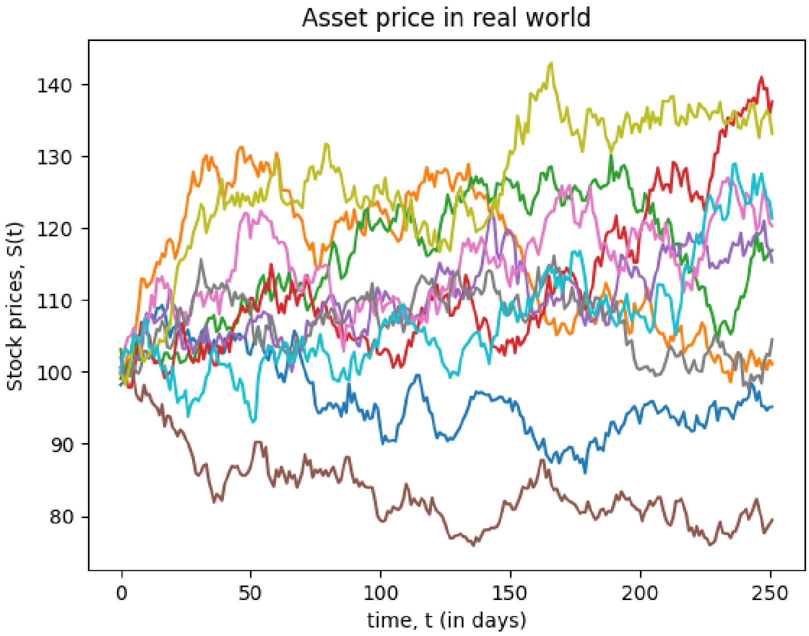
MA 374: Financial Engineering Lab
Lab 10

Aman Bucha
200123006

Question 1:

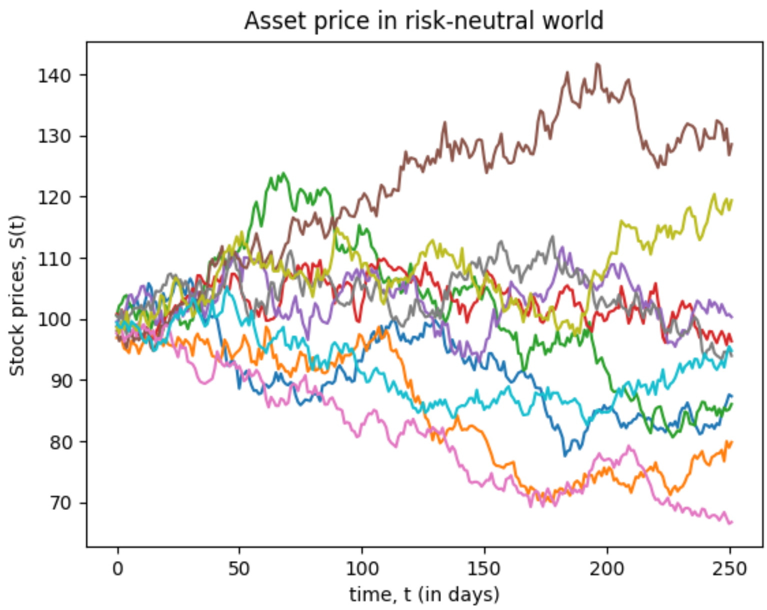
10 different paths of the asset price making use of GBM in real world is:



The evolution of the asset price in real world is governed by following differential equation:

$$dS = \mu S dt + \sigma S dW(t)$$

Again, 10 different paths of the asset price making use of GBM in risk-neutral world is:



The evolution of the asset price in the risk-neutral world is governed by following differential equation:

$$dS = rSdt + \sigma S dW^*(t)$$

where,

W* is a Brownian motion under risk-neutral probability

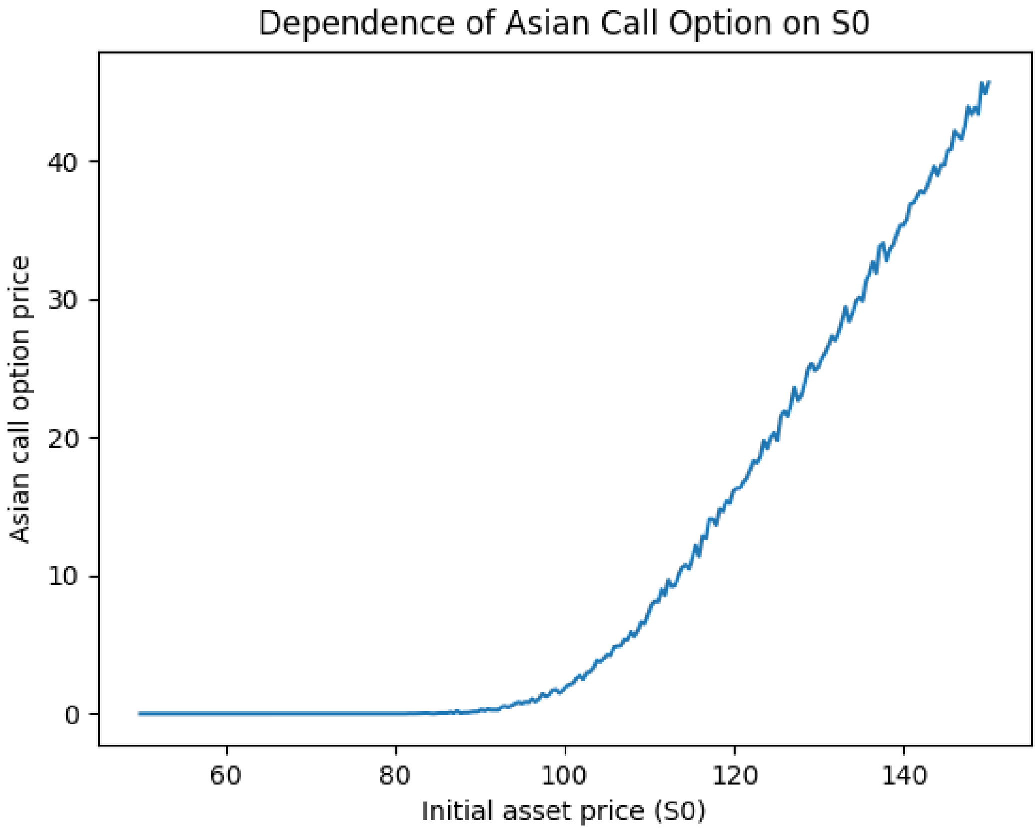
The prices of a six month fixed-strike Asian option with various strike prices are:

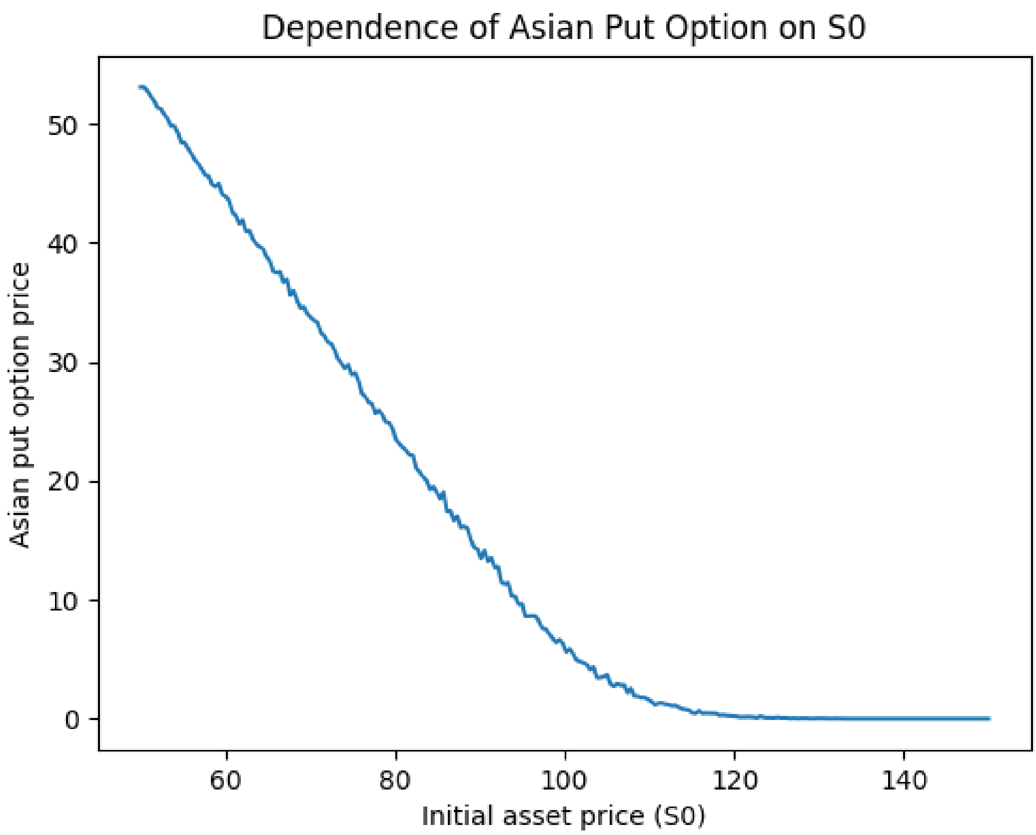
```
************** For K = 90 **************
Asian call option price                = 10.858681040880136
Variance in Asian call option price    = 58.26036082140784

Asian put option price                 = 0.2731084135551703
Variance in Asian put option price     = 1.3988165494129436
```

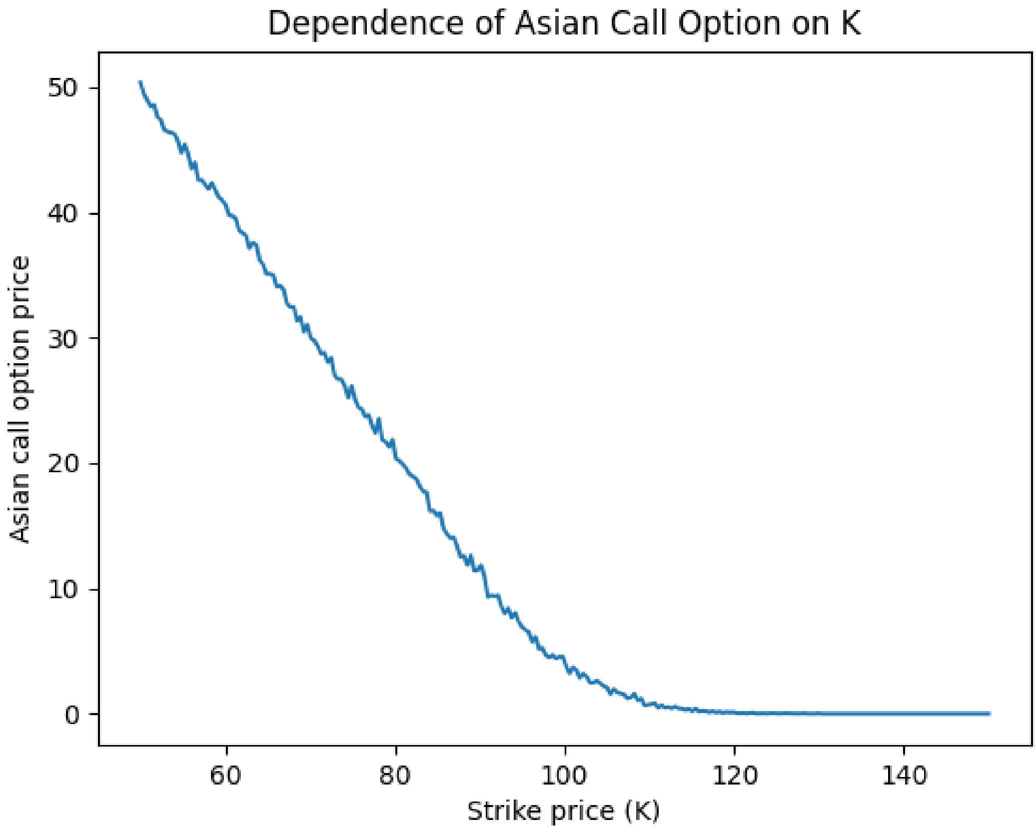
```
************** For K = 105 **************
Asian call option price                = 1.5395902000587407
Variance in Asian call option price    = 11.078701154469949

Asian put option price                 = 5.524384655109961
Variance in Asian put option price     = 31.43480739418062
```

