FRICTION

- It is a contact force which opposes relative motion.
- Static fric^n — the opposing force which comes into play when body tends to move over the surface of another but the actual motion has not yet started. It is the self adjusting force.
- Limiting fric^n — the maximum value of static fric^n upto which body does not slide.
- Kinetic fric^n — the fric^n acting on a body in motion. $[\mu_{static} > \mu_{kinetic}]$

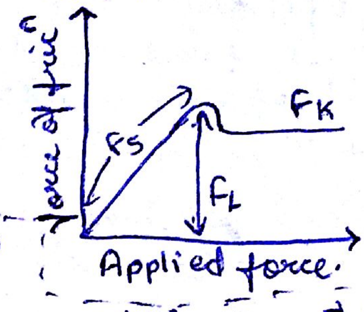
  Fric^n = μN [N — normal reac^n]

Note — check whether opposing force is less than or equal to static fric^n. — if it is less than opposing force itself is the fric^n's value.

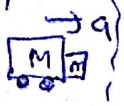
- Angle of fric^n is angle b/w fric^{n}al force & normal rx^n.
- If force or applied $F < F_L$ Body at rest.

  $F = F_L$ Body is just to slide

  $F > F_L$ Body start slide with some accelera^n

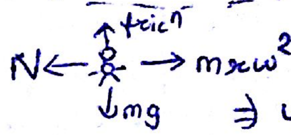


- Sticking of body (m) with accelerating cart (M),



$a_{min} = \frac{g}{\mu}$ & $F_{min} = (M+m)\frac{g}{\mu}$

- Sticking of person with wall of rotor,



$\Rightarrow \mu N = mg$

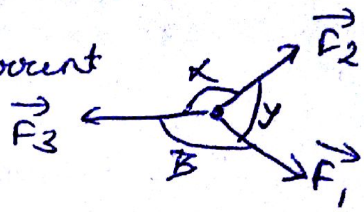
$\Rightarrow \mu m r \omega^2 = mg$

$\Rightarrow \omega_{min} = \sqrt{\frac{g}{\mu r}}$

LAWS OF MOTION

- Turning effect or torque is known as moment of force [ACW = +ve CW = -ve]
  $\tau = F r_\perp$ or $F_\perp r$ (where $r_\perp$ is perp. distance from axis of rotaⁿ)
- Coplanar forces is in eqb if $\Sigma \tau_{any\,axis} = 0$.
- Lami's theorem → an object is in eqb. under three concurrent force F_1, F_2, F_3 as in fig.

[Inertial frame, $a = 0$]



$$\frac{F_1}{\sin\alpha} = \frac{F_2}{\sin\beta} = \frac{F_3}{\sin\gamma}$$

- NLM → ① A body continues in its initial state of rest or motion with uniform velocity unless acted on by an unbalanced ext. force

② The rate of change of linear momentum of a body is directly proportional to the ext. force applied on the body & this change takes place in the direcⁿ of the applied force. $\left[\vec{F} \propto \frac{dP}{dt}\right]$

③ Force always occurs in pair, to every action their is always equal & opp. reacⁿ.

- $1N = 1Kg\text{-}m/s^2$ & $1\,dyne = 1\,gm\,cm/sec^2$ & $1N = 10^5$ Dyne & $1Kg\text{-}f = 9.8N$ & $1gm\text{-}f = 980$ dyne
- WORKING WITH NEWTON'S 1st & 2nd laws –
  → decide the system [it may be combinaⁿ of 2 or more block] they must have same acceleraⁿ.
  → make the list of all forces acting on the system [Indicate with magnitude & direcⁿ]
  → choose two mutually ⊥ axis in which is known or likely to have acceleraⁿ.

Note – If the string is having some mass, tension in it is diff. at diff. pts

- acceleraⁿ of the system is, $a = \dfrac{\text{Net. pulling force}}{\text{Total mass to be pulled.}}$

[Tension always occur in pair, it is included only when system is broken at one place.]

- Constraint – writing eqⁿ for sum of length from block to be constant & diff. – erentiating or (Tension × vel of block's & summation.) = 0.
- for 'm' not to be constant. $F = \frac{dP}{dt} = \frac{m\,dv}{dt} + \frac{v\,dm}{dt}$
- Pseudoforce – To apply newton's law in non inertial frame an imaginary force in the opp. direcⁿ of acceleraⁿ is applied (ma)
- Conservative force – If under action of force workdone in round trip is zero, path independent, then force is said to be conservative, otherwise non conservative.
- Impulse (I) → $I = \vec{F}t = \vec{P_f} - \vec{P_i}$ & Instanstaneous, $I = \int F\,dt = \Delta P$
  In F-t curve, Impulse is area b/w curve & time axis.
- Weighing machine measures apparent weight of body [or normal reacⁿ]
- Condiⁿ of body to be at rest relative to the inclined plane is acceleraⁿ = $g\tan\theta$ of wedge.
- $F_g : E_w : F_e : F_n = 1 : 10^{25} : 10^{36} : 10^{38}$

⊛ $F = ma = v\left(\frac{dm}{dt}\right)$

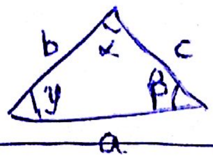
VECTOR ⇒ $A \cdot B = |A||B|\cos\theta$ \$ $A \cdot B = 0$ if vectors are perpen.

$A \times B = |A||B|\sin\theta\,\hat{n}$ \$ $A \times B = 0$ for $\parallel^{es}$ vector

$R = \sqrt{a^2+b^2+2ab\cos\theta}$ \$ $\alpha = \frac{B\sin\theta}{A+B\cos\theta}$ $[R = |A+B|]$ Note - for $|A-B|$ $\cos\theta$ becomes $\cos(180-\theta) = -\cos\theta$.

$R_{max} = |P+Q|$ \$ $R_{min} = |P-Q|$ \$ $|\vec{A}\times\vec{B}| = |\vec{A}-\vec{B}|$ ~~for~~ angle b/w them is $90°$.

• $|A \cdot B| = |A \times B|$ for $\theta = 45°$

• $\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c}$  ⇒ Lami's theorem.

KINEMATICS ⇒ • slope of distance - time graph = velocity

→ slope of v-t graph = accelera^n.

• $V = U + at$ • $S = \left(\frac{U+V}{2}\right)t$

• $S = ut + \frac{1}{2}at^2$

• $V^2 = U^2 + 2as$ • $\vec{S}_n = U + \frac{a}{2}(2n-1)$

• area under v-t graph - distance/displacement

• area under a-t graph - velocity.

• If a body starts from rest, having uniform accelera^n -

→ ratio of distance covered in 1s, 2s, 3s ... $= 1^2 : 2^2 : 3^2 : \dots$

→ ratio of dist^n covered in 1^{st}, 2^{nd} sec ... $= 1 : 3 : 5$

→ ratio of time in falling successive 1m $= \sqrt{1} : \sqrt{2}-\sqrt{1} : \sqrt{3}-\sqrt{2} \dots$

Eq^n of trajectory of projectile motion ⇒ $y = x\tan\theta - \frac{1}{2}\frac{gx^2}{u^2\cos^2\theta}$

• $H_{max} = \frac{v^2\sin^2\theta}{2g}$, $T = \frac{2v\sin\theta}{g}$, $H \cdot R = \frac{v^2\sin 2\theta}{g}$

⇒ Circular motion ⇒ $\Delta r = 2r\sin\theta/2$, $\Delta V = 2V\sin\theta/2$

• $S = r\theta$ [S = arc, θ = angular displacement]

• $V = r\omega$ [ω = angular velocity] $\omega = \frac{\theta}{t} = \frac{2\pi}{T}$

• $a = r\alpha$ [α = ang. accelera^n]

• $a_c = \frac{V^2}{R} = \omega^2 r$

WORK, ENERGY, POWER

- Work done by const. force $= \vec{F}\cdot\vec{s} = Fs\cos\theta$. [dot product] may be +ve, −ve, 0.
- Work done by variable force $\Rightarrow W = \int \vec{F}\cdot d\vec{s}$ [$1J = 1N \times 1m = 10^5 \text{dyne} \times 10^2 \text{cm} = 10^7 \text{erg}$]
- Work done = area under force displacement graph. | $1 e\cdot v = 1.6 \times 10^{-19} J$
- It is conservative force independent of path. | $1 \text{cal} = 4.18 J$ / $1 kWh = 3.6 \times 10^6 J$
- Spring force $\Rightarrow Kx$ [direc^n is decided (or sign) by direc^n of force \$ displacement]
- Work done $= \frac{1}{2}Kx^2$
- Work energy theorem $\Rightarrow$ work done $= \Delta KE = (KE)_f - (KE)_i$ $\left[KE = \frac{P^2}{2m}\right]$
- Potential energy $\Rightarrow U = -\int_{\infty}^{r} \vec{F}\cdot d\vec{r} = -W$
- Elastic PE $\Rightarrow U = \frac{1}{2}Kx^2$
- Gravito^{n}al PE $\Rightarrow U = -G\frac{m_1m_2}{r^2}$ \$ $\Delta U = \frac{mgh}{\left(1+\frac{h}{R}\right)}$ \$ $\left(1+\frac{h}{R}\right) \sim 1$ for $h <<<< R$
- $U_f - U_i = -W = (K_f - K_i) \times -1$
  $\Rightarrow U_f + K_f = U_i + K_i$ $[E = K + U]$
- Total mechanical energy is conserved. if $W_{\text{non conservative}} = 0$.
- $W_{\text{conservative}} + W_{\text{Non cons.}} + W_{\text{ext}} = K_f - K_i$ or $W_{nc} + W_{ext} = \Delta E_{\text{Total}}$

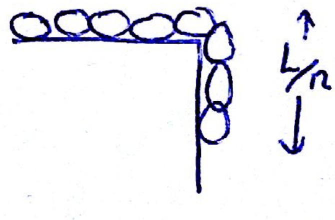
Stable	PE min.	$\frac{dU}{dx} = 0$	$\frac{d^2U}{dx^2} = +ve$	
Unstable	PE max.	$\frac{dU}{dx} = 0$	$\frac{d^2U}{dx^2} = -ve$	
Neutral	PE const	$\frac{dU}{dx} = 0$	$\frac{d^2U}{dx^2} = 0$	

- Work done in pulling a chain against gravity if $\frac{1}{n}$th part is hanging.
  $W = \frac{MgL}{2n^2}$ \$ Vel. while leaving table $= \sqrt{gL\left(1-\frac{1}{n^2}\right)}$
- Power $\Rightarrow P = \frac{dW}{dt} = \vec{F}\cdot\vec{V} = FV\cos\theta$.
- Work done = area under ~~of~~ P−T curve.

