– Q1) Find the solution of recurrence relation.

$$a_{n+1} - a_n = 1 \qquad \underline{a_0 = 1}$$

Sol^n The given recurrence relation is.

$$a_{n+1} - a_n = 1$$

$$E(a_n) - a_n = 1$$

$$(E-1)a_n = 1$$

The characteristic eq^n is given by

$$E - 1 = 0$$

$$\boxed{E = 1}$$

$$a_n^{(h)} = c_1 (1)^n \longrightarrow ①$$

$$a_n^{(p)} = \frac{1}{E-1}(1) \qquad E = \Delta + 1 \qquad E - 1 = \Delta$$

$$= \frac{1}{\Delta} n^{[0]}$$

$$= n^{[1]} = n$$

$$a_n = a_n^{(h)} + a_n^{(p)}$$

$$a_n = c_1 (1)^n + n. \longrightarrow ①$$

Set $n = 0$

$$a_0 = c_1 (1)^0$$

$$\boxed{1 = c_1}$$

from ①

$$a_n = (1)^n + n$$

$$\boxed{a_n = 1 + n}$$

Generating function Technique

Def^n of Generating function: Let $\{a_n\}$ be the Sequence, then the generating function for this sequence is given by

new

$$G(a, z) = \sum_{n=0}^{\infty} a_n z^n$$

$$G(a,z) = \sum_{n=0} a_n$$

$\{a, a, a, \ldots\}$

P1 $a_n = a \;\; \forall n \in W$

Solⁿ

$$G(a,z) = \sum_{n=0}^{\infty} a_n z^n$$

$$= \sum_{n=0}^{\infty} a z^n$$

$$= a \sum_{n=0}^{\infty} z^n$$

$$= a\left[z^0 + z^1 + z^2 + \cdots \infty\right]$$

$$= a\left[1 + z + z^2 + \cdots \infty\right]$$

$$= a(1-z)^{-1}$$

$$= \frac{a}{1-z}$$

$a_n = 2003 \;\; \forall n \in W$

$$G(a,z) = \frac{2003}{1-z}.$$

$a_n = 5$, $G(a,z) = \frac{5}{1-z}$

P2 $a_n = 2^n$ $n \in W$ find the generating function.

Solⁿ

$$G(a,z) = \sum_{n=0}^{\infty} a_n z^n.$$

$$= \sum_{n=0}^{\infty} 2^n z^n$$

$$= \sum_{n=0}^{\infty} (2z)^n$$

$$= (2z)^0 + (2z)^1 + (2z)^2 + \cdots \infty$$

$$= 1 + 2z + (2z)^2 + \cdots \infty$$

$$= (1-2z)^{-1}$$

$$= \frac{1}{1-2z}$$

$a_n = 2^n$, $G(a,z) = \frac{1}{1-2z}$

$a_n = (3)^n$ $G(a,z) = \frac{1}{1-3z}$

$\boxed{a_n = (-4)^n}$ $G(a,z) = \frac{1}{1+4z}$

$$= \frac{1}{\quad}$$

$1-(-4)z$

$a_n = \underline{c}(-4)^n$

$G(a,z) = \dfrac{c}{1+4z}$

Q: $a_n = \underline{b\,2^n}$ then what is the generating function.

Soln $G(a,z) = \sum_{n=0}^{\infty} a_n z^n$

$= \sum_{n=0}^{\infty} b\,2^n z^n$

$= b\left[\sum_{n=0}^{\infty} (2z)^n\right] = b\left[1+(2z)+(2z)^2+\cdots\infty\right]$

$= \dfrac{b}{1-2z}$

$G(a,z) = \sum_{n=0}^{\infty} a_n z^n$

$= a_0 z^0 + a_1 z^1 + a_2 z^2 + a_3 z^3 + a_4 z^4 + \cdots\infty$

$G(a,z) = \underline{a_0 + a_1 z^1} + a_2 z^2 + a_3 z^3 + \cdots\infty$

$[G(a,z)-a_0] = a_1 z^1 + a_2 z^2 + a_3 z^3 + \cdots\infty \longrightarrow (1)$

$[G(a,z)-a_0-a_1 z] = a_2 z^2 + a_3 z^3 + \cdots\infty \longrightarrow (2)$

$$\left[G(a,z) - a_0 - a_1 z\right] = a_2 z^2 + a_3 z^3 + \cdots \infty \longrightarrow (2)$$

(Q1) $a_{n+2} - 4a_{n+1} + 3a_n = 0$; $a_0 = 1$, $a_1 = 2$

Solve this recurrence relation by generating function method.

