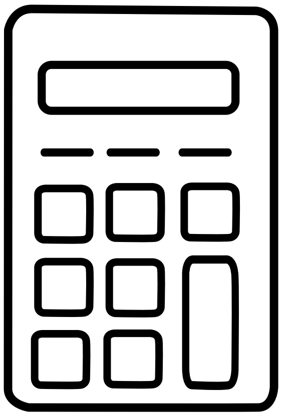
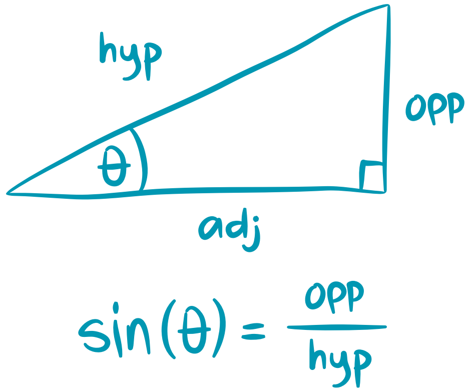
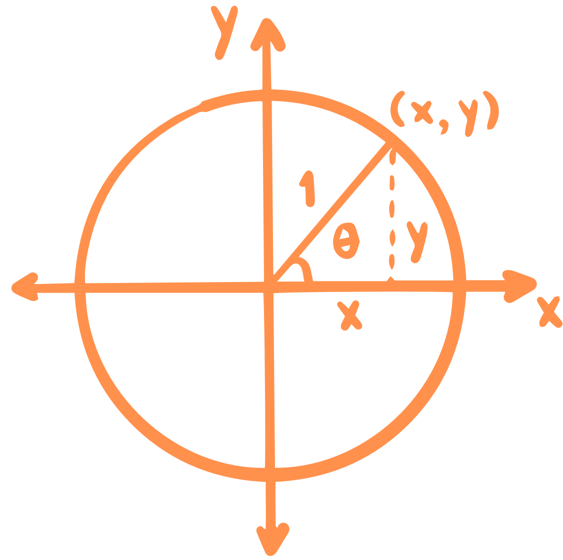
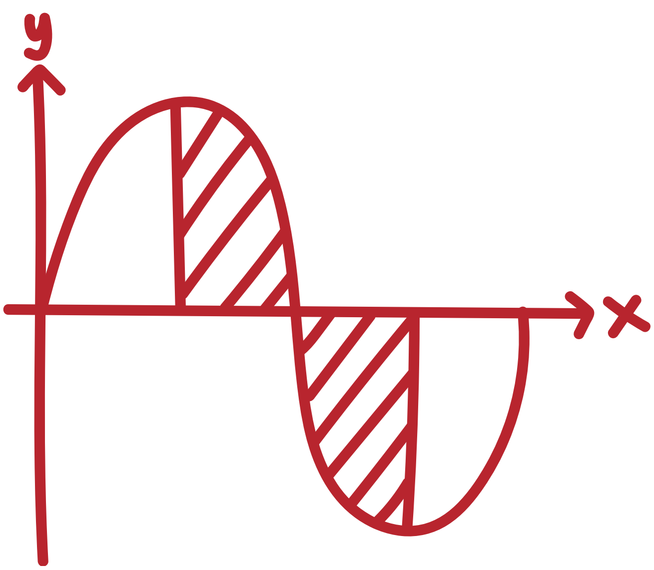
CENTRAL TEXAS COLLEGE

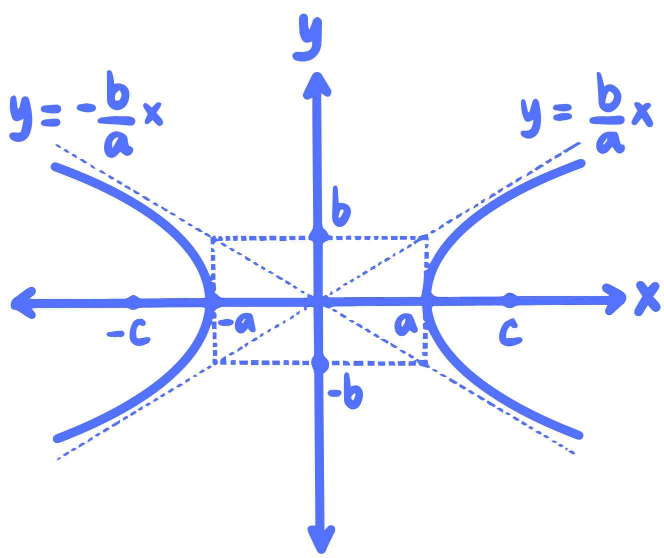






Mathematics Department Formula Book





9th Edition

Central Texas College

Mathematics Department

Formula Book

9th Edition

Prepared by:

Amanda Russell

Steven Burrow

Contemporary Math

Chapter 2

The Number of Subsets of a Set

A set of n elements has 2^n subsets.

Proper Subsets

A set of n elements has $2^n - 1$ proper subsets.

Chapter 3

Common English Expressions for p $\wedge$ q

Symbolic Statement	English Statement
p $\wedge$ q	p and q
p $\wedge$ q	p but q
p $\wedge$ q	p yet q
p $\wedge$ q	p nevertheless q

Common English Expressions for p $\rightarrow$ q

Symbolic Statement	English Statement
p $\rightarrow$ q	If p then q
p $\rightarrow$ q	q if p
p $\rightarrow$ q	p is sufficient for q
p $\rightarrow$ q	q is necessary for p
p $\rightarrow$ q	p only if q
p $\rightarrow$ q	Only if q, p.

Common English Expressions for $p \Leftrightarrow q$

Symbolic Statement	English Statement
$p \Leftrightarrow q$	p if and only if q
$p \Leftrightarrow q$	q if and only if p
$p \Leftrightarrow q$	If p then q, and if q then p.
$p \Leftrightarrow q$	p is necessary and sufficient for q
$p \Leftrightarrow q$	q is necessary and sufficient for p

Using the Dominance of Connectives

Statement	Dominant Connective is Bold	Statement with Grouping	Type of Statement
$p \to q \wedge \sim r$	$p \boldsymbol{\to} q \wedge \sim r$	$p \to (q \wedge \sim r)$	Conditional
$p \wedge q \to \sim r$	$p \wedge q \boldsymbol{\to} \sim r$	$(p \wedge q) \to \sim r$	Conditional
$p \Leftrightarrow q \to r$	$p \boldsymbol{\Leftrightarrow} q \to r$	$p \Leftrightarrow (q \to r)$	Biconditional
$p \to q \Leftrightarrow r$	$p \to q \boldsymbol{\Leftrightarrow} r$	$(p \to q) \Leftrightarrow r$	Biconditional
$p \wedge \sim q \to r \vee s$	$p \wedge \sim q \boldsymbol{\to} r \vee s$	$(p \wedge \sim q) \to (r \vee s)$	Conditional

Variations of the Conditional Statement

Name	Symbolic Form
Conditional	$p \to q$
Converse	$q \to p$
Inverse	$\sim p \to \sim q$
Contrapositive	$\sim q \to \sim p$

Negation of a Conditional Statement

$$\sim (p \to q) = p \wedge \sim q$$

De Morgan's Laws

1. $\sim (p \wedge q) = \sim p \vee \sim q$

2. $\sim (p \vee q) = \sim p \wedge \sim q$

Chapter 8

Finance

The percent formula, A = PB, mean A is P percent of B.

Sales Tax Amount = Tax Rate × Item's Cost

Discount Amount = Discount Rate × Original Price

The fraction for percent increase (or decrease) is: $\dfrac{\text{amount of increase or decrease}}{\text{original amount}}$

Simple Interest

$I = Prt$, Simple interest I when P is the principle and r is the annual rate for time t.

$A = P(1 + rt)$, The future value for simple interest.

Compound Interest

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

Continuous Compound Interest

$$A = Pe^{rt}$$

Present Value

$$P = \frac{A}{\left(1 + \frac{r}{n}\right)^{nt}}$$

Effective Annual Yield

$$Y = \left(1 + \frac{r}{n}\right)^{n} - 1$$

Sales Tax

$$\text{Sales Tax} = \text{Tax Rate} \times \text{Item Cost}$$

Calculating Federal Income Tax

1) Adjusted Gross Income = Gross Income − Adjustments

2) Taxable Income = Adjusted Gross Income − (Excemptions + Deductions)

3) Income Tax = Tax Computation − Tax Credits

Loan Payment Formula for Fixed Installment Loans

$$PMT = \frac{P\left(\frac{r}{n}\right)}{1 - \left(1 + \frac{r}{n}\right)^{-nt}}$$

Credit Card Average Daily Balance

$$\text{Credit Card Average Daily Balance} = \frac{\text{Sum of Unpaid Balances each day in Billing Period}}{\text{Number of days in billing period}}$$

Precent and Change

$$\text{Percent Increase} = \frac{\text{amount of increase}}{\text{original amount}}$$

$$\text{Percent Decrease} = \frac{\text{amount of decrease}}{\text{original amount}}$$

Chapter 10

Area Formulas

Trapezoid

$$A = \frac{1}{2}h(b_1 + b_2)$$

Rectangle

$A = bh$, The area of a rectangle (or, more generally, a parallelogram) of base b and height h.

Triangle

$$A = \frac{1}{2}bh, \quad \text{The area of a triangle of base } b \text{ and height } h.$$

Circle

$$A = \pi r^2, \quad \text{The area of a circle of radius } \mathbf{r}.$$

Volume Formulas

Prism

$$V = Bh, \quad \text{Volume of a prism } (B \text{ is the area of the base}).$$

Pyramid

$$V = \frac{1}{3}Bh, \quad \text{Volume of a pyramid } (B \text{ is the area of the base}).$$

Cube

$$V = a^3, \quad \text{The volume } V \text{ of a cube of edge } \mathbf{a}.$$

Rectangular Box

$$V = lwh, \quad \text{Volume } V \text{ of a rectangular box of length } \mathbf{l}, \text{ width } \mathbf{w}, \text{ and height } \mathbf{h}.$$

Circular Cylinder

$$V = \pi r^2 h, \quad \text{Volume } V \text{ of a circular cylinder of radius } \mathbf{r} \text{ and height } \mathbf{h}.$$

Circular Cone

$$V = \frac{1}{3}\pi r^2 h, \quad \text{Volume } V \text{ of a circular cone of radius } \mathbf{r}, \text{ height } \mathbf{h}, \text{ and slant height } \mathbf{s}.$$

Sphere

$$V = \frac{4}{3}\pi r^3, \quad \text{Volume } V \text{ of a sphere radius } \mathbf{r}.$$

Surface Area Formulas

Cube

$$S = 6a^2, \quad \text{The surface area } S \text{ of a cube of edge } a.$$

Rectangular Box

$S = 2(lw + lh + wh)$, Surface area S of a rectangular box of length **l**, width **w**, and height **h**.

Circular Cylinder

$S = 2\pi rh + 2\pi r^2$, Surface area S of a circular cylinder of radius **r** and height **h**.

Circular Cone

$S = \pi r^2 + \pi rs$, Surface area S of a circular cone of radius **r**, height **h**, and slant height **s**.

Sphere

$S = 4\pi r^2$, Surface area **S** of a sphere of radius **r**.

Sum of Polygon Angles

$S = (n - 2) \cdot 180°$, Sum of the measures of the angles of a polygon of **n** sides.

Circumference of a Circle

$C = \pi d = 2\pi r$, The perimeter (or circumference) of a circle of diameter **d** (or radius **r**).

Pythagorean Theorem

$$c^2 = a^2 + b^2,$$

The square of the hypotenuse **c** of a right triangle equal sum of squares of other two sides **a** and **b**.

Trigonometric Ratios

$$\sin A = \frac{\text{opposite side}}{\text{hypotenuse}}$$

$$\cos A = \frac{\text{adjacent side}}{\text{hypotenuse}}$$

$$\tan A = \frac{\text{opposite side}}{\text{adjacent side}}$$

Chapter 11

Permutations of n objects taken r at a time

$${}_nP_r = \frac{n!}{(n-r)!}, \text{ where } r \leq n$$

Distinguishable Permuations

With n_1 alike, n_2 alike, ..., n_k alike, the Distinguishable Permuations are:

$$\frac{n!}{n_1! \cdot n_2! \cdots n_k!}, \text{ where } n_1 + n_2 + \cdots + n_k = n$$

Combination of n objects taken r at a time

$${}_nC_r = \frac{n!}{(n-r)!r!}, \text{ where } r \leq n$$

Fundamental Counting Principle

If one event can occure in m ways, a second event can occur in n ways, and a third event can occur in p ways, and so on, then the sequence of events can occure in $m \times n \times p \times \cdots$ ways.

Probability

$P(\text{not } A) = 1 - P(A)$, probability of **complement** of event A.

$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, probability of event A **or** event B.

$P(A \text{ given } B) = \dfrac{P(A \text{ and } B)}{P(B)}$, probability of event A **given** event B.

$P(A \text{ and } B) = P(A) \cdot P(B)$, probability of event A **and** event B, where A and B are independent.

$P(A \text{ and } B) = P(A) \cdot P(B \text{ given that } A \text{ has occured})$, probability of event A **and** event B, where A and B are dependent.

Odds $f{:}u$ in favor of an event, where f is favorable ways and u is unfavorable ways that an event can occur.

$E = a_1p_1 + a_2p_2 + \cdots a_np_n$, expected value E, where a are the values that occur with probabilities p.

Theoretical Probability

$$P(E) \ = \ \frac{\text{number of outcomes in } E}{\text{total number of possible outcomes}} \ = \ \frac{n(E)}{n(S)}, \ \text{ event } E \text{ in sample space } S.$$

Empirical Probability

$$P(E) \ = \ \frac{\text{observered number of times } E \text{ occurs}}{\text{total number of observed occurances}}$$

$$\text{Probability of a permutation} = \frac{\text{number of ways the permuation can occur}}{\text{total number of possible permuations}}$$

Complement Formula: $P(E) \ + \ P(\text{not}E) \ = \ 1$

Chapter 12

Statistics

Mean of Data: $\bar{x} \ = \ \frac{\Sigma x}{n}$

Mean of Frequency Distribution: $\bar{x} \ = \ \frac{\Sigma(xf)}{n}$

$$\text{Midrange of Data} = \frac{\text{lowest data value } + \text{ highest data value}}{2}$$

Range = highest data value − lowest data value

$$\text{Standard Deviation} = \sqrt{\frac{\Sigma(\text{data item} - \text{mean})^2}{n \ - \ 1}} \ = \ \sqrt{\frac{\Sigma(x \ - \ \bar{x})^2}{n \ - \ 1}}$$

$$\text{Z-score} = \frac{\text{data item} - \text{mean}}{\text{standard deviation}}$$

$$\text{Position of the Median} = \frac{n \ + \ 1}{2}$$

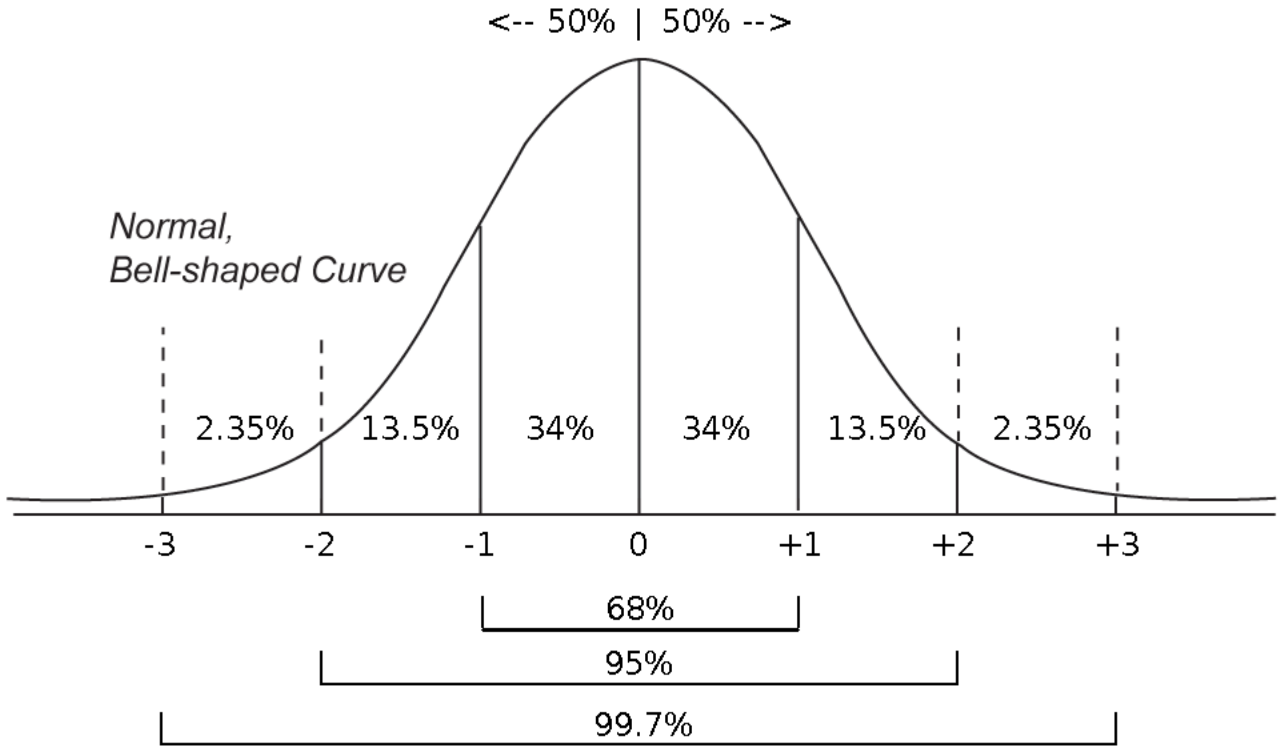
Mean $\bar{x}$: The sum of the set of data values divided by the number of values.

Median: The middle value when data values are ranked in order of magnitude. If there is an even number of values, it is the mean of the two middle values.

Mode: The value(s) that occur most often in the set of data.

Range: The difference between the greatest and least values in a set of data.

Empirical Rule



z-Score	Percentile	z-Score	Percentile	z-Score	Percentile	z-Score	Percentile
-4.0	0.003	-1.0	15.87	0.0	50.00	1.1	86.43
-3.5	0.02	-0.95	17.11	0.05	51.99	1.2	88.49
-3.0	0.13	-0.90	18.41	0.10	53.98	1.3	90.32
-2.9	0.19	-0.85	19.77	0.15	55.96	1.4	91.92
-2.8	0.26	-0.80	21.19	0.20	57.93	1.5	93.32
-2.7	0.35	-0.75	22.66	0.25	59.87	1.6	94.52
-2.6	0.47	-0.70	24.20	0.30	61.79	1.7	95.54
-2.5	0.62	-0.65	25.78	0.35	63.68	1.8	96.41
-2.4	0.82	-0.60	27.73	0.40	65.54	1.9	97.13
-2.3	1.07	-0.55	29.12	0.45	67.36	2.0	97.72
-2.2	1.39	-0.50	30.85	0.50	69.15	2.1	98.21
-2.1	1.79	-0.45	32.64	0.55	70.88	2.2	98.61
-2.0	2.28	-0.40	34.46	0.60	72.57	2.3	98.93
-1.9	2.87	-0.35	36.32	0.65	74.22	2.4	99.18
-1.8	3.59	-0.30	38.21	0.70	75.80	2.5	99.38
-1.7	4.46	-0.25	40.13	0.75	77.34	2.6	99.53
-1.6	5.48	-0.20	42.07	0.80	78.81	2.7	99.65
-1.5	6.68	-0.15	44.04	0.85	80.23	2.8	99.74
-1.4	8.08	-0.10	46.02	0.90	81.59	2.9	99.81
-1.3	9.68	-0.05	48.01	0.95	82.89	3.0	99.87
-1.2	11.51	0.0	50.0	1.0	84.13	3.5	99.98
-1.1	13.57					4.0	99.997

Regression Line (Line of Best Fit)

The best-fit lin associated with the n points $(x_1,\ y_1),\ (x_2,\ y_2),\ .\ .\ .\ ,\ (x_n,\ y_n)$ has the form:

$$y \ = \ mx \ + \ b$$

where:

$$\text{Slope} = m \ = \ \frac{n\Sigma(xy) \ - \ (\Sigma x)(\Sigma y)}{n(\Sigma x^2) \ - \ (\Sigma x)^2}$$

$$\text{Intercept} = b \ = \ \frac{\Sigma y \ - \ m(\Sigma x)}{n}$$

given:

$\Sigma xy \ = \ \text{sum of products} \ = \ x_1y_1 \ + \ x_2y_2 \ + \ \cdots \ + \ x_ny_n$

$\Sigma x \ = \ \text{sum of } x \text{ values} \ = \ x_1 \ + \ x_2 \ + \ \cdots \ + \ x_n$

$\Sigma y \ = \ \text{sum of } y \text{ values} \ = \ y_1 \ + \ y_2 \ + \ \cdots \ + \ y_n$

$\Sigma x^2 \ = \ \text{sum of squares of } x \text{ values} \ = \ x_1^2 \ + \ x_2^2 \ + \ \cdots \ + \ x_n^2$

Coefficient of Correlation

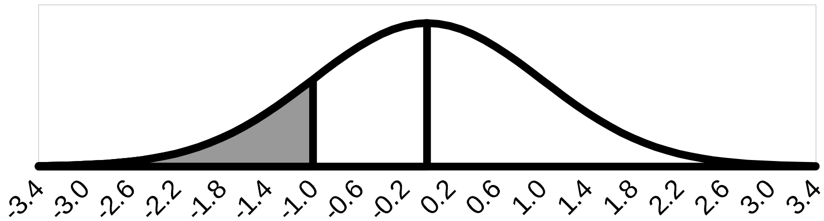
The **coefficient of correlation** for n points $(x_1,\ y_1),\ (x_2,\ y_2),\ .\ .\ .\ ,\ (x_n,\ y_n)$ is:

$$r \ = \ \frac{n\Sigma(xy) \ - \ (\Sigma x)(\Sigma y)}{\sqrt{n(\Sigma x^2) \ - \ (\Sigma x)^2}\sqrt{n(\Sigma y^2) \ - \ (\Sigma y)^2}}$$

The value of r is between -1 (a perfect negative correlation) and $+1$ (a perfect positive correlation). When r is close to or near 0, there is *no linear* correlation.

