Practice Set: 7

Q1: Explain the working of magnetic mirror machine with conservation laws. [CO1]

Q2: A plasma with an isotropic velocity distribution is placed in a magnetic mirror trap with mirror ratio $R_m = 4$. There are no collisions, so the particles in the loss cone simply escape, and the rest remain trapped. What fraction is trapped? [CO5]

$$R_m = 4 \Rightarrow \frac{1}{\sin^2\theta_m} = 4 \Rightarrow \sin\theta_m = \frac{1}{2}$$

$$\therefore \theta_m = \frac{\pi}{6}$$

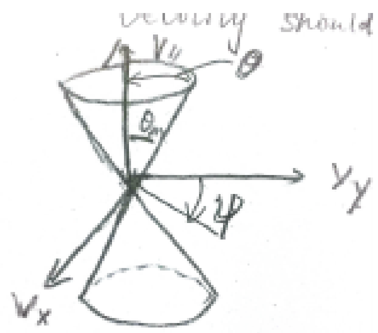

Figure 3

Since the velocity is isotropic distribution, the direction of velocity should distribute uniformly

$$\mathbf{d}\Omega = \sin\theta \mathbf{d}\theta \mathbf{d}\varphi$$

The total solid angle for a sphere is

$$\Omega_{total} = 4\pi$$

The solid angle for loss cone is

$$\Omega_{loss} = 2\int_0^{\frac{\pi}{6}} \sin\theta \mathbf{d}\theta \int_0^{2\pi} \mathbf{d}\varphi = (1 - \frac{\sqrt{3}}{2})4\pi$$

$\therefore$ the fraction of the trapped is $\frac{\sqrt{3}}{2}$.

Q3: Derive Boltzmann relation for electrons by using plasma fluid. [CO2]

$$\boxed{n = n_0 \exp\left(e\phi / KT_e\right)}$$

Q4: Define diamagnetic drift of plasma by using plasma fluid model and calculate the diamagnetic current. [CO3]

$$\mathbf{v}_{Di} = \frac{KT_i}{eB} \frac{n'}{n} \hat{\boldsymbol{\theta}} \quad \left(n' \equiv \frac{\partial n}{\partial r} < 0 \right) \qquad \mathbf{v}_{De} = -\frac{KT_e}{eB} \frac{n'}{n} \hat{\boldsymbol{\theta}}$$

$$\mathbf{j}_D = ne(\mathbf{v}_{Di} - \mathbf{v}_{De}) = (KT_i + KT_e) \frac{\mathbf{B} \times \nabla n}{B^2}$$

Q5: A cylindrically symmetric plasma column in a uniform **B**, has $n(r) = n_0 exp(-r^2/r_0^2)$ [CO5]

and $n_i = n_e = n_0 exp(e\phi/KT_e)$ (Boltzmann relation), then $\mathbf{v}_E$ and $\mathbf{v}_{De}$ are

a. Unequal and parallel
b. Unequal and opposite
c. Equal and parallel
d. Equal and opposite

Solution: From the given relations

$$n = n_0 e^{-r^2/r_0^2} = n_0 e^{e\phi/KT_e}$$

$$\phi = \frac{KT_e}{e} \ln\frac{n}{n_0} = \frac{KT_e}{e}\left(-\frac{r^2}{r_0^2}\right)$$

$$\mathbf{E} = -\frac{\partial\phi}{\partial r}\hat{\mathbf{r}} = \frac{KT_e}{e}\frac{2r}{r_0^2}\hat{\mathbf{r}}$$

$$\mathbf{v}_E = \frac{\mathbf{E}\times\mathbf{B}}{B^2} = -\frac{E_r}{B_z}\hat{\boldsymbol{\theta}} = -\hat{\boldsymbol{\theta}}\frac{KT_e}{eB}\frac{2r}{r_0^2}$$

$$\mathbf{v}_{De} = -\frac{\mathbf{B}\times\nabla p}{enB^2} = -\frac{KT_e}{eB}\frac{\partial n/\partial r}{n}\hat{\boldsymbol{\theta}} = -\hat{\boldsymbol{\theta}}\frac{KT_e}{eB}\frac{\partial}{\partial r}(\ln n)$$

$$= -\hat{\boldsymbol{\theta}}\frac{KT_e}{eB}\frac{\partial}{\partial r}\left(\frac{-r^2}{r_0^2}\right) = \hat{\boldsymbol{\theta}}\frac{KT_e}{eB}\frac{2r}{r_0^2} = -\mathbf{v}_E$$

Q6: Using the information given in the question 7, evaluate j_D in A/m^2 for $B = 0.4$ T, $n_0 = 10^{16}$ m^{-3}, $KT_e = KT_i = 0.25$ eV, $r = r_0 = 1$ cm. [CO5]

0.147

Solution: From question 7, we have $v_{De} = -v_E = -\frac{KT_e}{eB} \cdot \frac{2r}{r_0^2}$; and we know that $j_D = nev$,

and $n = n_0 e^{(-r^2/r_0^2)}$,

Now,

$$j_D = ne(v_{Di} - v_{De}) = -\hat{\theta}\, \frac{n_0(KT_e + KT_i)}{B} \cdot \frac{2r}{r_0^2} e^{-r^2/r_0^2}$$

$$j_D = \frac{(10^{16})(0.5)(1.6 \times 10^{-19})}{0.4(r_0^2/2r)(2.718)} = 0.147\,\mathrm{A/m^2}$$

Q7: An isothermal plasma is confined between the planes $x = \pm a$, in a magnetic field $\mathbf{B} = B_0 \hat{z}$. The density distribution is $n = n_0(1 - x^2/a^2)$, The diamagnetic drift velocity $\mathbf{v}_{De}$ for the electrons is [CO5]

a. $\hat{y} \frac{KT_e}{eB} \frac{2x}{a^2} (1 - \frac{x^2}{a^2})^{-1}$

b. $\hat{y} \frac{KT_e}{eB} \frac{2x}{a^2} (1 + \frac{x^2}{a^2})^{-1}$

c. $\hat{y} \frac{KT_e}{eB} \frac{2x}{a^2} (1 + \frac{x}{a^2})^{-1}$

d. $\hat{y} \frac{KT_e}{eB} \frac{2x}{a^2} (1 - \frac{x}{a^2})^{-1}$

Solution: The diamagnetic drift velocity in terms of charge density is

$$\mathbf{v}_{De} = -\frac{\gamma KT_e}{eB} \frac{\hat{z} \times \nabla n}{n}$$

In isothermal conditions, $\gamma = 1$

$$\nabla n = \hat{\mathbf{x}} \frac{\partial n}{\partial x} = -\frac{n_0 2x}{a^2} \hat{\mathbf{x}}$$

$$\mathbf{v}_{De} = \hat{\mathbf{y}} \frac{KT_e}{eB} \frac{2n_0}{a^2} \frac{x}{n_0} \left(1 - \frac{x^2}{a^2}\right)^{-1} = \hat{\mathbf{y}} \frac{KT_e}{eB} \frac{2x}{a^2} \left(1 - \frac{x^2}{a^2}\right)^{-1}$$