$W = \{0, 1, 2, 3, ----\}$

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Generating function

Let $\{a_n\}_{n \in W}$ be a sequence of real no's then the generating function is denoted by $G(a,z)$ and it is given by

$$\boxed{G(a,z) = \sum_{n=0}^{\infty} a_n z^n.}$$

$$G(a,z) = a_0 z^0 + a_1 z^1 + a_2 z^2 + a_3 z^3 + ----\infty$$

$$G(a,z) = a_0 + a_1 z^1 + a_2 z^2 + a_3 z^3 + ----\infty$$

(a) $G(a,z) - a_0 = a_1 z + a_2 z^2 + a_3 z^3 + -----\infty \longrightarrow (1)$

(b) $G(a,z) - a_0 - a_1 z = a_2 z^2 + a_3 z^3 + ------\infty \longrightarrow (2)$

P1 If $a_n = b \ \forall n \in W$

$\{a_0, a_1, a_2, a_3 ---\}$

$\{b, b, b, -----\}$

What is the generating function for a_n.

Sol^n: $G(a,z) = \sum_{n=0}^{\infty} a_n z^n$

$$= \sum_{n=0}^{\infty} b z^n.$$

$$\left((1-z)^{-1} = 1 + z + z^2 + ----\infty\right)$$

$$= b\left[\sum_{n=0}^{\infty} z^n\right]$$

$$= b\left[z^0 + z^1 + z^2 + -----\infty\right]$$

$$= b\left[1 + z + z^2 + ----\infty\right]$$

$$= b(1-z)^{-1} = \frac{b}{1-z}$$

$a_n = b \ \forall n \in W$

$G(a,z) = \frac{b}{1-z}$

$G(a,z) = \frac{2003}{1-z}$

$a_n = 2003 \ \forall n \in W.$

$\{a_0, a_1, a_2, a_3, -----\}$

$\{1, b, b^2, b^3, ----\}$

P2 ~~$G(a,z) = b^n$ then find~~

$a_n = b^n \ \forall n \in W$ find the generating function.

Sol^n $G(a,z) = \sum_{n=0}^{\infty} a_n z^n.$

$= \sum_{n=0}^{\infty} b^n z^n$

$= \sum_{n=0}^{\infty} (bz)^n.$

$= (bz)^0 + (bz)^1 + (bz)^2 + \cdots \infty$

$= 1 + (bz) + (bz)^2 + \cdots \infty$

$= (1-bz)^{-1}$

$G(a,z) = \frac{1}{1-bz}$

$a_n = b^n \quad \forall n \in \mathbb{N}$

$G(a,z) = \frac{1}{1-bz}$

—×—

$a_n = (3)^n \quad \forall n \in \mathbb{N}$

$G(a,z) = \frac{1}{1-3z}$

—×—

$a_n = (-3)^n$

$G(a,z) = \frac{1}{1-(-3)z}$

$= \frac{1}{1+3z}.$

If $a_n = ab^n$ then what is the generating function?

Sol^n: $a_n = ab^n$

$a_n = 5 \cdot 2^n$

$G(a,z) = \frac{5}{1-2z}$

$G(a,z) = \sum_{n=0}^{\infty} a_n z^n.$

$= \sum_{n=0}^{\infty} (ab^n) z^n.$

$= a \sum_{n=0}^{\infty} (bz)^n$

$= a\left[(bz)^0 + (bz)^1 + (bz)^2 + \cdots \infty\right]$

$= a\left[1 + bz + (bz)^2 + \cdots \infty\right]$

$= a(1-bz)^{-1}$

$= \frac{a}{1-bz}$

$G(a,z) = \frac{3}{1+3z}$

$a_n = 3(-3)^n.$

—×—

Q1) $a_{n+2} - 3a_{n+1} + 2a_n = 0$ find the generating function also find the Sol^n.

$a_0 = 0, \quad a_1 = 1$

$G(a,z) = \sum_{n=0}^{\infty} a_n z^n$

the Solⁿ. $a_0 = 0, \quad a_1 = 1$

$$G(a,z) = \sum_{n=0}^{\infty} a_n z^n$$

Solⁿ The given recurrence relation is.

$$a_{n+2} - 3a_{n+1} + 2a_n = 0$$

Multiply both sides by z^n.

