

OBJECTIVE TYPE QUESTIONS

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Choose the correct alternative

1. The general solution of the wave equation

$$\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2} \text{ is}$$

- (a) $y = (c_5 \cos c \sqrt{k} t + c_6 \sin c \sqrt{k} t) (c_7 \cos \sqrt{k} x + c_8 \sin \sqrt{k} x)$
 (b) $y = (c_5 \cos c \sqrt{k} t + \sin c \sqrt{k} t) (c_6 \cos \sqrt{k} x + c_7 \sin \sqrt{k} x)$
 (c) $y = (c_5 \cos c \sqrt{k} t + c_6 \sin c \sqrt{k} t) (c_7 \cos \sqrt{k} x + \sin \sqrt{k} x)$
 (d) $y = (c_5 \cos c \sqrt{k} t + c_6 \sin c \sqrt{k} t) (\cos \sqrt{k} x + \sin \sqrt{k} x)$

Ans. (a)

2. The general solution of the P.D.E.

$$\frac{\partial u}{\partial x} = a^2 \frac{\partial^2 u}{\partial x^2} \text{ is}$$

- (a) $u = \sum_{n=1}^{\infty} b_n (c_1 \cos pt + c_2 \sin pt) e^{p^2 c^2 x}$
 (b) $u = \sum_{n=1}^{\infty} b_n (c_1 \cos pt + c_2 \sin pt) e^{-p^2 c^2 x}$
 (c) $u = \sum_{n=1}^{\infty} b_n (c_1 \cos px + c_2 \sin px) e^{-p^2 c^2 t}$
 (d) $u = \sum_{n=1}^{\infty} b_n (c_1 \cos px + c_2 \sin px) e^{p^2 c^2 t}$

Ans. (c)

3. The general solution of two dimensional heat flow

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial t^2} = 0 \text{ is}$$

- (a) $u = (c_1 \cos px + c_2 \sin px) (e^{py} + e^{-py})$
 (b) $u = (c_1 \cosh px + c_2 \sinh px) (c_3 e^{py} + c_4 e^{-py})$
 (c) $u = (c_1 \cos py + c_2 \sin py) (c_3 e^{px} + c_4 e^{-px})$
 (d) $u = (c_1 \cos px + c_2 \sin px) (c_3 e^{py} + c_4 e^{-py})$

Ans. (d)

4. The formula of b_n in the equation

$$u = \sum_{n=1}^{\infty} b_n \sin \left(\frac{n\pi x}{l} \right) \text{ is}$$

- (a) $\frac{2}{l} \int_0^l u_0 \sin \left(\frac{n\pi x}{l} \right) dx$
 (b) $\frac{1}{l} \int_0^{2l} u_0 \sin \left(\frac{n\pi x}{l} \right) dx$
 (c) $\frac{1}{l} \int_0^l u_0 \sin \left(\frac{n\pi x}{l} \right) dx$
 (d) $\frac{2}{l} \int_0^{2l} u_0 \sin \left(\frac{n\pi x}{l} \right) dx$

Ans. (a)

5. The differential equation $Z_{xx} + x^2 Z_{yy} = 0$ is classified as:

- (a) Hyperbolic
 (c) Elliptic

- (b) Parabolic
 (d) None of these

(GBTU, 2011)

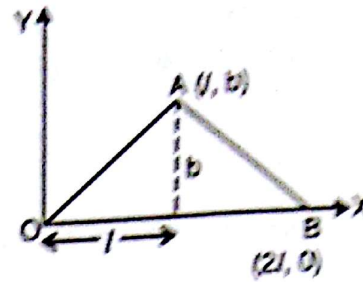
6. In the figure a string OAB is stretched to a height b , the equation of the OA portion of the string is

(a) $x = -\frac{by}{l}$

(b) $y = \frac{bx}{l}$

(c) $y = -\frac{by}{l}$

(d) $x = \frac{b}{l}y$



Ans. (b)

7. In the given figure write down OB portion of the string :

(a) $y = -\frac{b}{l}(x+l)$

(b) $y = -\frac{b}{l}(x-l)$

(c) $y = -\frac{b}{l}(x-2l)$

(d) $y = \frac{b}{l}(x-2l)$

Ans. (c)

8. If $u = x^2 + t^2$ is a solution of $c^2 \frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2}$, then the value of c is

(a) 1

(b) 2

(c) -2

(d) -1/2

Ans. (a)

9. Laplace's equation in polar coordinates is

(a) $\frac{\partial^2 u}{\partial r^2} + \frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial \theta^2} = 0$

(b) $\frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial \theta^2} = 0$

(c) $\frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} = 0$

(d) $\frac{\partial^2 u}{\partial r^2} + \frac{\partial u}{\partial r} + \frac{1}{r} \frac{\partial^2 u}{\partial \theta^2} = 0$

Ans. (b)

10. In one dimensional heat flow, the condition on temperature is

(a) temperature always increases.

(b) temperature decreases as time increases

(c) temperature always decreases

(d) temperature remains always non zero at all times.

Ans. (b)

11. If the ends $x = 0$ and $x = L$ are insulated in one dimensional heat flow problems, then the boundary conditions are

(a) $\frac{\partial u(0,t)}{\partial x} = 0, \frac{\partial u(L,t)}{\partial x} = 1$ at $t = 0$.

(b) $\frac{\partial u(0,t)}{\partial x} = 1, \frac{\partial u(L,t)}{\partial x} = 1$ at $t = 0$.

(c) $\frac{\partial u(0,t)}{\partial x} = 0, \frac{\partial u(L,t)}{\partial x} = 0$ for all t .

