Thapar Institute of Engineering & Technology, Patiala

Department of Electronics and Communication Engineering

UEC639 – Digital Communication

B. E. (Third Year): Semester-VI (ECE)

Tutorial-10 - Solution

Q1	Evaluate the probability of bit error performance of following system in the presence of additive white Gaussian channel in terms of energy per bit (E_b) to noise power spectral density (N_0): (a) Binary Phase Shift Keying (b) Binary Frequency Shift Keying (c) Quadrature phase shift keying
	Refer PPT uploaded on LMS
Q2	With the help of a suitable block diagram and the concept of orthonormal basis functions, explain the generation of the 16-PSK signal including its constellation diagram. Also, explain the receiver block diagram of the 16-PSK to estimate the transmitted message. Compare its bandwidth efficiency and bit error performance with BPSK, QPSK, and 32-PSK.
	Refer PPT uploaded on LMS
Q3	What is the relationship between bandwidth of BPSK and bandwidth of M-ary PSK system. Write the both bandwidths in terms of data rate R_b also.
	Band width of BPSK = $BW_{BPSK} = 2R_b$ Band width of M-ary PSK = $BW_{MaryPSK} = \frac{2R_b}{\log_2 M} = \frac{BW_{BPSK}}{\log_2 M}$
Q4	An 8-PSK system is used to transmit a certain R_b data rate through a channel having bandwidth B. If it is desired to transmit $2R_b$ data rate through same channel by using M-ary PSK, what will be the value of M?
	Band width of M-ary PSK = $BW_{8-aryPSK} = \frac{2R_b}{\log_2 8} = \frac{2R_b}{3} = B$ $BW_{M-aryPSK} = \frac{2\,2R_b}{\log_2 M} = \frac{2R_b}{3}$ $\log_2 M = 6$; $2^{\log_2 M} = 2^6 = 64$ M = 64
Q5	Consider a (6, 3) block code whose generator matrix is $G = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$ (a) Find code words of the message - 110 , 101 and 111 (b) Find H, the parity check matrix of the code (c) Compute the syndrome for the received vector $\boldsymbol{r} = \mathbf{0\,0\,1\,1\,1\,0}$, Is this a valid code

vector?

(a) Find code words of the message - 110 , 101 and 111

The code word (U) can be generated from the generation matrix (G) by using the following expression:

$$U = m\,G$$

Where, U is the code word, m is the message word, and G is the generation matrix.
G can be written as

$$\mathbf{G} = \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \\ \mathbf{V}_3 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{U}_4 = [1 \quad 1 \quad 0] \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \\ \mathbf{V}_3 \end{bmatrix} = 1 \cdot \mathbf{V}_1 + 1 \cdot \mathbf{V}_2 + 0 \cdot \mathbf{V}_3$$

$$= 1\ 1\ 0\ 1\ 0\ 0 + 0\ 1\ 1\ 0\ 1\ 0 + 0\ 0\ 0\ 0\ 0\ 0$$

$$= 1\ 0\ 1\ 1\ 1\ 0 \quad \text{(codeword for the message vector } 1\ 1\ 0)$$

Similarly, the code words for message 101 and 111 can be determine as

Message word	Code word
110	101110
101	011101
111	000111

(b) Find H, the parity check matrix of the code

To fulfill the orthogonally requirements for a systematic code, the H- matrix can be written as

$$H = [I_{n-k} \vdots P^T]$$

$$\begin{matrix} \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} \\ \mathbf{1} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{1} \\ \mathbf{1} & \mathbf{0} & \mathbf{1} \end{matrix}$$

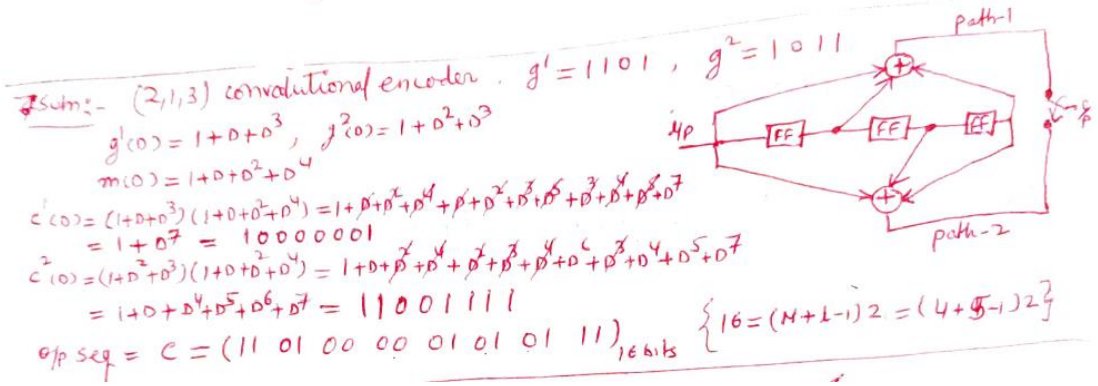
(c) Compute the syndrome for the received vector $r =$ 0 0 1 1 1 0, Is this a valid code vector?

$$\boldsymbol{S = r\,H^T}$$

$$\boldsymbol{S} = [\mathbf{0\ 0\ 1\ 1\ 1\ 0}] \begin{matrix} \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} \\ \mathbf{1} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{1} \\ \mathbf{1} & \mathbf{0} & \mathbf{1} \end{matrix}$$

S = [1 0 0]

Since, Syndrome –S is not equal to zero, hence this is not valid code vector.

