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## 1.0 Data Structures

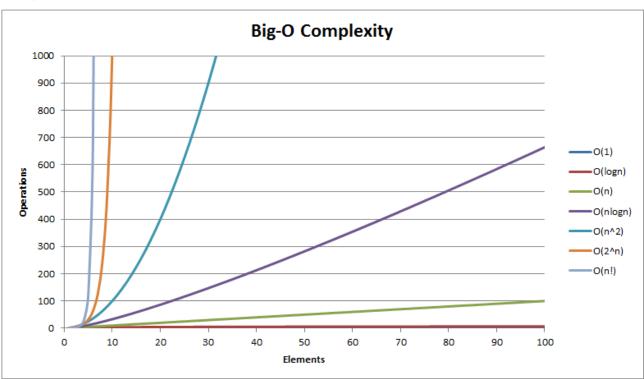
## 1.1 Overview

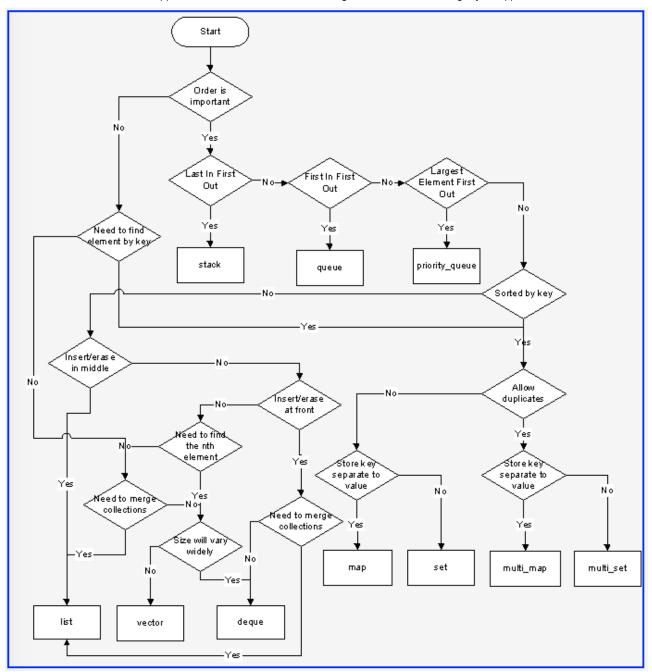


### **Data Structures**



# **Big-O Complexity Chart**





## 1.2 Vector std::vector

#### Use for

- Simple storage
- Adding but not deleting
- Serialization
- Quick lookups by index
- Easy conversion to C-style arrays
- Efficient traversal (contiguous CPU caching)

#### Do not use for

- Insertion/deletion in the middle of the list
- Dynamically changing storage
- Non-integer indexing

## **Time Complexity**

Operation	Time Complexity
Insert Head	0(n)
Insert Index	0(n)
Insert Tail	0(1)
Remove Head	0(n)
Remove Index	0(n)
Remove Tail	0(1)
Find Index	0(1)
Find Object	0(n)

```
std::vector<int> v;
// General Operations
//----
// Size
unsigned int size = v.size();
// Insert head, index, tail
v.insert(v.begin(), value);
                                   // head
v.insert(v.begin() + index, value);  // index
v.push_back(value);
                                    // tail
// Access head, index, tail
int head = v.front();  // head
head = v[0];
                         // or using array style indexing
int value = v.at(index);
                         // index
value = v[index];
                         // or using array style indexing
```

## 1.3 Deque std::deque

#### Use for

- Similar purpose of std::vector
- Basically std::vector with efficient push\_front and pop\_front

#### Do not use for

• C-style contiguous storage (not guaranteed)

#### **Notes**

- Pronounced 'deck'
- Stands for Double Ended Queue

```
// Access head, index, tail
int head = d.front();
                           // head
int value = d.at(index); // index
                       // tail
int tail = d.back();
// Size
unsigned int size = d.size();
// Iterate
for(std::deque<int>::iterator it = d.begin(); it != d.end(); it++) {
    std::cout << *it << std::endl;</pre>
}
// Remove head, index, tail
d.pop_front();
                              // head
d.erase(d.begin() + index);
                             // index
d.pop_back();
                              // tail
// Clear
d.clear();
```

# 1.4 List std::list and std::forward\_list

#### Use for

- Insertion into the middle/beginning of the list
- Efficient sorting (pointer swap vs. copying)

#### Do not use for

Direct access

## **Time Complexity**

Operation	Time Complexity
Insert Head	0(1)
Insert Index	O(n)
Insert Tail	0(1)
Remove Head	0(1)
Remove Index	0(n)

