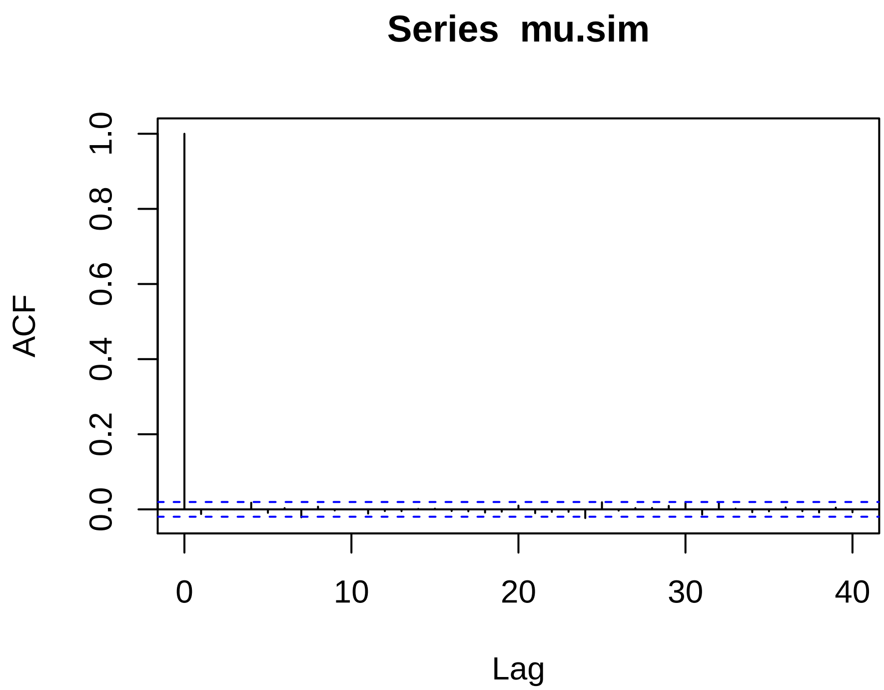
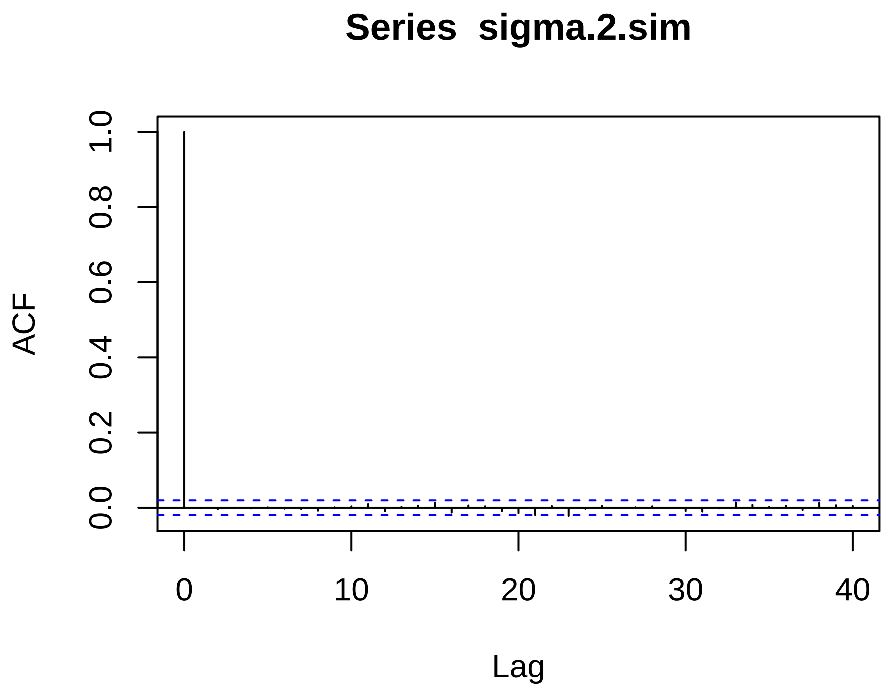
STAT 578 (Spring 2020) HW3 Solution