Statistics

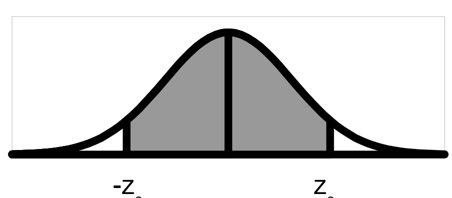
Standard Normal Distribution – Left Side



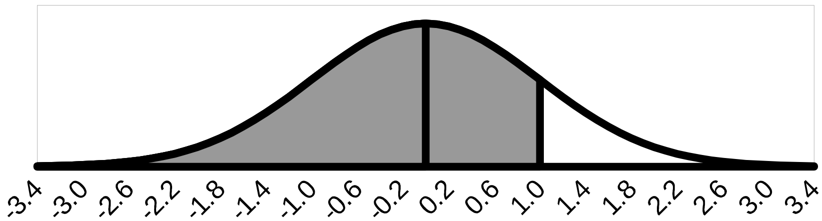
z	**.09**	**.08**	**.07**	**.06**	**.05**	**.04**	**.03**	**.02**	**.01**	**.00**
-3.4	.0002	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003
-3.3	.0003	.0004	.0004	.0004	.0004	.0004	.0004	.0005	.0005	.0005
-3.2	.0005	.0005	.0005	.0006	.0006	.0006	.0006	.0006	.0007	.0007
-3.1	.0007	.0007	.0008	.0008	.0008	.0008	.0009	.0009	.0009	.0010
-3.0	.0010	.0010	.0011	.0011	.0011	.0012	.0012	.0013	.0013	.0013
-2.9	.0014	.0014	.0015	.0015	.0016	.0016	.0017	.0018	.0018	.0019
-2.8	.0019	.0020	.0021	.0021	.0022	.0023	.0023	.0024	.0025	.0026
-2.7	.0026	.0027	.0028	.0029	.0030	.0031	.0032	.0033	.0034	.0035
-2.6	.0036	.0037	.0038	.0039	.0040	.0041	.0043	.0044	.0045	.0047
-2.5	.0048	.0049	.0051	.0052	.0054	.0055	.0057	.0059	.0060	.0062
-2.4	.0064	.0066	.0068	.0069	.0071	.0073	.0075	.0078	.0080	.0082
-2.3	.0084	.0087	.0089	.0091	.0094	.0096	.0099	.0102	.0104	.0107
-2.2	.0110	.0113	.0116	.0119	.0122	.0125	.0129	.0132	.0136	.0139
-2.1	.0143	.0146	.0150	.0154	.0158	.0162	.0166	.0170	.0174	.0179
-2.0	.0183	.0188	.0192	.0197	.0202	.0207	.0212	.0217	.0222	.0228
-1.9	.0233	.0239	.0244	.0250	.0256	.0262	.0268	.0274	.0281	.0287
-1.8	.0294	.0301	.0307	.0314	.0322	.0329	.0336	.0344	.0351	.0359
-1.7	.0367	.0375	.0384	.0392	.0401	.0409	.0418	.0427	.0436	.0446
-1.6	.0455	.0465	.0475	.0485	.0495	.0505	.0516	.0526	.0537	.0548
-1.5	.0559	.0571	.0582	.0594	.0606	.0618	.0630	.0643	.0655	.0668
-1.4	.0681	.0694	.0708	.0721	.0735	.0749	.0764	.0778	.0793	.0808
-1.3	.0823	.0838	.0853	.0869	.0885	.0901	.0918	.0934	.0951	.0968
-1.2	.0985	.1003	.1020	.1038	.1056	.1075	.1093	.1112	.1131	.1151
-1.1	.1170	.1190	.1210	.1230	.1251	.1271	.1292	.1314	.1335	.1357
-1.0	.1379	.1401	.1423	.1446	.1469	.1492	.1515	.1539	.1562	.1587
-0.9	.1611	.1635	.1660	.1685	.1711	.1736	.1762	.1788	.1814	.1841
-0.8	.1867	.1894	.1922	.1949	.1977	.2005	.2033	.2061	.2090	.2119
-0.7	.2148	.2177	.2206	.2236	.2266	.2296	.2327	.2358	.2389	.2420
-0.6	.2451	.2483	.2514	.2546	.2578	.2611	.2643	.2676	.2709	.2743
-0.5	.2776	.2810	.2843	.2877	.2912	.2946	.2981	.3015	.3050	.3085
-0.4	.3121	.3156	.3192	.3228	.3264	.3300	.3336	.3372	.3409	.3446
-0.3	.3483	.3520	.3557	.3594	.3632	.3669	.3707	.3745	.3783	.3821
-0.2	.3859	.3897	.3936	.3974	.4013	.4052	.4090	.4129	.4168	.4207
-0.1	.4247	.4286	.4325	.4364	.4404	.4443	.4483	.4522	.4562	.4602
-0.0	.4641	.4681	.4721	.4761	.4801	.4840	.4880	.4920	.4960	.5000

Critical Values

Level of Confidence c	z_c
0.80	1.28
0.90	1.645
0.95	1.96
0.99	2.575



Standard Normal Distribution – Right Side

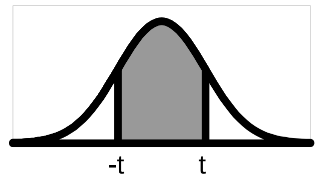


Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

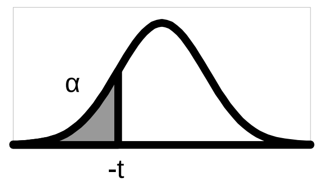
t-Distribution

d.f.						
	Level of Confidence, *c*	0.80	0.90	0.95	0.98	0.99
	One Tail, α	0.10	0.05	0.025	0.01	0.005
d.f.	**Two Tails, α**	0.20	0.10	0.05	0.02	0.01
1		3.078	6.314	12.706	31.821	63.657
2		1.886	2.920	4.303	6.965	9.925
3		1.638	2.353	3.182	4.541	5.841
4		1.533	2.132	2.776	3.747	4.604
5		1.476	2.015	2.571	3.365	4.032
6		1.440	1.943	2.447	3.143	3.707
7		1.415	1.895	2.365	2.998	3.499
8		1.397	1.860	2.306	2.896	3.355
9		1.383	1.833	2.262	2.821	3.250
10		1.372	1.812	2.228	2.764	3.169
11		1.363	1.796	2.201	2.718	3.106
12		1.356	1.782	2.179	2.681	3.055
13		1.350	1.771	2.160	2.650	3.012
14		1.345	1.761	2.145	2.624	2.977
15		1.341	1.753	2.131	2.602	2.947
16		1.337	1.746	2.120	2.583	2.921
17		1.333	1.740	2.110	2.567	2.898
18		1.330	1.734	2.101	2.552	2.878
19		1.328	1.729	2.093	2.539	2.861
20		1.325	1.725	2.086	2.528	2.845
21		1.323	1.721	2.080	2.518	2.831
22		1.321	1.717	2.074	2.508	2.819
23		1.319	1.714	2.069	2.500	2.807
24		1.318	1.711	2.064	2.492	2.797
25		1.316	1.708	2.060	2.485	2.787
26		1.315	1.706	2.056	2.479	2.779
27		1.314	1.703	2.052	2.473	2.771
28		1.313	1.701	2.048	2.467	2.763
29		1.311	1.699	2.045	2.462	2.756
30		1.310	1.697	2.042	2.457	2.750
31		1.309	1.696	2.040	2.453	2.744
32		1.309	1.694	2.037	2.449	2.738
33		1.308	1.692	2.035	2.445	2.733
34		1.307	1.691	2.032	2.441	2.728
35		1.306	1.690	2.030	2.438	2.724
36		1.306	1.688	2.028	2.434	2.719
37		1.305	1.687	2.026	2.431	2.715
38		1.304	1.686	2.024	2.429	2.712
39		1.304	1.685	2.023	2.426	2.708
40		1.303	1.684	2.021	2.423	2.704
45		1.301	1.679	2.014	2.412	2.690
50		1.299	1.676	2.009	2.403	2.678
60		1.296	1.671	2.000	2.390	2.660
70		1.294	1.667	1.994	2.381	2.648
80		1.292	1.664	1.990	2.374	2.639
90		1.291	1.662	1.987	2.368	2.632
100		1.290	1.660	1.984	2.364	2.626
500		1.283	1.648	1.965	2.334	2.586
1000		1.282	1.646	1.962	2.330	2.581
∞		1.282	1.645	1.960	2.326	2.576

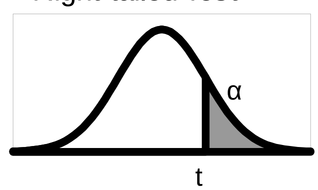
c-Confidence Interval



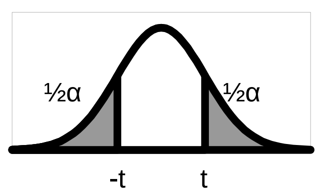
Left-tailed Test



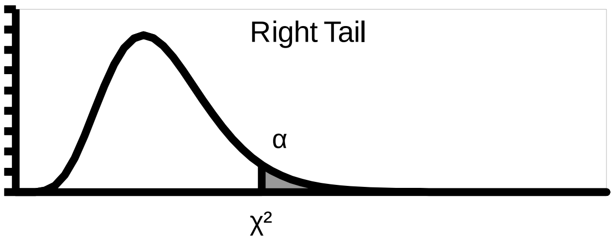
Right-tailed Test

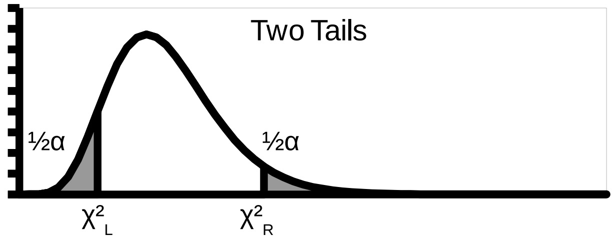


Two-tailed Test



Chi-Square Distribution





	α									
d.f.	0.995	0.99	0.975	0.95	0.90	0.10	0.05	0.025	0.01	0.005
1	—	—	0.001	0.004	0.016	2.706	3.841	5.024	6.635	7.879
2	0.010	0.020	0.051	0.103	0.211	4.605	5.991	7.378	9.210	10.597
3	0.072	0.115	0.216	0.352	0.584	6.251	7.815	9.348	11.345	12.838
4	0.207	0.297	0.484	0.711	1.064	7.779	9.488	11.143	13.277	14.860
5	0.412	0.554	0.831	1.145	1.610	9.236	11.070	12.833	15.086	16.750
6	0.676	0.872	1.237	1.635	2.204	10.645	12.592	14.449	16.812	18.548
7	0.989	1.239	1.690	2.167	2.833	12.017	14.067	16.013	18.475	20.278
8	1.344	1.646	2.180	2.733	3.490	13.362	15.507	17.535	20.090	21.955
9	1.735	2.088	2.700	3.325	4.168	14.684	16.919	19.023	21.666	23.589
10	2.156	2.558	3.247	3.940	4.865	15.987	18.307	20.483	23.209	25.188
11	2.603	3.053	3.816	4.575	5.578	17.275	19.675	21.920	24.725	26.757
12	3.074	3.571	4.404	5.226	6.304	18.549	21.026	23.337	26.217	28.300
13	3.565	4.107	5.009	5.892	7.042	19.812	22.362	24.736	27.688	29.819
14	4.075	4.660	5.629	6.571	7.790	21.064	23.685	26.119	29.141	31.319
15	4.601	5.229	6.262	7.261	8.547	22.307	24.996	27.488	30.578	32.801
16	5.142	5.812	6.908	7.962	9.312	23.542	26.296	28.845	32.000	34.267
17	5.697	6.408	7.564	8.672	10.085	24.769	27.587	30.191	33.409	35.718
18	6.265	7.015	8.231	9.390	10.865	25.989	28.869	31.526	34.805	37.156
19	6.844	7.633	8.907	10.117	11.651	27.204	30.144	32.852	36.191	38.582
20	7.434	8.260	9.591	10.851	12.443	28.412	31.410	34.170	37.566	39.997
21	8.034	8.897	10.283	11.591	13.240	29.615	32.671	35.479	38.932	41.401
22	8.643	9.542	10.982	12.338	14.041	30.813	33.924	36.781	40.289	42.796
23	9.260	10.196	11.689	13.091	14.848	32.007	35.172	38.076	41.638	44.181
24	9.886	10.856	12.401	13.848	15.659	33.196	36.415	39.364	42.980	45.559
25	10.520	11.524	13.120	14.611	16.473	34.382	37.652	40.646	44.314	46.928
26	11.160	12.198	13.844	15.379	17.292	35.563	38.885	41.923	45.642	48.290
27	11.808	12.879	14.573	16.151	18.114	36.741	40.113	43.195	46.963	49.645
28	12.461	13.565	15.308	16.928	18.939	37.916	41.337	44.461	48.278	50.993
29	13.121	14.256	16.047	17.708	19.768	39.087	42.557	45.722	49.588	52.336
30	13.787	14.953	16.791	18.493	20.599	40.256	43.773	46.979	50.892	53.672
40	20.707	22.164	24.433	26.509	29.051	51.805	55.758	59.342	63.691	66.766
50	27.991	29.707	32.357	34.764	37.689	63.167	67.505	71.420	76.154	79.490
60	35.534	37.485	40.482	43.188	46.459	74.397	79.082	83.298	88.379	91.952
70	43.275	45.442	48.758	51.739	55.329	85.527	90.531	95.023	100.425	104.215
80	51.172	53.540	57.153	60.391	64.278	96.578	101.879	106.629	112.329	116.321
90	59.196	61.754	65.647	69.126	73.291	107.565	113.145	118.136	124.116	128.299
100	67.328	70.065	74.222	77.929	82.358	118.498	124.342	129.561	135.807	140.169

Chapter 2

Class Width $= \dfrac{\text{Range of data}}{\text{Number of classes}}$ (round up to next convenient number)

Midpoint $= \dfrac{(\text{Lower class limit}) + (\text{Upper class limit})}{2}$

Relative Frequency $= \dfrac{\text{Class frequency}}{\text{Sample size}} = \dfrac{f}{n}$

Population Mean: $\mu \; = \; \dfrac{\Sigma x}{N}$

Sample Mean: $\bar{x} \; = \; \dfrac{\Sigma x}{N}$

Weighted Mean: $\bar{x} \; = \; \dfrac{\Sigma(x \cdot w)}{\Sigma w}$

Mean of Frequency Distribution: $\bar{x} \; = \; \dfrac{\Sigma(x \cdot f)}{n}$

Range = (Maximum entry) − (Minimum entry)

Population Variance: $\sigma^2 \; = \; \dfrac{\Sigma(x-\mu)^2}{N}$

Population Standard Deviation: $\sigma \; = \; \sqrt{\sigma^2} \; = \; \sqrt{\dfrac{\Sigma(x-\mu)^2}{N}}$

Sample Variance: $s^2 \; = \; \dfrac{\Sigma(x-\bar{x})^2}{n-1}$

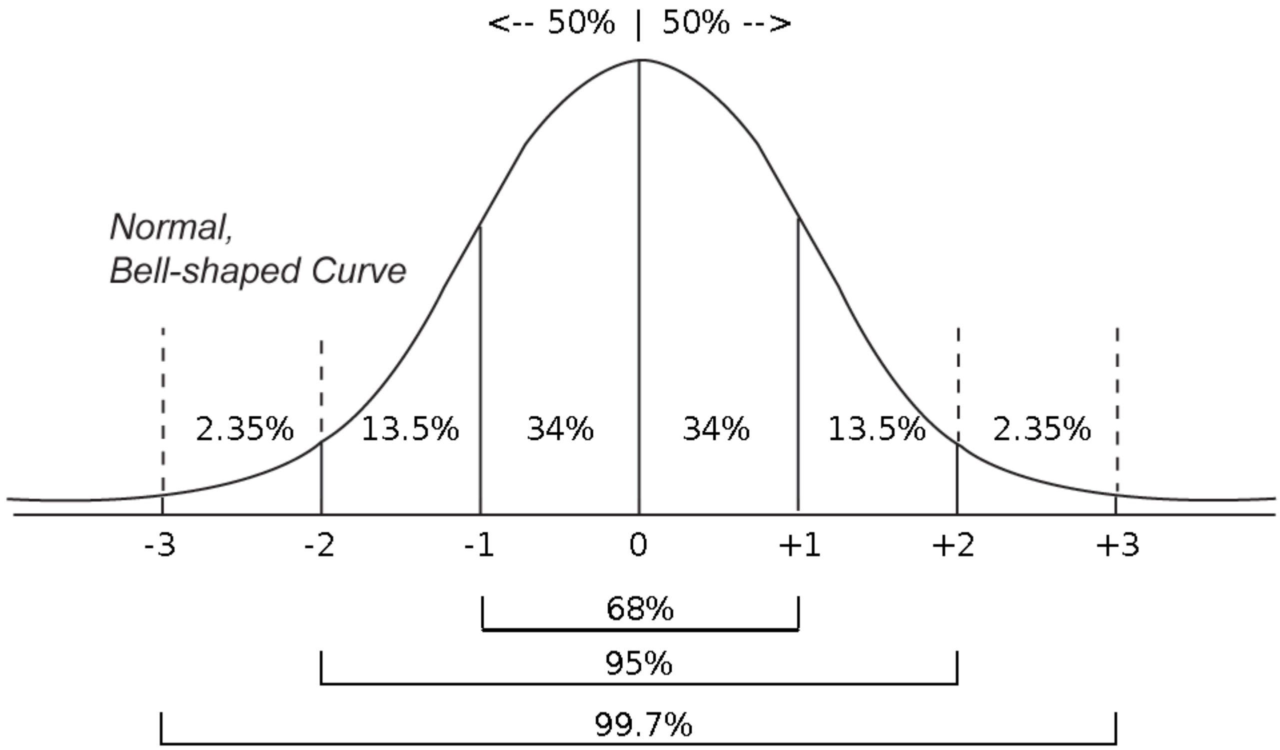
Sample Standard Deviation: $s \; = \; \sqrt{s^2} \; = \; \sqrt{\dfrac{\Sigma(x-\bar{x})^2}{n-1}}$

Empirical Rule (or 68-95-99.7 Rule) For data with a (symmetric) bell-shaped distribution:

1. About 68% of data lies within one standard deviation of the mean.

2. About 95% of the data lies within two standard deviations of the mean.

3. About 99.7% of the data lies within three standard deviations of the mean.



Chebychev's Theorem

The portion of any data set lying within k standard deviations ($k > 1$) of the mean is at least: $1 - \dfrac{1}{k^2}$

Sample Standard Deviation of a Frequency Distribution: $s = \sqrt{\dfrac{\Sigma(x-\bar{x})^2 f}{n-1}}$

Standard Score: $z = \dfrac{\text{Value} - \text{Mean}}{\text{Standard Deviation}} = \dfrac{x-\mu}{\sigma}$

Chapter 3

Classical (or Theoretical Probability): $P(E) = \dfrac{\text{Number of outcomes in event } E}{\substack{\text{Total number of outcomes}\\ \text{in sample space}}}$

Empirical (or Statistical) Probability: $P(E) = \dfrac{\text{Frequency of event } E}{\text{Total frequency}} = \dfrac{f}{n}$

Probability of a Complement: $P(E') = 1 - P(E)$

Probabilty of occurence of both events A and B:

$$P(A \text{ and } B) = P(A) \cdot P(B|A)$$

$$P(A \text{ and } B) = P(A) \cdot P(B) \text{ if } A \text{ and } B \text{ are independent}$$

Probability of occurence of either A or B or both:

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$P(A \text{ or } B) = P(A) + P(B) \text{ if } A \text{ and } B \text{ are mutually exclusive}$$

Permutations of n objects taken r at a time:

$$_nP_r = \frac{n!}{(n-r)!}, \text{ where } r \leq n$$

Distiguishable Permuations: n_1 alike, n_2 alike, ..., n_k alike:

$$\frac{n!}{n_1! \cdot n_2! \cdots n_k!}, \text{ where } n_1 + n_2 + \cdots + n_k = n$$

Combination of n objects taken r at a time:

$$_nC_r = \frac{n!}{(n-r)!r!}, \text{ where } r \leq n$$

Chapter 4

Mean of a Discrete Random Variable: $\mu = \Sigma x P(x)$

Variance of a Discrete Random Variable: $\sigma^2 = \Sigma(x-\mu)^2 P(x)$

Standard Deviation of a Discrete Random Variable: $\sigma = \sqrt{\sigma^2} = \sqrt{\Sigma(x-\mu)^2 P(x)}$

Expected Value: $E(x) = \mu = \Sigma x P(x)$

Binomial Probability of x successes in n trials:

$$P(x) = {_nC_x} p^x q^{n-x} = \frac{n!}{(n-x)!x!} p^x q^{n-x}$$

Population Parameters of a Binomial Distribution:

Mean: $\mu = np$ Variance: $\sigma^2 = npq$

Standard Deviation: $\sigma = \sqrt{npq}$

Geometric Distribution: The probability that the first success will occur on trial number x is

$$P(x) = p(q)^{x-1}, \text{ where } q = 1 - p.$$

Poisson Distribution: The probability of exactly x occurences in an interval is

$$P(x) = \frac{\mu^x e^{-\mu}}{x!}, \text{ where } e \approx 2.71828$$

and μ is the mean number of occurances per interval unit.

