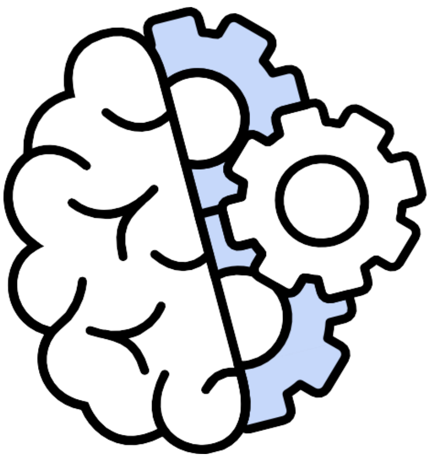
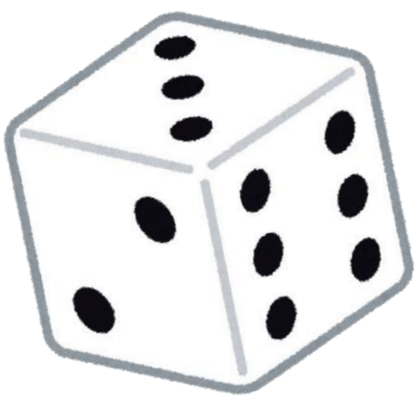
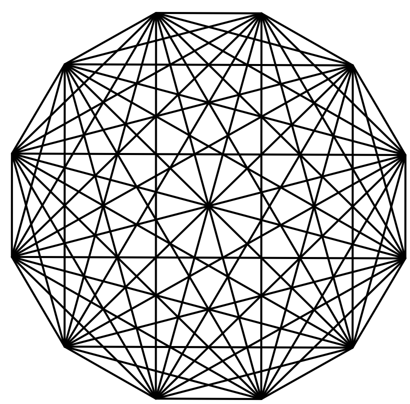
Contemporary Math Guide







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Contents

Introduction to Sets

What’s a Set?

Think of a set like a *box*. Inside this box, you can put things—any things you want. The stuff inside the box is called *elements*.

Example of a Set:

- Imagine you have a box, and you put in a red ball, a blue ball, and a green ball. We can call this set "The Set of Balls." So, your set looks like this: {red ball, blue ball, green ball}.

The Rules of Sets:

1. **No Repeats Allowed:** If you have two red balls, you only need to mention it once. So, even if you have {red ball, red ball, blue ball}, it’s still just {red ball, blue ball}.
2. **Order Doesn’t Matter:** If you mix up the order of the things in the box, it doesn’t change the set. So, {red ball, blue ball, green ball} is the same as {green ball, red ball, blue ball}.
3. **Special Brackets:** When we write down a set, we use curly brackets {}. This tells us that what’s inside is a set. So, {red ball, blue ball, green ball} means you have a set with three balls.

Equal Sets:

Two sets are *equal* when they have *exactly the same elements*. It doesn’t matter what order the elements are in or how they’re written—if the elements are the same, the sets are equal.

Example:

- Set A: {apple, banana, orange}
- Set B: {banana, orange, apple}

Even though the order is different, Set A and Set B have the same elements: apple, banana, and orange. So, **Set A equals Set B**.

Equivalent Sets:

Two sets are *equivalent* when they have the *same number of elements*, but the elements don't have to be the same.

Example:

- Set C: {car, bike, bus}
- Set D: {apple, banana, orange}

Set C has 3 elements (car, bike, bus) and Set D also has 3 elements (apple, banana, orange). The elements are different, but the number of elements is the same. So, **Set C is equivalent to Set D**.

How to Figure It Out:

1. **Check if the elements are exactly the same:**
    - If yes, the sets are *equal*.
    - If no, move to the next step.
2. **Check if they have the same number of elements:**
    - If yes, they are *equivalent*.
    - If no, they are neither equal nor equivalent.

Quick Recap:

- **Equal Sets:** Same elements.
- **Equivalent Sets:** Same number of elements, but they don't have to be the same elements.

What is a Subset?

Imagine you have a *big group of things* (that's a set), and you have a *smaller group of some of those things* (that's a subset).

Think of it Like a Basket of Fruits:

- **Big Basket (Set A):** {apple, banana, orange, grape}
- **Small Basket (Set B):** {apple, banana}

Set B is a *subset* of Set A because everything in the small basket (Set B) is also in the big basket (Set A). There's nothing in Set B that isn't in Set A.

Subset Symbols:

1. **Subset Symbol (⊆):**
   - This symbol tells us that one set is a smaller part of another set.
   - We write Set B ⊆ Set A to mean "Set B is a subset of Set A."
   - It's like saying "Everything in the small basket is also in the big basket."
2. **Proper Subset Symbol (⊂):**
   - This is used when the small set (Set B) is not only part of the big set (Set A) but also doesn't have all the stuff in Set A.
   - We write Set B ⊂ Set A to mean "Set B is a proper subset of Set A."
   - It's like saying "Set B has some, but not all, of the stuff in Set A."

Examples with Baskets:

- **Big Basket (Set A):** {apple, banana, orange, grape}
- **Small Basket (Set B):** {apple, banana}
- **Set B ⊆ Set A:** Set B is a subset of Set A (everything in B is in A).
- **Set B ⊂ Set A:** Set B is a proper subset because Set B doesn’t have orange or grape.

When Are They the Same?

- If **Set A** and **Set B** are the same, like both are {apple, banana, orange, grape}, then:
    - Set B ⊆ Set A (because they have the same stuff).
    - But we **cannot** use the ⊂ symbol because Set B isn’t missing anything from Set A. They’re equal.

Quick Recap:

- **⊆ (Subset):** Means "is part of." Everything in the small set is in the big set.
- **⊂ (Proper Subset):** Means "is a smaller part of." The small set is in the big set but doesn’t have everything the big set has.

Think of it like baskets of fruit: if all the fruit in one basket is in another, that’s a subset. If one basket has more types of fruit than the other, then the smaller one is a proper subset.

Sidenote:

To figure out how many subsets you can make from a set, use the formula 2^n, where n is the number of things in the set. Each thing has 2 choices: be in the group or not. So, you multiply 2 by itself for each thing. Now to find the proper subset it would be just $2^n - 1$

Venn Diagrams:

A *Venn diagram* is a picture that shows relationships between sets using circles. Each circle represents a set, and the way the circles overlap shows how the sets relate to each other.

Basic Set Operations and Symbols:

Union (∪, (AND)):

- **What It Means:** The *union* of two sets is a new set that contains everything in either set or both. It's like combining two groups together.
- **Symbol:** ∪
- **Example:** If Set A = {1, 2, 3} and Set B = {3, 4, 5}, then A ∪ B = {1, 2, 3, 4, 5}.
- **Venn Diagram:** The entire area covered by both circles represents the union.

Intersection (∩, (OR)):

- **What It Means**: The *intersection* of two sets is a new set that contains only the things that are in both sets at the same time.
- **Symbol:** ∩
- **Example:** If Set A = {1, 2, 3} and Set B = {3, 4, 5}, then A ∩ B = {3}.
- **Venn Diagram**: The overlapping area of the circles represents the intersection.

Complement (A' or A^c):

- **What It Means:** The *complement* of a set includes everything that is *not* in that set, within a larger "universal set" that contains all possible elements.
- **Symbol**: A' or A^c
- **Example:** If the universal set U = {1, 2, 3, 4, 5} and Set A = {1, 2, 3}, then A' = {4, 5}.

- **Venn Diagram:** The area outside the circle representing Set A represents the complement.

Putting It Together with Venn Diagrams:

- **Union (A ∪ B):** Color the entire area covered by both circles.
- **Intersection (A ∩ B):** Color only the overlapping part of the circles.
- **Difference (A – B):** Color the part of A’s circle that doesn’t touch B.
- **Complement (A’):** Color everything outside A’s circle.

Summary:

- **Union (∪):** Combine everything.
- **Intersection (∩):** Only what’s shared.
- **Difference (–):** What’s in one set but not the other.
- **Complement (A'):** Everything outside the set.

Simple Way to Remember:

1. **AND (∩):** Think of it as what’s *in common* or what both sets have. It’s the overlap.
    - **Example**: If someone asks you to bring fruits that are both in Set A *and* Set B, you only bring what's in both sets (like "bananas" in our example).
2. **OR (∪):** Think of it as *everything* from either set. It’s the combination.
    - **Example:** If someone asks you to bring fruits that are in Set A *or* Set B, you bring everything from both sets (all the fruits).

Key Points:

- **AND (∩):** Both together, only what’s in both sets.
- **OR (∪):** Either one, or both together, everything from either set.

Using these terms helps you figure out whether you're looking for overlap (AND) or the whole collection (OR) when working with sets!

Union and Intersection Combined:

- **Formula:** $|A \cup B| = |A| + |B| - |A \cap B|$
- **Meaning:** When you want to find the number of elements in the union of two sets A and B, you add the number of elements in each set, but you must subtract the ones that got counted twice (the intersection).
- **Example:**
  - If $|A| = 3$, $|B| = 3$, and $|A \cap B| = 2$, then: $|A \cup B| = 3 + 3 - 2 = 4$

Disjoint Sets:

- **Formula:** If A and B are disjoint, then $A \cap B = \emptyset$
- **Meaning:** Disjoint sets have no elements in common, so their intersection is an empty set.
- **Example:**
  - If Set A = {1, 2} and Set B = {3, 4}, then $A \cap B = \emptyset$

Summary of Formulas:

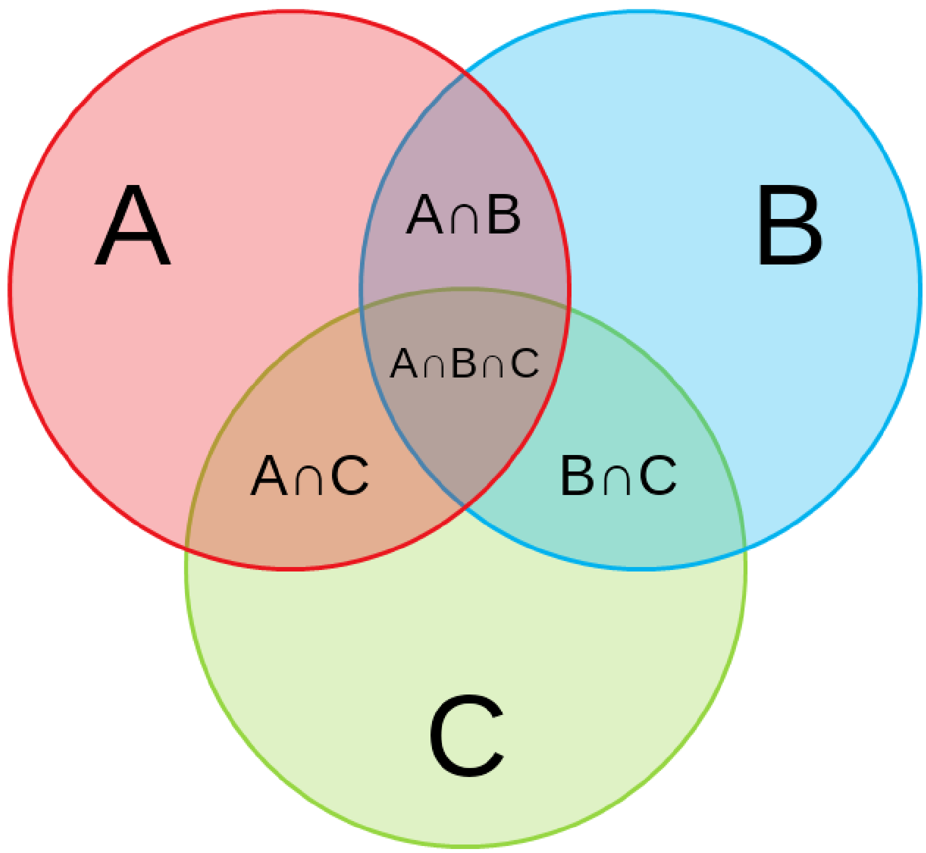
- **Intersection (AND):** $A \cap B$ - What's common in both sets.
- **Union (OR):** $A \cup B$ - Everything in either set.
- **Difference:** $A - B$ - What's in A but not in B.
- **Complement:** A' - Everything not in A.
- **Combined Union and Intersection:** $|A \cup B| = |A| + |B| - |A \cap B|$ - Counts all elements without double-counting

Three Sets in a Venn Diagram:

Imagine you have three groups (sets) of things. We can use three overlapping circles to represent these sets in a Venn diagram.

