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A General Model of Spatial Duopoly With Entry Barriers

by

Juhyun Yoon

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A GENERAL MODEL OF SPATIAL DUOPOLY WITH ENTRY BARRIERS

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Juhyun Yoon

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ABSTRACT

This study attempts to generalize the spatial competition model of duopoly with entry barriers. It assumes heterogeneous cost functions implicitly given, an arbitrary population distribution, and endogenous demand derived from the analysis of consumer behavior. In addition to these general assumptions, it also considers spatial price competition between firms with fixed location by assuming Cournot-Nash behavior; considers welfare implication along with profit maximizing behavior; compares the efficiency of autonomous operation by each firm versus joint operation by a single authority.

From the generalized assumptions, we found consistent implications for a spatial duopoly. Optimal price level under joint operation is higher than that from an autonomous operation when firms maximize their profits. The profit level achieved is greater for joint operation than for autonomous operation. If firms act to maximize social welfare, the resulting optimal price is lower than if profit maximization is sought. The profit level achieved from social welfare maximizing behavior is bounded at zero; thus, it results in less profit than does profit maximizing behavior. Social welfare is lowest when the two firms pursue profit maximization and are

jointly operated. Therefore, operational efficiency is maximized under joint operation regardless of the operational type. However, when the firms are pursuing profit maximization, joint operation is not desirable because increases in the firms' benefits result from reductions in consumers' satisfaction.

The investment problem for spatial duopolists expecting demand increases was analyzed as an extension of the basic model. Firms must choose a combination of price increases and capacity expansion to meet the increased demand. The optimal combination of price and capital stock was obtained via the calculus of variations.

Locational considerations are also attempted. Unfortunately, however, we could not derive general implications on locational behavior due to many complexities. This problem can be solved by computer simulations.

I. INTRODUCTION

Locational consideration has been applied to classical microeconomics much like time consideration has been developed in economic theory. The same goods transacted at different times can be treated as different goods with homogeneous characteristics except for the time factor. Similarly, the same goods transacted at different locations can be regarded as different goods with homogeneous characteristics except for their locations.[1)] These locational factors are caused mainly by the transportation cost of carrying goods from the production area to the consumption place.

Thus, perfect competition concept in classical microeconomics can not exist in a spatial context. The same goods being sold at different markets may have different delivered prices due to the different transportation costs that result from different distances between the producer and the consumer. That is, delivered prices are different even though factory prices are the same. Consumers tend to buy products which have the minimum delivered price: factory price plus transportation cost. A firm located near the market has a comparative advantage in the selling price due to a lower transportation cost. Thus each firm can enjoy a degree of monopolistic power within a certain range of its market area.

For this reason, Löschian type models and Greenhut-Ohta type of models usually treat firms spread out in space as spatial monopolists.[1] They assume that the market boundaries are fixed, and neglect the possibility that it can be changed, depending upon the behavior of other firms. Still, there exists some degree of spatial competition with the neighboring firms at the market boundary. From this perspective, spatial competition can be regarded as monopolistic competition rather than perfect competition.

Hotelling [1929] presented a model of two firms competing to sell a homogeneous product to customers spread evenly along a linear market. Since then, many urban economists have been interested in testing whether the results of classical microeconomics are consistent when locational choice is added to the analysis. Some researchers have argued that in a spatial context several results deviate from the conclusions of classical price theory. For example, Greenhut, Hwang, and Ohta [1975] have shown that some forms of spatial competition will result in higher prices than those realized under spatial monopoly. Capozza and Attaran [1976] have shown that some spatial comparative statics can differ from nonspatial results. And Capozza and Van Order [1977] showed conditions under which spatial considerations lead to deviations from the standard economics results. In particular, cost increases may lead to lower market equilibrium prices.

However, these results do not always hold in the general sense. As Greenhut-Hwang-Ohta [1975], Capozza and Van Order [1977] state, the results of spatial competition models are very sensitive to the assumptions used.

1. Statement of Problem

Based upon the treatment of market boundaries, one can separate spatial competition models into three categories: 1) Löschian type models, 2) Hotelling and Smithies type models, and 3) Greenhut-Ohta type models. However, consideration of spatial price competition exists only in the Hotelling and Smithies type models. Capozza and Van Order [1978] tried to generalize those models in a single framework.

Löschian type models assume that the market boundaries are fixed, and that each firm sets prices like a monopolist within their market area. The behavior of other firms is assumed to be exactly the same, and thus, there is no change in market boundary. These models consider the market shape with free entry.

Hotelling and Smithies type models assume that each firm considers competitors' prices to be fixed. However, many writers neglect other firms' reaction behavior. They only see the problem from a partial equilibrium perspective. As a result, these authors believe that Hotelling's model uses fixed prices in a duopoly setting, and analyzes the optimal locations for each firm.

These writers neglect the fact that Hotelling considered optimal pricing as well as optimal location. He assumed that each firm considered the other firm's behavior fixed only for that period. Therefore, if we consider reactions as Cournot-Nash behavior, then firms will adjust their prices depending on their competitor's resulting price under the assumption that locations are fixed. The adjustment process continues until optimal prices are obtained. When they consider the change in location, optimal prices will also change because the optimal prices are obtained under the assumption of fixed location of firms. As a result, firms tend to agglomerate in order to maximize profits. This is much different from the socially optimal market shape which minimizes the aggregated transportation costs.

Greenhut-Ohta type models assume that each firm takes the price at the market boundary (border price) as fixed.

Capozza and Van Order [1978] tried to generalize the three types of spatial competition models into one framework by representing their differences in the conjectural variations of firms. Thus, differences could be explained in a systematic way.

However, Capozza and Van Order did not actually expand the models, because they generalized them within their own framework of assumptions. They still assumed identical cost functions for firms, uniform population density, and a linear demand function; and only considered three specific forms of conjectural variation, based upon a partial equilibrium framework for the three models.

