UNIT-II

FUNDAMENTAL OF AC CIRCUITS

Lecture 9
Prepared By:
Krishan Arora
Assistant Professor and Head

The Basic Sine Wave Equation

The voltage produced by the previously described generator is:

$$e = E_m \sin \alpha \quad (V)$$

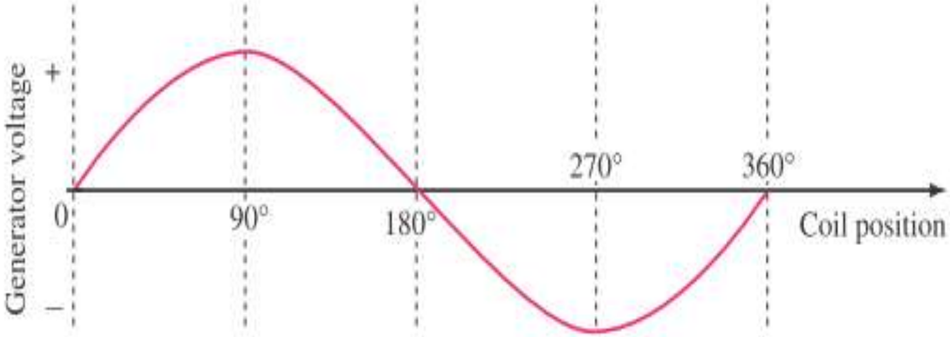


FIGURE 15–7 Coil voltage versus angular position.

- Em: the maximum coil voltage and
- α : the instantaneous angular position of the coil.

> - For a given generator and rotational velocity, Em is constant.)
> - Note that a 0° represents the horizontal position of the coil and that one complete cycle corresponds to 360°.

Angular Velocity (ω)

The rate at which the generator coil rotates is called its angular velocity

If the coil rotates through an angle of 30° in one second, its angular velocity is 30° per second.

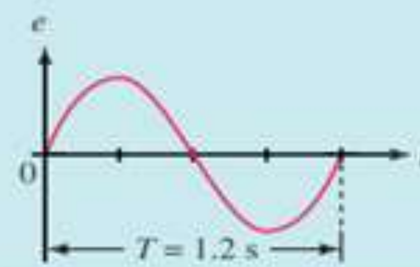
- When you know the angular velocity of a coil and the length of time that it has rotated, you can compute the angle through which it has turned using:

$$\alpha = \omega t$$

EXAMPLE 15–6 If the coil of Figure 15–23 rotates at $\omega = 300°/\text{s}$, how long does it take to complete one revolution?

Solution One revolution is 360°. Thus,

$$t = \frac{\alpha}{\omega} = \frac{360 \text{ degrees}}{300 \dfrac{\text{degrees}}{\text{s}}} = 1.2 \text{ s}$$

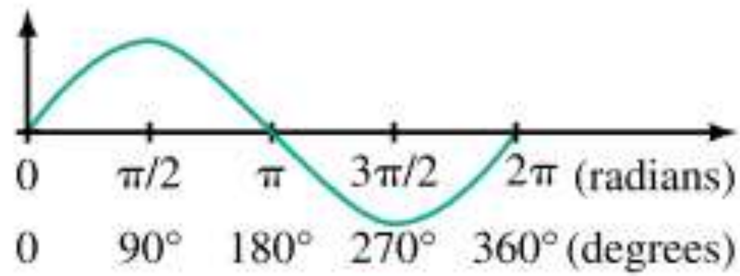


Since this is one period, we should use the symbol T. Thus, $T = 1.2$ s, as in Figure 15–25.

