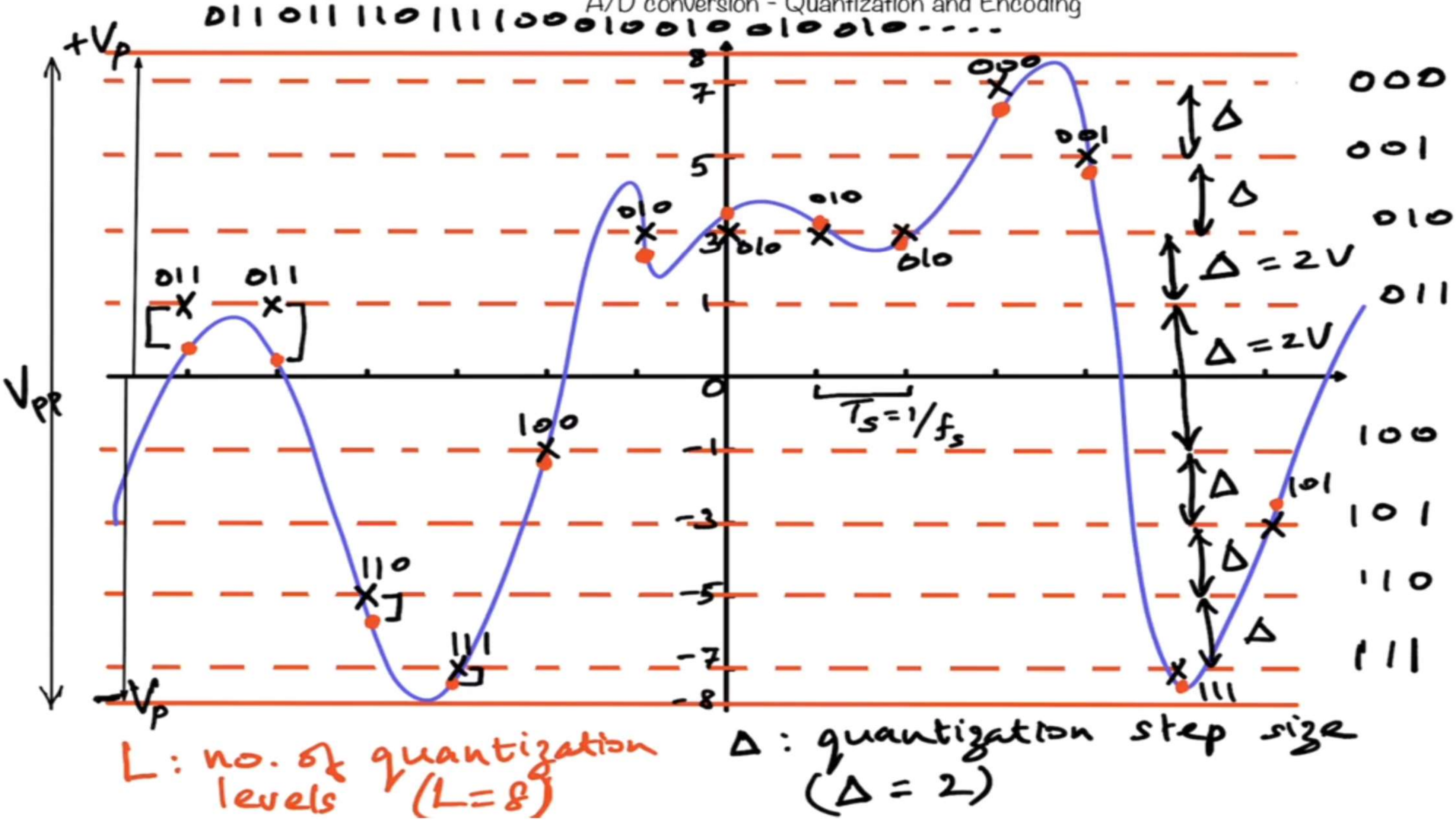
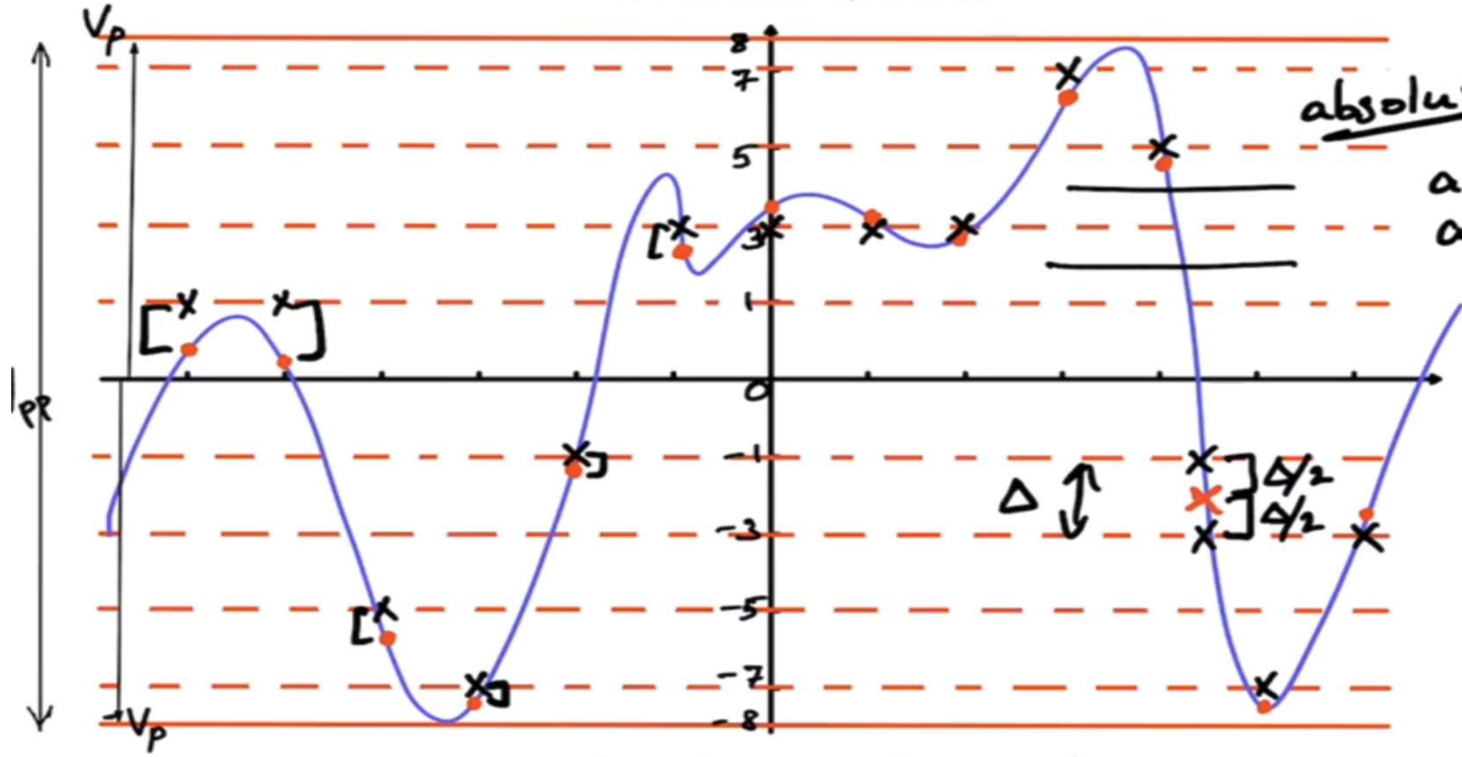
A/D conversion - Quantization and Encoding



