

## Unit 2

### Regular Expression

Conversion of regular expression to finite automata: -

(i)  $\phi \rightarrow (q_0)$

(ii)  $\epsilon / \Lambda \rightarrow (q_0)$

(iii)  $a + b \rightarrow (q_0 \xrightarrow{a, b} (q_0))$

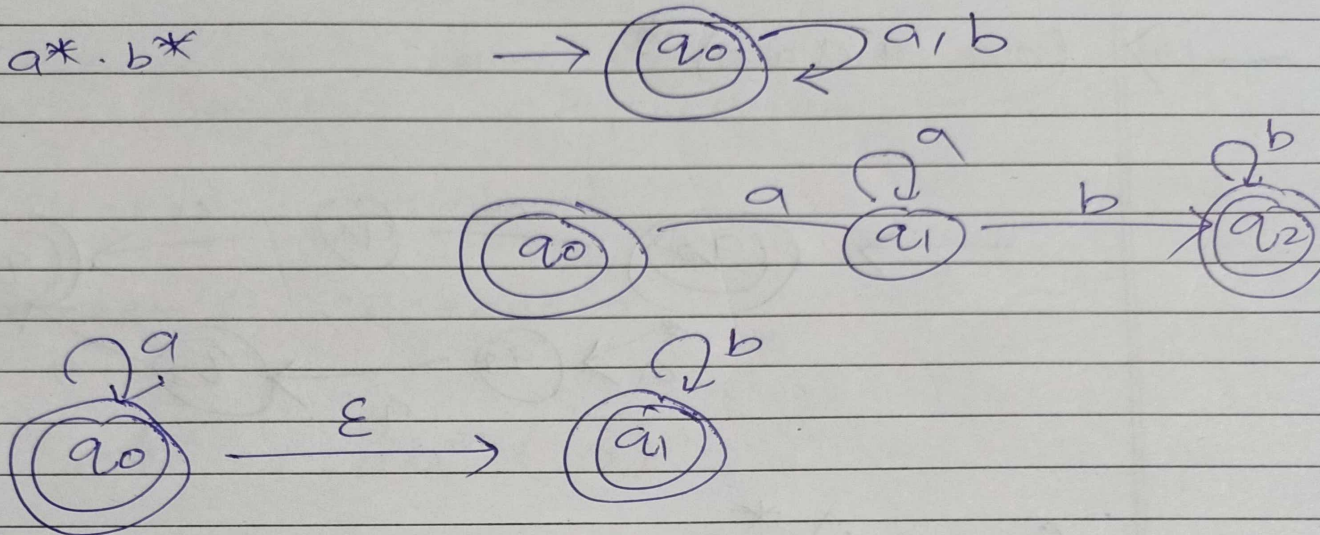
(iv)  $ab \rightarrow (q_0 \xrightarrow{a} q_1 \xrightarrow{b} (q_2))$

(v)  $(a + b)^* \rightarrow (q_0 \xrightarrow{a, b} q_0)$

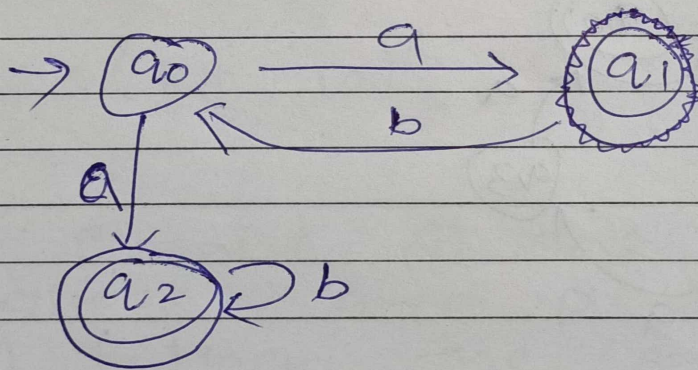
(vi)  $(ab)^* \rightarrow (q_0 \xrightarrow{a} q_1 \xrightarrow{b} q_0)$

