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Title: Explaining Probability in 3 levels

Level 1: Child

Hi everyone! Today, we are going to learn about probability!

When we say something is certain to happen, it means that such event is going to happen without a doubt. For example, we are certain that one plus one equals to two, or, two multiplied by five is ten.

Conversely, when we say something is impossible to happen, it means that there is no way that such event is going to happen. For instance, it is impossible that pigs can fly.

However, not all events in real life are strictly certain or impossible. For example, when we flip a coin, we are not sure which side of the coin will face upwards. Maybe it is the head side. Or maybe it is the tail side. When we are not sure if the event will happen or not, we can use the word "possible" to describe it. It is possible that we get head after flipping a coin.

All the examples mentioned just now are related to "probability". So, what is "probability"? Probability, is the chance that an event is going to happen.

Let us go back to the example of flipping a fair coin. We know that there are only two sides of the coin, which is the head and tail. If we toss it a lot of times, we could expect that about half the flips will be head and half the flips will be tails. So, the probability of coin landing heads is one half and the probability of coin landing tails is also one half.

The probability of an event to happen can be calculated as: the number of ways the event can happen, divided by, the total number of outcomes.

We can understand this concept using a probability line. A probability line is similar to a number line, except that it starts at 0 and ends at 1. A probability of zero means that the event is impossible to happen. A probability of one means that the event is certain to happen. A probability of one half means that there is an even chance the event will happen or not. For probabilities between zero and one half, the event is unlikely to happen. Similarly, for probabilities between one half and one, we say the event is likely to happen.

Let us look at an example to further understand probability.

We are given a black box and there are 5 red balls inside the box. Now, without looking at the box, we grab a ball out of the box.

What is the probability that we will get a red ball? Since there are 5 red balls inside the box, we have 5 ways to get a red ball. Also, the total number of balls inside the box is 5. Therefore, the required probability is 5 divided by 5, which is 1. So, we can conclude that we can certainly get a red ball.

Now, we put the red ball we just grabbed back into the box and put extra two yellow balls into the box. What is the probability that we will get a red ball? We still have 5 red balls as in the original situation. However, the total number of balls inside the ball has increased to 7 because we placed extra two balls into the box. Therefore, the required probability is 5 divided by 7. Since 5 over 7 is greater than one half, we can conclude that it is likely that we can get a red ball.

Level 2: Teen

In this video, we are going to talk about probability.

What is probability? What does this notation mean?

In this notation, A denotes an event and PA means the probability of the event occurring. To calculate the probability of an event A occurring, it equals to the number of favorable outcomes or outcomes lead to A, divided by, total number of possible outcomes.

To start with, we should perhaps discuss what a sample space is. A sample space is a set that contains all possible outcomes that can occur.

Let us try to understand the concept of sample space through the flipping coins scenario. When we flip a coin, we either get a head or a tail as an outcome. Therefore, the sample space is the set containing only a head and a tail.

What if we flip the coin twice? Let us observe the outcomes one by one. When we flip the first coin, we have either a head or a tail. Suppose we get a head in the first flip, during the second flip, we can get another two possibilities. Similarly, if we get a tail in the first flip, we can also get another two possibilities during the second flip. Notice that there are four possible outcomes. Hence, the sample space contains: head-head, head-tail, tail-head, and tail-tail.

The probability of an event occurring always lies between 0 and 1 inclusive. When the probability of an event is equal to zero, it means that the event will never happen. Meanwhile, if the probability of an event equals one, the event will always happen. In other words, event A has a one hundred percent chance of occurring.

If the probability of an event is point 6, it is equivalent to saying that there is a sixty percent chance that the event will occur. Also, it means that out of 100 trials, there are approximately 60 favorable outcomes.

There are two main rules in probability: addition rule and multiplication rule.

The addition rule states that for any events A and B. The probability of either A or B occurring is equal to the probability of A, plus the probability of B, minus the probability of A and B.

Let’s look at an example to understand the use of this rule.

Given that the probability that tomorrow will rain is 0.6, and the probability that tomorrow will snow is 0.2. Also, the probability that tomorrow will rain and snow is 0.3. Find the probability that tomorrow will rain or snow.

Let A be the event that tomorrow will rain, and its probability is 0.6. Let B be the event that tomorrow will snow, and its probability is 0.2. By the addition rule, we can easily calculate the probability of A or B is 0.6, plus 0.2. minus 0.3. So, the probability that tomorrow will rain or snow is 0.5.