Radian Measure

- In practice, q is usually expressed in radians per second,
- Radians and degrees are related by :

$$2\pi \text{ radians} = 360^\circ$$



(b) Cycle length scaled in degrees and radians

For Conversion:

$$\alpha_{\text{radians}} = \frac{\pi}{180^\circ} \times \alpha_{\text{degrees}}$$

$$\alpha_{\text{degrees}} = \frac{180^\circ}{\pi} \times \alpha_{\text{radians}}$$

Relationship between ω, T, and f

- Earlier you learned that one cycle of sine wave may be represented as either:

$$\alpha = 2\pi \text{ rads} \qquad t = T \text{ s}$$

- Substituting these into:

$$\alpha = \omega t$$

$$\omega T = 2\pi \quad \text{(rad)}$$

$$\omega = \frac{2\pi}{T} \quad \text{(rad/s)}$$

$$\omega = 2\pi f \quad \text{(rad/s)}$$

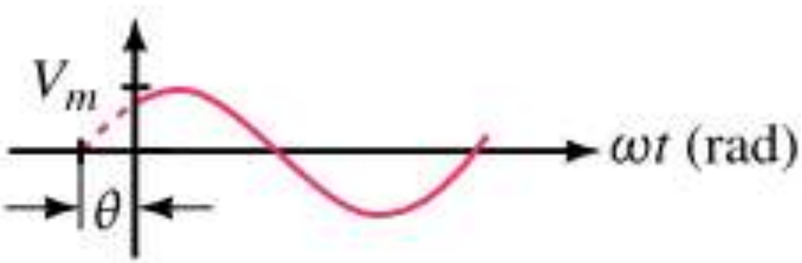
Sinusoidal Voltages and Currents as Functions of Time:

- We could replace the angle α as:

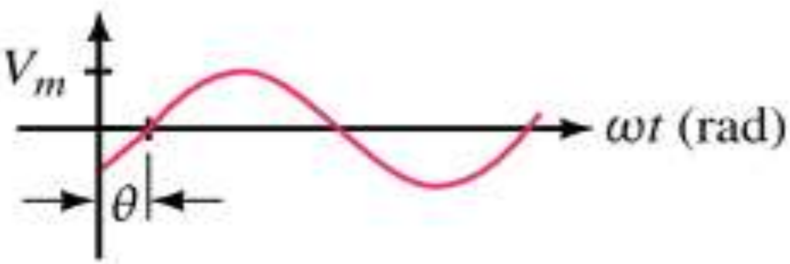
$$e = E_m \sin \omega t$$

Voltages and Currents with Phase Shifts

- If a sine wave does not pass through zero at t =0 s, it has a phase shift.
- Waveforms may be shifted to the left or to the right



(a) $v = V_m \sin(\omega t + \theta)$



(b) $v = V_m \sin(\omega t - \theta)$

Quick Quiz (Poll 1)

The time period or periodic time T of an alternating quantity is the time taken in seconds to complete

a. one cycle

b. alternation

c. none of the above

d. Half cycle

Quick Quiz (Poll 2)

The time period of an alternating quantity is 0.02 second. Its frequency will be

a. 25 Hz

b. 50 Hz

c. 100 Hz

d. 0.02 Hz

Quick Quiz (Poll 3)

The angular frequency of an alternating quantity is a mathematical quantity obtained by multiplying the frequency f of the alternating quantity by a factor

a. $\pi/2$

b. π

c. 2π

d. 4π

Introduction to Phasors

- A phasor is a rotating line whose projection on a vertical axis can be used to represent sinusoidally varying quantities.

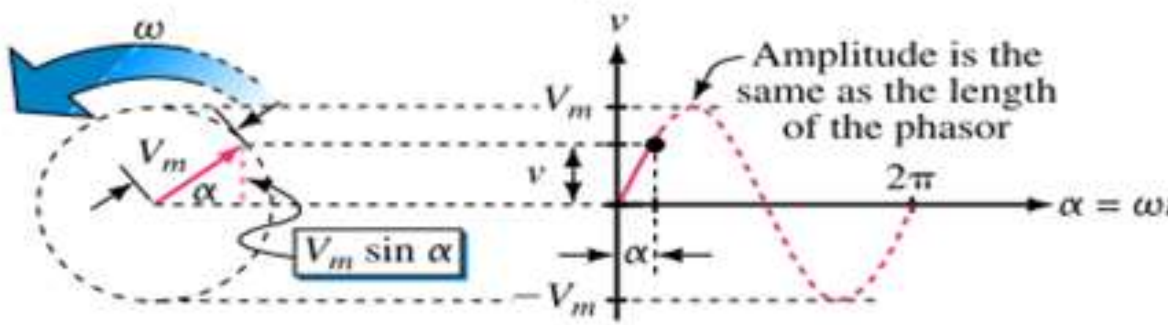
- To get at the idea, consider the red line of length **Vm** shown in Figure :

