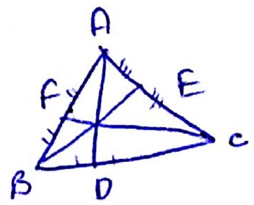
SOT

- $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$
- Nepier's formula, $\tan\left(\frac{B-C}{2}\right) = \frac{b-c}{b+c}\cot\frac{A}{2}$
- $\cos C = \frac{a^2+b^2-c^2}{2ab}$.
- Projecⁿ formulae: $a = b\cos C + c\cos B$
- Half angle formulas: $\sin\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{bc}}$ | $\cos\frac{A}{2} = \sqrt{\frac{s(s-a)}{bc}}$ | $\tan\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$
- $\Delta = \frac{1}{2}ac\sin B = \sqrt{s(s-a)(s-b)(s-c)} = rs$
- $r = (s-a)\tan\frac{A}{2} = 4R\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$
- Length of angle bisector $= \left(\frac{2bc}{b+c}\right)\cos\frac{A}{2}$.
- Image of orthocenter in any side of Δ lie on circum circle
- Dist. of orthocenter from vertex: $2R\cos C$
- Dist of orthocenter from side of Δ: $2R\cos B\cos C$
- Pedal Δ = Δgle formed by feet of altitudes on the side of Δ is called pedal Δ

  In an acute angled Δ, orthocenter of ΔABC is incenter of pedal Δ.
  sides of pedal Δ = $a\cos A$
  circum center of pedal Δ: $R' = R/2$.



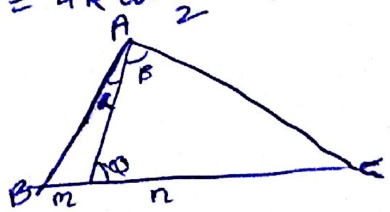
- Apollonius th: $AB^2 + AC^2 = 2(AD^2 + BD^2)$
- Escribed circle:

$r_1 = \frac{\Delta}{s-a} = s\tan\frac{A}{2} = 4R\sin\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$.

- Dist. b/w Incenter & excenter: $\frac{r}{\sin(B/2)\sin C/2}$
- Dist b/w excenters: $I_1I_2 = 4R\cos\frac{C}{2}$
- m-n theorem –

$(m+n)\cot\theta = m\cot\alpha - n\cot\beta$

$(m+n)\cot\theta = n\cot B - m\cot C$



$\cos A + \cos B + \cos C = 1 + 4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$

$r = 4R\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$, $R \geq 2r$

- ar(quad) = $\frac{1}{2}$ (prod. of diagonal) × (sine of included angle)
- In a cyclic quad, sum of pdt of opposite sides is equal to the product of diagonals.
- If the sum of opp side of quad is equal then only circle can be inscribed in it.

TRIGO

- $\tan A + \tan B + \tan C = \tan A \tan B \tan C$
- $\tan\frac{A}{2}\tan\frac{B}{2} + \tan\frac{B}{2}\tan\frac{C}{2} + \tan\frac{C}{2}\tan\frac{A}{2} = 1$
- $\sin 2A + \sin 2B + \sin 2C = 4\sin A \sin B \sin C$
- $\cos 2A + \cos 2B + \cos 2C = -1 - 4\cos A \cos B \cos C$

for Δ

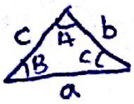
- $\cos A \cos(60-A) \cos(60+A) = \frac{1}{4}\cos 3A$ [also for sin $ tan]

: Regular Polygon → sum of internal angle $= (n-2)\pi$

Each angle $= \frac{(n-2)\pi}{n}$

In regular polygon circum center $ incenter are same.

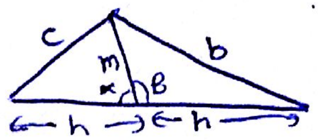
SOLUN OF $\triangle$

• LAW OF SINE : $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R = \frac{abc}{2\Delta}$  $ R be radius of circumcircle of $\triangle ABC$

• COSINE RULE : $\cos A = \frac{b^2+c^2-a^2}{2bc}$, $\cos B = \frac{a^2+c^2-b^2}{2ac}$, $\cos C = \frac{a^2+b^2-c^2}{2ab}$

Also, $\tan A = \frac{abc}{R(b^2+c^2-a^2)}$, $\tan B = \frac{abc}{R(a^2+c^2-b^2)}$ $ $\tan C = \frac{abc}{R(a^2+b^2-c^2)}$

• Projecⁿ formulae : $a = b\cos c + c\cos b$
$b = c\cos A + a\cos C$
$c = b\cos A + a\cos b$



• Appollonius theorem: $b^2+c^2 = 2(h^2+m^2) \Rightarrow h = \frac{a}{2}$ [using cosine rule]

• Napier's anology : $\tan\left(\frac{A-B}{2}\right) = \left(\frac{a-b}{a+b}\right)\cot\frac{C}{2}$, $\tan\left(\frac{B-C}{2}\right) = \left(\frac{b-c}{b+c}\right)\cot\frac{A}{2}$, $\tan\left(\frac{C-A}{2}\right) = \left(\frac{c-a}{c+a}\right)\cot\frac{B}{2}$
law of tangents.

• Mollweide's formula : $\frac{a+b}{c} = \frac{\cos\left(\frac{A-B}{2}\right)}{\sin\frac{C}{2}}$ $ $\frac{a-b}{c} = \frac{\sin\left(\frac{A-B}{2}\right)}{\cos\frac{C}{2}}$

• Area of triangle : $\Delta = \frac{1}{2}$ (Product of two sides) $\times$ (sine of including side)
$\Delta = \sqrt{s(s-a)(s-b)(s-c)}$ where s = semiperimeter.
$\Delta = \frac{abc}{4R}$ where R is circumradius
$\Delta = \frac{a^2\sin B\sin C}{2\sin(B+C)} = \frac{b^2}{2}\frac{\sin A\sin C}{\sin(A+C)} = \frac{c^2}{2}\frac{\sin A\sin B}{\sin(A+B)}$.

• Half angle formulae :- $\sin\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{bc}}$, $\sin\frac{B}{2} = \sqrt{\frac{(s-a)(s-c)}{ac}}$, $\sin\frac{C}{2} = \sqrt{\frac{(s-b)(s-a)}{ab}}$

$\left[\tan\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}\right]$ $\cos\frac{A}{2} = \sqrt{\frac{s(s-a)}{bc}}$, $\cos\frac{B}{2} = \sqrt{\frac{s(s-b)}{ca}}$, $\cos\frac{C}{2} = \sqrt{\frac{s(s-c)}{ab}}$

• Inradius : $r = \frac{\Delta}{s} = 4R\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2} = (s-a)\tan\frac{A}{2} = (s-b)\tan\frac{B}{2} = (s-c)\tan\frac{C}{2}$

* $r = \frac{a\sin\frac{B}{2}\sin\frac{C}{2}}{\cos\frac{A}{2}} = \frac{b\sin\frac{A}{2}\sin\frac{C}{2}}{\cos\frac{B}{2}} = \frac{c\sin\frac{A}{2}\sin\frac{B}{2}}{\cos\frac{C}{2}}$

* $\cos A + \cos B + \cos C = 1 + \frac{r}{R}$

• Excenter : $r_1 = \frac{\Delta}{s-a}$, $r_2 = \frac{\Delta}{s-b}$, $r_3 = \frac{\Delta}{s-c}$

$r_1 + r_2 + r_3 - r = 4R$

$\frac{1}{r^2} + \frac{1}{r_1^2} + \frac{1}{r_2^2} + \frac{1}{r_3^2} = \frac{a^2+b^2+c^2}{\Delta^2}$

$\frac{1}{bc} + \frac{1}{ca} + \frac{1}{ab} = \frac{1}{2Rr}$

$r_1 = s\tan\frac{A}{2}$, $r_2 = s\tan\frac{B}{2}$, $r_3 = s\tan\frac{C}{2}$

$r_1 = \frac{a\cos\frac{B}{2}\cos\frac{C}{2}}{\cos\frac{A}{2}}$, $r_2 = \frac{b\cos\frac{C}{2}\cos\frac{A}{2}}{\cos\frac{B}{2}}$

$\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} = \frac{1}{r}$.

$r_1r_2 + r_2r_3 + r_3r_1 = s^2$

$\Delta = 2R^2\sin A\cdot\sin B\sin C = 4Rr\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$ | $r_1r_2 + r_2r_3 + r_3r_1 = s^2$

$r_1 = 4R\sin\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$

$r_2 = 4R\cos\frac{A}{2}\sin\frac{B}{2}\cos\frac{C}{2}$

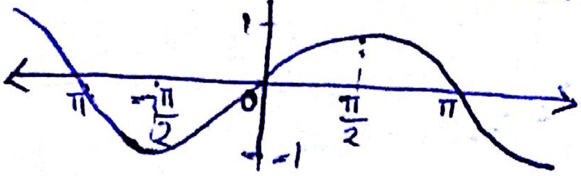
TRIGONOMETRY

- Sexagesimal system (degree) $1^\circ = \frac{1}{90}^{th}$ part of right angle $1^\circ = 60'$
- Centesimal system (grade) $1^g = \frac{1}{100}^{th}$ part of right angle $1^g = 100'$
- Circular system (radian) $1^c = \frac{\widehat{PC}}{AP}$

$$\frac{D}{90} = \frac{G}{100} = \frac{2R}{\pi}$$

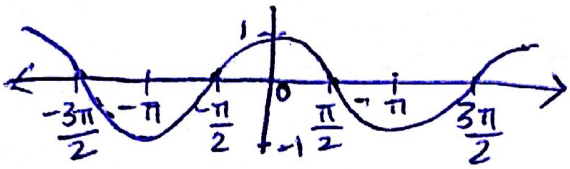
- Need of radian is to plot angle on real line $[\pi \sim 3.14]$
- Graphs →

$y = \sin x$



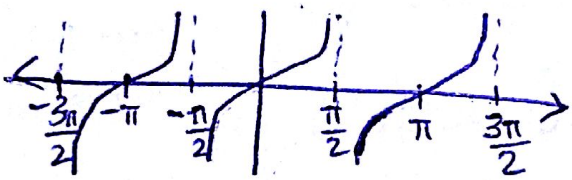
periodic in 2π

$y = \cos x$



periodic in 2π

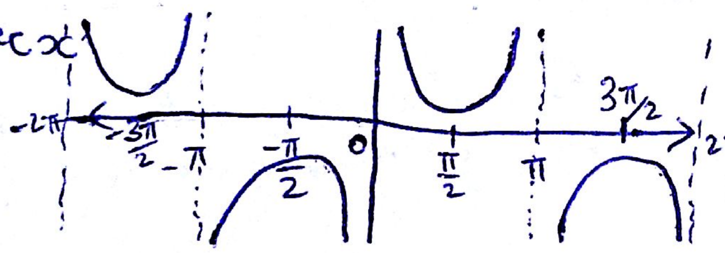
$y = \tan x$



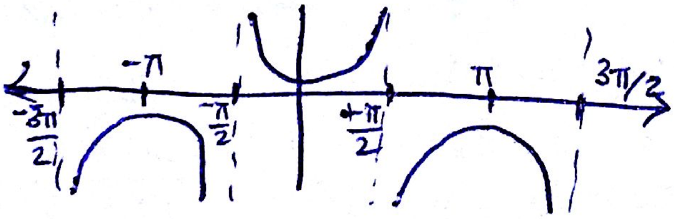
Periodic in π

$$-\sqrt{a^2+b^2} \le a\cos\theta + b\sin\theta \le \sqrt{a^2+b^2}$$

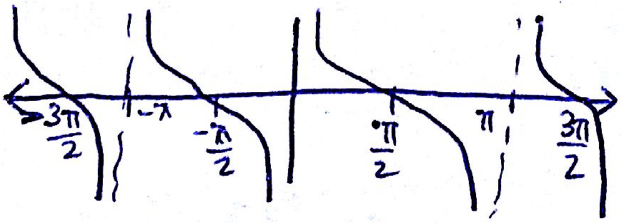
$y = \operatorname{cosec} x$



$y = \sec x$



$y = \cot x$



$$\sin 18 = \cos 72 = \frac{\sqrt{5}-1}{4}$$

$$\cos 36 = \sin 54 = \frac{\sqrt{5}+1}{4}$$

$$\sin 75 = \cos 15 = \frac{\sqrt{3}+1}{2\sqrt{2}}$$

$$\sin 15 = \cos 75 = \frac{\sqrt{3}-1}{2\sqrt{2}}$$

$$\tan 22.5 = \sqrt{2} - 1$$

Odd power →

- $\sin\theta = \sin\alpha$, $\theta = n\pi + (-1)^n\alpha$
- $\cos\theta = \cos\alpha$, $\theta = 2n\pi \pm \alpha$
- $\tan\theta = \tan\alpha$, $\theta = n\pi + \alpha$.

Even power →

- $\sin^2\theta = \sin^2\alpha$, $\theta = n\pi \pm \alpha$

Same for $\cos^2\theta = \cos^2\alpha$
$\tan^2\theta = \tan^2\alpha$.

- $\sin(A+B) = \sin A\cos B + \cos A\sin B$.
  $\sin(A-B) = \sin A\cos B - \cos A\sin B$.
  $\cos(A+B) = \cos A\cos B - \sin A\sin B$
  $\cos(A-B) = \cos A\cos B + \sin A\sin B$
  $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A\tan B}$
  $\tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A\tan B}$.
  $\sin(A+B)\sin(A-B) = \sin^2 A - \sin^2 B$
  $\cos(A+B)\cos(A-B) = \cos^2 A - \sin^2 B$.