Spatial consideration has been mainly analyzed in order to determine the market area or shape that results from competition among firms under free entry. Even though a movement of location is much freer compared to time concerns, movement is difficult due to the existence of large transaction costs (e.g., fixed cost, sunk cost, restricted availability of locational choice, vested right for the old location, etc.). In other words, free entry is very restricted in some industries. Given the consideration of locational factors, the perfect competition concept is difficult to support.

Moreover, there has been very little attention given to the welfare implications. We usually see public facilities with large fixed or sunk costs that create an entry barrier. The purpose of those firms is not only to make profits, but also to increase social welfare. Thus, we need to analyze the welfare aspects of spatial model.

2. Purpose of Study

Our purpose is to attempt to generalize the spatial competition model of duopoly firms, such as in Hotelling, with entry barriers. As mentioned earlier, entry barriers for some industries are easily encountered due to large fixed cost, scale economies, and sunk cost, etc. For this reason, interest is focused on a duopoly case rather than a free entry case.

Unlike Capozza and Van Order, this study will use several assumptions for the cost function, population distribution function, and demand function. Specifically, the model assumes:

i) heterogeneous cost functions, rather than identical zero costs or linear cost functions.

ii) an arbitrary population distribution function, rather than uniform population density functions.

iii) general form for the demand function, derived from an analysis of consumer behavior, rather than a perfectly inelastic demand or a linear demand function.

In addition to these more generalized assumptions, we will also consider:

iv) spatial price competition between firms within a general equilibrium framework, rather than ignoring the spatial price competition, considering it within a partial equilibrium framework.

v) welfare implications for consumers by analyzing firm's social welfare maximization behavior and their profit maximization behavior,[3]

vi) comparison of efficiency between autonomous operation by each firm and joint operation of all firms by a single authority. From this, policy implications are implied. In other words, the optimal pricing policies for autonomously operated duopoly firms with fixed locations, considering profit maximization behavior and social welfare maximization behavior, will be compared with the results from joint operation of firms by single authority, such as the monopoly case.

In addition to optimal short-run pricing policy, we will try to determine the optimal expansion plan of the duopolists, as well as the optimal long-run pricing policies. When the industry expects increased demand, firms encounter the strategic problem of choosing between a price adjustment and an adjustment of the capital stock level. The optimal combination will be derived by using the calculus of variations. In addition to the capacity expansion plan, locational behavior of duopoly firms will be analyzed as another extension of the basic model.

3. Plan of Study

The literature on spatial competition models, such as Loschian type models, Hotelling-Smithies type models, and Greenhut-Ohta models, is reviewed in chapter two. In addition, Capozza and Van Order's generalized model will be reviewed, and a misinterpretation of Hotelling's will be detailed, too. In chapter three, we consider the optimal pricing policies for various objectives, such as profit maximization and social welfare maximization. The chapter begins with the basic assumptions, model description, an analysis on consumer's behavior, and the market situation. In the second section, profit maximizing spatial duopoly firms with entry barriers and fixed locations will be analyzed. The welfare considerations of spatial competition are then analyzed. After that, the optimal results of firms' behavior under profit maximization and under welfare maximization, operated by autonomous authority versus by single authority for joint operation of two firms, are compared. In chapter four, the investment plans of spatial duopoly firms, combined with pricing policy, will be analyzed as an extension of the basic model, and the locational considerations will be tried in chapter five as the other extension. Finally a summary and conclusions are provided in chapter six.

NOTES

1) Arrow and Debreu [1954] proved the existence of a competitive equilibrium by regarding the same goods transacted at different location as totally different goods. There were several attempts to extend Arrow-Debreu framework in a spatial context such as Isard[1957], Isard and Ostroff [1958], Harris and Nadji [1985], Mazzoleni and Montesano [1958], etc.

2) Capozza and Van Order pointed out that "The traditional model of spatial competition follows the assumption in location theory that firms set price as if they are monopolists within their market area (Edwin Mills and Michael Las [1964], Martin Beckmann [1970], Greenhut and Ohta [1975], John Hartwick [1973], Nicholas Stern [1972] etc.)."

3) Actually, industries such as public service facilities (e.g., seaport, railroad, utility, airlines, etc.) have entry barriers due to large fixed cost or economies of scale, and they may pursue people's welfare maximization as well as firm's profit maximization.

II. REVIEW OF LITERATURE ON SPATIAL COMPETITION MODELS

In many papers on spatial competition, models are classified into three categories: 1) Lõschian models, 2) Hotelling-Smithies models, 3) Greenhut-Hwang-Ohta models. This classification is based upon assumptions concerning either the boundary condition or the reactions of other firms. Capozza and Van Order [1979] tried to generalize these types of models by incorporating the assumptions into the conjectural variations.

1. Lõschian Models

August Lõsch, the father of modern location theory, originally studied the behavior of firms in space under free entry in his famous book, <u>The Economics of Location</u>, published in 1954 in English. He assumed that:

- i) consumers are evenly distributed over an unbounded featureless plain.
- ii) individual demand is a function of delivered price, which is defined by the mill price plus the transportation cost from factory to the consumer.
- iii) firms are considered as spatial monopolists, and they maximize profits accordingly.

iv) the product of firms is differentiated only by distance from the factory to the consumer.

v) entry and exit by firms are free.

Due to free entry by firms, entry continues until all the spatial monopoly profits are eliminated. New competitors will enter the industry to pursue any excess profit. As this occurs, the demand curve faced by existing firms shifts to the left, as their market area shrinks, and profits are reduced. Market equilibrium occurs when there is no motive to enter or exit, i.e. when all firms are earning a zero profit. This is essentially Chamberlain's model of monopolistic competition--with spatial separations providing the product differentiation. Therefore, equilibrium is established when the demand curve facing each firm has shifted to a position of tangency with the long-run average cost curve.

As a result, the whole market area is filled with evenly distributed firms, each of which has a hexagonal market shape, like a bee hive. There is no price competition and no change in boundary, because each firm behaves in the same fashion as the other firms.

