Tutorial Sheet 6 Solution

UEC 502

Sol 1

$$\begin{aligned} V(z) &= X(z) + \frac{1}{2}z^{-1}V(z) \\ v(n) &= x(n) + \frac{1}{2}v(n-1) \\ Y(z) &= 2[3X(z) + V(z)] + 2z^{-1}V(z) \\ H(z) &= \frac{Y(z)}{X(z)} \\ &= \frac{8 - z^{-1}}{1 - 0.5z^{-1}} \\ h(n) &= 8(0.5)^n u(n) - (0.5)^{n-1}u(n-1) \end{aligned}$$

Sol2

$$\begin{aligned} H(z) &= 5 + \frac{3z^1}{1 + \frac{1}{3}z^{-1}} + \frac{1 + 2z^1}{1 - \frac{1}{2}z^{-1}} \\ h(n) &= 5\delta(n) + 3(-\frac{1}{3})^{n-1}u(n-1) + (\frac{1}{2})^n u(n) + 2(\frac{1}{2})^{n-1}u(n-1) \end{aligned}$$

Sol3

(a)

$$\begin{aligned} y(n) &= a_1 y(n-1) + a_2 y(n-2) + b_0 x(n) + b_1 x(n-1) + b_2 x(n-2) \\ H(z) &= \frac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}} \end{aligned}$$

(b)

$$\begin{aligned} H(z) &= \frac{1 + 2z^{-1} + z^{-2}}{1 + 1.5z^{-1} + 0.9z^{-2}} \\ \text{Zeros at } z &= -1, -1 \\ \text{Poles at } z &= -0.75 \pm j0.58 \end{aligned}$$

Since the poles are inside the unit circle, the system is stable.

$$\begin{aligned} H(z) &= \frac{1 + 2z^{-1} + z^{-2}}{1 - z^{-1} + 2z^{-2}} \\ \text{Zeros at } z &= -1, -1 \\ \text{Poles at } z &= 2, -1 \end{aligned}$$

Sol4

(a)

$$\begin{aligned} H(z) &= \frac{1+\frac{1}{3}z^{-1}}{1-\frac{3}{4}z^{-1}+\frac{1}{8}z^{-2}} \\ &= \frac{1+\frac{1}{3}z^{-1}}{(1-\frac{1}{2}z^{-1})(1-\frac{1}{4}z^{-1})} \\ &= \frac{\frac{10}{3}}{1-\frac{1}{2}z^{-1}}+\frac{-\frac{7}{3}}{1-\frac{1}{4}z^{-1}} \end{aligned}$$

Refer to fig 9.9-1

(b)