(vii)  $a b^* \rightarrow (q_0 \xrightarrow{a} (q_1 \xrightarrow{b} q_1))$

(8)  $a^* \cdot b^*$



(9)  $(ab)^* a b^*$



$$\phi \cdot 0 = 0$$

$$\phi \cdot 1 = 0$$

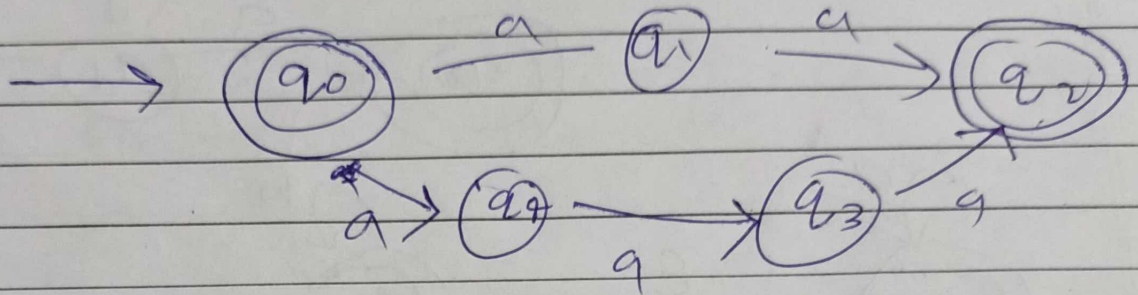
$$\epsilon \cdot 1 = 1$$

$$\epsilon \cdot 0 = 0$$

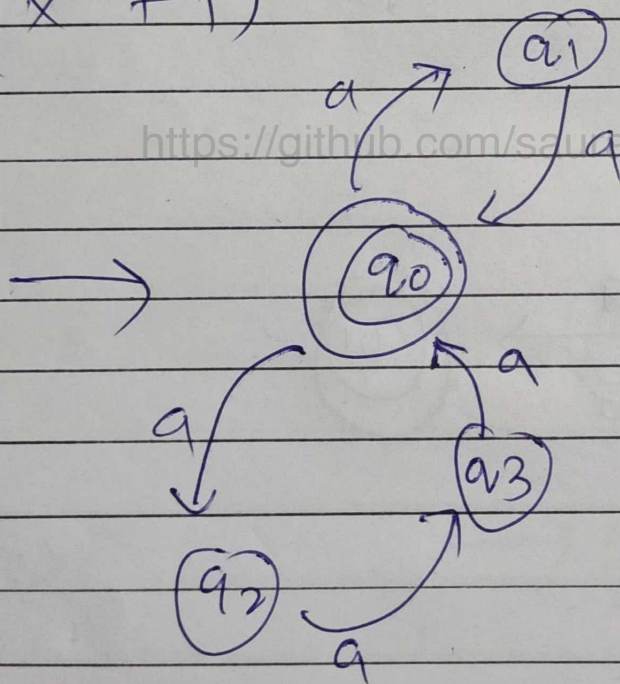
$$\epsilon \cdot (0+1) = 0+1$$



10)  $(aa + aqa)^*$



$(x + y)^*$

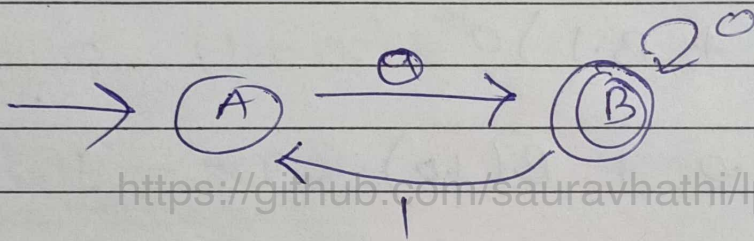


Conversion from Finite Automata to Regular expression :-

- (i) Direct Method
- (ii) Arden's Theorem

(i)

~~00~~  $00^*(10)^*$

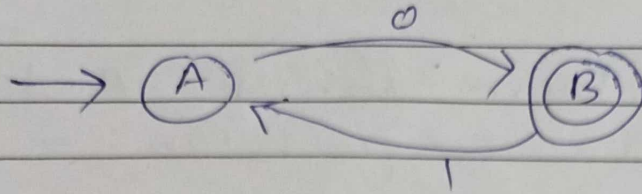


$0(0+10)^*$

- (ii) If  $P$  and  $Q$  are 2 regular expressions and  $P$  does not contain null then the equation  $R = Q + RP$  has a unique sol<sup>n</sup>  $R = QP^*$

$R$  is start





$$A = \epsilon + B \cdot 1 \quad \text{--- (i)}$$

$$B = A \cdot 0 \quad \text{--- (ii)}$$

(i) in (ii)

