Tutorial 4

A sinusoidal voltage is given by v= 20 sin ωt volts. (a) At what angle will the instantaneous value of voltage be the 10 V? (b) What is the maximum value of the voltage and at what angle?

Solution

(*a*) The angle can be determined by using the given equation,

$$10 = 20 \sin \omega t \Rightarrow \theta = \omega t = \sin^{-1}(10/20) = 30°$$

(*b*) The maximum value is $V_m = 20$ **V**. This occurs twice in one cycle when $\sin \omega t = \pm 1$, that is at the instants when $\omega t = 90°$ or $270°$.

Example 2

An alternating current of frequency 60 Hz has a maximum value of 12A (a) Write down the equation for its instantaneous value. (b) Calculate the value of current after 1/360 seconds. (c) Find the time taken to reach 9.6A for the first time.

Solution

(*a*) The angular frequency, $\omega = 2\pi f = 2\pi \times 60 = 377$ rad/s. Therefore, the equation for the instantaneous value is given as

$$i = 12 \sin 377t \text{ A} \qquad (i)$$

(*b*) The value of current at $t = 1/360$ second is

$$i = 12 \sin\left(377 \times \frac{1}{360} \times \frac{180^\circ}{\pi}\right) \text{A} = \mathbf{10.4\ A}$$

(*c*) Putting $i = 9.6$ A in Eq. (*i*), we have

$$9.6 = 12 \sin\left(377t \times \frac{180^\circ}{\pi}\right) \Rightarrow 377t \times \frac{180^\circ}{\pi} = 53.13^\circ$$

$$\therefore \qquad t = \frac{53.13\pi}{377 \times 180} = \mathbf{2.46\ ms}$$

Example 3

Determine the phase difference of the sinusoidal current i1= 4sin (100Πt + 30degree) Amp with respect to current i2= 6sin (100Πt) Amp. In terms of time and draw the phasor diagram to represent the two phasors.

Solution Comparing the expressions of the given sinusoids with the standard form, we get

$$\omega = 2\pi f = 100\pi \Rightarrow f = 50\ \text{Hz} \Rightarrow T = \frac{1}{f} = \frac{1}{50} = 0.02\ \text{s} = 20\ \text{ms}$$

Thus, the time for full revolution (360°) for either of the phasors is 20 ms. The time taken to cover 30° is

$$t = (20\ \text{ms}) \times \frac{30°}{360°} = \mathbf{1.67\ ms}$$

The phasor i_1 leads the phasor i_2 by 30° (or 1.67 ms). Taking i_2 as the reference phasor, the phasor i_1 can be drawn 30° leading i_2. The phasor diagram is drawn in Fig. 9.8, in terms of the maximum values of the two sinusoidal currents.

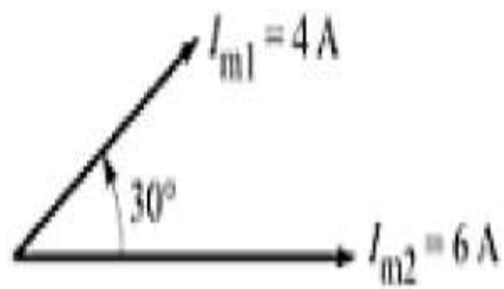

Fig. 9.8 *Phasor diagram.*

Example 4

In an ac circuit, the instantaneous voltage and current are given as

$$v = 55\sin \omega t \text{ V} \qquad \text{and} \qquad i = 6.1\sin(\omega t - \pi/5) \text{ A}$$

Determine the average power, the apparent power, the instantaneous power when ωt (in radians) equals 0.3, and the power factor in percentage.

Solution Here, the phase angle, $\theta = \pi/5$

The rms value of the voltage, $V = \dfrac{V_m}{\sqrt{2}} = \dfrac{55}{\sqrt{2}} = 38.89$ V

The rms value of the current, $I = \dfrac{I_m}{\sqrt{2}} = \dfrac{6.1}{\sqrt{2}} = 4.31$ A

Therefore, using Eq. 9.36, the average power is given as

$$P_{av} = VI\cos\theta = 38.89 \times 4.31 \times \cos \pi/5$$
$$= 167.62 \times 0.809 = \mathbf{135.6\ W}$$

The apparent power is

$$P_a = VI = 38.89 \times 4.31 = \mathbf{167.62\ VA}$$

Note that the apparent power is expressed in volt amperes (VA) and not in watts (W), since it is not a power in reality.

The instantaneous power at $\omega t = 0.3$ is given by Eq. 9.35, as

$$p = VI\cos\theta - VI\cos(2\omega t - \theta)$$
$$= 135.6 - 167.62 \times \cos(2 \times 0.3 - \pi/5) = \mathbf{-31.95\ VA}$$

The power factor is given as

$$pf = \cos\theta = \cos\pi/5 = 0.809 = \mathbf{80.9\ \%}$$

Example 5

Determine the average and rms value of current given by i= 10 +5 cos314t Amp.

Solution The given current is seen to be the combination of a dc current of 10 A and a sinusoidal current of peak value 5 A. The average value of the sinusoidal current being zero, the average of the overall current would be the same as the dc current. That is, I_{av} = 10 A.

The rms value can easily be found by adding the rms values of the two components on square basis,

$$I_{rms} = \sqrt{(10)^2 + \left(\frac{5}{\sqrt{2}}\right)^2} = \sqrt{112.5} = 10.6\ \mathrm{A}$$

IMPORTANT FORMULAE

- $f = \dfrac{1}{T}$
- Sinusoidal current, $i = I_m \sin \omega t$, where $\omega = 2\pi f$ or $\omega = \dfrac{2\pi}{T}$
- For a sinusoidal ac and for a full-wave-rectified wave,

$$I_{av} = \frac{2 I_m}{\pi}, \quad \text{and} \quad I_{rms} = \frac{I_m}{\sqrt{2}}.$$

- Form factor, $K_f = \dfrac{V_{rms}}{V_{av}} = 1.11$ (for sinusoidal waveform)
- Peak factor, $K_p = \dfrac{V_m}{V_{rms}} = 1.414$ (for sinusoidal waveform)

Property	*Resistance*	*Inductance*	*Capacitance*
Current	V/R	V/X_L	V/X_C
Frequency dependency	Independent	$X_L \propto f$	$X_C \propto (1/f)$
Power	$VI = I^2R = V^2/R$	Zero	Zero
Phase difference	0°	90° lagging	90° leading
Reactance	R	$jX_L = j\omega L = j2\pi fL$	$-jX_C = 1/j\omega C = 1/j2\pi fC$