

# Reinforcement Learning

- We still assume an MDP:
  - A set of states  $s \in S$
  - A set of actions (per state)  $A$
  - A model  $T(s,a,s')$
  - A reward function  $R(s,a,s')$
- Still looking for a policy  $\pi(s)$
- New twist: don't know  $T$  or  $R$ , so must try out actions
- Big idea: Compute all averages over  $T$  using sample outcomes



# The Story So Far: MDPs and RL

## Known MDP: Offline Solution

### Goal

Compute  $V^*$ ,  $Q^*$ ,  $\pi^*$

Evaluate a fixed policy  $\pi$

### Technique

Value / policy iteration

Policy evaluation

## Unknown MDP: Model-Based

### Goal

Compute  $V^*$ ,  $Q^*$ ,  $\pi^*$

Evaluate a fixed policy  $\pi$

### Technique

VI/PI on approx. MDP

PE on approx. MDP

## Unknown MDP: Model-Free

### Goal

Compute  $V^*$ ,  $Q^*$ ,  $\pi^*$

Evaluate a fixed policy  $\pi$

### Technique

Q-learning

Value Learning

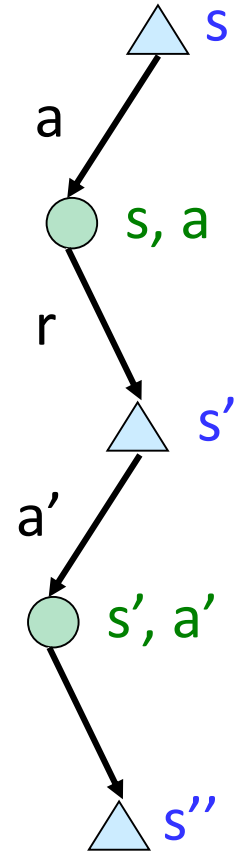
# Model-Free Learning

- Model-free (temporal difference) learning

- Experience world through episodes

$$(s, a, r, s', a', r', s'', a'', r'', s'''' \dots)$$

- Update estimates each transition  $(s, a, r, s')$
- Over time, updates will mimic Bellman updates



# Q-Learning

- We'd like to do Q-value updates to each Q-state:

$$Q_{k+1}(s, a) \leftarrow \sum_{s'} T(s, a, s') \left[ R(s, a, s') + \gamma \max_{a'} Q_k(s', a') \right]$$

- But can't compute this update without knowing T, R

- Instead, compute average as we go

- Receive a sample transition (s,a,r,s')
- This sample suggests

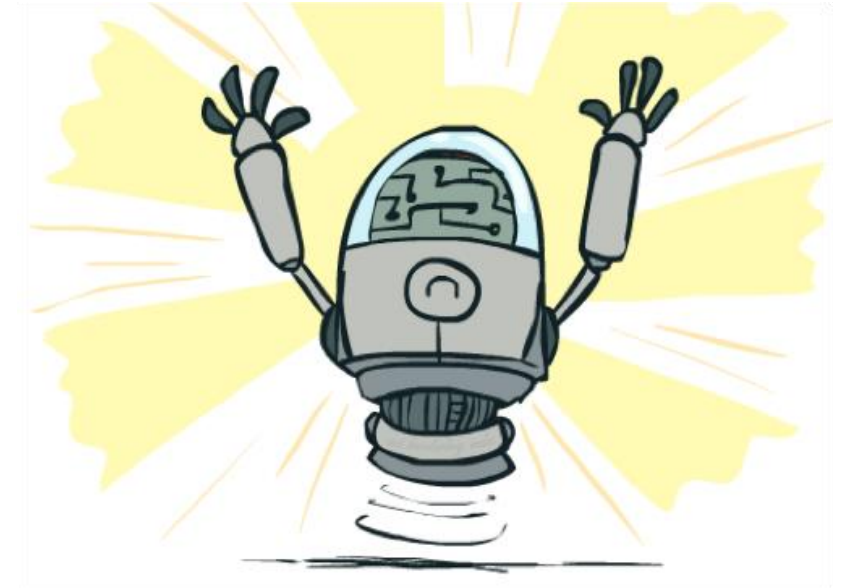
$$Q(s, a) \approx r + \gamma \max_{a'} Q(s', a')$$

- But we want to average over results from (s,a) (Why?)
- So keep a running average

$$Q(s, a) \leftarrow (1 - \alpha)Q(s, a) + (\alpha) \left[ r + \gamma \max_{a'} Q(s', a') \right]$$

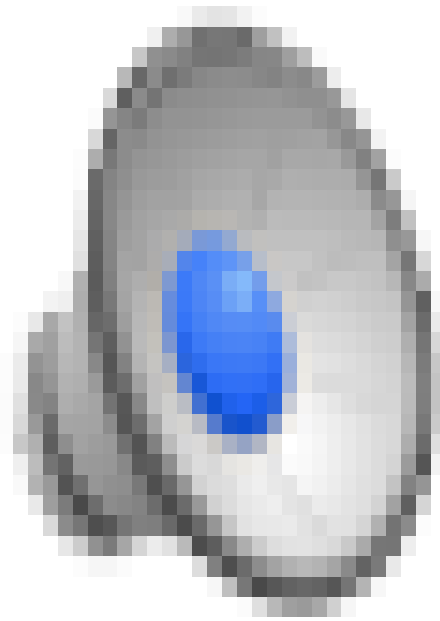
# Q-Learning Properties

- Amazing result: Q-learning converges to optimal policy -- even if you're acting suboptimally!
- This is called **off-policy learning**
- Caveats:
  - You have to explore enough
  - You have to eventually make the learning rate small enough
  - ... but not decrease it too quickly
  - Basically, in the limit, it doesn't matter how you select actions (!)

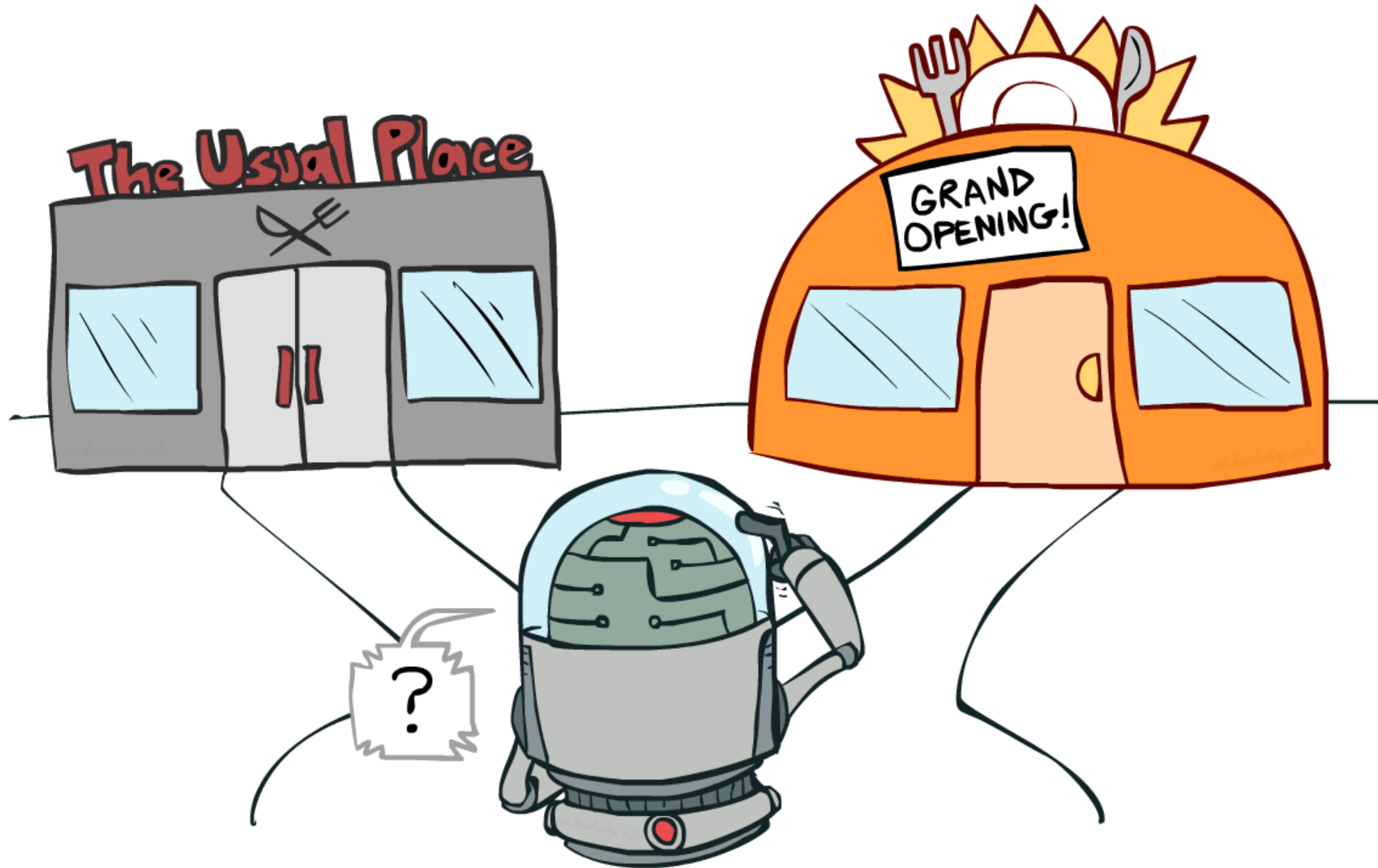


# Video of Demo Q-Learning Auto Cliff Grid

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# Exploration vs. Exploitation



