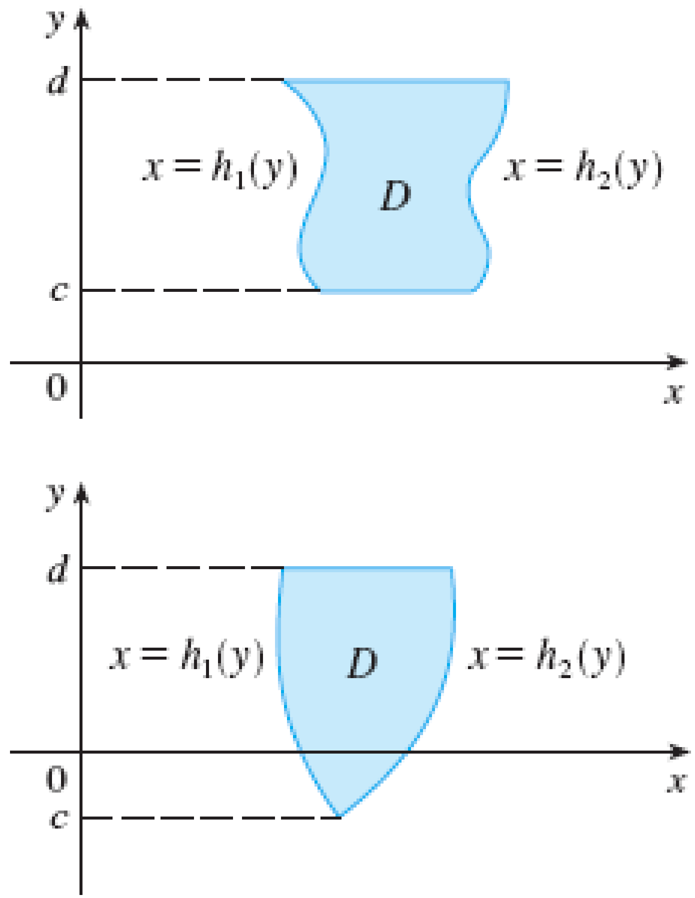


FIGURE 7

Some type II regions

$$\iint_D f(x, y)\,dA = \int_c^d \int_{h_1(y)}^{h_2(y)} f(x, y)\,dx\,dy$$

where *D* is a type II region given by Equation 4.

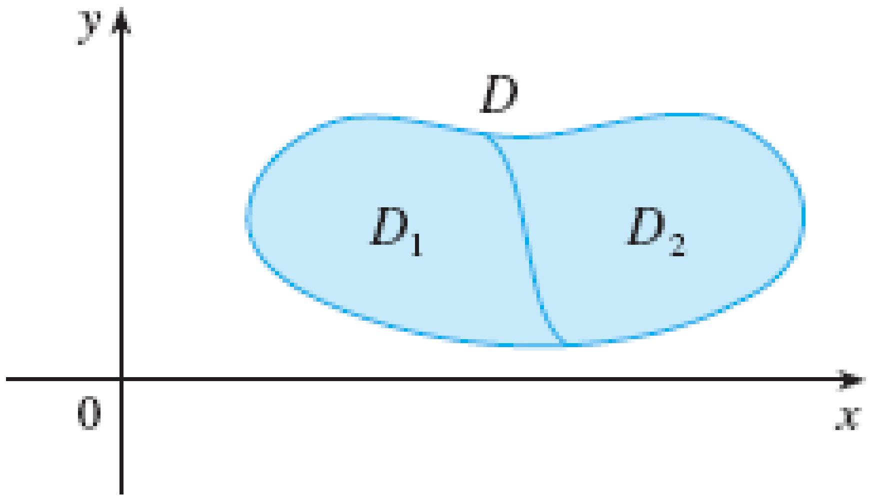


FIGURE 17

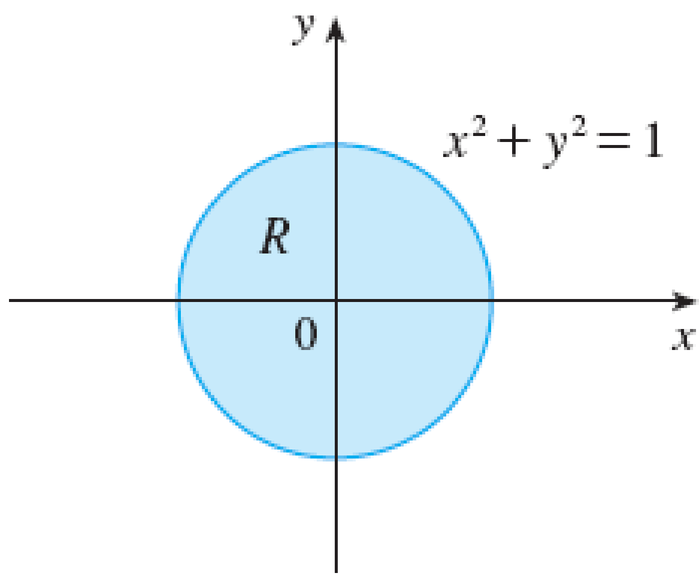
6. $\iint_D [f(x,y) + g(x,y)]dA = \iint_D f(x,y)dA + \iint_D g(x,y)dA$

7. $\iint_D cf(x,y)dA = c\iint_D f(x,y)dA$

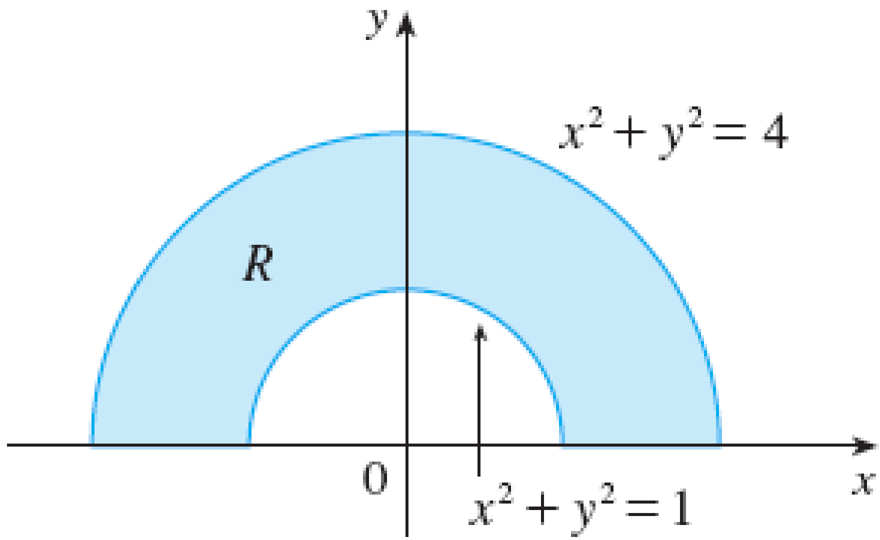
8. If f (x, y) ≥ g (x, y) for all (x, y) in D, then

$$\iint f(x,y)dA \geq \iint g(x,y)dA$$

9. $\iint_D f(x,y)dA = \iint_{D_1} f(x,y)dA + \iint_{D_2} f(x,y)dA$



(a) $R = \{(r, \theta) \mid 0 \leqslant r \leqslant 1, 0 \leqslant \theta \leqslant 2\pi\}$



(b) $R = \{(r, \theta) \mid 1 \leqslant r \leqslant 2, 0 \leqslant \theta \leqslant \pi\}$

