

$$y' = -ky \rightarrow y' + ky = 0$$

$$y(0) = y_0 \quad | \quad y(t_h) = \frac{y_0}{2}$$

(a)

$$I \neq: e^{\int k dt} \rightarrow e^{kt}$$

$$\therefore \int \frac{d}{dt} (y \cdot e^{kt}) dt = \int (0 \cdot e^{kt}) dt$$

$$y e^{kt} = C \rightarrow y = C e^{-kt}$$

$$y_0 = C e^{-k(0)} \rightarrow y_0 = C \rightarrow y(t) = y_0 e^{-kt}$$

$$\rightarrow y(t_h) = y_0 e^{-kt_h} \rightarrow \frac{y_0}{2} = y_0 e^{-kt_h} \rightarrow$$

$$\frac{1}{2} = e^{-kt_h} \rightarrow -kt_h = \ln\left(\frac{1}{2}\right) \rightarrow$$

$$t_h = \frac{\ln(1) - \ln(2)}{-k}$$

$$\Rightarrow t_h = \frac{\ln(2)}{k}$$

$$\frac{dS}{dt} = rS + d \rightarrow S' - rS = d$$
$$S(0) = S_0$$

(2)

IF: $e^{\int -r dt} \rightarrow e^{-rt}$

$$\therefore \int \frac{d}{dt} (S \cdot e^{-rt}) dt = \int (d \cdot e^{-rt}) dt$$

$$S e^{-rt} = -\frac{d}{r} e^{-rt} + C \rightarrow$$

$$S = -\frac{d}{r} + C e^{rt}$$

$$S_0 = -\frac{d}{r} + C e^{r(0)} \rightarrow S_0 = -\frac{d}{r} + C$$

$$\rightarrow C = S_0 + \frac{d}{r} \quad \therefore$$

$$S(t) = \left(S_0 + \frac{d}{r} \right) e^{rt} - \frac{d}{r}$$