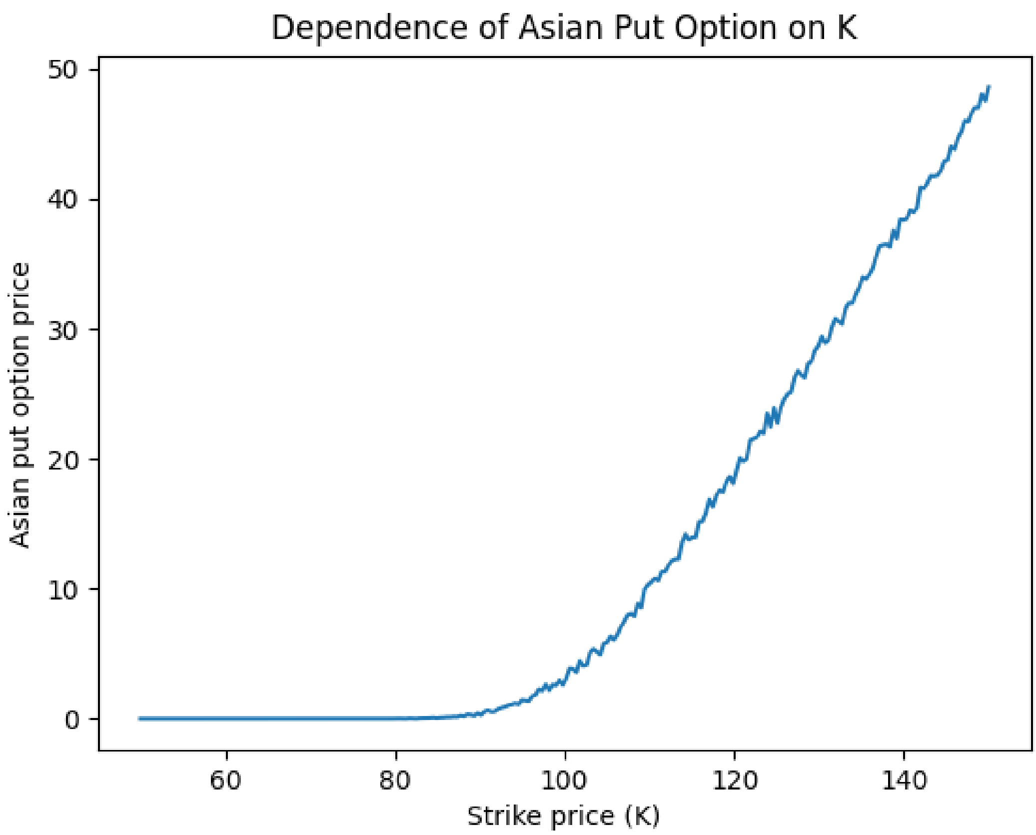
```
************** For K = 110 **************
Asian call option price                = 0.6172584016891576
Variance in Asian call option price    = 4.951312567452068

Asian put option price                 = 9.686886168995734
Variance in Asian put option price     = 50.61212880716365
```

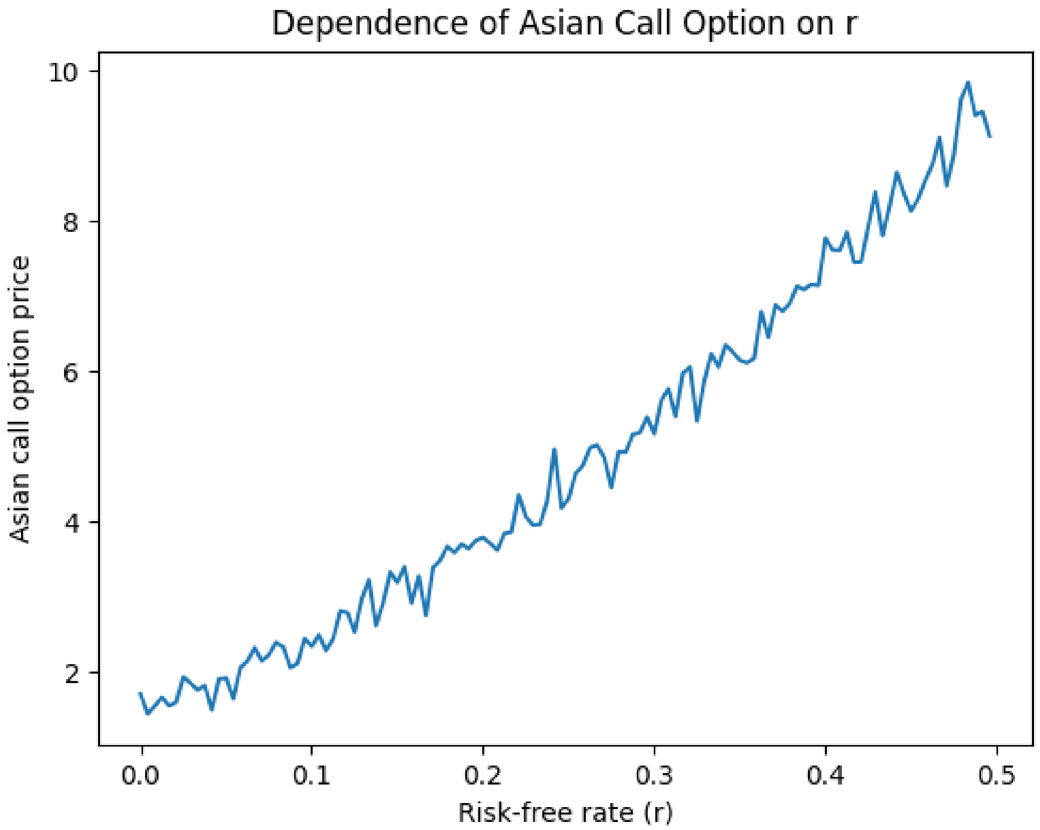
Sensitivity Analysis:


Dependence of Asian Call Option on S0
Asian call option price
0
10
20
30
40
60
80
100
120
140
Initial asset price (S0)

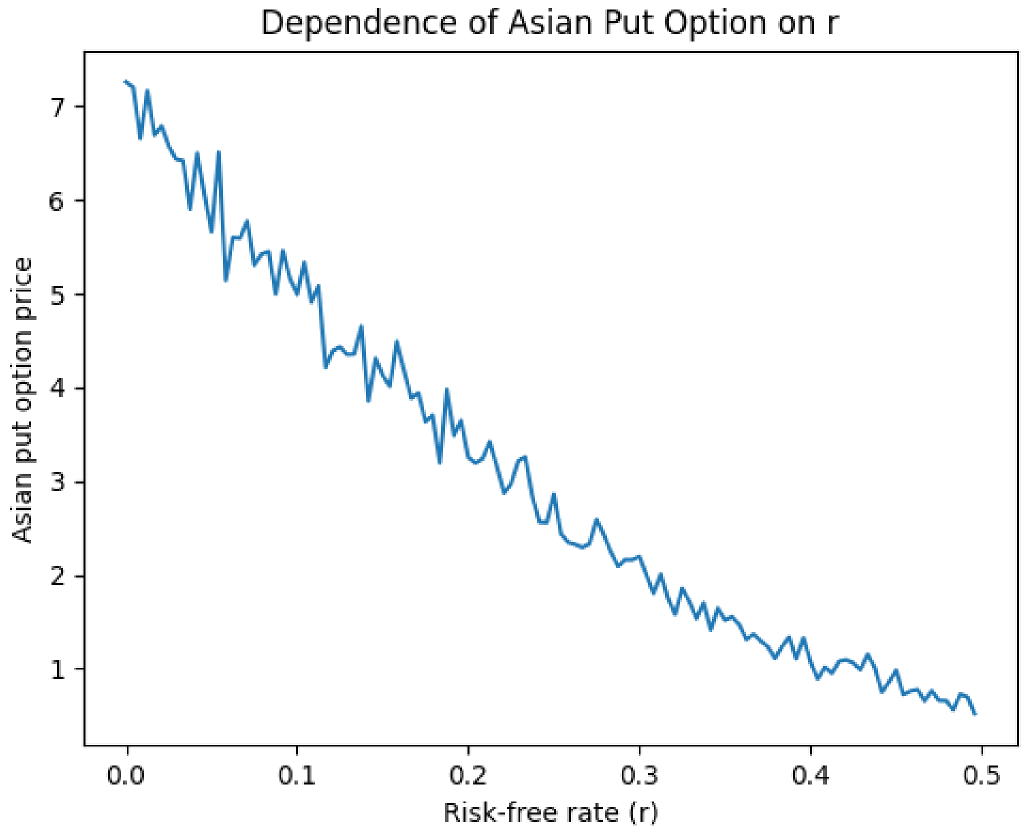

Dependence of Asian Put Option on S0
Asian put option price
0
10
20
30
40
50
60
80
100
120
140
Initial asset price (S0)