The vertical projection of this line (indicated in dotted red) is :

$$v = V_m \sin \alpha.$$

- By assuming that the phasor rotates at angular velocity of ω rad/s in the counterclockwise direction

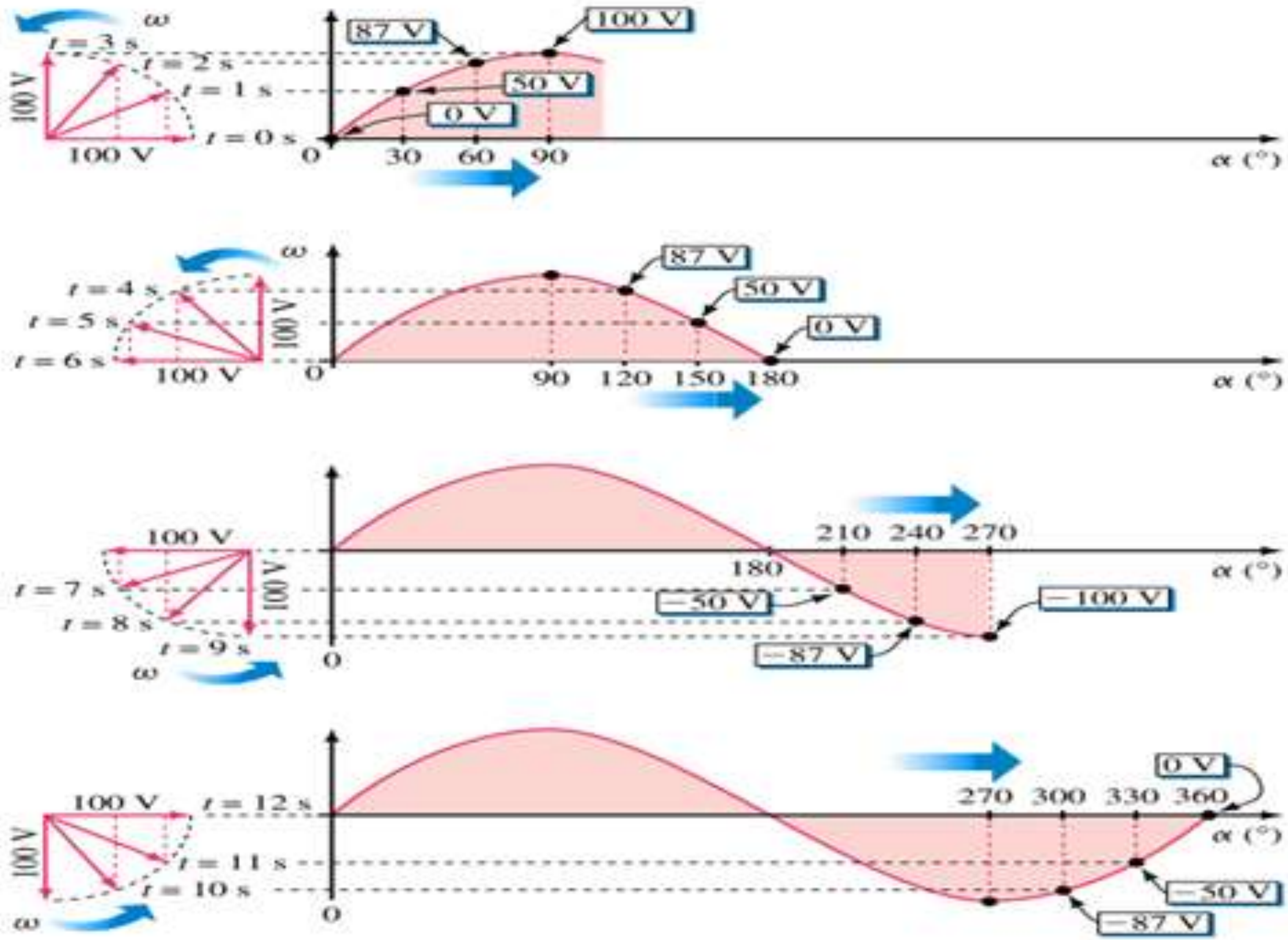
$$v = V_m \sin \omega t$$



(a) Phasor (b) Resulting sine wave

FIGURE 15–34 As the phasor rotates about the origin, its vertical projection creates a sine wave. (Figure 15–35 illustrates the process.)