However, the Löschian model provides implications for many spatial phenomena, including retail hierarchies and market configuration, Berry[1967]; welfare effects, Mills and Lav [1964], Denike and Parr[1970]; and the effect of transportation cost on market size, Hoover [1970], etc.

2. Hotelling-Smithies models

Hotelling [1929] tried to show the stability in spatial competition equilibrium and to determine the optimal locational behavior of profit-maximizing duopolists. He assumed:

i) consumers are uniformly distributed along a linear market area.

ii) consumers pay the transportation cost.

iii) the transportation cost per unit of product per unit of distance is constant.

iv) production costs for duopoly firms are zero.

v) demand is extremely inelastic and equal to one.

vi) no buyer has any preference for any particular seller except on the basis of delivered price.

vii) each firm behaves as if the other's price is fixed.

Under these conditions, he found profit maximizing price as a function of location; then determined the optimal location by substituting the optimal price into the condition of profit maximizing location. By considering the other firm's behavior simultaneously, Hotelling found the optimal locations, optimal prices, and the market boundary explicitly. The duopolists tend to gather in the center of the linear market--which differs from the solution to the minimization of social transport cost problem.

Lerner and Singer [1937] used slightly different demand assumptions from Hotelling's and tried to show the conditions under which Hotelling's result--tendency to aggregate of spatial firms--will be true. They assumed that there will be an upper limit on "demand price", up to which consumers will have a willingness to purchase the good. That is, consumers who live far enough away from the firm such that the delivered price exceeds the demand price will not purchase the good. Thus there is a perfectly inelastic demand up to the demand price, and a perfectly elastic zero demand above the demand price.

Smithies [1941] further developed the Lerner-Singer's modification by employing linear demand, with different elasticities at every point below the "demand price." This was one of the attempts to generalize the theory of spatial competition.

The Hotelling-Smithies type model is often used in research, as it is regarded as being more realistic than the others in that it deals with competitive reactions of duopoly firms. Many studies use Hotelling's model to test firms' locational behavior or to extend the spatial competition model. However, they tend to misinterpret Hotelling's work as one looking for optimal locations of duopoly firms under fixed prices. This tendency seems to be caused by the fact that most of the applied models are partial equilibrium analyses. They use a zero conjectural variation of the rival firm, i.e. the rival does not change its price during the period analyzed. However, the rival also assumes a zero conjectural variation of the other firm when maximizing profit under the assumption of Cournot behavior. Thus, the optimal prices of each duopolist depend upon their location--and optimal locations depend upon their prices. By solving these relations simultaneously, we can find the optimal locations and optimal prices of each firm.

As noted earlier, Hotelling first assumed fixed locations, and found optimal prices as a function of locations. He then found the optimal locations by substituting the optimal prices into optimality conditions for location. This provided optimal prices and locations, and, by default, the boundary. He could find these solutions explicitly because he considered the simplest case such as uniform population distribution, zero production cost, perfectly inelastic demand, etc. Therefore, Hotelling's model can be regarded as the simplest version of a general equilibrium model of spatial competition in a duopoly.

3. Greenhut-Ohta models

This model was originally designed to compare market sizes and shapes under spatial competition with free entry. It assumes:

i) buyers are evenly distributed over space.

ii) each buyer has the same linear demand curve with a unitary slope.

iii) marginal cost of production is zero.

iv) transportation cost per unit per mile is of a unit rate.

Greenhut-Ohta considers several kinds of market shapes, such as triangle, square, hexagon, and circle, but it does not consider rivals' behavior. The firm's delivered price at its market boundary is taken as fixed. In other words, the maximum price at the firm's boundary points is parametrically given. They stress Losch's theory that hexagonal market areas provide greater demands than the circle even under the condition of spatial competition, but they dispute Mills and Lav's claim that a circular market area of given size provides greater profits under competition than would a hexagon of the same size.

4. Capozza and Van Order's model

Capozza and Van Order [1978] tried to establish a generalized model of spatial competition by analyzing the assumption of conjectural variations from the above models. They expected the following to be characteristics of a reasonable spatial model:

1) As transport costs approach zero, perfect competition should be approached and price should approach marginal cost.
2) As fixed costs approach zero, concentrated production is less essential, spatial monopoly

power is diminished, and again, price should approach marginal cost.

3) As costs (fixed, marginal, or transportation) rise, price should rise.

4) As demand density rises, firms should be able to take advantage of economies-of-scale, and price should fall in the long-run.

5) As more firms enter the industry, competition should increase and price should fall.

In order to check these characteristics in the three models, their assumptions are generalized as follows:

i) firms have identical cost functions in a linear functional form.

ii) transport costs per mile are identical.

iii) consumers are evenly distributed in an unbounded plain, and have identical linear demand curves based upon delivered price.

iv) market area is circular, with free entry.

v) various conjectural variations of competing firms are used, according to the typical types of models.

Under these assumptions, they derived the equilibrium price and circular market radius according to each model's conjectural variation. In the Löschian models, conjectural variation is one, (i.e. $d\bar{p}/dp = 1$, where p stands for price of the firm and $\bar{p}$ stands for price reaction of the other firm), as Loschian assumed identical reactions by other firms. The conjectural variation is zero ($d\bar{p}/dp = 0$) for Hotelling-Smithies model since this was their assumption. The conjectural variation is minus one ($d\bar{p}/dp = -1$) for Greenhut-Ohta model, since this must hold in order to keep the market boundary price fixed.

The Löschian model exhibits all the perverse characteristics expected. Capozza and Van Order view the Loschian model as an extreme case defined by the price reaction of firms, and as a more acceptable model of collusive oligopoly. Hotelling-Smithies model does not satisfy the criteria they expected. However, in the absence of collusion, the zero conjectural variation assumption would appear to be a suitable approximation. Greenhut-Ohta model has the advantage that it does give plausible and unambiguous comparative statics results, and it handles the extreme cases properly.[1] However, in the context of spatial competition with mill pricing, the implied negative price reaction seems unlikely.