- $2\sin A\cos B = \sin(A+B) + \sin(A-B)$
- $2\cos A\sin B = \sin(A+B) - \sin(A-B)$
- $2\cos A\cos B = \cos(A+B) + \cos(A-B)$
- $2\sin A\sin B = \cos(A-B) - \cos(A+B)$
- $\sin C + \sin D = 2\sin\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$
- $\sin C - \sin D = 2\cos\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$
- $\cos C + \cos D = 2\cos\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$
- $\cos C - \cos D = 2\sin\left(\frac{C+D}{2}\right)\sin\left(\frac{D-C}{2}\right)$

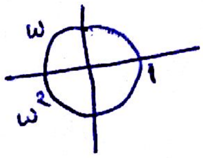
- $\sin(A+B+C) = \sin A\cos B\cos C + \cos A\sin B\cos C + \cos A\cos B\sin C - \sin A\sin B\sin C$
- $\cos(A+B+C) = \cos A\cos B\cos C - \cos A\sin B\sin C - \sin A\cos B\sin C - \sin A\sin B\cos C$
- $\tan(A+B+C) = \dfrac{\tan A + \tan B + \tan C - \tan A\tan B\tan C}{1 - \tan A\tan B - \tan B\tan C - \tan C\tan A}$

- $\sin 2\theta = 2\sin\theta\cos\theta$ \$ $\cos 2\theta = \cos^2\theta - \sin^2\theta = 1 - 2\sin^2\theta = 2\cos^2\theta - 1$
- $\tan(45 \pm \theta) = \dfrac{1 \pm \tan\theta}{1 \mp \tan\theta}$

  $\sin^2\theta = \dfrac{1-\cos 2\theta}{2}$ \$ $\cos^2\theta = \dfrac{1+\cos 2\theta}{2}$
- $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$
- $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$.
- $\sin 3\theta = 4\sin(60-\theta)\sin\theta\sin(60+\theta)$ → same for $\cos 3\theta$ \$ $\tan 3\theta$ (sin → cos, sin → tan)
- $\cos A\cos 2A\cos 2^2A\cos 2^3A \cdots\cdots \cos 2^{n-1}A = \dfrac{\sin 2^n A}{2^n \sin A}$
- $\sin\alpha + \sin(\alpha+\beta) + \sin(\alpha+2\beta) + \cdots + \sin(\alpha + \overline{n-1}\,\beta) = \dfrac{\sin\frac{n\beta}{2}}{\sin\frac{\beta}{2}}\sin\left(\alpha + (n-1)\frac{\beta}{2}\right)$
- $\cos\alpha + \cos(\alpha+\beta) + \cos(\alpha+2\beta) + \cdots + \cos[\alpha + (n-1)\beta] = \dfrac{\sin\frac{n\beta}{2}}{\sin\frac{\beta}{2}}\cos\left\{\alpha + (n-1)\frac{\beta}{2}\right\}$

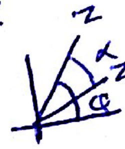
COMPLEX NO.

- $z = x + iy = |z|(\cos\theta + i\sin\theta) = |z|\,\text{cis}(\theta) = |z| e^{i\theta}$
- $\ln z = \ln|z| + i\theta$
- De moivre's theorem : $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$
- cube roots of unity : $1 + \omega + \omega^2 = 0$ | $\omega = \frac{1}{\omega^2}$
  $1 \times \omega \times \omega^2 = 1$

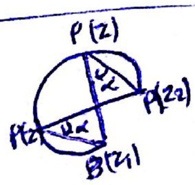


- Eqⁿ of line passing through pts z_1, z_2 : $\frac{z - z_1}{\bar{z} - \bar{z}_1} = \frac{z_1 - z_2}{\bar{z}_1 - \bar{z}_2}$
- Eqⁿ of line : $\bar{a}z + a\bar{z} + b = 0$
- Perp' dist 'd' = $\frac{|z_c\bar{a} + \bar{z}_c a + b|}{2|a|}$

- concept of Rotaⁿ : $\frac{z}{z'} = \frac{|z|}{|z'|} e^{i\alpha}$



- $\arg\left(\frac{z - z_1}{z - z_2}\right) = \alpha$ (fixed)



- circle : $|z - z_0| = r$
- Std Loci : $|z - z_1| = |z - z_2|$ ($\perp$ bisector)

  $|z - z_1| + |z - z_2| = |z_1 - z_2|$ [AP + PB = AB]

  $|z - z_1| - |z - z_2| = |z_1 - z_2|$

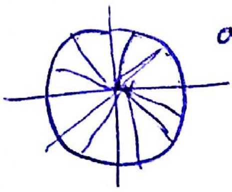
- $\arg\left(\frac{z - z_1}{z - z_2}\right) = \pm\frac{\pi}{2}$ circle.
- $\arg\left(\frac{z - z_1}{z - z_2}\right) = 0$ or π st. line
- $|z - z_0| = \left|\frac{\bar{a}z_0 + \bar{z}_0 a + r}{2|a|}\right|$

Ellipse $|z - z_1| + |z - z_2| = k$ [$k > |z_1 - z_2|$], $e = \frac{|z_1 - z_2|}{k}$

hyperb. $\big||z - z_1| - |z - z_2|\big| = k$ [$k < |z_1 - z_2|$] some e

circle $\left[\begin{array}{l} |z - z_1|^2 + |z - z_2|^2 = |z_1 - z_2|^2 \\ |z - z_1| = k|z - z_2| \quad (k \neq 1) \end{array}\right.$

- N^{th} root of unity : sum of the roots = 0
  Prod of roots = $(-1)^{n-1}$



all roots lie on circle of unit radius.

Series $ progression

- A.P. ⇒ $T_n = a+(n-1)d$ $ $S_n = \frac{n}{2}[2a+(n-1)d] = \frac{n}{2}[a+l]$
- If 3 nō's, a, b, c are in A.P. then, $2b = a+c$ $ nō's can be taken as $[(a-d),(a),(a+d)]$
- for 4 nōs, $(a-3d)$, $(a-d)$, $(a+d)$, $(a+3d)$ whose common diff is $\underline{2d}$
- G.P → $T_n = ar^{n-1}$ $ $S_n = \frac{a(r^n-1)}{r-1}$ for $r \neq 1$ ⇒ for $r=1$, $S_n = na$
- Sum of ∞ terms $= \frac{a}{1-r}$ for $r \in (-1,1)$
- for a, b, c in G.P, $b^2 = ac$.
- HM → if $a_1, a_2, a_3 \ldots$ are in HP. then $\frac{1}{a_1}, \frac{1}{a_2}, \frac{1}{a_3}, \ldots$ are in A.P.
- for a, b, c in H.P, $b = \frac{2ac}{a+c}$
- Means — $A.M = \frac{a_1+a_2+\cdots+a_n}{n}$ | $H.M = \frac{n}{\frac{1}{a_1}+\frac{1}{a_2}+\cdots+\frac{1}{a_n}}$

  $GM = (a_1 \cdot a_2 \cdot a_3 \cdot a_4 \cdots a_n)^{1/n}$

⊙ for 'n' AM inserted b/w nō's a $ b such that resultant seq is in A.P.

$a, a_1, a_2, a_3, \ldots, a_n, b$ [Total terms $= n+2$] ⇒ $b = a+(n+2-1)d$

or $d = \frac{b-a}{n+1}$ here, $A_1 = a+d$ $ $a_n = a+nd$

→ Sum of 'n' AM $= \frac{n}{2}(a+l) = \frac{n}{2}(a+d+b-d) = n\left(\frac{a+b}{2}\right) = n \times$ (AM of a $ b)

⊙ for 'n' GM inserted b/w nō's a $ b such that resulting seq. is in G.P

$a, G_1, G_2, \ldots, G_n, b$. [Terms $= n+2$] ⇒ $b = ar^{n+2-1}$ ⇒ $\frac{b}{a} = r^{n+1}$ ⇒ $r = \left(\frac{b}{a}\right)^{\frac{1}{n+1}}$

→ Product of 'n' GM $= ar \cdot ar^2 \cdot ar^3 \cdots ar^n = a^n r^{1+2+\cdots+n} = a^n r^{\frac{n(n+1)}{2}}$

$= a^n r^{\frac{n(n+1)}{2}} = a^n \left(\frac{b}{a}\right)^{n/2} = (\sqrt{ab})^n = [\text{GM of a \$ b}]^n$

⊙ for 'n' HM inserted b/w a $ b such that resulting seq is in H.P.

HP → $a, H_1, H_2, H_3, \ldots, H_n, b$ ⇒ $\frac{1}{a}, \frac{1}{H_1}, \frac{1}{H_2}, \ldots, \frac{1}{H_n}, \frac{1}{b}$ in AP.

⇒ $\frac{1}{b} = \frac{1}{a} + (n+2-1)d$ ⇒ $d = \frac{a-b}{ab(n+1)}$ $\left[\frac{1}{H_n} = \frac{1}{a} + nd\right]$

→ Sum of reciprocal of 'n' H.M. →

$\frac{1}{H_1}+\frac{1}{H_2}+\frac{1}{H_3}+\cdots+\frac{1}{H_n}$ ⇒ $S_n = \frac{n}{2}\left[\frac{1}{H_1}+\frac{1}{H_n}\right] = \frac{n}{2}\left[\frac{1}{a}+d+\frac{1}{b}-d\right] = \frac{n}{2}\left[\frac{1}{a}+\frac{1}{b}\right]$

⇒ $S_n = \frac{n}{2}\left[\frac{a+b}{ab}\right] = \frac{n}{\frac{2ab}{a+b}} = \frac{n}{\text{HM of a \$ b}}$.

- Sum of first 'n' natural nō's $= \frac{n(n+1)}{2}$.
- Sum of square of first 'n' natural nō's $= \frac{n(n+1)(2n+1)}{6}$
- Sum of cube of 1st 'n' natural nō's $= \left[\frac{n(n+1)}{2}\right]^2$.

- In general term of special sequence whose $T_n = an^3 + bn^2 + cn + d$ then we can find S_n by applying Σ on both side. $[\Sigma T_n = S_n]$
- AGP [combinaⁿ of AP & GP] :- if $a_1, a_2, a_3 \ldots$ are in AP & $b_1, b_2 \ldots$ are in GP.

then, $a_1 b_1 + a_2 b_2, a_3 b_3, \ldots$ are in AGP.

Its sum could be found as →

$$S = a_1 b_1 + a_2 b_2 + a_3 b_3 \ldots \ldots + a_n b_n$$

$$rS = \quad a_1 b_2 + a_2 b_3 \ldots \ldots + a_{n-1} b_n + a_n b_{n+1}$$

on sub.

$$S(1-r) = a_1 b_1 + d b_2 + d b_3 + \ldots + d b_n - a_n b_{n+1}$$

$$S(1-r) = a_1 b_1 + d\underbrace{[b_2 + b_3 + \ldots + b_n]}_{(n-1) \text{ terms in GP}} - a_n b_{n+1}$$

& for sum of infinite term if $r \in (-1,1)$ proceed & apply $S_\infty = \frac{a}{1-r}$

- Method of difference :- for a seq. $a_1, a_2, a_3, \ldots, a_n$ if, $(a_2 - a_1), (a_3 - a_2), (a_4 - a_3) \ldots$ are either in AP or GP then we could find its nth term & further apply Σ for S_n.

$$S = a_1 + a_2 + a_3 + \ldots \ldots + T_n$$

$$S = \quad a_1 + a_2 + \ldots \ldots + T_{n-1} + T_n$$

on subtracting

$$T_n = a_1 + \underbrace{[(a_2 - a_1) + (a_3 - a_2) + \ldots + (T_n - T_{n-1})]}_{\substack{\text{varies in AP or GP} \\ \text{no of terms} = (n-1)}}$$

- $T_n = an^2 + bn + c$ [1st diff. in A.P]
- $T_n = an^3 + bn^2 + cn + d$ [2nd diff. in AP]
- $T_n = ar^n + b$ [1st diff in GP]
- $T_n = ar^n + bn + c$ [2nd diff in GP]
- V_n method :- used where Σ can't be applied.

Ex → $T_n = \frac{1}{n(n+1)}$

$$T_n = \frac{(n+1) - (n)}{n(n+1)}$$

$$T_n = \frac{1}{n} - \frac{1}{n+1}$$

$$T_1 = \frac{1}{1} - \cancel{\frac{1}{2}}$$

$$T_2 = \cancel{\frac{1}{2}} - \cancel{\frac{1}{3}}$$

$$\vdots$$

$$T_n = \cancel{\frac{1}{n}} - \frac{1}{n+1}$$

$$\Rightarrow S_n = 1 - \frac{1}{n+1}$$

$$\Rightarrow S_n = \frac{n}{n+1}$$

⇒ if there are more than 3 terms in denominator then [greatest - least] [in T_n]

⇒ if their are more than 3 terms in numerator then [(greater+1)term - (least+1)term.] [in T_n]

- for $a_1, a_2, a_3, \ldots, a_n$, are any +ve real no's -

$$AM \geq GM \geq HM$$ equality hold when no's are equal

→ Used to find maxᵐ & minᵐ value.

Permutaⁿ $ Combinaⁿ

⊙ Permutaⁿ ⇒ both selecⁿ $ arrangements.

n – no. of objects
r – no. of selected objects.

- $^nP_r = \frac{\lfloor n}{\lfloor n-r}$ [$ $^nP_0 = 1$, $^nP_2 = n(n-1)$, $^nP_3 = n(n-1)(n-2)$
- $\lfloor 4 = 24$, $\lfloor 5 = 120$, $\lfloor 6 = 720$, $\lfloor 7 = 5040$ $ $\lfloor 0 = \lfloor 1 = 1$
- Principle of counting → and [both] → × (multiply), OR → + add
- Linear permutaⁿ (Linear arrangement) → without repeatiⁿ = ^nP_r
  → With repeatiⁿ = n^r
- Circular permutaⁿ (circular arrangement) → (one fixed) Unsymmetrical = $\lfloor n-1$ (for n objects) $ $\frac{^nP_r}{r}$ (r diff. objects) for diff things
  Symmetrical = $\frac{\lfloor n-1}{2}$ (for n objects) [like necklace, garland, Key rings] $ $\frac{^nP_r}{2r}$ (for r things out of n.)

- If out of 'n' objects 'p', 'q', ... are repeated then possible cases are $\frac{\lfloor n}{\lfloor p \lfloor q \cdots}$
  [MATHEMATICS → $\frac{\lfloor 11}{\lfloor 2 \lfloor 2 \lfloor 2}$]
- To find the rank of any word according to dictionary pattern. 1st numbering is done, then no. of words less than in left side are noted then write $\lfloor 0, \lfloor 1, \ldots$ from left side till needed [word get over] * add '1' to get ranking of the word.