What Does the Diagram Look Like?

- **Three circles** overlap in a way that creates multiple sections.
- Each circle represents a set (let's call them Set A, Set B, and Set C).
- The areas where the circles overlap represent different combinations of these sets.



Key Areas in a Three-Set Venn Diagram:

- **A Only:** The part of Set A's circle that doesn't touch Set B or Set C.
- **B Only:** The part of Set B's circle that doesn't touch Set A or Set C.
- **C Only:** The part of Set C's circle that doesn't touch Set A or Set B.
- **A ∩ B:** The overlap between Set A and Set B, but not Set C.
- **A ∩ C:** The overlap between Set A and Set C, but not Set B.
- **B ∩ C:** The overlap between Set B and Set C, but not Set A.
- **A ∩ B ∩ C:** The very center where all three circles overlap, representing elements that are in all three sets.

Set Operations with Three Sets:

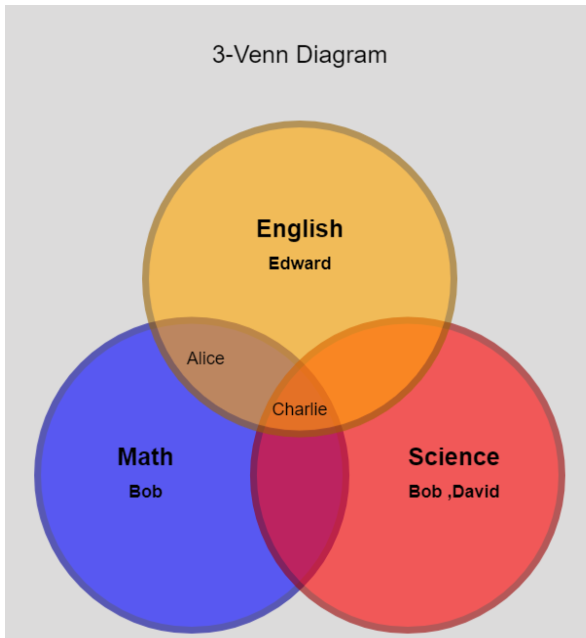
- **Union (A ∪ B ∪ C):**
    - **What It Means:** This includes *everything* in Set A, Set B, or Set C, or any combination of them.
    - **Venn Diagram:** This covers the entire area of all three circles.

- **Intersection (A ∩ B ∩ C):**
    - **What It Means:** This includes only the elements that are in *all three* sets.
    - **Venn Diagram:** This is the tiny area in the middle where all three circles overlap.

- **Intersection of Two Sets (A ∩ B, B ∩ C, A ∩ C):**
    - **What It Means:** This includes elements that are in *two sets at the same time*.
    - **Venn Diagram:** These are the overlapping areas between any two circles but *not* the third.

Understanding with an Example:
Let's say you have three sets:

- **Set A (Students who like Math):** {Alice, Bob, Charlie}
- **Set B (Students who like Science):** {Bob, Charlie, David}
- **Set C (Students who like English):** {Alice, Charlie, Edward}



- **A ∪ B ∪ C:** All students mentioned (everything in any of the sets): {Alice, Bob, Charlie, David, Edward}
- **A ∩ B ∩ C:** Students in common in all three sets: {Charlie}
- **A ∩ B:** Students in both A and B: {Bob, Charlie}

Summary:

- **Three sets** in a Venn diagram create different overlapping areas.
- **Union (∪):** Everything from all sets.
- **Intersection (∩):** Only what's common in all sets.

Venn diagrams with three sets help you visualize how these sets interact and how different operations combine or separate elements within them.

Survey Problems and Venn Diagrams:

When dealing with survey problems using Venn diagrams, you're typically looking at how different groups overlap or don't overlap. Here's how to approach these problems step by step:

Understanding the Basic Terms:

- **Universal Set (U):** This represents the total group you're studying. It includes everyone or everything in the survey.
- **Sets (A, B, C, etc.):** These are smaller groups within the universal set that share a particular characteristic.
- **Regions:** In the Venn diagram, different regions represent how these sets overlap.

Breaking Down the Venn Diagram:
In most survey problems with Venn diagrams:

- **Single Circle (Set A, Set B, etc.):** Represents one group.
- **Overlapping Area:** Shows where the sets intersect, meaning people or items that belong to more than one group.
- **Non-Overlapping Areas:** Represent people or items that belong only to one group.

Key Operations and Concepts:

- **Intersection (AND):** The overlap between sets. If you're asked how many people belong to both Set A and Set B, you're looking for the intersection.
- **Union (OR):** Everything in either set or both sets. If you're asked how many people belong to Set A or Set B (or both), you're looking for the union.
- **Complement:** This is everything *not* in a particular set. If you're asked how many people do not belong to Set A, you're looking for the complement of A.

Steps to Solve Survey Problems:

1. **Identify the Universal Set (U):** Determine the total number of respondents or items.
2. **Label Each Region:** Depending on how many sets there are, label each region in the Venn diagram. For example, in a diagram with three sets, you might have regions I, II, III, IV, etc.
3. **Fill in the Information:**
   - Start with the intersection of all sets if you know it (this is usually the most specific information).
   - Then fill in the areas where two sets overlap but not the third.
   - Finally, fill in the areas where only one set is involved.
4. **Subtract Where Necessary:** When calculating specific regions, you might need to subtract out areas you've already counted to avoid double-counting.

Example:

Let's say you have three sets in a survey:

- **Set A:** People who like apples.
- **Set B**: People who like bananas.
- **Set C:** People who like cherries.

the Venn diagram shows:

- 5 people like all three fruits (intersection of A, B, and C).
- 10 people like apples and bananas but not cherries (intersection of A and B, excluding C).
- 8 people like bananas and cherries but not apples (intersection of B and C, excluding A).
- 12 people like only apples.

You'd use this information to fill out each region in the Venn diagram, ensuring that when you add up all the regions, you get the total number of respondents.

Common Pitfalls:

- **Double-Counting:** Make sure each person or item is counted only once in the correct region.
- **Missing Regions:** Ensure that all regions of the Venn diagram are accounted for.
- **Using Complement Correctly:** Remember that the complement is everything outside the set you're focusing on.

LOGIC

Statements:

A *statement* is just a sentence that can be *clearly* classified as either true or false. It’s like making a claim about something.

Examples of Statements:

- "The sky is blue." (This can be true or false, depending on the context—like if it's daytime or nighttime.)
- "2 + 2 = 4." (This is true.)
- "All cats can fly." (This is false.)

Negations:

A *negation* is the opposite of a statement. If you know the original statement, the negation simply flips it from true to false or from false to true.

How to Negate a Statement:

- If your statement is "The sky is blue," the negation would be "The sky is not blue."
- If the statement is "2 + 2 = 4," the negation would be "2 + 2 ≠ 4."
- If the statement is "All cats can fly," the negation would be "Not all cats can fly" or "Some cats cannot fly."

Negations usually involve adding words like "not," "no," "none," or "some" to flip the meaning.

Quantified Statements:

Quantified statements involve quantities or amounts. They usually include words like "all," "some," or "none." These words tell us how much of something we’re talking about.

Common Quantifiers:

- **All:**
  - "All dogs are friendly."
  - This means every single dog is friendly.
- **Some:**
  - "Some dogs are friendly."
  - This means at least one dog is friendly, but not necessarily all of them.
- **None:**
  - "None of the dogs are friendly."
  - This means no dogs are friendly.

Negating Quantified Statements:

Negating quantified statements can be a bit tricky because you must switch the quantifier and the rest of the statement.

- **Negating "All":**
  - **Original:** "All dogs are friendly."
  - **Negation:** "Some dogs are not friendly."
  - This flips the meaning to say that not every dog is friendly—at least one isn't.
- **Negating "Some":**
  - **Original:** "Some dogs are friendly."
  - **Negation:** "No dogs are friendly."
  - This flips the meaning to say that there isn't even one friendly dog.
- **Negating "None":**
  - **Original:** "None of the dogs are friendly."
  - **Negation:** "Some dogs are friendly."
  - This flips the meaning to say that at least one dog is friendly.

Putting It All Together:

- **Identify the statement:** Determine if a sentence is a statement that can be true or false.
- **Find the negation:** Flip the truth of the statement by adding "not" or changing the meaning.
- **Deal with quantifiers:** Pay special attention to words like "all," "some," and "none" when negating statements.

Example to Summarize:

- **Statement:** "All birds can fly."
- **Negation:** "Some birds cannot fly." (This is true because penguins and ostriches are birds that can't fly.)
- **Quantified Statement:** "Some students like math."
- **Negation:** "No students like math." (This flips the meaning completely.)

Quick Recap:

- **Statements** can be true or false.
- **Negations** flip the truth of a statement.
- **Quantified Statements** use "all," "some," or "none" to talk about amounts, and you must switch these when negating.

What Are Compound Statements?
A *compound statement* is made when you combine two or more statements using words like "and," "or," "if... then," or "if and only if." These words are called *connectives* because they connect the statements together.

Basic Connectives and Their Symbols:

Here are the most common connectives you'll use:

1. **AND (∧):**
   - Meaning: Both statements must be true for the whole compound statement to be true.
   - Symbol: ∧ (this is called the "AND" symbol).
   - Example:
     - If p represents "I have a dog" and q represents "I have a cat," then p∧q means "I have a dog *and* I have a cat."
     - This is only true if *both* p(dog) and q (cat) are true.
2. **OR (∨):**
   - Meaning: At least one of the statements must be true for the whole compound statement to be true.
   - Symbol: ∨ (this is called the "OR" symbol).
   - Example:
     - If p is "I have a dog" and q is "I have a cat," then p∨q means "I have a dog *or* I have a cat."
     - This is true if *either* p (dog) or q (cat) is true, or both are true.
3. **NOT (¬):**
   - Meaning: This flips the truth value of a statement. If something is true, "NOT" makes it false, and vice versa.
   - Symbol: ¬ (this is called the "NOT" symbol).
   - Example: If p is "I have a dog," then ¬p means "I do *not* have a dog."

4. **IF... THEN (→):**
   - Meaning: This means "If the first statement is true, then the second one must also be true."
   - Symbol: → (this is called the "implication" or "if... then" symbol).
   - Example:
     - If p is "It rains" and q is "I will stay inside," then p→q means "If it rains, then I will stay inside."
     - This statement is considered false only if it rains (p is true) and you do *not* stay inside (q is false).
5. **IF AND ONLY IF (↔):**
   - Meaning: Both statements are either true together or false together.
   - Symbol: ↔ (this is called the "biconditional" or "if and only if" symbol).
   - Example:
     - If p is "I am a student" and q is "I have homework," then p↔q means "I am a student *if and only if* I have homework."
     - This is true if both p and q are either both true or both false.

Examples of Compound Statements:

Let's combine some statements using these connectives:

1. **AND (∧):**
   - **Statement:** "I have a dog" and "I have a cat."
   - **Symbol:** p∧q
   - This is true only if you have both a dog and a cat.
2. **OR (∨):**
   - **Statement:** "I have a dog" or "I have a cat."
   - **Symbol:** p∨q
   - This is true if you have either a dog, a cat, or both.

3. **IF... THEN (→):**
    - **Statement:** "If it rains, then I will stay inside."
    - **Symbol:** p→q
    - This is true unless it rains, and you do *not* stay inside.
4. **IF AND ONLY IF (↔):**
    - **Statement:** "I will go to the party if and only if my friend goes."
    - **Symbol:** p↔q
    - This is true only if both you and your friend either both go to the party or both do not go.
5. **NOT (¬):**
    - **Statement:** "I do not have a dog."
    - **Symbol:** ¬p
    - This is true if it’s false that you have a dog.

Quick Recap:

- **AND (∧):** True if both statements are true.
- **OR (∨):** True if at least one statement is true.
- **NOT (¬):** Flips the truth value.
- **IF... THEN (→):** True unless the first is true and the second is false.
- **IF AND ONLY IF (↔):** True when both statements are either both true or both false

What is a Truth Table?

A *truth table* is a tool used in logic to show all possible truth values for a given statement or combination of statements. It helps you see how the truth or falsehood of statements combines under different conditions.

