Program Analyis and Verification
Assignment 2
Abstract interpretation and Kildall's algorithm
Due date: Sep. 19th, 11.59 pm.

Interval analysis (IA) is a generalization of Constant Propagation (CP), in which every variable is mapped to an interval (or range) $[u, l]$. u and l can be positive or negative integers, or $-\infty$ or $+\infty$. u should be less than or equal to l. Let each IA element be a function from the set of all variables to intervals. That is, for IA do not use the set-of-pairs notation that we used in class for CP. Note, $[-\infty, \infty]$ is equivalent to *any value*, while $[u, u]$ is equivalent to the constant value u. If d is an IA element, you can use the notation $d(v)$ to refer to the interval associated with variable v, and can use the notation $d[v \mapsto [u, l]]$ to denote a function in which v is mapped to $[u, l]$ and all other variables are mapped to whatever they are in d.

Throughout this assignment, whenever we ask for a program, you should provide a short program, having at most 4-5 lines of code. Also, the program should not use any uninitialized variables.

Problem 1. Define transfer functions (as precise as possible) on the IA lattice for the statements "x := x + y" and "x := y + 1". For simplicity, you can assume that there are only two variables, x and y. Also define functions γ_I and α_I relating IA with the concrete lattice. All definitions should be compactly and mathematically stated, like in the class slides.

The α_I, γ_I that you state above should form a Galois connection, and the transfer functions you provide should be abstractions of the corresponding concrete transfer functions. However, you need not prove these properties explicitly.

Problem 2. Define a pair of functions γ_{CI}, α_{IC} between the IA and the CP lattices such that they form a Galois connection and such that the CP transfer functions are abstractions of the corresponding IA transfer functions that you defined for the problem above. (You need not prove these claims, just provide the definitions.) Also, show an IA fact d such that $\gamma_{CI}(\alpha_{IC}(d)) \sqsupset d$.

Problem 3. Show a simple program such that at some program point the γ_I image of the abstract JOP using IA is strictly a subset of the γ_C image of the abstract JOP using CP at the same point, where γ_C relates CP to the concrete lattice.

Problem 4. Demonstrate that the IA lattice (even with just one variable) is not of finite height. Show a simple program on which Kildall's algorithm will not terminate with the IA analysis.

Problem 5. Show a statement whose CP transfer function is not distributive, and demonstrate this non-distributivity. Show an example program where the CP abstract JOP at some program point is strictly more precise than the CP LFP solution at that point. Also show a statement whose CP transfer function is distributive.

Problem 6. Consider two lattices C and D and two functions α_{CD} and γ_{DC} such that these two form a Galois connection. Let C' be the subset of C consisting of elements that are γ_{DC} inages of the elements in D. Considering any three elements d_1, d_2 and $d_1 \sqcup d_2$ in D, prove that $\gamma_{DC}(d_1 \sqcup d_2)$ is the least upper bound of $\gamma_{DC}(d_1)$ and $\gamma_{DC}(d_2)$ in lattice C'. Your proof should not contain more than 7-8 steps.