# How to Explore?

- Several schemes for forcing exploration
  - Simplest: random actions ( $\epsilon$ -greedy)
    - Every time step, flip a coin
    - With (small) probability  $\epsilon$ , act randomly
    - With (large) probability  $1-\epsilon$ , act on current policy
  - Problems with random actions?
    - You do eventually explore the space, but keep thrashing around once learning is done
    - One solution: lower  $\epsilon$  over time
    - Another solution: exploration functions



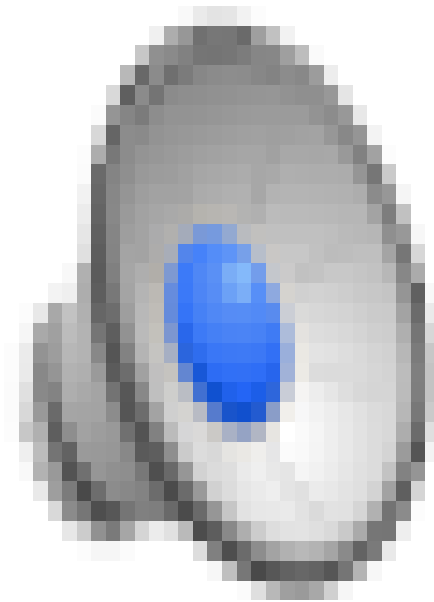
[Demo: Q-learning – manual exploration – bridge grid (L11D2)]

[Demo: Q-learning – epsilon-greedy -- crawler (L11D3)]



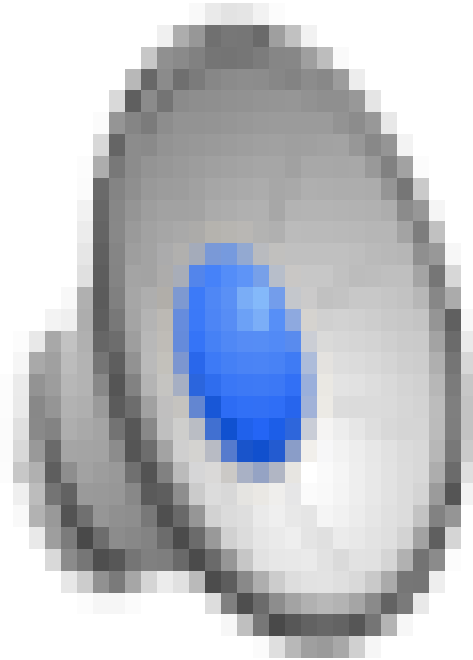
# Video of Demo Q-learning – Manual Exploration – Bridge Grid

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# Video of Demo Q-learning – Epsilon-Greedy – Crawler

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# Exploration Functions

- When to explore?

- Random actions: explore a fixed amount
- Better idea: explore areas whose badness is not (yet) established, eventually stop exploring

- Exploration function

- Takes a value estimate  $u$  and a visit count  $n$ , and returns an optimistic utility, e.g.  $f(u, n) = u + k/n$

Regular Q-Update:  $Q(s, a) \leftarrow_{\alpha} R(s, a, s') + \gamma \max_{a'} Q(s', a')$

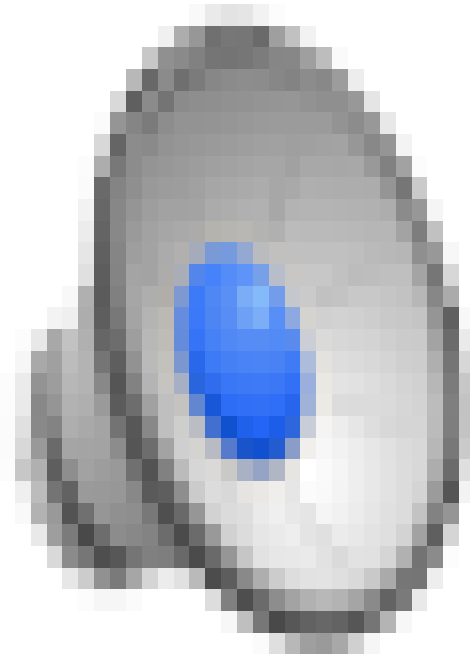
Modified Q-Update:  $Q(s, a) \leftarrow_{\alpha} R(s, a, s') + \gamma \max_{a'} f(Q(s', a'), N(s', a'))$

- Note: this propagates the “bonus” back to states that lead to unknown states as well!



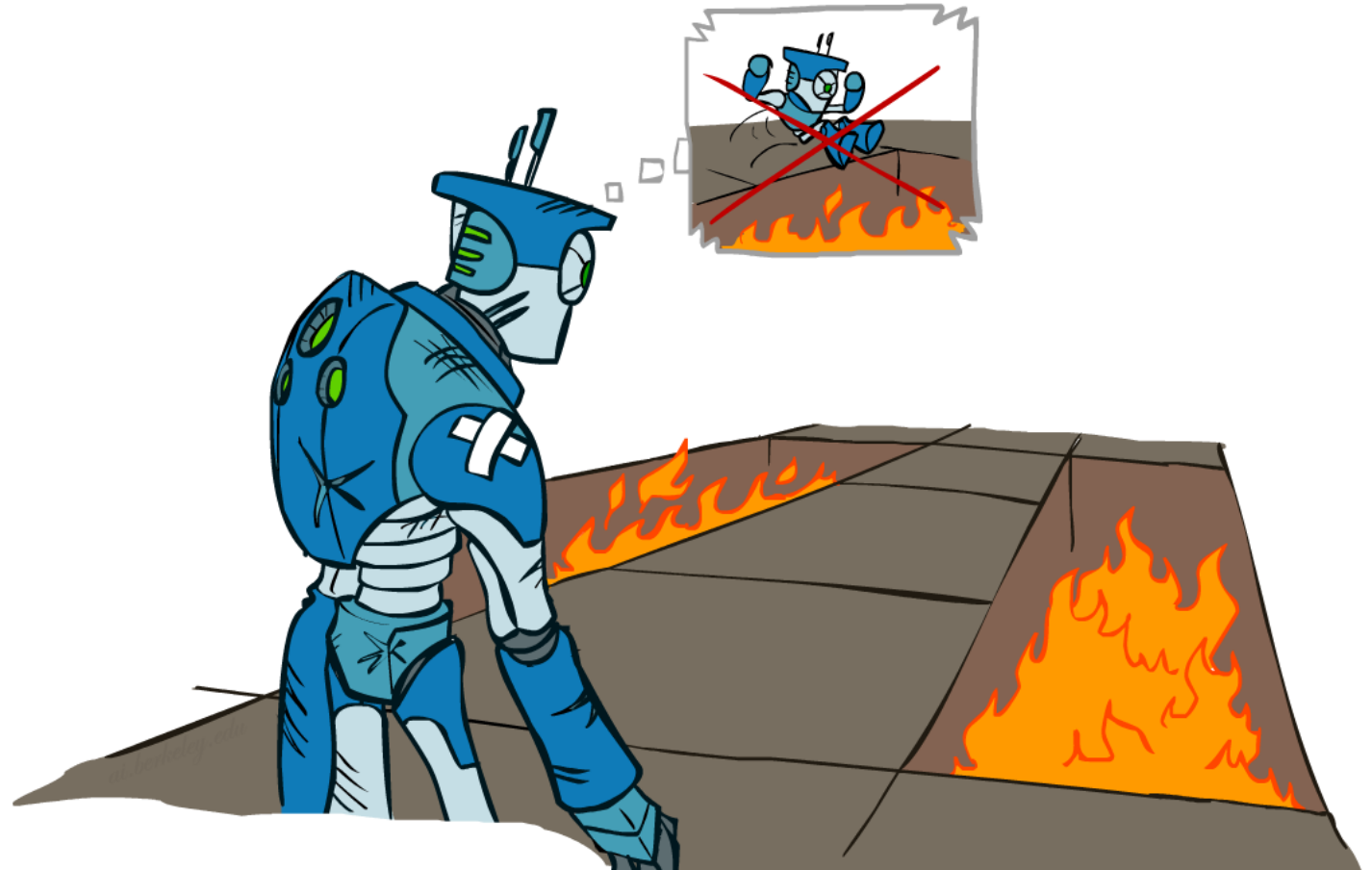
# Video of Demo Q-learning – Exploration Function – Crawler