Capozza and Van Order have developed an imperfect competition model that shows that the perverse results appear under relatively unusual circumstances. This provides a convenient way of interpreting imperfect competition models as an outgrowth of spatial consideration.

Capozza and Van Order only extended the three types of models by adding non-zero identical cost functions, linear demand curves, and various conjectural variations. They limited themselves by generalizing the models within their own framework of assumptions. In particular, they analyzed the spatial competition models within a partial equilibrium framework rather than a general equilibrium framework, and they did not address the welfare issue raised by the lack of perfect competition.

NOTE

1) Greenhut-Ohta model was developed in the context of zonal pricing by competitors rather than by mill pricing.

III. OPTIMAL PRICING OF DUOPOLY FIRMS UNDER SPATIAL COMPETITION

As we have seen in the previous chapter Hotelling-Smithies type models pay much attention to the reaction behavior of rival firms. The Hotelling-Smithies models dealt with price competition under several simplified assumptions such as inelastic demand, uniform population density, and zero production cost. Due to these assumptions, Hotelling could provide many implications for the locational behavior of firms. In this chapter, we will try to generalize the assumptions on demand, population distribution, production function, and cost function in order to study the spatial duopoly firms' behavior under various objectives, and compare the efficiency of autonomous versus joint operation of duopoly firms.

We will look at the optimality pricing conditions when the firms pursue profit maximization and social welfare maximization objectives under the presence of spatial competition. We will begin by analyzing individual consumer behavior to find the individual demands for the good produced by these duopoly firms. The market situation will be assessed when aggregating the market demands faced by each firm.

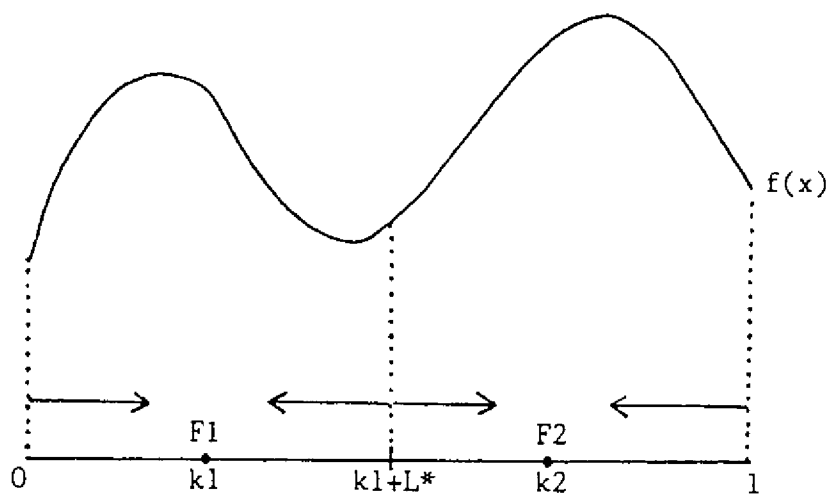
1. Model Description

Consider a linear country scaled 0 to 1 with two existing firms. Assume that consumers are identical except for their residential location, which is fixed with an arbitrary population density $f(x)$ for any location x in the range of [0,1]. Denote the locations of firms F1 and F2 by k1 and k2, respectively, as shown in [Figure 1].

Assuming that the products are equivalent, consumers will buy the good or service supplied by the nearest firm unless there exists a sufficient difference in prices. Effective consumer prices may differ due to either transportation costs from the factory to the market or travel costs from the residence to the firm. Consumers residing at any location in [0, k1] will purchase the good produced by the firm F1 and consumers residing at any location in [k2, 1] will purchase the good produced by the firm F2.

In the range of [k1, k2], we can find the location k1+L* where consumers are indifferent between buying the good from either F1 or F2. We can regard this location as the market boundary between the two firms. The location of k1+L* depends upon the factory price of each firm. In particular, k1+L* tends to move toward F2 when the factory price of F1 (denoted by p1) decreases relative

[Figure 1]. Firms' Location and Population Density



to the factory price of F2 (denoted by p2). Likewise, K1+L* tends to move toward F1 when p2 decreases relative to p1.[1)]

1) Consumer's Behavior

Assume that consumers are identical except for their residential locations, and that there are only two kinds of goods, one is produced by these duopoly firms (q) and the other is a composite of all other goods($\bar{q}$). Consumers will choose (q, $\bar{q}$) so as to maximize utility within their income constraints. Let us formulate consumer behavior as follows:

$$\max \quad U(q,\overline{q})$$
$$\text{s.t.} \quad p'q + \overline{q} = y$$

p' is the delivered price ($p' = p + cd$), which is defined as the sum of mill price(p) and transportation cost(cd), where c is transportation cost per unit distance per unit quantity of output, and d is distance from the firm to the residence, consumer's income is y, U is a utility function, and $\overline{q}$ is numeraire. The Lagrangian function can be written as

$$L(q, \overline{q}, \sigma; p', y) = U(q,\overline{q}) + \sigma\,(y - p'q - \overline{q})$$

where σ is the Lagrangian multiplier, and the first order conditions are

$$\delta L/\delta q = \delta U/\delta q - \sigma p' = 0 \qquad (1)$$
$$\delta L/\delta \overline{q} = \delta U/\delta \overline{q} - \sigma = 0 \qquad (2)$$
$$\delta L/\delta \sigma = y - p'q - \overline{q} = 0\ . \qquad (3)$$

Solving the F.O.C. simultaneously yields individual demand as a function of delivered price and income.

$$q^* = q(p',y), \quad \bar{q}^* = \bar{q}(p',y), \quad \sigma^* = \sigma(p',y) \tag{4}$$

Therefore, the indirect utility becomes

$$V(p',y) = U\,[q^*,\bar{q}^*] = U\,[q(p',y),\ \bar{q}(p',y)] \tag{5}$$

with $\delta V/\delta p' = -\sigma q < 0$ and $\delta V/\delta y = \sigma > 0$.[2] The maximum utility level decreases as the delivered price increases, and increases as income increases. We can then derive the following relations[3]

$$\delta V/\delta p = -\sigma q < 0 \tag{6-1}$$

$$\delta V/\delta c = -\sigma q d < 0 \tag{6-2}$$

$$\delta V/\delta d = -\sigma q c < 0 \tag{6-3}$$

which show the utility decreases as the mill price increases, as the unit transportation cost increases, or as the consumer lives farther away from the firm. Since our main interest is on the optimal pricing of duopoly firms, we will no longer consider the effect of a change in consumer income. Thus, we will denote the demand function as $q(p')$ rather than $q(p',y)$.

