Fractional Quantum Hall Effect

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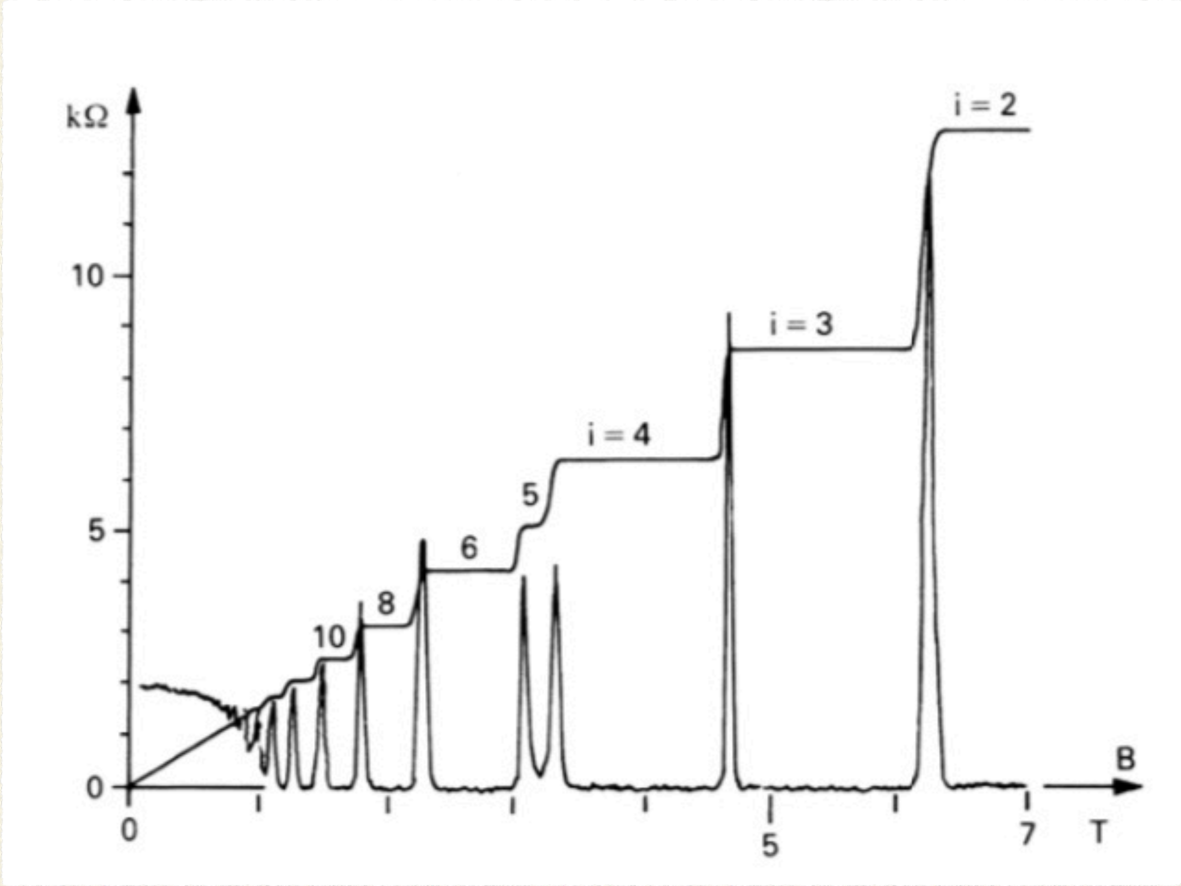
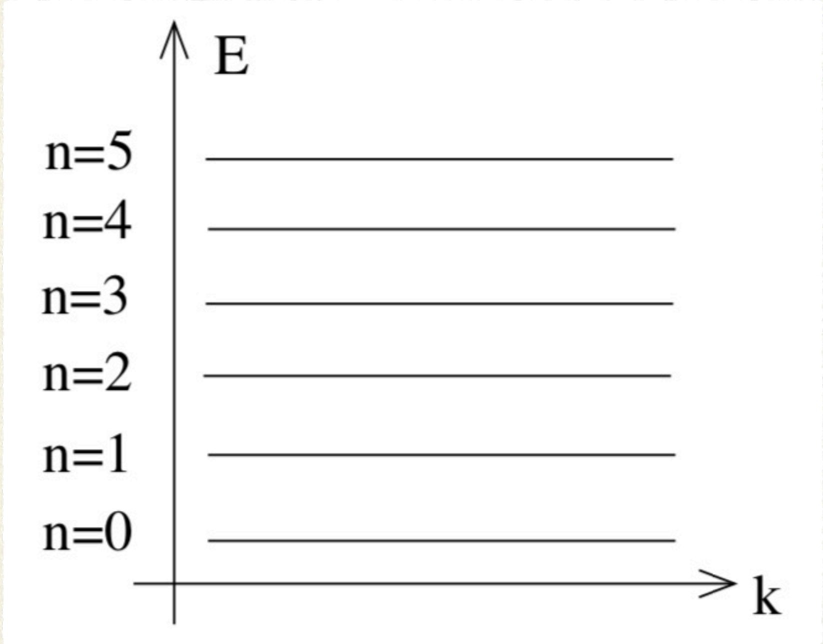
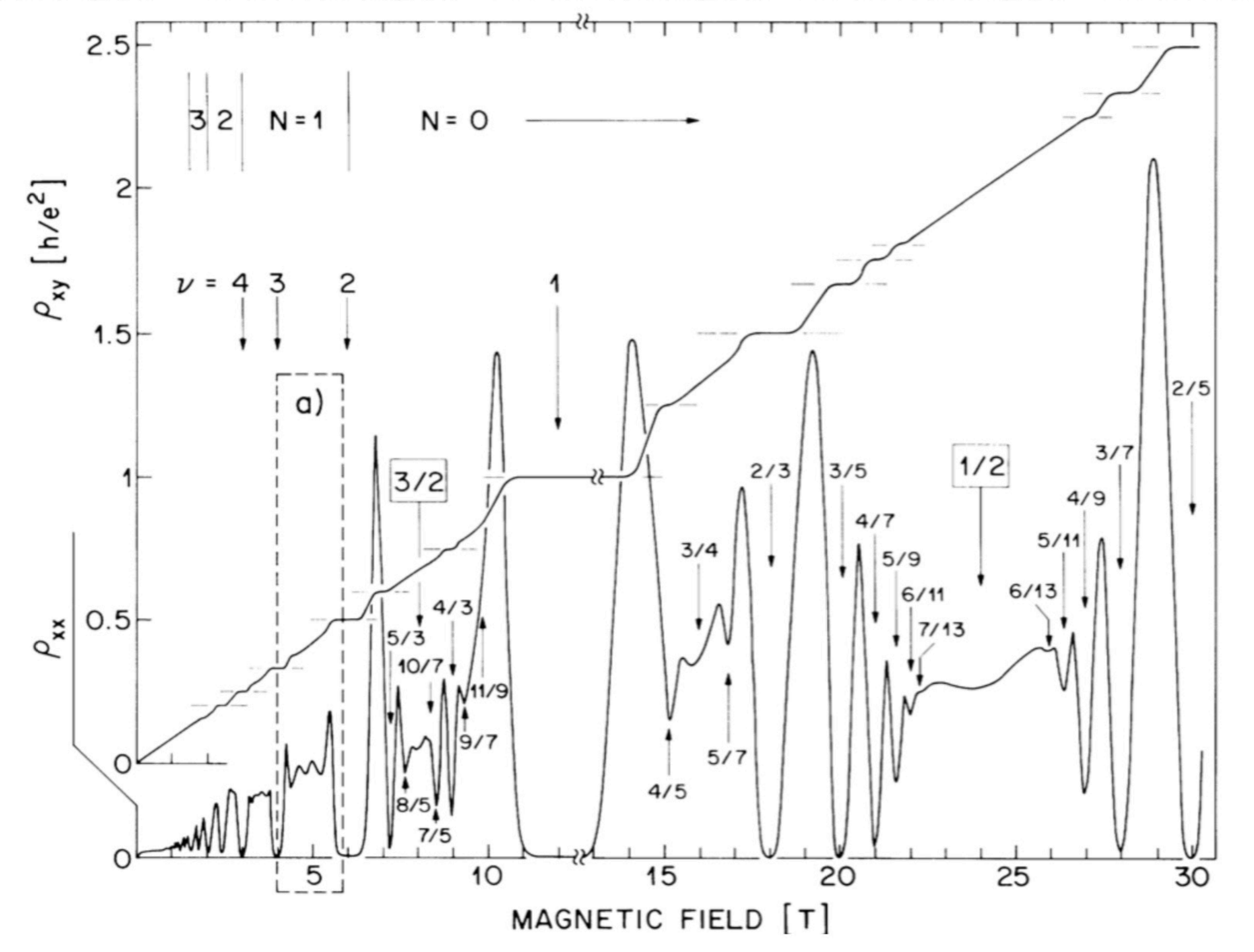
Integer Quantum Hall Effect

* $\sigma_{xy} = \dfrac{\nu e^2}{h}$ and below each plateau $\sigma_{xx} = 0$.
* 2D e⁻ gas in a magnetic field: Quantised Landau levels with spacing $\hbar\omega_c$ where $\omega_c = eB/m$ and degeneracy of each level is $\mathcal{N} = \dfrac{B \cdot A}{\phi_0}$ where $\phi_0 = h/e$(flux quanta).
* Important role of disorder and edge states.





