Complete the following questions. Try your best to do the problems on your own. If you get stuck ask a friend. Good luck ☺ .

Worksheet 6

Midterm 2 Practice

(Sections 2.1, 2.2, 2.3, 2.5, 2.7, 3.1)

1. Solve the following IVP's

   (a) $\begin{cases} \dfrac{dy}{dx} - \dfrac{3}{x}y & = x^3 \\ y(1) & = 4 \end{cases}$

   (b) $\begin{cases} \dfrac{dy}{dx} & = x^2 y \\ y(1) & = 1 \end{cases}$

2. The residual amount of C-14 present in the Lascaux charcoal samples was 15% of the original amount at the time the tree died. The half-life (the time required to decay to one-half of the original amount) of C-14 is approximately 5,600 years. The quantity Q of C-14 satisfies the decay equation

$$\frac{dQ}{dt} = -kQ.$$

   (a) Find the decay constant k if $Q(0) = Q_0$

   (b) Find the quantity $Q(t)$ at any time t

3. 20 year old Isabella has just started a new job, and has decided to try and quit smoking. With this new goal it was suggested they put their cigarette money into a back account. Because Isabella smoked almost 2 packs a day they were spending around $30.00 per week. If they make weekly deposits that pays annual interest of 10% compounded continuously,

   (a) What would the balance of Isabella's account in 5 years?

   (b) If Isabella keeps up with their goal, without slip ups, how much money would be in their account when they have their first child 15 years from when they started the account?

4. Chad always drinks a cup of tea before his 9:30 a.m. class. Suppose the tea is $210°F$ when it is freshly poured at 9:00 a.m., and 5 minutes later it cools to $180°F$ in a room of temperature $70°F$. However, he never drinks his tea until it cools to $135°F$. When can he drink his tea?

   (Hint: Use Newton's Law of Cooling $\begin{cases} \dfrac{dT}{dt} & = -k(T - M) \\ T(0) & = T_0. \end{cases}$

5. John and Mary have just built a new house, and the builder informs them that the insulation in the house provides for a *time constant* of 5 hours. Suppose that at midnight the

furnace fails with the outside temperature at the constant 10° and the inside temperature at 70°.
(Hint: Remember time constant $=\dfrac{1}{k}$.)

(a) Determine the initial-value problem that describes the future temperature inside the house.

(b) Using part (a), find $T(t)$.

(c) Determine the temperature inside the house at 5:00 a.m..

6. For the following DE's,

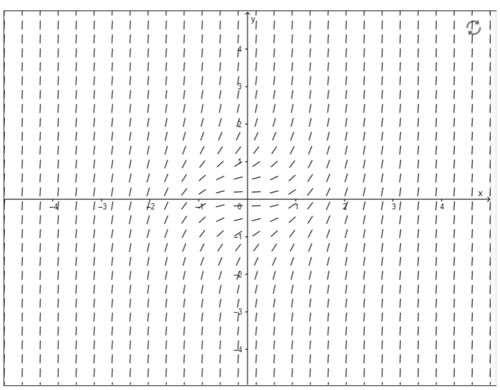
- Match each DE to their respective direction field. Explain why.
- Use the direction field of each DE to identify a constant (equilibrium) solution, if such solution exists.

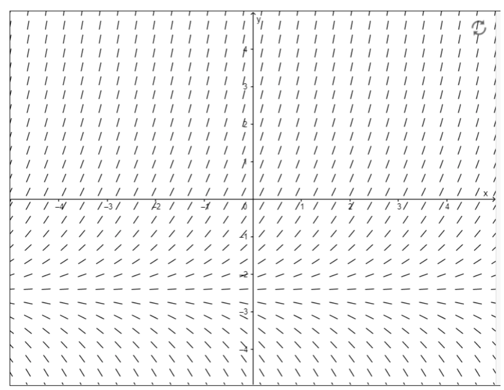
(a) $\dfrac{dy}{dx} = x^2 + y^2$

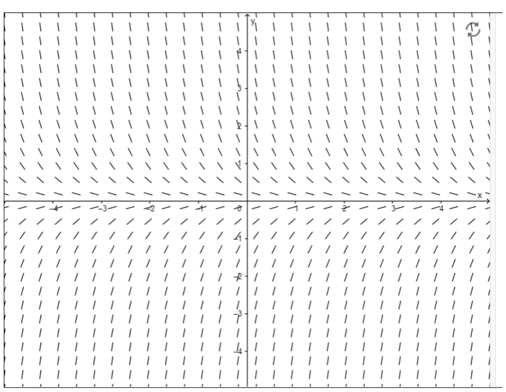
(b) $\dfrac{dy}{dx} = \dfrac{y}{x}$

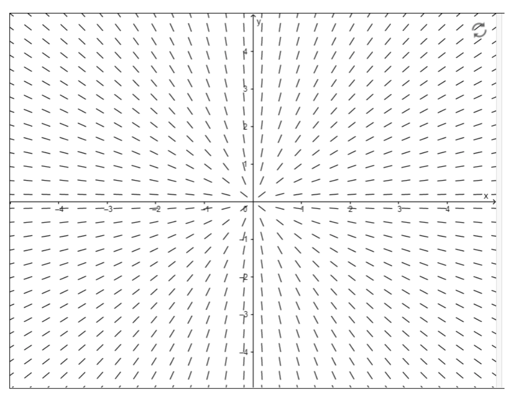
(c) $\dfrac{dy}{dx} = -2y$

(d) $\dfrac{dy}{dx} = y + 4$









7. For the following IVP's use the Euler method to numerically solve. Then solve the IVP's and compare the exact and approximate values.

(a) Use Euler's Method to approximate $y(0.5)$ for the following IVP. Use $h = 0.1$.

$$\begin{cases} \dfrac{dy}{dx} &= y \\ y(0) &= 1. \end{cases}$$

(b) Approximate $y(.45)$ using 10 steps on the interval $0 \leq x \leq \dfrac{3}{2}$ for the IVP

$$\begin{cases} \dfrac{dy}{dx} &= x + y \\ y(0) &= 1. \end{cases}$$

8. Determine whether the given equation is linear or nonlinear. If linear, classify as homogeneous or non-homogeneous and with constant or variable coefficients.
   (a) $y'' + xy' - y = \sin(x)$
   (b) $y'' - (x+1) + xy = \sin(y)$
   (c) $4y'' - xy = 0$

9. Verify that $y_1(x) = \sin(x)$ and $y_2(x) = \cos(x)$ are solutions of $y'' + y = 0$. Find two other solutions.

10. Find the specific function among the two-parameter family of functions that satisfies the specified initial conditions.

(a) $y(x) = c_1 + c_2 x$	$y(0) = 1$	$y'(0) = -1$
(b) $y(x) = c_1 e^{2x} + c_2 e^{-5x}$	$y(0) = 1$	$y'(0) = 0$
(c) $y(x) = c_1 e^{2x} + c_2 x e^{2x}$	$y(0) = 0$	$y'(0) = 2$

11. Determine the largest interval on which a unique solution of

$$\begin{cases} xy'' + \dfrac{x}{x-4} y' + y &= \sin(x) \\ y(1) &= 1 \\ y'(1) &= 0 \end{cases}$$

must exist.