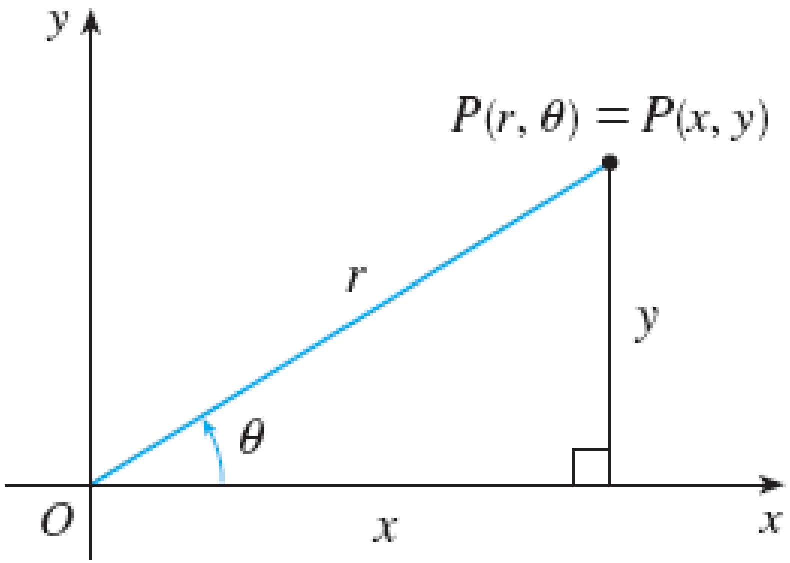


FIGURE 2

$$r^2=x^2+y^2 \quad x=r\cos\theta \quad y=r\sin\theta$$

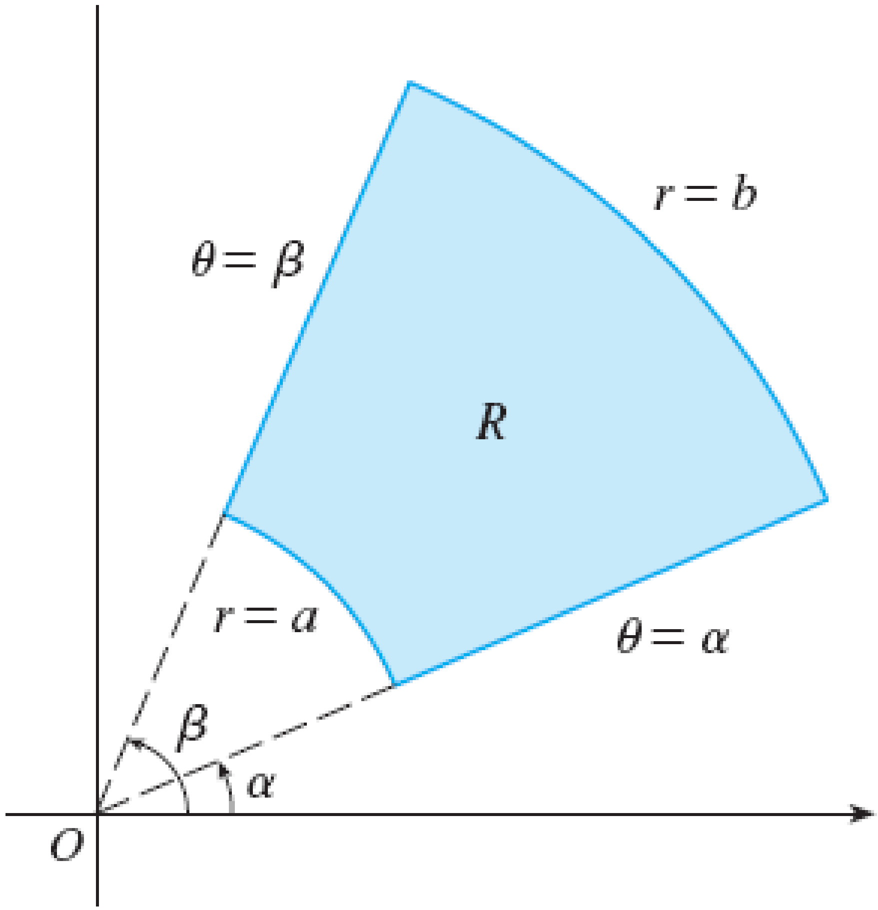


FIGURE 3 Polar rectangle

➢**CHANGE TO POLAR COORDINATES IN A DOUBLE INTEGRAL** If f is continuous on a polar rectangle R given by $0 \le a \le r \le b$, $\alpha \le \theta \le \beta$, where $0 \le \beta - \alpha \le 2\pi$, then

$$\iint_R f(x, y)\,dA = \int_\alpha^\beta \int_a^b f(r\cos\theta, r\sin\theta)\,r\,dr\,d\theta$$

3. If f is continuous on a polar region of the form

$$D = \{(r, \theta) \mid \alpha \le \theta \le \beta, h_1(\theta) \le r \le h_2(\theta)$$

then

$$\iint_D f(x, y)\,dA = \int_\alpha^\beta \int_{h_1(\theta)}^{h_2(\theta)} f(r\cos\theta, r\sin\theta)\, r\,dr\,d\theta$$

MCQ

Evaluate the double integral.

$$\int_0^2 \int_0^1 4x^2 y \, dy \, dx$$

a. 14/3

b. 15/3

c. 16/3

d. 4

MCQ

Find the value of ∬ xydxdy over the area bpunded by parabola $y=x^2$ and $x = -y^2$.

a) $\frac{1}{67}$
b) $\frac{1}{24}$
c) $-\frac{1}{6}$
d) $-\frac{1}{12}$

Answer: b

Explanation:

$$\int_0^1 \int_{-\sqrt{y}}^{-y^2} y.xdxdy = \frac{1}{2}\int_0^1 y[y^4 - y]dy = \frac{1}{2}[\frac{1}{6} - \frac{1}{3}] = -\frac{1}{12}$$

MCQ

Find the value of integral $\int_0^1 \int_{x^2}^{x} xy(x+y)dydx$.

a) $3/15$
b) $2/15$
c) $2/30$
d) $1/15$

Answer: b

Explanation: Given, F(x)=$\int_0^1 \int_{x^2}^{x} xy(x+y)dydx = \int_0^1 \int_{x^2}^{x} (x^2y+xy^2)dydx$

=$\int_0^1 [\frac{x^2y^2}{2} + \frac{xy^3}{3}]_x^{x^2} dx = \int_0^1 [\frac{x^3}{2} + \frac{x^4}{3} - \frac{x^4}{2} - \frac{x^5}{3}]dx = \frac{1}{2} + \frac{1}{3} - \frac{1}{2} - \frac{1}{5} = \frac{2}{15}$

➢**DEFINITION** The **triple integral** of f over the box B is

$$\iiint_B f(x, y, z)\,dV = \lim_{\max \Delta x_i, \Delta y_j, \Delta z_k \to 0} \sum_{i=1}^{l} \sum_{j=1}^{m} \sum_{k=1}^{n} f(x_{ijk}^*, y_{ijk}^*, z_{ijk}^*)\Delta V_{ijk}$$

if this limit exists.

$$\iiint_B f(x, y, z)\,dV = \lim_{l,m,n\to\infty} \sum_{i=1}^{l} \sum_{j=1}^{m} \sum_{k=1}^{n} f(x_i, y_j, z_k)\Delta V$$

➢**FUBINI'S THEOREM FOR TRIPLE INTEGRALS** If f is continuous on the rectangular box B=[a, b]$\times$ [c, d]$\times$[r, s], then

$$\iiint_B f(x, y, z)dV = \int_r^s \int_c^d \int_a^b f(x, y, z)dxdydz$$

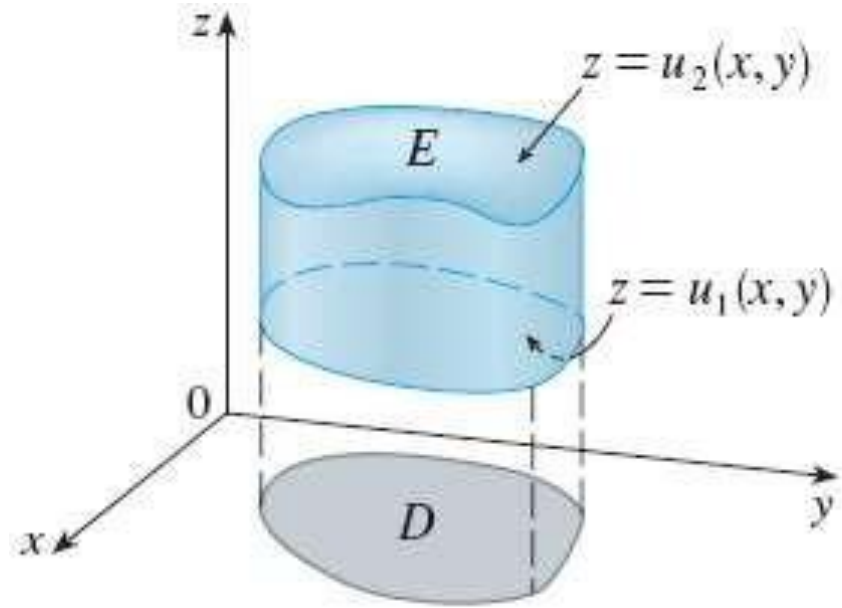


FIGURE 2
A type 1 solid region

$$\iiint_E f(x, y, z)\,dV = \iint_D \left[\int_{u_1(x,y)}^{u_2(x,y)} f(x, y, z)\,dz \right] dA$$

