Signatures of symmetry-protected topology in Entanglement Entropy for one-dimensional systems

UP 400 – BS Project

presented by

Aman Anand (4th year UG, 14807)

INDIAN INSTITUTE OF SCIENCE

भारतीय विज्ञान संस्थान

Introduction

Motivation and Objective

Model and Methodology

Results and Discussions

Conclusions and Future Work

What is Symmetry-protected topology?

Symmetry protected topological (SPT) order is a kind of order in zero temperature quantum mechanical states of matter that have symmetry and a finite energy gap. The SPT states are short-range entangled states with a symmetry.

The SPT order has the following defining properties:
(a) distinct SPT states with a given symmetry cannot be smoothly deformed into each other without a phase transition, if the deformation preserves the symmetry.
(b) however, they all can be smoothly deformed into the same trivial product state without a phase transition, if the symmetry is broken during the deformation.

Example- Haldane phase of odd-integer-spin chain.

What is Entanglement Entropy?

$$|\psi> = \frac{|00>+|11>}{\sqrt{2}} \neq (a_1|0>+b_1|1>)\otimes(a_2|0>+b_2|1>)$$

$$\rho_A = Tr_B[\rho]$$

$$S^n_{\mathrm{Renyi}} = \frac{1}{1-n}\log\left(Tr[\rho_A^n]\right)$$

$$S_{VN} = -Tr[\rho_A \ln \rho_A]$$

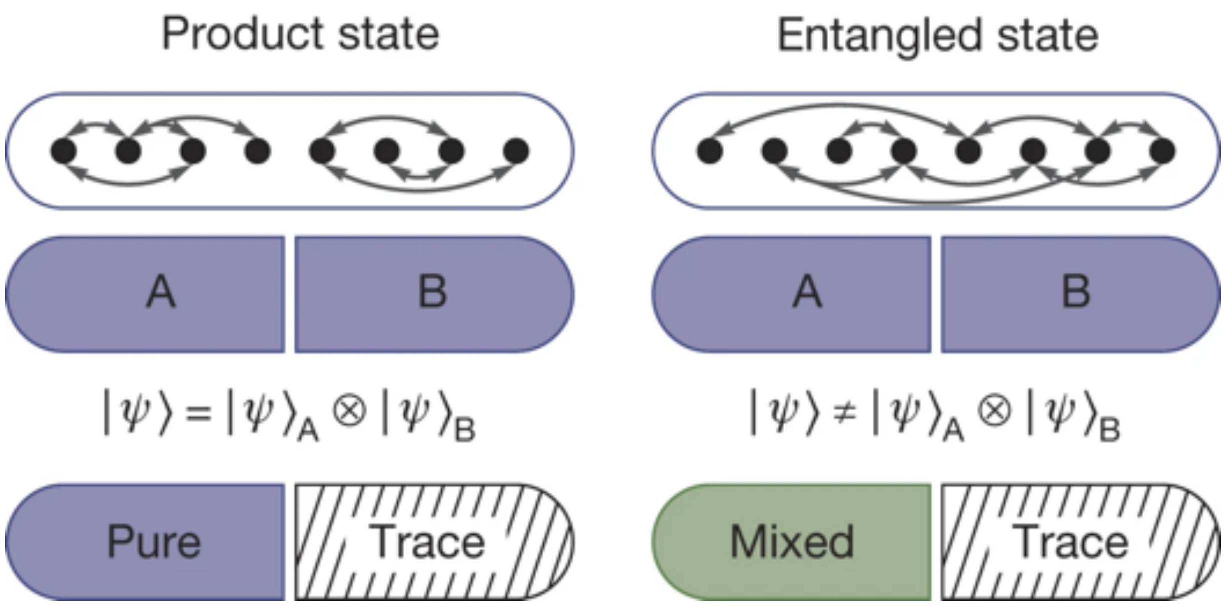


Image source: Nature **528,** 77–83 (2015)

Motivation

Objective

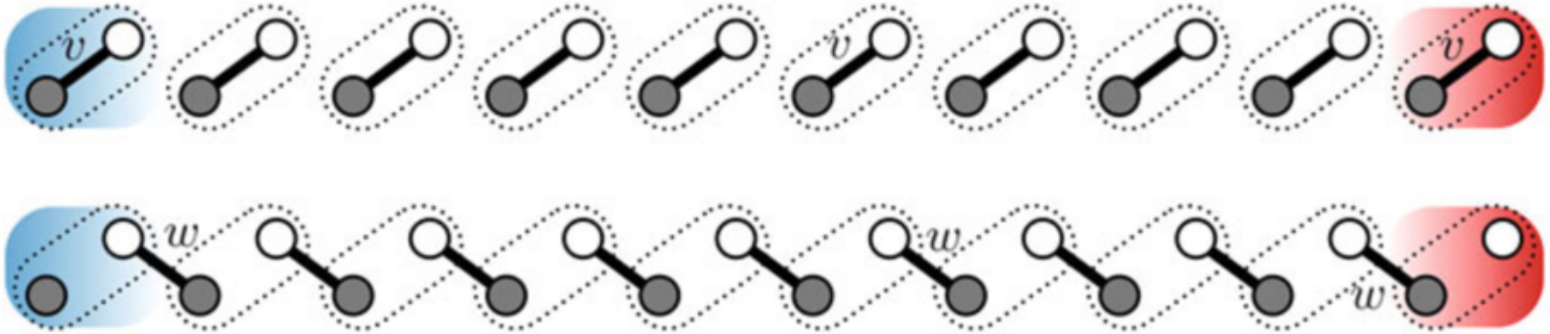
To calculate correlation matrix and using that to calculate the general time evolving entanglement entropy, where we further look for signatures of SPT

SSH Model

$$H = \sum_i \left[t_1 (a_i^\dagger b_i + b_i^\dagger a_i) + t_2 (a_{i+1}^\dagger b_i + b_i^\dagger a_{i+1}) + w_i a_i^\dagger a_i + w_i' b_i^\dagger b_i \right]$$

1) For $|t_1| < |t_2|$ the system is a topological insulator (TI).
2) For $|t_1| > |t_2|$ the system is topologically trivial
3) For $|t_1| = |t_2|$ there is a quantum critical point (QCP).

