Unit 6: Hypothesis Testing

1. Statistical Hypothesis is an assertion or conjecture concerning one or more populations.

2. Null Hypothesis: It refers to any hypothesis which we wish to test. A definite statement about the population parameter, i.e., Hypothesis of no difference is called Null Hypothesis. It is denoted by H_0.

3. Alternate Hypothesis: Any Hypothesis which is different (complementary) to the null Hypothesis. Rejection of H_0 leads to acceptance of an alternative hypothesis. It is denoted by H_1 or H_a.

$$(i)\ H_1 : \mu \neq \mu_0 \ (i.e.,\ \mu > \mu_0 \text{ or } \mu < \mu_0)$$
$$(ii)\ H_1 : \mu > \mu_0$$
$$(iii)\ H_1 : \mu < \mu_0$$

4. Test of a Statistical Hypothesis: It is a two action decision problem after the experimental sample values have been obtained, the two actions being the acceptance or rejection of the hypothesis under consideration.

- **Reject H_0** in favour of H_1 because of sufficient evidence in the data or
- **Fail to reject H_0** because of insufficient evidence in the data.

5. Types of Error

- **Type-I error:** Rejection of null hypothesis when it is true. It is denoted by α.

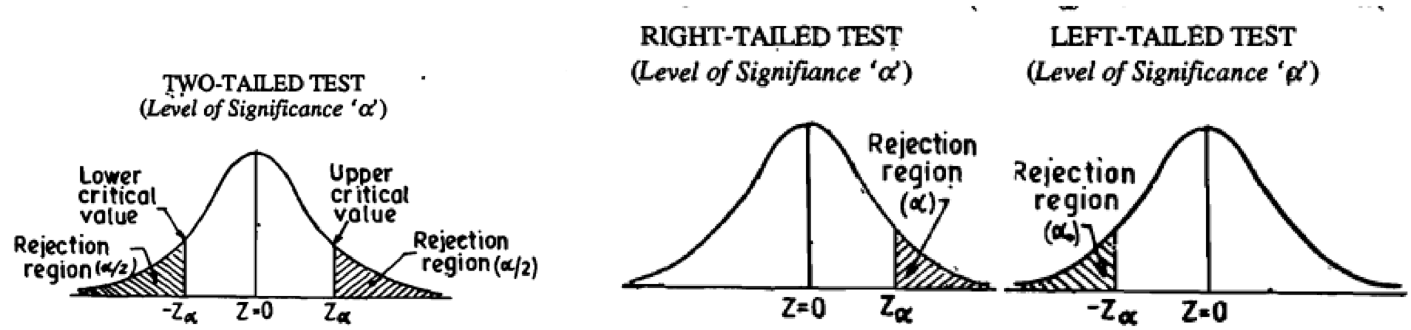
- **Type-II error:** Acceptance of null hypothesis when it is false. It is denoted by β.

The probability of committing both types of errors can be reduced by increasing the sample size.

6. Types of Tests:

- **One-tailed Test:** A test of any statistical hypothesis where the alternative hypothesis is one tailed (left-tailed or right tailed) is called one-tailed test.
- **Two-tailed Test:** A test of any statistical hypothesis where the alternative hypothesis is two-tailed.

7. Critical Region: Region of the outcome set where H_0 is rejected if the sample point falls in that region and is called critical region. Region of rejection of H_0 when it is true is known as critical region.



8. Level of Significance(α): Maximum probability with which we are prepared to reject H_0 when it is true. Total area of the region of rejection expressed as %age is called level of significance.

9. Critical value: The value of test statistic which separates the critical region and acceptance region. It depends upon:

- The level of significance used.
- The alternative hypothesis, whether it is two tailed or one-tailed.

Test of Significance for mean

(i) Z-Test (For larger value of n, almost all the distributions are very closely approximated by normal distribution, $n > 30$**)**

(ii) Student's t-Test (This test is applicable when sample size is small**)**

Representation of regions for Z −test

	Acceptance region	**Rejection region**
$H_1: \mu \neq \mu_0$ **Two tailed test**	$P\left(-z_\alpha \leq \frac{\bar{X}-\mu}{\frac{\sigma}{\sqrt{n}}} \leq z_\alpha\right) = 1-\alpha$ *or* $P(\lvert z\rvert < z_\alpha) = 1-\alpha$ **Or** $\lvert z_{calculated}\rvert < \lvert z_\alpha\rvert$	$P(\lvert z\rvert > z_\alpha) = \alpha$

$H_1: \mu > \mu_0$ **Right tailed test**	$P(z < z_\alpha) = 1 - \alpha$ **Or** $z_{calculated} < z_\alpha$	$P(z > z_\alpha) = \alpha$
$H_1: \mu < \mu_0$ **Left tailed test**	$P(z > -z_\alpha) = 1 - \alpha$ **Or** $z_{calculated} > -z_\alpha$	$P(z < -z_\alpha) = \alpha$

$X_1, X_2, X_3, \dots, X_n$ be a sample from a population with mean μ and variance σ^2.