$1 hp = 746$ watt
$1 MW = 10^6$ watt
$1 KW = 10^3$ watt

- In circular motion, W_{Tension} \$ $W_{\text{centripetal force}} = 0$, as force is $\perp$ to disp. $d\vec{s}$, hence W \$ $P = 0$
- $s = \frac{v^2}{2\mu g}$ [stopping distance]

$W = \frac{mgL}{2n^2}$



- $W = U = \frac{F^2}{2K}$ (in spring)

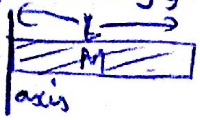
$1 \text{cal} = 4.2 J$

- $P^2 = 2m(KE)$ or $KE = \frac{P^2}{2m}$
- $\Delta P = F \times t$ = Impulse.
- Power $= F\cdot V = \frac{W}{t}$ $\left[P_{\text{pump or KE}} = \frac{1}{2}\rho A v^3\right]$ $v \to$ vel. of flow.
- $h_n = h\cdot e^{2n}$
- Dist. travelled before rebounding has stopped: $h\left(\frac{1+e^2}{1-e^2}\right)$

- $\frac{\Delta K}{K} = \left[1 - \frac{(m_1 - m_2)^2}{(m_1 + m_2)^2}\right]$
- When target is very light & at rest then after head on elastic collision it moves with double speed of projectile.
- In elastic collision of bodies having equal mass their vel. gets interch-
-anged

Rotational Motion.

- Motion of a rigid body ~~passing~~ about an axis passing through it or outside it is called rotational motion.
- $KE = \frac{1}{2} I \omega^2$ where, $I = \Sigma m_i r_i^2$ [moment of inertia] It give us distri-bu^n of mass ⊥ to axis of rotat^n.
- for rigid body, $I = \int r^2 \, dm$ | • Ring [⊗ = axis] | • Disc.
- Rod ⇒

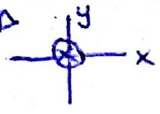


$I = \frac{mL^2}{3}$

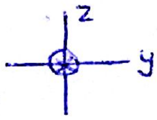
$I = mR^2$

$I = \frac{mR^2}{2}$

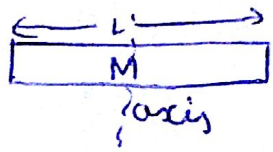
- Parallel axis theorem ⇒ $I = I_{cm} + mx^2$
  - I_{cm} → about com
  - x → dist. b/w two parallel axis
- Perp. axis theorem ⇒



$I_z = I_x + I_y$



$I_x = I_y + I_z$



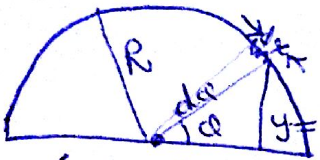
$\frac{mL^2}{12} = I$

- TORQUE → moment of force. $\vec{\tau} = \vec{r} \times \vec{F}$ [$\vec{\tau} \perp \vec{r}$ & $\vec{\tau} \perp \vec{F}$] $\tau = I\alpha$
  $= \vec{r} \vec{F} \sin\theta$,
  [ACW → +ve & CW → −ve]
- Moment arm or $r = \frac{\tau_{net}}{|F_{net}|}$ | $\tau \propto \frac{dL}{dt}$ L = ang. momentum
- $L = m v r_\perp$
- $\vec{L} = \vec{r} \times \vec{P}$ or $I\omega = L$ | for $\tau_{ext} = 0$, $L_i = L_f$
- $\vec{L} = m(\vec{r} \times \vec{v})$

Center of Mass

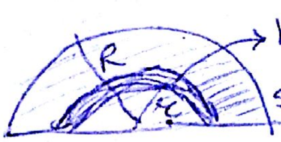
- It is the coordinate where whole mass of body is assumed to be confined.
- It is independent of origin
- $r_{cm} = \frac{\Sigma m_i r_i}{\Sigma m_i}$ [com does not change if net force on a body is zero or if no ext force acts on body]
- $r_{cm} = \frac{\int r dm}{\int dm}$

- Semi circular ring



cm $\left(0, \frac{2R}{\pi}\right)$

- Semi Circular disc



C.M $\left(0, \frac{4R}{3\pi}\right)$

- $X_{cm} = \frac{A_1 X_1 - A_2 X_2}{A_1 - A_2}$

$A \rightarrow$ area

$-ve$ sign = removed area.

$\Rightarrow$ v_{cm} & a_{cm} could be found by differentiating. r_{cm}.

Rocket: $v = v_0 - gt + v_r \ln\left(\frac{m_0}{m}\right)$ & $a = \frac{v_r dm}{m dt} - g$.

velocity = v

- $F_{thrust} = -V_r \frac{dm}{dt}$ [$\frac{dm}{dt}$ = rate of ejecⁿ of gases. V_r = vel. of ejecⁿ of gases w.r.t rocket] | mass rocket = m_0, after t time = m

- Mass density = mass per unit length,

- Uniform sphere, $I = \frac{2}{5} mR^2$

& spherical shell (I) $= \frac{2}{3} mR^2$

- Uniform right cone

$I = \frac{3}{10} mR^2$

[$I = \frac{mR^2}{2}$ & for half, $I = \frac{mR^2}{4}$]

- Radius of gyraⁿ (K) $\Rightarrow I = mK^2$

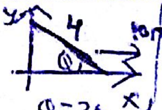
$\boxed{\omega_c = 2\omega_0}$

$x = 4\cos\theta$

$\frac{dx}{dt} = -4\sin\theta \frac{d\theta}{dt}$

$v = -4\sin\theta\, \omega$

$\omega = \frac{v}{-4\sin\theta} = \frac{-5}{2 \times \frac{1}{2}} = -5\,rad/sec$



then $I = \frac{ml^2 \sin^2\theta}{3}$

- $\alpha_{avg.} = \frac{\omega_f - \omega_i}{t_f - t_i}$

($L_{cm} = I\omega$)

- $\vec{L}_{about\ O} = \vec{L}_{com} + M(\vec{r}_0 \times \vec{v}_0)$

FLUID MECHANICS

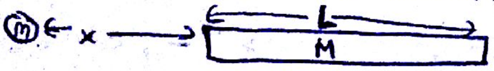
- Ideal lqd : Incompressible, non viscous in nature., $\rho = \frac{m}{V}$, R.D (or specific gravity) $= \frac{d_{subs.}}{d_{water}\,(at\ 4^\circ C)}$ → unitless
- $d_{water} = 1 g/cm^3$ or $1000 kg/m^3$
- for two lqd of densities ρ_1 & ρ_2 having masses m_1 & m_2 are mixed, $\rho = \frac{m}{V} = \frac{m_1 + m_2}{\frac{m_1}{\rho_1} + \frac{m_2}{\rho_2}}$ $\left[m_1 = m_2 \quad \frac{2\rho_1\rho_2}{\rho_1 + \rho_2} \right]$
- two lqd of density ρ_1 & ρ_2 with vol V_1 & V_2 are mixed, $\rho = \frac{m}{V} = \frac{\rho_1 V_1 + \rho_2 V_2}{V_1 + V_2}$ $\left[V_1 = V_2 \quad \rho = \frac{\rho_1 + \rho_2}{2} \right]$
- $\rho' = \frac{\rho}{1 + \gamma \Delta \theta}$ eff of temp on ρ, $\rho' = \frac{\rho}{1 - \frac{dP}{B}}$ eff. of press on ρ | $P = P_0 + \rho g h$ (abs press; P_0 → atm. press; $\rho g h$ gauge press) | Pascal's law, $P_1 = P_2$, $\frac{F_1}{A_1} = \frac{F_2}{A_2}$
- variaⁿ of press with depth :- $\frac{dP}{dy} = -\rho g$. | Press diff in accelerating fluid : $\frac{dP}{dx} = -\rho a_x$, $\frac{dP}{dy} = -\rho(g + a_y)$
- $\tan\theta = \frac{a}{g}$ | Archimedes principle BF $= V_{immersed}\, \rho_{lqd}\, g$. | App. weight : $Vg(\rho_S - \rho_L)$ | eqⁿ of continuity – $Av = const$
- Bernoulli's theorem : $P + \rho g h + \frac{1}{2}\rho v^2 = constant$ (Joule/m³)

$$\underbrace{\frac{P}{\rho g}}_{\text{press head}} + \underbrace{h}_{\text{gravitaⁿal head}} + \underbrace{\frac{v^2}{2g}}_{\text{vel. head}} = const$$

GRAVITATION

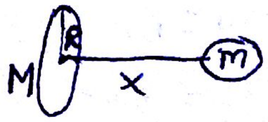
- Net gravitational force in a system is always zero. so syst. is in eqb. → center of mass does not change. ex- solar system

- NEWTON'S LAW OF GRAVITA^N ⇒ $F = \frac{G m_1 m_2}{r^2}$ [eq. & opp.] ⇒ always attractive.

