Quantum Teleportation

$$|\psi_{Alice}\rangle \otimes |\psi_{bridge}\rangle \otimes |\psi_{Bob}\rangle$$

* Objective: Teleport Alice's qubit to Bob

let $|\psi\rangle_{Alice} = \alpha|0\rangle + \beta|1\rangle$
where $\alpha, \beta \in \mathbb{C}$

In the program,
$|\psi_{Alice}\rangle = |-\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$

$$|0\rangle \rightarrow \boxed{X} \rightarrow \boxed{H} \rightarrow |\psi_{Alice}\rangle$$

$|\psi_{bridge}\rangle$: Remain with Alice but entangled with Bob's qubit.

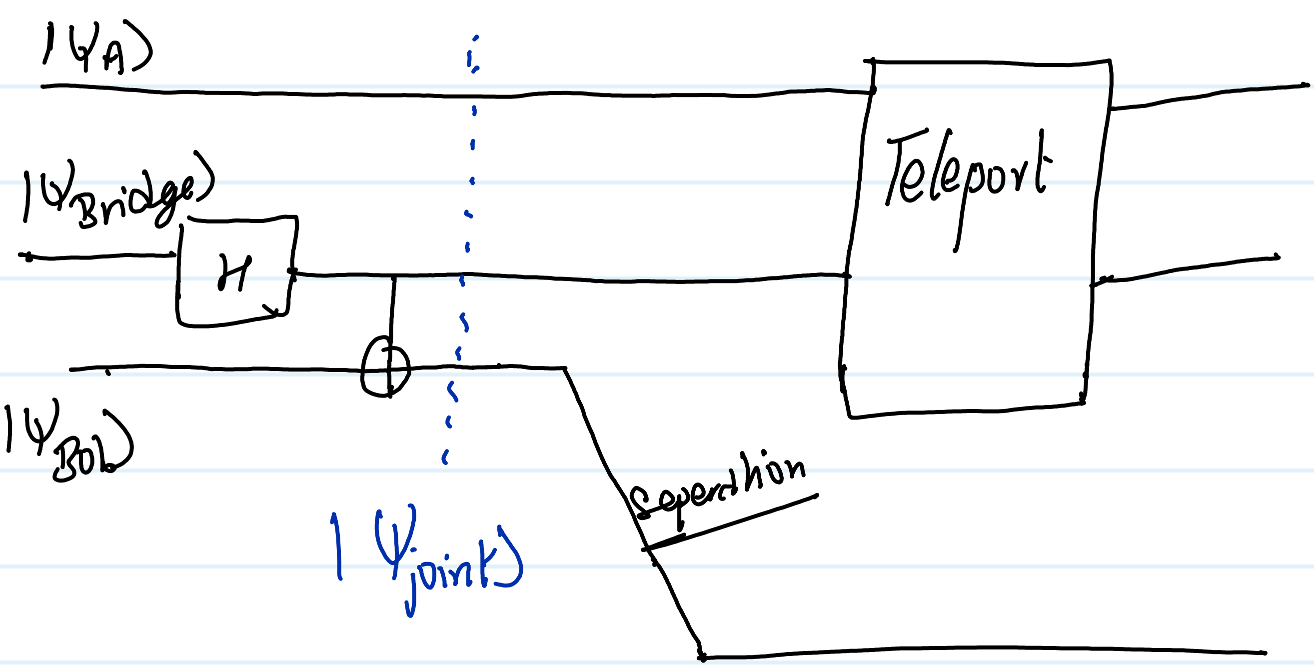
$$|\psi_{bridg}\,\psi_{Bob}\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$$

Encoding the Alice with bridge qubit.

$$|\psi_{joint}\rangle = |Alice\rangle \otimes |\psi_{bridge}\,\psi_{bob}\rangle$$

$$= \left(\alpha|0\rangle + \beta|1\rangle\right) \otimes \left(|00\rangle + |11\rangle\right)$$

$$= \alpha|000\rangle + \alpha|011\rangle + \beta|100\rangle + \beta|111\rangle$$

