E0 225: Homework 8

Deadline: 5 pm on December 22, 2020 (Tuesday)

Problem 1. For the following linear program:

$$\begin{aligned} \max \quad & x_1 - 2x_3 \\ \text{subject to} \quad & x_1 - x_2 \le 1 \\ & 2x_2 - x_3 \le 1 \\ & x_1, x_2, x_3 \ge 0 \end{aligned}$$

prove that the solution $(x_1, x_2, x_3) = (3/2, 1/2, 0)$ is optimal by analyzing the dual problem.

Problem 2. Given parameters $a_i, b_i, c_i \in \mathbb{R}$ as input, for $1 \le i \le m$, define

$$\mathcal{F} := \left\{ (x_1, x_2) \in \mathbb{R}^2 \mid a_i x_1 + b_i x_2 + c_i \le 0 \text{ for all } 1 \le i \le m \right\} \subset \mathbb{R}^2.$$

Develop a polynomial-time algorithm to find the largest (area wise) circle that can fit inside $\mathcal{F}$. The output of the algorithm should be the center of the circle and its radius.

[Recall that the distance between a point (p_1, p_2) and a line $ax_1 + bx_2 + c = 0$ (with nonzero a and b) is equal to $\frac{|ap_1 + bp_2 + c|}{\sqrt{a^2+b^2}}$].

Problem 3. Let $d_1 d_2 d_3 d_4$ be the last four digits of your IISc serial number. Provide a dry run of the Hungarian algorithm (specifically, the values of the dual variables and the tight sets) on the following graph. Here, the numbers in red are the associated edge weights.