

9) Let $A = \{1, 2, 3, 4, 6\}$. Let R be the relation on A define by $\{(a, b) : a \in A, b \in A, a \text{ divides } b\}$. Range of R is

- a. $\{1, 2, 3, 4, 6\}$ b. $\{1, 3, 4, 6\}$ c. $\{1, 2, 3, 6\}$ d. \emptyset

$\{(1, 2), (1, 3), (1, 4), (1, 6), (2, 4), (2, 6), (3, 6)\}$
Range = $\{1, 2, 3, 4, 6\}$

10) Let $A = \{x, y\}$ and $B = \{1, 2\}$. Find the number of relations from A into B .

- a. 16 b. 32 c. 64 d. 128

$2^m \cdot 2^n = 2^2 \cdot 2^2 = 2^4 = 16$

11) Let $A = \{1, 2, 3, 4\}$. Let R be the relation on A define by $\{(a, b) : a \in A, b \in A, a < b\}$. Which of the following is false

- a. R is not reflexive b. R is transitive c. R is transitive but not reflexive d. R is equivalence relation

$a < a$ True

12) The 'Subset' relation on set of sets is

- a. A partial ordering b. An equivalence relation c. Transitive and symmetric only d. Transitive and anti-symmetric only

$(P(A), \subseteq)$

$a < b, b < c$
 $a < c$

$a < a$
 $1 < 1$

$1 < 2$

$\cancel{2 < 1}$

$A \subseteq A$

$A \subseteq B, B \subseteq A$

$A \subseteq B, B \subseteq C$

Reflexive

$A = B$

Anti-sym

$A \subseteq C$

Transitive

Q1. Let $R = \{(a, b) | a \text{ and } b \text{ have common mother tongue}\}$ be a relation defined on the S where S is the set of all students of a campus. Then

- (a) R is not reflexive (b) R is not symmetric (c) R is not transitive (d) R is not antisymmetric

$a \& a$

$(a, a) \in R$

$a R b$ and $b R a$

(a, b)

Symmetric

$a = b$

Q5. Let $R = \{(a, b) \mid ab \geq 0\}$ be a relation which is defined on the set A where A is the set of Integers. Then

- (a) R is reflexive (b) R is symmetric (c) R is transitive (d) All of above

$$(a, a) \in R \Rightarrow a^2 \geq 0 \quad \forall a \in A$$

$$(a, b) \in R \quad ab \geq 0$$

$$ba \geq 0 \quad \text{since } a, b \in A$$

$$(b, a) \in R$$

$$(a, b) \in R \text{ and } (b, c) \in R$$

$$\uparrow \quad \uparrow$$

$$ab \geq 0, \quad bc \geq 0$$

Q12. Let R be a relation defined on the set A . Then two elements x and y of a set are said to be incomparable if

- (a) xRy or yRx (b) xRy and yRx (c) neither xRy nor yRx (d) None of these

if ~~not~~ comparable elements.

$a, b \in A$ aRb ~~or~~ bRa then we say they are comparable.

e.g. 3 and 5 under divisibility

$$\begin{array}{cc} 3 \nmid 5 & 5 \nmid 3 \\ \underline{3 \nmid 5} & \underline{5 \nmid 3} \end{array}$$

$$\underline{3} R \underline{6}$$

$$\underline{3} R \underline{6} \text{ but } \underline{6} \nmid \underline{3}$$

Q19. If $A = \{a, b, c\}$ and $B = \{e, f\}$ then total number of relations from A to B = ?

(a) 2^5

(b) 2^6

(c) 5^5

(d) 2^3

$m \cdot n = 3 \cdot 2$
 $2^{m \cdot n} = 2^{3 \cdot 2}$

Q20. The no. of non empty relations from $A = \{1, 2, 3\}$ to $B = \{a, b\}$ is

(a) 2^6

(b) 2^3

(c) $2^6 - 1$

(d) $2^5 - 1$

$= 2^6 = 64$

$(2^6 - 1)$

$\frac{m \cdot n}{2} - 1$ Total no of non-empty Relations.
 $\phi \rightarrow$ empty Relation is also a Relation.

$2^6 - 1$ no. of non-empty Relations.

$\rightarrow \times \leftarrow$

Q25. The minimal elements of the poset $(\{2, 4, 5, 10, 12, 20, 25\}, |)$ are

(a) 2 and 5

(b) 2, 4, 5

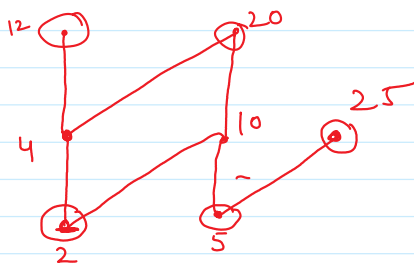
(c) 2

(d) None of these

Q26. Let $A = \{1, 2, 3, 4\}$. The relation R whose matrix is $\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$ is

$A = \{1, 2, 3, 4\}$ ✓

$A = \{1, 2, 3\}$



12, 20, 25 are Maximal elements
and 2, 5

$R = \{ (1,1), (1,3), (2,2), (2,4), (3,1), (4,3) \}$

a

	1	2	3	4
1	1	0	1	0
2	0	1	0	1
3	1	0	1	0
4	0	0	1	0

(d)None of these

(a) only symmetric

(b) only reflexive

(c) only transitive

✓ (d) an equivalence relation

$$R = \{ \underline{(1,2)}, \underline{(2,1)} \}$$

(a) R is reflexive

(b) R^T is symmetric

(c) D is not a theorem.

- $\text{col}(B)$ is transitive

$(a, a) \in R \quad \forall a \in A$ Reflexive)

~~(3, 1)~~ ~~(2, 4)~~
↑ ↑
C₀

$(a, b) \in R \implies (b, a) \in R$ Symmetric

$$(a, b) \in R \text{ and } (b, c) \in R \Rightarrow (a, c) \in R \text{ Transitive}$$

$$R = \{ \underline{(1,2)}, \underline{(2,1)}, \underline{(1,3)} \}$$

$$R = \{ (1,2), (2,$$

$$R = \{ (\underline{1}, \underline{2}), (\underline{3}, \underline{4}) \}$$

$(a,b), (b,c)$
 (a,c)

(a) only reflexive

(b)only symmetric

(c)only transitive

(d) an equivalence relation

$$R = \{(a, b) : \text{If } a \text{ and } b \text{ born on same day}\}$$

$$(\underline{a}, \underline{a})$$

(a, b)

$$\underbrace{(a, b) \in R} \quad \underbrace{(b, c) \in R}$$

Q35. Let $R = \{(a, b) : \text{If } a \text{ and } b \text{ belong to the same section K1500}\}$ defined on the A , where A is the set of persons, then R is
 (a) only reflexive (b) only symmetric (c) only transitive (d) an equivalence relation

$$R = \{ (a, b) : \text{If } a \text{ and } b \text{ belong to the same section } \underline{k \geq 0 \in \mathbb{N}} \}$$

(a, a) , (a, b) $(a, b) \in R$, $(b, c) \in R$

$$n(A \times A) = n(A) \times n(A) \\ = 3 \times 3 = 9 \checkmark$$

$$2^7 = 2^3 = 2^9$$

Q: if $A = \{a, b, c\}$ then $n(A \times A) = ?$

- (a) 4 (b) 16 (c) 8 (d) 9

Q: if $A = \{a, b, c\}$ then total number of relations from A to $A = ?$

$$\frac{mn}{2} = \frac{3.3}{2} = 1.65$$

- (a) 2^{16} ~~(b) 2^9~~ (c) 4^4 (d) 4^4

Q: Consider the given relation R , defined on $A = \{1, 2, 3, 4\}$

$R = \{(1,1), (1,2), (2,1), (2,2), (3,4), (4,1), (4,4)\}$, then which one of the following is true

- ~~(a)~~ R is reflexive ~~(b)~~ R is symmetric (c) R is not transitive (d) R is antisymmetric

Q: Which element of the *Poset* ($\{2,3,4,6,9,12,18,36,48,60,72\}, | \text{) is greatest}$

- (a) 48, 60, and 72 (b) 72 and 60 (c) 60 and 72 (d) ~~does not exist~~

Q: Which element/s of the Poset $(\{2,3,4,6,9,12,18,36,48,60,72\}, |)$ is/are Maximal

- (a) 72, 60 and 48 (b) 72, 60 (c) 60, 48 (d) 72, 48

Q: Which element/s of the Poset $(\{2,3,4,6,9,12,18,36,48,60,72\}, |)$ is/are minimal

- (a) 2 ✓ (b) 3, 2 (c) 3 (d) Do not exist

Q: Which elements of the Poset $(\{2,3,4,6,9,12,18,36,48,60,72\}, |)$ are upper bound of $\{36, 60\}$

