In general, the unit sample response $h_d(n)$ obtained from (8.2.17) is infinite in duration and must be truncated at some point, say at $n = M - 1$, to yield an FIR filter of length M. Truncation of $h_d(n)$ to a length $M - 1$ is equivalent to multiplying $h_d(n)$ by a "rectangular window," defined as

$$w(n) = \begin{cases} 1, & n = 0, 1, \ldots, M-1 \\ 0, & \text{otherwise} \end{cases} \tag{8.2.19}$$

Thus the unit sample response of the FIR filter becomes

$$\begin{aligned} h(n) &= h_d(n)w(n) \\ &= \begin{cases} h_d(n), & n = 0, 1, \ldots, M-1 \\ 0, & \text{otherwise} \end{cases} \end{aligned} \tag{8.2.20}$$

desired frequency response

$$H_d(\omega) = \begin{cases} 1e^{-j\omega(M-1)/2}, & 0 \le |\omega| \le \omega_c \\ 0, & \text{otherwise} \end{cases} \tag{8.2.26}$$

A delay of $(M-1)/2$ units is incorporated into $H_d(\omega)$ in anticipation of forcing the filter to be of length M. The corresponding unit sample response, obtained by evaluating the integral in (8.2.18), is

$$\begin{aligned} h_d(n) &= \frac{1}{2\pi} \int_{-\omega_c}^{\omega_c} e^{j\omega\left(n - \frac{M-1}{2}\right)} d\omega \\ &= \frac{\sin \omega_c \left(n - \frac{M-1}{2}\right)}{\pi \left(n - \frac{M-1}{2}\right)} \qquad n \neq \frac{M-1}{2} \end{aligned} \tag{8.2.27}$$

Clearly, $h_d(n)$ is noncausal and infinite in duration.

If we multiply $h_d(n)$ by the rectangular window sequence in (8.2.19), we obtain an FIR filter of length M having the unit sample response

$$h(n) = \frac{\sin \omega_c \left(n - \frac{M-1}{2}\right)}{\pi \left(n - \frac{M-1}{2}\right)} \qquad 0 \le n \le M-1 \quad n \neq \frac{M-1}{2} \tag{8.2.28}$$

Obtain filter coefficients

If M is selected to be odd, the value of $h(n)$ at $n = (M-1)/2$ is

$$h\left(\frac{M-1}{2}\right) = \frac{\omega_c}{\pi} \tag{8.2.29}$$

Solution of q3 →

$\delta = \min\{0.01, 0.001\} = 0.001$

$\Delta\omega = 0.2\pi$ ✓

$A = -20\log\delta = -20\log 0.001 = +60\,dB$ ✓

$\beta = 0.1102\,(A - 8.7) = 5.6533$ ✓

$$M \geq \frac{A - 8}{2.285\,\Delta\omega} \simeq 37$$

$\alpha = \frac{M-1}{2} = 18$ ✓ for $M = 37$

$h[n] = h_d[n]\,w[n] \qquad 0 \leq n \leq M-1$

Obtain M filter coefficients for the designed linear phase Bandpass filter.

Sol. 02 --- Oppenheim

7.3.1 Highpass Filter

The ideal highpass filter with generalized linear phase has the frequency response

$$H_{hp}(e^{j\omega}) = \begin{cases} 0, & |\omega| < \omega_c, \\ e^{-j\omega M/2}, & \omega_c < |\omega| \le \pi. \end{cases} \quad (7.65)$$

The corresponding impulse response can be found by evaluating the inverse transform of $H_{hp}(e^{j\omega})$, or we can observe that

$$H_{hp}(e^{j\omega}) = e^{-j\omega M/2} - H_{lp}(e^{j\omega}), \quad (7.66)$$

where $H_{lp}(e^{j\omega})$ is given by Eq. (7.56). Thus, $h_{hp}[n]$ is

$$h_{hp}[n] = \frac{\sin \pi(n - M/2)}{\pi(n - M/2)} - \frac{\sin \omega_c(n - M/2)}{\pi(n - M/2)}, \qquad -\infty < n < \infty. \quad (7.67)$$