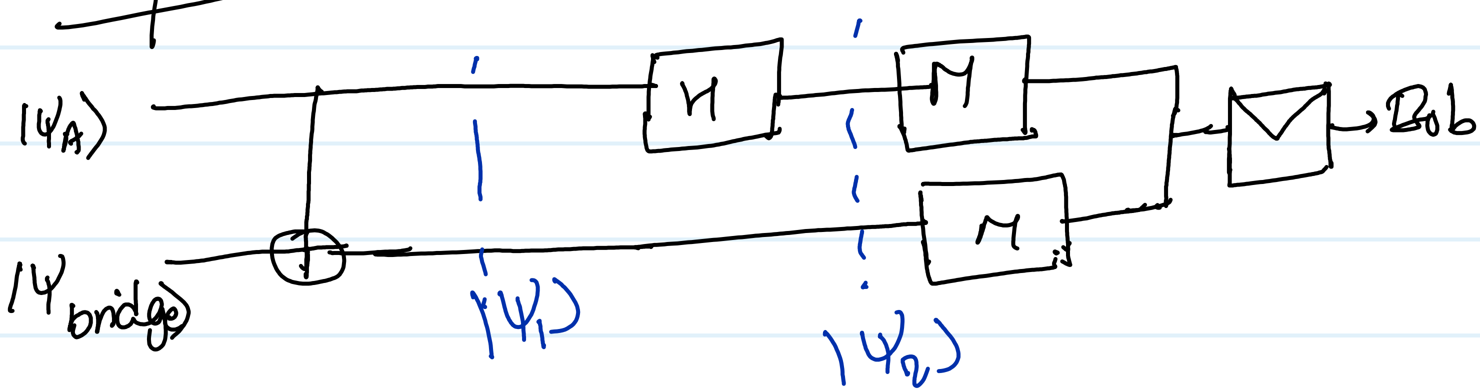


$|\psi_A \psi_{Bridg}\rangle \otimes |\psi_{Bob}\rangle$

$$|\psi_{joint}\rangle = \alpha|00\rangle \otimes |0\rangle + \alpha|01\rangle \otimes |1\rangle$$
$$\beta|10\rangle \otimes |0\rangle + \beta|11\rangle \otimes |1\rangle$$

Alice measure	$\vert\psi_{Bob}\rangle$
0 0	$\alpha\vert 0\rangle$
0 1	$\alpha\vert 1\rangle$
1 0	$\beta\vert 0\rangle$
1 1	$\beta\vert 1\rangle$

Teleport



$$|\psi_{joint}\rangle = |\psi_{Alice}\, \psi_{Bridg}\, \psi_{bob}\rangle$$

$$= \alpha|000\rangle + \alpha|011\rangle + \beta|100\rangle + \beta|111\rangle$$

$$|\psi_1\rangle = \alpha|000\rangle + \alpha|011\rangle + \beta|110\rangle + \beta|101\rangle$$

$$|\psi_2\rangle = \alpha(|0\rangle+|1\rangle) \otimes |00\rangle +$$
$$\alpha(|0\rangle+|1\rangle) \otimes |11\rangle +$$
$$\beta(|0\rangle-|1\rangle) \otimes |10\rangle +$$
$$\beta(|0\rangle-|1\rangle) \otimes |01\rangle$$

$$= \alpha(|00\rangle + |10\rangle) \otimes |0\rangle +$$
$$\alpha(|01\rangle + |11\rangle) \otimes |1\rangle +$$
$$\beta(|01\rangle - |11\rangle) \otimes |0\rangle +$$
$$\beta(|00\rangle - |10\rangle) \otimes |1\rangle$$

Let i.) Alice measure $|00\rangle$,

then, Bob measure :

$$|\psi_{bob}\rangle : \alpha|0\rangle + \beta|1\rangle$$

ii.) If $|\psi_{Alice}\ \psi_{Bridge}\rangle = |01\rangle$

then,

$$|\psi_{bob}\rangle = \alpha|1\rangle + \beta|0\rangle$$

Bob need to apply X gate to recover the original qubit

iii.) If $|\psi_{Alice}\, \psi_{Bridge}\rangle = |10\rangle$

then,

$|\psi_{bob}\rangle = \alpha|0\rangle - \beta|1\rangle$

Gate: Z.