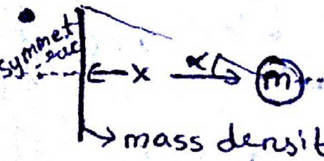
- 

⇒ $F = \frac{GMm}{x(x+L)}$

- B/w circular ring & pt mass on its axis at dist. 'x' ⇒



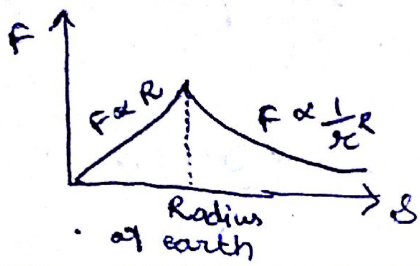
$F_{net} = \frac{GmMx}{(x^2+R^2)^{3/2}}$

- 

$F_{net.} = \frac{2GM\lambda \sin\alpha}{x}$

- Infinite length wire $= \frac{2GM\lambda}{x}$
- Semi circular $= \frac{2GM\lambda}{x}$
- $\frac{1}{4}^{th}$ circle $= \frac{\sqrt{2}\, GM\lambda}{x}$

- If mass is given -ve, then force is repulsive.
- Shell's theorem: If a pt lies on the surface of the spherical or cylindrical body or outside it then for these pts behave as pt mass.

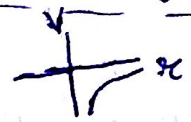
- 

$F \propto R$ till surface of earth

$F \propto \frac{1}{r^2}$ above surf. of earth

- Mass density = $\frac{\text{Mass}}{\text{Length}}$

V due to pt mass $= \frac{-GM}{r}$



- Gravitational field: space around pt mass. where a gravitational force can be experienced by other pt mass when placed in it.
- Gravitational field strength (E): It is defined as gravitational force per unit mass. (Vector quantity)

I or $E = \frac{F}{M} = \frac{Gm}{r^2}$ [for pt mass]

- Gravitational flux (ϕ) - no of gravitational field lines passing through an unit area perpendicularly. $\phi = \vec{E}\cdot\vec{dA} = \vec{E}\cdot\vec{dA}$

dA ⇒ cross sectional area.

- $\vec{E}\cdot \vec{ds} = 4\pi G m_0$ ⇒ E = gravitational field. ; ds = surface area of imaginary surface.

m_0 = mass inside imaginary surface

- $F = mE$
- Gravitational potential energy: $U = -W = -MEdx$ (h is height from surf. of earth) $\boxed{\Delta U = \frac{mgh}{1+h/R} = \frac{1}{2}mv^2}$
- Gravitational potential: gravitational P.E per unit mass. (V) $V = -\int I\,dx$
- $W = m\Delta V$
- Equipotential surface: these are the imaginary surface at which the potential remain same everywhere.

- E or $I = -\frac{dV}{dr}$

dv = potential diff. of two equipotential surface.

dr = perp. dist. b/w two equipotential surface.

- Effect of 'g' with height $\Rightarrow g' = g\left(\frac{R}{R+H}\right)^2$ & $g' = g\left(1-\frac{2h}{R}\right)$ for ~~h<<R~~ $h<<<R$
- Effect of 'g' with ~~height~~ depth $\Rightarrow g' = g\left(1-\frac{d}{R}\right)$ $\left[\frac{GMm}{r^2} = \frac{mv^2}{r}\right]$
- Effect of rota^n of earth $\Rightarrow g' = g - \omega^2 R \sin^2\theta$ { max^m – pole (0°), min^m → eq. (90°) }
- Self energy : Energy req. for forma^n of any rigid body. $[U = \int V\,dm]$
- satellite → body which revolve around the planets.
- Orbital velocity → $V = \sqrt{\frac{GM}{r}} \Rightarrow V \propto \frac{1}{\sqrt{r}}$ (r = dist from center of earth to orbit)

$V = \sqrt{\frac{GM}{R}} = \sqrt{gR} = 7.9\ Km/s$ $[r \sim R]$ (near earth's surface)

- Time period $\Rightarrow T = 2\pi\sqrt{\frac{r^3}{GM}}$ or $T = 84.6$ min for $r \sim R$

↳ min time period for any satellite.

- Energy → $PE = \frac{-GMm}{r}$, → $KE = \frac{GMm}{2r}$

∴ Total energy of satellite $= \frac{-GMm}{2r}$.

- Escape velocity → $V_e = \sqrt{\frac{2GM}{R}} = \sqrt{2gR} = 11.2\ Km/s$ for earth
- Geostationary satellite is launched from equatorial plane [T = 24hrs]
- $V_{escape} = \sqrt{2}\, V_{orbital}$
- Kepler's. Law :– ① All the planet revolve around sun in on elliptical orbit & the sun is at one of focci.

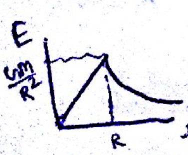
[In general case :– $g = \frac{4}{3}\pi G \rho R$]

② The radius vector joining sun & the planet cover equal area in equal interval of time. $\frac{dA}{dt} = \frac{L}{2m}$ (L → ang. momen, m → mass of planet)

③ When the planet move around the sun then the square of time period is proportional to the cube of semi major axis $T^2 \propto r^3$

for solid sphere, $E_i = \frac{GM}{R^3} r$

$E_o = \frac{GM}{r^2}$



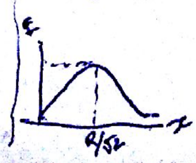
spherical shell

$E_i = 0$

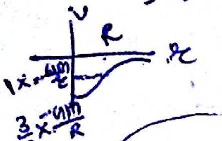
$E_o = \frac{GM}{r^2}$

axis of ring

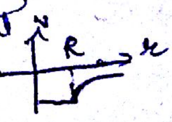
$E_x = \frac{GMx}{(R^2+x^2)^{3/2}}$



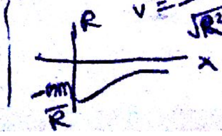
solid sphere, $V = \frac{-GM}{2R^3}(3R^2 - r^2)$



spherical shell



axis of ring $V = \frac{-GM}{\sqrt{R^2+x^2}}$



Simple harmonic Motion

$\left[F = -m\omega^2 x = -Kx, \quad K = m\omega^2 \text{ elastic const.}\right]$

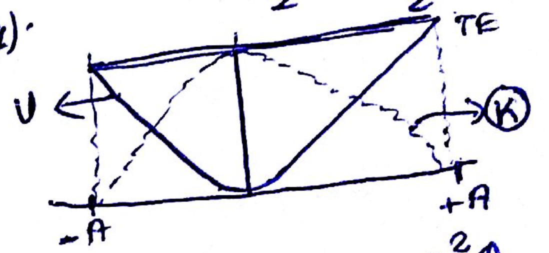
- $F = -Kx^n$, n is even no, force is always along $-ve$ x axis.
  , n is odd no, oscilla^n. about eq^b posi^n $x=0$.
  $n=1$, SHM

- $x = A\cos\underbrace{(\omega t + \phi)}_{\text{phase angle}}$, ϕ = initial phase at $t=0$, $T = \dfrac{2\pi}{\omega}$, $\nu = \dfrac{1}{T} = \dfrac{\omega}{2\pi}$

- $v = +\dfrac{dx}{dt} = -\omega A\sin(\omega t+\phi) \rightarrow \omega\sqrt{A^2-x^2}$, $a = \dfrac{dv}{dt} = -\omega^2 A(\cos\omega t+\phi) = -\omega^2 x$

- $K = \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2(A^2-x^2) = \frac{1}{2}K(A^2-x^2)$ $\Big\}$ • $F = -\dfrac{dU}{dx} = -Kx$, $\int dU = \int Kx\,dx$
  $U = \frac{1}{2}Kx^2 = \frac{1}{2}m\omega^2x^2$

- $T.E = \frac{1}{2}KA^2$ = does not oscillate (K $ U oscillate with 2ω freq).



- $\dfrac{d^2x}{dt^2} + \omega^2 x = 0$ [this eq^n's solu^n is sine or cosine fn of ωt]
  a

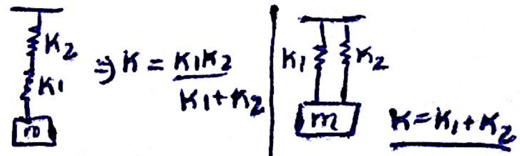
- find net force on displaced object : $\dfrac{F}{m} \Rightarrow -\omega^2 x = a$, $\alpha = -\omega^2\theta$
  Find ω then, $\omega = \dfrac{2\pi}{T}$ ✓

- or find total mechanical energy of sys ($\because$ const, in SHM) $\Rightarrow \dfrac{dE}{dt} = 0$
  $E \Rightarrow$ GPE + EPE + Elastic PE + Rota^{n}al KE, transla^n KE

- simple pendulum : $T = 2\pi\sqrt{\dfrac{l}{g}} = 2\pi\sqrt{\dfrac{l}{g}\left(1+\dfrac{\theta_0^2}{16}\right)}$ { if $T = 2$ sec $\rightarrow$ second pendulum

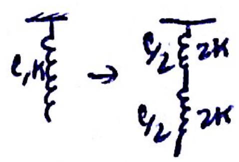
  if length of pendulum is large, $T = 2\pi\sqrt{\dfrac{1}{g\left(\frac{1}{l}+\frac{1}{R}\right)}}$ $(g \rightarrow g_{eff.})$

- Springblock system : $T = 2\pi\sqrt{\dfrac{m}{K}}$
  $m \rightarrow$ reduced mass.
  $K \rightarrow$ eqv force const.