A/D conversion - Quantization and Encoding

A/D conversion - Quantization



● denotes actual sample value

x denotes quantized sample value

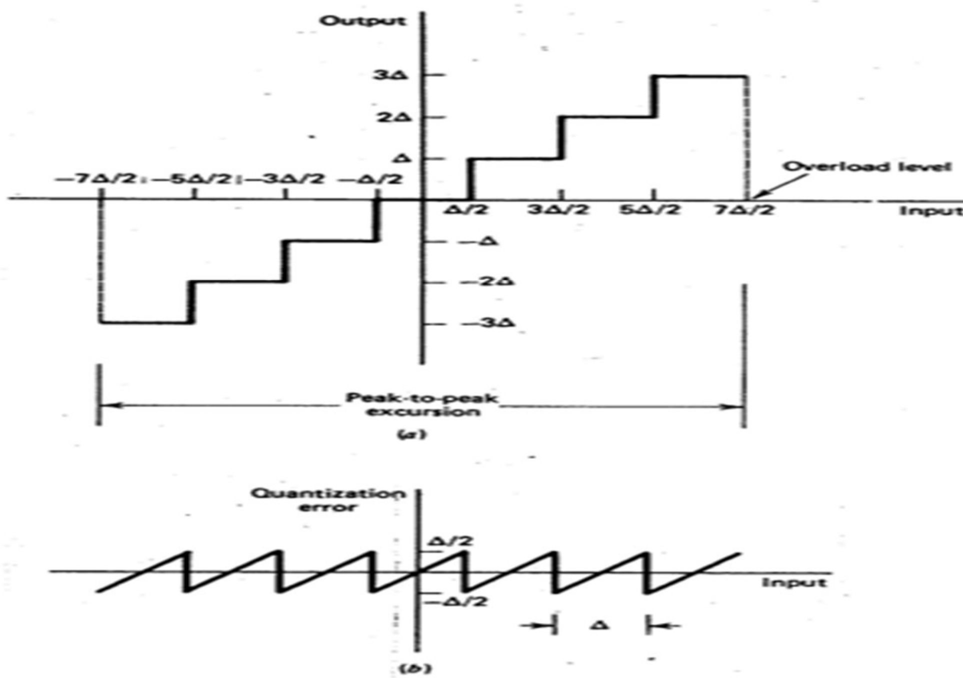
Quantization Error

absolute difference between actual sample value and quantized value

'e'

