Determining the Relationship Between Amplitude and Period of a Pendulum

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1 Abstract

This experiment sought to verify the theoretical relationship between the amplitude and the period of a pendulum. A mass was attached to the end of a string, which was hung from a support beam on the lab ceiling. The pendulum was displaced at various amplitudes and the time was measured for the pendulum to complete five oscillations. The experimental period was compared to the theoretical period for each set of conditions. The experimental period values were larger than the theoretical values, more so at larger amplitudes. Therefore, the results of this experiment were not accurate. Possible causes for the period varying from the theory include the measurement of the length, as well as the fact that the equation to determine the period for a harmonic oscillator is only accurate for small angles. However, this experiment yielded more accurate results at larger amplitudes.

2 Introduction

This experiment seeks to determine the relationship between the period and the amplitude of a pendulum. At small angles, less than 15 degrees, the "The force on a pendulum is a linear restoring force... so the pendulum will undergo simple harmonic motion."[1]. Figure 1 demonstrates the forces which act on a pendulum. For harmonic oscillation, the time to complete one full cycle is

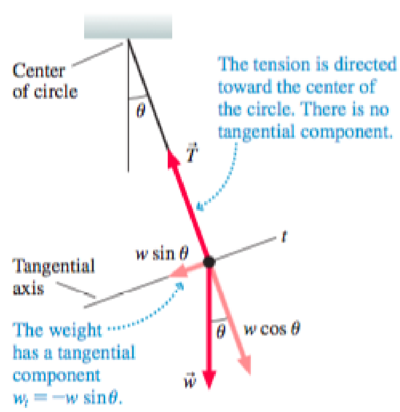

Figure 1: The forces on a pendulum [1].

called the period. The "period of a pendulum depends on the length and the free-fall acceleration", thus with amplitude variations at small angles it should remain constant when all other variables are held constant [1]. At small angles, the period is given by

$$T = 2\pi\sqrt{\frac{l}{g}}$$

being influenced only by l, the length of the pendulum, and g, the acceleration due to gravity [1]. In this experiment, the length of the pendulum was 1.85 m, and the acceleration due to gravity was 9.81 m/s^2.

$$T = 2\pi\sqrt{\frac{1.85}{9.81}} = 2.73s$$

At small angles, theoretically the period should be equal to 2.73. The theoretical angular velocity is calculated to be 2.30 rad/s.

3 Materials and Methods

Figure 2 demonstrates the design of the pendulum and measurement apparatus. A 200.06 gram mass was tied to one end of a string (m), and the other end was tied to a support beam on the ceiling of the lab. The length of the string (l), measured from the pivot point on the ceiling to the knot tied to the mass, was recorded as 1.85 meters. The mass of the string was accounted for, resulting in a total mass of 205.2 grams. A meter stick was placed on the ground parallel to the ceiling beam, to allow for the measurement of horizontal displacement (d).

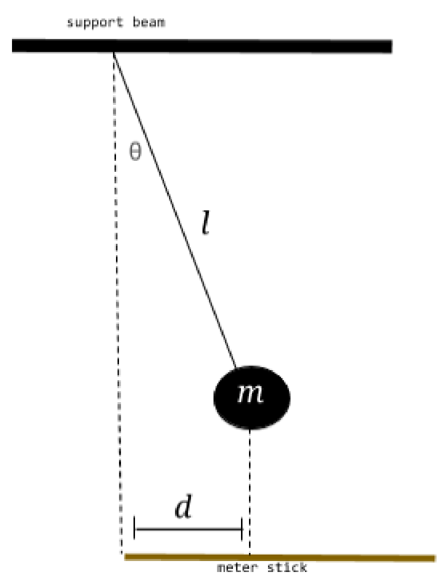

Figure 2: The experimental design.

The pendulum was displaced by the same student each time. Two different students recorded the time for the pendulum to complete five oscillations, using an online stopwatch, and the students' times were averaged and recorded [2]. Five trials were completed for each displacement. The process was repeated, varying the horizontal displacement (0.3, 0.4, 0.5, 0.6, 0.7, 0.8 and 0.9

meters). The mass and length of the pendulum were kept constant, allowing for the analysis of the relationship between the amplitude and the period.

