Probability

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Probability is a measure of likelihood that an event will occur.

Example: **Tossing a coin:** When a coin is tossed, there are two possible outcomes : either heads (H) or tails (T). We say that the probability of the coin landing H is ½. And the probability of the coin landing T is ½

TERMINOLOGY

Random Experiment: Experiments whose outcomes are unpredictable is known as Random Experiments. For example: Tossing a coin

Sample Space(S): It is the collection of all possible outcome of an experiment. Example: In tossing a coin one time S={H,T}

Event: The outcome of an experiment is known as Event. Mathematically we can say that event is a subset of sample space.

Example: Getting a head while tossing a coin one time is an event.

Types of Events:

1. **Complementary Event**: It is denoted by E^c or E'. It contains all the outcomes of sample space that is not in E.

For example If S={1,2,3,4,5,6} and E=[odd no.]={1,3,5}
Then E'=[Even number]={2,4,6}

2. Equally likely Events: If E and F are two events such that P(E)= P(F) then these are called Equally Likely events. For e.g. In Tossing a coin probability of coming up of head and tail is Equal.

3. Mutually Exclusive Event: Two or more events are said to be mutually exclusive if both cannot occur simultaneously in the same experiment.

Example: In a throw of single coin, either head can come or tail can come. There will be no common outcome in those events. It means E∩F=Φ (null Set)

4. Collective Exhaustive Events: If E and F are two events and both events gives complete sample space then these are called Exhaustive events.

E **U** F = S

5. Independent Events: Two events are said to be independent of each other when the happening of one event does not affect the happening of other event and vice versa. Here sample spaces are different for both cases.

When two events A and B are independent, the probability of both occurring is:

P(A and B)= P(A). P(B)

Probability: The probability of an event is defined as the ratio of no. of ways an event can happen to the no. of ways sample space can happen.

Let S be the sample space and let E be the event.
PROBABILITY= n(E)/n(S)

6. Dependent Events: Two **events** are **dependent** if the outcome or occurrence of the first affects the outcome or occurrence of the second so that the probability is changed.

When two events, A and B, are dependent, the probability of both occurring is:

P(A and B) = P(A) $\cdot$ **P(B|A)**

where **P(B|A)** is the **conditional probability** of an event B in relationship to an event A is the Probability that event B occurs given that event A has already occurred.

Example: The probability of choosing a jack on the second pick given that a queen was chosen on the first pick (without replacement)   is   called   a   *conditional   probability.*

Rules of Probability

1. $0 \leq P(E) \leq 1$

2. $P(S) = 1$ (Definite event)

3. $P(\Phi) = 0$ (Impossible event)

4. If A' denotes (not-A), then $P(A') = 1 - P(A)$.

5. For any events A and B we have :

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

But for mutually exclusive event **$P(A \cap B) = 0$**

For mutually exclusive events
$$P(A \cup B) = P(A) + P(B)$$

6. $P(A \cap B) = P(A) \cdot P(B/A) = P(B) \cdot P(A/B)$

7. $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C)$

$$-P(C \cap A) + P(A \cap B \cap C)$$

Coins

One coin {H,T}= 2

Two coins={HH,HT,TH,TT}= 4

Three Coin={HHH,TTT,HHT,TTH,HTH,THT,THH,HTT}= 8

Four Coins={HHHH,   TTTT

        HHHT,   TTTH

        HHTH,   TTHT

        HTHH,   THTT

        THHH,   HTTT,

        HHTT,   HTTT,

        HTTH,   HTHT

        TTHH,   THTH} =16

Question 1: In a simultaneous toss of 2 coins find the probability of 2 Tails?

[A] 1/2

[B] 1/4

[C] 2/3

[D] 3/4

Question 2: In a simultaneous toss of 2 coins find the probability of Exactly 1 Tail?

[A] 1/3

[B] 2/3

[C] 3/4

[D] 1/2

Question 3: In a simultaneous toss of 2 coins find the probability of No Tail?

[A] 1/3

[B] 1/4

[C] 1/2

[D] 2/3

Question 4: In a simultaneous toss of 2 coins find the probability of No head?

[A] 1/2

[B] 1/4

[C] 1/3

[D] 2/3

Question 5: Three coins are tossed simultaneously. Find the probability of all are heads.

[A] 1/4

[B] 1/8

[C] 2/3

[D] 5/8

Question 6: Three coins are tossed simultaneously. Find the probability of exactly two heads.

[A] 1/4

[B] 1/8

[C] 3/8

[D] 5/8

Question 7: Three coins are tossed simultaneously. Find the probability of at least two heads.

