Equations

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From the paper *Exclusive Photoproduction of Vector Meson of NLO from CGC* by Cai, Xiang, Wang, and Zhou:
Cai, Yanbing Xiang, Wenchang Wang, Mengliang Zhou, Daicui. (2020). Exclusive photoproduction of vector meson at next-to-leading order from color glass condensate. Chinese Physics C. 44. 074110. 10.1088/1674-1137/44/7/074110.
Under eq. 10, the following definitions are given:

$$r = x_\perp - y_\perp \tag{1}$$

$$r_1 = x_\perp - z_\perp \tag{2}$$

$$r_2 = z_\perp - y_\perp \tag{3}$$

where $x_\perp$ is the transverse coordinate of quark, $y_\perp$ is the transverse coordinate of the anti-quark, and $z_\perp$ is the transverse coordinate of the emitted gluon. r is passed in through $N(r, Y)$, so we aren't really concerned with what $x_\perp$ and $y_\perp$ are exactly, only that their difference equals r. In that case then, let $x_\perp = \frac{1}{2}r$, and $y_\perp = -\frac{1}{2}r$. Then, $r = x_\perp - y_\perp = \frac{1}{2}r - (-\frac{1}{2}r) = r$.
Plugging these into the values for r_1 and r_2, and simplifying expressions in terms of $z_\perp$ and r:

$$r_1 = x_\perp - z_\perp = \frac{1}{2}r - z_\perp \tag{4}$$

$$|r_1| = \sqrt{(\frac{1}{2}r)^2 + (z_\perp)^2 - 2(\frac{1}{2}r)(z_\perp)cos\theta} \tag{5}$$

$$|r_1| = \sqrt{\frac{1}{4}r^2 + z_\perp^2 rz_\perp cos\theta} \tag{6}$$

$$r_2 = z_\perp - y_\perp = \frac{1}{2}r + z_\perp \tag{7}$$

$$|r_2| = \sqrt{(\frac{1}{2}r)^2 + (z_\perp)^2 + 2(\frac{1}{2}r)(z_\perp)cos\theta} \tag{8}$$

$$|r_2| = \sqrt{\frac{1}{4}r^2 + z_\perp^2 + rz_\perp cos\theta} \tag{9}$$

Runge-Kutta Equations:

First order:

$$\begin{aligned} k_1 &= f(N(r,y)) \\ N(r,y+h) &= N(r,y) + hk_1 \end{aligned} \tag{10}$$

Second order:

$$\begin{aligned} k_1 &= f(N(r,y)) \\ k_2 &= f(N(r,y) + \frac{1}{2}hk_1) \\ N(r,y+h) &= N(r,y) + hk_2 \end{aligned} \tag{11}$$

Fourth order:

$$\begin{aligned} k_1 &= f(N(r,y)) \\ k_2 &= f(N(r,y) + \frac{1}{2}hk_1) \\ k_3 &= f(N(r,y) + \frac{1}{2}hk_2) \\ k_4 &= f(N(r,y) + hk_3 \\ N(r,y+h) &= N(r,y) + \frac{1}{6}h(k_1 + 2k_2 + 2k_3 + k_4) \end{aligned} \tag{12}$$