Now, I will introduce the terminology of “mutually exclusive events”. Two events, A and B, are mutually exclusive if they cannot occur at the same time, which means the probability of A and B is zero. For example, let A be the event that today is a weekday and B be the event that today is a weekend. Then A and B is the event that today is both a weekday and a weekend. However, such event is impossible to occur. Hence, the probability of A and B is zero. Therefore, we can conclude that A and B are mutually exclusive. If A and B are mutually exclusive, the addition rule can be further simplified as: The probability of either A or B occurring is equal to the probability of A, plus the probability of B.

The multiplication rule states that for any events A and B, the probability of A and B equal to the conditional probability of A given B, multiplied by the probability of B. For independent events A and B, the conditional probability of A given B equals to the probability of A, because the event B will not affect the occurrence of A. Therefore, if A and B are independent, the multiplication rule can be further simplified as: The probability of A and B is equal to the probability of A, multiplied by the probability of B.

Let us go back to the weather example to further understand the multiplication rule.

Given that the probability that tomorrow will rain is 0.6 and the probability that tomorrow will snow is 0.2. Also, the events that tomorrow will rain and tomorrow will snow are independent. Find the probability that tomorrow will rain and snow.

Similarly,

Let A be the event that tomorrow will rain, and its probability is 0.6. Let B be the event that tomorrow will snow, and its probability is 0.2. Since A and B are independent,

By the multiplication rule, probability of A and B is 0.6 times 0.2.

Hence, the probability that tomorrow will rain and snow is 0.12.

Level 3: College student

In this video, we are going to talk about conditional probability, and Bayes theorem.

To start with, let us review basic probability concepts. Probability is a numeric representation of the chance of an event occurring, which lies in between zero and one. The equation to calculate the probability of an event is: the number of ways the event can happen, divided by, the total number of outcomes.

A conditional probability can be denoted as follows: probability of event A, given that event B has already occurred. The equation to calculate this probability is: probability of A and B, divided by, probability of B. The intuition of this equation can be explained with a venn diagram.

Let’s look at the example below.

Let’s say the course S T A 2 9 9 has 30 students, all students are either doing a life science program or a physical science program or both. Given that 15 students are doing a program in life science, and 25 students are doing a program in physical science. What is the probability that a student is doing a life science program given that they are also doing a physical science program?

We can first draw the venn diagram. The sample space will be all 30 students. Let us name A as the event that the student is doing a life science program, and B as the event that the student is doing a physical science program. Notice that there is an overlapping area between the circles representing A , and B. This area belongs to students who are doing both programs. The number of students who are doing both programs can be calculated as 15, plus 25, minus 30, which is 10. So, the number of students who are doing only a life science program is 15, minus 10. Also, the number of students who are doing only a physical science program is 25, minus 10.

Given that a student is doing a physical science program, they are one of the 25 students. In addition to that, if a student is also doing a life science program, they must be one of the 10 students who do both programs. Therefore, the required conditional probability is 10, over 25.

How about the probability that a student is doing a physical science program given that they are also doing a life science program? Similarly, based on the “given” part, they are one of the 15 students who do a life science program. Besides, if a student is also doing a physical science program, they must be one of the 10 students who do both programs. Therefore, the required conditional probability is 10 over 15.

In fact, these two conditional probabilities can be related through Bayes theorem. The theorem states that for any two events, A and B with non-zero probabilities, the probability of A given B is: the probability of B given A, multiplied by the probability of A, divided by, the probability of B.

Here is a simple and neat proof of the theorem. The main point of this proof is to write out the definition of conditional probability by expanding it.

Let us look at the famous false positive example about Bayes theorem.

Suppose 2% of the population carries the virus Q, and there is a test with a five percent false positive rate. Assume that half of the population were tested positive. What is the probability that you were tested positive given that you do not carry the virus?

Let A be the event that you do not carry a virus. Since 2% of the population carries a virus, the probability that you carry a virus is 0.02. Hence the probability that you do not carry a virus, which is probability of A, is 1 minus 0.02 equals to 0.98.

Let B be the event that you were tested positive. So, the probability of B is 0.5.

This question is essentially asking for the probability of B given A. A five percent false positive rate is equivalent to, the probability of you do not carry a virus, given that you were tested positive, which is probability of A given B, equals 0.05.

Now, we have all the terms ready to apply Bayes theorem.

The probability of A given B is, the probability of B given A multiplied by the probability of A, divided by, the probability of B.

So, the you were tested positive given that you do not carry the virus is 0.05, times 0.98, divided by 0.5. In this case, the chance that you were tested positive given that you do not carry the virus is 0.098.