Chapter 5

Standard Score, or z-Score: $z = \dfrac{\text{Value} - \text{Mean}}{\text{Standard Deviation}} = \dfrac{x - \mu}{\sigma}$

Transforming a z-Score to an x-Value: $x = \mu + z\sigma$

Central Limit Theorem ($n \geq 30$ or population is normally distibuted):

Mean of the Sampling Distribution: $\mu_{\bar{x}} = \mu$

Variance of the Sampling Distribution: $\sigma_{\bar{x}}^2 = \dfrac{\sigma^2}{n}$

Standard Deviation of the Sampling Distribution (Standard Error): $\sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}}$

z-Score $= \dfrac{\text{Value} - \text{Mean}}{\text{Standard Error}} = \dfrac{\bar{x} - \mu_{\bar{x}}}{\sigma_{\bar{x}}} = \dfrac{\bar{x} - \mu}{\sigma/\sqrt{n}}$

Chapter 6

c-Confidence Interval for μ: $\bar{x} - E < \mu < \bar{x} + E$, where:

$E = z_c \dfrac{\sigma}{\sqrt{n}}$ if σ is known, and either the population is normally distributed or $n \geq 30$, or:

$E = t_c \dfrac{s}{\sqrt{n}}$ if σ is unknown, and either the population is normally distributed or $n \geq 30$.

Minimum Sample Size to Estimate μ:

$$n = \left(\frac{z_c \sigma}{E}\right)^2$$

Point Estimate for p**, the population proportion of successes:** $\hat{p} = \dfrac{x}{n}$

c**-Confidence Interval Population Proportion** p (when $np \geq 5$ and $nq \geq 5$):

$\hat{p} - E < p < \hat{p} + E$, where $E = z_c \sqrt{\dfrac{\hat{p}\hat{q}}{n}}$

Minimum Sample Size to Estimate p:

$$n = \hat{p}\hat{q}\left(\frac{z_c}{E}\right)^2$$

c**-Confidence Interval for Population Var** σ^2:

$$\frac{(n-1)s^2}{\chi_R^2} < \sigma^2 < \frac{(n-1)s^2}{\chi_L^2}$$

c**-Confidence Interval for Population Std Dev** σ:

$$\sqrt{\frac{(n-1)s^2}{\chi_R^2}} < \sigma < \sqrt{\frac{(n-1)s^2}{\chi_L^2}}$$

Chapter 7

Decision Rule:

1. $P \leq \alpha$, Reject H_0.

2. $P > \alpha$, Fail to Reject H_0.

z**-Test for a Mean** μ:

$z = \dfrac{\bar{x} - \mu}{\sigma/\sqrt{n}}$, if σ known, and either the population is normally distributed or $n \geq 30$.

***t*-Test for a Mean** μ:

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}}, \text{ for } \sigma \text{ unknown, and either the population is normally distributed or } n \geq 30.$$

(d.f. $= n - 1$)

***z*-Test for a Proportion** p (when $np \geq 5$ and $nq \geq 5$):

$$z = \frac{\hat{p} - \mu_{\hat{p}}}{\sigma_{\hat{p}}} = \frac{\hat{p} - p}{\sqrt{pq/n}}$$

Chi-Square Test for a Variance σ^2 **or Standard Deviation** σ:

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2} \text{ (d.f. } = n - 1)$$

Chapter 9

Correlation Coefficient:

$$r = \frac{n\Sigma xy - (\Sigma x)(\Sigma y)}{\sqrt{n\Sigma x^2 - (\Sigma x)^2}\sqrt{n\Sigma y^2 - (\Sigma y)^2}}$$

***t*-Test for Correlation Coefficient**:

$$t = \frac{r}{\sqrt{\frac{1 - r^2}{n - 2}}} \text{ (d.f. } = n - 2)$$

Equation of a Regression Line: $\hat{y} = mx + b$, where:

$$m = \frac{n\Sigma xy - (\Sigma x)(\Sigma y)}{n\Sigma x^2 - (\Sigma x)^2} \text{ and}$$

$$b = \bar{y} - m\bar{x} = \frac{\Sigma y}{n} - m\frac{\Sigma x}{n}$$

Coefficient of Determination:

$$r^2 = \frac{\text{Explained variation}}{\text{Total variation}} = \frac{\Sigma(\hat{y}_i - \bar{y})^2}{\Sigma(y_i - \bar{y})^2}$$

Standard Error of Estimate:

$$s_e = \sqrt{\frac{\Sigma(y_i - \hat{y}_i)^2}{n - 2}}$$

c-Prediction Interval for y:

$\hat{y} - E < y < \hat{y} + E$, where $E = t_c s_e \sqrt{1 + \dfrac{1}{n} + \dfrac{n(x_o - \bar{x})^2}{n\Sigma x^2 - (\Sigma x)^2}}$

(d.f. $= n - 2$)

Algebra

Chapter 1

Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Exponent Rules

1. **Product Rule:** $a^m \cdot \ a^n = a^{(m \cdot n)}$

2. **Quotient Rule:** $\dfrac{a^m}{a^n} = a^{m-n}$

3. **Power Rule:** $(a^m)^n = a^{(m \cdot n)}$

4.**Negative Exponent:** $a^{-2} = \dfrac{1}{a^2}$

Chapter 2

Composition of Two Functions

The composition of the function f with the function g is:

$$(f \circ g)(x) = f(g(x))$$

The domain of $f \circ g$ is the set of all x in the domain of g such that $g(x)$ is in the domain of f.

Lines and Slopes

The formula for Slope is:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

The Point-Slope form of a Line is:

$$y - y_1 = m \cdot (x - x_1)$$

The Slope-Intercept form of a Line is:

$$y = mx + b$$

Translation

Vertical Shift: $g(x) = f(x) \pm c$

Horizontal Shift: $g(x) = f(x \pm c)$

Reflection on the x-axis: $g(x) = -f(x)$

Reflection on the y-axis: $g(x) = f(-x)$

Vertical Stretch/Shrink: $g(x) = b \cdot f(x)$ if $b > 1$ it stretches if $0 < b < 1$ it shrinks

Horizontal Stretch/Shrink: $g(x) = f(b \cdot x)$ if $0 < b < 1$ it stretches if $b > 1$ it shrinks

Symmetry

Even Symmetry: $f(-x) = f(x)$ symmetric about the y-axis

Odd Symmetry: $f(-x) = -f(x)$ symmetric about the origin

Inverse Function

$$y = f^{-1}(x)$$

switch x and y, then solve for y, graphically it is a reflection about the $y = x$ line

Chapter 3

Repeated Zeros

A factor $(x - a)^k$, $k > 1$, yields a repeated zero $x = a$ of multiplicity k.

1. When k is odd, the graph *crosses* the x-axis at $x = a$.

2. When k is even, the graph *touches* the x-axis (but does not cross the x-axis) at $x = a$.

Vertical and Horizontal Asymptotes of a Rational Function

Let f be the rational function:

$$f(x) = \frac{N(x)}{D(x)} = \frac{a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0}{b_m x^m + b_{m-1}x^{m-1} + \cdots + b_1 x + b_0},$$ where $N(x)$ and $D(x)$ have no common factors.

1. The graph of f has vertical asymptotes at the zeros of $D(x)$.

2. The graph of f has one or no horizontal asymptote determined by comparing the degrees of $N(x)$ and $D(x)$.

   a. When $n < m$, the graph of f has the line $y = 0$ (the x-axis) as a horizontal asymptote.

   b. When $n = m$, the graph of f has the line $y = a_n/b_n$ (ratio of the leading coefficients) as a horizontal asymptote.

   c. When $n > m$ the graph of f has no horizontal asymptote.

Chapter 4

Formulas for Compound Interest

After t years, the balance A in an account with principle P and annual interest rate r (in decimal form) is given by the following formulas:

1. For n compoundings per year: $A = P\left(1 + \frac{r}{n}\right)^{nt}$

2. For continuous compounding: $A = Pe^{rt}$

Properties of Logarithms

Let b be a positive number such that $b \neq 1$, and let p be a real number.

1. **Product Rule:** $\log_b(M \cdot N) = \log_b(M) + \log_b(N)$

2. **Quotient Rule:** $\log_b\left(\frac{M}{N}\right) = \log_b(M) - \log_b(N)$

3. **Power Rule:** $\log_b(M^p) = p \cdot \log_b(M)$

4. **Change of Base Formula:** $\log_b(N) = \frac{\log_a(N)}{\log_a(b)}$

Exponential Growth and Decay Formula

$$A = A_0 e^{kt}$$

Half-Life Formula

$$ln(\frac{1}{2}) = kt$$

Chapter 11

The nth Term of an Arithmetic Sequence

The nth term of an arithmetic sequence has the form: $a_n \ = \ a_1 \ + \ (n-1)d$
where d is the common difference and a_1 is the first term.

The Sum of a Finite Arithmetic Sequence

The sum of a finite arithmetic sequence with n terms is: $S_n \ = \ \frac{n}{2}(a_1 + a_n)$

The nth Term of an Geometric Sequence

The nth term of an geometric sequence has the form: $a_n \ = \ a_1 r^{n-1}$

where r is the common ratio and a_1 is the first term.

The Sum of a Finite Geometric Sequence

The sum of a finite geometric sequence with n terms is: $S_n \ = \ \dfrac{a_1(1 \ - \ r^n)}{1 \ - \ r}$

The Sum of an Infinite Geometric Sequence

The sum of an infinite geometric sequence is: $S \ = \ \dfrac{a_1}{1 \ - \ r}$

Annuity Formula

$$A \ = \ \frac{P\left[\left(1 \ + \ \frac{r}{n}\right)^{nt} \ - \ 1\right]}{\left(\frac{r}{n}\right)}$$

Classical (or Theoretical Probability)

$$P(E) \ = \ \frac{\text{Number of outcomes in event } E}{\text{Total number of outcomes in sample space}}$$

Empirical (or Statistical) Probability

$$P(E) \;=\; \frac{\text{Frequency of event } E}{\text{Total frequency}} \;=\; \frac{f}{n}$$

Probability of a Complement

$$P(E') \;=\; 1 \;-\; P(E)$$

Probabilty of occurence of both events A and B

$$P(A \text{ and } B) = P(A) \cdot P(\text{B } |A)$$

$$P(A \text{ and } B) = P(A) \cdot P(B), \;\text{ if } A \text{ and } B \text{ are independent}$$

Probability of occurence of either A or B or both

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$P(A \text{ or } B) = P(A) + P(B), \;\text{ if } A \text{ and } B \text{ are mutually exclusive}$$

Permutations of n objects taken r at a time:

$${}_nP_r = \frac{n!}{(n-r)!}, \text{ where } r \leq n$$

Distinguishable Permuations

With n_1 alike, n_2 alike, ..., n_k alike, the Distinguishable Permuations are:

$$\frac{n!}{n_1! \cdot n_2! \cdots n_k!}, \text{ where } n_1 + n_2 + \cdots + n_k = n$$

Combination of n objects taken r at a time

$${}_nC_r = \frac{n!}{(n-r)!r!}, \text{ where } r \leq n$$

Trigonometry

The Six Trigonometric Functions

$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{y}{r} = \frac{1}{\csc\theta}$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{x}{r} = \frac{1}{\sec\theta}$$

$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{y}{x} = \frac{1}{\cot\theta} = \frac{\sin\theta}{\cos\theta}$$

$$\csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{r}{y} = \frac{1}{\sin\theta}$$

$$\sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{r}{x} = \frac{1}{\cos\theta}$$

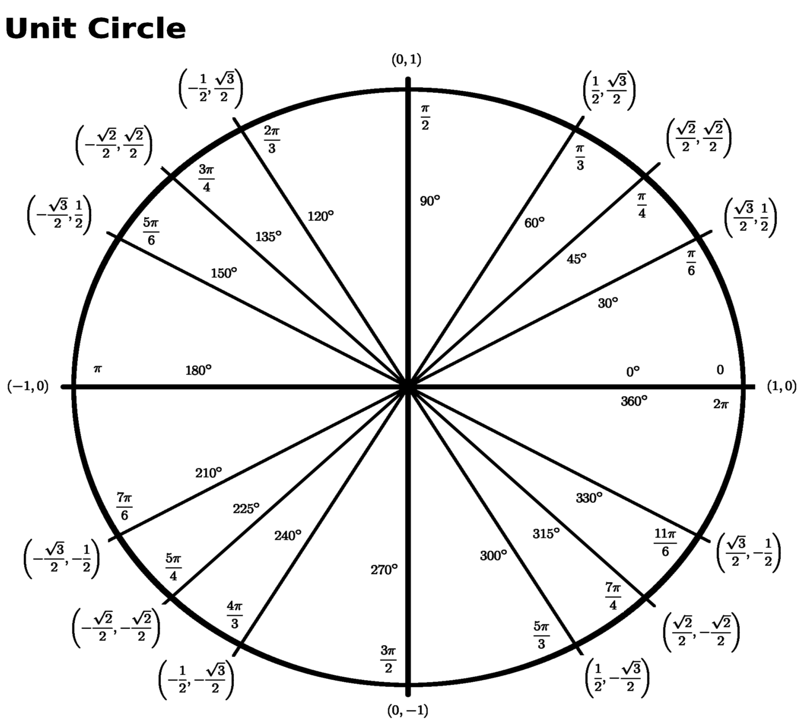
$$\cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{x}{y} = \frac{1}{\tan\theta} = \frac{\cos\theta}{\sin\theta}$$

Arc Length

$s = r\theta$

Area of a Sector

$A = \frac{1}{2}\pi r^2\theta$



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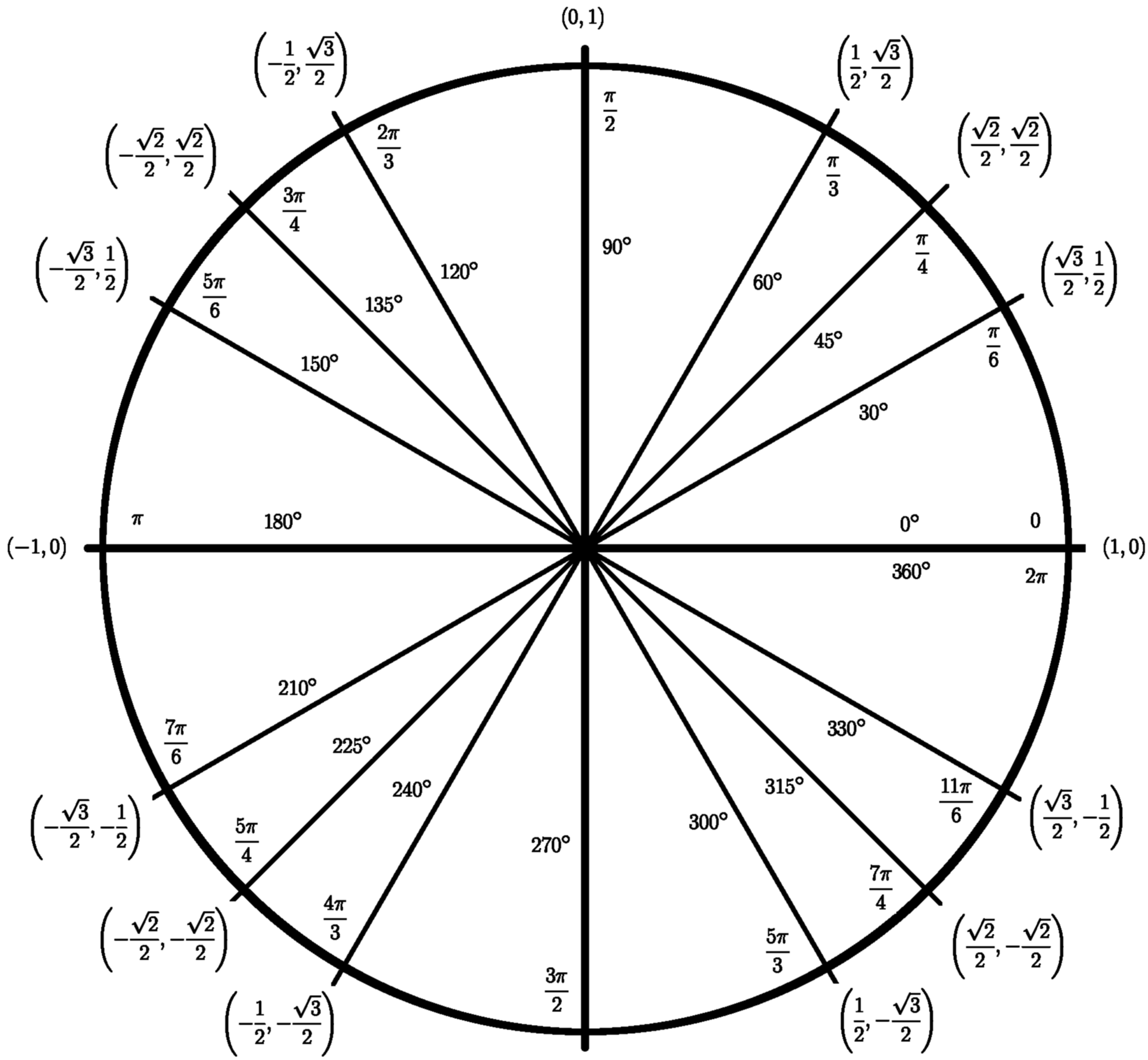
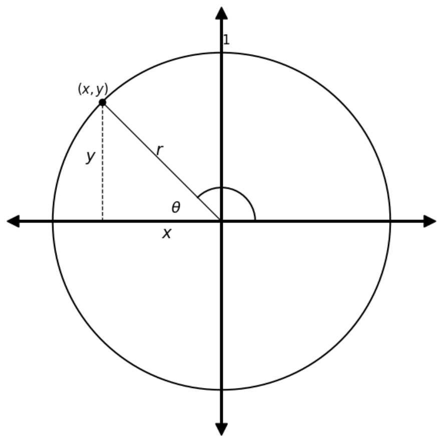
Pre-Calculus