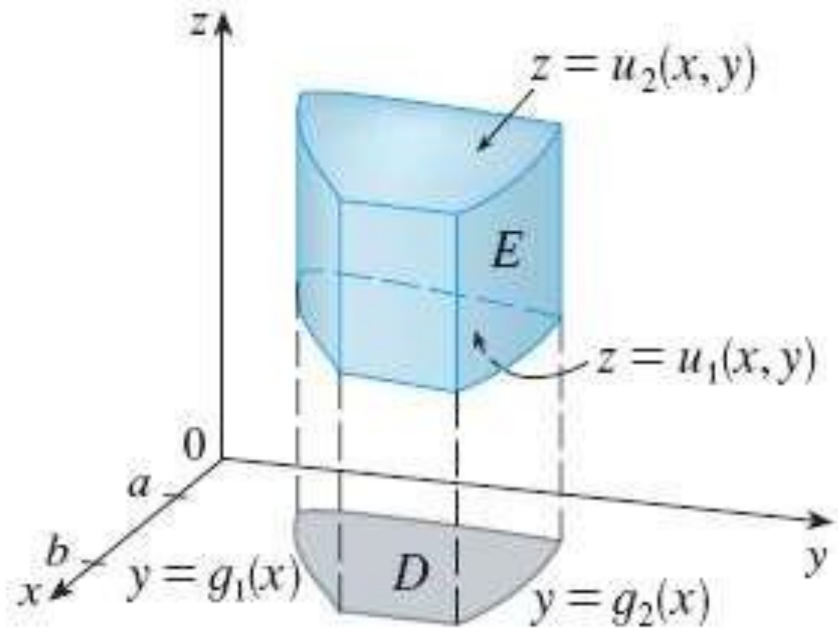


FIGURE 3
A type 1 solid region

$$\iint_E f(x, y, z)\,dV = \int_a^b \int_{g_1(x)}^{g_2(x)} \int_{u_1(x,y)}^{u_2(x,y)} f(x, y, z)\,dz\,dy\,dx$$

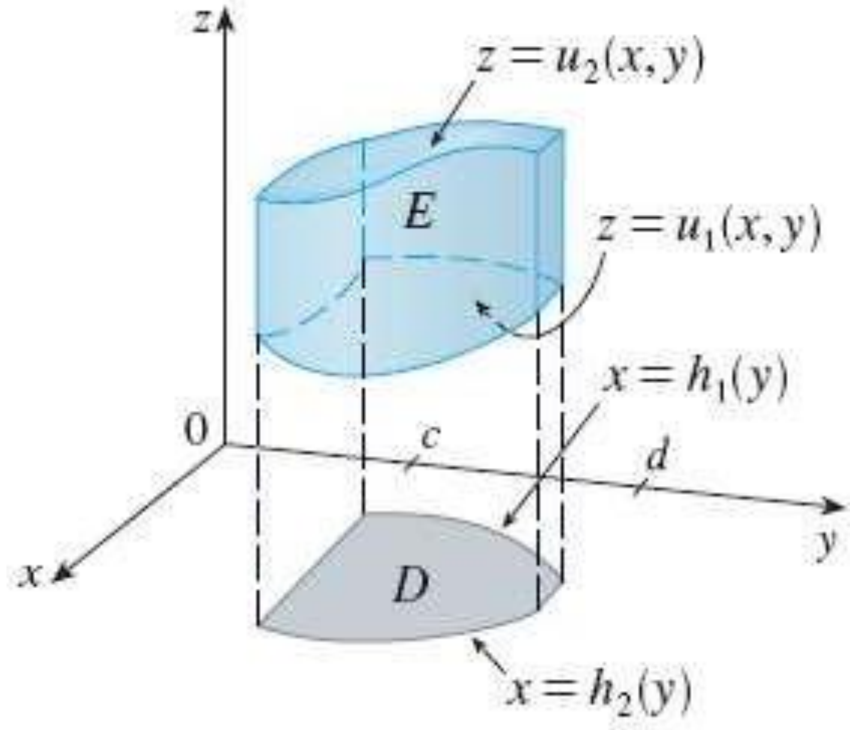


FIGURE 4

Another type 1 solid region

$$\iint_E f(x, y, z)\,dV = \int_c^d \int_{h_1(x)}^{h_2(x)} \int_{u_1(x,y)}^{u_2(x,y)} f(x, y, z)\,dz\,dx\,dy$$

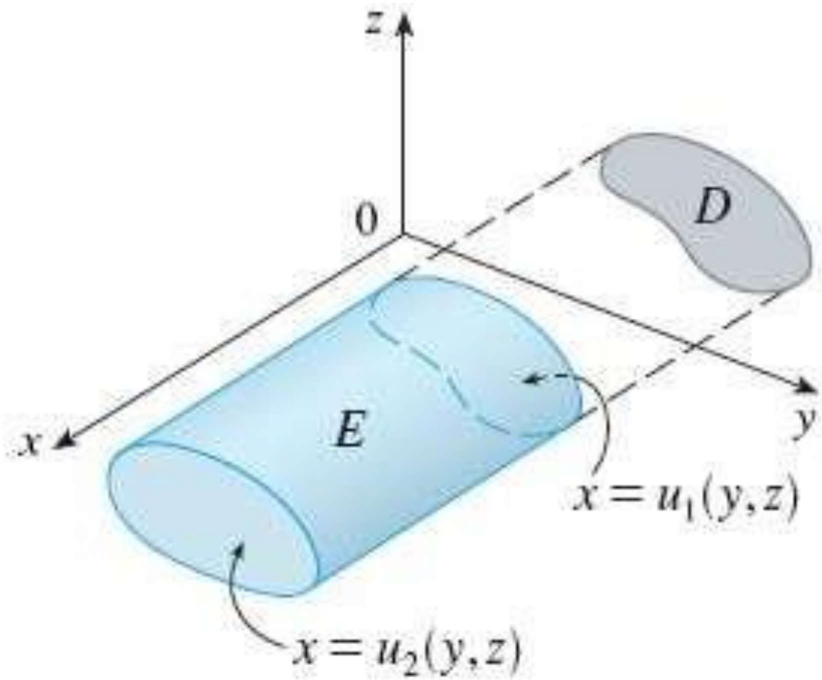


FIGURE 7
A type 2 region

$$\iiint_E f(x, y, z)\,dV = \iint_D \int_{u_1(y,z)}^{u_2(y,z)} f(x, y, z)\,dx\;dA$$

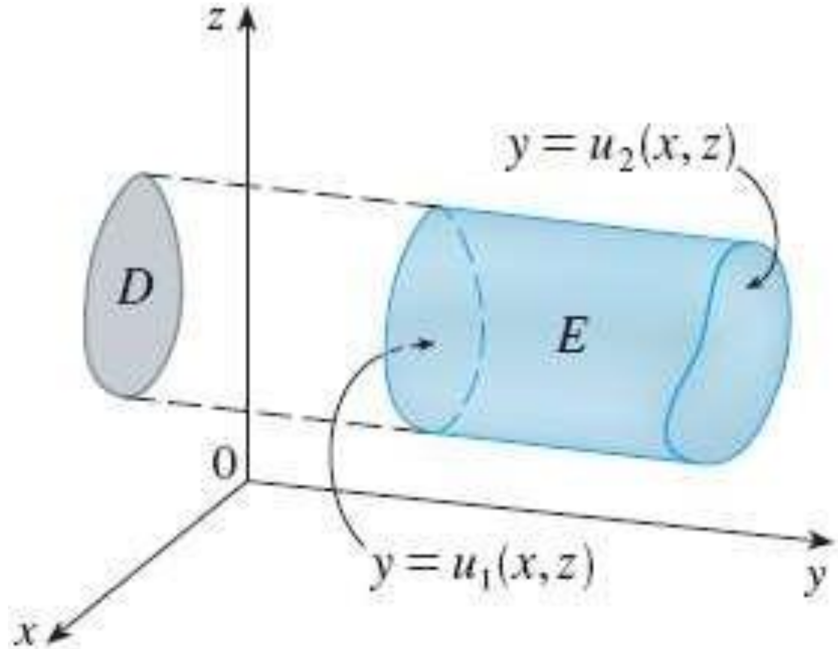


FIGURE 8
A type 3 region

$$\iiint_E f(x, y, z)\,dV = \iint_D \left[\int_{u_1(x,z)}^{u_2(x,z)} f(x, y, z)\,dy\right] dA$$