Critical values only for z-test

$z_\alpha \downarrow$ $\alpha \rightarrow$	1%	5%	10%
Two-tailed Test	± 2.58	± 1.96	± 1.645
Left tailed	-2.33	-1.645	-1.28
Right tailed	2.33	1.645	1.28

* $|Z_\alpha|$ for two-tailed test $= |Z_{\alpha/2}|$ for one-tailed test

Thus the significant or critical value of Z for a single-tailed test (left or right) at level of significance 'α' is same as the critical value of Z for a two-tailed test at level of significance '2α'.

Different regions for t-test will be in the form

	Critical region	Acceptance region
• For $H_1: \mu \neq \mu_0$,	$\lvert t\rvert > t_{\alpha,\nu}$	$\lvert t\rvert < t_{\alpha,\nu}$
• For $H_1: \mu < \mu_0$,	$t < -t_{\alpha,\nu}$	$t > -t_{\alpha,\nu}$

- **For H_1: $\mu > \mu_0$,** $\qquad\qquad$ $t > t_{\alpha,\nu}$ $\qquad\qquad$ $t < t_{\alpha,\nu}$

Tabulated value of t for ν degree of freedom at α level of significance for one-tailed test is the same as value of t for ν $d.f.$ at 2α level of significance for two tailed test.

Here ν represents degree of freedom.

- **Test of significance for single mean**
- **Test of significance for difference of means**

Test of significance for single mean $\;\boldsymbol{H_0: \mu = \mu_0}$

Z-Test for Single Mean

$$Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}}, \quad if\ \sigma\ is\ known$$

$$Z = \frac{\bar{X} - \mu}{s/\sqrt{n}}, \quad if\ \sigma\ is\ unknown,$$

$$Here\ s\ represents\ sample\ standard\ deviation.$$

t-Test for Single Mean: There are few assumptions for this test.

- The parent population from which the sample is drawn is normal.
- The sample observations are independent.
- The population standard deviation is unknown.

$$t = \frac{\bar{X} - \mu}{\frac{S}{\sqrt{n}}}\,, \qquad where\ S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$$

Also $(n-1)S^2 = ns^2$, so above can be written as $t = \frac{\bar{X}-\mu}{s/\sqrt{n-1}}$. $\nu = (n-1)$

Q1.

10.20 A random sample of 64 bags of white cheddar popcorn weighed, on average, 5.23 ounces with a standard deviation of 0.24 ounce. Test the hypothesis that $\mu = 5.5$ ounces against the alternative hypothesis, $\mu < 5.5$ ounces, at the 0.05 level of significance.

$H_0: \mu = 5.5$ $\quad n = 64, \bar{x} = 5.23, s = 0.24$

$H_1: \mu < 5.5$

z-test for single mean

Left tailed test, $\alpha = 5\%$

Critical Value $= -1.645$

$$Z = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{5.23 - 5.5}{0.24/\sqrt{64}} = -9$$

$Z_{cal} = -9$ $\quad Z_{critical} = -1.645$

$Z_{cal} < Z_{critical}$, H_0 is rejected.
H_1 is accepted.

Q2. The mean weekly sale of soap bars in departmental stores was 146.3 bars per store. After advertising campaign, the mean weekly sales in 22 stores for a typical week increased to 153.7 and showed a standard deviation of 17.2. Was the advertising campaign successful? [Given: $P\left((t_{21} > 1.721) = 0.05\right)$ or $P(|t_{21}| > 1.72) = 0.10$]

$H_0: \mu = 146.3$ $\quad n = 22, \bar{x} = 153.7, s = 17.2$

$H_1: \mu > 146.3$

t-test for single mean

Right tailed test, $\alpha = 5\%$, $d.f. = n - 1 = 21$

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n-1}} = \frac{153.7 - 146.3}{17.2/\sqrt{21}} = 1.971$$

$t_{cal} = 1.97$, $t_{critical} = 1.721$ at $\alpha = 5\%$, Right tailed, $d.f = 21$

$t_{cal} > t_{critical}$; H_0 is rejected.

Advertising campaign is successful.

