Thapar Institute of Engineering & Technology, Patiala

Department of Electronics and Communication Engineering

UEC639 – Digital Communication

B. E. (Third Year): Semester-VI (ENC)

Solution of Tutorial-1

Q1 Determine the Fourier transform of the following signal and its highest frequency component

$$x(t) = \frac{Sin(\,at)}{(\pi\, t)}$$

Solution

To determine the F.T of given function, we have to use duality property.

Let define a rectangular function as -

$$x(t) = p_a(t) = \begin{cases} 1 & |t| < a \\ 0 & |t| > a \end{cases}$$

The Fourier transform of this function is

$$X(\omega) = \int_{-\infty}^{\infty} p_a(t) e^{-j\omega t}\, dt = \int_{-a}^{a} e^{-j\omega t}\, dt = \frac{2 \sin a\omega}{\omega} = 2a \frac{\sin a\omega}{a\omega}$$

$$\mathscr{F}[p_a(t)] = \frac{2}{\omega} \sin a\omega$$

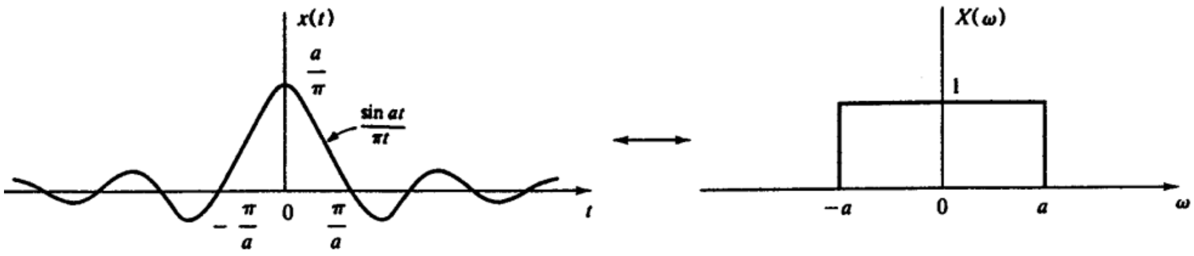
From duality property

$$\mathscr{F}\left[\frac{2}{t} \sin at\right] = 2\pi p_a(-\omega)$$

Thus, $$X(\omega) = \mathscr{F}\left[\frac{\sin at}{\pi t}\right] = \frac{1}{2\pi} \mathscr{F}\left[\frac{2}{t} \sin at\right] = p_a(-\omega) = p_a(\omega)$$

where, $p_a(\omega)$ is defined as

$$p_a(\omega) = \begin{cases} 1 & |\omega| < a \\ 0 & |\omega| > a \end{cases}$$



Q2	Find the Nyquist rate and Nyquist interval for the following signals

(i) $\boldsymbol{x_1(t) = 10\ Sin(2000\ \pi\ t)\ Cos(6000\ \pi\ t)}$

Solution

Expand the function x1(t) using trigonometric identity of 2 Sin(A) Cos(B)

$$x_1(t) = 10\ Sin(2000\ \pi\ t)\ Cos(6000\ \pi\ t)$$

$$x_1(t) = 5\ [\ Sin(8000\ \pi\ t) + \ Sin(4000\ \pi\ t)$$

Highest frequency component is $\omega_m = 2\ \pi\ f_m = 8000\ \pi$

$$f_m = 4000\ \ Hz$$

Nyquist Rate = 2 f_m = 8000 Hz = 8 kHz.

(ii) $\boldsymbol{x_2(t) = Sin(200\ \pi\ t)\ /\ \pi\ t}$

Solution

$$\frac{\sin\ at}{\pi t} \leftrightarrow p_a(\omega) = \begin{cases} 1 & |\omega| < a \\ 0 & |\omega| > a \end{cases}$$

Highest frequency component is = 100 Hz

Hence Nyquist rate = 2 * 100 = 200 Hz;

Nyquist Interval = 1/200 sec;

