MA374:Financial Engineering Lab: Lab 4
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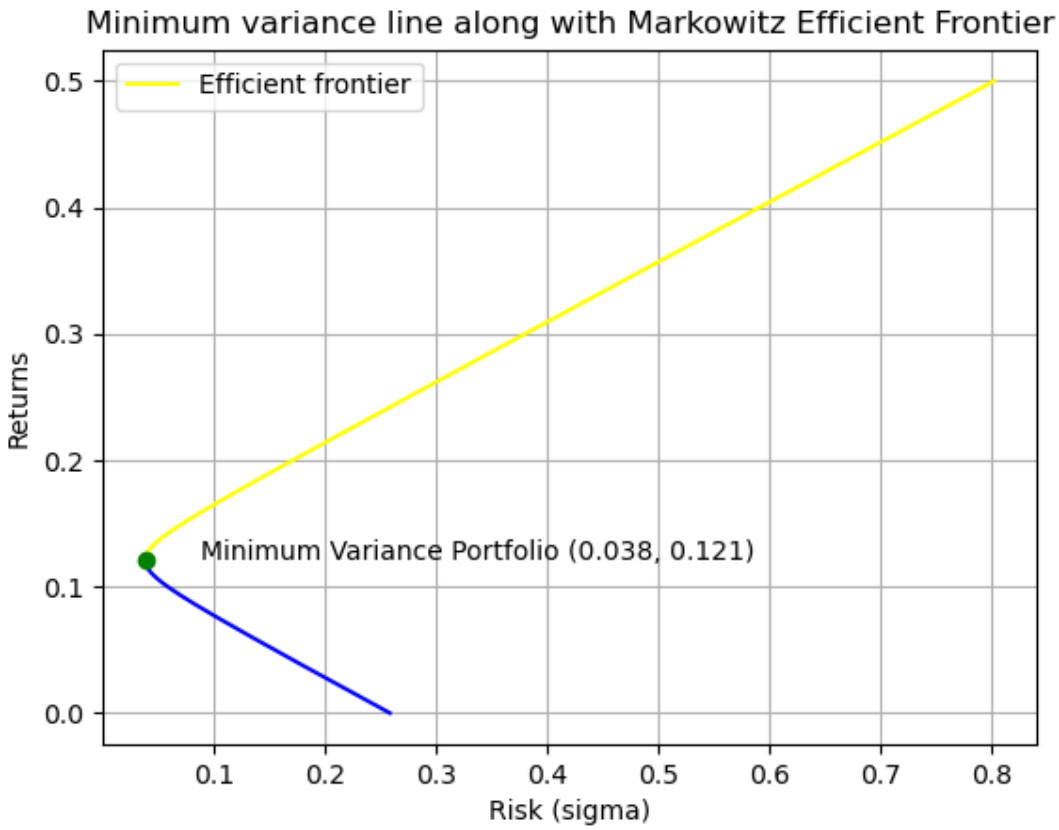
Question 1:

a) The Markowitz Efficient frontier is as follows:



Weights of the portfolio are obtained by the following formula:

$$w = \frac{\begin{vmatrix} 1 & uC^{-1}M^T \\ \mu_v & MC^{-1}M^T \end{vmatrix} uC^{-1} + \begin{vmatrix} uC^{-1}u^T & 1 \\ MC^{-1}u^T & \mu_v \end{vmatrix} MC^{-1}}{\begin{vmatrix} uC^{-1}u^T & uC^{-1}M^T \\ MC^{-1}u^T & MC^{-1}M^T \end{vmatrix}}$$

where, μ_v = return,

u = [1, 1, 1, ..., 1] (with same dimension as that of number of assets)

The value of σ_v^2 is calculated as per the following formula:

$$\sigma_v^2 = wCw^T$$

Taking the square root of that we will get risk of the portfolio.

Now, the weights of the minimum variance portfolio is :

$$w = \frac{\bar{u}C^{-1}}{uC^{-1}u^T}$$

Using all these formulas we can find the corresponding point on the minimum variance curve. Now, the efficient frontier is the one with higher expected return and lower standard deviation (lower risk). So, the points with higher return than the minimum variance portfolio point show the efficient frontier on the curve.(Denoted by yellow color).

b) The weights, return and risk of the portfolios for 10 different values on the efficient frontier:

```
For Part B:

+-----------------------------------------+----------+------------+
| Weights                                 |   Return |       Risk |
+=========================================+==========+============+
| [[0.23238779 0.47174332 0.2958689 ]]    | 0.161968 | 0.00898428 |
+-----------------------------------------+----------+------------+
| [[-0.48735321  0.75779423  0.72955899]] | 0.212257 | 0.03873    |
+-----------------------------------------+----------+------------+
| [[-0.97048555  0.94980836  1.02067719]] | 0.246015 | 0.0713889  |
+-----------------------------------------+----------+------------+
| [[-1.19138599  1.03760212  1.15378387]] | 0.261449 | 0.0897179  |
+-----------------------------------------+----------+------------+
| [[-1.3237168   1.09019514  1.23352166]] | 0.270696 | 0.101719   |
+-----------------------------------------+----------+------------+
| [[-2.17789045  1.42967441  1.74821604]] | 0.330378 | 0.197585   |
+-----------------------------------------+----------+------------+
| [[-2.23704258  1.45318359  1.78385899]] | 0.334511 | 0.205404   |
+-----------------------------------------+----------+------------+
| [[-3.06311669  1.7814951   2.2816216 ]] | 0.392231 | 0.330565   |
+-----------------------------------------+----------+------------+
| [[-3.10023388  1.7962468   2.30398708]] | 0.394824 | 0.336888   |
+-----------------------------------------+----------+------------+
| [[-3.10831545  1.79945871  2.30885675]] | 0.395389 | 0.338273   |
+-----------------------------------------+----------+------------+
```

