Unit 2 Problems

Tutorial 5

Example 1

An ac voltage is mathematically expressed as $v = 141.42\sin(157.08t + \Pi/2)$ volts. Find its (a) effective value (b) frequency (c) periodic time.

An ac voltage is mathematically expressed as

$$v = 141.42 \sin(157.08t + \pi/2) \text{ Volts}$$

$$v = V_m \sin(\omega t + \theta)$$

Effective value $= \frac{V_m}{\sqrt{2}} = \frac{141.42}{\sqrt{2}} = 100V$

$$\omega = 157.08$$

$$2\pi f = 157.08$$

$$f = \frac{157.08}{2\pi} = \underline{25 HZ}$$

$$T = \frac{1}{f} = \frac{1}{25} = \underline{0.04 \text{ sec}}$$

Example 2 Polar Notation Problem

- An AC current denoted by a phasor in complex plane as I= 4+j3 Amp. Is flowing through a resistor of 10 ohm . Determine the power consumed by the resistor.

$$r = \sqrt{a^2 + b^2} \qquad \text{and} \qquad \theta = \tan^{-1}\frac{b}{a}$$

Solution

Solution Let us first express the current **I** in the polar form,

$$\mathbf{I} = I_r + jI_i = 4 + j3 = \sqrt{4^2 + 3^2}\angle\tan^{-1}(3/4) = 5\angle 36.87°\ \text{A}$$

Thus, we find that the magnitude (the rms value) of the given current is 5 A. Therefore, the power consumed is

$$P = I^2R = 5^2 \times 10 = \mathbf{250\ W}$$

Problem on Rectangular and Polar calculations

Two phasors **A** and **B** are given as $\mathbf{A} = 3 + j1$, and $\mathbf{B} = 4 + j3$. Calculate the values of (*a*) $\mathbf{A} + \mathbf{B}$; (*b*) $\mathbf{A} - \mathbf{B}$; (*c*) $\mathbf{AB}$; (*d*) $\mathbf{A/B}$. Express the results in both polar and rectangular coordinates.

[**Ans.** (*a*) $7 + j4 = 8.06\angle 29.7°$;
(*b*) $-1 - j2 = 2.24\angle -116.57°$;
(*c*) $15.8\angle 55.3° = 8.99 + j12.99$;
(*d*) $0.632\angle -18.44° = 0.6 - j0.02$]

Addition subtraction and Multiplication

$$A = 3 + j1 \qquad B = 4 + j3$$

a) $A + B = 3 + j1 + 4 + j3 = 7 + j4 = 8.06\angle 29.7^\circ$

b) $A - B = 3 + j1 - 4 - j3 = -1 - j2 = 2.24\angle -116.57^\circ$

c) $AB = (3 + j1)(4 + j3) = 15.8\angle 55.3^\circ = 8.99 + j12.99$

d) $\dfrac{A}{B} = \dfrac{3 + j1}{4 + j3} = 0.632\angle -18.44^\circ = 0.6 - j0.02$

Addition subtraction and Multiplication

Addition, Subtraction and Multiplication For these operations, just use ordinary algebra plus two more rules: (1) keep real and imaginary parts separate, and (2) treat j^2 as -1. For example, whenever we add complex numbers, we add the real parts and the imaginary parts separately:

$$\mathbf{z}_1 + \mathbf{z}_2 = (3 + j4) + (-7 - j3) = (3 - 7) + j(4 - 3) = -4 + j1$$

and similarly for subtraction. Thus, complex numbers are added and subtracted like vectors in a plane. This is one of the few properties common between complex numbers and vectors.

