UNIT-II

Combinational Logic System

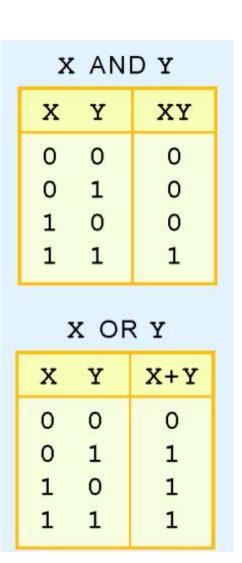
Lecture 12
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Introduction

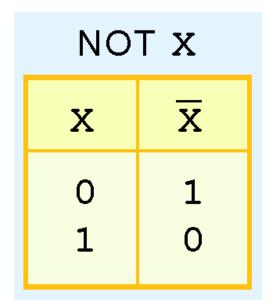
- In the middle of the twentieth century, computers were commonly known as "thinking machines" and "electronic brains."
 - Many people were fearful of them.
- Nowadays, we rarely ponder the relationship between electronic digital computers and human logic. Computers are accepted as part of our lives.
 - Many people, however, are still fearful of them.
- In this chapter, you will learn the simplicity that constitutes the essence of the machine.

- Boolean algebra is a mathematical system for the manipulation of variables that can have one of two values.
 - In formal logic, these values are "true" and "false."
 - In digital systems, these values are "on" and "off," 1 and
 0, or "high" and "low."
- Boolean expressions are created by performing operations on Boolean variables.
 - Common Boolean operators include AND, OR, and NOT.

- A Boolean operator can be completely described using a truth table.
- The truth table for the Boolean operators AND and OR are shown at the right.
- The AND operator is also known as a Boolean product. The OR operator is the Boolean sum.



- The truth table for the Boolean NOT operator is shown at the right.
- The NOT operation is most often designated by an overbar. It is sometimes indicated by a prime mark (') or an "elbow" (¬).



- A Boolean function has:
 - · At least one Boolean variable,
 - At least one Boolean operator, and
 - At least one input from the set {0,1}.
- It produces an output that is also a member of the set {0,1}.

Now you know why the binary numbering system is so handy in digital systems.

Quick Quiz (Poll 1)

Boolean algebra can be used ______

- a) For designing of the digital computers
- b) In building logic symbols
- c) Circuit theory
- d) Building algebraic functions

 The truth table for the Boolean function:

$$F(x,y,z) = x\overline{z} + y$$

is shown at the right.

 To make evaluation of the Boolean function easier, the truth table contains extra (shaded) columns to hold evaluations of subparts of the function.

$$F(x,y,z) = x\overline{z} + y$$

х	У	z	z	χĪ	xz+y
0	0	0	1	0	0
0	0	1	0	0	0
0	1	0	1	0	1
0	1	1	0	0	1
1	0	0	1	1	1
1	0	1	0	0	0
1	1	0	1	1	1
1	1	1	0	0	1

- As with common arithmetic, Boolean operations have rules of precedence.
- The NOT operator has highest priority, followed by AND and then OR.
- This is how we chose the (shaded) function subparts in our table.

$F(x,y,z) = x\overline{z} + y$

х	У	z	z	хz	x z +y
0	0	0	1	0	0
0	0	1	0	0	0
0	1	0	1	0	1
0	1	1	0	0	1
1	0	0	1	1	1
1	0	1	0	0	0
1	1	0	1	1	1
1	1	1	0	0	1

- Digital computers contain circuits that implement Boolean functions.
- The simpler that we can make a Boolean function, the smaller the circuit that will result.
 - Simpler circuits are cheaper to build, consume less power, and run faster than complex circuits.
- With this in mind, we always want to reduce our Boolean functions to their simplest form.
- There are a number of Boolean identities that help us to do this.