If spring has mass m_s and block's mass m, then —
$T = 2\pi\sqrt{\dfrac{m+\frac{m_s}{3}}{K}}$

$K \propto \dfrac{1}{l}$



KTG & Thermodynamics

- conducⁿ: Kernel vibraⁿ + free e^- [mainly in solid]
- convecⁿ: Density diff. (in fluids)
- Radiaⁿ: e^- transiⁿ in solids
- Assumptions of KTG — force of attracⁿ/repulsion among molecules (a) = 0
  - Vol. or size of molecule of gas (b) = 0
  - Uniform particle & vel. distribuⁿ [statistics Probability]
  - All molecules of gas follows NLM.
  - time of collision = 0. Also, energy loss = 0 } forces
  - $V_x = V_y = V_z$ [Equivalence of axes] } Force + Probability.
    No directed force
- Pressure $= \dfrac{\rho V_{rms}^2}{3}$ | V_{rms} (root mean square) $= \sqrt{\dfrac{v_1^2 + v_2^2 + \cdots + v_n^2}{N}}$ | $V_{ms} = \dfrac{v_1^2 + v_2^2 + \cdots + v_n^2}{N}$.
- $V_{rms} > V_{avg} > V_{mps}$. [R > A > M]
  $\sqrt{\dfrac{3RT}{M}}$ $\quad \sqrt{\dfrac{8RT}{\pi M}}$ $\quad \sqrt{\dfrac{2RT}{M}}$
- Avg. KE of one molecule $= \dfrac{3}{2} K_b T$ [K_b = Bolzmann's const. $K_b = \dfrac{R}{N_A} = 1.38 \times 10^{-23}\,JK^{-1}$]
  (depend only on T not on nature of gas)
- Rotaⁿal KE → disc-chalk conflict $(KE)_i \neq (KE)_f$
- Press. energy → movement of molecule in random direcⁿ
- Internal energy →
- Existence of atmosphere: if $V_{rms} <<< V_{escape}$ & vice versa.
  $V_{rms}^{H_2} = 1890\,m/s$ & $V_{escape} = 11{,}200\,m/s$
- Degree of freedom (F) – (no. of ways a gas can have KE) ⇒ mono atomic = $\gamma = 3$
  for solid → $\boxed{F = 6}$
  ⇒ Polyatomic gas → F = 6 ⇒ diatomic = $\gamma = 5$
  (high temp.) → F = 8 (at high temp.) $\gamma = 7$.
- Internal energy of gas → $U = \dfrac{F}{2} nRT$ & $(TE)_{1\,mole\,molecule} = \dfrac{F}{2} RT$
- Thermodynamic process → curve represent process
  → Pt. represent thermodynamic state.
  aim: conversion of heat to work in controlled manner.
- System = anything under observaⁿ
- Surrounding = Universe − system
- closed system = $\Delta mass = 0$
- Isolated system ⇒ Δm & $\Delta Q = 0$.

[$|slope|_{adiabatic} > |slope|_{isoth.}$ for same P & V.]

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• Thermodynamic state ⇒ Particle distribuⁿ : ρ (density)
Vel. distribuⁿ : V_{rms}.

Thermodynamic variable ⇒ $\underbrace{P / V / T}_{\text{Independent}} / V_{rms} / U$

$dQ = dU + dW$

dQ = heat given to system
dU = change in internal energy
dW = work done by system.

• $dQ = nCdT = ms\,dT$ [$C \rightarrow$ molar heat capacity, $s \rightarrow$ specific heat capacity]
$C \rightarrow$ depends on process & nature of gas.

$T \rightarrow$ thermodynamic condiⁿ of gas.

• $\Delta U = \frac{f}{2} nR\Delta T$ [$f \rightarrow$ nature of gas or storing capacity], ΔU = depends on thermodynamic state (not on process), $U \rightarrow$ total

• $dW = PdV$ & $W = \int PdV$.

• $\gamma = \frac{C_P}{C_V} = \frac{f+2}{f}$ or $\gamma = 1 + \frac{2}{f}$ ⇒ $\gamma > 1$ always $\because f > 0$

$C_V = \frac{fR}{2}$
$C_P = \left(\frac{f+2}{2}\right)R$
$C_P - C_V = R$

S.no	Variable	Isobaric	Isochoric	Isothermal	Adiabatic
①	C	$C_P = \left(\frac{f+2}{2}\right)R$	$C_V = \frac{fR}{2}$	$C = \frac{dQ}{ndT} = \infty$	$C = \frac{dQ}{ndT} = 0$
②	dQ	$nC_P dT$	$nC_V dT = dU$	$dQ = nCdT = \infty \times 0$ $dQ = dW$ [$dU = 0$]	$dQ = 0$.
③	dU	$n\frac{fR}{2}dT = nC_V dT$	$n\frac{fR}{2}dT = nC_V dT$	$nC_V dT$	$nC_V dT$
④	W (total)	$P\Delta V$	$W = 0$	$nRT \ln\frac{V_2}{V_1} = nRT\ln\frac{P_1}{P_2}$	$\frac{P_2V_2 - P_1V_1}{1-\gamma} = \frac{nR\Delta T}{1-\gamma}$
⑤	PV^x = const.	$x = 0$ PV^0 = const	$x = \infty$ PV^∞ = const.	$x = 1$ PV = const.	$x = \gamma = C_P/C_V$ PV^γ = const.
⑥	$\frac{dP}{dV} = \frac{-xP}{V}$	0	∞	$-\frac{P}{V}$	$-\frac{\gamma P}{V}$
	User friendly process	$\Delta P = 0$ moving wall Balloon	$\Delta V = 0$ rigid wall cylinder.	$\Delta T = 0$ Diathermal wall Very slow process	$\Delta Q = 0$ Adiabetic wall fast process.

• Adiabetic & isothermal graph will 100% intersect at single pt. draw tangent at that pt. measure angle from $(-\hat{x})$ direcⁿ, (more angle → adiabetic)

• Cyclic process ⇒ $W_T = 0$ ACW ⇒ −ve & CW ⇒ +ve

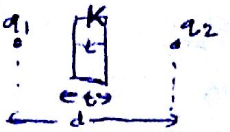
• Speed of sound wave : for fluid : $\sqrt{\frac{\beta}{\rho}}$, string : $\sqrt{\frac{T}{\mu}}$ [Isothermal, $\beta = P$]
$\beta = -\frac{VdP}{dV}$

ELECTROSTATICS

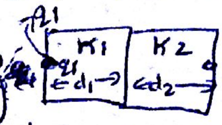
$1C = 3\times10^{9}$ stat C or esu

$\frac{1}{4\pi\varepsilon_0} = 9\times10^{9}$

- $F = \frac{1}{4\pi\varepsilon_0 K}\cdot\frac{q_1q_2}{r^2}$; $K\to$ dielectric constant. [air, $K=1$; conductor, $K=\infty$] ; $\varepsilon_0\to$ permittivity of vaccum
- coulombic force b/w two charge does not depend on medium but net electrostatic force b/w two charge always depend on medium.
- $y = \frac{ax}{(x^2+a^2)^{3/2}}$ (y_{max} at $x = a/\sqrt{2}$)
- for eqb, $\Sigma F=0$, $\Sigma \tau=0$ & use lami's theorem.
- $F_{q_1q_2} = \frac{1}{4\pi\varepsilon_0}\times\frac{q_1q_2}{(d-t+t\sqrt{K})^2}$



- $F_{q_1q_2} = \frac{1}{4\pi\varepsilon_0}\times\frac{q_1q_2}{(d_1\sqrt{K_1}+d_2\sqrt{K_2})^2}$



- $E = \lim_{q_0\to0}\frac{F}{q_0}$ [independent of q_0]

E_{actual} (ideal) > $E_{calculated}$ (non ideal) / Direcⁿ of electric field is away from +ve & toward −ve.

- $|\vec{F}| = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1q_2(\vec{r}_2-\vec{r}_1)}{|\vec{r}_2-\vec{r}_1|^3}$

$W_{ext} = \Delta U$; V & U → scalar sum

$W_{int} = -\Delta U$; F & E → vector sum

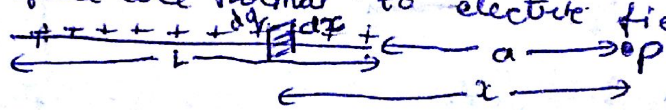
- $V = -\frac{W_{int}}{q_0}$, $V = \frac{Kq}{r}$

$U = qV$

- $\vec{E} = -\nabla V$ where $\nabla = \frac{d}{dx}\hat{i} + \frac{d}{dy}\hat{j} + \frac{d}{dz}\hat{k}$; In 1D → $E = -\frac{dV}{dr}$.

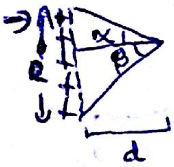
$dV = -\vec{E}\cdot d\vec{r}$

- On moving in the direcⁿ of electric field, potential decreases. $V_A - V_B = Ed\cos\theta$
- Equipotential surface are normal to electric field.
- Lineal charge → $V_\infty = 0$ ($a\to\infty$), $U = qV$



$E_{at\,P} = \frac{KQ}{a(a+l)}$, $V_P = \frac{KQ}{L}\ln\frac{a+l}{a}$

- Long charged wire →

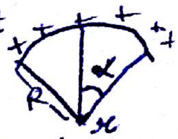


$E_x = \frac{K\lambda}{d}(\sin\alpha + \sin\beta)$

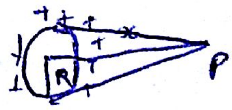
$E_y = \frac{K\lambda}{d}(\cos\alpha - \cos\beta)$

$= E = \frac{\lambda}{2\pi r\varepsilon_0}$

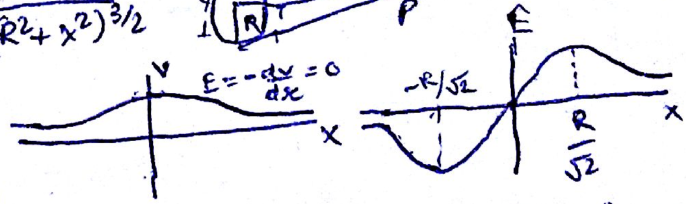
- If we take potential at ∞ distance to be zero, then potential at finite distance is not defined.
- Electric field is conservative field, hence conservaⁿ laws are valid, KE + PE = const
- Ring element $E_r = \frac{2K\lambda\sin\theta}{R}$, $V_r = \frac{KQ}{R}$



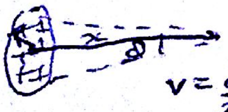
- On axis of ring → $V_P = \frac{KQ}{\sqrt{R^2+x^2}}$, $E_P = \frac{KQx}{(R^2+x^2)^{3/2}}$



E_{max} at $x = \pm\frac{R}{\sqrt{2}}$



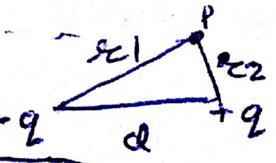
- Charged disc − $E = \frac{\sigma}{2\varepsilon_0}[1-\cos\theta]$, $\cos\theta = \frac{x}{\sqrt{R^2+x^2}}$



