L11 10 Feb 2021

10 February 2021 12:06

(a) ~~$y'' + 4y = 0$~~ (b) ~~$y'' - 8y = 0$~~

Q17. Let $f_1 = x, f_2 = x^4, f_3 = 1 + x, f_4 = 1$, then Wronskian $W(f_1, f_2, f_3, f_4) =$

(a) $6x$ (b) 2 (c) $3 - 4x$ (d) 0

$$W = \begin{vmatrix} x & x^4 & 1+x & 1 \\ 1 & 4x^3 & 1 & 0 \\ 0 & 12x^2 & 0 & 0 \\ 0 & 24x & 0 & 0 \end{vmatrix}_{4\times4} = 1 \begin{vmatrix} 1 & 4x^3 & 1 \\ 0 & 12x^2 & 0 \\ 0 & 24x & 0 \end{vmatrix} = 0$$

L.D

Show that the functions: $(1, \sin x, \cos x)$ are linearly independent.

$$W = \begin{vmatrix} 1 & \sin x & \cos x \\ 0 & \cos x & -\sin x \\ 0 & -\sin x & -\cos x \end{vmatrix} = 1\left(-\cos^2 x - \sin^2 x\right) = -1\left(\sin^2 x + \cos^2 x\right)$$
$$= -1 \neq 0$$

L.I

Abel's Formula to find Wronskian:

Let us consider a 2nd order homogeneous LDE:

$$a_0 y'' + a_1 y' + a_2 y = 0 \qquad (1)$$

Where $a_0 \neq 0$, a_1, a_2 are continuous on an interval I and y_1, y_2 be its linearly independent solutions, then Wronskian is given as:

Wronskian, $\boldsymbol{W = ce^{-\int\left(\frac{a_1}{a_0}\right)dx}}$ where c is a constant.

$$W = c \cdot e^{-\int\left(\frac{a_1}{a_0}\right)dx}$$

find Wronskian for: $y'' - 4y' + 4y = 0$

y_1, y_2 are L.I

$a_0 = 1, \quad a_1 = -4 \quad a_2 = 4.$

$$W = c \cdot e^{-\int \frac{-4}{1} dx} = c \cdot e^{\int 4dx} = c \cdot e^{4x}$$

Using Abel's formula, find Wronskian for: $y'' + y' + 4y = 0$

(A) $W = ce^{4x}$

(B) $W = ce^{-x}$

(C) $W = ce^{x}$

(D) $W = ce^{2x}$

$W = c\,e^{-\int \frac{1}{1} dx} = c\,e^{-x}$

$W = c\,e^{-\int \left(\frac{a_1}{a_0}\right) dx}$

$$y'' + a^2 y = 0, a \neq 0.$$

$a_0 = 1,\ a_1 = 0\ \ a_2 = a^2$

$W = c\,e^{-\int \frac{0}{1} dx} = c \cdot e^{0} = c$

Fundamental Solution or Basis

$\downarrow$

Solution and L.I

Show that $(1, x^2)$ form a set of fundamental solutions (basis) of homogeneous equation: $x^2 y'' - xy' = 0$.

$x^2 y'' - xy' = 0$

$y = 1$
$y = x^2$

$\left.\begin{array}{l} y = 1 \\ y' = 0 \\ y'' = 0 \end{array}\right\} \rightarrow$ $x^2(0) - x(0) = 0$, $0 = 0$ satisfied $\Rightarrow$ $y = 1$ is the sol.

$\left.\begin{array}{l} y = x^2 \\ y' = 2x \\ y'' = 2 \end{array}\right\} \rightarrow$ $\left.\begin{array}{l} x^2(2) - x(2x) = 0 \\ \cancel{2x^2} - \cancel{2x^2} = 0 \\ 0 = 0 \end{array}\right\}$ satisfied $\quad y = x^2$ is also the sol.

$W = \begin{vmatrix} 1 & x^2 \\ 0 & 2x \end{vmatrix} = 2x - 0 = 2x \neq 0 \quad$ if $x \neq 0$

L.I

$\{1, x^2\}$ Fundamental sol or Basis

CA-1 $\quad$ 19^{th} Feb 2021 $\quad$ Syllabus Unit-I

CA-1

19th Feb 2021
↓
Saturday

Syllabus Unit-I

MCQ

30 MCQ

↳ No Negative Marking

Oas

A → 19th

B → 20th