Kinship Structures (3)

Extensions of the kinship model

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Agenda

1. Time-variance
2. Two-sex models
3. Other extensions

Recap

1. What is the general form of the (time-invariant) kinship models?
2. What are the different ‘model specifications’ that we discussed yesterday?

Typology of kinship models

No	time	sex	state	reference
1	invariant	female	age	
2	variant	female	age	
3	invariant	two	age	
4	invariant	female	multiple	
5	variant	two	multiple	

Time-variance

Typology of kinship models

No	time	sex	state	reference
1	invariant	female	age	
2	**variant**	**female**	**age**	[1]
3	invariant	two	age	
4	invariant	female	multiple	
5	variant	two	multiple	

[1] Caswell, H., & Song, X. (2021).The formal demography of kinship. III. kinship dynamics with time-varying demographic rates. *Demographic Research*, *45*, 517–546

Time-variant kinship models

1. Demographic rates change over time
2. Past demographic change $\rightarrow$ contemporary kinship structures
3. E.g., mortality crises, baby booms
4. Estimates by age, period, and cohort

Recap: Time-invariant, one-sex model

The models are of the general form:

$$\underbrace{\mathbf{k}(x+1)}_{\substack{\text{age structure of kin}\\ \text{at Focal's age } x+1}} = \underbrace{\mathbf{U}\,\mathbf{k}(x)}_{\substack{\text{ageing and survival}\\ \text{of existing kin}}} + \underbrace{\begin{cases} \mathbf{0} \\ \mathbf{F}\,\mathbf{k}^{*}(x) \end{cases}}_{\substack{\text{new kin members}\\ \text{added to the population}}} .$$

where:

- $\mathbf{U}$ a matrix with survival probabilities in the subdiagonal
- $\mathbf{F}$ a matrix with fertility rates in the first row

Time-variant, one-sex model

The models are of the general form:

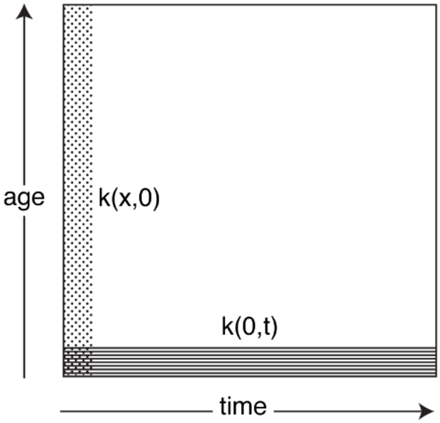
$$\underbrace{\mathbf{k}(x+1,t+1)}_{\substack{\text{age structure of kin}\\ \text{at Focal's age } x+1\\ \text{and time } t+1}} = \underbrace{\mathbf{U_t}\,\mathbf{k}(x,t)}_{\substack{\text{ageing and survival}\\ \text{of existing kin}}} + \underbrace{\begin{cases} \mathbf{0} \\ \mathbf{F_t}\,\mathbf{k}^*(x,t) \end{cases}}_{\substack{\text{new kin members}\\ \text{added to the population}}} .$$

where:

- $\mathbf{U_t}$ a matrix with time-variant survival probabilities in the subdiagonal
- $\mathbf{F_t}$ a matrix with time-variant fertility rates in the first row

Boundary conditions

Boundary conditions. The figure contains ages from 0 to ω and times from 0 to T. The boundary conditions correspond to $k(x, 0)$ for all x from 0 to ω and $\mathrm{k}(0, t)$ for all t from 0 to T



Boundary conditions

- specify the complete age vector at time $t = 0$

$$\mathbf{k}(x, 0) \qquad x = 0, ..., \omega.$$

- Specify the initial vector at each time

$$\mathbf{k}(0, t) \qquad t = 0, ..., \omega.$$

Daughters

Daughters ($\mathbf{a}$) are the result of the reproduction of Focal:

$$\underbrace{\mathbf{a}(x+1,t+1)}_{\substack{\text{age structure of daughters}\\ \text{at Focal's age } x+1}} = \underbrace{\mathbf{U_t}\,\mathbf{a}(x,t)}_{\substack{\text{ageing and survival}\\ \text{of existing daughters}}} + \underbrace{\mathbf{F_t}\,\mathbf{e}_x.}_{\substack{\text{new daughters}\\ \text{(subsidy)}}} \tag{1}$$

$$a(0) = \mathbf{0}.$$

where:

- $\mathbf{U_t}$ is a matrix with time-variant survival probabilities in the subdiagonal
- $\mathbf{F_t}$ is a matrix with time-variant fertility rates in the first row
- $\mathbf{F_t}\,\mathbf{e}_x$ is the subsidy vector
- e_x is the unit vector for age x
- $a(0)$ is the distribution of daughters at Focal's birth

Mothers

The population of mothers ($\mathbf{d}$) of Focal consists of at most a single individual:

$$\underbrace{\mathbf{d}(x+1,t+1)}_{\substack{\text{age structure of mothers}\\ \text{at Focal's age } x+1}} = \underbrace{\mathbf{U_t}\,\mathbf{d}(x,t)}_{\substack{\text{ageing and survival}\\ \text{of existing mothers}}} + \underbrace{\mathbf{0}.}_{\substack{\text{new mothers}\\ \text{(subsidy)}}} \tag{2}$$

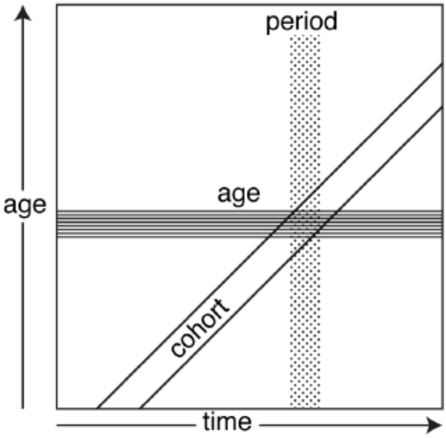
$$d(0,t+1) = \pi(t).$$

where:

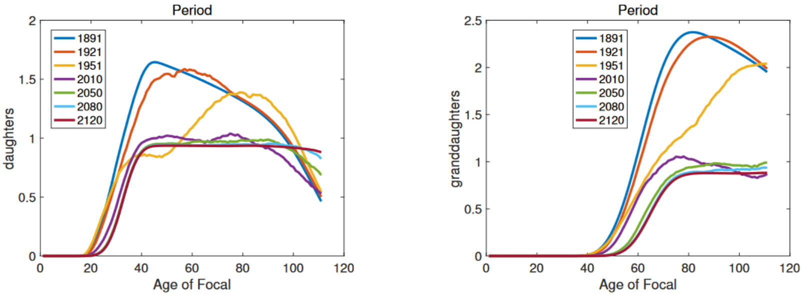
- $b(0,t+1)$ is the distribution of mothers at Focal's birth
- $\pi(t)$ is the distribution of ages of mothers in the population

Age, period, and cohort in kinship models

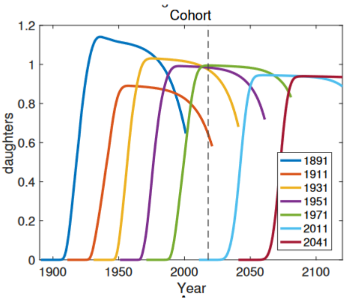
The period, cohort, and age dimensions of kinship development, within the age×time domain shown in Figure 2

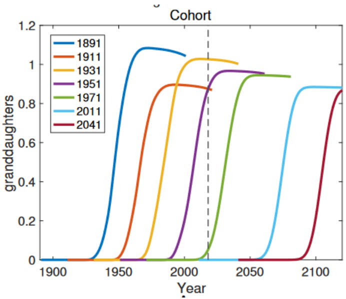


Period results for numbers of daughters and granddaughters in Sweden

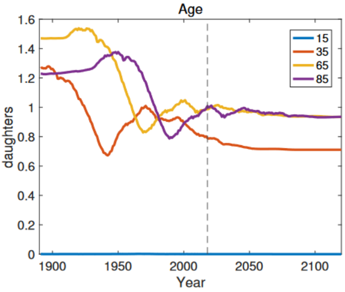


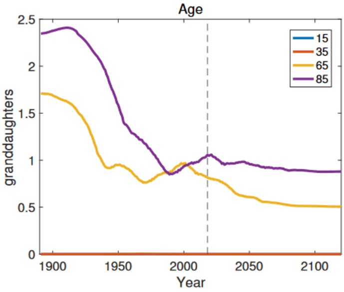
Cohort results for numbers of daughters and granddaughters in Sweden





Age results for numbers of daughters and granddaughters in Sweden





Discuss

1. What is the difference between the expected number of daughters calculated using (i) a time-invariant model, and (ii) the period dimension of the time-variant model?
2. What is the difference between the period and cohort results of the time-variant model?
3. How do the time-variant models deal with missing demographic data before a certain year (e.g., the UN only reports data starting in 1950)?