$$|e|_{max} = \frac{\Delta}{2}$$

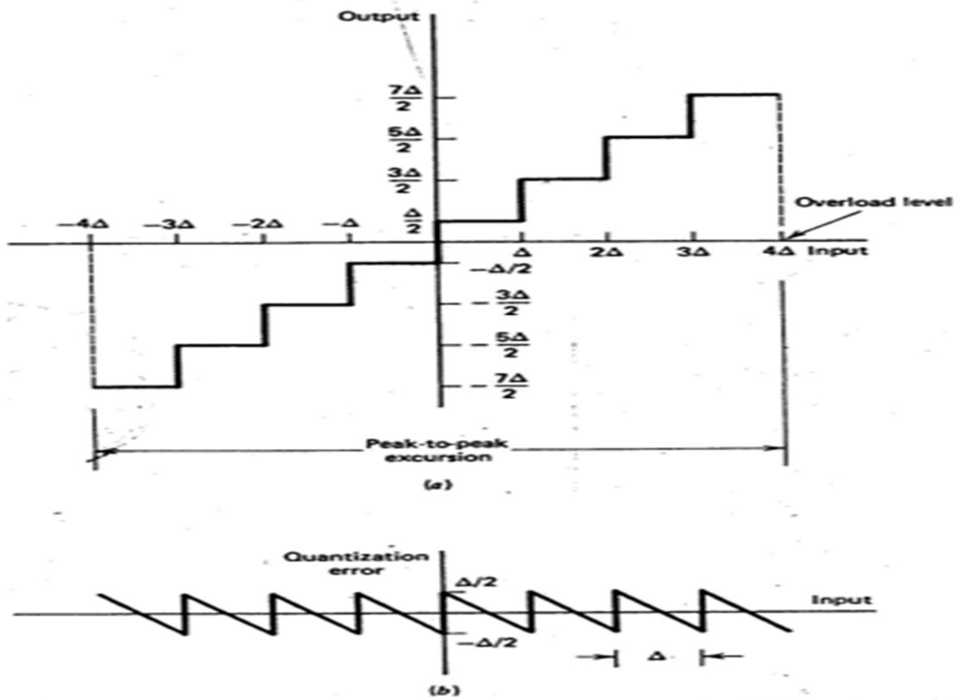
1) Mid-Thread type quantizer :-



(*a*) Transfer characteristic of quantizer of midtread type. (*b*) Variation of the quantization error with input.

* In midtread quantizer, the decision threshold of the quantizer are located at $\pm \delta/2, \pm 3\delta/2, \pm 5\delta/2, \cdots$, and the representation levels are located at $0, \pm\delta, \pm 2\delta, \cdots$, Where δ is the Step Size.
* A uniform quantizer characterized in this way is referred to as a Symmetric quantizer of the midtread type, because the origin lies in the middle of a thread of a Staircase.
* Quantization levels are odd Number.

Midriser type quantizer :-



(*a*) Transfer characteristic of quantizer of midriser type. (*b*) Variation of the quantization error with input.

* In midriser quantizer, the decision thresholds of the quantizer are located at $0, \pm\partial, \pm 2\partial, \cdots,$ & the representation levels are located at $\pm\partial/2, \pm 3\partial/2, \pm 5\partial/2, \cdots,$ where ∂ is the step size.
* A uniform quantizer characterized in this way is referred to as a symmetric quantizer of the midriser type, because the origin lies in the middle of a riser of the staircase.
* Quantization levels are even number.

Max. quantization error for any sample

$$|e|_{max} = \frac{\Delta}{2}$$

How to minimize quantization error?

Reduce Δ $\Rightarrow$ 'L' increases $\Rightarrow$ no. of bits per sample 'n' increases.

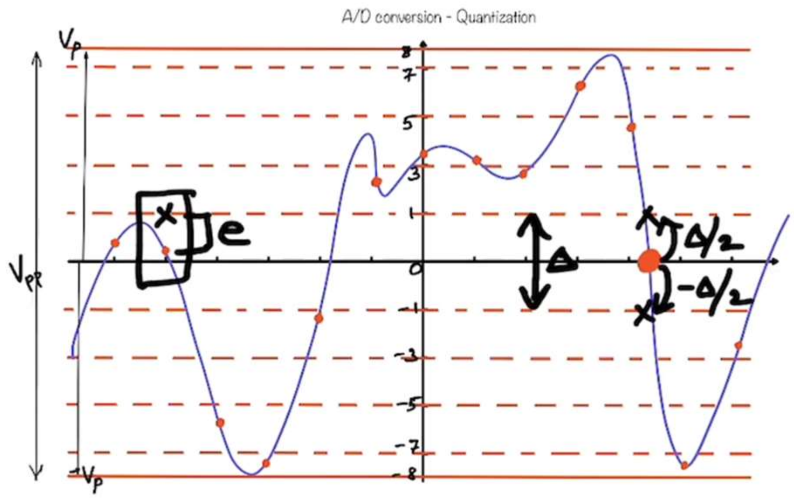
Effect on no. of bits per sample $\rightarrow$ increase in 'n'

10 samples/sec. $L = 8$ $n = 3$ bit rate $= 30$ bits/sec

" $L = 16$ $n = 4$ " $= 40$ bits/sec.

