Thapar Institute of Engineering & Technology, Patiala

Department of Electronics and Communication Engineering

UEC639 – Digital Communication

B. E. (Third Year): Semester-V (ENC)

Tutorial-6

Q1 Determine the total number of channels that can multiplex to achieve a bit rate of 906 kbps. Assume 5 bit PCM system and sampling frequency of 6 kHz and one additional bit for synchronization. (Answer = 30 channels)

Bit Rate = Rb = (n*N + 1) * fs
n = number of bit per sample = 5 (given)
N = number of channel multiplexed
fs = sampling frequency in Hz
Rb = bits per sec

906 * 1000 = (5*N + 1) * 6 *1000
The solution of this equation gives N = 30

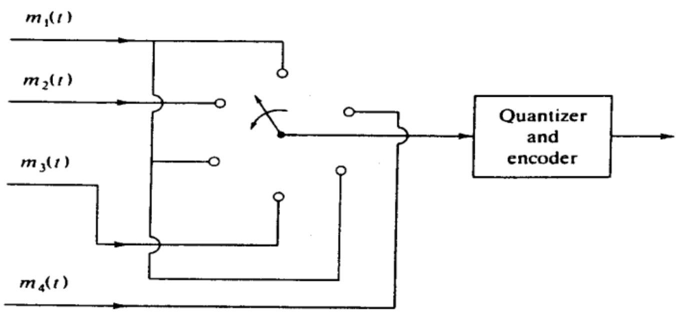
Q2 A signal $m_1(t)$ is band-limited to 3.6 kHz, and three other signals - $m_2(t), m_3(t), and\ m_4(t)$ are band-limited to 1.2 kHz each. These signals are to be transmitted using TDM.

(a) Setup a scheme for accomplishing the TDM with each signal sampled at its Nyquist rate.
(b) What must be the speed of the commutator (in samples per sec)?
(c) If L = 512 then what is the bit rate and transmission bandwidth?

(a)

Message	Bandwidth	Nyquist Rate
$m_1(t)$	3.6 kHz	7.2 kHz
$m_2(t)$	1.2 kHz	2.4 kHz
$m_3(t)$	1.2 kHz	2.4 kHz
$m_4(t)$	1.2 kHz	2.4 kHz

Commutator must have six poles, three poles of $m_1(t)$, one poles of each $m_2(t), m_3(t), and\ m_4(t)$.



(b) The speed of commutator is 2400 rotation per sec. This gives 7200 samples per sec of $m_1(t)$, 2400 samples per sec of $m_2(t)$, 2400 samples per sec of $m_3(t)$, 2400 samples per sec of $m_4(t)$. Total

	14400 samples/sec (c) If L = 512, then n = no of bits per sample = 9, then bit rate = Rb = 9 * 14400 bits/sec = 129.6 kbps; Transmission bandwidth = Rb/2 = 64.8 kHz
Q3	The T1 carrier system used in digital telephony multiplexes 24 voice channels based on 8-b PCM. Each voice signal is usually put through a low-pass filter with the cutoff frequency of about 3.4 kHz. The filtered voice signal is sampled at 8 kHz. In addition, a single bit is added at the end of the frame for the purpose of synchronization. Calculate (*a*) the duration of each bit, (*b*) the resultant transmission rate, and (*c*) the minimum required transmission bandwidth (Nyquist bandwidth).
	(*a*) With a sampling rate of 8 kHz, each frame of the multiplexed signal occupies a period of $$\frac{1}{8000} = 0.000125 \text{ s} = 125 \text{ microseconds} (\mu s)$$ Since each frame consists of twenty-four 8-b words, plus a single synchronizing bit, it contains a total of $$24(8) + 1 = 193 \text{ b}$$ Thus, the duration of each bit is $$T_b = \frac{125}{193} \mu s = 0.647 \mu s$$ (*b*) The resultant transmission rate is $$R_b = \frac{1}{T_b} = 1.544 \text{ Mb/s}$$ (*c*) From Eq. (*5.22*), the minimum required transmission bandwidth is $$f_{T1} = \frac{1}{2T_b} = 772 \text{ kHz}$$
Q4	**Given a set of signals** $$s_i(t) = 2\,Cos\left(2\pi f_c t + \frac{\pi\, i}{4}\right), \quad for\ i = 0, 1, \dots, 3$$ (i) What is the dimensionality, "N", of the space spanned by this set of signals? (ii) Find a set of orthonormal basis functions using Gram Schmidt Procedure to represent this set of signals. (iii) Draw the constellation diagram of this signal set (iv) Determine the norms of signal vector s_i **Solution is given for same signal but** $for\ i = 0, 1, \dots, 7$; **M = 8; The procedure for M = 4 is same.**