$$V(E) = \iiint_E dV$$

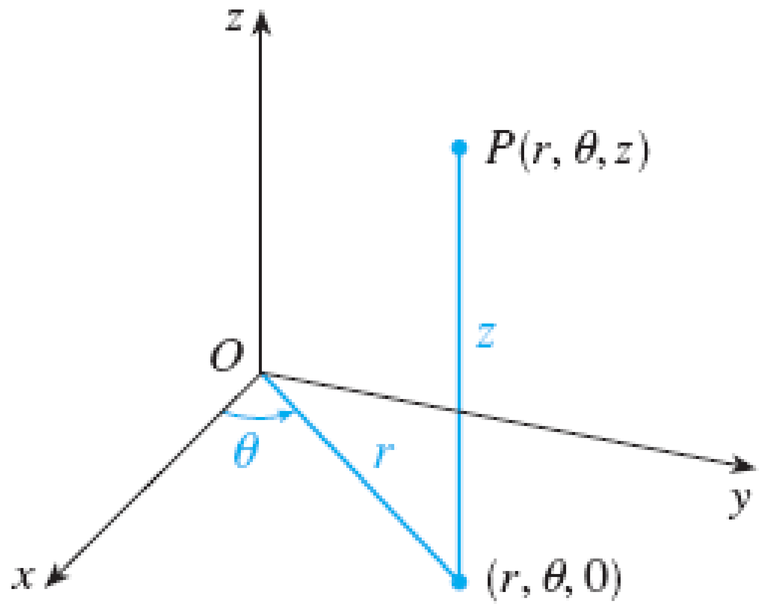


FIGURE 2

The cylindrical coordinates of a point

To convert from cylindrical to rectangular coordinates, we use the equations

1 $x = r\cos\theta \quad y = r\sin\theta \quad z = z$

whereas to convert from rectangular to cylindrical coordinates, we use

2. $r^2 = x^2 + y^2 \quad \tan\theta = \dfrac{y}{x} \quad z = z$

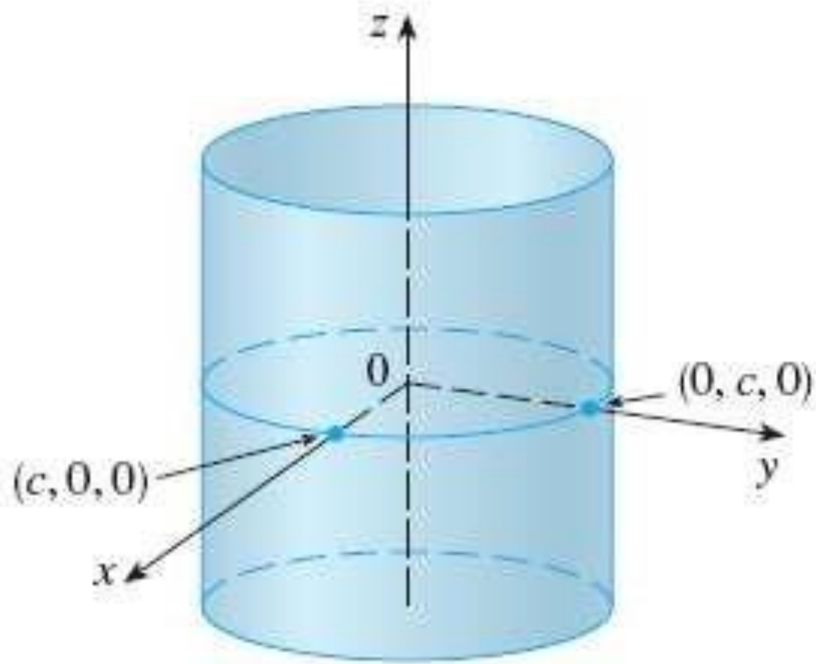


FIGURE 4

$r = c$, a cylinder

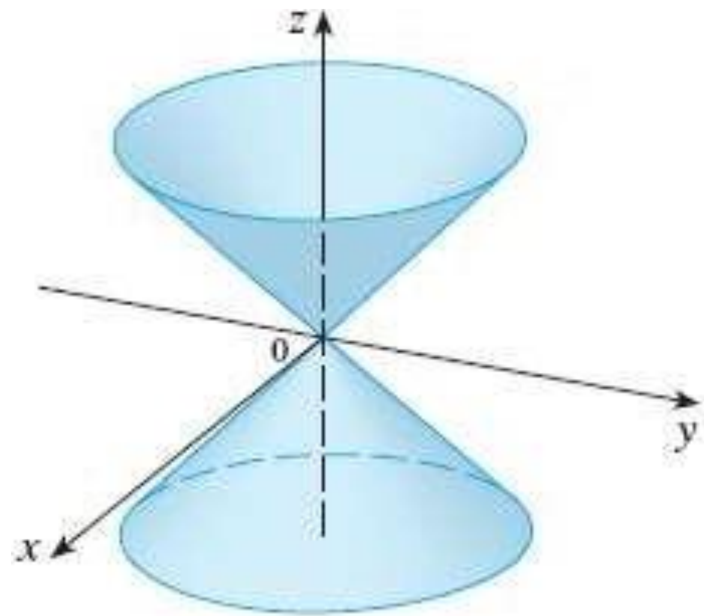


FIGURE 5

$z = r$, a cone

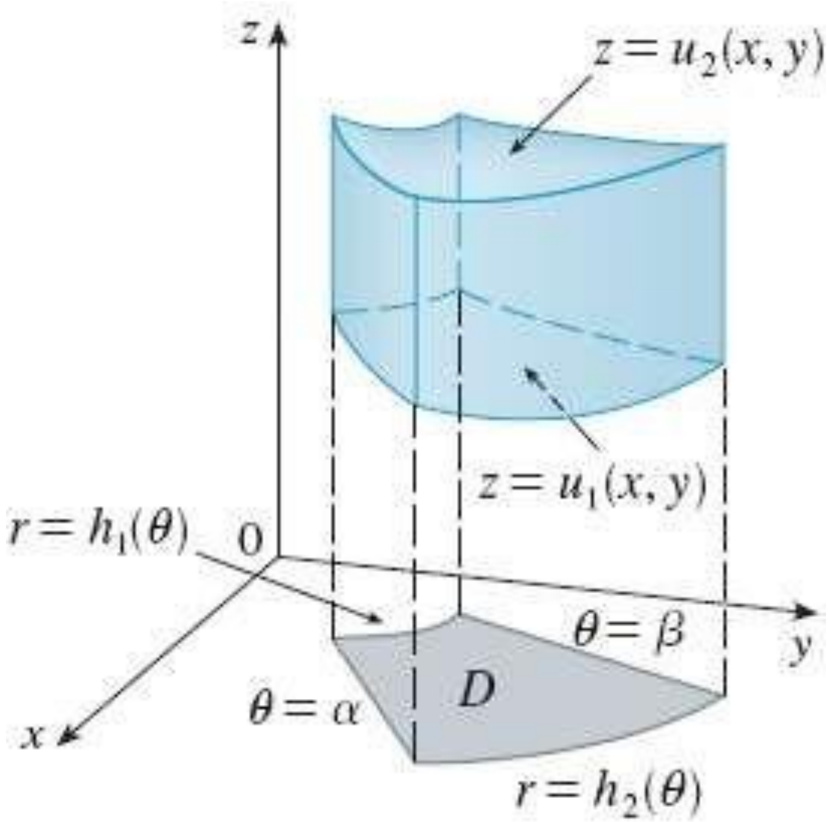


FIGURE 6

➢formula for triple integration in cylindrical coordinates.

$$\iiint_E f(x, y, z)\,dV = \int_{\alpha}^{\beta}\int_{h_1(\theta)}^{h_2(\theta)}\int_{u_1(r\cos\theta, r\sin\theta)}^{u_2(r\cos\theta, r\sin\theta)} f(r\cos\theta, r\sin\theta, z)\, r\,dz\,dr\,d\theta$$

