

matrix_factorizations_1

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1 Matrix Factorization Visualizations - Method 1 (Code from Homework 5)

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```
In [1]: import numpy as np
import matplotlib.pyplot as plt
import pandas as pd
```

```
In [80]: def grad_U(Ui, Yij, Vj, reg, eta):
        """
        Takes as input  $U_i$  (the  $i$ th row of  $U$ ), a training point  $Y_{ij}$ , the column
        vector  $V_j$  ( $j$ th column of  $V^T$ ),  $reg$  (the regularization parameter  $\lambda$ ),
        and  $eta$  (the learning rate).

        Returns the gradient of the regularized loss function with
        respect to  $U_i$  multiplied by  $eta$ .
        """

        grad = reg*Ui - Vj*(Yij - np.dot(Ui, Vj))

        return eta*grad

def grad_V(Vj, Yij, Ui, reg, eta):
    """
    Takes as input the column vector  $V_j$  ( $j$ th column of  $V^T$ ), a training point  $Y_{ij}$ ,
     $U_i$  (the  $i$ th row of  $U$ ),  $reg$  (the regularization parameter  $\lambda$ ),
    and  $eta$  (the learning rate).

    Returns the gradient of the regularized loss function with
    respect to  $V_j$  multiplied by  $eta$ .
    """

    grad = reg*Vj - Ui*(Yij - np.dot(Ui, Vj))

    return eta*grad
```

```

def get_err(U, V, Y, reg=0.0):
    """
    Takes as input a matrix Y of triples (i, j, Y_ij) where i is the index of a user,
    j is the index of a movie, and Y_ij is user i's rating of movie j and
    user/movie matrices U and V.

    Returns the mean regularized squared-error of predictions made by
    estimating Y_{ij} as the dot product of the ith row of U and the jth column of V.
    """

    error = 0.5*reg*(np.linalg.norm(U, 'fro')**2 + np.linalg.norm(V, 'fro')**2)

    for k in range(len(Y)):
        Yij = Y[k][2]
        i = Y[k][0] - 1
        j = Y[k][1] - 1
        error += 0.5*(Yij - np.dot(U[i,:], V[j,:]))**2

    return error / len(Y) # mean error

def train_model(M, N, K, eta, reg, Y, eps=0.0001, max_epochs=300):
    """
    Given a training data matrix Y containing rows (i, j, Y_ij)
    where Y_ij is user i's rating on movie j, learns an
    M x K matrix U and N x K matrix V such that rating Y_ij is approximated
    by (UV^T)_ij.

    Uses a learning rate of <eta> and regularization of <reg>. Stops after
    <max_epochs> epochs, or once the magnitude of the decrease in regularized
    MSE between epochs is smaller than a fraction <eps> of the decrease in
    MSE after the first epoch.

    Returns a tuple (U, V, err) consisting of U, V, and the unregularized MSE
    of the model.
    """

    # initialize entries of U and V uniformly and randomly in [-0.5, 0.5]
    U = np.random.rand(M, K) - 0.5*np.ones([M, K])
    V = np.random.rand(N, K) - 0.5*np.ones([N, K])

    epochs = 0
    rel_red = np.inf
    err = get_err(U, V, Y, reg)

    while epochs < max_epochs and rel_red > eps:
        # store previous error
        prev_err = np.copy(err)

```

```

    # shuffle training data
    order = np.random.permutation(len(Y))

    # perform SGD
    for o in order:
        i = Y[o][0] - 1
        j = Y[o][1] - 1
        Yij = Y[o][2]
        Ui = U[i,:]
        Vj = V[j,:]

        U[i,:] -= grad_U(Ui, Yij, Vj, reg, eta)
        V[j,:] -= grad_V(Vj, Yij, Ui, reg, eta)

    # compute error and loss reduction
    err = get_err(U, V, Y, reg)
    loss_red = err - prev_err

    # store initial loss reduction for comparison
    if epochs == 0:
        init_red = np.copy(loss_red)

    # compute relative loss reduction
    rel_red = loss_red/init_red
    epochs += 1

    return (U, V, err)

```

```

In [81]: # factorize using SGD as in Homework 5
Y_train = np.loadtxt('./data/train2.txt').astype(int)
Y_test = np.loadtxt('./data/test2.txt').astype(int)

M = max(max(Y_train[:,0]), max(Y_test[:,0])).astype(int) # users
N = max(max(Y_train[:,1]), max(Y_test[:,1])).astype(int) # movies
k = 20

regs = [10**-4, 10**-3, 10**-2, 10**-1, 0.3, 0.5, 0.7, 1]
eta = 0.03 # learning rate
E_ins = []
E_outs = []

# Use to compute Ein and Eout
for reg in regs:

    print("Training model with M = %s, N = %s, k = %s, eta = %s, reg = %s"%(M, N, k, eta, reg))
    U, V, e_in = train_model(M, N, k, eta, reg, Y_train)
    e_out = get_err(U, V, Y_test)

```

```

E_ins.append(e_in)
E_outs.append(e_out)

```

```

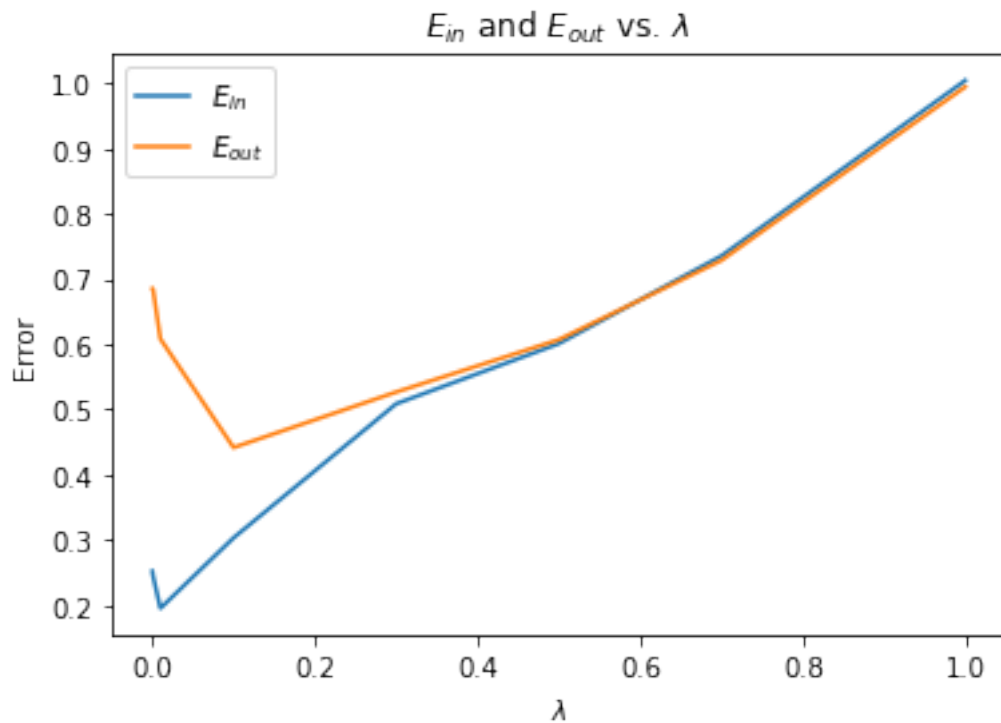
Training model with M = 943, N = 1664, k = 20, eta = 0.03, reg = 0.0001
Training model with M = 943, N = 1664, k = 20, eta = 0.03, reg = 0.001
Training model with M = 943, N = 1664, k = 20, eta = 0.03, reg = 0.01
Training model with M = 943, N = 1664, k = 20, eta = 0.03, reg = 0.1
Training model with M = 943, N = 1664, k = 20, eta = 0.03, reg = 0.3
Training model with M = 943, N = 1664, k = 20, eta = 0.03, reg = 0.5
Training model with M = 943, N = 1664, k = 20, eta = 0.03, reg = 0.7
Training model with M = 943, N = 1664, k = 20, eta = 0.03, reg = 1

```

```

In [82]: # Plot values of  $E_{in}$  across  $k$  for each value of  $\lambda$ 
plt.figure()
plt.plot(regs, E_ins)
plt.plot(regs, E_outs)
plt.title('$E_{in}$ and $E_{out}$ vs. $\lambda$')
plt.xlabel('$\lambda$')
plt.ylabel('Error')
plt.legend(['$E_{in}$', '$E_{out}$'])
plt.savefig('matrix_1_reg.png')

```



```

In [89]: E_outs

```

```
Out [89]: [0.6854140619862172,
          0.6836143075097076,
          0.6081313620287249,
          0.44147675467734,
          0.526633970830529,
          0.6070052718322477,
          0.7286440157051878,
          0.9949934500669039]
```

```
In [83]: # load data from cleaned files
movies = pd.read_csv('data/movies.csv')
data = pd.read_csv('data/data.csv').astype(int)
train_data = np.loadtxt('./data/data2.txt').astype(int)
```

```
In [84]: # train using best lambda on entire dataset
M = max(data['User']) # users
N = max(data['Movie']) # movies

U, V, e_in = train_model(M, N, k, eta, 0.1, train_data)