Unit Circle



$(0,1)$
$\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$
$\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$
$\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$
$(1,0)$
$\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$
$\left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$
$\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$
$(0,-1)$
$\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$
$\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$
$\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$
$(-1,0)$
$\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$
$\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$
$\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$
$\frac{\pi}{2}$ 90°
$\frac{\pi}{3}$ 60°
$\frac{\pi}{4}$ 45°
$\frac{\pi}{6}$ 30°
0 0°
2π 360°
$\frac{11\pi}{6}$ 330°
$\frac{7\pi}{4}$ 315°
$\frac{5\pi}{3}$ 300°
$\frac{3\pi}{2}$ 270°
$\frac{4\pi}{3}$ 240°
$\frac{5\pi}{4}$ 225°
$\frac{7\pi}{6}$ 210°
π 180°
$\frac{5\pi}{6}$ 150°
$\frac{3\pi}{4}$ 135°
$\frac{2\pi}{3}$ 120°



The Six Trigonometric Functions

$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{y}{r} = \frac{1}{\csc\theta}$$

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$$\csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{r}{y} = \frac{1}{\sin\theta}$$

$$\sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{r}{x} = \frac{1}{\cos\theta}$$

$$\cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{x}{y} = \frac{1}{\tan\theta} = \frac{\cos\theta}{\sin\theta}$$

Pythagorean Identities

$$\sin^2\theta + \cos^2\theta = 1$$

$$1 + \tan^2\theta = \sec^2\theta$$

$$1 + \cot^2\theta = \csc^2\theta$$

Cofunction Identities

$$\sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta \qquad \csc\left(\frac{\pi}{2} - \theta\right) = \sec\theta$$

$$\cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta \qquad \sec\left(\frac{\pi}{2} - \theta\right) = \csc\theta$$

$$\tan\left(\frac{\pi}{2} - \theta\right) = \cot\theta \qquad \cot\left(\frac{\pi}{2} - \theta\right) = \tan\theta$$

Even/Odd Identities

$$\sin(-\theta) = -\sin\theta \qquad \csc(-\theta) = -\csc\theta$$

$$\cos(-\theta) = \cos\theta \qquad \sec(-\theta) = \sec\theta$$

$$\tan(-\theta) = -\tan\theta \qquad \cot(-\theta) = -\cot\theta$$

Sum and Difference Formulas

$$\sin(u \pm v) = \sin u\cos v \pm \cos u\sin v$$

$$\cos(u \pm v) = \cos u\cos v \mp \sin u\sin v$$

$$\tan(u \pm v) = \frac{\tan u \pm \tan v}{1 \mp \tan u\tan v}$$

Double-Angle Formulas

$$\sin 2\theta = 2\sin\theta\cos\theta$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$$

$$\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}$$

Half-Angle Formulas

$$\sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1 - \cos\theta}{2}}$$

$$\cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1 + \cos\theta}{2}}$$

$$\tan\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1 - \cos\theta}{1 + \cos\theta}}$$

Power-Reducing Formulas

$$\sin^2\theta = \frac{1 - \cos 2\theta}{2}$$

$$\cos^2\theta = \frac{1 + \cos 2\theta}{2}$$

$$\tan^2\theta = \frac{1 - \cos 2\theta}{1 + \cos 2\theta}$$

Sum-to-Product Formulas

$$\sin u + \sin v = 2\sin\left(\frac{u+v}{2}\right)\cos\left(\frac{u-v}{2}\right)$$

$$\sin u - \sin v = 2\cos\left(\frac{u+v}{2}\right)\sin\left(\frac{u-v}{2}\right)$$

$$\cos u + \cos v = 2\cos\left(\frac{u+v}{2}\right)\cos\left(\frac{u-v}{2}\right)$$

$$\cos u - \cos v = -2\sin\left(\frac{u+v}{2}\right)\sin\left(\frac{u-v}{2}\right)$$

Product-to-Sum Formulas

$$\sin u\sin v = \frac{1}{2}[\cos(u-v) - \cos(u+v)]$$

$$\cos u\cos v = \frac{1}{2}[\cos(u-v) + \cos(u+v)]$$

$$\sin u\cos v = \frac{1}{2}[\sin(u+v) + \sin(u-v)]$$

$$\cos u\sin v = \frac{1}{2}[\sin(u+v) - \sin(u-v)]$$

Chapter 3

Formulas for Compound Interest

After t years, the balance A in an account with principle P and annual interest rate r (in decimal form) is given by the following formulas.

1. For n compoundings per year: $A = P\left(1 + \frac{r}{n}\right)^{nt}$

2. For continuous compounding: $A = Pe^{rt}$

Properties of Logarithms

Let a be a positive number such that $a \neq 1$, and let n be a real number. If u and v are positive real numbers, then the following properties are true.

Logarithms with Base a

1. Product Property: $\log_a(uv) = \log_a(u) + \log_a(v)$

2. Quotient Property: $\log_a\left(\frac{u}{v}\right) = \log_a(u) - \log_a(v)$

3. Power Property: $\log_a(u^n) = n \cdot \log_a(u)$

Exponential Growth and Decay

Exponential Growth and Decay: $A = A_0e^{kt}$

Chapters 4 and 5

Arc Length

For a circle of radius r, a central angle θ intercepts an arc of length s given by:

$$s = r\theta \text{ Length of circular arc}$$

where θ is measured in radians. Note that if $r = 1$, then $s = \theta$, and the radian measure of θ equals the arc length.

Linear and Angular Speeds

Consider a particle moving at a constant speed along a circular arc of radius r. If s is the length of the arc traveled in time t, then the **linear speed** v of the particle is:

$$\text{Linear speed } v = \frac{\text{arc length}}{\text{time}} = \frac{s}{t}$$

Moreover, if θ is the angle (in radian measure) corresponding to the arc length s, then the **angular speed** ω (the lowecase Greek letter omega) of the particle is:

$$\text{Angular speed } \omega = \frac{\text{central angle}}{\text{time}} = \frac{\theta}{t}$$

The connecting formula for linear speed, $\mathbf{v}$, and angular speed, $\boldsymbol{\omega}$, is:

$$v = \omega r$$

with radius r.

Area of a Sector

For a circle of radius r, the area A of a sector of the circle with central angle $\boldsymbol{\theta}$ is:

$$A = \frac{1}{2}\pi r^2\theta$$

where $\boldsymbol{\theta}$ is measured in radians.

Chapter 6

Law of Sines

If ABC is a triangle with sides a, b, and c, then:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Law of Cosines

Standard Form: $a^2 = b^2 + c^2 - 2bc\cos A$

Alternative Form: $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$

Area of a Triangle

$$K = \frac{1}{2}ab \cdot \sin(C)$$

Heron's Area Formula

$$K = \sqrt{s(s-a)(s-b)(s-c)} \text{ where } s = \frac{a+b+c}{2}$$

Component Form of a Vector

The component form of the vector with initial point $P(p_1, p_2)$ and the terminal point $Q(q_1, q_2)$ is given by:

$$\overrightarrow{PQ} = \langle q_1 - p_1,\ q_2 - p_2 \rangle = \langle v_1,\ v_2 \rangle = \mathbf{v}$$

The **magnitude** (or length) of $\mathbf{v}$ is given by:

$$|v| = \sqrt{(q_1 - p_1)^2 + (q_2 - p_2)^2} = \sqrt{v_1^2 + v_2^2}$$

If $|\mathbf{v}| = 1$, then $\mathbf{v}$ is a **unit vector**. Moreover, $|\mathbf{v}| = 0$ if and only if $\mathbf{v}$ is the zero vector $\mathbf{0}$.

Unit Vector

$$\mathbf{u} = \text{unit vector} = \frac{\mathbf{v}}{|\mathbf{v}|} = \left(\frac{1}{|\mathbf{v}|}\right)\mathbf{v} \text{ is unit vector in direction of } \mathbf{v}.$$

Definition of the Dot Product

The **dot product** of $\mathbf{u} = \langle u_1,\ u_2 \rangle$ and $\mathbf{v} = \langle v_1,\ v_2 \rangle$ is:

$$\mathbf{u} \cdot \mathbf{v} = u_1 v_1 + u_2 v_2$$

Projection Using the Dot Product

If $\mathbf{u}$ and $\mathbf{v}$ are nonzero vectors, then the projection of $\mathbf{u}$ onto $\mathbf{v}$ is:

$$\text{proj}_{\mathbf{v}}\mathbf{u} = \left(\frac{\mathbf{u} \cdot \mathbf{v}}{|\mathbf{v}|^2}\right)\mathbf{v}$$

Angle Between Two Vectors

If θ is the angle between two nonzero vectors $\mathbf{u}$ and $\mathbf{v}$, then:

$$\cos\theta = \frac{\mathbf{u} \cdot \mathbf{v}}{|\mathbf{u}|\ |\mathbf{v}|}$$

Work

The work W done by a *constant* force $\mathbf{F}$ acting along the line of motion of an object is given by:

$$W \;=\; \text{(magnitude of force)(distance)} \;=\; |\mathbf{F}|\;|\overrightarrow{PQ}|$$

Absolute Value of a Complex Number

The absolute value of the complex number $\mathbf{z = a + bi}$ is:

$$|a + bi| = \sqrt{a^2 + b^2}$$

Distance Formula in the Complex

The distance d between the points (a,b) and (s,t) in the complex plane is:

$$d = \sqrt{(s-a)^2 + (t-b)^2}$$

Midpoint Formula in the Complex Plane

The Midpoint of the line segment joining the points (a,b) and (s,t) in the complex plan is:

$$\text{Midpoint} = (\frac{a+s}{2}, \frac{b+t}{2})$$

Trigonometric Form of a Complex Number

The **trigonometric form** of the complex number $\mathbf{z = a + bi}$ is:

$$z \;=\; r(\cos\;\theta \;+\; i\sin\;\theta)$$

where $a \;=\; r\;\cos\;\theta$, $b \;=\; r\;\sin\;\theta$, $r \;=\; \sqrt{a^2\;+\;b^2}$, and $\tan\;\theta \;=\; b/a$. The number r is the **modulus** of z, and θ is called an **argument** of z.

Product and Quotient of Two Complex Numbers

Let $z_1 = r_1(\cos\ \theta_1 + i\sin\ \theta_1)$ and $z_1 = r_2(\cos\ \theta_2 + i\sin\ \theta_2)$ be complex numbers.

$$z_1 z_2 = r_1 r_2 \left[\cos(\theta_1 + \theta_2) + i\sin(\theta_1 + \theta_2)\right] \text{ is Product Rule.}$$

$$\frac{z_1}{z_2} = \frac{r_1}{r_2}\left[\cos(\theta_1 - \theta_2) + i\sin(\theta_1 - \theta_2)\right] \text{ is Quotient Rule.}$$

DeMoivre's Theorem

If $z = r(\cos\ \theta + i\sin\ \theta)$ is a complex number and n is a positive integer, then:

$$z^n = \left[r(\cos\ \theta + i\sin\ \theta)\right]^n = r^n(\cos n\theta + i\sin n\theta)$$

Finding *n*th Roots of a Complex Number

For a positive integer n, the complex number $z = r(\cos\ \theta + i\sin\ \theta)$ has exactly n distinct nth roots given by:

$$z_k = \sqrt[n]{r}\left(\cos\frac{\theta + 2\pi k}{n} + i\sin\frac{\theta + 2\pi k}{n}\right)$$

where $k = 0,\ 1,\ 2,\ .\ .\ .\ ,\ n - 1$

Chapter 10

Angle Between Two Lines

If two nonperpendicular lines have slopes m_1 and m_2, then the tangent of the angle between the two lines is:

$$\tan\ \theta = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right|$$

Distance Between a Point and a Line

The distance between the point $(x_1,\ y_1)$ and the line $Ax\ +\ By\ +\ C\ =\ 0$ is:

$$d\ =\ \frac{|Ax_1\ +\ By_1\ +\ C|}{\sqrt{A^2\ +\ B^2}}$$

Standard Equation of a Parabola

The **standard form of the equation of a parabola** with vertex at $(h,\ k)$ is as follows:

$(x\ -\ h)^2\ =\ 4p(y\ -\ k),\ p\ \neq\ 0$ has a vertical axis; directrix: $y\ =\ k\ -\ p$

$(y\ -\ k)^2\ =\ 4p(x\ -\ h),\ p\ \neq\ 0$ has a horizontal axis; directrix: $x\ =\ h\ -\ p$

The focus lies on the axis p units (*directed distance*) from the vector. If the vertex is at the origin, then the equation takes one of the following forms:

$$x^2\ =\ 4py \text{ for a vertical axis}$$

$$y^2\ =\ 4px \text{ for a horizontal axis}$$

Standard Equation of an Ellipse

The **standard form of the equation of an ellipse** with center $(h,\ k)$ and major and minor axes of lengths $2a$ and $2b$, respectively, where $0\ <\ b\ <\ a$, is:

$$\frac{(x\ -\ h)^2}{a^2}\ +\ \frac{(y\ -\ k)^2}{b^2}\ =\ 1 \text{ when major axis is horizontal}$$

$$\frac{(x\ -\ h)^2}{b^2}\ +\ \frac{(y\ -\ k)^2}{a^2}\ =\ 1 \text{ when major axis is vertical}$$

Standard Equation of a Hyperbola

The **standard form of the equation of a hyperbola** with center (h, k) is:

$$\frac{(x - h)^2}{a^2} - \frac{(y - k)^2}{b^2} = 1 \text{ when transverse axis is horizontal}$$

$$\frac{(y - k)^2}{a^2} - \frac{(x - h)^2}{b^2} = 1 \text{ when transverse axis is vertical}$$

$$\text{with } c^2 = a^2 + b^2$$

Asymptotes of a Hyperbola

The equations of the asymptotes of a hyperbola are:

$$y = k \pm \frac{b}{a}(x - h) \text{ when transverse axis is horizontal}$$

$$y = k \pm \frac{a}{b}(x - h) \text{ when transverse axis is vertical}$$

Eccentricity

$$e = \frac{c}{a}$$

Rotation of Axes to Eliminate an *xy*-Term

The general second-degree equation:

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$$

$$\cot 2\theta = \frac{A - C}{B}$$

$$x = x' \cos \theta - y' \sin \theta$$

$$y = x' \sin \theta + y' \cos \theta$$

Coordinate Conversion

The polar coordinates $(r,\ \theta)$ are related to the rectangular coordinates $(x,\ y)$ as follows:

Polar-to-Rectangular $\quad x = r\ \cos\ \theta \qquad y = r\ \sin\ \theta$

Rectangular-to-Polar $\quad \tan\ \theta = \dfrac{y}{x} \qquad r^2 = x^2 + y^2$

Polar Equations of Conics

The graph of a polar equation of the form:

1. $r = \dfrac{ep}{1 \pm e\cos\ \theta}$

2. $r = \dfrac{ep}{1 \pm e\sin\ \theta}$

is a conic, where $e > 0$ is the eccentricity and $|p|$ is the distance between the focus (pole) and the directrix.

Classification of Conics by Eccentricity

Let F be a fixed point (*focus*) and let D be a fixed line (*directrix*) in the plane. Let P be another point in the plane and let e (*eccentricity*) be the ratio of the distance between P and F to the distance between P and D. The collection of all points P with a given eccentricity is a conic.

1. The conic is an ellipse for $0 < e < 1$.

2. The conic is a parabola for $e = 1$.

3. The conic is a hyperbola for $e > 1$.

Classification of Conics in Algebraic Form

When a conic is in the form $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$, the determinant of the equation shows:

1. If $B^2 - 4AC > 0$, the conic is a hyperbola

2. If $B^2 - 4AC < 0$, the conic is an ellipse or circle.

3. If $B^2 - 4AC = 0$, the conic is a hyperbola

Calculus

Basic Differentiation Rules

1. $\frac{d}{dx}[cu] \;=\; cu'$

2. $\frac{d}{dx}[u \pm v] \;=\; u' \pm v'$

3. $\frac{d}{dx}[uv] \;=\; uv' + vu'$

4. $\frac{d}{dx}\left[\frac{u}{v}\right] \;=\; \frac{vu' - uv'}{v^2}$

5. $\frac{d}{dx}[c] \;=\; 0$

6. $\frac{d}{dx}[u^n] \;=\; nu^{n-1}u'$

7. $\frac{d}{dx}[x] \;=\; 1$

8. $\frac{d}{dx}[|u|] \;=\; \frac{u}{|u|}(u'),\ u \neq 0$

9. $\frac{d}{dx}[\ln u] \;=\; \frac{u'}{u}$

10. $\frac{d}{dx}[e^u] \;=\; e^u u'$

11. $\frac{d}{dx}[\log_a u] \;=\; \frac{u'}{(\ln a)u}$

12. $\frac{d}{dx}[a^u] \;=\; (\ln a)a^u u'$

13. $\frac{d}{dx}[\sin u] \;=\; (\cos u)u'$

14. $\frac{d}{dx}[\cos u] \;=\; -(\sin u)u'$

15. $\frac{d}{dx}[\tan u] \;=\; (\sec^2 u)u'$

16. $\frac{d}{dx}[\csc u] \;=\; -(\csc u \cot u)u'$

17. $\frac{d}{dx}[\sec u] \;=\; (\sec u \tan u)u'$

18. $\frac{d}{dx}[\cot u] \;=\; -(\csc^2 u)u'$

19. $\frac{d}{dx}[\arcsin u] \;=\; \frac{u'}{\sqrt{1-u^2}}$

20. $\frac{d}{dx}[\arccos u] \;=\; \frac{-u'}{\sqrt{1-u^2}}$

21. $\frac{d}{dx}[\arctan u] \;=\; \frac{u'}{1+u^2}$

22. $\frac{d}{dx}[\text{arccsc}\ u] \;=\; \frac{-u'}{|u|\sqrt{u^2-1}}$

23. $\frac{d}{dx}[\text{arcsec}\ u] \;=\; \frac{u'}{|u|\sqrt{u^2-1}}$

24. $\frac{d}{dx}[\text{arccot}\ u] \;=\; \frac{-u'}{1+u^2}$

25. $\frac{d}{dx}[\sinh\ u] \;=\; (\cosh\ u)u'$

26. $\frac{d}{dx}[\cosh\ u] \;=\; (\sinh\ u)u'$

27. $\frac{d}{dx}[\tanh\ u] \;=\; (\text{sech}^2\ u)u'$

28. $\frac{d}{dx}[\text{csch}\, u] \;=\; -(\text{csch}\, u \coth u)u'$

29. $\frac{d}{dx}[\text{sech}\, u] \;=\; -(\text{sech}\, u \tanh u)u'$

30. $\frac{d}{dx}[\coth\ u] \;=\; -(\text{csch}^2\ u)u'$

31. $\dfrac{d}{dx}\left[\sinh^{-1} u\right] \;=\; \dfrac{u'}{\sqrt{u^2+1}}$

32. $\dfrac{d}{dx}\left[\cosh^{-1} u\right] \;=\; \dfrac{u'}{\sqrt{u^2-1}}$

33. $\dfrac{d}{dx}\left[\tanh^{-1} u\right] \;=\; \dfrac{u'}{1-u^2}$

34. $\dfrac{d}{dx}\left[\operatorname{csch}^{-1} u\right] \;=\; \dfrac{-u'}{|u|\sqrt{1+u^2}}$

35. $\dfrac{d}{dx}\left[\operatorname{sech}^{-1} u\right] \;=\; \dfrac{-u'}{|u|\sqrt{1-u^2}}$

