Financial Engineering Lab-1

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Q1

Result:

```
PS C:\Users\amanb\OneDrive\Desktop\sem 6\fe lab\lab 1> python q1.py
+-----+---------------+--------------+
|   M |   call option |   put option |
+=====+===============+==============+
|   1 |       38.1676 |      19.9417 |
+-----+---------------+--------------+
|   5 |       34.9065 |      16.6806 |
+-----+---------------+--------------+
|  10 |       33.625  |      15.3991 |
+-----+---------------+--------------+
|  20 |       33.8594 |      15.6335 |
+-----+---------------+--------------+
|  50 |       33.9812 |      15.7553 |
+-----+---------------+--------------+
| 100 |       34.0112 |      15.7852 |
+-----+---------------+--------------+
| 200 |       34.0196 |      15.7937 |
+-----+---------------+--------------+
| 400 |       34.0191 |      15.7932 |
+-----+---------------+--------------+
```

M can be infinitely large, there is no constraint on its upper bound. This is because the only constraint binding us it of no-arbitrage principle

No Arbitrage Principle

$$u > R > d$$

$$\implies e^{\sigma\sqrt{\Delta t}+(r-\frac{1}{2}\sigma^2)\Delta t} > e^{r\Delta t} > e^{-\sigma\sqrt{\Delta t}+(r-\frac{1}{2}\sigma^2)\Delta t}$$

$$\implies \sigma\sqrt{\Delta t} - \frac{1}{2}\sigma^2\Delta t > 0 > -\sigma\sqrt{\Delta t} - \frac{1}{2}\sigma^2\Delta t$$

The second inequality always holds true. Thus,

$\sigma\sqrt{\Delta t} > \frac{1}{2}\sigma^2\Delta t$

$\implies \Delta t < 4/\sigma^2$

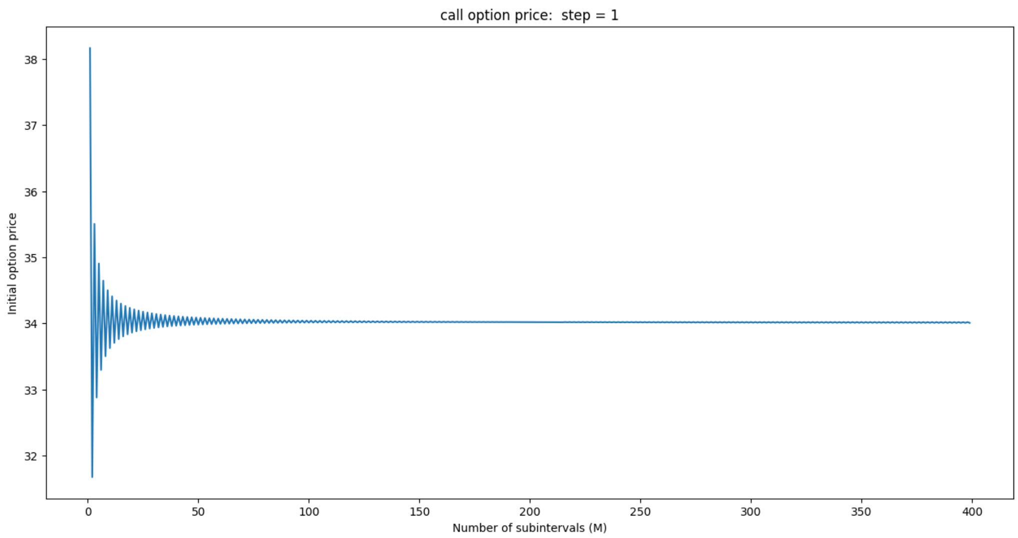
$\implies M > \frac{T\sigma^2}{4} = 0.1125$