(iii) $\boldsymbol{x_3(t) = \left(\frac{Sin(200\ \pi\ t)}{\pi\ t}\right)^2}$

Solution

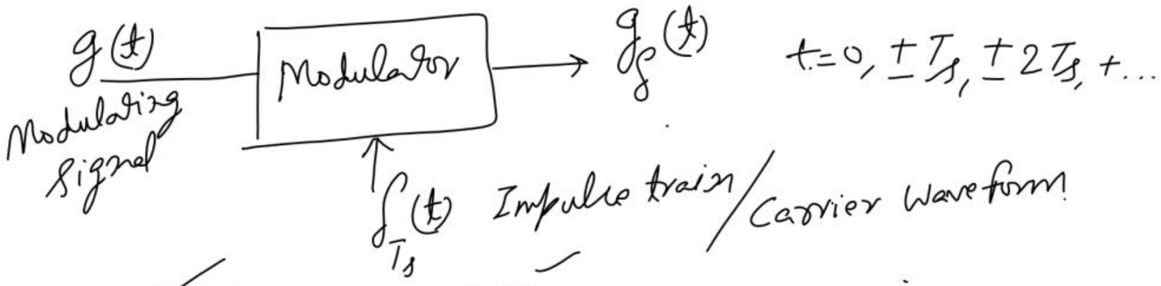
$$F[Sinc^2(at)] = \frac{1}{|a|}\ tri(\frac{f}{a})$$

The F.T of given signal is triangular function with highest frequency component 200 Hz.

Hence the Nyquist rate = 2 * 200 Hz = 400 Hz3

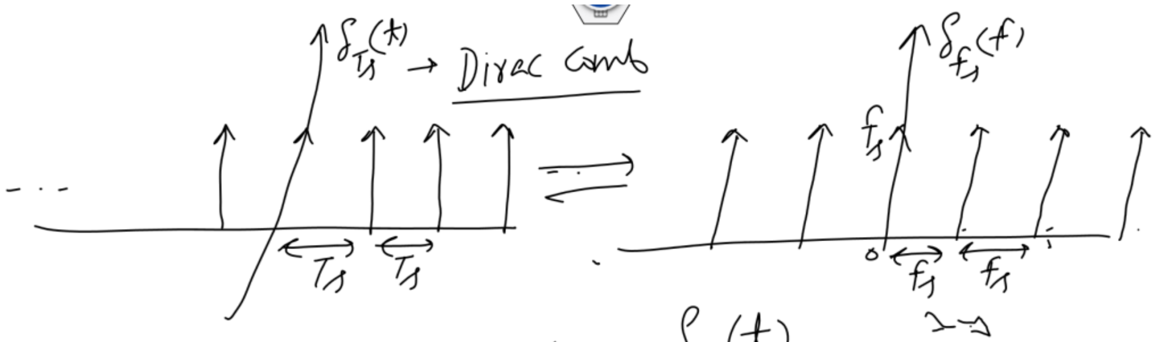
Q3	Express the sampling expression for low-pass signal in time-domain. Using this expression, derive the sampling expression for low-pass signal in frequency-domain.

$g(t) \rightarrow$ C.T Signal with finite energy



$$g(t) \rightleftharpoons G(f)$$

$$\delta_{T_s}(t) \rightleftharpoons \delta_{f_s}(f) = f_s \sum_{-\infty}^{\infty} \delta(f - m f_s)$$



$$g_\delta(t) = g(t) \times \delta_{T_s}(t)$$

$$g_\delta(t) = g(t) \times \sum_{-\infty}^{\infty} \delta(t - nT_s) \quad (t = nT_s)$$

$$\# \; \boxed{g_\delta(t) = \sum_{-\infty}^{\infty} g(nT_s)\, \delta(t - nT_s)}$$

Multiplication in time domain is equivalent to the Conv. of their respective F.T

$$g(t) \rightleftharpoons G(f) \qquad g_\delta(t) \rightleftharpoons G_\delta(f)$$

$$\delta_{T_1}(t) \rightleftharpoons \delta_{f_1}(f) = f_1 \sum \delta(f - mf_1)$$

$$G_\delta(f) = F[g_\delta(t)] = F[g(t) \times \delta_{T_1}(t)]$$

time domain multiplication

$$= G(f) \circledast f_1 \sum_{-\infty}^{\infty} \delta(f - mf_1)$$

freq. domain convolution

Conv.