$t_1 = v$ & $t_2 = w$



Methodology

The Entanglement Entropy was calculated using the correlation matrix.

$$C_{ij} = \mathrm{Tr}[\rho c_i^\dagger c_j] = \langle c_i^\dagger c_j \rangle = \langle \Psi | c_i^\dagger c_j | \Psi \rangle.$$

For non-interacting systems,

$$S_{Renyi}^{(n)} = \frac{1}{1-n} \mathrm{Tr}[\ln[(1-C)^n + C^n]].$$

$$S_{VN} = -\mathrm{Tr}[(1-\mathrm{C})\ln(1-\mathrm{C}) + (\mathrm{C})\ln(\mathrm{C})].$$

For the thermal case the correlation matrix is

$$C_{ij} = \sum_{k=1}^{N} U_{ki}^\dagger U_{jk} \frac{1}{e^{\beta E_k} + 1}.$$

For time evolving pure state the correlation matrix is

$$C_{ij}(t) = \sum_{\alpha\beta k} U_{i\beta} U_{j\alpha}^\dagger U_{k\beta} U_{k\alpha}^\dagger n_k e^{i(E_\beta - E_\alpha)t}.$$

Introduction

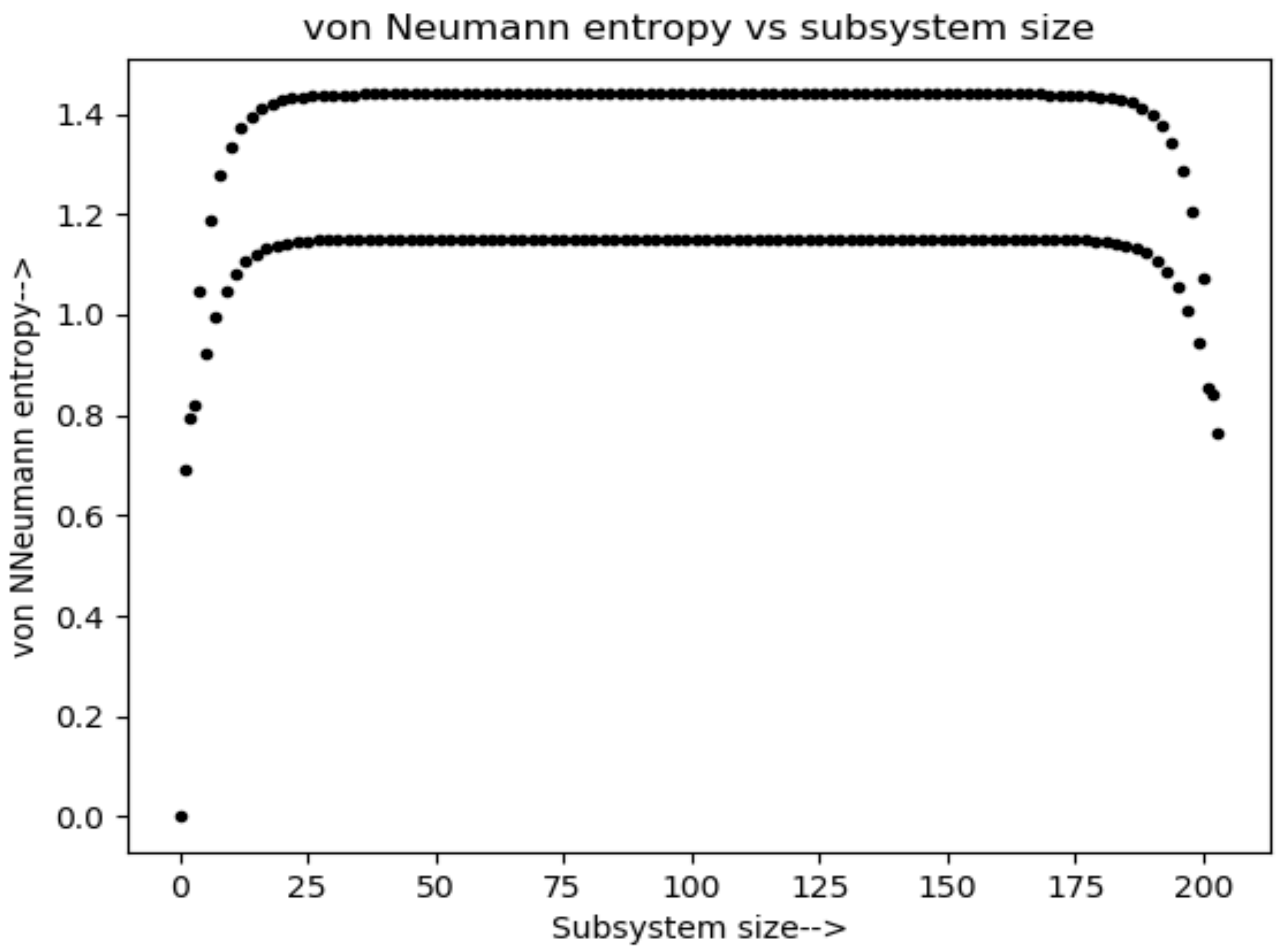
Motivation and Objective

Model and Methodology

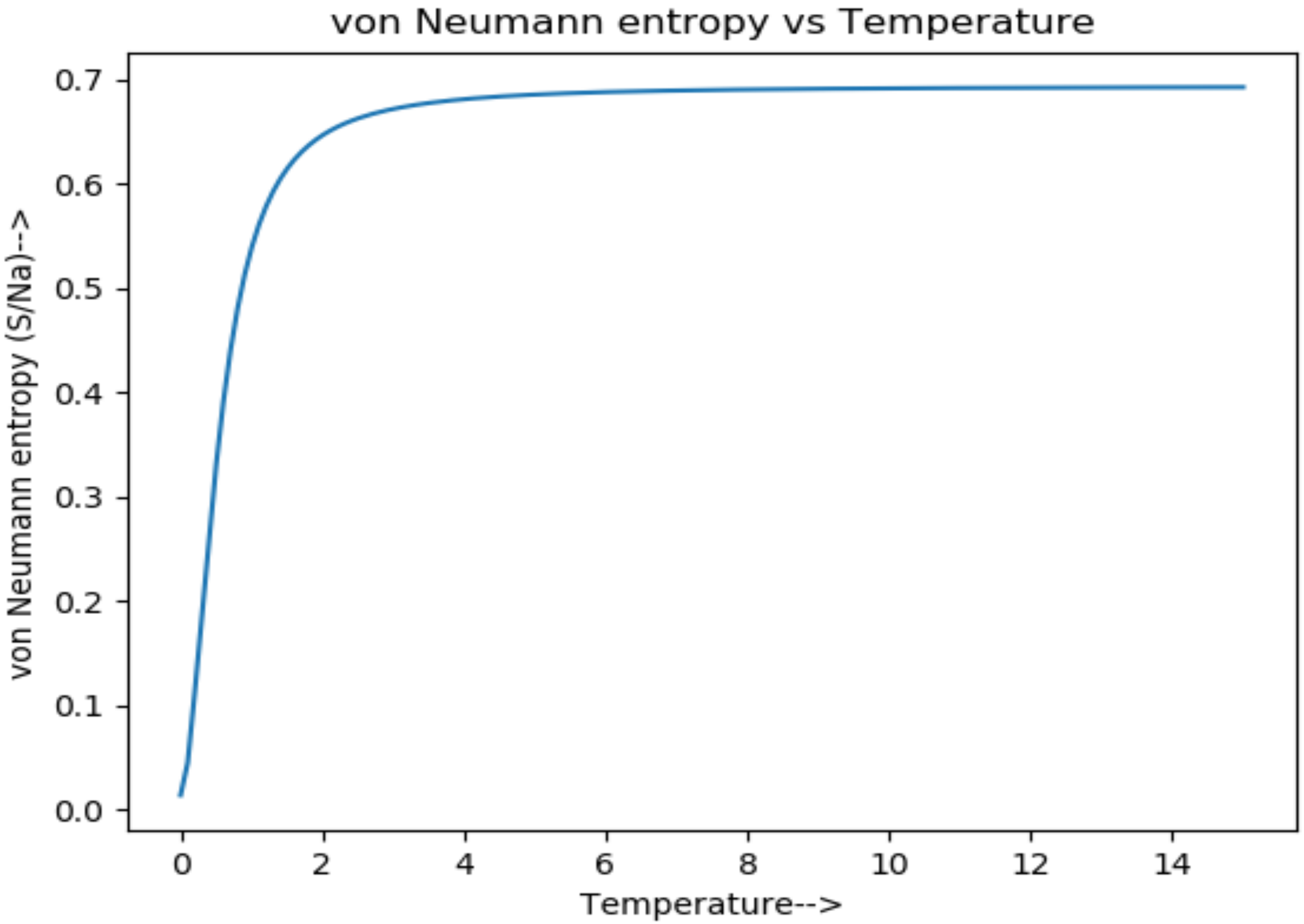
Results and Discussions

Conclusions and Future Work