4 Data

Table 1 lists the period, the calculated amplitude, calculated period, and the calculated angular velocity values for each horizontal displacement.

Table 1: Horizontal Displacement, Amplitude, Period, and Angular Velocity

Horizontal Displacement (m)	Amplitude (deg)	Experimental T (s)	Experimental ω (rad/s)
0.3	9.33	2.55	2.47
0.4	12.49	2.54	2.48
0.5	15.68	2.55	2.46
0.6	18.92	2.56	2.45
0.7	22.23	2.58	2.44
0.8	25.62	2.59	2.43
0.9	29.11	2.62	2.40

The average of the recorded times was divided by 5 to determine the period. The amplitude corresponding to each displacement was calculated, using the length of the string and the horizontal displacement

$$sin^{-1}(\frac{d}{l}) = \theta$$

The angular velocity was calculated using

$$\omega = \frac{2\pi}{T}$$

Figure 3 graphically represents the horizontal displacement versus experimental period values from Table 1. Standard error bars were added to account for systematic timing and measurement errors.

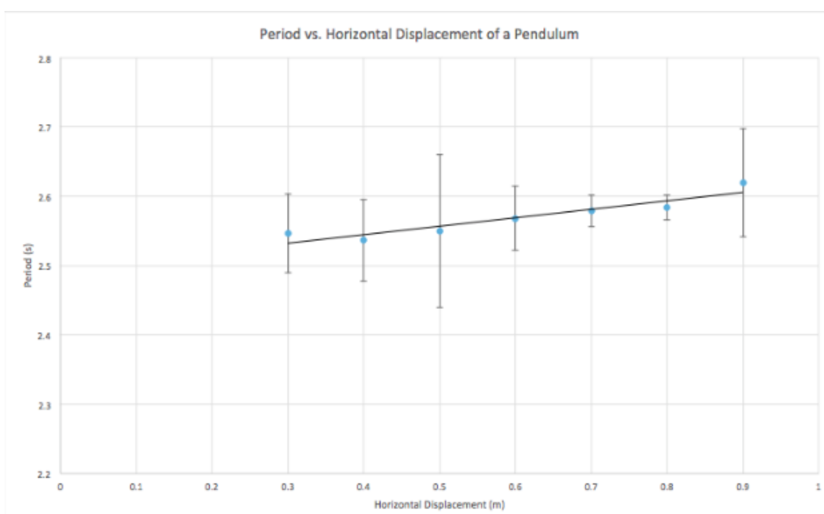

Figure 3: Graph of experimental period versus horizontal displacement with linear trend line.

The experimental period values are less than the theoretical period value, which was calculated to be 2.73 seconds. Figure 4 graphically represents the period versus the angular velocity for each amplitude tested. Standard error bars were added, along with a linear trend line. As the period decreases, the angular velocity appears to decrease.

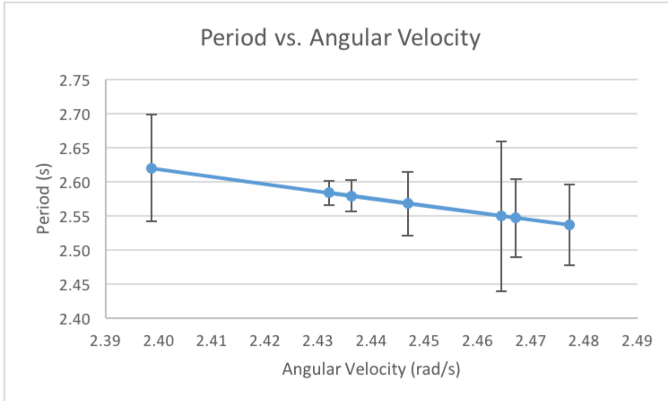

Figure 4: Graph of experimental period versus angular velocity.