After Interchanging the order of summation and conv.

$$G_\delta(f) = f_1 \sum_{-\infty}^{\infty} G(f) \circledast \delta(f - mf_1)$$

Now applying the property of delta funct.

$$\boxed{G_\delta(f) = f_1 \sum_{-\infty}^{\infty} G(f - mf_1)}$$

spectrum of sampled signal

Sampling in time domain $\longrightarrow$ Periodic spectrum in freq. domain

Q4 Derive the reconstruction equation (sampling theorem) to obtain the continuous-time signal from its sampled version. Discuss about the interpolation function.

Interpolation formula for reconstructing the original signal $g(t)$ from $g_\delta(t)$ $g(nT_s)$

$$G_\delta(f) = \sum_{-\infty}^{\infty} g(nT_s)\exp[-j2\pi f n T_s]$$

Sampled signal → $g_\delta(t) = \sum_{-\infty}^{\infty} g(nT_s)\,\delta(t-nT_s)$

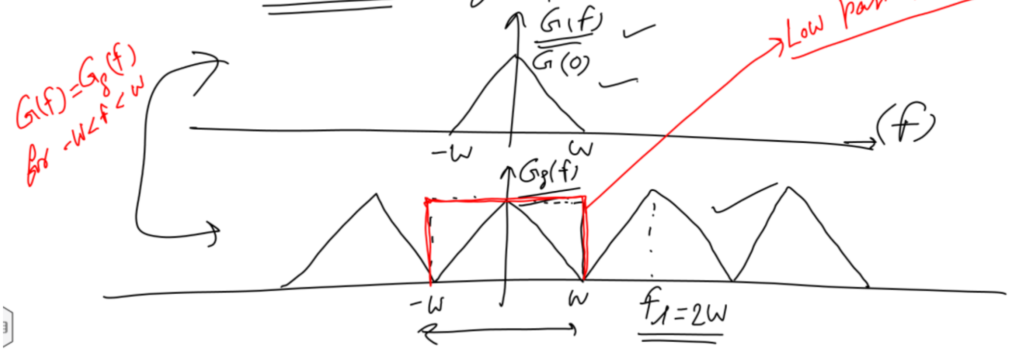
↓F.T ↓F.T

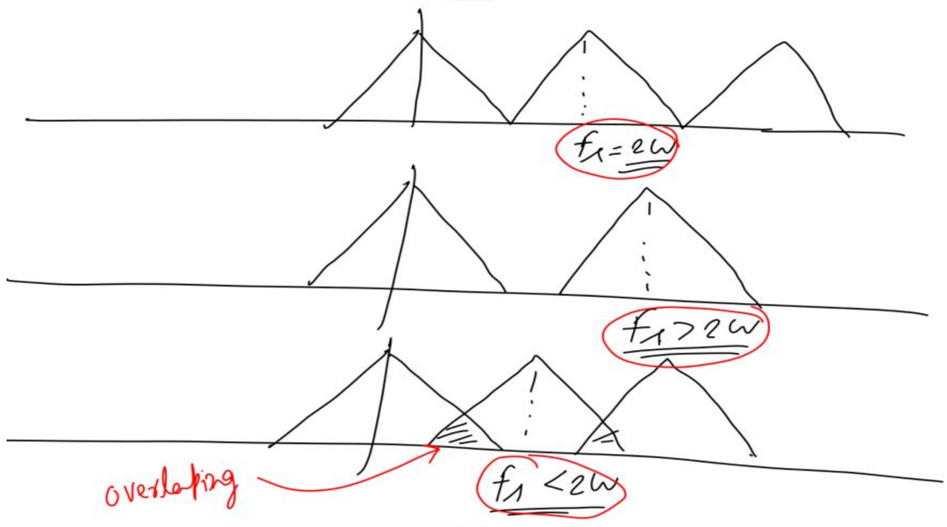
$$G_\delta(f) = \sum_{-\infty}^{\infty} g(nT_s)\exp(-j2\pi n f T_s)$$

