

Complete the following questions on a separate piece of paper. Show all work.  
 Do not use your notes unless absolutely necessary. If you use your notes please indicate  
 where.  
 Questions will be gone over at tutoring sessions. Please be prepared.

## Worksheet 1

### Algebraic and Calculus Preliminaries

1. Perform the indicated operation and reduce the answer to the lowest terms, if applicable.

(a)  $\frac{y}{y^2 - 2y + 1} - \frac{2}{y - 1} + \frac{3}{y + 2}$

(b)  $\frac{2x}{x^2 - 9} - \frac{1}{x + 3} - \frac{2}{x - 3}$

(c)  $\frac{4}{z + 2} - \frac{1}{z} + 1$

(d)  $\frac{x^2 - 5x - 14}{x^2 - 3x + 2} \cdot \frac{x^2 - 4}{x^2 - 14x + 49}$

(e)  $\frac{m^2 - 9}{m^2 + 5m + 6} \div \frac{3 - m}{m + 2}$

2. Solve each given equation. (Find only the real solutions)

(a)  $x + 5 = 14\frac{x}{2}$

(b)  $\frac{2x}{x + 1} = \frac{2x - 1}{x}$

(c)  $x^2 - x - 12 = 0$

(d)  $2x^2 + 4x = -1$

(e)  $3|x - 4| = 10$

3. Simplify each of the following expressions.

(a)  $\left( \frac{x^2 y^{-\frac{2}{3}}}{x^{-\frac{1}{2}} y^{-3}} \right)^2$

(b)  $e^{-3x} \left( -3x + \frac{x e^{-x}}{e^{2x}} \right)$

(c)  $\frac{x^{\frac{3}{2}} y^{\frac{7}{3}}}{x^3 y^{\frac{2}{9}}}$

4. Complete the following with differentiation techniques.

(a)  $f(x) = x^2 \ln x$

(b)  $g(x) = -\frac{1}{4x^2}$

- (c)  $h(x) = e^{-y^2+5y+1}$
- (d)  $j(x) = \frac{\cos(x)}{\sin(x)}$
- (e)  $k(x) = x^5 - 3x^2 + 9x$
- (f)  $l(x) = \frac{x^2 + 1}{x}$
- (g)  $d(x) = \cos(x)(\sin(x) + \cos(x))$
- (h)  $a(x) = \ln(x^2 + 1)$
- (i)  $r(x) = 4x^{\frac{3}{5}} + 7\sqrt{x} - 5$
- (j)  $t(x) = (2x^2 + 1)(x - 3x^{-3})$
- (k)  $s(x) = \frac{x^2 + x - 3}{x^3 - 3}$
- (l)  $w(x) = \sin(\cos(4x^4 + x^3 - 2x^2 + 7))$
- (m)  $x^3 + y^2 = 6xy$
- (n)  $y = \frac{xy}{x^2 + y^2}$
- (o)  $y\sqrt{x^2 - x} = \frac{1}{y^2} + \frac{7}{x^3}$

5. Integrate using u-substitution

- (a)  $\int y^5(1 + 2y^6)^7 dy$
- (b)  $\int \sin t(\cos t + 5)^7 dt$
- (c)  $\int_0^1 (2x^3 + 3x) \cos(x^4 + 3x^2) dx$

6. Integrate using integration by parts (IBP or IP).

- (a)  $\int xe^x dx$
- (b)  $\int e^x \sin(x) dx$
- (c)  $\int (x^2 + 3x + 1) \sin(x) dx$

## Worksheet 2

### Classifying ODE's and Implicit Differentiation/IVP's

7. Classify the following ordinary differentiation equations (ODE's) by Degree, Order, Linearity, Homogeneity, and variable or constant Coefficients.

(a)  $\frac{dy}{dx} + xy^2 = 1$

(b)  $x\frac{dy}{dx} + y = \sin(x)$

(c)  $e^x \frac{d^2y}{dx^2} + 2dydx + y = 0$

(d)  $y\frac{d^2y}{dx^2} + \frac{dy}{dx} + xy = 1$

(e)  $\frac{d^3y}{dx^3} + \frac{d^2y}{dx^2} + \frac{dy}{dx} + y = 0$

(f)  $\frac{d^3y}{dx^3} + y = \sin(x)$

(g)  $\frac{d^2w}{dt^2} - w^2 \frac{dw}{dt} + w = 0$

(h)  $\frac{d^2v}{dt^2} = t^2v$

(i)  $\frac{d^2y}{dt^2} + y^2 = 0$

8. Show that the following functions are solutions of their following respective ODE's.

(a)  $\frac{dy}{dx} = ay; \quad y = e^{ax}$

(b)  $\frac{dy}{dx} = y + e^x; \quad y = xe^x$

(c)  $\frac{dy}{dx} = \frac{x}{\sqrt{x^2 + a^2}}; \quad a \neq 0; \quad y = \sqrt{x^2 + a^2}$

(d)  $yy' = e^2x; \quad y^2 = e^2x$

(e)  $y' = \frac{xy}{x^2 + y^2}; \quad 2y^2 \ln y - x^2 = 0$

(f)  $y' + y = 0; \quad y(0) = 2; \quad y(x) = 2e^{-x}$

(g)  $y' = y^2; \quad y(0) = 0; \quad y(x) = 0$