Financial Engineering Lab 3

Aman Bucha

200123006

Question-1

Given: American Option with parameters:

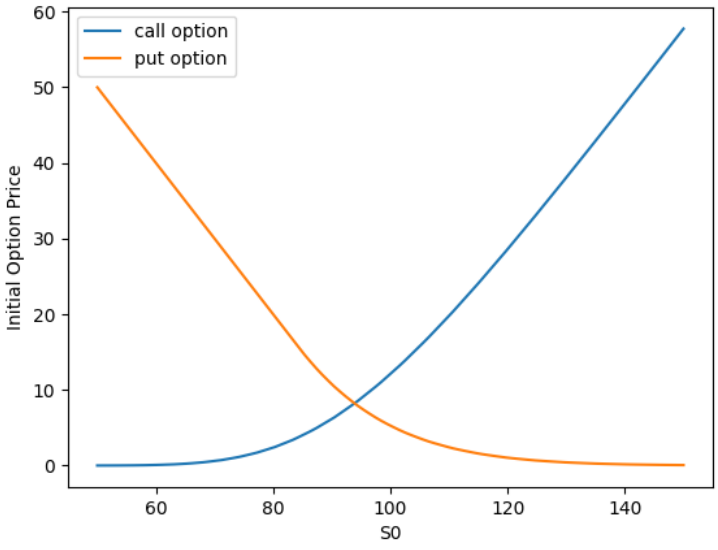
$S(0) = 100, K = 100, T = 1, M = 100, r = 8\%, \sigma = 20\%$

$u = e^{\sigma\sqrt{\Delta t}+(r-\frac{1}{2}\sigma^2)\Delta t}$

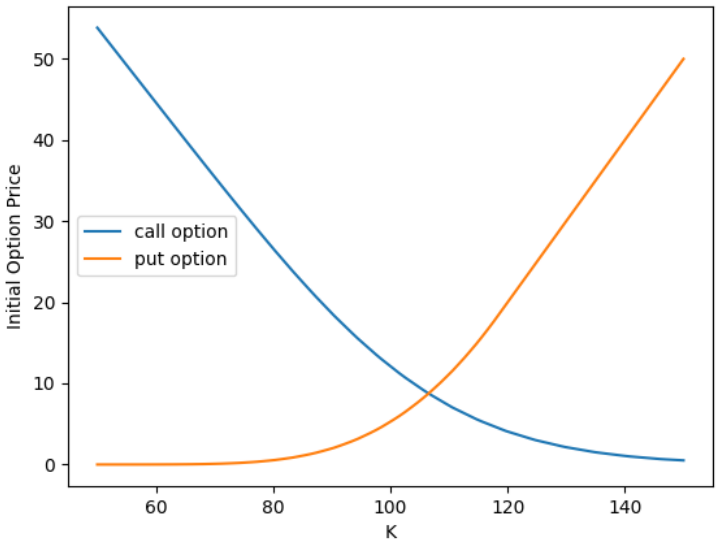
$d = e^{-\sigma\sqrt{\Delta t}+(r-\frac{1}{2}\sigma^2)\Delta t}$

```
[Running] python -u "c:\Users\amanb\OneDrive\Desktop\sem 6\fe lab\lab 3\q1.py"
initial call option value: 12.12304707401251
initial put option value: 5.27983714598916
```

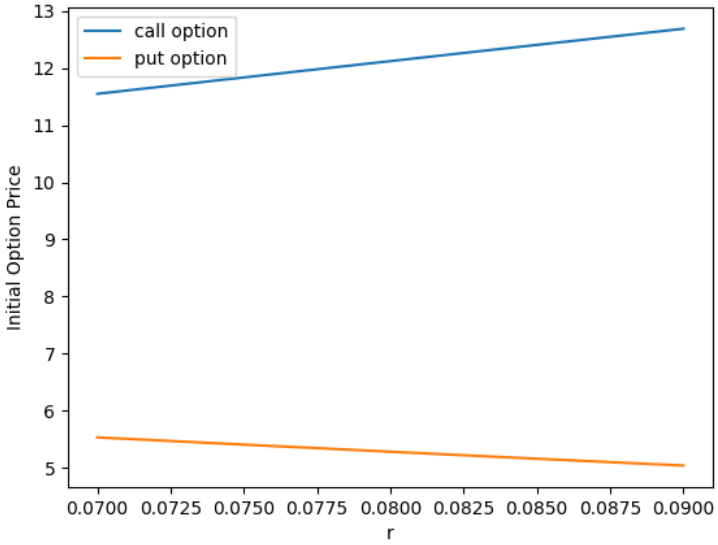
(a)



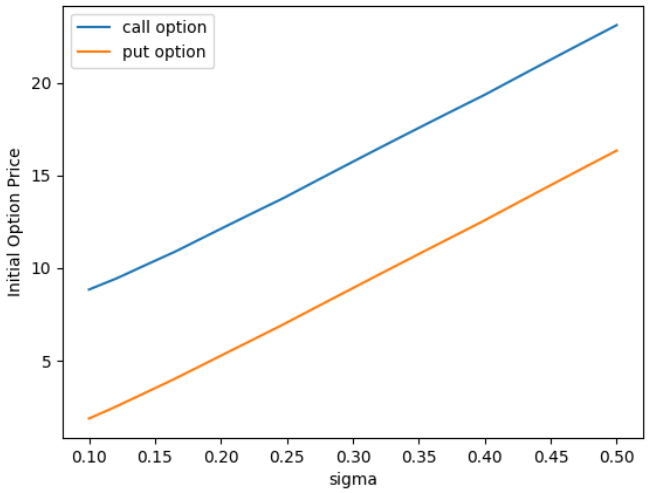
(b)


call option
put option
Initial Option Price
K
0
10
20
30
40
50
60
80
100
120
140

(c)

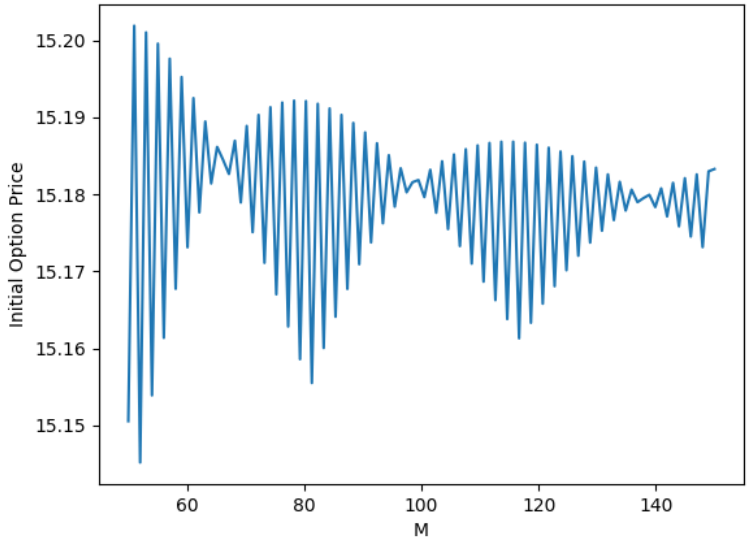

call option
put option
Initial Option Price
r
5
6
7
8
9
10
11
12
13
0.0700
0.0725
0.0750
0.0775
0.0800
0.0825
0.0850
0.0875
0.0900

(d)

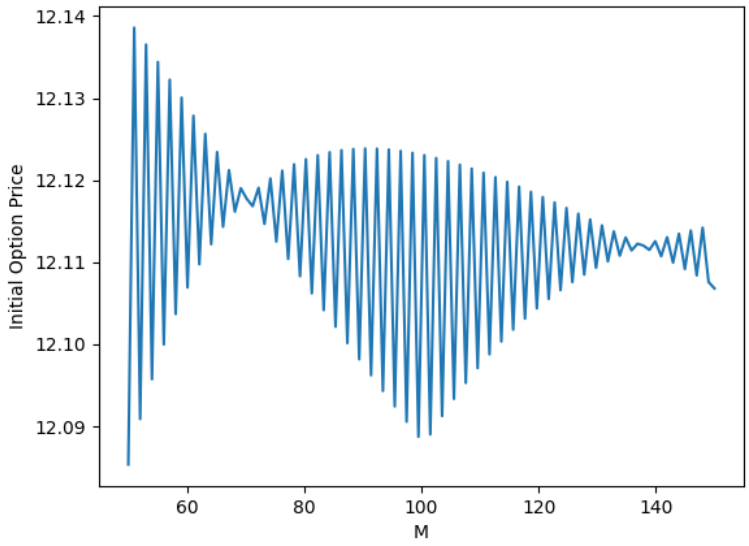


(e)

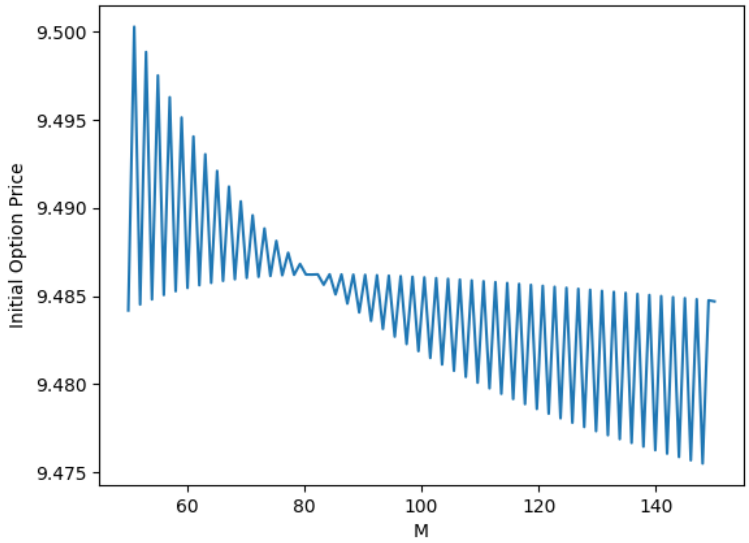
Call Option: K=95



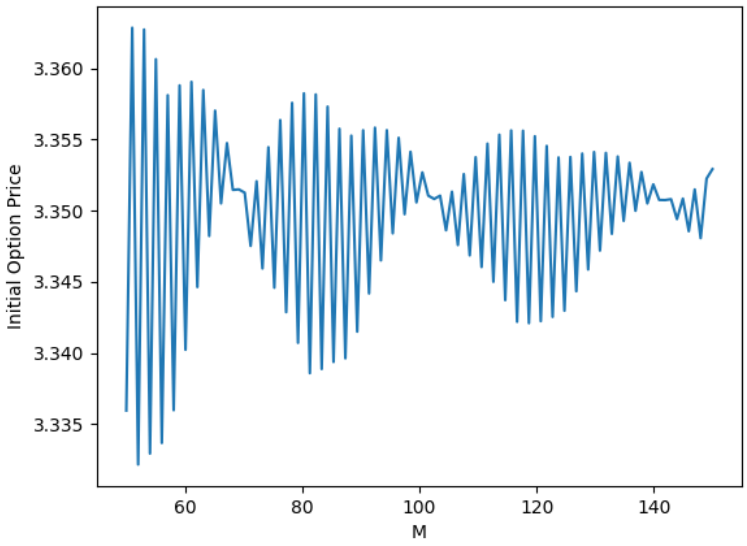
Call Option: K=100



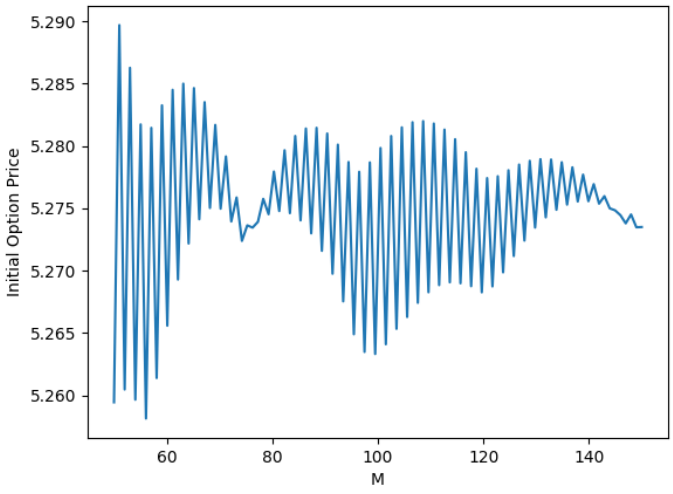
Call Option: K=105



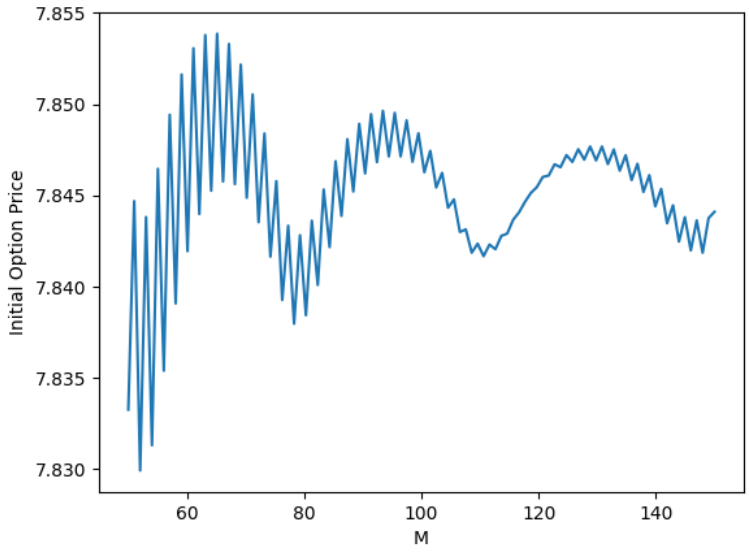
Put Option: K=95



Put Option: K=100



Put Option: K=105



Question-2

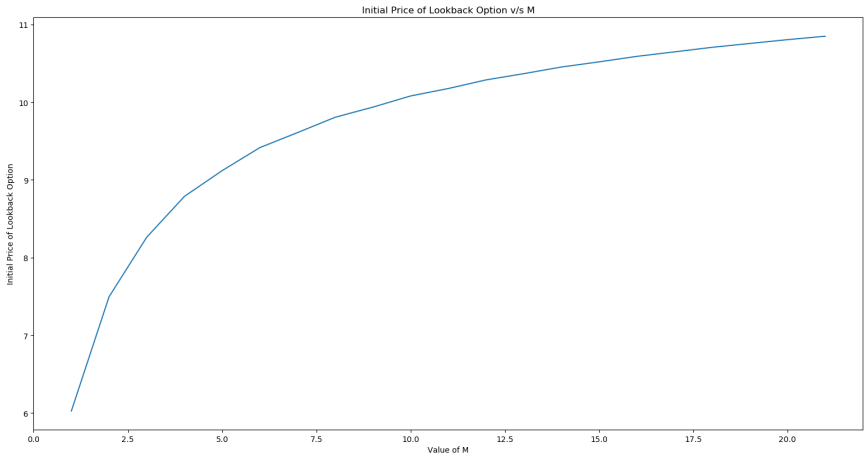
Given a lookback option with the following parameters:

$S(0) = 100, T = 1, M = 100, r = 8\%, \sigma = 20\%$

M	Initial Option Price	Computation Time (in seconds)
5	9.1193	0.0000925
10	10.080	0.0012662
20	10.80511	7.0800144
50	Not feasible	Not

The initial option price will converge as the value of M is increased.

The computation time increases exponentially as the value of M increases, therefore it becomes infeasible to calculate for larger values of M.



Prices of Option at all times for M=5

t=0	t=1	t=2	t=3	t=4	t=5
9.1193	9.0280	8.5481	7.4168	5.5016	0.0000
	9.5048	9.7991	9.9553	9.5714	11.1814
		7.1479	6.2019	4.6005	0.0000
		12.1687	8.3246	15.6319	19.4527
			7.1484	4.6005	0.0000
			17.5821	8.0036	9.3499
				6.6808	6.3745
				21.1881	25.3946
				4.6005	0.0000
				8.0036	9.3499
				3.8469	0.0000
				13.0714	16.2664
				3.8469	0.0000
				10.6809	13.5780
				10.6809	13.5780
				25.0512	29.4826
					0.0000
					9.3499
					0.0000
					16.2664
					0.0000

t=0	t=1	t=2	t=3	t=4	t=5
					7.8184
					5.3304
					21.2350
					0.0000
					7.8184
					2.9014
					18.8059
					2.9014
					18.8059
					18.8059
					32.1054

Question-3

Using Markov based optimization, we can now compute the option prices for larger values of M.

M	Initial Option Price	Computational Time (in seconds)
5	9.1193	0.000160
10	10.0806	0.000744
20	10.8051	0.017535
25	11.0034	0.05155
50	11.51086	3.11132

Therefore, the algorithm is efficient than the previous one.

Question-4

M	Initial European Call Price	Computational Time Without Markov (in seconds)	Computational Time with Markov (in seconds)
5	12.1632	0.00002	0.00002
10	12.2773	0.00040	0.00014
15	12.0520	0.01221	0.00020
20	12.1747	0.38390	0.00058
25	12.1367	12.26667	0.00105
50	12.0854	not feasible	0.00839

Here, the time complexity for the naïve approach is $O(2^n)$ whereas after using Markov based optimization (DP) the time complexity reduces to $O(n^2)$ thereby allowing us to calculate for larger values of M