Digital Communication (UEC-639)

Tutorial-3

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Question-1

Consider a signal $x(t) = 40\, Sin(200\,\pi\, t)$ is given as input to a PCM system. If 256 quantization levels are employed, then

(a) Determine the voltage between levels when there is no compression?

(b) Determine the smallest and largest effective separation between levels when compression is used with $\mu = 255$

Solution-1 (a)

No compression is the case of uniform quantization.
Hence, the voltage between levels is equal to the step size of quantize:
The step size $\Delta = 2 * \frac{V_{max}}{L} = 2 * \frac{40}{256} = 0.3125\ V$

Solution-1 (b)

With Compression (that is a non-uniform quantizing), the smallest effective separation between levels will be the one closest to the origin, and the largest effective separation between levels will be the one closest to $|x| = 1$

We know that:

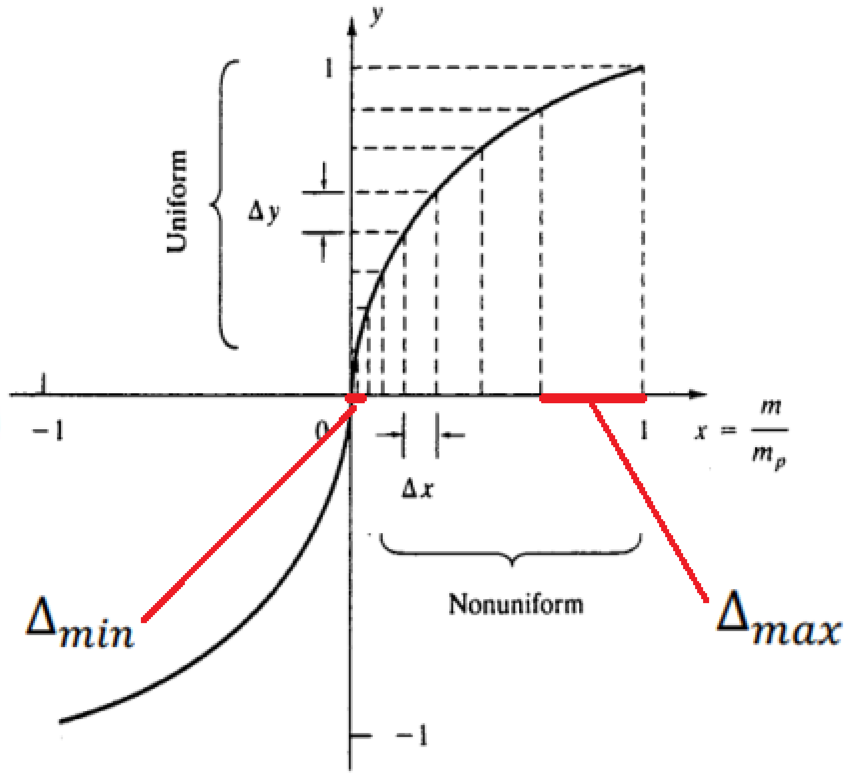
$$\frac{c(|x|)}{x_{max}} = y = \frac{ln\left(1 + \mu|x|/x_{max}\right)}{\ln(1+\mu)}$$

$$x_{max}=1$$

Let x_1 be the value of x corresponding to

(-128 to + 127)

$y = 1/127$, that is,



$$\frac{\ln(1+255\,|x_1|)}{\ln(256)} = \frac{1}{127}$$

Solving for $|x_1|$, we obtain

$$|x_1| = 1.75 \times 10^{-4}$$

Thus, the smallest effective separation between levels is given by

$$\begin{aligned}\Delta_{min} &= V_{max} * \ |x_1| \\ &= 40 * \ 1.75 \times 10^{-4} \\ &= 7\ mV\end{aligned}$$

The, largest effective levels is corresponds to x_{127}

Let x_{127} be the value of x corresponding to $y = 1 - \frac{1}{127} = \frac{126}{127}$, that is,

$$\frac{\ln(1 + 255\,|x_{127}|)}{\ln(256)} = \frac{126}{127}$$

Solving for $|x_{127}|$, we obtain

$$|x_{127}| = 0.957$$

The largest effective separation between levels is given by

$$\Delta_{max} = V_{max} * (1 - |x_{127}|) = 40 * (1 - 0.957) = 1.72\ V$$

Question-2

Consider an audio signal with spectral components from 300 to 3300 Hz. A PCM signal is generated with a sampling rate of 8000 samples per sec. The required output signal-to- quantizing-noise ratio is 30 dB.