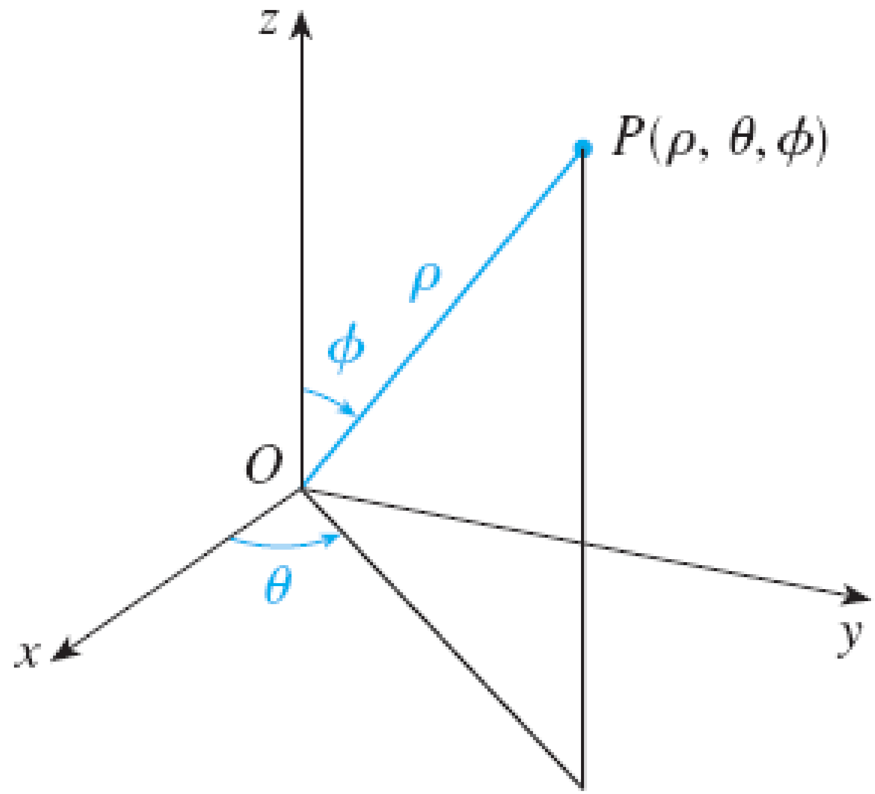


FIGURE 1

The spherical coordinates of a point

$$p \geq 0 \qquad 0 \leq \phi \leq \pi$$

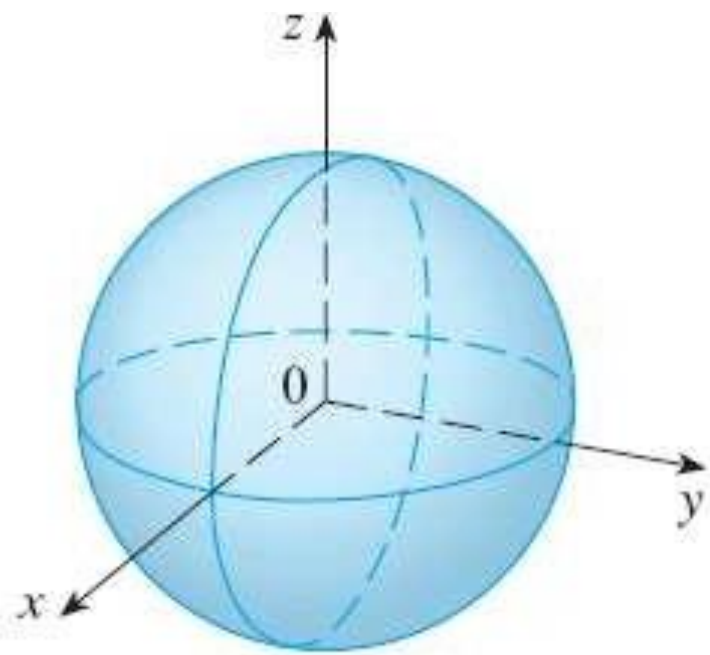


FIGURE 2 $\rho = c$, a sphere

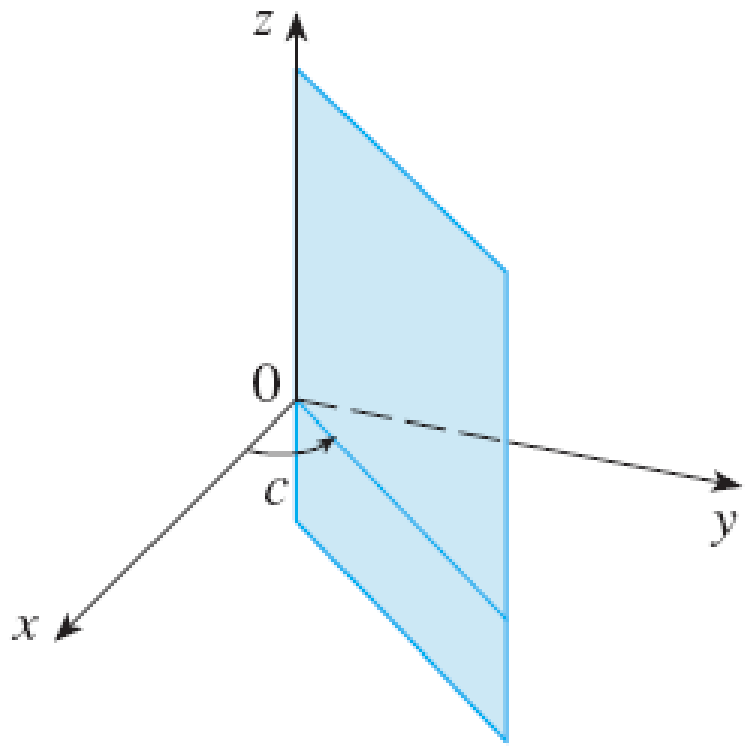
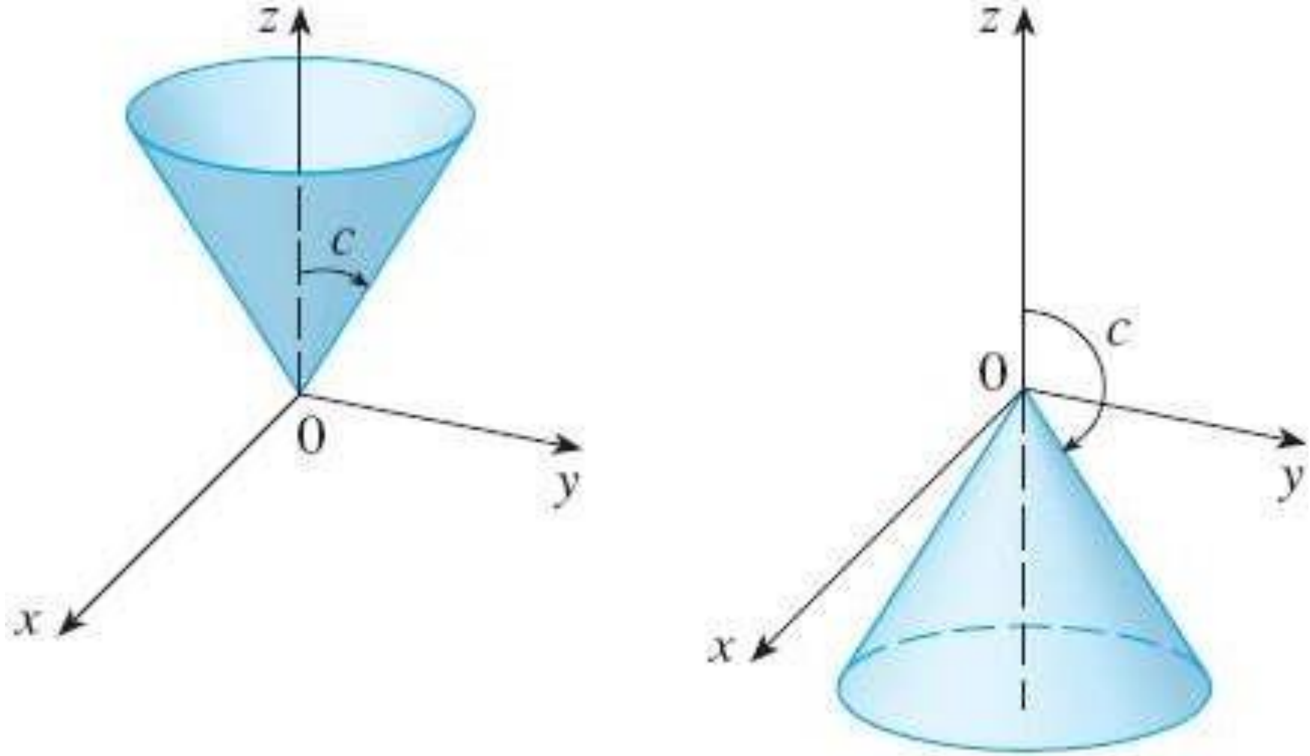