# note that U is Mxk and V is Nxk, so we transpose them to be consistent
# with the convention used in the Miniproject 2 pdf
U = U.T
V = V.T
```

```
In [85]: # mean-centering
V_mean = V.mean(axis=1)
for i in range(N):
    V[:,i] -= V_mean
for i in range(M):
    U[:,i] -= V_mean

# get SVD of V
A, S, B = np.linalg.svd(V)
```

```
In [86]: # project U, V into K-dimensional space
K = 2
U_tilde = np.matmul(A.T[:K], U)
V_tilde = np.matmul(A.T[:K], V)
```

```
In [129]: # visualize any 10 movies
ids = [11, 12, 71, 95, 98, 92, 578, 181, 172, 50]

titles = ['Seven', 'The Usual Suspects', 'The Lion King', 'Aladdin', 'The Silence of

x = [0]*10
y = [0]*10
for i in range(len(ids)):
    x[i] = V_tilde[0][ids[i]-1]
```

```

        y[i] = V_tilde[1][ids[i]-1]

fig, ax = plt.subplots()
plt.scatter(x, y)

for i, txt in enumerate(titles):
    ax.annotate(txt, (x[i], y[i]))

plt.title('Visualization of 10 Movies')
plt.xlabel('x')
plt.ylabel('y')
plt.savefig('matrix_1_a.png')

# run k-means to cluster points
from sklearn.cluster import KMeans

X = np.column_stack((x, y))
labels = KMeans(n_clusters=4, random_state=0).fit_predict(X)

fig, ax = plt.subplots()
plt.scatter(x, y, c=labels)

for i, txt in enumerate(titles):
    ax.annotate(txt, (x[i], y[i]))

plt.title('Visualization of 10 Movies')
plt.xlabel('x')
plt.ylabel('y')
plt.savefig('matrix_1_a_clusters.png')

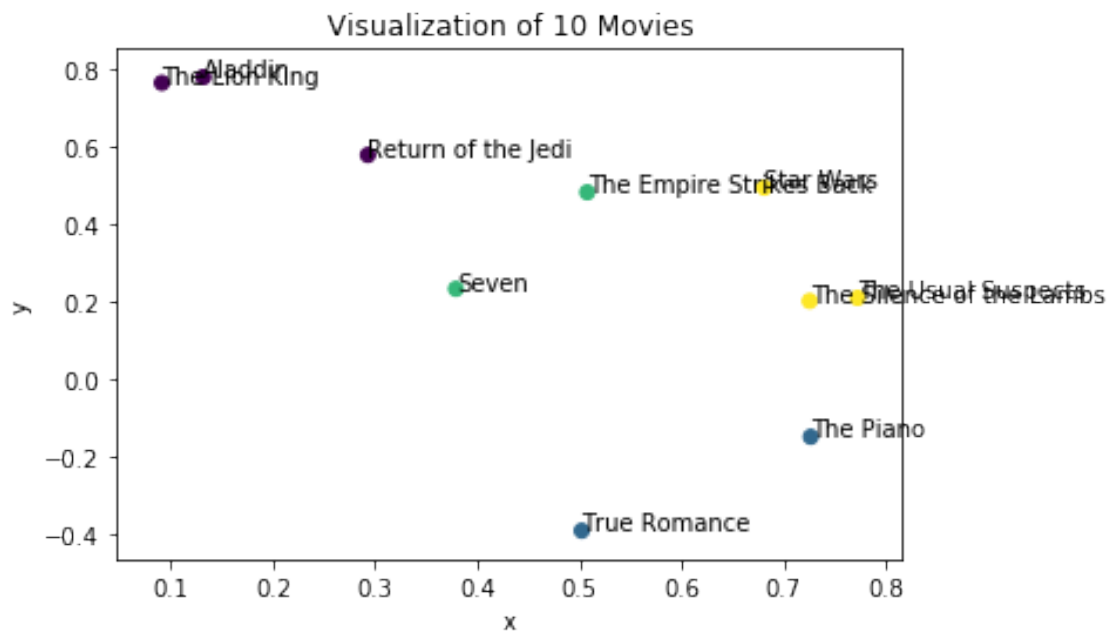
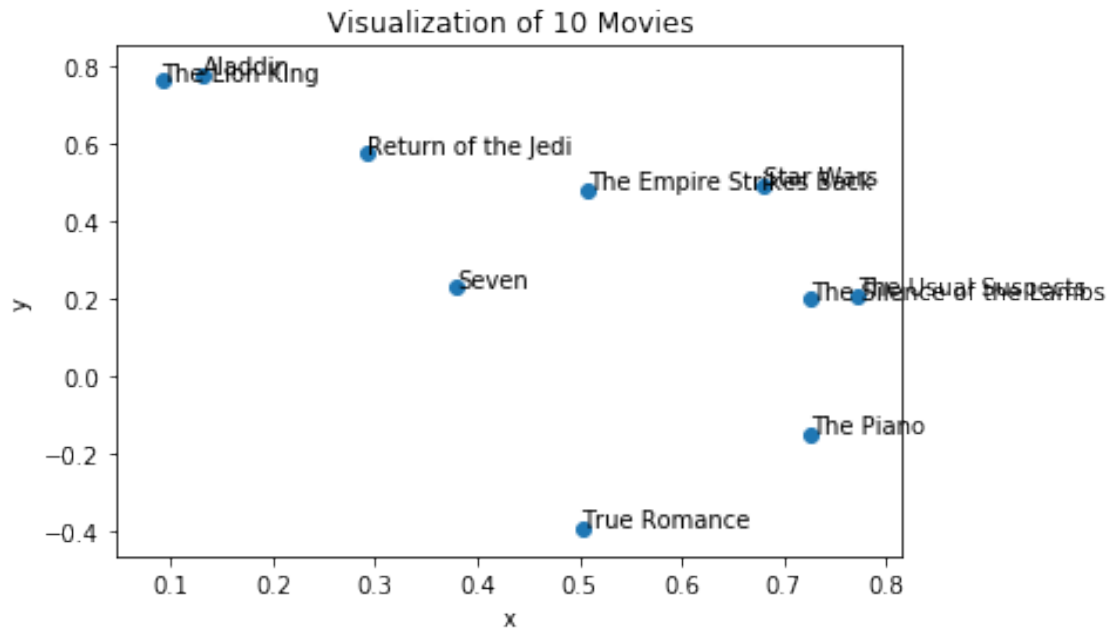
# visualize movies by their average ratings
ratings = []
for i in ids:
    ratings.append(movies['avg_rating'][i-1])

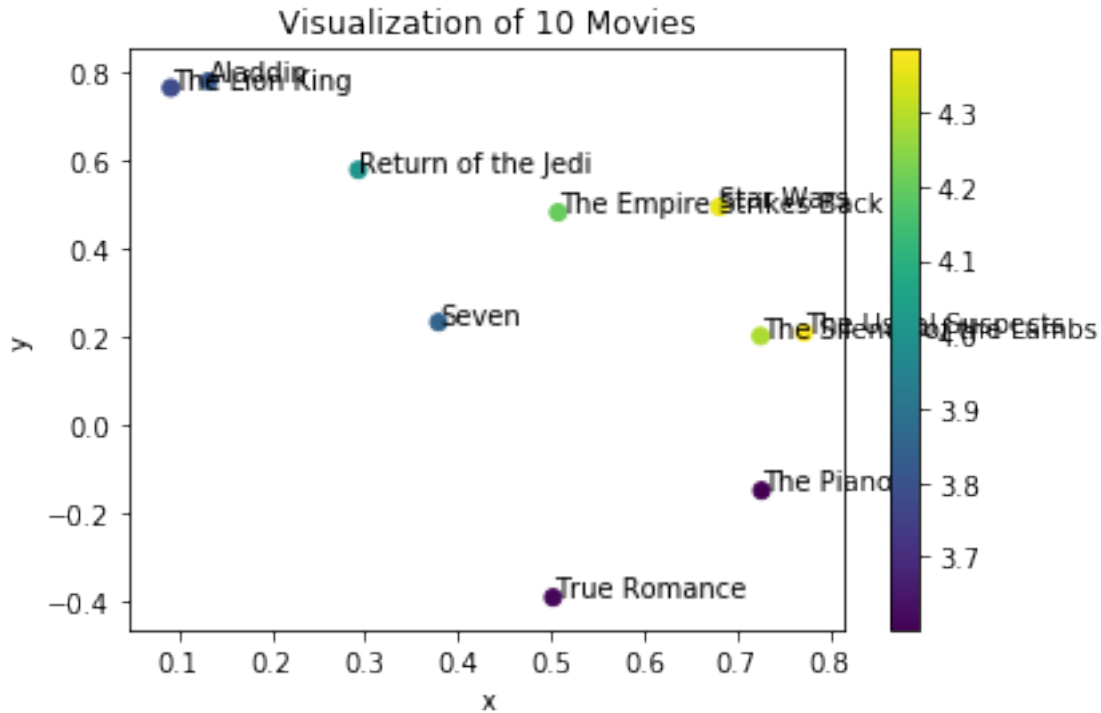
fig, ax = plt.subplots()
plt.scatter(x, y, c=ratings)

for i, txt in enumerate(titles):
    ax.annotate(txt, (x[i], y[i]))

plt.title('Visualization of 10 Movies')
plt.xlabel('x')
plt.ylabel('y')
plt.colorbar()
plt.savefig('matrix_1_a_ratings.png')

```