[A] 1/3

[B] 1/8

[C] 1/8

[D] 3/8

Question 8: Three coins are tossed simultaneously. Find the probability of no heads.

[A] 1/8

[B] 1/4

[C] 1/2

[D] 5/8

Question 9: Three coins are tossed simultaneously. Find the probability of at least 1 head and 1 tail.

[A] 1/8

[B] 1/4

[C] 3/4

[D] 5/8

Question 10: 4 coins are tossed simultaneously. Find the probability exactly 3 tails

[A] 1/8

[B] 1/4

[C] 1/2

[D] 5/8

Question 11: 4 coins are tossed simultaneously. Find the probability at least 1 tail.

[A] 1/16

[B] 1/4

[C] 1/2

[D] 15/16

DICE

When a dice is thrown= {1, 2 , 3, 4, 5, 6}
=6
When two dice are thrown, S =
{(1,1),(1,2),(1,3),(1,4),(1,5),(1,6)

(2,1),(2,2),(2,3),(2,4),(2,5),(2,6)

(3,1),(3,2),(3,3),(3,4),(3,5),(3,6)

(4,1),(4,2),(4,3),(4,4),(4,5),(4,6)

(5,1),(5,2),(5,3),(5,4),(5,5),(5,6)

(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)} = 36

Question 1: In a single throw of 2 Dice, What is the probability of a doublet?

[A] 1/3

[B] 1/36

[C] 1/6

[D] 1/12

Question 2: In a single throw of 2 Dice, What is the probability of getting sum equals to:

I.   5
II.   Multiple of 5
III.   7
IV.   Multiple of 3
V.   Greater than 9

Short Cut For Two Dice

SUM	2	3	4	5	6	7	8	9	10	11	12
Fav.	1	2	3	4	5	6	5	4	3	2	1

Question 3: In a simultaneous throw of 3 Dice find the probability of getting a total of 5.
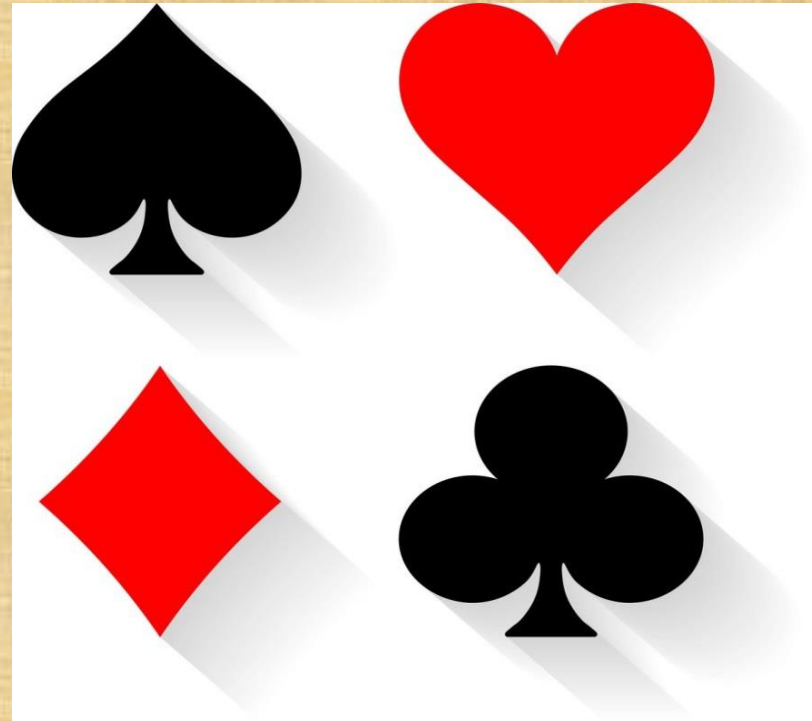
[A] 1/6

[B] 1/36

[C] 1/216

[D] 5/216

Cards

1. There are four Suits in a deck of Card viz. **Spade**, **Diamond**, **Heart**, **Club**.

2. **Red card=26** (13 Diamonds+ 13 Hearts)

3. 3. **Black Card= 26** (13 Clubs+13 Spades)

4. Every suit contains 13 cards viz. **Ace**, **2 to 10**, **Jack**, **Queen**, **King**

Face cards and Honored Card

Note: When ace is included in Face cards the combination is called Honored card.

Question 1: One card is drawn at random from the well shuffled pack of 52 cards. What is the probability of picking a black card?