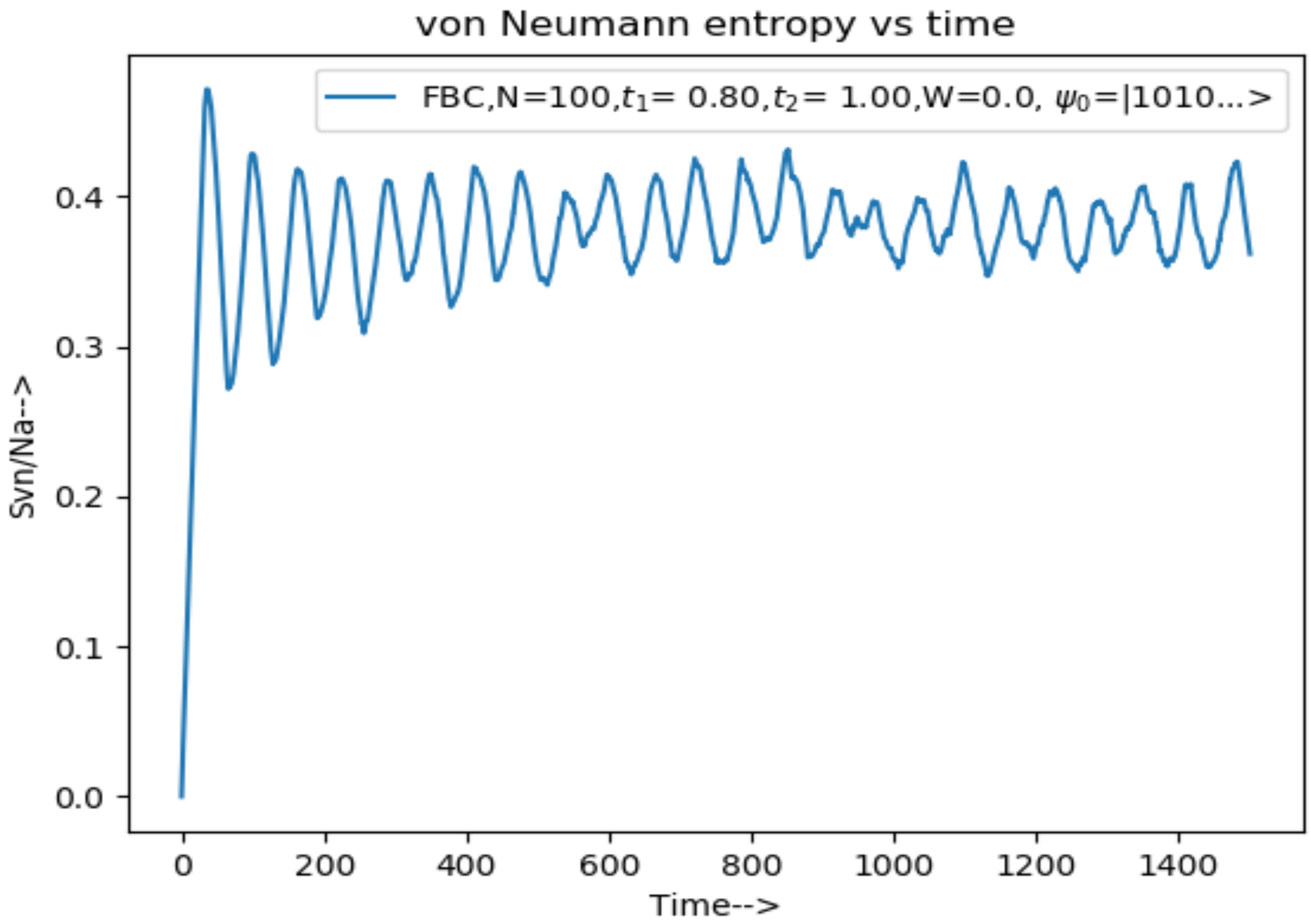
Entropy vs subsystem size



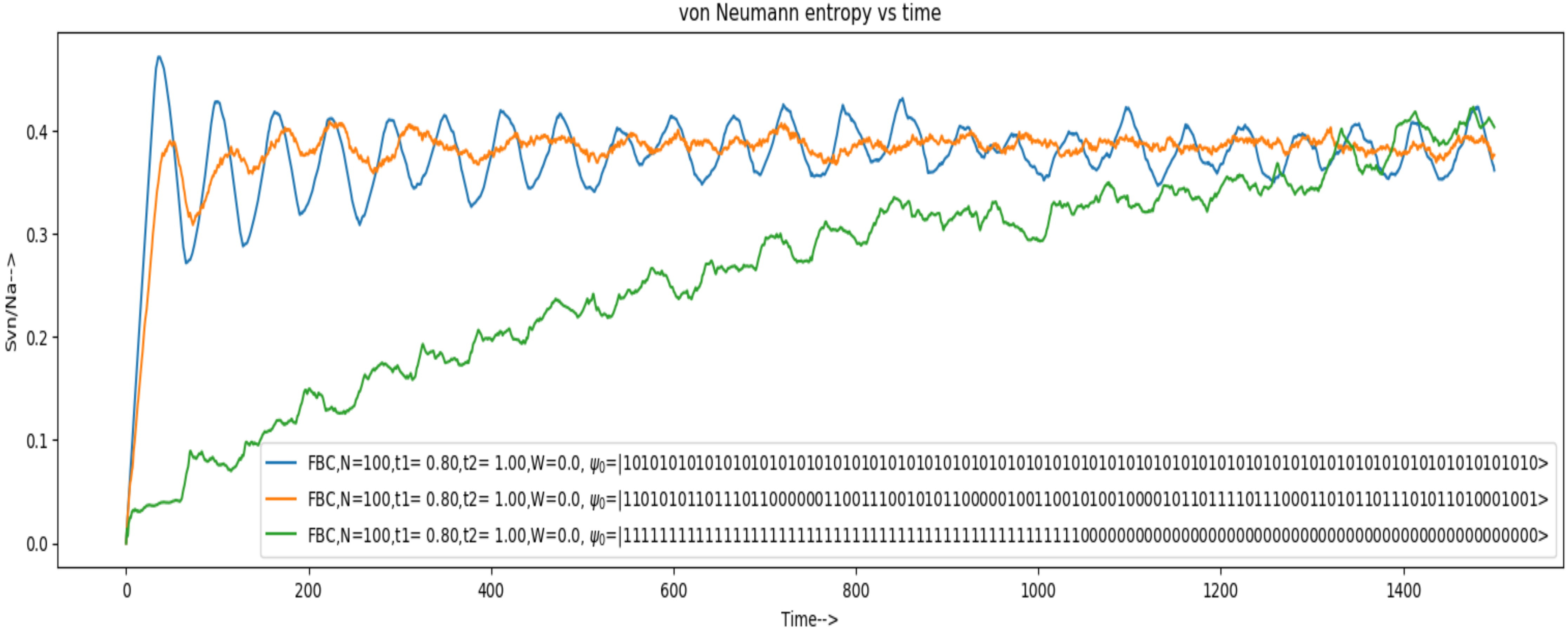
Entropy vs temperature



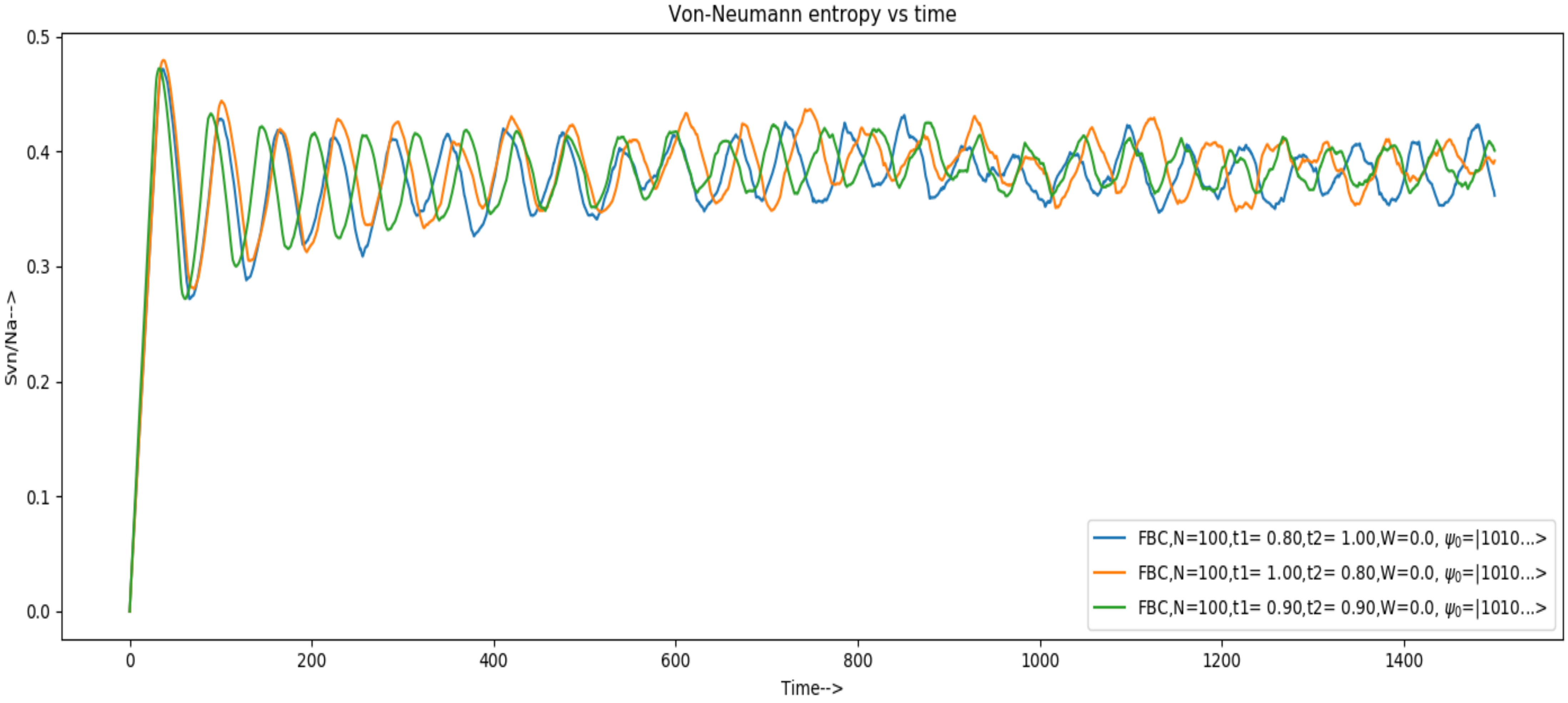
Entropy vs time

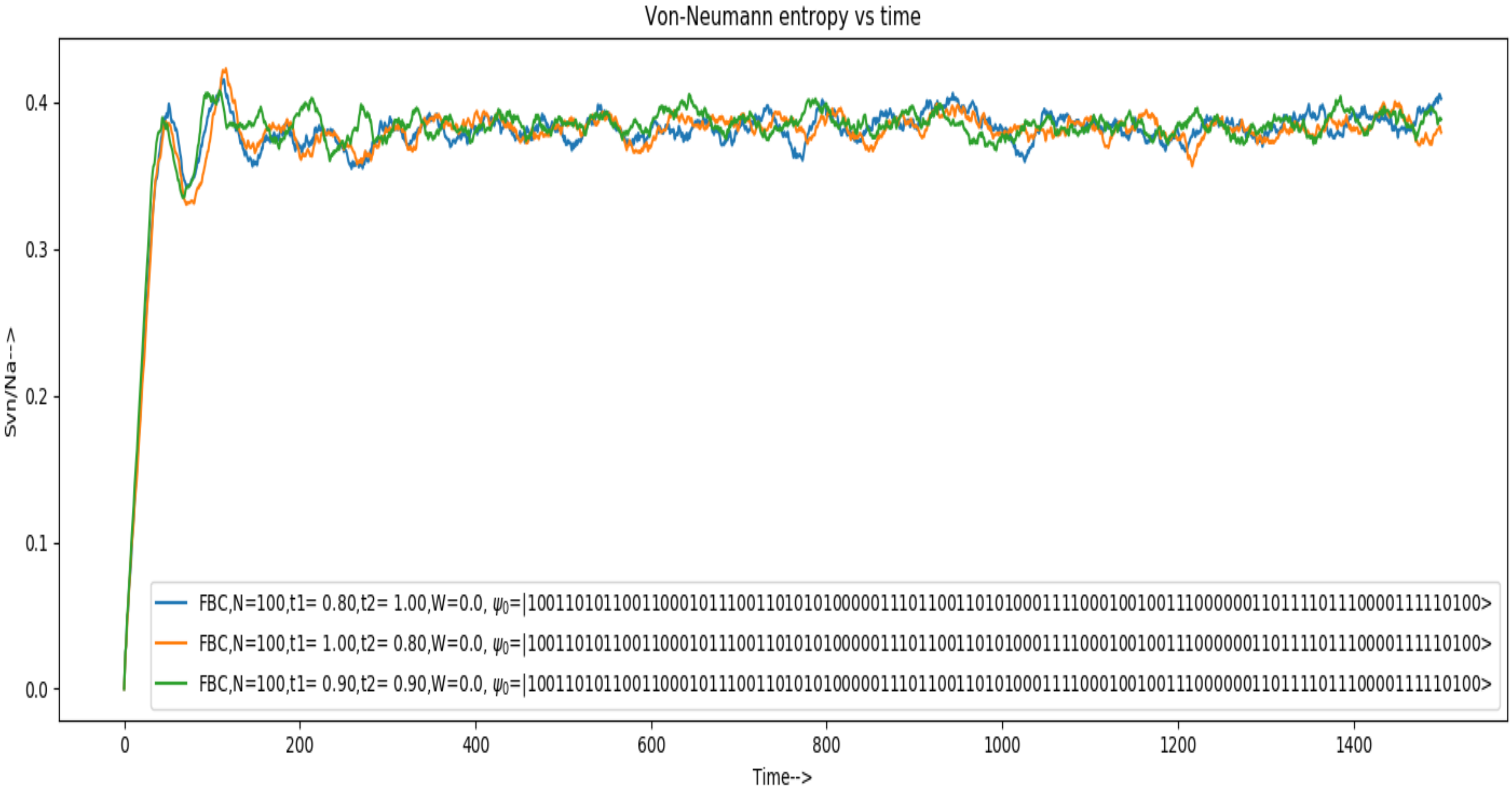


Effect of different pure states



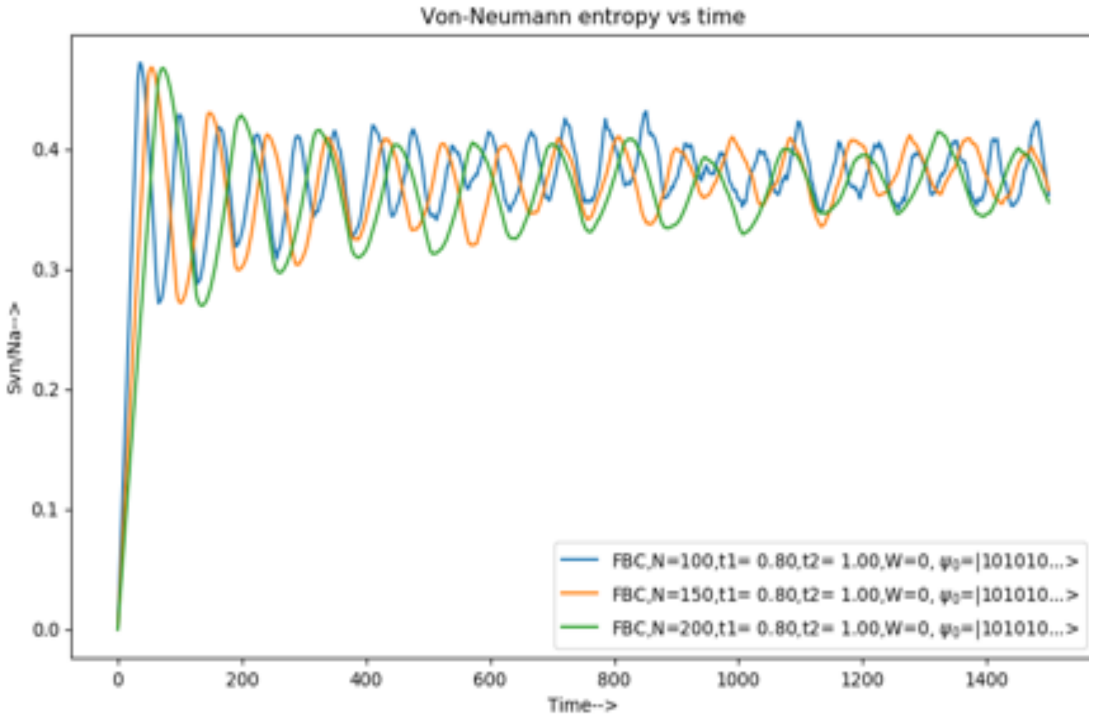
