Unit-1

Number Systems

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Analog vs Digital

Analog signal are time varying

Analog devices accepts value across a continuous range

Digital signal is modeled as accepting only one of two discrete value. High ‘1’ or Low ‘0’

Digital devices preferred over Analog

- Reproducibility of result
- Ease of design
- Flexibility
- Programmability
- Processing speed
- Economy
- Steadily advanced technology

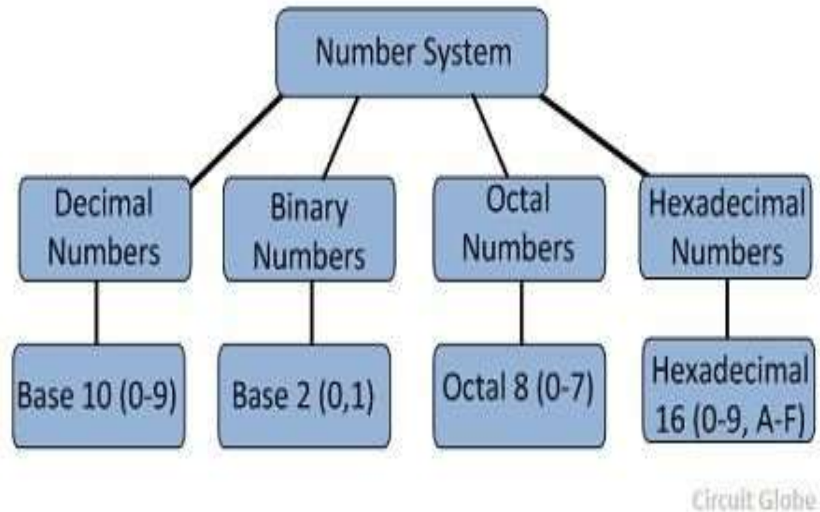
Most common digital devices are **Logic gate, Flip Flop**

Number System and Code

Digital system process binary digits 0 and 1

Base 10 is important for everyday business
Base 2 is important for processing of digital circuit
Base 8 and Base 16 provide convenient shortened representation for multibit number in a digital system

Binary	Decimal	Octal	3-Bit String	Hexadecimal	4-Bit String
0	0	0	000	0	0000
1	1	1	001	1	0001
10	2	2	010	2	0010
11	3	3	011	3	0011
100	4	4	100	4	0100
101	5	5	101	5	0101
110	6	6	110	6	0110
111	7	7	111	7	0111
1000	8	10	—	8	1000
1001	9	11	—	9	1001
1010	10	12	—	A	1010
1011	11	13	—	B	1011
1100	12	14	—	C	1100
1101	13	15	—	D	1101
1110	14	16	—	E	1110
1111	15	17	—	F	1111



Positional Number System

Digital system can understand positional number system

Value of a number is determined with help of **digit, position of the digit in the number and base of the number system**

Decimal positional number

$$
\begin{aligned}
(1234)_{10} &= 1000 + 200 + 30 + 4 \\
&= (1\times 1000) + (2\times 100) + (3\times 10) + (4\times 1) \\
&= (1\times 10^{3}) + (2\times 10^{2}) + (3\times 10^{1}) + (4\times 10^{0})
\end{aligned}
$$

Each weight is a power of 10.

Decimal point allow negative as well as positive power of 10

$$
\begin{aligned}
(5185.68)_{10} &= (5\times 1000) + (1\times 100) + (8\times 10) + (5\times 1) + (6\div 10) + (8\div 100) \\
&= (5\times 10^{3}) + (1\times 10^{2}) + (8\times 10^{1}) + (5\times 10^{0}) + (6\times 10^{-1}) + (8\times 10^{-2})
\end{aligned}
$$

In general a Number D in base r

$$D = a_n r^n + a_{n-1} r^{n-1} + \ldots\ldots. a_2 r^2 + a_1 r^1 + a_0 r^0 + a_{-1} r^{-1} + a_{-2} r^{-2} + \ldots\ldots. a_{-m} r^{-m}$$

Number conversion

Methods or techniques used to convert numbers from one base to another

Decimal to Other

Step 1 – Divide the decimal number to be converted by the value of the other base.

Step 2 – Get the remainder from Step 1 as (least significant digit) of new base number