[A] 1/3

[B] 1/2

[C] 1/4

[D] 1/13

Question 2: One card is drawn at random from the well shuffled pack of 52 cards. What is the probability of picking a Ace of spades or the jack of diamonds?

[A] 1/52

[B] 1/26

[C] 1/13

[D] 1/4

Question 3: One card is drawn at random from the well shuffled pack of 52 cards. What is the probability of picking an ace?

[A] 1/13

[B] 1/52

[C] 1/26

[D] 1/4

Question 4: One card is drawn at random from the well shuffled pack of 52 cards. What is the probability that the card is either a red card or a King?

[A] 5/13

[B] 7/13

[C] 9/13

[D] 1/52

Question 5: One card is drawn at random from the well shuffled pack of 52 cards. What is the probability that it is neither club nor queen?

[A] 4/13

[B] 5/13

[C] 7/13

[D] 9/13

Balls in a Bag OR marbles in a Bag

Questions 1: .A bag contains 5 red balls and 7 blue balls. Two balls are drawn at random without replacement, and then find the probability of that one is red and other is blue.

[A] 33/65

[B] 35/66

[C] 37/66

[D] 41/65

Questions 2: . A urn contains 4 red balls, 5 green balls and 6 white balls, if one ball is drawn at random, find the probability that it is neither red nor white.

[A] 1/3

[B] 1/4

[C] 1/5

[D] 2/3

Questions 3: . A bag contains 6 red balls and 7 white balls. Another bag contains 5 red balls and 3 white balls. One ball is selected from each. Find the probability that one ball is red and one is white?

[A] 53/104

[B] 47/104

[C] 63/104

[D] 51/104

Questions 4: In a bag there are 4 white, 4 red and 2 green balls. Two balls are drawn at random. What is the probability that at least one ball is of red color?

[A] 4/3

[B] 7/3

[C] 1/3

[D] 2/3

Questions 5: . A bag contains 2 red caps, 4 blue caps, 3 yellow caps and 5 green caps. If three caps are picked at random, what is the probability that none is green?

[A] 2/13

[B] 3/13

[C] 1/13

[D] 5/13

Questions 6: A bag contains 5 red and 7 white balls. Four balls are drawn out one by one and not replaced. What is the probability that they are alternatively of different colors?

[A] 7/99

[B] 11/99

[C] 14/99

[D] 19/99

Questions 7: A basket contains 5 red 4 blue 3 green marbles. If three marbles picked up random, What is the probability that either all are green or all are red?

[A] 1/20

[B] 7/20

[C] 3/20

[D] 9/20

Questions 8: A bag contains 3 red balls and 8 blacks ball and another bag contains 5 red balls and 7 blacks balls, one ball is drawn at random from either of the bag, find the probability that the ball is red.

[A] 93/264

[B] 95/264

[C] 91/264

[D] 97/264

Miscellaneous Questions:

Questions 1: A fair dice is rolled twice. The probability that an odd number will follow on even number is? **GATE-2005**

[A] 1/2

[B] 1/6

[C] 1/3

[D.] 1/4

Questions 2: An examination consists of two papers, Paper1 and 2. The probability of failing in Paper1 is 0.3 and that in paper 2 is 0.2. Given that a student has failed in paper2, the probability of failing in paper 1 is 0.6. the probability of failing in both the papers is? **GATE-2007**

[A] 0.5

[B] 0.18

[C.] 0.12

[D] 0.06

Bayes' Theorem:

Let S be the sample space and let E1,E2,E3,……En be n mutually exclusive and exhaustive events associated with a random experiment. If A is any event which occurs with E1 or E2 or E3…or En. then,

$$P(Ei/A) = \frac{P(Ei)P(A/Ei)}{\sum_{i=1}^{n} P(Ei)P(A/Ei)}$$

Question : A card from a pack of 52 cards is lost. From the remaining cards of the pack, two cards are drawn and are found to be both hearts. Find the probability of the lost card being a heart?

[A] 12/50

[B] 8/50

[C.] 11/50

[D] 9/50

Binomial Distribution:

If **p** is the probability of success of any event and **q** is the probability of failure of that event, then probability of event success **x times** in **n trials**(i.e. x success and n-x failure) is given by:

$$P(X=x) = n_{Cx}\ p^x\ q^{n-x}$$

Where X= Random variable, x= no. of success in n trials
P= probability of success, q= 1-p= probability of failure

Question: Find the probability of getting sum 9 exactly two in three times with a pair of dice.

Answer= 8/243

Thank you!