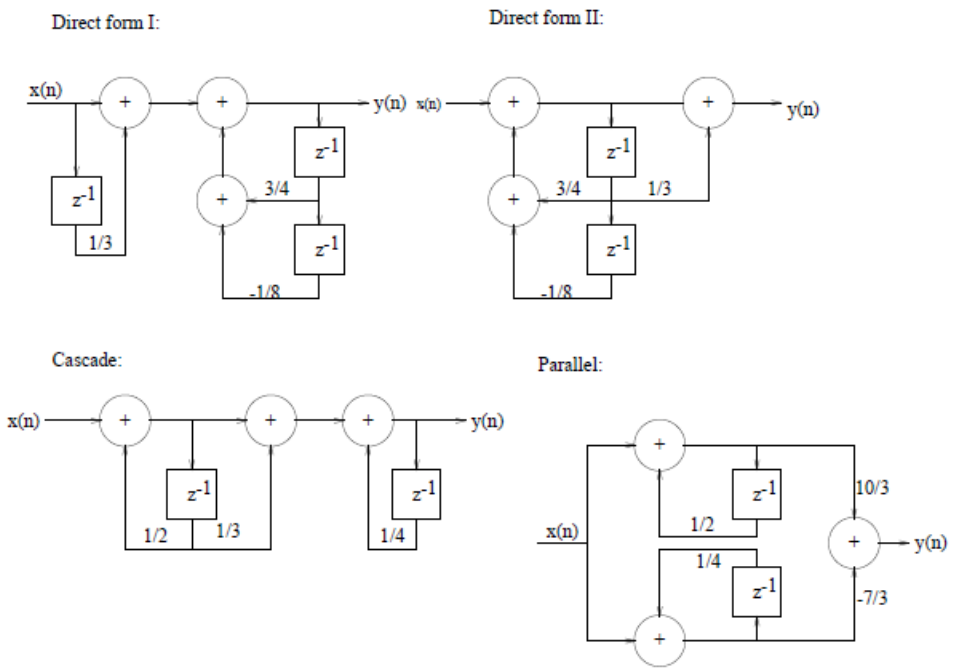

Figure 9.9-1:

$$\begin{aligned} H(z) &= \frac{0.7(1-0.36z^{-2})}{1+0.1z^{-1}-0.72z^{-2}} \\ &= \frac{0.7(1-0.6z^{-1})(1+0.6z^{-1})}{(1+0.9z^{-1})(1-0.8z^{-1})} \\ &= 0.35-\frac{0.1647}{1+0.9z^{-1}}-\frac{0.1853}{1-0.8z^{-1}} \end{aligned}$$

Refer to fig 9.9-2

(c)

$$H(z) = \frac{3(1+1.2z^{-1}+0.2z^{-2})}{1+0.1z^{-1}-0.2z^{-2}}$$

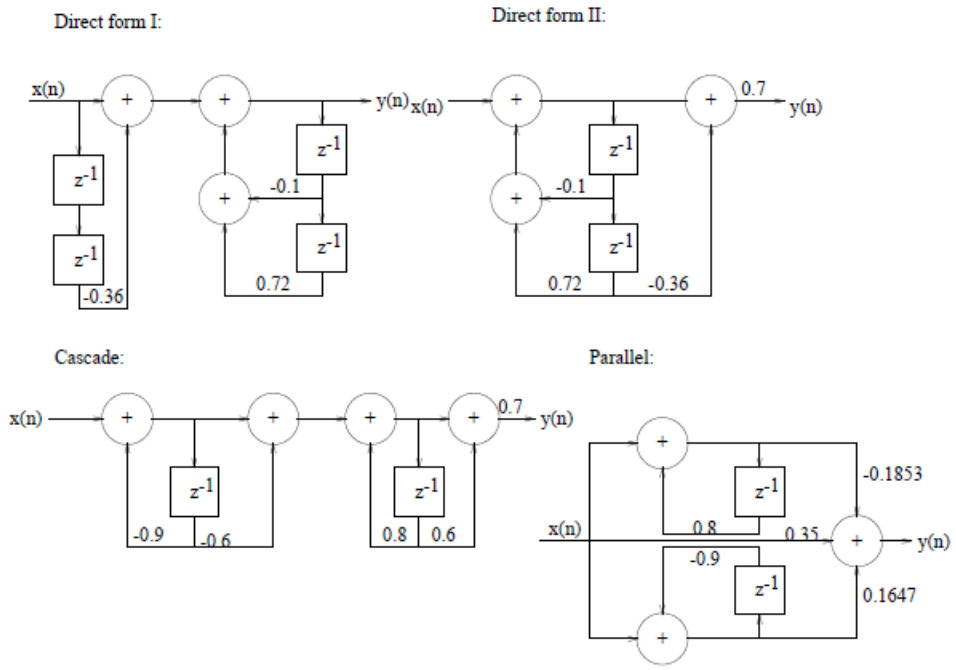

Figure 9.9-2:

$$
\begin{aligned}
&= \frac{3(1+0.2z^{-1})(1+z^{-1})}{(1+0.5z^{-1})(1-0.4z^{-1})} \\
&= -3 + \frac{7}{1-0.4z^{-1}} - \frac{1}{1+0.5z^{-1}}
\end{aligned}
$$