Tutorial 6

Example

When a two element series circuit is connected across an ac source of frequency 50 Hz, it offers an impedance Z = (10+j10) ohm. Find the values of two elements?

Solution

The nature of the impedance indicates that the circuit is inductive.

$$\mathbf{Z} = R + jX_L = (10 + j10)\,\Omega$$

$$R = \mathbf{10\,\Omega} \quad \text{and} \quad X_L = 10\,\Omega \;\Rightarrow\; L = \frac{X_L}{2\pi f} = \frac{10}{2\pi \times 50} = 0.0318\ \text{H} = \mathbf{31.8\ mH}$$

Example

When a two element parallel circuit is connected across an ac source of frequency 50 Hz, it offers an impedance Z = (10-j10) ohm. Find the values of two elements?

$$\mathbf{Y} = G + jB = (\text{conductance}) + j(\text{susceptance})$$

$$\mathbf{Y} = (G + jB) = \left(\frac{1}{R} + j\omega C\right)$$

Solution

Solution The nature of the impedance indicates that the circuit is capacitive. Since the circuit has two elements connected in parallel, we find the admittance of the circuit.

$$\mathbf{Y} = (G + jB) = \frac{1}{\mathbf{Z}} = \frac{1}{10 - j10} = \frac{1}{14.14\angle -45^\circ} = 0.0707\angle 45^\circ \text{ S} = (0.05 + j0.05) \text{ S}$$

$$\therefore \quad G = 0.05 \text{ S} \Rightarrow R = \frac{1}{G} = \frac{1}{0.05} = \mathbf{20\ \Omega} \quad \text{and} \quad B = 0.05 \text{ S} \Rightarrow C = \frac{B}{\omega} = \frac{0.05}{2\pi f} = \mathbf{159\ \mu F}$$

Example

- A circuit consists of a resistance R in series with a capacitive reactance of 60 ohm. Determine the value of R for which the power factor of the circuit is 0.8.

Solution

Solution The power factor, $\cos\theta = 0.8$. Therefore, we get

$$\sin\theta = \sqrt{1-\cos^2\theta} = \sqrt{1-(0.8)^2} = 0.6 \quad \Rightarrow \quad \tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{0.6}{0.8} = \frac{3}{4}$$

From the impedance triangle, we know that

$$\tan\theta = \frac{X_C}{R} \quad \Rightarrow \quad \frac{3}{4} = \frac{X_C}{R} \quad \text{or} \quad \frac{3}{4} = \frac{60}{R} \quad \Rightarrow \quad R = 80\ \Omega$$

Example

A series RLC circuit has R= 12 ohm, L = 0.15 H and C = 100 μF. It is connected to an ac source of voltage 100 V, whose frequency can be varied. Determine (a) the resonant frequency of the source at which the current supplied by it is maximum, (b) the value of this current.

Solution

Solution

(*a*) The current becomes maximum at the resonant frequency of the circuit,

$$f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi \times \sqrt{0.15 \times 100 \times 10^{-6}}} = \mathbf{41.09\ Hz}$$

(*b*) The maximum current supplied by the source is

$$I_0 = \frac{V}{R} = \frac{100}{12} = \mathbf{8.3\ A}$$

Example

- An iron choke takes 4 Amp current when connected to a 20 V dc supply. When connected to 65 V, 50 Hz ac supply, it takes 5 Amp. Current. Calculate (a) resistance and inductance of the coil (b) the power factor (c) the power drawn by the coil.

Solution

Solution

(*a*) The iron choke has both resistance R and inductance L. When connected to a dc supply, only its resistance is effective in limiting the current; when connected to an ac supply, its impedance becomes effective in limiting the current. Hence,

$$R = \frac{20\,\text{V}}{4\,\text{A}} = \mathbf{5\,\Omega} \quad \text{and} \quad Z = \frac{65\,\text{V}}{5\,\text{A}} = 13\,\Omega \;\Rightarrow\; X_L = \sqrt{Z^2 - R^2} = 12\,\Omega$$

$$\therefore \quad L = \frac{X_L}{2\pi f} = \frac{12}{2\pi \times 50} = 0.0382\,\text{H} = \mathbf{38.2\,mH}$$

(*b*) The power factor, $pf = \cos\theta = \dfrac{R}{Z} = \dfrac{5}{13} = \mathbf{0.38\ (lagging)}$

(*c*) The power, $P = VI\cos\theta = 65 \times 5 \times 0.38 = \mathbf{123.5\,W}$

Problem on Power and RC circuit

A current of 0.9 A flows through a series combination of a resistor of 120 Ω and a capacitor of reactance 250 Ω. Find the impedance, power factor, supply voltage, voltage across resistor, voltage across capacitor, apparent power, active power and reactive power.

Solution Taking current as the reference phasor, $\mathbf{I} = 0.9\angle 0^\circ$ A.

Impedance, $\mathbf{Z} = 120 - j250 = \mathbf{277.3\angle -64.4^\circ\ \Omega}$

Power factor, $pf = \cos\theta = \cos(-64.4^\circ) = \mathbf{0.432\ leading}$

Supply voltage, $\mathbf{V} = \mathbf{IZ} = (0.9\angle 0^\circ)(277.3\angle -64.4^\circ) = \mathbf{249.6\angle -64.4^\circ\ V}$

Voltage across resistor, $V_R = IR = (0.9\angle 0^\circ) \times 120 = \mathbf{108\angle 0^\circ\ V}$

Voltage across capacitor, $V_C = IX_C = (0.9\angle 0^\circ)(250\angle -90^\circ) = \mathbf{225\angle -90^\circ\ V}$

Apparent power, $P_{app} = VI = 249.6 \times 0.9 = \mathbf{224.6\ VA}$

Actual power, $P_a = VI\cos\theta = 249.6 \times 0.9 \times \cos 64.4^\circ = \mathbf{97.06\ W}$

Reactive power, $P_r = VI\sin\theta = 249.6 \times 0.9 \times \sin 64.4^\circ = \mathbf{202.58\ VAR}$