Dependence of Asian Call Option on K
Asian call option price
Strike price (K)
0
10
20
30
40
50
60
80
100
120
140

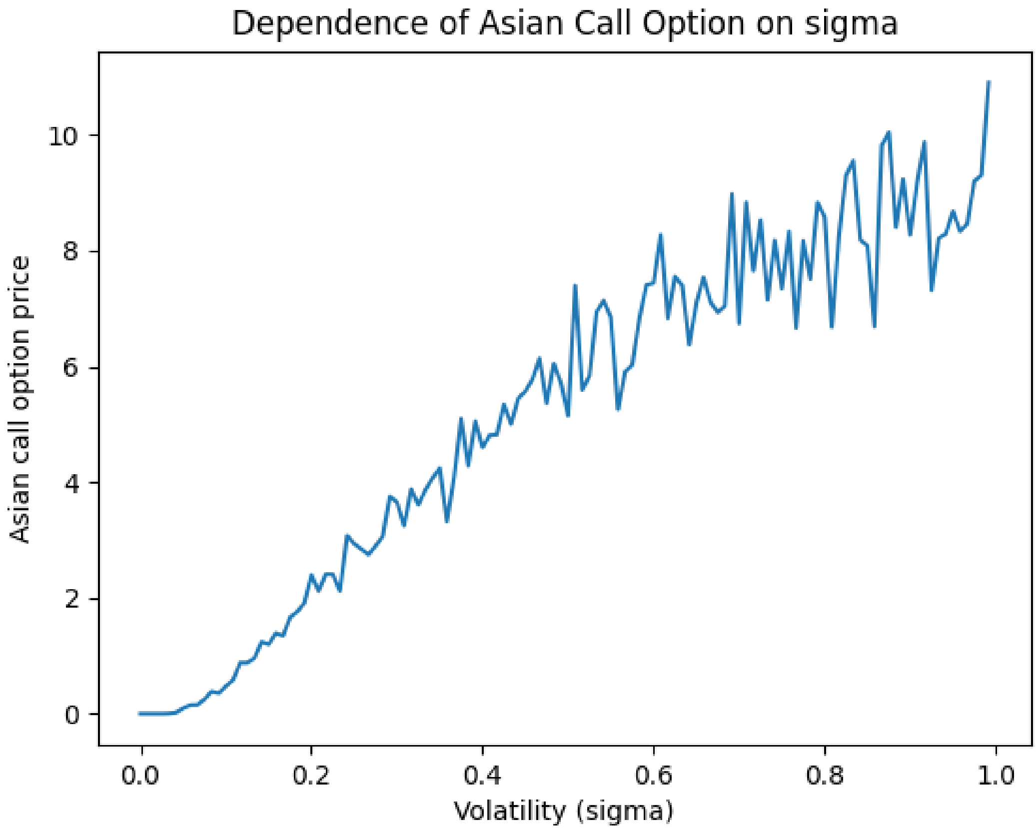

Dependence of Asian Put Option on K
Asian put option price
Strike price (K)
0
10
20
30
40
50
60
80
100
120
140

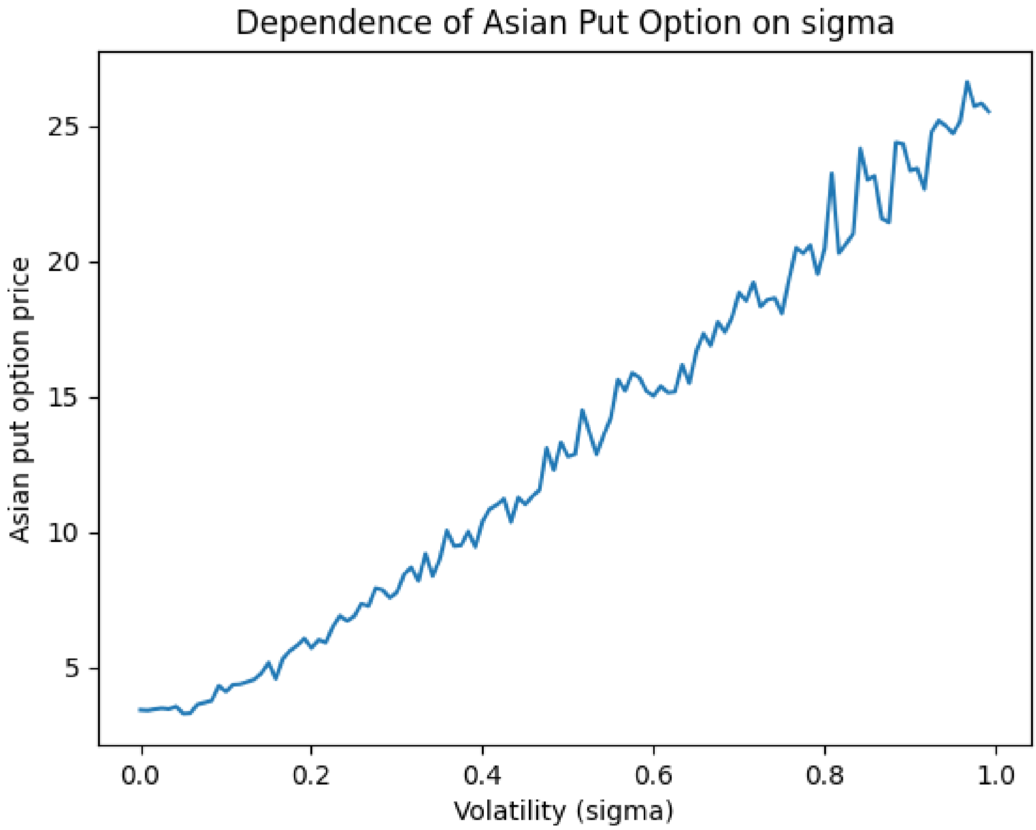


Dependence of Asian Call Option on r
Asian call option price
10
8
6
4
2
0.0
0.1
0.2
0.3
0.4
0.5
Risk-free rate (r)



Dependence of Asian Put Option on r
Asian put option price
7
6
5
4
3
2
1
0.0
0.1
0.2
0.3
0.4
0.5
Risk-free rate (r)


Dependence of Asian Call Option on sigma
Asian call option price
Volatility (sigma)
0
2
4
6
8
10
0.0
0.2
0.4
0.6
0.8
1.0


Dependence of Asian Put Option on sigma
Asian put option price
Volatility (sigma)
5
10
15
20
25
0.0
0.2
0.4
0.6
0.8
1.0

Observations:

1. The price of the call option increases while that of the put option decreases, with an increase in the initial asset price, S_0.