```
In [131]: # visualize 10 most popular movies
pop = data['Movie'].value_counts().head(10).index

for i in pop:
    print(movies[movies['ID'] == i]['Title'])

titles = ['Star Wars', 'Contact', ' Fargo', 'Return of the Jedi', 'Liar Liar', 'The E

x = [0]*10
y = [0]*10
for i in range(len(pop)):
    x[i] = V_tilde[0][pop[i]-1]
    y[i] = V_tilde[1][pop[i]-1]

# clustering
X = np.column_stack((x, y))
labels = KMeans(n_clusters=4, random_state=0).fit_predict(X)

# plot visualization
fig, ax = plt.subplots()
plt.scatter(x, y)

for i, txt in enumerate(titles):
    ax.annotate(txt, (x[i], y[i]))
```



```

plt.title('Visualization of 10 Most Popular Movies')
plt.xlabel('x')
plt.ylabel('y')
plt.savefig('matrix_1_b.png')

# plot clusters
fig, ax = plt.subplots()
plt.scatter(x, y, c=labels)

for i, txt in enumerate(titles):
    ax.annotate(txt, (x[i], y[i]))

plt.title('Visualization of 10 Most Popular Movies')
plt.xlabel('x')
plt.ylabel('y')
plt.savefig('matrix_1_b_clusters.png')

# visualize movies by their average ratings
ratings = []
for i in pop:
    ratings.append(movies['avg_rating'][i-1])

fig, ax = plt.subplots()
plt.scatter(x, y, c=ratings)

for i, txt in enumerate(titles):
    ax.annotate(txt, (x[i], y[i]))

plt.title('Visualization of 10 Most Popular Movies')
plt.xlabel('x')
plt.ylabel('y')
plt.colorbar()
plt.savefig('matrix_1_b_ratings.png')

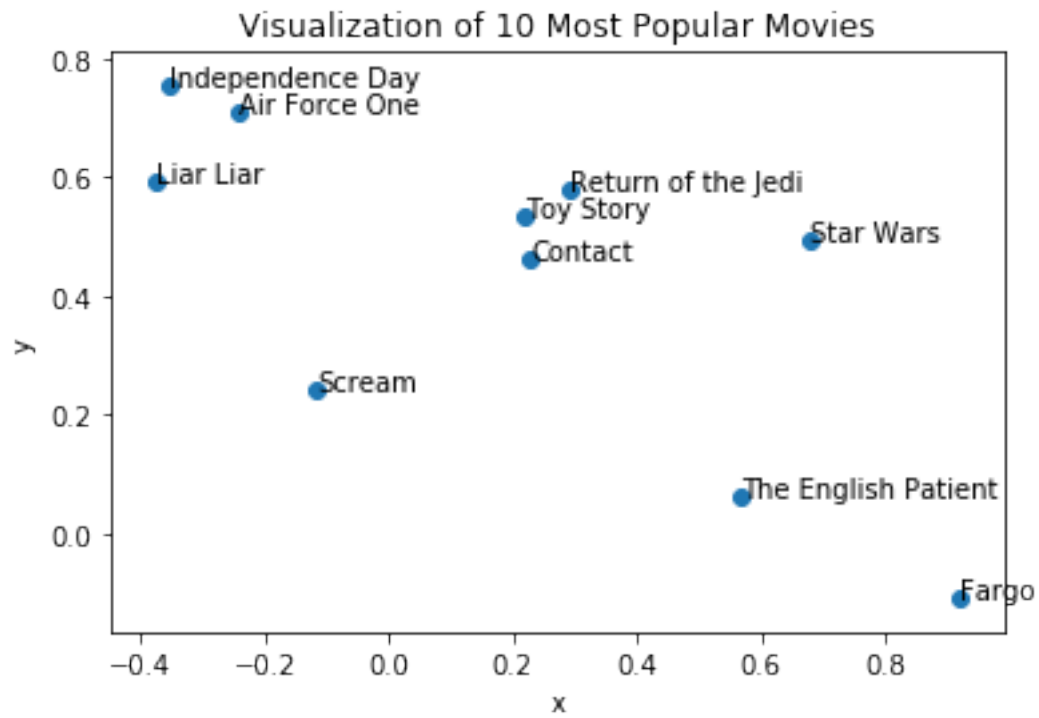
```

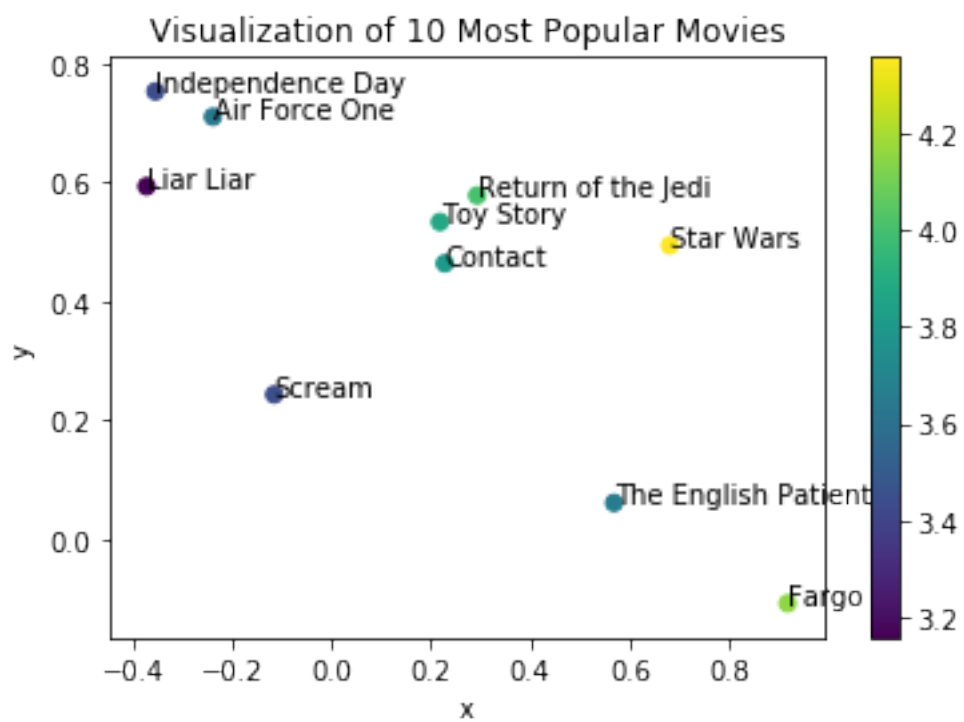
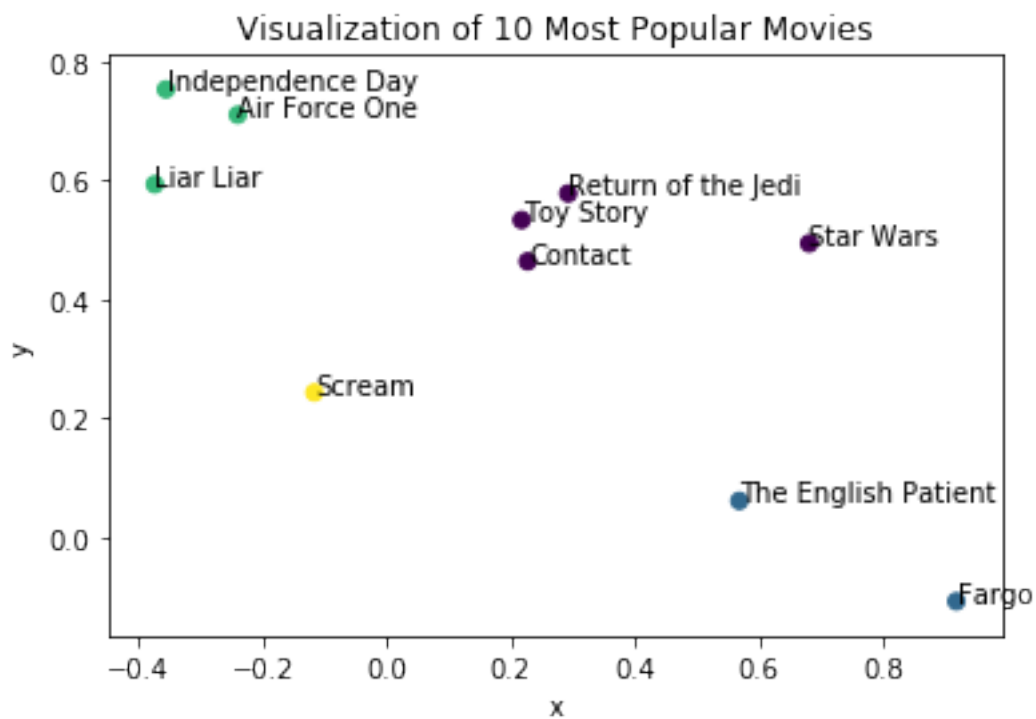
```