$\boxed{L = 2^n}$ $\boxed{n = \log_2 L}$

Average quantization noise power (N_q)



Quantization noise power for a particular sample is $\underline{\underline{e^2}}$

If the signal is uniformly distributed in the peak-peak range, then the quantization error 'e' is uniformly distributed in the interval $-\frac{\Delta}{2}$ to $+\frac{\Delta}{2}$

$$p(e) = \frac{1}{\frac{\Delta}{2} - \left(-\Delta/2\right)}$$

$$= \frac{1}{\frac{\Delta}{2} + \frac{\Delta}{2}} = \frac{1}{\Delta}$$

$$N_q = \int_{-\Delta/2}^{\frac{\Delta}{2}} e^2 \, p(e) \, de$$

$$N_q = \int_{-\Delta/2}^{\Delta/2} e^2 p(e)\, de = \int_{-\Delta/2}^{\Delta/2} e^2 \left(\frac{1}{\Delta}\right) de$$

$$= \frac{1}{\Delta} \int_{-\Delta/2}^{\Delta/2} e^2\, de = \frac{1}{\Delta} \left[\frac{e^3}{3}\right]_{-\Delta/2}^{\Delta/2}$$

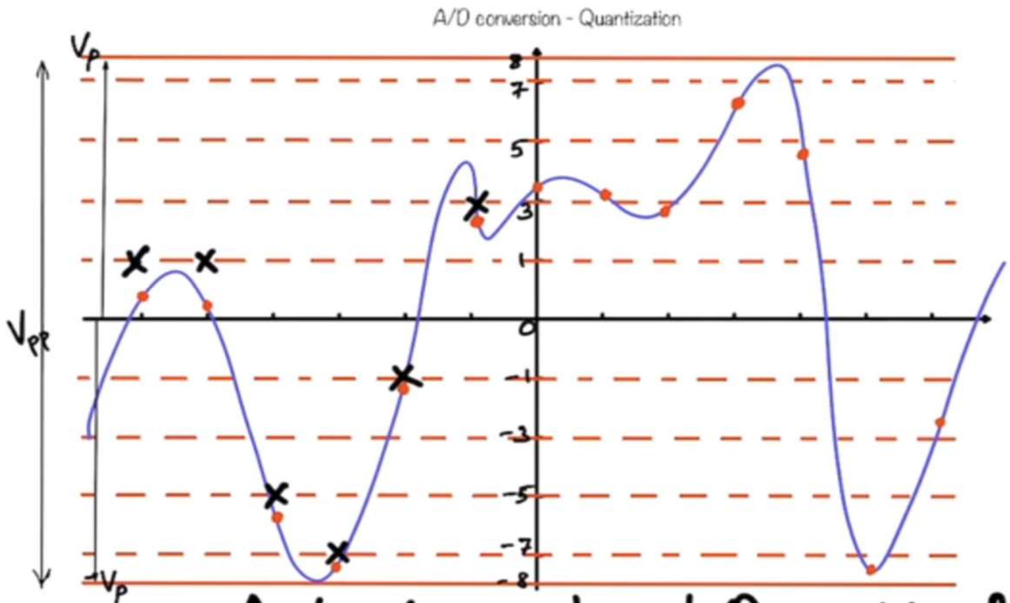
$$= \frac{1}{\Delta} \left[\frac{\Delta^3}{24} - \left(\frac{-\Delta^3}{24}\right)\right] = \frac{1}{\Delta} \left[\frac{\Delta^3}{24} + \frac{\Delta^3}{24}\right]$$

$$= \frac{1}{\Delta} \left(\frac{\Delta^3}{12}\right) = \frac{\Delta^2}{12}$$

$$\boxed{N_q = \frac{\Delta^2}{12}}$$

Avg. quant. noise power depends on the quant. step size Δ.

A/D conversion - Quantization



Quantization error/noise is non-reversible

$\therefore$ it is important to keep quant. error/noise as low as possible

Actual samples	Quantized values.
0.4 $\rightarrow$	1
0.25 $\rightarrow$	1

not possible to go from quantized value to actual value.

If $N_q \downarrow \Rightarrow \Delta \downarrow$

$L \uparrow \Rightarrow n \uparrow$

$\therefore$ Step Size $\Delta = \dfrac{2x_{max}}{L}$

$$\boxed{\Delta = \frac{2x_{max}}{2^N}} \longrightarrow (2)$$

$$\Delta = \frac{x_{max} - (-x_{max})}{L}$$

$$\boxed{\Delta = \frac{2x_{max}}{2^N}}$$

Substituting eq (2) in eq (1), we get

$$\sigma_Q^2 = \frac{(2x_{max}/2^N)^2}{12}$$

$$\sigma_Q^2 = \frac{4x_{max}^2 / 2^{2N}}{12} = \frac{\cancel{4}x_{max}^2}{2^{2N}} \times \frac{1}{\cancel{12}\,3}$$

$$\boxed{\sigma_Q^2 = \frac{1}{3} x_{max}^2 \cdot 2^{-2N}} \longrightarrow (3)$$

* Let 'p' denotes the average power of the message Signal $x(t)$, then the o/p SNR of a uniform quantizer is

$$\frac{S}{N} = \frac{\text{Signal power}}{\text{Noise power}} = \frac{P}{\sigma_Q^2}$$

$$= \frac{P}{\frac{1}{3} x_{max}^2\, 2^{-2N}} = \frac{3P}{x_{max}^2}\, 2^{2N}$$

$$\boxed{(SNR)_o = \frac{3P}{x_{max}^2} \cdot 2^{2N}} \longrightarrow (4)$$

* For Normalized I/p voltage $x_{max} = 1$ & Power $P \leq 1$.