3>1	5	2	4	
M	A	T	H	S
2	0	2	0	0
$\lfloor 4$	$\lfloor 3$	$\lfloor 2$	$\lfloor 1$	$\lfloor 0$
$2\lfloor 4$	0	$2\lfloor 2$	0	0

$48 + 4 = 52$

; and for repeatition of letters divide by factorial of no. of repeat letters.
Ex – Book

Rank = $52 + 1 = 53$

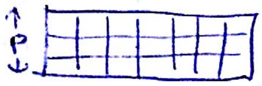
- To find unit digit of any no, write its value for some power $ apply the repeatition. [Ex → $3^1 = 3$, $3^2 = 9$, $3^3 = 7$, $3^4 = 1$] [$3^{109} = 3$, $(17)^{13} = 7^{13}$]
- To find no. of zeroes in last → $\lfloor x = \left[\frac{x}{5}\right] + \left[\frac{x}{5^2}\right] + \left[\frac{x}{5^3}\right] \cdots$ [Ex – $\left[\frac{36}{5}\right] + \left[\frac{36}{25}\right] + \cancel{\frac{36}{125}}$ = 8 neglect]
- To find exponent of any prime no → $\lfloor 34 = \left[\frac{34}{7}\right] + \cancel{\left[\frac{34}{72}\right]} = 4$
  $[7.2] = 7$; $[6.9] = 6$

⊙ Combinaⁿ ⇒ Only selecⁿ → No arrangements ⇒ $^nC_r = \frac{\lfloor n}{\lfloor n-r \lfloor r}$ or $^nC_r = \frac{^nP_r}{\lfloor r}$

- $^nC_r = {^nP_r}/{\lfloor r}$ $ $^nC_0 = 1$ $ $^nC_1 = n$ $ $^nC_2 = \frac{n(n-1)}{2 \cdot 1}$
- $^nC_r = {^nC_{n-r}}$; $^nC_n = 1$
- if $^nC_x = {^nC_y}$ then, $x = y$ or $x + y = n$.
- $\frac{^nC_r}{^nC_{r-1}} = \frac{n-r+1}{r}$
- $^nC_r = \frac{n}{r} \, {^{n-1}C_{r-1}}$
- $^nC_r = \frac{n(n-1)}{r(r-1)} \, {^{n-2}C_{r-2}}$
- $\frac{^nC_{r+1}}{^nC_r} = \frac{n-r}{r+1}$
- $^nC_r + {^nC_{r-1}} = {^{n+1}C_r}$
- $^nC_0 + {^nC_1} + \cdots + {^nC_n} = 2^n$
- $^nC_1 + {^nC_2} + \cdots + {^nC_n} = 2^n - 1$
- $^nC_0 + {^nC_2} + \cdots + {^nC_n} = 2^{n-1}$
  $= {^nC_1} + {^nC_3} + \cdots = 2^{n-1}$
- Permutaⁿ > Combinaⁿ

- Restricted selecⁿ's — if from 'n', 'r' objects are to be selected
  - → K are already selected then ${}^{n-K}C_{r-K}$
  - → K is never selected then ${}^{n-K}C_{r}$
- Sum of unit digits of all possible no. made from $(x, y, \ldots)$
  Ex – $(2,4,9)$ = 249, 294, 492, 429, 924, 942 is $(2+4+9)\underline{|2}$.
  – $(1,2,3,4,5) = (1+2+3+4+5)\underline{|4}$
- Sum of all possible no's made by $(a, b, c, \ldots)$
  Ex – $(1,2,3) \rightarrow (1+2+3)\underline{|3}\,[1+10+10^2] \rightarrow (1+2+3)\underline{|2} \times 111$
  – $(9,4,3,2,5) \rightarrow (9+4+3+2+5)\underline{|4} \times 11111$
- No. of divisors or factors. $24 = 2^3 \times 3^1 = (3+1)(1+1) = 8$
  total divisor = n $\qquad 144 = 2^4 \times 3^2 = (4+1)(2+1) = 15$
  Improper divisor = 2 [1 \$ the no. itself]
  Proper divisor = $n-2$
- Sum of divisors = $24 = 2^3 \times 3^1 = (2^0+2^1+2^2+2^3)(3^0+3^1) = 60$
- To find no. of ways in which given no. can be resolved as product of two factors → $\dfrac{(p+1)(q+1)\ldots+1}{2}$ Ex – $144 = 2^4\,3^2 = \dfrac{(4+1)(2+1)+1}{2} = 8$
- If not perfect sq. do not add 1. [for perfect sq.]
- Selecⁿ from 'n' distinct objects $= {}^nC_0 + {}^nC_1 + \ldots + {}^nC_n = 2^n$ \$ $2^n - 1$ for at least one.
- Selecⁿ from identical object = 1 [for n^{th} object] \$ no. of cases = $(n+1)$ [including no. selecⁿ]
- Selecⁿ of at least one of A_1 \$ $A_2, \ldots, A_n$ where A_1 are one kind of object, A_2 2ⁿᵈ type, A_3 third type ......
  then no. of ways $= (A_1+1)(A_2+1)\ldots(A_n+1) - 1$ [for no selecⁿ (−1)]
  \$ when both identical \$ distinct are pnt.
  $(A_1+1)(A_2+1)\ldots(A_n+1) \times 2^K - 1$ [K identical object]
- Combinaⁿ in geometry – for 'n' non collinear pts. we can make ${}^nC_2$ line seg.
  – for 'n' pts out of which 'm' are collinear → ${}^nC_2 - {}^mC_2 + 1$
  – no. of $\Delta = {}^nC_3$ [Not collinear]
  – for 'm' out of 'n' are collinear $= {}^nC_3 - {}^mC_3 + 0$
- No. of diagonals of 'n' sided polygon $= {}^nC_2 - n$
- No. of ∥gm out of m \$ n distinct ∥s line $= {}^mC_2 \times {}^nC_2$

← n →



no. of rect. $= \dfrac{np}{4}(n+1)(p+1)$

no. of sq. $= np + (n-1)(p-1) + (n-2)(p-2) - \ldots$

Ex – BANANA
2 1 3 1 3 1 (above letters)
3 0 2 0 1 0

$\underline{|5} \times \dfrac{3}{\underline{|2}\,\underline{|3}}\quad \dfrac{2}{\underline{|4}\,\underline{|3}\,\underline{|2}}\quad \dfrac{0}{\underline{|2}}\quad \dfrac{1}{\underline{|1}}\quad \dfrac{0}{\underline{|1}\,\underline{|0}}$

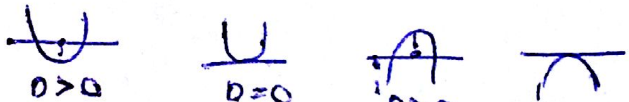
$\Rightarrow \dfrac{3}{\underline{|2}\,\underline{|3}} + \dfrac{2}{\underline{|2}\,\underline{|3}} + \dfrac{1}{\underline{|1}} + 1$

\$ add 1 for its Rank [$\underline{|3}, \underline{|2}, \underline{|1}, \underline{|0}$]

Quadratic eq^n

- $ax+by+c=0$, $\alpha, \beta \rightarrow$ roots, $\alpha+\beta = -\frac{b}{a}$, $|\alpha-\beta| = \frac{\sqrt{D}}{|a|}$, $\alpha \cdot \beta = \frac{c}{a}$
- $ax+by+c=0 = a(x-\alpha)(x-\beta)$, $\alpha, \beta = -\frac{b \pm \sqrt{b^2-4ac}}{2a}$ \$ $D = b^2-4ac$
  - $D>0$, real \$ distinct roots
  - $D=0$, real \$ equal roots $\Rightarrow$ If one root is $\alpha+i\beta$ then other is $\alpha-i\beta$.
  - $D<0$, imaginary roots $\rightarrow$ If $a+b+c=0$, one root is 1.
- A pair of root is said to be conjugate if and only if both product as well as sum are rational.
- If α, β are roots of $ax^2+bx+c=0$, then eq^n with root α^2+1, β^2+1 (symmetric) $\overset{put}{\Rightarrow}$ $x=\alpha^2+1$ or $\alpha = \pm\sqrt{x-1}$ in eq^n to get new eq^n.
- An eq^n $ax^2+bx+c=0$ is identity for $a=b=c=0$ [when satisfied by more than 2 values \$ true for all $x \in R$.]
- $Condi^n$ for common root $\rightarrow$ Both root common $\left(\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}\right)$
  $\rightarrow$ at least one root common $\rightarrow$ $(a_2c_1 - a_1c_2)^2 = (b_1c_2 - b_2c_1)(a_1b_2 - a_2b_1)$
- for eq^n $ax^3+bx^2+cx+d=0$ $(a \neq 0) = a(x-\alpha)(x-\beta)(x-\gamma)$
  $\alpha+\beta+\gamma = \frac{-b}{a}$, $\alpha \cdot \beta \cdot \gamma = \frac{-d}{a}$, $\alpha\beta+\beta\gamma+\gamma\alpha = \frac{c}{a}$.
- for $y = ax^2+bx+c \rightarrow$ parabolic \$ $y = a\left[\left(x+\frac{b}{2a}\right)^2 - \frac{D}{4a^2}\right]$ with vertex $\left(\frac{-b}{2a}, \frac{-D}{4a}\right)$
  ($\rightarrow$ always +ve for $D<0$ \$ $a=$ +ve)
- $Loca^n$ of roots of quad. eq^n —

Case ① Both root greater than γ
① $D \geq 0$
② $\gamma < -\frac{b}{2a}$
③ $a \times F(\gamma) > 0$



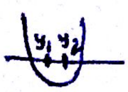
$D>0$	$D=0$	$D>0$	$D=0$
$a>0$	$a>0$	$a<0$	$a<0$
$F(\gamma)>0$	$F(\gamma)>0$	$F(\gamma)<0$	$F(\gamma)<0$.

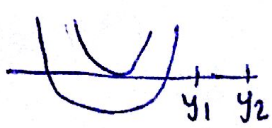
Case ② Both the root are less than γ then ① $D \geq 0$ ② $\gamma > -\frac{b}{2a}$ ③ $a \cdot F(\gamma) > 0$

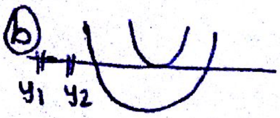
Case ③ γ lies b/w the root then
$D>0$
$a \times F(\gamma) < 0$.

$D>0$	$D>0$
$a>0$	$a<0$
$F(\gamma)<0$	$F(\gamma)>0$

- ① Both root lie b/w y_1 \$ y_2. (y_1, y_2) ⓐ $D \geq 0$ ⓑ $y_1 < \frac{-b}{2a} < y_2$ ⓒ $a \cdot F(y_1) > 0$, $a \cdot F(y_2) > 0$
② Exactly one root lie b/w (y_1, y_2) $\Rightarrow$ $F(y_1) \times F(y_2) < 0$
③ None root lie b/w $[y_1, y_2]$

ⓐ 
$\left\{\begin{array}{l}(i)\ a \cdot F(y_1) < 0 \\ (ii)\ a \cdot F(y_2) < 0\end{array}\right\}$ common.

ⓒ 
common
$\left\{\begin{array}{l}D>0 \\ \frac{-b}{2a} < y_1 \\ a \cdot F(y_1) > 0\end{array}\right.$

ⓑ 
$\left\{\begin{array}{l}(i)\ D \geq 0 \\ (ii)\ \frac{-b}{2a} > y_2 \\ (iii)\ a \cdot F(y_2) > 0\end{array}\right\}$ common.

④ $ax^2+bx+c > 0$ \$ $x \in (y, \infty)$ $a \neq 0$, $a, b, c \in R$.
 $a>0$, $D \geq 0$, $\frac{-b}{2a} \leq 0$
 $a>0$, $b>0$ $\left|y > \frac{-b}{2a}\right|$ $a \cdot F(y) \geq 0$

• If $y = \frac{a_1x^2 + b_1x + c_1}{a_2x^2 + b_2x + c_2} = \frac{A}{B}$ if: A & B have no common root → cross multiply & make Q.E in x then $D \geq 0$ to get range of y

if $y = \frac{a_1}{b_1}$, their exist a real value of x then we include it other wise exclude.

• Range of $y = \frac{ax+b}{cx+d}$, $c \neq 0$ & no common root. then range can be found by multiply checking coeff. of x [to exclude or not]

• If common terms are observed, remove range (y) for that x.

• A Q.E in two variable can be factorised if –

eq^n → $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$. [$2^{nd}$ degree general eq^n]

$abc + 2fgh - af^2 - bg^2 - ch^2 = 0$ & $h^2 - ab \geq 0$

☐ Coordinate geometry –

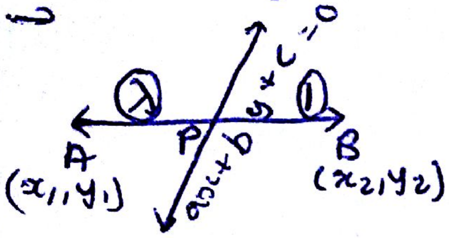
→ to find 3rd vertex (side) of equilateral Δ if 2 vertex are given.

$$\left(\frac{x_1+x_2 \mp \sqrt{3}(y_2-y_1)}{2},\ \frac{y_1+y_2 \pm \sqrt{3}(x_2-x_1)}{2}\right)$$

→ a Δ whose vertex are integer, can never be eq Δ.