Negation (NOT):

- **Symbol:** ¬
- **Meaning:** Negation flips the truth value of a statement. If something is true, its negation is false, and vice versa.

Truth Table for Negation:

Imagine p is a statement like "I have a dog."

p	¬p
T	F
F	T

- **Explanation:** If p (the statement "I have a dog") is true (T), then ¬p ("I do *not* have a dog") is false (F).
- If p is false (F), then ¬p is true (T).

Conjunction (AND):

- **Symbol:** ∧
- **Meaning:** Conjunction is true only when both statements are true.

Truth Table for Conjunction:

Imagine p is "I have a dog," and q is "I have a cat."

p	q	p∧q
T	T	T
T	F	F
F	T	F
F	F	F

- **Explanation:** p∧q ("I have a dog, *and* I have a cat") is true only when both p (dog) and q (cat) are true. If either one is false, the whole statement is false.

Disjunction (OR):

- **Symbol:** ∨
- **Meaning:** Disjunction is true if at least one of the statements is true.

Truth Table for Disjunction:

Imagine p is "I have a dog," and q is "I have a cat."

p	**q**	**p∨q**
T	T	T
T	F	T
F	T	T
F	F	F

- **Explanation:** p∨q ("I have a dog, *or* I have a cat") is true if either p (dog) or q (cat) is true, or if both are true. It’s only false if both p and q are false.

Putting It All Together:

- **Negation (NOT ¬):** Flips the truth value.
  - If p is true, ¬p is false, and vice versa.
- **Conjunction (AND ∧):** True only when *both* statements are true.
  - If p and q are both true, p∧q is true.
  - If either p or q is false, p∧q is false.
- **Disjunction (OR ∨):** True when *at least one* statement is true.
  - If either p or q is true (or both), p∨q is true.
  - If both p and q are false, p∨q is false.

Conditional (IF... THEN):

- **Symbol:** →
- **Meaning:** A conditional statement is an "if... then" statement. It says that if the first part (the "if" part) is true, then the second part (the "then" part) must also be true.

Truth Table for Conditional:

Imagine p is "It is raining," and q is "I will bring an umbrella."

p	q	p→q
T	T	T
T	F	F
F	T	T
F	F	T

Explanation:

- The statement p→q ("If it is raining, then I will bring an umbrella") is only *false* when it's raining (p is true), but you don't bring an umbrella (q is false).
- In every other case (when it's not raining, or if you bring an umbrella), the statement is considered true.

Biconditional (IF AND ONLY IF):

- **Symbol:** ↔
- **Meaning:** A biconditional statement is true when both parts are either both true or both false. It’s like saying, "These two statements go hand-in-hand; they’re either true together or false together."

Truth Table for Biconditional:

Imagine p is "I will study," and q is "I will pass the test."

- **Explanation:**
  - The statement p↔q ("I will study *if and only if* I will pass the test") is true when both p (studying) and q (passing the test) are either both true or both false.
  - It’s false if one is true and the other is false.

p	**q**	**p↔q**
T	T	T
T	F	F
F	T	F
F	F	T

Putting It All Together:

1. **Conditional (IF... THEN →):**
   - True unless the "if" part is true and the "then" part is false.
   - Example: If it’s raining and you don’t bring an umbrella, then the statement **is false.**
2. **Biconditional (IF AND ONLY IF ↔):**
   - True when both parts are the same (either both true or both false).
   - Example: If you study and pass the test, or if you don’t study and don’t pass, then the statement **is true.**

Equivalent Statements:

Equivalent statements are different ways of saying the same thing, even if they look different.

Example:

- **Original Conditional:** "If it rains, then I will bring an umbrella." ($p \rightarrow q$)
- **Contrapositive:** "If I don't bring an umbrella, then it isn't raining." ($\neg q \rightarrow \neg p$)

These two statements are equivalent—they always have the same truth value.

Variations of Conditional Statements:

There are four main variations of a conditional statement, and they each have their own meaning. Here's how to remember them:

- **Original Conditional:** $p \rightarrow q$
  - **Meaning:** "If p, then q."
  - **Example:** "If it rains, then I will bring an umbrella."

- **Converse:** $q \rightarrow p$
  - **Meaning:** "If q, then p."
  - **Example:** "If I bring an umbrella, then it is raining."
  - Memory Trick: Think of "conversing" (talking) and flipping the order—first you, then me.

- **Inverse:** $\neg p \rightarrow \neg q$
  - Meaning: "If not p, then not q."
  - Example: "If it doesn't rain, then I won't bring an umbrella."
  - Memory Trick: "Inverse" sounds like "invert," which means to flip in a different way—negate both parts.

- **Contrapositive:** ¬q→¬p
    - Meaning: "If not q, then not p."
    - Example: "If I don't bring an umbrella, then it isn't raining."
    - Memory Trick: The contrapositive combines the tricks for the converse and the inverse: flip the order *and* negate both parts.

How to Remember the Differences:

Here's a quick way to keep them straight:

- **Original (p→q):** "If this, then that."
- **Converse (q→p): C**onverse = **C**hange the order.
- **Inverse (¬p→¬q): I**nverse = **I**nvert by negating both.
- **Contrapositive (¬q→¬p): C**ontrapositive = **C**onverse + **I**nverse (flip and negate).

Why It's Important:

- **Only the contrapositive** is equivalent to the original statement. The converse and inverse are not always equivalent to the original, but they might be equivalent to each other.

Recap:

- **Original:** p→q
- **Converse:** q→p (Flip the order)
- **Inverse:** ¬p→¬q (Negate both parts)
- **Contrapositive:** ¬q→¬p (Flip the order *and* negate both parts)

Remembering these differences will help you understand how different statements relate to each other and when they really mean the same thing!

Example:

1. Setting Up the Statements:

Let's start with:

- p: "It is raining."
- q: "I will bring an umbrella."

Now, we'll build the truth tables for:

- **Original Conditional:** p→q
- **Converse:** q→p
- **Inverse:** ¬p→¬q
- **Contrapositive:** ¬q→¬p

Truth Table:

We'll set up the truth table step by step, showing the values for each statement.

p	q	p¬p	q¬q	p→q	q→p	¬p→¬q	¬q→¬p
T	T	F	F	T	T	T	T
T	F	F	T	F	T	T	F
F	T	T	F	T	F	F	T
F	F	T	T	T	T	T	T

Explanation of the Truth Table:

- **Original Conditional (p→q):**
  - True except when p is true and q is false (it's raining, but you don't bring an umbrella).
- **Converse (q→p):**
  - True if q is true and p is true, or if both are false. Not necessarily equivalent to the original.

- **Inverse (¬p→¬q):**
  - This is true unless ¬p is true (it's not raining) and ¬q is false (you do bring an umbrella). Not equivalent to the original but equivalent to the converse**.**
- **Contrapositive (¬q→¬p):**
  - Always matches the truth values of the original statement p→q. This is the only one that's always equivalent to the original conditional.

Key Takeaways:

- **Original and Contrapositive** are always equivalent.
- **Converse and Inverse** can be equivalent to each other but not to the original conditional.

Recap of the Truth Table:

- **Original Conditional (p→q):** Matches the contrapositive.
- **Converse (q→p):** Can differ from the original.
- **Inverse (¬p→¬q):** Matches the converse.
- **Contrapositive (¬q→¬p):** Always matches the original.

Negations of Conditional Statements:

First, let’s review a conditional statement, which is in the form:

- **Conditional Statement:** p→q
  - **Meaning:** "If p, then q." Negating a conditional statement is not as simple as just putting "not" in front of it. Instead, the negation of p→q is saying that the first part (p) happens, but the second part (q) doesn’t.
- **Negation of p→q:** p∧¬q
  - **Meaning:** "It is true that p happens, and it is also true that q does *not* happen."

Example:

- **Statement:** "If it rains (p), then I will bring an umbrella (q)."
- **Negation:** "It is raining (p), but I am not bringing an umbrella (¬q)."
- **Symbolically:** p→q becomes p∧¬q.

De Morgan’s Laws:

De Morgan’s Laws are rules that help us understand how to negate compound statements involving "and" (∧) and "or" (∨).

The Two Laws:

- **First Law:**
  - Negation of "AND":
  - ¬(p∧q) =¬p∨¬q
  - Meaning: The negation of "both p and q" is the same as saying "not p or not q".
    Example:
  - Original: "I have a dog p and a cat q."
  - Negation: "I do not have a dog ¬p, or I do not have a cat ¬q."

- **Second Law:**
    - Negation of "OR":
    - $\neg(p \vee q) = \neg p \wedge \neg q$
    - Meaning: The negation of "either p or q" is the same as saying "not p and not q."
      Example:
    - Original: "I will go to the party p or stay home q."
    - Negation: "I will not go to the party ¬p and I will not stay home ¬q."

How to Remember De Morgan's Laws:

- **Negating AND:** $\neg(p \wedge q)$ becomes "not p or not q."
    - *Memory Trick:* AND becomes OR when negated.
- **Negating OR:** $\neg(p \vee q)$ becomes "not p and not q."
    - *Memory Trick:* OR becomes AND when negated.

Summary with Examples:

- **Negating Conditionals:**
    - Original: "If it rains, then I will bring an umbrella." $(p \rightarrow q)$
    - Negation: "It is raining, but I am not bringing an umbrella." $(p \wedge \neg q)$
- **De Morgan's Laws:**
    - Negating AND: $\neg(p \wedge q) = \neg p \vee \neg q$
    - Example: "Not both a dog and a cat" means "Either no dog or no cat."
    - Negating OR: $\neg(p \vee q) = \neg p \wedge \neg q$
    - Example: "Neither party nor stay home" means "Not going to the party and not staying home."

Finance

Percent:

A *percent* is a way of expressing a number as a part of 100. It's like saying, "How many out of 100?"

Understanding Percent:

- **Example:** 25% means 25 out of 100, or 25 parts out of a total of 100 parts.

Formula:

- To convert a percentage to a decimal, divide by 100:
  - **Example:** 25% = 25/100 = 0.25
- To find a percentage of a number:
  - **Formula:** Part=Percent × Whole
  - **Example:** What is 25% of $80?
    $0.25 * 80 = 20 \text{ (So, 25\% of \$80 is \$20)}$

Sales Tax:

Sales tax is an additional amount of money added to the price of an item, usually expressed as a percentage of the original price.

How Sales Tax Works:

- **Example:** If you buy something that costs $100 and the sales tax is 8%, you'll pay 8% of $100 as tax.

Formula:

- To calculate the sales tax:
  - **Formula:** Sales Tax=Price × Tax Rate
  - **Example:** If the price is $100 and the tax rate is 8%, then:
    - $\text{Sales Tax} = 100 * 0.08 = 8$
    - Total cost = $100 (original price) + $8 (tax) = $108

Discounts:

A *discount* is a reduction in the original price, usually expressed as a percentage.

How Discounts Work:

- **Example:** If something costs $50 and there's a 20% discount, you pay less than the original price.

Formula:

- To calculate the discount amount:
  - **Formula:** Discount=Original Price × Discount Rate
  - **Example:** If the original price is $50 and the discount is 20%, then:
    - $\text{Discount} = 50 * 0.20 = 10$
    - New price = $50 (original price) - $10 (discount) = $40

Putting It All Together:

Let's say you're buying something that costs $200. There's a 10% discount and a 5% sales tax.

- **Calculate the Discount:**
  - $\text{Discount} = 200 * 0.10 = 20$
  - Price after discount = $200 - $20 = $180
- **Calculate the Sales Tax:**
  - $\text{Sales Tax} = 180 * 0.05 = 9$
  - Total cost = $180 + $9 = $189

Quick Recap:

- **Percent:** Part of a whole, out of 100.
- **Sales Tax:** Extra cost added, calculated as a percentage of the price.
- **Discount:** Price reduction, calculated as a percentage of the original price.

Understanding the Basics:

Income tax is the money you pay to the government based on how much money you earn. The more you earn, the more taxes you pay. The amount you pay depends on your income level and is calculated using tax rates.