FQHE



FQHE discovered in 1982. Nobel prize awarded in 1998.

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The Story

* People expected Wigner crystals to form at higher B fields or low filling factors.

* In IQHE we neglected Coulomb interaction and got our answer but for FQHE we need it. It turns out that $\hbar\omega_c >> E_{Coulomb} >> V_{disorder}$

* Because of the huge degeneracy we cannot do degenerate perturbation theory.

Laughlin's guess

* Laughlin 1983 tried to give an explanation for the filling factor of $\nu = \dfrac{1}{m}$

* Laughlin guessed the many body wave function as:
$$\psi(z_1, z_2, \ldots, z_N) = \prod_{i<j} (z_i - z_j)^m \ e^{-\sum_{i=1}^{n} \frac{|z_i|^2}{4l^2}}$$

* For small system size numerical verification gives a 99% overlap of the GS with Laughlin wave function. The reason for this accuracy is because it reduces the e- e- repulsion by making the probability go to zero as power ~ m when two electrons get closer.

Motivation for the wave function

* Two particles in lowest Landau level
* $\psi(z_1, z_2) = (z_1 + z_2)^M (z_1 - z_2)^m \; e^{-(|z_1|^2 + |z_2|^2)/4l^2}$
* Here M and m are the angular momentum states of COM and the relative coordinate.

* The $\nu = 1$, non-interacting IQHE
* $\psi(z_1, z_2, \ldots, z_N) = \prod_{i<j} (z_i - z_j) \; e^{-\sum_{i=1}^{n} \frac{|z_i|^2}{4l^2}}$
* The above expression can be easily derived by taking slater determinant of single particle states.

Plasma Analogy

- Comparing the wave function probability with $e^{-\beta U(\mathbf{z})}$ and setting $\beta = 2/m$

$$\implies U(z_i) = -m^2 \sum_{i<j} log\left(\frac{|z_i - z_j|}{l_B}\right) + \frac{m}{4l_B^2}\sum_{i=1}^{N}|z_i|^2$$

- From Maxwell's equation it can be seen that the first term is like the potential energy in 2D between two charges of charge q = -m and the second term is just the potential due to uniform charge density $\rho_0 = \dfrac{1}{2\pi l_B^2}$

- Now to minimise the energy we try to neutralise the background charge density, that gives the number density of charged particle is $n = \dfrac{1}{2\pi l_B^2}$

- The plasma becomes solid when m > 70

Quasi-holes

- The quasi-hole wave function can be written as:

$$\psi(z;\eta) = \prod_{j=1}^{M}\prod_{i=1}^{N}(z_i - \eta_j)\prod_{k<l}(z_k - z_l)^m \; e^{-\sum_{i=1}^{n}\frac{|z_i|^2}{4l^2}}$$

- Heuristic explanation of quasi-hole: If η was a dynamical variable, as opposed to a parameter, this is just the original wavefunction with an extra electron at position η. But because η is not a dynamical variable, but instead a parameter, it's really a Laughlin wavefunction that describes a deficit of a single electron at position η. This means that m holes act like a deficit of a single electron, so a single quasi-hole is $1/m$ of an electron.

Anyons and the Braid group

* Exchanging particles in 3D is trivial which results in particles called bosons and fermions but when $d = 2$ world lines of particles wind around one another. These particles are called anyons and follow fractional statistics. $\psi(r_1, r_2) = \ e^{i\alpha}\ \psi(r_2, r_1)$
* Mathematically, what's going on is that in dimensions d ≥ 3, the exchange of particles must be described by a representation of the permutation group. But, in d = 2 dimensions, exchanges are described as a representation of the braid group.
* The braiding can be done in terms of simplest operations R_i's. The operators satisfy: 1) $R_iR_j = R_jR_i \ |i - j| > 2$ and 2) $R_{i+1}R_iR_{i+1} = R_iR_{i+1}R_i$ (Yang-Baxter relation). The one dimensional representation of this group is $R_i = e^{i\alpha}$