Effect of phase transition

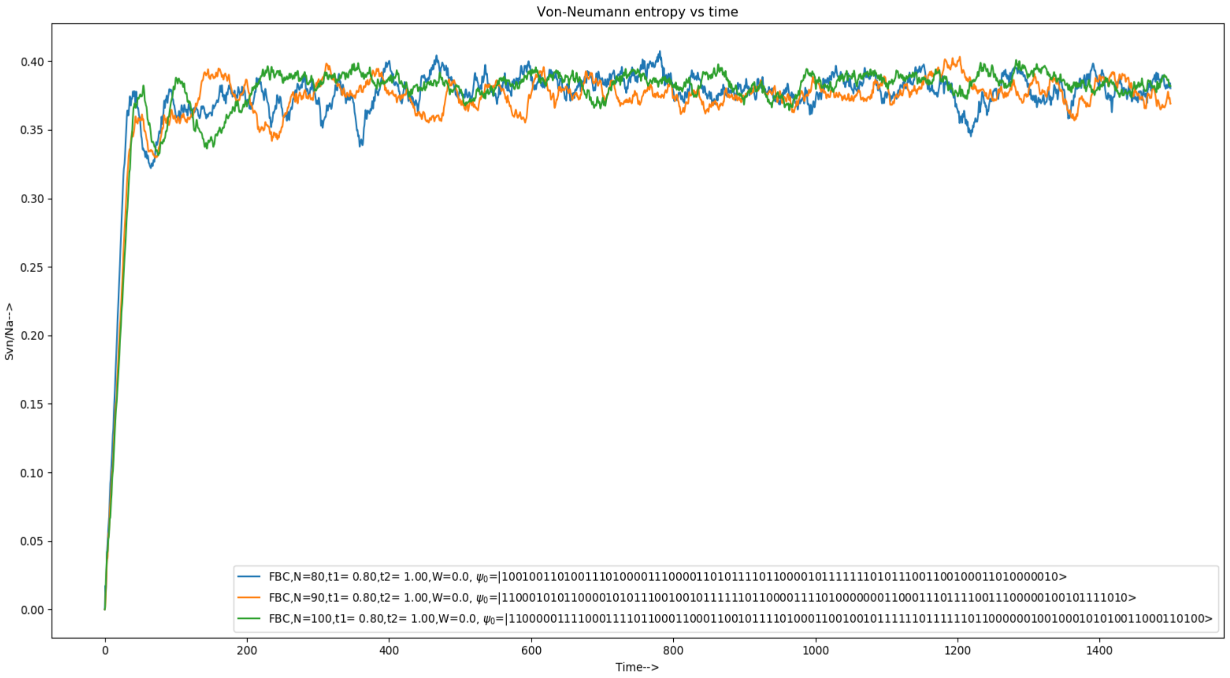




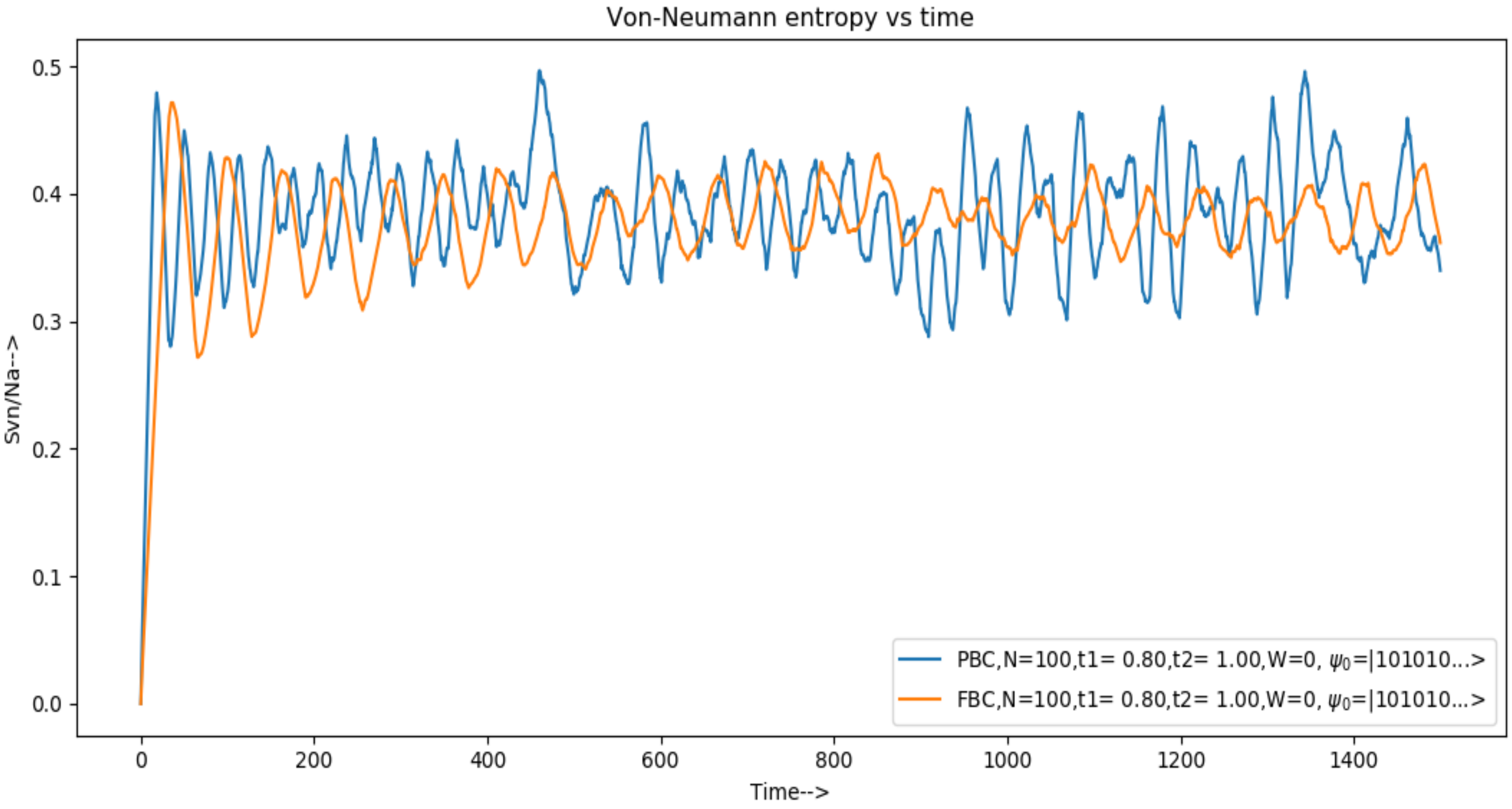
Von-Neumann entropy vs time
0.4
0.3
0.2
0.1
0.0
Svn/Na-->
0
200
400
600
800
1000
1200
1400
Time-->
FBC,N=100,t1= 0.80,t2= 1.00,W=0.0, ψ0=|1001101011001100010111001101010100000111011001101010001111000100100111000000110111101110000111110100>
FBC,N=100,t1= 1.00,t2= 0.80,W=0.0, ψ0=|1001101011001100010111001101010100000111011001101010001111000100100111000000110111101110000111110100>
FBC,N=100,t1= 0.90,t2= 0.90,W=0.0, ψ0=|1001101011001100010111001101010100000111011001101010001111000100100111000000110111101110000111110100>