(d) $\frac{\partial u(0,t)}{\partial x} = 0, \frac{\partial u(L,t)}{\partial x} = 1$ for all t .

Ans. (c)

12. The PDE $\frac{\partial^2 z}{\partial t^2} = c^2 \frac{\partial^2 z}{\partial x^2}$ is known as:

(a) wave equation

(b) heat equation

(c) Laplace equation

(d) none of these

Ans. (a)

Applications of Partial Differential Equations

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Indicate True or False for the following :

13. The small transverse vibrations of a string are governed by one dimensional heat equation $y_t = a^2 y_{xx}$.
(True/False) (U.P. II Semester, 2009) Ans. False
14. Two dimensional steady state heat flow is given by Laplace's equation $u_t = a^2(u_{xx} + u_{yy})$.
(True/False) (U.P. II Semester, 2009) Ans. False
15. $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$ is a two-dimensional wave equation. Ans. True
16. Radio equations are $V_{xx} = LCV_n$ and $I_{xx} = LCI_n$. Ans. True
17. The small transverse vibrations of a string are $y_t^2 = a^2 y_{xx}$. Ans. False

Fill in the Blanks

18. The general solution of the equation $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial t^2} = 0$ is
Ans. $(c_1 \cos px + c_2 \sin px)(c_3 \cos pt + c_4 \sin pt)$.
19. The general solution of the equation $\frac{\partial^2 z}{\partial x \partial y} = 0$ is
Ans. $f_1(x) + f_2(y)$.
20. The solution of $z(x, y)$ of the equation $\frac{\partial z}{\partial x} - y \frac{\partial z}{\partial y} = 0$ is
Ans. $f(x + \log y, z) = 0$.
21. The solution of $3x \frac{\partial z}{\partial x} - 5y \frac{\partial z}{\partial y} = 0$ is
Ans. $f(x^5 y^3, z)$
22. The solution of $\frac{\partial^2 z}{\partial x^2} = \sin(xy)$ is
Ans. $\frac{1}{y^2} \sin(xy) + x f_1(y) + f_2(y)$
23. The solution of $\frac{\partial u}{\partial t} = 2 \frac{\partial^2 u}{\partial x^2}$ if $u(0, t) = u(3, t) = 0$ and $u(x, 0) = 5 \sin 4\pi x - 8\pi x$ is
Ans. $(5 \sin 4\pi x e^{-32x^2 t} - 3 \sin 8\pi x e^{-12\pi^2 t})$

The solution to the P.D.E.

24. $3u_x + 2u_y = 0$ is where $u_x = \frac{\partial u}{\partial x}$, $u_y = \frac{\partial u}{\partial y}$ (U.P. II Semester 2009) Ans. $u(x, y) = ce^{\frac{k}{6}(2x-3y)}$
25. $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ is called
Ans. Laplace equation.
26. On solving $\frac{\partial u}{\partial x} + 2 \frac{\partial u}{\partial y} = 0$ by the method of separating of variable we suppose $u = \dots$
Ans. $u = XY$
27. In the separating of variable, we assume $u = XY$, then X is a function of... and Y is a function of
Ans. x only, y only.
28. Transverse vibration in one dimensional wave equation, the motion in horizontal direction is
Ans. zero.
29. The Laplace equation in two dimension is
Ans. $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$
30. D'Alembert's solution of the wave equation is
Ans. $y(x, t) = f(x + ct) + f(x - ct)$

31. The equation of steady state heat conduction in the rectangular-plate is

(GBTU, II Sem. 2011)

$$\text{Ans. } \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

Match the following

32. (i) $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$

(a) $\sum_{n=1}^{\infty} b_n (c_1 \cos px + c_2 \sin px) e^{-lp^2 c^2 t}$

(ii) $\frac{\partial^2 u}{\partial t^2} = a^2 \frac{\partial^2 u}{\partial x^2}$

(b) $A e^{k\left(\frac{x}{3} - \frac{y}{2}\right)}$

(iii) $3 \frac{\partial u}{\partial x} + 2 \frac{\partial u}{\partial y} = 0$

(c) $(c_5 \cos c\sqrt{k} t + c_6 \sin c\sqrt{k} t) (c_7 \cos \sqrt{k} x + c_8 \sin \sqrt{k} x)$

(iv) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial t^2} = 0$

(d) $(c_1 \cos px + c_2 \sin px) (c_3 e^{py} + c_4 e^{-py})$

Ans. (i) \rightarrow (c), (ii) \rightarrow (a), (iii) \rightarrow (b), (iv) \rightarrow (d).

Match the following equations

33. (i) One dimensional heat flow is

(a) $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$

(ii) The transverse vibration of a string is

(b) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$

(iii) Two dimensional heat flow is

(c) $\frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} + \frac{1}{r^2} \frac{\partial^2 u}{\partial \theta^2} = 0$

(iv) Two dimensional of heat flow in polar form is

(d) $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$

Ans. (i) \rightarrow (d), (ii) \rightarrow (a), (iii) \rightarrow (b), (iv) \rightarrow (c).

Choose the correct alternative :

1. The partial differential equation from $z = (a + x)^2 + y$ is

(i) $z = \frac{1}{4} \left(\frac{\partial z}{\partial x} \right)^2 + y$

(ii) $z = \left(\frac{\partial z}{\partial x} \right)^2 + y$

(iii) $z = \frac{1}{4} \left(\frac{\partial z}{\partial y} \right)^2 + y$

(iv) $z = \left(\frac{\partial z}{\partial y} \right)^2 + y$

2. The solution of $xy + yz = z$ is