49    Star Wars (1977)
Name: Title, dtype: object
257    Contact (1997)
Name: Title, dtype: object
99     Fargo (1996)
Name: Title, dtype: object
180    Return of the Jedi (1983)
Name: Title, dtype: object
292    Liar Liar (1997)
Name: Title, dtype: object
284    English Patient, The (1996)
Name: Title, dtype: object
286    Scream (1996)

```

Name: Title, dtype: object
0 Toy Story (1995)
Name: Title, dtype: object
298 Air Force One (1997)
Name: Title, dtype: object
120 Independence Day (ID4) (1996)
Name: Title, dtype: object





```

In [133]: # visualize 10 best movies
sorted_df = movies.sort_values(by='avg_rating', ascending=False)
best = sorted_df.head(10)['ID']

for i in best:
    print(movies[movies['ID'] == i]['Title'])

titles = ["Someone Else's America", 'Prefontaine', 'Aiqing wansui', 'Star Kid', 'Ent

x = [0]*10
y = [0]*10
count = 0
for i in best:
    x[count] = V_tilde[0][i-1]
    y[count] = V_tilde[1][i-1]
    count += 1

# clustering
X = np.column_stack((x, y))
labels = KMeans(n_clusters=3, random_state=0).fit_predict(X)

# plot visualization
fig, ax = plt.subplots()
plt.scatter(x, y)

for i, txt in enumerate(titles):
    ax.annotate(txt, (x[i], y[i]))

plt.title('Visualization of 10 Best Movies')
plt.xlabel('x')
plt.ylabel('y')
plt.savefig('matrix_1_c.png')

# plot clusters
fig, ax = plt.subplots()
plt.scatter(x, y, c=labels)

for i, txt in enumerate(titles):
    ax.annotate(txt, (x[i], y[i]))

plt.title('Visualization of 10 Best Movies')
plt.xlabel('x')
plt.ylabel('y')
plt.savefig('matrix_1_c_clusters.png')

# visualize movies by their average ratings
ratings = []
for i in best:

```

```

        ratings.append(movies['avg_rating'][i-1])

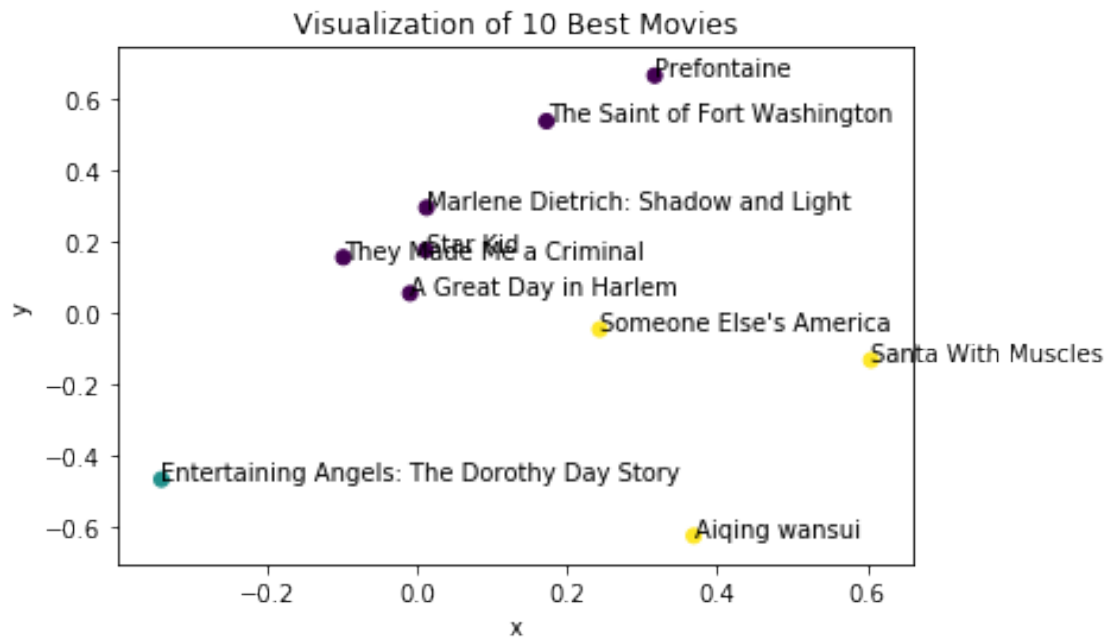
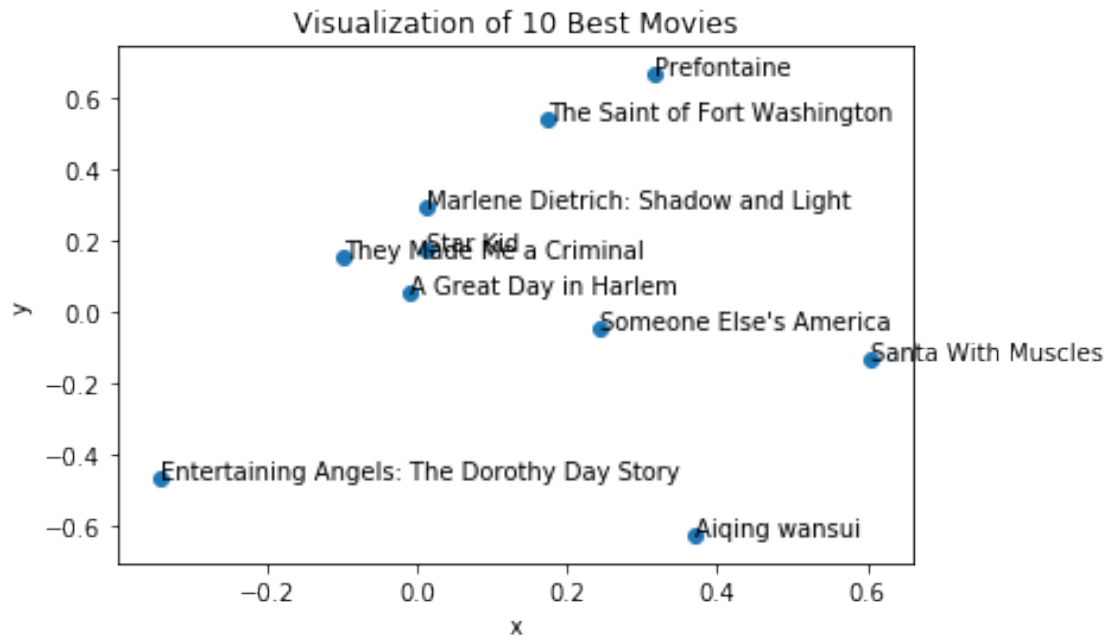
fig, ax = plt.subplots()
plt.scatter(x, y, c=ratings)

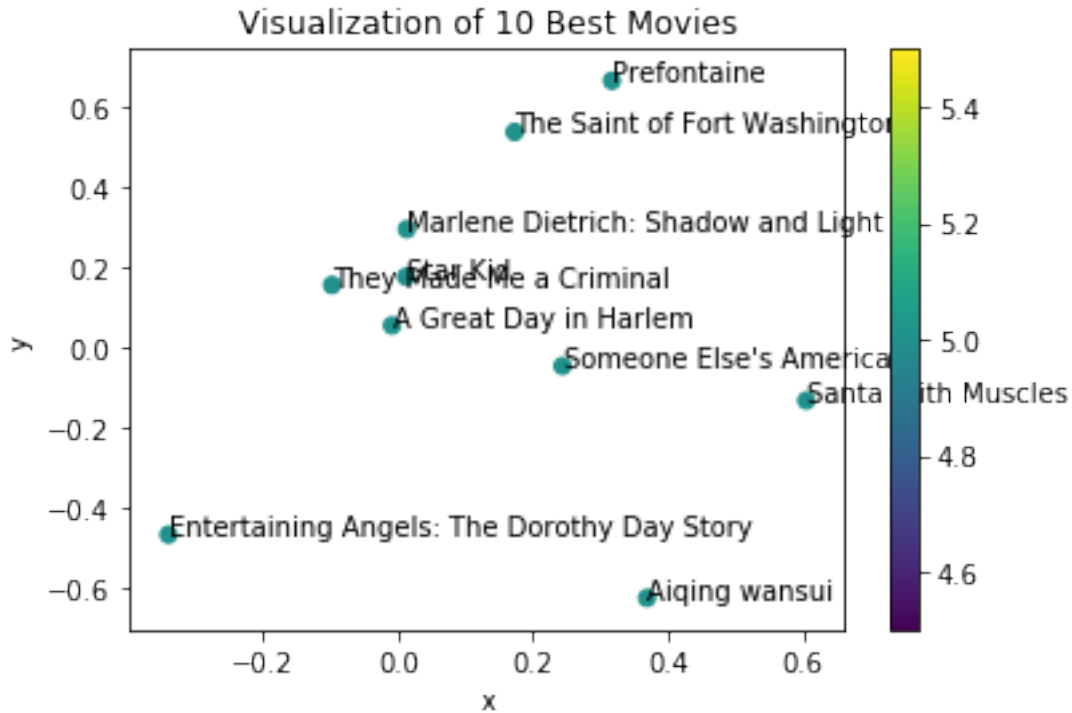
for i, txt in enumerate(titles):
    ax.annotate(txt, (x[i], y[i]))

plt.title('Visualization of 10 Best Movies')
plt.xlabel('x')
plt.ylabel('y')
plt.colorbar()
plt.savefig('matrix_1_c_ratings.png')

