CSE 460 Mobile Robotics - Lab 1

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***Abstract*— Lab 1 report for CSE 460 Mobile Robotic with Professor David Salda$\tilde{n}$a.**

I. Build a Simulator

For the first portion of the lab, I used the steps in Section 2.4 of the notes to build a simulator in Python. This included a circular trajectory as well as a line trajectory, where the difference in the code was the control policy. The code I experimented with to build a simulator can be found here: simulator.ipynb

II. Chapter 2 Exercises

The code for exercises 1 and 2 in chapter 2 can be found here: Ch.2 Exercises

A. Exercise 1

In exercise 1, the task is to find the control policy that causes the robot to move in an ellipse shape. The first step is to build the parametric equation that describes the ellipse. An ellipse rotating counter-clockwise can be represented as

$$\begin{bmatrix} -asin(t) + h \\ bcos(t) + k \end{bmatrix} \tag{1}$$

where a is the major axis, b is the minor axis, and (h,k) is the center of the ellipse. We are told a = 4, b = 2, and the center is at $\begin{bmatrix} 3 & 2 \end{bmatrix}^T$, all of which we can use to build p(t). We can also obtain the initial position by evaluating p(t) where t = 0.

$$p(t) = \begin{bmatrix} -4sin(t) + 3 \\ 2cos(t) + 2 \end{bmatrix} \tag{2}$$

$$p(0) = \begin{bmatrix} 3 \\ 4 \end{bmatrix} \tag{3}$$

The control function, u(t), is the first derivative of the position, p(t). This is the function that determines the motion of the robot.

$$u(t) = p(t)\frac{d}{dt} \tag{4}$$

$$u(t) = \begin{bmatrix} -4sin(t) + 3 \\ 2cos(t) + 2 \end{bmatrix} \frac{d}{dt} \tag{5}$$

$$u(t) = \begin{bmatrix} -4cos(t) \\ -2sin(t) \end{bmatrix} \tag{6}$$

Plugging u(t) into the control function of the point-robot simulator outputs the trajectory in Figure 1.

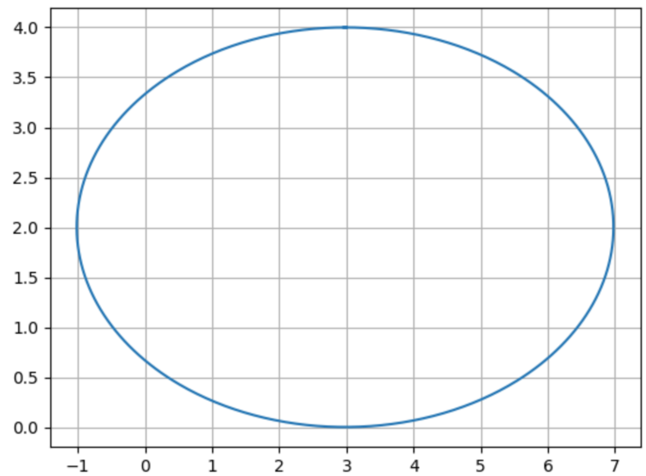

Fig. 1. Ch.2 Exercise 1

B. Exercise 2

In exercise 2, we need to rotate the major axis of the ellipse from exercise 1 by 30 degrees. To do so, we multiply the rotation matrix, R, by the position function, p(t), prior to shifting its center from the origin.

$$p(t) = \begin{bmatrix} -4sin(t) \\ 2cos(t) \end{bmatrix}, R = \begin{bmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{bmatrix} \tag{7}$$

Rotating about the center by 30 degrees gives us the new position function.

$$p(t) = \begin{bmatrix} cos(30) & -sin(30) \\ sin(30) & cos(30) \end{bmatrix} \begin{bmatrix} -4sin(t) \\ 2cos(t) \end{bmatrix} + x_0 \tag{8}$$

$$p(t) = \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} -4sin(t) \\ 2cos(t) \end{bmatrix} + x_0 \tag{9}$$

Performing matrix multiplication and substituting $\begin{bmatrix} 3 & 2 \end{bmatrix}^T$ for x_0 gives us:

$$p(t) = \begin{bmatrix} -2\sqrt{3}sin(t) - cos(t) + 3 \\ -2sin(t) + \sqrt{3}cos(t) + 2 \end{bmatrix} \tag{10}$$

To obtain our initial position, we evaluate p(0).

$$p(0) = \begin{bmatrix} 2 \\ \sqrt{3} + 2 \end{bmatrix} \tag{11}$$

We can obtain the control function u(t) by taking the first derivative of p(t).

$$u(t) = p(t)\frac{d}{dt} \tag{12}$$

$$u(t) = \begin{bmatrix} -2\sqrt{3}sin(t) - cos(t) + 3 \\ -2sin(t) + \sqrt{3}cos(t) + 2 \end{bmatrix} \frac{d}{dt} \tag{13}$$

$$u(t) = \begin{bmatrix} -2\sqrt{3}cos(t) + sin(t) \\ -2cos(t) - \sqrt{3}sin(t) \end{bmatrix} \tag{14}$$

Updating the control function and initial position in the simulator gives the output trajectory shown in Figure 2.

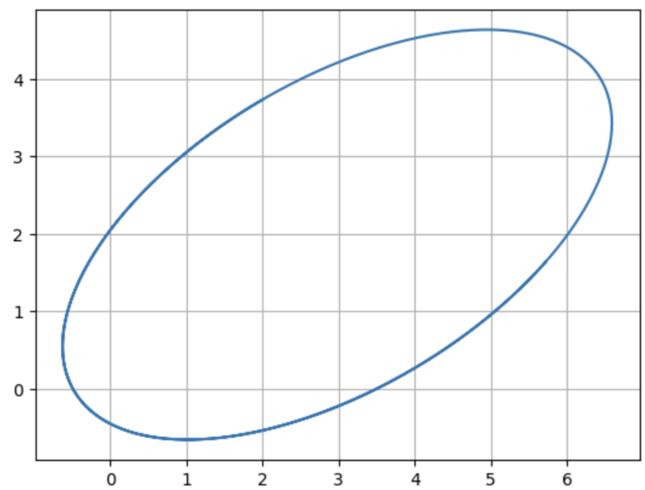

Fig. 2. Ch.2 Exercise 2

REFERENCES

[1] Amanda Baran, Github, https://github.com/amandabaran/robotics/tree/main/lab1