# Testing The Law of Conservation of Energy Using Projectile Motion

Iris Lin with Amanda Mengotto

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## 1 Abstract

It has been said that the Law of Conservation of Energy exists throughout the entire universe. However, when observing the world around us it can be difficult to find a demonstration of the law, as we cannot see the conversions of potential and kinetic energy. In this lab, the Law of Conservation of Energy was tested by rolling spheres down an inclined tube and observing the conversions of energy. The initial energy of the system is the Potential Energy of the sphere, and the final energy is the rotational and linear Kinetic Energies of the sphere. The horizontal distances the spheres traveled upon being projected out the end of the tube were used to analyze the law. Consistency and precision in the experimental horizontal distances allow us to conclude that the Law of Conservation of Energy holds true regardless of the mass, radius and material of the sphere. However, the experimental distances traveled were less than the theoretical distances which lead us to conclude that there was some type of error present. The friction in the tube, the potential oscillation of the spheres within the tube, and air resistance are possible explanations along with human error in measurement.

## 2 Introduction

The Law of Conservation of Energy states that "for an isolated system, energy is conserved—the final energy, including any change in thermal energy, equals the initial energy" [1]. In this experiment, the conversion from gravitational potential energy to translational kinetic energy and rotational kinetic energy was analyzed as a variety of solid spheres were rolled down an inclined tube and cast out the end of the tube and onto the floor. The potential energy at the top of the tube can be related to the total kinetic energy at the bottom end of the tube, as they should be equal.

$$PE_g = KE_{translational} + KE_{rotational}$$

This relationship can be used to determine the velocity of a sphere rolled down the incline at the moment it exits the tube. For a sphere, $I = \frac{2}{5}mr^2$ and $v = r\omega$, thus the velocity equation is derived as follows:

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$2gh = v^2 + (\frac{2}{5}r^2)\omega^2$$

$$2gh = \frac{7}{5}v^2$$

$$v = \sqrt{\frac{10}{7}gh}$$

The velocity can be used in coordination with the height of the tube and height of incline to predict the horizontal distance the projectile will travel. The experimental distances can be compared to the theoretical distances to assess whether the law of conservation of energy holds true for the system. The horizontal and vertical components of the projectile's path once it leaves the tube can be written as

$$x = v_0 t \qquad \text{and} \qquad y = \frac{1}{2}gt^2$$

where t is the time of travel, x is the horizontal distance, and y is the vertical height of the tube above the floor. The equations can be combined to determine the expected horizontal distance the sphere will travel.

$$x = v_0 \sqrt{\frac{2y}{g}}$$

The energy of the sphere can be assessed using measured values without time dependence.

# 3 Materials and Methods

A plastic tube was used to prevent the spheres from rolling off the edge of the table. The table was raised on one end using wood blocks to form an incline and the tube was secured to the table with tape. Carbon paper was taped over white paper on the floor in the region where the spheres were expected to land. Figure 1 demonstrates the experimental setup.

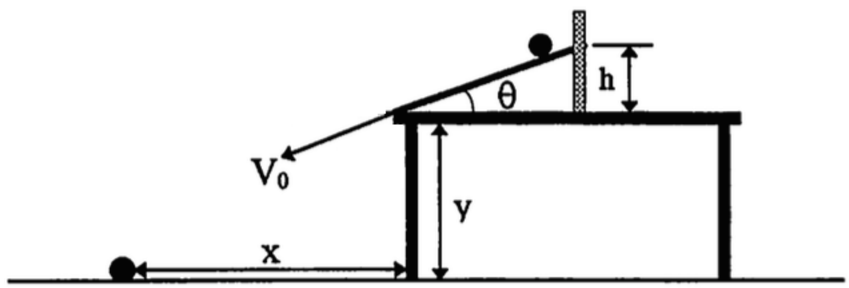


Figure 1: The experimental design.

The height of the table, $y$, was 0.76 m, the height of the tube, $h$, was 0.0644 m, and the length of the tube was 1.497 m. Using the formula for velocity derived above, the velocity at which the spheres exited the tube was calculated to be 0.95 m/s.

$$V_0 = \sqrt{\frac{10}{7}(9.8)(0.0644)} = 0.95 m/s$$

Because the radius and mass of the sphere are not included in the velocity or distance calculations, the distance traveled should remain constant for each sphere. The expected distance traveled was then determined to be 0.3741 m.

$$x = 0.95\sqrt{\frac{2(0.76)}{9.8}} = 0.3741 m$$

Six spheres of varying materials, masses, and radii were used. Table 1 below demonstrates the data recorded for each sphere. A micrometer was used to measure the radii of the spheres, and the masses were recorded using a balance. Each sphere was released from the center of the top of the plastic tube. The distance the sphere traveled on the ground from the table to the mark on the carbon paper was recorded. The spheres were released by the same student throughout the experiment. Each sphere was released five times and the distances for each trial were recorded and averaged.