$v = \frac{\sigma}{2\varepsilon_0}(\sqrt{R^2+x^2} - x)$; $V = \frac{\sigma}{2\varepsilon_0}(\sqrt{R^2+x^2} - x)$

- dipole moment = $q\times d$ → Potential at P ⇒ $V = \frac{KP\cos\theta}{r^2}$

$\tan\alpha = \frac{\tan\theta}{2}$



$E = \frac{KP}{r^3}\sqrt{3\cos^2\theta+1}$

- Electric force due to dipole → $F = \frac{PdE}{dx}$
- $\tau = P\times E = PE\sin\theta$, $W = PE[\cos\theta_1 - \cos\theta_2]$

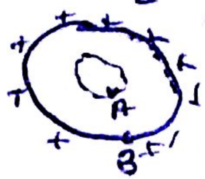
$I_{shell} = \frac{2}{5}MR^2$

- Electric field at centre of hemi spherical shell $= \frac{\sigma}{4\varepsilon_0}$  , $V = \frac{KQ}{R}$
- field at center  $E = \frac{\sigma_0 \cos\theta}{3\varepsilon_0}$
  $\sigma = \sigma_0 \cos\theta$
- solid hemisphere $= \frac{\rho R}{4\varepsilon_0}$
- Electric flux $= \oint \bar{E} \cdot d\bar{s}$, solid angle $\rightarrow \omega = \int \frac{ds\cos\theta}{r^2}$ [sphere, $\omega = 4\pi$ steradi
- Solid angle subtended by (spherical) closed surface at any internal pt is 4π steradi
  & outside is zero.
- $\oint \bar{E} \cdot d\bar{s} = \frac{q_{in}}{\varepsilon_0} = \phi$ $\{A = 2\pi R^2[1-\cos\theta]\}$ $4\pi \rightarrow \frac{q}{\varepsilon_0}$ $[4\pi]$
- $\omega = 2\pi[1-\cos\theta]$
- App of gauss →
  wire

$E \times 2\pi r l = \frac{\lambda l}{\varepsilon_0}$

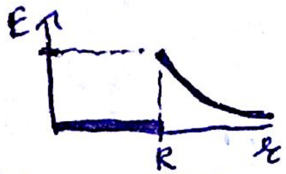
$E = \frac{\lambda_0}{2\pi r \varepsilon_0}$

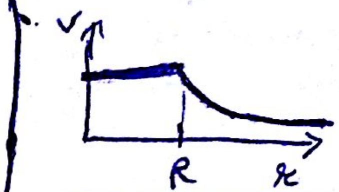
shell:



E
inside → 0
on → $\frac{KQ}{R^2}$
out → $\frac{KQ}{r^2}$

→ V
in & on $= \frac{KQ}{R}$
out $= \frac{KQ}{r}$





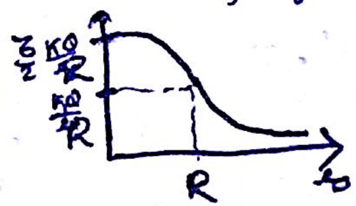
solid sphere

$E \cdot 4\pi r^2 = \frac{\rho \times \frac{4}{3}\pi r^3}{\varepsilon_0}$

$E = \frac{\rho r}{3\varepsilon_0} = \frac{Qr}{4\pi\varepsilon_0 R^3}$

for non uniform solid sphere, $q_{in} = \int \rho\, dV$ then, apply gauss's law.

$V_{in} = \frac{KQ}{2R^3}[3R^2 - r^2]$ $V_{out} = \frac{KQ}{r}$



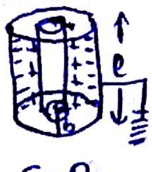
Capacitor

- $C = \frac{Q}{V} = \frac{dQ}{dV}$ = Farad(F) = capacitance = $\tan\theta$
- Energy ⇒ $U = W = \int q\,dV = \frac{1}{2}CV^2$ = area(under qV graph) $= \frac{Q^2}{2C} = \frac{1}{2}QV$
- Common potential, $V = \frac{Q_1+Q_2+Q_3}{C_1+C_2+C_3} = \frac{C_1V_1+C_2V_2+C_3V_3}{C_1+C_2+C_3}$ | Loss of energy → $\frac{C_1C_2(V_1-V_2)^2}{2(C_1+C_2)}$
- for sphere, $C = 4\pi\varepsilon_0 R$.
- for

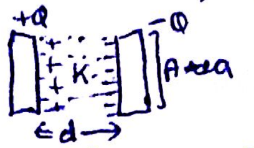
$C = \frac{Q}{V_A - V_B} = 4\pi\varepsilon_0\left(\frac{ab}{b-a}\right)$

- cylindrical capacitor

$C = \frac{2\pi\varepsilon_0 l}{\ln\left(\frac{b}{a}\right)}$



- parallel plate capacitor → $C = \frac{K\varepsilon_0 A}{d}$



$C = \frac{\varepsilon_0 A}{d - t + \frac{t}{K}}$

- Force b/w plates of capacitor : $QE = Q\frac{\sigma}{2\varepsilon_0} = \frac{Q^2}{2A\varepsilon_0}$
- Energy density : $\frac{\text{Energy}}{\text{Volume}} = \frac{\frac{1}{2}CV^2}{Ad} = \frac{1}{2}\varepsilon_0 E^2$

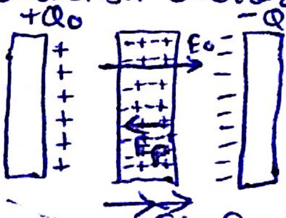
& Potential energy (u) $= \int$ Energy density $\cdot\, dV$ (vol.) $= \int \frac{1}{2}\varepsilon_0 E^2\, dV$ $[u \propto E^2]$

- T.E of sphere : $U = \frac{3}{5}\frac{KQ^2}{R}$ $\left[\int V dq\right]$ → $E_{inside} = \frac{KQ^2}{10R}$, $E_{out} = \frac{KQ^2}{2R}$

- DIELECTRIC : dipole mom. per unit vol. is said to be polarisaⁿ vector.

after touching, $Q_{free} = \frac{Q_0}{K}$ exp, $E_i = \frac{E_0}{K}$

$Q_P = Q_0\left[1 - \frac{1}{K}\right]$



Q_P ↓ charge due to polarisaⁿ

$E_i = E_0 - E_P$

⇒ If a charged capacitor is disconnected from the battery and a dielectric slab with same dimention is introduced b/w plates, then –
∵ charge is conserved, $C = KC_0$, $V = \frac{Q}{C} = \frac{V_0}{K}$, $E = \frac{E_0}{K}$ & $U = \frac{U_0}{K}$

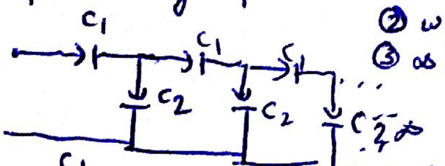
→ If battery is remained connected in above discussion, V = const, ⇒ E = const, $Q = KQ_0$, $C = KC_0$, $U = KU_0$

- If switch is closed, then in steady state, $W_{battery} = CV^2$.
  if polarity

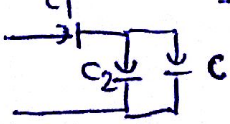
$E_{stored\ in\ C} = \frac{1}{2}CV^2$ & heat dev. in resistor $= \frac{1}{2}CV^2$

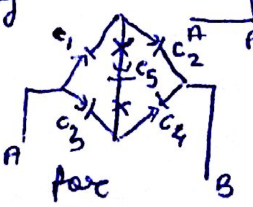
- combinaⁿ of capacitor : series (→ const. charge) : $\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots$, for n identical, $C = \frac{C_0}{n}$ & $C = \frac{C_1C_2}{C_1+C_2}$ for 2 C.
  : Parallel : $C = C_1 + C_2 + \ldots + C_n$ [const. potential].

- method for finding capacitance – ① naming and numbering ② wheat stone bridge ③ as ladder ④ method of symmetry

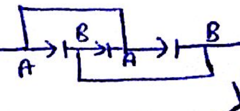


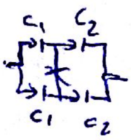
$C =$

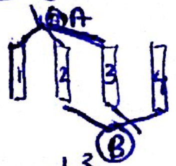


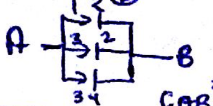


for $C_1C_4 = C_2C_3$

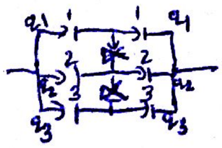


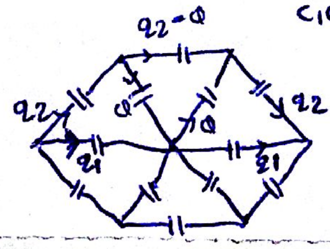




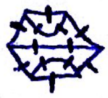


$C_{AB} = \frac{3\varepsilon_0 A}{d}$

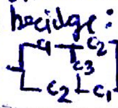




→



Unbalanced bridge :-



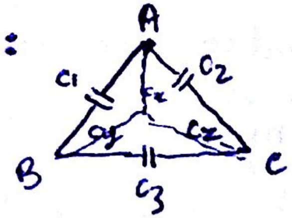
$C = \frac{C_1C_3 + C_2C_3 + 2C_1C_2}{C_1 + C_2 + 2C_3}$

- electric cell is a device which maintains a potential diff b/w two pts.
- Emf - $W_{by\ cell}$ in moving a unit charge in complete circuit including cell.
- Rate of flow of charge is said to be current $i = \frac{dq}{dt} = \tan\theta =$ slope of Q-t graph.
- $W_T = W_{int} + W_{ext}$. $\Rightarrow \mathcal{E} = ir + iR$ or $i = \frac{\mathcal{E}}{r+R}$ if current is opp, $\mathcal{E} = V - ir = i(R-r)$
- 

$c_1 = \frac{K_1 \varepsilon_0 A}{d_1}$, $C_2 = \frac{K_2 \varepsilon_0 A}{d_2}$, C_1 & $C_2 \to$ in series $C = \frac{C_1 C_2}{C_1 + C_2}$.