$$\therefore \quad SNR = \frac{3(1)}{(1)^2} \cdot 2^{2N}$$

$$(SNR) = 3 \cdot 2^{2N}$$

$$(SNR)_{dB} = 10\log_{10}(3 \cdot 2^{2N})$$

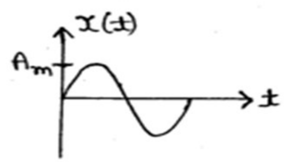
$$= 10\log_{10}(3) + 10\log_{10}(2^{2N})$$

$$= 4.8 + 20N\log_{10}(2)$$

$$\boxed{(SNR)_{dB} = 4.8 + 6N} \longrightarrow ⑤$$

Eq ⑤ is the Normalized Signal to quantization noise ratio in dB for any message Signal.

For Sinusoidal message Signal



* Let $x(t) = A_m \cos 2\pi f_m t$

$$\boxed{x_{max} = A_m}$$

∴ The power of this Signal is

$$P = \frac{V^2}{R}$$

V = rms value

$$V = \frac{A_m}{\sqrt{2}}$$

When R=1, the power 'p' is normalized

$$P = \frac{(A_m/\sqrt{2})^2}{R} = \frac{A_m^2}{2 \times 1}$$

$$\boxed{P = \frac{A_m^2}{2}}$$

W.K.T $(SNR)_o = \frac{3P}{x_{max}^2} \cdot 2^{2N}$ from eq ④

$$= \frac{3(\cancel{A_m^2}/2)}{\cancel{A_m^2}} \cdot 2^{2N}$$

$$(SNR)_o = \frac{3}{2} \cdot 2^{2N}$$

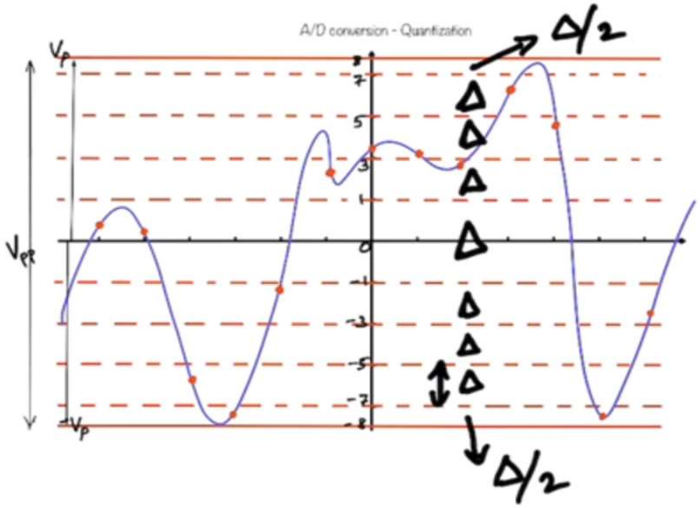
$$(SNR)_{dB} = 10\log_{10}\left(\frac{3}{2} \cdot 2^{2N}\right)$$

$$= 10\log_{10}(3/2) + 10\log_{10}(2^{2N})$$

$$= 1.76 + 20N\log_{10}(2)$$

$$\boxed{(SNR)_{dB} = 1.76 + 6.02N} \longrightarrow ⑥$$

* Eq ⑥ is Known as "6 dB rule" for uniform Quantization. This is because each additional bit of quantization level increases the Signal to Noise ratio by 6 dB.



Peak signal power (S_p)

$$S_p = \frac{V_p^2}{R} \qquad (R = 1\,\Omega)$$

$$S_p = V_p^2$$

$$V_{pp} = 2V_p \Rightarrow V_p = \frac{V_{pp}}{2}$$

$$V_{pp} = L\Delta$$

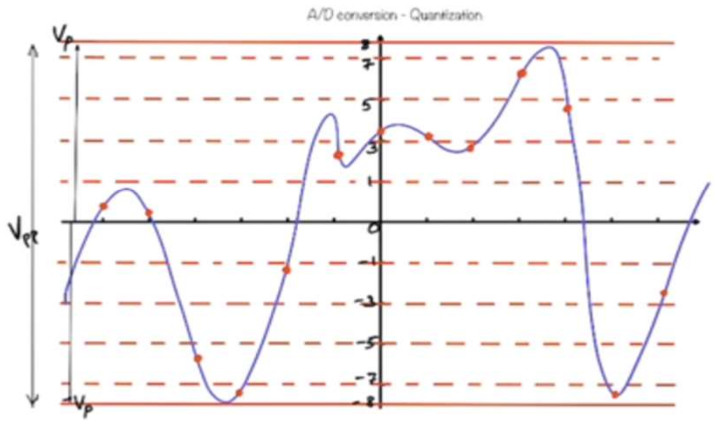
$$S_p = V_p^2 = \left(\frac{V_{pp}}{2}\right)^2 = \left(\frac{L\Delta}{2}\right)^2 = \frac{L^2\Delta^2}{4}$$

$$N_q = \Delta^2/12$$

Peak signal to avg. quant. noise power ratio (S_p/N_q)

$$\frac{S_p}{N_q} = \frac{\frac{L^2\Delta^2}{4}}{\frac{\Delta^2}{12}} = \frac{L^2\Delta^2}{4} \times \frac{12}{\Delta^2} = 3L^2$$