c)

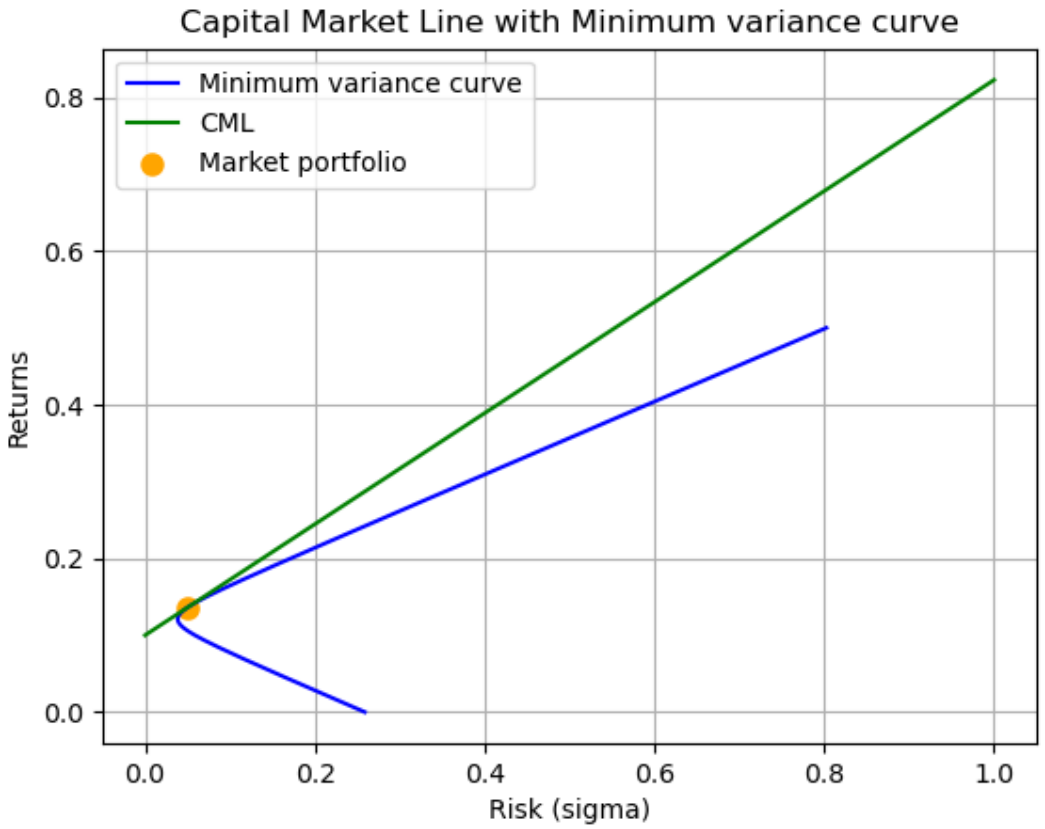
```
For Part C:

For 15% Risk:

Maximum Return is :  18.955479960806635  %
And the corresponding weights are :  [-0.16243566  0.62866033  0.53377534]

Minimum Return is :  5.2446841081023  %
And the corresponding weights are :  [ 1.79984338 -0.1512198  -0.64862357]
```

d)

```
For Part D:

For 18% return:

Minimum Risk is:  0.13056827100982518
And the corresponding weights are :  [-0.02568807  0.57431193  0.45137615]
```

e)

```
For Part E:

Market Portfolio Weights =  [0.59375  0.328125 0.078125]
Return =  0.13671875
Risk =  5.081128919221593  %
```




```
Equation of CML is:
y = 0.723 x + 0.100
```

