PS01 Graded Problems

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Problem 1.10: Word Vector Angle

- Set Up:
  - D = set of documents $\{d_1, \dots, d_{10}\}$
  - $W = \bigcup_{i=1} \text{words}(d_i)$ = "Lexicon", alphabetically ordered.
  - $f_{term}(t,d)$ = # of occurrences of w_t in d_d → "term frequency"
  - 'wordVecWords.txt' = W, elements on new lines. Len: $1651 = |W|$
  - 'wordVecV.mat' → $V \in \mathbb{N}^{1651 \times 10}$

$$V = \begin{array}{c} \text{word } 1 \\ \vdots \\ \text{word } 1651 \end{array} \begin{bmatrix} f_{term}(1,1) & \cdots & f_{term}(1,10) \\ \vdots & & \\ f_{term}(1651,1) & \cdots & f_{term}(1651,10) \end{bmatrix}$$

- Part A: Which pair of word vecs has smallest $\{$angle, distance$\}$ under ℓ_2? Why might they be the same/different?

$$v_d = \begin{bmatrix} f_{term}(1,d) \\ \vdots \\ f_{term}(1651,d) \end{bmatrix}$$

pdist(X) → squareform$(\cdot)$ → yields matrix w/ Euclidian dist b/w of X $\{row_i, row_j$ specified at index $i,j\}$ Find smallest.

transpose of V

pdist$(X$, 'cosine'$)$ yields $1-\cos(\theta_{x_i,x_j})$...

↳ Smallest angle would have $\cos \approx 1$

⇓

$1-\cos(\theta) \approx 0$

Therefore we search for

- Part B: Repeat with $\tilde{v}_d = \dfrac{v_d}{\sum_{t=1}^{|W|} f_{term}(t,d)} = \dfrac{v_d}{\text{sum}(v_d)}$ → Create new normalized version of W ☺

- Part C: $f_{doc}(t) = \sum_{d=1}^{D} \mathbb{I}[f_{term}(t,d) > 0]$ = How many docs does word t occur in? = "doc freq" → create $f\_doc = \begin{bmatrix} f_{doc}(1) \\ \vdots \\ f_{doc}(|W|) \end{bmatrix}$

$$w(t,d) = \frac{\overbrace{f_{term}(t,d)}^{V[t,d]}}{\underbrace{\sum_{t=1}^{|W|} f_{term}(t,d)}_{\text{len of doc } d}} \sqrt{\log\left(\frac{\overbrace{|D|}^{const=10}}{\underbrace{f_{doc}(t)}_{f_{doc}[t]}}\right)}$$

= term-frequency inverse document frequency score (TF-IDF) → Make array of zeros, iterate through.

now $w_d = \begin{bmatrix} w(1,d) \\ w(2,d) \\ \vdots \\ w(|W|,d) \end{bmatrix}$, $d \in [|D|]$

Repeat ~~2~~ 1 steps from part A. Smallest euclidian dist pair = (?, ?)

- Part D: Why use $w()$? What is it doing geometrically?

Problem 1.11: $f_i: \mathbb{R}^2 \to \mathbb{R}$, $i \in [3]$

- Part A: Write $\nabla f_i = \begin{bmatrix} \frac{\partial f_i}{\partial x} \\ \frac{\partial f_i}{\partial y} \end{bmatrix}$ for each

f_1: $\begin{bmatrix} 2 \\ 3 \end{bmatrix} = \nabla f_1$

@(1,1): $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$

@(1,0): $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$

$f_x x + f_y y = 0$

$\cancel{f_y} y = -\dfrac{f_x}{f_y} x$

f_2: $\nabla f_2 = \begin{bmatrix} 2x - y \\ 2y - x \end{bmatrix}$

$\nabla f_2(1,1) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$\nabla f_2(1,0) = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$

f_3: $\nabla f_3 = \begin{bmatrix} \cos(y-5) - (y-5)\cos(x-5) \\ -(x-5)\sin(y-5) - \sin(x-5) \end{bmatrix} = \begin{bmatrix} \cos(y-5) + (5-y)\cos(x-5) \\ (5-x)\sin(y-5) - \sin(x-5) \end{bmatrix}$

@(1,1): $\begin{bmatrix} \cos(-4) + 4\cos(-4) \\ 4\sin(-4) - \sin(-4) \end{bmatrix} = \begin{bmatrix} 6\cos(-4) \\ 3\sin(-4) \end{bmatrix}$

@(1,0): $\begin{bmatrix} \cos(-5) + 5\cos(-4) \\ 4\sin(-5) - \sin(-4) \end{bmatrix}$

- Part B: Make contour plots for each f_i in the range $-2 \leq x, y \leq 3.5$
  - ↳ Meshgrid: $[X, Y]$ = meshgrid(x, y) → use X, Y as sym vars to plot 3D funcs ☺ (vecs of x coords, y coords)
  - ↳ contour: Just generates contour plots.
  - ☑ ↳ Compute gradient @$(x,y) = (1,0)$ for each and plot on contour plots.
  - ☑ ↳ Draw on tangent line to contour @ (1,0).

Part C: Plot 3D linear approximation to each function @(1,0)

$ax + by + cz = d$

$z = -\dfrac{ax}{c} - \dfrac{by}{c} + d$