FIGURE 3 $\theta = c$, a half-plane



$0 < c < \pi/2$ $\qquad$ $\pi/2 < c < \pi$

FIGURE 4 $\phi = c$, a half-cone

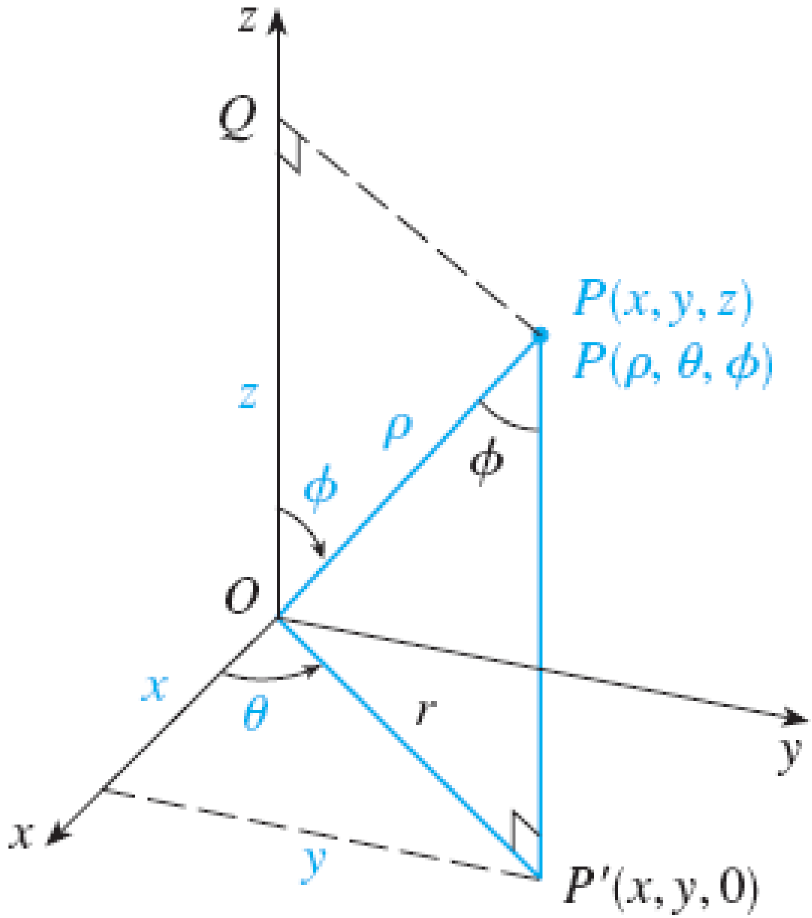


FIGURE 5

$$x = p\sin\phi\cos\theta \qquad y = p\sin\phi\sin\theta \qquad z = p\cos\phi$$

$$p^2 = x^2 + y^2 + z^2$$

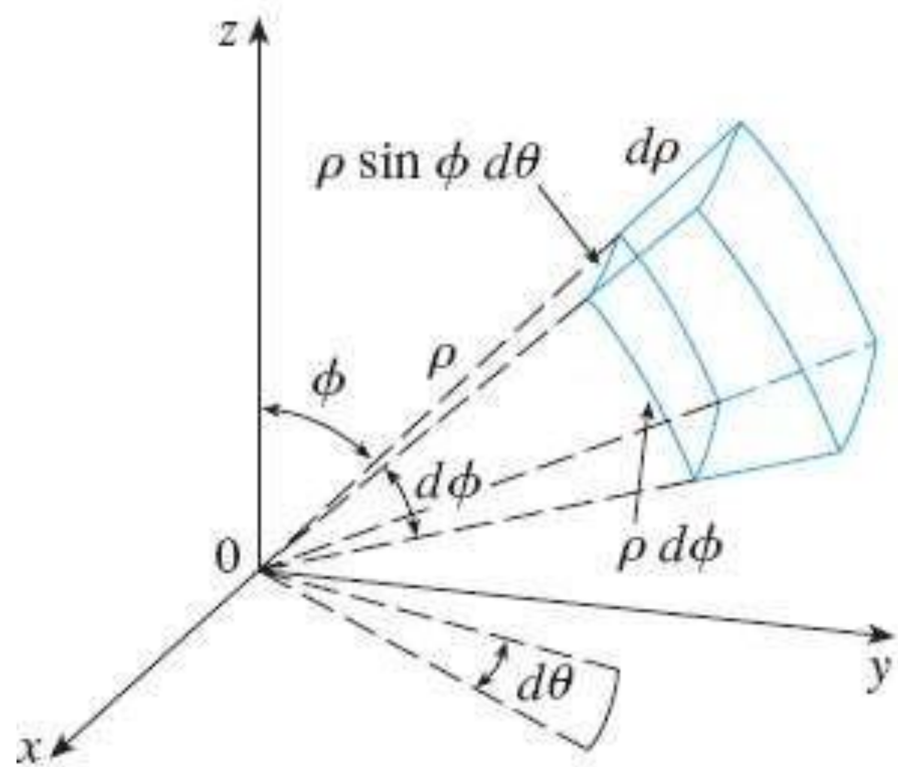


FIGURE 8

Volume element in spherical coordinates: dV=p²sinødpdΘd ø

➢Formula for triple integration in spherical coordinates

$$\iiint_E f(x, y, z)\,dV$$

$$= \int_c^d \int_\alpha^\beta \int_a^b f(p\sin\phi\cos\theta, p\sin\phi som\theta, p\cos\phi)\, p^2 \sin\phi\, dp\, d\theta\, d\phi$$

where E is a spherical wedge given by

$$E = \{(p, \theta, \phi) | a \leq p \leq b, \alpha \leq \theta \leq \beta, c \leq \phi \leq d\}$$

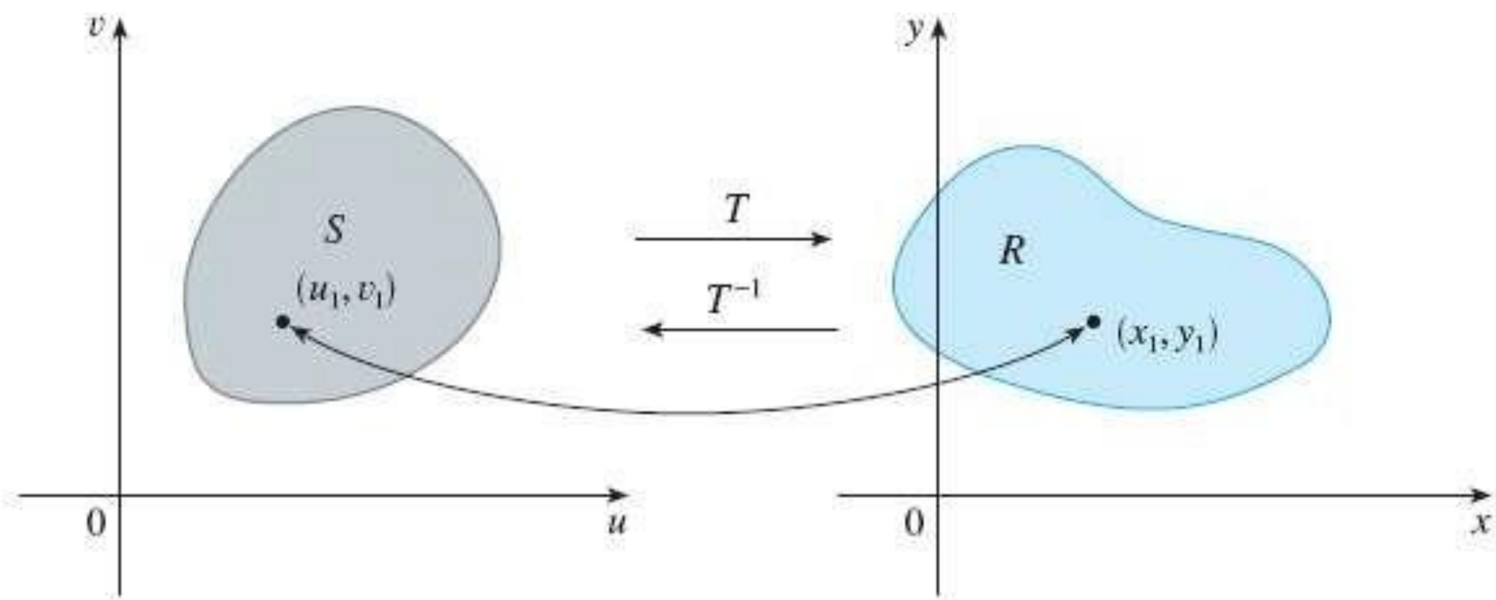
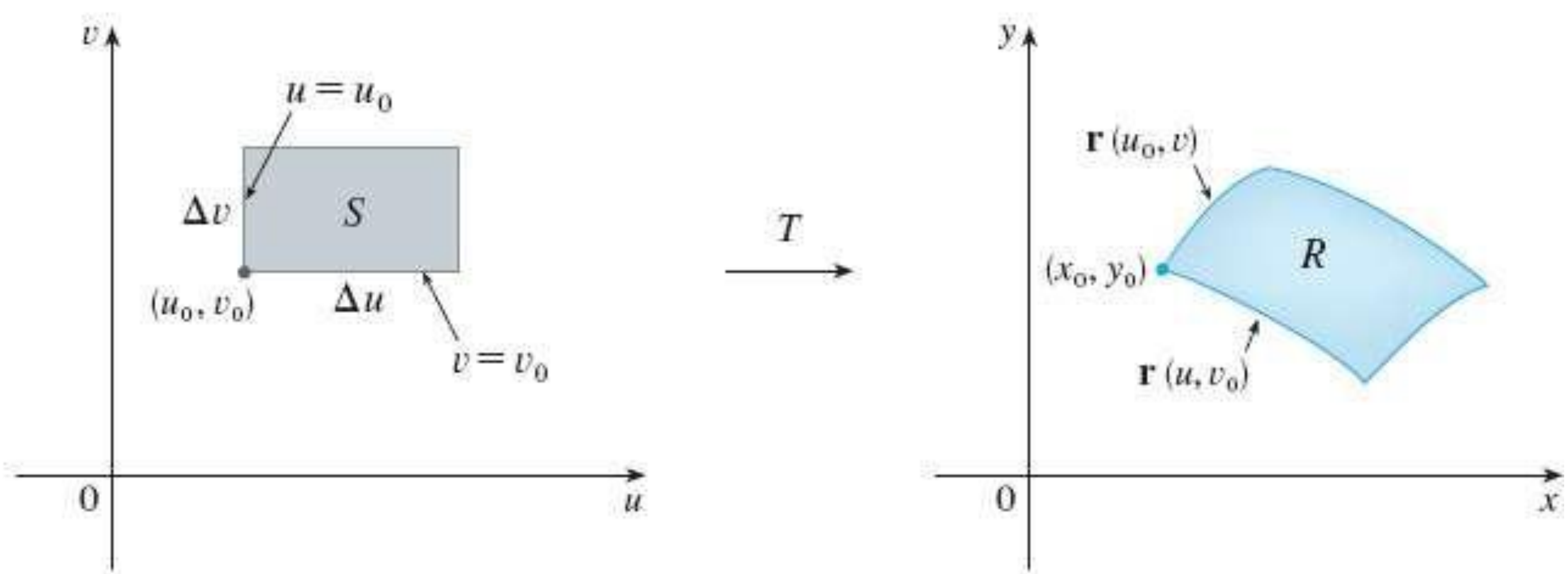