1588    Someone Else's America (1995)
Name: Title, dtype: object
1179    Prefontaine (1997)
Name: Title, dtype: object
1525    Aiqing wansui (1994)
Name: Title, dtype: object
1282    Star Kid (1997)
Name: Title, dtype: object
1637    Entertaining Angels: The Dorothy Day Story (1996)
Name: Title, dtype: object
1112    They Made Me a Criminal (1939)
Name: Title, dtype: object
1191    Marlene Dietrich: Shadow and Light (1996)
Name: Title, dtype: object
807     Great Day in Harlem, A (1994)
Name: Title, dtype: object
1456    Saint of Fort Washington, The (1993)
Name: Title, dtype: object
1489    Santa with Muscles (1996)
Name: Title, dtype: object

```



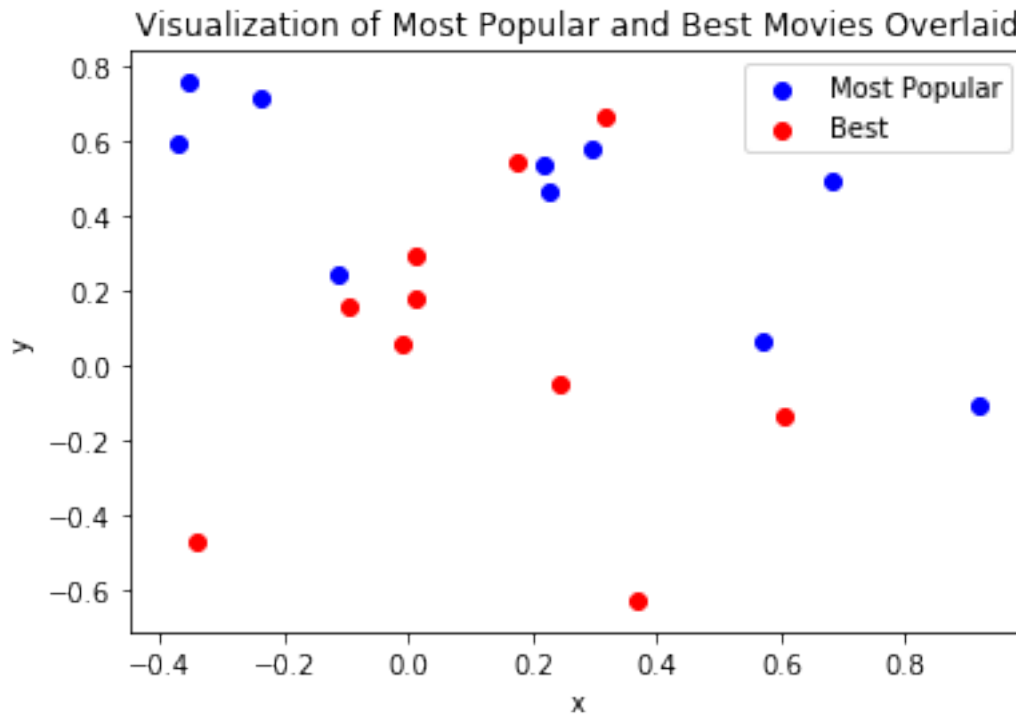


```
In [146]: # overlay best and most popular movies
x = [0]*20
y = [0]*20
count = 0

for i in pop:
    x[count] = V_tilde[0][i-1]
    y[count] = V_tilde[1][i-1]
    count += 1
for i in best:
    x[count] = V_tilde[0][i-1]
    y[count] = V_tilde[1][i-1]
    count += 1

fig, ax = plt.subplots()
plt.scatter(x[:10], y[:10], c='b')
plt.scatter(x[10:20], y[10:20], c='r')

plt.title('Visualization of Most Popular and Best Movies Overlaid')
plt.xlabel('x')
plt.ylabel('y')
plt.legend(['Most Popular', 'Best'])
plt.savefig('matrix_1_pop_best.png')
```



```
In [126]: # Visualize 10 animated movies
# get all animated movies
animated = movies[movies['Animation'] == 1]['ID'].head(10)

for i in animated:
    print(movies[movies['ID'] == i]['Title'])

titles = ['Toy Story', 'The Lion King', 'Aladdin', 'Snow White and the Seven Dwarfs']

x = [0]*10
y = [0]*10
count = 0

for i in animated:
    x[count] = V_tilde[0][i-1]
    y[count] = V_tilde[1][i-1]
    count += 1

# clustering
X = np.column_stack((x, y))
labels = KMeans(n_clusters=3, random_state=0).fit_predict(X)

# plot visualization
fig, ax = plt.subplots()
```



```

plt.scatter(x, y)

for i, txt in enumerate(titles):
    ax.annotate(txt, (x[i], y[i]))

plt.title('Visualization of 10 Animated Movies')
plt.xlabel('x')
plt.ylabel('y')
plt.savefig('matrix_1_d_1.png')

# plot clusters
fig, ax = plt.subplots()
plt.scatter(x, y, c=labels)

for i, txt in enumerate(titles):
    ax.annotate(txt, (x[i], y[i]))

plt.title('Visualization of 10 Animated Movies')
plt.xlabel('x')
plt.ylabel('y')
plt.savefig('matrix_1_d_1_clusters.png')

# visualize movies by their average ratings
ratings = []
for i in animated:
    ratings.append(movies['avg_rating'][i-1])

fig, ax = plt.subplots()
plt.scatter(x, y, c=ratings)

for i, txt in enumerate(titles):
    ax.annotate(txt, (x[i], y[i]))

plt.title('Visualization of 10 Animated Movies')
plt.xlabel('x')
plt.ylabel('y')
plt.colorbar()
plt.savefig('matrix_1_d_1_ratings.png')