- (a) 72, 60 and 48 (b) 3, 2 (c) 3 (d) Do not exist

Q: Which element/s of the Poset $(\{2, 3, 4, 6, 9, 12, 18, 36, 48, 60, 72\}, |)$ is/are lower bound of $\{6, 3\}$

- (a) 2, 3 (b) 6 (c) 3 (d) Do not exist

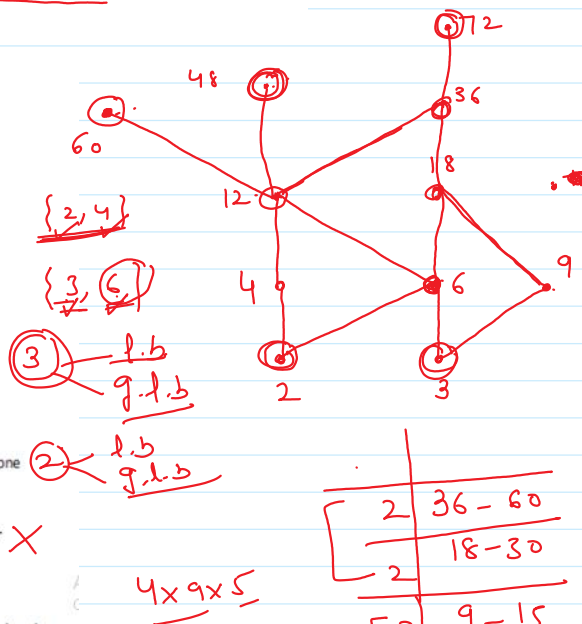
Q: The Hasse diagram of the Poset $(\{2, 3, 4, 6, 9, 12, 18, 36, 48, 60, 72\}, |)$ is

- (a) A Lattice (b) A bounded lattice (c) ~~Not a Lattice~~ (d) None of these

Q: Let R be a relation on the set of all integers defined by $R = \{(a, b) | a < b\}$. Then which one of following is true

- (a) R is reflexive ☒ (b) R is symmetric ☒ (c) R is transitive ☒ (d) R is partial order ☒

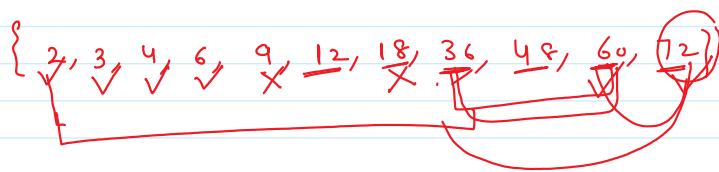
Q: Let R be a relation on the set of all students of K1420 defined by $R = \{(a, b) | a \text{ and } b \text{ are from same state}\}$. Then which one of following is true



(a) R is reflexive (b) R is symmetric ~~(c) R is transitive~~ (d) R is partial order ~~X~~

Q: Let R be a relation on the set of all students of K1420 defined by $R = \{(a, b) | a \text{ and } b \text{ are from same state}\}$ Then which one of following is true

(a) R is not reflexive (b) R is not symmetric (c) R is equivalence (d) R is partial order



$$36 = 6 \times 6 = 2 \times 3 \times 2 \times 3 = \left(\frac{2}{2}\right) \times 3^2$$

$$60 = 2 \times 2 \times 3 \times 5 = \left(\frac{2}{2}\right) \times 3 \times 5$$

$$\text{H.C.F} = \text{g.c.d} = \frac{2}{2} \times 3 = 12 \quad \text{g.l.b}$$

$$\begin{array}{r} 4 \times 9 \times 5 \\ \hline 36 \\ \hline 36 \times 3 \\ \hline 108 \end{array} \quad \begin{array}{r} 18-30 \\ \hline 2 \\ \hline 9-15 \\ \hline 3 \\ \hline 3-5 \\ \hline 1-5 \\ \hline 1 \end{array}$$

$$\{36, 60\}$$

(i) Reflexive
(ii) Anti-symmetric
(iii)

$$\{36, 60\}$$

$$2, 3, 4, 6, 12$$

lower bound.

lattice.

Let A be a non-empty set

(A, \leq) is called lattice.

if Supremum l.u.b and infimum g.l.b both exist for every pair of elements.