A/D conversion - Quantization



Average signal power (S_A)

Signal amplitude is uniformly distributed in the range $-V_p$ to $+V_p$

$$S_A = \int_{-V_p}^{V_p} v^2 p(v)\, dv$$

for uniform distribution $p(v) = \frac{1}{V_p - (-V_p)} = \frac{1}{2V_p}$

$$S_A = \int_{-V_p}^{+V_p} v^2 \left(\frac{1}{2V_p}\right) dv = \frac{1}{2V_p}\int_{-V_p}^{V_p} v^2\, dv = \frac{1}{2V_p}\left[\frac{v^3}{3}\right]_{-V_p}^{V_p}$$

$$= \frac{1}{2V_p}\left[\frac{V_p^3}{3} - \frac{(-V_p^3)}{3}\right] = \frac{1}{2V_p}\left[\frac{2V_p^3}{3}\right] = \frac{V_p^2}{3}$$

$$\boxed{S_A = \frac{V_p^2}{3}}$$

Since $V_p = \frac{V_{pp}}{2}$ and $V_{pp} = L\Delta$

$$S_A = \frac{\left(V_{pp}/2\right)^2}{3} = \frac{V_{pp}^2}{12} = \frac{(L\Delta)^2}{12} = \frac{L^2\Delta^2}{12}$$

Avg. signal to avg. quant. noise power ratio

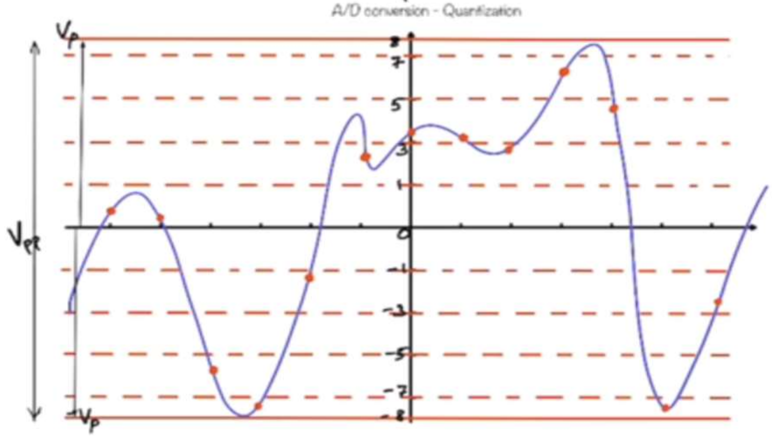
$$\frac{S_A}{N_q} = \frac{\frac{L^2\Delta^2}{12}}{\frac{\Delta^2}{12}} = \frac{L^2\cancel{\Delta^2}}{\cancel{12}} \times \frac{\cancel{12}}{\cancel{\Delta^2}} = L^2$$

$$\boxed{\frac{S_p}{N_q} = 3L^2} \qquad \left(\frac{S_p}{N_q}\right) = 3\left(\frac{S_A}{N_q}\right)$$

Mohammed Usman

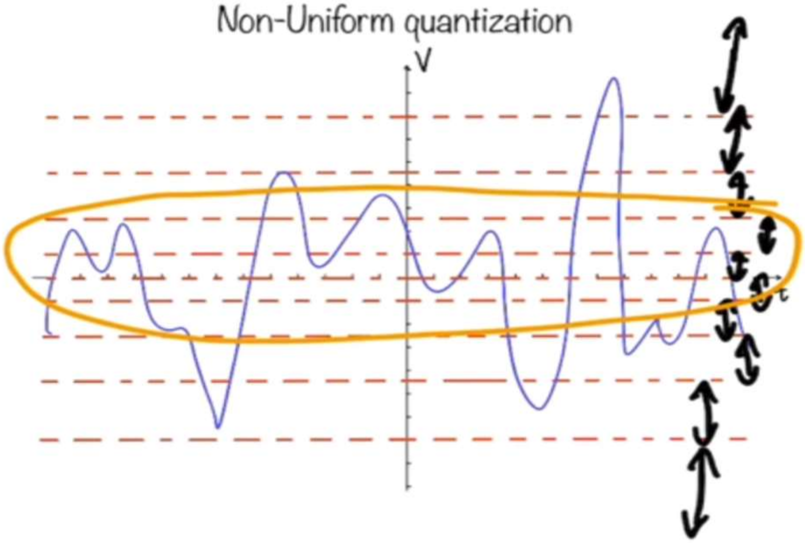
Quantization - practical considerations

Uniform quantization



step size 'Δ' is uniform (fixed) throughout the peak-peak range.