Q3. A sample of 900 members has a mean 3.4 cms and s.d. 2.61 cm. Is the sample from population of mean 3.25 cms and s.d. 2.61 cms? If the population is normal and its mean is unknown, find the 95% and 98% fiducial limits of true mean.

$n = 900$, $\bar{x} = 3.4$, $\sigma = 2.61$

$\mu = 3.25$,

$H_0: \mu = 3.25$ z-test for single mean

$H_1: \mu \neq 3.25$ (Two-tailed Test $\alpha = 5\%$, $Z_\alpha = \pm 1.96$)

$$Z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{3.4 - 3.25}{2.61/30} = 1.724$$

$|Z_{cal}| < |Z_\alpha|$; H_0 is accepted at 5%.

If the population is normal and its mean is unknown, find the 95% and 98% fiducial limits of true mean.

μ lies in th range $\bar{x} \pm Z_\alpha \left(\frac{\sigma}{\sqrt{n}}\right)$

95% confidence interval = 5% level of signific

$-1.96 < \frac{\bar{x}-\mu}{\sigma/\sqrt{n}} < 1.96$, μ lies in range $\bar{x} \pm 1.96 \frac{\sigma}{\sqrt{n}}$

$\left(3.4 \pm (1.96)\frac{(2.61)}{30}\right) = (3.229, 3.571)$

98% confidence interval = 2% level of significance

$\left(3.4 \pm (2.33)\frac{(2.61)}{}\right) = (3.198, 3.603)$

Test of significance for difference of means $\quad \boldsymbol{H_0: \mu_1 = \mu_2}$

- **Z-Test for difference of means**

$$Z = \frac{\overline{x_1} - \overline{x_2}}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}, \qquad if\ \sigma_1^2, \sigma_2^2\ are\ known\ and\ \sigma_1^2 \neq \sigma_2^2$$

$$Z = \frac{\overline{x_1} - \overline{x_2}}{\sigma\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}, \qquad if\ \sigma_1^2, \sigma_2^2\ are\ known\ and\ \sigma_1^2 = \sigma_2^2 = \sigma^2$$

$$Z = \frac{\overline{x_1} - \overline{x_2}}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}, \qquad if\ \sigma_1^2, \sigma_2^2\ are\ unknown\ and\ \sigma_1^2 \neq \sigma_2^2$$

$$Z = \frac{\overline{x_1} - \overline{x_2}}{s\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}, \qquad where\ s^2 = \frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2} \qquad if\ \sigma_1^2, \sigma_2^2\ are\ unknown\ and\ \sigma_1^2 = \sigma_2^2$$

- **t-Test for Difference of Means**

Assumptions for test

- The parent population from which the sample is drawn is normal.
- The population variances are equal and unknown.
- The two samples are random and independent of each other.

$$t = \frac{\overline{x_1} - \overline{x_2}}{S\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}, \qquad where\ S^2 = \frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2}$$

$$\nu = (n_1 + n_2 - 2)$$

Q4. A storekeeper wanted to buy a large quantity of light bulbs from two brands labelled one and two. He bought 100 bulbs from each brand and found by testing that brand one had mean lifetime of 1120 hours and the standard deviation of 75 hours; and brand two had mean life time of 1062 hours and standard deviation of 82 hours. Examine whether the difference of means is significant.

$n_1 = 100 \quad \overline{x}_1 = 1120 \quad s_1 = 75 \quad H_0: \mu_1 = \mu_2$

$n_2 = 100 \quad \overline{x}_2 = 1062 \quad s_2 = 82 \quad H_1: \mu_1 \neq \mu_2$

Z-test for difference of means, (Two tailed Test)

$$Z = \frac{580}{\sqrt{5625 + 6724}} = 5.219, \quad |Z_{cal}| > |Z_\alpha|$$

H_0 is rejected.

There is significant diff. b/w mean lifetime of bulbs of two given brands

Q5. A reading test is given to an elementary school class that consists of 12 Anglo-American children and 10 Mexican-American children. The results of the test are:

Angelo American	Mexican American
$\overline{x_1} = 74$	$\overline{x_2} = 70$
$s_1 = 8$	$s_2 = 10$

Is the difference between the means of the two groups significant at the 0.05 level?
[Given: $P\left((|t_{20}| > 2.086) = 0.05\right)$]

$n_1 = 12$ $\quad \overline{x}_1 = 74$ $\quad s_1 = 8$ $\qquad H_0: \mu_1 = \mu_2$