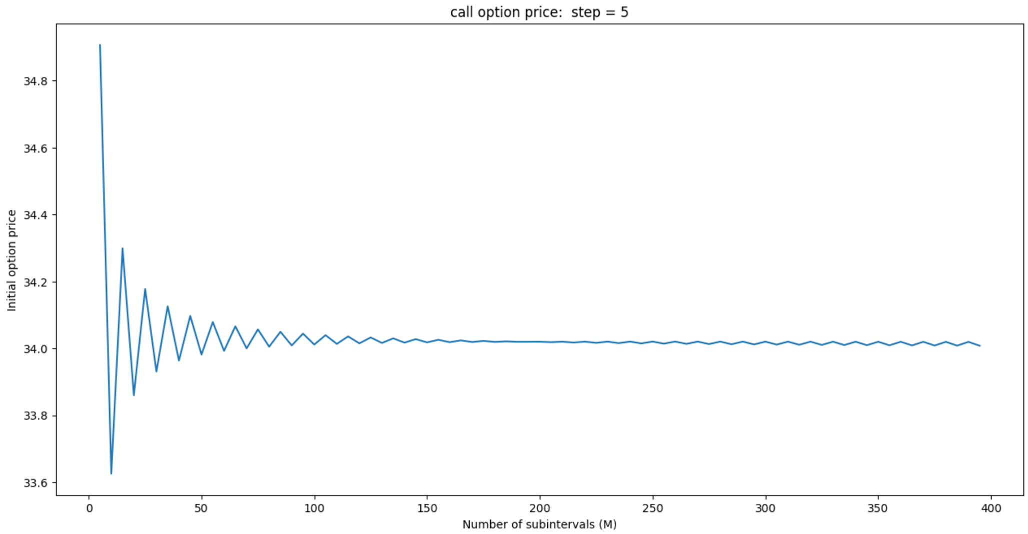
Hence there is no upper bound constraint on M

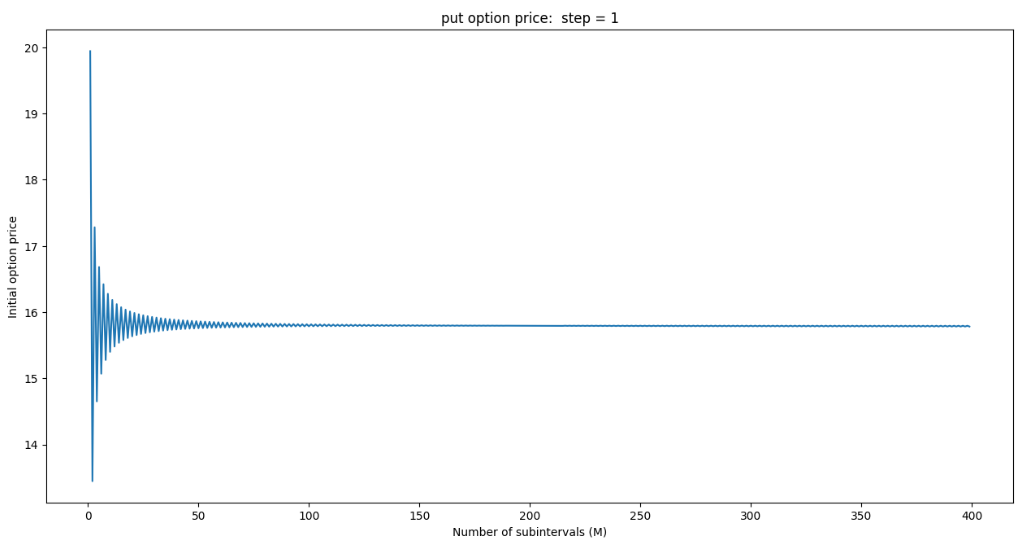
Note that since M is a natural number, there is no lower bound constraint on it too

