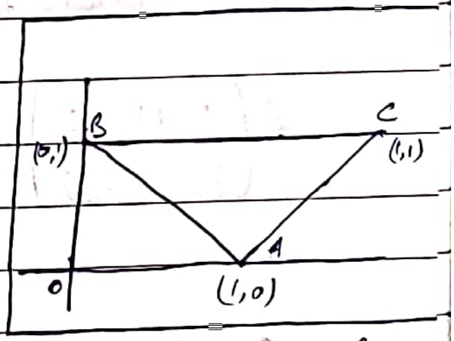
Aman Chauhan 1805158

① a) Composite transformation means when 2 or more transformation are performed on a figure to produce a new figure. Translation of points by the change of coordinate cannot be combined with other transformation by using simple matrix application. Such a combination is essential if we wish to rotate an image about a point other than origin by translation, rotation again translation. To combine three transformation into a single transformation, "homogeneous coordinates are used.

b) We can represent the given triangle, in term of homogeneous coordinates as-

$$ABC = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$



Suppose we made rotation in the counter clockwise direction then the transformation matrix for rotation $R_{45°}$ in terms of homogeneous coordinates system is:

$$R_{45°} = \begin{bmatrix} \cos 45 & \sin 45 & 0 \\ -\sin 45 & \cos 45 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The translation matrix, T_v, where $v = 1i + 0j$ is

$$T_v = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ t_x & t_y & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

Aman Chauhan 1805158

Now the rotation followed by translation can be computed as:

$$R_{45} \cdot T_v = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

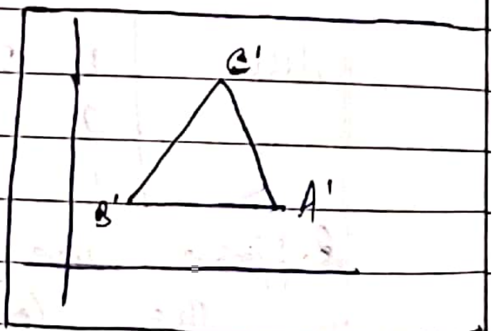
So the new coordinates A'B'C' of a given triangle ABC can be shown as-

$$[A'B'C'] = [ABC] \cdot R_{45} \cdot T_v$$

$$= \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} (1/\sqrt{2}+1) & 1/\sqrt{2} & 1 \\ (-1/\sqrt{2}+1) & 1/\sqrt{2} & 1 \\ 1 & \sqrt{2} & 1 \end{bmatrix}$$

implies that given triangle $A(1,0)$, $B(0,1)$, $C(1,1)$ be transformed into

$$A'\left(\frac{1}{\sqrt{2}}+1, \frac{1}{\sqrt{2}}\right), B'\left(\frac{-1}{\sqrt{2}}+1, \frac{1}{\sqrt{2}}\right), C'\left(1, \sqrt{2}\right)$$



Similarly we can obtain translation followed by rotation transformations as:

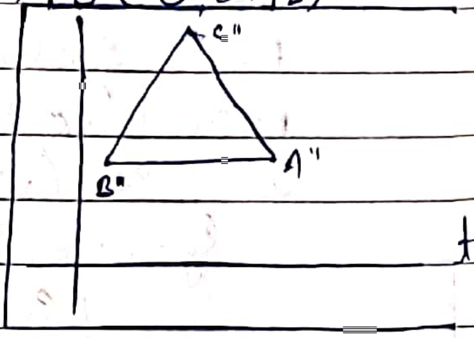
$$T_v \cdot R_{45} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & 1 \end{bmatrix}$$

And hence the new coordinates A'B'C' can be computed as: $[A''B''C''] = [ABC] \cdot R_{45} \cdot T_v$

