Relations

Let A and B be two non-empty sets, then relation R is a subset of $A \times B$

i.e., $\boxed{R} \subseteq A \times B$

eg $A = \{1, 2\}$, $B = \{a, b\}$

$A \times B = \{(1, a), (1, b), (2, a), (2, b)\}$

$R_1 = \{(1, a), (2, a)\}$ $R_1 \subseteq A \times B$.

$R_2 = \{(1, a), (1, b)\}$ $R_2 \subseteq A \times B$

$R_3 = \{(1, a)\}$ $R_3 \subseteq A \times B$

Total no. of relations from a set A to a set B.

$A = \{1, 2, 3\}$, $B = \{a, b\}$ $R = \{\underset{\times}{\_} \_ \_ \_ \_ \_\}$

$A \times B = \{(1, a), (1, b), (2, a), (2, b), (3, a), (3, b)\}$

Total no of Relations $= 2 \times 2 \times 2 \times 2 \times 2 \times 2$

$= 2^6$

$= 2^{3 \times 2}$

$= 2^{n(A) \times n(B)} = 2^{n(A \times B)}$

General formula:

Suppose set A has m elements and set B has n elements.

Then no of elements in $A \times B = m \times n$.

no. of Relations $= 2^{m \times n}$. $= 2^{mn}$.

no. of Relations from a set A to set A itself $= 2^{m \times m} = 2^{m^2}$

$R = \{(x, y) : x \in A, y \in B\}$

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Domain of Relation:

The Collection of all the first elements of the Coordinate pair of the relation is called the domain of the Relation..

$A = \{1, 2, 3\}$

$R = \{(1, 1), (1, 2), (1, 3), (2, 1)\}$

$D_R = \{1, 2\}$

Range of Relation.

The Collection of all the Second elements of the Coordinate pair of the Relation is called the Range of the relation.

$R_R = \{1, 2, 3\}$

Types of Relations:

(i) Reflexive relation :- Let A be a non-empty set. Define a Relation R on A, then R is called reflexive relation if.

$$(a, a) \in R \quad \forall \ a \in A$$

e.g: $A = \{1, 2, 3\}$

$R = \{(1, 2), (1, 3), (2, 2), (3, 3)\}$

As $1 \in A$ but $(1, 1) \notin R$.

This relation is not reflexive relation,

— x —

eg2 $A = \{1, 2, 3\}$

$R = \{(1, 1), (2, 2), (3, 3), \ldots\}$

As $\forall a \in A \quad (a, a) \in R$

This Relation is reflexive

— x —

Counting of no. of Reflexive relations:

$|A|$ 1 2 3 ----- n.

Counting of no. of Reflexive relations:

Consider a set A having n elements.

$A = \{1, 2, ----, n\}$

$R = \{\underbrace{(1,1), (2,2), (3,3), ---, (n,n)}, ---\}$

$A \times A$	1	2	3	----	n
1	(1,1)	(1,2)	(1,3)	---	(1,n)
2	(2,1)	(2,2)	(2,3)	---	(2,n)
3	(3,1)	(3,2)	(3,3)	---	(3,n)
⋮					
n	(n,1)	(n,2)	(n,3)	---	(n,n)

$(n^2 - n)$

Total no of Reflexive Relations $= \underbrace{2 \times 2 \times ---- \times 2}_{(n^2-n \text{ times})}$

$= \boxed{2^{n^2-n}}$

e.g: If A Set A has 3 elements, then how many reflexive relations are there?

Soln: Total no. of reflexive relations $= 2^{n^2-n}$

$= 2^{3^2-3}$

$= 2^{9-3} = 2^6 = 64$

Symmetric relation: Let A be a non-empty set, Define a relation R on A then this relation is called Symmetric relation if $(a,b) \in R$ then $(b,a) \in R$

$A = \{1, 2, 3\}$

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$R = \{(1,1), (1,2), (1,3), (3,1)\}$

$(2,2) \notin R$ $\quad$ R is not reflexive.

also $(1,2) \in R$ but $(2,1) \notin R$

This relation is not symmetric

— X

$A = \{1, 2, 3\}$

$R = \{(1,1), (2,2), (3,3)\}$

This relation is reflexive and symmetric

— X —

Counting of no. of Symmetric Relations.

$A = \{1, 2, 3, ---, n-1, n\}$

A \ A	1	2	3	----	n-2	n-1	n	
1	(1,1)	(1,2)	(1,3)	----	(1, n-2)	(1, n-1)	(1, n)	1
2	(2,1)	(2,2)	(2,3)	----	(2, n-2)	(2, n-1)	(2, n)	2
3	(3,1)	(3,2)	(3,3)	---	(3, n-2)	(3, n-1)	(3, n)	3
⋮								
n-2	(n-2, 1)	(n-2, 2)	(n-2, 3)	----	(n-2, n-2)	(n-2, n-1)	(n-2, n)	
n-1	(n-1, 1)	(n-1, 2)	(n-2, 3)	---	(n-1, n-2)	(n-1, n-1)	(n-1, n)	
n	(n, 1)	(n, 2)	(n, 3)	---	(n, n-2)	(n, n-1)	(n, n)	
	1	1						

Counting of Symmetric Relations.

$= \{2\} \times \{2 \times 2\} \times \{2 \times 2 \times 2\}$

$\times --- \times \{2 \times 2 \times --- \times 2\}$

$= 2^1 \times 2^2 \times 2^3 \times --- 2^n$

$= 2^{1+2+3+---+n} = 2^{\Sigma n} = \boxed{2^{\frac{n(n+1)}{2}}}$

Q1 If A set A has 3 elements, then find no of symmetric relations

Q1 If A set A has 3 elements, then no of symmetric relations

Soln, no of symmetric relations $= 2^{n(n+1)/2}$

$= 2^{\frac{3 \times \cancel{4}^{2}}{\cancel{2}}} = 2^{6} = 64$

— X —