$n_2 = 10$ $\quad \overline{x}_2 = 70$ $\quad s_2 = 10$ $\qquad H_1: \mu_1 \neq \mu_2$ (Two tailed test t-test $\alpha = 5\%$. d.f. = 20)

Two independent samples, d.f = $n_1 + n_2 - 2$

$$t = \frac{\overline{x}_1 - \overline{x}_2}{S\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}, \quad S^2 = \frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2}$$

$$t = \frac{74 - 70}{9.42\sqrt{\frac{1}{12} + \frac{1}{10}}} \qquad S^2 = \frac{12(64) + 10(100)}{20} = 88.4$$

$S = 9.42$

$t = 0.99$ $\qquad t_\alpha = 2.09$

$|t_{cal}| < |t_\alpha|$; H_0 is accepted

There is no significant diff b/w mean scores.

Q6. 50 pieces of two types of thread were tested under similar conditions. Type A thread had an average tensile strength of 86.7 kg with standard deviation of 6.28 kg, while type B thread had an average tensile strength of 77.8 kg with standard deviation of 5.61 kg. Is it reasonable to believe that average tensile strength of thread A exceeds tensile strength of thread B by at least 12 kg at 5% level of significance?

$n_1 = n_2 = 50$, $\bar{x}_1 = 86.7$, $\bar{x}_2 = 77.8$

$s_1 = 6.28$, $s_2 = 5.61$

$H_0: \mu_1 - \mu_2 = 12$

$H_1: \mu_1 - \mu_2 > 12$

(z-test for difference of means
Right tailed test, $\alpha = 5\%$)

$$z = \frac{\bar{x}_1 - \bar{x}_2 - 12}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{86.7 - 77.8 - 12}{\sqrt{\frac{(6.28)^2}{50} + \frac{(5.61)^2}{50}}} = -2.603$$

$z_{cal} = -2.603$, $z_{critical} = 1.645$

$z_{cal} < z_{critical}$, H_0 is accepted.

Paired t-test

When two samples are not independent but sample observations are paired together.

- Sample size is same.
- Sample observations are not independent.

$$H_0: \mu_1 = \mu_2$$

$$d_i = x_{i2} - x_{i1}$$

$$t = \frac{\bar{d}}{S_d/\sqrt{n}} = \frac{\bar{d}}{s_d/\sqrt{n-1}} \qquad , \nu = (n-1)$$

Q7. Eleven school boys were given a test in Statistics. They were given a month's tuition and a second test was held at the end of it. Do the marks give evidence that the students have benefited by the extra coaching?

Boys	1	2	3	4	5	6	7	8	9	10	11
Marks in	23	20	19	21	18	20	18	17	23	16	19

1st test											
Marks in 2nd test	24	19	22	18	20	22	20	20	23	20	18
$d_i = x_{i2} - x_{i1}$	1	-1	3	-3	2	2	2	3	0	4	-1

Given: $[P(t_{10} > 1.812) = 0.05]$

$H_0: \mu_Y = \mu_X$

$H_1: \mu_Y > \mu_X$

(Right tailed test, $\alpha = 5\%$)

Paired t-test, $d.f = 11 - 1 = 10$

$\bar{d} = 1.09$

$s^2 = E(d^2) - (E(d))^2$

$s^2 = 5.27 - (1.09)^2$

$s^2 = 4.08, \quad s = 2.019$

$$t = \frac{\bar{d}}{s/\sqrt{n-1}} = \frac{1.09}{2.019/\sqrt{10}} = 1.7072$$

$t_\alpha = 1.81$, $\quad t < t_\alpha$; H_0 is accepted.

There is no significant diff b/w marks before & after coaching.

Test of significance for difference of variance

- **F-Test**

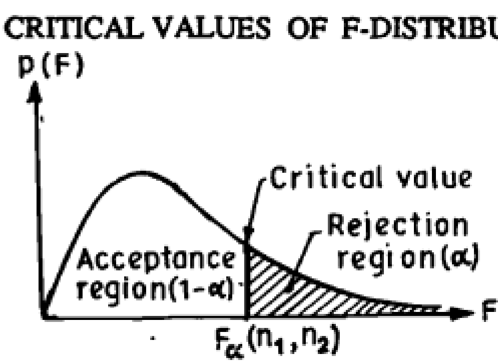
It is used to test the equality of variance of the populations from which two samples have been drawn.

$$F = \frac{{S_X}^2}{{S_Y}^2}$$

F follows F –distribution with $(v_X,\ v_Y)$ degree of freedom.

