Paramagnetism and Diamagnetism

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August 2024

1 Introduction

2 Pauli Paramagnetism

First note energy of up spins(+) (it's up wrt to the S_z component along +z-direction) and down spins(-) are given as $\varepsilon_{\pm} = \mp \mu_s B$, with $\mu_s = g_s \mu_B m_s$ (where $\mu_B = e\hbar/2mc$ is the Bohr magneton). Each electron contributes to $\pm \mu_B/V$ (depending on if its in the direction or opposite direction of H). This makes our magnetization density as

$$M = -\mu_B(n_+ - n_-) \tag{1}$$

Since the magnetic field splits the degenerate energy levels into two. If the DOS when B = 0, is $g(\varepsilon)$. Then DOS for the up and down spins will be $g_{\pm}(\varepsilon) = g(\varepsilon \mp \mu_B H)/2$. So, we get

$$n_{\pm} = \int d\varepsilon g_{\pm}(\varepsilon) f(\varepsilon) \tag{2}$$

and the chemical potential is fixed by

$$n = n_{+} + n_{-}. (3)$$

Since, the energy ε_{\pm} is very small wrt ε_{F} . We can hence expand the DOS as

$$g_{\pm}(\varepsilon) = \frac{1}{2}g(\varepsilon \mp \mu_B H) = \frac{1}{2}g(\varepsilon) \mp \mu_B H g'(\varepsilon). \tag{4}$$

Hence, when we plug this we get the same relation for chemical potential as without the magnetic field. Hence, we use the same μ as for metals. Our magnetization density now becomes

$$M = \mu_B^2 H \int g'(\varepsilon) f(\varepsilon) d\varepsilon. \tag{5}$$

Doing integration by parts and taking $f'(\varepsilon) = \delta(\varepsilon - \varepsilon_F)$, gives

$$M = \mu_B^2 H g(\varepsilon_F). \tag{6}$$

So, our susceptibility becomes independent of temperature

$$\chi = \frac{\partial M}{\partial H} = \mu_B^2 g(\varepsilon_F). \tag{7}$$