→ In any Δ, $AB^2 + AC^2 = 2(AD^2 + BD^2)$ [D is midpt. of BC]

→



$$\frac{AP}{PB} = \frac{\lambda}{1} = \frac{-(ax_1+by_1+c)}{(ax_2+by_2+c)}$$

→ Nine pt. circle → mid pt. of O & C, Also Radius of nine pt O $= \frac{1}{2} \times$ circum radius

→ ar(Δ) formed by line $y = m_1x + c_1$, $y = m_2x + c_2$, $y = m_3x + c_3$ is

$$\Delta = \frac{1}{2}\left|\frac{(c_2-c_3)^2}{m_2-m_3} + \frac{(c_3-c_1)^2}{m_3-m_1} + \frac{(c_1-c_2)^2}{m_1-m_2}\right|$$

& for $a_1x + b_1y + c_1 = 0$ & so on.
$$\Delta = \frac{1}{2|C_1C_2C_3|}\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$ constant in eqn

$C_1 = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$, $C_2 \begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix}$ & similarly C_3

→ orthocenter of Δ formed by pts → $\left(\alpha, \frac{1}{\alpha}\right)$, $\left(\beta, \frac{1}{\beta}\right)$, $\left(\gamma, \frac{1}{\gamma}\right)$ is $\left(\frac{-1}{\alpha\beta\gamma}, -\alpha\beta\gamma\right)$

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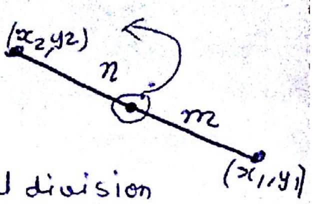
Straight line

- distance b/w two pts $\Rightarrow d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$
- Internal division $\Rightarrow \left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$
- External division $\Rightarrow \left(\frac{mx_2-nx_1}{m-n}, \frac{my_2-ny_1}{m-n}\right)$



- when we assume Ratio $K:1$ $\Rightarrow$ K is +ve for internal division & -ve for external division.
- $ar(\triangle) = \frac{1}{2}\left| x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_1)\right|$ (ar must be taken in mode)
- for $\triangle ABC$, with sides $a, b, c \Rightarrow a+b > c$, $b+c > a$, $a+c > b$ OR $|a-b| < c$, $|b-c| < a$, $|a-c| < b$
- In eq. $\triangle$, G I H O are at same pt.
- Median is a line joining vertex with mid pt. of opp. side [divide in $2\triangle$'s of equal area]
- circumcenter is a pt equidistant from all three vertex or pt. of intersecⁿ of perp. bisector of sides.

[for right $\triangle$, circumcenter lies on mid pt. of hypotenuse]

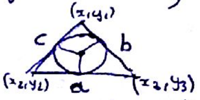
- Altitude – a line drawn through vertex perp. to opp. side.
- Orthocenter – All three altitude are concurrent & that concurrent pt is known as orthocenter.

[In rt $\triangle$, orthocenter is the vertex where rt angle is formed]

- In any $\triangle$ (except equilateral), orthocenter —2— centroid —1— circumcenter
- centroid → intersecting pt. of median. $\Rightarrow G\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$
- Incenter → equidistant from sides of $\triangle$ & pt. of intersecⁿ of Internal angle bisector.

(for Isosceles $\triangle$, internal angle bisector is perp.)

$$I\left[\frac{ax_1+bx_2+cx_3}{a+b+c}, \frac{ay_1+by_2+cy_3}{a+b+c}\right]$$

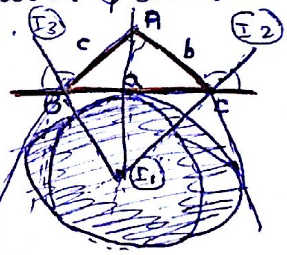


- Excentre → Intersecⁿ pt. of one of the internal angle bisector & external angle bisector of other two vertex angle.

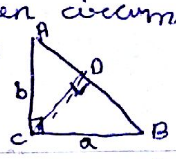
Three excentres are possible

$$I_1 = \left[\frac{-ax_1+bx_2+cx_3}{-a+b+c}, \frac{-ay_1+by_2+cy_3}{-a+b+c}\right]$$

[for I_2 → -sign shift to b, I_3 → -ve shift c]

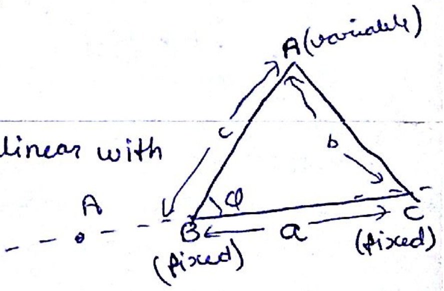


- If sum of square of vertex [i.e. $(x_1, y_1) \Rightarrow x_1^2+y_1^2$] is same then circumcenter is $(0,0)$.
- In any rt. $\triangle$,



$$\frac{AD}{DB} = \frac{b^2}{a^2}$$

- $|b-c| \le a$ equality holds if pt A is collinear with B & C and divide externally
- $b+c \ge a$ equality hold for A collinear with B & C & divide internally.
- $\cos\varphi = \dfrac{a^2+c^2-b^2}{2ac}$



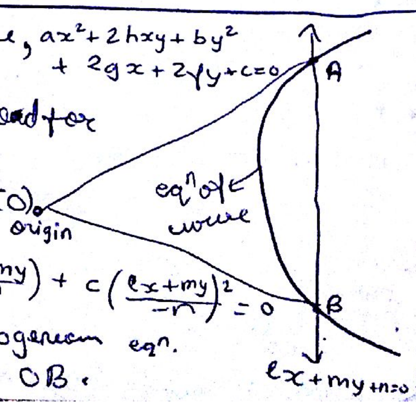
P.T.O

- for A & B be the intersecting pt of a curve, $ax^2+2hxy+by^2+2gx+2fy+c=0$ & a st. line $lx+my+n=0$ then method for finding pair of st. line OA & OB

Step① $\Rightarrow \frac{lx+my}{-n} = 1$

Step② $ax^2+2hxy+by^2+2gx\left(\frac{lx+my}{-n}\right)+2fy\left(\frac{lx+my}{-n}\right)+c\left(\frac{lx+my}{-n}\right)^2=0$

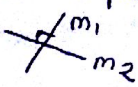
Step③ after solving it is of 2nd degree homogeneous eqn. whose pairs could be known which represent OA & OB.

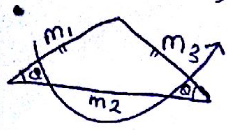


- Slope ⇒ angle made with +ve x axis ⇒ $m = \tan\theta$ [slope of vertical line is not defined] ⇒ [$x_2 \neq x_1$]

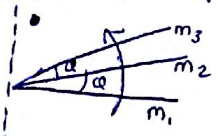
$m = \tan\theta = \frac{y_2 - y_1}{x_2 - x_1}$

- Slope of parallel lines are equal & product of slope of two line perp. to each other ⇒ $m_1 m_2 = -1$
- for two lines with slope m_1, m_2 inclined at angle θ with each other, acute angle = $\tan\theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$

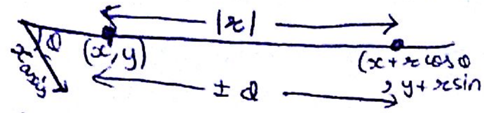


- 

$\frac{m_2 - m_1}{1 + m_1 m_2} = \frac{m_3 - m_2}{1 + m_2 m_3}$

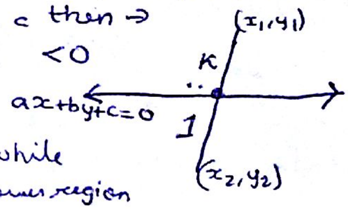
- 

$\frac{m_2 - m_1}{1 + m_1 m_2} = \frac{m_3 - m_2}{1 + m_2 m_3}$

- $\frac{x}{a} + \frac{y}{b} = 1$, where a & b are intercept on x- & y-axis.
- $ax + by + c = 0$, where slope of line is $\frac{-a}{b}$ $\left[\frac{-\text{coeff. of } x}{\text{coeff. of } y}\right]$
- $x\cos\theta + y\sin\theta = p$, where p is perp. dist. from origin. θ = angle made by x axis.
- 

'r' is the algebric distance b/w fixed pts A & B. $\frac{x - x_1}{\cos\theta} = \frac{y - y_1}{\sin\theta} = r$.

- unique soln → $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ & no soln or parallel = $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ & same line = $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$
- perp. dist of a line from origin ⇒ $p = \frac{|c|}{\sqrt{a^2 + b^2}}$ & dist. b/w two parallel line $= \frac{|c_2 - c_1|}{\sqrt{a^2 + b^2}}$
- perp. dist. from (x_1, y_1) to line $ax + by + c = 0$ ⇒ $d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$
- (α, β) is mirror image of pt. (x_1, y_1) w.r.t $ax + by + c = 0$ then ⇒ $\frac{\alpha - x_1}{a} = \frac{\beta - y_1}{b} = \frac{-2(ax_1 + by_1 + c)}{a^2 + b^2}$
- $\frac{\alpha - x_1}{a} = \frac{\beta - y_1}{b} = \frac{-(ax_1 + by_1 + c)}{a^2 + b^2}$ where (α, β) is foot of perp from pt (x_1, y_1) to line.
- If pts (x_1, y_1), (x_2, y_2) lie on same side w.r.t $ax + by + c$ then ⇒ $(ax_1 + by_1 + c)(ax_2 + by_2 + c) > 0$ and for opp. side < 0
- $K = \frac{-(ax_1 + by_1 + c)}{ax_2 + by_2 + c}$ where $K : 1$ is ratio.



- Locn of (α, β) w.r.t $ax + by + c = 0$ ⇒ upper region = $\frac{a\alpha + b\beta + c}{b} > 0$ while less than 0 for lower region
- Family of line → If all the lines passes through a fixed pts.
  Ex → $a_1x + b_1y + c_1 + \lambda(a_2x + b_2y + c_2) = 0$ [making $\lambda = 0$ for some pair (x, y)]
- Eqn of angle bisector → for two intersecting line $a_1x + b_1y + c_1 = 0$ & $a_2x + b_2y + c_2 = 0$

$$\frac{a_1h + b_1K + c_1}{\sqrt{a_1^2 + b_1^2}} = \pm \frac{a_2h + b_2K + c_2}{\sqrt{a_2^2 + b_2^2}}$$ [Both bisectors are perp. to eachother]

If θ is angle b/w the bisector & given line. $|\tan\theta| = 1$ & $\theta = 45°$

$|\tan\theta| < 1$ then, corresp. angle is acute but for $|\tan\theta| > 1$, obtuse

- necessary condin for a second degree general eqn represent pair of st. lines.
⇒ $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represent pair of st line.
when $abc + 2fgh - af^2 - bg^2 - ch^2 = 0$ & $h^2 \geq ab$
[Pair can be found by making eqn quad in either x^2 or y^2 of eqn & then using quad. formula]

- for 2nd degree homogeneous eqn, $g = f = c = 0$, $ax^2 + 2hxy + by^2 = 0$
let the eqns be $y = m_1 x$ & $y = m_2 x$ then m_1, m_2 are roots of quad $bm^2 + 2hm + a = 0$ hence, $m_1 + m_2 = \frac{-2h}{b}$ & $m_1 m_2 = \frac{a}{b}$
- If θ is angle b/w line then, $\tan\theta = \left| \frac{2\sqrt{h^2 - ab}}{a + b} \right|$
[$a + b = 0$ for line perp. to each other]
- $\frac{x^2 - y^2}{a - b} = \frac{xy}{h}$ is eqn of angle bisector of homogeneous eqns pair ⇒

- $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$, $\tan\theta = \left| \frac{2\sqrt{h^2 - ab}}{a + b} \right|$

Soln of pair of st lines
⇒ $2ax + 2hy + 2g = 0$ [diff ① w.r.t x] y = const.
⇒ $2hx + 2by + 2f = 0$ [diff ① w.r.t y] x = cons.
will represent intersn pt [not necessarily same pair]

(P.T.O)

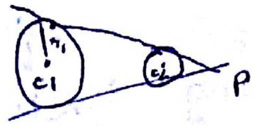
CIRCLE

• If a line intersect circle, → intersect − $r > p$
touch − $r = p$
lie outside − $r < p$

p = perp. dist.

• 
$d(c_1, c_2) = r_1 + r_2$


$d(c_1, c_2) = |r_1 - r_2|$


$\frac{PC_1}{PC_2} = \frac{R_1}{R_2}$

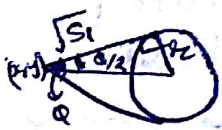
• 
$PA \times PB = PC \times PD = r^2 - OP^2$

• $(x-\alpha)^2 + (y-\beta)^2 = r^2$, r = radius, (α, β) is center.

• $x^2 + y^2 + 2gx + 2fy + c = 0$ → center = $(-g, -f)$
→ radius = $\sqrt{g^2 + f^2 - c}$

• Length of tangent drawn from a ext. pt (x_1, y_1) → $\sqrt{x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c} = \sqrt{S_1}$

• $\tan\frac{\theta}{2} = \frac{r}{\sqrt{S_1}}$, where '$\theta$' is angle b/w two tangent.



• A pt. lie → outside circle → $S > 0$
on circle → $S = 0$
in circle → $S < 0$
where, $S = x_1^2 + y_1^2 + 2gx + 2fy + c$

• $(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$.

(x_1, y_1) (x_2, y_2)

$\boxed{m_1 \times m_2 = -1}$

• A line ⊥ to tangent is called normal.
• Normal always passes through center of circle.
• To find eqⁿ of tang. assume eqⁿ $y = mx + c$ & perp. dist from center = radius
solve for m to get 2 values.
• Angle b/w two circle − angle b/w tangent at intersecting pt of circle,
where $|r_1 - r_2| < d < r_1 + r_2$.

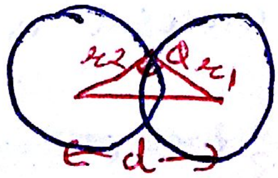
$\cos\theta = \frac{r_1^2 + r_2^2 - d^2}{2 r_1 r_2}$ [Cosine rule.]

• Normal through tangent passes through resp. center.
for angle $\theta = 90°$ [orthogonal intersecⁿ]

$d^2 = r_1^2 + r_2^2$

Or

$2g_1 g_2 + 2f_1 f_2 = c_1 + c_2$.



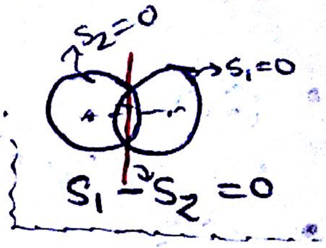
• Slope is also known as gradient.

- Radical axis – Locus of a ~~xxx~~ pt. from where length of tangent drawn to two circle is equal.

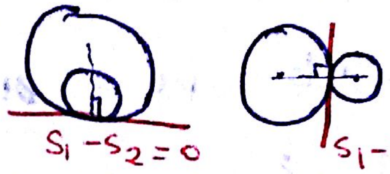
$\sqrt{S_1} = \sqrt{S_2}$ or $S_1 = S_2$ [a straight line]

this straight line is $\perp$ to line joining their center.

- It represent common chord for intersecting circles.