Some Important points to be taken care of

- $H_0: \sigma_X^2 = \sigma_Y^2$
- Under H_0, F is always 1. Our aim is to find how far any observed value of F can differ from unity as a result of sampling.
- Higher S is kept in numerator, so we should always make $F > 1$.
- Since F cannot be negative, so it is always right tailed test.
- If $F < F_{\nu_1,\nu_2}(\alpha)$, then the difference between σ_X^2 , ${\sigma_Y}^2$ is insignificant.
- For reading F-distribution table, degree of freedom corresponding to higher S will be seen in top row and degree of freedom corresponding to lower S will be seen in left column.



- The critical region for
  1. Left tailed alternative $\sigma_1^2 < \sigma_2^2$ is $F < F_{\nu_1,\nu_2}(1-\alpha)$.
  2. Right tailed alternative $\sigma_1^2 > \sigma_2^2$ is $F > F_{\nu_1,\nu_2}(\alpha)$.
  3. Two tailed test $\sigma_1^2 \neq \sigma_2^2$ is $F < F_{\nu_1,\nu_2}(1-\alpha/2)$, $F > F_{\nu_1,\nu_2}(\alpha/2)$.
- $F_{\nu_1,\nu_2}(\alpha) = \frac{1}{F_{\nu_2,\nu_1}(1-\alpha).}$

Q8. In one sample of 8 observations, the sum of the squares of deviations of the sample values from the sample mean was 84.4 and in other sample of 10 observations it was 102.6. Test whether this difference is significant at 5% level of significance. Given $F_{0.05}\ (7,\ 9) = 3.29,\ F_{0.05}(9,\ 7) = 3.68$.

$n_1 = 8 \qquad \sum_{i=1}^{8} (x_i - \bar{x})^2 = 84.4 \qquad H_0 : \sigma_1^2 = \sigma_2^2$

$n_2 = 10 \qquad \sum_{j=1}^{10} (y_i - \bar{y})^2 = 102.6 \qquad H_1 : \sigma_1^2 > \sigma_2^2$

(Right Tailed test)

$S_x^2 = \frac{84.4}{7} = 12.057, \quad S_y^2 = \frac{102.6}{9} = 11.4$

$\nu = 7 \rightarrow$ Row

$\nu = 9 \rightarrow$ Column

$F = \frac{12.057}{11.4}$

$F_{cal} = 1.057$

$F_{0.05}(7, 9) = 3.29$

H_0 is accepted

There is insignificant diff b/w Pop. variances.

Q9. Two random samples gave the following data:

	Size	Mean	Variance
Sample 1	8	9.6	1.2
Sample 2	11	16.5	2.5

Can we conclude that the two samples have been drawn from the same normal population? [Given $P(F_{(7,10)} > 3.29) = 0.05$, $P(F_{(10,7)} > 3.65) = 0.05$, $P(|t_{17}| > 2.11) = 0.05$]

$n_1 = 8, \quad \bar{x}_1 = 9.6, \quad s_1^2 = 1.2$

$n_2 = 11, \quad \bar{x}_2 = 16.5, \quad s_2^2 = 2.5$

$H_0: \sigma_1^2 = \sigma_2^2$

$H_1: \sigma_2^2 > \sigma_1^2$ (Right tailed test)

$$S_1^2 = \frac{n_1}{n_1-1} s_1^2 = \frac{8}{7}(1.2) = 1.371$$

$$S_2^2 = \frac{n_2}{n_2-1} s_2^2 = \frac{11}{10}(2.5) = 2.75$$

$$F = \frac{S_2^2}{S_1^2} = \frac{2.75}{1.371}$$

$$F = 2.006$$

Critical value: $F_{10,7}(5\%) = 3.65$, $F_{cal} < F_{critical}$

H_0 is accepted.

There is insignificant difference b/w Population variances.

So t-test is valid.

$H_0': \mu_1 = \mu_2$

$H_1': \mu_1 \neq \mu_2$ (Two-tailed test, $\alpha = 5\%$, d.f. $= 8+11-2 = 17$)

$$t = \frac{\overline{x}_1 - \overline{x}_2}{S\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}}, \quad S^2 = \frac{n_1 s_1^2 + n_2 s_2^2}{n_1+n_2-2}$$

$$S^2 = \frac{(8)(1.2) + (11)(2.5)}{17} = 2.182$$

$$S = 1.477$$

$$t = \frac{9.6 - 16.5}{1.477\sqrt{\frac{1}{8}+\frac{1}{11}}} = -10.053,$$

$t_{critical} = 2.11$ two-tailed test, $\alpha = 5\%$; d.f. = 17

$|t_{cal}| > |t_{critical}|$; H_0 is rejected

There is significant diff b/w means.

