Test of Significance for mean

(i) Z-Test (For larger value of n, almost all the distributions are very closely approximated by normal distribution, $n > 30$**)**

(ii) Student's t-Test (This test is applicable when sample size is small**)**

1. **Test of significance for single mean**
2. **Test of significance for difference of means**

1. **Test of significance for single mean** $\boldsymbol{H_0: \mu = \mu_0}$

- **Z-Test for Single Mean**

$$Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}}, \quad if\ \sigma\ is\ known$$

$$Z = \frac{\bar{X} - \mu}{s/\sqrt{n}}, \quad if\ \sigma\ is\ unknown,$$

$$Here\ s\ represents\ sample\ standard\ deviation.$$

- **t-Test for Single Mean**: There are few assumptions for this test.
  - The parent population from which the sample is drawn is normal.
  - The sample observations are independent.
  - The population standard deviation is unknown.

$$t = \frac{\bar{X} - \mu}{\frac{S}{\sqrt{n}}}, \quad where\ S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$$

Also $(n-1)S^2 = ns^2$, so above can be written as $t = \frac{\bar{X}-\mu}{s/\sqrt{n-1}}$. $\nu = (n-1)$

2. Test of significance for difference of means $\boldsymbol{H_0: \mu_1 = \mu_2}$

- **Z-Test for difference of means**

$$Z = \frac{\overline{x_1} - \overline{x_2}}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}, \quad if\ \sigma_1^2, \sigma_2^2\ are\ known\ and\ \sigma_1^2 \neq \sigma_2^2$$

$$Z = \frac{\overline{x_1} - \overline{x_2}}{\sigma\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \,, \qquad if\ \sigma_1^2\,, \sigma_2^2\ \ are\ known\ and\ \sigma_1^2 = \sigma_2^2 = \sigma^2$$

$$Z = \frac{\overline{x_1} - \overline{x_2}}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} \,, \qquad if\ \sigma_1^2\,, \sigma_2^2\ \ are\ unknown\ and\ \sigma_1^2 \neq \sigma_2^2$$

$$Z = \frac{\overline{x_1} - \overline{x_2}}{s\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \,, \qquad where\ s^2 = \frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2} \qquad if\ \sigma_1^2\,, \sigma_2^2\ \ are\ unknown\ and\ \sigma_1^2 = \sigma_2^2$$

- **t-Test for Difference of Means**

  Assumptions for test
  - The parent population from which the sample is drawn is normal.
  - The population variances are equal and unknown.
  - The two samples are random and independent of each other.

$$t = \frac{\overline{x_1} - \overline{x_2}}{S\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \,, \qquad where\ S^2 = \frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2}$$

$$\nu = (n_1 + n_2 - 2)$$

- **Paired t-test**

When two samples are not independent but sample observations are paired together.

- Sample size is same.
- Sample observations are not independent.

$$H_0 : \mu_1 = \mu_2$$

$$d_i = x_{i2} - x_{i1}$$

$$t = \frac{\bar{d}}{S_d/\sqrt{n}} = \frac{\bar{d}}{s_d/\sqrt{n-1}} \qquad , \nu = (n-1)$$