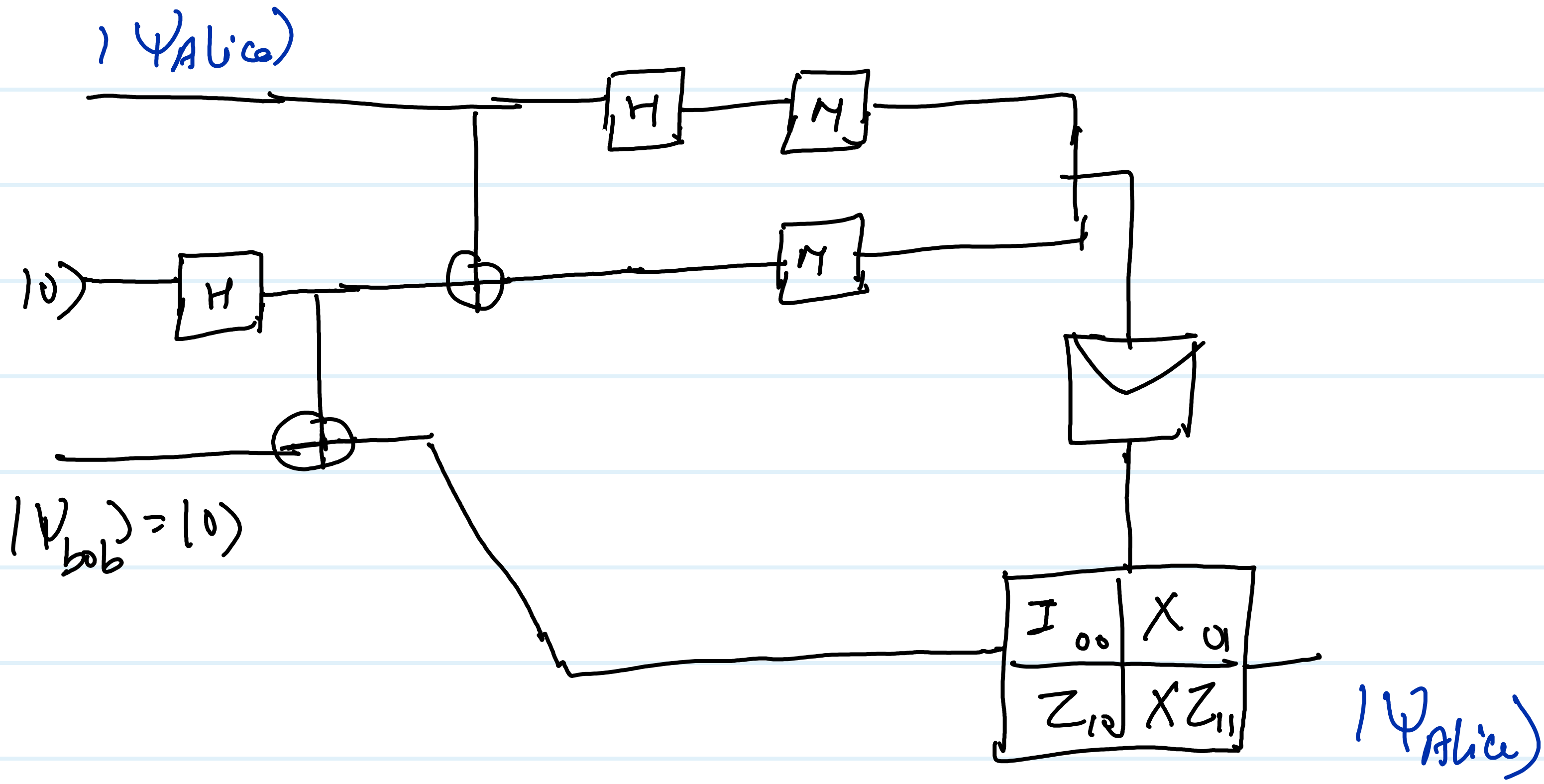
iv.) If $|\psi_{Alice}\, \psi_{Bob}\rangle = |11\rangle$

$|\psi_{bob}\rangle = \alpha|1\rangle - \beta|0\rangle$

Gate: XZ

I 00	X 01
Z 10	XZ 11

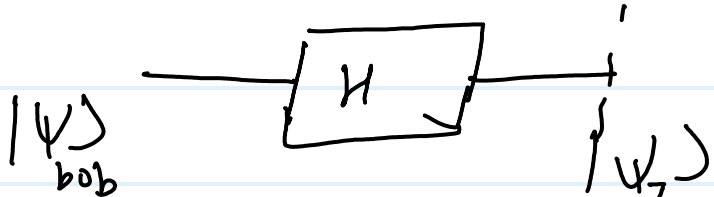
Circuit



I use $|-\rangle$ as Alice's qubit.