So samples are not taken for same normal population.

Q10. Pumpkins were grown under two experimental conditions. Two random samples of 11 and 9 pumpkins show s.d. of their weights as 0.8 and 0.5 resp. Assuming that the weight distributions are normal, test the hypothesis that true variances are equal, against the alternative that they are not, at 10% level. [Given $P(F_{10,8} \geq 3.35) = 0.05$ and $P(F_{8,10} \geq 3.07) = 0.05$]

Solution. We want to test *Null Hypothesis*, $H_0 : \sigma_X^2 = \sigma_Y^2$, against the *Alternative Hypothesis*, $H_1 : \sigma_X^2 \neq \sigma_Y^2$ (Two-tailed).

We are given :

$$n_1 = 11,\ n_2 = 9,\ s_X = 0.8 \text{ and } s_Y = 0.5.$$

Under the null hypothesis, $H_0 : \sigma_X = \sigma_Y$, the statistic

$$F = \frac{S_X^2}{S_Y^2}$$

follows F-distribution with $(n_1 - 1, n_2 - 1)$ d.f.

Now $$n_1 s_X^2 = (n_1 - 1) S_X^2$$

$\therefore$ $$S_X^2 = \left(\frac{n_1}{n_1 - 1}\right) s_X^2 = \left(\frac{11}{10}\right) \times (0.8)^2 = 0.704$$

Similarly, $$S_Y^2 = \left(\frac{n_2}{n_2 - 1}\right) s_Y^2 = \left(\frac{9}{8}\right) \times (0.5)^2 = 0.28125$$

$\therefore$ $$F = \frac{0.704}{0.28125} = 2.5$$

The significant values of F for two tailed test at level of significance $\alpha = 0{\cdot}10$ are :

$$\left.\begin{array}{r} F > F_{10,8}(\alpha/2) = F_{10,8}(0{\cdot}05) \\ \text{and } F < F_{10,8}(1-\alpha/2) = F_{10,8}(0{\cdot}95) \end{array}\right\} \qquad \ldots(*)$$

We are given the tabulated (significant) values :

$$P[F_{10,8} \geq 3{\cdot}35] = 0{\cdot}05 \quad \Rightarrow \quad F_{10,8}(0{\cdot}05) = 3{\cdot}35 \qquad \ldots(**)$$

Also $\quad P[F_{8,10} \geq 3{\cdot}07] = 0{\cdot}05 \quad \Rightarrow \quad P\left[\frac{1}{F_{8,10}} \leq \frac{1}{3{\cdot}07}\right] = 0{\cdot}05$

$$\Rightarrow \quad P[F_{10,8} \leq 0{\cdot}326] = 0{\cdot}05 \quad \Rightarrow \quad P[F_{10,8} \geq 0{\cdot}326] = 0{\cdot}95 \ldots(***)$$

Hence from (*), (**) and (***), the critical values for testing $H_0 : \sigma_X^2 = \sigma_Y^2$, against $H_1 : \sigma_X^2 \neq \sigma_Y^2$ at level of significance $\alpha = 0{\cdot}10$ are given by :

$$F > 3{\cdot}35 \quad \text{and} \quad F < 0{\cdot}326 = 0{\cdot}33$$

Since, the calculated value of F (=2·5) lies between 0·33 and 3·35, it is not significant and hence null hypothesis of equality of population variances may be accepted at level of significance $\alpha = 0{\cdot}10$.

Chi-Square ($\mathcal{X}^2$) Test

Uses of $\mathcal{X}^2$ distribution

- To test the goodness of fit.

Goodness of Fit Test

It is used to test the significance of the discrepancy between theory and the experimental values.

$$\mathcal{X}^2 = \sum_{i=1}^{n}\left[\frac{(f_i - e_i)^2}{e_i}\right], \qquad \sum_{i=1}^{n} f_i = \sum_{i=1}^{n} e_i$$

It follows chi-square distribution. Here n represents no. of classes.

If $\chi^2 < \chi_\nu^2(\alpha)$, then H_0 is accepted.

Some conditions for this test

- Individual frequencies must not be too small. In case $f_i < 5$, it is combined with the neighboring frequencies so that the combined frequency is ≥ 5.
- Degree of freedom $\nu = n$ −no. of constraints.

Q11. The following data give the number of air-craft accidents that occurred during the various days of week.

Day	Mon	Tues	Wed	Thu	Fri	Sat
Frequency	15	19	13	12	16	15

Test whether the accidents are uniformly distributed over the week.
Given $P(\chi^2 > 11.07) = 0.05$ at 5 degree of freedom.

