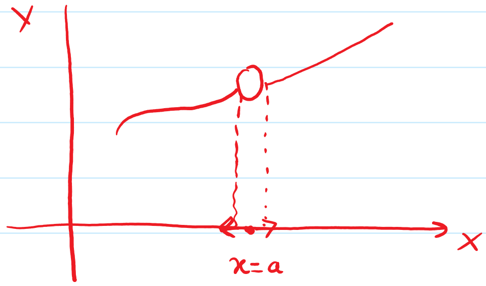
limit of function of one variable.



$$\underset{x \to a-}{\mathrm{lt}} f(x) = \text{L.H.L}$$

$$\underset{x \to a+}{\mathrm{lt}} f(x) = \text{R.H.L.}$$

if $\underset{x \to a+}{\mathrm{lt}} f(x) = \underset{x \to a-}{\mathrm{lt}} f(x)$, and finite

Then we say that limit of the function exists.

limit of Function of two variables

We say that the limit of function of two variables exists if it exist along all the paths and this limit should be equal along all the paths.

Q1
$$\lim_{\substack{x\to 1\\ y\to 2}} \frac{2x^2y}{x^2+y^2+1}$$

$$= \frac{2(1)^2(2)}{(1)^2+(2)^2+1} = \frac{4}{6} = \frac{2}{3} \text{ Ans.}$$

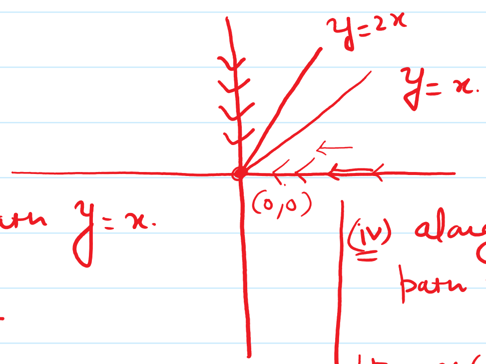
Q2
$$\lim_{(x,y)\to(0,0)} (x^2+y^2)$$

$$= 0^2+0^2$$

$$= 0 \text{ Ans}$$

— × —

Q
$$\lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2}$$



(i) along x-axis ($y=0$)

$$\lim_{x\to 0} \frac{x(0)}{x^2+0^2} = 0 \checkmark$$

(ii) along y-axis ($x=0$)

$$\lim_{y\to 0} \frac{0(y)}{0^2+y^2} = 0 \checkmark$$

(iii) along the path $y=x$.

$$\lim_{x\to 0} \frac{x(x)}{x^2+x^2}$$

$$= \lim_{x\to 0} \frac{x^2}{x^2+x^2}$$

$$= \lim_{x\to 0} \frac{\cancel{x^2}}{2\cancel{x^2}} = \lim_{x\to 0}\left(\frac{1}{2}\right) = \frac{1}{2} \checkmark$$

(iv) along the path $y=2x$

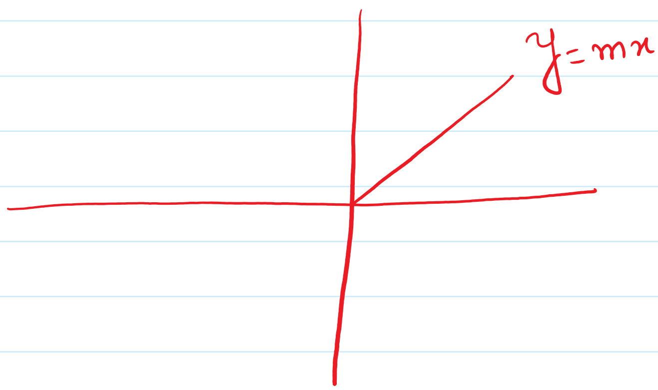
$$\lim_{x\to 0} \frac{x(2x)}{x^2+(2x)^2}$$

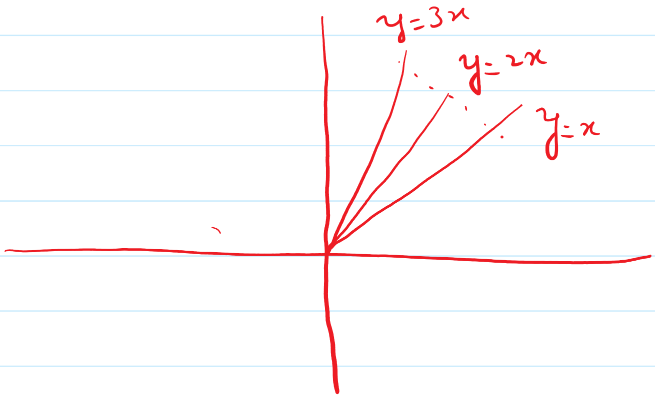
$$= \lim_{x\to 0} \frac{2x^2}{x^2+4x^2}$$

$$= \lim_{x\to 0} \frac{2\cancel{x^2}}{5\cancel{x^2}}$$

$$= 2/5 \checkmark$$

limit doesn't exist

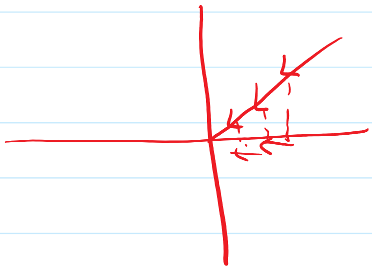




Q3 $\displaystyle \underset{(x,y)\to(0,0)}{Lt} \frac{xy}{x^2+y^2}$

deg(num) = deg(deno) always take the path $y=mx$ to solve the question.