36. $\dfrac{d}{dx}\left[\coth^{-1} u\right] \;=\; \dfrac{u'}{1-u^2}$

Basic Integration Formulas

1. $\displaystyle\int kf(u)du \;=\; k\int f(u)du$

2. $\displaystyle\int [f(u) \pm g(u)]du \;=\; \int f(u)du \pm \int g(u)du$

3. $\displaystyle\int du \;=\; u + C$

4. $\displaystyle\int u^n du \;=\; \frac{u^{n+1}}{n+1} + C$

5. $\displaystyle\int \frac{du}{u} \;=\; \ln|u| + C$

6. $\displaystyle\int e^u du \;=\; e^u + C$

7. $\displaystyle\int a^u du \;=\; \left(\frac{1}{\ln a}\right) a^u + C$

8. $\displaystyle\int \sin u\; du \;=\; -\cos u + C$

9. $\displaystyle\int \cos u\; du \;=\; \sin u + C$

10. $\displaystyle\int \tan u\; du \;=\; -\ln|\cos u| + C$

11. $\displaystyle\int \csc u\; du \;=\; -\ln|\csc u + \cot u| + C$

12. $\displaystyle\int \sec u\; du \;=\; \ln|\sec u + \tan u| + C$

13. $\displaystyle\int \cot u\; du \;=\; \ln|\sin u| + C$

14. $\displaystyle\int \sec^2 u\; du \;=\; \tan u + C$

15. $\displaystyle\int \csc^2 u\; du \;=\; -\cot u + C$

16. $\displaystyle\int \sec u\;\tan u\; du \;=\; \sec u + C$

17. $\displaystyle\int \csc u \cot u\; du \;=\; -\csc u + C$

18. $\displaystyle\int \frac{du}{\sqrt{a^2 - u^2}} \;=\; \text{arcsin}\frac{u}{a} + C$

19. $\displaystyle\int \frac{du}{a^2 + u^2} \;=\; \frac{1}{a}\text{arctan}\frac{u}{a} + C$

20. $\displaystyle\int \frac{du}{u\sqrt{u^2 - a^2}} \;=\; \frac{1}{a}\text{arcsec}\frac{|u|}{a} + C$

Integration Tables

Forms Involving u^n

1. $\displaystyle\int u^n du \;=\; \frac{u^{n+1}}{n+1} + C,\ n \neq -1$

2. $\displaystyle\int \frac{1}{u} du \;=\; \ln|u| + C$

Forms Involving $a + bu$

3. $\displaystyle\int \frac{u}{a+bu} du \;=\; \frac{1}{b^2}(bu - a\ln|a+bu|) + C$

4. $\displaystyle\int \frac{u}{(a+bu)^2} du \;=\; \frac{1}{b^2}\left(\frac{a}{a+bu} \;+\; \ln|a+bu|\right) + C$

5. $\displaystyle\int \frac{u}{(a+bu)^n} du \;=\; \frac{1}{b^2}\left[\frac{-1}{(n-2)(a+bu)^{n-2}} \;+\; \frac{a}{(n-1)(a+bu)^{n-1}}\right] + C$

6. $\displaystyle\int \frac{u^2}{a+bu} du \;=\; \frac{1}{b^3}\left[-\frac{bu}{2}(2a - bu) \;+\; a^2\ln|a+bu|\right] + C$

7. $\displaystyle\int \frac{u^2}{(a+bu)^2} du \;=\; \frac{1}{b^3}\left(bu \;-\; \frac{a^2}{a+bu} \;-\; 2u\ln|a+bu|\right) + C$

8. $\displaystyle\int \frac{u^2}{(a+bu)^3} du \;=\; \frac{1}{b^3}\left[\frac{2a}{a+bu} \;-\; \frac{a^2}{2(a+bu)^2} \;+\; \ln|a+bu|\right] + C$

9. $\displaystyle\int \frac{u^2}{(a+bu)^n} du \;=\; \frac{1}{b^3}\left[\frac{-1}{(n-3)(a+bu)^{n-3}} \;+\; \frac{2a}{(n-2)(a+bu)^{n-2}} \;-\; \frac{a^2}{(n-1)(a+bu)^{n-1}}\right] + C,\ n \neq 1, 2, 3$

10. $\displaystyle\int \frac{1}{u(a+bu)} du \;=\; \frac{1}{a}\ln\left|\frac{u}{a+bu}\right| + C$

11. $\displaystyle\int \frac{1}{u(a+bu)^2} du \;=\; \frac{1}{a}\left(\frac{1}{a+bu} \;+\; \frac{1}{a}\ln\left|\frac{u}{a+bu}\right|\right) + C$

12. $\displaystyle\int \frac{1}{u^2(a+bu)} du \;=\; -\frac{1}{a}\left(\frac{1}{u} \;+\; \frac{b}{a}\ln\left|\frac{u}{a+bu}\right|\right) + C$

13. $\displaystyle\int \frac{1}{u^2(a+bu)^2} du \;=\; -\frac{1}{a^2}\left[\frac{a+2bu}{u(a+bu)} \;+\; \frac{2b}{u}\ln\left|\frac{u}{a+bu}\right|\right) + C$

Forms Involving $a + bu + cu^2,\ b^2 \neq 4ac$

14.i. $\displaystyle\int \frac{1}{a+bu+cu^2} du \text{ when } b^2 < 4ac \;=\; \frac{2}{\sqrt{4ac-b^2}}\arctan\frac{2cu+b}{\sqrt{4ac-b^2}} + C$

14.ii. $\displaystyle\int \frac{1}{a+bu+cu^2} du \text{ when } b^2 > 4ac \;=\; \frac{1}{\sqrt{b^2-4ac}}\ln\left|\frac{2cu+b-\sqrt{b^2-4ac}}{2cu+b+\sqrt{b^2-4ac}}\right| + C$

15. $\displaystyle\int \frac{u}{a+bu+cu^2} du \;=\; \frac{1}{2c}\left(\ln|a+bu+cu^2| \;-\; b\int \frac{1}{a+bu+cu^2} du\right)$

Forms Involving $\sqrt{a+bu}$

16. $\displaystyle\int u^n\sqrt{a+bu}du \;=\; \frac{2}{b(2n+3)}\left[u^n(a+bu)^{3/2} \;-\; na\int u^{n-1}\sqrt{a+bu}du\right]$

17.i. $\displaystyle\int \frac{1}{u\sqrt{a+bu}}du \text{ when } a>0 \;=\; \frac{1}{\sqrt{a}}\ln\left|\frac{\sqrt{a+bu}-\sqrt{a}}{\sqrt{a+bu}+\sqrt{a}}\right|+C$

17.ii. $\displaystyle\int \frac{1}{u\sqrt{a+bu}}du \text{ when } a<0 \;=\; \frac{2}{\sqrt{-a}}\arctan\sqrt{\frac{a+bu}{-a}}+C$

18. $\displaystyle\int \frac{1}{u^n\sqrt{a+bu}}du \;=\; \frac{-1}{a(n-1)}\left[\frac{a+bu}{u^{n-1}} \;+\; \frac{(2n-3)b}{2}\int \frac{1}{u^{n-1}\sqrt{a+bu}}du\right],\; n\neq 1$

19. $\displaystyle\int \frac{\sqrt{a+bu}}{u}du \;=\; 2\sqrt{a+bu} \;+\; a\int \frac{1}{u\sqrt{a+bu}}du$

20. $\displaystyle\int \frac{\sqrt{a+bu}}{u^n}du \;=\; \frac{-1}{a(n-1)}\left[\frac{(a+bu)^{3/2}}{u^{n-1}} \;+\; \frac{(2n-5)b}{2}\int \frac{\sqrt{a+bu}}{u^{n-1}}du\right],\; n\neq 1$

21. $\displaystyle\int \frac{u}{\sqrt{a+bu}}du \;=\; \frac{-2(2a-bu)}{3b^2}\sqrt{a+bu}+C$

22. $\displaystyle\int \frac{u^n}{\sqrt{a+bu}}du \;=\; \frac{2}{(2n+1)b}\left(u^n\sqrt{a+bu} \;-\; na\int \frac{u^{n-1}}{\sqrt{a+bu}}du\right)$

Forms Involving $a^2 \pm u^2,\; a>0$

23. $\displaystyle\int \frac{1}{a^2+u^2}du \;=\; \frac{1}{a}\arctan\frac{u}{a}+C$

24. $\displaystyle\int \frac{1}{u^2-a^2}du \;=\; -\int \frac{1}{a^2-u^2}du \;=\; \frac{1}{2a}\ln\left|\frac{u-a}{u+a}\right|+C$

25. $\displaystyle\int \frac{1}{(a^2\pm u^2)^n}du \;=\; \frac{1}{2a^2(n-1)}\left[\frac{u}{(a^2\pm u^2)^{n-1}} \;+\; (2n-3)\int \frac{1}{(a^2\pm u^2)^{n-1}}du\right],\; n\neq 1$

Forms Involving $\sqrt{u^2\pm a^2},\; a>0$

26. $\displaystyle\int \sqrt{u^2\pm a^2}du \;=\; \frac{1}{2}(u\sqrt{u^2\pm a^2} \;\pm\; a^2\ln|u \;+\; \sqrt{u^2\pm a^2}|)+C$

27. $\displaystyle\int u^2\sqrt{u^2\pm a^2}du \;=\; \frac{1}{8}[u(2u^2\pm a^2)\sqrt{u^2\pm a^2} \;-\; a^4\ln|u \;+\; \sqrt{u^2\pm a^2}|]+C$

28. $\displaystyle\int \frac{\sqrt{u^2+a^2}}{u}du \;=\; \sqrt{u^2+a^2} \;-\; a\ln\left|\frac{a+\sqrt{u^2+a^2}}{u}\right|+C$

29. $\displaystyle\int \frac{\sqrt{u^2-a^2}}{u}du \;=\; \sqrt{u^2-a^2} \;-\; a\,\mathrm{arcsec}\frac{|u|}{a}+C$

30. $\displaystyle\int \frac{\sqrt{u^2\pm a^2}}{u^2}du \;=\; \frac{-\sqrt{u^2\pm a^2}}{u} \;+\; \ln|u \;+\; \sqrt{u^2\pm a^2}|+C$

31. $\displaystyle\int \frac{1}{\sqrt{u^2\pm a^2}}du \;=\; \ln|u \;+\; \sqrt{u^2\pm a^2}|+C$

32. $\displaystyle\int \frac{1}{u\sqrt{u^2+a^2}}du \;=\; \frac{-1}{a}\ln\left|\frac{a+\sqrt{u^2+a^2}}{u}\right|+C$

33. $\displaystyle\int \frac{1}{u\sqrt{u^2-a^2}}du \;=\; \frac{1}{a}\mathrm{arcsec}\frac{|u|}{a}+C$

34. $\displaystyle\int \frac{u^2}{\sqrt{u^2\pm a^2}}du \;=\; \frac{1}{2}(u\sqrt{u^2\pm a^2} \;\mp\; a^2\ln|u \;+\; \sqrt{u^2\pm a^2}|)+C$

35. $\displaystyle\int \frac{1}{u^2\sqrt{u^2 \pm a^2}}du \;=\; \mp\frac{\sqrt{u^2 \pm a^2}}{a^2 u} + C$

36. $\displaystyle\int \frac{1}{(u^2 \pm a^2)^{3/2}}du \;=\; \frac{\pm u}{a^2\sqrt{u^2 \pm a^2}} + C$

Forms Involving $\sqrt{a^2 - u^2},\ a > 0$

37. $\displaystyle\int \sqrt{a^2 - u^2}du \;=\; \frac{1}{2}\left(u\sqrt{a^2 - u^2} \;+\; a^2\arcsin\frac{u}{a}\right) + C$

38. $\displaystyle\int u^2\sqrt{a^2 - u^2}du \;=\; \frac{1}{8}\left[u(2u^2 - a^2)\sqrt{a^2 - u^2} \;+\; a^4\arcsin\frac{u}{a}\right] + C$

39. $\displaystyle\int \frac{\sqrt{a^2 - u^2}}{u}du \;=\; \sqrt{a^2 - u^2} \;-\; a\ln\left|\frac{a + \sqrt{a^2 - u^2}}{u}\right| + C$

40. $\displaystyle\int \frac{\sqrt{a^2 - u^2}}{u^2}du \;=\; \frac{-\sqrt{a^2 - u^2}}{u} \;-\; \arcsin\frac{u}{a} \;+\; C$

41. $\displaystyle\int \frac{1}{\sqrt{a^2 - u^2}}du \;=\; \arcsin\frac{u}{a} + C$

42. $\displaystyle\int \frac{1}{u\sqrt{a^2 - u^2}}du \;=\; \frac{-1}{a}\ln\left|\frac{a + \sqrt{a^2 - u^2}}{u}\right| + C$

43. $\displaystyle\int \frac{u^2}{\sqrt{a^2 - u^2}}du \;=\; \frac{1}{2}\left(-u\sqrt{a^2 - u^2} \;+\; a^2\arcsin\frac{u}{a}\right) + C$

44. $\displaystyle\int \frac{1}{u^2\sqrt{a^2 - u^2}}du \;=\; \frac{-\sqrt{a^2 - u^2}}{a^2 u} + C$

45. $\displaystyle\int \frac{1}{(a^2 - u^2)^{3/2}}du \;=\; \frac{u}{a^2\sqrt{a^2 - u^2}} + C$

Forms Involving $\sin u$ **or** $\cos u$

46. $\displaystyle\int \sin u\ du \;=\; -\cos u \;+\; C$

47. $\displaystyle\int \cos u\ du \;=\; \sin u \;+\; C$

48. $\displaystyle\int \sin^2 u\ du \;=\; \frac{1}{2}(u \;-\; \sin u\cos u) + C$

49. $\displaystyle\int \cos^2 u\ du \;=\; \frac{1}{2}(u \;+\; \sin u\cos u) + C$

50. $\displaystyle\int \sin^n u\ du \;=\; -\frac{\sin^{n-1} u\cos u}{n} \;+\; \frac{n-1}{n}\int \sin^{n-2} u\ du$

51. $\displaystyle\int \cos^n u\ du \;=\; \frac{\cos^{n-1} u\sin u}{n} \;+\; \frac{n-1}{n}\int \cos^{n-2} u\ du$

52. $\displaystyle\int u\sin u\ du \;=\; \sin u \;-\; u\cos u \;+\; C$

53. $\displaystyle\int u\cos u\ du \;=\; \cos u \;+\; u\sin u \;+\; C$

54. $\displaystyle\int u^n \sin u \, du = -u^n \cos u + n \int u^{n-1} \cos u \, du$

55. $\displaystyle\int u^n \cos u \, du = u^n \sin u - n \int u^{n-1} \sin u \, du$

56. $\displaystyle\int \frac{1}{1 \pm \sin u} du = \tan u \mp \sec u + C$

57. $\displaystyle\int \frac{1}{1 \pm \cos u} du = -\cot u \pm \csc u + C$

58. $\displaystyle\int \frac{1}{\sin u \cos u} du = \ln|\tan u| + C$

Forms Involving $\tan u$, $\cot u$, $\sec u$, or $\csc u$

59. $\displaystyle\int \tan u \, du = -\ln|\cos u| + C$

60. $\displaystyle\int \cot u \, du = \ln|\sin u| + C$

61. $\displaystyle\int \sec u \, du = \ln|\sec u + \tan u| + C$

62. $\displaystyle\int \csc u \, du = \ln|\csc u - \cot u| + C = -\ln|\csc u + \cot u| + C$

63. $\displaystyle\int \tan^2 u \, du = -u + \tan u + C$

64. $\displaystyle\int \cot^2 u \, du = -u - \cot u + C$

65. $\displaystyle\int \sec^2 u \, du = \tan u + C$

66. $\displaystyle\int \csc^2 u \, du = -\cot u + C$

67. $\displaystyle\int \tan^n u \, du = \frac{\tan^{n-1} u}{n-1} - \int \tan^{n-2} u \, du, \; n \neq 1$

68. $\displaystyle\int \cot^n u \, du = -\frac{\cot^{n-1} u}{n-1} - \int \cot^{n-2} u \, du, \; n \neq 1$

69. $\displaystyle\int \sec^n u \, du = \frac{\sec^{n-2} u \tan u}{n-1} + \frac{n-2}{n-1} \int \sec^{n-2} u \, du \;\; n \neq 1$

70. $\displaystyle\int \csc^n u \, du = -\frac{\csc^{n-2} u \cot u}{n-1} + \frac{n-2}{n-1} \int \csc^{n-2} u \, du \;\; n \neq 1$

71. $\displaystyle\int \frac{1}{1 \pm \tan u} du = \frac{1}{2}(u \pm \ln|\cos u \pm \sin u|) + C$

72. $\displaystyle\int \frac{1}{1 \pm \cot u} du = \frac{1}{2}(u \mp \ln|\sin u \pm \cos u|) + C$

73. $\displaystyle\int \frac{1}{1 \pm \sec u} du = u + \cot u \mp \csc u + C$

74. $\displaystyle\int \frac{1}{1 \pm \csc u} du = u - \tan u \pm \sec u + C$

Forms Involving Inverse Trigonometric Functions

75. $\int \arcsin u \; du \; = \; u \arcsin u \; + \; \sqrt{1-u^2} \; + \; C$

76. $\int \arccos u \; du \; = \; u \arccos u \; - \; \sqrt{1-u^2} \; + \; C$

77. $\int \arctan u \; du \; = \; u \arctan u \; - \; \frac{1}{2} \ln \sqrt{1+u^2} \; + \; C$

78. $\int \text{arccot } u \; du \; = \; u \text{ arccot } u \; + \; \frac{1}{2} \ln \sqrt{1+u^2} \; + \; C$

79. $\int \text{arcsec } u \; du \; = \; u \text{ arcsec } u \; - \; \ln |u \; + \; \sqrt{u^2-1}| \; + \; C$

80. $\int \text{arccsc } u \; du \; = \; u \text{ arccsc } u \; + \; \ln |u \; + \; \sqrt{u^2-1}| \; + \; C$

Forms Involving e^u

81. $\int e^u du \; = \; e^u \; + \; C$

82. $\int u e^u du \; = \; (u-1)e^u \; + \; C$

83. $\int u^n e^u du \; = \; u^n e^u \; - \; n \int u^{n-1} e^u du$

84. $\int \frac{1}{1+e^u} du \; = \; u \; - \; \ln(1 \; + \; e^u) \; + \; C$

85. $\int e^{au} \sin bu \; du \; = \; \frac{e^{au}}{a^2+b^2}(a \sin bu \; - \; b \cos bu) \; + \; C$

86. $\int e^{au} \cos bu \; du \; = \; \frac{e^{au}}{a^2+b^2}(a \cos bu \; + \; b \sin bu) \; + \; C$

Forms Involving $\ln u$

87. $\int \ln u \; du \; = \; u(-1 \; + \; \ln u) \; + \; C$

88. $\int u \ln u \; du \; = \; \frac{u^2}{4}(-1 \; + \; 2 \ln u) \; + \; C$

89. $\int u^n \ln u \; du \; = \; \frac{u^{n+1}}{(n+1)^2}[-1 \; + \; (n+1) \ln u] \; + \; C, \; n \neq 1$

90. $\int (\ln u)^2 du \; = \; u[2 \; - \; 2 \ln u \; + \; (\ln u)^2] \; + \; C$

91. $\int (\ln u)^n du \; = \; u(\ln u)^n \; - \; n \int (\ln u)^{n-1} du$

Forms Involving Hyperbolic Functions

92. $\int \cosh u \; du \; = \; \sinh u \; + \; C$

93. $\int \sinh u \; du \;=\; \cosh u \;+\; C$

94. $\int \text{sech}^2 \; u \; du \;=\; \tanh u \;+\; C$

95. $\int \text{csch}^2 \; u \; du \;=\; -\coth u \;+\; C$

96. $\int \text{sech} \; u \; \tanh u \; du \;=\; -\text{sech} \; u \;+\; C$

97. $\int \text{csch} \; u \; \coth u \; du \;=\; -\text{csch} \; u \;+\; C$

Forms Involving Inverse Hyperbolic Functions (in logarithmic form)

98. $\int \frac{du}{\sqrt{u^2 \pm a^2}} \;=\; \ln(u \;+\; \sqrt{u^2 \pm a^2}) + C$

99. $\int \frac{du}{a^2 - u^2} \;=\; \frac{1}{2a} \ln \left| \frac{a+u}{a-u} \right| + C$

100. $\int \frac{du}{u\sqrt{a^2 \pm u^2}} \;=\; -\frac{1}{a} \ln \frac{a \;+\; \sqrt{a^2 + u^2}}{|u|} + C$

Triangle

$h = a \sin \theta$

$\text{Area} = \frac{1}{2}bh$

Law of Cosines: $c^2 = a^2 + b^2 - 2ab \cos \theta$

Right Triangle

Pythagorean Theorem: $c^2 = a^2 + b^2$

Equilateral Triangle

$h = \frac{\sqrt{3}s}{2}$

$\text{Area} = \frac{\sqrt{3}s^2}{4}$

Parallelogram

$\text{Area} = bh$

Trapezoid

$\text{Area} = \frac{h}{2}(a + b)$

Circle

$\text{Area} = \pi r^2$

$\text{Circumference} = 2\pi r$

Sector of Circle

$\text{Area} = \dfrac{\theta r^2}{2}$ (θ in radians)

$s = r\theta$ (θ in radians)

Circular Ring

Using p = average radius, and w = width of ring: $\text{Area} = \pi(R^2 - r^2) = 2\pi p w$

Sector of Circular Ring

Using p = average radius, w = width of ring, and θ in radians: $\text{Area} = \theta p w$

Ellipse

$\text{Area} = \pi ab$

$\text{Circumference} = 2\pi\sqrt{\dfrac{a^2 + b^2}{2}}$

Cone

Using A = area of base: $\text{Volume} = \dfrac{Ah}{3}$

Right Circular Cone

$\text{Volume} = \dfrac{\pi r^2 h}{3}$

$\text{Lateral Surface Area} = \pi r\sqrt{r^2 + h^2}$

Frustum of Right Circular Cone

$\text{Volume} = \dfrac{\pi(r^2 + rR + R^2)h}{3}$

$\text{Lateral Surface Area} = \pi s(R + r)$

Right Circular Cylinder

Volume $= \pi r^2 h$

Lateral Surface Area $= 2\pi rh$

Sphere

Volume $= \dfrac{4}{3}\pi r^3$

Surface Area $= 4\pi r^2$

Wedge

Using $A =$ area of upper face, and $B =$ area of base: $A = B\ sec\ \theta$

Chapter 3

Rolle's Theorem

Let f be continuous on the closed interval $[a,\ b]$ and differentiable on the open interval $(a,\ b)$. If $f(a) \ = \ f(b)$, then there is at least one number c in $(a,\ b)$ such that $f'(c) \ = \ 0$.