Q12. A bird watcher sitting in a park has spotted a number of birds belonging to 6 categories. The exact classification is given below:

Category	1	2	3	4	5	6
Frequency	6	7	13	17	6	5

Test at 5% level of significance whether or not the data is compatible with the assumption that this particular park is visited by birds belonging to these six categories in the proportion 1:1:2:3:1:1.[Given that $P(\chi^2 \geq 11.07) = 0.05$ for 5 degree of freedom.

Category	f_i	e_i	$(f_i - e_i)^2/e_i$
1	6	6	0
2	7	6	1/6
3	13	12	1/12
4	17	18	1/18
5	6	6	0
6	5	6	1/6

$x+x+2x+3x+x+x=54$

$x=6$

$$\chi^2_{cal} = 0.472$$

$$\chi^2_{tab}(5\%, d.f = 5) = 11.07$$

H_0 is accepted.

Data is consistent with the hypothesis that birds of six categories are in the proportion 1:1:2:3:1:1.

Q13. Given below is the number of male and female births in 1000 families having five children:

No. of male births	0	1	2	3	4
No. of female births	5	4	3	2	1
No. Of families	40	300	250	200	30

Test whether the given data is consistent with the hypothesis that the binomial law holds if the chance of a male birth is equal to that of female birth.

No Male births	f_i	$e_i = \frac{1000}{32} \times 5c_r$
0	40	31.25
1	300	156.25
2	250	312.5
3	200	312.5
4	30	156.25
5	180	31.25

$$\text{Binomial} = {}^nC_r\, p^x q^{n-r}$$

$$e_i = 1000 \times {}^5C_r \left(\frac{1}{2}\right)^r \left(\frac{1}{2}\right)^{5-r}$$

$$e_i = 1000 \times {}^5C_r \times \left(\frac{1}{2}\right)^5$$

$$d.f. = 6-1 = 5$$

$$\chi^2_{cal} = \frac{(40-31.25)^2}{31.25} + \frac{(300-156.25)^2}{156.25} + \frac{(250-312.5)^2}{312.5}$$

$$+ \frac{(200-312.5)^2}{312.5} + \frac{(30-156.25)^2}{156.25} + \frac{(180-31.25)^2}{31.25}$$

$$\chi^2_{cal} = 2.45 + 132.25 + 12.5 + 40.5 + 102.01 + 708.05 = 997.76$$

$$\chi^2_{tab}(\alpha = 5\%,\ d.f. = 5) = 11.07 \quad , \quad \chi^2_{cal} > \chi^2_{tab}$$

H_0 is rejected, Male + Female births are not equally probable.

Q14. When the first proof of 392 pages of a book of 1200 pages was read, the distribution of printing mistakes were found to be as follows.

No. of mistakes in a page:	0	1	2	3	4	5	6
No. of pages:	275	72	30	7	5	2	1

Fit the Poisson distribution and test the goodness of fit.

[Given: $P(\chi^2 > 5.99) = 0.05$ for 2 degree of freedom.]

$$P(X = r) = \frac{e^{-\lambda t}\lambda t^{r}}{r!} \qquad e_i = 392 \times \frac{e^{-\lambda t}\lambda t^{r}}{r!}$$

X = no. of printing mistakes per page.

$r = 0, 1, \ldots, 6$

λt = average no. of mistakes per page. $\qquad \lambda t = \dfrac{\text{Total no of mistakes}}{\text{Total no of pages.}}$

$$\lambda t = \frac{0+72+60+21+20+10+6}{392} = 0.482$$

$$e^{-\lambda t} = e^{-0.482} = 0.6174$$

no of mistakes	f_i	$e_i = \frac{392 \times (0.6174) \times (0.482)}{x!}$
0	275	242.02 ✓
1	72	116.65 ✓
2	30	28.11 ✓
3	7	4.52
4	5	0.54
5	2	0.05
6	1	0.004
3–6	15	5.114 ✓

* $\sum f_i = \sum e_i$
* $\lambda t_e = \lambda t_o$
* If expected frequency goes below 5, then we merge frequencies to make more than 5.

$$\chi^2_{cal} = \frac{(275-241.86)^2}{241.86} + \frac{(72-116.58)^2}{116.58} + \frac{(30-28.09)^2}{28.09} + \frac{(15-5.104)^2}{5.104}$$

$$\chi^2_{cal} = 4.471 + 17.121 + 0.127 + 19.11 = 40.82$$

$$d.f = 4 - \underset{\downarrow}{1}_{\sum f_i = \sum e_i} - \underset{\downarrow}{1}_{\lambda t_o = \lambda t_e} = 2$$

$\chi^2_{cal} > \chi^2_{tab}$; H_0 is rejected.