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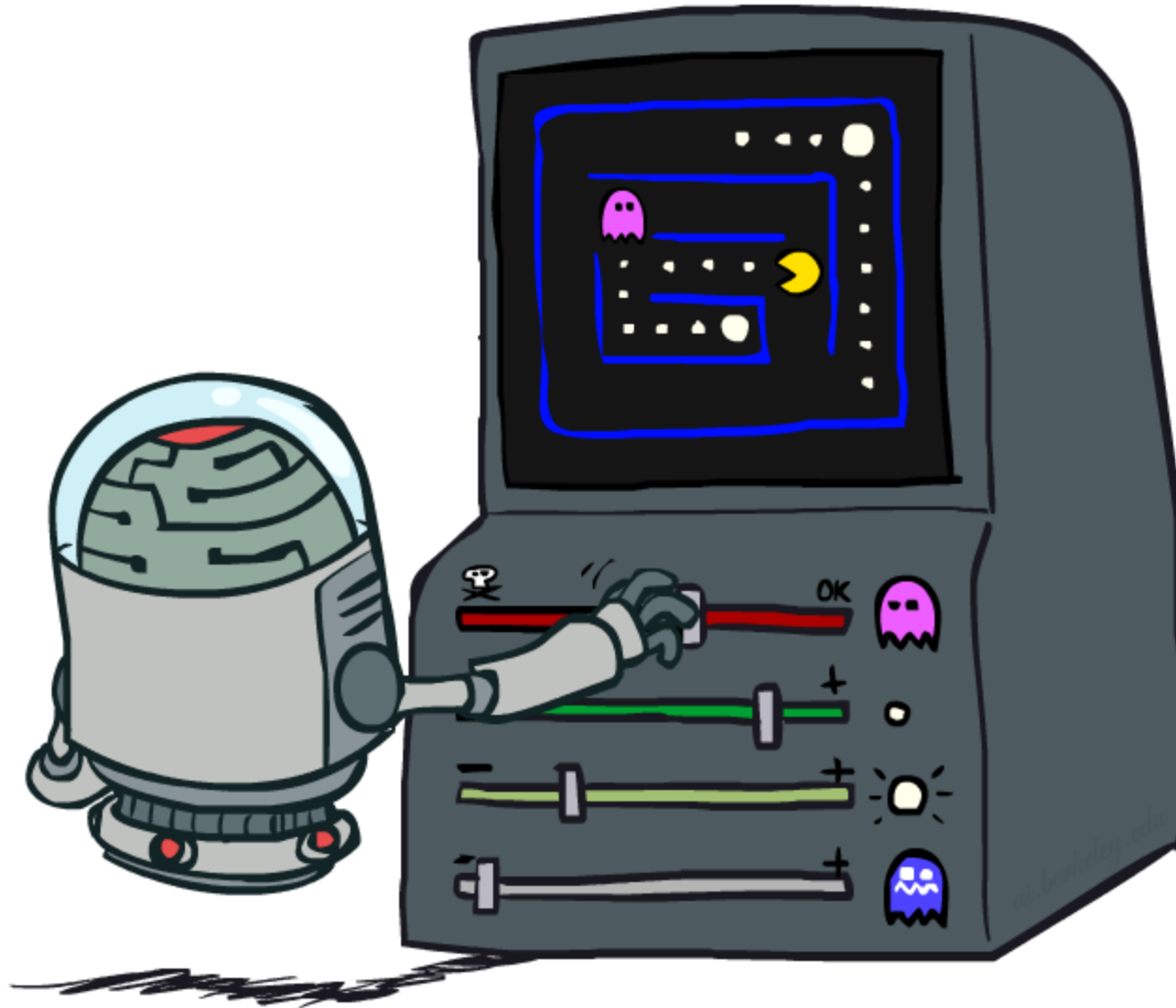


# Regret

- Even if you learn the optimal policy, you still make mistakes along the way
- Regret is a measure of your total mistake cost: the difference between your (expected) rewards, including youthful suboptimality, and optimal (expected) rewards
- Minimizing regret goes beyond learning to be optimal – it requires optimally learning to be optimal
- Example: random exploration and exploration functions both end up optimal, but random exploration has higher regret

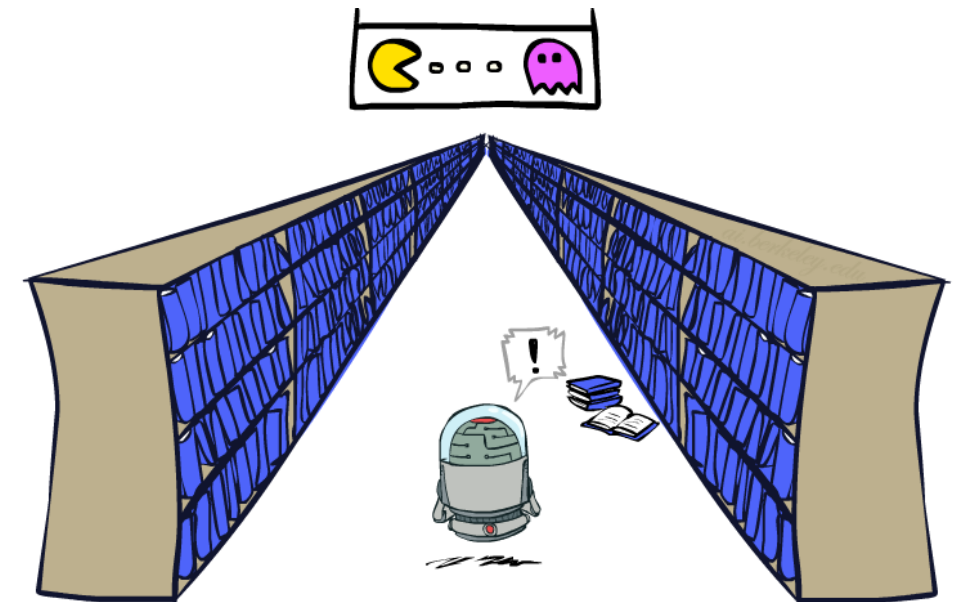
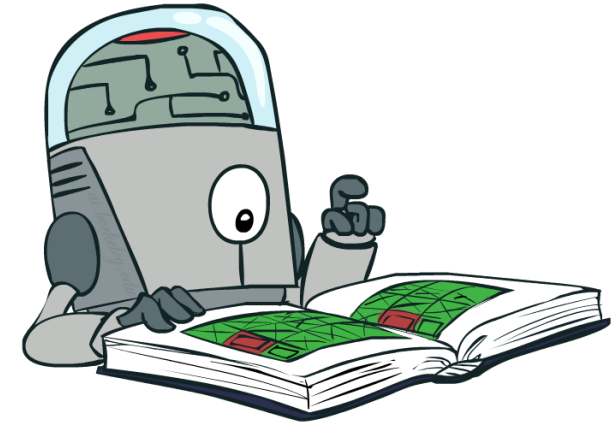


# Approximate Q-Learning



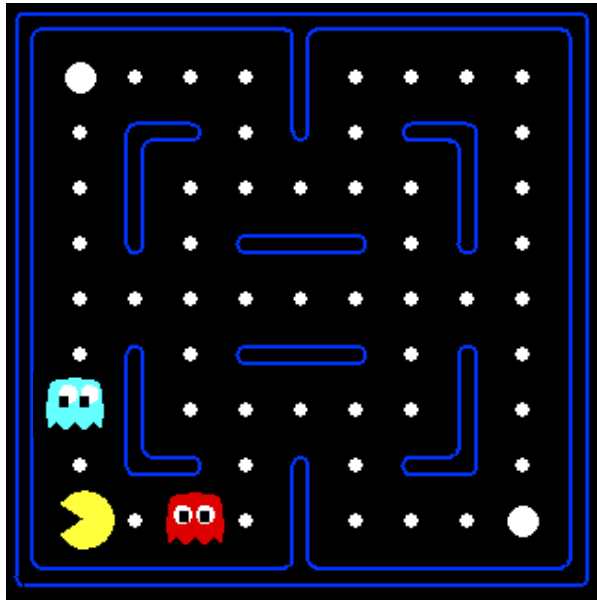
# Generalizing Across States

- Basic Q-Learning keeps a table of all q-values
- In realistic situations, we cannot possibly learn about every single state!
  - Too many states to visit them all in training
  - Too many states to hold the q-tables in memory
- Instead, we want to generalize:
  - Learn about some small number of training states from experience
  - Generalize that experience to new, similar situations
  - This is a fundamental idea in machine learning, and we'll see it over and over again

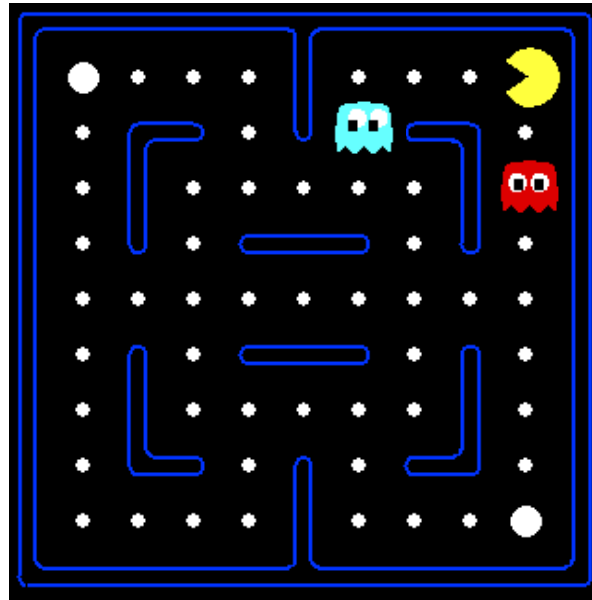


# Example: Pacman

Let's say we discover through experience that this state is bad:



In naïve q-learning, we know nothing about this state:



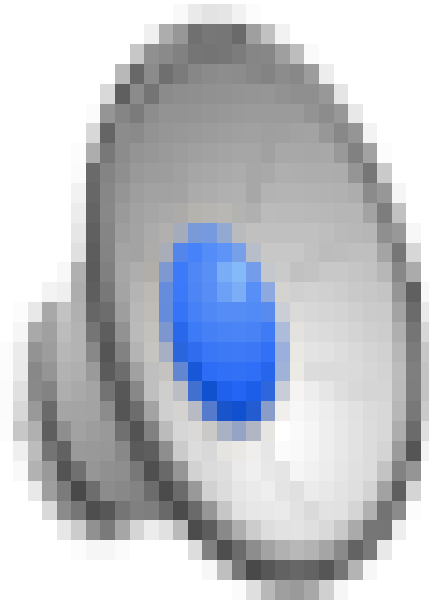
Or even this one!