Break

Two-sex models

Typology of kinship models

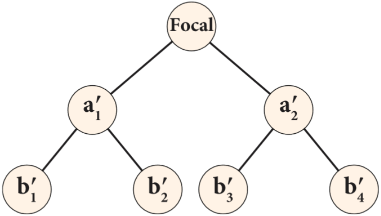
No	time	sex	state	reference
1	invariant	female	age	
2	variant	female	age	
3	**invariant**	**two**	**age**	[2]
4	invariant	female	multiple	
5	variant	two	multiple	

[2] Caswell, H. (2022). The formal demography of kinship IV: Two-sex models and their approximations. *Demographic Research*, *47*, 359–396

Why do we need two-sex models?

1. Differential survival and reproduction for women and men
2. These differences are not stable: change over time
3. More pronounced in some settings
4. Estimate male and female kin for male and female Focals

Two-sex model (descendants)



$$\tilde{\mathbf{a}} = \begin{pmatrix} \mathbf{a'}_1 \\ \mathbf{a'}_2 \end{pmatrix}$$

where

- $\tilde{\mathbf{a}}$ is a block-structured matrix of the expected number of living offspring
- $\mathbf{a'}_1$ is the expected number of living sons
- $\mathbf{a'}_2$ is the expected number of living daughters

Offspring (sons and daughters)

Children ($\tilde{\mathbf{a}}$) are the result of the reproduction of Focal:

$$\underbrace{\tilde{\mathbf{a}}(x+1)}_{\substack{\text{age structure of offspring} \\ \text{at Focal's age } x+1}} = \underbrace{\tilde{\mathbf{U}}\,\tilde{\mathbf{a}}(x)}_{\substack{\text{ageing and survival} \\ \text{of existing offspring}}} + \underbrace{\tilde{\mathbf{F}}\,\tilde{\phi}(x).}_{\substack{\text{new offspring} \\ \text{(subsidy)}}} \tag{3}$$

$$\tilde{a}(0) = \mathbf{0}.$$

where:

- $\tilde{\mathbf{U}}$ is a block-structured matrix of survival probabilities
- $\tilde{\mathbf{F}}$ is a block-structured matrix of fertility rates
- $\tilde{\mathbf{F}}\,\tilde{\phi}(x)$ is the subsidy vector
- $\tilde{\phi}(x)$ is the state vector of a Focal of specified sex
- $\tilde{a}(0)$ is the distribution of offspring at Focal's birth

Blocks-structured input matrices

For mortality:

$$\tilde{\mathbf{U}} = \begin{pmatrix} \mathbf{U}_f & \mathbf{0} \\ \mathbf{0} & \mathbf{U}_m \end{pmatrix}$$

For fertility:

$$\tilde{\mathbf{F}} = \begin{pmatrix} \bar{\alpha}\mathbf{F}_f & \bar{\alpha}\mathbf{F}_m \\ \alpha\mathbf{F}_f & \bar{\alpha}\mathbf{F}_m \end{pmatrix}$$

where

- $\mathbf{U}_f$ is a matrix with female survival probabilities in the subdiagonal
- $\mathbf{F}_f$ is a matrix with female fertility rates in the first row
- α is the proportion males among offspring
- $\bar{\alpha}$ is $1 - \alpha$

Parents

The population of parents ($\tilde{\mathbf{d}}$) of Focal consists of at most a single individual:

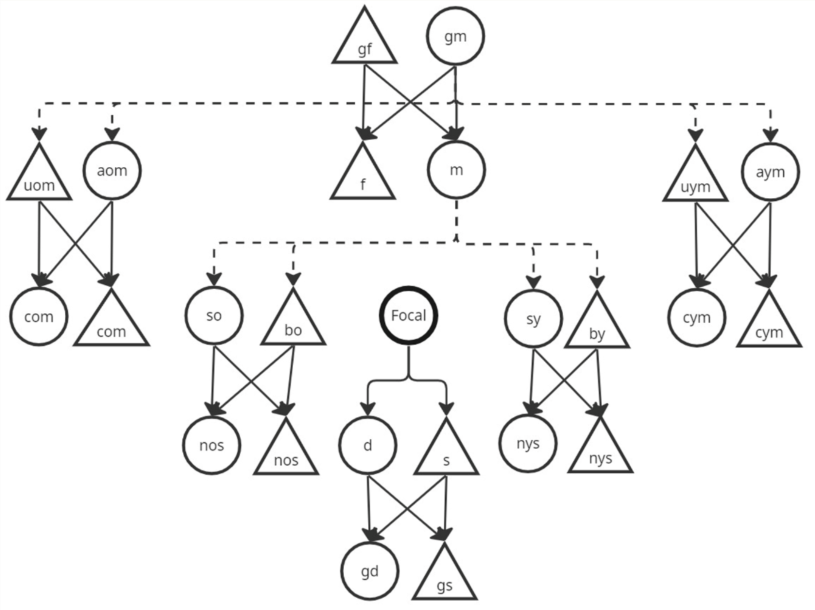
$$\underbrace{\tilde{\mathbf{d}}(x+1)}_{\substack{\text{age structure of mothers}\\ \text{at Focal's age } x+1}} = \underbrace{\tilde{\mathbf{U}}\,\tilde{\mathbf{d}}(x)}_{\substack{\text{ageing and survival}\\ \text{of existing mothers}}} + \underbrace{\mathbf{0}.}_{\substack{\text{new mothers}\\ \text{(subsidy)}}} \tag{4}$$

$$\tilde{d}(0) = \tilde{\pi}.$$

where:

- $\tilde{b}(0)$ is the distribution of parents at Focal's birth
- $\tilde{\pi}$ is the distribution of ages of parents in the population

Two-sex kin estimation

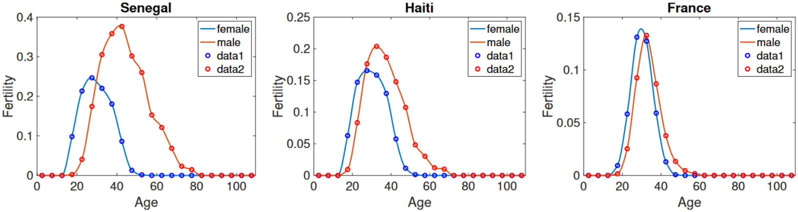


Data requirements

$$\begin{aligned}
\mathbf{U}_{\mathrm{f}},\ \mathbf{U}_{\mathrm{m}} &= \text{female and male survival matrices} \\
\mathbf{F}_{\mathrm{f}}.\ \mathbf{F}_{\mathrm{m}} &= \text{female and male fertility matrices} \\
\boldsymbol{\pi}_{\mathrm{f}},\ \boldsymbol{\pi}_{\mathrm{m}} &= \text{distribution of ages at maternity and paternity} \\
\alpha &= \text{proportion males among offspring} \\
\bar{\alpha} &= 1-\alpha
\end{aligned}$$

Male and female fertility

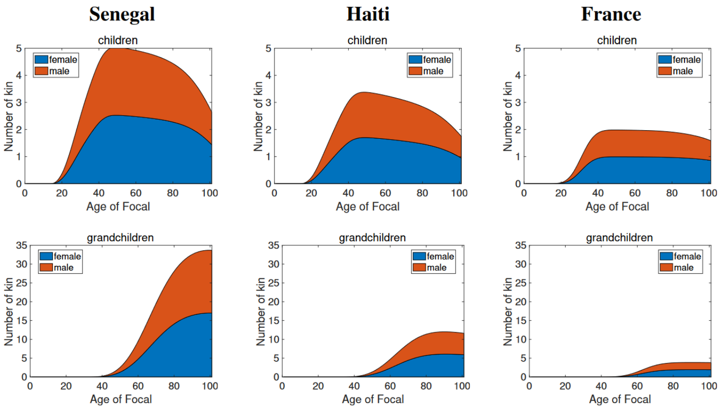
Figure 5: **The observed (circles) and interpolated (lines) age-specific fertility rates for Senegal (2013), Haiti (2010), and France (2012). Based on data from Schoumaker (2019).**



