Portfolio Design

ECE367 PS05 Problem 5.7 -- Aman Bhargava

Problem Setup

- $p \in \mathbb{R}^n$; $\sum_{i=1}^n p_i = 1$. $p_i \geq 0 \; \forall \; i \in [n]$
- Each stock has expected returns in $\bar{x} \in \mathbb{R}^n$.
- $\mathbb{E}[(\bar{x}_i - x_i)^2] = \Sigma_{ii}$
- $\mathbb{E}[(x - x_i)(x - x_j)] \rightarrow \Sigma_{ij}$

- Variance of investment p is $p^T \Sigma p$. We wish to minimize variance of investment with minimum expected return $r_{\min}$.

Part A: See Graph Below

Part B: See Graph Below

The primary benefit of diversification is to lower risk. The less correlated the investments are, the more they reduce risk. That being said, one can achieve higher expected values if they pursue less diversified (and therefore more risky) strategies, though it is questionable whether or not the incremental expected value is worth the incremental risk.

In [1]:
```
# IMPORT BOX #

using LinearAlgebra
using Plots
using JuMP, Ipopt
import GLPK

using Convex, SCS
```

In [2]:
```
x̄ = [1.1; 1.35; 1.25; 1.05]

Σ = [0.2 -0.2 -0.12 0.02;
     -0.2 1.4 0.02 0;
     -0.12 0.2 1 -0.4;
     0.02 0 -0.4 0.2]

Σ+=transpose(Σ)
Σ*=0.5
```

Out[2]:
```
4×4 Array{Float64,2}:
  0.2   -0.2    -0.12   0.02
 -0.2    1.4     0.11   0.0
 -0.12   0.11    1.0   -0.4
  0.02   0.0    -0.4    0.2
```

In [3]:
```julia
# Let us first make the Convex.jl module available

function get_var(r)
    # Generate random problem data
    m = 4;  n = 4

    x = Variable(4)
    r_imposed = r

    # This can be done by: minimize(objective, constraints)
    problem = minimize(sum(quadform(x, Σ)), [sum(x) == 1, x >= 0, sum(x̄.*x) >= r

    # Solve the problem by calling solve!
    solve!(problem, SCS.Optimizer)
    return problem.optval, evaluate(x)
end
```

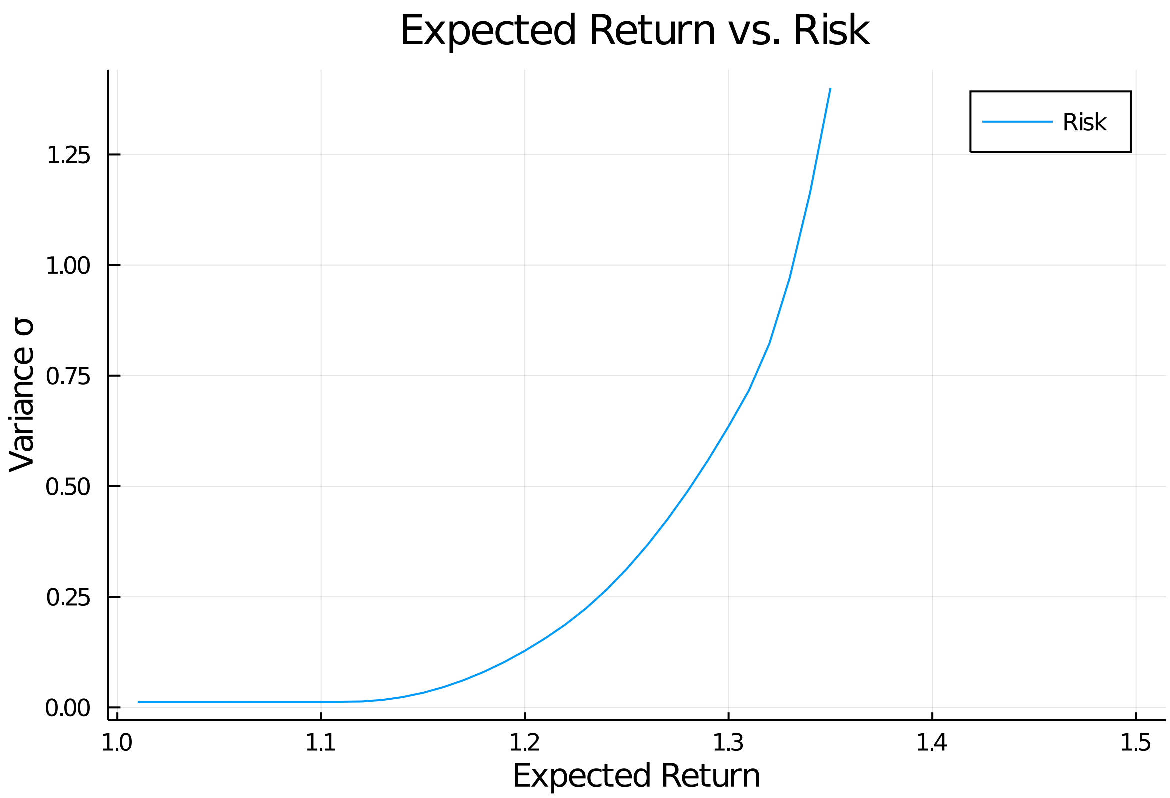
Out[3]: get_var (generic function with 1 method)

