Q1) Solve the recurrence relation;

$$a_n - 6a_{n-1} + 11a_{n-2} - 6a_{n-3} = 0$$

Its order $= n-(n-3) = \cancel{n}-\cancel{n}+3 = 3$

Its characteristic eqns are

$$\alpha^3 - 6\alpha^2 + 11\alpha - 6 = 0 \qquad (\pm 1, \pm 2, \pm 3, \pm 6)$$

Let $[\alpha = 1]$ $\qquad \alpha - 1 = 0$

$$(1)^3 - 6(1)^2 + 11(1) - 6 = 0$$

$$1 - 6 + 11 - 6 = 0$$

$$12 - 12 = 0$$

$$0 = 0$$

$\Rightarrow \alpha = 1$ is the root of the given eqn.

1	1	−6	11	−6
		1	−5	6
	1	−5	6	0

$$(\alpha-1)(\alpha^2 - 5\alpha + 6) = 0$$

$$(\alpha-1)(\alpha^2 - 3\alpha - 2\alpha + 6) = 0$$

$$(\alpha-1)(\alpha(\alpha-3) - 2(\alpha-3)) = 0$$

$$(\alpha-1)(\alpha-2)(\alpha-3) = 0$$

$$\alpha = 1, 2, 3$$

$$\boxed{a_n = c_1(1)^n + c_2(2)^n + c_3(3)^n}$$

$$a_{n+3} + 6a_{n+2} + 11a_{n+1} + 6a_n = 0$$

$$\left[a_n = c_1(-1)^n + c_2(-2)^n + c_3(-3)^n \quad \underline{\underline{\text{Ans.}}} \right]$$

Its degree $= (\cancel{n}+3) - \cancel{n} = 3.$

$\therefore$ Its characteristic eqn is

$$\alpha^3 + 6\alpha^2 + 11\alpha + \underline{6} = 0 \quad \left[\pm 1, \ \pm 2, \ \pm 3, \ \pm 6 \right]$$

$\boxed{\alpha = -1}$ $\quad \alpha + 1 = 0$

$$(-1)^3 + 6(-1)^2 + 11(-1) + 6 = 0$$

$$-1 + \underline{6} - 11 + \underline{6} = 0$$

$$12 - 12 = 0$$

$$\boxed{0 = 0}$$

-1	1	6	11	6
		-1	-5	-6
	1	5	6	0

$$(\alpha+1)(\alpha^2 + 5\alpha + 6) = 0$$

$$(\alpha+1)(\alpha^2 + 3\alpha + 2\alpha + 6) = 0$$

$$(\alpha+1)\left[\alpha(\alpha+3) + 2(\alpha+3)\right] = 0$$

$$(\alpha+1)(\alpha+2)(\alpha+3) = 0$$

$$\alpha = -1, -2, -3$$

$$a_n = c_1(-1)^n + c_2(-2)^n + c_3(-3)^n$$

— x —

Factorial polynomials

Factorial poly.	General poly.
$x^{(2)}$	x^2

① $x^{(0)} = x^0 = 1$

② $x^{(1)} = x$

③ $x^{(2)} = x(x-1)$

We want to express x^2 in factorial polynomials.

$$x^{(2)} = x^2 - x$$

$$x^{(2)} = x^2 - x^{(1)} \qquad (x = x^{(1)})$$

$$\boxed{x^2 = x^{(2)} + x^{(1)}}$$

We next want to express x^3 as factorial polynomial

$$x^{(3)} = x(x-1)(x-2)$$

$$= (x^2 - x)(x-2)$$

$$= x^3 - 2x^2 - x^2 + 2x$$

$$x^{(3)} = x^3 - 3x^2 + 2x$$

$$x^3 = x^{(3)} + 3x^2 - 2x$$

$$= x^{(3)} + 3\left[x^{(2)} + x^{(1)}\right] - 2x^{(1)}$$

$$x = x + x$$

$$= x^{(3)} + 3\left[x^{(2)} + x^{(1)}\right] - 2x^{(1)}$$

$$= x^{(3)} + 3x^{(2)} + 3x^{(1)} - 2x^{(1)}$$

$$\boxed{x^3} = \left\{ \underbrace{x^{(3)} + 3x^{(2)} + x^{(1)}} \right\}$$

$$\boxed{\begin{aligned} x^{(0)} &= 1 \\ x &= x^{(1)} \\ x^2 &= x^{(2)} + x^{(1)} \\ x^3 &= x^{(3)} + 3x^{(2)} + x^{(1)} \end{aligned}}$$

Shift operator: Let $f(x)$ be a continuous function defined in $[a, b]$, then the operator E is called shift operator and it is defined as.