The Mean Value Theorem

If f is continuous on the closed interval $[a,\ b]$ and differentiable on the open interval $(a,\ b)$, then there exists a number c in $(a,\ b)$ such that:

$$f'(c) \ = \ \frac{f(b) \ - \ f(a)}{b \ - \ a}$$

Newton's Method for Approximating the Zeros of a Function

Let $f(c) \ = \ 0$, where f is differentiable on an open interval containing c. Then, to approximate c, use these steps:

Make an initial estimate x_1 that is close to c. (A graph is helpful.)

Determine a new approximation:

$$x_{n+1} \ = \ x_n \ - \ \frac{f(x_n)}{f'(x_n)}$$

When $|x_n \ - \ x_{n+1}|$ is within the desired accuracy, let x_{n+1} serve as the final approximation. Otherwise, return to Step 2 and calculate a new approximation.

Each successive application of this procedure is called an **iteration**.

Chapter 4

Summation Formulas

1. $\sum_{i=1}^{n} c = cn$

2. $\sum_{i=1}^{n} i = \dfrac{n(n+1)}{2}$

3. $\sum_{i=1}^{n} i^2 = \dfrac{n(n+1)(2n+1)}{6}$

4. $\sum_{i=1}^{n} i^3 = \dfrac{n^2(n+1)^2}{4}$

Mean Value Theorem for Integrals

If f is continuous on the closed interval [a,b], then there exists a number c in the closed interval [a,b] such that:

$$\int_a^b f(x)\,dx = f(c) \cdot (b-a)$$

Definition of the Average Value of a Function on an Integral

If f is integrable on the closed interval [a,b] then the average value of f on the interval is:

$$\frac{1}{b-a}\int_a^b f(x)\,dx$$

Chapter 5

L'Hôpital's Rule

Let f and g be functions that are differentiable on an open interval $(a,\ b)$ containing c, except possibly at c itself. Assume that $g'(x)\ \neq\ 0$ for all x in $(a,\ b)$, except possibly at c itself. If the limit of $f(x)/g(x)$ as x approaches c produces the indeterminate form 0/0, then:

$$\lim_{x\to c}\frac{f(x)}{g(x)}\ =\ \lim_{x\to c}\frac{f'(x)}{g'(x)}$$

provided the limit on the right exists (or is infinite). This result also applies when the limit of $f(x)/g(x)$ as x approaches c produces any one of the indeterminate forms $\infty/\infty,\ (-\infty)/\infty,\ \infty/(-\infty),$ or $(-\infty)/(-\infty)$.

Hyperbolic Identities

$$\cosh^2 x\ -\ \sinh^2 x\ =\ 1$$

$$\tanh^2 x\ +\ \operatorname{sech}^2 x\ =\ 1$$

$$\coth^2 x\ -\ \operatorname{csch}^2 x\ =\ 1$$

$$\sinh^2 x\ =\ \frac{-1\ +\ \cosh 2x}{2}$$

$$\cosh^2 x\ =\ \frac{1\ +\ \cosh 2x}{2}$$

$$\sinh 2x\ =\ 2\sinh x\cosh x$$

$$\cosh 2x\ =\ \cosh^2 x\ +\ \sinh^2 x$$

$$\sinh(x\ +\ y)\ =\ \sinh x\cosh y\ +\ \cosh x\sinh y$$

$$\sinh(x - y) = \sinh x \cosh y - \cosh x \sinh y$$

$$\cosh(x + y) = \cosh x \cosh y + \sinh x \sinh y$$

$$\cosh(x - y) = \cosh x \cosh y - \sinh x \sinh y$$

Inverse Hyperbolic Functions

$$\sinh^{-1} x = \ln(x + \sqrt{x^2 + 1}),\ \text{Domain: } (-\infty,\ \infty)$$

$$\cosh^{-1} x = \ln(x + \sqrt{x^2 - 1}),\ \text{Domain: } [-1,\ \infty)$$

$$\tanh^{-1} x = \frac{1}{2}\ln\frac{1 + x}{1 - x},\ \text{Domain: } (-1,\ 1)$$

$$\coth^{-1} x = \frac{1}{2}\ln\frac{x + 1}{x - 1},\ \text{Domain: } (-\infty,\ -1) \cup (1,\ \infty)$$

$$\operatorname{sech}^{-1} x = \ln\frac{1 + \sqrt{1 - x^2}}{x},\ \text{Domain: } (0,\ 1]$$

$$\operatorname{csch}^{-1} x = \ln\left(\frac{1}{x} + \frac{\sqrt{1 + x^2}}{|x|}\right),\ \text{Domain: } (-\infty,\ 0) \cup (0,\ \infty)$$

Differentiation and Integration Involving Inverse Hyperbolic Functions

Let u be a differentiable function of x.

$$\frac{d}{dx}[\sinh^{-1} u] = \frac{u'}{\sqrt{u^2 + 1}}$$

$$\frac{d}{dx}[\cosh^{-1} u] = \frac{u'}{\sqrt{u^2 - 1}}$$

$$\frac{d}{dx}[\tanh^{-1} u] = \frac{u'}{1 - u^2},\ |x| < 1$$

$$\frac{d}{dx}[\coth^{-1} u] = \frac{u'}{1 - u^2},\ |x| > 1$$

$$\frac{d}{dx}[\text{sech}^{-1}u] = \frac{-u'}{u\sqrt{1 - u^2}}$$

$$\frac{d}{dx}[\text{csch}^{-1}u] = \frac{-u'}{|u|\sqrt{1 + u^2}}$$

$$\int \frac{du}{\sqrt{u^2 \pm a^2}} = \ln(u + \sqrt{u^2 \pm a^2}) + C$$

$$\int \frac{du}{a^2 - u^2} = \frac{1}{2a}\ln\left|\frac{a + u}{a - u}\right| + C$$

$$\int \frac{du}{u\sqrt{a^2 \pm u^2}} = -\frac{1}{a}\ln\frac{a + \sqrt{a^2 \pm u^2}}{|u|} + C$$

Chapter 6

Euler's Method

Given that $y' = F(x, y)$ and a fixed distance h, approximate the curve y by starting at the point (x_0, y_0) and repeating the process:

$$x_1 = x_0 + h \qquad y_1 = y_0 + hF(x_0, y_0)$$

$$x_2 = x_1 + h \qquad y_2 = y_1 + hF(x_1, y_1)$$

$$\vdots$$

$$x_n = x_{n-1} + h \qquad y_n = y_{n-1} + hF(x_{n-1}, y_{n-1})$$

Exponential Growth and Decay Model

If y is a differentiable function of t such that $y > 0$ and $y' = ky$ for some constant k, then:

$$y = Ce^{kt}$$

where C is the **initial value** of y, and k is the **proportionality constant**. **Exponential growth** occurs when $k > 0$, and **exponential decay** occurs when $k < 0$.

Logisitic Differential Equation

$$\frac{dy}{dt} = ky\left(1 - \frac{y}{L}\right)$$

$$y = \frac{L}{1 + be^{-kt}}$$

Solution of a First-Order Linear Differential Equation

An integrating factor for the first-order linear differential equation:

$$y' + P(x)y = Q(x)$$

is $u(x) = e^{\int P(x)dx}$. The solution of the differential equation is:

$$ye^{\int P(x)dx} = \int Q(x)e^{\int P(x)dx}dx + C$$

Differential Equation for Electrical Circuits

$$L\frac{dI}{dt} + RI = E$$

where I is current, R is resistance, L is inductance, and E is electromotive force (voltage).

Soluble Concentrate Formula

Consider a tank that at time $t = 0$ contains v_0 gallons of solution of which, by weight, q_0 pounds is soluble concentrate. Another solution containing q_1 pounds of the concentrate per gallon is running into the tank at the rate of r_1 gallons per minute. The solution in the tank is kep well stirred and is withdrawn at the rate of r_2 gallons per minute. Let Q be the amount of concentrate (in pounds) in the solution at any time t.

$$\frac{dQ}{dt} + \frac{r_2 Q}{v_0 + (r_1 - r_2)t} = q_1 r_1$$

Chapter 6

Areas of a Region Between Two Curves

If f and g are continuous on $[a,\ b]$ and $g(x)\ \leq\ f(x)$ for all x in $[a,\ b]$, then the area of the region bounded by the graphs of f and g and the vertical lines $x\ =\ a$ and $x\ =\ b$ is:

$$A\ =\ \int_a^b [f(x)\ -\ g(x)]dx$$

The Disk/Washer Method

$$V\ =\ \pi\int_a^b ([R(x)]^2\ -\ [r(x)]^2)dx$$

Volumes of Solids with Known Cross Sections

1. For cross sections of area $A(x)$ taken perpendicular to the x-axis:

$$\text{Volume}\ =\ \int_a^b A(x)dx$$

2. For cross sections of area $A(y)$ taken perpendicular to the y-axis:

$$\text{Volume}\ =\ \int_c^d A(y)dy$$

The Shell Method

To find the volume of a solid of revolution with the **shell method**, use one of the formulas below.

Horizontal Axis of Revolution $\quad\text{Volume}\ =\ V\ =\ 2\pi\int_c^d p(y)h(y)dy$

Vertical Axis of Revolution $\quad\text{Volume}\ =\ V\ =\ 2\pi\int_a^b p(x)h(x)dx$

Definition of Arc Length

Let the function $y = f(x)$ represent a smooth curve on the interval $[a, b]$. The **arc length** of f between a and b is:

$$s = \int_a^b \sqrt{1 + [f'(x)]^2} dx$$

Similarly, for a smooth curve $x = g(y)$, the **arc length** of g between c and d is:

$$s = \int_c^d \sqrt{1 + [g'(x)]^2} dy$$

Definition of the Area of a Surface of Revolution

Let $y = f(x)$ have a continuous derivative on the interval $[a, b]$. The area S of the surface of revolution formed by revolving the graph of f about a horizontal or vertical axis is:

$$S = 2\pi \int_a^b r(x) \sqrt{1 + [f'(x)]^2} dx \quad y \text{ is a function of } x$$

where $r(x)$ is the distance between the graph of f and the axis of revolution. If $x = g(y)$ on the interval $[c, d]$, then the surface area is:

$$S = 2\pi \int_c^d r(y) \sqrt{1 + [g'(y)]^2} dy \quad x \text{ is a function of } y$$

where $r(y)$ is the distance between the graph of g and the axis of revolution.

Theorem of Pappus

Let R be a region in a plane and let L be a line in the same plane such that L does not intersect the interior of R. If r is the distance between the centroid of R and the line, then the volume V of the solid of revolution formed by revolving R about the line is:

$$V = 2\pi r A$$

where A is the area of R. (Note that $2\pi r$ is the distance travelled by the centroid as the region is revolved about that line.)

Definition of Work Done by a Variable Force

If an object is moved along a straight line by a continuously varying force $F(x)$, then the **work** W done by the force as the object is moved from:

$$x = a \text{ to } x = b$$

is given by:

$$W = \lim_{\|\Delta\|\to 0} \sum_{i=1}^{n} \Delta W_i = \int_a^b F(x)dx$$

Laws of Physics

Hookes Law: F=kd

Newton's Law of Gravitation: $F = G\dfrac{m_1 m_2}{d^2}$

Moments and Center of Mass: Two-Dimensional System

Let the point masses $m_1,\ m_2,\ ...,\ m_n$ be located at $(x_1, y_1),\ (x_2, y_2),\ ...,\ (x_n, y_n)$.

1. The moment about the y-axis is: $M_y = m_1x_1 + m_2x_2 + \cdots + m_nx_n$
2. The moment about the x-axis is: $M_x = m_1y_1 + m_2y_2 + \cdots + m_ny_n$
3. The center of mass $(\bar{x},\ \bar{y})$, or center of gravity, is:

$$\bar{x} = \frac{M_y}{m} \text{ and } \bar{y} = \frac{M_x}{m}$$

where $m = m_1 + m_2 + \cdots + m_n$ is the total mass of the system.

Moments and Center of Mass of a Planar Lamina

Let f and g be continuous functions such that $f(x) \geq g(x)$ on $[a,\ b]$, and consider the planar lamina of uniform density ρ bounded by the graphs of $y = f(x)$, $y = g(x)$, and $a \leq x \leq b$.

1. The **moments about the x- and y-axes** are:

$$M_x = \rho\int_a^b \left[\frac{f(x) + g(x)}{2}\right][f(x) - g(x)]dx$$

$$M_y = \rho\int_a^b x[f(x) - g(x)]dx$$

2. The **center of mass** $(\bar{x},\ \bar{y})$ is given by $\bar{x} = \dfrac{M_y}{m}$ and $\bar{y} = \dfrac{M_x}{m}$, where $m = \rho\displaystyle\int_a^b [f(x) - g(x)]dx$ is the mass of the lamina.

Definition of Force Exterted by a Fluid

The **force** F **exerted by a fluid** of constant weight-density w (per unit of volume) against a submerged vertical plane region from $y = c$ to $y = d$ is:

$$F = w \lim_{\|\Delta\| \to 0\|} \sum_{i=1}^{n} h(y_i)L(y_i)\Delta y = w \int_c^d h(y)L(y)dy$$

where $h(y)$ is the depth of the fluid at y and $L(y)$ is the horizontal length of the region at y.

Chapter 7

Integration by Parts

If u and v are functions of x and have continuous derivatives, then:

$$\int udv = uv - \int vdu$$

Integrals Involving Sine-Cosine Products with Different Angles

Integrals involving the products of sines and cosines of two *different* angles occur in many applications. In such instances, you can use the following product-to-sum identities:

$$\sin(mx)\sin(nx) = \frac{1}{2}(\cos[(m - n)x] - \cos[(m + n)x])$$

$$\sin(mx)\cos(nx) = \frac{1}{2}(\sin[(m - n)x] + \sin[(m + n)x])$$

$$\cos(mx)\cos(nx) = \frac{1}{2}(\cos[(m - n)x] + \cos[(m + n)x])$$

Special Integration Formulas $(a > 0)$

1. $\int \sqrt{a^2 - u^2}du = \frac{1}{2}\left(a^2 \arcsin\frac{u}{a} + u\sqrt{a^2 - u^2}\right) + C$

2. $\int \sqrt{u^2 - a^2}du = \frac{1}{2}\left(u\sqrt{u^2 - a^2} - a^2 \ln|u + \sqrt{u^2 - a^2}|\right) + C,\ u > a$

3. $\int \sqrt{u^2 + a^2}du = \frac{1}{2}\left(u\sqrt{u^2 + a^2} - a^2 \ln|u + \sqrt{u^2 + a^2}|\right) + C,\ u > a$

Decomposition of $N(x)/D(x)$ into Partial Fractions

1. Linear factors: For each factor of the form $(px + q)^m$, the partial fraction decomposition must include the following sum of m fractions.

$$\frac{A_1}{(px + q)} + \frac{A_2}{(px + q)^2} + \cdots + \frac{A_m}{(px + q)^m}$$

2. Quadratic factors: For each factor of the form $(ax^2 + bx + c)^n$, the partial fraction decomposition must include the following sum of n fractions.

$$\frac{B_1x + C_1}{ax^2 + bx + c} + \frac{B_2x + C_2}{(ax^2 + bx + c)^2} + \cdots + \frac{B_nx + C_n}{(ax^2 + bx + c)^n}$$

The Trapezoidal Rule

Let f be continuous on $[a, b]$. The Trapezoidal Rule for approximating $\int_a^b f(x)dx$ is:

$$\int_a^b f(x)dx \approx \frac{b - a}{2n}[f(x_0) + 2f(x_1) + 2f(x_2) + \cdots + 2f(x_{n-1}) + f(x_n)]$$

Moreover, as $n \to \infty$, the right-hand side approaches $\int_a^b f(x)dx$

Simpson's Rule

Let f be continuous on $[a,\ b]$ and let n be an even integer. Simpson's Rule for approximating $\displaystyle\int_a^b f(x)dx$ is:

$$\int_a^b f(x)dx \approx \frac{b - a}{3n}[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \cdots\ 4f(x_{n-1}) + f(x_n)]$$

Moreover, as $n \to \infty$, the right-hand side approaches $\displaystyle\int_a^b f(x)dx$

Error in Trapezoidal and Simpson's Rules

$|E| \leq \dfrac{(b - a)^3}{12n^2}[\max|f''(x)|],\ a \leq x \leq b$ for Trapezoidal Rule.

Moreover, if f has a continuous fourth derivative on $[a,\ b]$, then the error E in approximating $\displaystyle\int_a^b f(x)dx$ by Simpson's Rule is:

$|E| \leq \dfrac{(b - a)^5}{180n^4}[\max|f^{(4)}(x)|],\ a \leq x \leq b$ for Simpson's Rule.

Chapter 10

Standard Equation of a Parabola

The **standard form** of the equation of a parabola with vertex $(h,\ k)$ and directrix $y = k - p$ is:

$$(x - h)^2 = 4p(y - k), \ \ \text{vertical axis}$$

For directrix $x = h - p$ the equation is:

$$(y - k)^2 = 4p(x - h), \ \ \text{horizontal axis}$$

The focus lies on the axis p units (directed distance) from the vertex. The coordinates of the focus are as follows:

$$(h,\ k + p), \ \ \text{vertical axis}$$

and

$$(h + p,\ k), \ \ \text{horizontal axis}$$

Standard Equation of an Ellipse

The standard form of the equation of an ellipse with center $(h,\ k)$ and major and minor axes of lengths $2a$ and $2b$, where $a > b$, is:

$$\frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1, \ \ \text{major axis is horizontal}$$

or:

$$\frac{(x - h)^2}{b^2} + \frac{(y - k)^2}{a^2} = 1, \ \ \text{major axis is vertical}$$

with $c^2 = a^2 - b^2$

Definition of Eccentricity of an Ellipse

The **eccentricity** e of an ellipse is given by the ratio:

$$e = \frac{c}{a}$$

Standard Equation of a Hyperbola

The standard form of the equation of a hyperbola with center at (h, k) is:

$$\frac{(x - h)^2}{a^2} - \frac{(y - k)^2}{b^2} = 1, \text{ transverse axis is horizontal}$$

or:

$$\frac{(y - k)^2}{a^2} - \frac{(x - h)^2}{b^2} = 1, \text{ transverse axis is vertical}$$

where $c^2 = a^2 + b^2$

Parametric Form of the Derivative

If a smooth curve C is given by the equations:

$$x = f(t) \text{ and } y = g(t)$$

then the slope of C at (x, y) is:

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}, \ \frac{dx}{dt} \neq 0$$

Arc Length in Parametric Form

If a smooth curve C is given by $x = f(t)$ and $y = g(t)$ such that C does not intersect itself on the interval $a \le t \le b$ (except possibly at the endpoints), then the arc length of C over the interval is given by:

$$s = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt = \int_a^b \sqrt{[f'(t)]^2 + [g'(t)]^2} dt$$

Area of a Surface of Revolution

If a smooth curve C given by $x = f(t)$ and $y = g(t)$ does not cross itself on an interval $a \le t \le b$, then the area S of the surface of revolution formed by revolving C about the coordinate axes is given by the following:

1. $S = 2\pi \int_a^b g(t) \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt,$ revolution about x-axis

2. $S = 2\pi \int_a^b f(t) \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt,$ revolution about y-axis

Coordinate Conversion

The polar coordinates $(r,\ \theta)$ of a point are related to the rectangular coordinates $(x,\ y)$ of the point as follows.