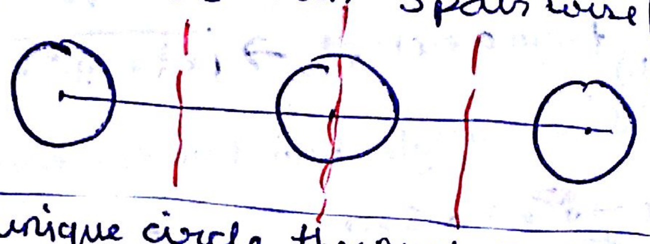


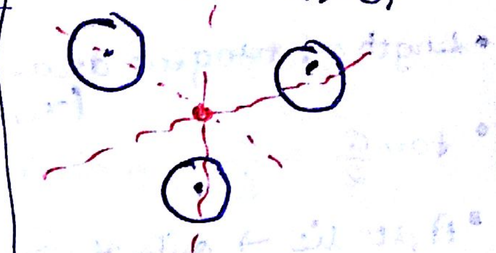
- Radical axis represent eq^n of common tangent for circles touching each other.



- The pair wise 3 radical axis drawn to three ~~xxx~~ circles in a plane are always be concurrent for non collinear center

- If center are collinear then 3 pair wise radical axis are parallel to each other





- We can pass a unique circle through 3 non collinear pts.
- family of variable circle passing through intersecting pt of circle $S=0$ & Line $L=0$ where λ is a parameter. $S+\lambda L=0$.
- Family of variable circle passing through intersec^n pts of 2 circle is given by $S_1 + \lambda(S_1-S_2)=0$ or $S_2+\lambda(S_1-S_2)=0$ ($\lambda \Rightarrow$ parameter)
- Family of variable circle passing through (x_1,y_1) & (x_2,y_2) is – $(x-x_1)(x-x_2)+(y-y_1)(y-y_2)+\lambda L=0$ where L is line joining (x_1,y_1) & (x_2,y_2)
- DIRECTOR CIRCLE – Locus of a pt. from where two perp. tangent can be drawn to a curve, is called director circle.
  - It is concentric with given circle & radius is $\sqrt{2}$ times of given circle
- Eq^n of tangent touching the line at given pt on line. $\Rightarrow$ $(x-x_1)^2+(y-y_1)^2+\lambda L=0$

- Parametric form of circle $x^2+y^2=a^2 \Rightarrow (a\cos\theta, a\sin\theta)$
- shifted $[(x-x_1)^2+(y-y_1)^2 = a^2] = (x_1+a\cos\theta, y_1+a\sin\theta)$
- tangent $\Rightarrow x^2+y^2=a^2 \Rightarrow y = mx \pm a\sqrt{m^2+1}$ & shifted $x \to (x-x_1)$, $y \to (y-y_1)$
- Eq^n of tangent ~~to~~ curve $S=0$ is given by $T=0$, (x_1,y_1) = pt of contact
- Eq^n of pair of tangent from pt(x_1,y_1) to $S=0$. $S \cdot S_1 = T^2$
- Eq^n of chord of contact from ext. pt (x_1,y_1) $T=0$
- Eq^n of chord of contact with mid pt. (x_1,y_1) $T=S_1$

$S_1 \to$ put value (x_1,y_1) in eq^n of circle

S = eq^n of curve

$T=0 \Rightarrow x^2 \to xx_1$

$y^2 \to yy_1$

$2x \to x+x_1$

$2y \to y+y_1$

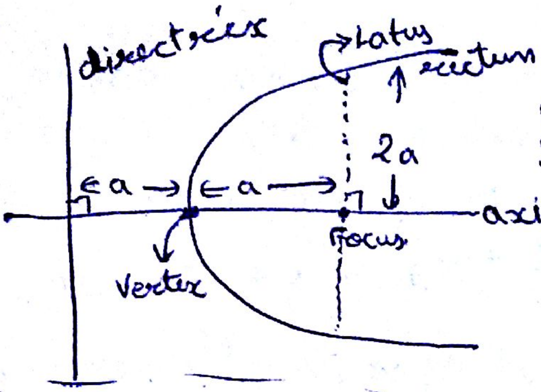
$2xy \to xy_1+y_1x$

CONIC SECTION - Parabola.

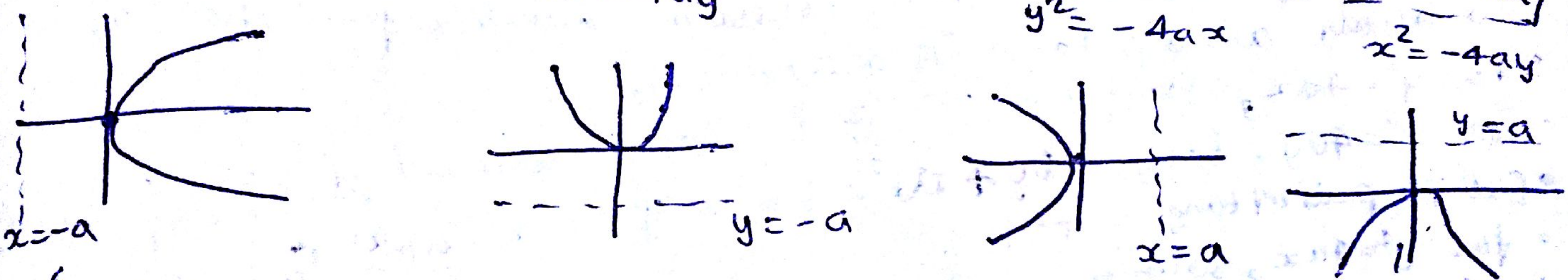
- fixed pt → focus
- fixed line → directrix
- const. ratio - eccentricity. → $e=1$ ⇒ parabola
  → $e \in (0,1)$ ⇒ ellipse
  ⇒ $e > 1$ = hyperbola.

- $$\sqrt{(x-x_1)^2+(y-y_1)^2} = \frac{e\,|ax+by+c|}{\sqrt{a^2+b^2}}$$

$\Delta = abc + 2fgh - af^2 - bg^2 - ch^2 \neq 0$
& $h^2 - ab = 0$ parabola
$h^2 - ab < 0$ ellipse $h^2 - ab > 0$ Hyperbola

- 

→ Chord :- intersect parabola at 2 distinct pts.
→ focal chord :- passes through focus.
→ Double ordinate :- chord ∥'s to latus rectum
→ Latus rectum :- chord ⊥ to axis & passing through focus [length = 4a]

- 

- (⊥ dist of (x,y) from axis of parabola$)^2$ = $4A$(⊥ dist. of (x,y) from tangent at vertex)

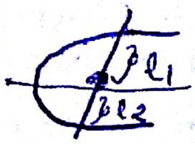
- Parametric form →

	$y^2=4ax$	$y^2=-4ax$	$x^2=4ay$	$x^2=-4ay$
x	at^2	$-at^2$	$2at$	$2at$
y	$2at$	$2at$	at^2	$-at^2$

Note: for $(y-k)^2 = 4a(x-h)$ | $x = h + at^2$, $y = k + 2at$.

- If $P(t_1)$ & $Q(t_2)$ are end pt. of a chord of a parabola $y^2 = 4ax$ which subtend 90° at vertex. Then $t_1 t_2 = -4$
- If $P(t_1)$ & $Q(t_2)$ are end pts of focal chord of parabola $y^2 = 4ax$ then $t_1 t_2 = -1$

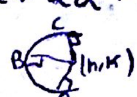
- The harmonic mean of segment of focal chord of any parabola is equal to semi latus rectum (2a)



$$\boxed{2a = \frac{2 l_1 l_2}{l_1 + l_2}}$$

- Loca^n of pt w.r.t parabola, Put focus in parabola to get sign for interior region while opp. sign for outer region.
- Tangent's eq^n $\Rightarrow T = 0$ $[x^2 \Rightarrow xx_1,\ 2xy = x_1y + y_1x$ & $2x = x + x_1]$
- for $y^2 = 4ax$, tang $\Rightarrow t \cdot y = x + at^2$ [pt. of contact $= (at^2, 2at)$]
- Coordinate of pt. of intersec^n of tangents of parabola $y^2 = 4ax$ is $[at_1t_2,\ a(t_1 + t_2)]$
- The tangent drawn at the end pts of focal chord of a parabola intersect at 90° & its x-coordinate lie on directrix.
- For any parabola the foot of perp. drawn from focus to any tangent on parabola is pt of intersec^n of tang. at vertex with tangent at that pt. [orig. tangent]
- For any parabola, the segment of tang. b/w pt. of contact & directrix always subtends right angle at the focus.
- For any parabola, a circle whose diameter as a focal chord of parabola always touch its directrix.
- For $y^2 = 4ax$, tong. $\Rightarrow y = mx + \frac{a}{m}$ [m = slope & contact pt $\left(\frac{a}{m^2}, \frac{2a}{m}\right)$]
- For $x^2 = 4ay$, tang. $\Rightarrow$ $\cancel{ty + at^2} = tx$ $= \boxed{y = mx - am^2}$ [contact $(2am, am^2)$]
- Ext. pt. pair of tang. $\Rightarrow T^2 = S \cdot S_1$
- for $y^2 = 4ax$, solving $y = mx + \frac{a}{m}$ for m' a quad. of 2 roots.
- angle b/w pair of line (tang.) $\Rightarrow \tan\theta = \left|\frac{2\sqrt{h^2 - ab}}{a + b}\right|$
- area of Δ by pt A, B, C on parabola $\cancel{y^2=4ax}$, area $= a^2(t_1 - t_2)(t_2 - t_3)(t_3 - t_1)$
- If $S = 0$ is eq^n of conic, then soln of $\frac{dS}{dx} = 0$ & $\frac{dS}{dy} = 0$ will give center of conic.
- Eq^n of normal for $y^2 = 4ax$ $\Rightarrow y + tx = 2at + at^3$

  Shortest length of normal chord $= 6\sqrt{3}a$ [also 90° at vertex]
- $t_2 = -t_1 - \frac{2}{t_1}$ for normal at $P(t_1)$ meet at $Q(t_2)$ again for para. $y^2 = 4ax$
- $y^2 = 4ax \Rightarrow y = mx - 2am - am^3$ [pt. $(at^2, 2at) \xrightarrow{t \to -m} (am^2, -2am)$] $-m \leftarrow m$ for $y^2 = -4ax$
- $x^2 = 4ay \Rightarrow y = mx + 2a + \frac{a}{m^2}$ [pt$_{contact}\left(\frac{-2a}{m}, \frac{a}{m^2}\right)$]
- Co-normal pts: A, B, C



- $y^2 = 4ax \Rightarrow am^3 + m(2a - h) + k = 0$ $\quad m \to m_1, m_2, m_3$

  $m_1m_2m_3 = \frac{-k}{a}$

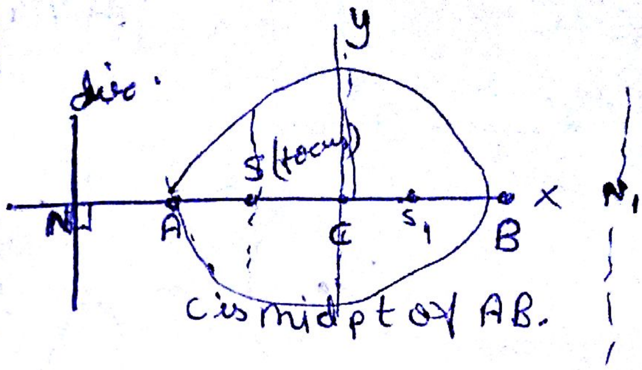
  $m_1m_2 + m_2m_3 + m_3m_1 = \frac{2a - h}{a}$

  $m_1 + m_2 + m_3 = 0$

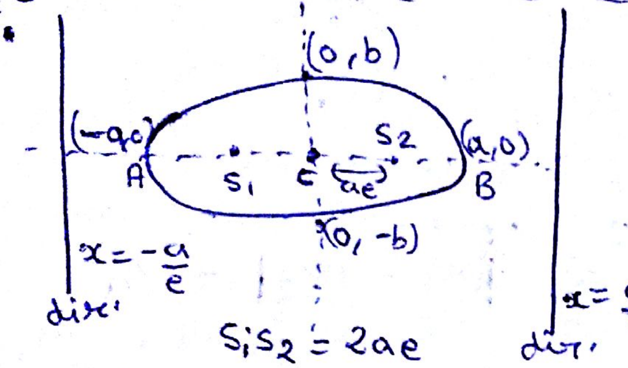
  Centroid of ΔABC - on axis of parabola
- three normals can be drawn from a pt. if + dist. from tong. at vertex is greater or eq. to $2a$ [on parabola side]
- to find slope :- $\frac{dy}{dx}$ of any curve.
- to find mirror image of (α, β) w.r.t $x = y$ is (β, α)
- a circle passing through three co-normal pts also passes through the vertex of given parabola.

ELLIPSE

- $CS = ae$ \$ $CN = a/e$
- Eq^n $\rightarrow$ $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ $\left[\begin{matrix} b<a \\ b^2 = a^2(1-e^2)\end{matrix}\right]$

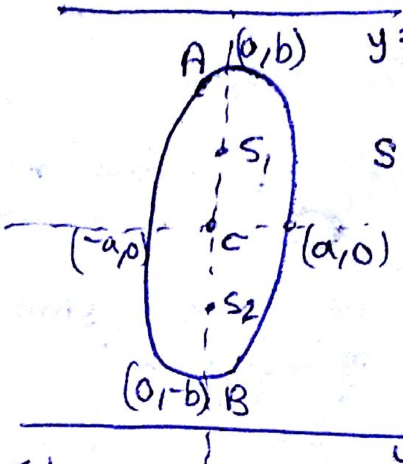


\$ for $a < b$ $\left[a^2 = b^2(1-e^2)\right]$



$S_1S_2 = 2ae$

$a > b$. $S_1(-ae, 0)$
$S_2(ae, 0)$



$S_1 = (0, be)$

$S_2(0, -be)$

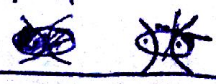
$S_1S_2 = 2be$.