$$E(f(x)) = f(x+h)$$

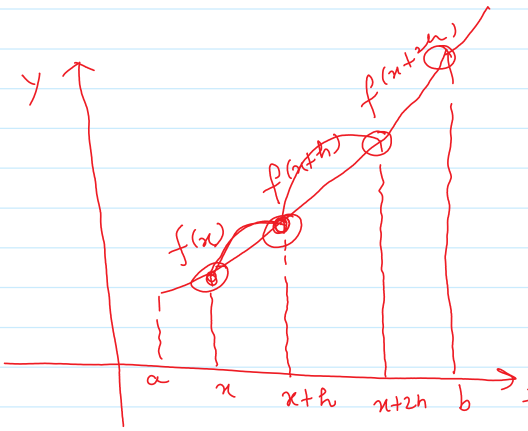
$$E^2(f(x)) = E\left[E(f(x))\right]$$

$$= E\left[f(x+h)\right]$$

$$E^2\left[f(x)\right] = f(x+2h)$$

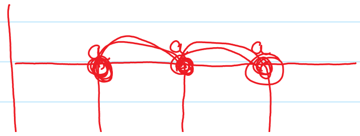
$$\vdots$$

$$E^n\left[f(x)\right] = f(x+nh)$$

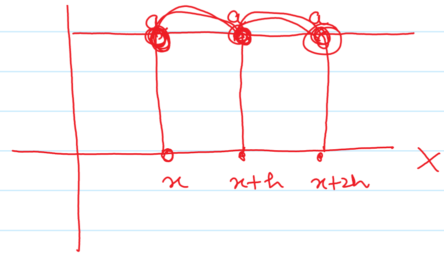


$f(x) = 2003$

$E(\underline{2003}) = ?$



$E(2003) = ?$



$E(2003) = 2003$

$E^2(2003) = E(E(2003))$

$= 2003$

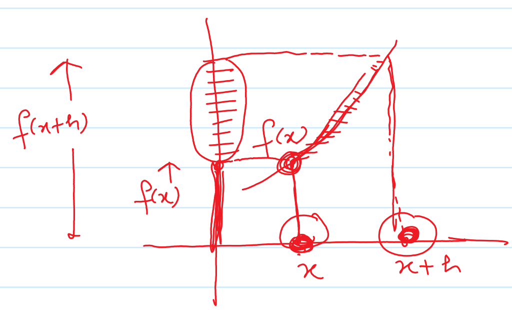
$$\boxed{E^n(2003) = 2003}$$

In general $E(a) = a$

② Forward operator:- Let $f(x)$ be defined for all $x \in [a,b]$ and Δ is the forward operator which is defined as.

$$\Delta f(x) = f(x+h) - f(x)$$

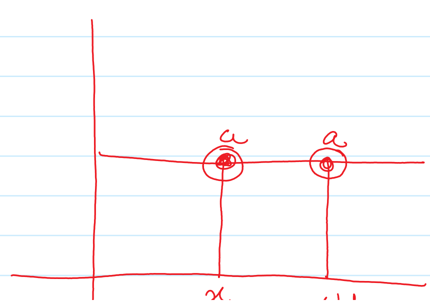
$\Delta(a) =$



$\Delta(a) = f(x+h) - f(x)$

$= a - a = 0$

$\Delta(a) = 0$

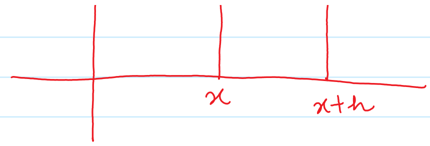


2.

$\Delta(a) = 0$

$\Delta^2(a) = 0$

$\Delta^3(a) = 0$



Relationship between E and Δ.

$\Delta f(x) = f(x+h) - f(x)$

$= E(f(x)) - f(x)$

$\Delta f(x) = (E-1) f(x)$

$\Delta = E - 1$

$\boxed{E = \Delta + 1}$

$E = \Delta + 1$

Elephant

other animal.

$\boxed{E = \Delta + 1}$

Prove that Δ doesn't behave like derivative operator when it is applied on general polynomial.

$\rightarrow a_{n+2} + a_{n+1} + a_n = 0$

$\rightarrow a_n + a_{n-1} + a_{n-2} = 0$

$\boxed{h=1}$

① $\Delta(1) = 0$

② $\Delta(x)$

$f(x) = x$

$\Delta f(x) = f(x+h) - f(x)$

$= f(x+1) - f(x)$

$= \cancel{x}+1 - \cancel{x}$

$\boxed{\Delta f(x) = 1}$

$\Delta x = 1$

Δx^2

$f(x) = x^2$

$\Delta f(x) = f(x+1) - f(x)$

$= (x+1)^2 - x^2$

$= \cancel{x^2}+1+2x - \cancel{x^2}$

$\boxed{\Delta x^2 = 2x+1}$

Clearly Δ doesn't behave like derivative operator

—x—

Note

Δ behaves like derivative operator when applied on factorial polynomials.

$\Delta(1) = 0$

$\Delta x^{(1)} = \Delta x = 1$

$\Delta x^{(2)} = \Delta x(x-1)$

$= \Delta(x^2 - x)$

$f(x) = x^2 - x$

$\Delta f(x) = f(x+1) - f(x)$

$= [(x+1)^2 - (x+1)] - [x^2 - x]$

$= \cancel{x^2} + \cancel{1} + 2x - \cancel{x} - \cancel{1} - \cancel{x^2} + \cancel{x}$

$\Delta x^{(2)} = 2x = 2x^{(1)}$

$\Delta x^{(3)} = 3x^{(2)}$

—x—