2. The price of the call option decreases while that of the put option increases, with an increase in the strike prices, K.

3. The price of the call option increases while that of the put option decreases, with an increase in the risk free interest, r

4. The price of both call and put option increases with an increase in the volatility.

5. There appears to be some fluctuations in the plots, which we try to minimise using the variance reduction schemes, in the next question.

Question 2:

The prices of a six month fixed-strike Asian option with various strike prices, after performing variance reduction are:

```
*************** For K = 90 ***************
Asian call option price                = 11.091485184660582
Variance in Asian call option price    = 50.303830526814316

Asian put option price                 = 0.24515698059727795
Variance in Asian put option price     = 1.2012522257345026
```

```
*************** For K = 105 ***************
Asian call option price                = 1.609334874424664
Variance in Asian call option price    = 9.7307353292109

Asian put option price                 = 5.619259034464727
Variance in Asian put option price     = 25.36891623473016
```

```
*************** For K = 110 ***************
Asian call option price                = 0.637067999964191
Variance in Asian call option price    = 4.4204369331232884

Asian put option price                 = 9.862709994988204
Variance in Asian put option price     = 37.41935258185477
```

Observations:

The price of both call and put options obtained using both with and without variance reduction, are comparable. The respective variances are compared in the following table:

1. For Call Price:

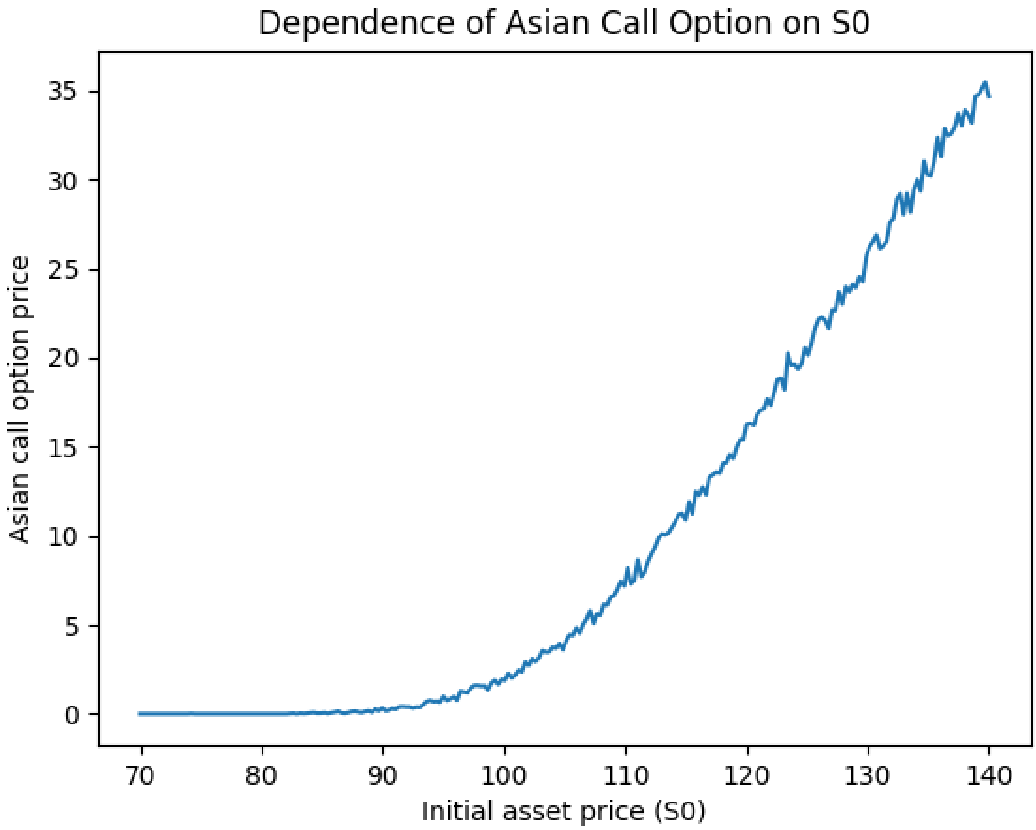
Serial No.	Strike Price (K)	Variance(Without Reduction)	Variance(With Reduction)
1	95	58.26036082140784	50.303830526814316
2	105	11.078701154469949	9.7307353292109
3	110	4.951312567452068	4.4204369331232884

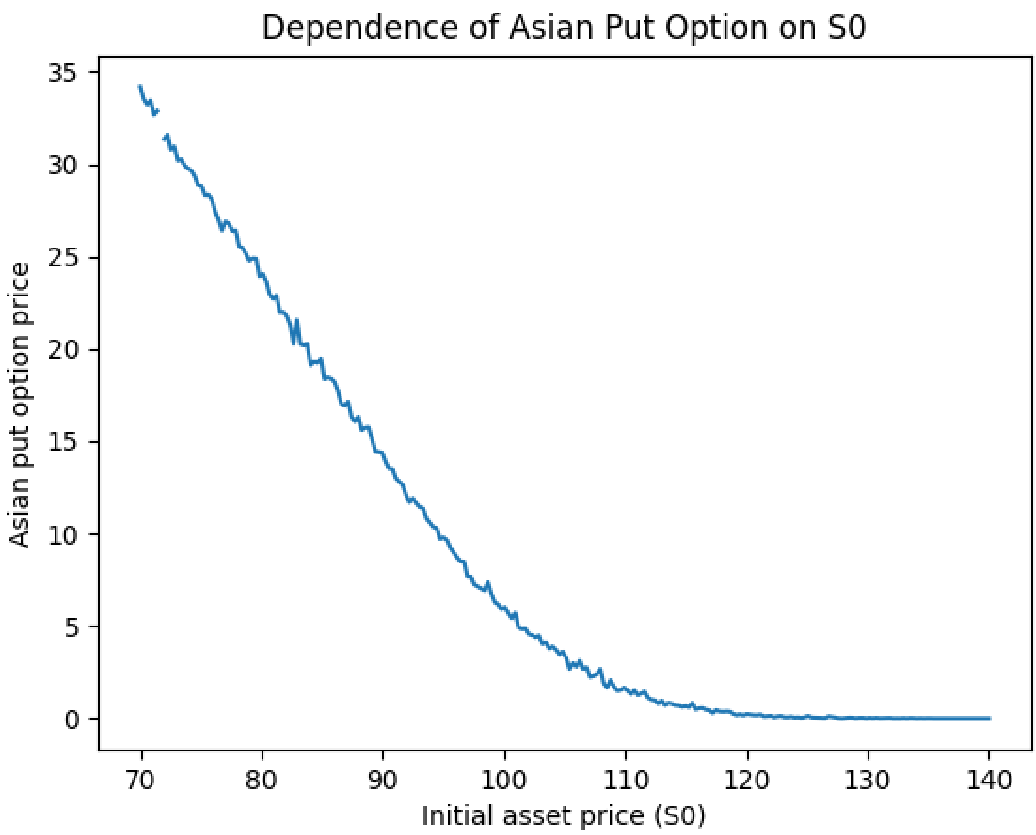
2. For Put Price:

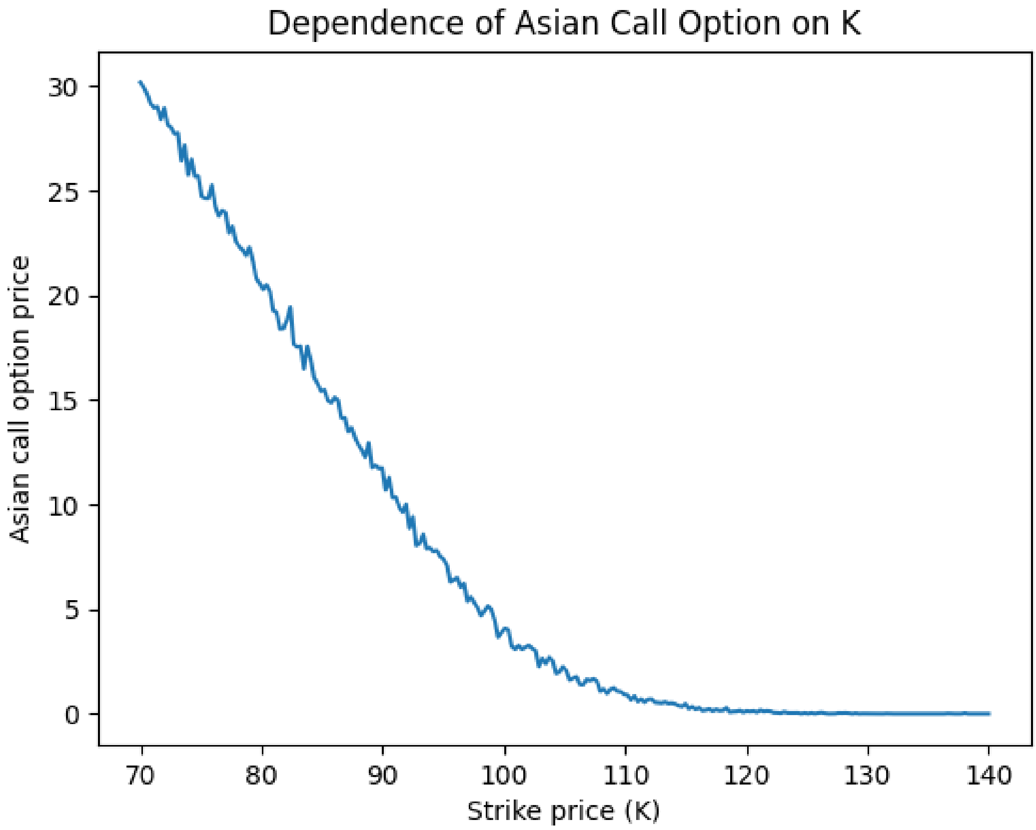
Serial No.	Strike Price (K)	Variance(Without Reduction)	Variance(With Reduction)
1	95	1.3988165494129436	1.2012522257345026
2	105	31.43480739418062	25.36891623473016
3	110	50.61212880716365	37.41935258185477

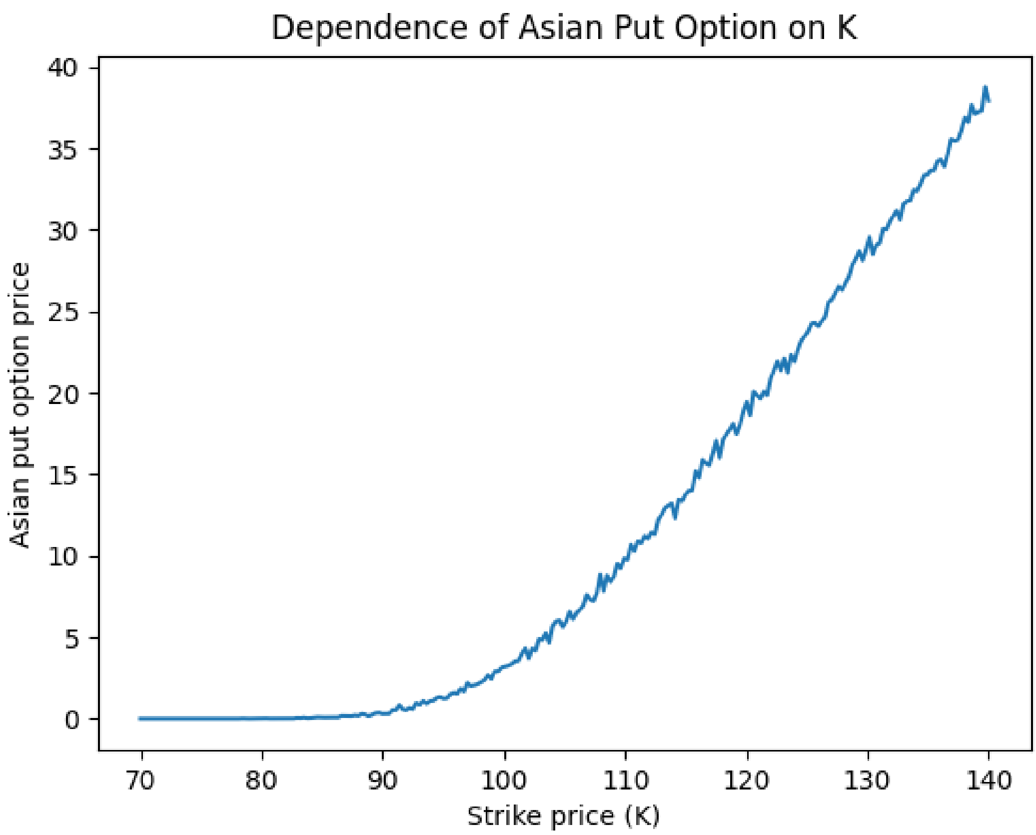
So, we can clearly observe that the variance reduction is successful, and we have reduced the variance in calculating the option prices.

