UNIT-IV

Fundamentals of semiconductor devices and digital circuits

Lecture 26

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Boolean Algebra

- Sometimes it is more economical to build a circuit using the complement of a function (and complementing its result) than it is to implement the function directly.
- DeMorgan's law provides an easy way of finding the complement of a Boolean function.
- Recall DeMorgan's law states:

$$\overline{(xy)} = \overline{x} + \overline{y} \quad \text{and} \quad \overline{(x+y)} = \overline{x}\,\overline{y}$$

Boolean Algebra

- DeMorgan's law can be extended to any number of variables.
- Replace each variable by its complement and change all ANDs to ORs and all ORs to ANDs.
- Thus, we find the the complement of:

$$F(X,Y,Z) = (XY) + (\overline{X}Z) + (Y\overline{Z})$$

is:

$$\overline{F}(X,Y,Z) = \overline{(XY) + (\overline{X}Z) + (Y\overline{Z})}$$

$$= \overline{(XY)}\,\overline{(\overline{X}Z)}\,\overline{(Y\overline{Z})}$$

$$= (\overline{X}+\overline{Y})(X+\overline{Z})(\overline{Y}+Z)$$

Boolean Algebra

- Through our exercises in simplifying Boolean expressions, we see that there are numerous ways of stating the same Boolean expression.
  - These "synonymous" forms are *logically equivalent.*
  - Logically equivalent expressions have identical truth tables.
- In order to eliminate as much confusion as possible, designers express Boolean functions in *standardized* or *canonical* form.

Boolean Algebra

- There are two canonical forms for Boolean expressions: sum-of-products and product-of-sums.
  - Recall the Boolean product is the AND operation and the Boolean sum is the OR operation.
- In the sum-of-products form, ANDed variables are ORed together.
  - For example: $F(x,y,z) = xy + xz + yz$
- In the product-of-sums form, ORed variables are ANDed together:
  - For example: $F(x,y,z) = (x+y)(x+z)(y+z)$

Boolean Algebra

- It is easy to convert a function to sum-of-products form using its truth table.
- We are interested in the values of the variables that make the function true (=1).
- Using the truth table, we list the values of the variables that result in a true function value.
- Each group of variables is then ORed together.

$F(x,y,z) = x\bar{z}+y$

x	y	z	$x\bar{z}+y$
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

Boolean Algebra

- The sum-of-products form for our function is:

$$F(x,y,z) = \bar{x}y\bar{z}+\bar{x}yz+x\bar{y}\bar{z}+xy\bar{z}+xyz$$

We note that this function is not in simplest terms. Our aim is only to rewrite our function in canonical sum-of-products form.

$$F(x,y,z) = x\bar{z}+y$$

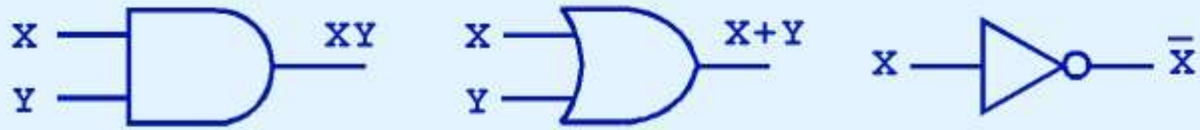
x	y	z	$x\bar{z}+y$
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

Logic Gates

- We have looked at Boolean functions in abstract terms.
- In this section, we see that Boolean functions are implemented in digital computer circuits called gates.
- A gate is an electronic device that produces a result based on two or more input values.
  - In reality, gates consist of one to six transistors, but digital designers think of them as a single unit.
  - Integrated circuits contain collections of gates suited to a particular purpose.

Logic Gates

- The three simplest gates are the AND, OR, and NOT gates.



X AND Y

X	Y	XY
0	0	0
0	1	0
1	0	0
1	1	1

X OR Y

X	Y	X+Y
0	0	0
0	1	1
1	0	1
1	1	1

NOT X

X	$\overline{X}$
0	1
1	0

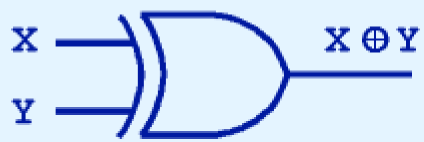
- They correspond directly to their respective Boolean operations, as you can see by their truth tables.

Logic Gates

- Another very useful gate is the exclusive OR (XOR) gate.
- The output of the XOR operation is true only when the values of the inputs differ.

X XOR Y

X	Y	X ⊕ Y
0	0	0
0	1	1
1	0	1
1	1	0



Note the special symbol ⊕ for the XOR operation.