Approximations for two-sex kinship models

1. Androgynous fertility
   - Assume that $\mathbf{F}_m = \mathbf{F}_f$
2. GKP factors
   - Run one-sex model and multiply resulting kinship structure by a 'GKP factor'
   - daughters $\times$ 2, granddaughters $\times$ 4, great-granddaughters $\times$ 8, mothers $\times$ 2, grandmothers $\times$ 4, great-grandmothers $\times$ 8, sisters $\times$ 2, nieces $\times$ 4, aunts $\times$ 4, and cousins $\times$ 8

Expected number of female and male kin in three countries



Discuss

1. Why do we need the ‘androgynous’ and ‘GKP factor’ approximations for two-sex kinship models?
2. Which of the two do you think is better? Can you think of other possible ‘approximations’?

Other extensions

Typology of kinship models

No	time	sex	state	reference
1	invariant	female	age	
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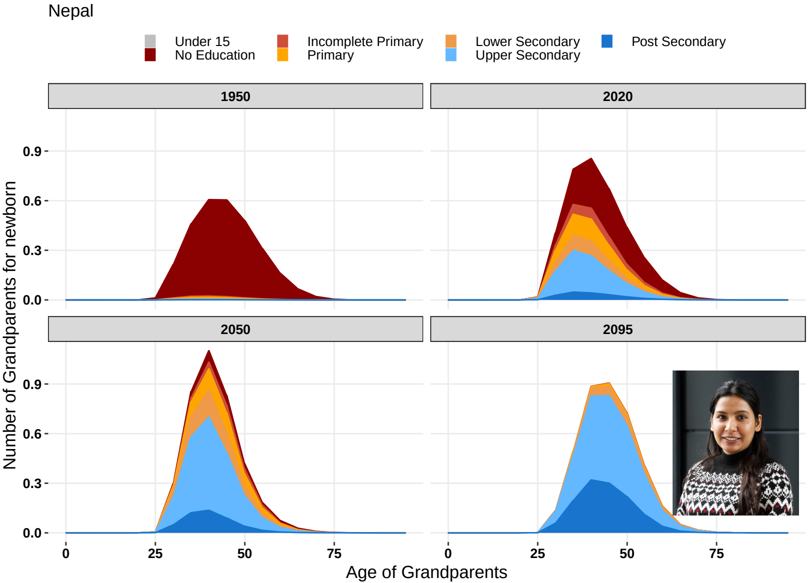
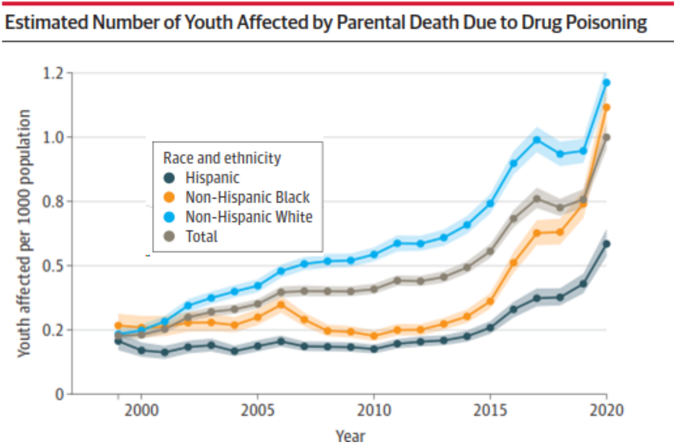
4	invariant	female	multiple	
5	**variant**	two	**multiple**	[3]

[3] Williams, I., Alburez-Gutierrez, D., & DemoKin Team. (2023). *DemoKin: 1.0.3*. https://CRAN.R-project.org/package=DemoKin

Multistate kinship models



Kin loss by cause of death[4]



[4] Schlüter, B.-S., Alburez-Gutierrez, D., Bibbins-Domingo, K., Alexander, M. J., & Kiang, M. V. (2024). Youth Experiencing Parental Death Due to Drug Poisoning and Firearm Violence in the US, 1999-2020. *JAMA*. https://doi.org/10.1001/jama.2024.8391

Discuss

1. Do you see any application of the kinship models to your own work?
2. Which model specification would be more appropriate for this?