In case of divisibility.

$$\rightarrow \text{g.l.b} = \text{H.C.F}(a, b)$$

$$\rightarrow \text{l.u.b} = \text{l.c.m}(a, b)$$

if g.l.b and l.u.b both exist for every pair of elements

$\{2, 3, 5\}, 1$ IS it a lattice.

$$\text{g.l.b}(2, 3) = \text{H.C.F}(2, 3) = 1 \notin \{2, 3, 5\}$$

So it is not a lattice.

$$\{ \{1, 2, 3, 4\}, 1 \}$$

$$\{ \{1, 2, 4\}, 1 \}$$

$$\text{H.C.F}(1, 2) = 1, \quad \text{l.c.m}(1, 2) = 2$$

$$\text{H.C.F}(2, 4) = 2, \quad \text{l.c.m}(2, 4) = 4$$

$$\text{H.C.F}(1, 4) = 1, \quad \text{l.c.m}(1, 4) = 4$$

$$\left\{ \{ \underline{1, 2, 3, 4} \}, (1) \right\}$$

$$\text{l.c.m}(3,4) = 12 \notin \{1, 2, 3, 4\}$$

It is not a lattice.

Reflexive, AntiSymmetric, transitive

Partial order relation.

— X —

(P(S), \subseteq) Then it is a lattice

(i) g.l.b (A, B) = (A \cap B)

$$A = \{1, 2\}$$

(ii) l.u.b (A, B) = (A \cup B)

$$\underline{P(A)} = \{ \underline{\emptyset}, \underline{\{1\}}, \underline{\{2\}}, \underline{\{1, 2\}} \}$$

— X —

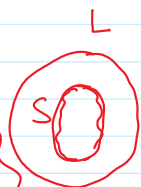
Sub-lattice

Let L be a Lattice.

and S be a subset of L

if S is itself a Lattice

then it is called sub-lattice ✓



$$\left. \begin{array}{l} \emptyset \cap \{1\} = \emptyset \checkmark \\ \{1\} \cap \{2\} = \emptyset \checkmark \\ \{2\} \cap \emptyset = \emptyset \checkmark \\ \{1\} \cap \{1, 2\} = \{1\} \checkmark \\ \{2\} \cap \{1, 2\} = \{2\} \checkmark \end{array} \right\}$$

$$\emptyset \cup \{1\} = \{1\} \checkmark$$

$$\{1\} \cup \{2\} = \{1, 2\} \checkmark$$

$$\{2\} \cup \emptyset = \{2\} \checkmark$$

$$\{1\} \cup \{1, 2\} = \{1, 2\} \checkmark$$

$$\{2\} \cup \{1, 2\} = \{1, 2\} \checkmark$$

$$L = \{ \{1, 2, 4\}, 1 \}$$

$$\underline{S} = \{ \underline{\{1, 2\}}, 1 \}$$

$$\text{g.l.b } \{ \underline{1, 2} \} = \underline{1}$$

$$\text{l.u.b } \{ \underline{1, 2} \} = \underline{2}$$

Q1) What is

$$(S) \subseteq L$$

\Rightarrow S is a sub-lattice

$$A = \{1, 2, 3\}$$

$$P(A) = \{ \emptyset, \underline{\{1\}}, \underline{\{2\}}, \underline{\{1, 2\}}, \underline{\{1, 3\}}, \underline{\{2, 3\}}, \underline{\{1, 2, 3\}} \}$$

$$\{1, 2\} \cap \{1, 3\} = \underline{\{1\}}$$

$$\{1, 2\} \cup \{1, 3\} = \underline{\{1, 2, 3\}}$$

(P(A), \subseteq) It is a lattice

$$B = \{1, 2\}$$

$$(B) \subseteq (A)$$

$(P(x), \mathbb{C})$ It is a lattice

Tower of Hanoi Problem.

What is the recurrence relation of
Tower of Hanoi problem.

$$\begin{bmatrix} a_3 & \textcircled{3} & 2^3 - 1 = 7 \\ a_4 & \textcircled{4} & 2^4 - 1 = 15 \\ a_5 & 5 & 2^5 - 1 = 31 \end{bmatrix}$$