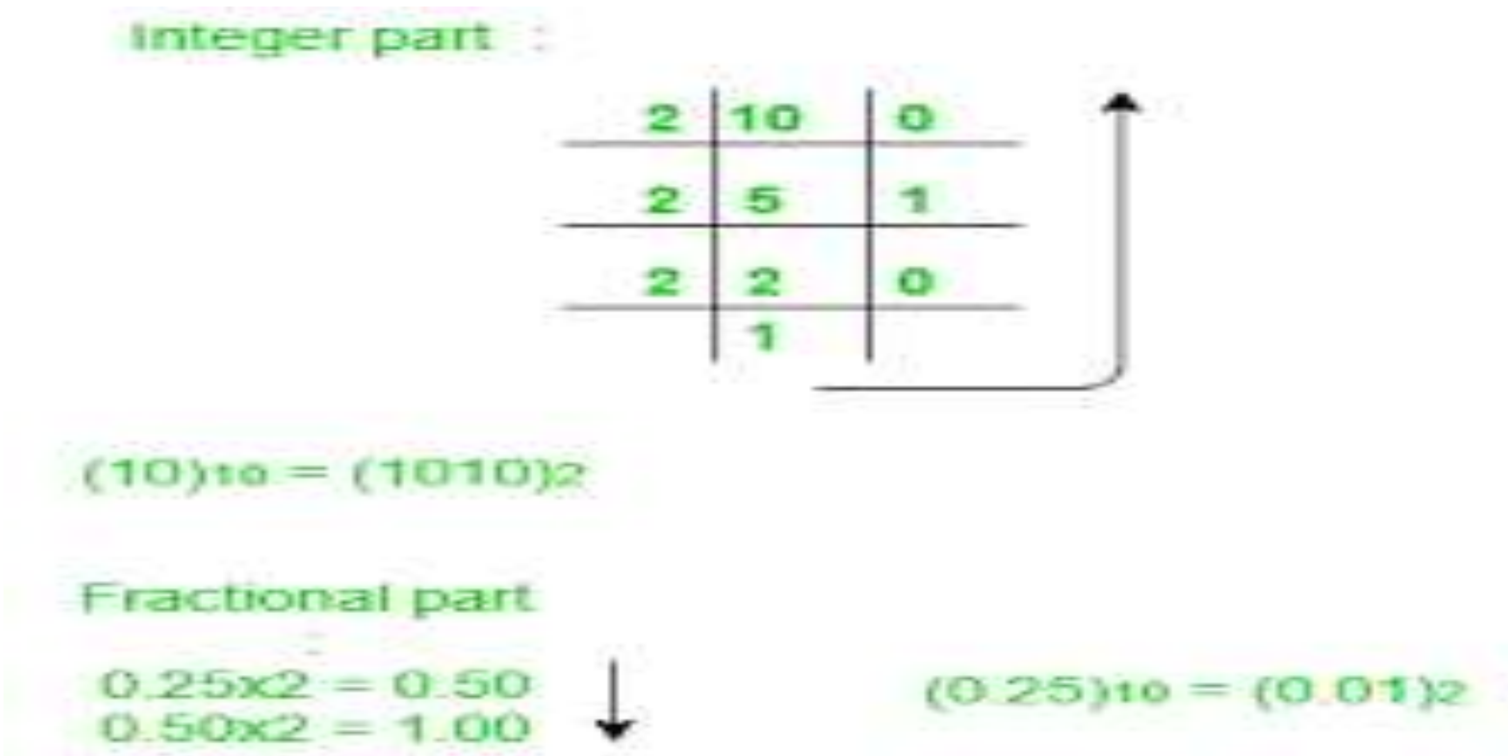
Step 3 – Divide the quotient of the previous divide by the new base.

Step 4 – Record the remainder from Step 3 as the next digit

Repeat Steps 3 and 4, getting remainders until the quotient becomes zero

The last remainder thus obtained will be the Most Significant bit(MSB) of the new base number.

Example


Integer part :
2 | 10 | 0
2 | 5 | 1
2 | 2 | 0
1
(10)10 = (1010)2
Fractional part
0.25x2 = 0.50
0.50x2 = 1.00
(0.25)10 = (0.01)2

Decimal to Hexadecimal

$$(3509)_{10} = (DB5)_{16}$$

Divisor	Quotient	Remainder
16	3509	5
16	219	11
16	13	13
	0	

Decimal to Octal

$$(569)_{10} = (1071)_8$$

		Remainders
8	569	
8	71	1
8	8	7
8	1	0
	0	1

Read in reverse order

$0.342_{10} = ?_8$

$0.342 \times 8 = 2.736 \quad (.2_8)$

$0.736 \times 8 = 5.888 \quad (.25_8)$

$0.888 \times 8 = 7.104 \quad (.257_8)$

$0.104 \times 8 = 0.832 \quad (.2570_8)$

$0.342_{10} \approx 0.2570_8$ it's an approximation

Other Base System to Decimal System

Step 1 – Determine positional value of each digit

Step 2 – Multiply the obtained position values by the digits in the corresponding columns.

Step 3 – Sum the products calculated in Step 2.

