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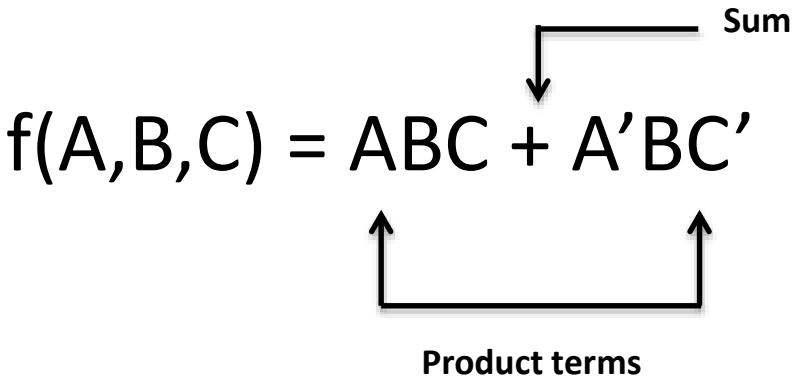
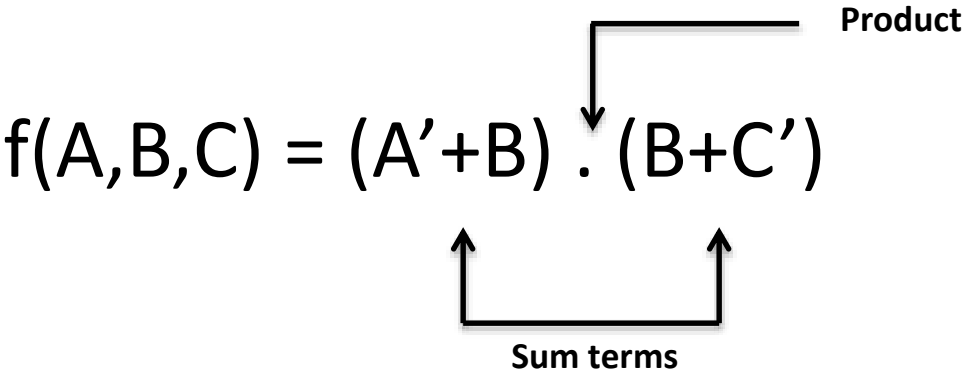
Sum of Product

- The sum-of-products (**SOP**) form is a method (or form) of simplifying the Boolean expressions of logic gates.
- Sum and product derived from the symbolic representations of the OR and AND functions.
- OR (+) , AND (.) , addition and multiplication.



Product of Sum

- When two or more sum terms are multiplied by a Boolean OR operation.
- Sum terms are defined by using OR operation and the product term is defined by using AND operation.



Standard SOP and POS Forms

- The canonical forms are the special cases of SOP and POS forms.
- These are also known as standard SOP and POS forms.

Canonical Form

- In SOP or POS form, all individual terms do not involve all literals.
- For example AB + A’BC the first product term do not contain literal C.
- If each term in SOP or POS contain all literals then the expression is known as standard or canonical form.

Canonical Form

- Each individual term in the POS form is called **Maxterm**.
- Each individual term in the SOP form is called **Minterm**.
- In Minterm, we look for the functions where the output results is “1”.
- while in Maxterm we look for function where the output results is “0”.
- We perform **Sum of minterm** also known as Sum of products (SOP) .
- We perform **Product of Maxterm** also known as Product of sum (POS).

Convert SOP to standard SOP form

Step 1: Find the missing literal in each product term if any.

Step 2: And each product term having missing literals with terms form by ORing the literal and its complement.

Step 3: Expends the term by applying, distributive law and reorder the literals.

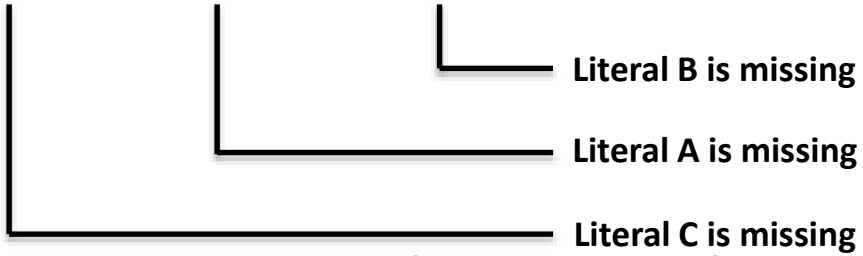
Step 4: Reduce the repeated product terms. Because A + A = A (Theorem 1a).

Example:

$$f(A,B,C) = AB + BC + AC$$

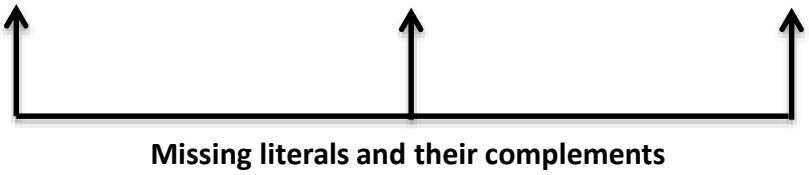
Step 1: Find the missing literals in each product term.

$$f(A,B,C) = AB + BC + AC$$



Step 2: AND the product term with missing literal + its complement.

$$f(A,B,C) = AB \cdot (C+C') + BC \cdot (A+A') + AC \cdot (B+B')$$



Step 3: Expends the term and reorder the literals.

f (A,B,C) = AB . (C+C') + BC . (A+A') + AC . (B+B')

Expand & Reorder:

ABC + ABC' + ABC + A'BC + ABC + AB'C

Step 4: Omit repeated product terms.

f(A,B,C)=ABC + ABC' + ABC + A'BC + ABC + AB'C

f(A,B,C)= ABC + ABC' + A'BC + AB'C

Convert POS to standard POS form

Step 1: Find the missing literal in each sum term if any.