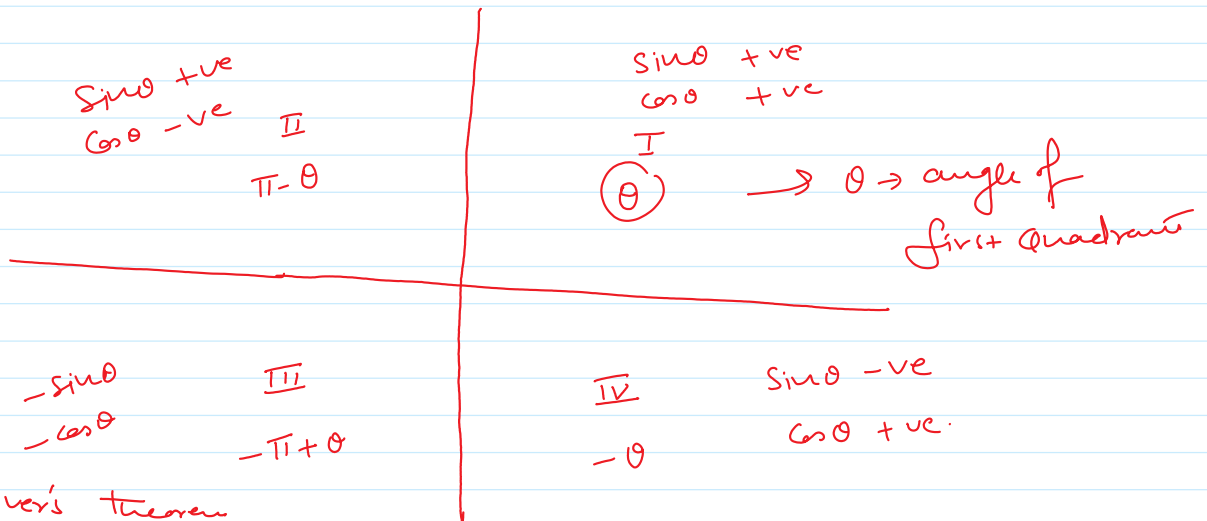
$$a_4 - 2a_3 = 1$$

$$a_5 - 2a_4 = 1$$

$$a_n - 2a_{n-1} = 1$$

$$a_n = 2a_{n-1} + 1$$

$$a_{\textcircled{1}} = 1$$



De Moivre's theorem

$$(\cos \theta + i \sin \theta)^{\textcircled{n}} = (\cos n\theta + i \sin n\theta)$$

$$a_{\textcircled{n}} + a_{n-1} + a_{\textcircled{n-2}} = 0$$

$$x^2 + x + 1 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-1 \pm \sqrt{1 - 4(1)(1)}}{2} = \frac{-1 \pm \sqrt{-3}}{2}$$

$$= \frac{-1 \pm i\sqrt{3}}{2}$$

$$a_n = C_1 \left(\frac{-1 + i\sqrt{3}}{2} \right)^{\textcircled{n}} + C_2 \left(\frac{-1 - i\sqrt{3}}{2} \right)^{\textcircled{n}}$$



$$= -\frac{1}{2} \pm i \frac{\sqrt{3}}{2}$$

$$a_n = c_1 \left(-\frac{1}{2} + i \frac{\sqrt{3}}{2} \right)^n + c_2 \left(-\frac{1}{2} - i \frac{\sqrt{3}}{2} \right)^n$$



$$-\frac{1}{2} + i \frac{\sqrt{3}}{2} = r(\cos \theta + i \sin \theta)$$

$$\cos \theta = -\frac{1}{2}, \quad \sin \theta = \frac{\sqrt{3}}{2}$$

$$r \cos \theta = -\frac{1}{2} \rightarrow (1)$$

$$r \sin \theta = \frac{\sqrt{3}}{2} \rightarrow (2)$$

Sq and adding (1) & (2)

$$r^2 = \frac{1}{4} + \frac{3}{4}$$

$$r^2 = 1$$

$$r = 1$$

$$\theta = \pi - \left(\frac{\pi}{3} \right)$$

$$= \frac{3\pi - \pi}{3} = \frac{2\pi}{3} \checkmark$$

$$-\frac{1}{2} + i \frac{\sqrt{3}}{2} = \left(\cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3} \right)$$

$$\left(-\frac{1}{2} + i \frac{\sqrt{3}}{2} \right)^n = \left(\cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3} \right)^n$$

$$\left(-\frac{1}{2} + i \frac{\sqrt{3}}{2} \right)^n = \cos \frac{2n\pi}{3} + i \sin \frac{2n\pi}{3}$$

$$\left(-\frac{1}{2} - i \frac{\sqrt{3}}{2} \right)^n = \cos \frac{2n\pi}{3} - i \sin \frac{2n\pi}{3}$$

$$a_n = c_1 \left(-\frac{1}{2} + i \frac{\sqrt{3}}{2} \right)^n + c_2 \left(-\frac{1}{2} - i \frac{\sqrt{3}}{2} \right)^n$$

$$= c_1 \left(\cos \frac{2n\pi}{3} + i \sin \frac{2n\pi}{3} \right) + c_2 \left(\cos \frac{2n\pi}{3} - i \sin \frac{2n\pi}{3} \right)$$

