MA374 Financial Engineering Lab
Assignment - 9

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Question - 1

I collected option price data of five stocks for the duration of 28-Feb-2023 to 30-Mar-2021 from the NIFTY50 index. The stocks considered are:-

- ***AsianPaint***
- ***ICICI BANK***
- ***Tech Mahindra (TECHM)***
- ***Wipro***
- ***UPL***

Question - 2

Theory for Part - (b)

In order to find the implied volatility we note that the call option price according to the BSM formulae is given by the following relations:-

$$C(t,s)=sN(d_1)-ke^{-r\tau}N(d_2)$$

$$d_1=\frac{1}{\sigma\sqrt{(\tau)}}(\log(\frac{s}{k})+(r+\sigma^2/2)\tau)$$

$$d2=d1-\sigma\sqrt{(\tau)}$$

$$\tau=T-t$$

$$N(x)=\frac{1}{\sqrt{2\Pi}}\int_{-\infty}^{x}e^{-\frac{x^2}{2}}$$

Now from the data collected, we know the value of C(t,s) lets say P hence we need to find the roots of the equation $f(\sigma)=0$ *where*

$$f(\sigma) = sN(d_1) - ke^{-r\tau}N(d_2) - P$$

Now we use Newton method in order to estimate the value of sigma . We use:-

$$\sigma_{n+1} = \sigma_n - \frac{f(\sigma_n)}{f'(\sigma_n)} \quad \sigma_0 = 0.3$$

Note that f′(x) is the Vega of the call option and can be found to be equal to

$$s\sqrt{\tau}N'(d1)$$

Theory for Part - (c)

I estimated historical volitility using the following technique:-

We thus observe $S(t_0), \ldots, S(t_n)$, and in order to estimate σ we use the fact that S has a log-normal distribution. Let us therefore define $\xi_1, \ldots, \xi_n$ by

$$\xi_i = \ln\left(\frac{S(t_i)}{S(t_{i-1})}\right).$$

From (5.15) we see that $\xi_1, \ldots, \xi_n$ are independent, normally distributed random variables with

$$E[\xi_i] = \left(\alpha - \frac{1}{2}\sigma^2\right)\Delta t,$$

$$Var[\xi_i] = \sigma^2 \Delta t.$$

Using elementary statistical theory we see that an estimate of σ is given by

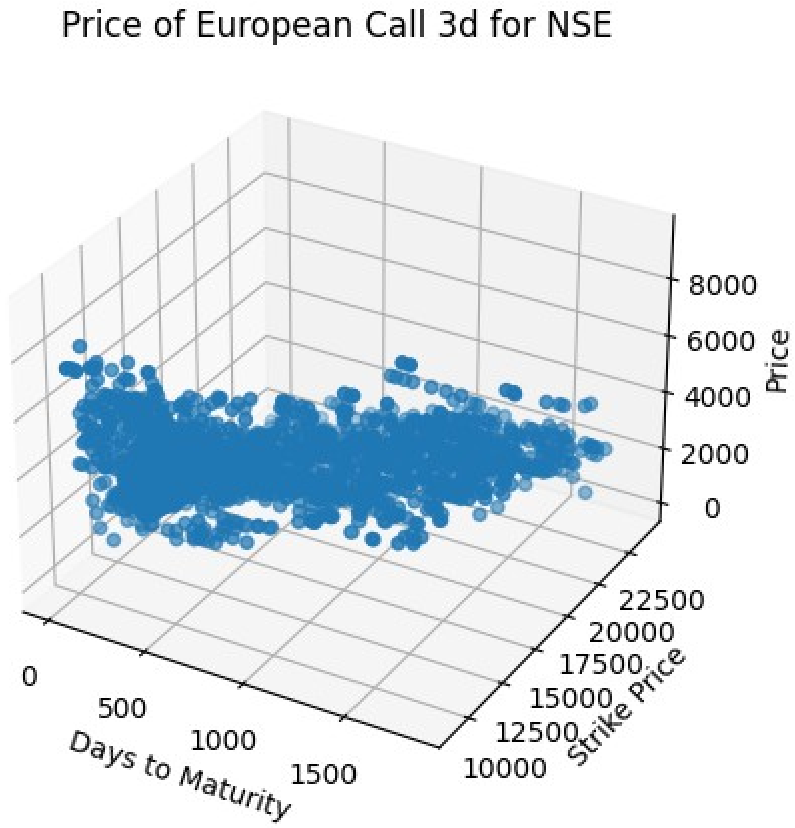
$$\sigma^\star = \frac{S_\xi}{\sqrt{\Delta t}},$$

I am attaching the required results for part a,b and c of the 2nd problem for each stock from the next page:-

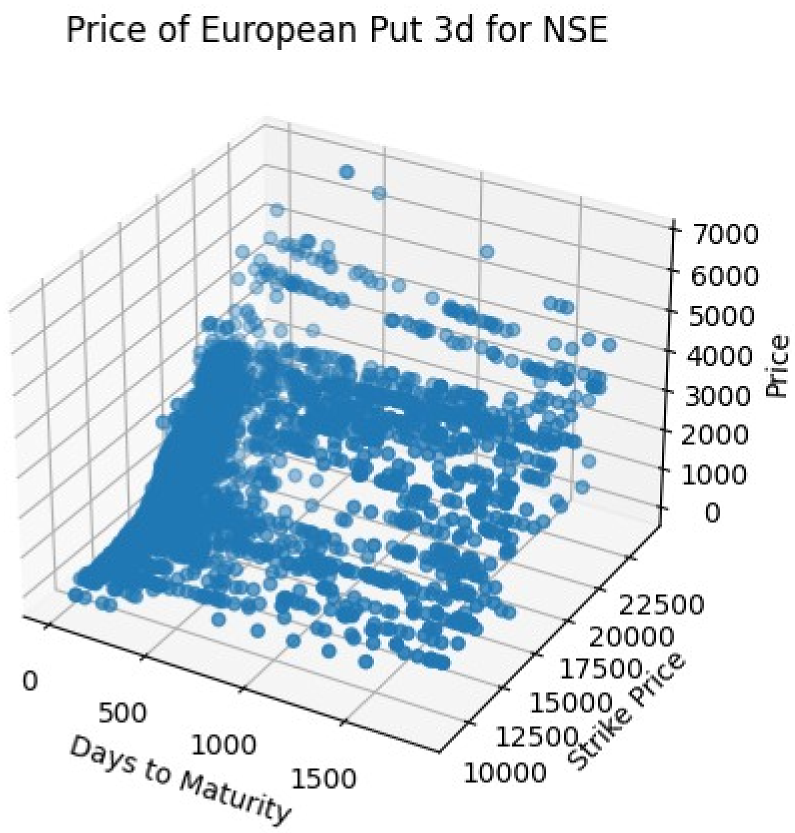
NOTE - OBSERVATIONS FOR ALL THE PARTS ARE IN THE LAST PAGES

NIFTY

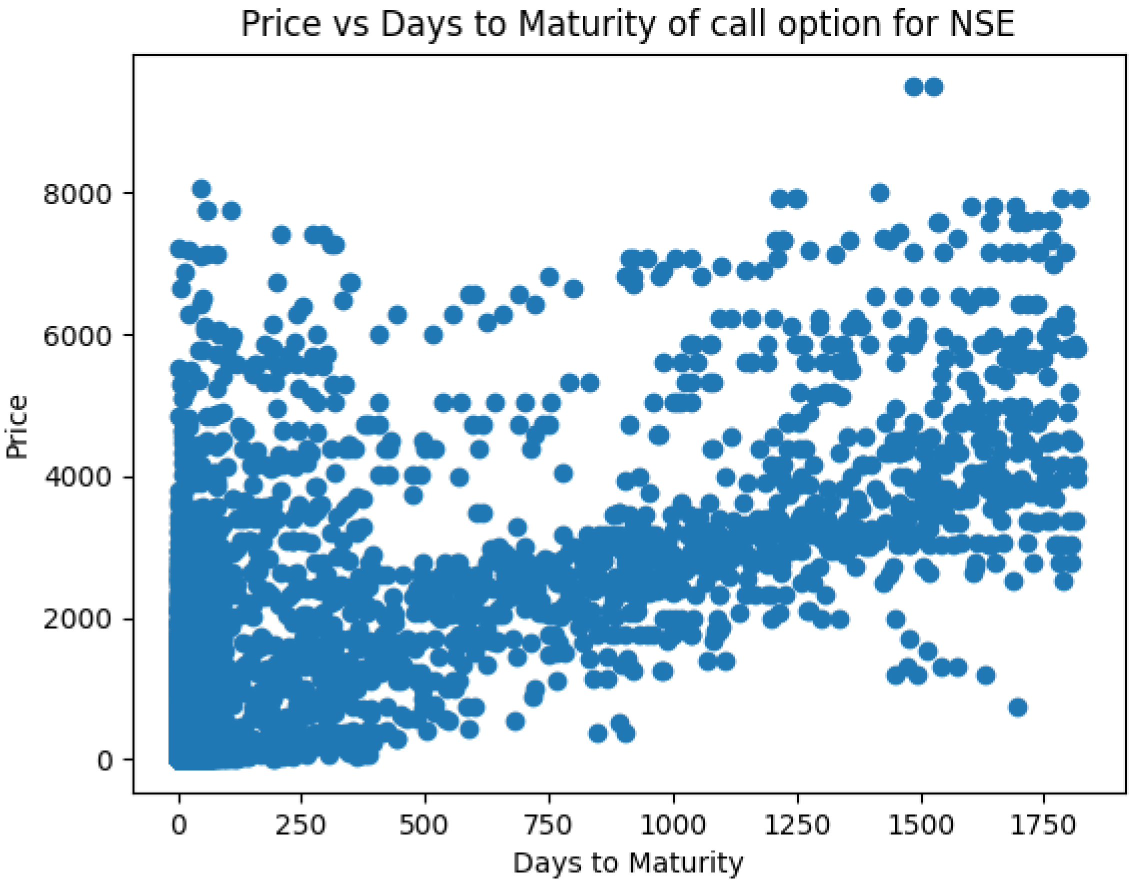
Part - (a)

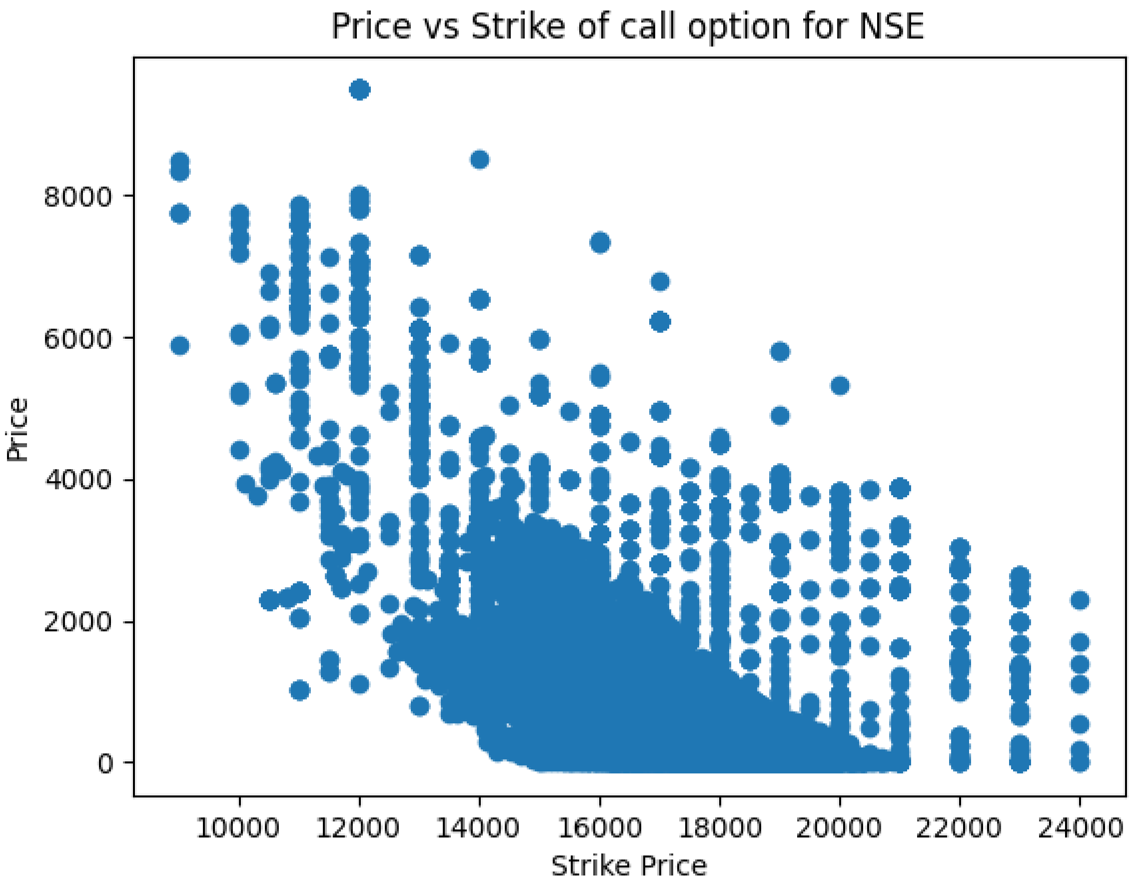


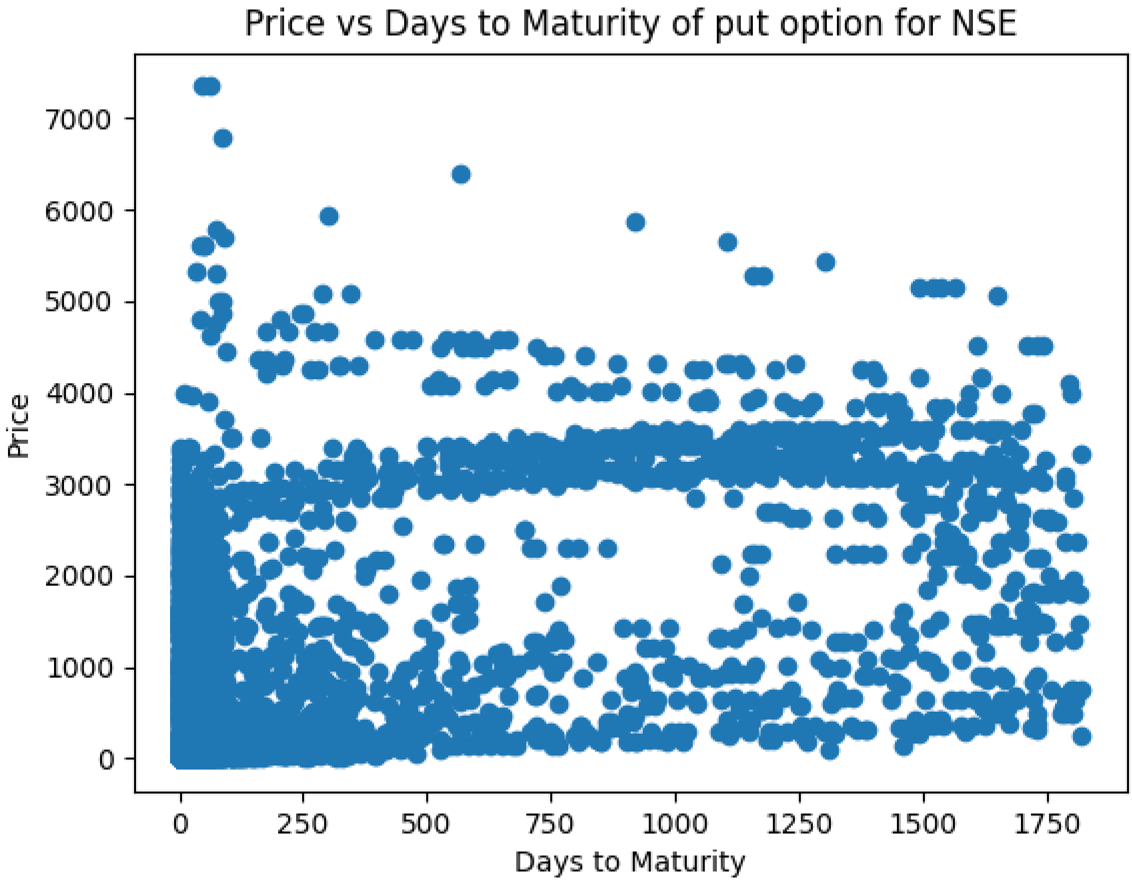
Price of European Call 3d for NSE
8000
6000
4000
2000
0
Price
22500
20000
17500
15000
12500
10000
Strike Price
0
500
1000
1500
Days to Maturity

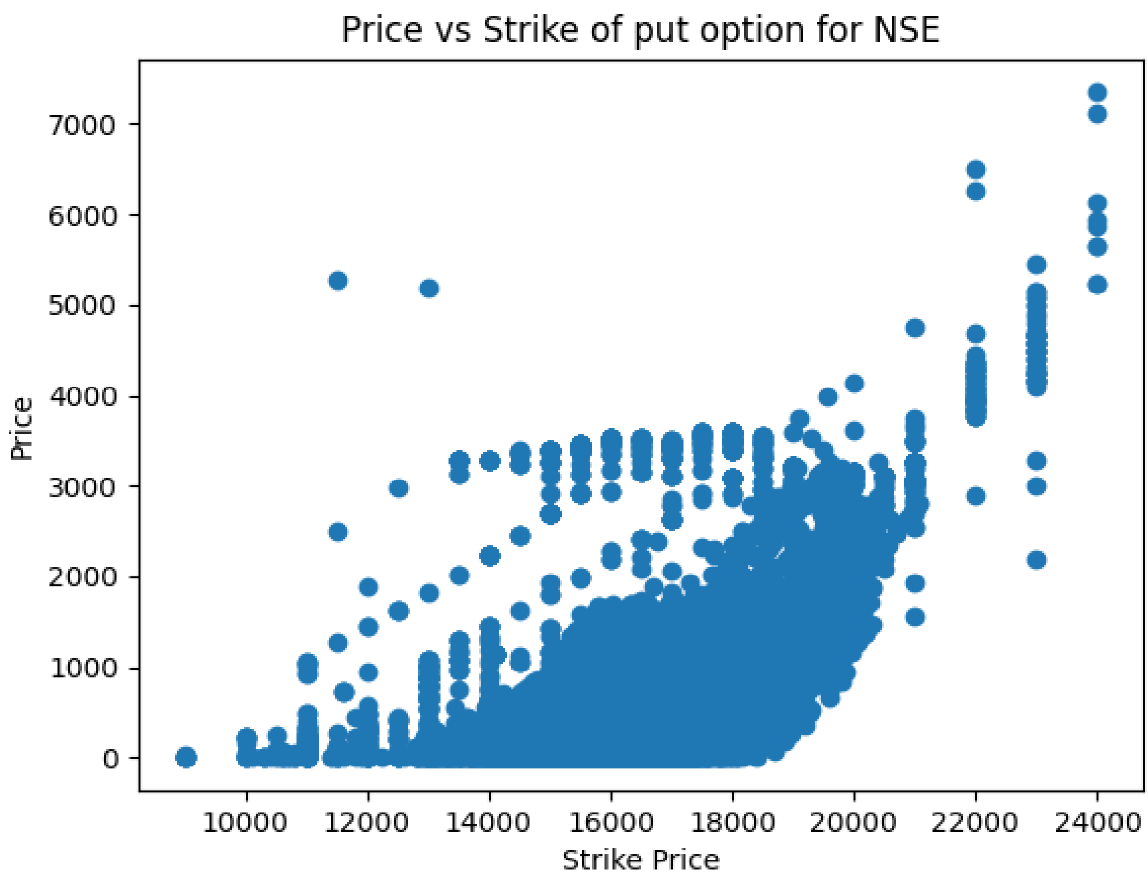


Price of European Put 3d for NSE
7000
6000
5000
4000
3000
2000
1000
0
Price
22500
20000
17500
15000
12500
10000
Strike Price
0
500
1000
1500
Days to Maturity

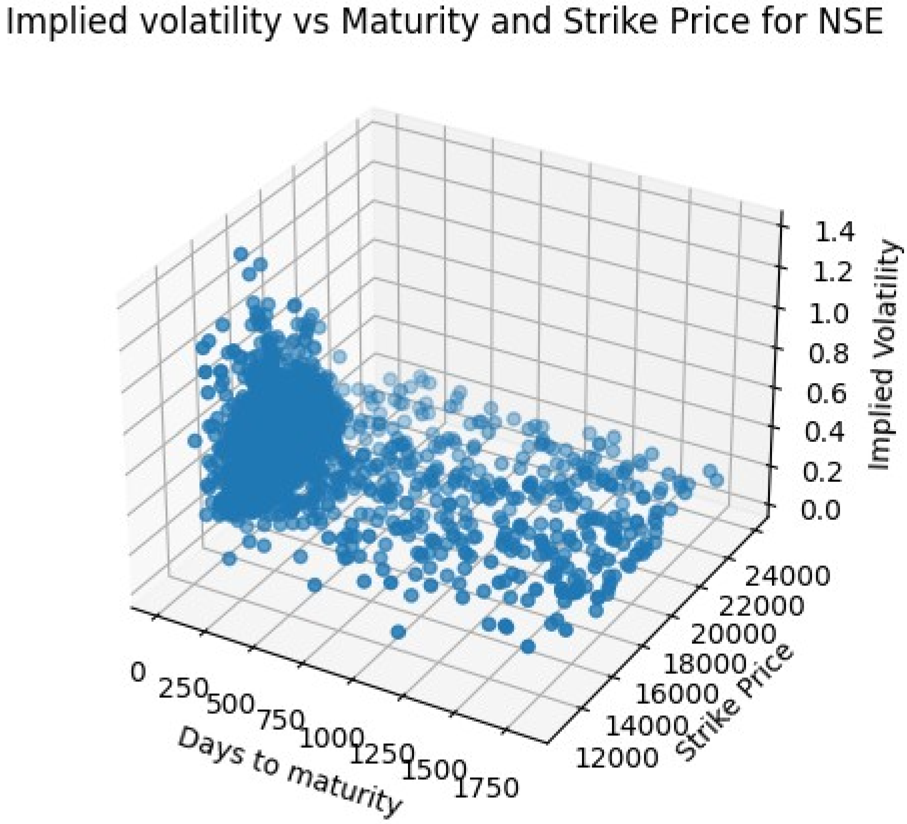

Price vs Days to Maturity of call option for NSE
Price
Days to Maturity
0
2000
4000
6000
8000
0
250
500
750
1000
1250
1500
1750


Price vs Strike of call option for NSE
Price
Strike Price
0
2000
4000
6000
8000
10000
12000
14000
16000
18000
20000
22000
24000

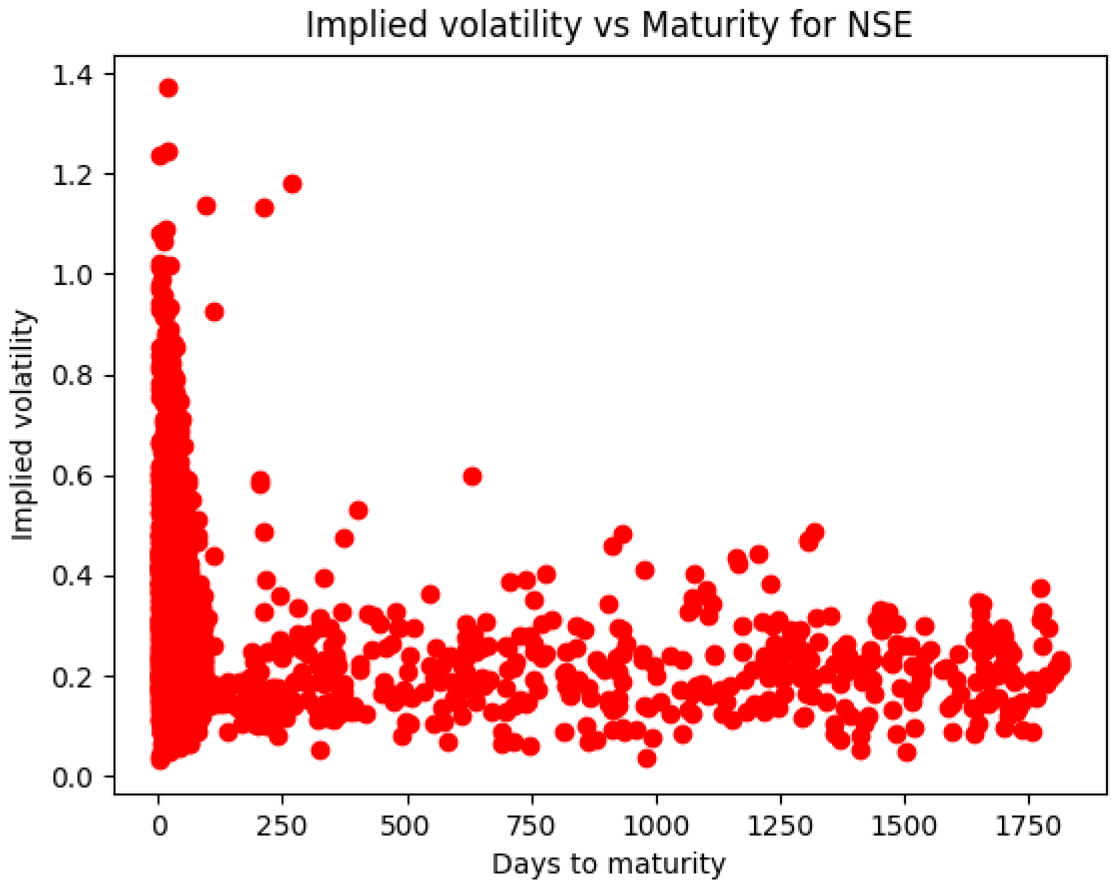

Price vs Days to Maturity of put option for NSE
7000
6000
5000
4000
3000
2000
1000
0
Price
0
250
500
750
1000
1250
1500
1750
Days to Maturity


Price vs Strike of put option for NSE
7000
6000
5000
4000
3000
2000
1000
0
Price
10000
12000
14000
16000
18000
20000
22000
24000
Strike Price

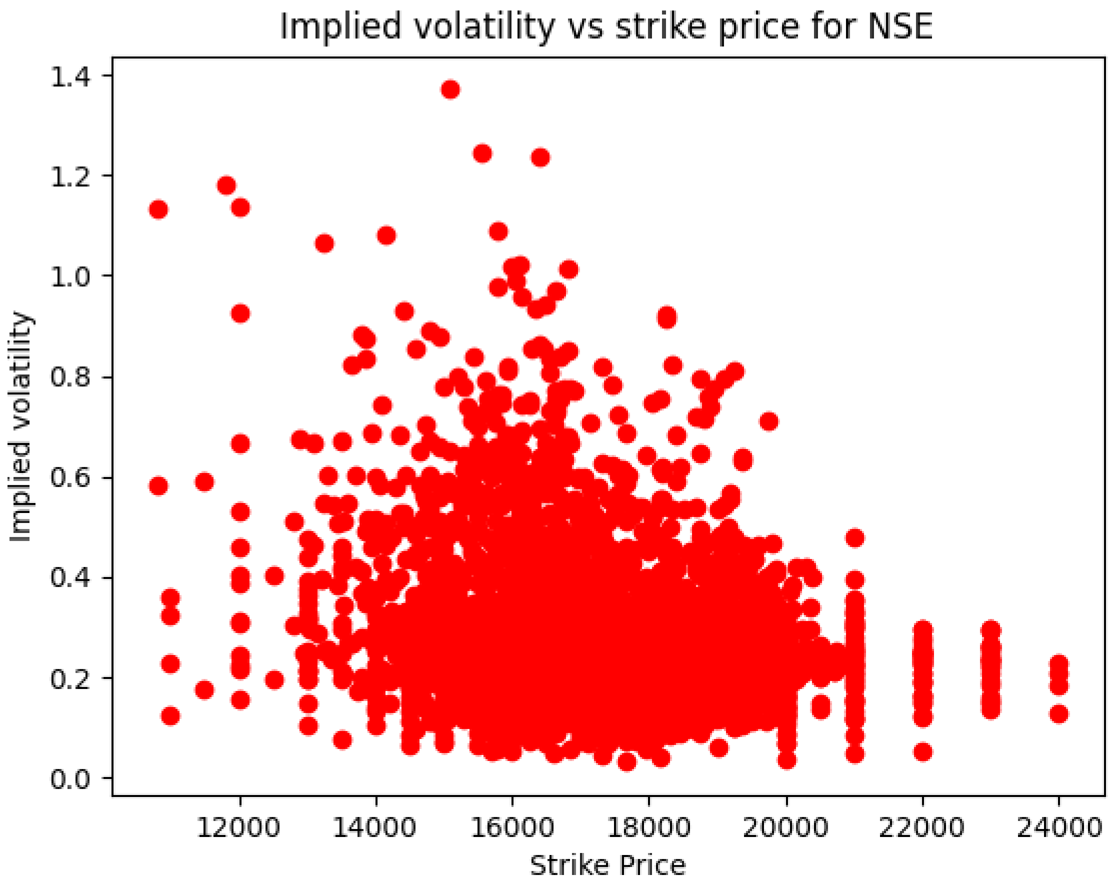
Part - (b)



Implied volatility vs Maturity and Strike Price for NSE
Implied Volatility
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
24000
22000
20000
18000
16000
14000
12000
Strike Price
0
250
500
750
1000
1250
1500
1750
Days to maturity

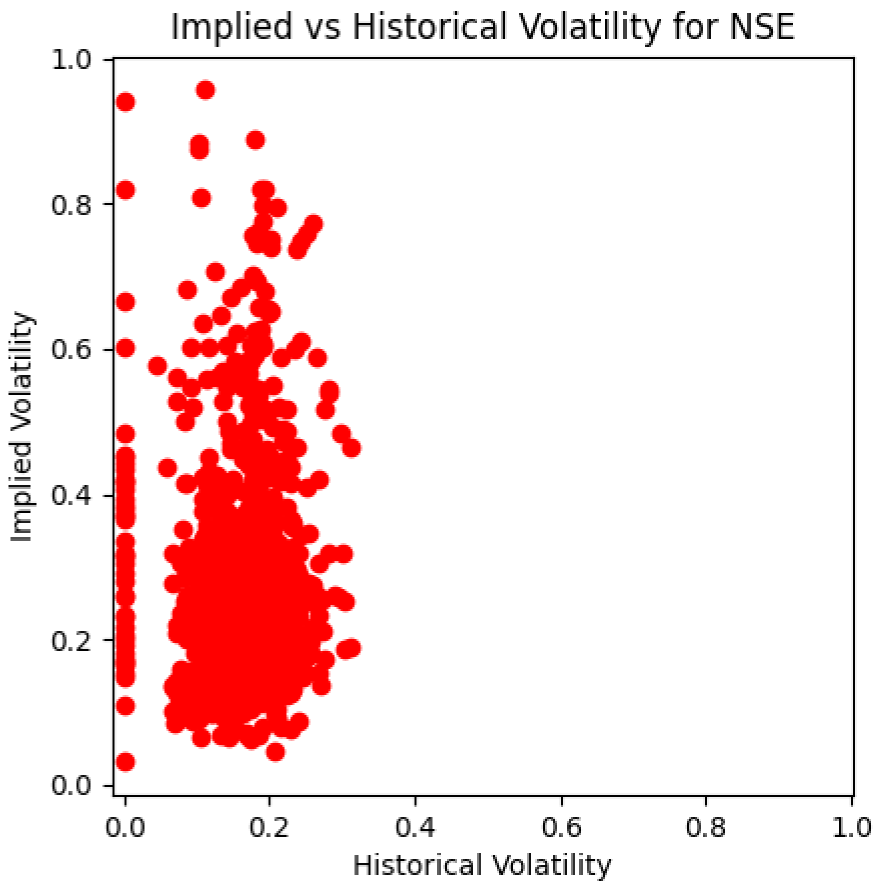


Implied volatility vs Maturity for NSE
Implied volatility
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
0
250
500
750
1000
1250
1500
1750
Days to maturity

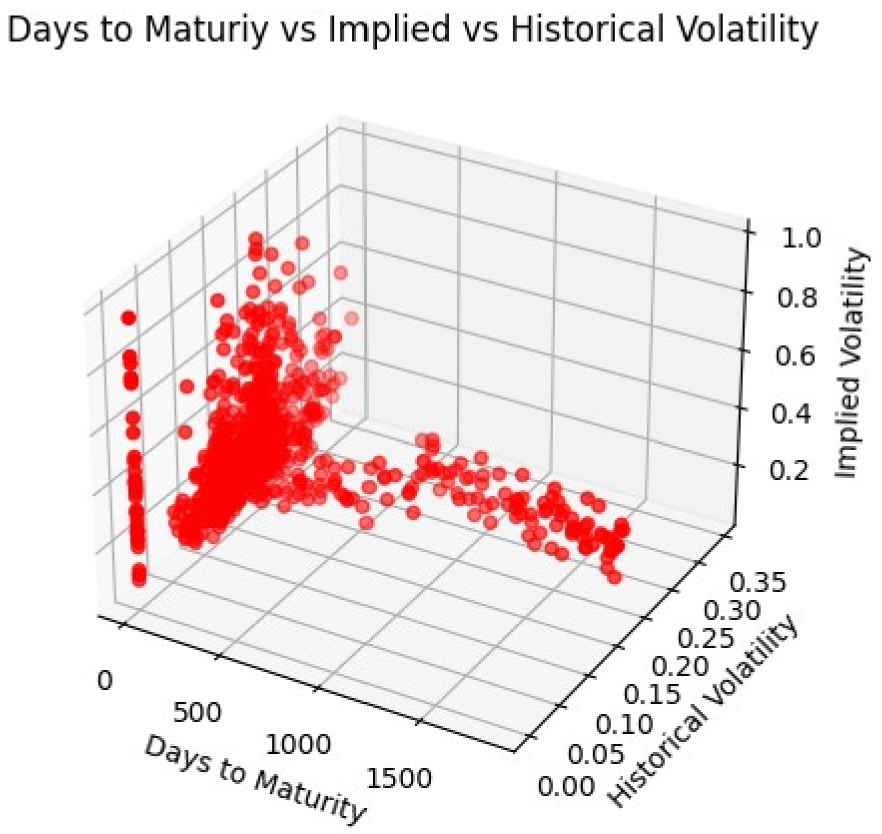


Implied volatility vs strike price for NSE
Implied volatility
Strike Price
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
12000
14000
16000
18000
20000
22000
24000

Part - (c)



Implied vs Historical Volatility for NSE
Implied Volatility
Historical Volatility
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0

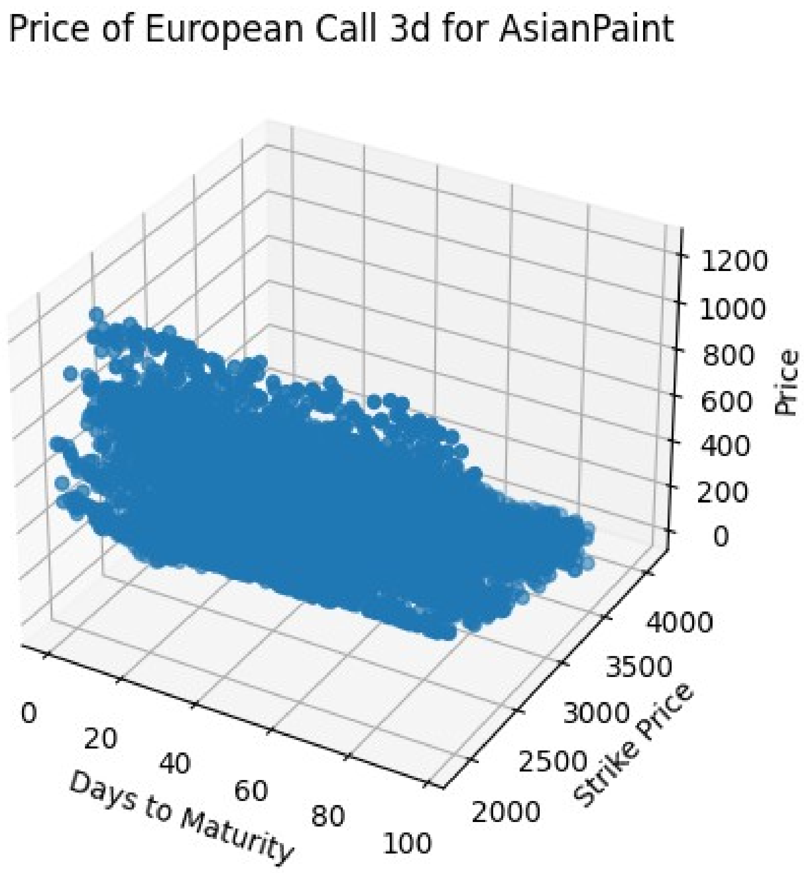


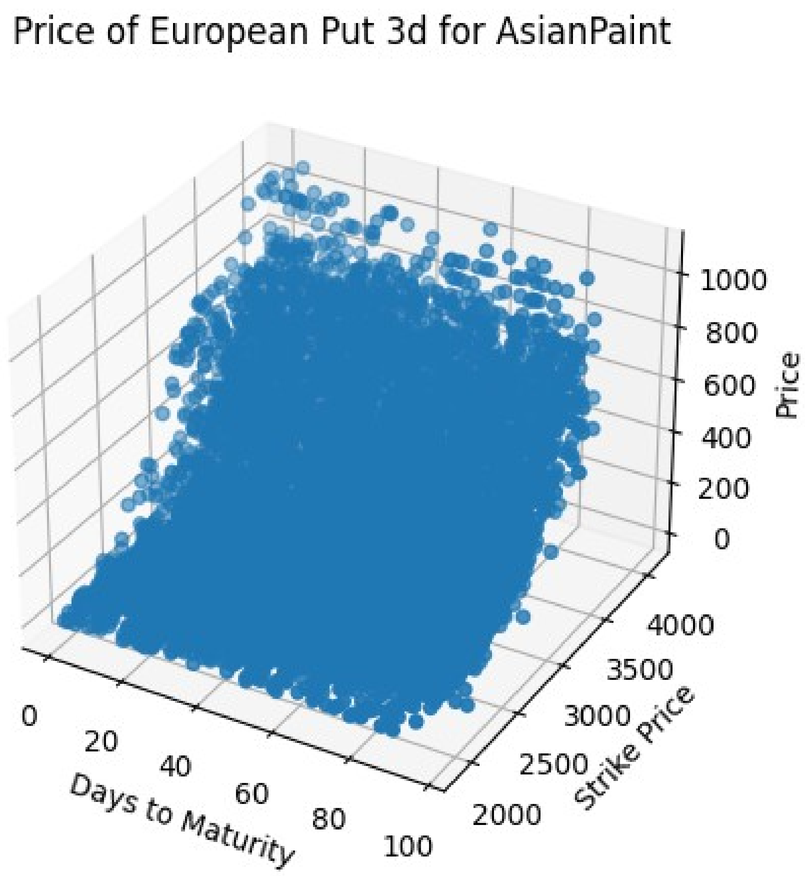
I am attaching the screenshot of tabulated values of implied and historical volatility of 15 randomly chosen points from my dataset for comparison

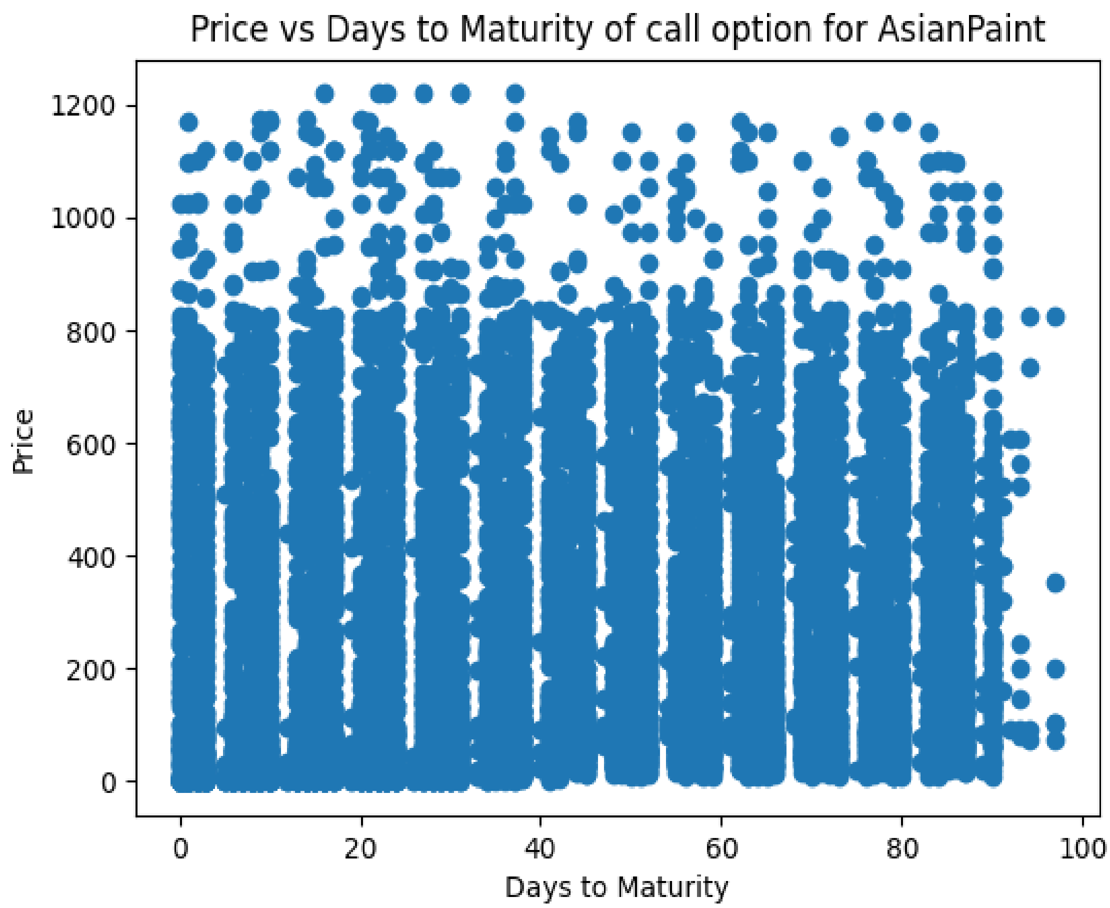
Days to Maturity	Implied Volatility	Historical Volatility
49	0.31479	0.241481
22	0.183026	0.163722
685	0.313886	0.173189
20	0.0684638	0.0808166
45	0.185711	0.206274
14	0.707889	0.169567
66	0.163289	0.201515
20	0.130874	0.151308
1477	0.185332	0.211289
31	0.284065	0.188904
20	0.624073	0.177512
40	0.111245	0.126631
28	0.130062	0.126892
1632	0.222686	0.206519
1080	0.144066	0.223667

ASIANPAINT

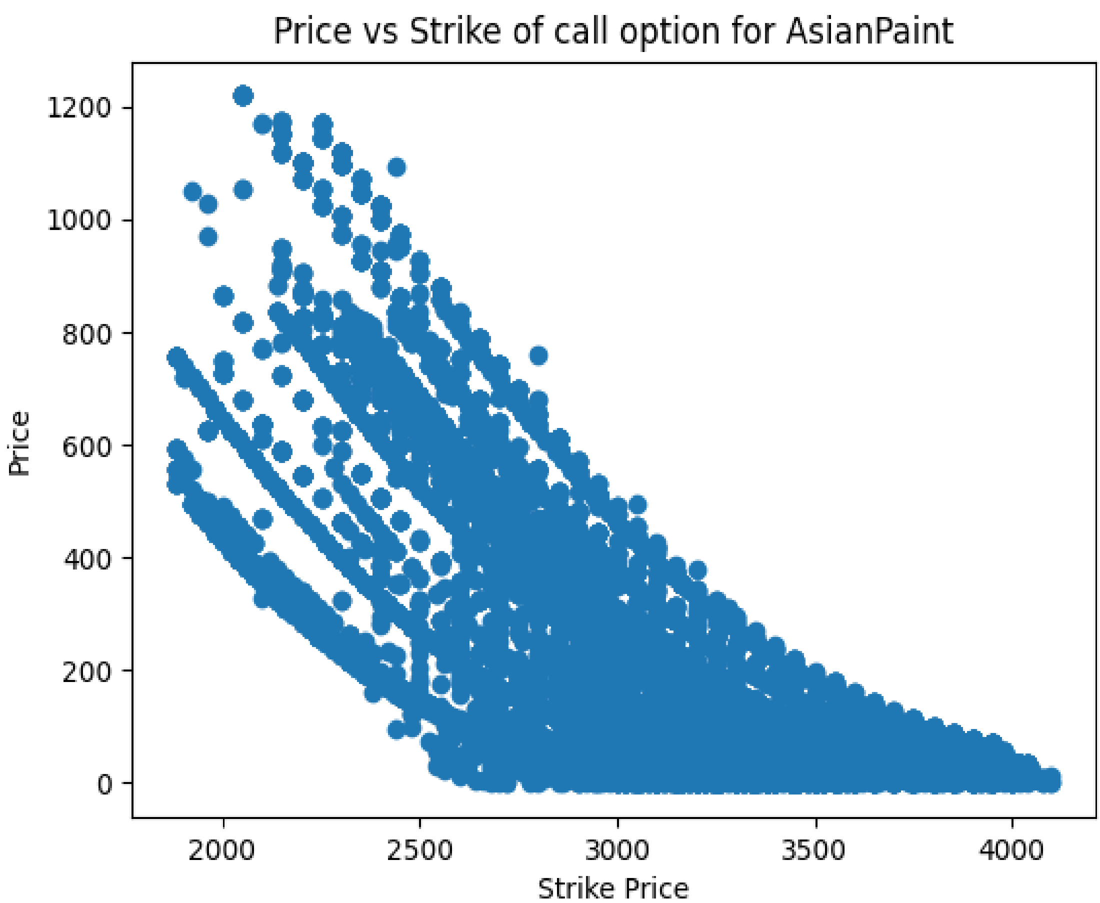
Part - (a)



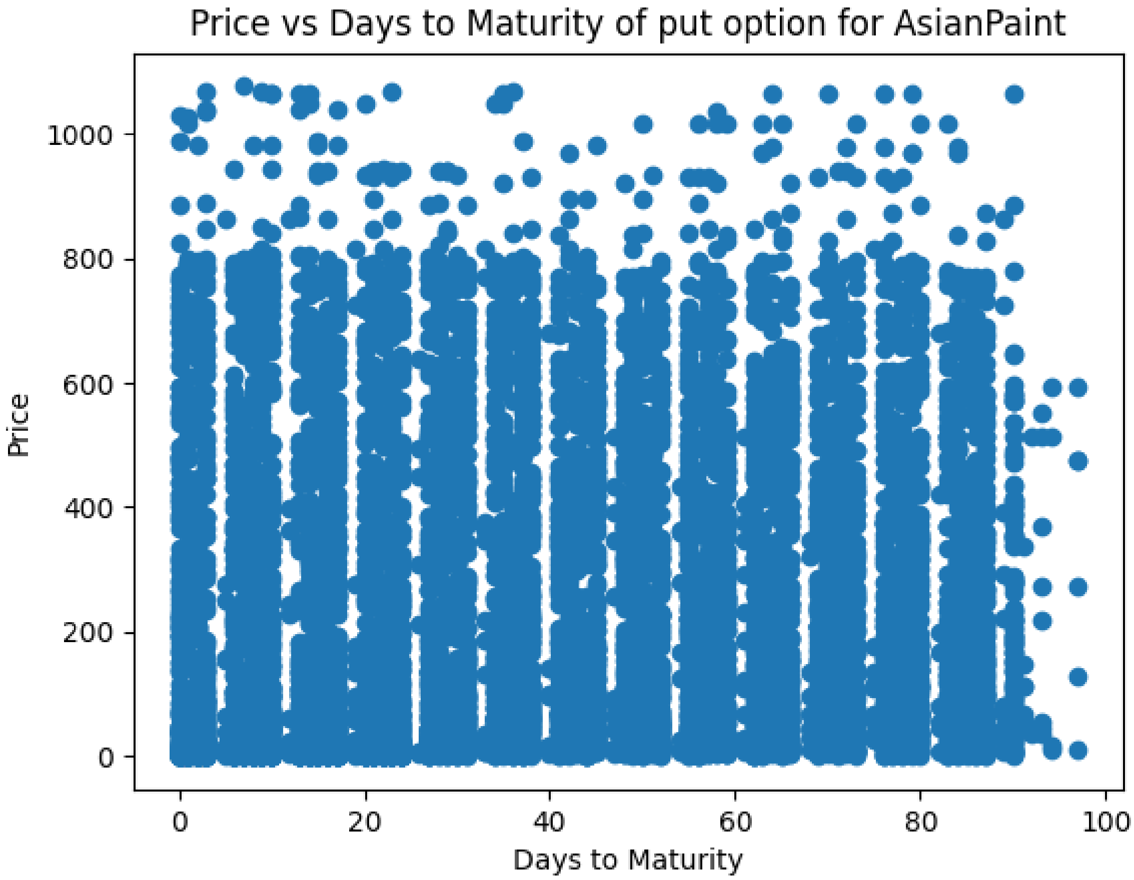




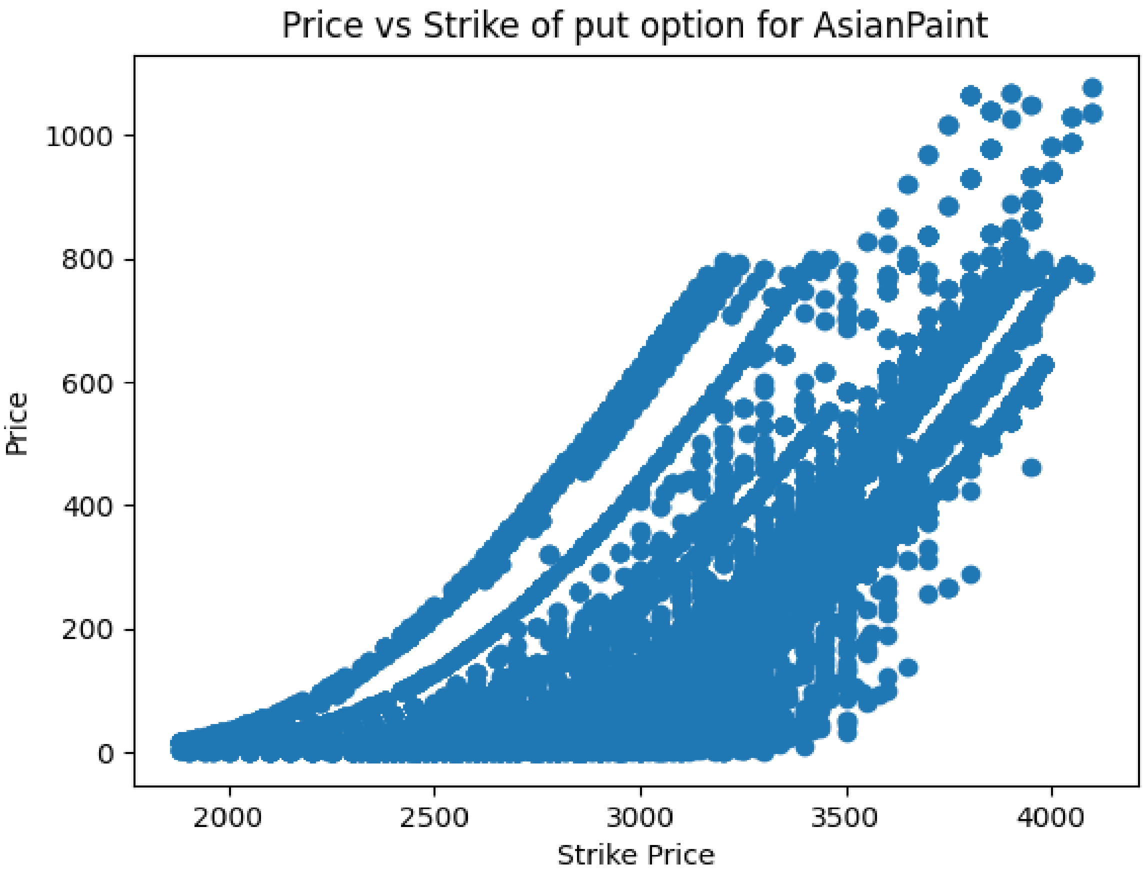
Price vs Days to Maturity of call option for AsianPaint
Price
Days to Maturity
0
200
400
600
800
1000
1200
0
20
40
60
80
100



Price vs Strike of call option for AsianPaint
Price
Strike Price
0
200
400
600
800
1000
1200
2000
2500
3000
3500
4000

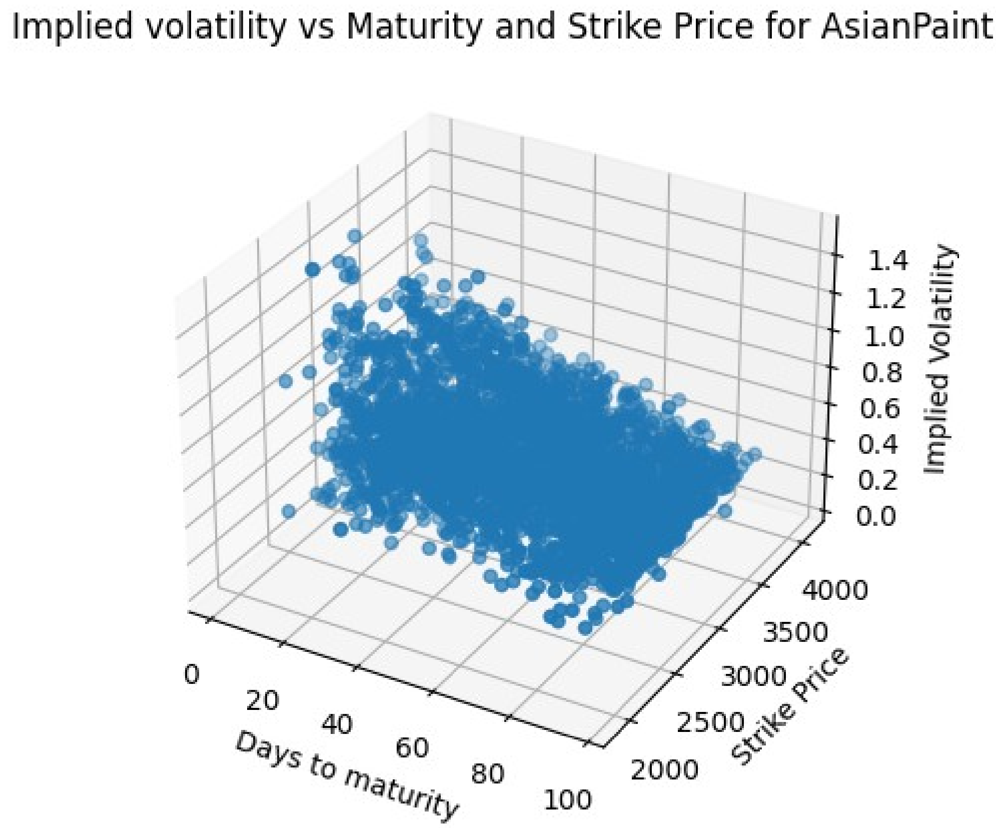


Price vs Days to Maturity of put option for AsianPaint
Price
Days to Maturity
0
200
400
600
800
1000
0
20
40
60
80
100

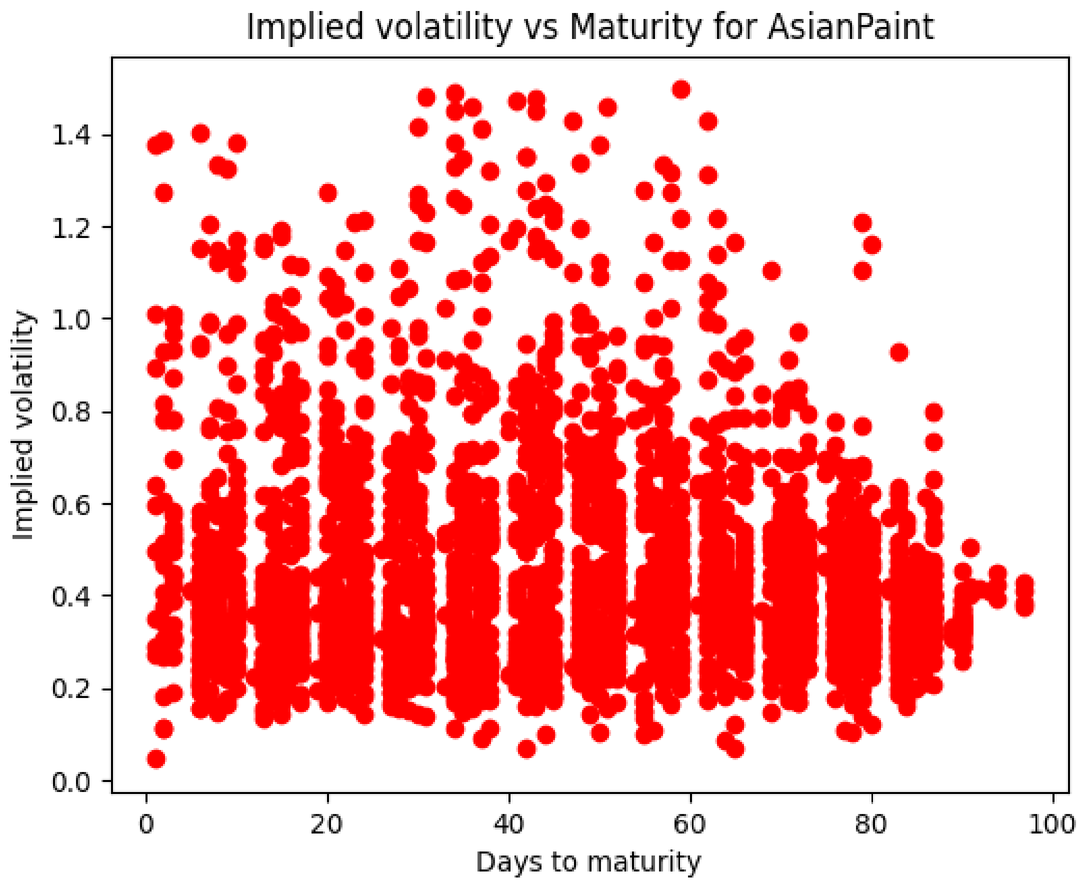


Price vs Strike of put option for AsianPaint
Price
Strike Price
0
200
400
600
800
1000
2000
2500
3000
3500
4000

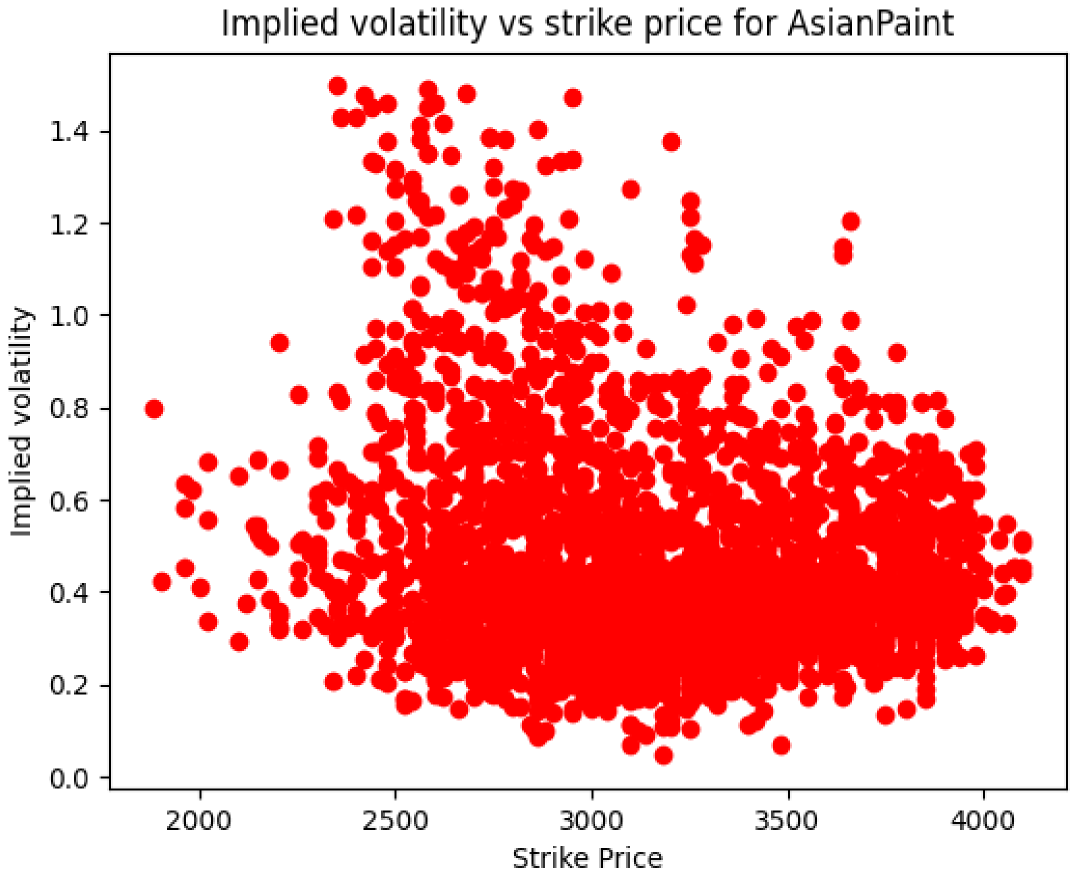
Part - (b)



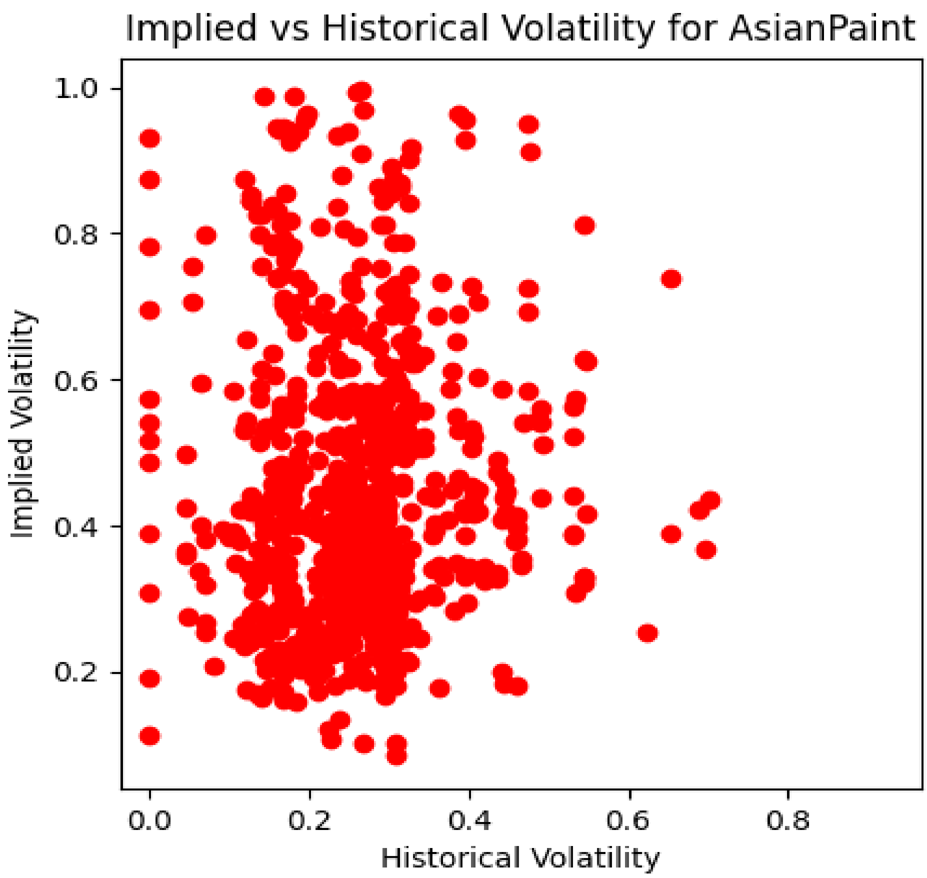
Implied volatility vs Maturity and Strike Price for AsianPaint
Implied Volatility
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
4000
3500
3000
2500
2000
Strike Price
0
20
40
60
80
100
Days to maturity

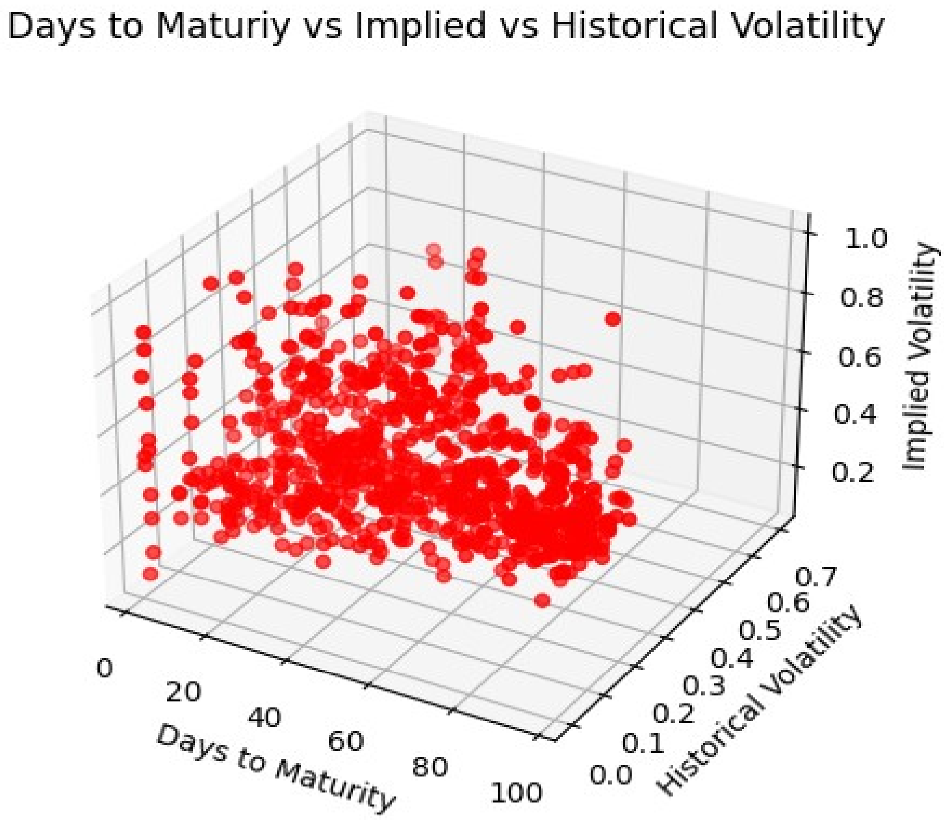


Implied volatility vs Maturity for AsianPaint
Implied volatility
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
0
20
40
60
80
100
Days to maturity


Implied volatility vs strike price for AsianPaint
Implied volatility
Strike Price
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
2000
2500
3000
3500
4000

Part - (c)


Implied vs Historical Volatility for AsianPaint
Implied Volatility
Historical Volatility
0.2
0.4
0.6
0.8
1.0
0.0
0.2
0.4
0.6
0.8

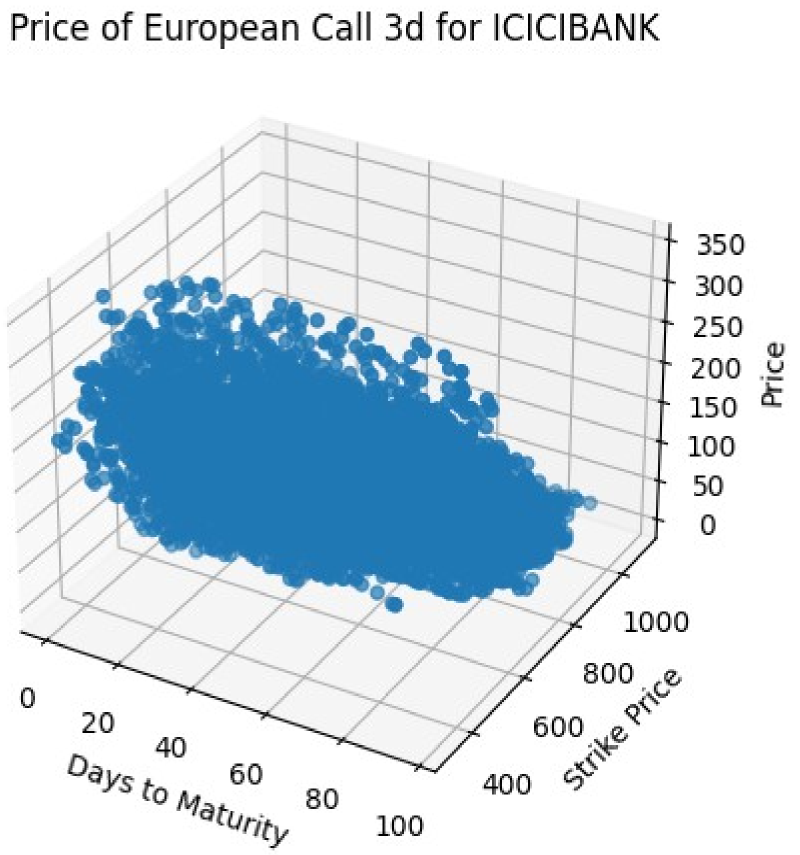


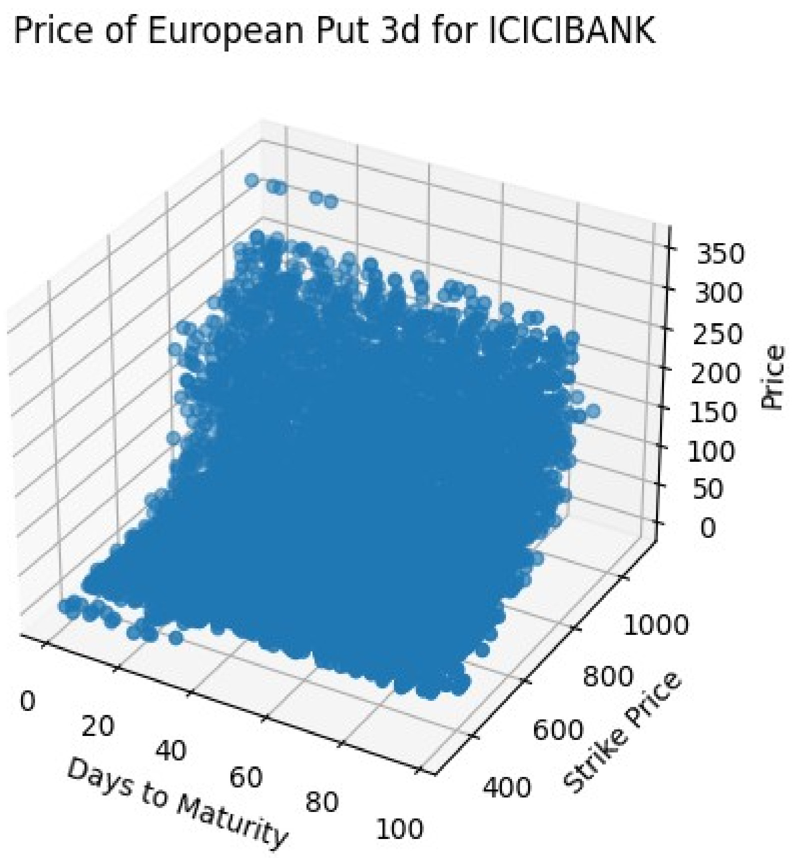
I am attaching the screenshot of tabulated values of implied and historical volatility of 15 randomly chosen points from my dataset for comparison

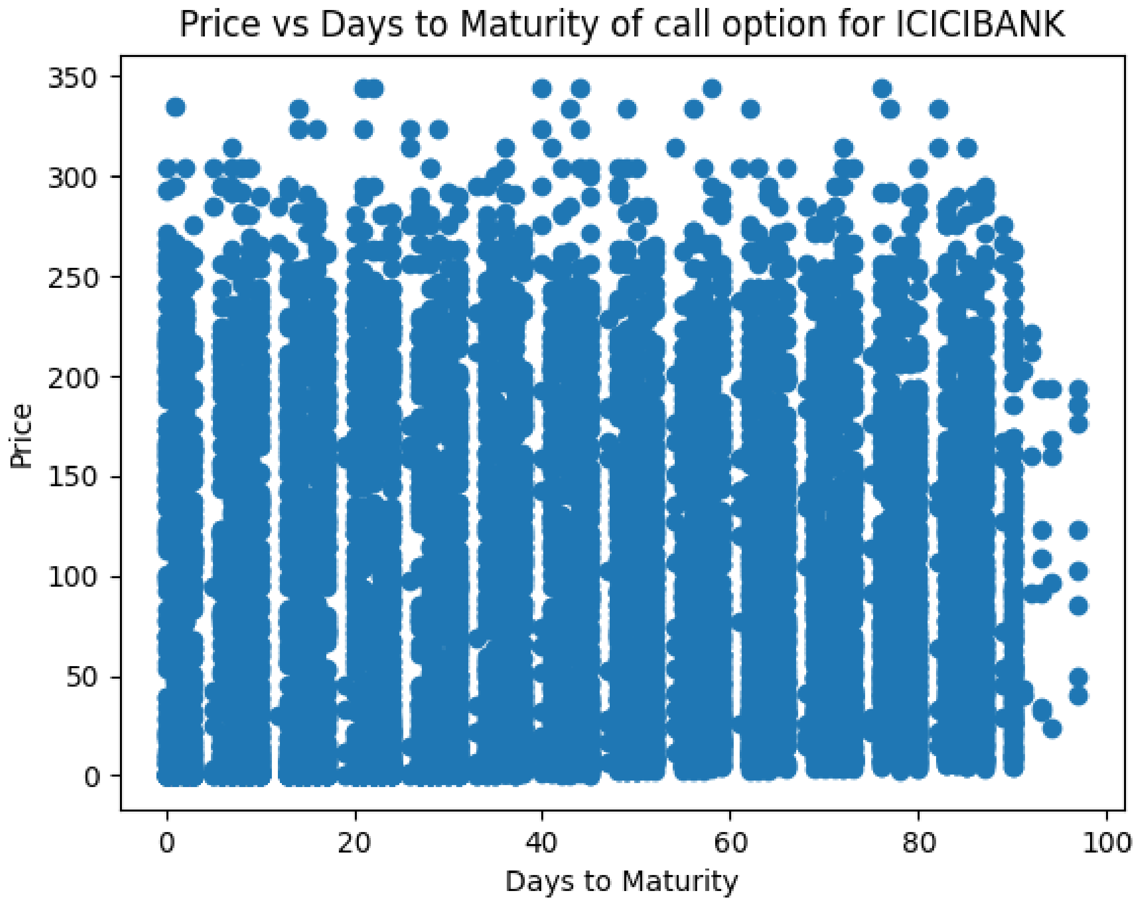
Days to Maturity	Implied Volatility	Historical Volatility
10	0.274481	0.23789
9	0.26427	0.116551
71	0.363259	0.284244
50	0.304293	0.303694
36	0.443901	0.253476
37	0.283395	0.251273
73	0.382945	0.229682
45	0.182955	0.445291
10	0.528045	0.253519
72	0.348765	0.256945
86	0.384529	0.247126
23	0.282801	0.22978
77	0.302678	0.250189
13	0.94362	0.167518
79	0.350486	0.248919

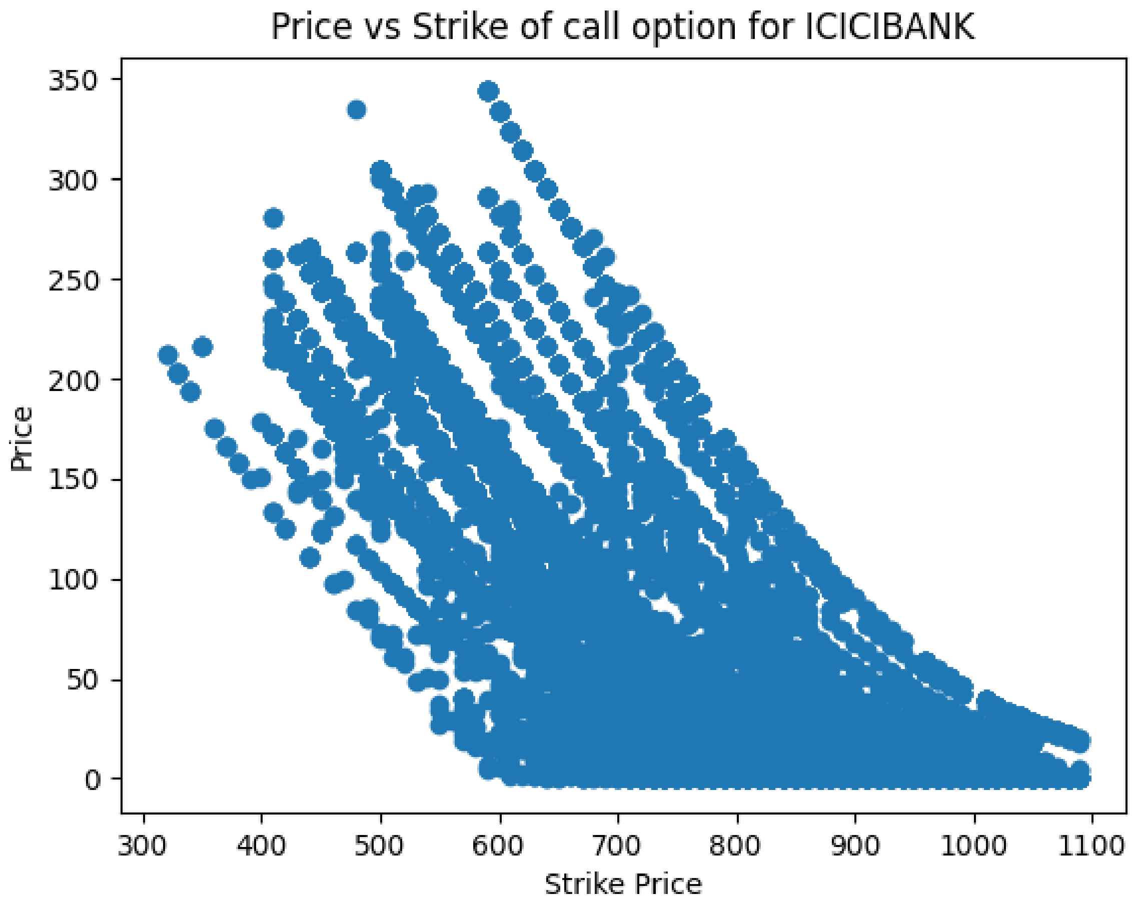
ICICIBANK

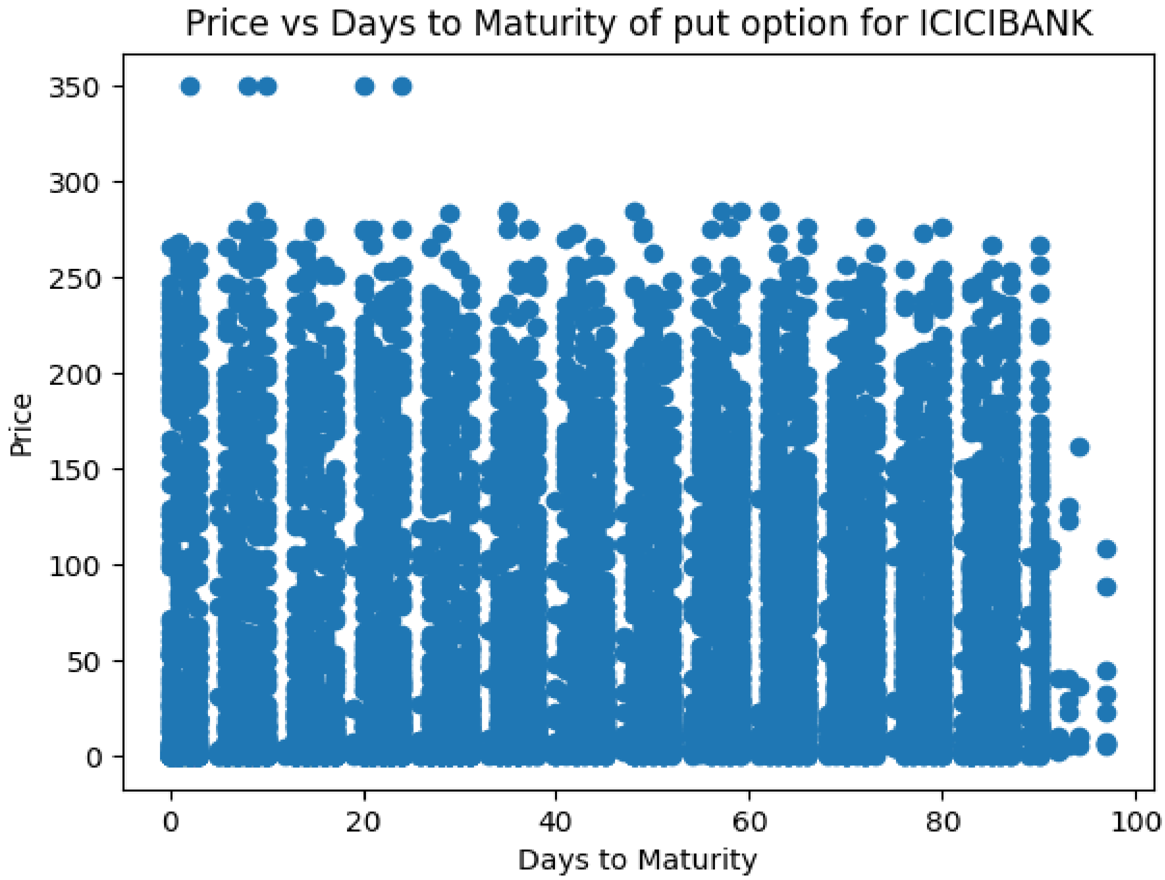
Part - (a)

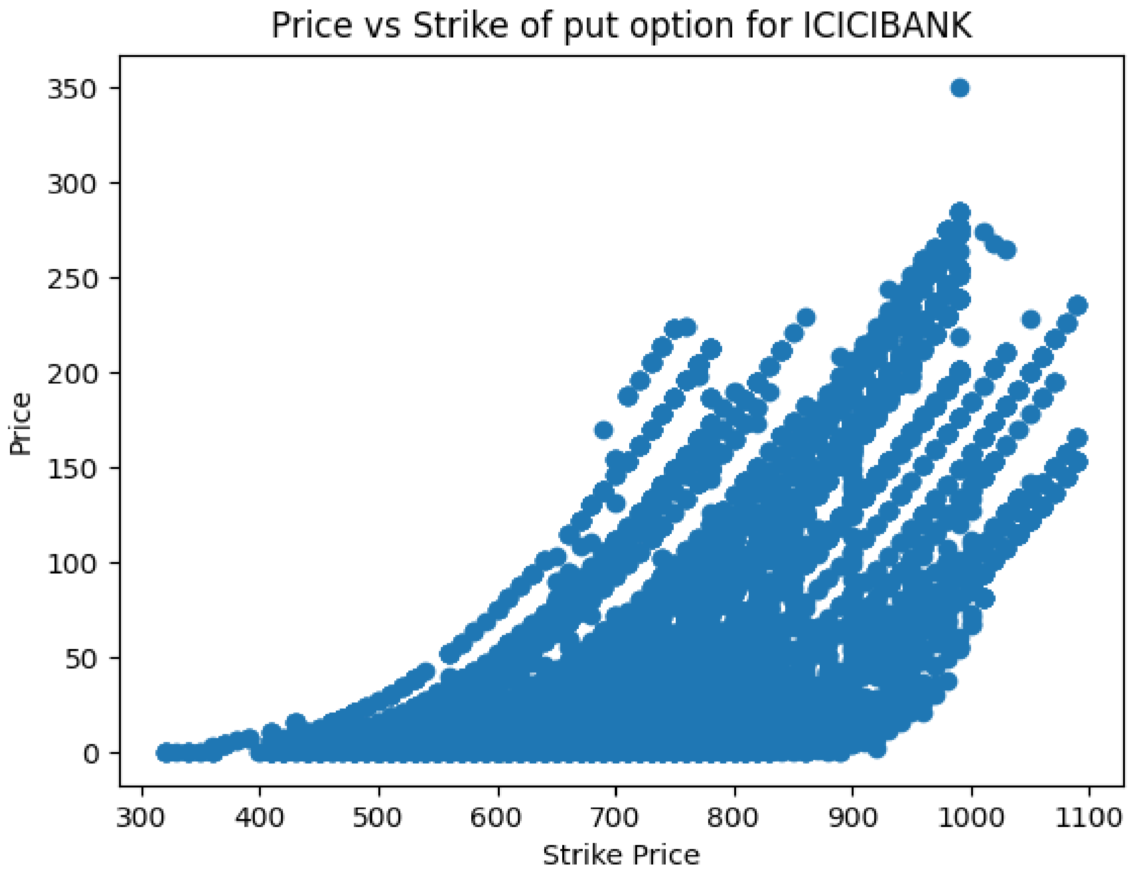





Price vs Days to Maturity of call option for ICICIBANK
Price
Days to Maturity
0
50
100
150
200
250
300
350
0
20
40
60
80
100

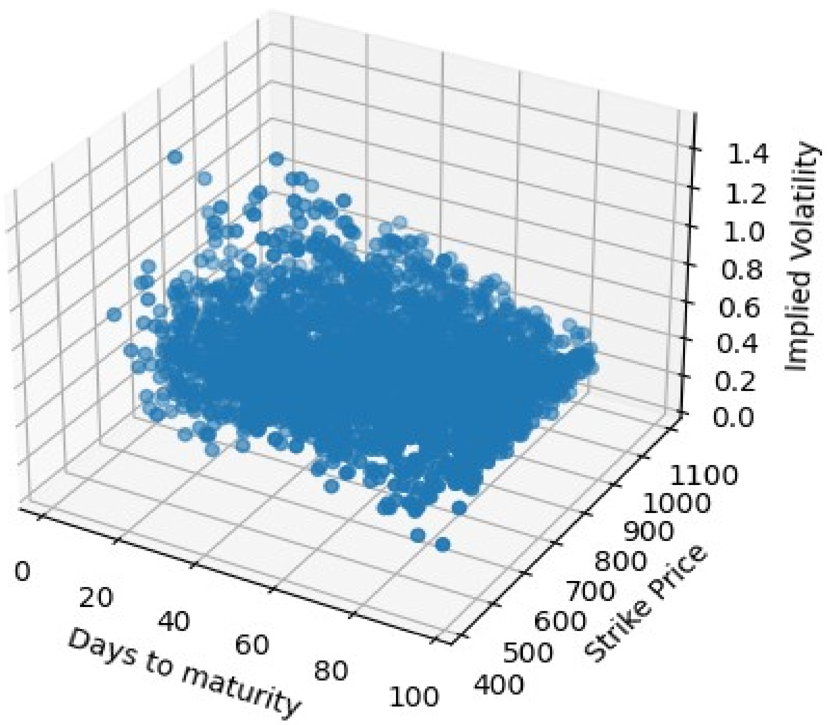

Price vs Strike of call option for ICICIBANK
Price
Strike Price
0
50
100
150
200
250
300
350
300
400
500
600
700
800
900
1000
1100


Price vs Days to Maturity of put option for ICICIBANK
Price
Days to Maturity
0
50
100
150
200
250
300
350
0
20
40
60
80
100

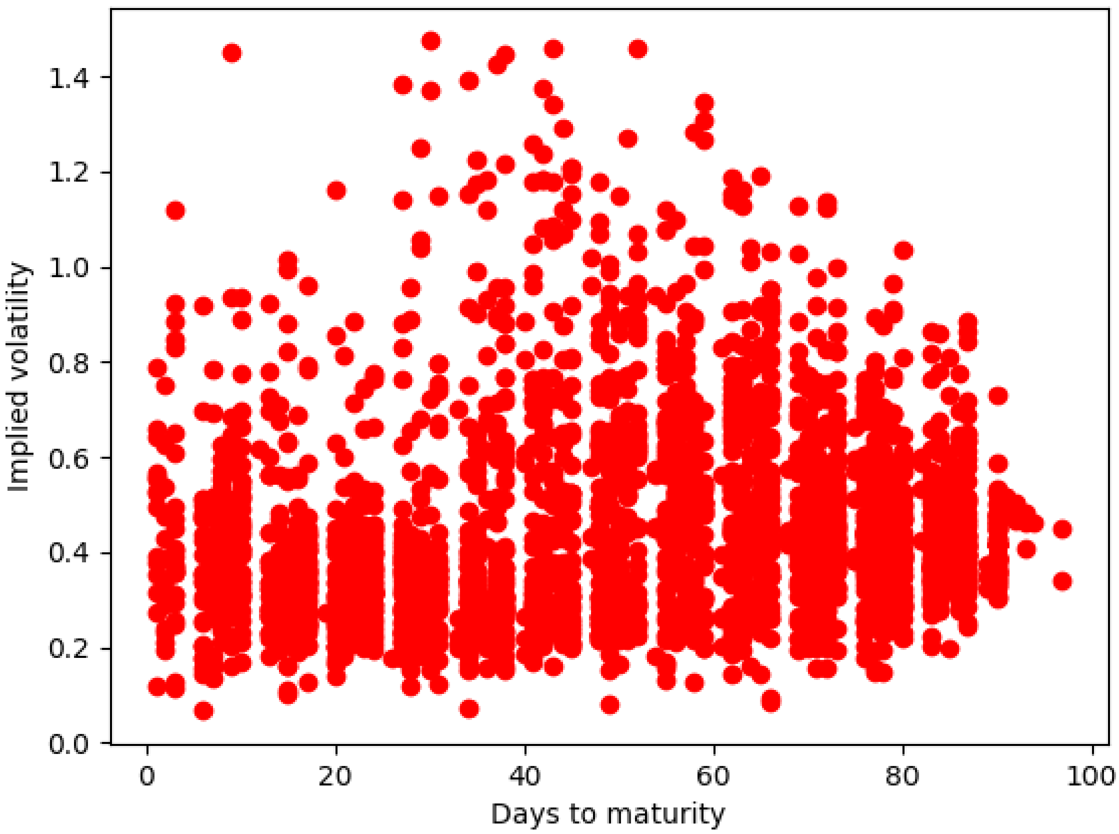

Price vs Strike of put option for ICICIBANK
Price
Strike Price
0
50
100
150
200
250
300
350
300
400
500
600
700
800
900
1000
1100

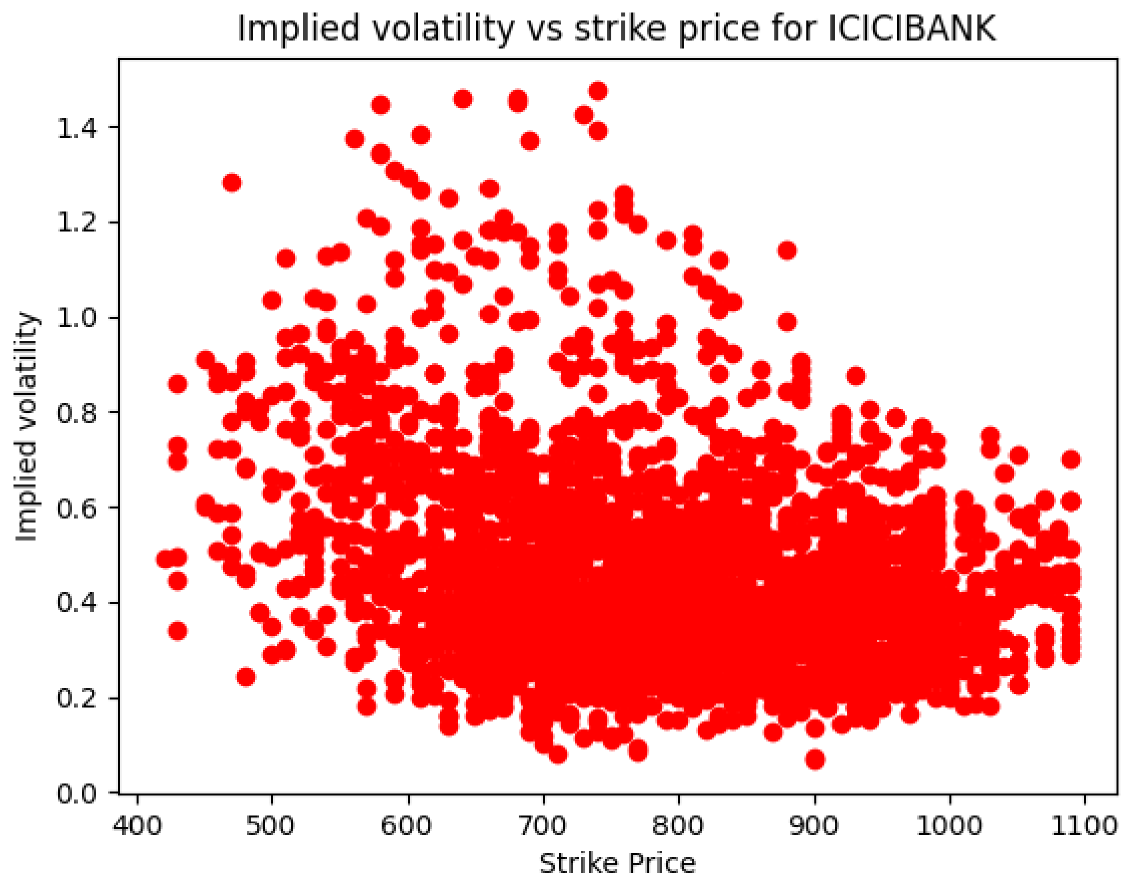
Part - (b)

Implied volatility vs Maturity and Strike Price for ICICIBANK

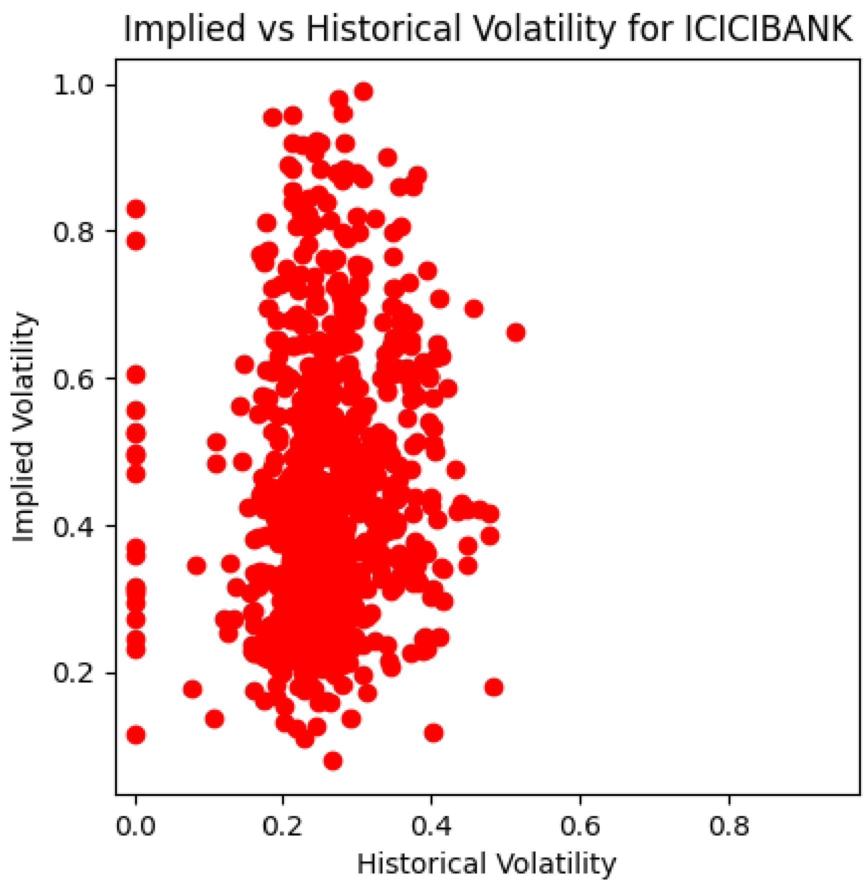


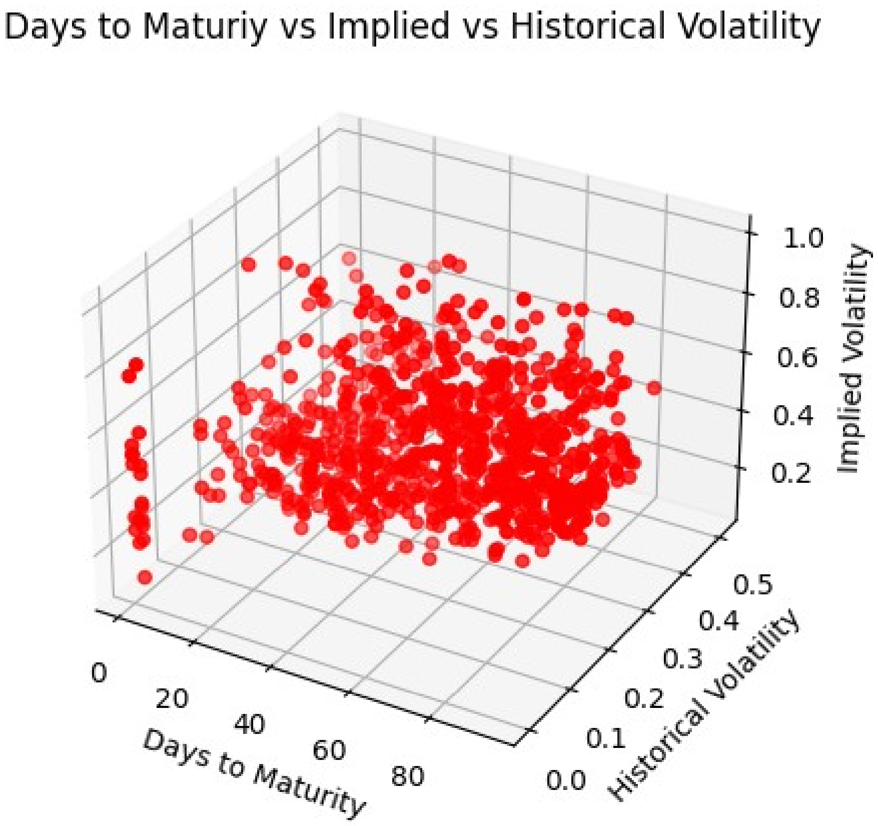
Implied volatility vs Maturity for ICICIBANK




Implied volatility vs strike price for ICICIBANK
Implied volatility
Strike Price
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
400
500
600
700
800
900
1000
1100

Part - (c)


Implied vs Historical Volatility for ICICIBANK
Implied Volatility
Historical Volatility
0.2
0.4
0.6
0.8
1.0
0.0
0.2
0.4
0.6
0.8



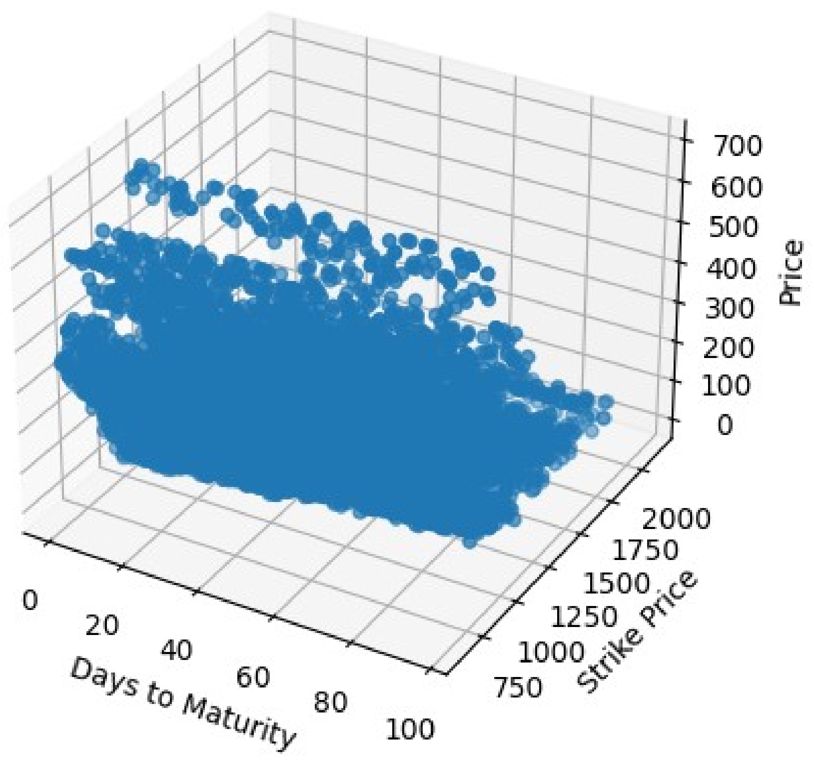
I am attaching the screenshot of tabulated values of implied and historical volatility of 15 randomly chosen points from my dataset for comparison

Days to Maturity	Implied Volatility	Historical Volatility
45	0.470872	0.2785
58	0.468119	0.224011
73	0.361824	0.238577
52	0.600145	0.246222
72	0.645183	0.352548
77	0.430063	0.266562
90	0.378527	0.270779
84	0.495415	0.344458
42	0.214641	0.288461
43	0.245449	0.192463
72	0.406273	0.218255
83	0.355435	0.208854
50	0.681007	0.223067
57	0.42564	0.233021
72	0.367191	0.237319

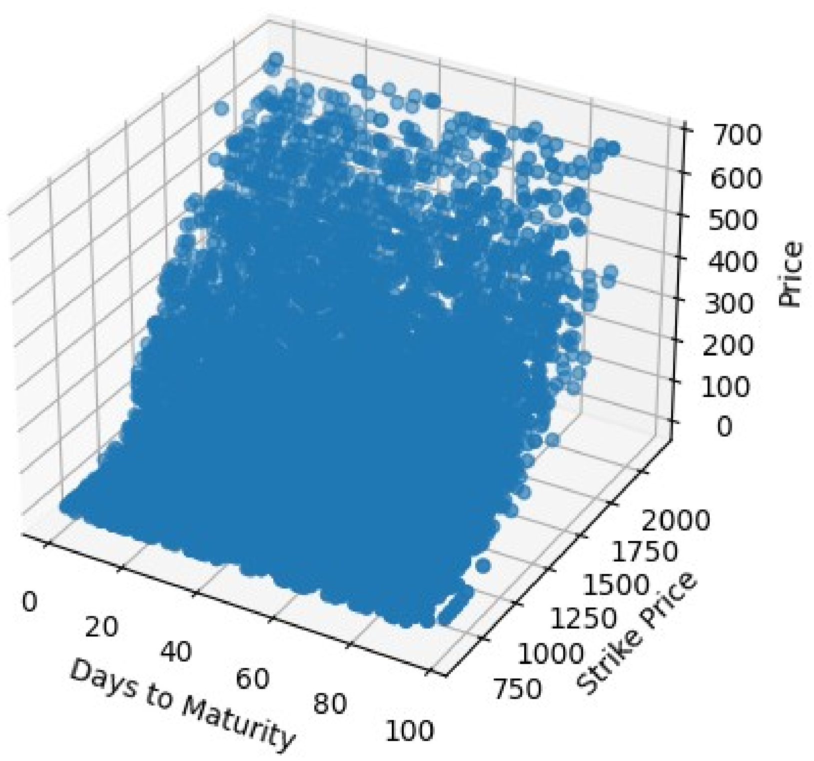
TECHM

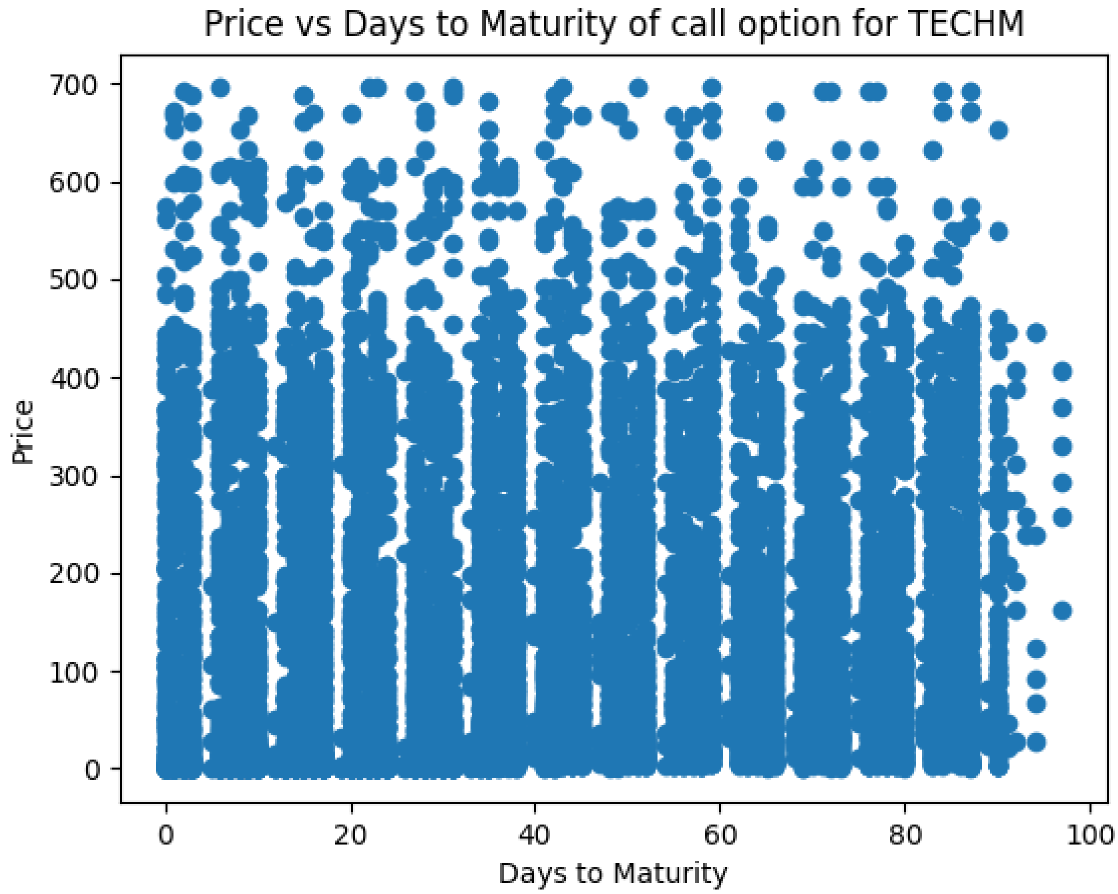
Part - (a)

Price of European Call 3d for TECHM

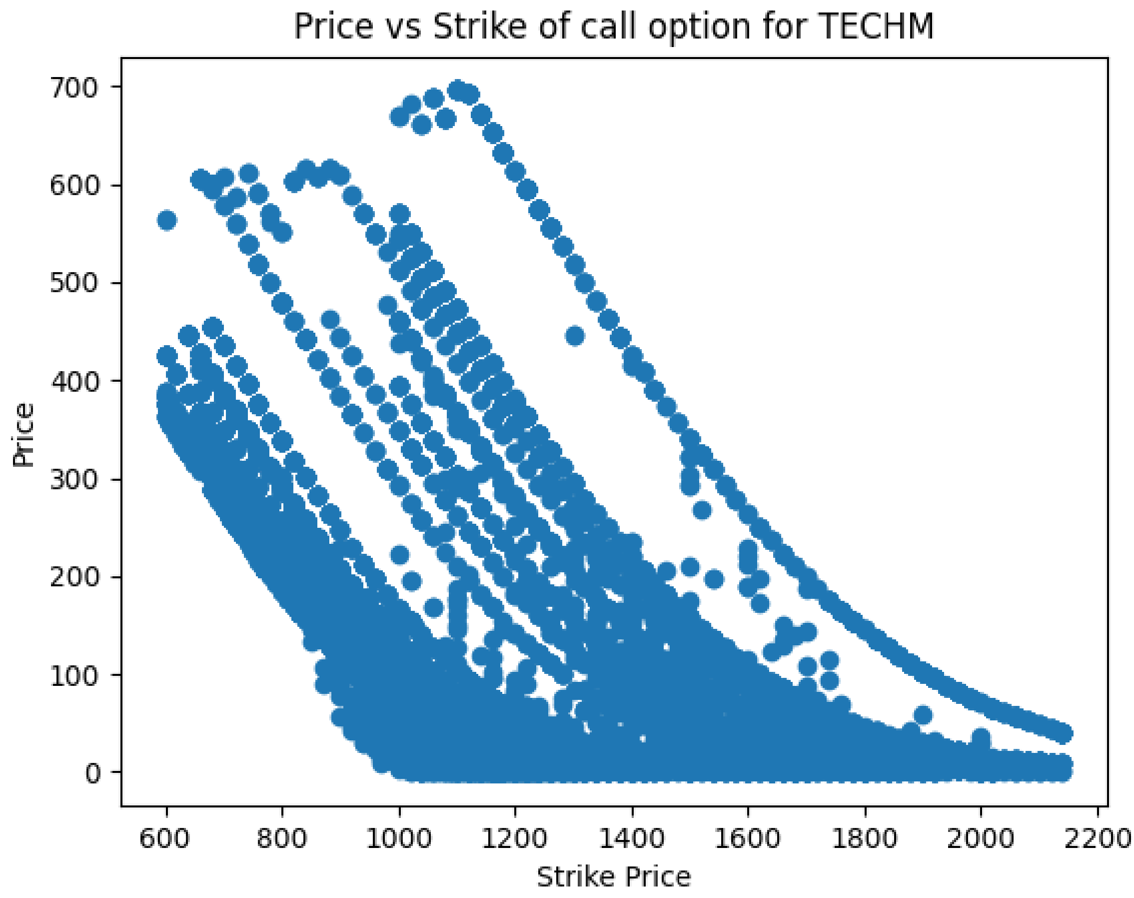


Price of European Put 3d for TECHM

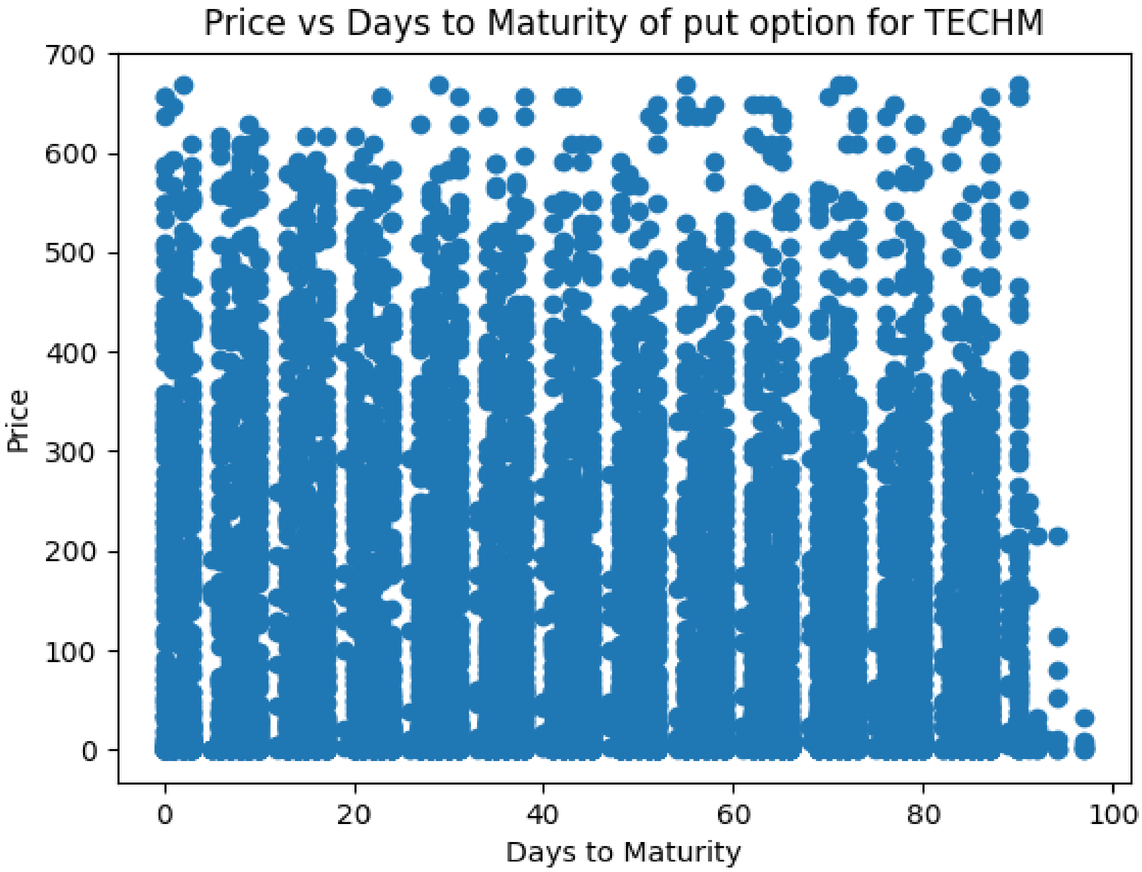




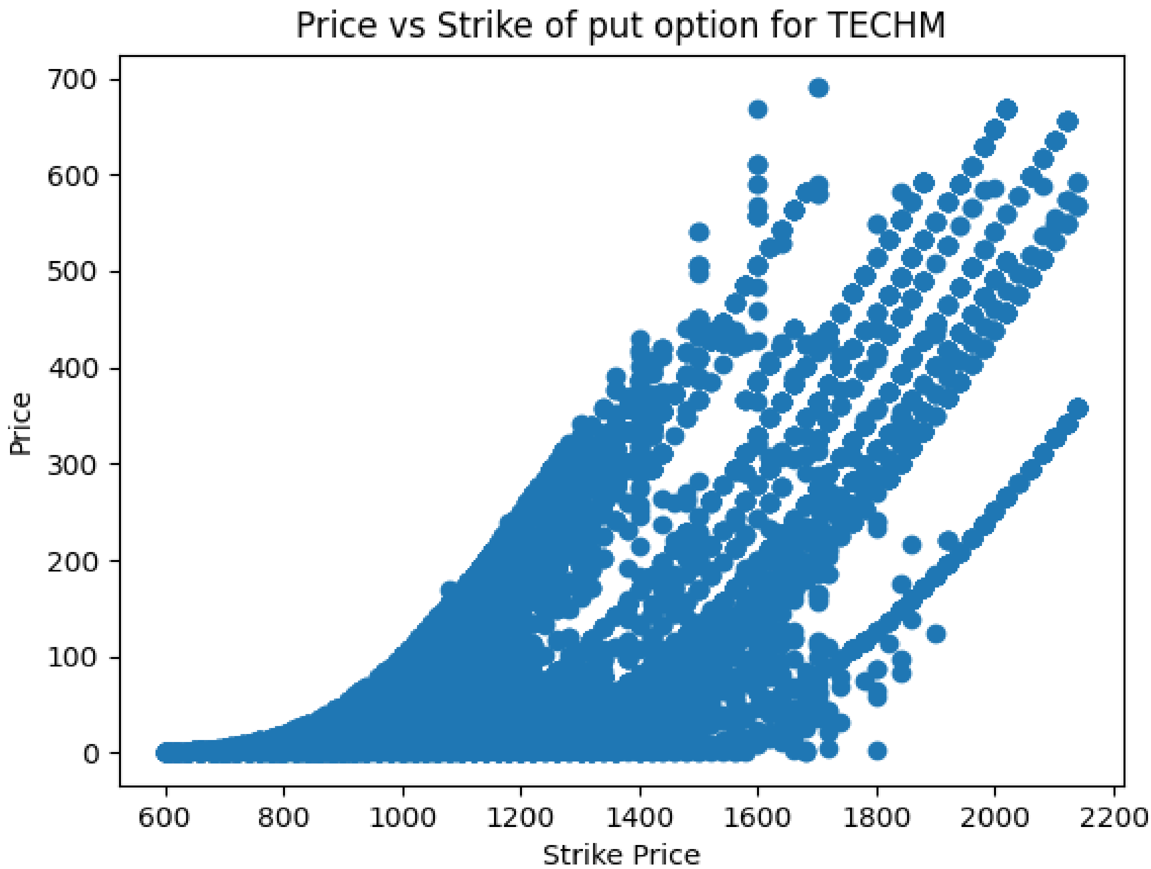
Price vs Days to Maturity of call option for TECHM
Price
Days to Maturity
0
100
200
300
400
500
600
700
0
20
40
60
80
100



Price vs Strike of call option for TECHM
Price
Strike Price
0
100
200
300
400
500
600
700
600
800
1000
1200
1400
1600
1800
2000
2200



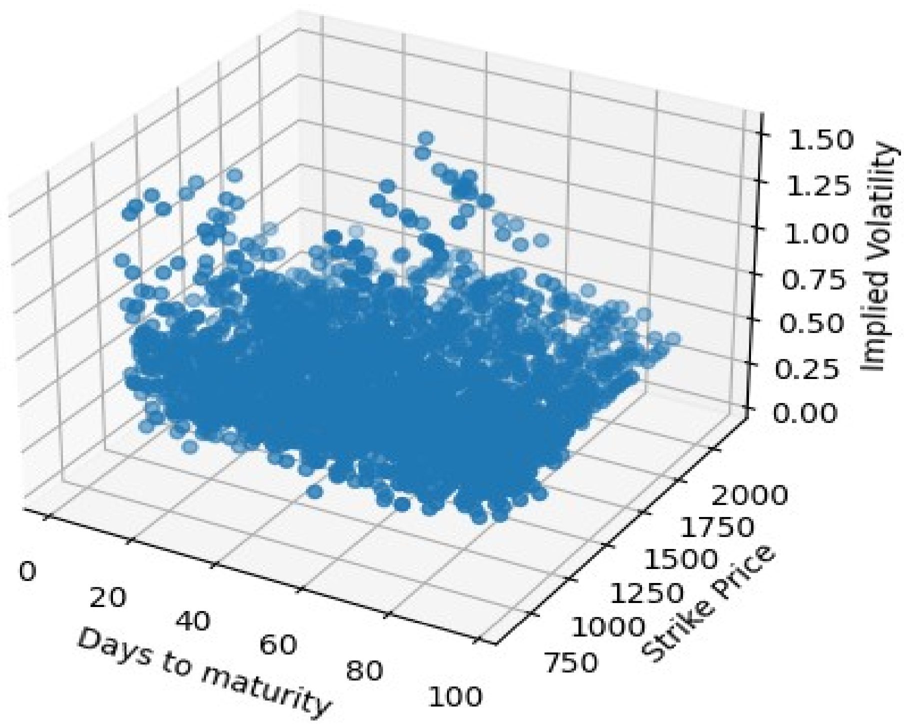
Price vs Days to Maturity of put option for TECHM
Price
Days to Maturity
0
100
200
300
400
500
600
700
0
20
40
60
80
100



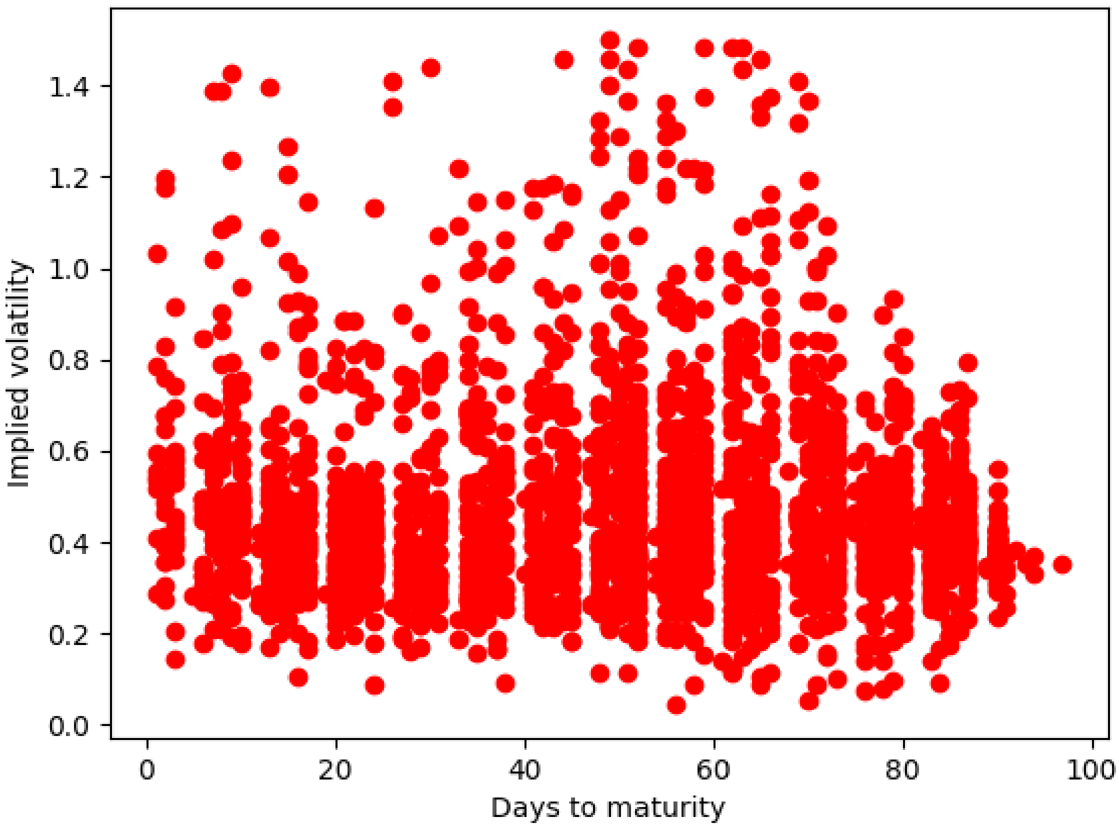
Price vs Strike of put option for TECHM
Price
Strike Price
0
100
200
300
400
500
600
700
600
800
1000
1200
1400
1600
1800
2000
2200

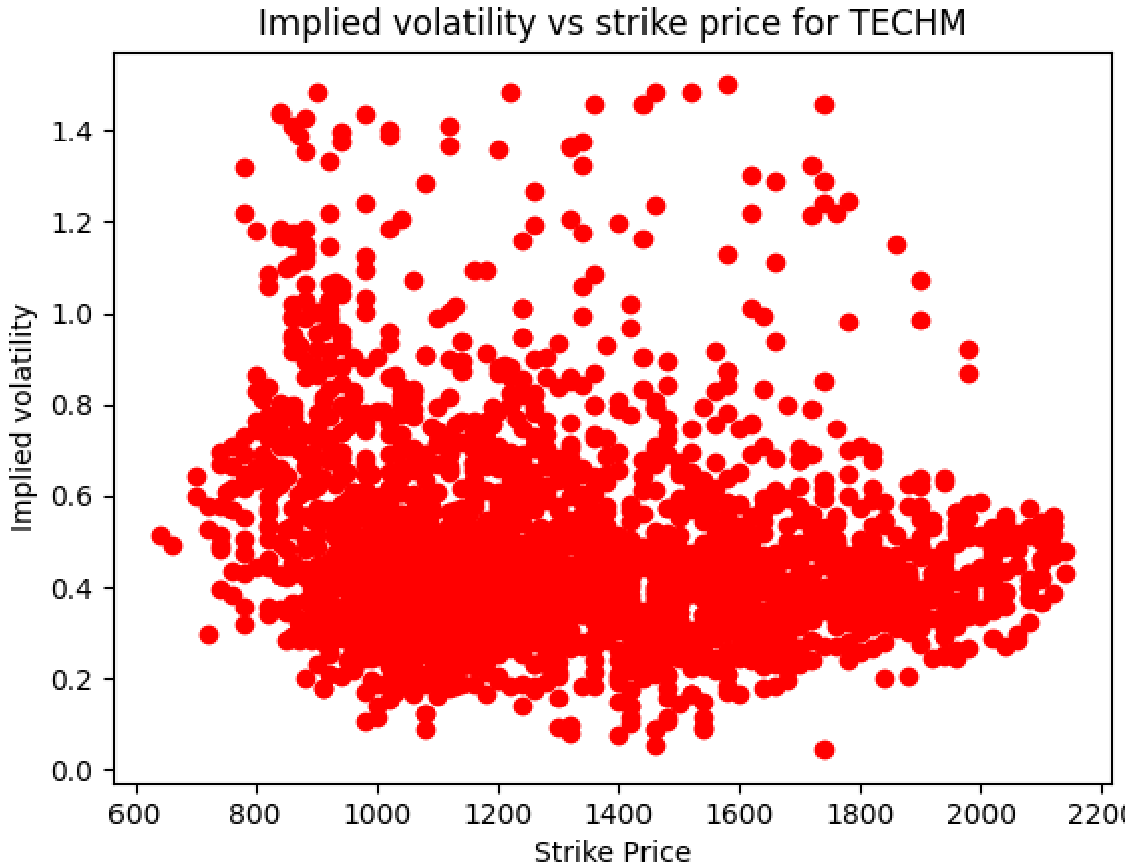
Part - (b)

Implied volatility vs Maturity and Strike Price for TECHM



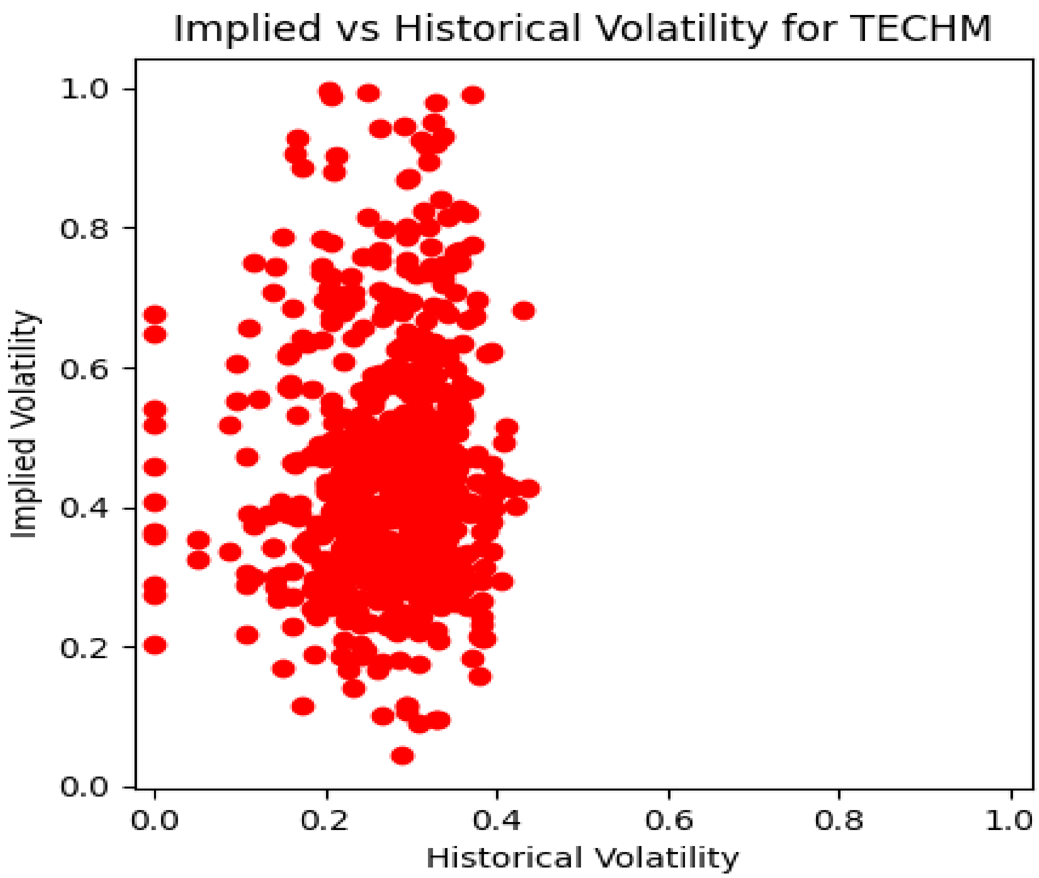
Implied volatility vs Maturity for TECHM



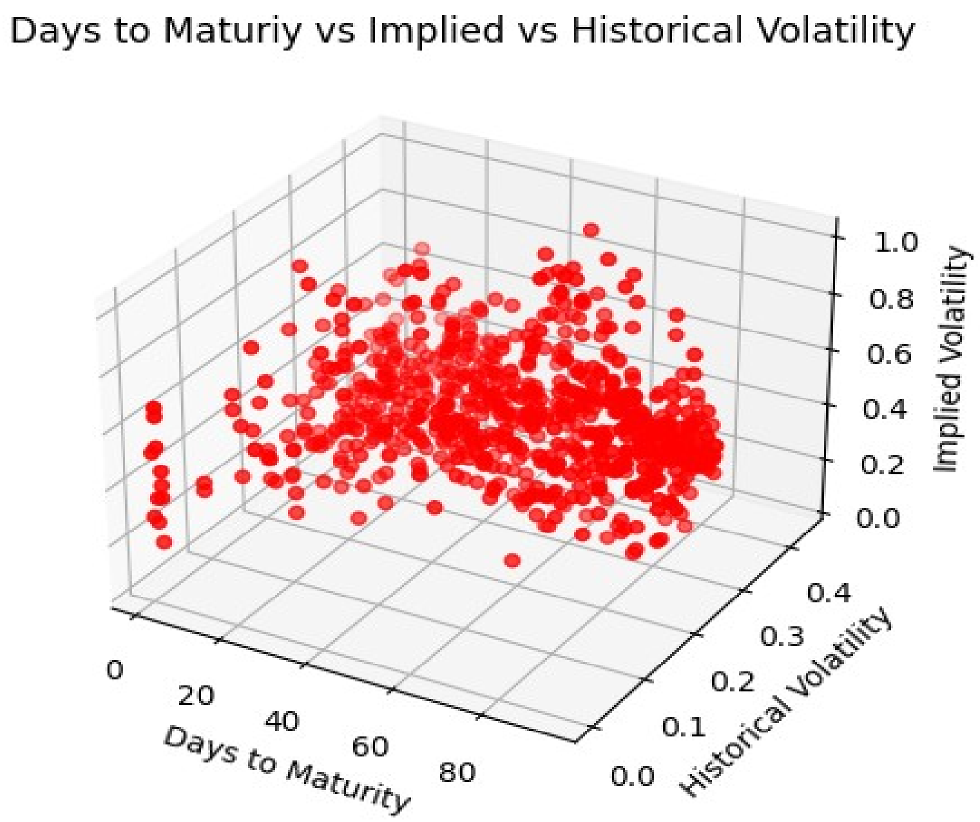


Implied volatility vs strike price for TECHM
Implied volatility
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
600
800
1000
1200
1400
1600
1800
2000
2200
Strike Price

Part - (c)



Implied vs Historical Volatility for TECHM
Implied Volatility
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0
Historical Volatility

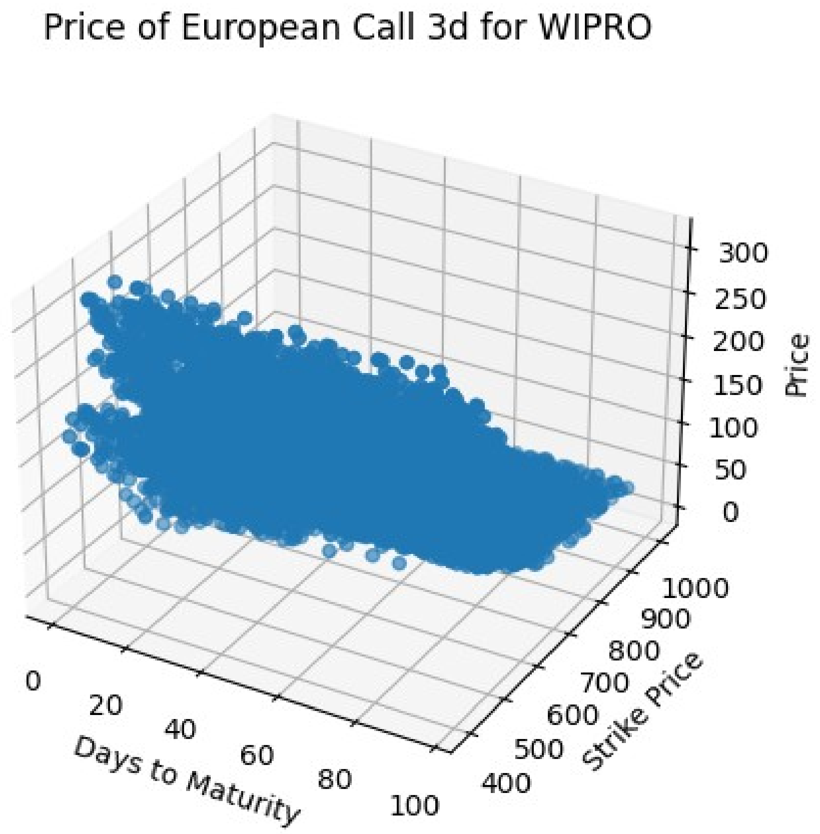


I am attaching the screenshot of tabulated values of implied and historical volatility of 15 randomly chosen points from my dataset for comparison

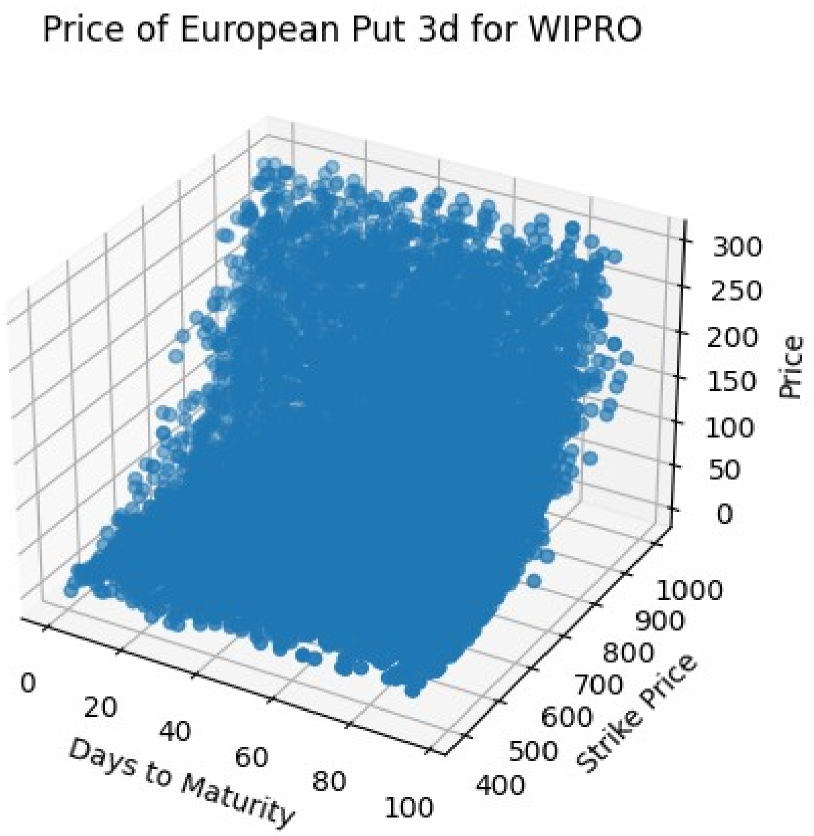
Days to Maturity	Implied Volatility	Historical Volatility
22	0.319486	0.333773
34	0.289979	0.364248
83	0.359387	0.324488
75	0.467421	0.317117
48	0.475013	0.377968
62	0.183427	0.371214
69	0.302145	0.208366
83	0.45204	0.271975
55	0.46983	0.257616
55	0.436723	0.215882
3	0.203922	0
86	0.392496	0.242984
10	0.518777	0.087616
43	0.243869	0.382347
8	0.269105	0.240378

WIPRO

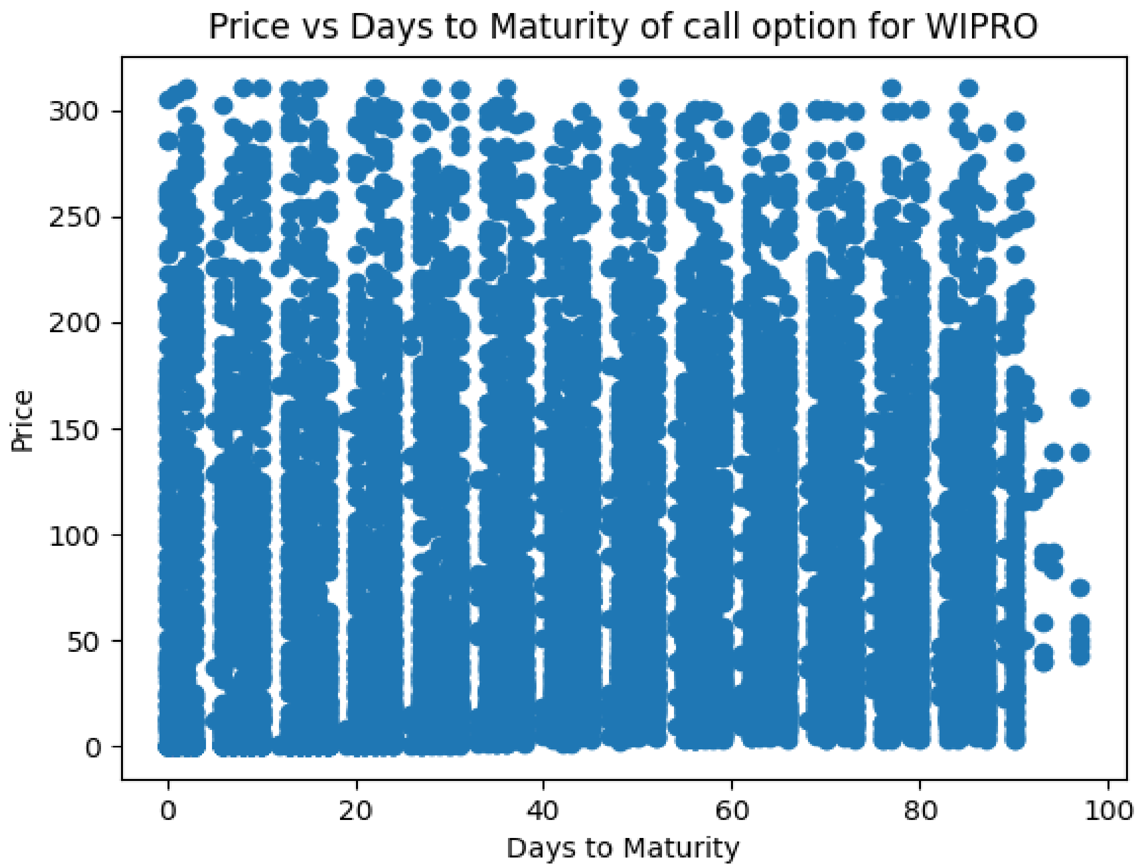
Part - (a)

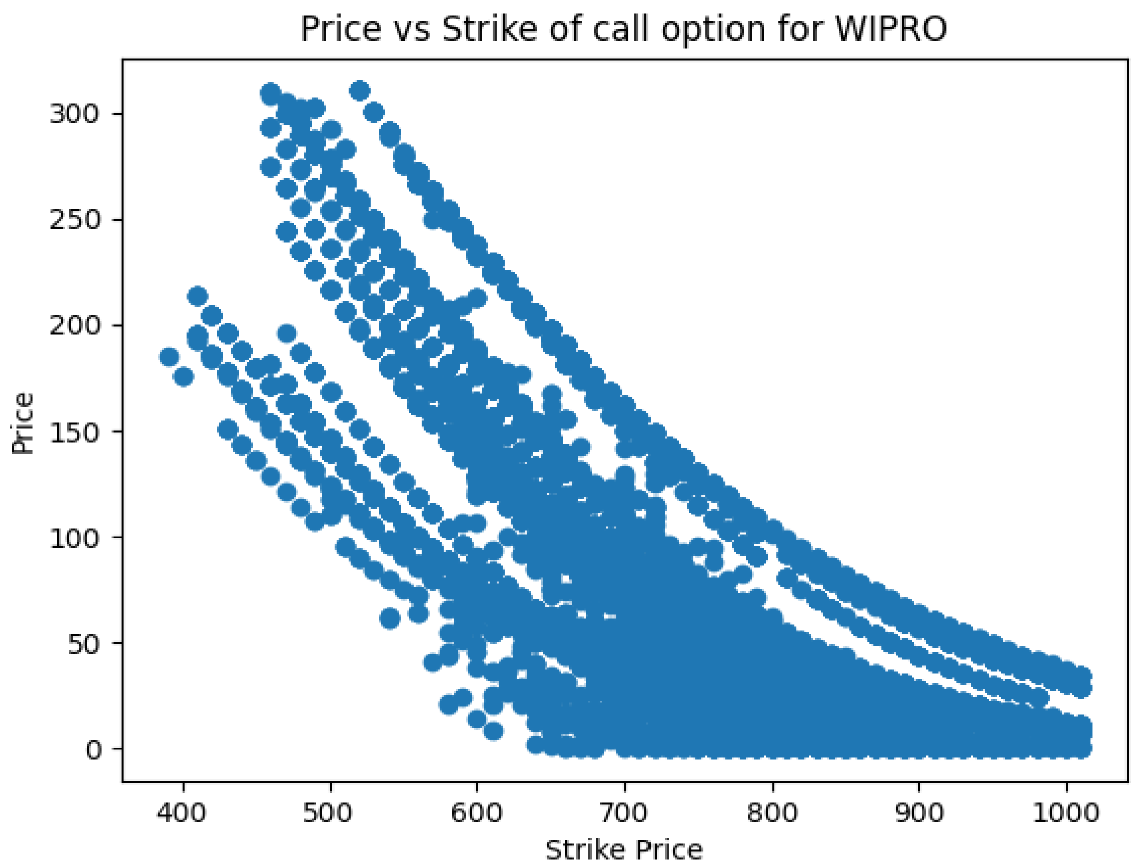


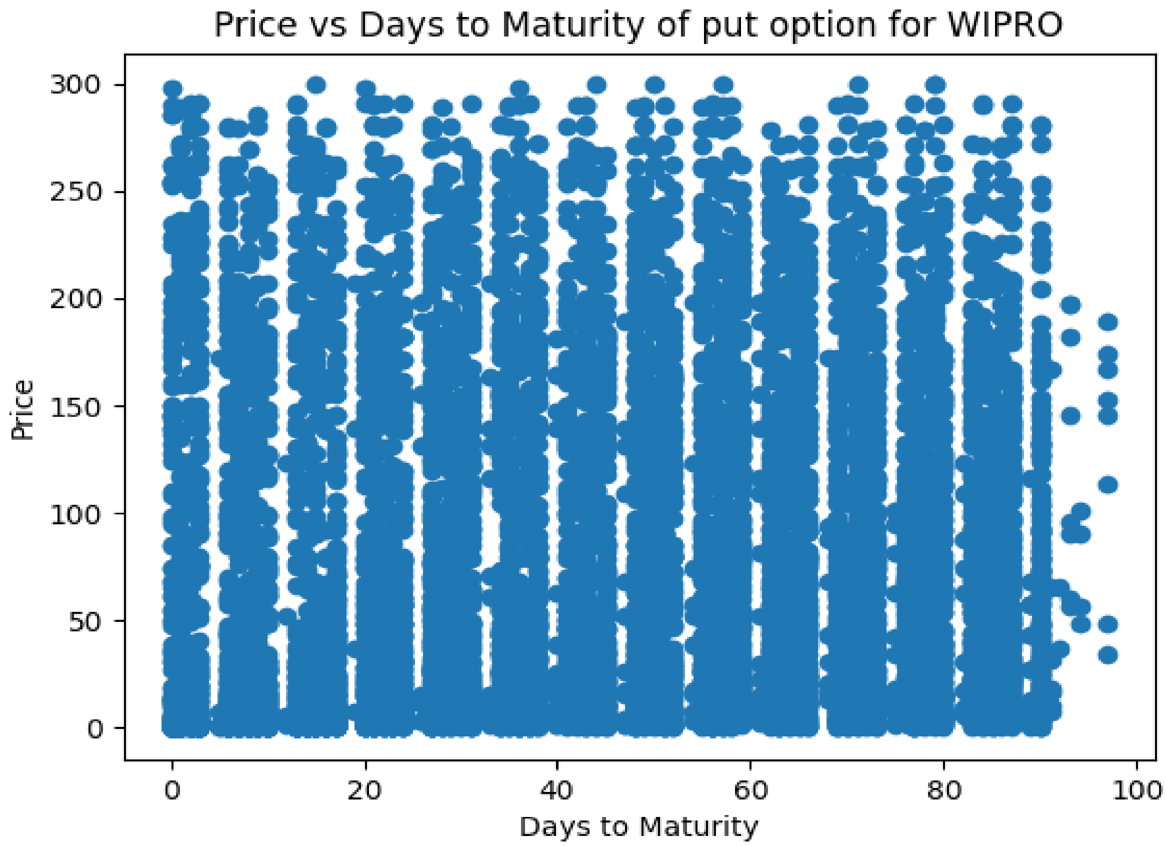
Price of European Call 3d for WIPRO
Price
0
50
100
150
200
250
300
Days to Maturity
0
20
40
60
80
100
Strike Price
400
500
600
700
800
900
1000



Price of European Put 3d for WIPRO
Price
0
50
100
150
200
250
300
Days to Maturity
0
20
40
60
80
100
Strike Price
400
500
600
700
800
900
1000


Price vs Days to Maturity of call option for WIPRO
Price
Days to Maturity
0
50
100
150
200
250
300
0
20
40
60
80
100

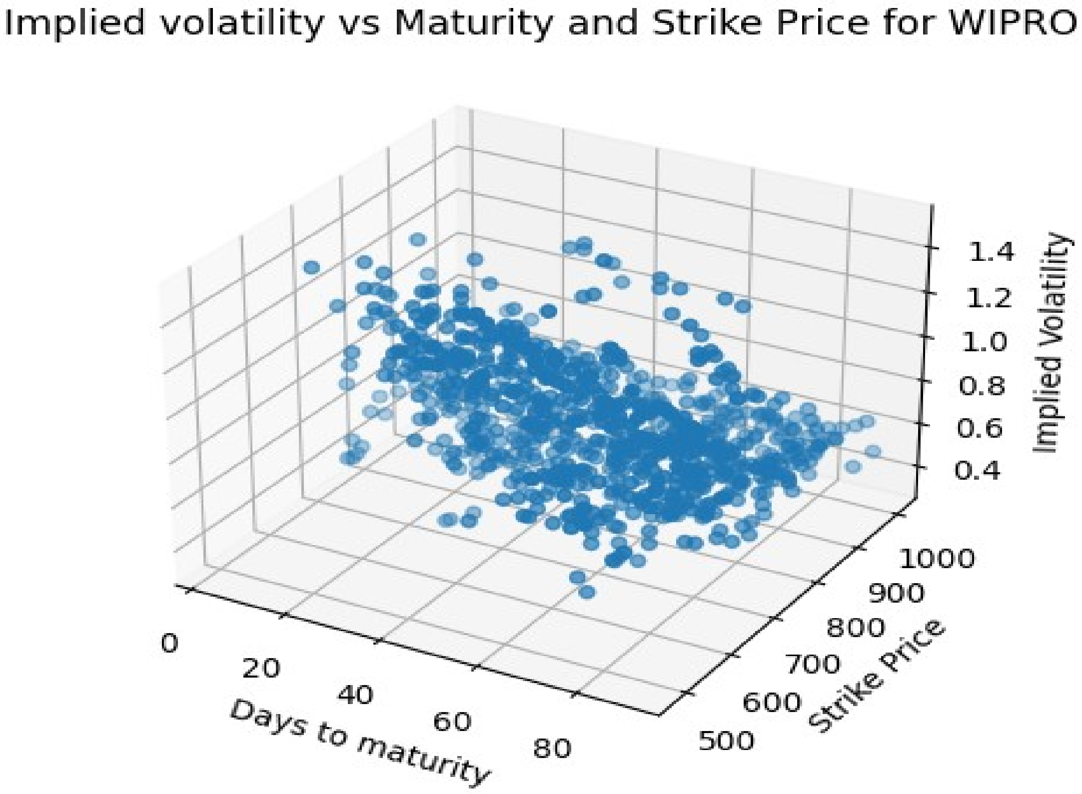

Price vs Strike of call option for WIPRO
Price
Strike Price
0
50
100
150
200
250
300
400
500
600
700
800
900
1000



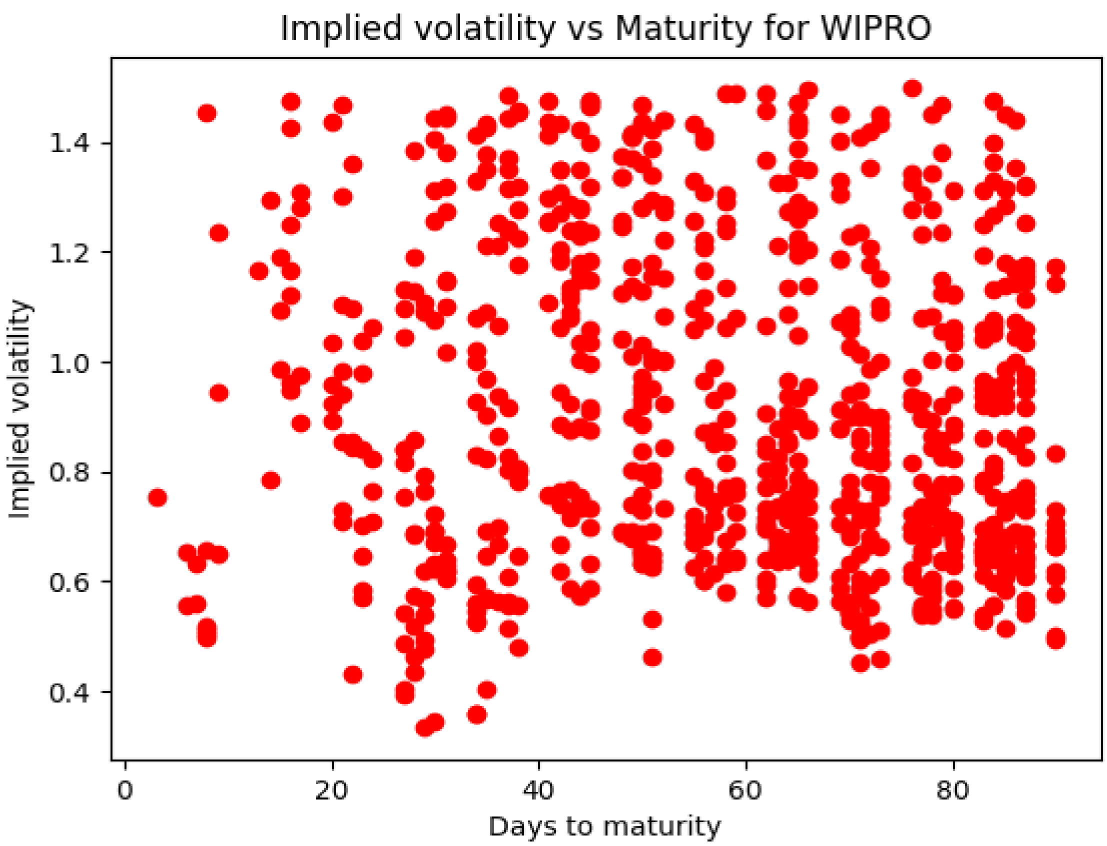
Price vs Days to Maturity of put option for WIPRO
Price
Days to Maturity
0
50
100
150
200
250
300
0
20
40
60
80
100

Price vs Strike of put option for WIPRO
Price
Strike Price
0
50
100
150
200
250
300
400
500
600
700
800
900
1000

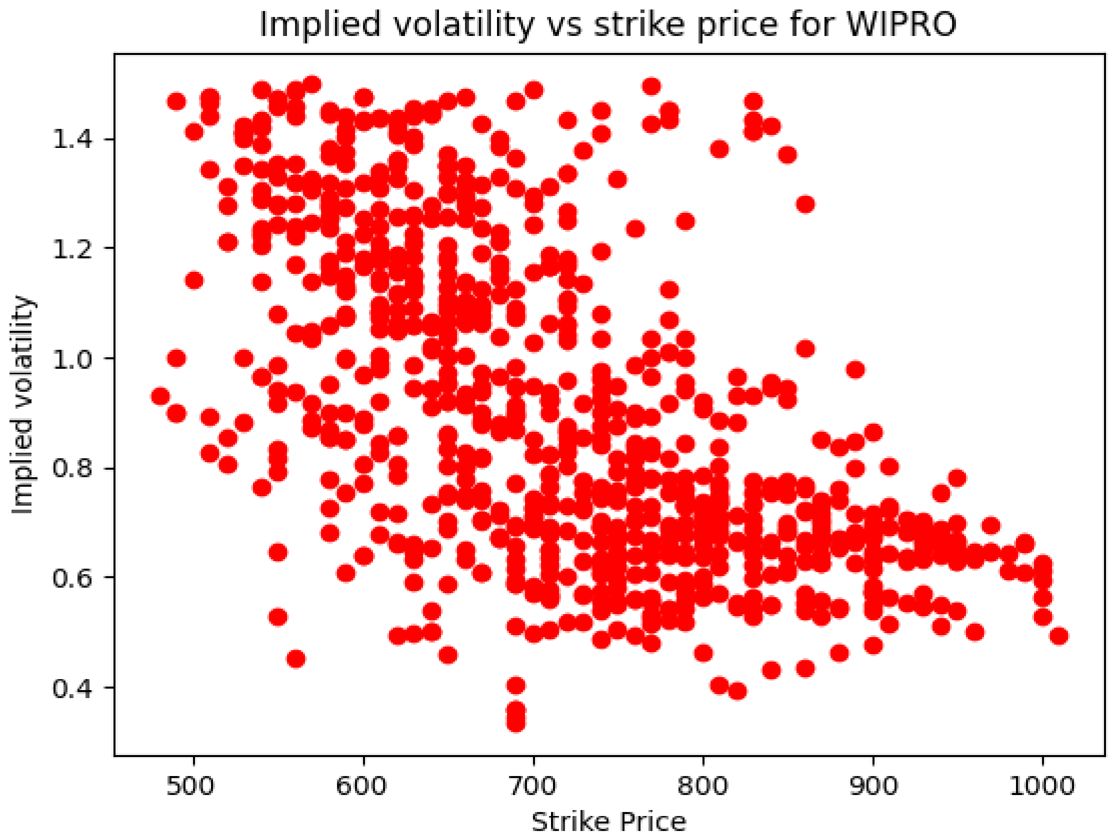
Part - (b)



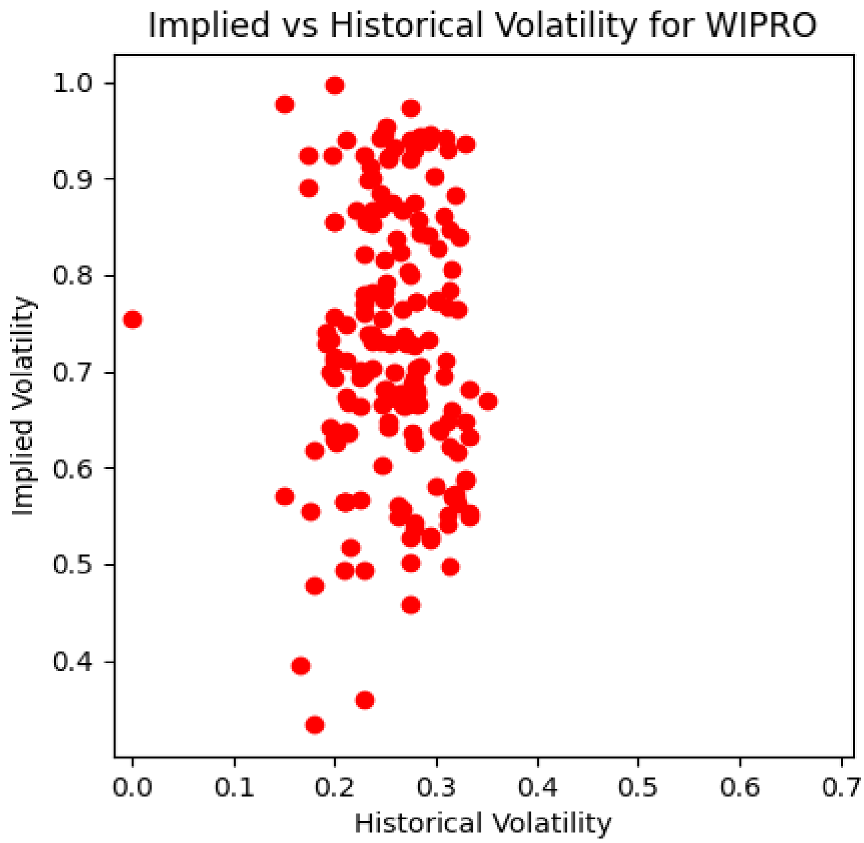
Implied volatility vs Maturity and Strike Price for WIPRO
1.4
1.2
1.0
0.8
0.6
0.4
Implied Volatility
1000
900
800
700
600
500
Strike Price
0
20
40
60
80
Days to maturity

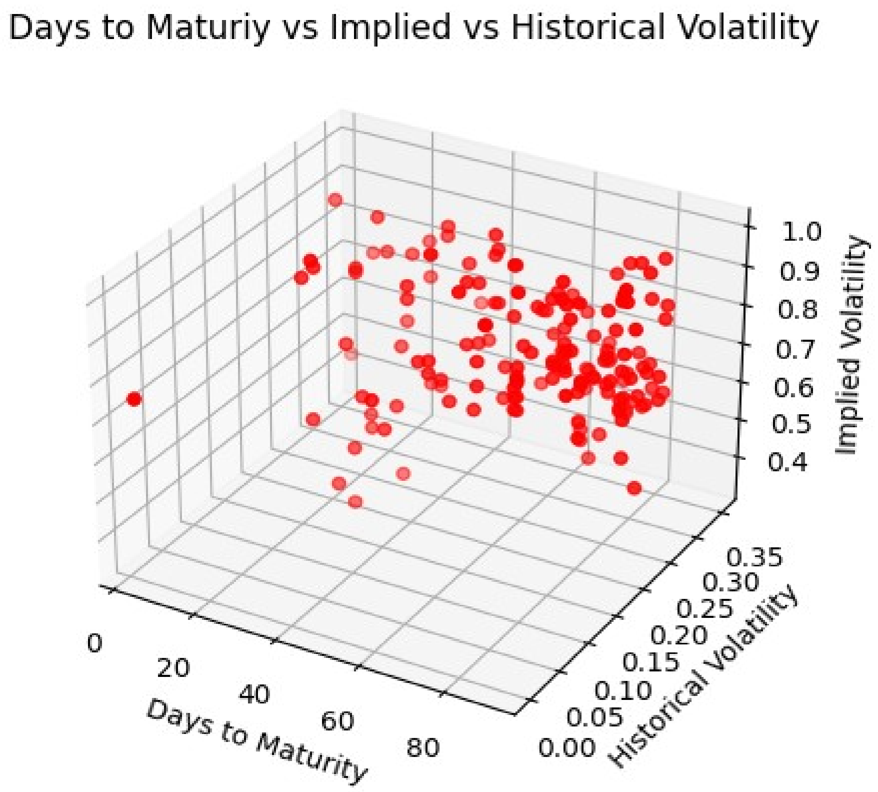


Implied volatility vs Maturity for WIPRO
1.4
1.2
1.0
0.8
0.6
0.4
Implied volatility
0
20
40
60
80
Days to maturity


Implied volatility vs strike price for WIPRO
Implied volatility
Strike Price
1.4
1.2
1.0
0.8
0.6
0.4
500
600
700
800
900
1000

Part - (c)


Implied vs Historical Volatility for WIPRO
Implied Volatility
Historical Volatility
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7

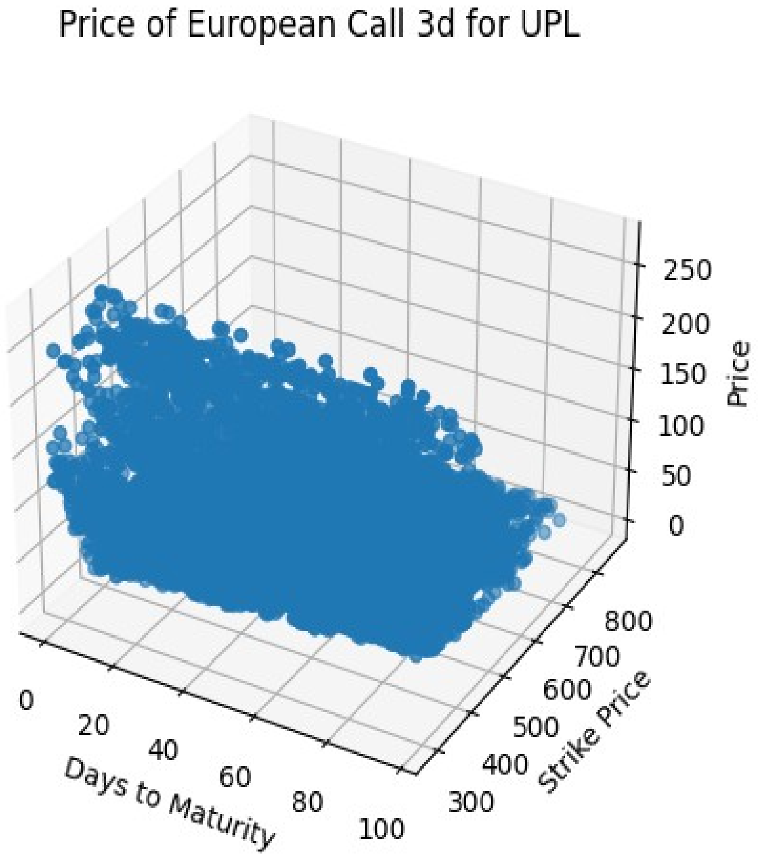


I am attaching the screenshot of tabulated values of implied and historical volatility of 15 randomly chosen points from my dataset for comparison

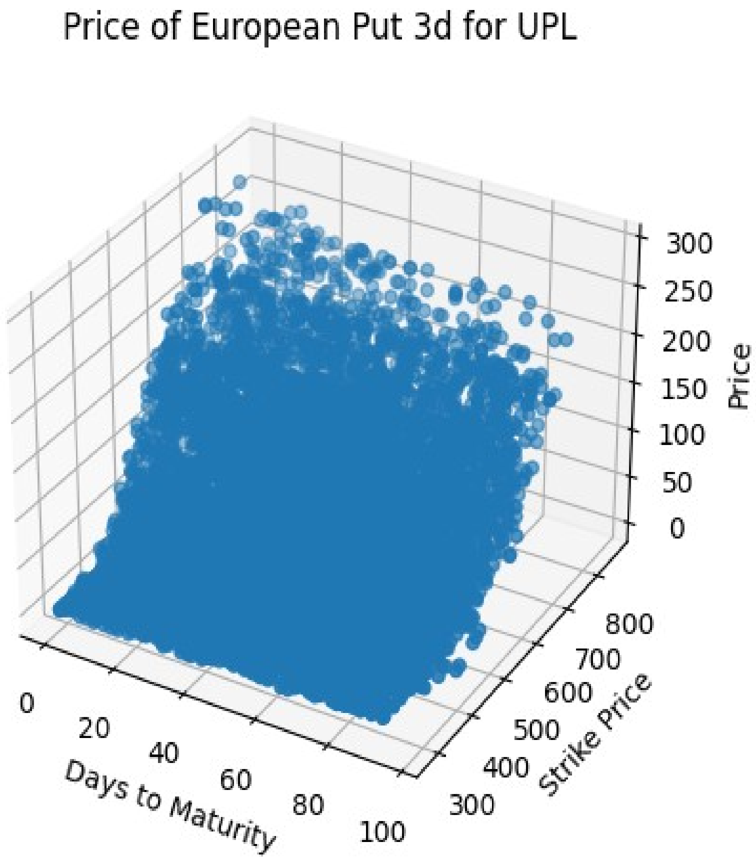
Days to Maturity	Implied Volatility	Historical Volatility
64	0.711515	0.211015
85	0.664432	0.269213
76	0.92391	0.228765
36	0.668793	0.351533
70	0.632869	0.332992
37	0.555074	0.175512
85	0.727479	0.278195
90	0.493661	0.228104
34	0.52876	0.294071
70	0.553727	0.332992
73	0.665423	0.279991
8	0.496968	0.314719
80	0.711881	0.309707
83	0.861767	0.306976
50	0.730381	0.245435

UPL

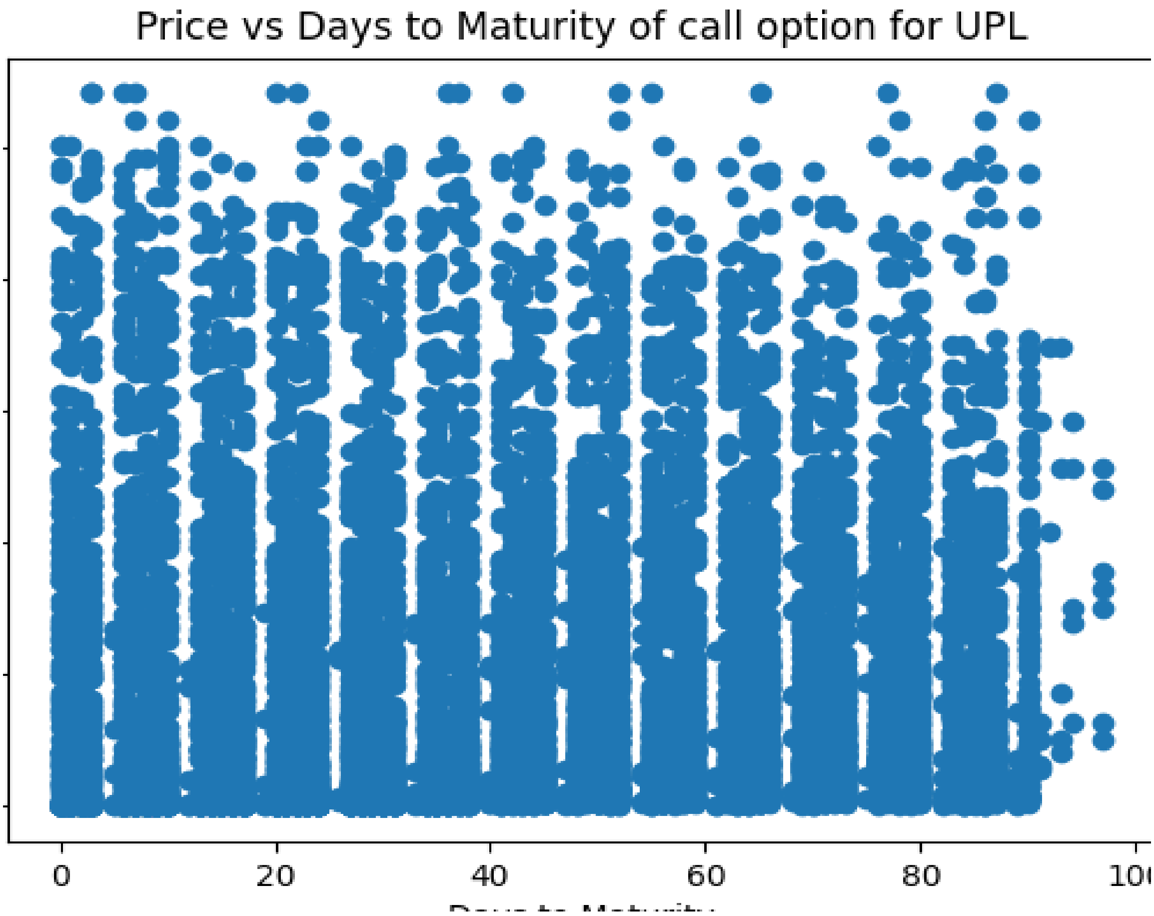
Part - (a)



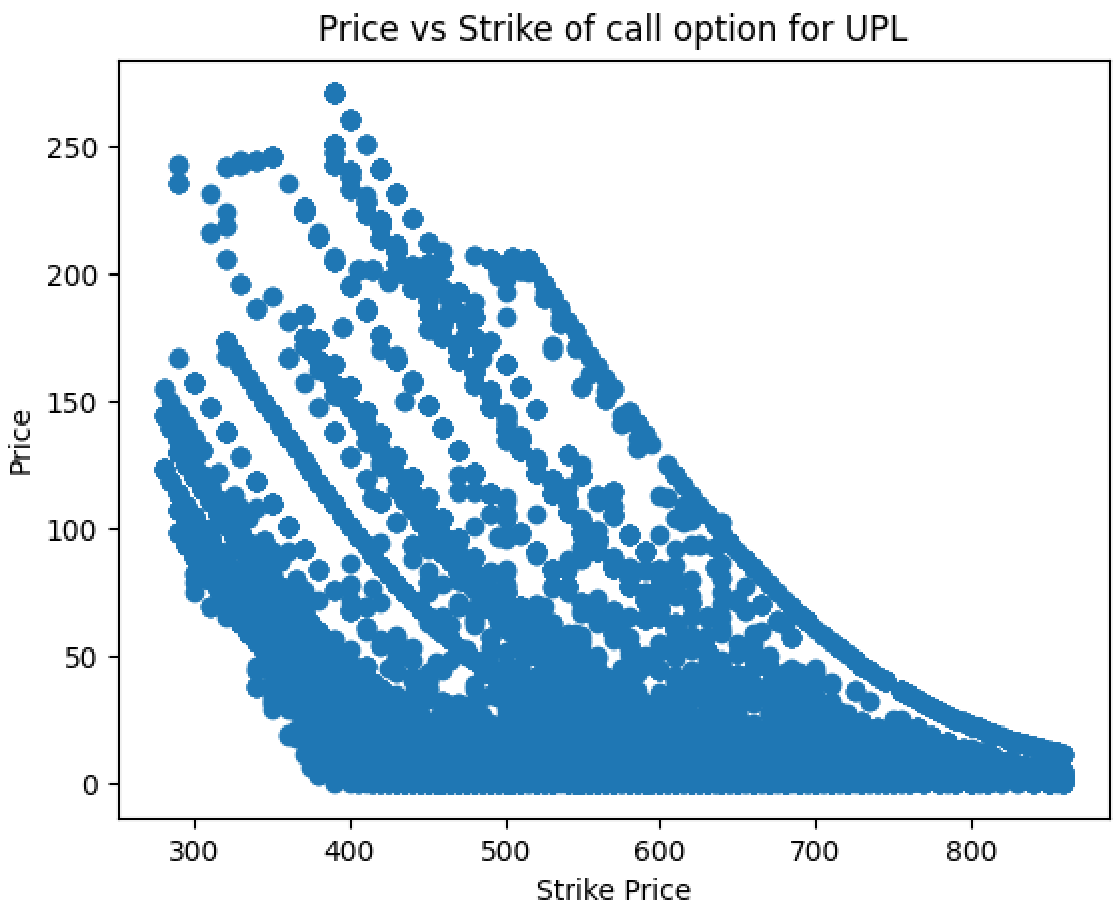
Price of European Call 3d for UPL
Price
0
50
100
150
200
250
800
700
600
500
400
300
Strike Price
0
20
40
60
80
100
Days to Maturity



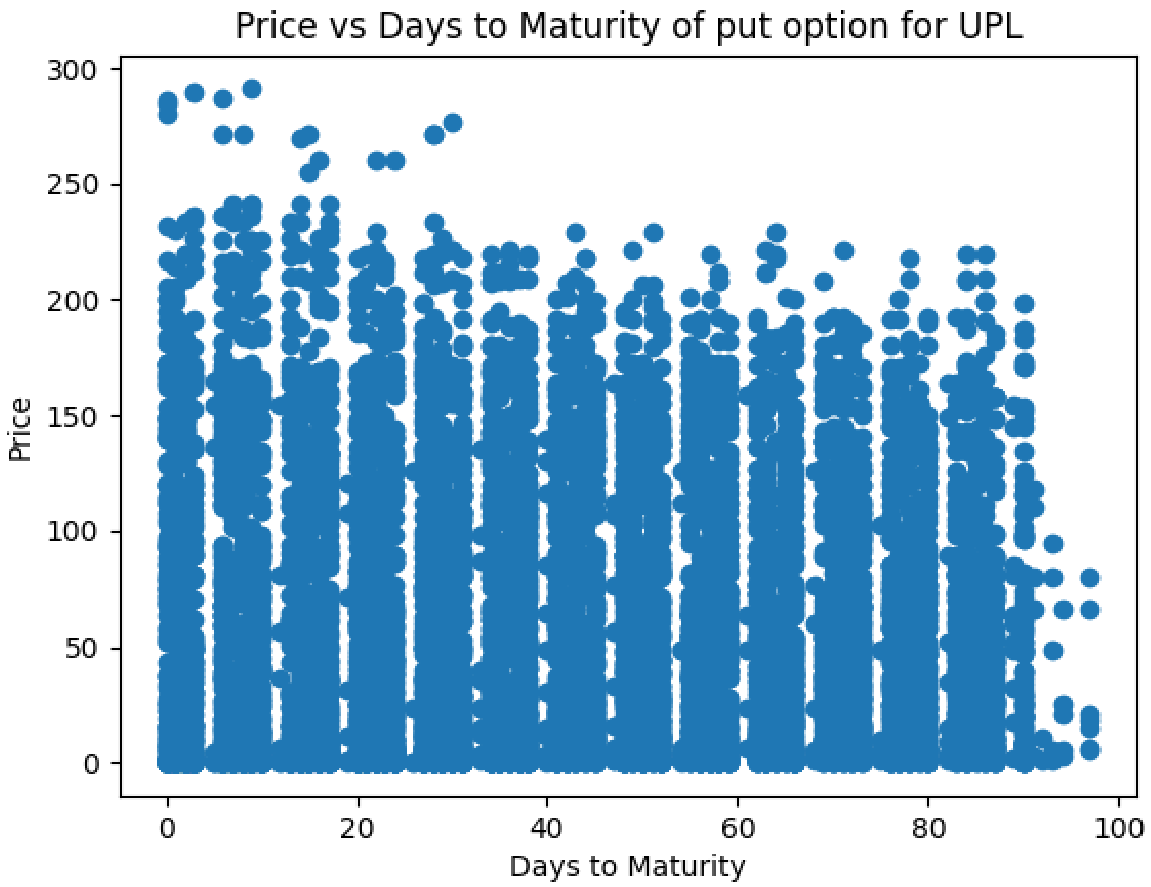
Price of European Put 3d for UPL
Price
0
50
100
150
200
250
300
800
700
600
500
400
300
Strike Price
0
20
40
60
80
100
Days to Maturity

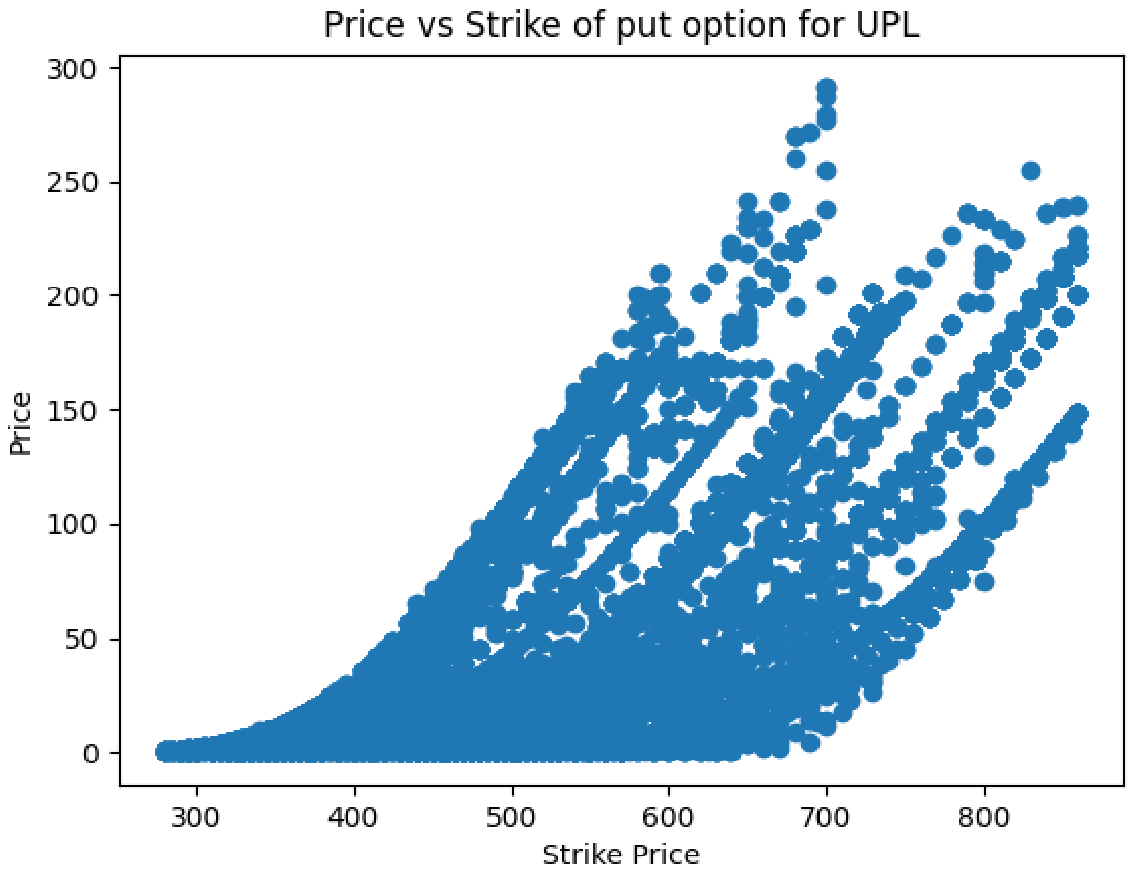


Price vs Days to Maturity of call option for UPL
0
20
40
60
80
100

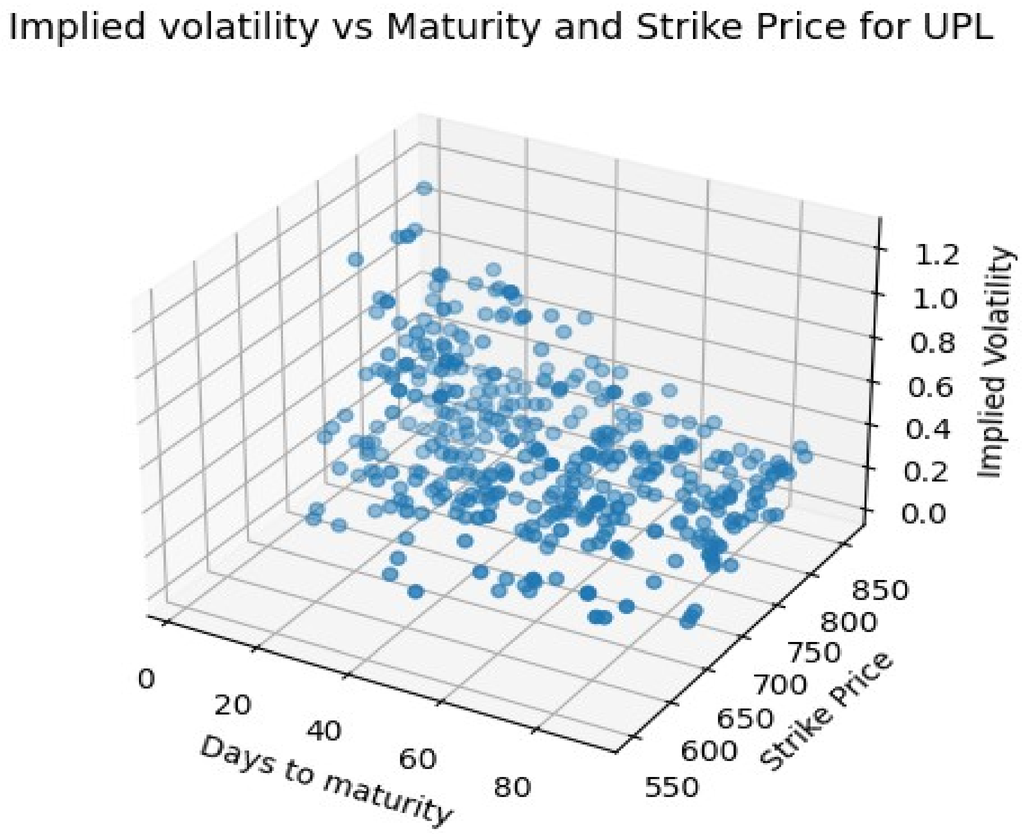


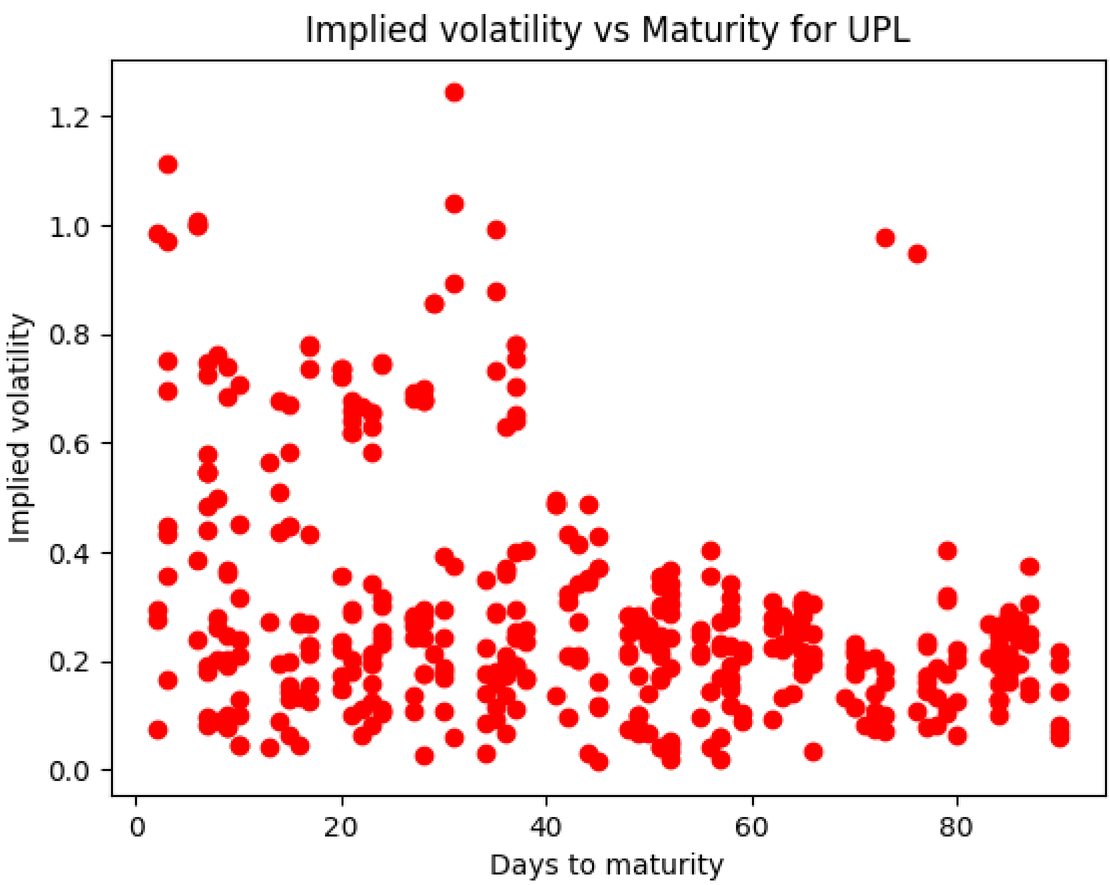
Price vs Strike of call option for UPL
Price
250
200
150
100
50
0
300
400
500
600
700
800
Strike Price

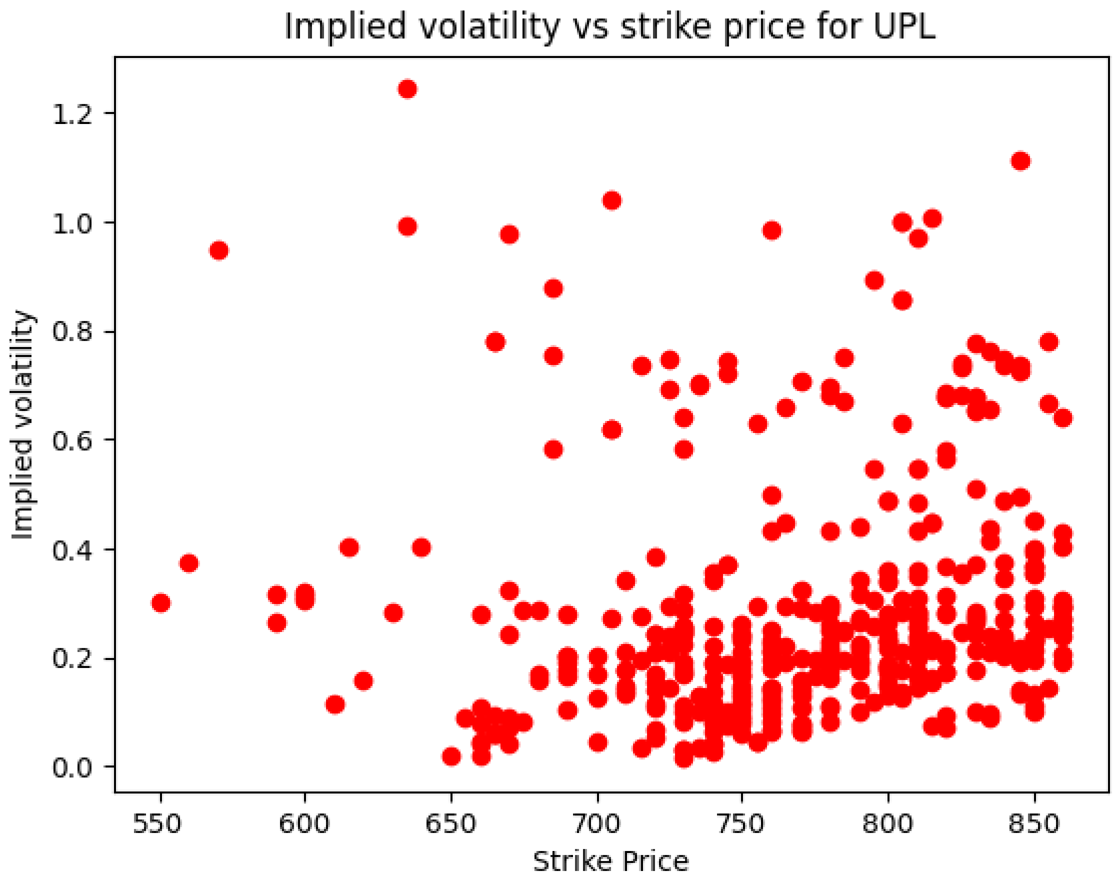

Price vs Days to Maturity of put option for UPL
Price
Days to Maturity
0
50
100
150
200
250
300
0
20
40
60
80
100


Price vs Strike of put option for UPL
Price
Strike Price
0
50
100
150
200
250
300
300
400
500
600
700
800

Part - (b)

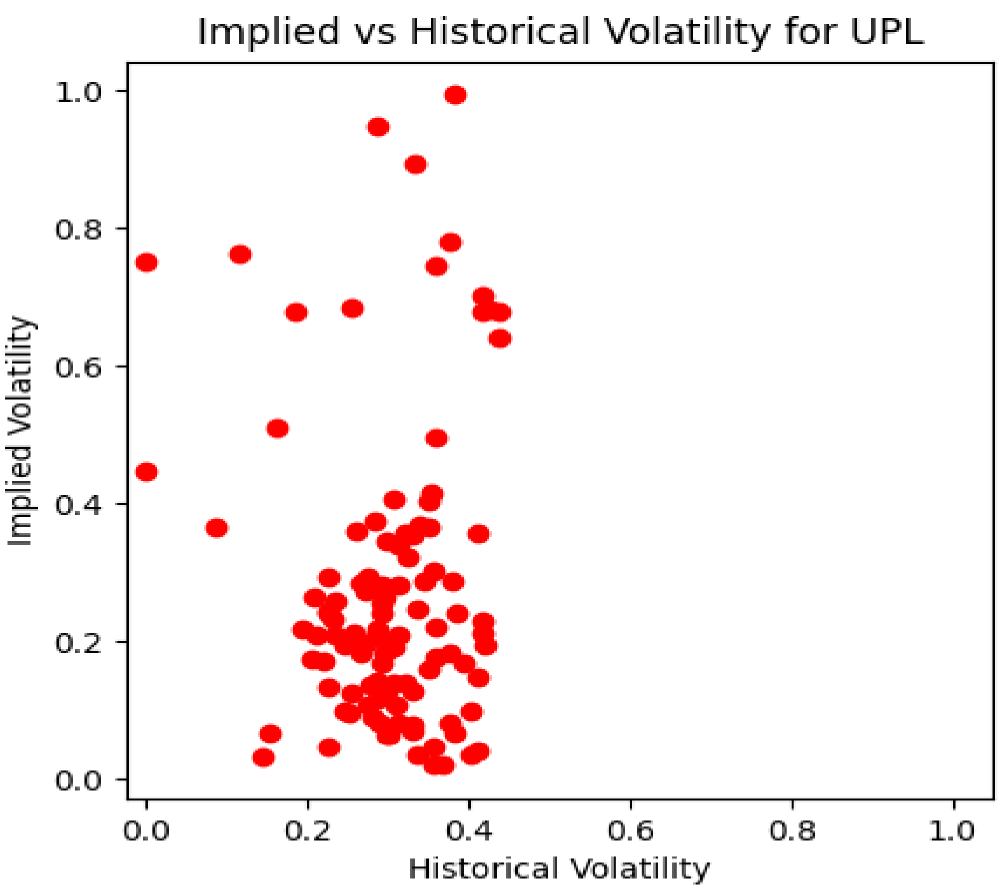

Implied volatility vs Maturity and Strike Price for UPL
Implied Volatility
0.0
0.2
0.4
0.6
0.8
1.0
1.2
Strike Price
550
600
650
700
750
800
850
Days to maturity
0
20
40
60
80


Implied volatility vs Maturity for UPL
Implied volatility
0.0
0.2
0.4
0.6
0.8
1.0
1.2
Days to maturity
0
20
40
60
80

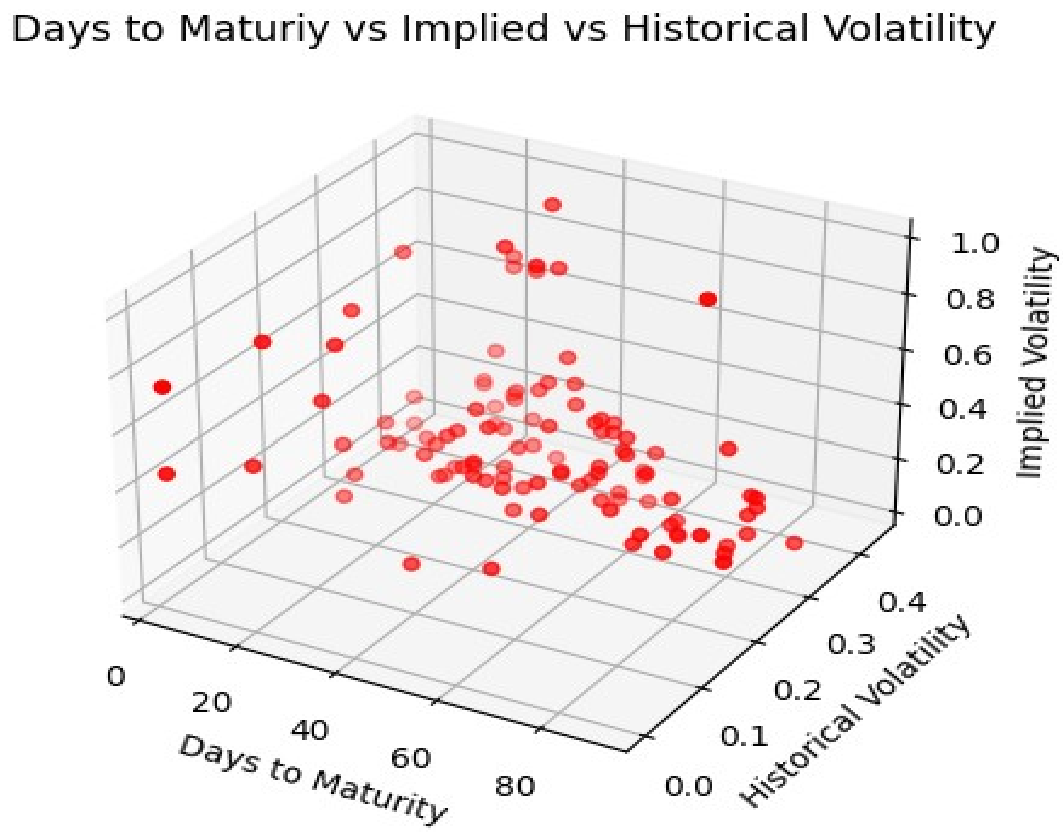


Implied volatility vs strike price for UPL
1.2
1.0
0.8
0.6
0.4
0.2
0.0
Implied volatility
550
600
650
700
750
800
850
Strike Price

Part - (c)



Implied vs Historical Volatility for UPL
1.0
0.8
0.6
0.4
0.2
0.0
Implied Volatility
0.0
0.2
0.4
0.6
0.8
1.0
Historical Volatility



I am attaching the screenshot of tabulated values of implied and historical volatility of 15 randomly chosen points from my dataset for comparison

Days to Maturity	Implied Volatility	Historical Volatility
41	0.496075	0.360069
37	0.24284	0.22702
10	0.239619	0.22725
51	0.302407	0.356029
90	0.0710777	0.331322
45	0.369818	0.339566
48	0.209381	0.286679
85	0.281847	0.293822
3	0.751566	0
71	0.0825181	0.290985
27	0.10653	0.312332
84	0.264617	0.210079
90	0.218469	0.19542
15	0.128776	0.331092
35	0.993704	0.384296

Observations

Some important points

- *Since the number of data points is extremely large I have used random sampoling to sample about 10000 data points for plotting and about 5000 points for volatility analysis*
- *The code for implied volatility takes about 1 minute to run*
- *The data shown for implied vs historical volatility on output only comprises of about 15 points but the complete data is saved in the historical volatility folder for each stock as a csv file*
- ***On running the code there might be some runtime warnings which correspond to the divergence of Newton's method at certain points casuing overflow or underflow. I have rejected these data points in this experiment.***

Observations for part(a)

- *We can clearly observe that the price of call option decreases with the strike price whereas for put option it increases. This is consistent with the pricing formulae of the classical Black Scholes model.*
- *But the plot for call option and put option with maturity doesn't exactly match with our expectations. The general trend is that the price of call option tends to increase while that of put option tends to decrease with an increase in maturity period.*
- *This may be due to the fact that that the market is affected due to many other factors and also that we are considering options of varying strike prices hence the general trend is disturbed.*

Observations for part(b)

- *Theoretically, the implied volatility is generally a convex function of strike price, and the curve so formed is known as the Volatility Smile. This behaviour is prominent in stocks such as* ***ASIANPAINT,ICICIBANK,TECHM*** *whereas for the other stock we do not see evidence of such behaviour*
- *The volatility generally tends to decrease for larger maturity values, but for some of the above plots this nature is not observed.*

Observations for part(c)

- *On observing the scatter plot of implied and historical volatility we can see that they are somewhat positively correlated and the values are quite similar to each other in most of the cases*
- *However the implied volatility in general reaches higher values as compared to the historical volatility which is mostly in the range (0,0.3). The implied volatiltiy on the other hand as certain outliers which take value of about 1.5.*
- *The plot for historical vs implied volatility very well captures this relation. Other plots show the dependence of different types of volatility with varying strike price and maturity.*