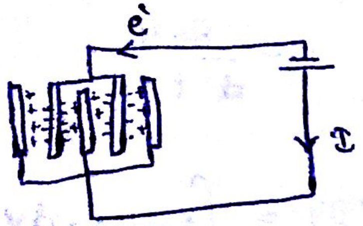
$C_1 = \frac{K_1 \varepsilon_0 A_1}{d}$, $C_2 = \frac{K_2 \varepsilon_0 A_2}{d}$ $C = C_1 + C_2$.

- star delta method :



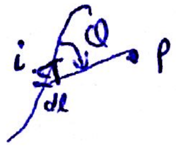
$C_x = C_1 + C_2 + \frac{C_1 C_2}{C_3}$, $C_y = C_1 + C_3 + \frac{C_1 C_3}{C_2}$

$C_z = C_2 + C_3 + \frac{C_2 C_3}{C_1}$

- 

Magnetic effect of current

- **Biot savart law** :- magnetic field due to current element idl :-



⊗ inward

⊙ outward

$$dB = \frac{\mu_0}{4\pi} \cdot \frac{i\,dl \sin\theta}{r^2}, \quad \mu_0 = 4\pi \times 10^{-7}$$

$$\vec{dB} = \frac{\mu_0}{4\pi} \cdot \frac{i(\vec{dl} \times \vec{r})}{r^3}$$

$dB \perp dl \perp r$

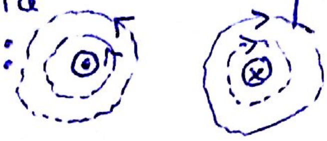
$\vec{dB} \cdot \vec{dl} = 0$

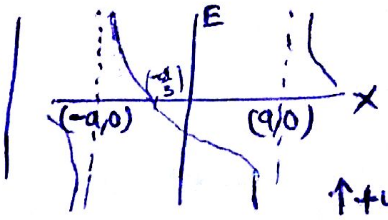
- magnetic field due to st current : $B = \dfrac{\mu_0 i}{4\pi d}(\sin\alpha + \sin\beta)$

circular current $= \dfrac{\mu_0 i}{4\pi r}\theta$ [θ in radian]

zero.

- Magnetic lines of force :





↑ +ve

↓ −ve

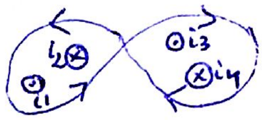
- on axis of ring : $B_1 = \dfrac{\mu_0 i R^2}{2(R^2+x^2)^{3/2}}$

for n turns, $B_n = \dfrac{\mu_0 N i R^2}{2(R^2+x^2)^{3/2}}$

- Ampere's circuital law :-

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 i_{inside}$$

$\oint \vec{B}\vec{dl} = \mu_0(i_1 - i_2 + i_3)$

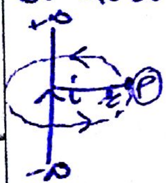


$\oint \vec{B} \cdot \vec{dl} = \mu_0(i_1 - i_2 - i_3 + i_4)$

$\oint \vec{B}\vec{dl} = \mu_0(i_1 + i_2 + i_3)$

→ Application —

① due to st current

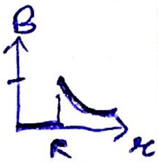


$B \cdot 2\pi r = \mu_0 i$

$B = \dfrac{\mu_0 i}{2\pi r}$.

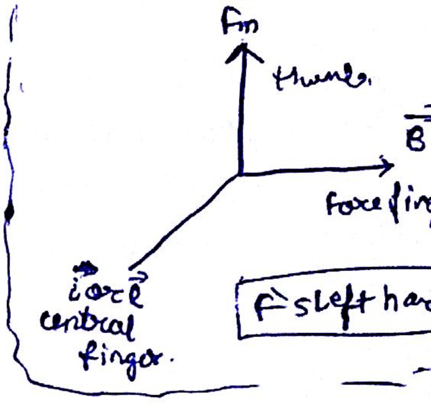
② due to current carrying pipe — (hollow conductor)

$$B = \begin{cases} 0, & r < R \\ \dfrac{\mu_0 i}{2\pi r}, & r \geq R \end{cases}$$



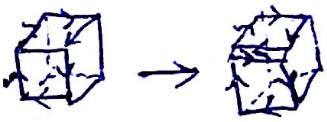
DCP : Magnetics

- $F_m = q(\vec{v} \times \vec{B}) = qvB\sin\theta$ [$1T = 10^4$ gauss] $\rightarrow$ q +ve, F along $\vec{v} \times \vec{B}$; q −ve, F opp to $\vec{v} \times \vec{B}$ $\left[\vec{a} \perp \vec{b} \text{ or } \vec{a} \cdot \vec{b} = 0 \right.$
- $\vec{F_m} \perp \vec{v}$ or $\vec{F_m} \perp \vec{ds}$ or $W_{\vec{F_m}} = 0$
- $\frac{q}{m} \Rightarrow$ specific charge (α). | $qvB = \frac{mv^2}{r} \Rightarrow$ circle [$\theta = 90°$] ; $v \rightarrow v\sin\phi$, $\theta \neq 90°$) pitch: $v\cos\theta\, t$
- $\vec{F_m} = i(\vec{l} \times \vec{B})$ (for closed polygon, $\vec{dl}$ net $= 0$, $\vec{F_m} = 0$) (vector addi^n = 0)
- The net magnetic force acting on any closed current loop in uniform magnetic field = 0.



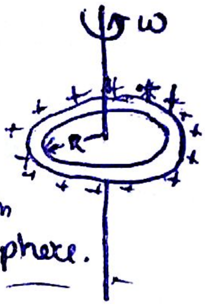
- magnetic moment, $|\vec{M}| = NiA$ (N → no of turns; A → area of cross sec^n)

direc^n from south to North or by right hand thumb rule

-


$M = \sqrt{2}$ (due to 1 loop) $= \sqrt{2}(il^2)$

- $\frac{M}{L} = \frac{q}{2m}$ (M → magnetic moment; L → angular momentum) for disc, ring & sphere.



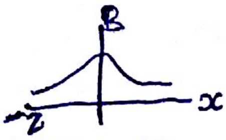
⊗ inward

⊙ outward.

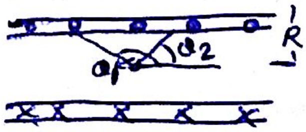
- $\vec{\tau} = \vec{M} \times \vec{B}$
- $\tau = MB\sin\theta = NiAB\sin\theta$.
- $d\vec{B} = \frac{\mu_0}{4\pi} \cdot \frac{i(\vec{dl} \times \hat{r})}{r^2}$
- $B = \frac{\mu_0}{4\pi} \cdot \frac{i\theta}{R}$ [arc of circle subs. θ at centre]
- $\oint \vec{B} \cdot \vec{dl} = \mu_0 i_{net}$
- Long current carrying wire :- $B_0 = \frac{\mu_0 i}{2\pi r}$, $B_i = \left(\frac{\mu_0 i}{2\pi R^2}\right) r$
- $B \cdot l = \mu_0 (nli)$ solenoid

$\Rightarrow B_{st.wire} = \frac{\mu_0 i}{4\pi d}(\sin\alpha + \sin\beta)$

$\Rightarrow$ axis of ring or circular coil $\Rightarrow B = \frac{\mu_0 N i R^2}{2(R^2+x^2)^{3/2}}$



$B_{solenoid} = \frac{\mu_0 n i}{2}(\cos\theta_2 - \cos\theta_1)$



at centre, $\theta_1 = \pi$, $\theta_2 = 0$; $B_c = \mu_0 n i$ | ends, $\theta_1 = \pi/2$, $\theta_2 = 0$; $B_e = \frac{\mu_0 n i}{2}$

Toroid :

$B \cdot 2\pi r = \mu_0 (Ni)$

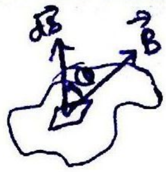
$B = \frac{\mu_0 N i}{2\pi r}$

$\vec{B} = \frac{\mu_0}{4\pi} \cdot \frac{q}{r^3}(\vec{v} \times \vec{r})$

$V = \frac{E_0}{B_0}$ $\qquad c = \frac{1}{\sqrt{\epsilon_0 \mu}}$

- $\vec{\tau} = \vec{M} \times \vec{B}$ (M → dipole moment)
- $\vec{M} = Ni\vec{A}$, $F = m\vec{B}$ (m → pole st.), $M = md$

ELECTROMAGNETIC INDUCTION → DCP



- $d\phi = \vec{B}\cdot d\vec{S}$ → unit weber (wb)
- $\oint \vec{B}\cdot d\vec{S} = 0$
- Faraday's law, $e = -N\dfrac{d\phi}{dt}$

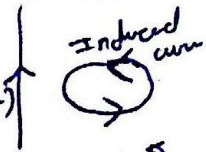
Also, $e = -\dfrac{d\phi}{dt}$, $i = \dfrac{1}{R}\left(\dfrac{-d\phi}{dt}\right)$, $dq = i\,dt = \dfrac{1}{R}(-d\phi)$

- LENZ's law: the direcⁿ of any magnetic inducⁿ effect is such as to affect the cause of effect

→ RIN = if a loop is placed in right of st. current carrying conductor and $i\uparrow$ then induced current = ACW (N)

→ ⊗IN

⊗ increasing

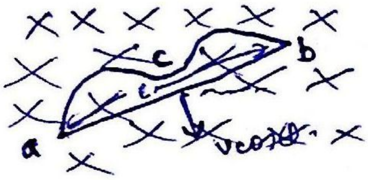


- Motional emf :→ $F_m = -e(\vec{V}\times\vec{B})$, $eE = eVB$.

$\Delta V = El = Blv$

- $i = \dfrac{Bvl}{R}$, $F_m = Bil = \dfrac{B^2l^2v}{R}$, $P = FV = \dfrac{B^2l^2v^2}{R}$ | $|e| = \left|\dfrac{-d\phi}{dt}\right| = \dfrac{d}{dt}(B\cdot S) = \dfrac{d}{dt}(Blx)$

$e = Blv$.