$$a_{n+2} z^n - 3a_{n+1} z^n + 2a_n z^n = 0$$

$$\sum_{n=0}^{\infty} a_{n+2} z^n - 3 \sum_{n=0}^{\infty} a_{n+1} z^n + 2 \sum_{n=0}^{\infty} a_n z^n = 0$$

$$\left[a_2 z^0 + a_3 z^1 + a_4 z^2 + \cdots \infty\right] - 3\left[a_1 z^0 + a_2 z^1 + a_3 z^2 + \cdots \infty\right] + 2G(a,z) = 0$$

$$\left[a_2 + a_3 z + a_4 z^2 + \cdots \infty\right] - 3\left[a_1 + a_2 z + a_3 z^2 + \cdots \infty\right] + 2G(a,z) = 0$$

$$\frac{z^2}{z^2}\left[a_2 + a_3 z + a_4 z^2 + \cdots \infty\right] - \frac{3z}{z}\left[a_1 + a_2 z + a_3 z^2 + \cdots \infty\right] + 2G(a,z) = 0$$

$$\frac{1}{z^2}\left[a_2 z^2 + a_3 z^3 + a_4 z^4 + \cdots \infty\right] - \frac{3}{z}\left[a_1 z + a_2 z^2 + a_3 z^3 + \cdots \infty\right] + 2G(a,z) = 0$$

$$\frac{1}{z^2}\left[G(a,z) - a_0 - a_1 z\right] - \frac{3}{z}\left[G(a,z) - a_0\right] + 2G(a,z) = 0$$

$$\frac{\left[G(a,z) - a_0 - a_1 z\right] - 3z\left(G(a,z) - a_0\right) + 2z^2 G(a,z)}{z^2} = 0$$

$$\boxed{a_0 = 0, \quad a_1 = 1}$$

$$1G(a,z) - a_0 - a_1 z - 3z\left(G(a,z) - a_0\right) + 2z^2 G(a,z) = 0$$

$$\left[1 - 3z + 2z^2\right] G(a,z) - a_0 - a_1 z + 3a_0 z = 0$$

$$\left[1 - 3z + 2z^2\right] G(a,z) - z = 0$$

$$G(a,z) = \frac{z}{2z^2 - 3z + 1}$$

$2z - 1 = 0$

$\boxed{z = 1/2}$

$$= \frac{z}{2z^2 - 2z - z + 1}$$

$z - 1 = 0$

$\boxed{z = 1}$

$$= \frac{z}{2z(z-1)-1(z-1)}$$

$z-1=0$ $\quad \boxed{z=1}$

$2z-1=0$

$2z=1$

$\boxed{z=1/2}$

$$= \frac{z}{(2z-1)(z-1)}$$

$$= \frac{1/2}{(2z-1)\left(\frac{1}{2}-1\right)} + \frac{1}{(2-1)(z-1)}$$

$$= \frac{\cancel{1/2}}{-\cancel{1/2}(2z-1)} + \frac{1}{z-1}$$

$$G(a,z) = \frac{1}{1-(2)z} - \frac{1}{1-(1)z}$$

$$\boxed{a_n = (2)^n - (1)^n}$$

Q $\quad a_n - 5a_{n-1} + 6a_{n-2} = 0 \qquad a_0 = 1, \quad a_1 = 2$

$0-2=-2$

Sol^n $\quad a_n z^n - 5a_{n-1}z^n + 6a_{n-2}z^n = 0$

$n-(n-2) = \cancel{n}-\cancel{n}+2$
$= 2$

$$\sum_{n=2}^{\infty} a_n z^n - 5\sum_{n=2}^{\infty} a_{n-1} z^n + 6\sum_{n=2}^{\infty} a_{n-2} z^n = 0$$

$$\left[a_2 z^2 + a_3 z^3 + \cdots \infty\right] - 5\left[a_1 z^2 + a_2 z^3 + \cdots \infty\right] + 6\left[a_0 z^2 + a_1 z^3 + \cdots \infty\right] = 0$$

$$\left[a_2 z^2 + a_3 z^3 + \cdots \infty\right] - 5z\left[a_1 z + a_2 z^2 + \cdots \infty\right] + 6z^2\left[a_0 + a_1 z + \cdots \infty\right] = 0$$

$$\left[G(a,z) - a_0 - a_1 z\right] - 5z\left[G(a,z) - a_0\right] + 6z^2 G(a,z) = 0$$

$$\left[1 - 5z + 6z^2\right] G(a,z) - a_0 - a_1 z + 5a_0 z = 0$$

$$\left[1 - 5z + 6z^2\right] G(a,z) - 1 - 2z + 5z = 0$$

$$\left[6z^2 - 5z + 1\right] G(a,z) - 1 + 3z = 0$$

$$(6z^2-5z+1)\,G(a,z) = 1-3z$$

$$\boxed{G(a,z) = \frac{1-3z}{(6z^2-5z+1)}} = \frac{(1-3z)}{6z^2-3z-2z+1}$$

$$= \frac{(1-3z)}{3z(2z-1)-1(2z-1)} = \frac{(1-3z)}{(3z-1)(2z-1)}$$

$$= \frac{-\cancel{(3z-1)}}{\cancel{(3z-1)}(2z-1)} = \frac{1}{1-2z}$$

$$G(a,z) = \frac{1}{1-2z}$$

$$\boxed{a_n = (2)^n}$$