1. (a)

```r
acf(mu.sim)
```




```r
acf(sigma.2.sim)
```

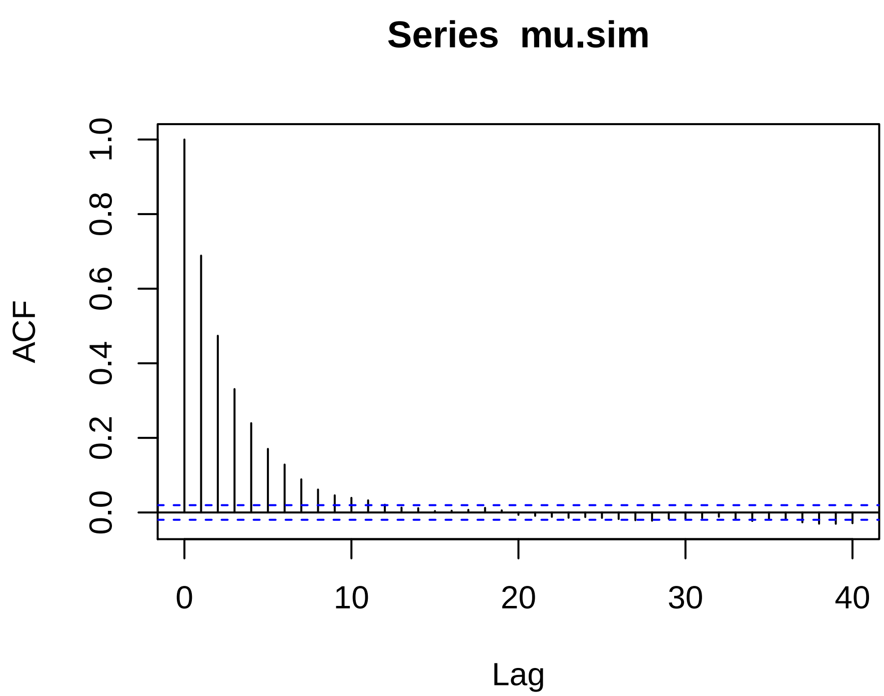


(b) (i) rho = 0.03

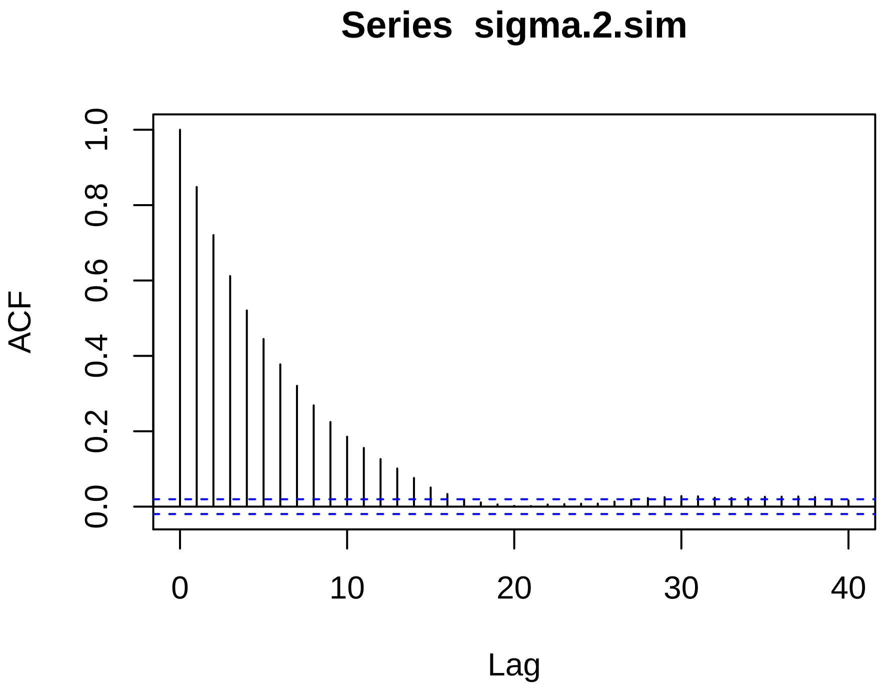
(ii)

```
## [1] 0.3536742
```

```r
acf(mu.sim)
```



```
acf(sigma.2.sim)
```



(c) The Gibbs sampler exhibited faster mixing as the autocorelations for both μ and σ^2 decay much faster in the Gibbs sampler.