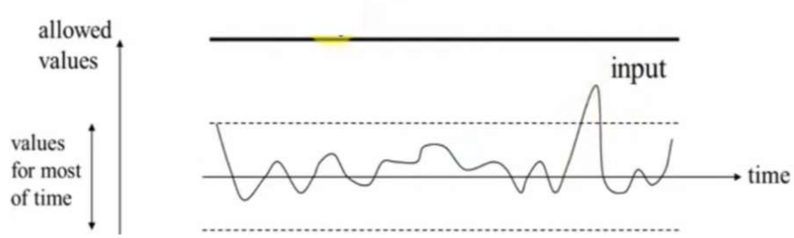
Non-Uniform quantization



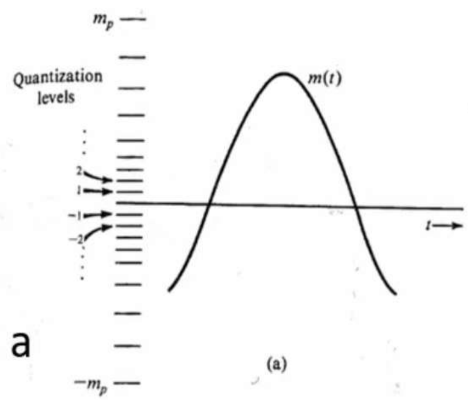
- Practical signals have most of the amplitude components in the lower amplitude region
- The sensitivity to quantization error is more pronounced at lower amplitudes than at higher amplitudes.
- A-law and μ-law for non-uniform quantization.

Companding: Non-uniform quantization

- SNR is an indication of the quality of the received signal, ideally we would like to have constant SNR
- Unfortunately, the SNR is directly proportional to the signal power, which varies from talker to talker
- The signal power can also vary because of the connecting circuits
- SNR vary even for the same talker, when the person speaks softly
- Smaller amplitudes pre-dominate in speech and larger amplitude much less frequent.
- This means the SNR will be low most of the time

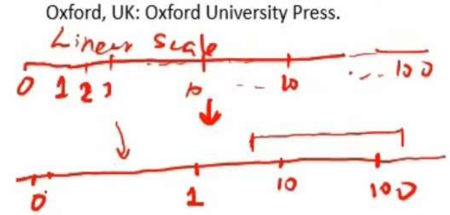


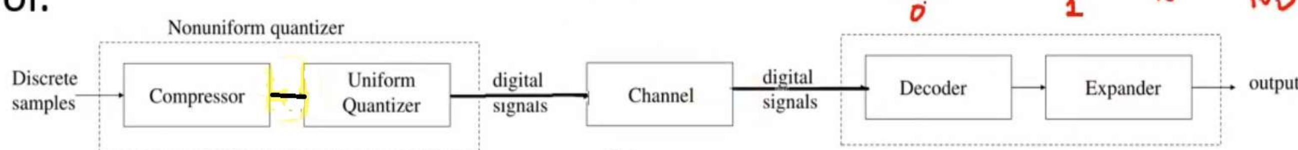
- The root of this difficulty is that the quantization steps are of uniform value
- The quantization noise is directly proportional to the square of the step size. $\overline{q^2} = \frac{\Delta v^2}{12} = \frac{m_p^2}{3L^2}$
- The problem can be solved by using smaller steps for smaller amplitudes as shown in fig. (a)
- The same result can be obtained by first compressing a signal and then using uniform quantization fig. (b)
- The compressed samples are restored to their original values at receiver by using an expander
- The compressor and expander together are called compandor.



(a)

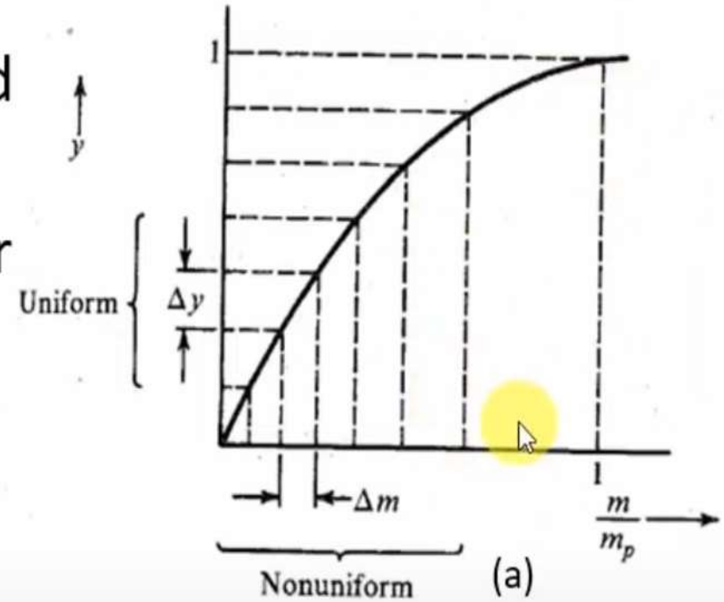
Lathi, B. P., & Ding, Z. (2009). Modern digital and analog communication systems (4th ed.). Oxford, UK: Oxford University Press.