Which is obtained by the following formula:

$$\mu = \frac{\mu_M - \mu_{rf}}{\sigma_M}\,\sigma + \mu_{rf}$$

where,

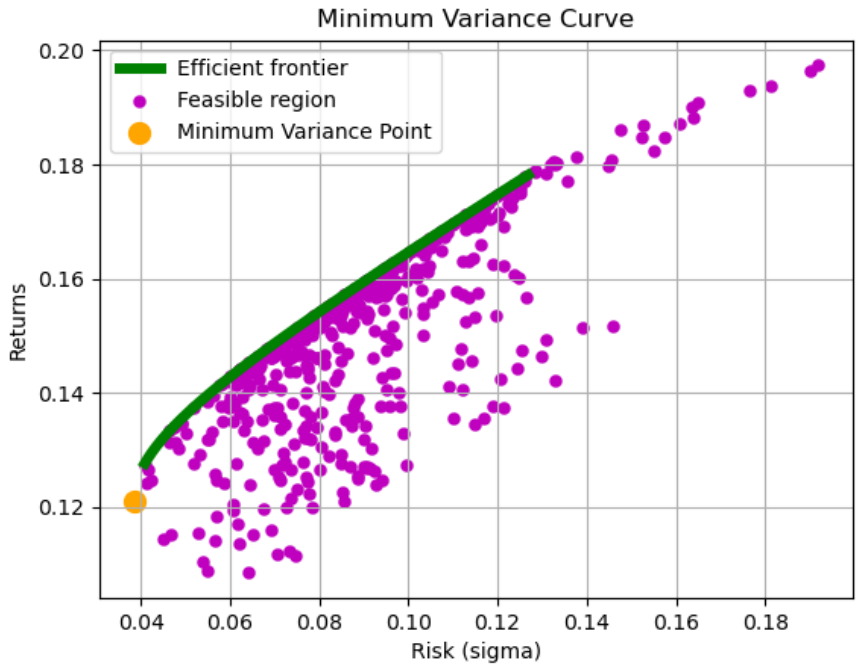
μ_M = return corresponding to market portfolio
μ_{rf} = risk free return
σ_M = risk corresponding to market portfolio

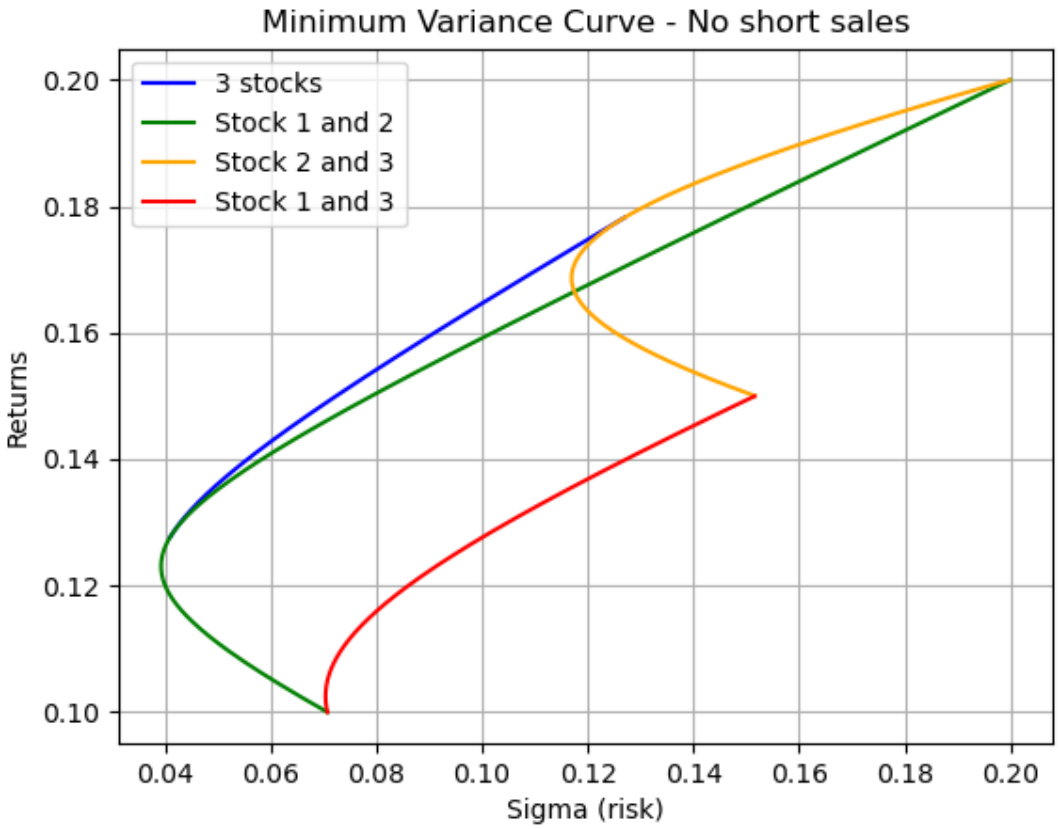
f)