Operation	Time Complexity
Remove Tail	0(1)
Find Index	0(n)
Find Object	O(n)

```
std::list<int> 1;
//-----
// General Operations
//----
// Insert head, index, tail
1.push front(value);
                                    // head
1.insert(l.begin() + index, value);  // index
1.push_back(value);
                                    // tail
// Access head, index, tail
int head = 1.front();
                                                          // head
int value = std::next(l.begin(), index);
                                                          // index
int tail = 1.back();
                                                          // tail
// Size
unsigned int size = 1.size();
// Iterate
for(std::list<int>::iterator it = 1.begin(); it != 1.end(); it++) {
   std::cout << *it << std::endl;</pre>
}
// Remove head, index, tail
1.pop_front();
                             // head
1.erase(l.begin() + index); // index
                             // tail
1.pop_back();
// Clear
1.clear();
//-----
// Container-Specific Operations
// Splice: Transfer elements from list to list
       splice(iterator pos, list &x)
```

```
// splice(iterator pos, list &x, iterator i)
// splice(iterator pos, list &x, iterator first, iterator last)
l.splice(l.begin() + index, list2);

// Remove: Remove an element by value
l.remove(value);

// Unique: Remove duplicates
l.unique();

// Merge: Merge two sorted lists
l.merge(list2);

// Sort: Sort the list
l.sort();

// Reverse: Reverse the list order
l.reverse();
```

## 1.5 Map std::map and std::unordered\_map

#### Use for

- Key-value pairs
- Constant lookups by key
- Searching if key/value exists
- Removing duplicates
- std::map
  - Ordered map
- std::unordered\_map
  - Hash table

### Do not use for

Sorting

### **Notes**

- Typically ordered maps (std::map) are slower than unordered maps (std::unordered\_map)
- Maps are typically implemented as binary search trees

### **Time Complexity**

#### std::map

Operation	Time Complexity
Insert	O(log(n))
Access by Key	0(log(n))
Remove by Key	O(log(n))
Find/Remove Value	O(log(n))

#### std::unordered\_map

Operation	Time Complexity
Insert	0(1)
Access by Key	0(1)
Remove by Key	0(1)
Find/Remove Value	

```
std::map<std::string, std::string> m;
//-----
// General Operations
//-----
// Insert
m.insert(std::pair<std::string, std::string>("key", "value"));
// Access by key
std::string value = m.at("key");
// Size
unsigned int size = m.size();
// Iterate
for(std::map<std::string, std::string>::iterator it = m.begin(); it != m.end(); it++
   std::cout << *it << std::endl;</pre>
}
// Remove by key
m.erase("key");
```

```
// Clear
m.clear();

//-----
// Container-Specific Operations
//-----
// Find if an element exists by key
bool exists = (m.find("key") != m.end());

// Count the number of elements with a certain key
unsigned int count = m.count("key");
```

## 1.6 Set std::set

#### Use for

- Removing duplicates
- Ordered dynamic storage

#### Do not use for

- Simple storage
- Direct access by index

### **Notes**

• Sets are often implemented with binary search trees

## **Time Complexity**

Operation	Time Complexity
Insert	0(log(n))
Remove	0(log(n))
Find	0(log(n))

```
std::set<int> s;
// General Operations
//-----
// Insert
s.insert(20);
// Size
unsigned int size = s.size();
// Iterate
for(std::set<int>::iterator it = s.begin(); it != s.end(); it++) {
   std::cout << *it << std::endl;</pre>
}
// Remove
s.erase(20);
// Clear
s.clear();
//-----
// Container-Specific Operations
//-----
// Find if an element exists
bool exists = (s.find(20) != s.end());
// Count the number of elements with a certain value
unsigned int count = s.count(20);
```

## 1.7 Stack std::stack

#### Use for

- First-In Last-Out operations
- Reversal of elements

## **Time Complexity**

Operation	Time Complexity
Push	0(1)

Operation	Time Complexity
Рор	0(1)
Тор	0(1)

## **Example Code**

# 1.8 Queue std::queue

### Use for

- First-In First-Out operations
- Ex: Simple online ordering system (first come first served)
- Ex: Semaphore queue handling
- Ex: CPU scheduling (FCFS)

#### **Notes**

• Often implemented as a std::deque

```
std::queue<int> q;
```

```
//-----
// General Operations
//-----
// Insert
q.push(value);

// Access head, tail
int head = q.front();  // head
int tail = q.back();  // tail

// Size
unsigned int size = q.size();

// Remove
q.pop();
```

## 1.9 Priority Queue std::priority\_queue

#### Use for

- First-In First-Out operations where priority overrides arrival time
- Ex: CPU scheduling (smallest job first, system/user priority)
- Ex: Medical emergencies (gunshot wound vs. broken arm)