(a) What is the minimum number of uniform quantizing levels and bits per sample needed?

(b) Determine the minimum system bandwidth required.

(c) Repeats parts (a) to (c) when a $\mu - law$ compander is used with $\mu = 255$.

Solution-2 (a)

We know that:

$$\left(\frac{S}{N_q}\right)_{0\ \text{dB}} = 1.76 + 20\log L \geqslant 30$$

$$\log L \geqslant \frac{1}{20}(30 - 1.76) = 1.412 \rightarrow L \geqslant 25.82$$

Thus, the minimum number of uniform quantizing levels needed is 26.

We know that:

$$n = [\log_2 L] = [\log_2 26] = [4.7] = 5\ \text{b per sample}$$

The minimum number of bits per sample is 5.

Solution-2 (b)

the minimum required system bandwidth is

$$f_{\text{PCM}} = \frac{n}{2} f_s$$

$$= \frac{5}{2}(8000) = 20\,000\,\text{Hz} = 20\,\text{kHz}$$

Solution-2 (c)

$$\left(\frac{S}{N_q}\right)_{0\ \text{dB}} = 20 \log L - 10.1 \geqslant 30$$

$$\log L \geqslant \frac{1}{20}(30 + 10.1) = 2.005 \rightarrow L \geqslant 101.2$$

Thus, the minimum number of quantizing levels needed is 102.

$$n = [\log_2 L] = [6.67] = 7$$

The minimum number of bits per sample is 7.

The minimum bandwidth required for this case is

$$f_{PCM} = \frac{n}{2} f_s$$

$$= \frac{7}{2}(8000) = 28\,000\,\mathrm{Hz} = 28\,\mathrm{kHz}$$

Question-6

A DM system is designed to operate at 3 times the Nyquist rate for a signal with a 6 kHz bandwidth. The quantizing step size is 250 mV.

(a) Determine the maximum amplitude of a 3-kHz input sinusoidal signal for which delta modulator does not show slope overload.

(b) Determine the post filtered output signal-to-quantizing -noise ratio in dB.

Solution-6

$$x(t) = A\,Sin\,(2\,\pi\,f_m\,t\,)$$

Max slope?

$$\textbf{max}\left|\frac{dx(t)}{dt}\right| = A\,2\,\pi\,f_m$$

$$= A\,(2\,\pi)(3*10^3)$$

To ensure no overloading, we have

$$\frac{\Delta}{T_s} \geq \left.\frac{dx(t)}{dt}\right|_{\max}$$

where Δ is the quantization step–size.

The maximum allowable amplitude of the input signal is

$$A_{max} = \frac{\Delta}{\omega_m T_s}$$

Fs=1/Ts=2*Fm

Fs`=3(Fs)=3*2*6*10^3

$$= \frac{250\,mV}{2\,\pi\,3*10^3}\,3*2*6*10^3$$

$$= 477.7\,mV$$

Now, the post filtered output signal-to-quantizing-noise ratio is given as

$$(\mathrm{SNR})_o = \left(\frac{S}{N_q}\right)_o = \frac{3\, f_s^3}{8\, \pi^2\, f_m^2\, f_M}$$

where f_M is the frequency of post reconstruction filter ($f_M \geq f_m$)

For maximum SNR we select, $f_M = f_m$

$$SNR = \frac{(\,3 * [\,3 * 2 * 6 * 10^3]^3)}{8\, \pi^2\, (3 * 10^3)^3}$$

$$= 65.72$$

SNR in dB = 10 * log10(65.72) = 18.18 dB

Thanks !