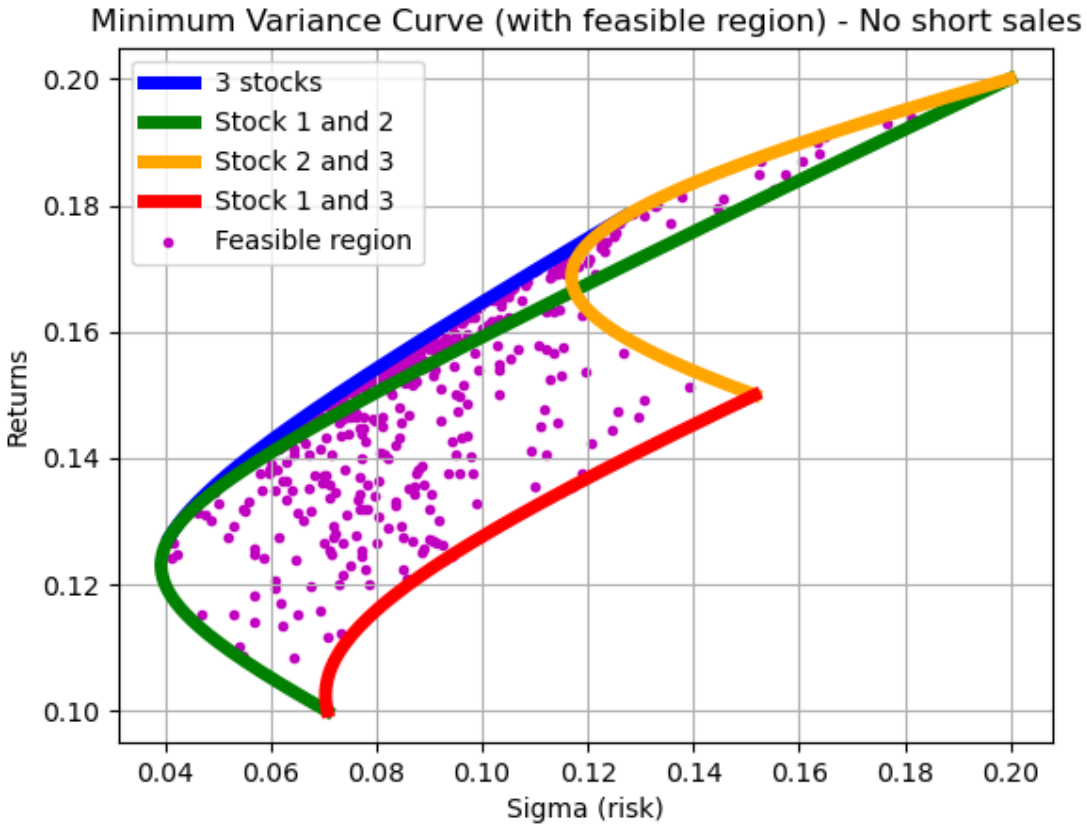
```
For Part F:

Risk = 10.0  %
Risk-free weights = -0.9680665771282883
Risky Weights = [1.16853953 0.64577185 0.1537552 ]
Returns= 0.17226494462892933


Risk = 25.0  %
Risk-free weights = -3.9201664428207224
Risky Weights = [2.92134883 1.61442961 0.384388  ]
Returns = 0.2806623615723234
```

Question 2:

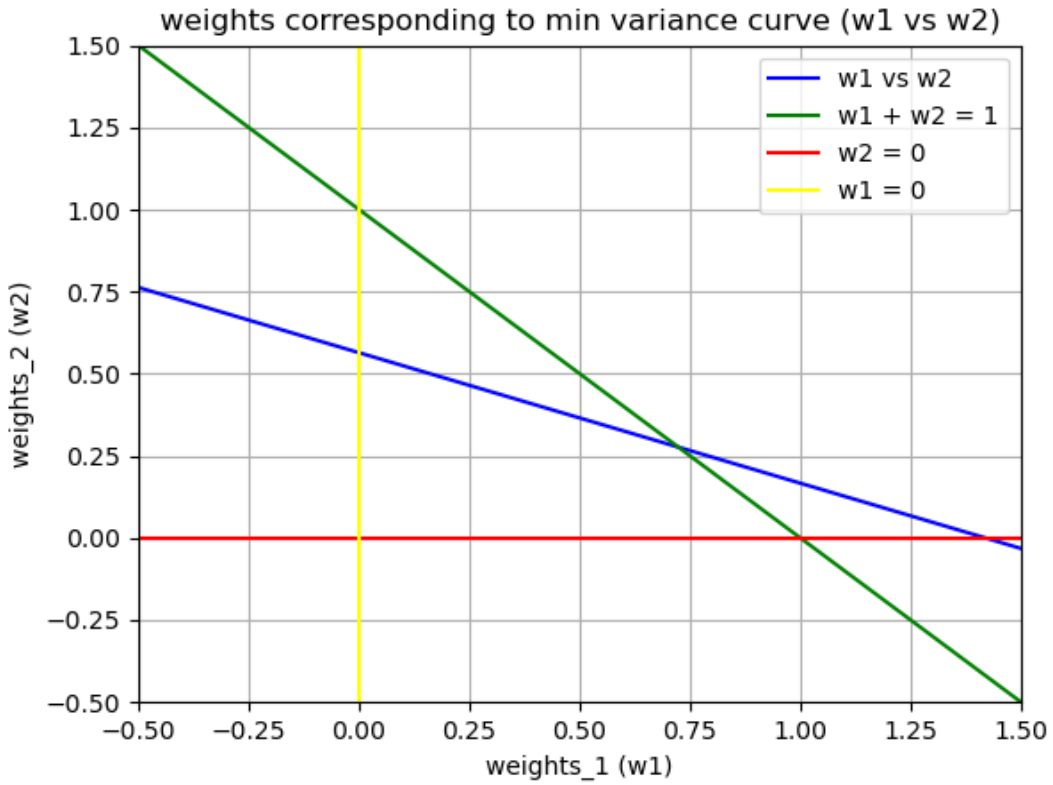
The various plots (assuming short sales are not allowed, i.e. weights are non-negative) are as follows:



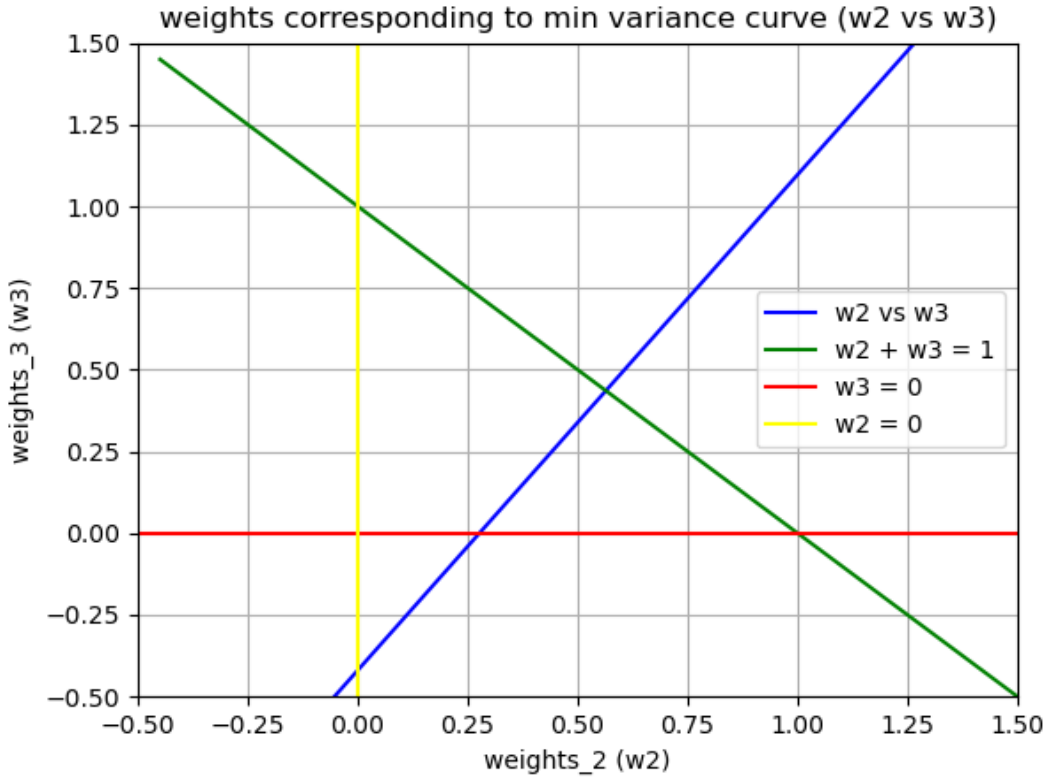

Minimum Variance Curve - No short sales
3 stocks
Stock 1 and 2
Stock 2 and 3
Stock 1 and 3
Returns
Sigma (risk)
0.10
0.12
0.14
0.16
0.18
0.20
0.04
0.06
0.08
0.10
0.12
0.14
0.16
0.18
0.20


Minimum Variance Curve (with feasible region) - No short sales
3 stocks
Stock 1 and 2
Stock 2 and 3
Stock 1 and 3
Feasible region
Returns
Sigma (risk)
0.10
0.12
0.14
0.16
0.18
0.20
0.04
0.06
0.08
0.10
0.12
0.14
0.16
0.18
0.20

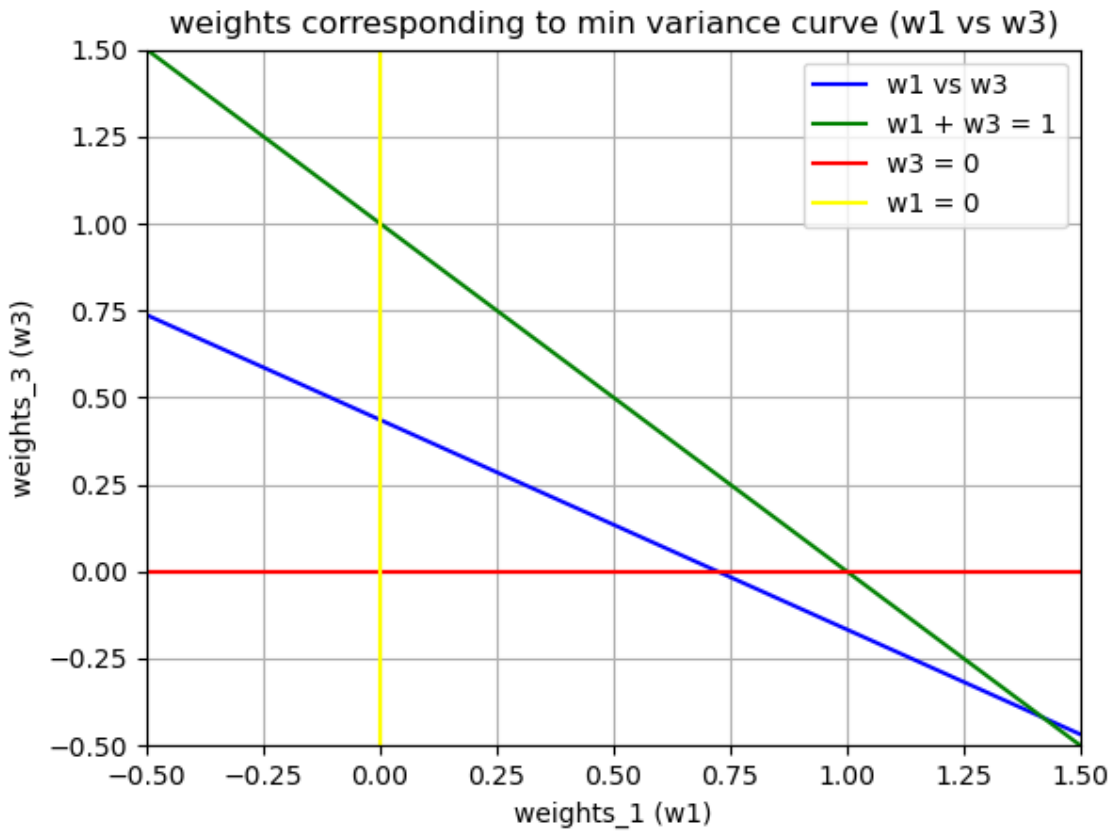
The plot for the weights corresponding to the minimum variance curve are:




```
Eqn of line w1 vs w2 is:
w2 = -0.40 w1 + 0.56
```



```
Eqn of line w2 vs w3 is:
w3 = 1.52 w2 + -0.42
```

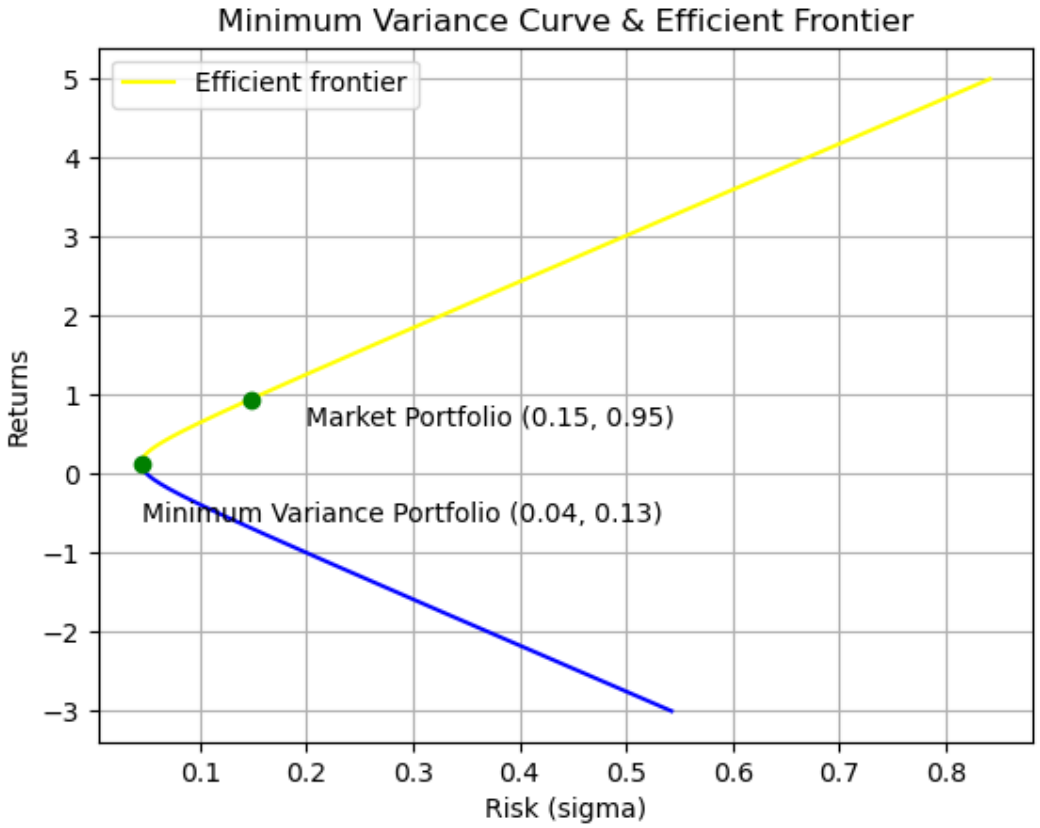



```
Eqn of line w1 vs w3 is:
w3 = -0.60 w1 + 0.44
```

Question 3:

The data for the stocks had been collected for the time period between 01/01/2018 to 01/12/2022 on monthly basis (total 60 data points). The companies considered are Apple, Amazon, Facebook, Google, IBM, Intel, Microsoft, Netflix, Nike, and Tesla. The monthly return was obtained as the difference in stock prices between beginning of 2 consecutive months. Then annual return was calculated suitably:

a) The Markowitz efficient frontier is:

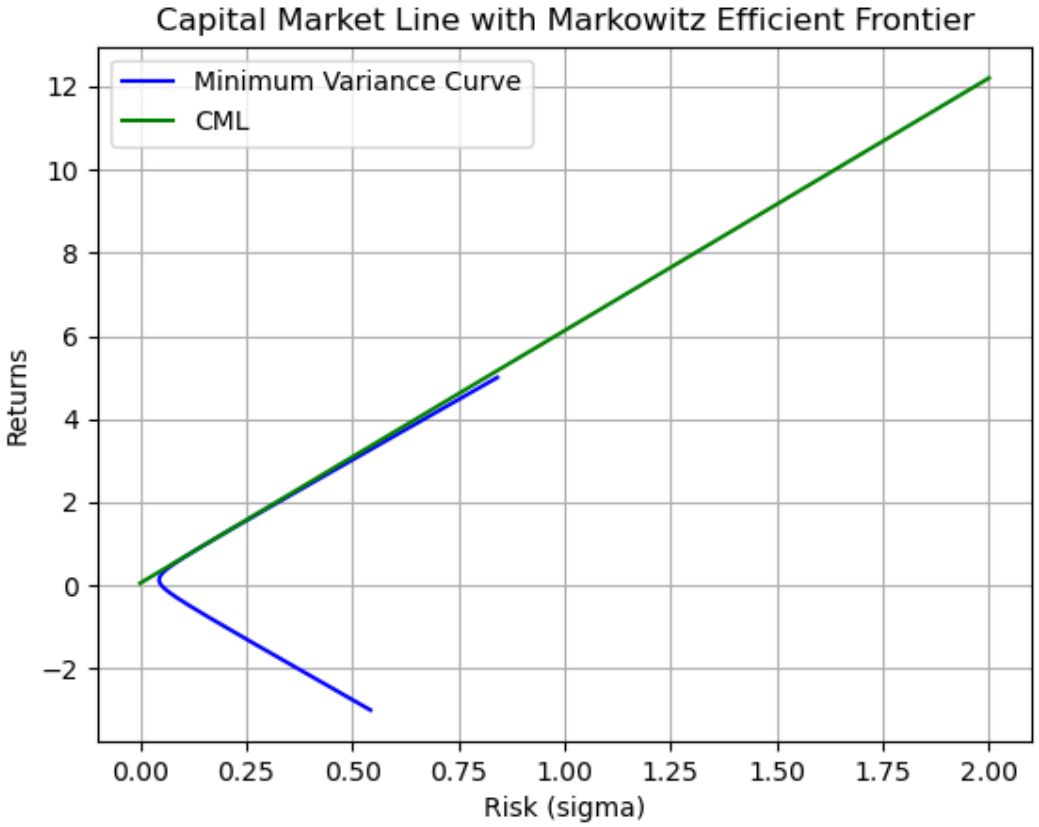


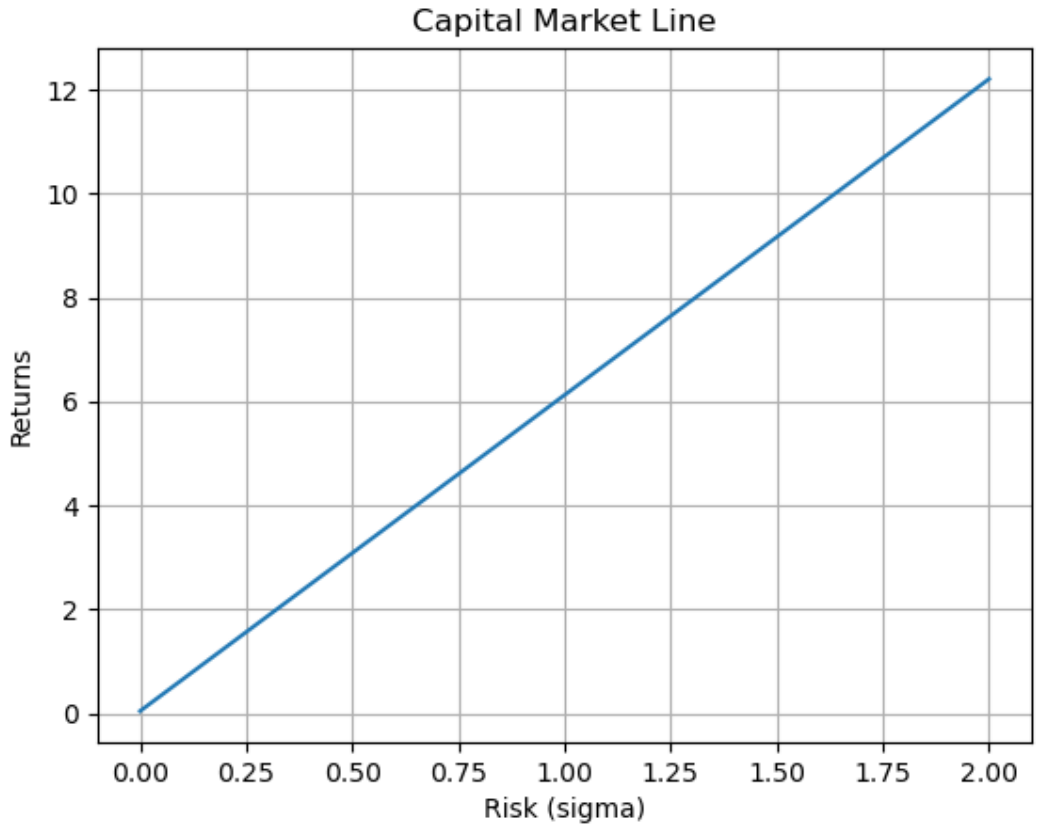
b)

```
Market Portfolio Weights        =  [ 0.22761242  1.13992789 -0.21802614 -0.27195818 -1.86085521 -0.2067485
  1.09153062  0.28576137  0.7953939   0.01736182]
Return                          =  0.9494417828642224
Risk                            =  14.801199750535993  %
```

c)

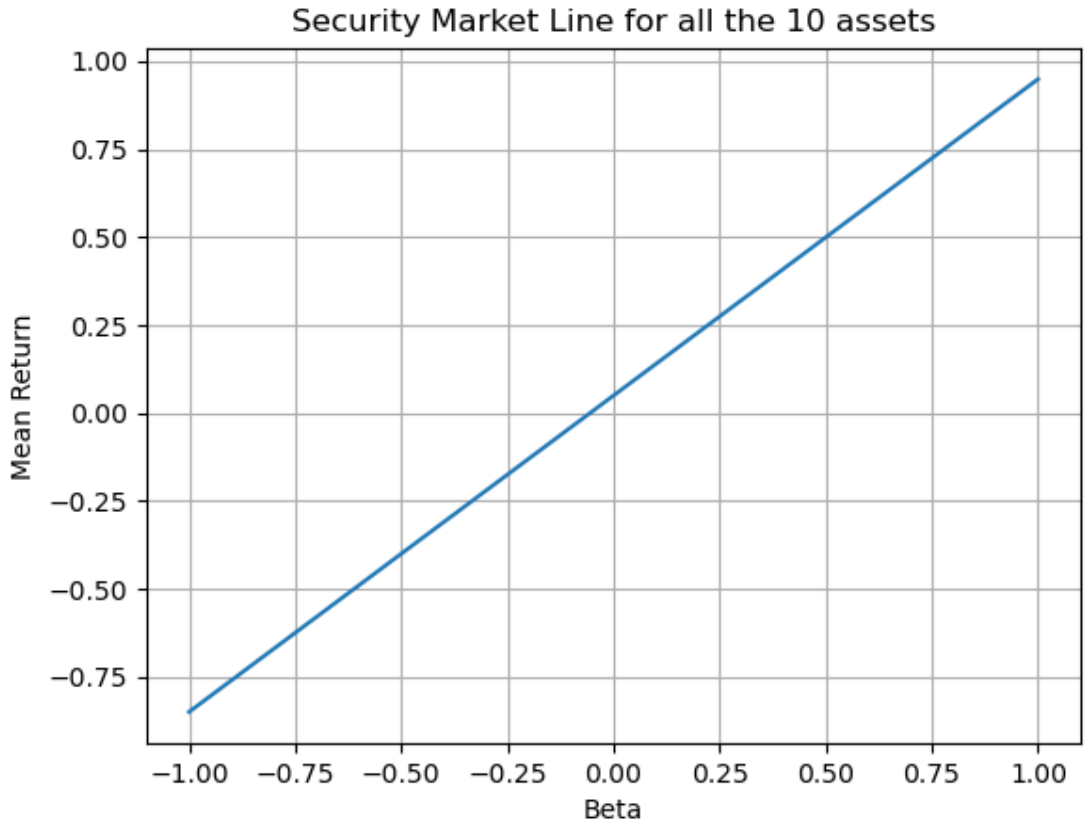
```
Equation of CML is:
y = 6.08 x + 0.05
```


Capital Market Line with Markowitz Efficient Frontier
Minimum Variance Curve
CML
Returns
Risk (sigma)
12
10
8
6
4
2
0
−2
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00


Capital Market Line
Returns
Risk (sigma)
12
10
8
6
4
2
0
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00

d)

```
Eqn of Security Market Line is:
mu = 0.90 beta + 0.05
```



The Security market line is obtained using the following formula:

$$\mu = (\mu_M - \mu_{rf})\beta + \mu_{rf}$$

where,

μ_M = return corresponding to market portfolio

μ_{rf} = risk free return