2) Market Situation

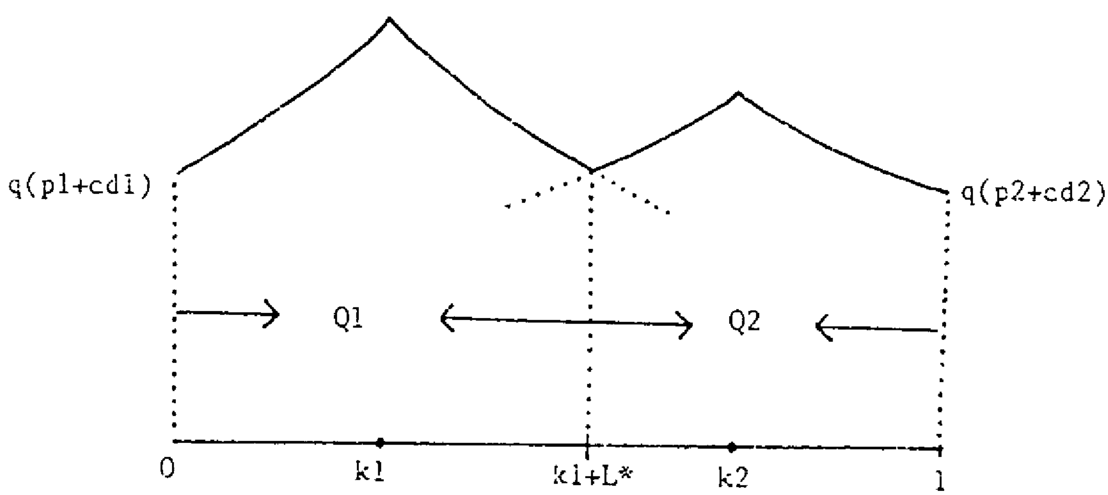
Consumers residing at a certain location x within a range of $[0, k1+L^*]$ will purchase the good (q) produced by the firm F1 in the amount of $q(p') = q(p1+c|k1-x|)$, where $|k1-x|$ is the distance from x to F1. Consumers residing at x within $[k1+L^*,1]$ will purchase the good (q) produced by the firm F2 in the amount of $q(p') = q(p2+c|k2-x|)$, where $|k2-x|$ is the distance from x to F2.

At the market boundary $k1+L^*$, consumers can purchase the amount $q(p') = q(p1+cL^*) = q[p2+c(k2-k1+L^*)]$ from either F1 or F2. Thus, the delivered prices are equal at $k1+L^*$: $p1 + cd1 = p2 + cd2$, where (d1, d2) represent the distance from each firm to the boundary. By solving the relation $p1+cL^* = p2+c(k2-k1-L^*)$, we can find the value of L^*.

$$L^* = [p2-p1+c(k2-k1)] / 2c . \tag{7}$$

Therefore, the market demands for firms F1 and F2, (Q1, Q2), are derived by the aggregation of individual demands at each location weighted by the population density, $f(x)$, over the ranges of $[0, k1+l^*]$ and $[k1+l^*, 1]$, respectively.[4]

[Figure 2] Individual Demand (q) and Market Demand (Q)



$$Q1(p1,p2) = \int_{o}^{k1+L*} q(p1+c\,|k1-x|\,)\ f(x)\ dx \tag{8}$$

$$= \int_{o}^{k1} q(p1+ck1-cx)\ f(x)\ dx$$

$$+ \int_{k1}^{k1+L*} q(p1+cx-ck1)\ f(x)\ dx$$

$$Q2(p1,p2) = \int_{k1+L*}^{1} q(p2+c\,|k2-x|\,)\ f(x)\ dx \tag{9}$$

$$= \int_{k1+L*}^{k2} q(p2+ck2-cx)\ f(x)\ dx$$

$$+ \int_{k2}^{1} q(p2+cx-ck2)\ f(x)\ dx$$

The derived market demands for each firm are functions of mill prices p1 and p2, because the L* in market boundary depends upon mill prices. We can then derive the following relations.

$$\delta Q1/\delta p1 = \int_{o}^{k1+L^*} (\delta q/\delta p1)\ f(x)\ dx$$

$$- (1/2c)\ q[(p1+p2)/2+c(k2-k1)/2]\ f(k1+L^*)$$

$$< 0 \qquad (10\text{-}1)$$

$$\delta Q1/\delta p2 = (1/2c)\ q[(p1+p2)/2+c(k2-k1)/2]\ f(k1+L^*)$$

$$> 0 \qquad (10\text{-}2)$$

$$\delta Q2/\delta p2 = \int_{k1+L^*}^{1} (\delta q/\delta p2)\ f(x)\ dx$$

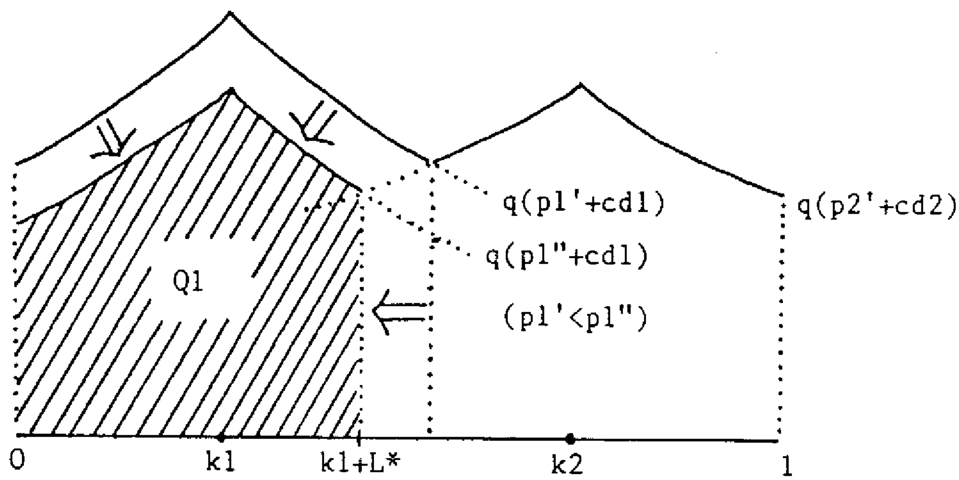
$$- (1/2c)\ q[(p1+p2)/2+c(k2-k1)/2]\ f(k1+L^*)$$

