Practice Set: 8

Q1: What is plasma fluid model?. [CO1]

Q2: Explain electron plasma oscillations by using the plasma fluid model. [CO3]

$$\frac{\partial^2 n_1}{\partial t^2} + \omega_p^2 n_1 = 0$$

where the **electron plasma frequency** is:

$$\omega_p = \sqrt{\frac{e^2 n_0}{m\varepsilon_0}}$$

Q3: In a plasma of density $n = 7 \times 10^{18}\ m^{-3}$, the plasma frequency (in GHz) is $23.74\ GHz$ [CO3]

Solution: The plasma frequency is given by

$$\omega = \sqrt{\frac{ne^2}{m_e \epsilon_0}}$$

$$\omega = \sqrt{\frac{7\times10^{18}\times(1.6\times10^{-19})^2}{9.1\times10^{-31}\times8.85\times10^{-12}}} = 1.491 \times 10^{11} \text{ rad/s}$$

$$f = \frac{\omega}{2\pi} = \frac{1.491\times10^{11}}{2\times3.14} = 2.374 \times 10^{10}\ Hz = 23.74\ GHz$$

Q4: The plasma density in the lower ionosphere has been measured during satellite re-entry to be about 3×10^{12} m^3 at 50 km altitude. What is the plasma frequency there in MHz? [CO3]

15.5

Solution: We know that the plasma frequency is given by

$$\omega = \sqrt{\frac{ne^2}{m_e \epsilon_0}}$$

$$\omega = \sqrt{\frac{3\times10^{12}\times(1.6\times10^{-19})^2}{9.1\times10^{-31}\times8.85\times10^{-12}}} = 9.765 \times 10^7 \quad \text{rad/s}$$

$$f = \frac{\omega}{2\pi} = \frac{9.765\times10^7}{2\times3.14} = 1.554 \times 10^7\, Hz = 15.54\, MHz$$

Q5: Explain the dispersion relation for electron plasma waves (Langmuir Waves).

$$\omega^2 = \omega_{pe}^2 + \frac{k_B T_e}{m_e} k^2$$

Q6: Electron plasma waves are propagated in a uniform plasma with $KT_e = 100$ eV, $n = 10^{16}$ m^{-3}, and $B = 0$. If the frequency f is 1.1 GHz, what is the wavelength in cm? [CO4]

According to the dispersion relation

$$\omega^2 = \omega_{pe}^2 + \frac{3KT_e}{m_e}k^2$$

Now we calculate the unknown value in the equation

$$\omega_{pe} \approx 2\pi\sqrt{n} = 5.62 \times 10^9 rad/sec$$

$$\omega = 2\pi f = 6.908 \times 10^9 rad/sec$$

$$\frac{3kT_e}{m_e} = \frac{3 \times 100 \times 1.6E{-}19}{9.109 \times 10^{-31}} = 5.27 \times 10^{13}$$

$$\therefore k^2 = \frac{\omega^2 - \omega_{pe}^2}{\frac{3kT_e}{m_e}} = 3.05 \times 10^5 \Rightarrow k = 552.377$$

$$\Rightarrow \lambda = \frac{2\pi}{k} = 1.14 \times 10^{-2} m$$