$$= (c_1 + c_2) \cos \frac{2n\pi}{3} + (i c_1 - i c_2) \sin \frac{2n\pi}{3}$$

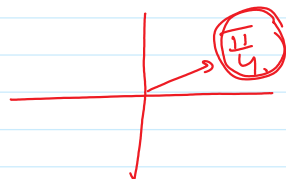
$$= \left(A \cos \frac{2n\pi}{3} + B \sin \frac{2n\pi}{3} \right)$$

$$r^n (A \cos n\theta + B \sin n\theta)$$

$$\rightarrow c_1 (1+i)^n + c_2 (1-i)^n$$

$$\rightarrow r^n [A \cos n\theta + B \sin n\theta]$$

$$(1-i)^n [A \cos \frac{n\pi}{4} + B \sin \frac{n\pi}{4}]$$



$$1+i = r(\cos \theta + i \sin \theta)$$

$$1+i = r(\cos \theta + i \sin \theta)$$

$$r \cos \theta = 1 \rightarrow (1)$$

$$r \cos \theta = 1 \rightarrow (1)$$

$$r \sin \theta = 1 \rightarrow (2)$$

$$r \sin \theta = 1 \rightarrow (2)$$

$$r^2 = (1)^2 + (1)^2$$

$$r^2 = (1)^2 + (1)^2$$

$$r^2 = 2$$

$$r^2 = 1+1$$

$$r = \sqrt{2}$$

$$r = \sqrt{2}$$

$$\cos \theta = \frac{1}{\sqrt{2}}, \quad \sin \theta = \frac{1}{\sqrt{2}}$$

$$\cos \theta = \frac{1}{\sqrt{2}}, \quad \sin \theta = \frac{1}{\sqrt{2}}$$

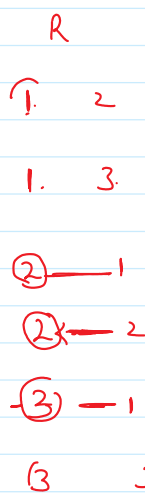
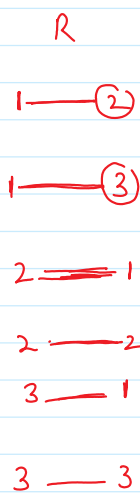
$$M_R = \begin{matrix} & \begin{matrix} 1 & 2 & 3 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} & \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \end{matrix}$$

M_{R^2}

R^2

$$R = \{ (1,2), (1,3), (2,1), (2,2), (3,1), (3,3) \}$$

$$R \circ R = \{ \}$$



$$R \circ R = \left\{ \begin{matrix} \underline{(1,1)}, \underline{(1,2)}, \underline{(1,3)}, \\ \underline{(2,2)}, \underline{(2,3)}, \underline{(2,1)}, \\ \underline{(3,2)}, \underline{(3,3)}, \underline{(3,1)} \end{matrix} \right\}$$

$$M_{R^2} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Φ Δe^{6x}

$$\begin{aligned} \Delta f(x) &= f(x+h) - f(x) & h=1 \\ &= e^{6(x+h)} - e^{6x} \\ &= e^{6x} (e^{6h} - 1) = e^{6x} (e^6 - 1) \end{aligned}$$

(a) $e^{6x} (e^{6h} - 1)$

(b) $e^{6x} (e^{xh} - 1)$

(c) $e^{6x} (e^6 - 1)$

(d) $e^{6x} (e^x - 1)$

Φ

the value of $\Delta(\log x)$

(a) ✓ $\log(1 + \frac{h}{x})$

(b) $\log(1 + \frac{h}{x})$

(c) $\log(x + \frac{h}{x})$

(d) $\log(x + \frac{h}{x})$

$$\begin{aligned}\Delta f(x) &= f(x+h) - f(x) \\ &= \log(x+h) - \log x \\ &= \log\left(\frac{x+h}{x}\right) \\ &= \log\left(1 + \frac{h}{x}\right)\end{aligned}$$

$x \rightarrow (x+h)$

$\Phi = \textcircled{5} \cdot \underline{\underline{E(3x-2)}}$

(a) $5(x+5h) - 2$

$3(x+5h) - 2$

(b) ✓ $3(x+5h) - 2$

(c) $5(x+5h) - 2$

(d) $2(x+2h) - 2$

$R = \{ (\underline{1}, \underline{2}), (\underline{2}, \underline{3}), (\underline{3}, \underline{1}) \}$