Gross Income, Adjusted Gross Income, and Taxable Income:

- Gross Income: This is the total amount of money you earn in a year before anything is taken out for taxes or other deductions. It includes your salary, interest from savings, and any other income.
    - **Example from the Image:** The woman's gross income was her wages of $29,800 plus $3,100 in interest, which totals $32,900.
- Adjusted Gross Income (AGI): This is your gross income after subtracting certain adjustments, like contributions to retirement plans or health savings accounts. These adjustments reduce your income, so you pay less tax.
    - **Example:** The woman contributed $1,900 to her retirement plan. So, her AGI is $32,900 - $1,900 = $31,000.
- Taxable Income: This is what you actually pay taxes on. You find it by taking your adjusted gross income and subtracting exemptions and deductions (like standard deductions or personal exemptions).
    - **Example:** The woman's taxable income was $31,000 - ($4,050 + $7,600) = $19,350.

Marginal Tax Rates:

Taxes are calculated using marginal tax rates. This means your income is divided into chunks, and each chunk is taxed at a different rate.

- **Example:** If you have a taxable income of $78,000:
    - The first chunk (up to $9,275) is taxed at 10%.
    - The next chunk (from $9,276 to $37,650) is taxed at 15%.
    - The last chunk (from $37,651 to $78,000) is taxed at 25%.

You add up the taxes from each chunk to get your total tax.

Calculating Tax Owed:

Let's break down the calculation step-by-step using the example of the single man with $78,000 in taxable income.

- **First Marginal Rate:**
    - The first $9,275 is taxed at 10%.
    - $\text{Tax: } 0.10 * 9275 = \$927.50$
- **Second Marginal Rate:**
    - The next chunk ($9,276 to $37,650) is taxed at 15%.
    - $\text{Tax: } 0.15 * (37651 - 9275) = \$4{,}256.25$
- **Third Marginal Rate:**
    - The remaining income ($37,651 to $78,000) is taxed at 25%.
    - $\text{Tax: } 0.25 * (78000 - 37650) = \$10{,}087.50$
- **Total Tax Owed:**
    - Add up the taxes from each chunk:
    - $927.50 (first chunk) + $4,256.25 (second chunk) + $10,087.50 (third chunk) = $15,271.25 total tax owed.

Summary of Key Points:

- **Gross Income:** Total income before deductions.
- **Adjusted Gross Income (AGI):** Gross income minus specific deductions.
- **Taxable Income:** AGI minus exemptions and deductions.
- **Marginal Tax Rates:** Tax rates applied to different portions (chunks) of your income.

What is Simple Interest?

Simple interest is the extra money you pay when you borrow money or the extra money you earn when you lend money. It's called "simple" because it's only calculated on the original amount you borrowed or invested (called the *principal*), not on any interest that builds up over time.

The Simple Interest Formula:

The formula to calculate simple interest is:

$$I = P * r * t$$

Where:

- **I** is the interest (the extra money paid or earned).
- **P** is the principal (the original amount of money).
- **r** is the interest rate (as a decimal).
- **t** is the time (usually in years).

Example 1: Calculating Simple Interest

Let's say you borrow $3,500 at a 6.5% interest rate for 3 months.

- **Convert the Interest Rate:**
  - 6.5% as a decimal is 0.065
- **Convert the Time:**
  - 3 months is $\frac{3}{12}$ years, which is 0.25 years.
- **Plug into the Formula:**
  - $I = 3500 * 0.065 * 0.25$
  - $I = 56.88$

So, the interest you owe after 3 months is $56.88.

Future Value with Simple Interest:

When you borrow money, you need to pay back the original amount plus the interest. The total amount you pay back is called the *future value*.

The formula to find the future value is:

$$A = P(1 + rt)$$

Where:

- **A** is the future value (total amount you owe).
- The other variables are the same as before.

Example 2: Calculating Future Value

You borrow $7,500 at a 5% interest rate for 7 years.

- **Convert the Interest Rate:**
  - 5% as a decimal is 0.05
- **Plug into the Formula:**
  - $A = 7500(1 + 0.05(7))$
  - $A = 7500(1.35) = \$10{,}125$

So, after 7 years, you owe $10,125.

Finding the Interest Rate

Sometimes, you might know the future value and want to find out the interest rate. You can rearrange the future value formula to solve for the interest rate (r).

Example 3: Finding the Interest Rate

You borrowed \$4,500, and after 6 years, you need to pay back \$7,200. What's the interest rate?

- **Start with the Formula:**
  - $7200 = 4500(1 + r(6))$
- **Solve for r:**
  - First, divide both sides by 4500: $\frac{7200}{4500} = 1 + 6r$
  - Simplify: $1.6 = 1 + 6r$
  - Subtract 1 from both sides: $0.6 = 6r$
  - Divide by 6: $r = 0.1$

So, the interest rate is 10% (because 0.1 as a decimal is 10%).

Summary

- **Simple Interest (I):** Extra money earned or paid, calculated with $\mathrm{I} = \mathrm{Prt}$
- **Future Value (A):** Total amount to pay back or receive, calculated with $A = P(1 + rt)$
- **Finding the Interest Rate:** Rearrange the future value formula to solve for r

What is Compound Interest?

Compound interest is interest that is calculated not just on the original amount of money (called the *principal*), but also on the interest that has already been added. In other words, you earn interest on your interest.

The Compound Interest Formula:

The formula for compound interest is:

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

Where:

- **A** is the amount of money accumulated after n years, including interest.
- **P** is the principal amount (the initial money you deposit or borrow).
- **r** is the annual interest rate (in decimal form).
- **n** is the number of times that interest is compounded per year.
- **t** is the number of years the money is invested or borrowed for.

Example 1: Compounding Semiannually

Let's say you deposit $7,000 into a savings account with a 7% annual interest rate, compounded semiannually (twice a year), for 6 years.

- **Identify the Variables:**
  - Principal (P) = $7,000
  - Interest rate (r) = 7% or 0.07 as a decimal
  - Time (t) = 6 years
  - Compounding periods per year (n) = 2 (since it's semiannual)
- **Plug into the Formula:**

$$A = 7000\left(1 + \frac{0.07}{2}\right)^{(2)(6)}$$

- **Calculate:**
    - $\frac{0.07}{2} = 0.035$ 0.072=0.035\frac{0.07}{2} = 0.03520.07=0.035
    - $A = 7000(1 + 0.035)^{12}$
    - $A = 7000(1.035)^{12}$
    - $A \approx 10{,}577.48$

So, after 6 years, your investment would grow to approximately \$10,577.48.

How to Find the Interest Earned

To find the interest earned, subtract the original principal from the amount accumulated:

Interest Earned=$A - P = 10{,}577.48 - 7{,}000 = 3{,}577.48$

So, the interest earned over 6 years is \$3,577.48.

Example 2: Different Compounding Frequencies

Let's say you have \$30,000 and you invest it at a 5.5% annual interest rate for 5 years, but you want to see how different compounding frequencies affect the final amount.

- **Semiannually (n=2):**

$$A = 30{,}000\left(1 + \frac{0.055}{2}\right)^{(2)(5)} = 30{,}000\ (1.0275)^{10} \approx \$39{,}349.53$$

- **Quarterly (n=4):**

$$A = 30{,}000\left(1 + \frac{0.055}{4}\right)^{(4)(5)} = 30{,}000\ (1.01375)^{20} \approx \$39{,}421.99$$

- **Monthly (n=12):**

$$A = 30{,}000\left(1+\frac{0.055}{12}\right)^{(12)(5)} = 30{,}000\,(1.0045833)^{60} \approx \$39{,}471.11$$

Summary of Key Points:

- **Compound Interest** means you earn interest on both the principal and the interest that has been added.
- **The More Frequent the Compounding, the More Money You Earn.** Compounding monthly will give you more interest than compounding semiannually or quarterly.
- **Use the Formula:** $A = P\left(1+\frac{r}{n}\right)^{nt}$ to calculate the future value of an investment with compound interest

What is an Installment Loan?

When you buy a car and don't have all the money upfront, you usually take out a loan. An *installment loan* means you borrow a certain amount of money (called the principal) and then pay it back over time in regular monthly payments. Each payment covers part of the principal and some interest.

The Loan Payment Formula:

To figure out how much you'll pay each month, you use this formula:

$$PMT = \frac{P\left(\frac{r}{n}\right)}{1 - \left(1 + \frac{r}{n}\right)^{-nt}}$$

Where:

- **PMT** is the monthly payment.
- **P** is the loan amount (principal).
- **r** is the annual interest rate (as a decimal).
- **n** is the number of payments per year.
- **t** is the number of years the loan lasts.

Example 1: Calculating Monthly Payments

Let's say you borrow $9,000 for 6 years at an annual interest rate of 7%. The loan is repaid monthly.

- **Identify the Variables:**
  - P=9,000
  - r=0.07
  - n=12 (since payments are monthly)
  - t=6

- **Plug into the Formula:**

$$PMT = \frac{9{,}000(\frac{0.07}{12})}{1-\left(1+\frac{0.07}{12}\right)^{(-12)(6)}}$$

- **Calculate:**

$$PMT \approx \$153.44$$

So, the monthly payment is approximately $153.44.

Finding the Total Interest Paid

To find out how much total interest you pay, you need to calculate the total amount paid over the life of the loan and subtract the original loan amount.

- **Total Amount Paid:**
  - Total Payments=$PMT * n * t$
  - Total Payments=$153.44 * 12 * 6 = 11{,}047.68$
- **Total Interest Paid:**
  - Total Interest=Total Payments−P
  - Total Interest=11,047.68−9,000=2,047.68

So, you pay $2,047.68 in interest over the life of the loan.

Comparing Loans

If you're deciding between different loan options, you can use the formula to compare monthly payments and total interest for each option.

Example 2: Comparing Two Loans

- **Loan A:** $18,000 for 3 years at 5.1% interest.
  - $$PMT = \frac{18000\left(\frac{0.051}{12}\right)}{1-\left(1+\frac{0.051}{12}\right)^{(-12)(3)}} \approx 540.28$$
  - **Total Amount Paid**=Monthly Payment* Number of Payments $540.28(36) = \$19450.08$
  - **Total interest:** 19,450.08−18,000=1,450.08
- **Loan B:** $18,000 for 5 years at 5.2% interest.
  - **Calculate using the same steps as for Loan A.**

By comparing the monthly payments and total interest, you can decide which loan is better for your budget.

Summary of Key Points:

- **Installment Loan:** Borrow money and repay in regular monthly payments.
- **Loan Payment Formula:** Use the formula to calculate how much you'll pay each month.
- **Total Interest Paid:** Calculate by subtracting the original loan amount from the total amount paid overtime.
- **Comparing Loans:** Look at both the monthly payment and the total interest to make the best financial decision.

What is a Mortgage?

A *mortgage* is a long-term loan you take out to buy a house. It's like borrowing money from the bank to pay for the house, and then paying back the bank over time, usually over 15, 20, or 30 years.

Down Payment:

Before you get a mortgage, you usually need to make a *down payment*. This is the amount of money you pay upfront, out of your pocket, and it's usually a percentage of the total price of the house.

- **Example:** If you want to buy a house that costs $220,000 and the bank requires a 10% down payment:
    - Down Payment = $0.10 \times 220{,}000 = 22{,}000$
    - So, you pay $22,000 upfront.

Mortgage Amount:

The *mortgage* is the amount you still need to borrow after the down payment.

- **Example:** Using the house price from above:
    - Mortgage Amount = Sale Price - Down Payment
    - Mortgage Amount = $220,000 - $22,000 = $198,000

Monthly Payments:

You repay the mortgage through *monthly payments* over many years. Each payment includes part of the money you borrowed (the principal) and interest (the cost of borrowing the money).

The Loan Payment Formula:

To calculate your monthly payment, use this formula:

$$PMT = \frac{P\left(\frac{r}{n}\right)}{1 - \left(1 + \frac{r}{n}\right)^{-nt}}$$

Where:

- **P** = Mortgage Amount (principal).
- **r** = Annual interest rate (as a decimal).
- **n** = Number of payments per year (usually 12 for monthly payments).
- **t** = Number of years for the loan.

Example: Calculating Monthly Payments

Let's say you're buying a house for $195,000, with a 10% down payment, and you're taking a 30-year mortgage at 7.5% interest.

- **Calculate Down Payment:**
  - $0.10 \times 195{,}000 = 19{,}500$
  - So, you pay $19,500 upfront.

- **Calculate Mortgage Amount:**
  - $195{,}000 - 19{,}500 = 175{,}500$
  - You borrow $175,500 from the bank.