$a < b$

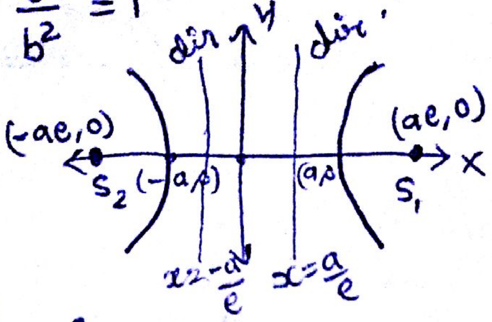
- Length of major axis $= 2a$ \$ Length of minor axis $= 2b$.
- Major axis is line joining focci of ellipse., \$ its $\perp$ bis. is minor axis.
- Intersec^n pt of major \$ minor axis is called center of ellipse.
- Length of latus rectum $= \dfrac{2b^2}{a}$ | • pts on ellipse $\Rightarrow (a\cos\theta, b\sin\theta)$.
- $PS_1 + PS_2 = 2a$ \$ $PS_1 + PS_2 > S_1S_2$, where P is any pt on ellipse.
- area enclosed by ellipse $= \pi ab$. | Foot of $\perp$ drawn from focci to any tang. of ellipse is always on its auxilary circle.
- $\dfrac{\text{Sq. of dist. of Pt. P from minor axis}}{\text{Sq. of length of semi major axis}} + \dfrac{\text{Sq. of dist. of Pt. P from major axis}}{\text{Sq. of length of semi minor axis}} = 1$
- The harmonic mean of the segment of focal chord of any conic is equal to semi latus rectum of that conic.
- Auxillary circle – A circle whose end pt of diameter as end pt of major axis.
- $\dfrac{x}{a}\cos\left(\dfrac{\alpha+\beta}{2}\right) + \dfrac{y}{b}\sin\left(\dfrac{\alpha+\beta}{2}\right) = \cos\left(\dfrac{\alpha-\beta}{2}\right)$ a line joining any 2 pts on ellipse.
- Tangent: $\dfrac{x}{a}\cos\theta + \dfrac{y}{b}\sin\theta = 1$ [at pt $(a\cos\theta, b\sin\theta)$], $T = 0$ or $y = mx \pm \sqrt{a^2m^2 + b^2}$
- Director circle – Locus of pt. from where $\perp$ tang. can be drawn to ellipse. center $\rightarrow$ concentric \$ radius $= \sqrt{a^2+b^2}$. [Independent of inclination of ellipse]
- The product of length of perp. to any tang. of ellipse from both focci is always equal to square of length of semi-minor axis.
- The portion of tang. intersected b/w pt of contact \$ directrix always subtend $90°$ at corres. focus.
- Normal :- $\dfrac{y-y_1}{y_1/b^2} = \dfrac{x-x_1}{x_1/a^2}$ | $ax\sec\theta - by\csc\theta = a^2 - b^2$ | $y = mx \pm \dfrac{m(a^2-b^2)}{\sqrt{a^2+b^2m^2}}$

HYPERBOLA

If ellipse $ hyperbola are confocal then they intersect at 90° $ vice versa



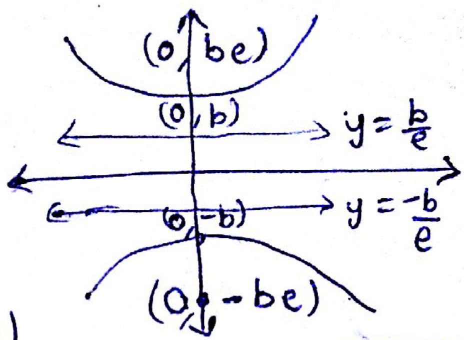
• $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$



Symmetric about x $ y axis

$b^2 = a^2(e^2-1)$

• $\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$



$a^2 = b^2(e^2-1)$

• Line joining focci of hyperbola → transverse axis $ its ⊥ bis. → conj. axis.

• $x^2 + y^2 = a^2$ auxilary circle

Also, Latus rectum (length) $= \frac{2b^2}{a}$

• for pt. P on hyperbola, $|PS_1 - PS_2| = 2a$. S_1 $ S_2 focus $ P is locus of pt. on hyperbola.

• Pt on hyperbola → $(a\sec\theta, b\tan\theta)$ [θ = eccentric angle]

• Chord's eq^n → $\frac{x}{a}\cos\left(\frac{\alpha-\beta}{2}\right) - \frac{y}{b}\sin\left(\frac{\alpha+\beta}{2}\right) = \cos\left(\frac{\alpha+\beta}{2}\right)$

• tangent : T = 0 , $\frac{x}{a}\sec\theta - \frac{y}{b}\tan\theta = 1$ | in slope form : $y = mx \pm \sqrt{a^2m^2 - b^2}$

• Eq^n of director circle → $x^2+y^2 = a^2-b^2$ [a > b] $ center (0,0) $ radius $= \sqrt{a^2-b^2}$

• Normal : $\frac{x-x_1}{x_1/a^2} = \frac{y-y_1}{y_1/-b^2}$ ⇒ $ax\cos\theta + by\cot\theta = a^2+b^2$ | $y = mx \pm \frac{m(a^2+b^2)}{\sqrt{a^2-b^2m^2}}$

• the por^n of tang. b/w pt. of contact $ directrix always subtend 90° at focus.

• Product of length of ⊥ drawn from focci to tang. is always equal to b^2.

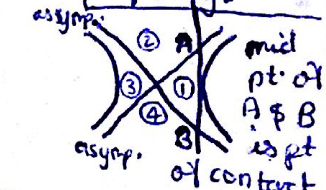
• foot of ⊥ drawn from focci to any tang. is eq^n of its auxillary circle.

• A line which touches the curve at ∞ is called asymtoles. Eq^n ⇒ $y = \pm\frac{b}{a}x$

• Asymtols always intersect at centre of hyperbola $ if θ is angle b/w asymp. then, $\tan\frac{\theta}{2} = \frac{b}{a}$

• The mid pt. of por^n of tang. b/w asymptoles of hyperbola is always pt. of contact of tang.

• If any line intersect hyperbola at pt. A $ B $ intersect their asymptoles at pt. C $ D, then mid pt of A, B is also mid pt of CD



• Put focus $ get sign for diff. region int) ext (int.

• from ① $ ③ two distinct tang. can be drawn to same side

• from ② $ ④ two distinct tang can be drawn to diff. branch [opp. side]

• If asymtoles are at right angle then given hyperbola is rectangular hyperbola. Also, a = b $ $e = \sqrt{2}$ [Ex → $x^2 - y^2 = a^2$]

• For rota^n of axis, $X = x\cos\theta + y\sin\theta$ $ $Y = -x\sin\theta + y\cos\theta$

	x	y
X	$\cos\theta$	$\sin\theta$
Y	$-\sin\theta$	$\cos\theta$

θ → +ve → anticlockwise
θ → −ve → clockwise

• for 45° rota^n in clockwise direc^n [θ = −45°] $xy = c^2 = \frac{a^2}{2}$

• Rect. hyper. in which → $a = b = c\sqrt{2}$

• transverse axis ⇒ x = y $ Vertex = (±c, ±c)

• focci → $(\pm\sqrt{2}c, \pm\sqrt{2}c)$ • directrix → $x+y = \pm\sqrt{2}c$

• parametric form → (x, y) → $(ct, \frac{c}{t})$ x → ct , y = c/t

area of Δ formed by asymptotes $ tang to rect. hyp. is a^2

• tang → $x_1y + y_1x = 2c^2$ → $\frac{x}{t} + ty = 2c$ | Normal → $y - y_1 = \frac{x_1}{y_1}(x - x_1)$ ⇒ $y - \frac{c}{t} = t^2(x - ct)$

• If normal at P(t_1) to curve $xy = c^2$ intersect at Q(t_2) then → $t_1^3 t_2 = -1$

• If (α_1, β_1) $ (α_2, β_2) are center of circle $ rect. hyper. resp. intersecting at 4 pt (x_i, y_i), i = 1, 2, 3, 4 then, $\frac{x_1+x_2+x_3+x_4}{4} = \frac{\alpha_1+\alpha_2}{2}$ $ $\frac{y_1+y_2+y_3+y_4}{4} = \frac{\beta_1+\beta_2}{2}$

• Conjugate hyperbola → a $ b interchanged. Also, $\frac{1}{e_1^2} + \frac{1}{e_2^2} = 1$

• If S = 0 is eq^n of hyperbola $ S + λ = 0 is eq^n of its asymptotes, then λ can be found by Δ = 0, $\Delta = abc + 2fgh - af^2 - bg^2 - ch^2$

• If S′ = 0 is eq^n of conj. hyperbola, then → $\frac{S'+S}{2} = S + \lambda$ ⇒ $S' = S + 2\lambda = 0$

LIMITS

- $\lim_{x\to a} f(x) = \lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x)$

Indeterminate : $\frac{\infty}{\infty}$ $0\times\infty$ 0^0 ∞^0 1^∞, $\infty-\infty$

- $\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} \ldots\ldots (-1 < x \le 1)$

$\log(1-x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} - \frac{x^5}{5} \ldots\ldots (-1 < x \le 1)$

$e^x = \frac{x^0}{\underline{|0}} + \frac{x^1}{\underline{|1}} + \frac{x^2}{\underline{|2}} + \frac{x^3}{\underline{|3}} + \ldots\ldots$

$a^x = 1 + x(\ln a) + \frac{x^2}{\underline{|2}}(\ln a)^2 + \ldots\ldots$

- Limits can be solved by :- factoriza^n

Rationaliza^n

- $\lim_{x\to\infty} \frac{1}{x} = 0$

$\lim_{x\to 0} \frac{\log(1+x)}{x} = 1$

$\lim_{x\to 0} \frac{a^x-1}{x} = \ln a$

$\lim_{x\to 0} \frac{e^x-1}{x} = 1$

$\sin x = x - \frac{x^3}{\underline{|3}} + \frac{x^5}{\underline{|5}} - \frac{x^7}{\underline{|7}} \ldots$

$\cos x = 1 - \frac{x^2}{\underline{|2}} + \frac{x^4}{\underline{|4}} - \frac{x^6}{\underline{|6}} + \ldots$

$\tan x = x + \frac{x^3}{3} + \frac{2}{15}x^5 + \ldots$

$\lim_{x\to a} \frac{x^n - a^n}{x-a} = na^{n-1}$

$\lim_{x\to 0} \frac{(1+x)^n - 1}{x} = n$

$\lim_{\theta\to 0} \frac{\sin\theta}{\theta} = 1$ $\lim_{x\to 0} \frac{\sin^{-1}x}{x} = 1$

$\lim_{\theta\to 0} \frac{\tan\theta}{\theta} = 1$ $\lim_{x\to 0} \frac{\tan^{-1}x}{x} = 1$

Form 0^0, ∞^0 $L = e^{\lim_{x\to a} g(x)\log f(x)}$ $\left[L = \lim_{x\to a} [f(x)]^{g(x)}\right]$

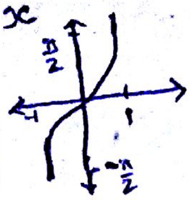
Form 1^∞ $L = e^{\lim_{x\to a} (f(x)-1)\cdot g(x)}$

L'HOPITAL'S Rule: if $\lim_{x\to a} \frac{f(x)}{g(x)}$ takes $\frac{0}{0}$ or $\frac{\infty}{\infty}$ then $\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}$

Function

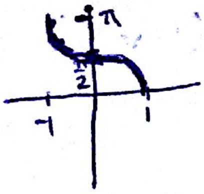
- Every fn is relaⁿ but every relaⁿ is not fn.
- Distincⁿ b/w relaⁿ & Fn → Vertical line test.
- $y = \sin^{-1}x$

Domain : $[-1, 1]$
Range : $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$



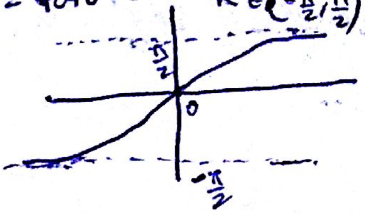
- $y = \cos^{-1}x$

$D : [-1, 1]$
$R : [0, \pi]$

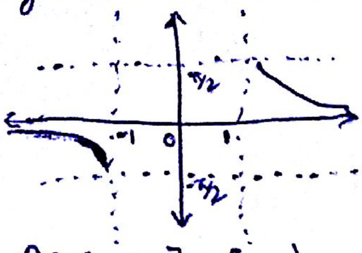


- $y = \tan^{-1}x$

$D \in R$
$R \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$



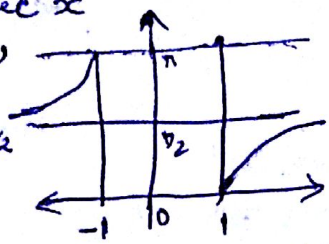
- $y = \text{cosec}^{-1}x$



$D : (-\infty, 1] \cup [1, \infty)$
$R : \left[-\frac{\pi}{2}, \pi/2\right] - \{0\}$

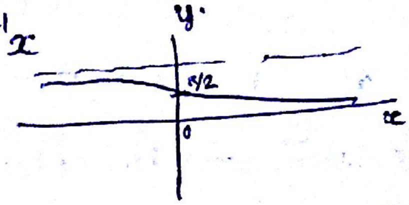
- $y = \sec^{-1}x$

$D : (-\infty, -1] \cup [1, \infty)$
$R : [0, \pi] - \pi/2$

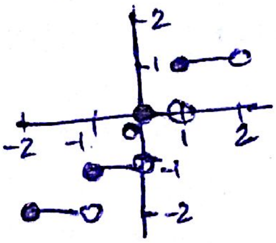


- $y = \cot^{-1}x$

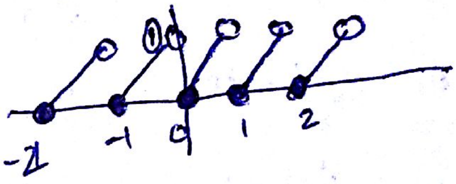
$D \in R$
$R \in (0, \pi)$



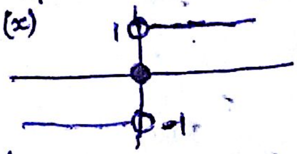
- Greatest Integer Fn (floor value fn)



- fracⁿal part Fn.