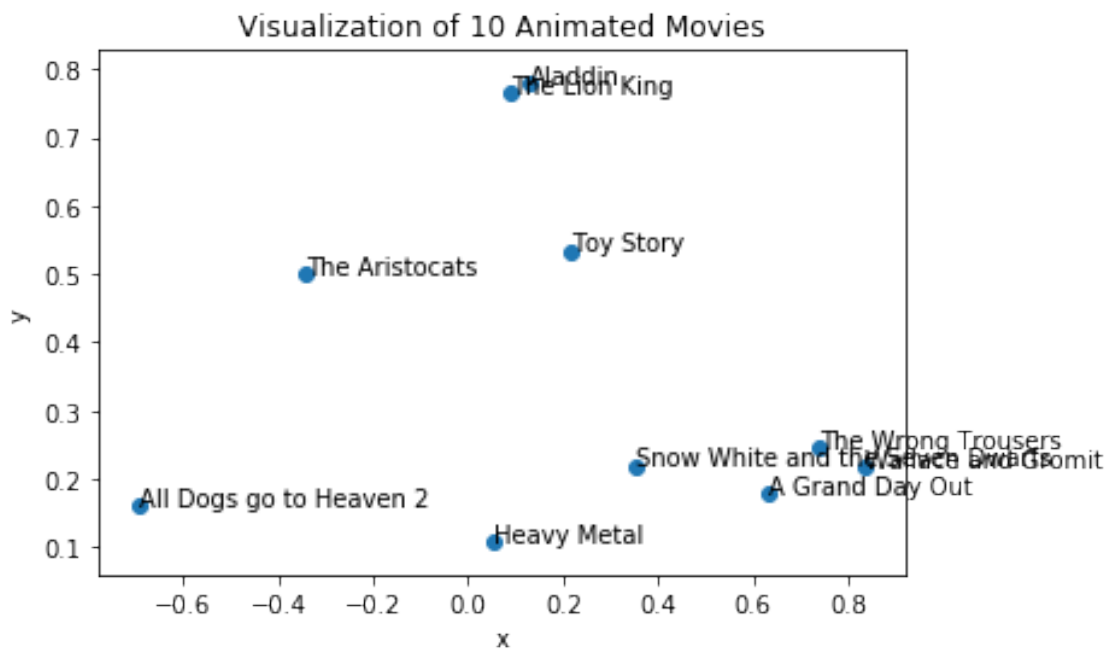
```

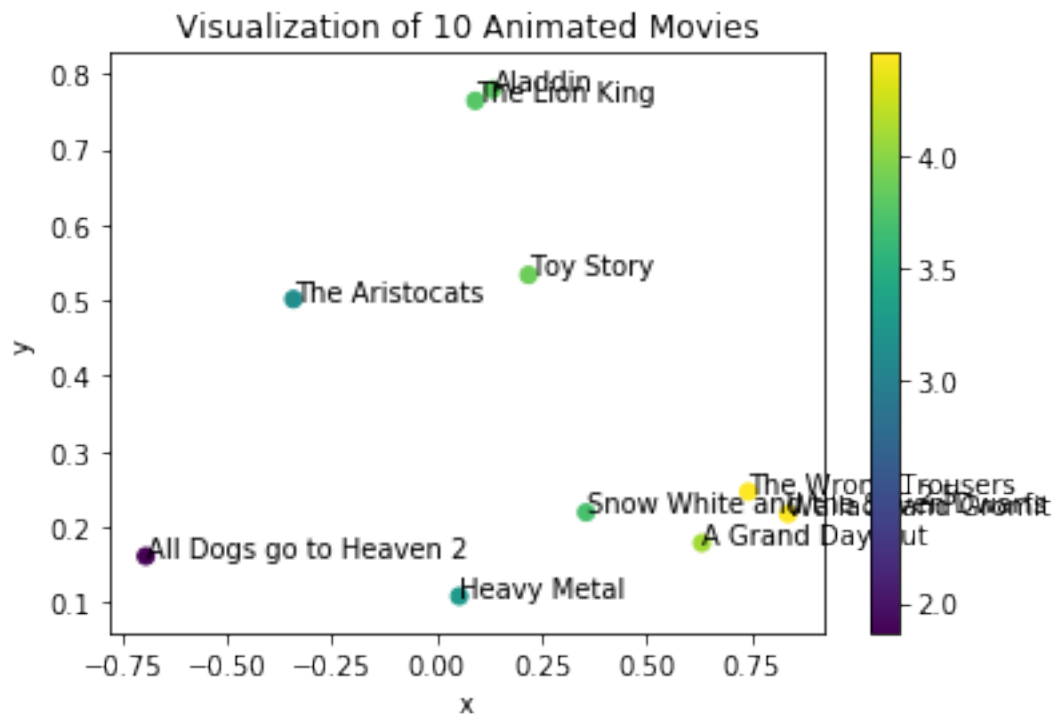
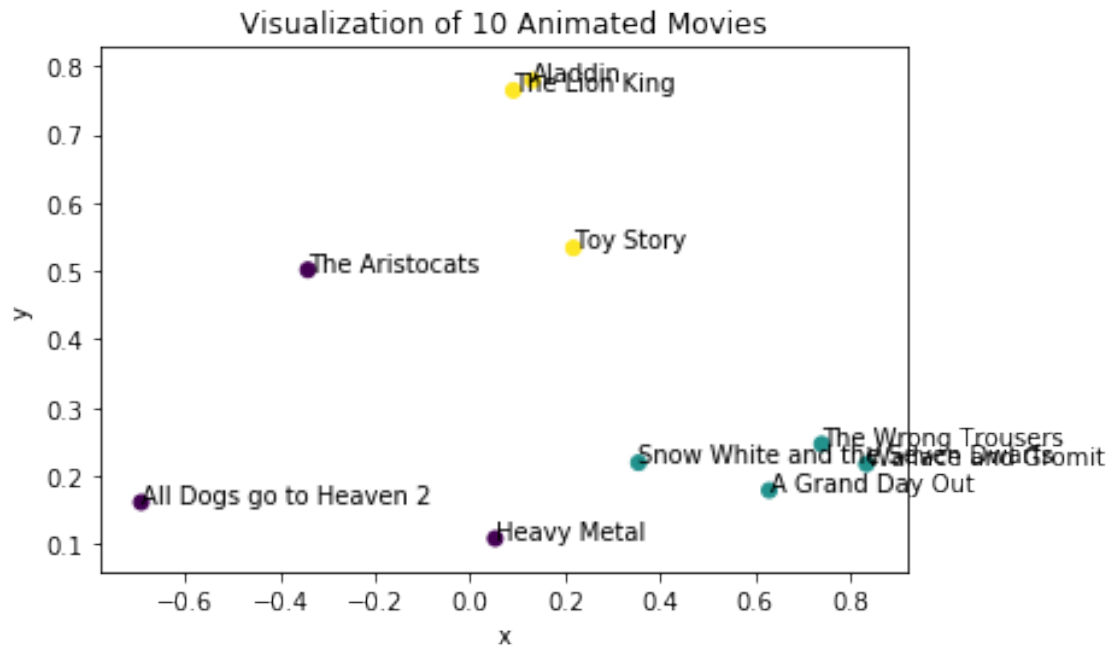
```

0    Toy Story (1995)
Name: Title, dtype: object
70   Lion King, The (1994)
Name: Title, dtype: object
94   Aladdin (1992)
Name: Title, dtype: object
98   Snow White and the Seven Dwarfs (1937)
Name: Title, dtype: object
100  Heavy Metal (1981)

```

Name: Title, dtype: object
 101 Aristocats, The (1970)
 Name: Title, dtype: object
 102 All Dogs Go to Heaven 2 (1996)
 Name: Title, dtype: object
 113 Wallace & Gromit: The Best of Aardman Animatio...
 Name: Title, dtype: object
 168 Wrong Trousers, The (1993)
 Name: Title, dtype: object
 188 Grand Day Out, A (1992)
 Name: Title, dtype: object





```
In [125]: # Visualize 10 Western movies
          # get all Western movies
```

```

western = movies[movies['Western'] == 1]['ID'].head(10)

for i in western:
    print(movies[movies['ID'] == i]['Title'])

titles = ['Legends of the Fall', 'Maverick', 'Dances with Wolves', 'The Good, The Bad & Ugly']

x = [0]*10
y = [0]*10
count = 0

for i in western:
    x[count] = V_tilde[0][i-1]
    y[count] = V_tilde[1][i-1]
    count += 1

# clustering
X = np.column_stack((x, y))
labels = KMeans(n_clusters=3, random_state=0).fit_predict(X)

# plot visualization
fig, ax = plt.subplots()
plt.scatter(x, y)

for i, txt in enumerate(titles):
    ax.annotate(txt, (x[i], y[i]))

plt.title('Visualization of 10 Western Movies')
plt.xlabel('x')
plt.ylabel('y')
plt.savefig('matrix_1_d_2.png')

# plot clusters
fig, ax = plt.subplots()
plt.scatter(x, y, c=labels)

for i, txt in enumerate(titles):
    ax.annotate(txt, (x[i], y[i]))

plt.title('Visualization of 10 Western Movies')
plt.xlabel('x')
plt.ylabel('y')
plt.savefig('matrix_1_d_2_clusters.png')

# visualize movies by their average ratings
ratings = []
for i in western:
    ratings.append(movies['avg_rating'][i-1])

```

```

fig, ax = plt.subplots()
plt.scatter(x, y, c=ratings)

for i, txt in enumerate(titles):
    ax.annotate(txt, (x[i], y[i]))

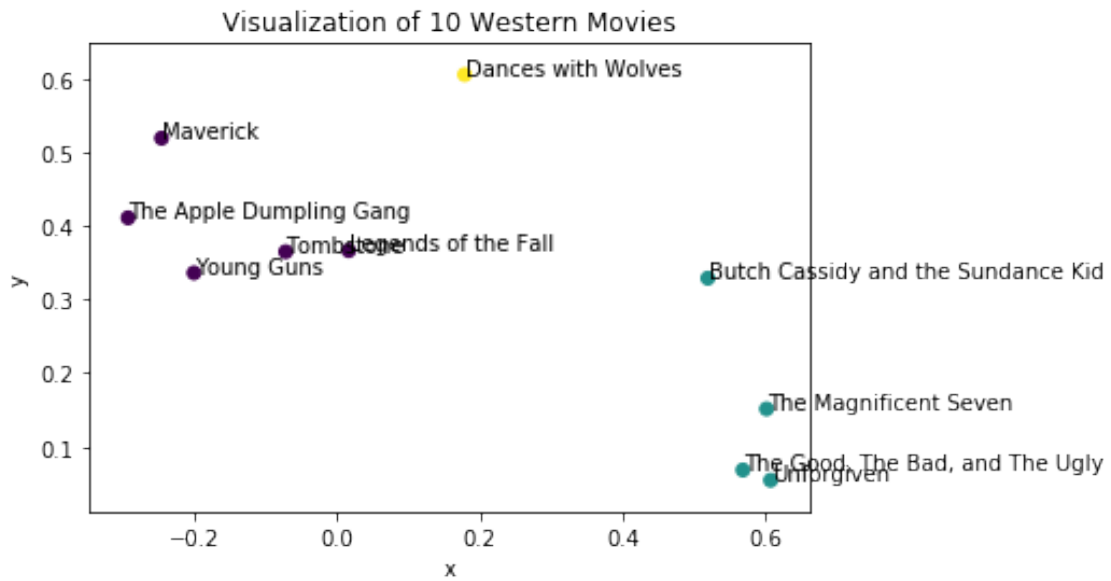
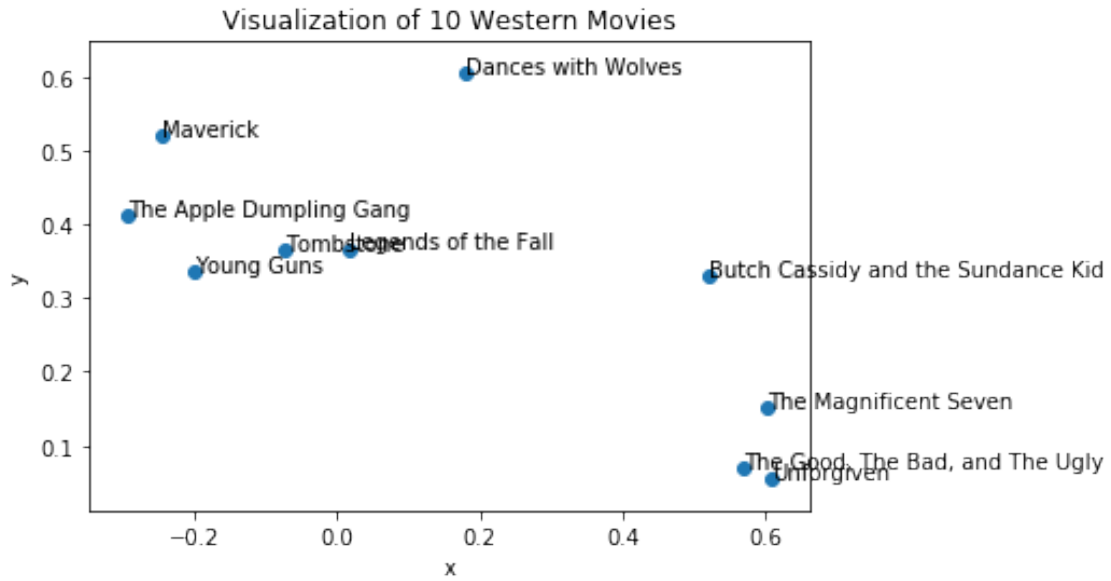
plt.title('Visualization of 10 Western Movies')
plt.xlabel('x')
plt.ylabel('y')
plt.colorbar()
plt.savefig('matrix_1_d_2_ratings.png')