$e = \int_b^a (\vec{V}\times\vec{B})\cdot d\vec{l}$.

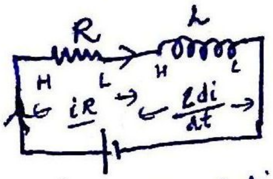
$e_{acb} = e_{ab} = (\text{length of } ab)\times V_{\perp}\times B$

- $e = \int_0^l B x \omega\, dx = \dfrac{B\omega l^2}{2}$

⇒ self inductance: $e \propto \dfrac{di}{dt}$, $e = -L\dfrac{di}{dt}$

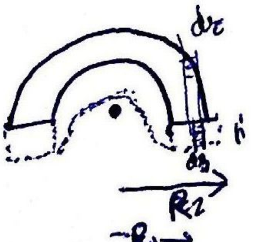
$N\phi_B \propto i$ ⇒ $L = \dfrac{N\phi_B}{i}$ [unit: henry]

$N\phi_B = Li$ or $L = \left|\dfrac{-e}{di/dt}\right|$



(KVL) $E - iR - L\dfrac{di}{dt} = 0$

- Inductance of solenoid:
  $B = \mu_0 \dfrac{N}{l} i$
  $Li = N\phi$ & $\phi = BS$ ⇒ $L = \dfrac{\mu_0 N^2 S}{l} = \mu_0 n^2 V$ → vol



$\phi_B = \int B\cdot ds = \int_{R_1}^{R_2} \dfrac{\mu_0 N i}{2\pi r} h\,dr$. and $Li = N\phi$

$L = \dfrac{\mu_0 N^2 h}{2\pi} \ln\left|\dfrac{R_2}{R_1}\right|$

$U_{Inductor} = \dfrac{1}{2}LI^2$

magnetic energy density:

$u = \dfrac{U\cdot\text{energy}}{\text{vol}} = \dfrac{1}{2}\dfrac{B^2}{\mu_0}$ $\left[u = \dfrac{1}{2}\epsilon_0 E^2\right.$

ELECTROMAGNETIC WAVES

- $\oint B\cdot dl = \mu_0 (i_c + i_d)$ [disp. current due to change in magnetic field, conduc^n current due to flow of charge]

  $\left(i_d = \varepsilon_0 \frac{d\phi}{dt} \right)$

- $\phi_E = EA = \left(\frac{q}{A\varepsilon_0}\right)A = \frac{q}{\varepsilon_0}$, $\frac{d\phi}{dt} = \frac{1}{\varepsilon_0}\frac{dq}{dt}$, $i_d = \varepsilon_0 \frac{d\phi_E}{dt} = \varepsilon_0\left(\frac{1}{\varepsilon_0} \times \frac{dq}{dt}\right) = i_c$ [$i_d = i_c$]

- Maxwell's eq^n →
  - $\oint E\cdot ds = \frac{q_{in}}{\varepsilon_0}$ [Gauss's law for electricity]
  - $\oint E\cdot dl = -\frac{d\phi_B}{dt}$ [Faraday's law]
  - → $\oint B\cdot ds = 0$ [gauss's law for magnetism]
  - $\oint B\cdot dl = \mu_0(i_c + i_d)$ [Ampere-maxwell's law]

- $E_x = E_0 \sin(\omega t - Kz)$, $B_y = B_0 \sin(\omega t - Kz)$] thus EMW travel in direc^n of $E \times B$ [$K = \frac{2\pi}{\lambda}$, $\omega = cK$

- $c = \frac{E_0}{B_0} = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$ \$ $V = \frac{1}{\sqrt{\varepsilon \mu}}$ | • Energy density $= \frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2}\frac{B^2}{\mu_0}$ (50% + 50%) | $\Delta P = \frac{E}{c}$ (comp. absorp^n) | $\Delta P = \frac{2E}{c}$ [comp reflection]

- $I = \frac{1}{2}\varepsilon_0 E^2 c$ | Visible light 4000 Å to 7000 Å

V	B	G	Y	O	R
4000 to 4500	4500-5200	5200 - 5600	5600-6000	6000-6250	6250-7000

-

Type	λ	ν
Radio waves	> 0.1 m	3×10^9 Hz
micro waves	0.1 m to 1 mm	3×10^9 Hz to 3×10^{11} Hz
Infrared	1 mm to 7000 Å	3×10^{11} Hz to 4.3×10^{14} Hz
Visible light	7000 Å to 4000 Å	4.3×10^{14} Hz to 7.5×10^{14} Hz
U.V. rays	4000 Å to 10 Å	7.5×10^{14} Hz to 3×10^{17} Hz
X-rays	10 Å to 0.01 Å	3×10^{17} Hz to 3×10^{20} Hz
γ rays.	< 0.01 Å	> 3×10^{20} Hz

$\lambda \downarrow$ \$ $\nu \uparrow$

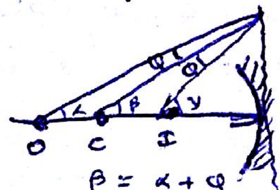
- γ rays – these higher freq. radia^n's are produced in nuclear reac^n's
  – emitted by radioactive nuclei
  – used in medicines to destroy cancer cells
- X-Rays – rapid decelera^n of e^-'s that bombard a heavy metal target
  – electronic transi^n b/w energy levels.
  – used as diagnostic tool in medicine and as a treatment of certain form of cancers
- UV rays ⇒ special lamp \$ hot bodies, vit-D
  – skin cancer, dep. by cfc, freons
- Visible light – Photosynthesis need visible light.
- Infrared – heat waves produced by hot bodies, in most of material H_2O molecules are prt.
  – used for early detec^n of tumors.
  – also used to obs growth of crop \$ for military.
- Microwaves – generated by oscilla^n's in device called klystron.
  ⇒ microwave ovens. [this raises the temp. of any food containing H_2O]
- Radio waves – generated when charges are accelerating through conducting wires.
  – generated by L-C oscilla^n's and are used in radio, TV \$ communica^n sys.
  – generated by ionosphere

(*) short radio waves are reflected by ionosphere

RAY OPTICS

$\mu_0 = 4\pi \times 10^{-7}$ H/m
$\varepsilon_0 = 8.85 \times 10^{-12}$ F/m

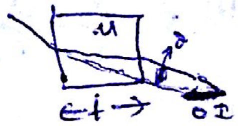
- If incident ray is rotated by θ then reflected ray is rotated by 2θ.
- The min length of plane mirror to see one's full height is $H/2$, $\delta = 180 - 2i$
- A man standing exactly at midway b/w a wall & mirror and to see full height of wall behind him, min length of mirror $= \frac{H}{3}$.
- assumed to be paraxial ray – rays close to Principal axis and make small angle with it
- $\frac{1}{F} = \frac{1}{v} + \frac{1}{u}$, $m = \frac{-v}{u}$, $f = R/2$, $P(D) = \frac{-1}{F(m)}$, $V_I = m^2 V_0$



$\beta = \alpha + \theta$
$\gamma = \beta + \theta$
$2\beta = \gamma + \alpha$

- $\frac{\sin i_1}{\sin i_2} = {}_1\mu_2 = \frac{\mu_2}{\mu_1} = \frac{\lambda_1}{\lambda_2} = \frac{v_1}{v_2}$, $\delta = |i - r|$
- R to D, $d_{app} = \frac{d_{actual}}{\mu}$, D to R, $d_{app} = \mu \cdot d_{actual}$
- shift due to glass slab → Normal shift,

$$OI = \left(1 - \frac{1}{\mu}\right)t.$$



→ lateral shift: $d = t\left(1 - \frac{\cos i}{\sqrt{\mu^2 - \sin^2 i}}\right)\sin i$

for small angle, $d = ti\left(\frac{\mu - 1}{\mu}\right)$

- $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$.

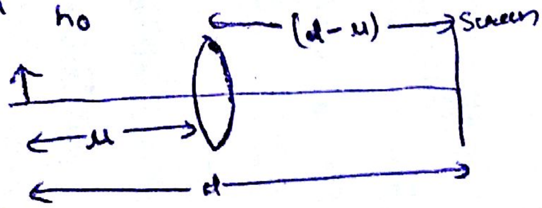
$m = \left(\frac{\mu_1}{\mu_2}\right)\left(\frac{v}{u}\right)$

$\mu_1\theta_1 = \mu_2\theta_2$
$\theta_1 = \alpha + \beta$
$\beta = \theta_2 + \gamma$
diminish θ_1 & put in

- Lens maker's formula ⇒ $\frac{1}{F} = \left(\frac{\mu_2}{\mu_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

($\mu_2 < \mu_1$ opp nature)

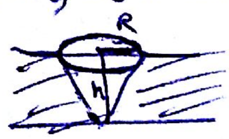
- $F_{water} = 4 \cdot F_{air}$
- $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ or $m = \frac{v}{u} = \frac{h_I}{h_0}$
- Displacement method:



$O^2 = I_1 I_2$
obj length ↓ , Image length ↓

- $\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$ (two thin lens)
- TIR, $\mu_D \sin i = \mu_R \sin r$, $\theta_c = \sin^{-1}\left(\frac{\mu_R}{\mu_D}\right)$ [for $\mu_R = 1$, $\theta_c = \sin^{-1}\left(\frac{1}{\mu}\right)$]
- crown glass $\theta_c = 42°$

$R = \frac{h}{\sqrt{\mu^2 - 1}}$

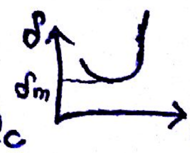


- Refracⁿ through prism → $A = r_1 + r_2$, $\delta = (i_1 + i_2) - A$ but A & i are small ∴ $\delta = (\mu - 1)A$
- for min devia^n, $\mu = \frac{\sin i}{\sin r}$, $r_1 = r_2 = \frac{A}{2} = r$, $\delta_m = 2i - A$

$i = \frac{A + \delta_m}{2}$

$$\mu = \frac{\sin\left(\frac{A + \delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

and condiⁿ for no emergence, $A > 2\theta_c$



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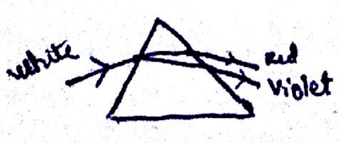