- **Calculate Monthly Payment:**
  - $P = 175{,}500$
  - $r = 0.075$ (7.5% as a decimal)
  - $n = 12$ (monthly payments)
  - $t = 30$ (loan term is 30 years)

$$PMT = \frac{175500\left(\frac{0.075}{12}\right)}{1-\left(1+\frac{0.075}{12}\right)^{(-12)(30)}} \approx 1227$$

So, the monthly payment is approximately $1,227.

Total Interest Paid Over 30 Years

To find out how much interest you pay over the life of the loan, you calculate the total amount paid and subtract the original loan amount.

- **Total Payments:**
  - $PMT * n * t = 1227 * 12 * 30 = 441720$
  - **You pay a total of $441,720 over 30 years.**
- **Total Interest:**
  - Total Interest = Total Payments - Mortgage Amount
  - **441,720−175,500=266,220**

So, you pay $266,220 in interest over 30 years.

Summary of Key Points:

- **Mortgage:** A loan to buy a house, paid back over time.
- **Down Payment:** The upfront payment, usually a percentage of the house price.
- **Monthly Payments:** Regular payments that cover the loan amount and interest.
- **Total Interest:** The extra money you pay for borrowing the money, calculated over the life of the loan.

What is a Credit Card?

A *credit card* lets you buy things now and pay for them later. It's like borrowing money from the bank every time you use the card, and then paying it back over time. But, if you don't pay it back quickly, the bank charges you extra money called *interest*.

How Credit Card Interest Works:

Credit card interest is usually high, so if you don't pay off your balance (the amount you owe) every month, you can end up paying a lot more than what you originally spent.

The Average Daily Balance Method:

Most credit cards calculate interest using the *Average Daily Balance Method*. Here's how it works:

1. **Calculate the Average Daily Balance:**
   - Add up all the unpaid balances you had each day during the billing period.
   - Divide this total by the number of days in a billing period.
   - Formula for Average Daily Balance

$$= \frac{Sum\ of\ unpaid\ balances\ for\ each\ day\ in\ the\ billing\ period}{Number\ of\ days\ in\ the\ billing\ period}$$

2. **Calculate Interest Using the Simple Interest Formula:**
   - $I = P * r * t$
   - **I =** Interest charged
   - **P =** Average daily balance
   - **r =** Monthly interest rate (usually given as a percentage)
   - **t =** Number of months (usually 1 month)

Example: Calculating Credit Card Interest

Let's say you have a credit card with the following transactions:

- **Previous balance:** $6,370
- **New charges:**
    - March 7: $70
    - March 12: $60
    - March 21: $290
- **Payment:** March 5: $450
- **Interest Rate:** 1.5% of the average daily balance
- The credit card requires a $10 minimum monthly balance if the balance due at the end of the billing period is less than $360. Otherwise, the minimum monthly payment is $\frac{1}{36}$ of the balance due at the end of the billing period

Steps to Calculate Interest:

1. **Unpaid Balance After Each Transaction:**
    - March 1: $6,370
    - March 5: $6,370 - $450 = $5,920
    - March 7: $5,920 + $70 = $5,990
    - March 12: $5,990 + $60 = $6,050
    - March 21: $6,050 + $290 = $6,340
2. **Calculate the Average Daily Balance:**
    - Multiply each unpaid balance by the number of days it stayed the same.
    - Sum these amounts and divide by the number of days in the billing period (e.g., 31 days).
      Calculation:
    - March 1 - 4: $6,370 × 4 days = $25,480
    - March 5 - 6: $5,920 × 2 days = $11,840
    - March 7 - 11: $5,990 × 5 days = $29,950
    - March 12 - 20: $6,050 × 9 days = $54,450

- March 21 - 31: \$6,340 × 11 days = \$69,740
  Total = \$191,460
- $Average\ Daily\ Balance = \frac{191460}{31} \approx 6176.13$

3. **Calculate Interest:**
   - **Interest:** $I = 6176.13(0.015)(1) = 92.64$

4. Balance Due and Minimum Payment:

Once you have the interest, you can find out how much you owe (balance due) and how much you need to pay at a minimum (minimum payment).

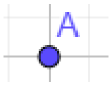
- **Balance Due:**
  - Add the interest to the last balance from the billing period.
  - **Example:** \$6,340 (March 31 balance) + \$92.64 (interest) = \$6,432.64
- **Minimum Payment:**
  - If the balance is above a certain amount (e.g., \$360), the minimum payment is usually a small percentage of the balance.
  - **Example:** $\frac{1}{36}(6432.64) \approx 179$

So, the minimum payment due by April 9 is \$179.

Summary of Key Points:

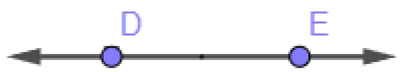
- **Credit Cards:** Borrow money to make purchases, pay back later, but watch out for interest charges.
- **Average Daily Balance:** The method most credit cards use to calculate how much interest you owe.
- **Interest Charges:** Can add up quickly if you don't pay off your balance in full each month.
- **Minimum Payment:** The smallest amount you must pay each month to avoid late fees, but it's better to pay more to reduce interest.

Geometry

Points 

- **What is a Point?**
    - A *point* is like a dot on a piece of paper. It has no size—no length, width, or height. It simply shows a location. Think of it as a place where something happens, but it doesn't take up any space.
    - We usually label points with a capital letter, like **A**, **B**, or **C**.

Lines

- **What is a Line?** 

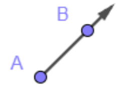
    - A *line* is a straight path that goes on forever in both directions. Imagine a super long road that never ends. It has no thickness, just length.
    - A line is made up of an infinite number of points, all lined up in a row.
    - We name a line using two points on it, like **line AB** or **line BA**. You can also see it written with a little line symbol above the letters: $\overleftrightarrow{AB}\ or\ \overleftrightarrow{BA}$

Line Segments

- **What is a Line Segment?** 

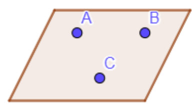
    - A *line segment* is just a piece of a line. It has two endpoints, meaning it starts at one point and ends at another.
    - Think of it like a stick—it's straight, but it doesn't go on forever.
    - We name a line segment with its endpoints like segment AB or segment BA, write it as $\overline{AB}$

Rays

- **What is a Ray?** 

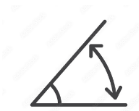
  - A *ray* starts at one point and goes on forever in one direction, like a laser beam that starts somewhere and never stops.
  - We name a ray starting with its endpoint like ray AB (it starts at A and passes through B), written as $\overrightarrow{AB}$

Planes

- **What is a Plane?** 

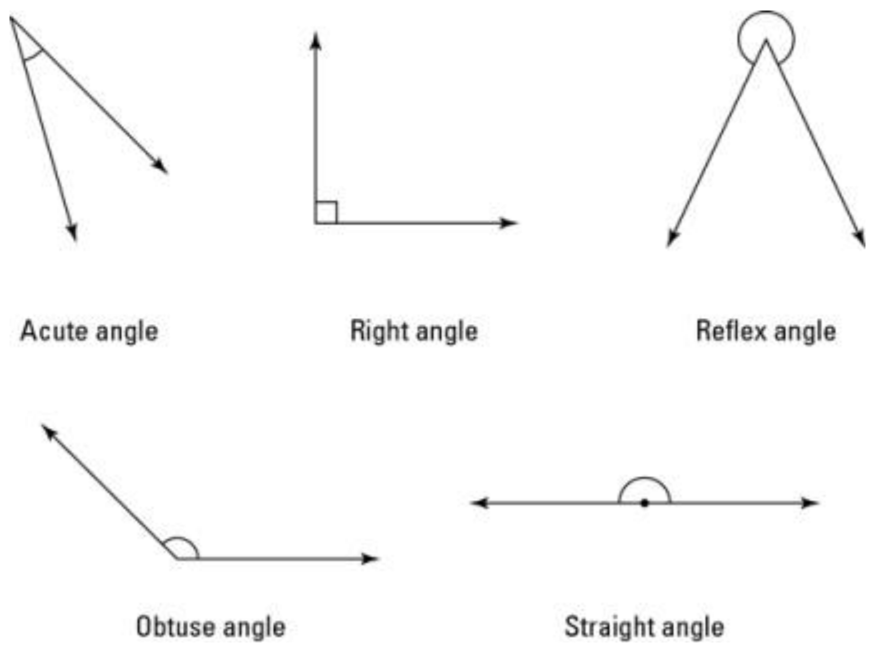
  - A *plane* is a flat surface that goes on forever. Imagine a giant sheet of paper that never ends.
  - It has length and width, but no thickness—just like a line, but in two dimensions.
  - We usually name a plane with a single capital letter or by naming three points that are not all on the same line, like **plane ABC**.

Angles

- **What is an Angle?** 
  - An *angle* is made when two rays start from the same point. The point where they start is called the *vertex* of the angle.
  - Angles are like the corners of a shape. The size of an angle tells us how "open" or "closed" the corner is.

Types of Angles:

- **Acute Angle:** Less than 90 degrees (like a sharp corner).
- **Right Angle:** Exactly 90 degrees (like the corner of a square).
- **Obtuse Angle:** More than 90 degrees but less than 180 degrees (like a wide-open door).
- **Straight Angle:** Exactly 180 degrees (like a straight line)



Summary:

- **Points**: No size, just a location.
- **Lines**: Infinite in length, no thickness.
- **Line Segments**: Part of a line, with two endpoints.
- **Rays**: Start at one point, go on forever in one direction.
- **Planes**: Flat surfaces that extend forever.
- **Angles**: Formed by two rays starting from the same point; types include acute, right, obtuse, and straight.

Triangles are one of the most basic shapes in geometry. They are polygons with three sides and three angles.

Parts of a Triangle

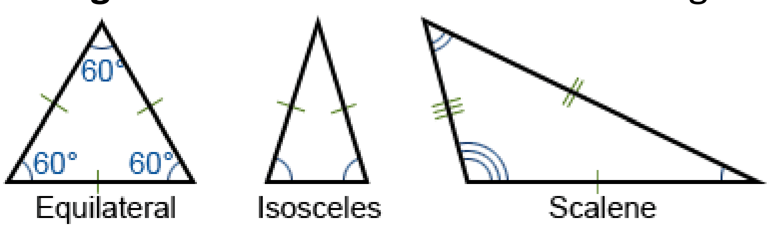
- **Sides**: The three straight lines that make up the triangle.
- **Angles**: The space between two sides. Each triangle has three angles.

Types of Triangles by Angles

- **Acute Triangle**: All three angles are less than 90°.
- **Right Triangle**: One of the angles is exactly 90°.
- **Obtuse Triangle**: One of the angles is greater than 90°.

Types of Triangles by Sides

- **Equilateral Triangle**: All three sides are of equal length, and all three angles are 60°.
- **Isosceles Triangle**: Two sides are of equal length, and the angles opposite these sides are equal.
- **Scalene Triangle**: All three sides and all three angles are different.

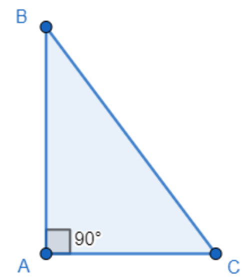


The Sum of the Angles

- The sum of the angles in any triangle is always 180°.
- For example, if one angle is 60° and another is 80°, the third must be 40° because 60° + 80° + 40° = 180°.

The Pythagorean Theorem (For Right Triangles)

- In a right triangle, the side opposite the right angle is called the hypotenuse, and the other two sides are called legs.
- The Pythagorean Theorem States $a^2 + b^2 = c^2$ where c is the length of the hypothenuse, and a and b are the lengths of the other two sides



Similar Triangles

- Triangles are similar if their corresponding angles are equal, and their corresponding sides are proportional.
- If two triangles are similar, you can set up a proportion to find unknown side lengths.

Understanding Triangles: Angles and Sides

- **Finding the Angles**:
    - If you know the measure of one or two angles, you can always find the third by subtracting from 180°.
- **Pythagorean Theorem**:
    - If you know the two sides of the right triangle, you can use $a^2 + b^2 = c^2$ to find third side

- **Proportional Sides**:
    - If triangles are similar, you can use proportions to find missing side lengths.