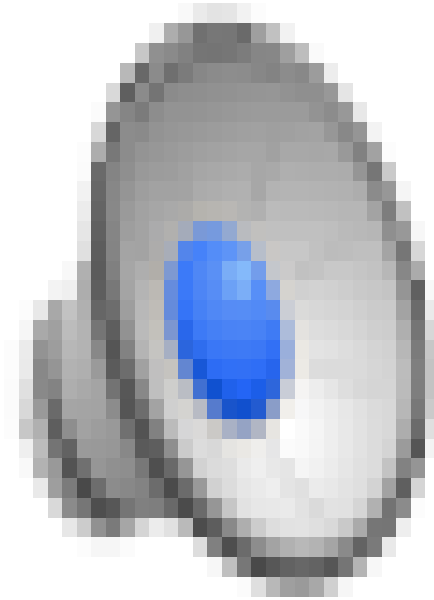
# Video of Demo Q-Learning Pacman – Tiny – Watch All

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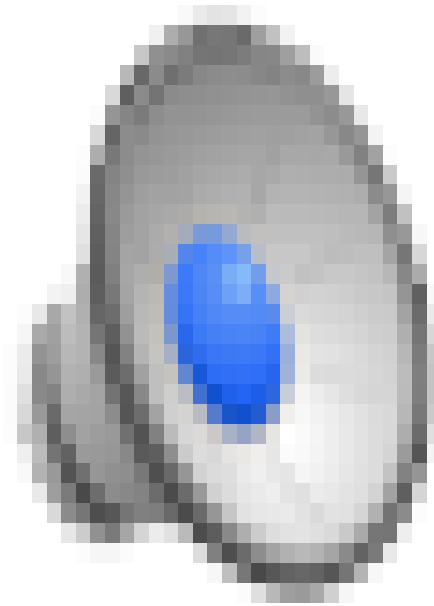
# Video of Demo Q-Learning Pacman – Tiny – Silent Train

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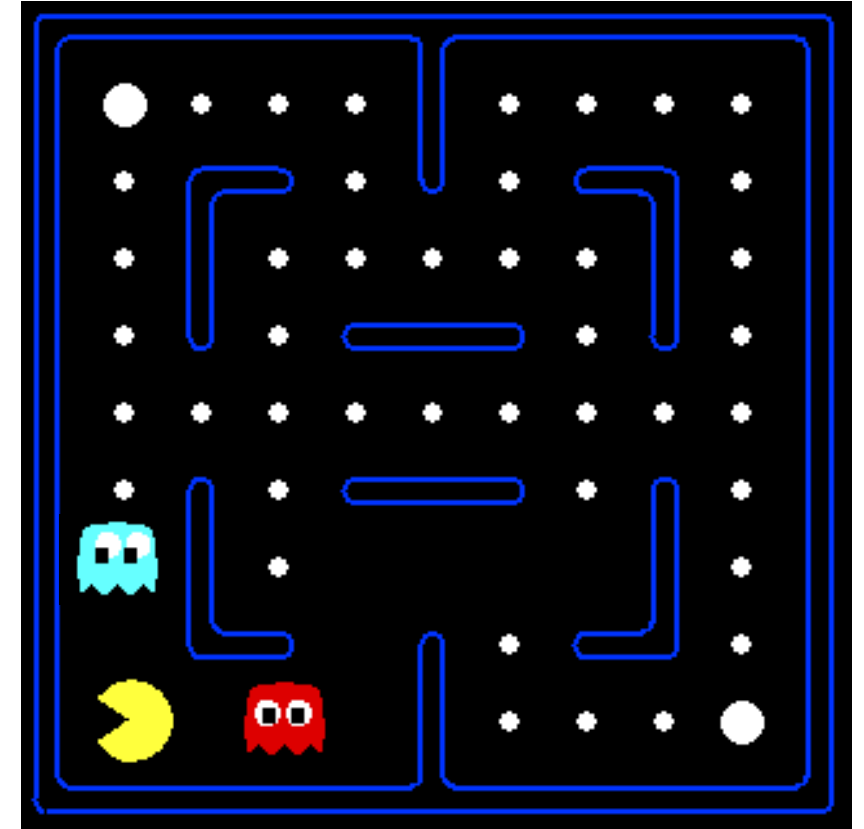
# Video of Demo Q-Learning Pacman – Tricky – Watch All

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# Feature-Based Representations

- Solution: describe a state using a vector of features (properties)
  - Features are functions from states to real numbers (often 0/1) that capture important properties of the state
  - Example features:
    - Distance to closest ghost
    - Distance to closest dot
    - Number of ghosts
    - $1 / (\text{dist to dot})^2$
    - Is Pacman in a tunnel? (0/1)
    - ..... etc.
    - Is it the exact state on this slide?
  - Can also describe a q-state  $(s, a)$  with features (e.g. action moves closer to food)



# Linear Value Functions

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- Using a feature representation, we can write a q function (or value function) for any state using a few weights:

$$V(s) = w_1 f_1(s) + w_2 f_2(s) + \dots + w_n f_n(s)$$

$$Q(s, a) = w_1 f_1(s, a) + w_2 f_2(s, a) + \dots + w_n f_n(s, a)$$

- Advantage: our experience is summed up in a few powerful numbers
- Disadvantage: states may share features but actually be very different in value!

# Approximate Q-Learning

$$Q(s, a) = w_1 f_1(s, a) + w_2 f_2(s, a) + \dots + w_n f_n(s, a)$$

- Q-learning with linear Q-functions:

transition =  $(s, a, r, s')$

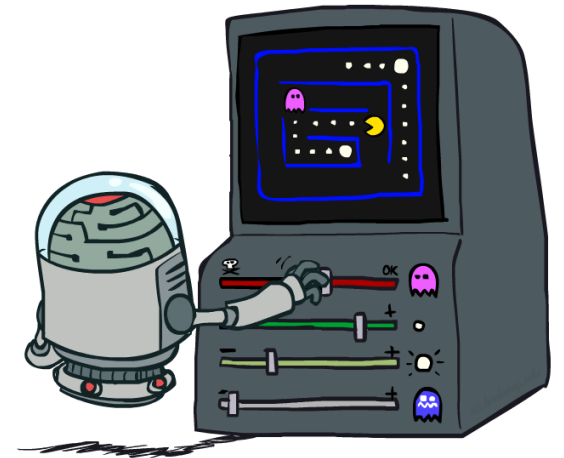
difference =  $\left[ r + \gamma \max_{a'} Q(s', a') \right] - Q(s, a)$

$Q(s, a) \leftarrow Q(s, a) + \alpha [\text{difference}]$

$w_i \leftarrow w_i + \alpha [\text{difference}] f_i(s, a)$

Exact Q's

Approximate Q's



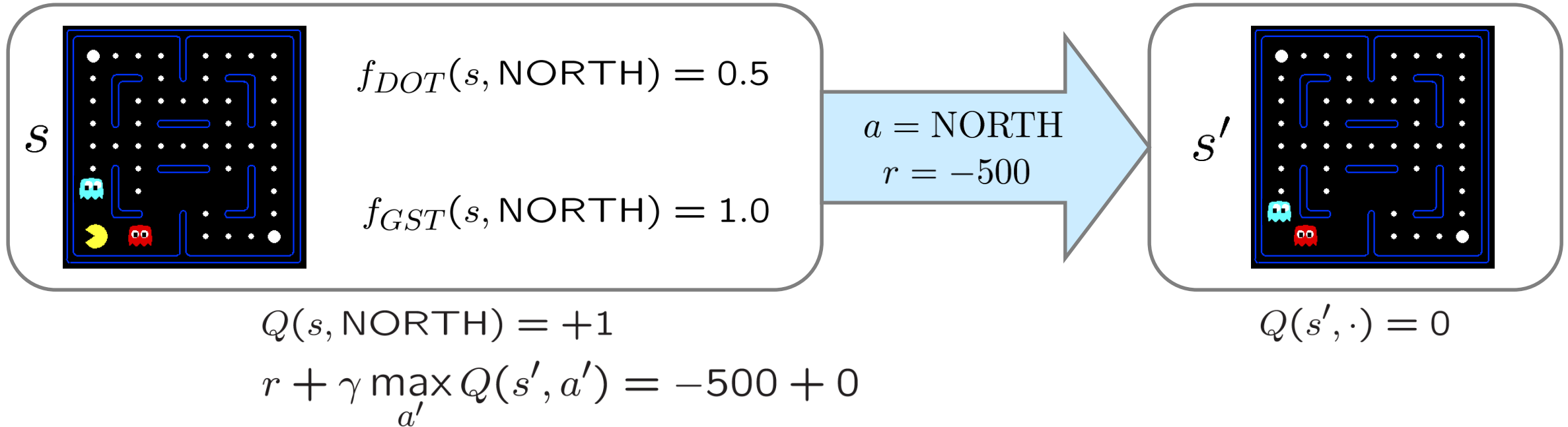
- Intuitive interpretation:

- Adjust weights of active features
- E.g., if something unexpectedly bad happens, blame the features that were on: disprefer all states with that state's features

- Formal justification: online least squares

# Example: Q-Pacman

$$Q(s, a) = 4.0 f_{DOT}(s, a) - 1.0 f_{GST}(s, a)$$



difference = -501

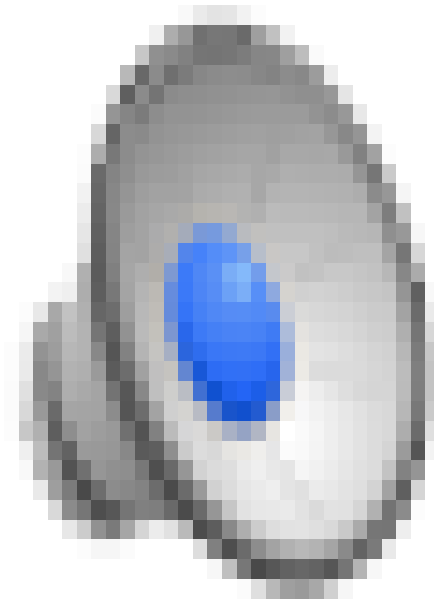
$w_{DOT} \leftarrow 4.0 + \alpha [-501] 0.5$

$w_{GST} \leftarrow -1.0 + \alpha [-501] 1.0$

$$Q(s, a) = 3.0 f_{DOT}(s, a) - 3.0 f_{GST}(s, a)$$

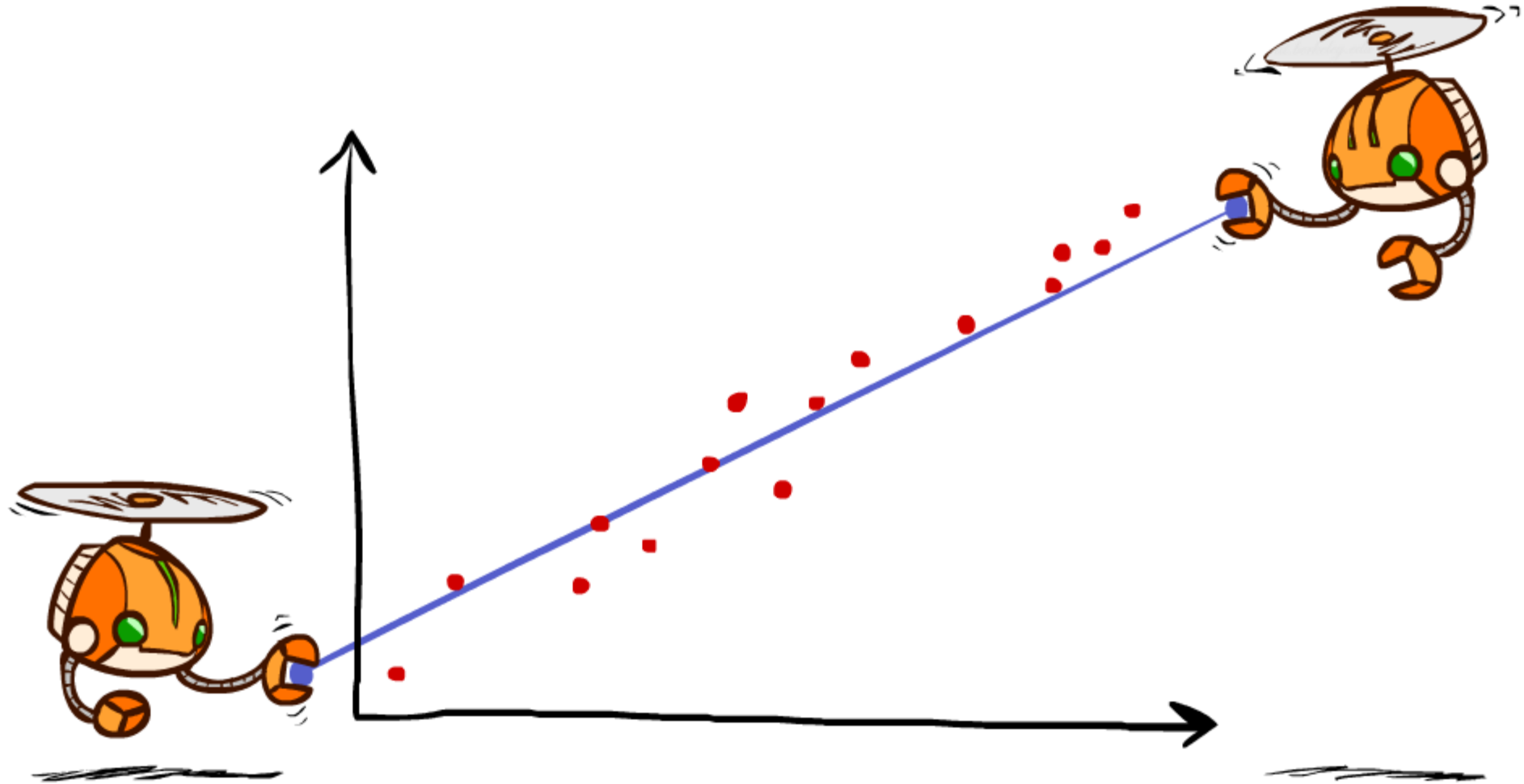
# Video of Demo Approximate Q-Learning -- Pacman

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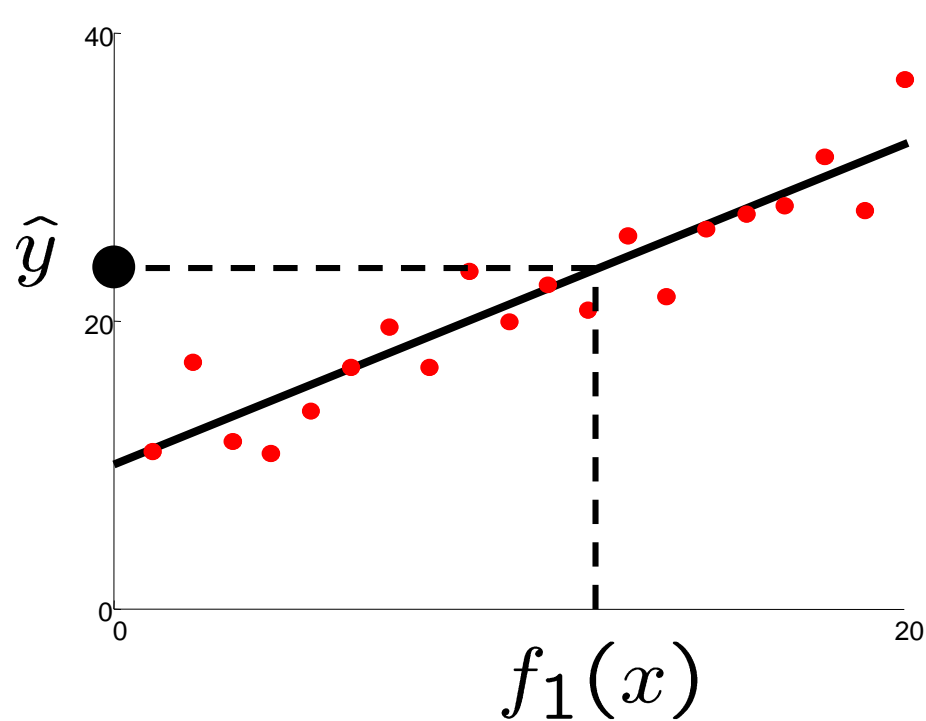




# Q-Learning and Least Squares

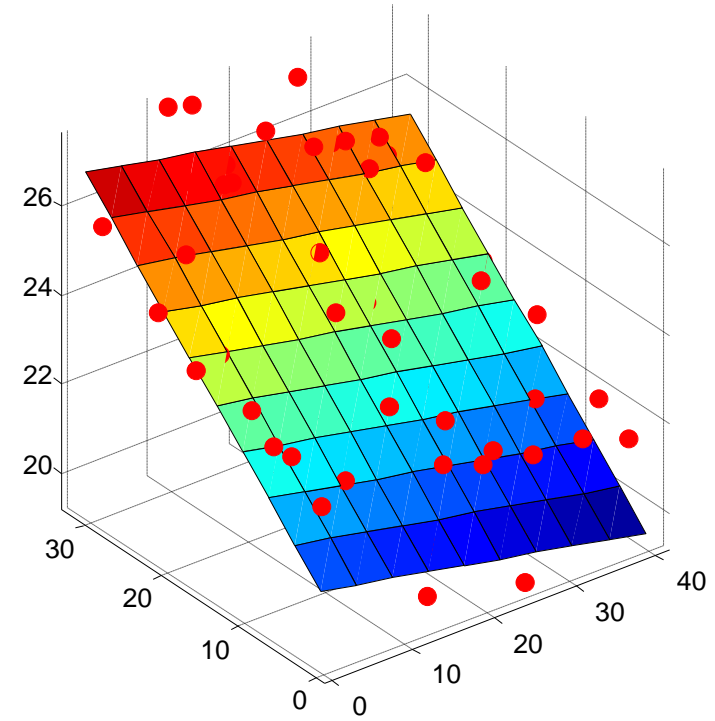


# Linear Approximation: Regression\*



Prediction:

$$\hat{y} = w_0 + w_1 f_1(x)$$

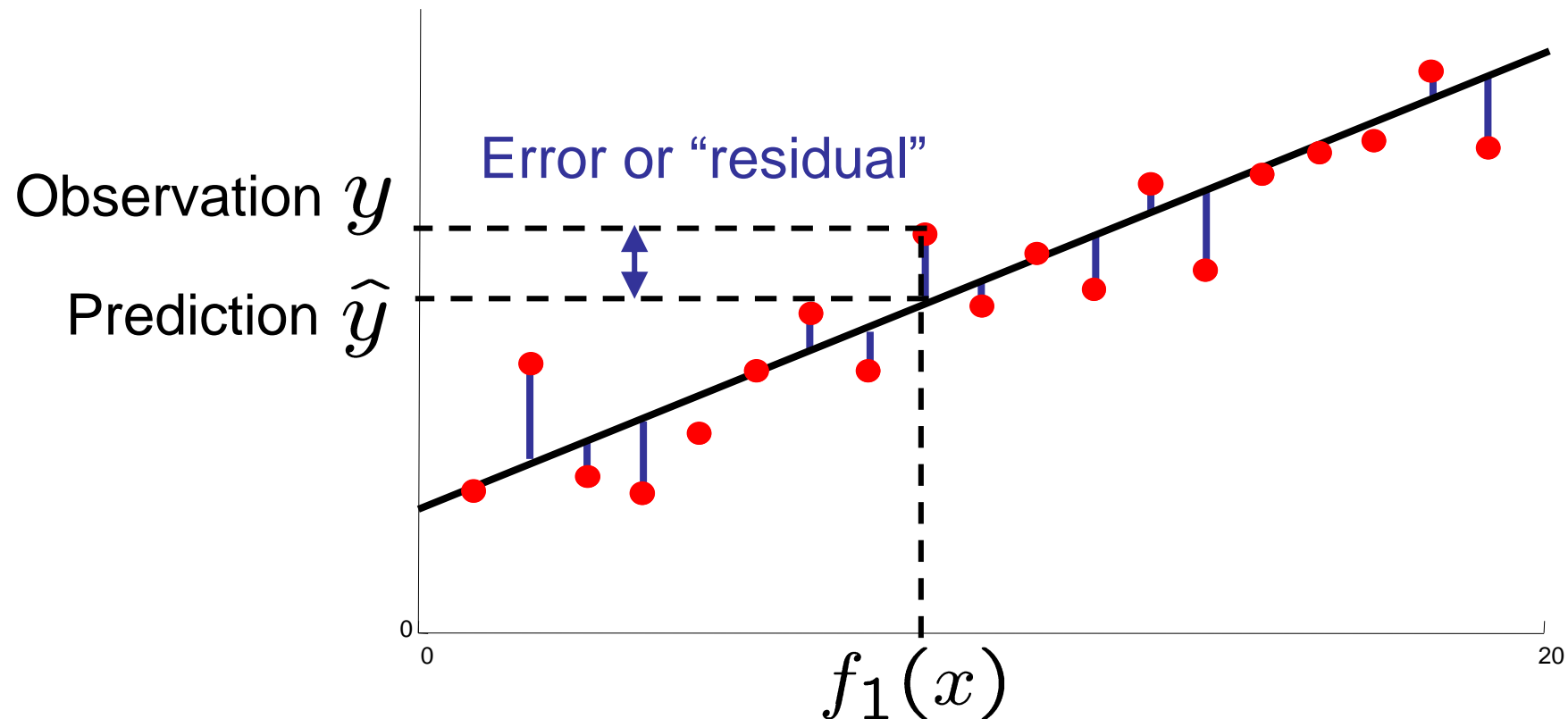


Prediction:

$$\hat{y}_i = w_0 + w_1 f_1(x) + w_2 f_2(x)$$

# Optimization: Least Squares\*

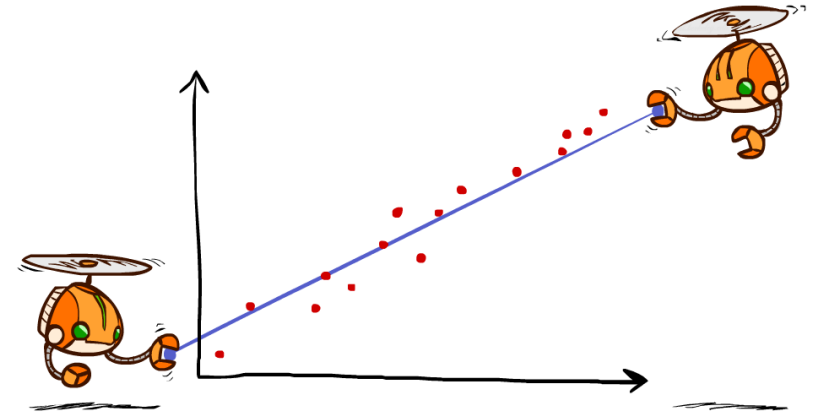
$$\text{total error} = \sum_i (y_i - \hat{y}_i)^2 = \sum_i \left( y_i - \sum_k w_k f_k(x_i) \right)^2$$



# Minimizing Error\*

Imagine we had only one point  $x$ , with features  $f(x)$ , target value  $y$ , and weights  $w$ :

$$\begin{aligned}\text{error}(w) &= \frac{1}{2} \left( y - \sum_k w_k f_k(x) \right)^2 \\ \frac{\partial \text{error}(w)}{\partial w_m} &= - \left( y - \sum_k w_k f_k(x) \right) f_m(x) \\ w_m &\leftarrow w_m + \alpha \left( y - \sum_k w_k f_k(x) \right) f_m(x)\end{aligned}$$



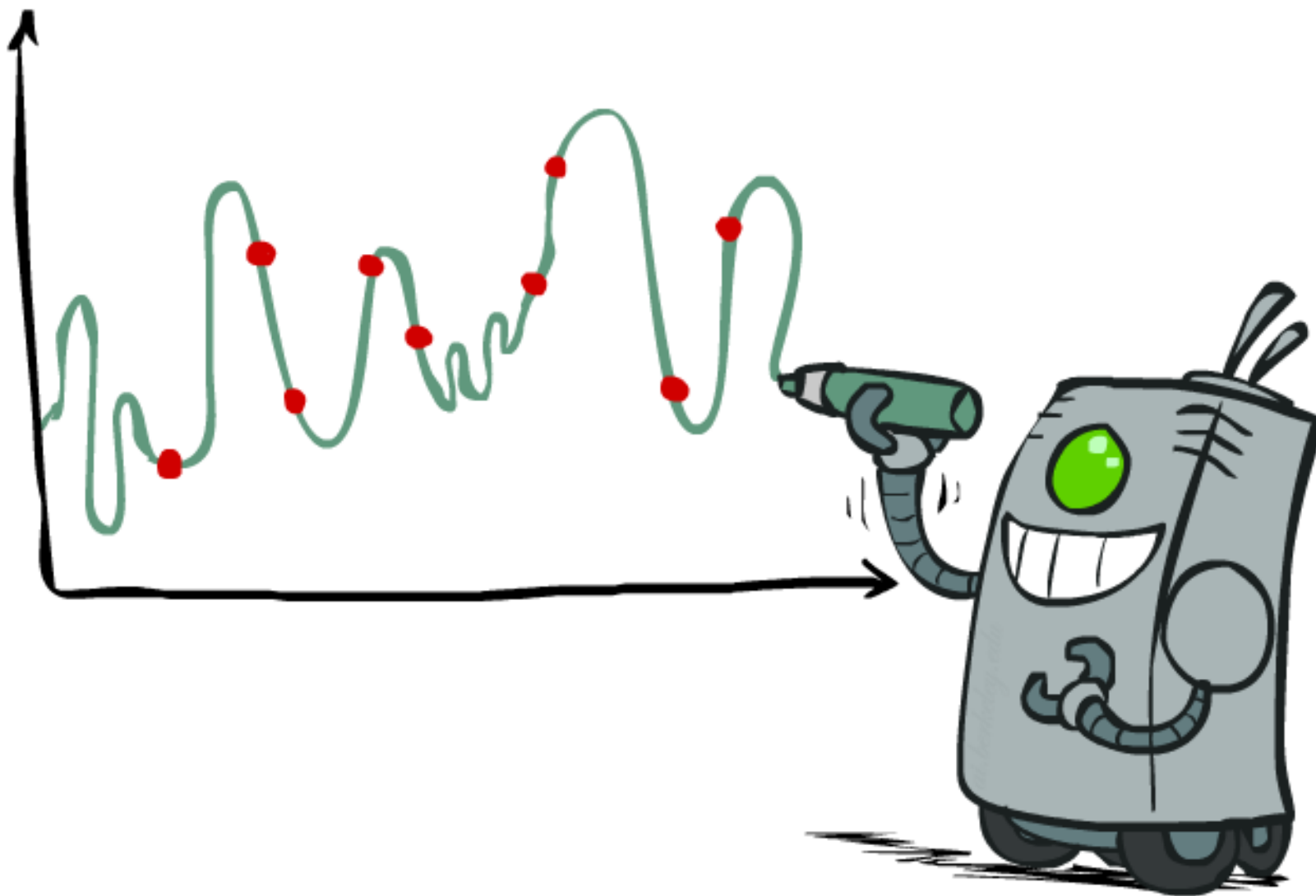
Approximate q update explained:

$$w_m \leftarrow w_m + \alpha \left[ \underbrace{r + \gamma \max_{a'} Q(s', a')}_{\text{“target”}} - \underbrace{Q(s, a)}_{\text{“prediction”}} \right] f_m(s, a)$$