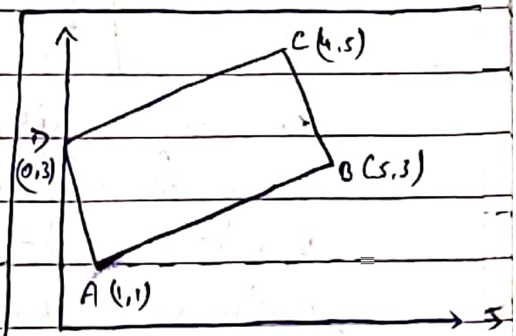
$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & 1 \end{bmatrix} = \begin{bmatrix} 2/\sqrt{2} & 2/\sqrt{2} & 0 \\ 0 & 2/\sqrt{2} & 1 \\ 1/\sqrt{2} & 3/\sqrt{2} & 1 \end{bmatrix}$$

Aman Chauhan 1805158

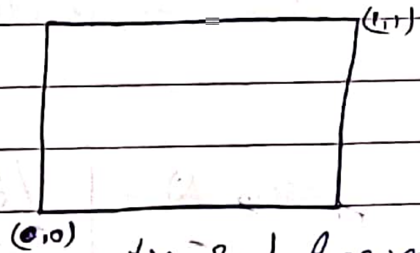
Thus in this case the given triangle ABC is transformed into $A''(2/\sqrt{2}, 2/\sqrt{2})$, $B''(0, 2/\sqrt{2})$, $C''(1/\sqrt{2}, 1/\sqrt{2})$



(2) a) Currently we observe that window edges are not || to coordinates axis. Consequently we will first rotate the window regarding A hence it is aligned along with the axis.



Window Region



Viewport Region

Now $\tan\alpha = (3-1)/(5-1) = 1/2$

$\therefore \sin\alpha = 1/\sqrt{5}$ & $\cos\alpha = 2/\sqrt{5}$

Now, we are rotating the rectangle in clockwise direction. Consequently α is $-ve$ which is $-\alpha$

The rotation matrix about A(1,1)
$$\begin{bmatrix} \cos\alpha & -\sin\alpha & (1-\cos\alpha)x_p + \sin\alpha y_p \\ \sin\alpha & \cos\alpha & (1-\cos\alpha)y_p + \sin\alpha y_p \\ 0 & 0 & 1 \end{bmatrix}$$

$$[T_{R,\theta}]_A = \begin{bmatrix} 2/\sqrt{5} & 1/\sqrt{5} & \left(\frac{1-3}{\sqrt{5}}\right) \\ -1/\sqrt{5} & 2/\sqrt{5} & \left(\frac{1-1}{\sqrt{5}}\right) \\ 0 & 0 & 1 \end{bmatrix}$$

The 'x' extent of rotated window is length of AX: $\sqrt{4^2+2^2} = 2\sqrt{5}$

Aman Chauhan 1805158

As same, the y extent is length of AZ that is, $\sqrt{1^2+2^2} = \sqrt{5}$

For scaling the rotated window to the normalization viewport we calculate S_x & S_y as, S_x = (viewport x extent)/(window x extent) = $1/2\sqrt{5}$

S_y = (viewport y extent)/(window y extent) = $1/\sqrt{5}$

$$\begin{bmatrix} S_x & 0 & -S_x \cdot xw_{min} + xv_{min} \\ 0 & S_y & -S_y \cdot xw_{in} + yv_{min} \\ 0 & 0 & 1 \end{bmatrix}$$

As in expression, the common form of transformation matrix showing mapping of a window to a viewport: [T] = N

within this problem [T] may be termed as N as this is case of normalisation transformation with, $xw_{min} = 1$, $xv_{min} = 0$

$yw_{min} = 1$, $yv_{min} = 0$

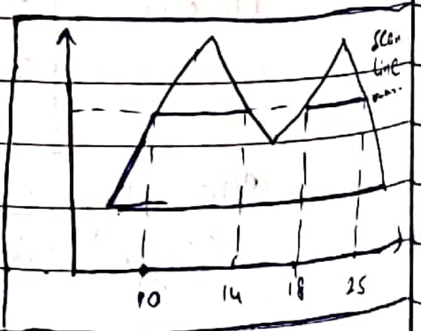
$S_x = 1/2\sqrt{5}$, $S_y = 1/\sqrt{5}$

So, $N = \begin{bmatrix} 1/2\sqrt{5} & 0 & (-1/2)(1/\sqrt{5}) + 0 \\ 0 & 1/\sqrt{5} & (-1/\sqrt{5})1 + 0 \\ 0 & 0 & 1 \end{bmatrix}$