FIGURE 1



v
$u = u_0$
Δv
S
(u_0, v_0)
Δu
$v = v_0$
0
u
T
y
$\mathbf{r}\,(u_0, v)$
R
(x_0, y_0)
$\mathbf{r}\,(u, v_0)$
0
x

FIGURE 3

- **DEFINITION** The **Jacobian** of the transformation T given by x= g (u, v) and y= h (u, v) is

$$\frac{\partial(x, y)}{\partial(u, v)} = \begin{vmatrix} \dfrac{\partial x}{\partial u} & \dfrac{\partial x}{\partial v} \\ \dfrac{\partial y}{\partial u} & \dfrac{\partial y}{\partial v} \end{vmatrix} = \frac{\partial x}{\partial u}\frac{\partial y}{\partial v} - \frac{\partial x}{\partial v}\frac{\partial y}{\partial u}$$

➢**CHANGE OF VARIABLES IN A DOUBLE INTEGRAL** Suppose that T is a C^1 transformation whose Jacobian is nonzero and that maps a region S in the uv-plane onto a region R in the xy-plane. Suppose that f is continuous on R and that R and S are type I or type II plane regions. Suppose also that T is one-to-one, except perhaps on the boundary of . S. Then

$$\iint_R f(x, y)\,dA = \iint_S f(x(u,v), y(u,v)) \left|\frac{\partial(x, y)}{\partial(u,v)}\right| du\,dv$$

Let T be a transformation that maps a region S in uvw-space onto a region R in xyz-space by means of the equations

$$x=g\,(u, v, w) \quad y=h\,(u, v, w) \quad z=k\,(u, v, w)$$

The **Jacobian** of T is the following 3X3 determinant:

12.

$$\frac{\partial(x, y, z)}{\partial(u, v, w)} = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} & \frac{\partial x}{\partial w} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} & \frac{\partial y}{\partial w} \\ \frac{\partial z}{\partial u} & \frac{\partial z}{\partial v} & \frac{\partial z}{\partial w} \end{vmatrix}$$

13. $$\iiint_R f(x, y, z)\, dV$$

$$= \int_S \int f(x(u, v, w), y(u, v, w), z(u, v, w)) \left| \frac{\partial(x, y, z)}{\partial(u, v, w)} \right| du\, dv\, d$$

$$\int_0^1 \int_x^{2x} \int_{x-2}^{x+2} dy\, dz\, dx$$

$$= \int_0^1 \int_x^{2x} \left[y \right]_{x-2}^{x+2} dz\, dx = \int_0^1 \int_x^{2x} (\cancel{x}+2) - (\cancel{x}-2)\, dz\, dx$$

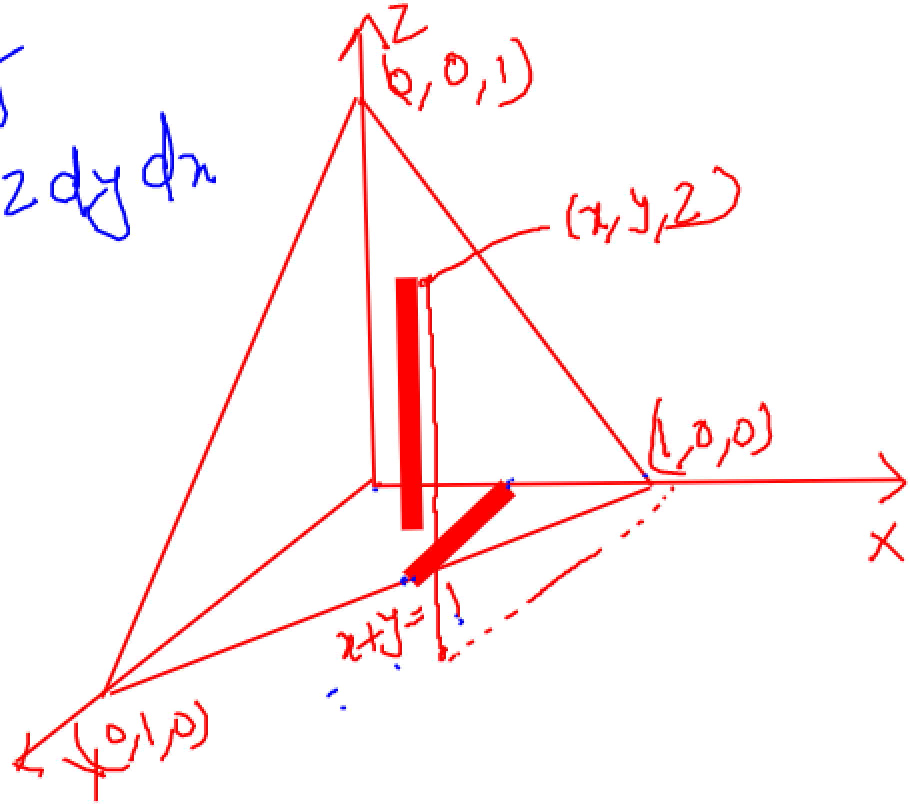
$$= 2 \int_0^1 \left(\int_x^{2x} z\, dz \right) dx = \cancel{2} \int_0^1 \left[\frac{z^2}{\cancel{2}} \right]_x^{2x} dx$$

$$= \int_0^1 (4x^2 - x^2)\, dx = 3 \int_0^1 x^2\, dx$$

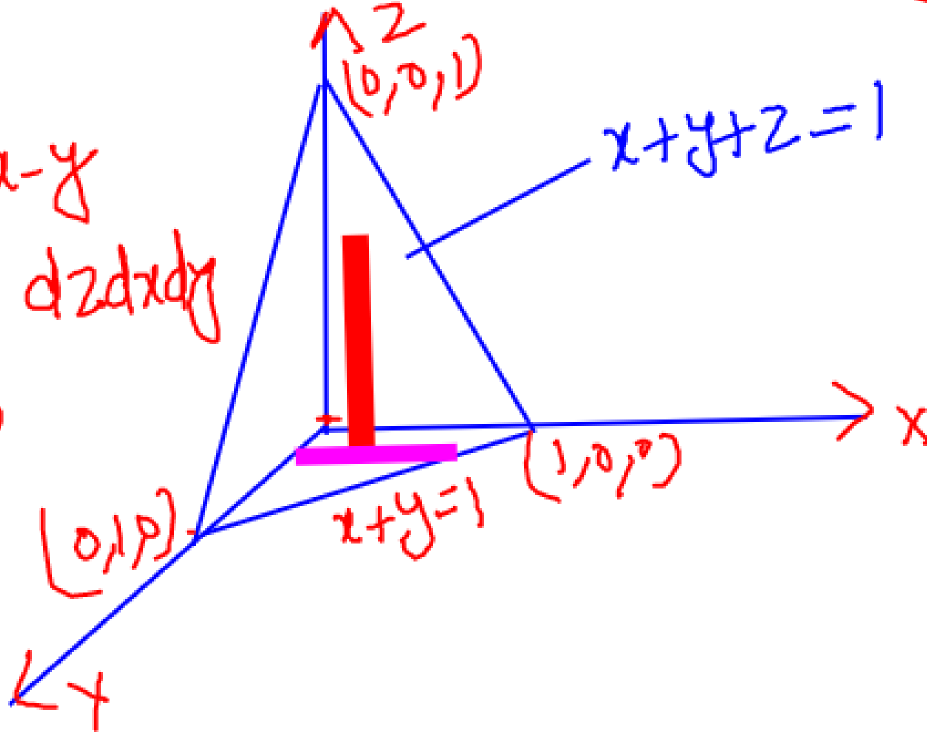
$$= \cancel{3} \left[\frac{x^3}{\cancel{3}} \right]_0^1 = 1$$