$R^{-1} = \{ (2,1), (3,2), (1,3) \}$

$\rightarrow R_1 - R_2$

$\rightarrow R_1 \cup R_2$

$\rightarrow R_2 - R_1$

$$\begin{aligned}
 G(a, z) &= \frac{z}{1-7z+10z^2} \\
 &= \frac{z}{\underbrace{1-5z-2z+10z^2}_{(1-5z)(1-2z)}} \\
 &= \frac{z}{(1-5z)(1-2z)} \\
 &= \frac{z}{(1-2z)(1-\frac{5}{2}z)} + \frac{\frac{1}{5}}{(1-\frac{2}{5}z)(1-5z)} \\
 &= -\frac{1}{3} \frac{1}{1-2z} + \frac{1}{3} \frac{1}{1-5z} \\
 &= -\frac{1}{3} (2)^n + \frac{1}{3} (5)^n.
 \end{aligned}$$

$1-2z=0 \Rightarrow z=\frac{1}{2}$
 $1-5z=0 \Rightarrow z=\frac{1}{5}$

$G(a, z) = \frac{1}{1-az} \Rightarrow a_n = (a)^n$

$a_n = 3^n + 5^n \quad \checkmark$

$$\begin{aligned}
 G(a, z) &= \frac{1}{(1-3z)} - \frac{1}{(1-5z)} \\
 &= \frac{1-5z+1-3z}{(1-3z)(1-5z)} = \frac{2-8z}{(1-3z)(1-5z)} \\
 &= \frac{2(1-4z)}{(1-3z)(1-5z)} \quad \text{Ans}
 \end{aligned}$$

$\textcircled{1} \quad \downarrow \quad 31, 127, 499$
 $b_{n-1} \quad b_n$

$\textcircled{1} \quad \times \quad b_{n+1} = 5b_{n-1} + 3$
 $= 5(1) + 3 = 8 \quad \times$

$\textcircled{3} \quad \checkmark \quad b_n = 4b_{n-1} + 3$
 $= 4(1) + 3 = 7 \quad \checkmark$

$\textcircled{1} \quad \downarrow \quad 31, 127, 499$
 $b_{n-1} \quad b_n$

$\textcircled{2} \quad \times \quad b_{n+1} = 5b_{n-1} + 7$
 $= 5(1) + 7 = 12 \quad \times$

$\textcircled{4} \quad \times \quad b_n = b_{n-1} + 1$

- ✓

q $\textcircled{4} + \textcircled{0}n + \textcircled{15}n^2 + \textcircled{10}n^3 + \textcircled{25}n^5 + \textcircled{16}n^6 + \dots$

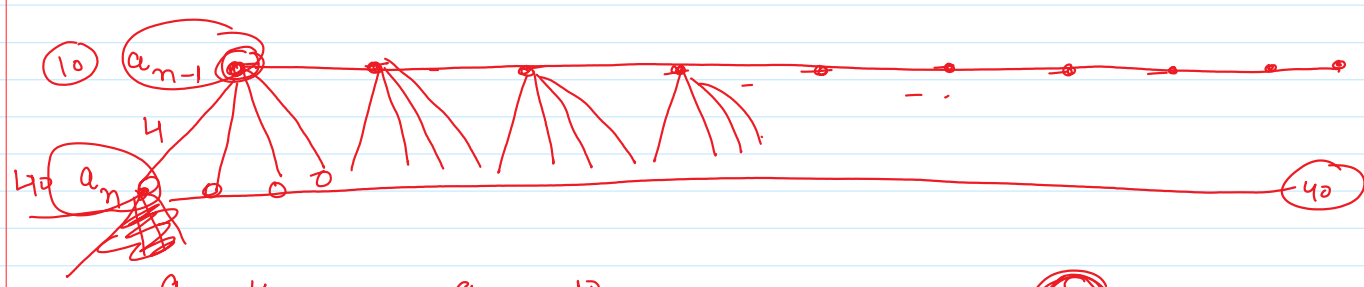
(a) $\times \frac{10}{\cancel{4}}, 4, 0, 16, 25$