Similarly, multiplication of $\mathbf{z}_1$ and $\mathbf{z}_2$ is

$$\begin{aligned}\mathbf{z}_1\mathbf{z}_2 &= (3 + j4)(-7 - j3) = 3(-7) + j4(-j3) + 3(-j3) + j4(-7) \\ &= -21 - j^2 12 - j9 - j28 = -21 + 12 - j9 - j28 = (-21 + 12) - j(9 + 28) \\ &= -9 - j37\end{aligned}$$

Division

Division and Conjugation Division requires a trick to get the results in standard form:

$$\frac{\mathbf{z}_1}{\mathbf{z}_2} = \frac{3+j4}{-7-j3} = \frac{3+j4}{-7-j3} \times \frac{-7+j3}{-7+j3}$$

$$= \frac{(3)(-7)+(j4)(j3)+(3)(j3)+(j4)(-7)}{(-7)^2-(j3)^2} = \frac{-21-12+j9-j28}{49+9}$$

$$= \frac{-33-j19}{58} = -\frac{33}{58} - j\frac{19}{58}$$

Problem on Representation of sin wave equations

Obtain the sum of the three voltages,

$$v_1 = 147.3\cos(\omega t + 98.1^\circ)\text{ V},\quad v_2 = 294.6\cos(\omega t - 45^\circ)\text{ V}\quad \text{and}\quad v_3 = 88.4\sin(\omega t + 135^\circ)\text{ V}$$

Solution We plot the above phasors in complex plane, in terms of their peak values. First, we write the voltages in terms of sine functions. Since, $\sin(90^\circ + \theta) = \cos\theta$, we can write

$$v_1 = 147.3\sin(90^\circ + \omega t + 98.1^\circ)\text{ V} = 147.3\sin(\omega t + 188.1^\circ)\text{ V}$$

$$v_2 = 294.6\sin(90^\circ + \omega t - 45^\circ)\text{ V} = 294.6\sin(\omega t + 45^\circ)\text{ V}$$

and

$$v_3 = 88.4\sin(\omega t + 135^\circ)\text{ V}$$

Problem on XL and XC calculation

(a) What reactance will be offered (i) by an inductor of 0.2 H, (ii) by a capacitance of 10 µF, to an ac voltage source of 10V, 100 Hz? (b) What, if the frequency of the source is changed to 140 Hz?

Solution

(*a*) (*i*) $X_L = 2\pi f L = 2\pi \times 100 \times 0.2 = \mathbf{125.66\ \Omega}$

(*ii*) $X_C = \dfrac{1}{2\pi f C} = \dfrac{1}{2\pi \times 100 \times 10 \times 10^{-6}} = \mathbf{159.15\ \Omega}$

(*b*) (*i*) $X_L = 2\pi f L = 2\pi \times 140 \times 0.2 = \mathbf{175.9\ \Omega}$

(*ii*) $X_C = \dfrac{1}{2\pi f C} = \dfrac{1}{2\pi \times 140 \times 10 \times 10^{-6}} = \mathbf{113.7\ \Omega}$

Problem on Resonance Frequency

A coil having resistance & inductance of 5Ω & $32mH$, respectively is connected in series with a 796-pF capacitor. Determine resonance frequency of the circuit

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

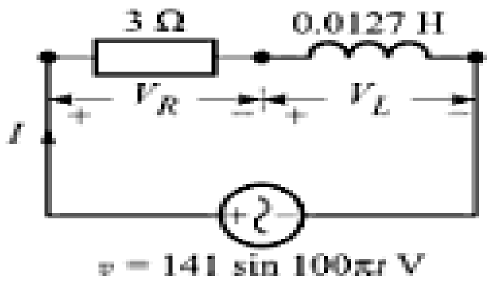
$$= \frac{1}{2\pi\sqrt{32\times10^{-3}\times796\times10^{-12}}}$$

$$= 31.53 \text{ KHZ}$$

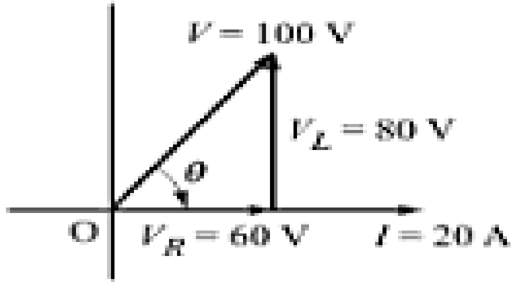
Problem on Series RL circuit

For the series *RL* circuit shown in Fig. 10.2*a*,

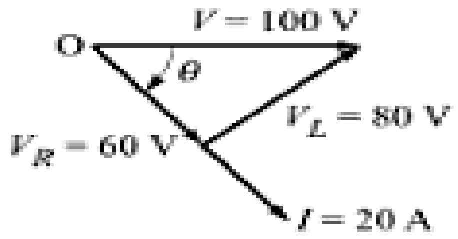
(*a*) Calculate the rms value of the steady state current and the relative phase angle.
(*b*) Write the expression for the instantaneous current.
(*c*) Find the average power dissipated in the circuit.
(*d*) Determine the power factor.
(*e*) Draw the phasor diagram.