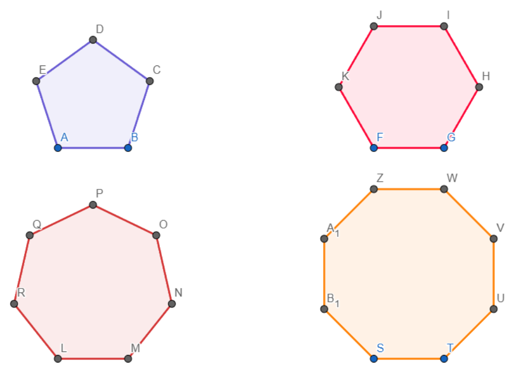
Polygons

A polygon is a closed shape formed by three or more straight lines called sides. The sides meet at points called vertices. Examples of polygons include triangles (3 sides), quadrilaterals (4 sides), pentagons (5 sides), and so on.

Regular Polygons: A regular polygon is a polygon where all sides are equal in length, and all angles are equal in measure.

Naming Polygons:

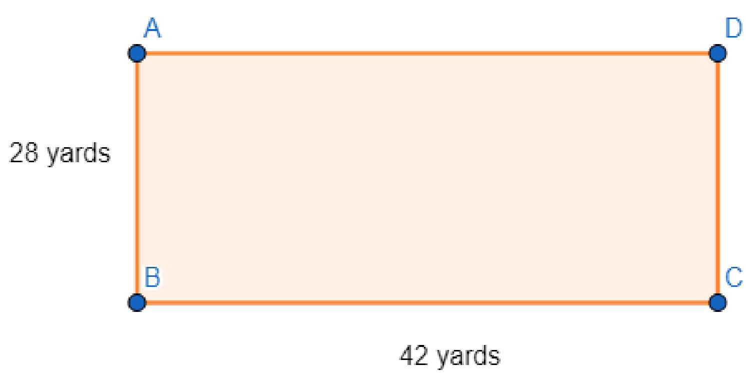
- Triangle: 3 sides
- Quadrilateral: 4 sides
- Pentagon: 5 sides
- Hexagon: 6 sides
- Heptagon: 7 sides
- Octagon: 8 sides



Perimeter

The perimeter is the total distance around the outside of a polygon. To find the perimeter, you simply add up the lengths of all the sides.

For example: If you have a rectangular field with a length of 42 yards and a width of 28 yards, the perimeter P is calculated as:



$$P = 2L + 2W = 2(42) + 2(28) = 84 + 56 = 140\ yards$$

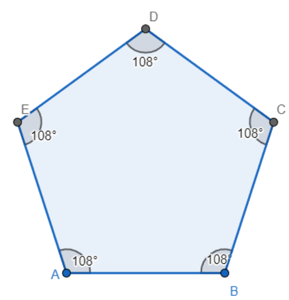
Sum of the Measures of a Polygon's Angles

The sum of the interior angles of a polygon depends on the number of sides. For any polygon with n sides, the sum of the interior angles can be calculated using the formula:

$$\text{Sum of interior angles} = (n-2) * 180°$$

Example: For a pentagon (5 sides), the sum of the interior angles is:

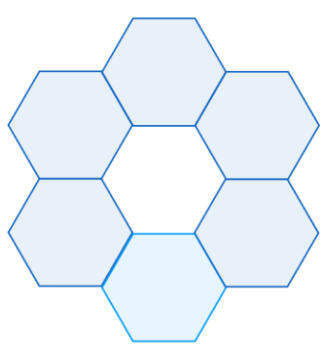
$$(5-2) * 180° = 540°$$



Tessellations

A tessellation is a pattern of shapes that completely covers a plane without any gaps or overlaps. The shapes used in a tessellation are often regular polygons. The angles at each vertex of a tessellation must add up to $360°$ for it to fit together perfectly.

Example of a tessellation: Imagine covering a floor with tiles shaped like hexagons. Each hexagon has angles that sum up to $720°$, and the angles where the hexagons meet must add up to $360°$, making a perfect tessellation.



Recap:

- **Polygons** are closed shapes with straight sides.
- **Perimeter** is the distance around a polygon.
- The **sum of a polygon's interior angles** is $(n - 2) * 180°$
- **Tessellations** are patterns of polygons that cover a plane without gaps or overlaps.

Understanding Area

Area is the measure of the amount of space inside a two-dimensional shape. Think of it as how much surface the shape covers. We calculate the area for different shapes using specific formulas.

Area of a Parallelogram:

- **Formula**: $\text{Area} = \text{Base} * \text{height} = \text{bh}$
- **Example:** if the base is 60 cm and the height is 53 cm, the area would be: $Area = 60\ cm * 53\ cm = 3180\ cm^2$

Area of a Trapezoid:

- **Formula**: $\text{Area} = \frac{1}{2}(base1 + base2)(height)$
- **Example:** For bases of 22 m and 14m and a height of 8 m, the area would be $A = \frac{1}{2}(22 + 14)(8) = 144m^2$

Area of a Triangle:

- **Formula**: $\text{Area} = \frac{1}{2}(\text{base})(\text{height})$
- **Example:** With a base of 8.9 yds and a height of 6.8 yds. $Area = \frac{1}{2}(8.9)(6.8) = 30.26\ yds^2$

Understanding Circumference

Circumference is the distance around a circle, like the perimeter of a polygon.

- **Formula for Circumference**:
  - When you know the radius: $C = 2\pi r$
  - When you know the diameter: $C = \pi d$
  - **Example:** If the radius of a circle is 5 cm, then the circumference is: $C = 2\pi(5) = 10\pi \approx 31.42$

Volume

- **Volume** is the amount of space inside a three-dimensional object, like how much water a box can hold.
- The **formula for volume** depends on the shape of the object:
    - **Rectangular Solid (Box):** Volume=length × width × height
    - **Cube (All sides equal):** Volume=side × side × side=s^3
    - **Cylinder:** Volume=π × $radius^2$× height
    - **Pyramid:** Volume=$\frac{1}{3}$ $(Base\ Area)(height)$
    - **Cone:** Volume=$\frac{1}{3}(radius^2)(height)$
    - **Sphere:** Volume=$\frac{4}{3}\pi(radius^3)$

Surface Area

- **Surface Area** is the total area of all the outer surfaces of a three-dimensional object, like how much wrapping paper you would need to cover a gift.
- The **formula for surface area** also varies depending on the shape:
    - **Cube:** $\text{Surface Area} = 6\text{s}^2$
    - **Rectangular Solid:** $\text{Surface Area} = 2\text{lw} + 2\text{lh} + 2\text{wh}$ (where l is the length, w is width, and h is height)
    - **Cylinder:** $\text{Surface Area} = 2\pi\text{r}^2 + 2\pi\text{rh}$ (where r is radius and h is height)

PROBABILTY

What is The Fundamental Counting Principle?

Imagine you're picking an outfit to wear. You have:

- 3 different shirts
- 2 different pants

To figure out how many different outfits you can make, you multiply the number of shirts by the number of pants: 3 shirts ×2 pants=6 outfits This means you have 6 different outfit combinations to choose from.

Another Example

Let's say you're at a restaurant, and you can choose:

- 5 appetizers
- 4 main courses
- 3 desserts

To find out how many different meal combinations you can create, multiply: 5 appetizers × 4 main courses × 3 desserts=
60 meal combinations So, there are 60 different ways to mix and match your meal.

Key Idea

Whenever you want to figure out how many different combinations or outcomes are possible by making choices, you just multiply the number of options for each choice together.

A Bigger Example

Suppose you want to create a special code with:

- The first letter being either A or B (2 choices)
- The second part being any letter from A to Z (26 choices)
- The third part being any number from 0 to 9 (10 choices)

To find out how many different codes you can make: $2 * 26 * 10 = 520$ different codes

Summary

- **Multiply** the number of choices you have for each step. The total will give you the number of different combinations or outcomes possible.

What is a Permutation?

Imagine you have a few items, like books, and you want to arrange them on a shelf. A *permutation* is just a fancy word for a specific way to arrange those items.

Important Rules for Permutations:

1. **Order Matters**: The order in which you place the items matters. For example, arranging books A, B, and C in the order "ABC" is different from "CAB" or "BCA".
2. **No Repeating**: Each item can only be used once in each arrangement. So, if you have three books, each book can only appear once in any arrangement.

Example:

Suppose you have 3 different books: Book 1, Book 2, and Book 3. How many ways can you arrange these 3 books on a shelf?

- **First spot**: You have 3 choices (Book 1, Book 2, or Book 3).
- **Second spot**: After placing the first book, you have 2 choices left.
- **Third spot**: Only 1 book is left for the last spot.

So, the number of possible arrangements (permutations) is: $3 \times 2 \times 1 = 6$

This means there are 6 different ways to arrange the 3 books.

Factorial (!):

When you see a number followed by an exclamation point, like "3!", it means "3 factorial," which is a way to multiply all whole numbers from the number down to 1.

- **$3! = 3 \times 2 \times 1 = 6$**

Permutations Formula:

If you have a group of n items and want to arrange r of them, the formula to find the number of permutations is:

$$nPr = \frac{n!}{(n-r)!}$$

Example with Formula:

If you have 5 books and want to arrange 3 of them on a shelf, the number of possible arrangements is:

$$5P3 = \frac{5!}{(5-3)!} = \frac{5!}{2!} = \frac{5 \ * 4 \ * 3 \ * 2 * 1}{2 * 1} = \frac{120}{2} = 60$$

So, there are 60 ways to arrange 3 books out of 5.

Permutations with Repeating Items:

If some items are identical (like the letters in "CALCULUS"), you use a slightly different formula to account for the repeats:

Number of Permutations=$\frac{n!}{p_1! * p_2! * \cdots * p_k!}$

Where $p_1, p_2, \ldots$ are the numbers of each identical item.

Example:

For the word "CALCULUS" (which has 8 letters with some repeats):

$$\frac{8!}{2! * 2! * 2!} = \frac{40320}{8} = 5040$$

So, there are 5040 distinct ways to arrange the letters in "CALCULUS."

What is a Combination?

A **combination** is a way to choose a group of items from a larger set, where the order in which you choose them **does not matter**. For example, imagine you have 3 different fruits: an apple, a banana, and an orange. If you want to choose 2 fruits to make a fruit salad, the combination of apple and banana is the same as the combination of banana and apple because order doesn't matter in combinations.

How Combinations are Different from Permutations:

- **Permutations** care about the order: The order in which items are arranged is important.
- **Combinations** don't care about the order: The order in which items are selected doesn't matter.

How to Calculate Combinations:

The formula to calculate the number of combinations is:

$$nCr = \frac{n!}{r!(n-r)!}$$

- **n** is the total number of items to choose from.
- **r** is the number of items you are choosing.
- **!** means "factorial," which is the product of all positive integers up to that number. For example, $5! = 5\times4\times3\times2\times1=120$

Example:

If you want to know how many different ways you can choose 2 fruits from 3 (apple, banana, orange), you will use the combination formula:

$$3C2 = \frac{3!}{2!\,(3-2)!} = \frac{3 * 2 * 1}{2(1) * 1} = \frac{6}{2} = 3$$

This means there are 3 different combinations:

- Apple and Banana
- Apple and Orange
- Banana and Orange

So, in combinations, the order of picking doesn't matter—Apple and Banana is the same as Banana and Apple.

When to Use Combinations:

Use combinations when you are interested in groups of items and the order does not matter, like selecting a team from a group of people, or picking a set of lottery numbers.

What is Probability?

- **Probability** is just a way to measure how likely something is to happen. For example, when you flip a coin, you want to know the chance (probability) of getting heads or tails.

Important Terms:

- **Experiment**: Any action where the outcome is uncertain, like flipping a coin or drawing a card.
- **Sample Space**: The set of all possible outcomes. For a coin flip, the sample space is {Heads, Tails}.
- **Event**: Any specific outcome or set of outcomes you're interested in. For example, getting heads in a coin flip is an event.

How to Calculate Probability

- **Identify the total number of possible outcomes** (this is the size of your sample space).
- **Count how many of those outcomes are the ones you care about** (these are the successful outcomes for your event).
- **Divide the number of successful outcomes by the total number of possible outcomes**.

The formula looks like this:

$$\text{Probability of an event } \big(P(E)\big) = \frac{Number\ of\ successful\ outcomes}{Total\ Number\ of\ possible\ outcomes}$$

Example:

- **Coin Flip**: You flip a coin. The sample space is {Heads, Tails}.
  - There are 2 possible outcomes.
  - If you want to know the probability of getting heads, there's 1 head in the sample space.
  - So, the probability of getting heads is $\frac{1}{2}$.