2. (a) (i)

```
library(rjags)
```

```
## Loading required package: coda
## Linked to JAGS 4.3.0
## Loaded modules: basemod,bugs
```

```
d <- read.table("~/UIUC/STAT578_20Spring/HW3/polls2016.txt",
                header=TRUE)
d$sigma <- d$ME/2
d$poll <- NULL; d$ME <- NULL

inits <- list(list(mu=100,  tau=100, .RNG.name="base::Wichmann-Hill", .RNG.seed=12),
              list(mu=100,  tau=0.01, .RNG.name="base::Wichmann-Hill", .RNG.seed=34),
              list(mu=-100, tau=100, .RNG.name="base::Wichmann-Hill", .RNG.seed=56),
              list(mu=-100, tau=0.01, .RNG.name="base::Wichmann-Hill", .RNG.seed=78))
m1 <- jags.model("~/UIUC/STAT578_20Spring/HW3/polls20161.bug",
                 d,
                 inits,
                 n.chains=4)
```

```
## Compiling model graph
##    Resolving undeclared variables
##    Allocating nodes
## Graph information:
##    Observed stochastic nodes: 7
##    Unobserved stochastic nodes: 9
```

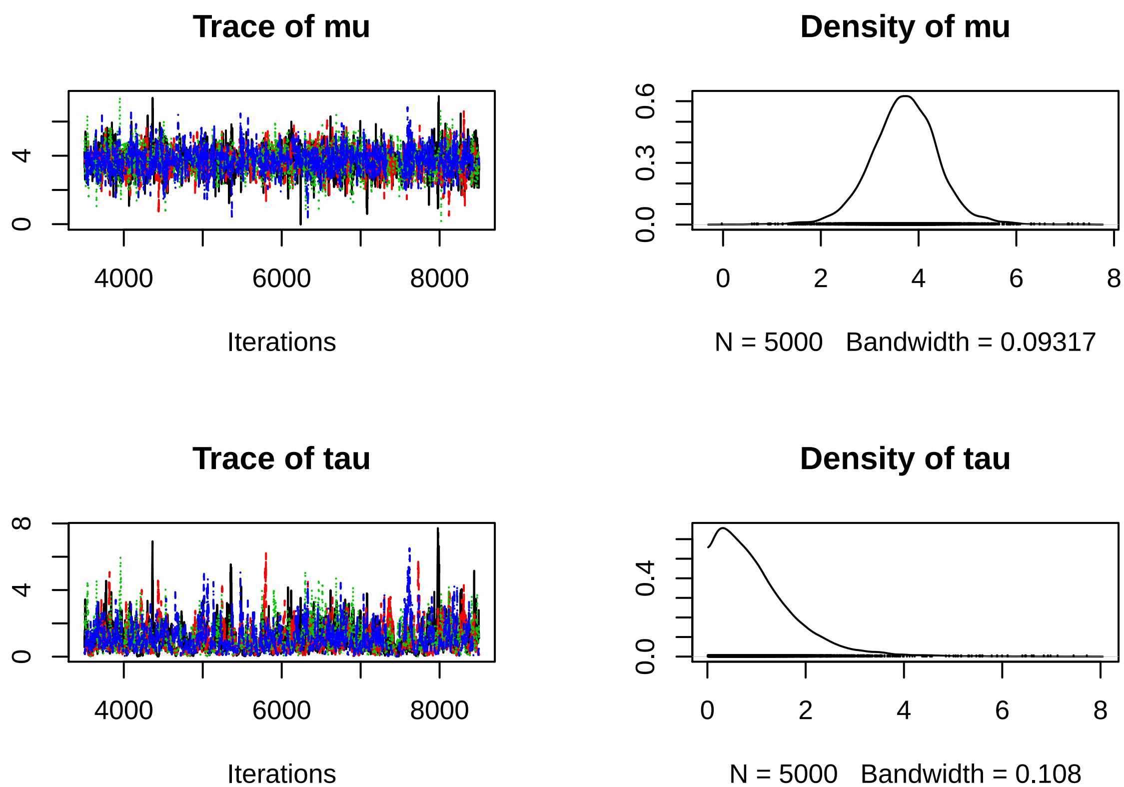
```
##    Total graph size: 42
##
## Initializing model
```