(a) The circuit.



(b) Phasor diagram.



(c) Phasor diagram redrawn.

Fig. 10.2 *A series RL circuit.*

$$\mathbf{V} = V\angle 0^\circ = \frac{V_m}{\sqrt{2}}\angle 0^\circ = \frac{141}{\sqrt{2}}\angle 0^\circ = 100\angle 0^\circ = 100 + j0 \text{ volts}$$

The impedance, $\mathbf{Z} = R + j\omega L = 3 + j100\pi \times 0.0127 = 3 + j4 = 5\angle 53.1^\circ$ ohms

$\therefore$ Current, $\mathbf{I} = \dfrac{\mathbf{V}}{\mathbf{Z}} = \dfrac{100\angle 0^\circ}{5\angle 53.1^\circ} = \mathbf{20\angle{-53.1^\circ}\ A}$

Thus, the rms value of the steady state current is 20 A, and the phase angle is 53.1° lagging.

(*b*) The expression for the instantaneous current can be written as

$$i = 20\sqrt{2}\sin(100\pi t - 53.1^\circ) = \mathbf{28.28\sin(100\pi t - 53.1^\circ)\ A}$$

(*c*) Average power, $P = VI\cos\theta = 100 \times 20 \times \cos 53.1^\circ = \mathbf{1200\ W}$

Or, $$P = I^2R = (20)^2\,3 = \mathbf{1200\ W}$$

(*d*) $pf = \cos\theta = \cos 53.1^\circ =$ **0.6 lagging**. Alternatively,

$$pf = \frac{\text{Average power}}{\text{Apparent power}} = \frac{P}{VI} = \frac{1200}{100 \times 20} = \mathbf{0.6\ lagging}$$

(*e*) Taking the current as reference, the phasor diagram is drawn in Fig. 10.2*b*, where

$$I = 20\text{ A};\quad V_R = IR = 20 \times 3 = 60\text{ V};\quad V_L = IX_L = 20 \times 4 = 80\text{ V}\quad \text{and}\quad V = 100\text{ V}$$

The same phasor diagram is redrawn in Fig. 10.2*c*, by rotating it clockwise by an angle 53.1°, so that the applied voltage becomes the reference phasor.

Problem on Power and RC circuit

A current of 0.9 A flows through a series combination of a resistor of 120 Ω and a capacitor of reactance 250 Ω. Find the impedance, power factor, supply voltage, voltage across resistor, voltage across capacitor, apparent power, active power and reactive power.

Solution Taking current as the reference phasor, $\mathbf{I} = 0.9\angle 0°$ A.

Impedance,	$\mathbf{Z} = 120 - j250 = \mathbf{277.3\angle -64.4°\ \Omega}$
Power factor,	$pf = \cos\theta = \cos(-64.4°) = \mathbf{0.432\ leading}$
Supply voltage,	$\mathbf{V} = \mathbf{IZ} = (0.9\angle 0°)(277.3\angle -64.4°) = \mathbf{249.6\angle -64.4°\ V}$
Voltage across resistor,	$V_R = IR = (0.9\angle 0°) \times 120 = \mathbf{108\angle 0°\ V}$
Voltage across capacitor,	$V_C = IX_C = (0.9\angle 0°)(250\angle -90°) = \mathbf{225\angle -90°\ V}$
Apparent power,	$P_{app} = VI = 249.6 \times 0.9 = \mathbf{224.6\ VA}$
Actual power,	$P_a = VI\cos\theta = 249.6 \times 0.9 \times \cos 64.4° = \mathbf{97.06\ W}$
Reactive power,	$P_r = VI\sin\theta = 249.6 \times 0.9 \times \sin 64.4° = \mathbf{202.58\ VAR}$