Q1.

$$s_i(t) = 2\cos\left(2\pi f_c t + \frac{\pi i}{4}\right) \quad \text{for } i = 0, 1, \ldots 7$$

Stage-1 : To find the value of "N"

Substitute the value of "i" in $s_i(t)$, we get

$s_0(t) = 2\cos(2\pi f_c t) = 2\cos(\theta) \qquad \because \theta = 2\pi f_c t$

$s_1(t) = 2\cos(\theta + \frac{\pi}{4})$

$s_2(t) = 2\cos(\theta + \frac{\pi}{2}) = -2\sin(\theta)$

$s_3(t) = 2\cos(\theta + 3\pi/4)$

$s_4(t) = 2\cos(\theta + \pi) = -2\cos(\theta)$

$s_5(t) = 2\cos(\theta + 5\frac{\pi}{4})$

$s_6(t) = 2\cos(\theta + 3\pi/2) = 2\sin\theta$

$s_7(t) = 2\cos(\theta + 7\pi/4)$

Now, we have to check whether these 8-signals are linear independent or not.

we will start from M^{th} signal $\rightarrow s_7(t)$

$s_7(t) = 2\cos(\theta + 7\pi/4) = 2\cos\theta\cos(7\pi/4) - 2\sin\theta\sin(7\pi/4)$

$s_7(t) = \frac{1}{\sqrt{2}}\left[2\cos\theta + 2\sin\theta\right] = \frac{1}{\sqrt{2}}\left[s_0(t) + s_6(t)\right]$

$s_7(t)$ can be written as a linear combination of $s_0(t)$ & $s_6(t)$

$\left[s_6(t) = 2\sin(\theta) = -s_2(t) \right]$

$$\boxed{\cos(A+B) = \cos A\cos B - \sin A\sin B}$$

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$$s_5(t) = 2\cos(\theta + 5\pi/4)$$
$$= 2\cos\theta\cos(5\pi/4) - 2\sin\theta\sin(5\pi/4)$$
$$= \frac{1}{\sqrt{2}}\left[-2\cos\theta + 2\sin\theta\right]$$

\# $$s_5(t) = \frac{1}{\sqrt{2}}\left[s_4(t) + s_6(t)\right]$$

\# $$s_4(t) = -2\cos\theta$$
$$s_4(t) = -s_0(t)$$

$$s_3(t) = 2\cos(\theta + 3\pi/4)$$
$$s_3(t) = 2\cos\theta\cos(3\pi/4) - 2\sin\theta\sin(3\pi/4)$$
$$= \frac{1}{\sqrt{2}}\left[-2\cos\theta - 2\sin\theta\right]$$

\# $$s_3(t) = \frac{1}{\sqrt{2}}\left[s_4(t) + s_2(t)\right]$$
$$s_3(t) = \frac{1}{\sqrt{2}}\left[s_2(t) - s_0(t)\right]$$

\# $$s_2(t) = -2\sin\theta$$

\# $$s_1(t) = 2\cos(\theta + \pi/4) = 2\left[\cos\theta\cos\frac{\pi}{4} - \sin\theta\sin\frac{\pi}{4}\right]$$
$$s_1(t) = \frac{1}{\sqrt{2}}\left[s_0(t) + s_2(t)\right]$$

\# $$s_0(t) = 2\cos\theta$$

After obervating all 8 signals, we can say that all these signals can be written with the help of two Basis function.