(ii)

```r
update(m1, 2500)  # burn-in
x1 <- coda.samples(m1, c("mu","tau"), n.iter=5000)
```

(iii) From the trace plots of mu and tau, there do not seem to be convergence issues.

```r
plot(x1, smooth=FALSE)
```



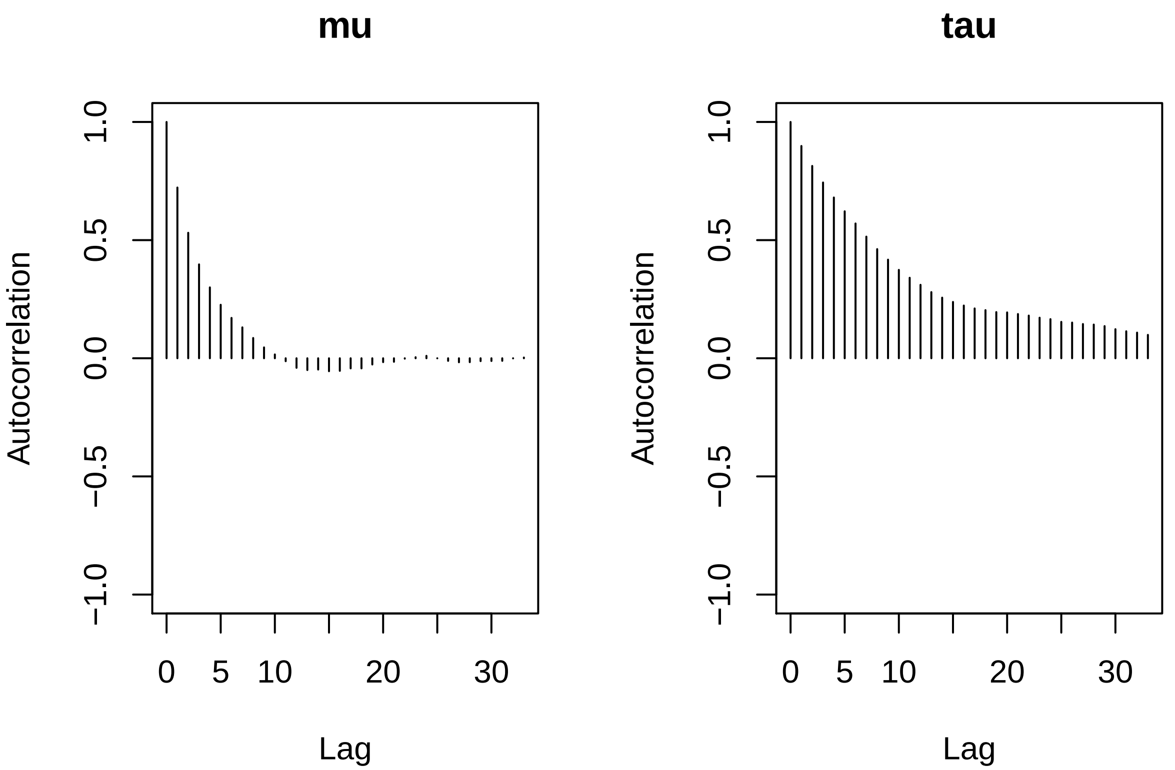
(iv) From the Gelman-Rubin statistics, there do not seem to be any convergence issues. Both Gelman-Rubin statistics are very close to 1.

```r
gelman.diag(x1, autoburnin=FALSE)
```

```
## Potential scale reduction factors:
##
##     Point est. Upper C.I.
## mu           1       1.00
## tau          1       1.01
##
## Multivariate psrf
##
## 1
```

(v) mu mixes much faster than tau. The autocorrelation for mu decreases to close to 0 after 20 iterations. The autocorrelation for tau is higher, and decreases to less than 0.25 after 20 iterations.

```
autocorr.plot(x1[[1]])
```



(vi) The effective sample sizes are considered adequate, since they are above 400, the recommended value from the lectures.

```
effectiveSize(x1)
```

```
##        mu       tau
## 2357.2008  787.2083
```

