E0 225: Homework 10

Deadline: 9 pm on January 24, 2021 (Sunday)

Problem 1. Considering the problem of computing a maximum-cardinality matching (in general graphs), establish the best possible approximation ratio for the following algorithm.

Input: Graph $G = (V, E)$
Output: A subset of edges $M \subseteq E$.

1: Initialize $M = \emptyset$
2: **while** there exists an edge $\widehat{e} \in E \setminus M$ such that $M \cup \{\widehat{e}\}$ is a matching **do**
3: Update $M \leftarrow M \cup \{\widehat{e}\}$
4: **end while**
5: **return** M

Note that the algorithm considers the edges in an arbitrary order.

Problem 2. Let $\mathcal{I}$ be any instance of the maximum-coverage problem specified over n elements and with subsets $S_1, S_2, \ldots, S_m \subseteq U$; here $|U| = n$ and let $k \in \mathbb{Z}_+$ be the cardinality constraint in $\mathcal{I}$. Write Opt to denote the value of an optimal solution in $\mathcal{I}$ (i.e., the size of the union of any k given subsets is at most Opt).
Develop a polynomial-time algorithm that—given $\mathcal{I}$ as input—finds $2k \log n$ sets (among the given ones) whose union is of size at least Opt.