- Signum fn : $y = \text{sgn}(x)$



- Fn — one one
  - many one (Horiz. line test)

- Range = codomain ⇒ ONTO fn
  if not, into fn

→ one one + onto : Bijective

Even fn : $F(x) = F(-x)$
Odd Fn $F(x) = -F(-x)$

LCM of p & q ($p = \frac{m}{n}$, $q = \frac{r}{s}$) $= \frac{\text{LCM}(m, r)}{\text{HCF}(n, s)}$

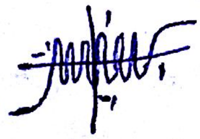
- Periodic fn : $F(x+T) = F(x)$
- $\sin^n x$, $\cos^n x$, $\text{cosec}^n x$, $\sec^n x$ $\xrightarrow{\text{Period}}$ n odd − 2π, even − π
- $\tan^n x$ & $\cot^n x$ → π (always)

- Typical funcⁿ :→ $F(x+y) = F(x) \cdot F(y)$, $F(x) = a^x$
  - → $F(x+y) = F(x-y) = \frac{F(x)}{F(y)}$, $F(x) = a^x$
  - → $F(x) + F(y) = F(xy)$, $F(x) = \log_a x$
  - → $F(x) - F(y) = F\left(\frac{x}{y}\right)$, $F(x) = \log_a x$
  - → $F(x) \cdot F\left(\frac{1}{x}\right) = F(x) + F\left(\frac{1}{x}\right)$, $F(x) = \pm x^n + 1$

Continuity & Differentiability | Methods of Diffⁿ

• Continuity : $\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) = f(a)$ [Graph is broken at any pt]

Type of discontinuity.

• Removable  • Non removable $[x]$, asymptote oscillaⁿ's $f(x) = \sin \frac{\pi}{x}$ 

continuity of fn : $[x]$

$\text{sgn}\, x$

$\lim_{n \to \infty} a^n$

Intermediate value theorem.

Rational & irrational.

Composite fn.

• Differentiability : $f'(a^+) = f'(a^-) = f'(a)$

Non diff. fn → a corner
a discontinuity
a vertical tangent
using graph

Differentiable → cont's also.

Cont's → may or may not be diff ble

not cont's → non diff ble

Diffⁿ of Implicit fn : $-\dfrac{\text{diff wrt } x \text{, } y \text{ as const}}{\text{diff wrt } y \text{, } x \text{ as const}}$

• $\frac{d}{dx} x^n = n x^{n-1}$

$\frac{d}{dx} e^x = e^x$

$\frac{d}{dx} (a^x) = a^x \ln a$

$\frac{d}{dx} (\ln|x|) = \frac{1}{x}$

$\frac{d}{dx} (\log_a x) = \frac{1}{x} \log_a e$

$\frac{d}{dx} (\sin x) = \cos x$

$\frac{d}{dx} (\cos x) = -\sin x$

$\frac{d}{dx} (\tan x) = \sec^2 x$

$\frac{d}{dx} (\cot x) = -\text{cosec}^2 x$

$\frac{d}{dx} (\sec x) = \sec x \tan x$

$\frac{d}{dx} (\text{cosec}\, x) = -\text{cosec}\, x \cot x.$

$\frac{d}{dx} (\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$

$\frac{d}{dx} (\cos^{-1} x) = \frac{-1}{\sqrt{1-x^2}}$

$\frac{d}{dx} (\tan^{-1} x) = \frac{1}{1+x^2}$

$\frac{d}{dx} (\cot^{-1} x) = \frac{-1}{1+x^2}$

$\frac{d}{dx} (\sec^{-1} x) = \frac{1}{x\sqrt{x^2-1}}$

$\frac{d}{dx} (\text{cosec}^{-1} x) = \frac{-1}{x\sqrt{x^2-1}}$

$\frac{d}{dx} [f_1(x) f_2(x)] = f_1(x) f_2'(x) + f_1'(x) f_2(x)$

$\frac{d}{dx} \left[\frac{f_1(x)}{f_2(x)}\right] = \frac{f_1'(x) f_2(x) - f_1(x) f_2'(x)}{[f_2(x)]^2}$

$\sqrt{a^2-x^2}$	$x = a\sin\theta$ or $a\cos\theta$.
$\sqrt{a^2+x^2}$	$x = a\tan\theta$ or $a\cot\theta$
$\sqrt{x^2-a^2}$	$x = a\sec\theta$ or $a\,\text{cosec}\,\theta$
$\sqrt{\frac{a+x}{a-x}}$ or $\sqrt{\frac{a-x}{a+x}}$	$x = a\cos 2\theta.$
$\sqrt{(a-x)(x-b)}$ or $\sqrt{\frac{a-x}{x-b}}$	$x = a\cos^2\theta + b\sin^2\theta.$

take log and diff for exponents & product comp. fn's

APPLICATION OF DERIVATIVE

- $\frac{dy}{dx} = \tan\theta =$ slope of curve.
- angle b/w two curve $\Rightarrow \tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

(orthogonal if $m_1 m_2 = -1$)

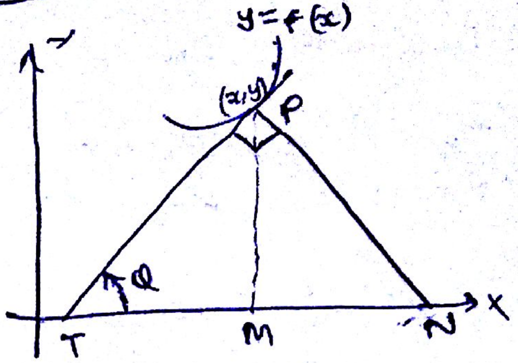
$a = \frac{dv}{dt} = \frac{d^2x}{dx^2}$ \$ $v = \frac{dx}{dt}$.

Length of tangent : PT

Normal : PN $\left[\tan\theta = \frac{dy}{dx}\right]$

subtang. : TM

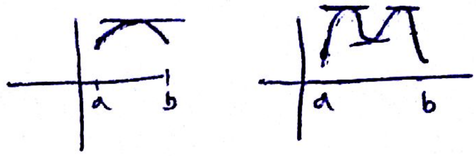
sub normal : Mn



Approxima^n

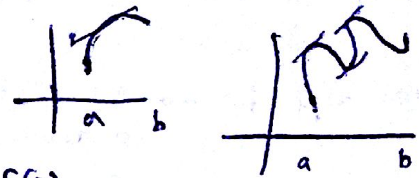
- Mean Value theorems. –

① Rolle's theorem : fn – cont's in $[a,b]$, diff. in (a,b), $f(a) = f(b)$ } then at least one pt $f'(c) = 0$



② Lagrange's mean value theorem : fn conts in $[a,b]$, diff in (a,b) } at least one pt c where $f'(c) = \frac{f(b) - f(a)}{b - a}$

$(a < c < b)$



③ Cauchy's mean value theorem : both $f(x), g(x) \rightarrow$ cont's $[a,b]$, diff (a,b)

$$\frac{f'(c)}{g'(c)} = \frac{f(b) - f(a)}{g(b) - g(a)}$$

- for a curve $f(x)$ if $f'(x) > 0$ ↑ fn, $f'(x) < 0$ ↓ fn.
- At pt of inflec^n concavity → convexity or vice versa; $f''(x) = 0$
- for maxima $f''(x) < 0$, \$ for minima, $f''(x) > 0$.

$f(a+h) < f(a) > f(a-h)$; $f(a+h) > f(a) < f(a-h)$

Area of circular sector $= \frac{1}{2} r^2 \theta$

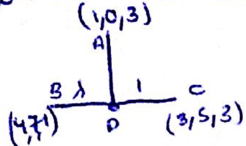
Check value at end pts of range $[a,b]$ at a \$ b

3-D

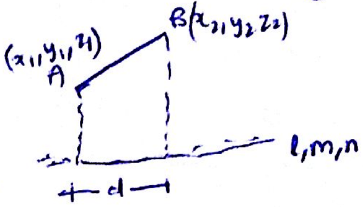
- distance formula : $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$
- division formula : $\left(\dfrac{m_1x_2 \pm m_2x_1}{m_1 \pm m_2}, \dfrac{m_1y_2 \pm m_2y_1}{m_1 \pm m_2}, \dfrac{m_1z_2 \pm m_2z_1}{m_1 \pm m_2}\right)$ $+ \Rightarrow$ internal, $- \Rightarrow$ external
- $\alpha, \beta, \gamma \Rightarrow$ DA's,
  $\cos\alpha \; \cos\beta, \cos\gamma \Rightarrow$ DC's, $l^2+m^2+n^2=1$
  $\;\;l \quad\; m \quad\; n$
  $\sin^2\alpha+\sin^2\beta+\sin^2\gamma=2.$
  $\cos 2\alpha+\cos 2\beta+\cos 2\gamma=-1$

fixed distance $\Rightarrow l, m, n$
DR of $\vec{AB} \Rightarrow \vec{B}-\vec{A}$
DC of $\vec{AB} \Rightarrow \dfrac{\vec{B}-\vec{A}}{r}$

- Cross method can be used in 3-D geom.
- Foot of $\perp$

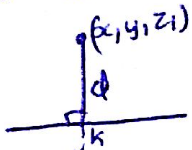


DR[AD] $\times$ DR(BC) $= 0$



- Projec^n of l on m, $|l(x_2-x_1)+m(y_2-y_1)+n(z_2-z_1)|$
- Projec^n of line on coordinate axes = DR's of line.
- 3-D: Plane, line, sphere

- General eq^n of plane $\rightarrow ax+by+cz+d=0$
- Intercept form. $\rightarrow \dfrac{x}{a}+\dfrac{y}{b}+\dfrac{z}{c}=1$
- $d = \dfrac{|ax_1+by_1+cz_1+d|}{\sqrt{a^2+b^2+c^2}}$



- $K \Rightarrow \dfrac{x-x_1}{a}=\dfrac{y-y_1}{b}=\dfrac{z-z_1}{c}=\dfrac{-(ax_1+by_1+cz_1+d)}{a^2+b^2+c^2}$

Normal form = $lx+my+nz=p$.
Pt form. = $a(x-x_1)+b(y-y_1)+c(z-z_1)=0$
dist b/w plane = $\dfrac{|d_2-d_1|}{\sqrt{a^2+b^2+c^2}}$

- Eq^n of plane passing through 3 pts :-

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix}=0.$$

- Condi^n of coplanarity of 4 pts :-

$$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \\ x_4-x_1 & y_4-y_1 & z_4-z_1 \end{vmatrix}=0$$

Family of Planes : $P_1+\lambda P_2=0.$

- Angle bisector of two plane : $\dfrac{a_1x+b_1y+c_1z+d_1}{\sqrt{a_1^2+b_1^2+c_1^2}} = \pm \dfrac{a_2x+b_2y+c_2z+d_2}{\sqrt{a_2^2+b_2^2+c_2^2}}$

$\Rightarrow$ ① make d_1 & d_2 +ve.
② $a_1a_2+b_1b_2+c_1c_2>0$ + obtuse
<0 − obtuse
& + ve sign contains origin.

- Angle b/w line & plane –
  $\cos(90-\theta) = \dfrac{al+bm+cn}{\sqrt{a^2+b^2+c^2}\sqrt{l^2+m^2+n^2}}$

- St. line in 3-D : $a_1x+b_1y+c_1z+d_1=0=a_2x+b_2y+c_2z+d_2=0.$
  symm. form : $\dfrac{x-x_1}{a}=\dfrac{y-y_1}{b}=\dfrac{z-z_1}{c}=r$

pt assumed on line –
$x=x_1+ar$
$y=y_1+br$
$z=z_1+cr.$

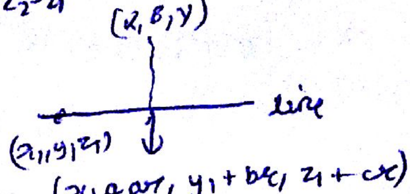
general form $\rightarrow$ symm. form : put $z=0$, $\left.\begin{aligned} a_1x+b_1y+d_1=0 \\ a_2x+b_2y+d_2=0. \end{aligned}\right\} \begin{aligned} x=\alpha \\ y=\beta \end{aligned}$

$\dfrac{x-\alpha}{l}=\dfrac{y-\beta}{m}=\dfrac{z-0}{n}$ $\left\{\begin{aligned} la_1+mb_1+nc_1=0 \\ la_2+mb_2+nc_2=0 \end{aligned}\right.$ solve by cross elimina^n for l, m, n.

- Eq^n of line passing through two given pts : $\dfrac{x-x_1}{x_2-x_1} = \dfrac{y-y_1}{y_2-y_1} = \dfrac{z-z_1}{z_2-z_1} = r$

- $\dfrac{x-x_1}{a} = \dfrac{y-y_1}{b} = \dfrac{z-z_1}{c} = r$

  then pdt of DR. = 0.