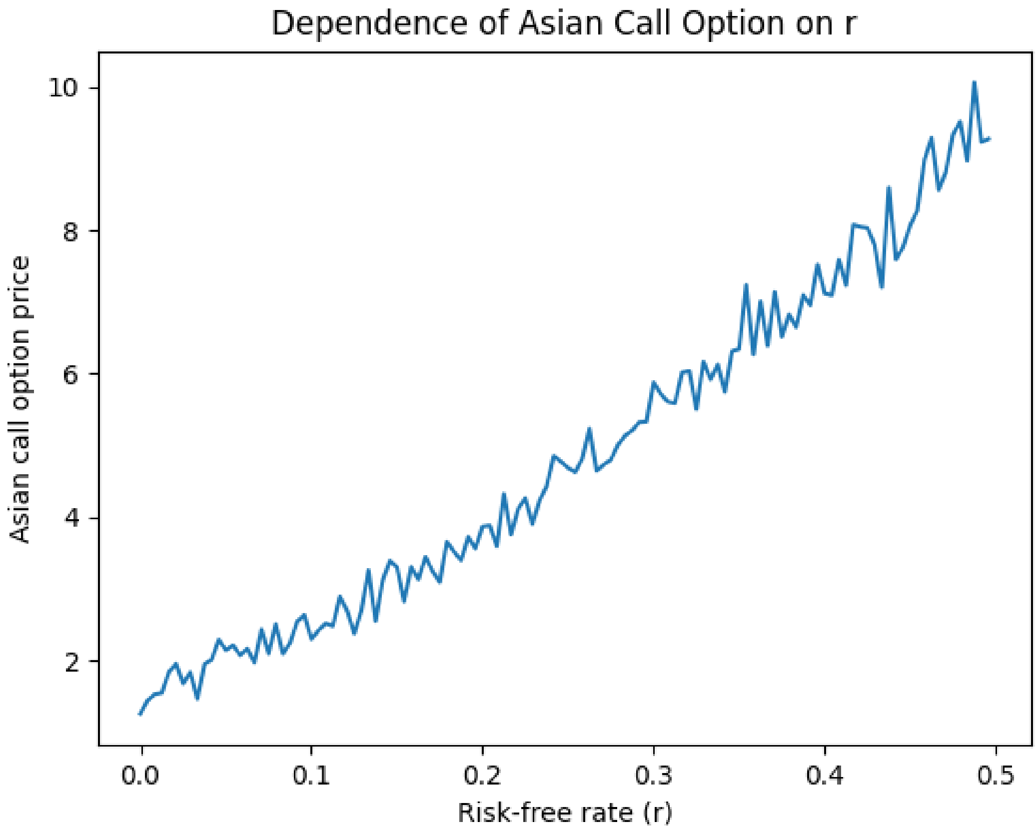
Sensitivity Analysis after performing Variance Reduction:

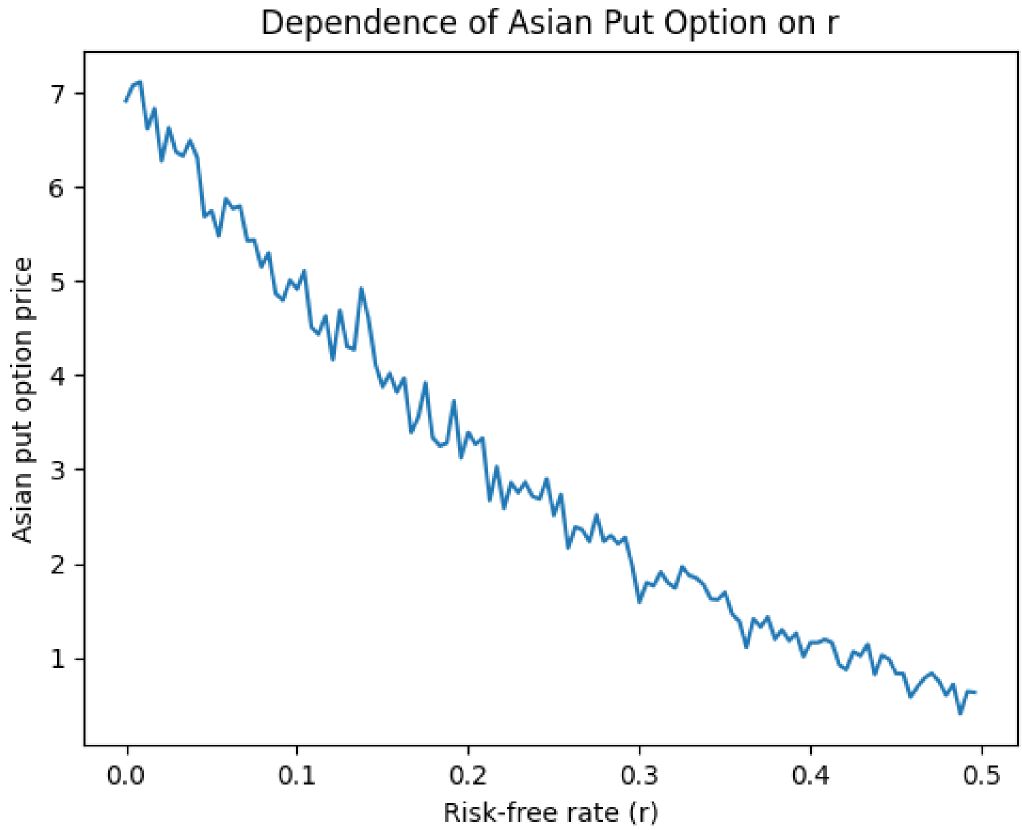

Dependence of Asian Call Option on S0
Asian call option price
Initial asset price (S0)
0
5
10
15
20
25
30
35
70
80
90
100
110
120
130
140