Logic Gates

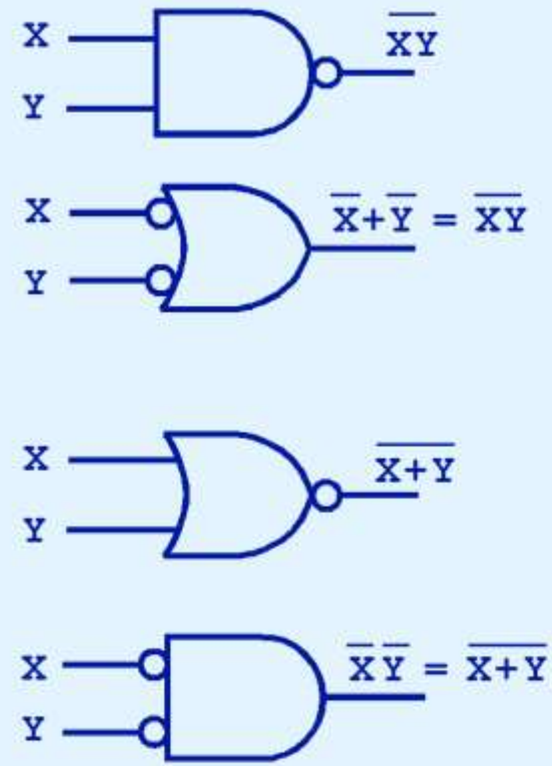
- NAND and NOR are two very important gates. Their symbols and truth tables are shown at the right.