- condi^n for coplanar or intersecting line –

$$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix} = 0$$

for $\neq 0$, neither $\parallel^{l}$ nor intersecting
⇓
skew line or non coplanar

- dist b/w two noncoplanar or skew lines –

$$d = \frac{\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix}}{\sin\theta}$$

$\sin\theta = \pm\sqrt{(l_1m_2 - l_2m_1)^2 + (m_1n_2 - m_2n_1)^2 + (n_1l_2 - n_2l_1)^2}$

OR $\left.\begin{aligned} \vec{r} &= \vec{a}_1 + \lambda\vec{b}_1 \\ \vec{r} &= \vec{a}_2 + \lambda\vec{b}_2 \end{aligned}\right\} \to \left|\dfrac{(\vec{b}_1 \times \vec{b}_2)\cdot(\vec{a}_2 - \vec{a}_1)}{|\vec{b}_1 \times \vec{b}_2|}\right|$

- sphere : general : $(x-\alpha)^2 + (y-\beta)^2 + (z-\gamma)^2 = r^2$.

  std : $x^2+y^2+z^2 + 2ux + 2vy + 2wz + d = 0$ $[C = (-u, -v, -w)$, $r = \sqrt{u^2+v^2+w^2-d}\,]$

  $(x-x_1)(x-x_2) + (y-y_1)(y-y_2) + (z-z_1)(z-z_2) = 0$

VECTORS

angle bisector of $\vec{a}$ & $\vec{b}$ $\quad k\left(\frac{\vec{a}}{|\vec{a}|} + \frac{\vec{b}}{|\vec{b}|}\right)$

- Projec^n of $\vec{a}$ on $\vec{b}$: $\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$, $\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}$, $\sin\theta = \left|\frac{\vec{a}\times\vec{b}}{|a||b|}\right|$
- $(|\vec{a}+\vec{b}+\vec{c}|)^2 = a^2+b^2+c^2+2[a\cdot b+b\cdot c+c\cdot a]$
- scalar triple pdt : $(\vec{a}\times\vec{b})\cdot\vec{c} = \vec{a}\cdot(\vec{b}\times\vec{c}) = [\vec{a}\ \vec{b}\ \vec{c}] = \begin{vmatrix}\vec{a}\\ \vec{b}\\ \vec{c}\end{vmatrix}$
- scalar pdt of 4 vect. : $(\vec{a}\times\vec{b})\cdot(\vec{c}\times\vec{d}) = \begin{vmatrix}\vec{a}\cdot\vec{c} & \vec{a}\cdot\vec{d}\\ \vec{b}\cdot\vec{c} & \vec{b}\cdot\vec{d}\end{vmatrix}$
- vector triple pdt : $(\vec{a}\times\vec{b})\times\vec{c} = (\vec{c}\cdot\vec{a})\vec{b} - (\vec{c}\cdot\vec{b})\vec{a}$

  $(\vec{b}\times\vec{c})\times\vec{a} = (\vec{a}\cdot\vec{b})\vec{c} - (\vec{a}\cdot\vec{c})\vec{b}$
- vector pdt of 4 vectors : $\underbrace{(\vec{a}\times\vec{b})}_{\vec{\lambda}}\times(\vec{c}\times\vec{d}) \Rightarrow (\vec{\lambda}\cdot\vec{d})\vec{c} - (\vec{\lambda}\cdot\vec{c})\vec{d}$

  $\Rightarrow [(\vec{a}\times\vec{b})\cdot\vec{d}]\vec{c} - [(\vec{a}\times\vec{b})\cdot\vec{c}]\vec{d}$

  $\Rightarrow [\vec{a}\ \vec{b}\ \vec{d}]\vec{c} - [\vec{a}\ \vec{b}\ \vec{c}]\vec{d}$
- Reciprocal system : $[\vec{a}\ \vec{b}\ \vec{c}][\vec{a}'\ \vec{b}'\ \vec{c}'] = 1$

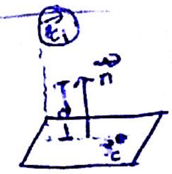
  $\vec{a}' = \frac{\vec{b}\times\vec{c}}{[\vec{a}\ \vec{b}\ \vec{c}]}$
- Vol. of parallelopiped : $[a\ b\ c]$
- Vol of tetrahedron : $\frac{1}{6}[a\ b\ c]$

ar($\triangle$) $= \frac{1}{2}|\vec{AB}\times\vec{AC}|$

ar($\square$) $= |\vec{a}\times\vec{b}|$ [$\vec{a}, \vec{b}$ sides]

ar($\square$) $= \frac{1}{2}|\vec{c}\times\vec{d}|$ [$\vec{c}, \vec{d}$ diagonal]

⇒ VECTOR APPLICATION IN 3-D :-



Plane : $\vec{r}\cdot\vec{n} = d$, dist. of any pt to the plane ⇒ $\left|\frac{\vec{r}_1\cdot\vec{n} - d}{|\vec{n}|}\right|$

pt form: $(\vec{r}-\vec{r}_1)\cdot\vec{n} = 0$

- eq^n of plane passing through r_1, r_2, r_3 $[r-r_1\quad r_2-r_1\quad r_3-r_1] = 0$
- condi^n of coplanarity of 4 pts : $[r_2-r_1\quad r_3-r_1\quad r_4-r_1] = 0$.

Angle b/w plane : $\frac{\vec{n}_1\cdot\vec{n}_2}{|\vec{n}_1||\vec{n}_2|}$

Sphere : $|\underbrace{\vec{r}\cdot\vec{c}}_{\text{center}}| = \underbrace{a}_{\text{rad}}$ | diam. opp pts. $(\vec{r}-\vec{a})\cdot(\vec{r}-\vec{b}) = 0$

line : $\vec{r} = \vec{a} + \lambda\vec{b}$, if passes through origin then $\vec{r} = \lambda\vec{b}$

angle b/w two lines $\longrightarrow \cos\theta = \frac{\vec{b}_1\cdot\vec{b}_2}{|\vec{b}_1||\vec{b}_2|}$

- angle b/w plane & line :-

  $\vec{r}\cdot\vec{n} = d \qquad \vec{r} = \vec{a}+\lambda\vec{b}$

  $\cos(90-\theta) = \frac{\vec{b}_1\cdot\vec{n}_1}{|\vec{b}_1||\vec{n}_1|}$

- dist. b/w two skew lines (non coplanar) –

  $\vec{r} = \vec{a}_1 + \lambda\vec{b}_1$ $\qquad$ if coplanar, $(\vec{b}_1\times\vec{b}_2)\cdot(\vec{a}_2-\vec{a}_1) = 0$

  $\vec{r} = \vec{a}_2 + \lambda\vec{b}_2$

  shortest distance : $\left|\frac{(\vec{b}_1\times\vec{b}_2)\cdot(\vec{a}_2-\vec{a}_1)}{|\vec{b}_1\times\vec{b}_2|}\right|$

Diff. ans. to get quesn

Indefinite Integraⁿ

- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$ $n \neq -1$
- $\int \frac{1}{x} dx = \log|x| + C.$
- $\int e^x dx = e^x + c.$
- $\int a^x dx = \frac{a^x}{\log a} + c.$
- $\int \sin x\, dx = -\cos x + c.$
- $\int \cos x\, dx = \sin x + c.$
- $\int \tan x\, dx = \ln|\sec x| + c$
- $\int \cot x\, dx = \ln|\sin x| + c$
- $\int \sec x\, dx = \ln|\sec x + \tan x| + c = \ln\left|\tan\left(\frac{\pi}{4} + \frac{x}{2}\right)\right| + c$

- $\int \sec^2 x\, dx = \tan x + c$
- $\int \operatorname{cosec}^2 x\, dx = -\cot x + c$
- $\int \sec x \tan x\, dx = \sec x + c.$
- $\int \operatorname{cosec} x \cot x\, dx = -\operatorname{cosec} x + c$
- $\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x + c$ or $-\cos^{-1} x + c.$
- $\int \frac{dx}{1+x^2} = \tan^{-1} x$ or $-\cot^{-1} x + c.$
- $\int \frac{dx}{x\sqrt{x^2-1}} = \sec^{-1} x + c$ or $-\operatorname{cosec}^{-1} x + c.$

Integraⁿ by-

simplificaⁿ
substituⁿ
trigonometric substituⁿ
division by $\cos^2\theta$.
addiⁿ or subtracⁿ (const)
Integraⁿ by part
Integraⁿ by cancellaⁿ
Partial fracⁿ
using formula.

$\Rightarrow \int \frac{p\cos x + q\sin x + r}{a\cos x + b\sin x + c} dx,$ express, N^r as $\lambda D^r + \mu \frac{d}{dx} D^r + \gamma$

- $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a} + C$
- $\int \frac{dx}{\sqrt{x^2+a^2}} = \log\left(x + \sqrt{x^2+a^2}\right) + c$
- $\int \frac{dx}{\sqrt{x^2-a^2}} = \log\left|x + \sqrt{x^2-a^2}\right| + c$
- $\int e^{ax} \sin bx\, dx = \frac{e^{ax}}{a^2+b^2}(a\sin bx - b\cos bx) + c$
- $\int e^{ax} \cos bx\, dx = \frac{e^{ax}}{a^2+b^2}(b\sin bx + a\cos bx) + c$
- $\int \sqrt{a^2+x^2}\, dx = \frac{x}{2}\sqrt{a^2+x^2} + \frac{a^2}{2} \ln\left|x + \sqrt{a^2+x^2}\right| + c$
- $\int \sqrt{x^2-a^2}\, dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2} \ln\left|x + \sqrt{x^2-a^2}\right| + c$
- $\int \sqrt{a^2-x^2}\, dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2} \sin^{-1}\left(\frac{x}{a}\right) + c$
- $\int \frac{dx}{x^2+a^2} = \frac{1}{a} \tan^{-1}\frac{x}{a} + c$
- $\int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln\left|\frac{x-a}{x+a}\right| + c$
- $I_n = \int \sec^n x\, dx = \frac{\sec^{n-2} x \tan x}{n-1} + \left(\frac{n-2}{n-1}\right) I_{n-2}$
- $I_n = \int \tan^n x\, dx = \frac{\tan^{n-1} x}{n-1} - I_{n-2}$
- $I_n = \int \cot^n x\, dx = -\frac{\cot^{n-1} x}{n-1} - I_{n-2}.$

$\int \frac{(ax^2+b)\,dx}{x^4+px^2+q}$ put $\left(x \pm \frac{1}{x}\right) = t$

ILATE: $uv_1 - u'v_2 + u''v_3 - u'''v_4 + \dots$

for, $\int \frac{1}{L_1\sqrt{L_2}} dx$, $\int \frac{L_1}{\sqrt{L_2}} dx$, $\int \frac{\sqrt{L_2}}{L_1} dx$, $L_2 = t^2$ (linear Fn.)

for $\int \frac{1}{L\sqrt{Q}} dx$, $\frac{1}{t} = $ Linear

for $\int \frac{dx}{(ax^2+d)\sqrt{cx^2+d}}$ $\left(x = \frac{1}{t}\right)$

$\int \frac{dx}{Q\sqrt{L}} \Rightarrow L = t^2$

$\cancel{\int \frac{dx}{Q_1\sqrt{Q_2}} \Rightarrow x = \frac{1}{t}}$

$\int [x f'(x) + f(x)]\, dx = x f(x) + c.$

$\int e^x (f(x) + f'(x)) = e^x f(x) + c$

form: $\sqrt{\sec^2 x \pm a}$, $\sqrt{\operatorname{cosec}^2 x \pm a}$, $\sqrt{\tan^2 x \pm a}$ etc.

Rationalise seperate & solve.

$\int \frac{dx}{x\sqrt{ax^n+b}}$ $ax^n + b = t.$

$\int \frac{dx}{(a+cx^2)^{3/2}}$ $x = \frac{1}{t}$

$\int \frac{x\,dx}{(a+2bx+cx^2)^{3/2}}$ $x = \frac{1}{t}.$

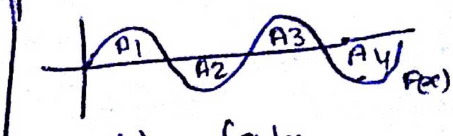
Definate Integrals & area.

- $\sum x^3 = \left[\frac{n(n+1)}{2}\right]^2$

$\sin\alpha + \sin(\alpha+\beta) + \sin(\alpha+2\beta) + \cdots + \sin(\alpha+\overline{n-1}\,\beta) = \dfrac{\sin\left(\frac{n\beta}{2}\right)\sin\left[\alpha+\frac{(n-1)\beta}{2}\right]}{\sin\left(\frac{\beta}{2}\right)}$

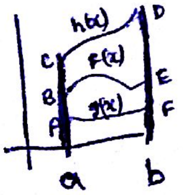
$\cos\alpha + \cos(\alpha+\beta) + \cdots + \cos(\alpha+\overline{n-1}\,\beta) = \dfrac{\sin\frac{n\beta}{2}}{\sin\frac{\beta}{2}}\cos\left(\alpha+\overline{n-1}\,\frac{\beta}{2}\right)$

- $1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} \cdots = \frac{\pi^2}{12}$
- $1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \cdots = \frac{\pi^2}{6}$
- $1 + \frac{1}{3^2} + \frac{1}{5^2} + \cdots = \frac{\pi^2}{8}$
- $\frac{1}{2^2} + \frac{1}{4^2} + \frac{1}{6^2} + \cdots = \frac{\pi^2}{24}$



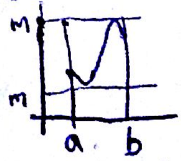
$y = F(x)$ $\int y\,dx = A_1 - A_2 + A_3 - A_4$.

- $\lim_{n\to\infty} \frac{1}{n}\sum_{r=1}^{n} F\left(\frac{r}{n}\right) = \int_0^1 F(x)\,dx.$
- $\int_a^b F(x)dx = -\int_b^a F(x)dx = \int_a^c F(x)dx + \int_c^b F(x)dx$. ($c$ may lie anywhere)
- $\int_a^b F(x)dx = \int_a^b F(a+b-x)dx$
- $\int_0^{2a} F(x)dx = \int_0^a F(x)dx + \int_0^a F(2a-x)$
- $\int_0^{\pi/2} \log\sin x\,dx = \int_0^{\pi/2}\log\cos x\,dx = -\frac{\pi}{2}\log 2$
- $\int_0^{2a} F(x)dx = \begin{cases} 0 & \text{if } F(2a-x) = -F(x) \quad \text{odd} \\ 2\int_0^a F(x)dx. & \text{if } F(2a-x) = F(x) \quad \text{even} \end{cases}$
- $\int_0^{2a} F(x)dx = \int_0^a \{F(a-x) + F(a+x)\}dx$
- $\int_{-a}^{a} F(x)dx = \begin{cases} 0 & \text{if } F(x) \text{ is odd} \\ 2\int_0^a F(x)dx & F(x) \text{ is even} \end{cases}$
- $\int_a^b F(x)dx = (b-a)\int_0^1 F[(b-a)x+a]dx$
- $\int_0^{nT} F(x)dx = n\int_0^T F(x)dx$, T = time period of Fn.
- if $F(t)$ is odd fn, $\phi(x) = \int_a^x F(t)dt$ is even fn.
- LEIBNITZ's Rule: $\frac{d}{dx}\left\{\int_{u(x)}^{v(x)} F(t)dt\right\} = F[v(x)]\frac{dv(x)}{dx} - F[u(x)]\frac{du(x)}{dx}$ | F cont's $[a,b]$, $u(x), v(x)$ diff'ble

- 

$\int_a^b g(x)dx \le \int_a^b F(x)dx \le \int_a^b h(x)dx$

- $\int_a^b m\,dx \le \int_a^b F(x)dx \le \int_a^b M\,dx$



- $\int_0^{\pi/2} \sin^m x\cos^n x\,dx = \dfrac{\Gamma\frac{m+1}{2}\,\Gamma\frac{n+1}{2}}{2\,\Gamma\frac{m+n+2}{2}}$ $\left[\Gamma(n+1) = n\Gamma(n),\ \Gamma\frac{1}{2} = \sqrt{\pi},\ \Gamma(1) = 1\right]$ $(\Gamma(n+1) = n!)$