(b)

- The input-output characteristics of compressor are shown in fig. (a)
- The horizontal axis is normalized input signal and the vertical axis is the output signal y.
- The compressor maps the input signal into larger increments
- Hence the interval Δm contains large number of steps when m is small
- The quantization noise is small for smaller input signal
- Thus loud talker and stronger signals are penalized with higher noise steps in order to compensate the soft talker and weak signals



(a)

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Compression Laws

- There are two laws regarding compressions

1. μ-law

This law is used in North America and Japan

$$y = \frac{1}{\ln(1+\mu)} \ln\left(1 + \frac{\mu m}{m_p}\right) \qquad 0 \le \frac{m}{m_p} \le 1$$

2. A-law

This law is used in the rest of the word

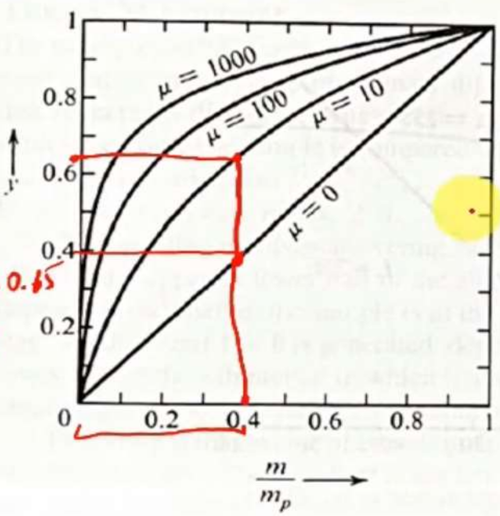
$$y = \begin{cases} \dfrac{A}{1+\ln A}\left(\dfrac{m}{m_p}\right) & 0 \le \dfrac{m}{m_p} \le \dfrac{1}{A} \\ \dfrac{1}{1+\ln A}\left(1 + \ln \dfrac{Am}{m_p}\right) & \dfrac{1}{A} \le \dfrac{m}{m_p} \le 1 \end{cases}$$

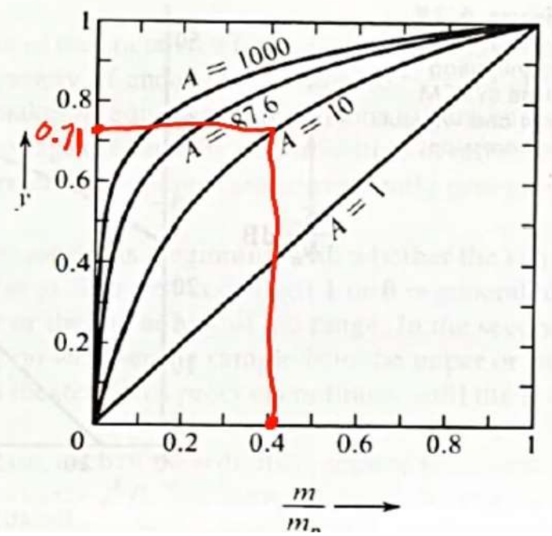
$n = 8$

$L = 2^n = 2^8 = \underline{256}$

$L_q = 256 \times 0.4$
$= 102$

$L_{q, \mu=10, \frac{m}{m_p}=0.4} = 256 \times 0.65$
$= 166$





$L = 256$

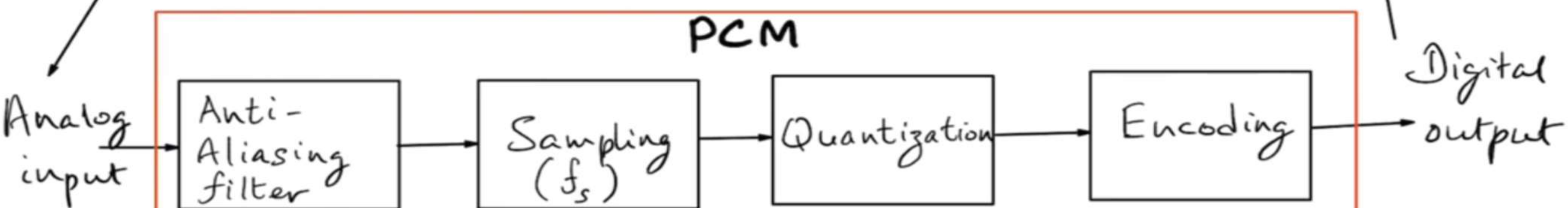
$L_{q, A=10, \frac{m}{m_p}=4} =$

256×0.71

$= \underline{181}$

Lathi, B. P., & Ding, Z. (2009). Modern digital and analog communication systems (4th ed.). Oxford, UK: Oxford University Press.

Pulse Code Modulation (PCM)



LPF

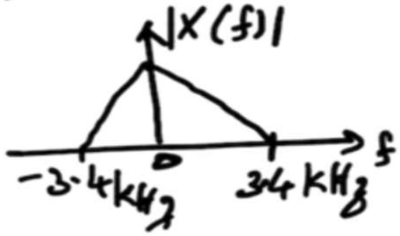
$f_c \simeq f_s/2$

8000 samples/sec

$L = 256$

$n = 8$ bits/sample

PCM for speech signals.



Source bit rate

$$R_b = n f_s = 8 \times 8000$$
$$= 64000 \text{ bits/sec}$$
$$= 64 \text{ kbps}$$