Effect of system size

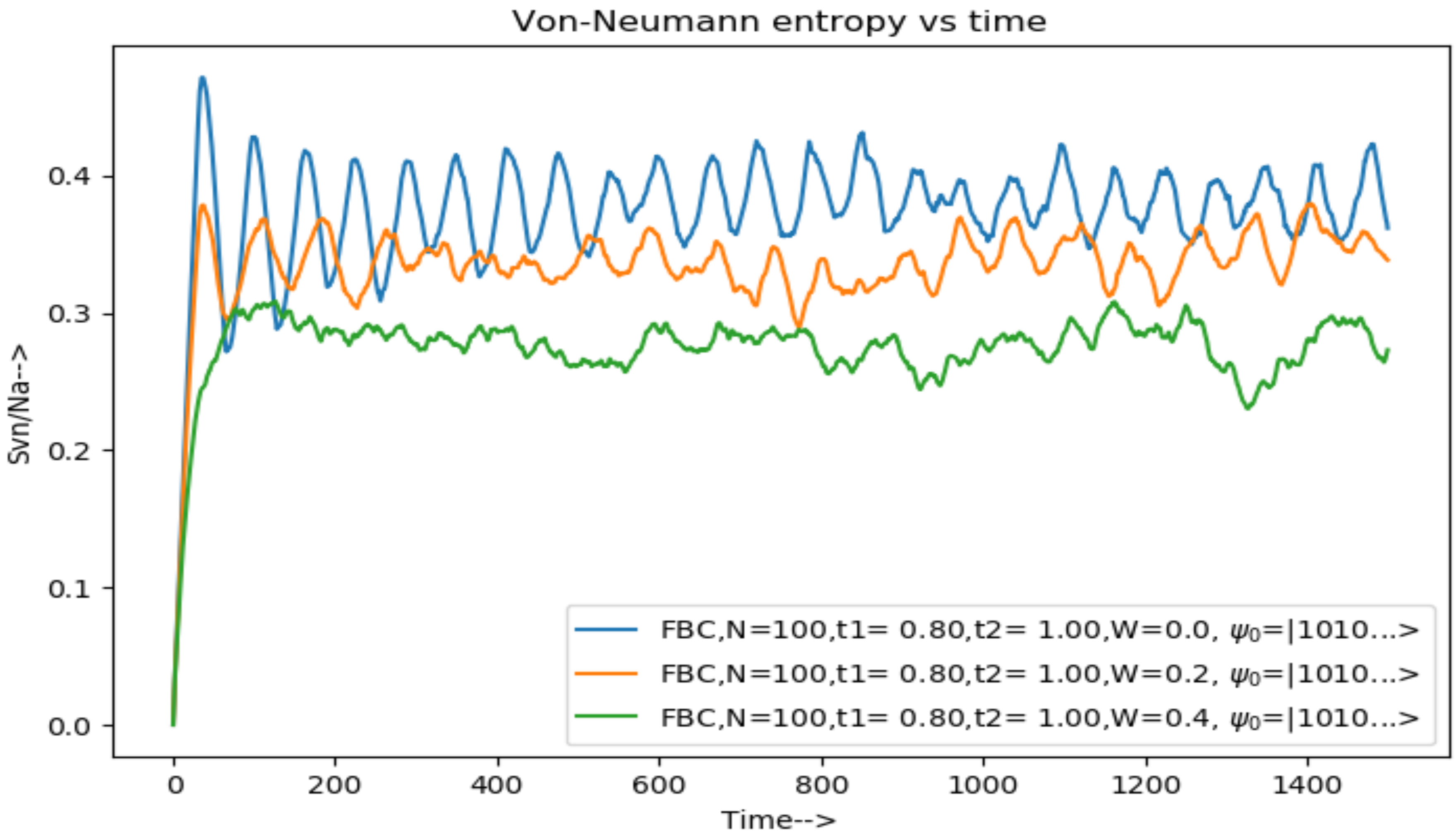




Effect of boundary condition



Effect of disorder



Conclusions and Future Work

Conclusions

- The Entanglement Entropy is not a good measure for finding signature of symmetry protected topology and one must look for the signatures in low energy states.
- The boundary conditions and system size don't cause any significant change in the entangled entropy time evolution.
- Disorder causes Anderson localisation which leads to decrement of the overall entropy.

Future Work

- Taking low energy states and looking at their time evolution with phase transitions.
- Will the system approximately thermalise?
- How will the results change when one introduces interaction.
- Looking into similar models like the Kitaev chain.

Thank You