Introduction to Phasors



EXAMPLE 15–15 Draw the phasor and waveform for current $i = 25 \sin \omega t$ mA for $f = 100$ Hz.

Solution The phasor has a length of 25 mA and is drawn at its $t = 0$ position, which is zero degrees as indicated in Figure 15–36. Since $f = 100$ Hz, the period is $T = 1/f = 10$ ms.

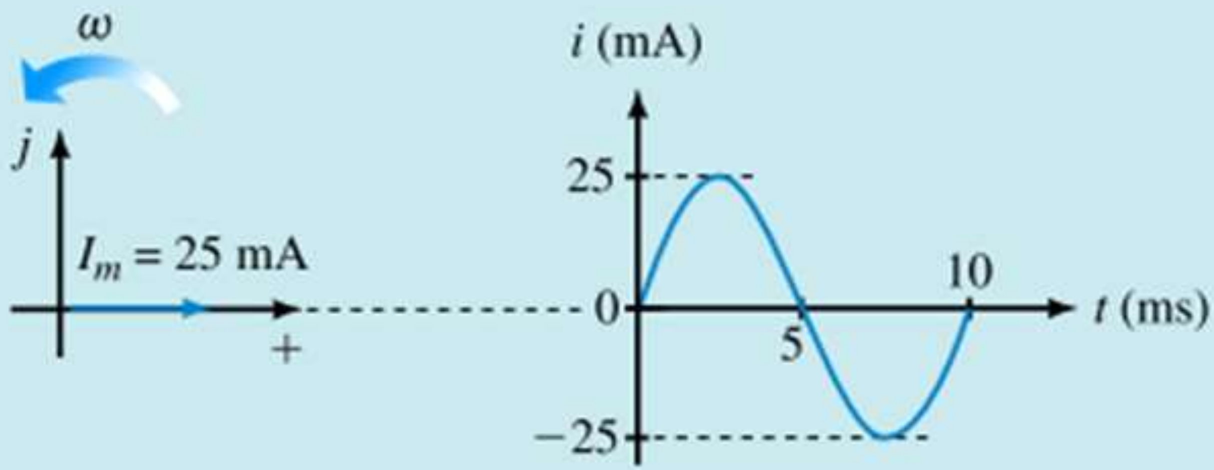


FIGURE 15–36 The reference position of the phasor is its $t = 0$ position.