Polar-to-Rectangular: $x = r \cdot \cos\theta$ and $y = r \cdot \sin\theta$

Rectangular-to-Polar: $\tan\theta = \frac{y}{x}$ and $r^2 = x^2 + y^2$

Polar Equations of Conics

The graph of a polar equation of the form:

$$r = \frac{ed}{1 \pm e \cdot \cos\theta} \text{ or } r = \frac{ed}{1 \pm e \cdot \sin\theta}$$

is a conic, where $e > 0$ is the eccentricity and $|d|$ is the distance between the focus at the pole and its corresponding directrix.

Slope in Polar Form

If f is a differentiable function of θ, then the *slope* of the tangent line to the graph of $r = f(\theta)$ at the point $(r,\ \theta)$ is:

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{f(\theta)\cos\theta + f'(\theta)\sin\theta}{-f(\theta)\sin\theta + f'(\theta)\cos\theta}$$

provided that $dx/d\theta \neq 0$ at $(r,\ \theta)$.

Area in Polar Coordinates

If f is continuous and nonnegative on the interval $[\alpha, \beta]$, $0 < \beta - \alpha \leq 2\pi$, then the area of the region bounded by the graph of $r = f(\theta)$ between the radial lines $\theta = \alpha$ and $\theta = \beta$ is:

$$A = \frac{1}{2}\int_{\alpha}^{\beta} [f(\theta)]^2 d\theta = \frac{1}{2}\int_{\alpha}^{\beta} r^2 d\theta,\ 0 < \beta - \alpha \leq 2\pi$$

Arc Length of a Polar Curve

Let f be a function whose derivative is continuous on an interval $\alpha \leq \theta \leq \beta$. The length of the graph of $r = f(\theta)$ from $\theta = \alpha$ to $\theta = \beta$ is:

$$s = \int_{\alpha}^{\beta} \sqrt{[f(\theta)]^2 + [f'(\theta)]^2} d\theta = \int_{\alpha}^{\beta} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} d\theta$$

Area of a Surface of Revolution

Let f be a function whose derivative is continuous on an interval $\alpha \leq \theta \leq \beta$. The area of the surface formed by revolving the graph $r = f(\theta)$ from $\theta = \alpha$ to $\theta = \beta$ about the indicated line is as follows:

1. $S = 2\pi \int_{\alpha}^{\beta} f(\theta)\sin(\theta) \cdot \sqrt{[f(\theta)]^2 + [f'(\theta)]^2} d\theta,$ about the polar axis

2. $S = 2\pi \int_{\alpha}^{\beta} f(\theta)\cos(\theta) \cdot \sqrt{[f(\theta)]^2 + [f'(\theta)]^2} d\theta,$ about the line $\theta = \frac{\pi}{2}$

Classification of Conics by Eccentricity

Let f be a fixed point (*focus*) and let D be a fixed line (*directrix*) in the plane. Let P be another point in the plane and let e (*eccentricity*) be the ratio of the distance between P and F to the distance between P and D. The collection of all points P with a given eccentricity is a conic:

1. The conic is an ellipse for $0 < e < 1$.

2. The conic is a parabola for $e = 1$.

3. The conic is a hyperbola for $e > 1$.

Classification of Conics in Algebraic Form

When a conic is in the form $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$, the determinant of the equation shows:

1. If $B^2 - 4AC > 0$, the conic is a hyperbola

2. If $B^2 - 4AC < 0$, the conic is an ellipse or circle.

3. If $B^2 - 4AC = 0$, the conic is a hyperbola

Test	**Series**	**Convergence**	**Divergence**	**Comment**
nth-Term	$\sum_{n=1}^{\infty} a_n$		$\lim_{n\to\infty} a_n \neq 0$	Test cannot show convergence.
Geometric Series	$\sum_{n=1}^{\infty} ar^n$	$0 < \lvert r \rvert < 1$	$\lvert r \rvert \geq 1$	Sum: $S = \frac{a}{1-r}$
Telescoping Series	$\sum_{n=1}^{\infty} (b_n - b_{n+1})$	$\lim_{n\to\infty} b_n = L$		Sum: $S = b_1 - L$
p-Series	$\sum_{n=1}^{\infty} \frac{1}{n^p}$	$p > 1$	$0 < p \leq 1$	
Alternating Series	$\sum_{n=1}^{\infty} (-1)^{n-1} a_n$	$0 < a_{n+1} \leq a_n$ and $\lim_{n\to\infty} a_n = 0$		Remainder: $\lvert R_N \rvert \leq a_{N+1}$
Integral	$\sum_{n=1}^{\infty} a_n,\ a_n = f(n) > 0$	$\int_1^{\infty} f(x)dx$ converges	$\int_1^{\infty} f(x)dx$ diverges	Remainder: $0 < R_N < \int_N^{\infty} f(x)dx$
Root	$\sum_{n=1}^{\infty} a_n$	$\lim_{n\to\infty} \sqrt[n]{\lvert a_n \rvert} < 1$	$\lim_{n\to\infty} \sqrt[n]{\lvert a_n \rvert} > 1$ or $= \infty$	Test inconclusive when $\lim_{n\to\infty} \sqrt[n]{\lvert a_n \rvert} = 1$
Ratio	$\sum_{n=1}^{\infty} a_n$	$\lim_{n\to\infty} \left\lvert \frac{a_{n+1}}{a_n} \right\rvert < 1$	$\lim_{n\to\infty} \left\lvert \frac{a_{n+1}}{a_n} \right\rvert > 1$ or $= \infty$	Test inconclusive when $\lim_{n\to\infty} \left\lvert \frac{a_{n+1}}{a_n} \right\rvert = 1$
Direct Comparison	$\sum_{n=1}^{\infty} a_n$	$0 < a_n \leq b_n$ and $\sum_{n=1}^{\infty} b_n$ converges	$0 < b_n \leq a_n$ and $\sum_{n=1}^{\infty} b_n$ diverges	
Limit Comparison	$\sum_{n=1}^{\infty} a_n$	$\lim_{n\to\infty} \frac{a_n}{b_n} = L > 0$ and $\sum_{n=1}^{\infty} b_n$ converges	$\lim_{n\to\infty} \frac{a_n}{b_n} = L > 0$ and $\sum_{n=1}^{\infty} b_n$ diverges	

Chapter 11

Alternating Series Remainder

If a convergent alternating series satisfies the condition $a_{n+1} \leq a_n$, then the absolute value of the remainder R_N involved in approximating the sum S by S_N is less than (or equal to) the first neglected term. That is,

$$|S - S_N| = |R_N| \leq a_{N+1}$$

Definition of Taylor and Maclaurin Series

If a function f has derivatives of all orders at $x = c$, then the series:

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(c)}{n!}(x - c)^n = f(c) + f'(c)(x - c) + \cdots + \frac{f^{(n)}(c)}{n!}(x - c)^n + \cdots$$

is called the **Taylor series for *f(x)* at *c***. Moreover, if $c = 0$, then the series is the **Maclaurin series for *f***.

Taylor's Theorem

If a function f is differentiable through order $n + 1$ in an interval I containing c, then, for each x in I, there exists z between x and c such that:

$$f(x) = f(c) + f'(c)(x - c) + \frac{f''(c)}{2!}(x - c)^2 + \cdots + \frac{f^{(n)}(c)}{n!}(x - c)^n + R_n(x)$$

where:

$$R_n(x) = \frac{f^{(n+1)}(z)}{(n+1)!}(x - c)^{n+1}$$

Power Series for Elementary Functions

$$\frac{1}{x} = 1-(x-1)+(x-1)^2-(x-1)^3+(x-1)^4- \cdots +(-1)^n(x-1)^n+ \cdots, \text{ Converges: } 0 < x < 2$$

$$\frac{1}{1+x} = 1-x+x^2-x^3+x^4-x^5+ \cdots +(-1)^n x^n + \cdots, \text{ Converges: } -1 < x < 1$$

$$\ln x = (x-1) - \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3} - \frac{(x-1)^4}{4} + \cdots + \frac{(-1)^{n-1}(x-1)^n}{n} + \cdots, \text{ Converges: } 0 < x \leq 2$$

$$e^x = 1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\frac{x^4}{4!}+\frac{x^5}{5!}+ \cdots + \frac{x^n}{n!} + \cdots, \text{ Converges: } -\infty < x < \infty$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} - \cdots + \frac{(-1)^n x^{2n+1}}{(2n+1)!} + \cdots, \text{ Converges: } -\infty < x < \infty$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \cdots + \frac{(-1)^n x^{2n}}{(2n)!} + \cdots, \text{ Converges: } -\infty < x < \infty$$

$$\arctan x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \frac{x^9}{9} - \cdots + \frac{(-1)^n x^{2n+1}}{2n+1} + \cdots, \text{ Converges: } -1 \leq x \leq 1$$

$$\arcsin x = x + \frac{x^3}{2\cdot 3} + \frac{1\cdot 3x^5}{2\cdot 4\cdot 5} + \frac{1\cdot 3\cdot 5x^7}{2\cdot 4\cdot 6\cdot 7} + \cdots + \frac{(2n)!x^{2n+1}}{(2^n n!)^2(2n+1)} + \cdots, \text{ Converges: } -1 \leq x \leq 1$$

$$(1+x)^k = 1+kx+\frac{k(k-1)x^2}{2!}+\frac{k(k-1)(k-2)x^3}{3!}+\frac{k(k-1)(k-2)(k-3)x^4}{4!}+\cdots, \text{ Converges: } -1 < x < 1^*,$$

* Could also converge at $x = \pm 1$ depending on k.

Function Comparisons as Limit goes to Infinity

$$\ln(n) \ll \sqrt[p]{n} \ll n \ll n^p \ll b^n \ll n! \ll n^n$$

Chapter 12

Angle Between Two Vectors

If θ is the angle between two nonzero vectors $\mathbf{u}$ and $\mathbf{v}$, where $0 \leq \theta \leq \pi$, then:

$$\cos\theta = \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\| \; \|\mathbf{v}\|}$$

Projection Using the Dot Product

If $\mathbf{u}$ and $\mathbf{v}$ are nonzero vectors, then the projection of $\mathbf{u}$ onto $\mathbf{v}$ is:

$$\text{proj}_{\mathbf{v}}\mathbf{u} = \left(\frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{v}\|^2}\right)\mathbf{v}$$

Definition of Cross Product of Two Vectors in Space

Let:

$\mathbf{u} = u_1\mathbf{i} + u_2\mathbf{j} + u_3\mathbf{k}$ and $\mathbf{v} = v_1\mathbf{i} + v_2\mathbf{j} + v_3\mathbf{k}$

be vectors in space. The **cross product** of $\mathbf{u}$ and $\mathbf{v}$ is the vector:

$$\mathbf{u} \times \mathbf{v} = (u_2v_3 - u_3v_2)\mathbf{i} - (u_1v_3 - u_3v_1)\mathbf{j} + (u_1v_2 - u_2v_1)\mathbf{k}$$

Geometric Property of the Triple Scalar Product

The volume V of the parallelpiped with vectors $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ as adjacent edges is:

$$V = |\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w})|$$

Standard Equation of a Plane in Space

The plane containing the point $(x_1,\ y_1,\ z_1)$ and having normal vector:

$$\mathbf{n}\ =\ \langle a,\ b,\ c\rangle$$

can be represented by the **standard form** of the equation of a plane:

$$a(x\ -\ x_1)\ +\ b(y\ -\ y_1)\ +\ c(z\ -\ z_1)\ =\ 0.$$

Surface of Revolution

If the graph of a radius function r is revolving about one of the coordinates axes, then the equation of the resulting surface of revolution has one of the forms listed below.

1. Revolved about the x-axis: $y^2 + z^2 = [r(x)]^2$

2. Revolved about the y-axis: $x^2 + z^2 = [r(y)]^2$

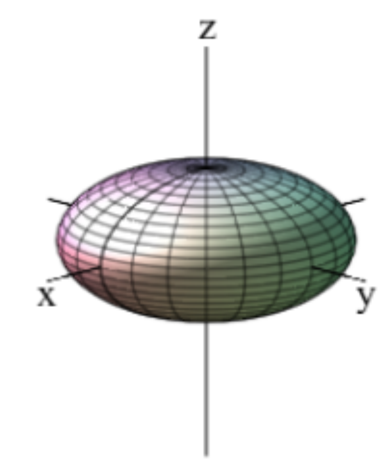
3. Revolved about the z-axis: $x^2 + y^2 = [r(z)]^2$

Ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

Trace	Plane
Ellipse	Parallel to xy-plane
Ellipse	Parallel to xz-plane
Ellipse	Parallel to yz-plane

The surface is a sphere when: $a = b = c \neq 0$.

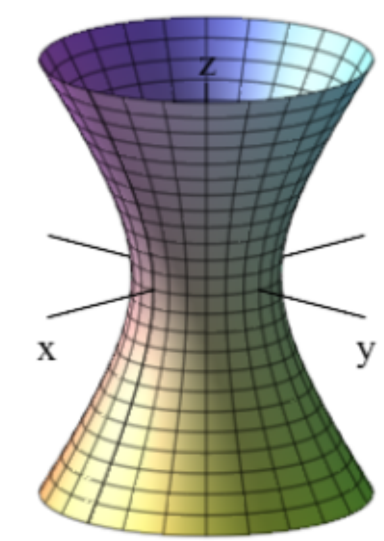


Hyperboloid of One Sheet

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$$

Trace	Plane
Ellipse	Parallel to xy-plane
Hyperbola	Parallel to xz-plane
Hyperbola	Parallel to yz-plane

The axis of the hyperboloid corresponds to the variable whose coefficient is negative.

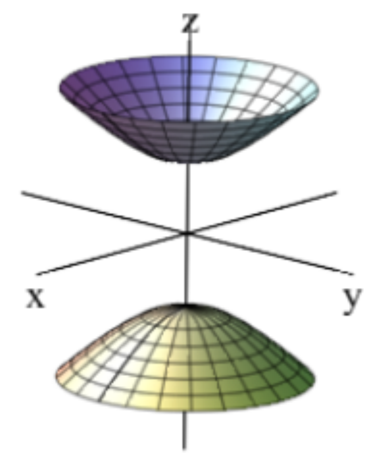


Hyperboloid of Two Sheets

$$\frac{z^2}{c^2} - \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Trace	Plane
Ellipse	Parallel to xy-plane
Hyperbola	Parallel to xz-plane
Hyperbola	Parallel to yz-plane

The axis of the hyperboloid corresponds to the variable whose coefficient is positive.
There is no trace in the coordinate plane perpendicular to this axis.

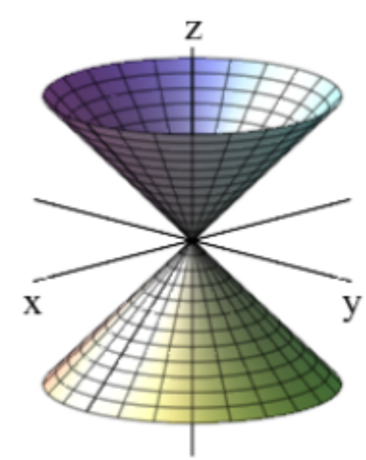


Elliptic Cone

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 0$$

Trace	Plane
Ellipse	Parallel to xy-plane
Hyperbola	Parallel to xz-plane
Hyperbola	Parallel to yz-plane

The axis of the cone corresponds to the variable whose coefficient is negative.
The traces in the coordinate planes parallel to this axis are intersecting lines.

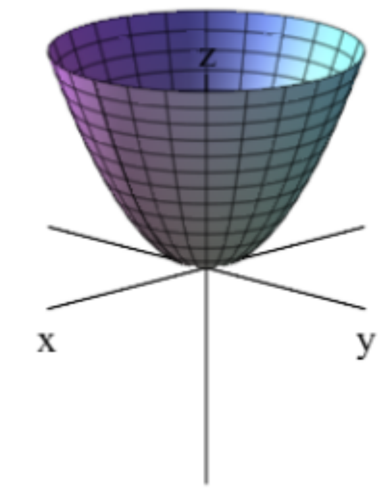


Elliptic Paraboloid

$$z = \frac{x^2}{a^2} + \frac{y^2}{b^2}$$

Trace	Plane
Ellipse	Parallel to *xy*-plane
Parabola	Parallel to *xz*-plane
Parabola	Parallel to *yz*-plane

The axis of the paraboloid corresponds to the variable raised to the first power.

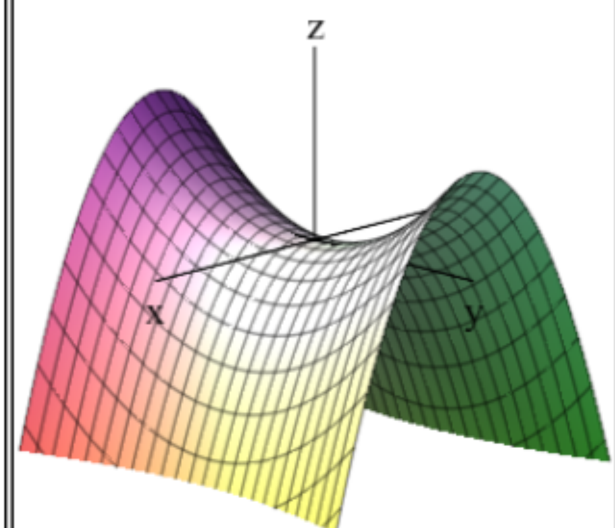


Hyerbolic Paraboloid

$$z = \frac{y^2}{b^2} - \frac{x^2}{a^2}$$

Trace	Plane
Hyperbola	Parallel to *xy*-plane
Parabola	Parallel to *xz*-plane
Parabola	Parallel to *yz*-plane

The axis of the paraboloid corresponds to the variable raised to the first power.



Distance Between a Point and a Plane

The distance between a plane and a point Q (not in the plane) is:

$$D = \|\text{proj}_{\mathbf{n}}\overrightarrow{PQ}\| = \frac{|\overrightarrow{PQ} \cdot \mathbf{n}|}{\|\mathbf{n}\|}$$

where P is a point in the plane and n is normal to the plane.

Distance Between a Point and a Line in Space

The distance between a point Q and a line in space is:

$$D = \frac{\|\overrightarrow{PQ} \times \mathbf{u}\|}{\|\mathbf{u}\|}$$

where $\mathbf{u}$ is a direction vector for the line and P is a point on the line.

The Cylindrical Coordinate System

In a **cylindrical coordinate system**, a point P in space is represented by an ordered triple $(r,\ \theta,\ z)$:

1. $(r,\ \theta)$ is a polar representation of the projection of P in the xy-plane.

2. z is the directed distance from $(r,\ theta)$ to P.

To convert from rectangular to cylindrical coordinates (or vice versa), use the conversion guidelines for polar coordinates listed below:

Cylindrical-to-Rectangular $x = r\cos\theta,\ y = r\sin\theta,\ z = z$

Rectangular-to-Cylindrical $r^2 = x^2 + y^2,\ \tan\theta = \dfrac{y}{x},\ z = z$

The Spherical Coordinate System

In a **spherical coodinate system**, a point P in space is represented by an ordered triple $(\rho,\ \theta\ \ \phi)$, where ρ is the lowercase Greek letter *rho* and ϕ is the lowercase Greek letter *phi*.

1. ρ is the distance between P and the origin, $\rho\ \geq\ 0$

2. θ is the same angle used in cylindrical coordinates for $r\ \geq\ 0$

3. ϕ is the angle *between* the positive z-axis and the line segment $\overrightarrow{OP}$, $0\ \leq\ \phi\ \leq\ \pi$

Note that the first and third coordinates, ρ and ϕ are nonnegative.