$a_0 + \textcircled{a_1}x + a_2x^2 + a_3x^3 + a_4x^4 + \dots$

(b) $\times \frac{0}{\cancel{4}}, 4, 15, 10, 16, 25$

$\textcircled{4}, \textcircled{15}, \textcircled{10}, \textcircled{0}, 25, 16$

(d) $\checkmark \frac{4}{\cancel{10}}, 15, 25$



$a_n = 40, a_{n-1} = 10$

$a_n = 4 \times 10$
 $a_n = 4 a_{n-1}$

$a_k = \textcircled{a^k}$
 $G(a, z) = \sum_{k=0}^{\infty} a_n z^k$
 $= \sum$

(a) $\textcircled{a_{n+2} - 5 a_{n+1} + 6 a_n = 0}, \textcircled{a_0=1, a_1=1}$

(a) $\frac{x-4}{1-x}$

(b) $\frac{1}{1-5x+6x^2}$

(c) $\frac{1}{1-x}$

(a)

$$\frac{x-4}{1-5x+6x^2}$$

(b)

$$\frac{1}{1-5x+6x^2}$$

(c)

$$\frac{1}{1-5x+6x^2}$$

$$\frac{1}{1-3z-2z+6z^2}$$

$$= \frac{1}{(1-3z)-2z(1-3z)}$$

$$= \frac{1}{(1-2z)(1-3z)} = \frac{1}{(1-2z)(1-\frac{3}{2})} + \frac{1}{(1-\frac{2}{3})(1-3z)}$$

$$a_0 = 1, a_1 = 1$$

$$= \frac{1}{(1-2z)(-\frac{1}{2})} + \frac{1}{(\frac{1}{3})(1-3z)}$$

$$\gamma(a, z) = \frac{-2}{1-2z} + \frac{3}{1-3z}$$

$$a_n = -2(2)^n + 3(3)^n$$

$$a_0 = -2 + 3 = 1, a_1 = -4 + 9 = 5$$

$$a_n = 6a_{n-1} + 10^n$$

$$a_n - 6a_{n-1} = 10^n$$

~~(E-6)~~

$$E(a_n - 6a_{n-1}) = E(10^n)$$

$$a_{n+1} - 6a_n = 10^{n+1}$$

$$Ea_{n+1} - 6Ea_n = 10^{n+1}$$

$$(E-6)a_n = 10^{n+1}$$

$$E-6=0$$

$$E=6$$

$$a_n^{(h)} = C_1(6)^n$$

$$a_n^{(p)} = \frac{1}{(E-6)} 10^{n+1}$$

$$= \frac{1}{10-6} 10^{n+1}$$

$$= \frac{10}{4} (10^n)$$

$$= \frac{5}{2} (10)^n$$

$$C_1(6)^n + \frac{5}{2} (10)^n$$

10, 10, 10, 10, - - - -

$$\gamma(a, z) =$$

$$\frac{10}{1-z}$$

$$\gamma(a, z) = \frac{\text{Constant}}{1-z}$$

10, 10, 10, ...

$$C(a, z) =$$

$$\frac{10}{1-z}$$

$$C(a, z) = \frac{\text{Constant}}{1-z}$$

Q

$$y_{k+2} + 8y_{k+1} + 15y_k = \frac{1}{101}$$

(a)

$$\frac{8}{2424}$$

(b)

$$\frac{15}{2424}$$

(c)

$$\frac{24}{2424}$$

(d) $\frac{1}{2424}$

$$P.S =$$

$$\frac{1}{E^2 + 8E + 15}$$

$$\frac{1}{101}$$

$$a^n$$

$$= \frac{1}{(1+8+15)} \cdot \frac{1}{101}$$

$$= \frac{1}{24} \times \frac{1}{101} = \frac{1}{2424}$$

$$a_n = 2a_{n-1} + 1$$

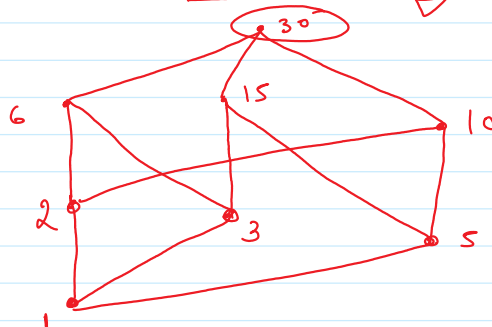
$$a_n - 2a_{n-1} = 1$$

linear - non-homogeneous

$(D_{30}, /)$

$D_{30} \rightarrow$ Divisors of 30.

1, 2, 3, 5, 6, 10, 15, 30



Maximal element = 30

Minimal element = 1

greatest element = 30

least element = 1

$$A = \{1, 2, 3\}$$

$$R_1 = \{(1,1), (2,2), (3,3), (1,2)\}$$

$$R_2 = \{(1,1), (2,2), (3,3), (3,1)\}$$

$$R_1 \cap R_2 = \{(1,1), (2,2), (3,3)\} \quad \text{relative}$$


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