(b) (i)

```
model {

  for (j in 1:length(y)) {
    y[j] ~ dnorm(theta[j], 1/sigma[j]^2)
    theta[j] ~ dnorm(mu, 1/tau^2)
  }

  mu ~ dunif(-1000,1000)
  logtau ~ dunif(-100,100)

  tau <- exp(logtau)

}
```

(ii)

```
inits <- list(list(mu=100,  logtau=log(100), .RNG.name="base::Wichmann-Hill", .RNG.seed=12),
              list(mu=100,  logtau=log(0.01), .RNG.name="base::Wichmann-Hill", .RNG.seed=34),
              list(mu=-100, logtau=log(100), .RNG.name="base::Wichmann-Hill", .RNG.seed=56),
              list(mu=-100, logtau=log(0.01), .RNG.name="base::Wichmann-Hill", .RNG.seed=78))
```

```
m2 <- jags.model("~/UIUC/STAT578_20Spring/HW3/polls20162.bug",
                 d,
                 inits,
                 n.chains=4)
```

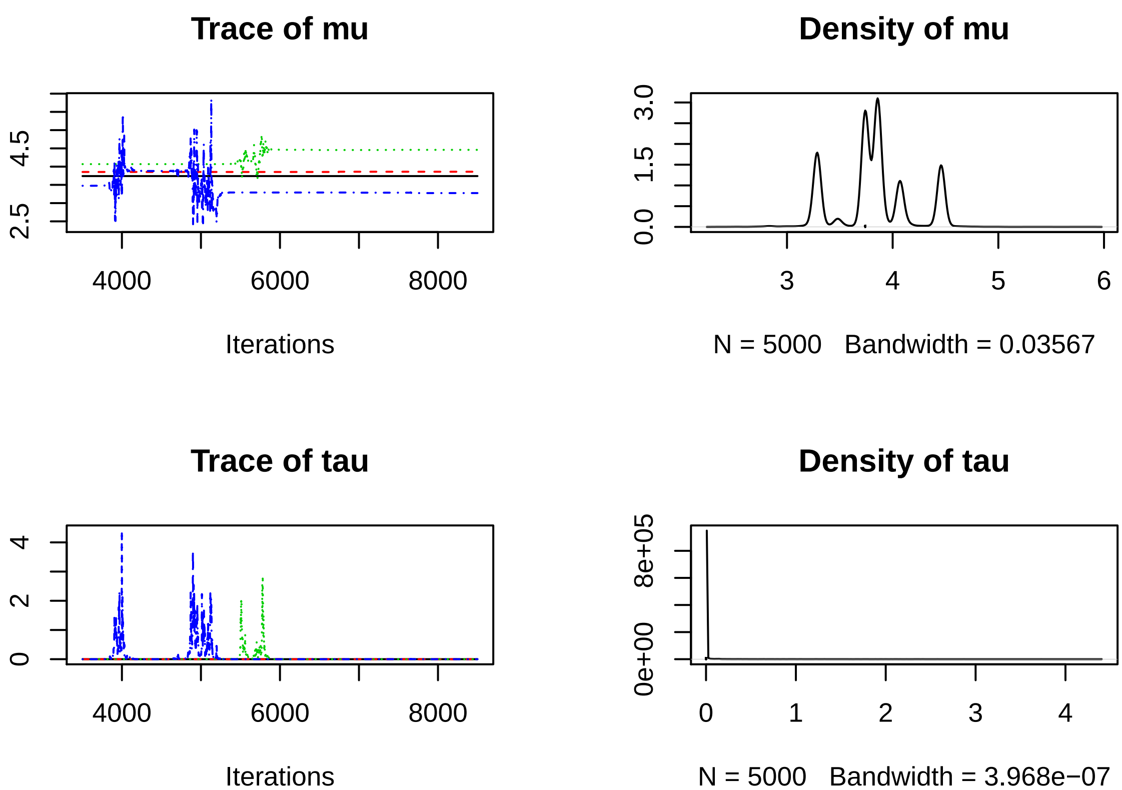
```
## Compiling model graph
##    Resolving undeclared variables
##    Allocating nodes
## Graph information:
##    Observed stochastic nodes: 7
##    Unobserved stochastic nodes: 9
##    Total graph size: 44
##
## Initializing model
```