Spherical-to-Rectangular

$$x\ =\ \rho\sin\phi\cos\theta,\ y\ =\ \rho\sin\phi\sin\theta,\ z\ =\ \rho\cos\phi$$

To convert from one system to the other, use the conversion guidelines listed below.

Rectangular-to-Spherical

$$\rho^2\ =\ x^2\ +\ y^2\ +\ z^2,\ \tan\theta\ =\ \frac{y}{x},\ \phi\ =\ \arccos\left(\frac{z}{\sqrt{x^2\ +\ y^2\ +\ z^2}}\right)$$

To change coordinates between cylindrical and spherical systems, use the conversion guidelines listed below.

Spherical-to-Cylindrical ($r\ \geq\ 0$)

$$r^2\ =\ \rho^2\sin^2\phi,\ \theta\ =\ \theta,\ z\ =\ \rho\cos\phi$$

Cylindrical-to-Spherical $(r \geq 0)$

$$\rho = \sqrt{r^2 + z^2},\ \theta = \theta,\ \phi = \arccos\left(\frac{z}{\sqrt{r^2 + z^2}}\right)$$

Chapter 13

Arc Length of a Space Curve

If C is a smooth curve given by $\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j} + z(t)\mathbf{k}$ on an interval $[a, b]$, then the arc length of C on the interval is:

$$s = \int_a^b \sqrt{[x'(t)]^2 + [y'(t)]^2 + [z'(t)]^2}dt = \int_a^b \|\mathbf{r}'(t)\|dt$$

Summary of Velocity, Acceleration, and Curvature

Unless noted otherwise, let C be a curve (in the plane or in space) given by the position vector:

$$\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j},\ \text{Curve in the plane}$$

$$\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j} + z(t)\mathbf{k},\ \text{Curve in space}$$

where x, y, and z are twice-differentiable functions of t.

Velocity vector, speed, and acceleration vector

$$\mathbf{v}(t) = \mathbf{r}'(t),\ \text{velocity vector}$$

$$\|\mathbf{v}(t)\| = \frac{ds}{dt} = \|\mathbf{r}'(t)\|,\ \text{speed}$$

$$\mathbf{a}(t) = \mathbf{r}''(t),\ \text{acceleration vector}$$

$\mathbf{a}(t) = a_{\mathbf{T}}\mathbf{T}(t) + a_{\mathbf{N}}\mathbf{N}(t) = \dfrac{d^2s}{dt^2}\mathbf{T}(t) + K\left(\dfrac{ds}{dt}\right)^2\mathbf{N}(t)$, K is curvature and $\dfrac{ds}{dt}$ is speed

Unit tangent vector and principle unit normal vector

$$\mathbf{T}(t) = \frac{\mathbf{r}'(t)}{\|\mathbf{r}'(t)\|}, \text{ Unit tangent vector}$$

$$\mathbf{N}(t) = \frac{\mathbf{T}'(t)}{\|\mathbf{T}'(t)\|}, \text{ Principle unit normal vector}$$

Components of acceleration

$$a_{\mathbf{T}} = \mathbf{a} \cdot \mathbf{T} = \frac{\mathbf{v} \cdot \mathbf{a}}{\|\mathbf{v}\|} = \frac{d^2 s}{dt^2}, \text{ Tangential component of acceleration}$$

$$a_{\mathbf{N}} = \mathbf{a} \cdot \mathbf{N} = \frac{\|\mathbf{v} \times \mathbf{a}\|}{\|\mathbf{v}\|} = \sqrt{\|\mathbf{a}\|^2 - a_{\mathbf{T}}^2}, \text{ Normal component of acceleration}$$

$$a_{\mathbf{N}} = K\left(\frac{ds}{dt}\right)^2, \ K \text{ is curvature and } \frac{ds}{dt} \text{ is speed}$$

Formulas for curvature in the plane

$$K = \frac{|y''|}{[1 + (y')^2]^{3/2}}, \ C \text{ given by } y = f(x)$$

$$K = \frac{|x'y'' - y'x''|}{[(x')^2 + (y')^2]^{3/2}}, \ C \text{ given by } x = x(t), \ y = y(t)$$

Formulas for curvature in the plane or in space

$$K = \|\mathbf{T}'(s)\| = \|\mathbf{r}''(s)\|, \ s \text{ is arc length parameter}$$

$$K = \frac{\|\mathbf{T}'(t)\|}{\|\mathbf{r}'(t)\|} = \frac{\|\mathbf{r}'(t) \times \mathbf{r}''(t)\|}{\|\mathbf{r}'(t)\|^3}, \ t \text{ is general parameter}$$

$$K = \frac{\mathbf{a}(t) \cdot \mathbf{N}(t)}{\|\mathbf{v}(t)\|^2}$$

Cross product formulas apply only to curves in space.

Alternative Formula for Principle Unit Normal Vector

$$\mathbf{N} = \frac{(\mathbf{v}\cdot\mathbf{v})\mathbf{a} - (\mathbf{v}\cdot\mathbf{a})\mathbf{v}}{\|(\mathbf{v}\cdot\mathbf{v})\mathbf{a} - (\mathbf{v}\cdot\mathbf{a})\mathbf{v}\|}$$

Chapter 14

Increments and Differentials

In this section, the concepts of increments and differentials are generalized functions of two or more variables. Recall that for $y = f(x)$, the differential of y was defined as:

$$dy = f'(x)dx$$

Similar terminology is used for a function of two variables, $z = f(x, y)$. That is, Δx and Δy are the **increments of *x* and *y***, and the **increment of *z*** is:

$$\Delta z = f(x + \Delta x, y + \Delta y) - f(x, y)$$

Definition of Total Differential

If $z = f(x, y)$, and Δx and Δy are increments of x and y, then the **differentials** of the independent variables x and y are:

$$dx = \Delta x \text{ and } dy = \Delta y$$

and the **total differential** of the dependent variable z is:

$$dz = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy = f_x(x, y)dx + f_y(x, y)dy$$

Chain Rule: Implicit Differentiation

If the equation $F(x, y) = 0$ defines y implicitly as a differentiable function of x, then:

$$\frac{dy}{dx} = -\frac{F_x(x, y)}{F_y(x, y)},\ F_y(x, y) \neq 0$$

If the equation $F(x, y, z) = 0$ defines z implicitly as a differentiable function of x and y, then:

$$\frac{\partial z}{\partial x} = -\frac{F_x(x, y, z)}{F_z(x, y, z)} \text{ and } \frac{\partial z}{\partial y} = -\frac{F_y(x, y, z)}{F_z(x, y, z)},\ F_z(x, y, z) \neq 0$$

Alternative Form of the Directional Derivative

If f is a differentiable function of x and y, then the directional derivative of f in the direction of the unit vector **u** is:

$$D_{\mathbf{u}} f(x,\ y) = \nabla f(x,\ y)) \cdot \mathbf{u}$$

Second Partials Test

Let f have continuous second partial derivatives on an open region containing a point $(a,\ b)$ for which:

$$f_x(a,\ b) = 0 \text{ and } f_y(a,\ b) = 0$$

To test for relative extrema of f, consider the quantity:

$$d = f_{xx}(a,\ b) f_{yy}(a,\ b) - [f_{xy}(a,\ b)]^2$$

1. If $d > 0$ and $f_{xx}(a,\ b) > 0$, then f has a **relative minimum** at $(a,\ b)$

2. If $d > 0$ and $f_{xx}(a,\ b) < 0$, then f has a **relative maximum** at $(a,\ b)$

3. If $d < 0$, then $(a,\ b,\ f(a, b))$ is a **saddle point**

4. The test is inconclusive if $d = 0$

Sum of the Squared Errors

$$S = \sum_{i=1}^{n}[f(x_i) - y_i]^2$$

Least Squares Regression Line

The **least squares regression line** for $\{(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)\}$ is given by $f(x) = ax + b$, where:

$$a = \frac{n\sum_{i=1}^{n} x_i y_i - \sum_{i=1}^{n} x_i \sum_{i=1}^{n} y_i}{n\sum_{i=1}^{n} x_i^2 - \left(\sum_{i=1}^{n} x_i\right)^2} \text{ and } b = \frac{1}{n}\left(\sum_{i=1}^{n} y_i - a\sum_{i=1}^{n} x_i\right)$$

Lagrange's Theorem

Let f and g have continuous first partial derivatives such that f has an extremum at a point (x_0, y_0) on the smooth constraint curve $g(x, y) = c$. If $\nabla g(x_0, y_0) \neq \mathbf{0}$, then there is a real number λ such that:

$$\nabla f(x_0, y_0) = \lambda \nabla g(x_0, y_0)$$

Chapter 15

Definition of Mass of a Planar Lamina of Variable Density

If ρ is a continuous density function on the lamina corresponding to a plane region R, then the mass m of the lamina is given by:

$$m = \iint_R \rho(x, y)\,dA, \quad \text{variable density}$$

Moments and Center of Mass of a Variable Density Planar Lamina

Let ρ be a continuous density function on the planar lamina R. The **moments of mass** with respect to the x- and y-axes are:

$$M_x = \iint_R y\rho(x,\ y)dA \text{ and}$$

$$M_y = \iint_R x\rho(x,\ y)dA$$

If m is the mass of the lamina, then the **center of mass** is:

$$(\bar{x},\ \bar{y}) = \left(\frac{M_y}{m},\ \frac{M_x}{m}\right)$$

If R represents a simple plane region rather than a lamina, then the point $(\bar{x},\ \bar{y})$ is called the **centroid** of the region.

Moments of Inertia

$$I_x = \iint_R (y^2)\rho(x,\ y)dA \text{ and } I_y = \iint_R (x^2)\rho(x,\ y)dA$$

Definition of Surface Area

If f and its first partial derivatives are continuous on the closed region R in the xy-plane, then the **area of the surface** $\boldsymbol{S}$ given by $z = f(x,\ y)$ over R is defined as:

$$\text{Surface area} = \iint_R dS = \iint_R \sqrt{1 + [f_x(x,\ y)]^2 + [f_y(x,\ y)]^2}dA$$

Definition of the Jacobian

If $x = g(u,\ v)$ and $y = h(u,\ v)$, then the **Jacobian** of x and y with respect to u and v, denoted by $\partial(x,\ y)/\partial(u,\ v)$, is:

$$\frac{\partial(x,\ y)}{\partial(u,\ v)} = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix} = \frac{\partial x}{\partial u}\frac{\partial y}{\partial v} - \frac{\partial y}{\partial u}\frac{\partial x}{\partial v}$$

Change of Variables for Double Integrals

Let R be a vertically or horizontally simple region in the xy-plane, and let S be a vertically or horizontally simple region in the uv-plane. Let T from S to R be given by $T(u,\ v) = (x,\ y) = (g(u,v),\ h(u,v))$, where, g and h have continuous first partial derivatives. Assume that T is one-to-one expect possibly on the boundary of S. If f is continuous on R, and $\partial(x,\ y)/\partial(u,\ v)$, is nonzero on S,

$$\iint_R f(x,\ y)dxdy = \iint_S f(g(u,v),\ h(u,v)) \left|\frac{\partial(x,\ y)}{\partial(u,\ v)}\right| dudv$$

Chapter 16

Definition of Curl of a Vector Field

The curl of $\mathbf{F}(x,\ y,\ z) = M\mathbf{i} + N\mathbf{j} + P\mathbf{k}$ is:

$$\text{curl}\,\mathbf{F}(x,\ y,\ z) = \nabla \times \mathbf{F}(x,\ y,\ z) = \left(\frac{\partial P}{\partial y} - \frac{\partial N}{\partial z}\right)\mathbf{i} - \left(\frac{\partial P}{\partial x} - \frac{\partial M}{\partial z}\right)\mathbf{j} + \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right)\mathbf{k}$$

If curl $\mathbf{F} = \mathbf{0}$, then $\mathbf{F}$ is said to be **irrotational**.

Test for Conservative Vector Field in Space

Suppose that M, N, and P have continuous first partial derivatives in an open sphere Q in space. The vector field:

$$\mathbf{F}(x,\ y,\ z)\ =\ M\mathbf{i}\ +\ N\mathbf{j}\ +\ P\mathbf{k}$$

is conservative if and only if:

$$\text{curl}\ \mathbf{F}(x,\ y,\ z)\ =\ \mathbf{0}$$

That is, **F** is conservative if and only if:

$$\frac{\partial P}{\partial y}\ =\ \frac{\partial N}{\partial z},\ =\ \frac{\partial P}{\partial x}\ =\ \frac{\partial M}{\partial z},\ \text{and}\ \frac{\partial N}{\partial x}\ =\ \frac{\partial M}{\partial y}$$

Definition of Divergence of a Vector Field

The **divergence** of $\mathbf{F}(x,\ y)\ =\ M\mathbf{i}\ +\ N\mathbf{j}$ is:

$$\text{div}\ \mathbf{F}(x,\ y)\ =\ \nabla\ \cdot\ \mathbf{F}(x,\ y)\ =\ \frac{\partial M}{\partial x}\ +\ \frac{\partial N}{\partial y},\ \text{Plane}$$

The **divergence** of $\mathbf{F}(x,\ y,\ z)\ =\ M\mathbf{i}\ +\ N\mathbf{j}\ +\ P\mathbf{k}$ is:

$$\text{div}\ \mathbf{F}(x,\ y,\ z)\ =\ \nabla\ \cdot\ \mathbf{F}(x,\ y,\ z)\ =\ \frac{\partial M}{\partial x}\ +\ \frac{\partial N}{\partial y}\ +\ \frac{\partial P}{\partial z},\ \text{Space}$$

If div $\mathbf{F}\ =\ 0$, then **F** is said to be **divergence free**.

Definition of the Line Integral of a Vector Field

Let **F** be a continuous vector field defined on a smooth curve C given by:

$$\mathbf{r}(t),\ a\ \leq\ t\ \leq\ b$$

The **line integral** of **F** on C is given by:

$$\int_C \mathbf{F}\ \cdot\ d\mathbf{r}\ =\ \int_C \mathbf{F}\ \cdot\ \mathbf{T}ds\ =\ \int_a^b \mathbf{F}(x(t),\ y(t),\ z(t))\ \cdot\ \mathbf{r}'(t)dt$$

Fundamental Theorem of Line Integrals

Let C be a piecewise smooth curve lying in an open region R and given by:

$$\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j},\ a \le t \le b$$

If $\mathbf{F}(x, y) = M\mathbf{i} + N\mathbf{j}$ is conservative in R, and M and N are continuous in R, then:

$$\int_C \mathbf{F} \cdot d\mathbf{r} = \int_C \nabla f \cdot d\mathbf{r} = f(x(b), y(b)) - f(x(a), y(a))$$

where f is a potential function of F. That is $\mathbf{F}(x, y) = \nabla f(x, y)$

Green's Theorem

Let R be a simply conneted region with a piecewise smooth boundary C, oriented counterclockwise (that is, C is traversed *once* so that the region R always lies to the *left*). If M and N have continuous first partial derivatives in an open region containing R, then:

$$\int_C M dx + N dy = \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \right) dA$$

Line Integral for Area

If R is a plane region bounded by a piecewise smooth simple closed curve C, oriented counterclockwise, then the area of R is given by:

$$A = \frac{1}{2} \int_C x dy - y dx$$

Area of a Parametric Surface

Let S be a smooth parametric surface:

$$\mathbf{r}(u,\ v) \ = \ x(u,\ v)\mathbf{i} \ + \ y(u,\ v)\mathbf{j} \ + \ z(u,\ v)\mathbf{k}$$

defined over an open region D in the uv-plane. If each point on the surface S corresponds to exactly one point in the domain of D, then the **surface area** of S is given by:

$$\text{Surface area} \ = \ \iint_S dS \ = \ \iint_D \|\mathbf{r}_u \ \times \ \mathbf{r}_v\| dA$$

where:

$$\mathbf{r}_u \ = \ \frac{\partial x}{\partial u}\mathbf{i} \ + \ \frac{\partial y}{\partial u}\mathbf{j} \ + \ \frac{\partial z}{\partial u}\mathbf{k} \text{ and } \mathbf{r}_v \ = \ \frac{\partial x}{\partial v}\mathbf{i} \ + \ \frac{\partial y}{\partial v}\mathbf{j} \ + \ \frac{\partial z}{\partial v}\mathbf{k}$$

Evaluating a Surface Integral

Let S be a surface with equation $z \ = \ g(x,\ y)$, and let R be its projection onto the xy-plane. If g, g_x, and g_y are continuous on R and f is continuous on S, then the surface intergral of f over S is:

$$\iint_S f(x,\ y,\ z)dS \ = \ \iint_R f(x,\ y,\ g(x,y))\sqrt{1 \ + \ [g_x(x,\ y)]^2 \ + \ [g_y(x,\ y)]^2}dA$$

Summary of Line and Surface Integrals

Line Integrals

$$ds \ = \ \|\mathbf{r}'(t)\|dt \ = \ \sqrt{[x'(t)]^2 \ + \ [y'(t)]^2 \ + \ [z'(t)]^2}dt$$

$$\int_C f(x,\ y,\ z)ds \ = \ \int_a^b f(x(t),\ y(t),\ z(t))ds, \ \text{ Scalar form}$$

$$\int_C \mathbf{F} \ \cdot \ d\mathbf{r} \ = \ \int_C \mathbf{F} \ \cdot \ \mathbf{T}ds \ = \ \int_a^b \mathbf{F}(x(t),\ y(t),\ z(t)) \ \cdot \ \mathbf{r}'(t)dt, \ \text{ Vector form}$$

Surface Integrals $[z = g(x, y)]$

$dS = \sqrt{1 + [g_x(x, y)]^2 + [g_y(x, y)]^2}dA$

$\iint_S f(x, y, z)dS = \displaystyle\iint_R f(x, y, g(x,y))\sqrt{1 + [g_x(x, y)]^2 + [g_y(x, y)]^2}dA,$ Scalar form

$\iint_S \mathbf{F} \cdot \mathbf{N}dS = \displaystyle\iint_R \mathbf{F} \cdot [-g_x(x, y)\mathbf{i} - g_y(x, y)\mathbf{j} + \mathbf{k}]dA,$ Vector form (upward normal)

Surface Integrals (parametric form)

$dS = \|\mathbf{r}_u(u, v) \times \mathbf{r}_v(u, v)\|dA$

$\displaystyle\iint_S f(x, y, z)dS = \iint_D f(x(u,v), y(u,v), z(u,v))dS,$ Scalar form

$\displaystyle\iint_S \mathbf{F} \cdot \mathbf{N}dS = \iint_D \mathbf{F} \cdot (\mathbf{r}_u \times \mathbf{r}_v)dA,$ Vector form

The Divergence Theorem

Let Q be a solid region bounded by a closed surface S oriented by a unit normal vector directed outward from Q. If $\mathbf{F}$ is a vector field whose component functions have continuous first partial derivatives in Q, then:

$$\iint_S \mathbf{F} \cdot \mathbf{N}dS = \iiint_Q \text{div } \mathbf{F}dV$$

Stoke's Theorem

Let S be an oriented surface with unit normal vector $\mathbf{N}$, bounded by a piecewise smooth simple closed curve C with a positive orientation. If $\mathbf{F}$ is a vector field whose component functions have continuous first partial derivatives on an open region containing S and C, then:

$$\int_C \mathbf{F} \cdot d\mathbf{r} = \iint_S (\text{curl } \mathbf{F}) \cdot \mathbf{N} dS$$

Summary of Integration Formulas

Fundamental Theorem of Calculus

$$\int_a^b F'(x)dx = F(b) - F(a)$$

Fundamental Theorem of Line Integrals

$$\textstyle\int_C \mathbf{F} \cdot d\mathbf{r} = \displaystyle\int_C \nabla f \cdot d\mathbf{r} = f(x(b),\ y(b)) - f(x(a),\ y(a))$$

Green's Theorem

$$\textstyle\int_C M dx + N dy = \displaystyle\iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dA = \int_C \mathbf{F} \cdot \mathbf{T} ds = \int_C \mathbf{F} \cdot d\mathbf{r} = \iint_R (\text{curl } \mathbf{F}) \cdot \mathbf{k} dA$$

$$\int_C \mathbf{F} \cdot \mathbf{N} ds = \iint_R \text{div } \mathbf{F} dA$$

Divergence Theorem

$$\iint_S \mathbf{F} \cdot \mathbf{N} dS = \iiint_Q \text{div } \mathbf{F} dV$$

Stoke's Theorem

$$\int_C \mathbf{F} \cdot d\mathbf{r} = \iint_S (\text{curl } \mathbf{F}) \cdot \mathbf{N} dS$$