5 Results

As the angular velocity increases, the period decreases, as expected. As the horizontal displacement increases, the experimental period also increases at larger angles. The experimental observations regarding period and amplitude support the theoretical relationship at small angles, as the period only changes by 0.01 second for angles less than 16 degrees. However, at larger angles the period increases as the amplitude increases. As the amplitude increases, the theoretical equation used to calculate the period begins to lose its accuracy due to the fact that it involves the small angle approximation, $sin(\theta) \approx \theta$. It was expected that the experimental period would align with the theoretical period at small angles, and error would be more prevalent as the amplitude became larger. However, the experimental period values appear to be more accurate at larger angles. The period versus horizontal displacement graph is shown on a larger scale in Figure 5. While the period did increase from 2.54 seconds to 2.62 seconds, the difference between the most extreme values is only 0.08 seconds.
The slope of this trend line can be calculated as 0.2, which is not negligible.

$$\frac{\Delta y}{\Delta x} = \frac{(2.58 - 2.56)}{(0.7 - 0.6)} = 0.2$$

5.1 Length Measurement

A possible source of error lies in the measurement of the length of the pendulum. To determine what length would make the theoretical period equal to the experimental periods, the theoretical period can be set equal to the equation and the equation can be solved for the length, l.

$$2.76 = 2\pi \sqrt{\frac{l}{9.81}}$$

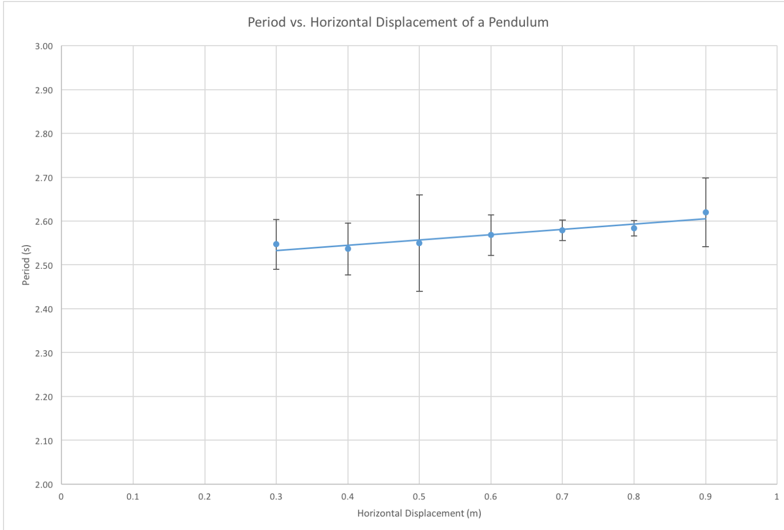

Figure 5: Graph of experimental period versus horizontal displacement at a larger scale.

$$0.4393 = \sqrt{\frac{l}{9.81}}$$
$$0.1930 = \frac{l}{9.81}$$
$$1.37 = l$$

To result in experimental period values that are equal to the theoretical period values at small angles, the experimental length must be 0.48 meters shorter than the experimental length measured. It is unlikely that errors in measuring the length were this significant. Therefore, the measurement of the pendulum's length is not the primary explanation for the increased experimental period values.

5.2 Distributed Mass

It was suspected that because a portion of the total mass of the pendulum was distributed throughout the string, the relationship between period and amplitude was affected. However, the acceleration due to gravity is the same regardless of the mass hanging on the pendulum. The driving force of the pendulum is gravity, therefore its angular velocity and period should be the same for all masses.

6 Conclusion

The experimental data supports the theoretical relationship between period and amplitude at small angles, as the period does not significantly vary at angles below 16 degrees. However, the experimental period values are smaller than the theoretical period value, and diverge from the theory more at smaller angles. The results found in this experiment are precise, however they are not accurate. It is clear that there is a direct relationship between the amplitude and the period for angles larger than 16 degrees. As the amplitude increases, the angular velocity increases, and thus the period increases.

7 Acknowledgements

The authors acknowledge the assistance of additional lab partner Calvin Celebuski.

8 Notes and References

1. Knight, Randall, Brian Jones, and Stuart Field. *College Physics: A Strategic Approach.* 3rd ed. Glenview, IL: Pearson Education, 2015.
2. "Online Stop Watch", Online Timer, accessed December 3, 2015, http://stopwatch.online-timers.com/online-stopwatch