So here we compose rotation & transformation N to get viewing transformation

$$NR = \begin{bmatrix} 1/2\sqrt{5} & 0 & -1/\sqrt{5} \\ 0 & 1/\sqrt{5} & -1/\sqrt{5} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2/\sqrt{5} & 1/\sqrt{5} & (1-3/\sqrt{5}) \\ -1/\sqrt{5} & 2/\sqrt{5} & (1-2/\sqrt{5}) \\ 0 & 0 & 1 \end{bmatrix}$$

b) The intersection of scanline and polygon is noted and the color is filled accordingly. Moving along scan line scanning intersection with edge of polygon



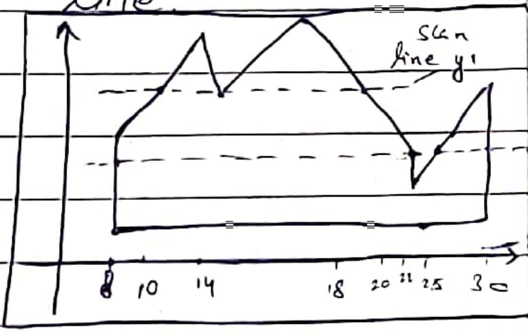
Aman Chauhan 1805158

Scan line intersect with each edge of polygon. Name each intersection point of polygon.

Let $P_1 = 10$, $P_2 = 14$, $P_3 = 18$, $P_4 = 25$.

$\therefore$ The points we get is (10, 14) (18, 25)

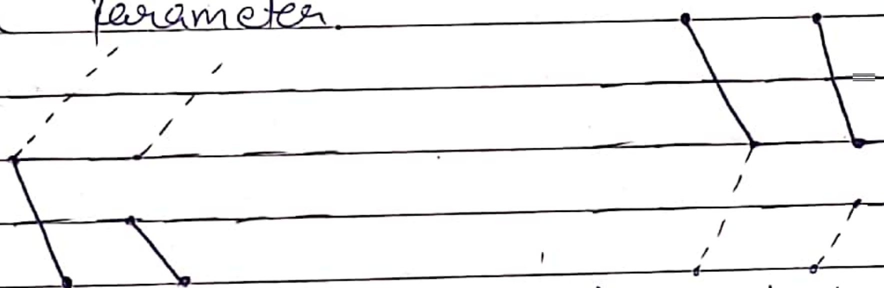
but sometimes there are some edge that need special treatment. These are for which points edge sharing line lies on opposite side of scan line.



Hence, for scanline y-pairs are (10, 14), (14, 20)

for scan line y' -pairs are (8, 22) (25, 30)

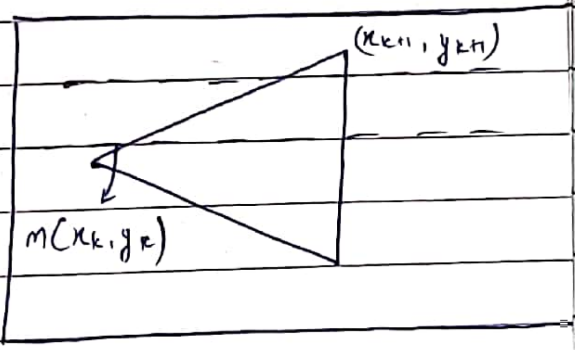
Adjust the endpoint if value for a polygon as we process edges in order around the polygon parameter.



a) y coordinate of upper endpoint of current edge is dec. by 1

b) y coordinate of upper endpoint of next edge is dec. by 1

$$m = \frac{y_{k+1} - y_k}{x_{k+1} - x_k}, \quad m = \frac{1}{x_{k+1} - x_k}$$



$x_{k+1} = x_k + 1/m$

at k^{th} step

$x_k = x_0 + k/m$