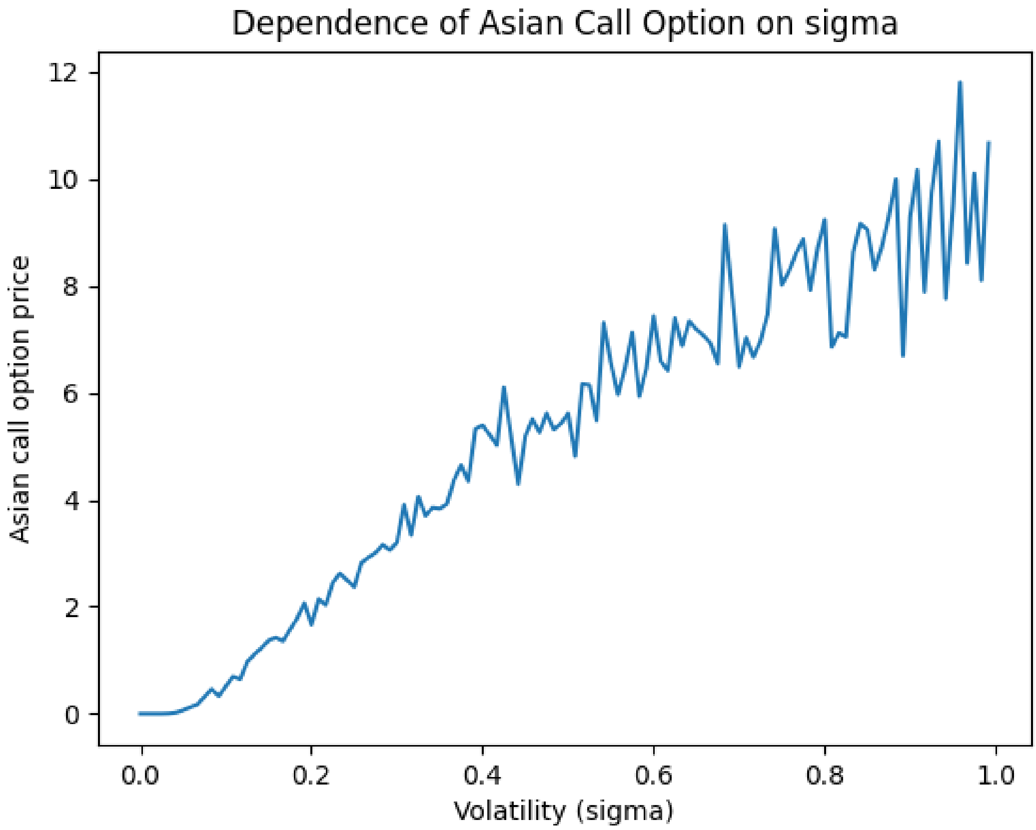

Dependence of Asian Put Option on S0
Asian put option price
Initial asset price (S0)
0
5
10
15
20
25
30
35
70
80
90
100
110
120
130
140


Dependence of Asian Call Option on K
Asian call option price
Strike price (K)
0
5
10
15
20
25
30
70
80
90
100
110
120
130
140

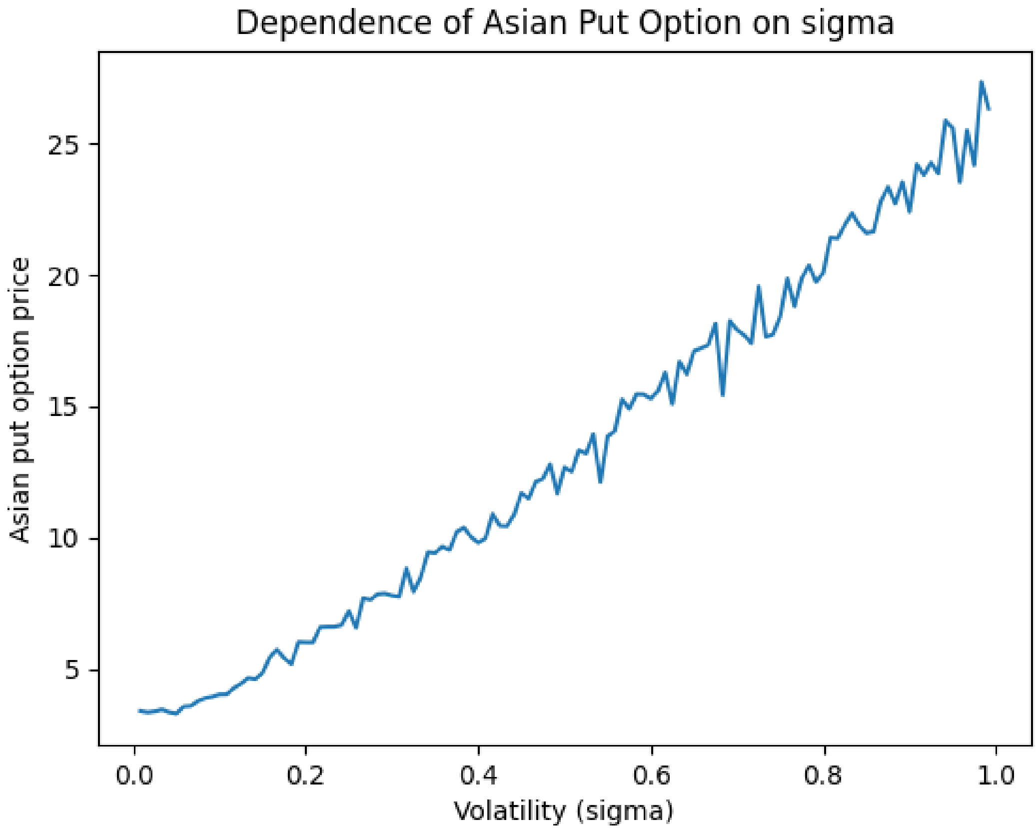

Dependence of Asian Put Option on K
Asian put option price
Strike price (K)
0
5
10
15
20
25
30
35
40
70
80
90
100
110
120
130
140


Dependence of Asian Call Option on r
Asian call option price
Risk-free rate (r)
0.0
0.1
0.2
0.3
0.4
0.5
2
4
6
8
10


Dependence of Asian Put Option on r
Asian put option price
Risk-free rate (r)
0.0
0.1
0.2
0.3
0.4
0.5
1
2
3
4
5
6
7



Dependence of Asian Call Option on sigma
Asian call option price
Volatility (sigma)
0
2
4
6
8
10
12
0.0
0.2
0.4
0.6
0.8
1.0



Dependence of Asian Put Option on sigma
Asian put option price
Volatility (sigma)
5
10
15
20
25
0.0
0.2
0.4
0.6
0.8
1.0

Variance Reduction Scheme:

The method of Control Variates is used as the variance reduction technique. This method exploits the information about the errors in estimation of known quantities to reduce the error in the estimation of the unknown quantity.

The price of a standard European Put Option, with the payoff of max[(K – S(T), 0)], and a standard European Call Option, with the payoff of max[(S(T) – K, 0)], is taken as the Control Variable respectively.

Let $Y_1, Y_2, \dots Y_n$ be the output from the n replications of the simulations, and $X_1, X_2, \dots X_n$ be the corresponding output using the control variable.

For any fixed b, we calculate following:

$$\bar{Y_i}(b) = Y_i - b(X_i - E(X))$$

We can show that the estimator obtained in this manner, called Control Variate Estimator, is an unbiased estimator.

Now, we calculate using an optimal value of b, which minimizes the variance of our estimator. We calculate following term:

$$b_n = \frac{\sum_1^n (X_i - \bar{X})(Y_i - \bar{Y})}{\sum_1^n (X_i - \bar{X})^2}$$

Using strong law of large numbers, we can conclude that b* is our required value, where $b_n \to b^*$, with probability 1

Hence using this optimal value of b, we can calculate the Control Variate Estimator, which achieves variance reduction. The variance reduction ratio depends on the correlation between the quantity Y and the control X.

Observations:

1. Earlier, we have quantitatively demonstrated that the variance reduction is achieved. This claim is even more supported by the constructed plots.

2. On careful analysis, the fluctuations in the plots seem to be less than the case when variance reduction was not applied. So, the scheme achieves its goal.

3. The nature of the plots is consistent with our expectations, which is explained in the last question.