$\mu \downarrow$ with $\uparrow \lambda$

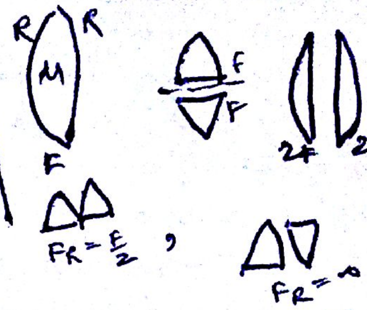
- $\delta_V - \delta_R$ = angular dispersⁿ [$\delta = (\mu - 1)A$]

dispersive Power or $\omega = \frac{\mu_V - \mu_R}{\mu_Y - 1} = \frac{\text{ang dispers}^n}{\text{avg dispers}^n}$

- disp. without avg devia^n (condi^n for) $\Rightarrow$ $\dfrac{\mu'_y - 1}{\mu_y - 1} = \dfrac{A}{A'}$ 

- Avg devia^n without dispers^n $\Rightarrow$

$$\frac{A}{A'} = \frac{(\mu'_y - 1)\,\omega'}{(\mu_y - 1)\,\omega} = \frac{\mu'_v - \mu'_r}{\mu_v - \mu_r}$$



- Eye is most sensitive to yellow - green light.
- Photographic plate is more sensitive to blue $ least to Red,
- for two mirror inclined at angle θ, $n = \underbrace{\dfrac{360}{\theta}}_{} - 1$ (even) | $n = \underbrace{\dfrac{360}{\theta}}_{odd}$ $ not on bisector $ $\dfrac{360}{\theta} - 1$ for on bisector.

- Rayleigh : Intensity of scattering $\propto \dfrac{1}{\lambda^4}$
- for achromatism, $\dfrac{\omega_1}{f_1} + \dfrac{\omega_2}{f_2} = 0$
- Myopia (short sightedness)  | • Hypermetropia 

  II

- Presbyopia - both near $ far objects are not clearly visible. [bifocal lens]
- Astigmatism - eye cannot see object in two ⊥ direc^n clearly. [cylindrical lens]

Interference & diffrac^n of light

- Condi^n for interference – coherent source
  - same λ or ν
  - equality of Amp.

$A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\phi}$

$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$

- Young's double slit exp – $\Delta x = S_2P - S_1P = d\sin\theta$.

constructive: $\Delta x = n\lambda$ $(0, \pm 1, \pm 2)$, $d\sin\theta = n\lambda$.

destructive: $\Delta x = (2n-1)\frac{\lambda}{2}$ $(\pm 1, \pm 2)$, $d\sin\theta = (2n-1)\frac{\lambda}{2}$

$\because \sin\theta = \frac{y}{D}$ or $\theta = \frac{y}{D}$

$y_{bright} = \frac{n\lambda D}{d}$

$y_{dark} = \frac{(2n-1)\lambda D}{2d}$

Fringe width $(\omega) = \frac{\lambda D}{d}$

- $\Delta\phi = \frac{2\pi}{\lambda}\Delta x$, $I = 4I_0\cos^2\phi/2$, $\Delta x = (\mu - 1)t$ [Path diff produced by slap]
- optical path length $= \mu t$
- shift of fringe : $\frac{(\mu-1)tD}{d}$ [in glass slab] $\rightarrow$ shift is independent of λ $\rightarrow$ no. of fringes shifted $= \frac{(\mu-1)t}{\lambda}$

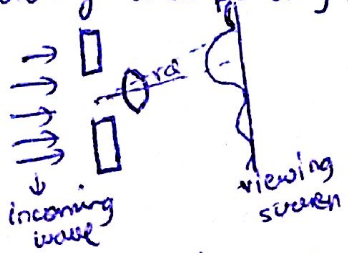
- $\frac{I_{max}}{I_{min}} = \frac{\sqrt{I_1} + \sqrt{I_2}}{\sqrt{I_1} - \sqrt{I_2}}$

- Interference in thin films $\rightarrow$ constructive $= 2\mu t = (n - \frac{1}{2})\lambda$

destructive $= 2\mu t = n\lambda$

- Diffrac^n – prop. of spreading when passing through aperture or by sharp edges.

- Fraunhofer diffrac^n

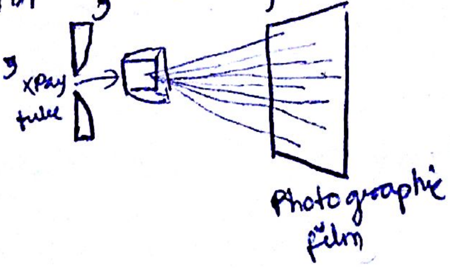


- Diffrac^n from narrow slits, accor. to Huygen's principle, each portion of the slit act as a source of light wave. $\frac{a}{2}\sin\theta = \frac{\lambda}{2}$, $\sin\theta = \frac{\lambda}{a}$ (for 2 parts), 4 parts $\rightarrow \sin\theta = \frac{2\lambda}{a}$

6 parts $\rightarrow \sin\theta = \frac{3\lambda}{a}$

generally, $\sin\theta = \frac{m\lambda}{a}$ $(m = \pm 1, \pm 2, \pm 3 \dots)$

- Diffrac^n grating : $d\sin\theta = n\lambda$, its Resolving power : $R = \frac{\lambda}{\lambda_2 - \lambda_1} = \frac{\lambda}{\Delta\lambda}$ $\left(\lambda = \frac{\lambda_1 + \lambda_2}{2}\right)$
- diffrac^n of xRay by crystals,

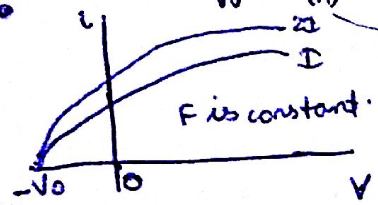


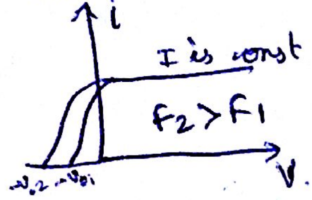
$$\boxed{2d\sin\theta = n\lambda}$$

Bragg's law

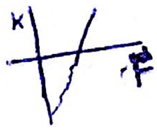
Modern Physics

$1\,Fm = 10^{-15}\,m$

- $E(\text{in eV}) = \dfrac{12375}{\lambda(\text{in Å})}$ • $E = Pc$
- $(\text{Pressure})_{rad} = \dfrac{I}{c}$, $F_c = IS$, $P_{rad} = \dfrac{I}{c}$ for totally absorbed & $\dfrac{2I}{c}$ for totally reflected
- $\lambda = \dfrac{h}{p} = \dfrac{h}{mv} = \dfrac{h}{\sqrt{2m(KE)}} = \dfrac{h}{\sqrt{2mqV}}$ (de broglie wavelength) & $\lambda(\text{in Å}) = \sqrt{\dfrac{150}{V(\text{volt})}}$
- $m_n v_n r_n = \dfrac{nh}{2\pi}$, $\dfrac{mv_n^2}{r_n} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{e^2}{r_n^2}$, $E_i - E_f = \dfrac{hc}{\lambda}$, $Rhc = 13.6\ eV$
- Total no. of emission from excited state n_1 to $n_2 = \dfrac{(n_1-n_2)(n_1-n_2+1)}{2}$
- λ_{min} of X Ray for max energy $\Rightarrow \lambda_m(\text{Å}) = \dfrac{hc}{eV} = \dfrac{12375}{V(\text{volt})}$, Moseley's law $\rightarrow \sqrt{f} = a(Z-b)$
- 



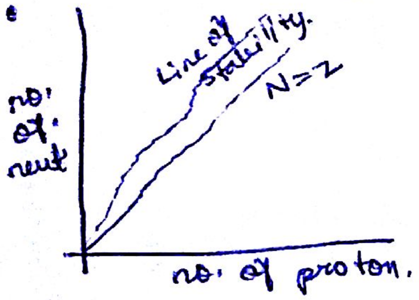
$K_{max} = hf - W$



for x ray

$\dfrac{1}{\lambda} = \underset{\downarrow}{(Z-b)}\left(\dfrac{1}{n_1^2} - \dfrac{1}{n_2^2}\right)$

$b=1$ for K series

- 

$^{A}_{Z}X = {}^{A-4}_{Z-2}Y + {}^{4}_{2}He$ [α decay]

[β decay] $n \rightarrow p + e^- +$ antineutrino
$p \rightarrow n + e^+ +$ neutrino

Positron (e^+) emission from a nucleus ↓ at. no by 1

- Electron capture: $^{A}_{Z}X + e^- \rightarrow {}^{A}_{Z-1}Y + \nu$
- Radioactivity: $-\dfrac{dN}{dt} = \underset{\substack{\downarrow \\ \text{decay constant}}}{\lambda} N$, $N = N_0 e^{-\lambda t}$, $t_{avg} = \dfrac{1}{\lambda}$, $t_{1/2} = \dfrac{0.693}{\lambda} = \dfrac{\ln 2}{\lambda}$ | $R = -\dfrac{dN}{dt}$

$1\,Bq = 1\,dps$, $1\,Ci = 3.7\times10^{10}\,dps$, $1\,Rd = 10^6\,dps$

- After 'n' half lives, no of nuclei left $= \dfrac{N_0}{2^n}$, fracn left $= \dfrac{1}{2^n}$, % left $= \dfrac{100}{2^n}$
- no of nuclei decayed after time 't': $N_t = N_0(1 - e^{-\lambda t})$
- ΔN is no. of nuclei decayed in time Δt, $\Delta N = \lambda N \Delta t$ [when $\Delta t \lll t_{1/2}$]
- $E = mc^2$, $1u = 931.5\,MeV$ & $c^2 = 931.5\,MeV/u$
- Nuclear fission (divide & conquer): atom bomb
- Nuclear fusion: hydrogen bomb $[{}^{2}_{1}H + {}^{2}_{1}H \rightarrow {}^{3}_{2}He + {}^{1}_{0}n + 3.3\,MeV]$