 Most Boolean identities have an AND (product) form as well as an OR (sum) form. We give our identities using both forms. Our first group is rather intuitive:

Identity	AND	OR
Name	Form	Form
Identity Law Null Law Idempotent Law Inverse Law	$1x = x$ $0x = 0$ $xx = x$ $x\overline{x} = 0$	$0 + x = x$ $1 + x = 1$ $x + x = x$ $x + \overline{x} = 1$

 Our second group of Boolean identities should be familiar to you from your study of algebra:

Identity	AND	OR
Name	Form	Form
Commutative Law Associative Law Distributive Law	xy = yx (xy) z = x (yz) x+yz = (x+y) (x+z)	x+y = y+x $(x+y)+z = x + (y+z)$ $x(y+z) = xy+xz$

- Our last group of Boolean identities are perhaps the most useful.
- If you have studied set theory or formal logic, these laws are also familiar to you.

Identity Name	AND Form	OR Form
Absorption Law DeMorgan's Law	$x(x+y) = x$ $(\overline{xy}) = \overline{x} + \overline{y}$	$x + xy = x$ $\overline{(x+y)} = \overline{x}\overline{y}$
Double Complement Law	$(\overline{\overline{x}}) = x$	

We can use Boolean identities to simplify the function:

as follows: $F(X,Y,Z) = (X + Y)(X + \overline{Y})(X\overline{Z})$

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(X + Y) (X + \overline{Y}) (X\overline{Z})
                                   Idempotent Law (Rewriting)
 (X + Y) (X + Y) (X + Z)
                                   DeMorgan's Law
 (XX + XY + XY + YY) (X + Z)
                                   Distributive Law
((X + YY) + X(Y + \overline{Y}))(\overline{X} + Z)
                                   Commutative & Distributive Laws
((X + 0) + X(1))(\overline{X} + Z)
                                   Inverse Law
  X(\overline{X} + Z)
                                   Idempotent Law
  XX + XZ
                                   Distributive Law
   0 + XZ
                                   Inverse Law
                                   Idempotent Law
     XZ
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Simplifying Logic Expressions

Find the minimum sum-of-products representation for the boolean function

$$A + \overline{AC} + B$$
.

We first write the sum-of-products representation:

$$A + \overline{AC} + B = A + (\overline{A} + \overline{C}) + B$$

$$= A + (A + \overline{C}) + B$$

$$= A + A + \overline{C} + B$$

$$= A + \overline{C} + B.$$

Here, $A + A + \overline{C} + B$ is in a sum-of-products form. The minimum sum-of-products form, however, is $A + \overline{C} + B$.

Simplifying Logic Expressions

$$Z = \overline{X}Y + X\overline{Y} + XY$$

The following sequence of simplifications show that this expression for Z is equivalent to X+Y:

$$Z = \overline{X}Y + X\overline{Y} + XY$$

$$= \overline{X}Y + X(\overline{Y} + Y)$$

$$= \overline{X}Y + X \cdot 1$$

$$= \overline{X}Y + X$$

$$= Y + X$$

Name	AND form	OR form
Identity law	1A = A	0 + A = A
Null law	0A = 0	1 + A = 1
Idempotent law	AA = A	A + A = A
Inverse law	$A\overline{A} = 0$	A + A = 1
Commutative law	AB = BA	A + B = B + A
Associative law	(AB)C = A(BC)	(A + B) + C = A + (B + C)
Distributive law	A + BC = (A + B)(A + C)	A(B + C) = AB + AC
Absorption law	A(A + B) = A	A + AB = A
De Morgan's law	$\overline{AB} = \overline{A} + \overline{B}$	$\overline{A + B} = \overline{A}\overline{B}$

Quick Quiz (Poll 2)

Applying DeMorgan's theorem to the expression ABC, we get _____.

- A. $\overline{A} + \overline{B} + \overline{C}$
- $B. \overline{A+B+C}$
- C. $A + \overline{B} + C\overline{C}$
- D. A(B + C)

Quick Quiz (Poll 3)

• AC + ABC = AC

A True

B False