Table 1: Spheres used and properties measured.

| Name of Sphere | Radius (m) | Mass (kg) |
|---|---|---|
| Large Black Rubber | 0.0124 | 0.008 |
| Large Purple Plastic | 0.0122 | 0.0191 |
| Large Metal | 0.0127 | 0.0661 |
| Small Black Plastic | 0.00952 | 0.0041 |
| Small Rusty Metal | 0.00952 | 0.00284 |
| Small Wooden | 0.00953 | 0.0024 |

Three of the spheres had similar radii that were approximately 0.012 m and three of the spheres had radii that were approximately 0.0095 m. The independent variables were the radius, mass, and material of the spheres. The dependent variable was the linear distance the spheres traveled once exiting the tube, labeled as $x$ in Figure 1.

# 4 Data

Table 2 demonstrates the average distance each sphere traveled, calculated from the distances from five trials for each sphere. It also lists the expected distance for each sphere for comparison.

Table 2: Average distances the spheres traveled.

| Type of Sphere | Distance Traveled (m) | Expected Distance (m) |
|---|---|---|
| Large Black Rubber | 0.2774 | 0.3741 |
| Large Purple Plastic | 0.2828 | 0.3741 |
| Large Metal | 0.2836 | 0.3741 |
| Small Black Plastic | 0.2740 | 0.3741 |
| Small Rusty Metal | 0.2798 | 0.3741 |
| Small Wooden | 0.2384 | 0.3741 |

Figure 2 displays a bar graph comparing the experimental distances traveled to the expected distance. Standard deviation error bars are included to account for small errors in measurement. It is clear that the experimental distances traveled are all less than the theoretical horizontal distance each sphere was expected to have traveled. The distance the small wooden sphere traveled is even less than the distance the other five spheres traveled.