Step 2: OR each sum term having missing literals with terms form by ANDing the literal and its complement.

Step 3: Expends the term by applying, distributive law and reorder the literals.

Step 4: Reduce the repeated product terms. Because A + A = A (Theorem 1a).

Example:

$$f(A,B,C) = (A + B) \cdot (B + C) \cdot (A + C)$$

Step 1: Find the missing literals in each sum term.

$$f(A,B,C) = (A + B) \cdot (B + C) \cdot (A + C)$$

Literal B is missing

Literal A is missing

Literal C is missing

Step 2: OR the sum term with missing literal . its complement.

$$f(A,B,C) = (A + B)+(C.C') + (B + C)+(A.A') + (A + C)+(B.B')$$

Missing literals and their complements

Step 3: Expends the term and reorder the literals.

f (A,B,C) = (A + B)+(C.C') + (B + C)+(A.A') + (A +C)+(B.B')

Expand & Reorder:

f(A,B,C)=(A+B+C).(A+B+C').(A+B+C).(A'+B+C).(A+B+C).(A+B'+C)

Step 4: Omit repeated sum terms.

f(A,B,C)=(A+B+C).(A+B+C').(A+B+C).(A'+B+C).(A+B+C).(A+B'+C)

f(A,B,C)=(A+B+C).(A+B+C').(A'+B+C).(A+B'+C)

Minterms

- Minterms are AND terms with every variable present in either true or complemented form.
- Given that each binary variable may appear normal (e.g., x) or complemented (e.g.x'), there are 2^n m
- Example: Two variables (X and Y)produce 2 x 2 = 4 combinations:

  XY(both normal)

  XY'(X normal, Y complemented)

  X'Y(X complemented, Y normal)

  X'Y'(both complemented)

Thus there are four minterms of two variables.

Maxterms

- Maxterms are OR terms with every variable in true or complemented form.
- Given that each binary variable may appear normal (e.g., x) or complemented (e.g. x'), there are 2^n maxterms for n variables.
- Example: Two variables (X and Y) produce 2x2=4 combinations:

  X+Y(both normal)

  X+Y'(x normal, y complemented)

  X'+Y(x complemented, y normal)

  X'+Y'(both complemented)

A	B	C	Minterms	Maxterms
0	0	0	$A'B'C' = m_0$	$A+B+C = M_0$
0	0	1	$A'B'C = m_1$	$A+B+C' = M_1$
0	1	0	$A'BC' = m_2$	$A+B'+C = M_2$
0	1	1	$A'BC = m_3$	$A+B'+C' = M_3$
1	0	0	$AB'C' = m_4$	$A'+B+C = M_4$
1	0	1	$AB'C = m_5$	$A'+B+C' = M_5$
1	1	0	$ABC' = m_6$	$A'+B'+C = M_6$
1	1	1	$ABC = m_7$	$A'+B'+C' = M_7$

Minterms:

1. $f(A,B,C) = A'B'C' + A'BC' + A'BC + ABC$

   $= m_0 + m_2 + m_3 + m_7$

   $= \Sigma m(0,2,3,7)$

2. $f(A,B,C) = A'B'C + A'BC + AB'C + ABC$

   $= m_1 + m_3 + m_5 + m_7$

   $= \Sigma m(1,3,5,7)$

3. $f(A,B,C) = A'B'C' + A'BC' + A'BC + ABC'$

   $= m_0 + m_2 + m_3 + m_6$

   $= \Sigma m(0,2,3,6)$

Maxterms:

1. $f(A,B,C) = (A+B+C).(A+B'+C).(A+B'+C')+(A'+B'+C')$

   $= M_0 + M_2 + M_3 + M_7$

   $= \Pi M(0,2,3,7)$

2. $f(A,B,C)= (A+B+C').(A+B'+C').(A+B'+C').(A'+B'+C')$

   $= M_1 + M_3 + M_5 + M_7$

   $= \Pi M\ (1,3,5,7)$

3. $f(A,B,C)= (A+B+C).(A+B'+C).(A+B'+C').(A'+B'+C)$

   $= M_0 + M_2 + M_3 + M_6$

   $= \Pi M\ (0,2,3,6)$

SOP-POS

A	B	C	F
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

SOP Expression

Write AND term for each input combination produces HIGH
Write the input variables for 1 and compliment for 0.
OR the AND terms to obtain the output function.

F(SOP) = A'BC + AB'C + ABC ' + ABC

POS Expression

Write OR term for each input combination produces LOW
Write the input variables for 0 and complement for 1
AND the OR terms to obtain the output function

F(POS)= F = (A + B + C) (A + B + C ') (A + B' + C) (A' + B + C)

POS is compliment of SOP

SOP-POS Conversion

- Convert the SOP expression to an equivalent POS expression:

$$\overline{A}\,\overline{B}\,\overline{C} + \overline{A}\,B\overline{C} + \overline{A}\,BC + A\overline{B}\,C + ABC$$

  - The evaluation is as follows:

$$000+010+011+101+111$$

  - There are 8 possible combinations. The SOP expression contains five of these, so the POS must contain the other 3 which are: 001, 100, and 110.

$$(A+B+\overline{C})(\overline{A}+B+C)(\overline{A}+\overline{B}+C)$$

Quick Quiz (Poll 1)

- min term when x=0, y=0 and z=1
- A) x'y'z
- B) x+y+z’

Quick Quiz (Poll 2)

- Y=AB+BC+AC is
- A) SOP
- B) POS