Therefore, the dimension of signal space = N = 2

$N = 2$

STAGE-2

Step 2.1

Energy in first signal $= E_0 = \int_0^T s_0^2(t)\,dt$

$s_0(t) = 2\cos(2\pi f_c t) = A\cos(2\pi f_c t)$

$\because A = 2 \; ; \quad E_0 = \frac{A^2 T}{2} \Rightarrow \sqrt{2T}$

First Basis function $= \phi_0(t) = \frac{s_0(t)}{\sqrt{E_0}} = \frac{\cancel{A}\cos(2\pi f_c t)}{\cancel{A}\sqrt{T/2}}$

$\boxed{\phi_0(t) = \sqrt{\frac{2}{T}}\cos(2\pi f_c t)}$

$s_{00} = \sqrt{E_0} = \sqrt{\frac{A^2 T}{2}} = \sqrt{\frac{4T}{2}} = \sqrt{2T}$

$\boxed{s_{00} = \sqrt{2T}}$

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Step 2.2

$$s_{ij} = \int_0^T s_i(t)\, \phi_j(t)\, dt$$

$$s_{10} = \int_0^T s_1(t)\, \phi_0(t)\, dt$$

$s_1(t) = 2\cos(2\pi f_c t + \pi/4)$ $\quad \phi_0(t) = \sqrt{\frac{2}{T}} \cos(2\pi f_c t)$

$$\boxed{s_{10} = \sqrt{T}}$$

Energy in $s_1(t) = E_1 = \int_0^T s_1^2(t)\, dt$

$$\boxed{E_1 = 2T}$$

$$\phi_1(t) = \frac{2\cos\left(\theta + \frac{\pi}{4}\right) - \sqrt{T}\sqrt{\frac{2}{T}}\cos(\theta + \pi}{\sqrt{2T - T}}$$

$$\phi_1(t) = \frac{s_1(t) - s_{10}\, \phi_0(t)}{\sqrt{E_1 - s_{10}^2}}$$

Applying $\cos(A+B) = \cos A \cos B - \sin A \sin B$,

$\boxed{\phi_1(t) = -\sqrt{\frac{2}{T}} \sin\theta}$

$$\phi_i(t) = \frac{g_i(t)}{\sqrt{\int_0^T g_i^2(t)\, dt}}$$

$$\boxed{g_i(t) = s_i(t) - \sum_{j=1}^{i-1} s_{ij}\, \phi_j(t)}$$

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Now using $s_{ij} = \int_0^T s_i(t)\,\phi_j(t)\,dt$

determine

$$s_{11} = \int_0^T s_1(t)\,\phi_1(t)\,dt$$

$$s_{11} = -\int_0^T 2\cos\left(\theta + \frac{\pi}{4}\right)\sqrt{\frac{2}{T}}\,\sin(\theta)\,dt \qquad \because \theta = 2\pi f_c t$$

on solving, we get

$$\boxed{s_{11} = \sqrt{T}} \quad \#$$

Similarly we can determine other coefficients as:

$(s_{00},\ s_{01})$ $\quad (\sqrt{2T},\ 0)$

$(s_{10},\ s_{11})$ $\quad (\sqrt{T},\ \sqrt{T})$ $\qquad \phi_0(t) = \sqrt{\frac{2}{T}}\cos(2\pi f_c t)$

$(s_{20},\ s_{21})$ $\quad (0,\ \sqrt{2T})$ $\qquad \phi_1(t) = -\sqrt{\frac{2}{T}}\sin(2\pi f_c t)$