X NAND Y

X	Y	X NAND Y
0	0	1
0	1	1
1	0	1
1	1	0

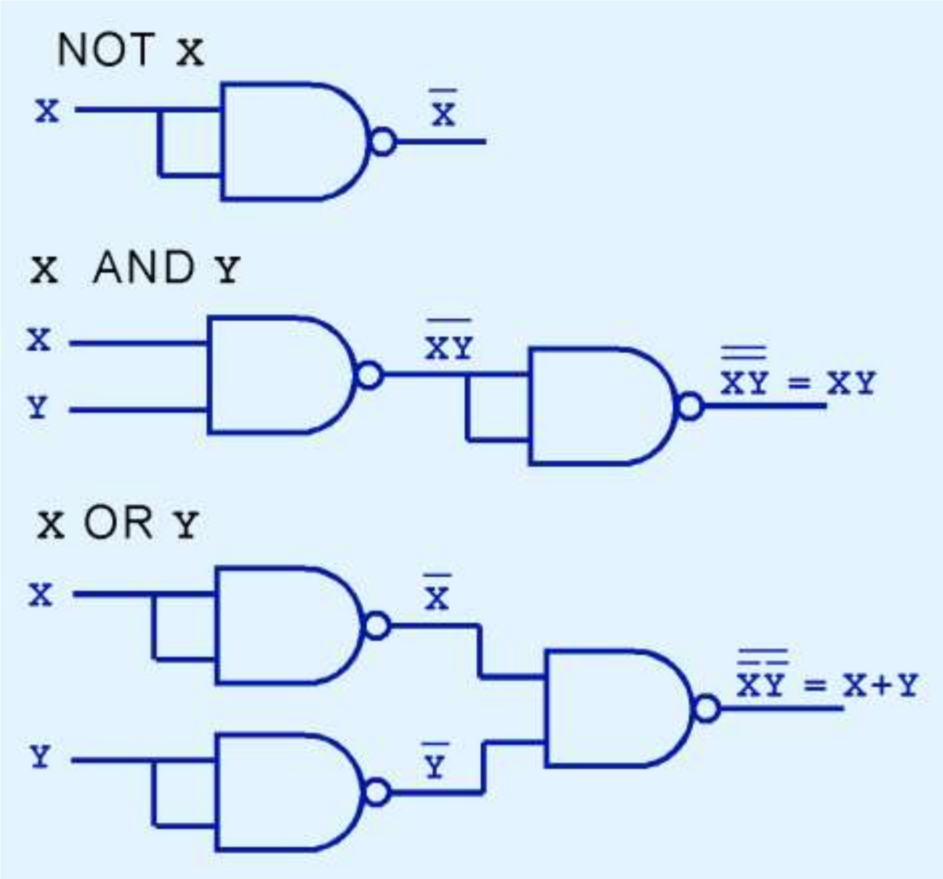
X NOR Y

X	Y	X NOR Y
0	0	1
0	1	0
1	0	0
1	1	0



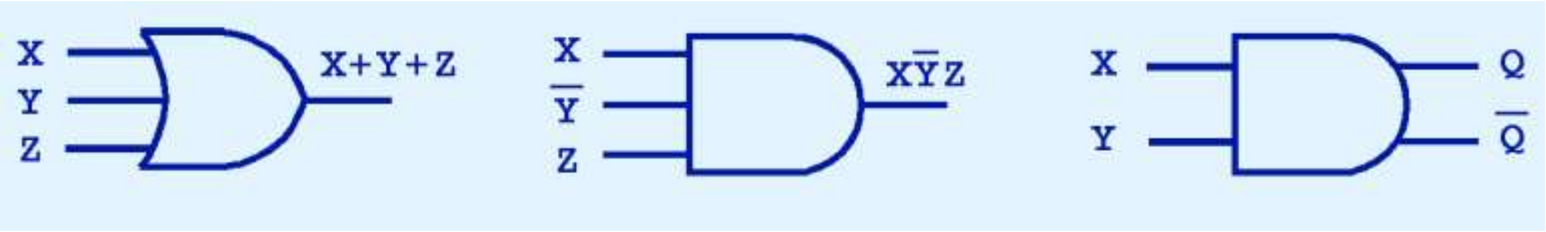
Logic Gates

- NAND and NOR are known as *universal gates* because they are inexpensive to manufacture and any Boolean function can be constructed using only NAND or only NOR gates.



Logic Gates

- Gates can have multiple inputs and more than one output.
  - A second output can be provided for the complement of the operation.
  - We'll see more of this later.



Quick Quiz (Poll 1)

- Electronic circuits that operate on one or more input signals to produce standard output _______

  a) Series circuits
  b) Parallel Circuits
  c) Logic Signals
  d) Logic Gates

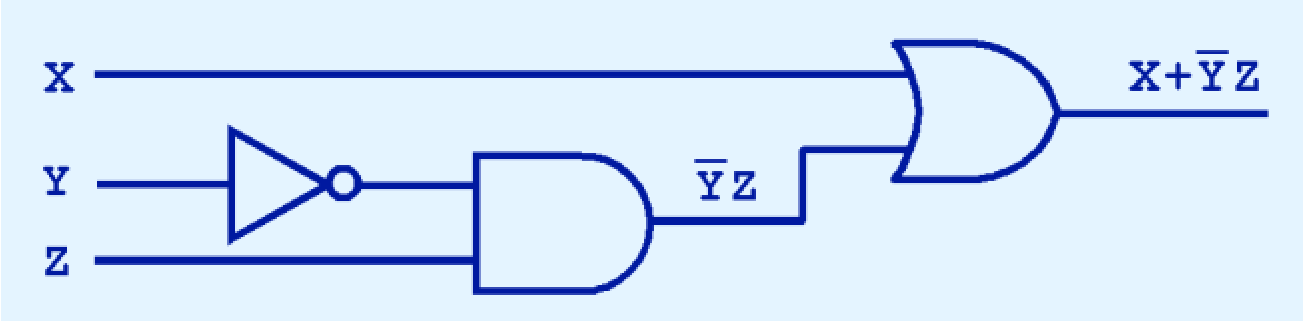
Quick Quiz (Poll 2)

- A ____________ gate gives the output as 1 only if all the inputs signals are 1.

  a) AND
  b) OR
  c) EXOR
  d) NOR

Digital Components

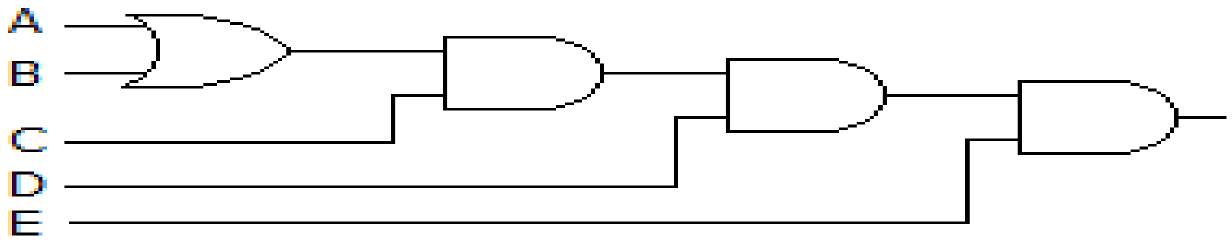
- The main thing to remember is that combinations of gates implement Boolean functions.
- The circuit below implements the Boolean function: $F(X,Y,Z) = X+\overline{Y}Z$



We simplify our Boolean expressions so that we can create simpler circuits.

Quick Quiz (Poll 3)

Derive the Boolean expression for the logic circuit shown below:



A. $C(A + B)DE$

B. $\left[C(A + B)D + \overline{E}\right]$

C. $\left[\left[C(A + B)D\right]\overline{E}\right]$

D. $ABCDE$

Quick Quiz (Poll 4)

- The universal gate that can be used to implement any Boolean expression is ___________

  a) NAND
  b) EXOR
  c) OR
  d) AND