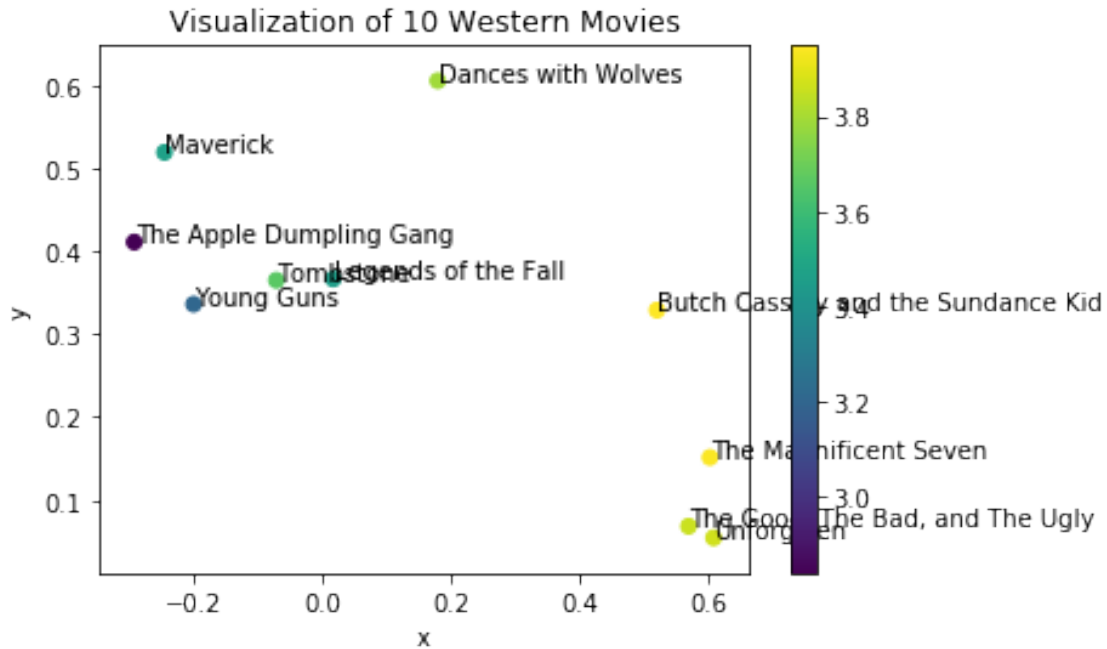
```

```

50    Legends of the Fall (1994)
Name: Title, dtype: object
72    Maverick (1994)
Name: Title, dtype: object
96    Dances with Wolves (1990)
Name: Title, dtype: object
176   Good, The Bad and The Ugly, The (1966)
Name: Title, dtype: object
202   Unforgiven (1992)
Name: Title, dtype: object
231   Young Guns (1988)
Name: Title, dtype: object
411   Apple Dumpling Gang, The (1975)
Name: Title, dtype: object
431   Butch Cassidy and the Sundance Kid (1969)
Name: Title, dtype: object
466   Tombstone (1993)
Name: Title, dtype: object
505   Magnificent Seven, The (1954)
Name: Title, dtype: object

```





```
In [124]: # Visualize 10 horror movies
horror = movies[movies['Horror'] == 1]['ID'].head(10)

for i in horror:
    print(movies[movies['ID'] == i]['Title'])

titles = ['From Dusk Till Dawn', 'Robert A. Heinlein"s The Puppet Masters', 'Heavy Metal']

x = [0]*10
y = [0]*10
count = 0

for i in horror:
    x[count] = V_tilde[0][i-1]
    y[count] = V_tilde[1][i-1]
    count += 1

# clustering
X = np.column_stack((x, y))
labels = KMeans(n_clusters=3, random_state=0).fit_predict(X)

# plot visualization
fig, ax = plt.subplots()
plt.scatter(x, y)
```

```

for i, txt in enumerate(titles):
    ax.annotate(txt, (x[i], y[i]))

plt.title('Visualization of 10 Horror Movies')
plt.xlabel('x')
plt.ylabel('y')
plt.savefig('matrix_1_d_3.png')

# plot clusters
fig, ax = plt.subplots()
plt.scatter(x, y, c=labels)

for i, txt in enumerate(titles):
    ax.annotate(txt, (x[i], y[i]))

plt.title('Visualization of 10 Horror Movies')
plt.xlabel('x')
plt.ylabel('y')
plt.savefig('matrix_1_d_3_clusters.png')

# visualize movies by their average ratings
ratings = []
for i in horror:
    ratings.append(movies['avg_rating'][i-1])

fig, ax = plt.subplots()
plt.scatter(x, y, c=ratings)

for i, txt in enumerate(titles):
    ax.annotate(txt, (x[i], y[i]))

plt.title('Visualization of 10 Horror Movies')
plt.xlabel('x')
plt.ylabel('y')
plt.colorbar()
plt.savefig('matrix_1_d_3_ratings.png')