- **Deck of Cards**: You draw one card from a standard deck of 52 cards.
    - There are 52 possible outcomes.
    - If you want to know the probability of drawing a king, there are 4 kings in a deck.
    - So, the probability of drawing a king is $\frac{4}{52} = \frac{1}{13}$

Types of Probability:

- **Theoretical Probability**: When all outcomes are equally likely, and you calculate the probability based on possible outcomes (like the examples above).
- **Empirical Probability:** When you calculate probability based on actual data from experiments. For example, if you roll a die 100 times and get a 6, 15 times, the empirical probability of rolling a 6 is $\frac{15}{100} = \frac{3}{20}$

Quick Recap:

- **Probability** is a way to measure how likely something is to happen.
- **Calculate probability** by dividing the number of ways your event can happen by the total number of possible outcomes.
- **Theoretical probability** is based on possible outcomes; **Empirical probability** is based on actual data.

Fundamental Counting Principle

- **What It Is**: If you have multiple choices to make, and each choice can be made independently of the others, you just multiply the number of options for each choice to get the total number of possible outcomes.
- **Example**: If you have 3 shirts and 2 pairs of pants, the total number of outfits you can make is $3 \times 2 = 6$.

Permutations

- **What It Is**: A permutation is an arrangement of items where the order *does* matter.
- **Example**: If you have 3 books and want to know in how many ways you can arrange them on a shelf, you are looking for the number of permutations. For 3 books, the number of permutations is $3!$ (which is $3 \times 2 \times 1 = 6$).

Combinations

- **What It Is**: A combination is a selection of items where the order *doesn't* matter.
- **Example**: If you want to choose 2 books from a set of 3, and the order doesn't matter, you are looking for the number of combinations. For 3 books taken 2 at a time, the number of combinations is calculated using the combination formula.

Applying Probability

- **Probability with Permutations**: If you're looking at the probability of specific ordered outcomes, you consider permutations.
    - **Example**: If you want to know the probability that out of 6 jokes, a specific one is told first and another specific one is told last, you'd use permutations to find the number of favorable outcomes and divide it by the total number of possible permutations.

- **Probability with Combinations**: If the order doesn't matter, and you want to know the probability of selecting a group of items, you use combinations.
    - **Example**: In a lottery, the numbers can be drawn in any order, so you use combinations to find the total number of possible selections and calculate the probability.

Key Idea

- **Order Matters?** If yes, think permutations.
- **Order Doesn't Matter?** Think combinations.

Events Involving "Not"

- **What does "Not" mean?**
  - Imagine you're playing a game, and there's a chance you could win. The event "Not winning" is the opposite of the event "Winning."
- **Formula for "Not" Events:**
  - The probability of "Not" happening is just 1 minus the probability of it happening.
  - **Formula:** P (Not E) =1−P(E)
  - **Example:** If the probability of drawing a queen from a deck of cards is $\frac{1}{13}$, then the probability of **not** drawing a queen is $1 - \frac{1}{13} = \frac{12}{13}$

Events Involving "Or"

- **What does "Or" mean?**
  - "Or" means you’re interested in either one event happening or another event happening (or both).
- **Types of "Or" Events:**
  - **Mutually Exclusive Events:**
    - These are events that cannot happen at the same time. If you draw one card, it can’t be both a king and a queen.
    - Formula for Mutually Exclusive Events:
      P(A or B)=P(A)+P(B)
    - **Example:** The probability of drawing a king or a queen from a deck is $\frac{4}{52} + \frac{4}{52} = \frac{8}{52} = \frac{2}{13}$
  - **Not Mutually Exclusive Events:**
    - These are events that can happen at the same time. A card could be both a queen and black.
    - Formula for Not Mutually Exclusive Events:
      $P(A\ or\ B) = P(A) + P(B) - P(A\ and\ B)$

- Example: If you want the probability of drawing a card that is either a queen or a black card, you'd add **the** probabilities but subtract the overlap (where the card is both a queen and black).

Odds

- **What are Odds?**
  - Odds are another way of talking about the likelihood of an event, but instead of using fractions or percentages, we talk about the ratio of success to failure.
  - **Formula for Odds:**
    - **Odds in favor:** Odds in favor of E$= \frac{P(E)}{P(Not\ E)}$
    - **Odds against:** Odds against E$= \frac{P(Not\ E)}{P(E)}$
- Example: If the probability of winning a game is $\frac{1}{4}$, the odds in favor are $\frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3}$
  - This means for every 1 time you win, you lose 3 times.

Independent Events ("And" Probability)

- **What are Independent Events?**
  - Imagine flipping a coin twice. The outcome of the first flip (heads or tails) doesn't affect the outcome of the second flip. These are independent events because what happens in one doesn't change what happens in the other.
- **How do you find the probability of two independent events both happening?**
  - You multiply the probability of each event.
  - For example, if you want the probability of getting heads on both flips, you multiply the chance of heads on the first flip by the chance of heads on the second flip:

$$P(heads\ and\ heads) = P(heads) * P(heads) = \frac{1}{2} * \frac{1}{2} = \frac{1}{4}$$

Dependent Events (Conditional Probability)

- **What are Dependent Events?**
  - Now, think about picking a candy from a box with 5 chocolate-covered cherries out of 20 candies. If you pick one and eat it, the total number of candies left is now 19, and the chances of picking another chocolate-covered cherry changes. This is because your first pick affects your second pick—these are dependent events.

- **How do you find the probability of two dependent events happening?**
  - You first find the probability of the first event, and then multiply it by the probability of the second event happening, **given** that the first event has already happened.

- Example: P (two chocolate cherries) = P(first chocolate)×P(second chocolate given first was chocolate)=$\frac{1}{4} * \frac{4}{19} = \frac{1}{19}$

Conditional Probability

- **What is Conditional Probability?**
  - It's the probability of an event happening, given that another event has already happened. It's like asking, "What are the chances of this happening now that I know this other thing has already happened?"
  - The formula for conditional probability is:

$$P(B|A) = \frac{P(A\, and\, B)}{P(A)}$$

  - This means you're finding the probability of B happening when you know A has already happened.

Key Formulas to Remember:

- **Independent Events:** $P(A\ and\ B) = P(A) * P(B)$
- **Dependent Events:** $P(A\ and\ B) = P(A) * P(B\ given\ A)$
- **Conditional Probability:** $P(B|A) = \frac{P(A\, and\, B)}{P(A)}$

Expected Value

Expected value is a concept used in probability to predict what might happen on average in the long run. It gives you an idea of what you can expect over time if you repeat an experiment or a game many times.

How to Calculate Expected Value:

1. **List All Possible Outcomes**: Write down all the different results that could happen.
2. **Assign Probabilities**: Each outcome has a probability, which is the chance of that outcome happening.
3. **Multiply**: For each outcome, multiply the value (what you get or lose) by its probability.
4. **Add**: Add up all those values. This sum is your expected value.

Example 1: Rolling a Die

If you roll a fair six-sided die, the outcomes are 1, 2, 3, 4, 5, and 6. Each number has a 1/6 chance of appearing.

To find the expected value, you multiply each number by its probability (1/6) and then add them up:

$$E = \frac{1}{6} * 1 + \frac{1}{6} * 2 + \frac{1}{6} * 3 + \frac{1}{6} * 4 + \frac{1}{6} * 5 + \frac{1}{6} * 6 = \frac{7}{2} = 3.5$$

So, the expected value is 3.5. This means that if you roll the die many times, the average of all the numbers rolled will be around 3.5.

Example 2: Insurance Policy

Imagine you have an insurance policy that costs $50 and pays out $200,000 if you become seriously ill. The probability of this happening might be 1 in 5,000.

- Then expected payout for the insurance company is calculated as:

$$Expected\ Payout = 200000 * \frac{1}{5000} = \$40$$

The company expects to pay out $40 per policy on average, but since they charge $50, they expect to make a $10 profit per policy.

Key Takeaways:

- **Expected Value** helps you understand the average outcome over time.
- It is calculated by multiplying each possible outcome by its probability and then summing them up.

Population and Sample:

- **Population:** Imagine you're trying to understand something about a large group, like all the TV households in the country. This entire group is called the population.
- **Sample:** Since it's impossible to talk to every single household, you take a smaller group from this population, called a sample. This sample should represent the larger population as accurately as possible.

Random Sampling:

- **Random Sample:** This is like drawing names from a hat. Everyone in the population has an equal chance of being chosen. For example, if you want to know how students feel about a course, you could randomly select a few students to ask instead of asking everyone.

Frequency Distributions:

- **Frequency Distribution:** After you gather data from your sample, you need to organize it to make sense of it. A frequency distribution is a table that shows how often each different value in a set of data occurs.
  - **Data Item:** This is just a single piece of information, like a student's stress rating.
  - **Frequency:** This tells you how many times a particular data item appears. For instance, if 42 students gave a stress rating of 6, the frequency for 6 is 42.

Grouped Frequency Distributions:

- **Grouped Frequency Distribution:** When you have a lot of data, you might want to group the data into ranges (like 0-10, 11-20, etc.). This helps simplify the data and makes it easier to see patterns.
    - **Class Limits:** These define the range for each group (e.g., 0-10 is one class, 11-20 is another).
    - **Class Width:** This is the size of each class or group. If one class goes from 0 to 10, the width is 10.

Graphs:

- **Stem-and-Leaf Plot:** This is a simple way to organize data and see how it's distributed. The "stem" is usually the tens digit, and the "leaf" is the ones digit. For example, the number 52 would have 5 as the stem and 2 as the leaf.

Measures of Central Tendency
refer to different ways of finding the "center" or "average" of a set of data. Here are the most common ones:

Mean (Average):

- To find the mean, you add up all the numbers in the data set and then divide by the total number of items.
- Formula: Mean = $\frac{\sum x}{n}$
  - $\sum x$ is the sum of all data values.
  - n is the number of data values.

    Example: If you have the numbers 2, 4, 6, 8, and 10, the mean would be:

$$Mean = \frac{2+4+6+8+10}{5} = \frac{30}{5} = 6$$

Median:

- The median is the middle value in a data set when the numbers are arranged in order.
- If there's an odd number of data points, the median is the middle one.
- If there’s an even number of data points, the medium is the average of the two middle numbers.
- **Example:** If you have the numbers 3,5,7,9, and 11, the medium is 7. If the numbers are 3,5,7,9,11, and 13 then the medium is $\frac{7+9}{2} = 8$

Mode:

- The mode is the number that appears most frequently in a data set.
- A data set can have more than one mode if multiple numbers appear with the same highest frequency, or it can have no mode if no number repeats.
- Example: In the set 1, 2, 2, 3, 4, the mode is 2. In the set 1, 2, 3, 4, all numbers appear once, so there is no mode.

Midrange:

- The midrange is found by adding the smallest and largest numbers in the data set and dividing by 2.
- It gives a simple estimate of the center
- **Example:** For the numbers 10 and 90, the midrange is $\frac{10+90}{2} = 50$

Each of these measures gives us a different way to understand the "center" or "average" of a data set, and they are used depending on what kind of data you have and what you want to understand from it.

Dispersion

is about how spread out the data is. Imagine you have a set of numbers, like test scores from a class. Measures of dispersion tell you how much these scores differ from each other. Here are the key measures:

Range:

- This is the simplest measure of dispersion. It's the difference between the highest and lowest values in your data.
- Formula: Range=Highest Value−Lowest Value
- Example: If the highest score is 100 and the lowest is 60, the range is 40.

Standard Deviation:

- Standard deviation is a bit more complicated. It tells you how much each number in the set deviates, or differs, from the mean (average) of the set.
- **Steps to Calculate Standard Deviation:**
  1. **Find the mean** of the data set.
  2. **Subtract the mean** from each data point to find the deviation for each point.
  3. **Square** each deviation (this makes them positive numbers).
  4. **Sum up** all the squared deviations.
  5. **Divide** this sum by the number of data points minus one (this is called the variance).
  6. **Take the square root** of the variance to get the standard deviation.
- **Formula**: Standard Deviation= $\sqrt{\frac{\sum(data\ points - mean)^2}{n-1}}$

    - **Example:** If most scores are close to the mean, the standard deviation is small. If they are spread out, the standard deviation is large.

