Tutorial No.:– 07

question no.–01:– Use rectangular window to design a symmetric lowpass linear-phase FIR filter. The desired frequency response is

$$H_{lp}(e^{jw}) = H_d(w) = \begin{cases} 1\, e^{-jw(M-1)/2} & ; \ 0 \leq |w| \leq w_c \\ \text{zero} & ; \ \text{otherwise} \end{cases}$$

Follow the appropriate procedure to obtain the unit sample response of filter $h(n)$ of length M with cutoff frequency w_c. [choose $M=25$ and $w_c = 3\pi/4$]

Hint:–
$h(n) = h_d(n) \times W(n) \Rightarrow$ Filter coefficients
$W(n)$ = Window function
$h_d(n)$ = inverse DTFT of desired frequency response $H_d[e^{jw}]$

For the window functions, refer to table–A (enclosed)

question no.–02:– Use rectangular window to design a symmetric highpass linear-phase FIR filter. The desired frequency response is

$$H_{hp}(e^{jw}) = H_d(w) = \begin{cases} \text{zero} & ; \ |w| < w_c \\ 1\, e^{-jwM/2} & ; \ w_c \leq |w| \leq \pi \end{cases}$$

Follow the appropriate procedure to obtain the unit sample response of filter $h(n)$ of length $(M+1)$ with cutoff frequency w_c. [choose $M=24$ and $w_c = 3\pi/4$]

Hint:– $H_{hp}(e^{jw}) = e^{-jwM/2} - H_{lp}(e^{jw})$

For the window functions, refer to table – B (enclosed) question no. - 03 :- Use Kaiser window to design a symmetric bandpass linear-phase FIR filter. The desired impulse response in the discrete-time domain is

$$h_{bpf}(n) = h_d(n) = \frac{\sin\left[\omega_1\left(n - \frac{(M-1)}{2}\right)\right]}{\pi\left\{n - \frac{(M-1)}{2}\right\}} - \frac{\sin\left[\omega_2\left(n - \frac{(M-1)}{2}\right)\right]}{\pi\left\{n - \frac{(M-1)}{2}\right\}}$$

with $\omega_1 > \omega_2$

Design specifications are as follows

upper cutoff frequency $= \omega_{c_1} = \omega_1 = 0.7\pi$

lower cutoff frequency $= \omega_{c_2} = \omega_2 = 0.3\pi$

stopband edge frequency $= \omega_{s_i}$

stopband ripple $= \delta_s = 0.001$

passband ripple $= \delta_p = 0.01$

passband edge frequency $= \omega_{p_i}$

transition width $= |\Delta\omega| = 0.2\pi = |\omega_{p_i} - \omega_{s_i}|$

& $\omega_{c_i} = (\omega_{p_i} + \omega_{s_i})/2$

Hint :- Kaiser window specifications are

$$w[n] = \begin{cases} \dfrac{I_0\left[\beta\left\{1 - \left[\frac{(n-\alpha)}{\alpha}\right]^2\right\}^{1/2}\right]}{I_0[\beta]} ; & \text{for } 0 \leq n \leq M-1 \\ \text{zero} ; & \text{otherwise} \end{cases}$$

$\alpha = \frac{(M-1)}{2}$; $I_0(x)$ represents the zeroth-order modified Bessel function of first-kind

$\delta = \text{Min}\{\delta_p, \delta_s\}$ and $A = -20\log_{10}(\delta)$ dB

$$\beta = \begin{cases} 0.1102\,(A-8.7) & \text{for } A > 50\,dB \\ 0.5842\,(A-21)^{0.4} + 0.07886\,(A-21) & \text{for } 21 \leq A \leq 50\,dB \\ \text{zero} & \text{for } A < 21\,dB \end{cases}$$

$$M \geq \left\lceil \frac{A-8}{2.285\,\Delta\omega} \right\rceil$$

$I_0(x) =$ `besseli(0, x)` in Matlab

Obtain filter coefficients $h(n)$ for an FIR filter of length M. **Length of the designed bandpass filter is M.**

Question no. – 04:– The system function of a narrow bandreject filter or a narrow band stop filter is

$$H[z] = b_0 \left[\frac{1 - 2(\cos\omega_0)z^{-1} + z^{-2}}{1 - 2r(\cos\omega_0)z^{-1} + r^2 z^{-2}} \right]$$

Plot the pole-zero diagram in z-domain for $\omega_0 = \pi/4$ and $r = 0.85$.

