

⑤ In this question, we have total 5 samples out of which [3+, 2-]

$$\text{Now, Entropy}(S) = -p_{+} \log_2 p_{+} - p_{-} \log_2 p_{-}$$

$$= -\frac{3}{5} \log_2 \frac{3}{5} - \frac{2}{5} \log_2 \frac{2}{5} = 0.91$$

Now for find gain we need to find their entropy first.

$$\text{Entropy}(A_{\text{high}}) = -\frac{1}{2} \log_2 \frac{1}{2} - \frac{1}{2} \log_2 \frac{1}{2} = 1$$

$$\text{Entropy}(A_{\text{normal}}) = -\frac{2}{3} \log_2 \frac{2}{3} - \frac{1}{3} \log_2 \frac{1}{3} = 0.91$$

$$\text{Now, Gain}(S, A) = \text{Entropy}(S) - \sum_{v \in \{high, normal\}} \frac{|S_v|}{|S|} \text{Entropy}(S_v)$$

$$= 0.91 - \frac{2}{5} \times 1 - \frac{3}{5} \times 0.91 = 0.020$$

$$\boxed{\text{Gain}(S, A) = 0.020}$$

$$\text{Entropy}(B_{\text{hot}}) = -\frac{1}{2} \log_2 \frac{1}{2} - \frac{1}{2} \log_2 \frac{1}{2} = 1$$

$$\text{Entropy}(B_{\text{mild}}) = -\frac{1}{2} \log_2 \frac{1}{2} - \frac{1}{2} \log_2 \frac{1}{2} = 1$$

$$\text{Entropy}(B_{\text{cool}}) = \frac{1}{1} \log_2 1 - 0 \log_2 0 = 0$$

$$\text{Now, Gain}(S, B) = \text{Entropy}(S) - \sum_{v \in \{hot, mild, cool\}} \frac{|S_v|}{|S|} \text{Entropy}(S_v)$$

$$= 0.91 - \frac{2}{5} \times 1 - \frac{2}{5} \times 1 - \frac{1}{5} \times 0 = 0.171$$

$$\boxed{\text{Gain}(S, B) = 0.171}$$