Paper: Hawkes Models and Their Applications

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Keyword self-exciting point process

Hawk process

$$R(t) = \sum_{k, S_k < t} Y_k = f(\{x_k\}, s_k < t)$$

Hawkes process - the one arrival creates a heightened chance of further arrivals in the near future.

Two applications: seismology and in finance

- marked forces fed back by the increase and decrease of stocks

Stoichiometric process

$$\begin{aligned} X &\to 0 \text{ death} \\ 0 &\to X \text{ birth} \end{aligned}$$

or

$$\begin{aligned} X &\to 2X \\ X &\to 0 \end{aligned}$$

like in fission or budding. Or

$$\begin{aligned} 2X &\to 3X \\ X &\to 0 \end{aligned}$$

Or a sexual process

$$\begin{aligned} X + Y &\to 2X + Y \\ X + Y &\to X + 2Y \\ X &\to 0 \\ Y &\to 0 \end{aligned}$$

There is a package of julia that generates SSA, DE, langevin equation by simple input of the chemical equations. Explore only for birth and death process.

Hawkes process - self-excitation, occurrence of one event may trigger a series of similar events.

Explosion of popularity is attributed to financial applications.

Rate of arrivals depend in a particular way on past history.

random variable called ork associated with each arrival.

Classical Hawkes Process

Counting and Point Processes

$$N(t) = \sum_i^{\infty} \mathcal{I}\{T_i \le t\} = \sum_{T_{i \le t}} 1$$

T_i jump times are also called arrival times
$T_0 = 0$,
$E_i := T_i - T_{i-1}$ are called the interarrival times.

homogenous Poisson process has

$$E_i \sim \exp(\lambda), E(E_i) = \frac{1}{\lambda} \wedge E\left(\frac{N_i}{t}\right)$$

Conditional Intensity and Compensators

$$\lambda^*(t) = \lim_{\Delta \to 0} \frac{E(N_{t+\Delta} - N_t | \mathcal{H}_t)}{\Delta}$$

Poisson process with rate function $\lambda(t)$, homogeneous if $\lambda(t) = \lambda$ does not depend on t.

Compensator

$$\Lambda = \int_0^t \lambda_s^* ds$$

unique predictable process for which $N_t - \Lambda_t$ isa local martingale.

Hawkes Process and the Self-Exciting Propertiy

Hawkes process

a counting process N_t whose conditional intensity process for $t >]0$ is

$$\lambda_t^* = \lambda + \sum_{T_i < t} \mu(t - T_i)$$

$\lambda > 0$ the background arrival rate and μ.

For a special excitation function $\mu(t) = \alpha \exp(-\beta t)$, exponentially decaynig intensity λ_t^*.

punctuated equilibrium

The Immigration-Birth View and Stationarity

Immigrants represents exogenous shocks to the systems

An immigrant arriving at times s creates a new Poisson process of births or offspring with intensity $\mu(t - s)$ for $t > s$.

Transforms: The transformation of $\delta(t - t_0)$ is the kerner $K(s; t = t_0)$. The question of transformation is what is the diract delta series that reproduces your time series signal.

Laplace transforms is usually used in circuits, specially in dissipative systems.

Each of this births generates more offspring. Hawkes process can be represented by the chemical equation.

An arrival at time s will generate a Possion(η) number of first-generation offspring over $t \geq s$. This η, called the branching ratio.

the expected number of offspring in all generations is $\sum_{k=1}^{\infty} \eta^k = \frac{\eta}{1-\eta}$ if $\eta < 1$. The condition $\eta < 1$ is necessary and sufficient for the process to be stationary and have a finite mean.

Liapunov exponent

$$\lim_{t \to \infty} \frac{N_t}{t} = \frac{\lambda}{1 - \eta} \quad \text{almost surely}$$

Omori law and Omori-Utus law

an aftershock model. The aftershock rate as approximately

$$K(t + c)^{-1} (K, c > 0)$$

modified Omori law

$$\mu(t) = K(t + c)^{-p} (p > 1)$$

μ is also called th epower-law kernal since its normalized form

$$v(t) = \frac{p - 1}{c} \left(1 + \frac{t}{c}\right)^{-p}$$