Find the volume of the region bounded by $x=0$, $y=0$, $z=0$, $x+y+z=1$

$$\text{vol} = \int_0^1 \int_{y=0}^{1-x} \int_{z=0}^{1-x-y} dz\,dy\,dx$$



Find the volume of the region bounded by $x=0$, $y=0$, $\underbrace{x+y+z=1}$



$$\text{Vol} = \int_{0}^{1}\int_{x=0}^{1-y}\int_{z=0}^{1-x-y} dz\,dx\,dy$$

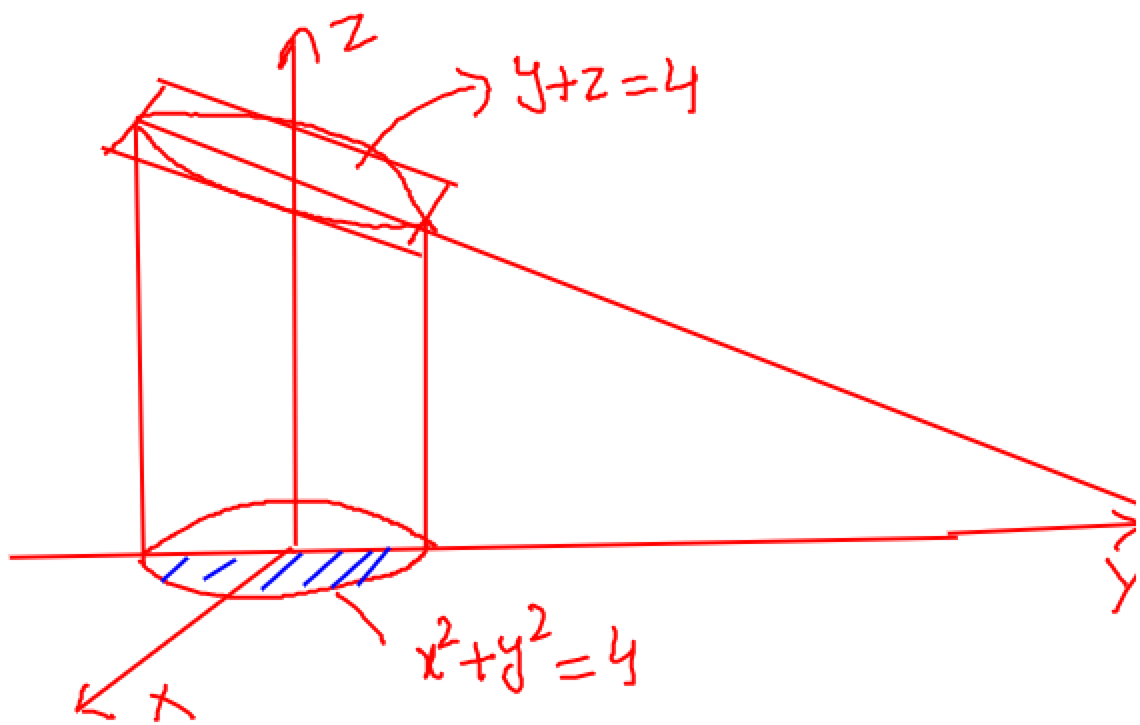
$0 \le z \le 1-x-y$

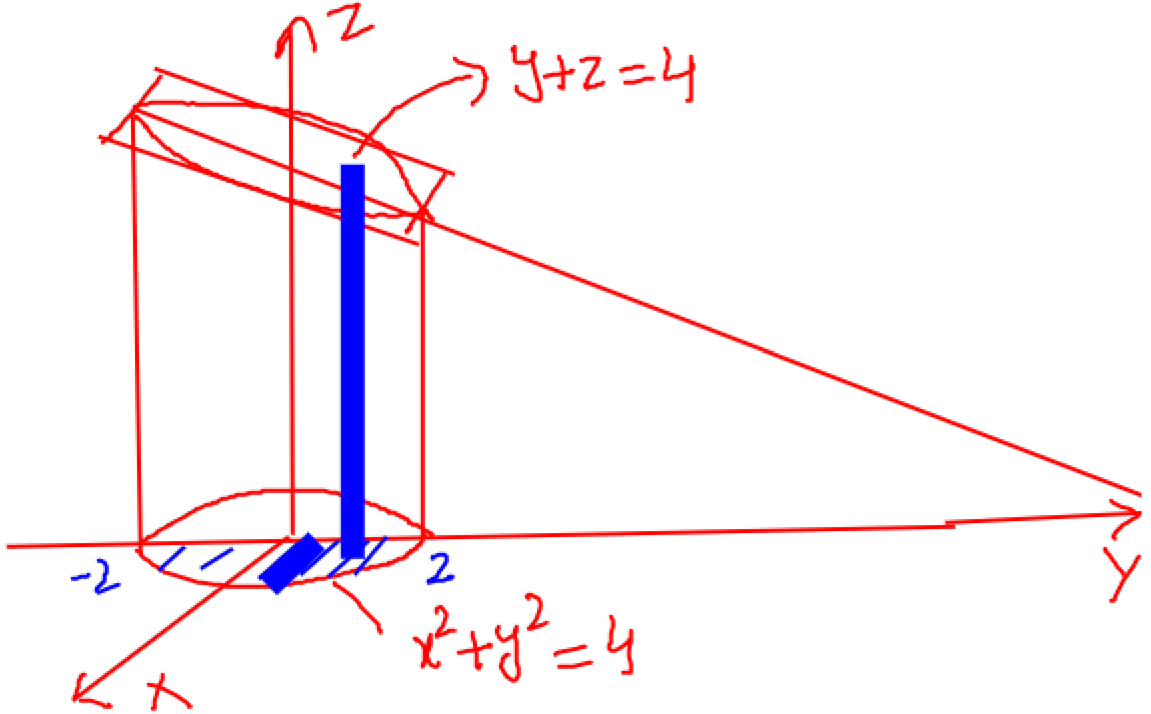
$0 \le y \le 1-x$

$0 \le x \le 1$

Find the volume bounded by the cylinder $x^2+y^2=4$ and the planes $y+z=4$ and $z=0$.

Find the volume bounded by the cylinder $x^2+y^2=4$ and the planes $y+z=4$ and $z=0$.



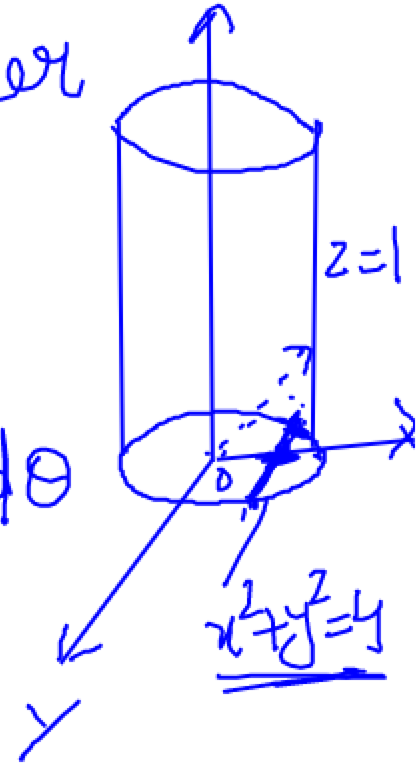


z
y+z=4
-2
2
y
x
x²+y²=4

$$\text{Volume} = 2\int_{-2}^{2}\int_{0}^{\sqrt{4-y^2}}\int_{z=0}^{4-y} dz\,dx\,dy$$

Find the volume of cylinder

$x^2+y^2=4, \quad z=1$



$$Vol = \int_{\theta=0}^{2\pi}\int_{r=0}^{2}\int_{z=0}^{1} r\,dz\,dr\,d\theta$$

$$Vol = \int_{-2}^{2}\int_{y=-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\int_{z=0}^{1} dz\,dy\,dx$$

$x^2+y^2=4$

$y^2 = 4-x^2$

$y=\pm\sqrt{4-x^2}$

Evaluate $\displaystyle\iiint_E 6z^2\,dV$ where E is the region below $4x+y+2z=10$ in the first octant.

Ans $\dfrac{625}{2}$

THANK

YOU…….