

Master Theorem Worksheet

This is a worksheet to help you master solving recurrence relations using the Master Theorem. For each recurrence, either give the asymptotic solution using the Master Theorem (state which case), or else state that the Master Theorem doesn't apply. You should be able to go through these 25 recurrences in 10 minutes.

Problem 1-1. $T(n) = 3T(n/2) + n^2$

$$\begin{aligned} a &= 3 & c_{\text{crit}} &= \log_3 2 \approx 0.631 \\ b &= 2 & & \\ c &= 2 & c &> c_{\text{crit}} \end{aligned}$$

for $\frac{3}{4} \leq k < 1$:

$$\frac{3n^2}{4} \leq kn^2$$

Case 3

$$T(n) = \Theta(n^2)$$

Problem 1-2. $T(n) = 7T(n/2) + n^2$

$$\begin{aligned} a &= 7 & c_{\text{crit}} &= \log_2 7 \\ b &= 2 & & \\ c &= 2 & c &< c_{\text{crit}} \end{aligned}$$

Case 1

$$T(n) = \Theta(n^{\log_2 7})$$

Problem 1-3. $T(n) = 4T(n/2) + n^2$

$$\begin{aligned} a &= 4 & c_{\text{crit}} &= \log_2 4 = 2 \\ b &= 2 & & \\ c &= 2 & c &= c_{\text{crit}} \\ & & k &= 0 \end{aligned}$$

Case 2

$$T(n) = \Theta(n^2 \log n)$$

Problem 1-4. $T(n) = 3T(n/4) + n \lg n$

$$\begin{aligned} a &= 3 & c &= 1 > c_{\text{crit}} \\ b &= 4 & k &= 1 \end{aligned}$$

$$\frac{3n}{4} \log\left(\frac{n}{4}\right) \leq kn \log n$$

Case 3

$$T(n) = \Theta(n \log n)$$

Problem 1-5. $T(n) = 4T(n/2) + \lg n$

$$\begin{aligned} a &= 4 & c &= 0 < c_{\text{crit}} \\ b &= 2 & k &= 1 \end{aligned}$$

Case 1

$$T(n) = \Theta(n^2)$$

$$c_{\text{crit}} = \log_2 4 = 2$$

Problem 1-6. $T(n) = T(n-1) + n$

Does not apply

Problem 1-7. $T(n) = 4T(n/2) + n^2 \lg n$

$$a=4 \quad C=2$$

$$b=2 \quad k=1$$

$$T(n) = \Theta(n^2 \log^2 n)$$

Case 2

$$C_{\text{crit}} = \log_2 4 = 2 = C_{\text{crit}}$$

→ Problem 1-8. $T(n) = 5T(n/2) + n^2 \lg n$

$$a=5 \quad C=2 < C_{\text{crit}}$$

$$b=2 \quad k=1$$

$$T(n) = \Theta(n^{\log_2 5})$$

Case 1

$$C_{\text{crit}} = \log_2 5$$

Problem 1-9. $T(n) = 3T(n/3) + n/\lg n$

$$a=3 \quad C=1$$

$$b=3 \quad k=-1$$

$$C_{\text{crit}} = \log_3 3 = 1 = C_{\text{crit}}$$

Problem 1-10. $T(n) = 2T(n/4) + c$

$$a=2 \quad C=0 < C_{\text{crit}}$$

$$b=4$$

case 1

$$T(n) = \Theta(\sqrt{n})$$

$$C_{\text{crit}} = \log_4 2 = \frac{1}{2}$$

Problem 1-11. $T(n) = T(n/4) + \lg n$

$$a=1 \quad C=0$$

$$b=4 \quad k=1$$

$$T(n) = \Theta(\log^2 n)$$

case 2

$$C_{\text{crit}} = \log_4 1 = 0$$

Problem 1-12. $T(n) = T(n/2) + T(n/4) + n^2 =$

Not in master theorem form

Does not apply

Problem 1-13. $T(n) = 2T(n/4) + \lg n$

$$a=2 \quad C=0 < C_{\text{crit}}$$

$$b=4 \quad k=1$$

case 1

$$T(n) = \Theta(\sqrt{n})$$

$$C_{\text{crit}} = \log_4 2 = \frac{1}{2}$$

Problem 1-14. $T(n) = 3T(n/3) + n \lg n$

$$a=3 \quad C=1$$

$$b=3 \quad k=1$$

Case 2

$$T(n) = \Theta(n \log^2 n)$$

$$C_{\text{crit}} = \log_3 3 = 1$$

Problem 1-15. $T(n) = 8T((n - \sqrt{n})/4) + n^2$

Does not apply

Problem 1-16. $T(n) = 2T(n/4) + \sqrt{n}$

$a=2$ $c = \frac{1}{2} = c_{crit}$

$b=4$

Case 2

$T(n) = \Theta(n^{\frac{1}{2}} \log n)$

$c_{crit} = \frac{1}{2}$

Problem 1-17. $T(n) = 2T(n/4) + n^{0.51}$

$a=2$ $c = 0.51 > c_{crit}$

$b=4$

$\frac{2}{4^{0.51}} n^{0.51} \leq k n^{0.51} \checkmark$

Case 3

$T(n) = \Theta(n^{0.51})$

$c_{crit} = \frac{1}{2}$

Problem 1-18. $T(n) = 16T(n/4) + n!$

$a=16$ $16 \frac{n!}{4} \leq k n! \checkmark$

$b=4$

Case 3

$T(n) = \Theta(n!)$

$c_{crit} = 2$

$c = ?$

Problem 1-19. $T(n) = 3T(n/2) + n$

$a=3$ $c = 1 < c_{crit}$

$b=2$

Case 1

$T(n) = \Theta(n^{\log_2 3})$

$c_{crit} = \log_2 3$

Problem 1-20. $T(n) = 4T(n/2) + cn$

$a=4$ $c=1 < c_{crit}$

$b=2$

Case 1

$T(n) = \Theta(n^2)$

$c_{crit} = 2$

Problem 1-21. $T(n) = 3T(n/3) + n/2$

$a=3$ $c=1 = c_{crit}$

$b=3$

$k=0$

Case 2

$T(n) = \Theta(n \log n)$

$c_{crit} = 1$

Problem 1-22. $T(n) = 4T(n/2) + n/\lg n$

$a=4$ $c=1 < c_{crit}$ Case 1

$b=2$ $k=-1$

$c_{crit} = 2$

$T(n) = \Theta(n^2)$

Problem 1-23. $T(n) = 7T(n/3) + n^2$

$$a = 7 \quad c = 2 > c_{\text{crit}}$$

$$b = 3$$

$$c_{\text{crit}} = \log_3 7$$

$$\frac{7n^2}{9} \leq kn^2 \checkmark$$

Case 3

$$T(n) = \Theta(n^2)$$

Problem 1-24. $T(n) = 8T(n/3) + 2^n$

$$a = 8$$

$$b = 3$$

$$c_{\text{crit}} = \log_3 8$$

$$8(2^{\frac{n}{3}}) \leq K(2^n) \checkmark$$

Case 3

$$T(n) = \Theta(2^n)$$

Problem 1-25. $T(n) = 16T(n/4) + n$

$$a = 16$$

$$b = 4$$

$$c_{\text{crit}} = \log_4 16 = 2$$

$$c = 1 < c_{\text{crit}}$$

Case 1

$$T(n) = \Theta(n^2)$$

how
do I
determine
c?