Plot its magnitude response in terms of

$$|H(\omega)| \text{ Vs } \omega.$$

for this narrow bandreject filter.

Table --- A

WINDOW FUNCTIONS FOR FIR FILTER DESIGN

Name of window	Time-domain sequence, $h(n), 0 \le n \le M-1$
Bartlett (triangular)	$1 - \dfrac{2\left\lvert n - \frac{M-1}{2}\right\rvert}{M-1}$
Blackman	$0.42 - 0.5\cos\dfrac{2\pi n}{M-1} + 0.08\cos\dfrac{4\pi n}{M-1}$
Hamming	$0.54 - 0.46\cos\dfrac{2\pi n}{M-1}$
Hanning	$\dfrac{1}{2}\left(1 - \cos\dfrac{2\pi n}{M-1}\right)$
Kaiser	$\dfrac{I_0\left[\alpha\sqrt{\left(\frac{M-1}{2}\right)^2 - \left(n - \frac{M-1}{2}\right)^2}\right]}{I_0\left[\alpha\left(\frac{M-1}{2}\right)\right]}$
Lanczos	$\left\{\dfrac{\sin\left[2\pi\left(n - \frac{M-1}{2}\right)\Big/(M-1)\right]}{2\pi\left(n - \frac{M-1}{2}\right)\Big/\left(\frac{M-1}{2}\right)}\right\}^L \quad L > 0$
Tukey	$1, \quad \left\lvert n - \frac{M-1}{2}\right\rvert \le \alpha\frac{M-1}{2} \qquad 0 < \alpha < 1$ $\dfrac{1}{2}\left[1 + \cos\left(\dfrac{n - (1+\alpha)(M-1)/2}{(1-\alpha)(M-1)/2}\pi\right)\right]$ $\alpha(M-1)/2 \le \left\lvert n - \frac{M-1}{2}\right\rvert \le \frac{M-1}{2}$

Table ---B

Properties of Commonly Used Windows

Some commonly used windows are shown in Figure 7.21.[6] These windows are defined by the following equations:

Rectangular

$$w[n] = \begin{cases} 1, & 0 \le n \le M, \\ 0, & \text{otherwise} \end{cases} \tag{7.47a}$$

Bartlett (triangular)

$$w[n] = \begin{cases} 2n/M, & 0 \le n \le M/2, \\ 2 - 2n/M, & M/2 < n \le M, \\ 0, & \text{otherwise} \end{cases} \tag{7.47b}$$

Hanning

$$w[n] = \begin{cases} 0.5 - 0.5\cos(2\pi n/M), & 0 \le n \le M, \\ 0, & \text{otherwise} \end{cases} \tag{7.47c}$$

Hamming

$$w[n] = \begin{cases} 0.54 - 0.46\cos(2\pi n/M), & 0 \le n \le M, \\ 0, & \text{otherwise} \end{cases} \tag{7.47d}$$

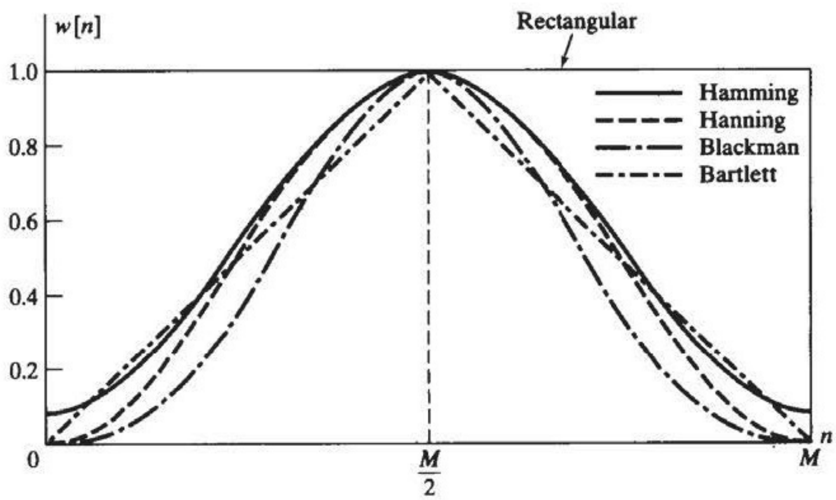

Figure 7.21 Commonly used windows.

Blackman

$$w[n] = \begin{cases} 0.42 - 0.5\cos(2\pi n/M) + 0.08\cos(4\pi n/M), & 0 \le n \le M, \\ 0, & \text{otherwise} \end{cases} \tag{7.47e}$$