$$< 0 \qquad (10\text{-}3)$$

$$\delta Q2/\delta p1 = (1/2c)\ q[(p1+p2)/2+c(k2-k1)/2]\ f(k1+L^*)$$

$$> 0 \qquad (10\text{-}4)$$

[Figure 3] Change in Market Demand Caused By Price Change



These equations show that market demand faced by each firm decreases as the firm increases its mill price, or as the rival decreases its mill price. A change in mill price will affect the individual demand within a market region as well as the market boundary. These then lead to a change in the aggregate demand facing each firm. For example, when the demand schedule of consumers residing at any location between [Ø,1] with mill prices (p1, p2) is q(p'=p+cd), as shown in [Figure 3], then F1's market demand will be decreased, and the market boundary will shift toward F1 if p1 is increased relative to p2.

2. Profit Maximizing Firms' Behavior

Let us analyze the firms' behavior based on the market demands derived in the previous section. We will consider firm's objective to be profit maximization in this section, while welfare objectives will be treated in the next section. We need firms' profit functions in order to find their optimal pricing policies under spatial competition. Using the market demands and assuming heterogeneous implicit cost functions, the profit functions are found:

$$\pi 1(p1,p2) = p1\ Q1(p1,p2) - C1[Q1(p1,p2)] \qquad (11)$$

$$\pi 2(p1,p2) = p2\ Q2(p1,p2) - C2[Q2(p1,p2)] \qquad (12)$$

where $\pi 1$, $\pi 2$: profit functions of each firm

p1, p2 : mill prices of firms

Q1, Q2 : market demand functions

C1, C2 : cost functions.

In this framework, each firm's behavior is directly affected by its rival's behavior. This occurs because the demand facing each firm is related to the other firm's pricing policy, through its effect on the market boundary. We assume Cournot-Nash behavior by firms in their price competition. In other words, each firm determines its price under the assumption that the price of the competitor is fixed at its optimal level. Now we will consider each firm's optimization behavior.

1) Firm F1's Behavior

When its objective is to maximize profit, we can formulate firm F1's behavior as follows:

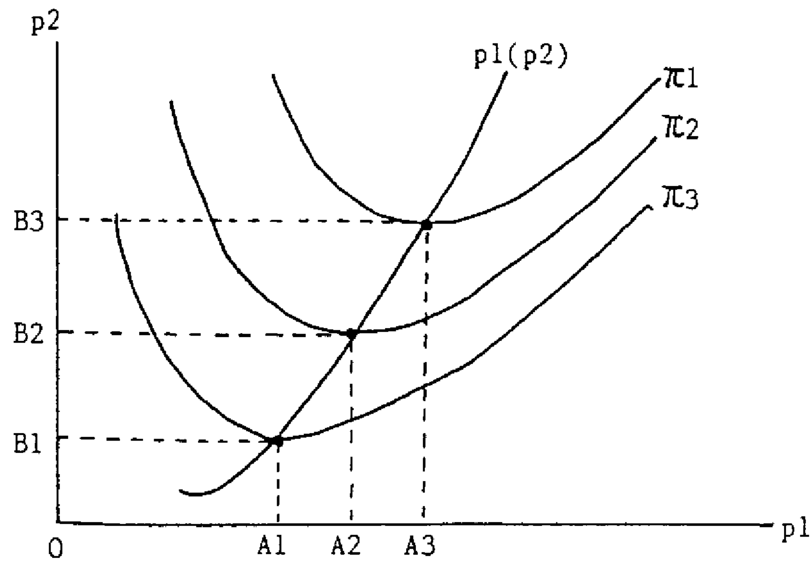
$$\max \quad \pi 1 = p1\ Q1(p1,p2) - C1[Q1(p1,p2)].$$

Partial derivation of $\pi 1$ with respect to p1 yields the following first order condition.

$$\begin{aligned} \delta\pi 1/\delta p1 &= Q1(p1,p2) + p1\ (\delta Q1/\delta p1) \\ &\quad - (\delta C1/\delta Q1)(\delta Q1/\delta p1) \\ &= Q1(p1,p2) + (p1 - MC1)\ (\delta Q1/\delta p1) \\ &= 0 \end{aligned} \tag{3}$$

Solving this equation leads to F1's reaction function, which is dependent upon firm F2's behavior, p1=p1(p2), as shown in [Figure 4]. We can draw U-shaped iso-profit curves for firm F1 by considering the F1's profit

[Figure 4] F1's Reaction Curve under Profit Maximization



functions corresponding to various fixed levels of p2.[5] For the various profit levels, $\pi_1 < \pi_2 < \pi_3$, we can draw corresponding U-shaped iso-profit curves as shown in [Figure 4], where profits increase in an upward direction. The optimal pricing of F1 occurs at the lowest point on an iso-profit curve for any given fixed p2. For example, when F2's price is fixed at B1, then F1 will choose A1, where it can get maximum level of profit π_1 under the situation of p2=B1, and when F2's price is determined at B2, then F1 will choose A2 in order to get maximum level of profit π_2, and so forth. The loci of these optimal prices forms the reaction function of F1, p1=p1(p2), which is identical to the one derived from the first order condition in equation (3).