#### **Notes**

Often implemented as a std::vector

```
std::priority_queue<int> p;

//------
// General Operations
//-----
// Insert
p.push(value);

// Access
int top = p.top(); // 'Top' element
// Size
unsigned int size = p.size();
```

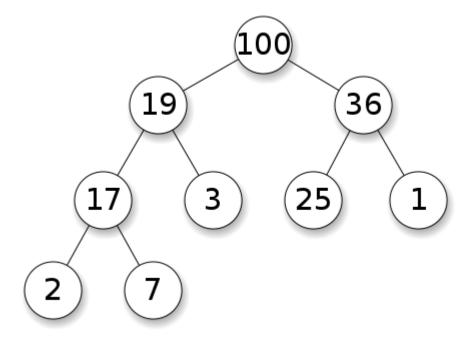
// Remove
p.pop();

## 1.10 Heap std::priority\_queue

#### **Notes**

- A heap is essentially an instance of a priority queue
- A min heap is structured with the root node as the smallest and each child subsequently larger than its parent
- A max heap is structured with the root node as the largest and each child subsequently smaller than its parent
- A min heap could be used for Smallest Job First CPU Scheduling
- A max heap could be used for Priority CPU Scheduling

### Max Heap Example (using a binary tree)

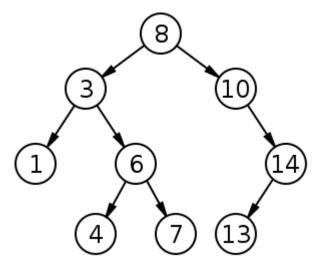


# 2.0 Trees

# 2.1 Binary Tree

- A binary tree is a tree with at most two (2) child nodes per parent
- Binary trees are commonly used for implementing O(log(n)) operations for ordered maps, sets, heaps, and binary search trees
- Binary trees are sorted in that nodes with values greater than their parents are inserted to the right, while nodes with values less than their parents are inserted to the left

### **Binary Search Tree**



## 2.2 Balanced Trees

- Balanced trees are a special type of tree which maintains its balance to ensure O(log(n)) operations
- When trees are not balanced the benefit of log(n) operations is lost due to the highly vertical structure
- Examples of balanced trees:
  - AVL Trees
  - Red-Black Trees

## 2.3 Binary Search

#### Idea:

- 1. If current element, return
- 2. If less than current element, look left
- 3. If more than current element, look right
- 4. Repeat

#### **Data Structures:**

- Tree
- Sorted array

## Space:

0(1)

### **Best Case:**

0(1)

### **Worst Case:**

• 0(log n)

### Average:

• 0(log n)

### Visualization:



# 2.4 Depth-First Search

#### Idea:

- 1. Start at root node
- 2. Recursively search all adjacent nodes and mark them as searched
- 3. Repeat

### **Data Structures:**

- Tree
- Graph

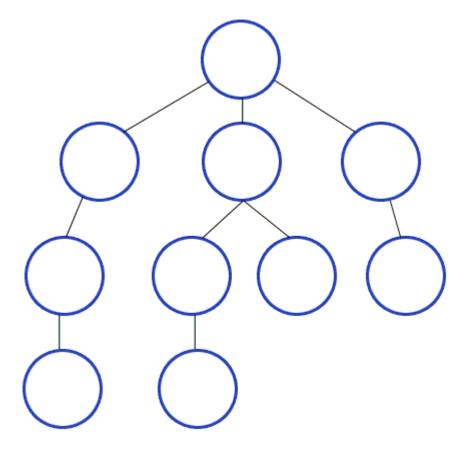
### Space:

• O(V), V = number of verticies

#### Performance:

• O(E), E = number of edges

## Visualization:



# 2.5 Breadth-First Search

### Idea:

- 1. Start at root node
- 2. Search neighboring nodes first before moving on to next level

### **Data Structures:**

- Tree
- Graph

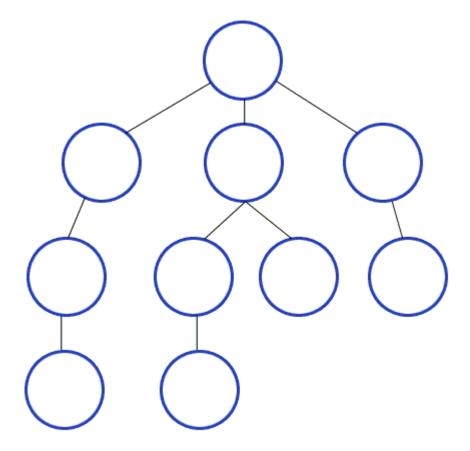
## Space:

O(V), V = number of verticies

#### Performance:

• O(E), E = number of edges

### Visualization:



# 3.0 NP Complete Problems

# 3.1 NP Complete

- NP Complete means that a problem is unable to be solved in polynomial time
- NP Complete problems can be verified in polynomial time, but not solved

# 3.2 Traveling Salesman Problem

## 3.3 Knapsack Problem

Implementation

# 4.0 Algorithms

## 4.1 Insertion Sort

#### Idea

- 1. Iterate over all elements
- 2. For each element:
  - Check if element is larger than largest value in sorted array
- 3. If larger: Move on
- 4. If smaller: Move item to correct position in sorted array

#### **Details**

- Data structure: Array
- Space: 0(1)
- Best Case: Already sorted, 0(n)
- Worst Case: Reverse sorted, 0(n^2)
- **Average**: 0(n^2)

## **Advantages**

- Easy to code
- Intuitive
- Better than selection sort and bubble sort for small data sets
- Can sort in-place

### Disadvantages

Very inefficient for large datasets

#### Visualization

## 6 5 3 1 8 7 2 4

## 4.2 Selection Sort

#### Idea

- 1. Iterate over all elements
- 2. For each element:
  - o If smallest element of unsorted sublist, swap with left-most unsorted element

#### **Details**

- Data structure: Array
- Space: 0(1)
- Best Case: Already sorted, 0(n^2)
- Worst Case: Reverse sorted, 0(n^2)
- **Average**: 0(n^2)

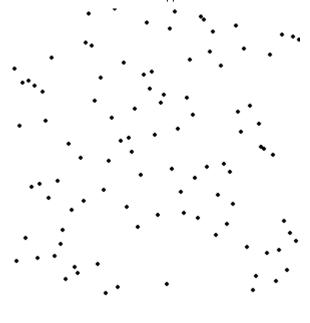
### **Advantages**

- Simple
- Can sort in-place
- Low memory usage for small datasets

# Disadvantages

• Very inefficient for large datasets

#### Visualization



## 4.3 Bubble Sort

### Idea

- 1. Iterate over all elements
- 2. For each element:
  - o Swap with next element if out of order
- 3. Repeat until no swaps needed

## **Details**

• Data structure: Array

• Space: 0(1)

• Best Case: Already sorted 0(n)

• Worst Case: Reverse sorted, 0(n^2)

• **Average**: 0(n^2)

### **Advantages**

• Easy to detect if list is sorted

### Disadvantages

- Very inefficient for large datasets
- Much worse than even insertion sort

#### Visualization

6 5 3 1 8 7 2 4

# 4.4 Merge Sort

#### Idea

- 1. Divide list into smallest unit (1 element)
- 2. Compare each element with the adjacent list
- 3. Merge the two adjacent lists
- 4. Repeat

#### **Details**

• Data structure: Array

• **Space**: O(n) auxiliary

• Best Case: O(nlog(n))

• Worst Case: Reverse sorted, O(nlog(n))

• Average: O(nlog(n))

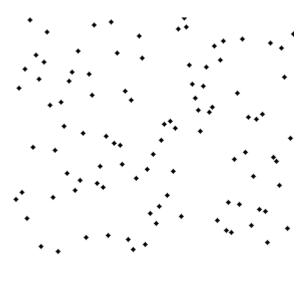
## **Advantages**

- High efficiency on large datasets
- Nearly always O(nlog(n))
- Can be parallelized
- Better space complexity than standard Quicksort

## Disadvantages

- Still requires O(n) extra space
- Slightly worse than Quicksort in some instances

## Visualization



6 5 3 1 8 7 2 4

# 4.5 Quicksort

#### Idea

1. Choose a **pivot** from the array

- 2. Partition: Reorder the array so that all elements with values *less* than the pivot come before the pivot, and all values *greater* than the pivot come after
- 3. Recursively apply the above steps to the sub-arrays

#### **Details**

• Data structure: Array

• **Space**: 0(n)

• Best Case: O(nlog(n))

• Worst Case: All elements equal, 0(n^2)

• Average: O(nlog(n))

### **Advantages**

- Can be modified to use O(log(n)) space
- Very quick and efficient with large datasets
- Can be parallelized
- Divide and conquer algorithm

### Disadvantages

- Not stable (could swap equal elements)
- Worst case is worse than Merge Sort

## **Optimizations**

- Choice of pivot:
  - o Choose median of the first, middle, and last elements as pivot
  - o Counters worst-case complexity for already-sorted and reverse-sorted

### Visualization