Q6	Consider a (2,1,3) convolution encoder with $g^{(1)}$ = (1 1 0 1), $g^{(2)}$ = (1 0 1 1). a) Draw the encoder block diagram. Find the code word corresponding to the information sequence (11101) using transform-domain approach.
	 Soln:- (2,1,3) convolutional encoder. $g^1 = 1101$, $g^2 = 1011$ $g^1(D) = 1+D+D^3$, $g^2(D) = 1+D^2+D^3$ $m(D) = 1+D+D^2+D^4$ $C^1(D) = (1+D+D^3)(1+D+D^2+D^4) = 1+D+D^2+D^4+D+D^2+D^3+D^5+D^3+D^4+D^5+D^7$ $= 1+D^7 = 10000001$ $C^2(D) = (1+D^2+D^3)(1+D+D^2+D^4) = 1+D+D^2+D^4+D^3+D^4+D^5+D^6+D^3+D^4+D^5+D^7$ $= 1+D+D^4+D^5+D^6+D^7 = 11001111$ o/p seq = C = (11 01 00 00 01 01 01 11)$_{16\ bits}$ $\{16 = (M+L-1)2 = (4+5-1)2\}$

Complementary Error Function Table

x	erfc(x)	x	erfc(x)	x	erfc(x)	x	erfc(x)	x	erfc(x)	x	erfc(x)	x	erfc(x)
0	1.000000	0.5	0.479500	1	0.157299	1.5	0.033895	2	0.004678	2.5	0.000407	3	0.00002209
0.01	0.988717	0.51	0.470756	1.01	0.153190	1.51	0.032723	2.01	0.004475	2.51	0.000386	3.01	0.00002074
0.02	0.977435	0.52	0.462101	1.02	0.149162	1.52	0.031587	2.02	0.004281	2.52	0.000365	3.02	0.00001947
0.03	0.966159	0.53	0.453536	1.03	0.145216	1.53	0.030484	2.03	0.004094	2.53	0.000346	3.03	0.00001827
0.04	0.954889	0.54	0.445061	1.04	0.141350	1.54	0.029414	2.04	0.003914	2.54	0.000328	3.04	0.00001714
0.05	0.943628	0.55	0.436677	1.05	0.137564	1.55	0.028377	2.05	0.003742	2.55	0.000311	3.05	0.00001608
0.06	0.932378	0.56	0.428384	1.06	0.133856	1.56	0.027372	2.06	0.003577	2.56	0.000294	3.06	0.00001508
0.07	0.921142	0.57	0.420184	1.07	0.130227	1.57	0.026397	2.07	0.003418	2.57	0.000278	3.07	0.00001414
0.08	0.909922	0.58	0.412077	1.08	0.126674	1.58	0.025453	2.08	0.003266	2.58	0.000264	3.08	0.00001326
0.09	0.898719	0.59	0.404064	1.09	0.123197	1.59	0.024538	2.09	0.003120	2.59	0.000249	3.09	0.00001243
0.1	0.887537	0.6	0.396144	1.1	0.119795	1.6	0.023652	2.1	0.002979	2.6	0.000236	3.1	0.00001165
0.11	0.876377	0.61	0.388319	1.11	0.116467	1.61	0.022793	2.11	0.002845	2.61	0.000223	3.11	0.00001092
0.12	0.865242	0.62	0.380589	1.12	0.113212	1.62	0.021962	2.12	0.002716	2.62	0.000211	3.12	0.00001023
0.13	0.854133	0.63	0.372954	1.13	0.110029	1.63	0.021157	2.13	0.002593	2.63	0.000200	3.13	0.00000958
0.14	0.843053	0.64	0.365414	1.14	0.106918	1.64	0.020378	2.14	0.002475	2.64	0.000189	3.14	0.00000897
0.15	0.832004	0.65	0.357971	1.15	0.103876	1.65	0.019624	2.15	0.002361	2.65	0.000178	3.15	0.00000840
0.16	0.820988	0.66	0.350623	1.16	0.100904	1.66	0.018895	2.16	0.002253	2.66	0.000169	3.16	0.00000786
0.17	0.810008	0.67	0.343372	1.17	0.098000	1.67	0.018190	2.17	0.002149	2.67	0.000159	3.17	0.00000736
0.18	0.799064	0.68	0.336218	1.18	0.095163	1.68	0.017507	2.18	0.002049	2.68	0.000151	3.18	0.00000689
0.19	0.788160	0.69	0.329160	1.19	0.092392	1.69	0.016847	2.19	0.001954	2.69	0.000142	3.19	0.00000644
0.2	0.777297	0.7	0.322199	1.2	0.089686	1.7	0.016210	2.2	0.001863	2.7	0.000134	3.2	0.00000603