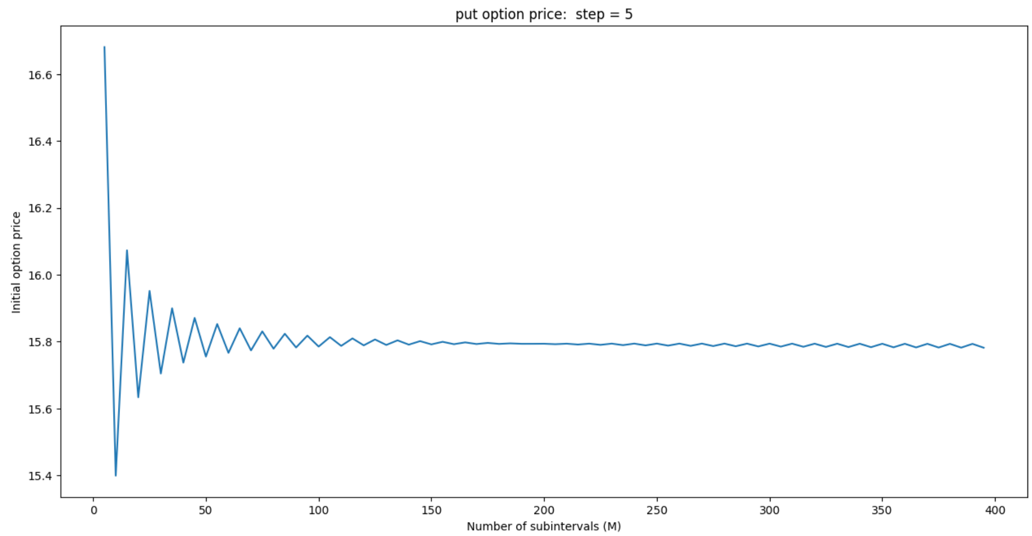
Q2

Graphs:




call option price: step = 5
Initial option price
Number of subintervals (M)
34.8
34.6
34.4
34.2
34.0
33.8
33.6
0
50
100
150
200
250
300
350
400


put option price: step = 1
Initial option price
Number of subintervals (M)
20
19
18
17
16
15
14
0
50
100
150
200
250
300
350
400



Observations:

In each case, the graphs converge to a particular value.

For call option, the convergent price is 34.019

For put option, the convergent price is 15.793

Q3

Code Output:

```
PS C:\Users\amanb\OneDrive\Desktop\sem 6\fe lab\lab 1> python q3.py
call  option:
+--------+----------------------------------------------------------------------------------------------------------------------+
|   Time | call option                                                                                                          |
+========+======================================================================================================================+
|    0   | [33.859]                                                                                                             |
+--------+----------------------------------------------------------------------------------------------------------------------+
|    0.5 | [15.096, 31.893, 59.959]                                                                                             |
+--------+----------------------------------------------------------------------------------------------------------------------+
|    1   | [5.155, 13.47, 29.804, 57.7, 100.663]                                                                                |
+--------+----------------------------------------------------------------------------------------------------------------------+
|    1.5 | [1.125, 4.121, 11.767, 27.573, 55.295, 98.439, 160.611]                                                              |
+--------+----------------------------------------------------------------------------------------------------------------------+
|    3   | [0.0, 0.0, 0.0, 0.118, 1.236, 6.149, 19.725, 46.976, 91.193, 154.842, 242.03, 359.934, 519.1]                        |
+--------+----------------------------------------------------------------------------------------------------------------------+
|    4.5 | [0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 8.149, 36.251, 83.951, 149.15, 237.159, 355.959, 516.323, 732.792, 1024.993, 1419.425] |
+--------+----------------------------------------------------------------------------------------------------------------------+
put  option:
+--------+---------------------------------------------------------------------------------------------------------------+
|   Time | put option                                                                                                    |
+========+===============================================================================================================+
|    0   | [15.634]                                                                                                      |
+--------+---------------------------------------------------------------------------------------------------------------+
|    0.5 | [24.673, 15.487, 8.479]                                                                                       |
+--------+---------------------------------------------------------------------------------------------------------------+
|    1   | [35.965, 24.983, 15.269, 8.004, 3.504]                                                                        |
+--------+---------------------------------------------------------------------------------------------------------------+
|    1.5 | [48.305, 36.97, 25.271, 14.963, 7.436, 2.998, 0.942]                                                          |
+--------+---------------------------------------------------------------------------------------------------------------+
|    3   | [78.228, 72.358, 64.433, 53.855, 40.533, 25.955, 13.222, 4.958, 1.236, 0.172, 0.009, 0.0, 0.0]                |
+--------+---------------------------------------------------------------------------------------------------------------+
|    4.5 | [95.534, 93.129, 89.883, 85.502, 79.587, 71.603, 60.825, 46.278, 26.64, 8.281, 0.602, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0] |
+--------+---------------------------------------------------------------------------------------------------------------+
```

Observations:

- As time increases, we find that the maximum option price increases and the minimum option prices decreases (or remains constant)
- We find that at time t, an option price can take $1 + t/\Delta t$ values