Shifted Sine Waves Phasor Representation

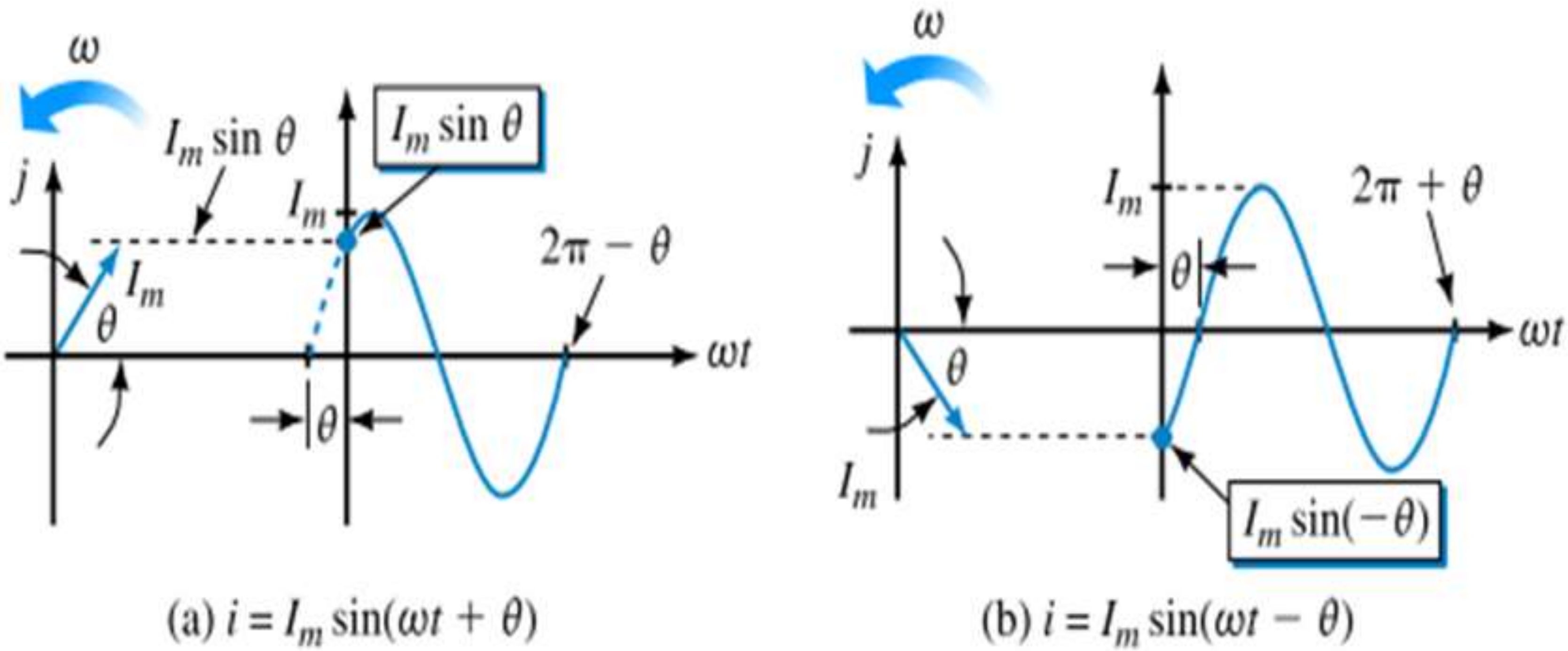


FIGURE 15–37 Phasors for shifted waveforms. Angle θ is the position of the phasor at $t = 0$ s.

The equation for an ac voltage is given as $v = 0.04 \sin(2000t + 60°)$ V. Determine the frequency, the angular frequency, and the instantaneous voltage when $t = 160\ \mu s$. What is the time represented by a 60° phase angle?

Solution Comparing the given equation with the general equation given in Eq. 9.3, we get

$$\omega = 2\pi f = \mathbf{2000\ rad/s} \qquad \text{and} \qquad f = \frac{\omega}{2\pi} = \frac{2000}{2\pi} = \mathbf{318.3\ Hz}$$

The instantaneous value of the voltage at $t = 160\ \mu s$ is

$$v = 0.04 \sin(2000 \times 160 \times 10^{-6}\ \text{rad} + 60°)\ \text{V} = 0.04 \sin(0.32\ \text{rad} + 60°)\ \text{V}$$

$$= 0.04 \sin\left(0.32 \times \frac{180°}{\pi} + 60°\right)\ \text{V} = 0.04 \sin(18.3° + 60°)\ \text{V} = 0.0392\ \text{V} = \mathbf{39.2\ mV}$$

$$\text{Time period, } T = \frac{1}{f} = \frac{1}{318.3} = 3.14\ \text{ms}$$

Thus, full-cycle of 360° corresponds to 3.14 ms. Therefore, the angle 60° corresponds to

$$t = \frac{60°}{360°} \times 3.14\ \text{ms} = \mathbf{0.52\ ms}$$

Quick Quiz (Poll 4)