$(s_{30}\ \ s_{31})$ $\quad (-\sqrt{T},\ \sqrt{T})$

$(s_{40}\ \ s_{41})$ $\quad (-\sqrt{2T},\ 0)$

$(s_{50}\ \ s_{51})$ $\quad (-\sqrt{T},\ -\sqrt{T})$

$(s_{60}\ \ s_{61})$ $\quad (0,\ -\sqrt{2T})$

$(s_{70}\ \ s_{71})$ $\quad (\sqrt{T},\ -\sqrt{T})$

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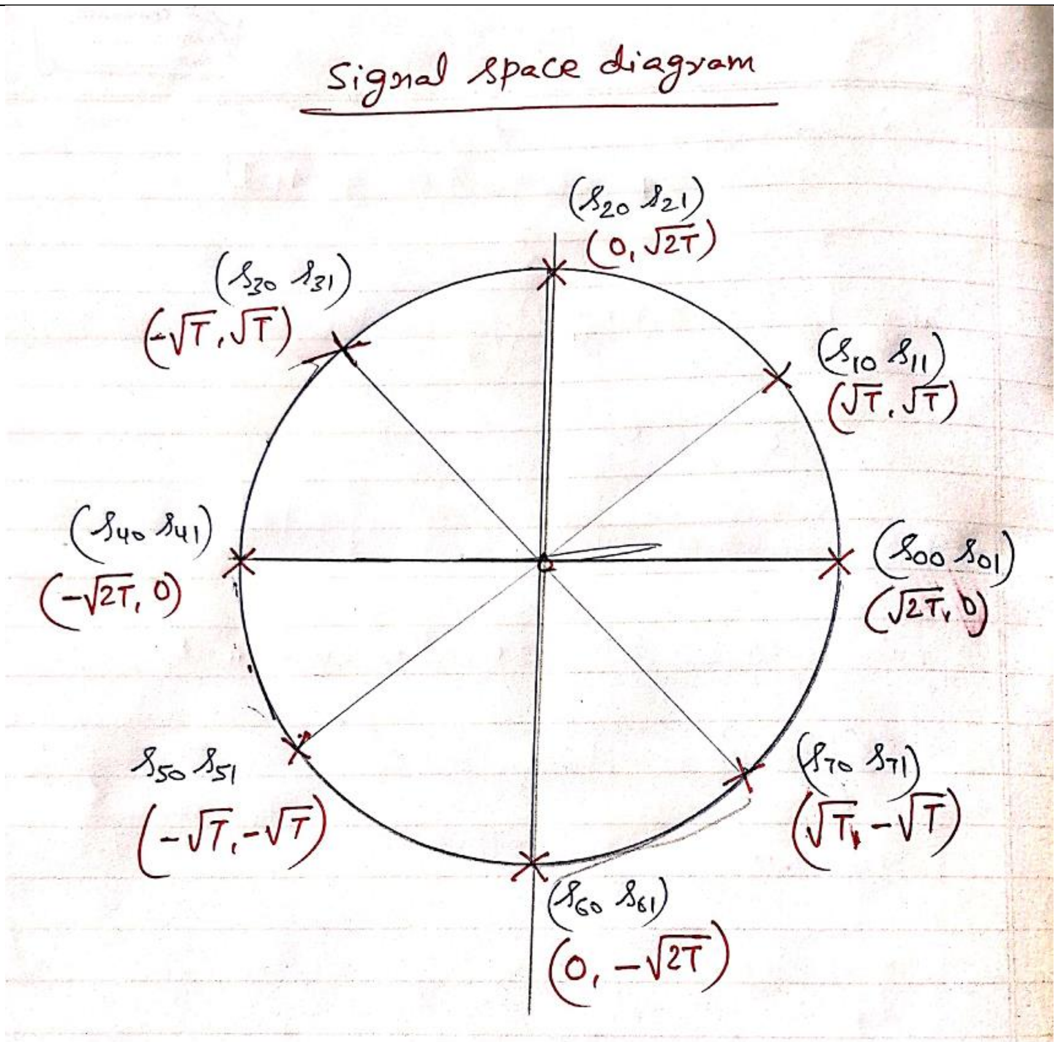
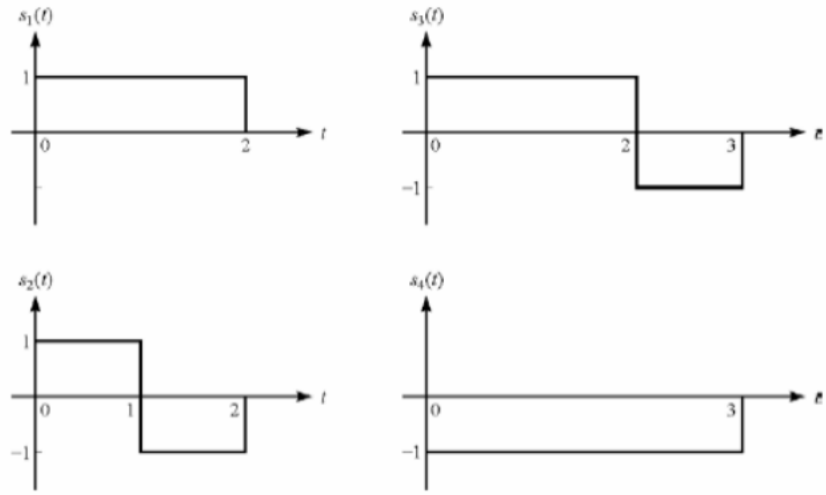


Q5 Apply Gram-Schmidt orthogonalization procedure to obtain the orthonormal basis functions require to represent the following function. Then, determine the vector representations of the signals and determine the signal energies.



	Refer the Lecture ppt for the solution