To design an FIR approximation to the highpass filter, we can proceed in a manner

$$h[n] = \quad h_{hp}[n] \quad w[n],$$ ***Obtain filter coefficients***

Linear-Phase Lowpass Filter

The desired frequency response is defined as

$$H_{lp}(e^{j\omega}) = \begin{cases} e^{-j\omega M/2}, & |\omega| < \omega_c, \\ 0, & \omega_c < |\omega| \le \pi, \end{cases} \quad (7.56)$$

where the generalized linear phase factor has been incorporated into the definition of the ideal lowpass filter. The corresponding ideal impulse response is

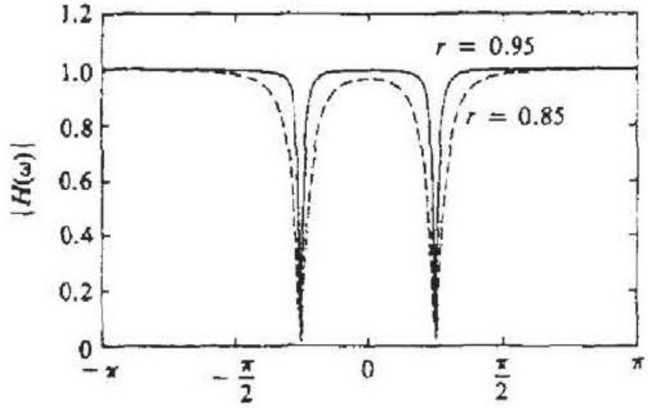
$$h_{lp}[n] = \frac{1}{2\pi} \int_{-\omega_c}^{\omega_c} e^{-j\omega M/2} e^{j\omega n} d\omega = \frac{\sin[\omega_c(n - M/2)]}{\pi(n - M/2)} \quad (7.57)$$

for $-\infty < n < \infty$. It is easily shown that $h_{lp}[M - n] = h_{lp}[n]$, so if we use a symmetric window in the equation

$$h[n] = \frac{\sin[\omega_c(n - M/2)]}{\pi(n - M/2)} w[n], \quad (7.58)$$

then a linear-phase system will result.

Sol4 -- J.G. Proakis



To create a null in the frequency response of a filter at a frequency ω_0, we simply introduce a pair of complex-conjugate zeros on the unit circle at an angle ω_0. That is,

$$z_{1,2} = e^{\pm j\omega_0}$$

Thus the system function for an FIR notch filter is simply

$$\begin{aligned} H(z) &= b_0(1 - e^{j\omega_0}z^{-1})(1 - e^{-j\omega_0}z^{-1}) \\ &= b_0(1 - 2\cos\omega_0 z^{-1} + z^{-2}) \end{aligned} \tag{4.5.30}$$

As an illustration. Fig. 4.51 shows the magnitude response for a notch filter having a null at $\omega = \pi/4$.

The problem with the FIR notch filter is that the notch has a relatively large bandwidth, which means that other frequency components around the desired null are severely attenuated. To reduce the bandwidth of the null, we can resort to a more sophisticated, longer FIR filter designed according to criteria described in Chapter 8. Alternatively, we could, in an ad hoc manner, attempt to improve on the frequency response characteristics by introducing poles in the system function.

Suppose that we place a pair of complex-conjugate poles at

$$p_{1,2} = re^{\pm j\omega_0}$$

The effect of the poles is to introduce a resonance in the vicinity of the null and thus to reduce the bandwidth of the notch. The system function for the resulting filter is

$$H(z) = b_0 \frac{1 - 2\cos\omega_0 z^{-1} + z^{-2}}{1 - 2r\cos\omega_0 z^{-1} + r^2 z^{-2}} \tag{4.5.31}$$

The magnitude response $|H(\omega)|$ of the filter in (4.5.31) is plotted in Fig. 4.52 for $\omega_0 = \pi/4$, $r = 0.85$, and for $\omega_0 = \pi/4$, $r = 0.95$. When compared with the frequency response of the FIR filter in Fig. 4.51, we note that the effect of the poles is to reduce the bandwidth of the notch.

In addition to reducing the bandwidth of the notch, the introduction of a pole in the vicinity of the null may result in a small ripple in the passband of the filter due to the resonance created by the pole. The effect of the ripple can be reduced by introducing additional poles and/or zeros in the system function of the notch filter. The major problem with this approach is that it is basically an ad hoc, trial-and-error method.