Poisson distribution is not fitting the observed frequencies

Imp points

1.

$$Z = \frac{X-\mu}{\sigma}.$$

$$\chi^2 = \left(\frac{X-\mu}{\sigma}\right)^2.$$

$$\chi^2 = \sum_{i=1}^{n} Z_i^2$$

2. $$F = \frac{\chi_1^2/\nu_1}{\chi_2^2/\nu_2}$$

3.

14·3·2. Standard Error. The standard deviation of the sampling distribution of a statistic is known as its *Standard Error*, abbreviated as S.E. The standard errors of some of the well-known statistics, *for large samples*, are given below, where n is the sample size, σ^2 the population variance, and P the population proportion, and $Q = 1 - P$; n_1 and n_2 represent the sizes of two independent random samples respectively drawn from the given population (s).

S. No.	*Statistic*	*Standard Error*
1.	Sample mean : $\bar{x}$	$\sigma/\sqrt{n}$
2.	Observed sample proportion 'p'	$\sqrt{PQ/n}$
3.	Sample *s.d.* : s	$\sqrt{\sigma^2/2n}$
4.	Sample variance : s^2	$\sigma^2\sqrt{2/n}$
5.	Sample quartiles	$1{\cdot}36263\ \sigma/\sqrt{n}$
6.	Sample median	$1{\cdot}25331\ \sigma/\sqrt{n}$
7.	Sample correlation coefficient (r)	$(1-\rho^2)/\sqrt{n}$, ρ being the population correlation coefficient
8.	Sample moment : μ_3	$\sigma^3\sqrt{96/n}$
9.	Sample moment : μ_4	$\sigma^4\sqrt{96/n}$
10.	Sample coefficient of variation (v)	$\dfrac{v}{\sqrt{2n}}\sqrt{1+\dfrac{2v^3}{10^4}} \simeq \dfrac{v}{\sqrt{2n}}$
11.	Difference of two sample means : $(\bar{x}_1 - \bar{x}_2)$	$\sqrt{\dfrac{\sigma_1^2}{n_1}+\dfrac{\sigma_2^2}{n_2}}$
12.	Difference of two sample *s.d.*'s : $(s_1 - s_2)$	$\sqrt{\dfrac{\sigma_1^2}{2n_1}+\dfrac{\sigma_2^2}{2n_2}}$
13.	Difference of two sample proportions : $(p_1 - p_2)$	$\sqrt{\dfrac{P_1Q_1}{n_1}+\dfrac{P_2Q_2}{n_2}}$

Utility of Standard Error. S.E. plays ...
theory and forms th...

Practice questions

Q15. The nicotine content of two samples of tobacco were found to be as follows:

Sample A: 24 27 26 21 25

Sample B: 27 30 28 31 22 36

Can it be said that two samples come from the same normal population?

Q16. The scores of 10 candidates prior and after training are given below:

Prior: 84 48 36 37 54 69 83 96 90 65

After: 90 58 56 49 62 81 84 86 84 75 Is the training effective?

Q17. Samples of students were drawn from the two universities and from their weight in kg, mean and standard deviations are calculated. Test the significance of the difference between the means.

	Mean	S.D.	Sample Size
University A	55	10	400
University B	57	15	100

Q18. Among 64 off springs of a certain cross between guinea pigs, 34 were red, 10 were black and 20 were white. According to genetic model these numbers should be in the ratio 9:3:4. Are the data consistent with model at 5% level of significance?

Q19. A sample of 400 students is found to have a mean height of 67.47 inches. Can it be reasonably regarded as a sample from a large population with mean height 67.39 inches and S.D 1.3 inches?

Q20. In two groups of ten children each increases in weight due to two different diets in the same period, were in kg.

Group A: 8 5 7 8 3 2 7 6 5 7

Group B: 3 7 5 6 5 4 4 5 3 6

Find whether the variances are significantly different.

Q21. The 9 items of a sample have the following values

45, 47, 50, 52, 48, 47, 49, 53, 51

Test the hypothesis that the mean of population is 47.5

Q22. Two independent groups of 10 children were tested to find how many digits they could repeat from memory after hearing them. The results are as follows:

Group A	8	6	5	7	6	8	7	4	5	6
Group B	10	6	7	8	6	9	7	6	7	7

Is the difference between the mean scores of the two groups significant?