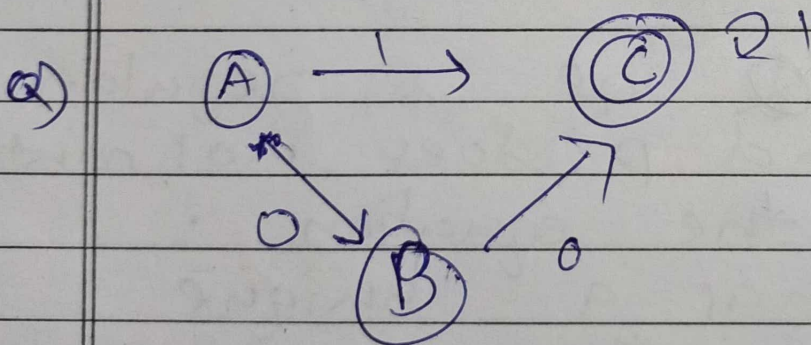
$$B = (\epsilon + B \cdot 1) \cdot 0$$

$$B = \epsilon \cdot 0 + B(1 \cdot 0)$$

<https://github.com/sauravhathi/lpu-cse>

$$B = 0 + B(1 \cdot 0)$$

$$B = 0(10)^*$$



$$+ (00 + 1)^*$$

$$11^* + 001^*$$

$$(1+00)^*$$

$$A = \cancel{B \cdot 0} \epsilon$$

$$B = A \cdot 0$$

$$C = A \cdot 1 + B \cdot 0 + C \cdot 1$$

$$B = \epsilon \cdot 0 \Rightarrow B = 0$$

$$C = \epsilon \cdot 1 + 0 \cdot 0 + C \cdot 1$$

$$C = 1 + 00 + C \cdot 1$$

$$C = (1 + 00)^* 1$$

Null - NFA

$$\delta: Q \times \Sigma \rightarrow 2^Q \quad \text{NFA}$$

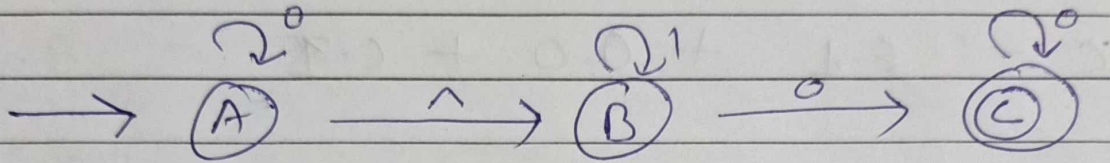
$$\delta: Q \times (\Sigma \cup \Lambda) \rightarrow 2^Q \quad \epsilon\text{-NFA}$$

Removal of null :-

- (i) If there exist null between A and B states then copy the transitions of B to A and remove the null



- (ii) If A is initial state make B also as initial state.
- (iii) If B is the final state make A also as the final state



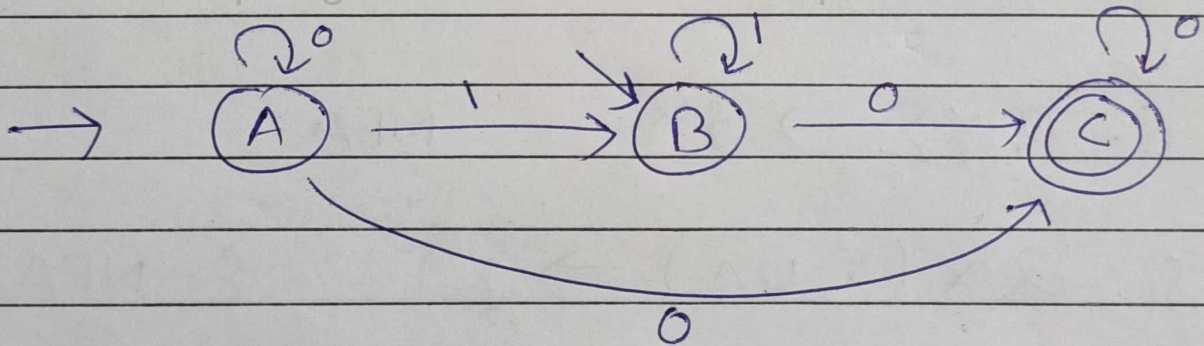
$$\delta(B, 0) \rightarrow C$$

$$\delta(B, 1) \rightarrow B$$

$$\delta(A, 0) \rightarrow C$$

$$\delta(A, 1) \rightarrow B$$

<https://github.com/sauravhathi/lpu-cse>





## Pumping Lemma:-

$L = a^n \cdot b^n$  ;  $n \geq 0$  Not Regular

$L = a^n \cdot b^m$  ;  $n > m$  Not Regular

$L = a^n \cdot b^m$  ;  $n, m \geq 0$  Regular

There is no memory in finite automata therefore in a language there is a requirement of memory, then the language is not regular.