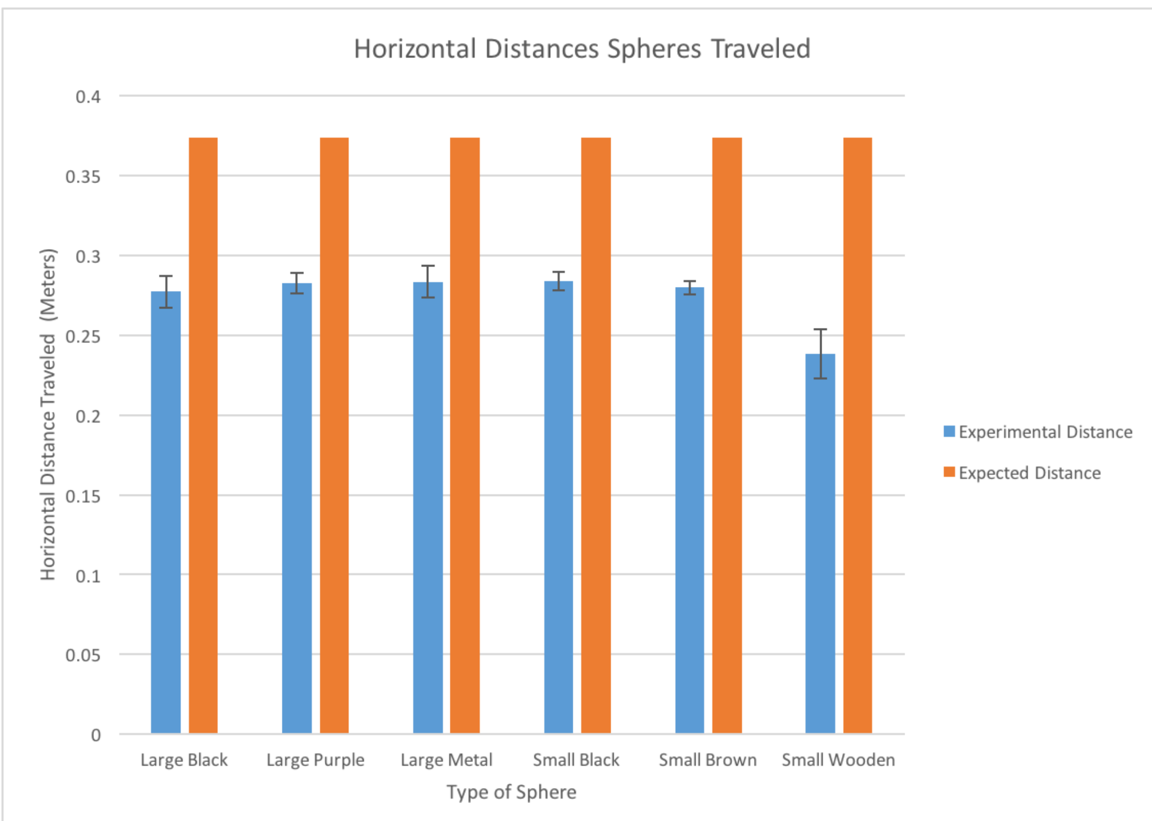


Figure 2: Graph comparing the theoretical and experimental distances traveled for each sphere.

# 5 Results

It is clear that the horizontal distance traveled after exiting the tube was less than the expected distance for each sphere. For the small wooden sphere, the distance traveled was much less than the expected distance traveled. There are several possible explanations for this deviation from the theoretical horizontal distance. The first possible error is that the wooden sphere may have been a shell rather than a solid sphere. Next, we can explore the effects of friction on the spheres. Finally, the path the spheres traveled may not have been exactly straight and this extra distance traveled side to side within the tube may have altered the velocity at which the spheres exited the tube.

## 5.1 Hollow Wooden Sphere

This could be due to the fact that the small wooden sphere is hollow rather than solid. To test this, we can recalculate the distance for the small wooden sphere using the Moment of Inertia formula for hollow spheres, $I = \frac{2}{3}mr^2$. The final velocity for the sphere would then be given by

$$V_0 = \sqrt{\frac{6}{5}(g)(h)} = \sqrt{\frac{6}{5}(9.8)(0.0644)} = 0.87m/s$$

This velocity can be used to calculate the expected distance of the wooden spherical shell.

$$x = 0.87\sqrt{\frac{2(0.76)}{9.8}} = 0.3426m$$

This distance is less than the expected distance for the solid spheres. Accounting for this error would reduce the gap between the experimental and theoretical distances traveled but would not completely explain it. For all spheres, there must be another factor not accounted for.

## 5.2 Friction

The experimental distances may be less than the theoretical distances due to the friction in the tube. If rolling friction were present, the final velocity as the spheres exit the tube would be less than the theoretical final velocity calculated. If the velocity were to decrease, the theoretical horizontal distance traveled would also decrease. It is difficult to determine the coefficient of rolling friction and difficult to quantify the effect of friction on the experimental results. However, it is possible that friction is a component of the explanation as to why the experimental results deviate from the theoretical distance.

## 5.3 Oscillation

In our experiment, we observed that the spheres did not always travel in a straight path through the tube. Instead, it was observed that the spheres travelled side to side slightly before settling on a straight path. This caused the spheres to exit the end of the tube at different points, rather than simply being projected straight out the end. Although this would not significantly affect the height of the spheres upon exiting the tube, it could have affected the displacement of the sphere as only the horizontal distance was analyzed. The spheres did not consistently land in a straight line from the tube, sometimes traveling to the left or right. This component could have compromised the distances, causing the experimental distances recorded to be slightly less than the theoretical distance.

# 6 Conclusion

Five of the six spheres, omitting the small wooden sphere, traveled very similar distances. The largest horizontal distance traveled was 0.2836 m and the smallest aside from the wooden sphere was 0.2740 m, resulting in a range of 0.0096 m. This demonstrates that the sphere's surface material, mass, and radius do not have a significant affect on the horizontal distance traveled. However, the distribution of the mass and uniform density do have an affect on the horizontal distance traveled. The small range in experimental distances traveled supports the Law of Conservation of Energy. The experimental data is precise, however it is not accurate as it is not very close to the theoretical horizontal distance calculated using energy principles. This can be attributed to a variety of causes, including but not limited to uneven mass distribution, rolling friction, air resistance, and the spheres oscillating down the tube from side to side.

## 7 Acknowledgements

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## 8 Notes and References


1. Knight, Randall, Brian Jones, and Stuart Field. *College Physics: A Strategic Approach.* 3rd ed. Glenview, IL: Pearson Education, 2015.