Soln: The given recurrence relation is. $G(a,z) = \sum_{n=0}^{\infty} a_n z^n$

$$a_{n+2} - 4a_{n+1} + 3a_n = 0$$

Multiply both sides by z^n

$$a_{n+2} z^n - 4a_{n+1} z^n + 3a_n z^n = 0$$

Taking Summation where n varies from 0 to ∞

$$\sum_{n=0}^{\infty} a_{n+2} z^n - 4 \sum_{n=0}^{\infty} a_{n+1} z^n + 3 \sum_{n=0}^{\infty} a_n z^n = 0$$

$$\left[a_2 z^0 + a_3 z^1 + a_4 z^2 + \cdots \infty\right] - 4\left[a_1 z^0 + a_2 z + a_3 z^2 + \cdots \infty\right] + 3G(a,z) = 0$$

$$\left[a_2 + a_3 z + a_4 z^2 + \cdots \infty\right] - 4\left[a_1 + a_2 z + a_3 z^2 + \cdots \infty\right] + 3G(a,z) = 0$$

$$\frac{z^2}{z^2}\left[a_2 + a_3 z + a_4 z^2 + \cdots \infty\right] - \frac{4}{z} z\left[a_1 + a_2 z + a_3 z^2 + \cdots \infty\right] + 3G(a,z) = 0$$

$$\frac{1}{z^2}\left[a_2 z^2 + a_3 z^3 + a_4 z^4 + \cdots \infty\right] - \frac{4}{z}\left[a_1 z + a_2 z^2 + a_3 z^3 + \cdots \infty\right] + 3G(a,z) = 0$$

$$\frac{1}{z^2}\left[G(a,z) - a_0 - a_1 z\right] - \frac{4}{z}\left[G(a,z) - a_0\right] + 3G(a,z) = 0$$

$$\frac{\left[G(a,z)-a_0-a_1 z\right]-4z\left[G(a,z)-a_0\right]}{z^2}+z^2 3G(a,z)=0.$$

$$\left(G(a,z)-a_0-a_1 z\right)-4z\left[G(a,z)-a_0\right]+3z^2 G(a,z)=0$$

$$\left[1-4z+3z^2\right]G(a,z)-a_0-a_1 z+4a_0 z=0$$

$$\left[1-4z+3z^2\right]G(a,z)-1-2z+4z=0$$

$$(1-4z+3z^2)\,G(a,z)-1+2z=0$$

$$(3z^2-4z+1)\,G(a,z)=1-2z$$

$$G(a,z)=\frac{1-2z}{3z^2-4z+1}=\frac{(1-2z)}{3z^2-3z-z+1}$$

$$=\frac{(1-2z)}{3z(z-1)-1(z-1)}$$

$3z-1=0$

$3z=1$

$z=1/3$

$$=\frac{(1-2z)}{(3z-1)(z-1)}$$

$z-1=0$

$z=1$

$$=\frac{\left(1-\frac{2}{3}\right)}{(3z-1)\left(\frac{1}{3}-1\right)}+\frac{(1-2)}{(3-1)(z-1)}$$

$G(a,z)=\frac{1}{1-2z}$, $a_n=(2)^n$.

$$=\frac{\left(\frac{1}{3}\right)}{(3z-1)\left(-\frac{2}{3}\right)}+\frac{(-1)}{2(z-1)}$$

$$=\frac{-1}{2(3z-1)}-\frac{1}{2}\,\frac{1}{z-1}$$

$$G(a,z)=\frac{1}{2(1-3z)}+\frac{1}{2}\,\frac{1}{1-z}$$

$$a_n=\frac{1}{2}(3)^n+\frac{1}{2}(1)^n$$ Ans.

Q obtain only generating function from the following recurrence relation.

$\sum_{n=0}^{\infty}$ ✗ $\quad a_{-2}$ ✗

$$a_n - 3a_{n-1} + 2a_{n-2} = 0 \qquad a_0 = 1, \quad a_1 = 2$$

$n-(n-2) = \cancel{n}-\cancel{n}+2$
$= 2$

Multiply both sides by z^n.

$$a_n z^n - 3a_{n-1} z^n + 2a_{n-2} z^n = 0$$

$$\sum_{n=2}^{\infty} a_n z^n - 3\sum_{n=2}^{\infty} a_{n-1} z^n + 2\sum_{n=2}^{\infty} a_{n-2} z^n = 0$$

$$\left[a_2 z^2 + a_3 z^3 + a_4 z^4 + \cdots \infty\right] - 3\left[a_1 z^2 + a_2 z^3 + \cdots\right] + 2\left[a_0 z^2 + a_1 z^3 + a_2 z^4 + \cdots\right] = 0$$

$$\left[G(a,z) - a_0 - a_1 z\right] - 3z\left[a_1 z + a_2 z^2 + \cdots \infty\right] + 2z^2\left[a_0 + a_1 z + a_2 z^2 + \cdots \infty\right] = 0$$

$$\left[G(a,z) - a_0 - a_1 z\right] - 3z\left[G(a,z) - a_0\right] + 2z^2 G(a,z) = 0$$

$$\left[1 - 3z + 2z^2\right] G(a,z) - a_0 - a_1 z + 3z a_0 = 0$$

$$\left[1 - 3z + 2z^2\right] G(a,z) - 1 - 2z + 3z = 0$$

$$(2z^2 - 3z + 1)\, G(a,z) - 1 + z = 0$$

$$(2z^2 - 3z + 1)\, G(a,z) = 1 - z$$

$$G(a,z) = \frac{1-z}{2z^2 - 3z + 1}$$

$$= \frac{1-z}{2z^2 - 2z - z + 1}$$

$$= \frac{(1-z)}{2z(z-1) - 1(z-1)}$$

$$= \frac{(1-z)}{(2z-1)(z-1)}$$

$$= -\frac{\cancel{(z-1)}}{(2z-1)\cancel{(z-1)}}$$

$$= -\frac{1}{2z-1}$$

$$G(a,z) = \frac{1}{1-2z}$$

$$\boxed{a_n = (2)^n}$$