Revised Model

$$\begin{aligned}
Pr(y_{i,u,v} = 1) &= \text{probit}(\eta_{i,u,v}) \\
\eta_{i,u,v} &= \underbrace{b_{i0} + b_{i1}age_i + b_{i2}open_i}_{\text{random effects}} + \underbrace{\beta_1 sex_i + \beta_2 \mathbb{1}(u,v)}_{\text{fixed effects}} + \epsilon_{i,u,v} \\
&\sim N(b_{i0} + b_{i1}age_i + b_{i2}open_i + \beta_1 sex_i + \beta_2 \mathbb{1}(u,v), \sigma^2)
\end{aligned}$$

$$\begin{aligned}
b_{ij} &\overset{iid}{\sim} N(\mu_j, \Sigma_j) \ j = 0, 1, 2 \\
\mu_j &\sim N(\lambda_j, \phi_j) \\
\Sigma_j &\sim IG(a_j, b_j) \\
\beta_k &\sim N(\theta_k, \psi_k) \ k = 1, 2 \\
\epsilon_{i,u,v} &\overset{iid}{\sim} N(0, \sigma^2) \\
\sigma^2 &\sim IG(\alpha, \beta)
\end{aligned}$$

Joint Posterior

$$\pi(\underbrace{\beta_1, \beta_2, \mu_j, \Sigma_j, \sigma^2}_{\Theta}, b_{ij}, \tilde{Z} | \tilde{Y}, \tilde{X})$$

$$\propto \pi(\Theta) \prod_{i=1}^{n} \prod_{j=0}^{2} N(b_{ij}; \mu_j, \Sigma_j) \prod_{m=1}^{M} \{\mathbb{1}(z_m > 0)\mathbb{1}(y_m = 1) + \mathbb{1}(z_m < 0)\mathbb{1}(y_m = 0)\} \times N(z_m; \eta_m, 1)$$

n : number of subjects

M : number of (i, u, v) triads

Full Conditionals

$$\pi(\beta_1|-) \propto \pi(\beta_1) \prod_{m=1}^{M} N(D_m; \beta_1 sex_m, \sigma^2)$$

$$D_m = z_m - (b_{i0} + b_{i1} age_i + b_{i2} open_i + \beta_2 \mathbb{1}(u, v))$$

$$\pi(\beta_1|-) \sim N(\frac{\theta_1 \sigma^2 + \psi_1 \sum_{m=1}^{M} sex_m \cdot D_m}{\sigma^2 + \psi_1 \sum_{m=1}^{M} sex_m^2}, \frac{\psi_1 \sigma^2}{\sigma^2 + \psi_1 \sum_{m=1}^{M} sex_m^2})$$

Similarly, $$\pi(\beta_2|-) \sim N(\frac{\theta_2 \sigma^2 + \psi_2 \sum_{m=1}^{M} \mathbb{1}_m \cdot D_m}{\sigma^2 + \psi_2 \sum_{m=1}^{M} \mathbb{1}_m^2}, \frac{\psi_2 \sigma^2}{\sigma^2 + \psi_2 \sum_{m=1}^{M} \mathbb{1}_m^2})$$

$$\pi(b_{i0}|-) \sim N(\frac{\mu_0 \sigma^2 + \Sigma_0 \sum_{l=1}^{L} D_l}{\sigma^2 + \mu_0 L}, \frac{\mu_0 \sigma^2}{\sigma^2 + \mu_0 L})$$

l : index of observations where i is fixed

$$\pi(\mu_j|-) \propto \pi(\mu_j) \prod_{i=1}^{n} N(b_{ij}; \mu_j, \Sigma_j)$$

$$\sim N(\frac{\lambda_j \Sigma_j + \phi_j \sum_{i=1}^{n} b_{ij}}{\Sigma_j + n\phi_j}, \frac{\phi_j \Sigma_j}{\Sigma_j + n\phi_j})$$

$$\pi(\Sigma_j|-) \propto \pi(\Sigma_j) \prod_{i=1}^{n} N(b_{ij}; \mu_j, \Sigma_j)$$

$$\sim IG(a_j + \frac{n}{2}, b_j + \frac{1}{2} \sum_{i=1}^{n} (b_{ij} - \mu_j)^2)$$