E0 208: Assignment 1

Due Date : March 10th, 11:59pm, 2021

1. **(10 marks)** Let $CH(P)$ denote the convex hull of a set of points P in the plane. Show that the following statements are equivalent.
   - CH(P) is the intersection of all convex objects containing P
   - CH(P) is the smallest convex object containing P.
     Note: X is called the "smallest" convex object containing P if there does not exist another convex object Y containing P, where $Y \subset X$

2. **(10 marks)** Answer whether the following statements are true or false. If the statement is true, give a proof. If it is false, give a counterexample.
   Let A and B be convex objects.
   (a) **(5 mark)** $A \cap B$ is convex.
   (b) **(5 mark)** $A \cup B$ is convex.

3. (a) **(10 marks)** Give a Clockwise(p,q,r) test to determine whether the turn at q, going from p to q to r is a clockwise/right turn. Let $p = (p_x, p_y), q = (q_x, q_y), r = (r_x, r_y)$ be the coordinates of the points.
   (b) **(10 marks)** In each iteration of Jarvis March algorithm for computing convex hull, we found the point with minimum angle. In practice, computing angles can be expensive. Show that the next point can be found by just using the Clockwise(p,q,r) (clockwise test) (without computing any angles).

4. **(15 marks)** Let Q be a simple polygon(edges of polygon do not intersect) with n vertices. Design a $O(n)$ time algorithm to check if Q is a convex polygon. Argue the correctness of your algoithm.
   Assume that the polygon Q is stored in an array containing the ordered vertices of Q, starting with the topmost vertex.

5. **(15 marks)** Let us design a $O(n \log n)$ incremental algorithm to compute the convex hull of a pointset P with n points.

   (a) **(10 marks)** Given a convex polygon Q with m vertices and a point r that is outside Q, describe a $O(\log m)$-time algorithm to compute the lower and upper tangents from r to Q. Assume that the convex polygon Q is stored in an array that contains the vertices of Q in a counterclockwise order.

   (b) **(5 marks)** Using (a), design a $O(n \log n)$-time incremental algorithm to compute convex hull of P. The algorithm inserts points of P one by one in an appropriate order and updates the convex hull of points inserted so far.

6. **(15 marks)** The convex layers of a point set P are defined by repeatedly computing the convex hull of P and removing its vertices from P, until P is empty. Describe an algorithm to compute the convex layers of a given set of n points in the plane in $O(n^2)$ time.
   Note: It is enough to describe the algorithm. Do not have to write pseudocode.

7. **(15 marks)** In Chan's ultimate convex hull algorithm we guessed the value of h as 3^{2^i} in ith iteration. What would happen if we had guessed h as 3^i?