“target”

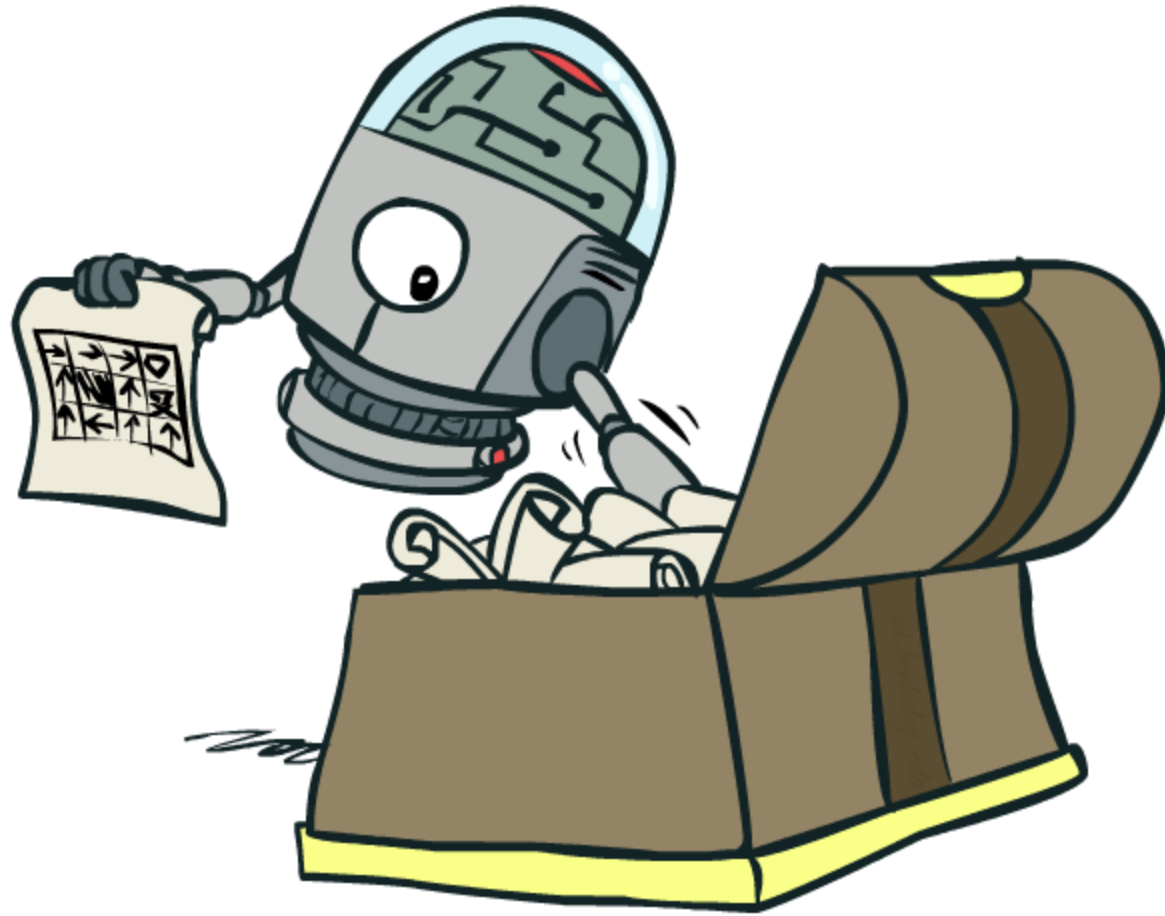
“prediction”

# Overfitting: Why Limiting Capacity Can Help\*



# Policy Search

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# Policy Search

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- Problem: often the feature-based policies that work well (win games, maximize utilities) aren't the ones that approximate  $V$  /  $Q$  best
  - E.g. your value functions from project 2 were probably horrible estimates of future rewards, but they still produced good decisions
  - Q-learning's priority: get Q-values close (modeling)
  - Action selection priority: get ordering of Q-values right (prediction)
  - We'll see this distinction between modeling and prediction again later in the course
- Solution: learn policies that maximize rewards, not the values that predict them
- Policy search: start with an ok solution (e.g. Q-learning) then fine-tune by hill climbing on feature weights

# Policy Search

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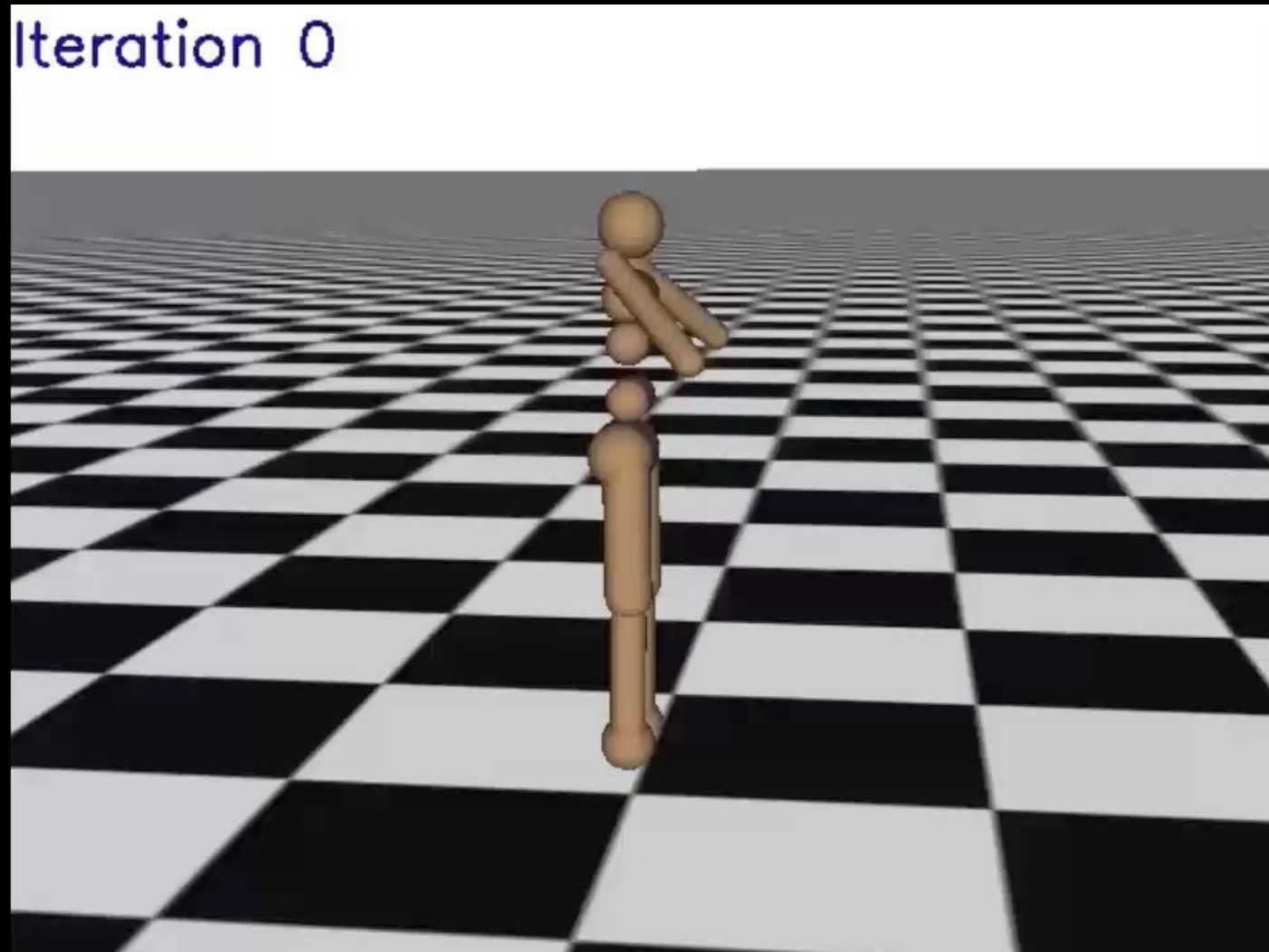
- Simplest policy search:
  - Start with an initial linear value function or Q-function
  - Nudge each feature weight up and down and see if your policy is better than before
- Problems:
  - How do we tell the policy got better?
  - Need to run many sample episodes!
  - If there are a lot of features, this can be impractical
- Better methods exploit lookahead structure, sample wisely, change multiple parameters...