Why It Matters

- **Range** gives you a quick sense of how spread out your data is, but it only considers the extremes.
- **Standard Deviation** gives a fuller picture by considering every data point, showing you how much the values typically differ from the mean.

Normal Distribution

Normal distribution is a way to describe how data is spread out in many real-world situations, like heights, test scores, or measurement errors. It is also known as the "bell curve" because of its shape.

Key Points:

Shape: The normal distribution is symmetrical, which means it looks the same on both sides of the center. The highest point is in the middle, where the mean, median, and mode are all equal.

68-95-99.7 Rule:

- About 68% of the data falls within 1 standard deviation of the mean.
- About 95% of the data falls within 2 standard deviations of the mean.
- About 99.7% of the data falls within 3 standard deviations of the mean.

Standard Deviation and z-Scores:

- Standard Deviation tells you how spread out the data is from the mean.
- **Z-Score:** This shows how many standard deviations a particular data point is from the mean. If the z-score is positive, the data point is above the mean. If it is negative, the data point is below the mean
- **Formula:** $\mathrm{zScore} = \frac{\text{data item} - \text{mean}}{\text{standard deviation}}$

Percentiles:

- A percentile tells you how much of the data is below a particular point. For example, if you're in the 90th percentile, that means you're above 90% of the data.

Margin of Error:

- This is a way to express the uncertainty in a sample statistic. It tells you about the range within which the true population parameter is likely to fall.
- Formula: $\pm \frac{1}{\sqrt{n}} * 100$

A distribution can be "skewed" when it isn't symmetrical like a normal (bell-shaped) distribution. Skewness tells us if the data is leaning more to one side.

Symmetric Distribution:

- In a symmetric distribution, the data is evenly spread out on both sides of the center. The mean, median, and mode are all in the middle.

Skewed Right (Positive Skew):

- If a distribution is skewed to the right, most of the data is on the left side, and the tail (the thin end) stretches to the right.
- Example: Incomes in a country where most people earn low to moderate amounts, but a few earn very high incomes. The "few" high incomes pull the tail to the right.
- In this case, the mean is usually greater than the median.

Skewed Left (Negative Skew):

- If a distribution is skewed to the left, most of the data is on the right side, and the tail stretches to the left.
- Example: Test scores where most students did well, but a few scored very low. The "few" low scores pull the tail to the left.
- In this case, the mean is usually less than the median.

Visualizing Skewness:

- **Right Skew**: Imagine a hill where the left side is steeper, and the right side is a gentle slope. Most of the weight is on the left, but the curve drags out to the right.
- **Left Skew**: Imagine the opposite, where the right side is steeper, and the left side is a gentle slope. Most of the weight is on the right, but the curve drags out to the left.

A **normal distribution** is a type of distribution that is symmetric and bell-shaped. It means most of the data points (like test scores, heights, etc.) are around the average, with fewer points farther away.

Z-Scores

A **z-score** tells you how far a data point is from the average (mean), measured in standard deviations. Think of it as a way to figure out if a particular number is high, low, or average.

Formula: $\text{zScore} = \frac{\text{data item} - \text{mean}}{\text{standard deviation}}$

- **Positive z-score**: The data point is above the mean.
- **Negative z-score**: The data point is below the mean.
- **z = 0**: The data point is exactly at the mean.

How to Use Z-Scores

You can use z-scores to find out what percentage of data falls below or above a certain value.

Percentiles

- A **percentile** is a way to rank data points. For example, if you're in the 90th percentile, it means you scored higher than 90% of the other scores.

Example: Finding the Percentage of Data Below a Certain Point

Formula: $\text{zScore} = \frac{\text{data item} - \text{mean}}{\text{standard deviation}}$

- **Use a z-score table** (Attach on page 99/100) to find the percentile. This tells you what percentage of the data is below your data point.
    - Example: If you have a z-score of 1.5, you look it up on the table, and it might say 93.32%. This means 93.32% of the data points are below your value.

Example: Finding the Percentage of Data Above a Certain Point

- **Find the z-score** using the same formula.
- **Find the percentile** using the z-score table.
- **Subtract this percentile from 100%** to find the percentage of data above your point.
    - **Example:** If your z-score is 1.5, and the table says 93.32%, then 100% - 93.32% = 6.68% of the data is above your value.

Example: Finding the Percentage Between Two Data Points

- **Find the z-scores** for both data points.
- **Use the z-score table** to find the percentiles for both.
- **Subtract the smaller percentile from the larger one** to find the percentage of data between those points.
    - **Example:** If one z-score is 1.5 (93.32%) and another is 0.5 (69.15%), then 93.32% - 69.15% = 24.17% of the data is between these two points.
    -

Reading Z-Scores from a Table

When using a z-score table:

- Find your z-score (like 1.2 or -1.5).
- Read across the row to find the corresponding percentile

Scatter Plots:

- Imagine you have two sets of numbers (like years of education and scores on a test). A scatter plot helps you see if there's a relationship between these two sets.
- You plot each pair of numbers as a point on a graph. For example, if someone has 12 years of education and scores a 5 on a test, you'd put a point where 12 (education) and 5 (score) meet.

Correlation:

- **Positive Correlation:** As one number goes up, the other tends to go up too. Think of it like when you study more, your grades usually go up.
- **Negative Correlation:** As one number goes up, the other goes down. For example, the more you watch TV, the less time you have to study.
- **No Correlation:** The numbers don't follow any clear pattern; they seem random.

Regression Lines:

- A regression line is like drawing the best-fitting straight line through your scatter plot. It helps predict one set of numbers based on the other.
- For example, if you know how many hours someone studied, the regression line can help predict their test score.

Correlation Coefficient (r):

- This is a number between -1 and 1 that tells you how strong the relationship is:
    - **r = 1** means perfect positive correlation (as one goes up, the other always goes up).
    - **r = -1** means perfect negative correlation (as one goes up, the other always goes down).
    - **r = 0** means no correlation.

Formulas:

- To find the correlation coefficient r by hand: $r = \frac{n(\sum xy)-(\sum x)(\sum y)}{\sqrt{n(\sum x^2)-(\sum x)^2}\sqrt{n(\sum y^2)-(\sum y)^2}}$
- The equation of the regression line is $y = mx + b$
- Where:
    - m is the slope of the line, showing how much y changes for a unit change in x.
    - b is the y-intercept, the point where the line crosses the y-axis.

** Note: Instead of calculating it I would use the online platform GeoGebra to calculated for you. Step by step guide is on page 100-101 **

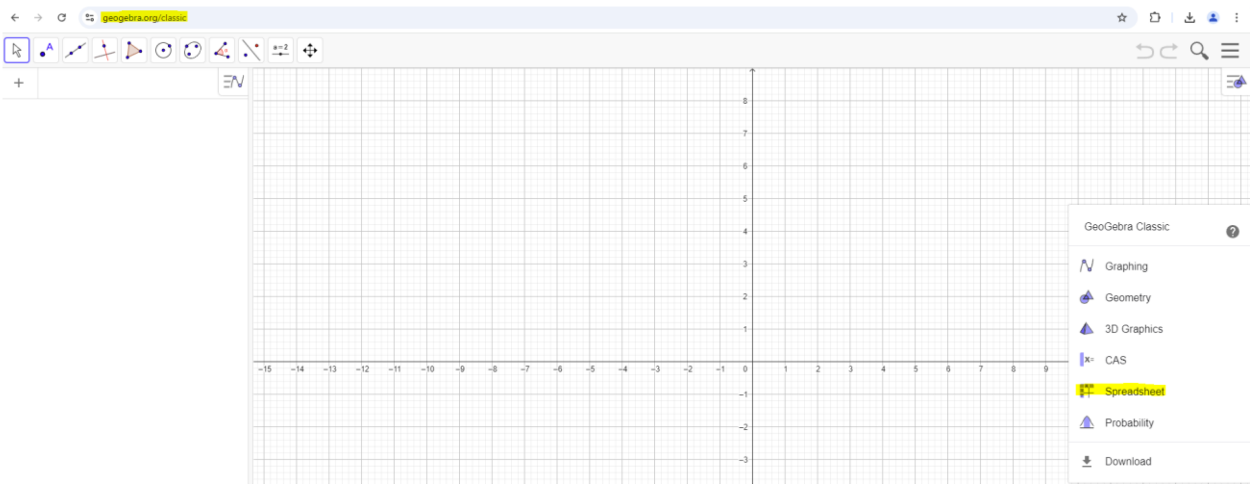
References

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

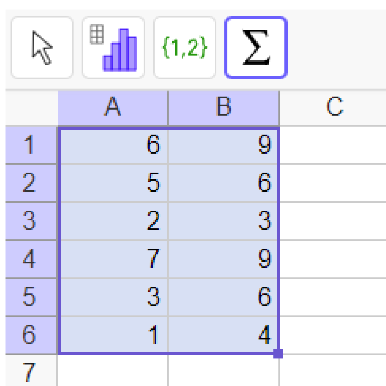
z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

How to find Correlation on GeoGebra

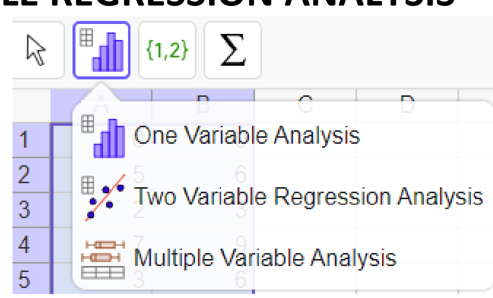
Step 1: Go to GeoGebra Classic and click the spreadsheet tab on right side



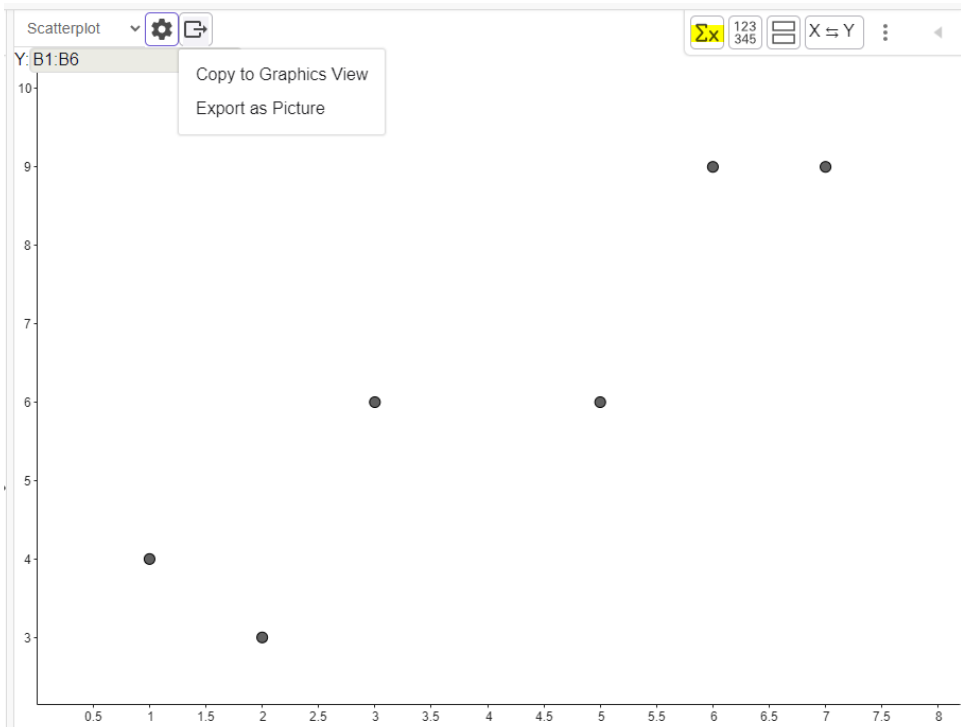
Step 2: Fill in your data points in the spreadsheet and then highlight your data



Step 3: Click on that second box on top that looks like bar graph and choose **TWO VARIABLE REGRESSION ANALYSIS**



Step 4: Once you hit that button on the right-hand side you get you graph and then you will see the symbol $\sum x$ click on that



Step 5: Read the Statistics is gives you

Statistics

MeanX	4
MeanY	6.1667
Sx	2.3664
Sy	2.4833
r	0.9189
ρ	0.9122
Sxx	28
Syy	30.8333
Sxy	27

Correlation is the symbol r