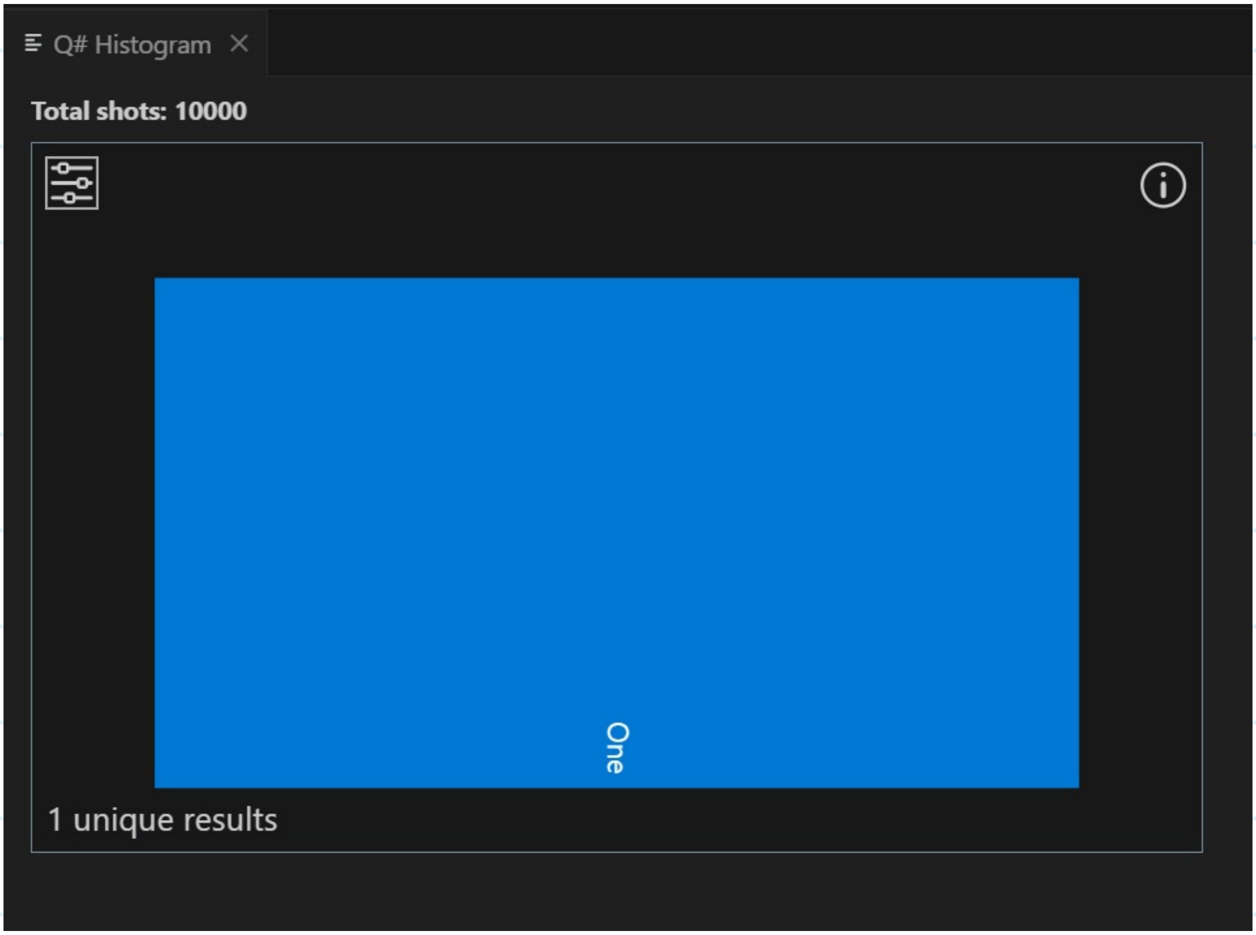
Verification

To verify $|-\rangle$ I apply H



If $|-\rangle$ teleported successfully, then $|\psi_3\rangle = |1\rangle$

Result



* I performed this experiment, and received only one unique result.
* The result state that our teleportation implementation is correct