Sol^n $\displaystyle \underset{(x,y)\to(0,0)}{Lt} \frac{xy}{x^2+y^2}$



Let $(x,y)\to(0,0)$ along the path $y=mx$

$$\underset{x\to 0}{Lt} \frac{x(mx)}{x^2+(mx)^2} = \underset{x\to 0}{Lt} \frac{mx^2}{x^2+m^2x^2}$$

$$= \underset{x\to 0}{Lt} \frac{m\cancel{x^2}}{\cancel{x^2}(1+m^2)} = \frac{m}{1+m^2}$$

As we change the value of m value of limit also change

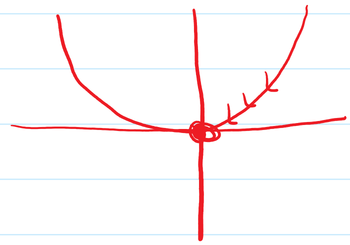
$\therefore$ we can say that limit doesn't exist.

Q2 Find the limit of the following function.

$$\lim_{(x,y)\to(0,0)} \frac{2x^2y}{x^4+y^2}$$

(Here, deg of num $\neq$ deg of deno. But we can make their degrees same.)

Let $(x,y)\to(0,0)$ along the path $\underbrace{y=mx^2}$



$$\lim_{x\to 0} \frac{2x^2(mx^2)}{x^4+(mx^2)^2}$$

$$= \lim_{x\to 0} \frac{2mx^4}{x^4+m^2x^4} = \lim_{x\to 0} \frac{2m\cancel{x^4}}{\cancel{x^4}(1+m^2)}$$

$$= \lim_{x\to 0} \frac{2m}{1+m^2} = \frac{2m}{1+m^2}$$

$m=1$

$m=2$

$\therefore$ limit doesn't exist.

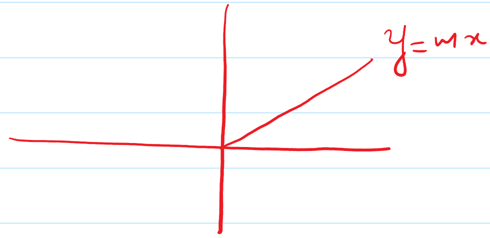
$\frac{4}{5}$ $\qquad$ $\frac{2}{2}=1$

Q $$\lim_{(x,y)\to(0,0)} \frac{x^3-y^3}{x^2+y^2}$$

(In this case, It is not possible to make degree of num and deno equal)

then always take the path $y=mx$

Let $(x,y)\to(0,0)$ along the path $y=mx$



$$= \lim_{x\to 0} \frac{x^3-(mx)^3}{x^2+(mx)^2}$$

$$= \lim_{x\to 0} \frac{\cancel{x^3}(1-m^3)}{\cancel{x^2}(1+m^2)}$$

$$= \lim_{x \to 0} \frac{\cancel{x}(1-m^3)}{\cancel{x^2}(1+m^2)}$$

$$= \lim_{x \to 0} x \frac{(1-m^3)}{1+m^2} = 0$$

Here we say that limit exists.

Evaluate $\lim_{(x,y)\to(0,0)} \frac{xy^3}{x^2+y^6}$

Let $(x,y)\to(0,0)$ along the path $x = my^3$

$$= \lim_{y\to 0} \frac{(my^3)(y^3)}{m^2y^6+y^6} = \lim_{y\to 0} \frac{m\cancel{y^6}}{\cancel{y^6}(m^2+1)} = \frac{m}{m^2+1}$$

So limit doesn't Exist

Continuity of function of two variables

A function $f(x,y)$ is said to be Continuous at the point (a,b) if $\lim_{(x,y)\to(a,b)} f(x,y) = f(a,b)$

N1 if limit doesn't exist, then function is not Continuous.

N2 $\lim_{(x,y)\to(a,b)} f(x,y) \neq f(a,b)$

then function is not continuous.

Q Show that the function

$$f(x,y) = \begin{cases} x^2+2y & (x,y) \neq (1,2) \\ 0 & (x,y) = (1,2) \end{cases}$$

is discontinuous at (1,2).

Sol^n $\lim_{(x,y)\to(1,2)} f(x,y) = \lim_{(x,y)\to(1,2)} (x^2+2y) = 1^2+2\times 2 = 1+4=5$

$f(1,2) = 0$

$\lim_{(x,y)\to(1,2)} f(x,y) \neq f(1,2)$

$\Rightarrow$ function is not continuous at (1,2)