In [ ]:
```julia
rs = (1:50)/100 .+1
risk = zeros(50)

xmat = zeros(50, 4) # Matrix to store portfolio compositions

for i = 1:50
    risk[i], xmat[i,:] = get_var(rs[i])
end
```

Plot A

In [5]:
```julia
plot(rs, risk, labels="Risk")
title!("Expected Return vs. Risk")
xlabel!("Expected Return")
ylabel!("Variance σ")
```

Out[5]:

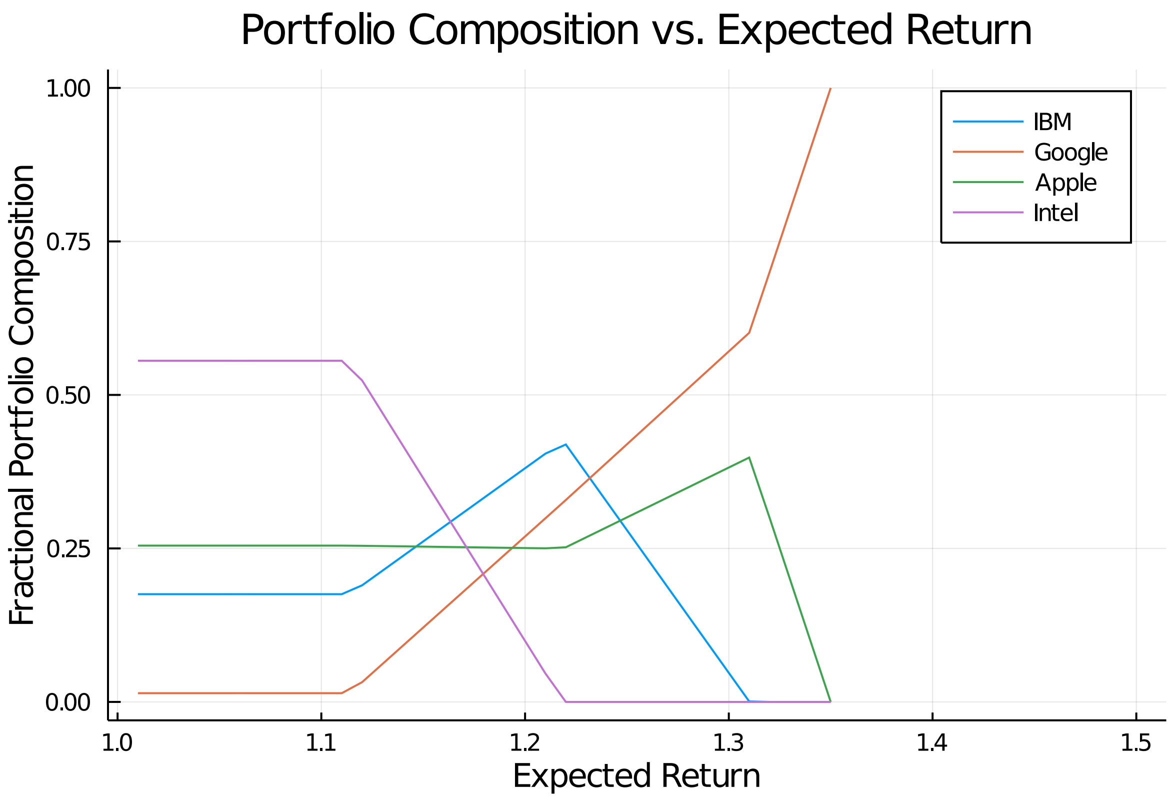


Plot B

In [6]:

```
ls = ["IBM", "Google", "Apple", "Intel"]

plot(rs, xmat[:,1], label=ls[1])
plot!(rs, xmat[:,2], label=ls[2])
plot!(rs, xmat[:,3], label=ls[3])
plot!(rs, xmat[:,4], label=ls[4])

title!("Portfolio Composition vs. Expected Return")
xlabel!("Expected Return")
ylabel!("Fractional Portfolio Composition")
```

Out[6]:



Portfolio Composition vs. Expected Return
Fractional Portfolio Composition
1.00
0.75
0.50
0.25
0.00
1.0
1.1
1.2
1.3
1.4
1.5
Expected Return
IBM
Google
Apple
Intel

In [ ]:

In [ ]:

In [ ]:

In [ ]:

In [ ]: