Interprocedural Analysis: Sharir-Pnueli's Call-Strings Approach

Deepak D'Souza

Department of Computer Science and Automation
Indian Institute of Science, Bangalore.

13 September 2021

Outline

Handling programs with procedure calls

How would we extend an abstract interpretation to handle programs with procedures?

```
main(){
  x := 0;
  f();
  g();
  print x;
}
```

```
f(){
  x := x+1;
  return;
}
```

```
g(){
  f();
  return;
}
```

Handling programs with procedure calls

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main(){
  x := 0;
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  g();
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}
```

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f(){
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g(){
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```

Question: what is the collecting state before the `print x` statement in `main`?

Handling programs with procedure calls

How would we extend an abstract interpretation to handle programs with procedures?

```
main(){
  x := 0;
  f();
  g();
  print x;
}
```

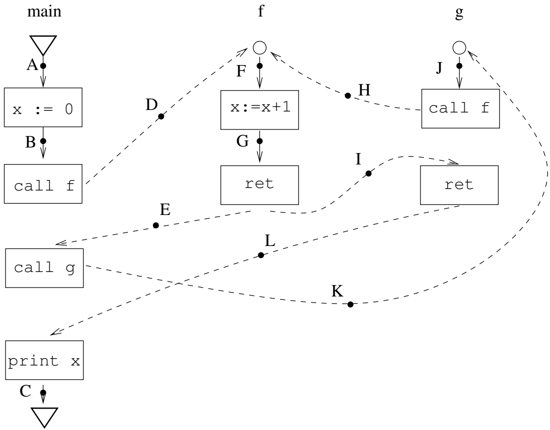
```
f(){
  x := x+1;
  return;
}
```

```
g(){
  f();
  return;
}
```

Question: what is the collecting state before the `print x` statement in `main`? Answer: $x \mapsto 2$.

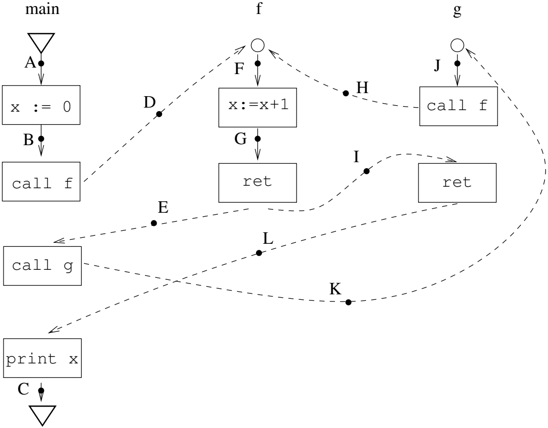
Handling programs with procedure calls

- Add extra edges
  - call edges: from call site (`call p`) to start of procedure (`p`)
  - ret edges: from return statement (in `p`) to point after call sites ("ret sites") (`call p`).



Handling programs with procedure calls

- Assume only global variables.
- Transfer functions for call/return edges?



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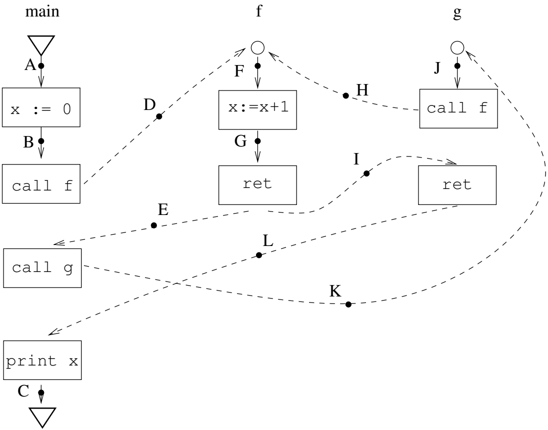
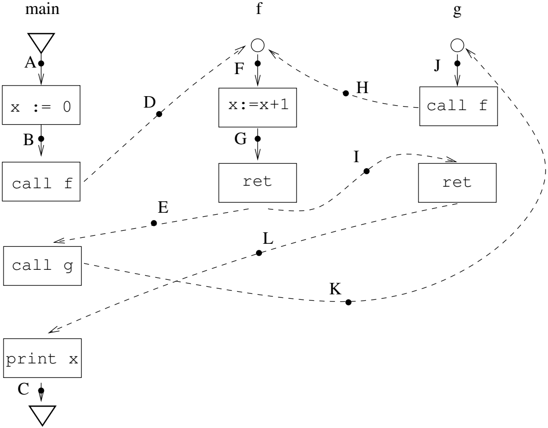


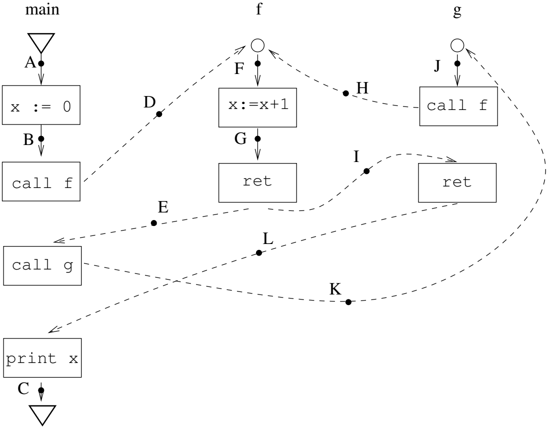
Handling programs with procedure calls

- Assume only global variables.
- Transfer functions for call/return edges? Identity function
- Now compute JOP in this extended control-flow graph.



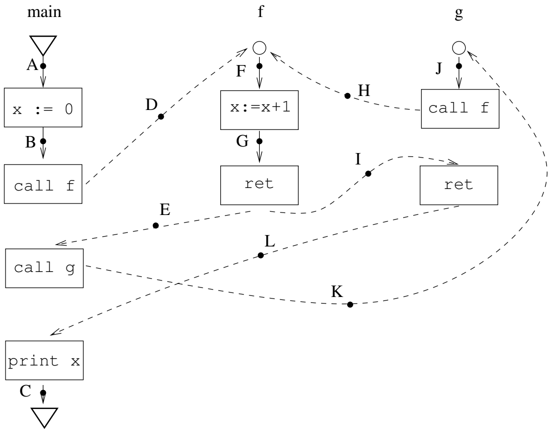
Problem with JOP in this graph

Ex. 1. Actual collecting state at C?



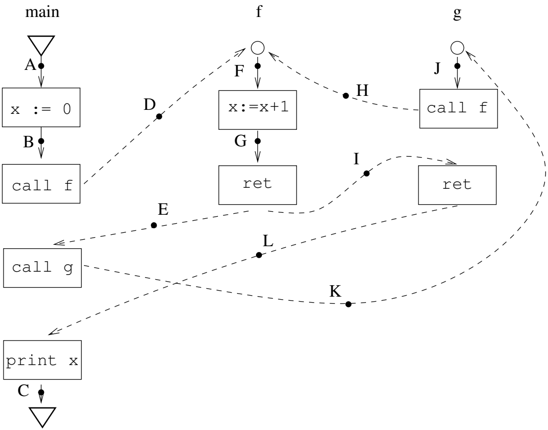
Problem with JOP in this graph

Ex. 1. Actual collecting state at C? $\{x \mapsto 2\}$.



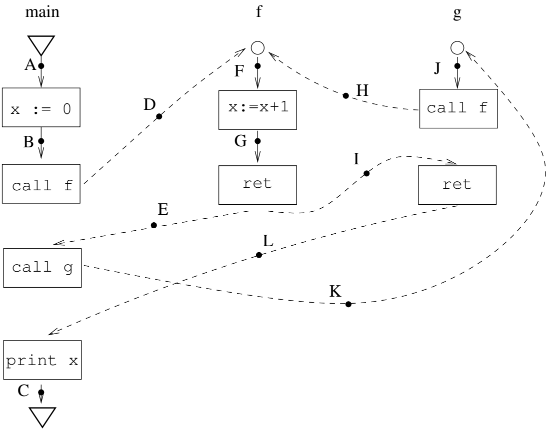
Problem with JOP in this graph

Ex. 1. Actual collecting state at C? $\{x \mapsto 2\}$.
Ex. 2. JOP at C using collecting analysis?



Problem with JOP in this graph

Ex. 1. Actual collecting state at C? $\{x \mapsto 2\}$.
Ex. 2. JOP at C using collecting analysis? $\{x \mapsto 1,\ x \mapsto 2,\ x \mapsto 3, \ldots\}$.



Problem with JOP in this graph

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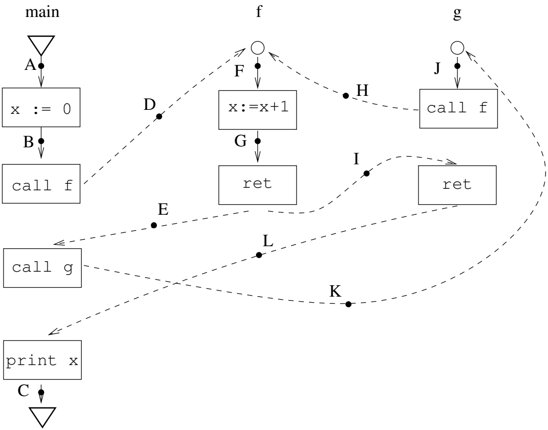
- JOP is sound but very imprecise.
- Reason: Some paths don't correspond to executions of the program: Eg. ABDFGILC.



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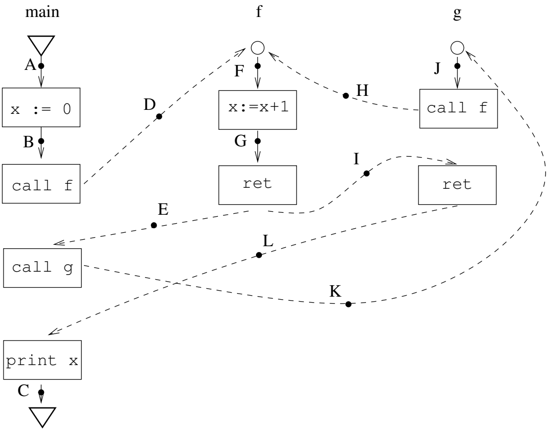


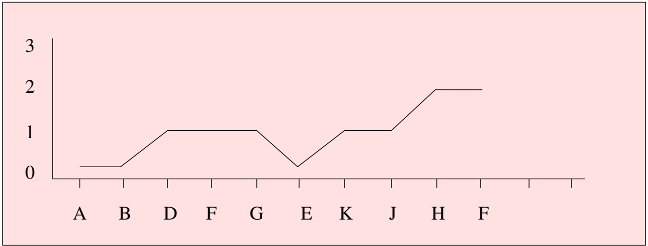
What we want is Join over "Interprocedurally-Valid" Paths (JVP).

Interprocedurally valid paths and their call-strings

- Informally a path ρ in the extended CFG G' is inter-procedurally valid if every return edge in ρ "corresponds" to the most recent "pending" call edge.
- For example, in the example program the ret edge E corresponds to the call edge D.
- The call-string of a valid path ρ is a subsequence of call edges which have not been "returned" as yet in ρ.
- For example, $cs(ABDFGEKJHF)$ is "KH".

Interprocedurally valid paths and their call-strings

- A path $\rho = ABDFGEKJHF$ in $IVP_{G'}$ for example program:



- Associated call-string $cs(\rho)$ is KH.
- For $\rho = ABDFGEK$ $cs(\rho) = K$.
- For $\rho = ABDFGE$ $cs(\rho) = \epsilon$.

Interprocedurally valid paths and their call-strings

More formally: Let ρ be a path in G'. We define when ρ is interprocedurally valid (and we say $\rho \in IVP(G')$) and what is its call-string $cs(\rho)$, by induction on the length of ρ.

- If $\rho = \epsilon$ then $\rho \in IVP(G')$. In this case $cs(\rho) = \epsilon$.
- If $\rho = \rho' \cdot N$ then $\rho \in IVP(G')$ iff $\rho' \in IVP(G')$ with $cs(\rho') = \gamma$ say, and one of the following holds:
  1. N is neither a call nor a ret edge.
     In this case $cs(\rho) = \gamma$.
  2. N is a call edge.
     In this case $cs(\rho) = \gamma \cdot N$.
  3. N is ret edge, and γ is of the form $\gamma' \cdot C$, and N corresponds to the call edge C.
     In this case $cs(\rho) = \gamma'$.
- We denote the set of (potential) call-strings in G' by Γ. Thus $\Gamma = \mathcal{C}^*$, where $\mathcal{C}$ is the set of call edges in G'.

Join over interprocedurally-valid paths (JVP)

- Let P be a given program, with extended CFG G'.
- Let $path_{I,N}(G')$ be the set of paths from the initial point I to point N in G'.
- Let $\mathcal{A} = ((D, \leq), f_{MN}, d_0)$ be an abstract interpretation for P.
- Then we define the join over all interprocedurally valid paths (JVP) at point N in G' to be:

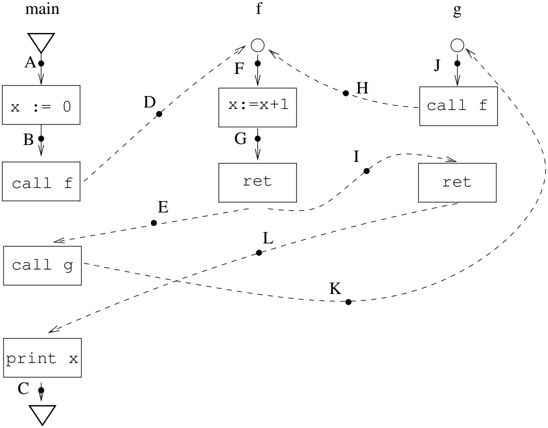
$$\bigsqcup_{\rho \,\in\, path_{I,N}(G') \cap IVP(G')} f_\rho(d_0).$$

Sharir and Pnueli's approaches to interprocedural analysis

Micha Sharir and Amir Pnueli: Two approaches to interprocedural data flow analysis, in *Program Flow Analysis: Theory and Applications* (Eds. Muchnick and Jones) (1981).

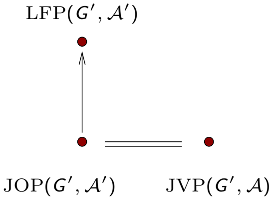
One approach to obtain JVP: Call-Strings

- Find JOP over same graph, but modify the abs int.
- Modify transfer functions for call/ret edges to detect and invalidate invalid edges.
- Augment underlying data values with some information for this.
- Natural thing to try: "call-strings".



Overall plan

- Define an abs int $\mathcal{A}'$ which extends given abs int $\mathcal{A}$ with call-string data.
- Show that JOP of $\mathcal{A}'$ on G' coincides with JVP of $\mathcal{A}$ on G'.
- Use Kildall (or any other technique) to compute LFP of $\mathcal{A}'$ on G'. This value over-approximates JVP of $\mathcal{A}$ on G'.



Call-string abs int $\mathcal{A}'$: Lattice $(D', \leq')$

- Elements of D' are maps $\xi : \Gamma \to D$

ξ :

ϵ	c_1	$c_1 c_2$	$c_1 c_2 c_2$	....
d_0	d_1	d_2	d_3	....

- Ordering on D': $\leq'$ is the pointwise extension of $\leq$ in D.
- That is $\xi_1 \leq' \xi_2$ iff for each $\gamma \in \Gamma$, $\xi_1(\gamma) \leq \xi_2(\gamma)$.

$\xi_1 \sqcup \xi_2$:

ϵ	c_1	$c_1 c_2$	$c_1 c_2 c_2$	....
$d_0 \sqcup e_0$	$d_1 \sqcup e_1$	$d_2 \sqcup e_2$	$d_3 \sqcup e_3$	....

ξ_1 :

ϵ	c_1	$c_1 c_2$	$c_1 c_2 c_2$	....
d_0	d_1	d_2	d_3	....

ξ_2 :

ϵ	c_1	$c_1 c_2$	$c_1 c_2 c_2$	....
e_0	e_1	e_2	e_3	....

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ξ_1 :

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d_0	d_1	d_2	d_3	....

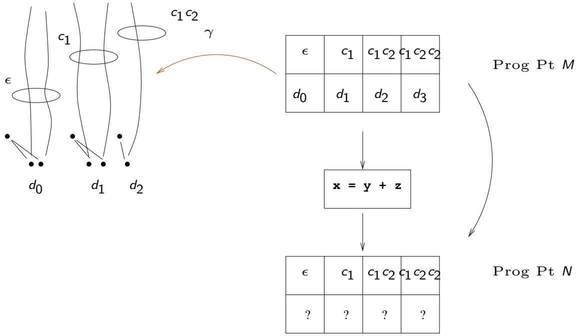
ξ_2 :

ϵ	c_1	$c_1 c_2$	$c_1 c_2 c_2$	....
e_0	e_1	e_2	e_3	....

- Check that $(D', \leq')$ is also a complete lattice.

Meaning of abstract values in $\mathcal{A}$'

- A call-string table ξ at program point N represents the fact that, for each call-string γ, there are some initial paths with call-string γ reaching N, and the join of the abstract states (obtained by propagating d_0) along these paths is $\xi(\gamma)$.
- The transfer functions of $\mathcal{A}'$ should keep this meaning in mind.



Call-string abs int $\mathcal{A}'$: Initial value ξ_0

- Initial value ξ_0 is given by

$$\xi_0(\gamma) = \begin{cases} d_0 & \text{if } \gamma = \epsilon \\ \bot & \text{otherwise.} \end{cases}$$

ξ_0 :

ϵ	c_1	$c_1 c_2$	$c_1 c_2 c_2$	....
d_0	$\bot$	$\bot$	$\bot$	....

Call-string abs int $\mathcal{A}'$: transfer functions

- Transfer functions for non-call/ret edge N:

$$f'_{MN}(\xi) = f_{MN} \circ \xi.$$

- Transfer functions for call edge N:

$$f'_{MN}(\xi) = \lambda\gamma. \begin{cases} \xi(\gamma') & \text{if } \gamma = \gamma' \cdot N \\ \bot & \text{otherwise} \end{cases}$$

- Transfer functions for ret edge N whose corresponding call edge is C:

$$f'_{MN}(\xi) = \lambda\gamma.\xi(\gamma \cdot C)$$

- Transfer functions f'_{MN} is monotonic (distributive) if each f_{MN} is monotonic (distributive).

Transfer functions f'_{MN} for example program

- Non-call/ret edge B:

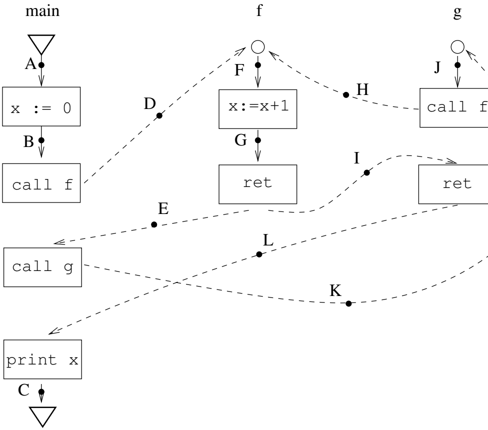
$$\xi_B = f_{AB} \circ \xi_A.$$

- Call edge D:

$$\xi_D(\gamma) = \begin{cases} \xi_B(\gamma') & \text{if } \gamma = \gamma' \cdot D \\ \bot & \text{otherwise} \end{cases}$$

- Return edge E:

$$\xi_E(\gamma) = \xi_G(\gamma \cdot D).$$

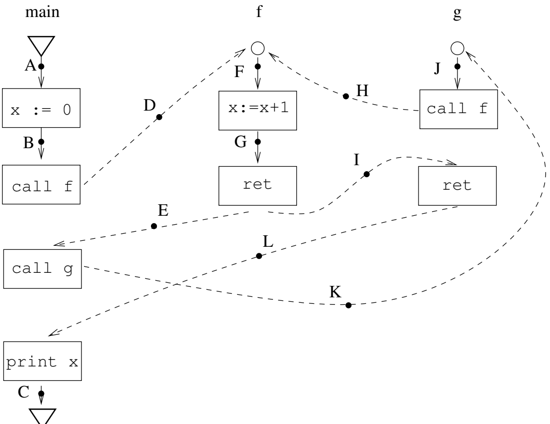


Exercise 1

Let $\mathcal{A}$ be the standard collecting state analysis. For brevity, represent a set of concrete states as $\{0, 1\}$ (meaning the 2 concrete states $x \mapsto 0$ and $x \mapsto 1$). Assume an initial value $d_0 = \{0\}$.

Show the call-string tagged abstract states (in the lattice $\mathcal{A}'$) along the paths

1. ABDFGEKJHFGIL (interprocedurally valid)
2. ABDFGIL (interprocedurally invalid).



Correctness claim

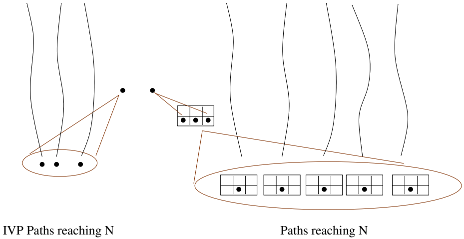
Assumption on $\mathcal{A}$: Each transfer function satisfies $f_{MN}(\perp) = \perp$.

Claim

Let N be a point in G'. Then

$$JVP_{\mathcal{A}}(N) = \bigsqcup_{\gamma \in \Gamma} JOP_{\mathcal{A}'}(N)(\gamma).$$

Proof: Use following lemmas to prove that LHS dominates RHS and vice-versa.



Correctness claim: Lemma 1

Lemma 1

Let ρ be a path in $IVP_{G'}$. Then

$$f'_{\rho}(\xi_0) = \lambda\gamma. \begin{cases} f_{\rho}(d_0) & \mathit{if}\, \gamma = cs(\rho) \\ \perp & \text{otherwise.} \end{cases}$$

ϵ	c_1	$cs(\rho)$	$c_1 c_2 c_2$	....
$\perp$	$\perp$	d	$\perp$	....

Proof: by induction on the length of ρ.

Correctness claim: Lemma 2

Lemma 2

Let ρ be a path not in $IVP_{G'}$. Then

$$f'_\rho(\xi_0) = \lambda\gamma.\bot.$$

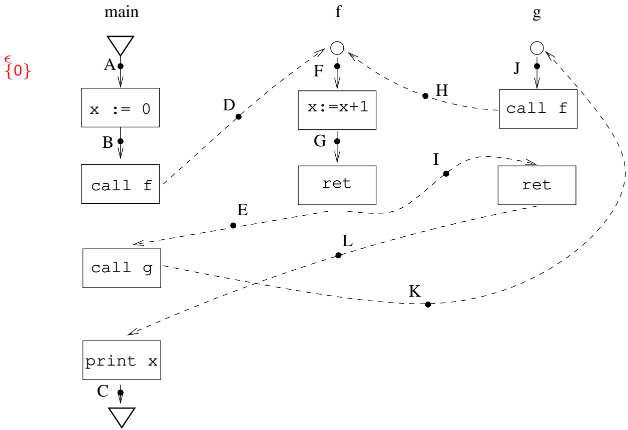
ϵ	c_1	c_2	$c_1 c_2 c_2$	....
$\bot$	$\bot$	$\bot$	$\bot$	....

Proof:

- ρ must have an invalid prefix.
- Consider smallest such prefix $\alpha \cdot N$. Then it must be that α is valid and N is a return edge not corresponding to $cs(\alpha)$.
- Using previous lemma it follows that $f'_{\alpha \cdot N}(\xi_0) = \lambda\gamma.\bot$.
- But then all extensions of α along ρ must also have transfer function $\lambda\gamma.\bot$.

Exercise 2

Use Kildall's algo to compute the LFP of the $\mathcal{A}'$ analysis for the example program. Start with initial value $d_0 = \{0\}$.



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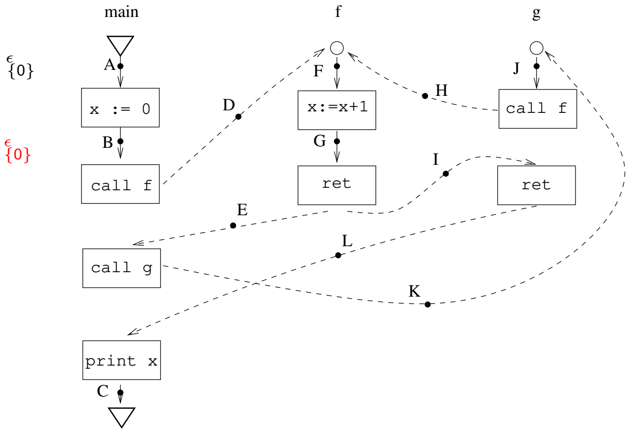

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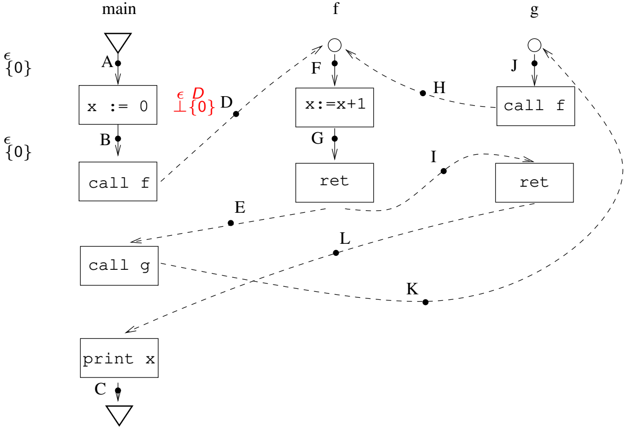

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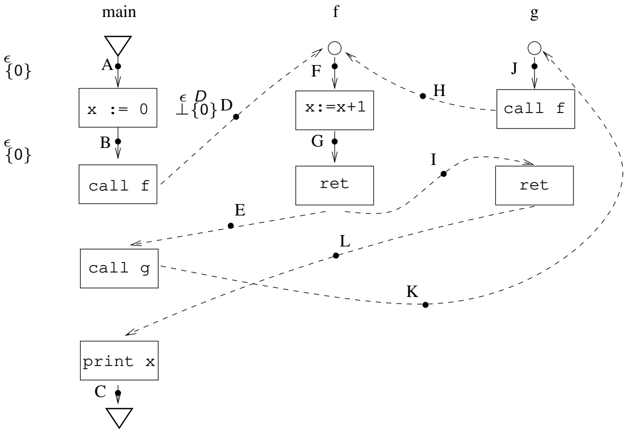
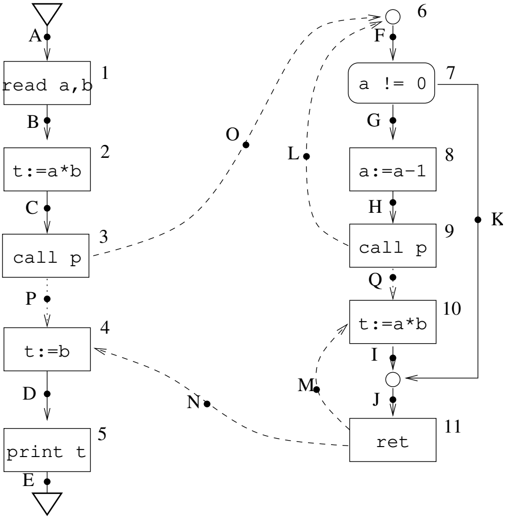


Computing JOP for abs int $\mathcal{A}'$

- Problem is that D' is infinite in general (even if D were finite). So we cannot use Kildall's algo to compute an over-approximation of JOP (it may not terminate when the program has recursive procedures).

Available expressions

- An expresssion (like "$a * b$") is available along an execution if there is a point where the expression is evaluated and thereafter none of the constituent variables (like a and b) are written to.
- An expression is available at a point N in a program, if along every execution reaching N, the expression is available.
- Is $a * b$ available at program point N?



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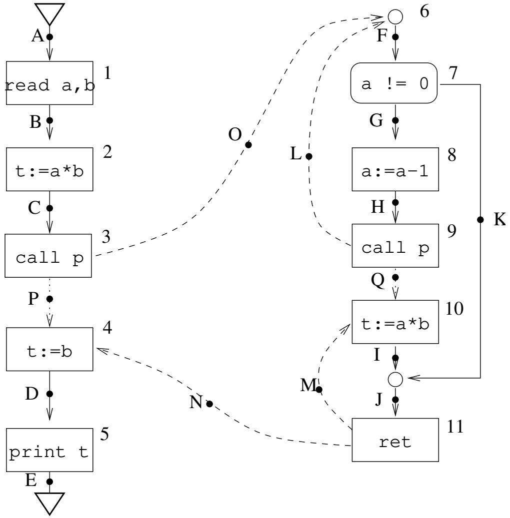
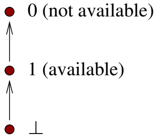
- An expression is available at a point N in a program, if along every execution reaching N, the expression is available.
- Is $a * b$ available at program point N? Yes.

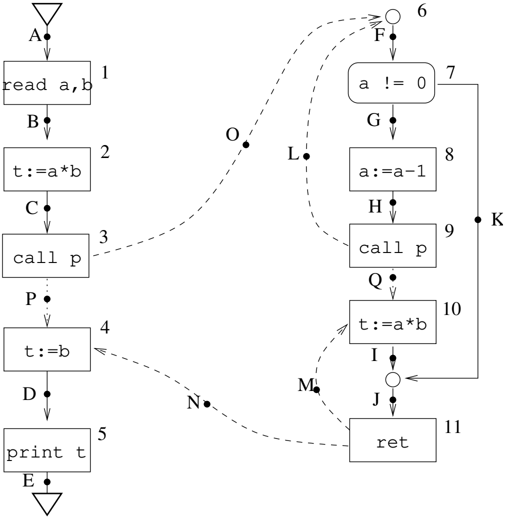


Available expressions analysis



Lattice for Av-Exp analysis for $a * b$.

- "0" concretizes to the set $States \times \{A, NA\}$; while "1" concretizes to $States \times \{A\}$. "⊥" concretizes to $\emptyset$.
- JOP of analysis says $a * b$ is not available at program point N.



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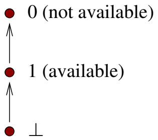
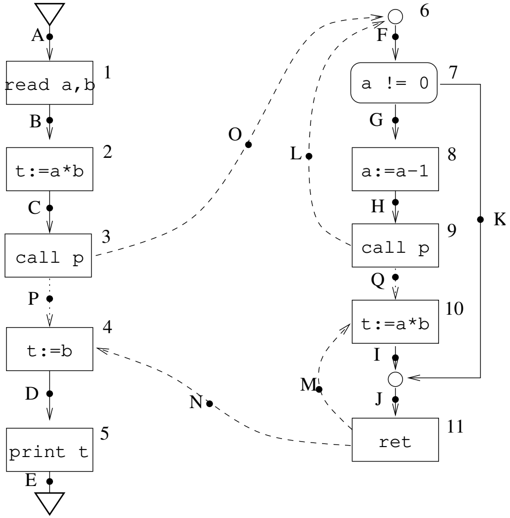


Lattice for Av-Exp analysis for $a * b$.

- "0" concretizes to the set $States \times \{A, NA\}$; while "1" concretizes to $States \times \{A\}$. "$\perp$" concretizes to $\emptyset$.
- JOP of analysis says $a * b$ is not available at program point N.
- JVP says it is available.

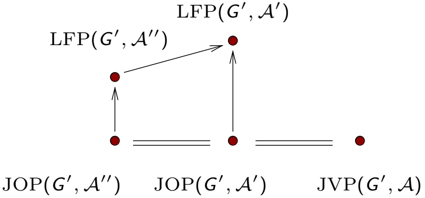


Computing JOP for abs int $\mathcal{A}'$

- We give two methods to bound the number of call-strings we need to consider, when underlying lattice $(D, \leq)$ is finite.
  - Give a bound on largest call-string needed.
  - Use "approximate" call-strings.

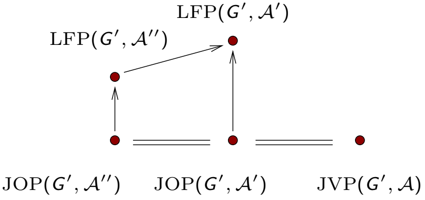
Bounded call-string method for finite underlying lattice D

- Possible to bound length of call-strings Γ we need to consider.
- For a number l, we denote the set of call-strings (for the given program P) of length at most l, by Γ_l.
- Define a new analysis $\mathcal{A}''$ (M-bounded call-string analysis) in which call-string tables have entries only for Γ_M for a certain constant M, and transfer functions ignore entries for call-strings of length more than M.
- We will show that $\mathrm{JOP}(G', \mathcal{A}'') = \mathrm{JOP}(G', \mathcal{A}')$.



LFP of $\mathcal{A}''$ is more precise than LFP of $\mathcal{A}'$

- Consider any fixpoint V' (a vector of tables) of $\mathcal{A}'$.
- Truncate each entry of V' to (call-strings of) length M, to get V''.
- Clearly V' dominates V''.
- Further, observe that V'' is a post-fixpoint of the transfer functions for $\mathcal{A}''$.
- By Knaster-Tarski characterisation of LFP, we know that V'' dominates LFP($\mathcal{A}''$).



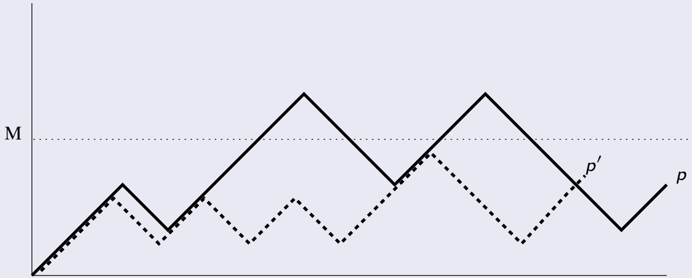
Sufficiency (or safety) of bound

Let k be the number of call sites in P.

Claim

For any path p in $IVP(r_1, N)$ with a prefix q such that $|cs(q)| > k|D|^2 = M$ there is a path p' in $IVP(r_1, N)$ with $|cs(q')| \leq M$ for each prefix q' of p', and $f_p(d_0) = f_{p'}(d_0)$.

Paths with bounded call-strings



Proving claim

Claim

For any path p in $IVP(r_1, N)$ such that for some prefix q of p, $|cs(q)| > M = k|D|^2$, there is a path p' in $IVP_{\Gamma_M}(r_1, N)$ with $f_{p'}(d_0) = f_p(d_0)$.

- Sufficient to prove:

Subclaim

For any path p in $IVP(r_1, N)$ with a prefix q such that $|cs(q)| > M$, we can produce a smaller path p' in $IVP(r_1, N)$ with $f_{p'}(d_0) = f_p(d_0)$.

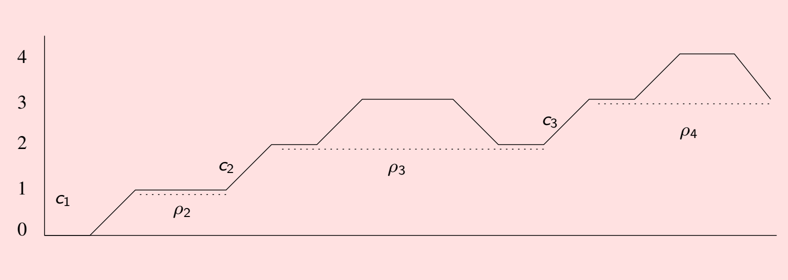
- ...since if $|p| \leq M$ then $p \in IVP_{\Gamma_M}$.

Proving subclaim: Path decomposition

A path ρ in $IVP(r_1, n)$ can be decomposed as

$$\rho_1 \| (c_1, r_{p_2}) \| \rho_2 \| (c_2, r_{p_3}) \| \sigma_3 \| \cdots \| (c_{j-1}, r_{p_j}) \| \rho_j.$$

where each ρ_i $(i < j)$ is a valid and complete path from r_{p_i} to c_i, and ρ_j is a valid and complete path from r_{p_j} to n. Thus $c_1, \ldots, c_{j-1}$ are the unfinished calls at the end of ρ.



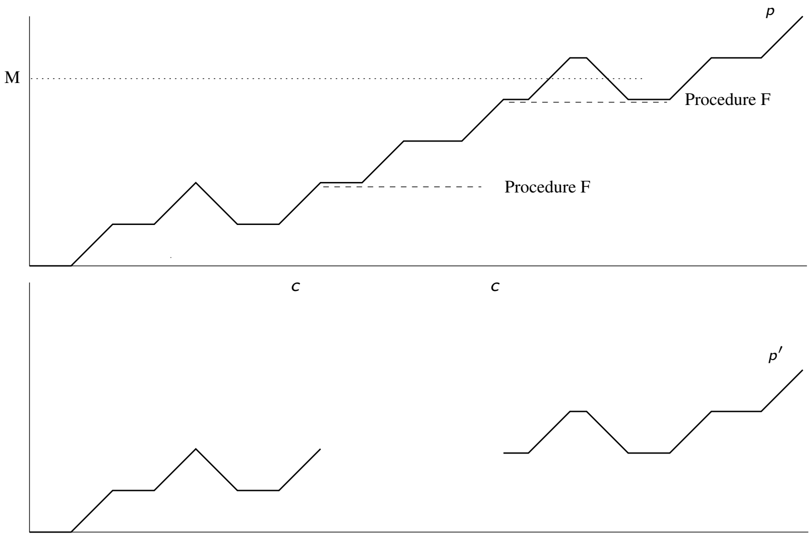
Proving subclaim

- Let p_0 be the first prefix of p where $|cs(p_0)| > M$.
- Let decomposition of p_0 be

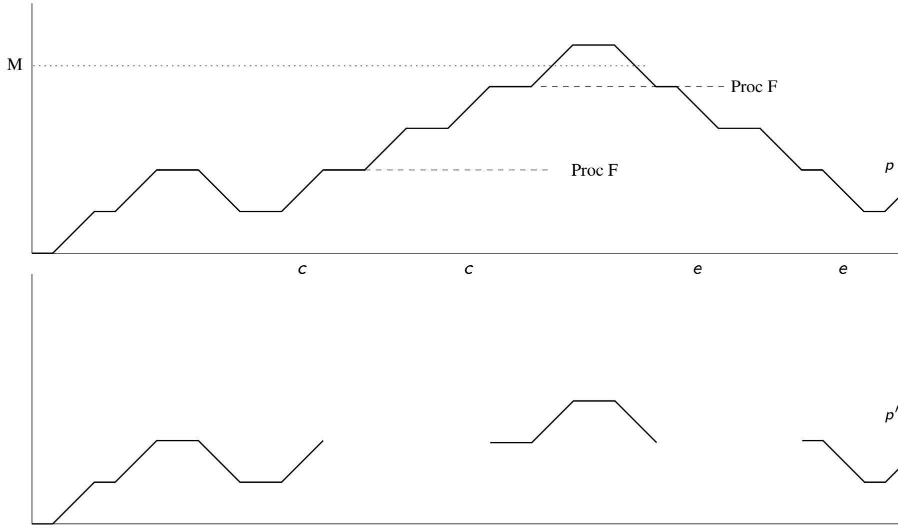
$$\rho_1 \| (c_1, r_{p_2}) \| \rho_2 \| (c_2, r_{p_3}) \| \sigma_3 \| \cdots \| (c_{j-1}, r_{p_j}) \| \rho_j .$$

- Tag each unfinished-call c in p_0 by $(c, f_{q \cdot c}(d_0), f_{q \cdot cq' e}(d_0))$ where e is corresponding return of c in p.
- If no return for c in p tag with $(c, f_{q \cdot c}(d_0), \perp)$.
- Number of distinct such tags is $k \cdot |D|^2$.
- So there are two calls qc and $qcq'c$ with same tag values.

Proving subclaim – tag values are ⊥



Proving subclaim – tag values are not ⊥



Approximate (suffix) call-string method

Idea:

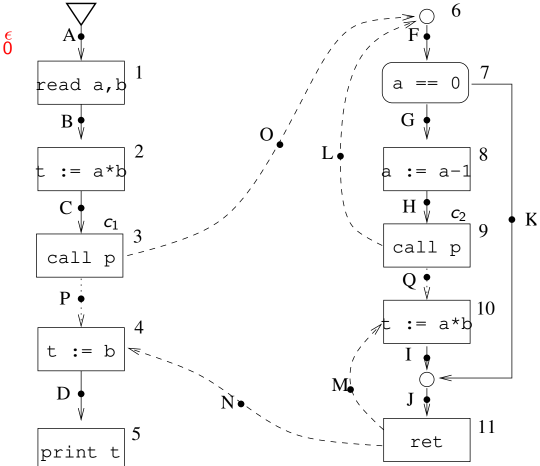
- Consider only call-strings of up to length $\leq l$.
- For $l = 2$, call strings can be of the form "c_1" or "$c_1 c_2$" etc. So each table ξ is now a finite table.
- Transfer functions for non-call/ret edges remain same.
- Transfer functions for call edge C: Shift γ entry to $\gamma \cdot C$ if $|\gamma \cdot C| \leq l$; else shift it to $\gamma' \cdot C$ where γ is of the form $A \cdot \gamma'$, for some call A.
- Transfer functions for ret edge N:
  - If $\gamma = \gamma' \cdot C$ and N corresponds to call edge C, then shift $\gamma' \cdot C$ entry to $A \cdot \gamma'$ which are "feasible" at the return site;

From Sharir-Pnueli 1981, p136 of typed version

string. As long as the length of a call string is less than j, update it as in Section 4. However, if q is a call string of length j, then, when appending to it a call edge, discard the first component of q and add the new call block to its end. When appending a return edge, check if it matches the last call in q and, if it does, delete this call from q and add to its start all possible call blocks which call the procedure containing the first call in q. This approximation may be termed a <u>call-string suffix approximation</u>.

Exercise: approximate call-strings

Assume approximate call-string length of 2. Use Kildall's algo to compute the ξ table values for the example program. Start with initial value $d_0 = 0$.



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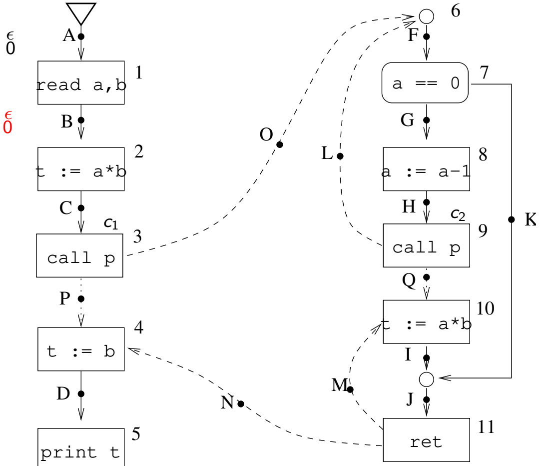

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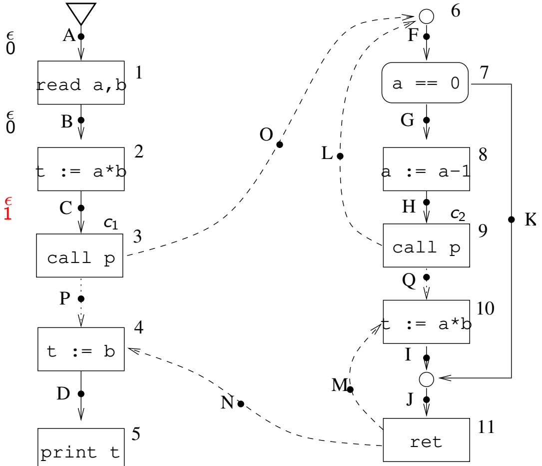

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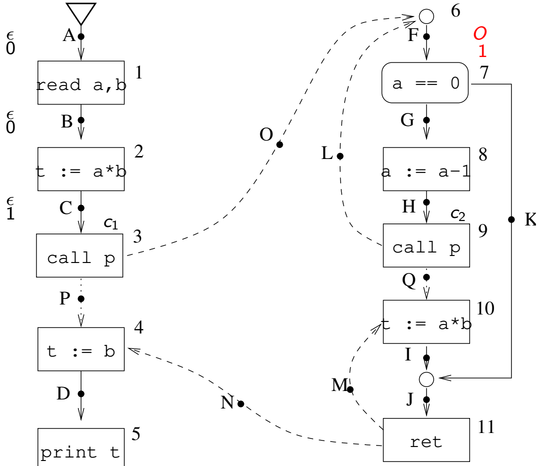

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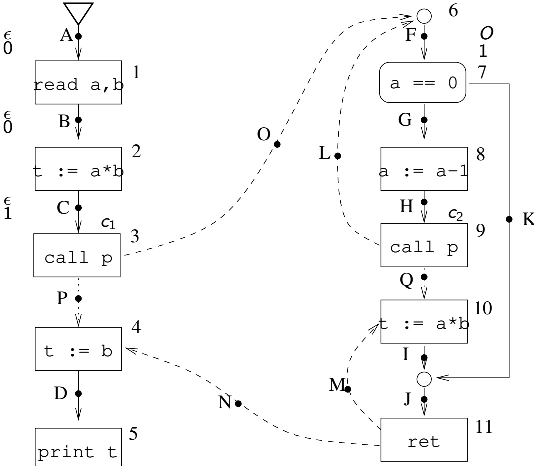
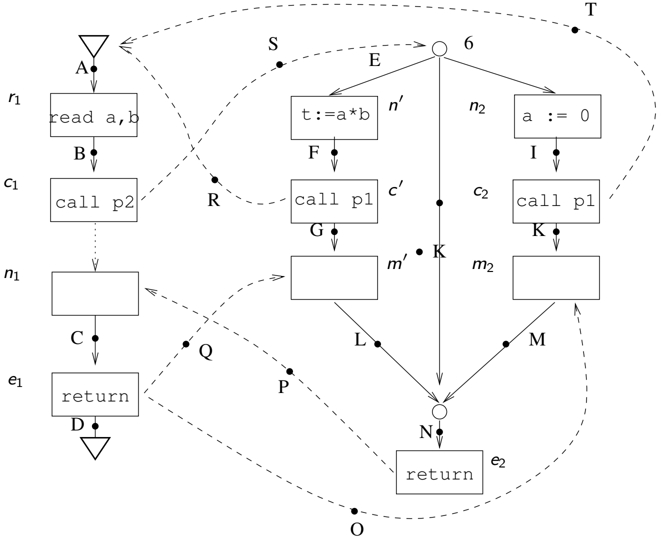


Example


read a,b
call p2
return
t:=a*b
call p1
a := 0
call p1
return
r_1
c_1
n_1
e_1
n'
c'
m'
n_2
c_2
m_2
e_2
6
A
B
C
D
E
F
G
I
K
K
L
M
N
O
P
Q
R
S
T

Transfer functions f'_{MN} for Example 2

- Non-call/ret edge C:

$$\xi_C = f_{BC} \circ \xi_B.$$

- Call edge O:

$$\xi_O(\gamma) = \begin{cases} \xi_C(\gamma') & \text{if } \gamma = \gamma' \cdot O \\ \perp & \text{otherwise} \end{cases}$$

- Return edge N:

$$\xi_N(\gamma) = \xi_J(\gamma \cdot O).$$