2) Firm F2's Behavior

Now, consider firm F2's behavior. Similarly to the case of F1, F2's problem is

$$\max \quad \pi 2 = p2\ Q2(p1,p2) - C2[Q2(p1,p2)] \ .$$

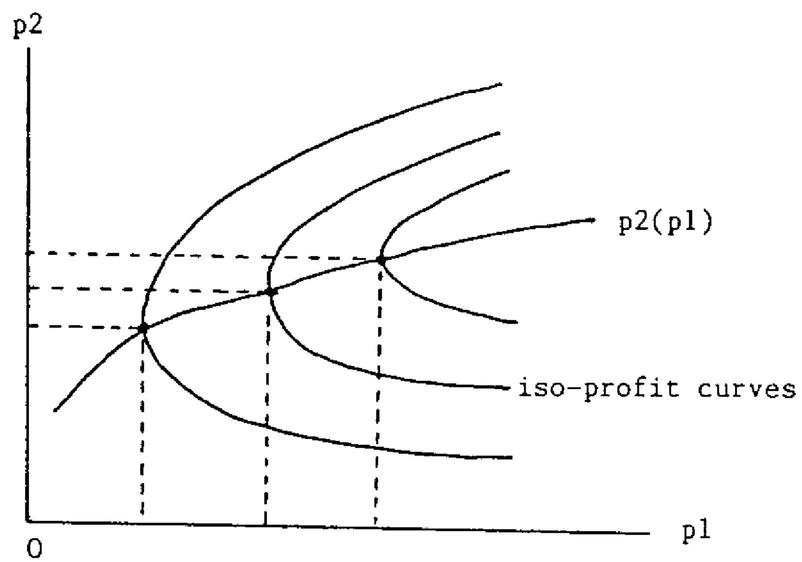
The optimality condition becomes

$$\begin{aligned} \delta\pi 2/\delta p2 &= Q2(p1,p2) + (p2-MC2)\ \delta Q2/\delta p2 \\ &= 0 \ . \end{aligned} \tag{14}$$

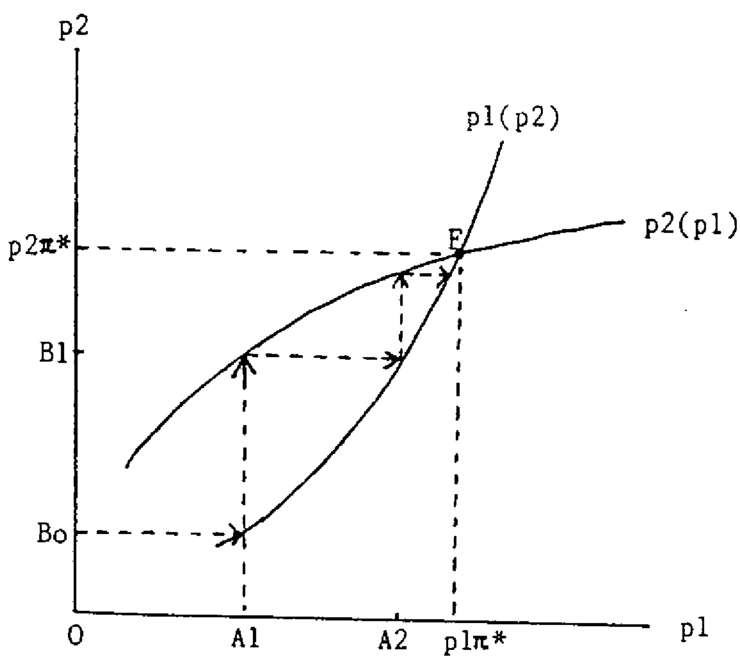
The reaction function for firm F2, p2=p2(p1), will be derived by solving this first order condition. [Figure 5] shows this result, which is similar to [Figure 4].

When we analyze the duopolists' behavior at the same time (i.e., solving the optimality conditions, equations (13) and (14), simultaneously), we obtain the optimal price for each firm (p1*, p2*). From the reaction curves, p1=p1(p2) and p2=p2(p1), in [Figure 4] and

[Figure 5] F2's Reaction Curve under Profit Maximization



[Figure 6] Price Reaction Process



[Figure 5], we can find the optimal prices (p1*, p2*) as shown in [Figure 6]. For example, assume that p2=Bo in [Figure 6]. Firm F1 will choose its price, p1=A1, according to its reaction function p1(p2)=p1(Bo)=A1. When F1 sets its price at p1=A1, then firm F2 will choose p2=B1 according to its reaction function p2(p1)=p2(A1)=B1. F1 will then choose p1=A2 under the situation of p2=B1. This price reaction process will continue until a Cournot-Nash equilibrium point E is reached, where p1* = p1(p2*) and p2* = p2(p1*).

3. Welfare Maximizing Firms' Behavior

In the previous section, we looked into firms' profit maximization behavior. We now consider the case where the firms' objective is to maximize social welfare, and compare the behavioral differences resulting from these different objectives.

