Drude model

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1 Introduction

Drude model considers solid as nucleus fixed but free electrons moving. The momentum of an electron is described by

$$\frac{d\mathbf{p}}{dt} = -\frac{\mathbf{p}}{\tau} + \mathbf{f}(t) \tag{1}$$

2 Plasmons

We find that this model supports collective excitations of the whole system, which we call plasmon. First we study the behaviour of this system under a time varying electric field of the form $\mathbf{E} = \mathbf{E}(\omega)e^{-i\omega t}$. Note a time varying field also creates a time varying magnetic field, but the induced magnetic field is very small because of the factor v/c.

$$\frac{d\mathbf{p}}{dt} = -\frac{\mathbf{p}}{\tau} - e\mathbf{E}(\omega)e^{-i\omega t} \tag{2}$$

Take $\mathbf{p} = \mathbf{p}(\omega)e^{-i\omega t}$

$$\implies -i\omega\mathbf{p}(\omega) = -\frac{\mathbf{p}(\omega)}{\tau} - e\mathbf{E}(\omega) \tag{3}$$

$$\mathbf{p}(\omega) = \frac{e\tau\mathbf{E}(\omega)}{i\omega\tau - 1} \tag{4}$$

Since, $\mathbf{j} = -ne\mathbf{p}/m$

$$\implies \mathbf{j}(\omega) = \frac{ne^2\tau}{m}\frac{\mathbf{E}(\omega)}{1 - i\omega\tau} \tag{5}$$

If the space variation of electric field is much larger than the mean free path, then current density at that point satisfies the DC Ohm's law at that point, i.e.,

$$\mathbf{j}(\mathbf{r}, \omega) = \sigma(\omega)\mathbf{E}(\mathbf{r}, \omega) \tag{6}$$

This gives us the frequency dependent conductivity as

$$\sigma(\omega) = \frac{\sigma_0}{1 - i\omega\tau}, \quad \sigma_0 = \frac{ne^2\tau}{m}. \tag{7}$$

To find the frequency-dependent dielectric constant, we use Maxwell's equations

$$-\nabla^2 \mathbf{E} = \nabla \times (\nabla \times \mathbf{E}) = \nabla \times \left(-\frac{1}{c}\frac{\partial \mathbf{H}}{\partial t} \right) = \frac{i\omega}{c} \nabla \times \mathbf{H} = \frac{i\omega}{c} \left(\frac{4\pi}{c} \mathbf{j} + \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} \right) \quad (8)$$

$$\implies -\nabla^2 \mathbf{E} = \frac{i\omega}{c} \left(\frac{4\pi}{c} \sigma(\omega) \mathbf{E} - \frac{i\omega}{c} \mathbf{E} \right) = \frac{\omega^2}{c^2} \left(1 + \frac{4\pi i \sigma}{\omega} \right) \mathbf{E} \quad (9)$$

This gives us wave equation of the form

$$-\nabla^2 \mathbf{E} = \frac{\omega^2}{c^2} \epsilon(\omega) \mathbf{E}, \quad \epsilon(\omega) = \left(1 + \frac{4\pi i \sigma}{\omega} \right). \quad (10)$$

In the limit of high-frequency $\omega\tau \gg 1$

$$\epsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2}, \quad \omega_p^2 = \frac{4\pi n e^2}{m}. \quad (11)$$