Unit - I

Proofs and logic.

Propositions

Our discussion begins with an introduction to the basic building blocks of logic—propositions. A **proposition** is a declarative sentence (that is, a sentence that declares a fact) that is either true or false, but not both.

EXAMPLE 1 All the following declarative sentences are propositions.

Extra Examples

1. Washington, D.C., is the capital of the United States of America. (True)
2. Toronto is the capital of Canada. (False)
3. $1 + 1 = 2$. (True)
4. $2 + 2 = 3$. (False)

Propositions 1 and 3 are true, whereas 2 and 4 are false.

Ans: (1) As Statement 1 is true, ∴ It is a preposition.

(2) As Statement 2 is false, ∴ It is a preposition.

(3)

EXAMPLE 2 Consider the following sentences. Are these prepositions

1. What time is it?

EXAMPLE 2 Consider the following sentences. Are these prepositions

1. What time is it?
2. Read this carefully.
3. $x + 1 = 2$.
4. $x + y = z$.

Ans (1) As this is not a declarative Sentence, ∴ It is not a preposition.

(2) This is not a preposition.

(3) if $x = 1$ $x+1 = 2$ True.]
$x \neq 1$ $x+1 = 2$ False

Its prepositin can be true and false both ∴ It is not a prepositin.

(4) This is not a prepositin.

—X—

~p $\overline{p}$

DEFINITION 1 Let p be a proposition. The *negation of* p, denoted by $\neg p$ (also denoted by $\overline{p}$), is the statement

"It is not the case that p."

~p : "It is not the case that p"

The proposition $\neg p$ is read "not p." The truth value of the negation of p, $\neg p$, is the opposite of the truth value of p.

p : "Michael's pc runs Linux"

EXAMPLE 3 Find the negation of the proposition

"Michael's PC runs Linux"

Extra Examples

and express this in simple English.

Solution: The negation is

"It is not the case that Michael's PC runs Linux."

This negation can be more simply expressed as

"Michael's PC does not run Linux."

~p: "It is not the case that Michael's pc runs Linux"

—X—

~p : "Michael's pc doesn't runs Linux"

—X

EXAMPLE 4 Find the negation of the proposition

"Vandana's smartphone has at least 32GB of memory"

and express this in simple English.

Solution: The negation is

"It is not the case that Vandana's smartphone has at least 32GB of memory."

This negation can also be expressed as

"Vandana's smartphone does not have at least 32GB of memory"

~p : "It is not the case that Vandana's Smartphone has at least 32 GB of memory"

~p : Vandana's Smartphone does not have at least 32 GB of memory

—x—

TABLE 1 The Truth Table for the Negation of a Proposition.

p	$\neg p$
T	F
F	T

Negation of a proposition.

p	~p
T	F
F	T

2

no. of raws generated in any table = $2^{\text{no. of variables}}$

$= 2^1 = 2$

DEFINITION 2 Let p and q be propositions. The *conjunction* of p and q, denoted by $p \wedge q$, is the proposition "p and q." The conjunction $p \wedge q$ is true when both p and q are true and is false otherwise.

EXAMPLE 5 Find the conjunction of the propositions p and q where p is the proposition "Rebecca's PC has more than 16 GB free hard disk space" and q is the proposition "The processor in Rebecca's PC runs faster than 1 GHz."

DEFINITION 3 Let p and q be propositions. The *disjunction* of p and q, denoted by $p \vee q$, is the proposition "p or q." The disjunction $p \vee q$ is false when both p and q are false and is true otherwise.

TABLE 2 The Truth Table for the Conjunction of Two Propositions.

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

TABLE 3 The Truth Table for the Disjunction of Two Propositions.

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

DEFINITION 4 Let p and q be propositions. The *exclusive or* of p and q, denoted by $p \oplus q$, is the proposition that is true when exactly one of p and q is true and is false otherwise.

TABLE 4 The Truth Table for the Exclusive Or of Two Propositions.

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

"Students who have taken calculus or computer science, but not both, can enroll in this class."