0.21	0.766478	0.71	0.315335	1.21	0.087045	1.71	0.015593	2.21	0.001776	2.71	0.000127	3.21	0.00000564
0.22	0.755704	0.72	0.308567	1.22	0.084466	1.72	0.014997	2.22	0.001692	2.72	0.000120	3.22	0.00000527
0.23	0.744977	0.73	0.301896	1.23	0.081950	1.73	0.014422	2.23	0.001612	2.73	0.000113	3.23	0.00000493
0.24	0.734300	0.74	0.295322	1.24	0.079495	1.74	0.013865	2.24	0.001536	2.74	0.000107	3.24	0.00000460
0.25	0.723674	0.75	0.288845	1.25	0.077100	1.75	0.013328	2.25	0.001463	2.75	0.000101	3.25	0.00000430
0.26	0.713100	0.76	0.282463	1.26	0.074764	1.76	0.012810	2.26	0.001393	2.76	0.000095	3.26	0.00000402
0.27	0.702582	0.77	0.276179	1.27	0.072486	1.77	0.012309	2.27	0.001326	2.77	0.000090	3.27	0.00000376
0.28	0.692120	0.78	0.269990	1.28	0.070266	1.78	0.011826	2.28	0.001262	2.78	0.000084	3.28	0.00000351
0.29	0.681717	0.79	0.263897	1.29	0.068101	1.79	0.011359	2.29	0.001201	2.79	0.000080	3.29	0.00000328
0.3	0.671373	0.8	0.257899	1.3	0.065992	1.8	0.010909	2.3	0.001143	2.8	0.000075	3.3	0.00000306
0.31	0.661092	0.81	0.251997	1.31	0.063937	1.81	0.010475	2.31	0.001088	2.81	0.000071	3.31	0.00000285
0.32	0.650874	0.82	0.246189	1.32	0.061935	1.82	0.010057	2.32	0.001034	2.82	0.000067	3.32	0.00000266
0.33	0.640721	0.83	0.240476	1.33	0.059985	1.83	0.009653	2.33	0.000984	2.83	0.000063	3.33	0.00000249
0.34	0.630635	0.84	0.234857	1.34	0.058086	1.84	0.009264	2.34	0.000935	2.84	0.000059	3.34	0.00000232
0.35	0.620618	0.85	0.229332	1.35	0.056238	1.85	0.008889	2.35	0.000889	2.85	0.000056	3.35	0.00000216
0.36	0.610670	0.86	0.223900	1.36	0.054439	1.86	0.008528	2.36	0.000845	2.86	0.000052	3.36	0.00000202
0.37	0.600794	0.87	0.218560	1.37	0.052688	1.87	0.008179	2.37	0.000803	2.87	0.000049	3.37	0.00000188
0.38	0.590991	0.88	0.213313	1.38	0.050984	1.88	0.007844	2.38	0.000763	2.88	0.000046	3.38	0.00000175
0.39	0.581261	0.89	0.208157	1.39	0.049327	1.89	0.007521	2.39	0.000725	2.89	0.000044	3.39	0.00000163
0.4	0.571608	0.9	0.203092	1.4	0.047715	1.9	0.007210	2.4	0.000689	2.9	0.000041	3.4	0.00000152
0.41	0.562031	0.91	0.198117	1.41	0.046148	1.91	0.006910	2.41	0.000654	2.91	0.000039	3.41	0.00000142
0.42	0.552532	0.92	0.193232	1.42	0.044624	1.92	0.006622	2.42	0.000621	2.92	0.000036	3.42	0.00000132
0.43	0.543113	0.93	0.188437	1.43	0.043143	1.93	0.006344	2.43	0.000589	2.93	0.000034	3.43	0.00000123
0.44	0.533775	0.94	0.183729	1.44	0.041703	1.94	0.006077	2.44	0.000559	2.94	0.000032	3.44	0.00000115
0.45	0.524518	0.95	0.179109	1.45	0.040305	1.95	0.005821	2.45	0.000531	2.95	0.000030	3.45	0.00000107
0.46	0.515345	0.96	0.174576	1.46	0.038946	1.96	0.005574	2.46	0.000503	2.96	0.000028	3.46	0.00000099
0.47	0.506255	0.97	0.170130	1.47	0.037627	1.97	0.005336	2.47	0.000477	2.97	0.000027	3.47	0.00000092
0.48	0.497250	0.98	0.165769	1.48	0.036346	1.98	0.005108	2.48	0.000453	2.98	0.000025	3.48	0.00000086
0.49	0.488332	0.99	0.161492	1.49	0.035102	1.99	0.004889	2.49	0.000429	2.99	0.000024	3.49	0.00000080