Complex Fourier series representation of $G_\delta(f)$

Let $g(t) \Rightarrow$ finite energy signal with $G(f)$ defined for $-W<f<W$

$G(f)=0$ for $|f|\geq W$





$$G_\delta(f) = \sum_{-\infty}^{\infty} g(nT_1)\exp\left(\frac{-j2\pi nf}{f_1}\right)$$

Let $f_1 = 2W$

$$\boxed{G_\delta(f) = \sum_{-\infty}^{\infty} g\left(\frac{n}{2W}\right)\exp\left[\frac{-j\pi nf}{2W}\right]}$$

$$G_\delta(f) = f_1 \sum_{-\infty}^{\infty} G(f - mf_1) = 2W \sum_{-\infty}^{\infty} G(f - mf_1)$$

\# $$\boxed{G(f) = \frac{1}{2W} G_\delta(f) \quad -W < f < W}$$

$$G(f) = \frac{1}{2W}\sum_{-\infty}^{\infty} g\left(\frac{n}{2W}\right)\exp\left(\frac{-j\pi nf}{2W}\right)$$

$$G(f) = F[g(t)]$$

$$g(t) = \int_{-\infty}^{\infty} G(f)\, e^{j2\pi ft}\, df \quad \rightarrow \text{Inverse F.T of } G(f)$$

$$g(t) = \int_{-W}^{W} \frac{1}{2W}\sum_{-\infty}^{\infty} g\left(\frac{n}{2W}\right)\exp\left(\frac{-j\pi nf}{W}\right)\exp(j2\pi ft)\, df$$

Interchanging the order of Integration and summ.

$$g(t) = \sum_{-\infty}^{\infty} g\left(\frac{n}{2W}\right)\frac{1}{2W}\int_{-W}^{W}\exp\left[j2\pi f\left(t - \frac{n}{2W}\right)\right] df$$

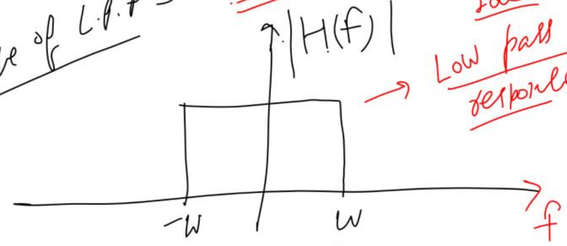
$$g(t) = \sum_{-\infty}^{\infty} g\left(\frac{n}{2W}\right)\frac{\sin(2\pi Wt - n\pi)}{(2\pi Wt - n\pi)}$$

$$\operatorname{sinc}(x) = \frac{\sin(\pi x)}{\pi x} = \begin{cases} 1 & x = 0 \\ 0 & x = \pm 1, \pm 2, \ldots \end{cases}$$

\# $$\boxed{g(t) = \sum_{-\infty}^{\infty} g\left(\frac{n}{2W}\right)\operatorname{sinc}(2Wt - n)} \rightarrow \text{Interpolation formula for reconstruction}$$

I/P $\rightarrow g(n/2W)$

Impulse response of L.P.F = sinc function in time domain



$\rightarrow$ Ideal Low pass filter response

Sol 4

The bandpass sampling theorem states that if a bandpass signal $m(t)$ has a spectrum of bandwidth $\omega_B(= 2\pi f_B)$ and an upper frequency limit $\omega_u(= 2\pi f_u)$, then $m(t)$ can be recovered from $m_s(t)$ by bandpass filtering if $f_s = 2f_u/k$, where k is the largest integer not exceeding f_u/f_B. All higher sampling rates are not necessarily usable unless they exceed $2f_u$.

Sol 5	(a) $$m(t) = 10\cos 2000\pi t \cos 8000\pi t$$ $$= 5\cos 6000\pi t + 5\cos 10000\pi t$$ $$f_M = 5000 \text{ Hz} = 5 \text{ kHz}$$ Thus, $f_s = 2f_M = 10$ kHz. (b) $f_u = f_M = 5$ kHz and $f_B = (5-3) = 2$ kHz. $$\frac{f_u}{f_B} = \frac{5}{2} = 2.5 \rightarrow k = 2$$ Based on the bandpass sampling theorem, $$f_s = \frac{2f_u}{k} = 5 \text{ kHz}$$