Binary Number – 11101_2

Calculating Decimal Equivalent –

Step	Binary Number	Decimal Number
Step 1	11101_2	$((1 \times 2^4) + (1 \times 2^3) + (1 \times 2^2) + (0 \times 2^1) + (1 \times 2^0))_{10}$
Step 2	11101_2	$(16 + 8 + 4 + 0 + 1)_{10}$
Step 3	11101_2	29_{10}

32 16 8 4 2 1

1 0 0 1 1 1 =32 + 4 +2 +1 = (39)10

Octal to Decimal

$$(2754)_8 = (2\times 8^3) + (7\times 8^2) + (5\times 8^1) + (4\times 8^4)$$

$$= 1024 + 448 + 40 + 4$$

$$= 1516_{10}$$

Hexadecimal to Decimal

$$(54.D2)^{16} = (5\times 16^1) + (4\times 16^0) + (13\times 16^{-1}) + (2\times 16^{-2})$$

$$= 80 + 4 + 0.8125 + 0.0078125$$

$$= 84.8203125_{10}$$

Binary to Octal

Step 1 − Divide the binary digits into groups of three (starting from the right).
Step 2 − Convert each group of three binary digits to one octal digit.

Binary Number − 10101_2

Calculating Octal Equivalent −

Step	Binary Number	Octal Number
Step 1	10101_2	010 101
Step 2	10101_2	2_8 5_8
Step 3	10101_2	25_8

- **Octal to Binary**

Step 1 − Convert each octal digit to a 3 digit binary number.
Step 2 − Combine all the resulting binary groups (of 3 digits each) into a single binary number

Octal Number − 25_8

Calculating Binary Equivalent −

Step	Octal Number	Binary Number
Step 1	25_8	010_2 101_2
Step 2	25_8	010101_2

Binary to Hexadecimal

Step 1 − Divide the binary digits into groups of four (starting from the right).
Step 2 − Convert each group of four binary digits to one hexadecimal symbol.

Binary Number − 10101_2

Calculating hexadecimal Equivalent −

Step	Binary Number	Hexadecimal Number
Step 1	10101_2	0001 0101
Step 2	10101_2	15_{16}

- **Hexadecimal to Binary**

Step 1 − Convert each hexadecimal digit to a 4 digit binary number.
Step 2 − Combine all the resulting binary groups (4 digits each) into a single binary number.

Hexadecimal Number − 15_{16}

Calculating Binary Equivalent −

Step	Hexadecimal Number	Binary Number
Step 1	15_{16}	0001_2 0101_2
Step 2	15_{16}	00010101_2

Practice Question

$$if \;\; (25)_x = (37)_{10} \quad find \;\; x$$

$$2x + 5 = 37$$

$$2x = 32$$

$$x = 16$$

Quick Quiz (Poll 1)

$73_x = 54_y$ Possible value of x and y

(A) 8 , 16

(B) 10 , 12

(C) 9 , 13

(D) 8 , 11

Solution

Given $73_x = 54_y$

$$7*x^1 + 3*x^0 = 5*y^1 + 4*y^0$$

$$7x + 3 = 5y + 4$$

put the value of option one by one and remember that base-value can't be less than given numbers in eq.

we are trying with option d

$$7*8 + 3 = 5*11 + 4$$

$$56 + 3 = 55 + 4$$

$$59 = 59$$, so ans is d.

Quick Quiz (Poll 2)

- The representation of octal number (532.2)8 in decimal is _________

a) (346.25)10
b) (532.864)10
c) (340.67)10
d) (531.668)10

- Answer: a
  Explanation: Octal to Decimal conversion is obtained by multiplying 8 to the power of base index along with the value at that index position.
  $(532.2)8 = 5 * 8^2 + 3 * 8^1 + 2 * 8^0 + 2 * 8^{-1} =$
  $(346.25)10$

Practice Question

$$If\ \sqrt{(41)_r} = (7)_{10} \quad find\ value\ of\ r$$

$$H\,\text{int}\,s : Convert\,LHS\ and\ RHS\ in\ same\ format$$

$$Square\ on\ both\ side$$

$$(41)_r = (49)_{10}$$

$$4r + 1 = 49$$

$$r = {}^{48}\!\big/_{4}$$

$$r = 12$$

Example

$$If\ \sqrt{(224)_r} = (13)_r \quad find\ value\ of\ r$$

$$(224)_r = 169_r$$

$$2r^2 + 2r + 4 = r^2 + 6r + 9$$

$$r^2 - 4r - 5 = 0$$

$$r = 5, -1$$

$$r = 5$$

Practice Question

$(4021.5)_5 = (\underline{\hspace{4cm}})_{10}$

$(B65F)_{16} = (\underline{\hspace{4cm}})_{10}$

$(630.4)_8 = (\underline{\hspace{4cm}})_{10}$

$(0.6875)_{10} = (\underline{\hspace{5cm}})_2$

$(0.513)_{10} = (\underline{\hspace{5cm}})_8$

$(306.D)_{16} = (\underline{\hspace{4.5cm}})_8$

$(10110001101011.11110010)_2 = (\underline{\hspace{4cm}})_{16}$

$(108)_{10} = (\underline{\hspace{2cm}})_{16}$

$(\underline{\hspace{4.5cm}})_2 = (\underline{\hspace{3cm}})_{10} = (576)_8 = (\underline{\hspace{2.5cm}})_{16}$