1 The Fundamental Concepts of Quantum Mechanics

2 The Quantum Mechanical law of motion

2.1

Given $L = (m/2)\dot{x}^2$, from Euler-Lagrange(E.L.) equation

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} = 0$$

$\implies m\ddot{x} = 0 \implies \dot{x} = constant = v$. Now since $S_{cl} = \int_{t_a}^{t_b} L \cdot dt$
$\implies S_{cl} = (m/2)v^2(t_b - t_a)$, we can get v from the initial conditions as follows: $x(t) = vt + c$, also $x(t_a) = x_a$ and $x(t_b) = x_b$.

$$\implies v = \frac{x_a - x_b}{t_a - t_b}, \text{so } S_{cl} = \frac{m}{2}\frac{(x_a - x_b)^2}{t_a - t_b}$$

2.2

Given $L = (m/2)(\dot{x}^2 - \omega^2 x^2)$, from Euler-Lagrange(E.L.) equation
$\implies \ddot{x} = -\omega^2 x \implies x(t) = A\text{sin}(\omega(t - t_a)) + B\text{sin}(\omega(t_b - t))$
$\implies x_a = B\text{sin}(\omega(t_b - t_a))$ and $x_b = A\text{sin}(\omega(t_b - t_a))$
Since, $S_{cl} = \int_a^b L \cdot dt = (m/2)\int_a^b (\dot{x}^2 - \omega^2 x^2) \cdot dt$, doing IBP
$= (m/2)\Big[\dot{x}x|_a^b - \int_a^b \ddot{x}x \cdot dt - \int_a^b \omega^2 x^2 \cdot dt\Big]$
$= (m/2)\Big[\dot{x}(t_b)x_b - \dot{x}(t_a)x_a\Big]$
$= (m/2)\Big[(A\omega\text{cos}(\omega T) - B\omega)x_b - (A\omega - B\omega\text{cos}(\omega T))x_a\Big]$
$= \frac{m\omega}{2\text{sin}(\omega T)}\Big[(x_a^2 + x_b^2)\text{cos}(\omega T) - 2x_a x_b\Big]$

2.3

Given $L = (m/2)\dot{x}^2 + fx$, from Euler-Lagrange(E.L.) equation
$\implies m\ddot{x} = f \implies x(t) = (f/2m)(t - t_a)(t - t_b) + x_a\frac{(t - t_b)}{(t_a - t_b)} + x_b\frac{(t - t_a)}{(t_b - t_a)}$
Since, $S_{cl} = \int_a^b L \cdot dt = (m/2)\int_a^b (\dot{x}^2 + (2f/m)x) \cdot dt$, doing IBP
$= (m/2)\Big[\dot{x}x|_a^b - \int_a^b \ddot{x}x \cdot dt + \int_a^b (2f/m)x \cdot dt\Big]$
$= (m/2)\Big[\dot{x}x|_a^b + \int_a^b (f/m)x \cdot dt\Big]$
$= (m/2)\Big[\dot{x}(t_b)x_b - \dot{x}(t_a)x_a + \int_a^b (f/m)x \cdot dt\Big]$

Putting all the values and doing all the simplification yields,

$$S_{cl} = \frac{m(x_b - x_a)^2}{2T} + \frac{fT(x_b + x_a)}{2} - \frac{f^2T^3}{24m}.$$

2.4

From equation (2.6) we know that

$$\delta S = \left[\delta x \frac{\partial L}{\partial \dot{x}}\right]_{t_a}^{t_b} - \int_{t_a}^{t_b} \delta x \left[\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x}\right] \cdot dt$$

The E.L. equation simplifies this to

$$\delta S = \left[\delta x \frac{\partial L}{\partial \dot{x}}\right]_{t_a}^{t_b} = \delta x_b \left(\frac{\partial L}{\partial \dot{x}}\right)_{x=x_b} - \delta x_a \left(\frac{\partial L}{\partial \dot{x}}\right)_{x=x_a}$$

This gives us the variation at end points as

$$\left(\frac{\partial L}{\partial \dot{x}}\right)_{x=x_b} = +\frac{\partial S_{cl}}{\partial x_b} \quad \text{and,} \quad \left(\frac{\partial L}{\partial \dot{x}}\right)_{x=x_a} = -\frac{\partial S_{cl}}{\partial x_a}.$$

2.5

Since, $\delta S_{cl} = \int_{t_a+\delta t_a}^{t_b+\delta t_b} L(\alpha) \cdot dt - \int_{t_a}^{t_b} L(0) \cdot dt$ for a p

$$\implies \delta S_{cl} = L(t_b)\delta t_b - L(t_a)\delta t_a + \int_{t_a}^{t_b} \delta L \cdot dt$$

2.6

checkerboard problem

3 Developing the concepts with special example

3.1