It is a negation test to test to check whether the language is regular or not.

Statement  $\rightarrow$  If  $L$  is a regular language with any integer  $n$  such that  $w \in L$  where  $|w| \geq n$  and  $w = xyz$  then it must follow

- (i)  $|xy| \leq n$  (ii)  $|y| \geq 1$



(iii) For all  $i \geq 0$  ;  $xy^iz \in L$

$$L = a^n b^n ; n \geq 0$$

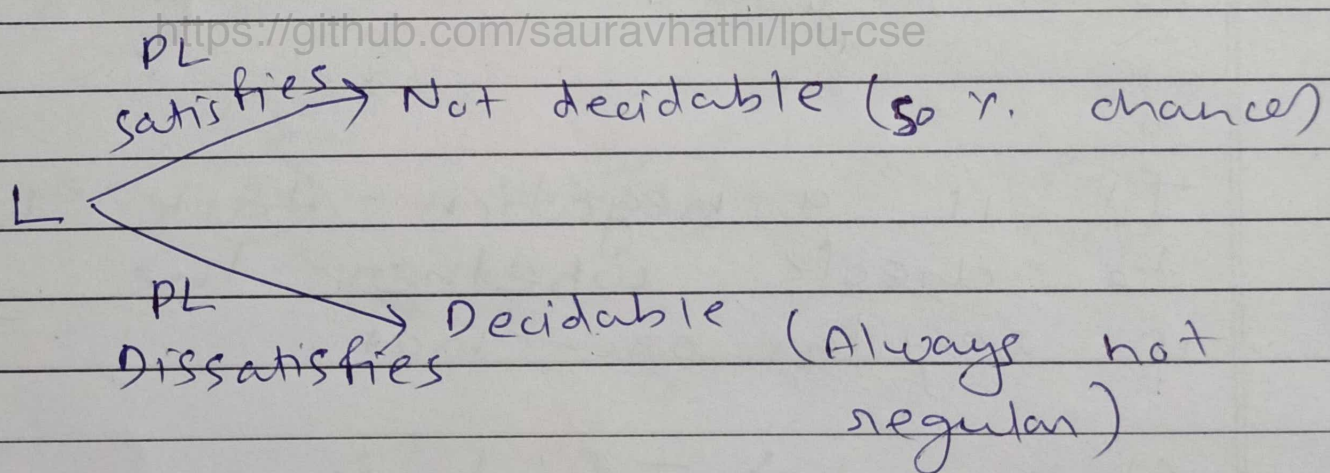
$$W = a a a b b b$$

$$n = 3$$

$$|W| = 6$$

$$W = \frac{a \ a}{x} \ \frac{a}{y} \ \frac{b \ b \ b}{z}$$

$$\frac{a a}{x} \ \frac{a^i}{y} \ \frac{b b b}{z}$$



\* Properties of Regular Expression :-

i) Associativity

$$R_1 + (R_2 + R_3) = (R_1 + R_2) + R_3$$

$$R_1 \cdot (R_2 \cdot R_3) = (R_1 \cdot R_2) \cdot R_3$$

associative property holds for both union and concatenation

(ii) Commutative

$$R_1 + R_2 = R_2 + R_1$$

$$R_1 \cdot R_2 \neq R_2 \cdot R_1$$

$$a \cdot b \neq b \cdot a$$

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Commutative property holds only for union.

(iii) Idempotent

$$R_1 + R_1 = R_1$$

$$R_1 \cdot R_1 \neq R_1$$

Idempotent property holds only for union.



## Closure Properties

(i) Union (ii) Concatenation

(iii) Kleene

$$a^* = \{\epsilon, a, aa, aaa, \dots\}$$

$$a^+ = \{a, aa, aaa, \dots\}$$

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