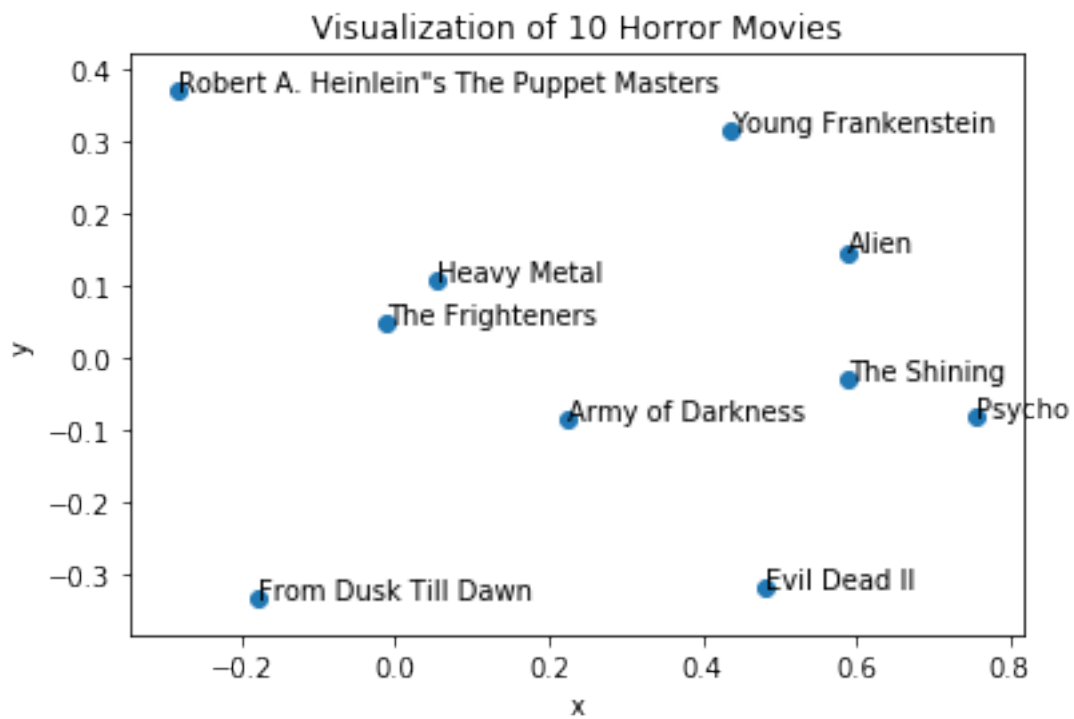
```

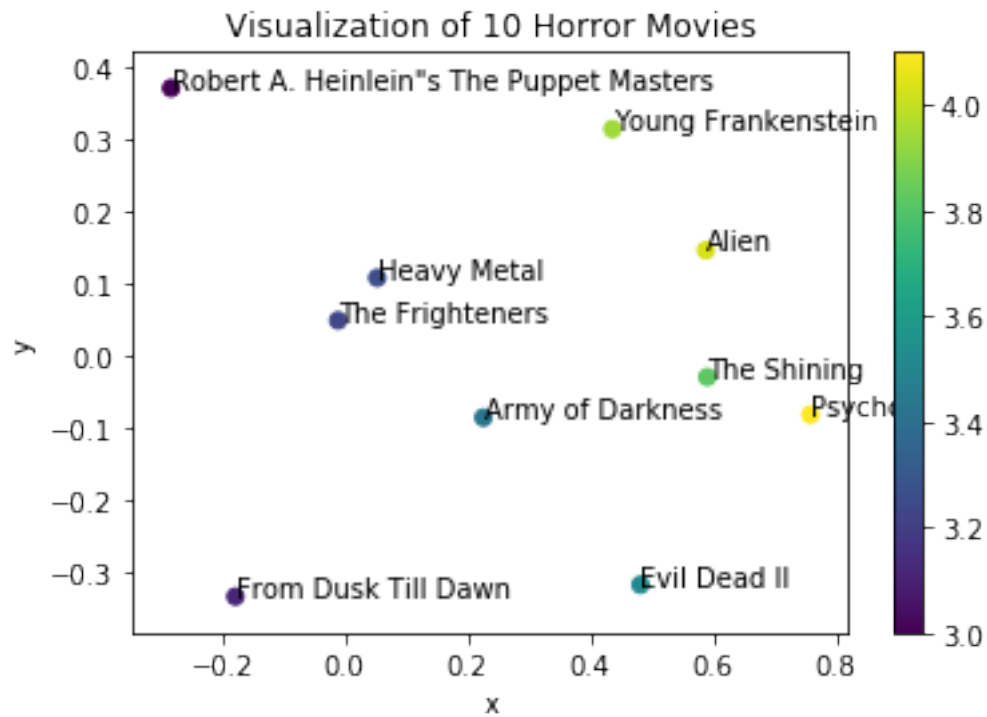
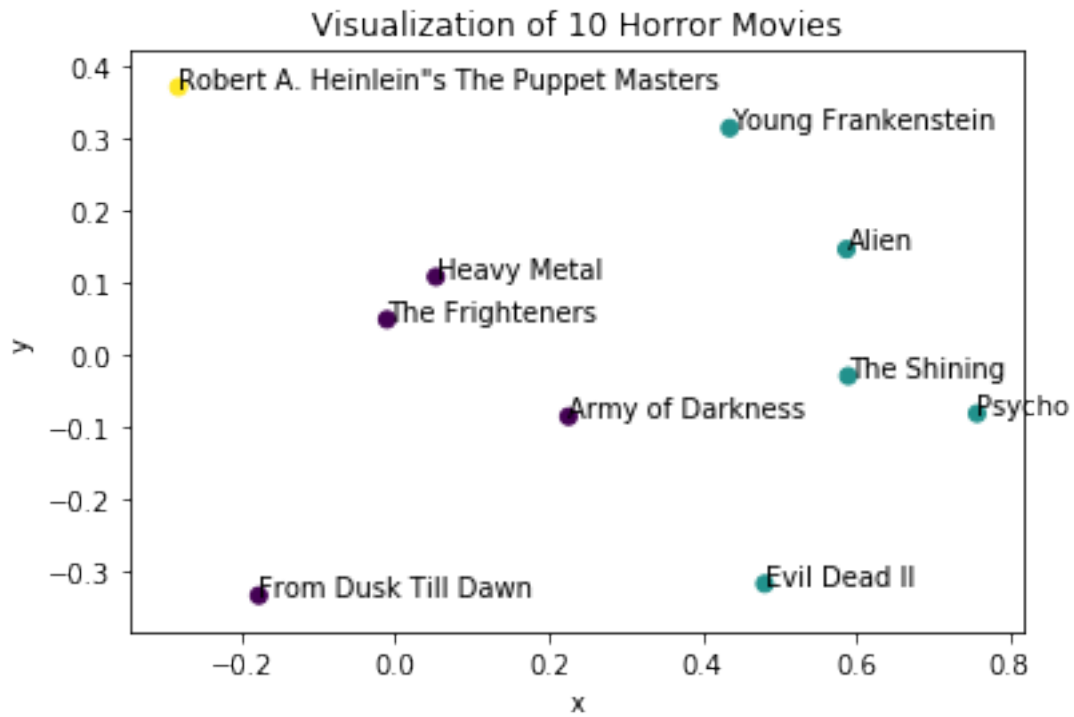
```

16    From Dusk Till Dawn (1996)
Name: Title, dtype: object
83    Robert A. Heinlein's The Puppet Masters (1994)
Name: Title, dtype: object
100   Heavy Metal (1981)
Name: Title, dtype: object
122   Frighteners, The (1996)
Name: Title, dtype: object
182   Alien (1979)
Name: Title, dtype: object
183   Army of Darkness (1993)

```


Name: Title, dtype: object
184 Psycho (1960)
Name: Title, dtype: object
199 Shining, The (1980)
Name: Title, dtype: object
200 Evil Dead II (1987)
Name: Title, dtype: object
207 Young Frankenstein (1974)
Name: Title, dtype: object





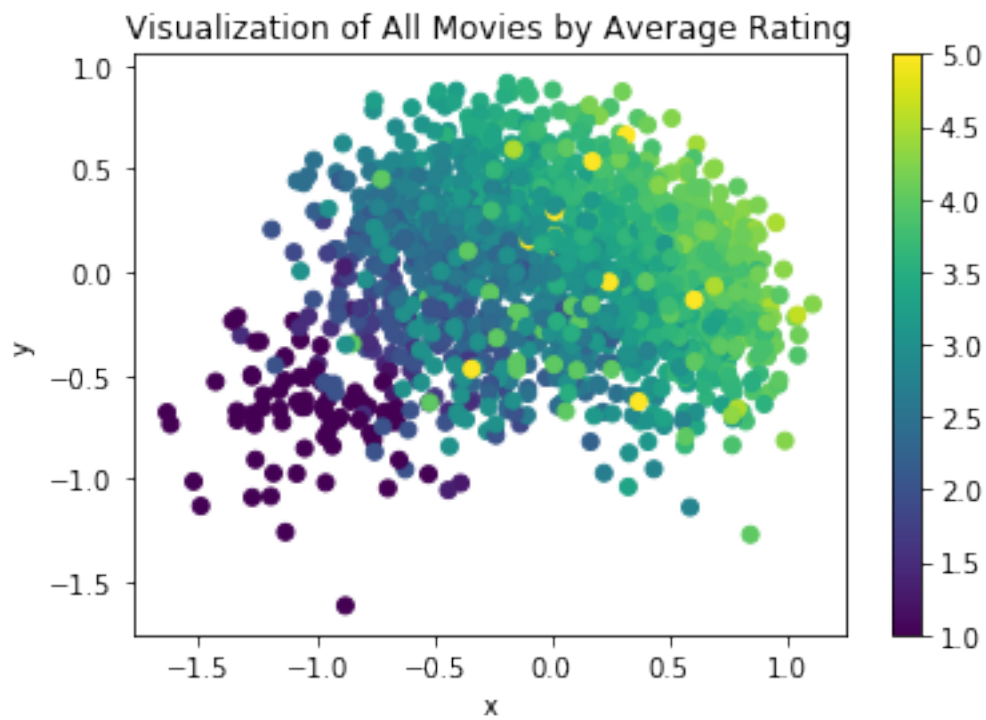
```

In [142]: # visualization of all movies by average rating
ratings = []
for i in range(len(movies)):
    ratings.append(movies['avg_rating'][i])

fig, ax = plt.subplots()
plt.scatter(V_tilde[0], V_tilde[1], c=ratings)

plt.title('Visualization of All Movies by Average Rating')
plt.xlabel('x')
plt.ylabel('y')
plt.colorbar()
plt.savefig('matrix_1_avg_rating.png')

```



```

In [145]: x = [0]*30
y = [0]*30
count = 0

for i in horror:
    x[count] = V_tilde[0][i-1]
    y[count] = V_tilde[1][i-1]
    count += 1
for i in western:
    x[count] = V_tilde[0][i-1]

```

```

        y[count] = V_tilde[1][i-1]
        count += 1
    for i in animated:
        x[count] = V_tilde[0][i-1]
        y[count] = V_tilde[1][i-1]
        count += 1

fig, ax = plt.subplots()
plt.scatter(x[:10], y[:10], c='b')
plt.scatter(x[10:20], y[10:20], c='r')
plt.scatter(x[20:30], y[20:30], c='g')

plt.title('Visualization of Animated, Western, and Horror Movies Overlaid')
plt.xlabel('x')
plt.ylabel('y')
plt.legend(['Horror', 'Western', 'Animated'])
plt.savefig('matrix_1_genres.png')

```

