Understanding formalisms in GR

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Motivation

Why use Landau-Lifshitz formalism?

- Relating the gothic metric and the energy-momentum tensor allows **perturbations** in the maximally symmetric space-time and **find solutions** imposing the harmonic gauge condition $\partial_\nu \mathfrak{g}^{\mu\nu} = 0$ that are otherwise in terms of **non integrals**.
- The potentials are expressed as $h^{\mu\nu} = \eta^{\mu\nu} - \mathfrak{g}^{\mu\nu}$ and h is the **trace-reversed** metric perturbation.
- It is the **first step** to understand other formalisms such as the 3+1 decomposition and the tetrad formalism.
- It directly gives a **wave form** which is used to model gravitational waves through $\Box_g h^{\mu\nu}$.
- It is the **best** formalism suited to a Post-Newtonian expansion [3].

Algorithm

Applying the formalism in the Mathematica framework

Structure of the code

PRELIMINARIES →

- Load package
- Define manifold
- Set function ApplyRule@

EINSTEIN TENSOR →

- Define Einstein tensor in terms of the Gothic metric
- Perturb the gothic metric

$$\mathfrak{g}^{\mu\nu} = \sqrt{-g}\, g^{\mu\nu}$$

$$h^{\mu\nu} = \eta^{\mu\nu} - \mathfrak{g}^{\mu\nu}$$

ASSIGNING COMPONENTS

- Set chart
- Define ansatz for the gothic metric
- Establish components for the gothic metric, perturbation tensor and the Einstein tensor

$$\mathfrak{g}^{00} = \sqrt{-g}\, a(r)$$

$$\mathfrak{g}^{0j} = 0$$

$$\mathfrak{g}^{jk} = \sqrt{-g}(\eta_{jk} - b(r)\Omega_j\Omega_i)$$

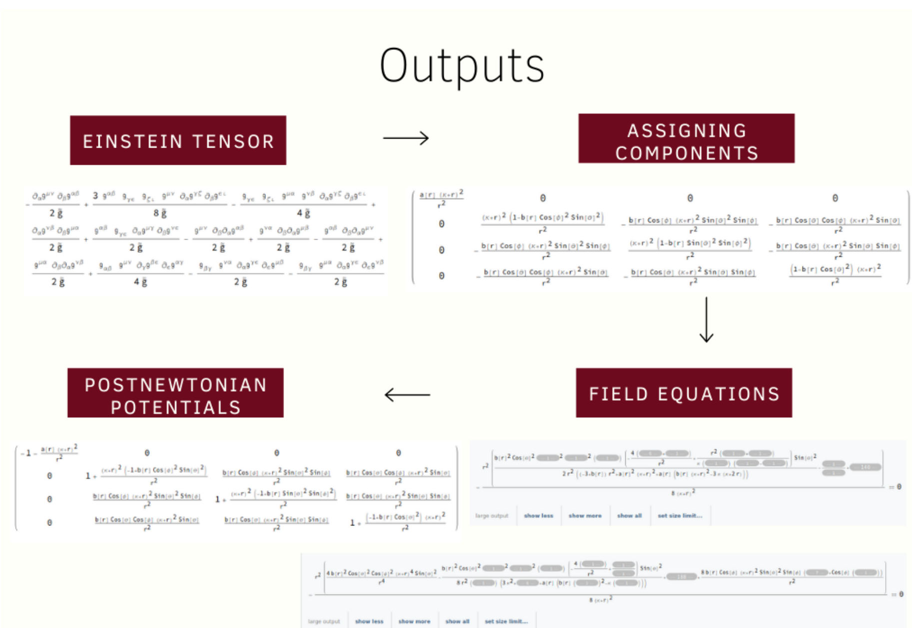
← FIELD EQUATIONS

- Consider an empty space
- Obtain each component of the Einstein tensor written in terms of the scalar functions
- Solve for the scalar functions

POSTNEWTONIAN POTENTIALS ←

- The first element of the perturbed gothic metric is the newtonian potential
- Multipole expansion of the field equations

Outputs


Outputs
EINSTEIN TENSOR
ASSIGNING COMPONENTS
FIELD EQUATIONS
POSTNEWTONIAN POTENTIALS
large output
show less
show more
show all
set size limit...

Future work

- Manually **simplify** the Einstein tensor to solve for the scalar functions
- Write the Ricci tensor in terms of the gothic metric to **vary the action** and find the field equations (Useful for f(R) models)
- Generate codes for a **3+1** decomposition and the **tetrad** formalism
- Find out why the metric in harmonic coordinates **looks like** a 3+1 decomposition
- Find out **similarities** between the formalisms

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