A phasor is

A. A line which represents the magnitude and phase of an alternating quantity

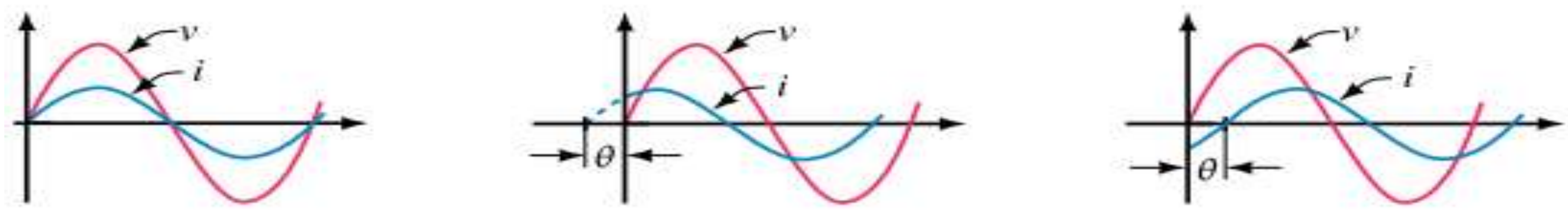
B. A line representing the magnitude and direction of an alternating quantity

C. A colored tag or band for distinction between different phases of a 3-phase supply

D. An instrument used for measuring phases of an unbalanced 3-phase load

Phasor Difference

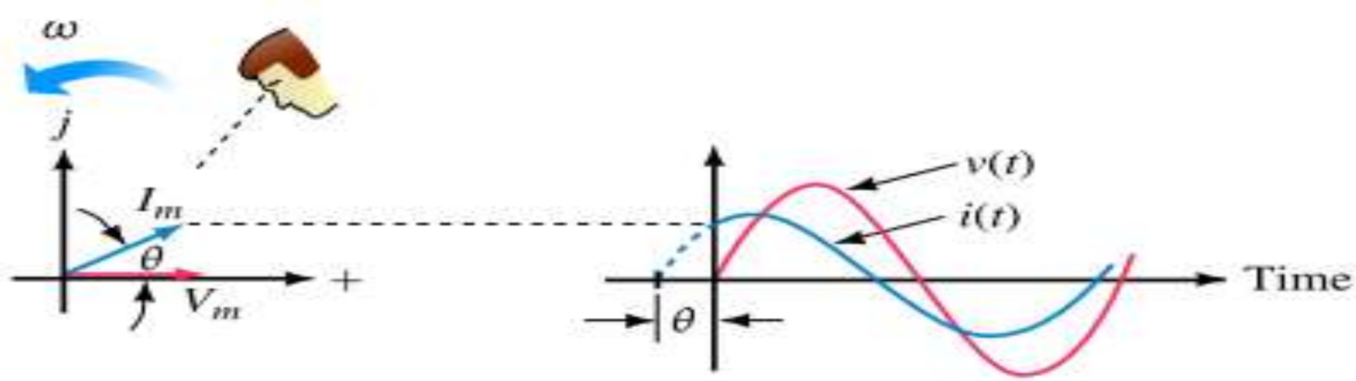
- Phase difference refers to the angular displacement between different waveforms of the same frequency.



(a) In phase (b) Current leads (c) Current lags

FIGURE 15–40 Illustrating phase difference. In these examples, voltage is taken as reference.

- The terms lead and lag can be understood in terms of phasors. If you observe phasors rotating as in Figure, the one that you see passing first is leading and the other is lagging.



(a) I_m leads V_m (b) Therefore, $i(t)$ leads $v(t)$

Quick Quiz (Poll 5)

If the phase angle ϕ is positive then the phase difference is said to a ___________ phase difference

A additive

B Leading

C Lagging

D None of the above.

Quick Quiz (Poll 6)

If the phase difference between them is equal to zero, the two AC voltages (or any two AC quantities) are said to be ____________.

A in phase

B out of phase

C in phase opposition

D None of the above