REYES Nuclear Physics Mentoring Week 3

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1 Introduction

This week's discussion began with the discussion on scattering amplitude. Scattering Amplitude $\mathbb{M}$ characterizes probability of an interaction to occur.

$i\mathbb{M} \propto \langle final|\mathbb{S}-1|initial\rangle$

Where $\mathbb{S}$ is known as S matrx. And we subtract 1 from the S matrix to remove identity which means the condition of no interaction between the particles.

The S-matrix or scattering matrix relates the initial state and the final state of a physical system undergoing a scattering process.

1.1 Scattering Theory

Model independent features of scattering amplitudes are:

1. Spacetime Symmetry -Loretnz Invariance
2. Internal Symmetry -Flavour,Baryon Number
3. probability conservation -The S matrix is unnitary operator i.e $\mathbb{S}^{\dagger}\mathbb{S} = I$
4. causality - Amplitudes are boundary values of analytic functions in complex energy plane.
5. CPT Symmetry - Relates particle—anti-particles in scattering processes.

1.1.1 Probability Conservation

The S matrix is unnitary operator i.e $\mathbb{S}^{\dagger}\mathbb{S} = I$. After some work we can show that(in a limited energy region) $Im\mathbb{M} = \rho|\mathbb{M}|^2$.

ρ is known as phase space kinematic function. Characterizes on-shell scattering of two-particles.

$$\rho = \frac{\xi q^{\star}}{8\pi E^{\star}}$$

also $q^{\star} = \frac{1}{2}\sqrt{E^{\star 2} - 4m^2}$ with $\xi = \begin{cases} \frac{1}{2} & identical \\ 1 & otherwise \end{cases}$

1.1.2 Phase Shift

At a fixed energy, amplitude determined by magnitude and phase (2 real numbers). Therefore scattering amplitude, $\mathbb{M} = |\mathbb{M}|e^{i\delta}$

Imposing unitarity $Im\mathbb{M} = \rho|\mathbb{M}^2|$ we get $|\mathbb{M}| = \frac{1}{\rho}sin\delta$. such that $\mathbb{M} = \frac{1}{\rho}e^{i\delta}sin\delta$

Here δ is the phase shift.

1.1.3 K-Matrix

We know that $Im\mathbb{M} = \rho|\mathbb{M}|^2$

This implies that $Im\mathbb{M}^{-1} = -\rho$

$\implies \mathbb{M}^{-1} = K^{-1} - i\rho$

$\mathbb{M} = K\dfrac{1}{1 = i\rho K}$

Note: K matrix can be related to phase shift by $K = \rho cot\delta$

2 Exercises

2.1 Exercise 1

Plot ρ(phase space) for identical particles in the range $1.8 \leq \dfrac{E*}{m} \leq 3.2$

Solution:

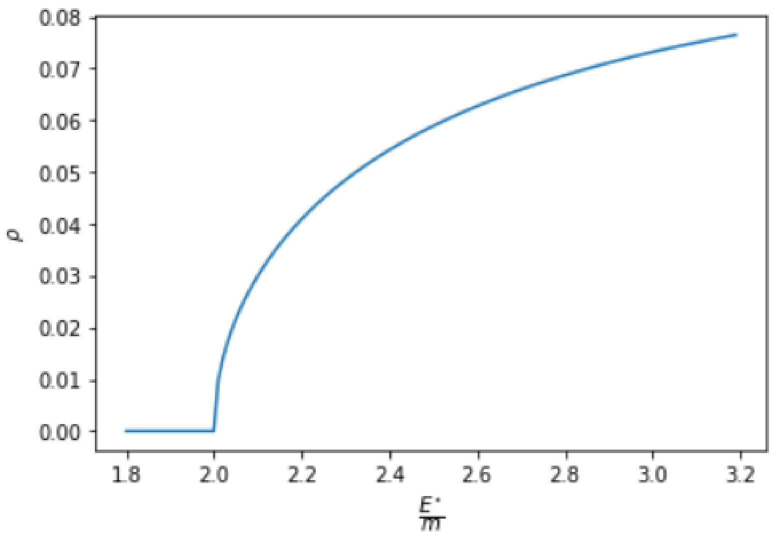
$\rho = \dfrac{\xi q^\star}{8\pi E^\star}$

For identical particles : $\xi = 1/2$

and $q^\star = \dfrac{1}{2}\sqrt{E^{\star 2} - 4m^2}$

This makes it $\rho = \dfrac{1}{32\pi}\dfrac{\sqrt{E^\star - 4m^2}}{E^\star}$

$\rho = \dfrac{1}{32\pi}\sqrt{1 - 4(\dfrac{m}{E^\star})^2}$



Listing 1: Code used

```
import matplotlib.pyplot as plot
import numpy as np
import cmath as cm

def f(x):
```

```
    return (1/32*cm.pi)*cm.sqrt(1 - (4*x**(-2)))

f = np.vectorize(f)
x = np.arange(1.8,3.2,0.01)
y = f(x)
plt.plot(x,np.real(y))
plt.xlabel(r'$\dfrac{E^{\star}}{m}$')
plt.ylabel(r'$\rho$')
```

2.2 Exercise 2

Derive the phase shift representation for the scattering amplitude.

We know that $\mathbb{M} = |\mathbb{M}|e^{i\delta}$, using euler's formula, can be expanded as

$$\mathbb{M} = |\mathbb{M}|(cos\delta + isin\delta)$$

.

We also know that

$$Im\mathbb{M} = \rho|\mathbb{M}|^2]$$

So from the above two relations we get that

$$|\mathbb{M}|sin\delta = \rho|\mathbb{M}|^2 \implies \rho = \frac{sin\delta}{|\mathbb{M}|}$$

Substituting $\mathbb{M} = |\mathbb{M}|e^{i\delta}$ in the above result we get:

$$\mathbb{M} = \frac{1}{\rho}e^{i\delta}sin\delta$$

2.3 Exercise 3

Show that $Im\mathbb{M}^{-1} = -\rho$

Ww know that $\mathbb{M} = |\mathbb{M}|e^{i\delta}$.

Right multiplication by $\mathbb{M}^{-1}$.

$$\mathbb{M}\mathbb{M}^{-1} = e^{i\delta}|\mathbb{M}|\mathbb{M}^{-1} = I$$

$$\implies \mathbb{M}^{-1} = \frac{I}{e^{i\delta}|\mathbb{M}|} = \frac{1}{e^{i\delta|\mathbb{M}|}} = \frac{e^{-i\delta}}{|\mathbb{M}|}$$

$$\mathbb{M}^{-1} = \frac{e^{-i\delta}}{|\mathbb{M}|} = \frac{cos(-\delta) + isin(-\delta)}{|\mathbb{M}|} = \frac{\rho(cos\delta - isin\delta)}{sin\delta} \implies Im\mathbb{M}^{-1} = -\rho$$

2.4 Exercise 4

Show that $K^{-1} = \rho cot\delta$

$$\mathbb{M}^{-1} = \frac{\rho cos\delta - i\rho sin\delta}{sin\delta} = K^{-1} - \rho i \implies K^{-1} = \frac{\rho cos\delta}{sin\delta} = \rho cot\delta$$