(iii)

```
update(m2, 2500)  # burn-in
x2 <- coda.samples(m2, c("mu","tau"), n.iter=5000)
```

(iv) From the trace plots of mu and tau under the new prior, there seems to be major convergence issues for both tau and mu. Not all of the chains are sampling from the same region and fairly evenly over all iterations.

```
plot(x2, smooth=FALSE)
```



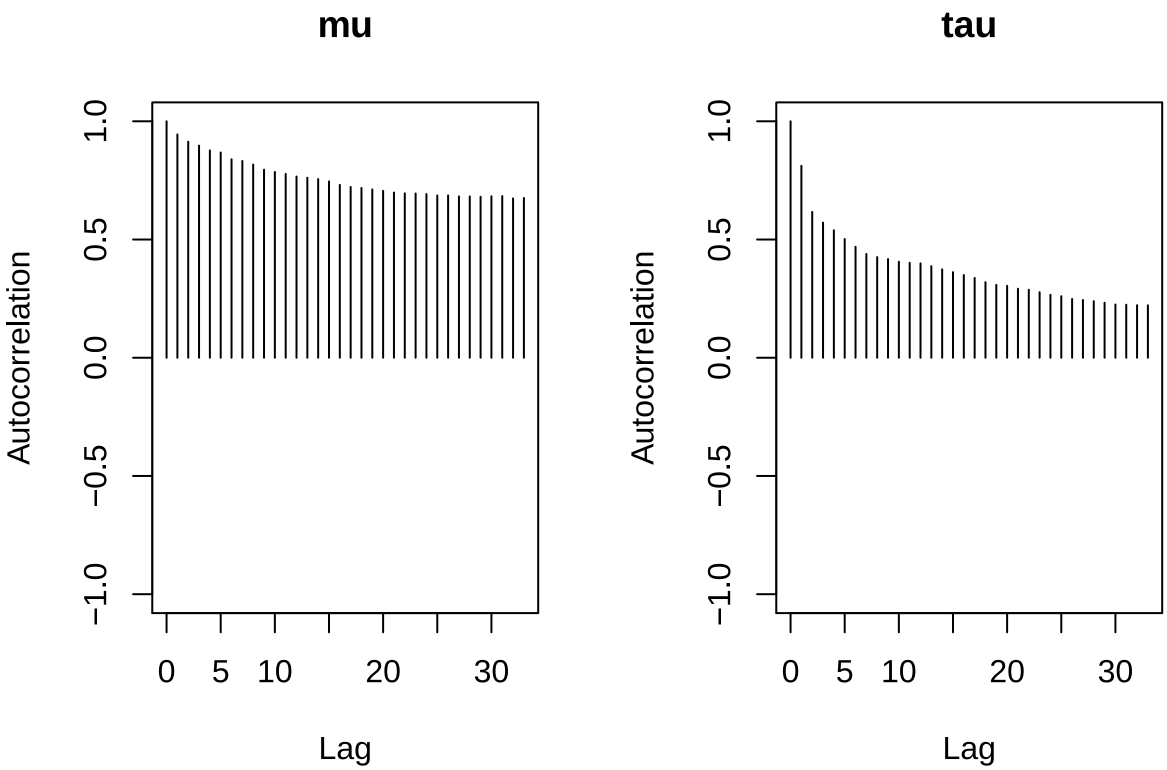
(v) The Gelman-Rubin statistics for both tau and mu are greater than 1.1, which indicates convergence issues.

```
gelman.diag(x2, autoburnin=FALSE)
```

```
## Potential scale reduction factors:
##
##     Point est. Upper C.I.
## mu        2.95       7.89
## tau       1.22       1.51
##
## Multivariate psrf
##
## 2.73
```

(vi) From the autocorrelation plots, we see that mu and tau both mix more slowly than in part (a).

```
autocorr.plot(x2[[1]])
```



(vii) The convergence issues can be explained by the prior we place upon tau. The flat prior on logtau does not translate to a flat prior on tau when it is transformed back. Half of the taus drawn from this implied prior distribution are very close to 0, while the other half are very large. This causes the large jumps we observe in our trace plots, as sometimes the taus are tiny, and sometimes they are enormous. If we place on improper flat prior on logtau, then the posterior would be improper. It should not be entirely surprising that we see convergence problems when approximating this prior.