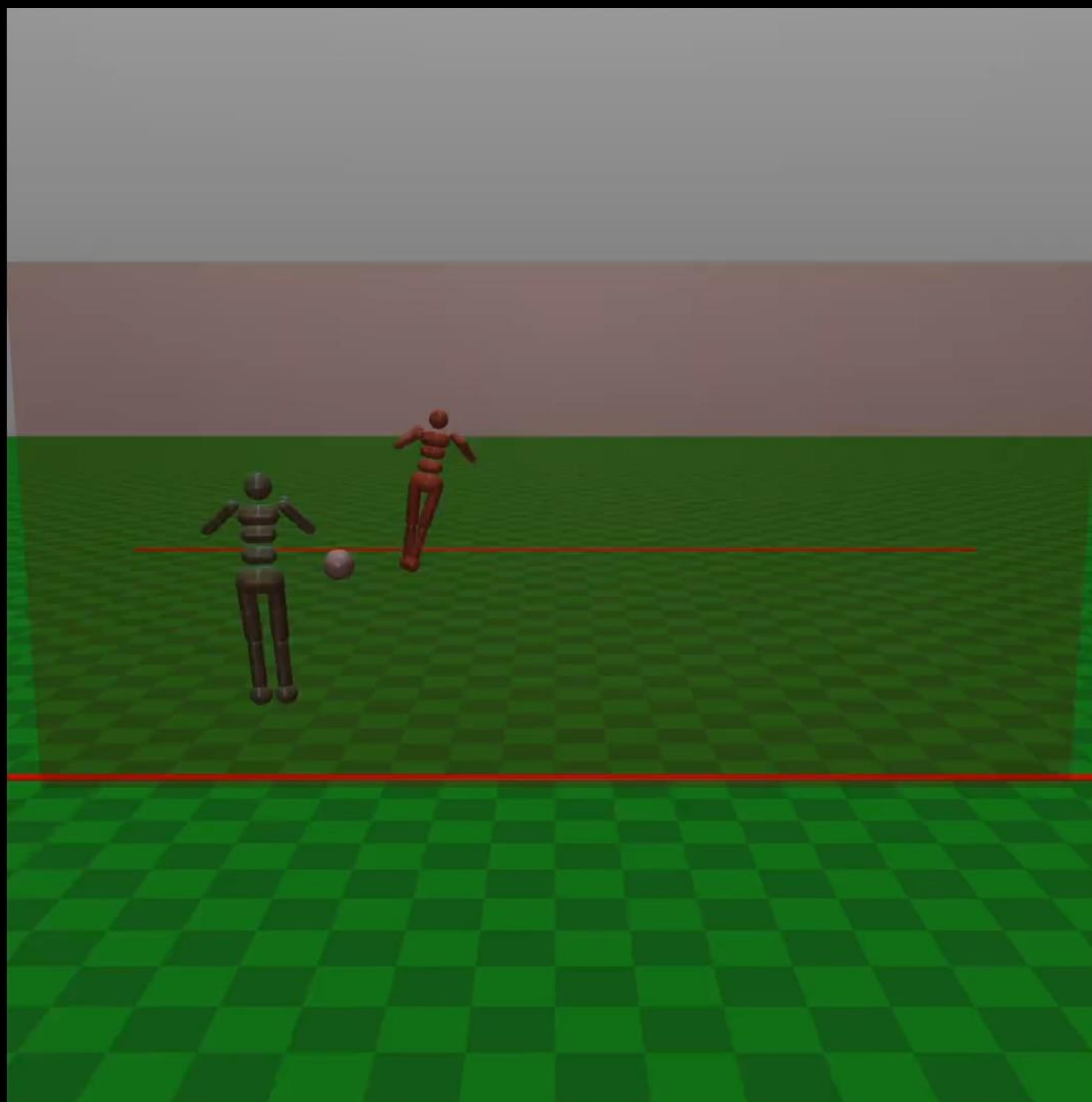
# RL: Helicopter Flight



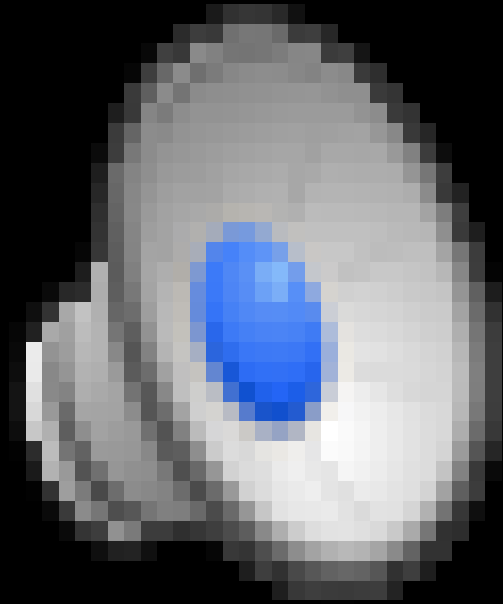
# RL: Learning Locomotion



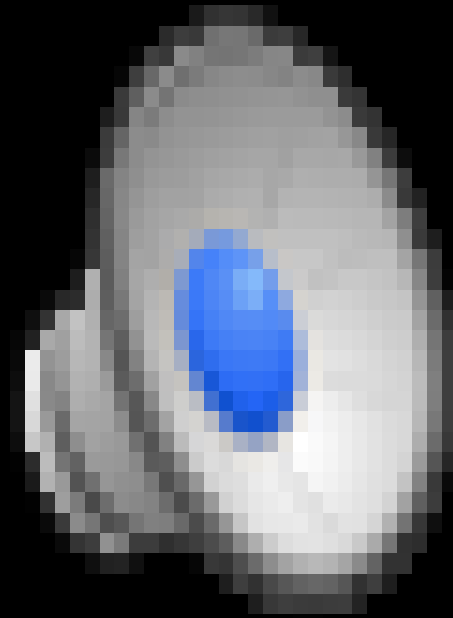
# RL: Learning Soccer



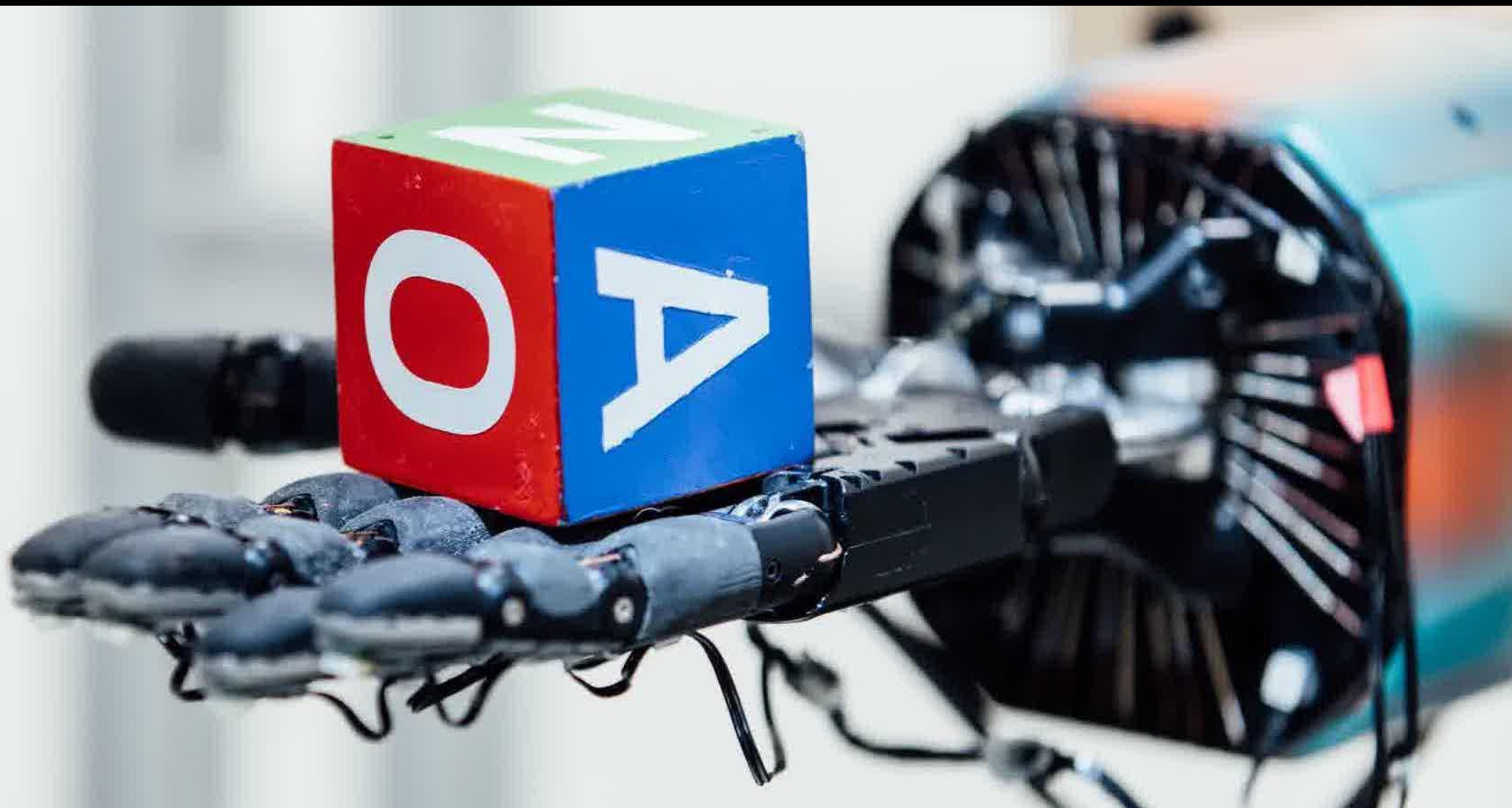
# RL: Learning Manipulation



# RL: NASA SUPERball



# RL: In-Hand Manipulation





# Conclusion

- We're done with Part I: Search and Planning!
- We've seen how AI methods can solve problems in:
  - Search
  - Constraint Satisfaction Problems
  - Games
  - Markov Decision Problems
  - Reinforcement Learning
- Next up: Part II: Uncertainty and Learning!