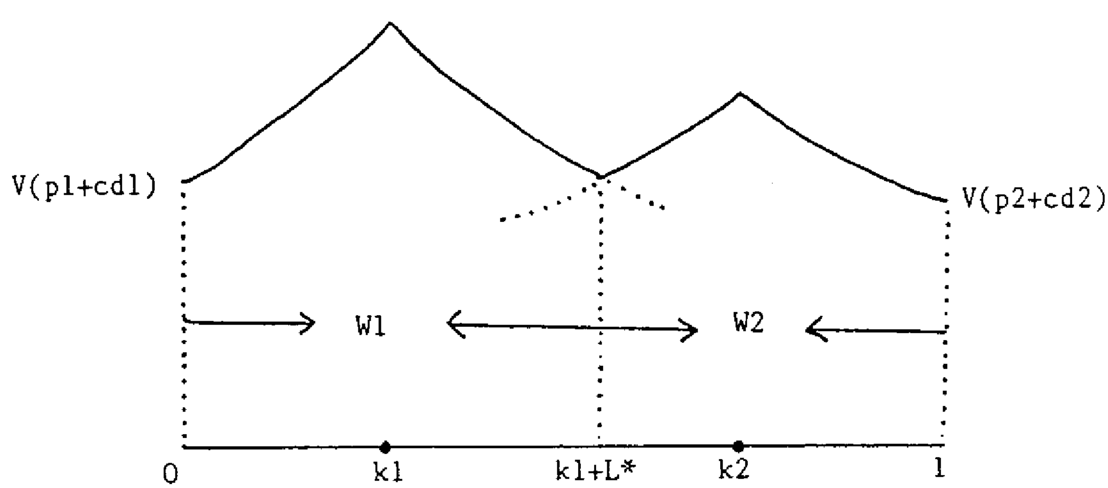
As derived in section 1, a consumer's indirect utility V(p') is dependent upon the delivered price of the good under the assumption of fixed income. Increases in either mill price, unit transportation cost, or distance from the firm to the residence will cause decreases in maximum utility as shown in equation (6). When we define the social welfare function faced by each firm as the aggregated individual indirect utility level at each location over a specified market region, then the social welfare function considered by each firm becomes

$$W1(p1,p2) = \int_{o}^{k1+L^*} V(p1+cd1)\ f(x)\ dx \qquad (15)$$

$$W2(p1,p2) = \int_{k1+L^*}^{1} V(p2+cd2)\ f(x)\ dx \qquad (16)$$

where V(p') represents the indirect utility level of the consumers residing at each location, as defined in equation (5) and shown in [Figure 7]. Since the market

[Figure 7] Indirect Utility and Social Welfare



boundary (k1+L*) varies with prices, social welfare functions are dependent upon firms' prices. We can then derive the following relations between the welfare level and firms' prices:

$$\delta W1/\delta p1 = \int_{o}^{k1+L*} (\delta V/\delta p1) f(x)dx$$

$$- (1/2c)V[(p1+p2)/2+c(k2-k1)/2]\ f(k1+L*)$$

$$= - \int_{o}^{k1+l*} \sigma(p')\ q(p')\ f(x)\ dx$$

$$-(1/2c)\ V[(p1+p2)/2+c(k2-k1)/2]\ f(k1+L*)$$

$$< \emptyset \qquad (17\text{-}1)$$

$$\delta W1/\delta p2 = (1/2c)\ V[(p1+p2)/2+c(k2-k1)/2]\ f(k1+L*)$$

$$> \emptyset \qquad (17\text{-}2)$$

$$\delta W2/\delta p2 = \int_{k1+L*}^{1} (\delta V/\delta p2)\ f(x)\ dx$$

$$- (1/2c)\ V[(p1+p2)/2+c(k2-k1)/2]\ f(k1+L*)$$

$$= - \int_{k1+L*}^{1} \sigma(p')\ q(p')\ f(x)\ dx$$

$$- (1/2c)\ V[(p1+p2)/2+c(k2-k1)/2]\ f(k1+L^*)$$

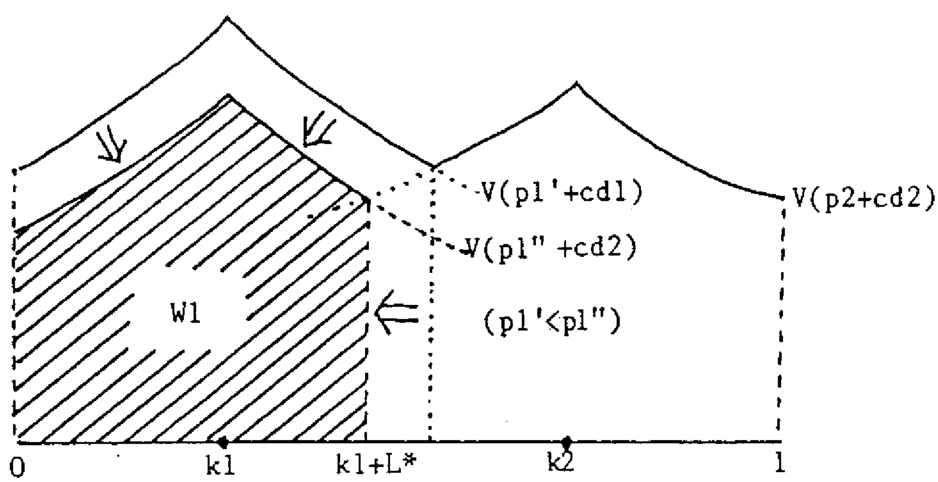
$$< 0 \qquad (17\text{-}3)$$

$$\delta W2/\delta p1 = (1/2c)\ V[(p1+p2)/2+c(k2-k1)/2]\ f(k1+L^*)$$

$$> 0 \qquad (17\text{-}4)$$

These relations show that a change in aggregate consumer welfare results in two ways from a change in price. An increase in a firm's price reduces the individual demand

[Figure 8] Change in Social Welfare Caused by Price Change



$q(p')$ within its market region due to $(\delta q/\delta p') < 0$; and it increases the firm's relative price, causing shrinkage in its market region. For example, when the indirect utility of consumers residing at any location between $[0,1]$ with mill prices $(p1,p2)$ is $V(p')$ as in [Figure 8], and p1 is increased relative to p2, then social welfare provided by firm F1 will be decreased and the market boundary will shift toward F1.

1) Firm F1's Behavior

We now analyze the firm's behavior based on the definitions and characteristics of the social welfare functions derived above. The behavior of firm F1 can be formulated as

$$\max \; W1(p1,p2)$$
$$\text{s.t. } \pi 1(p1,p2) \geq 0$$

where $W1(p1,p2)$ is the social welfare function faced by F1, as defined in equation (15), and $\pi 1(p1,p2)$ is its profit function, as defined in equation (11). The Lagrangian function is

$$L1(p1,\alpha 1;p2) = W1(p1,p2) + \alpha 1\ \pi 1(p1,p2)$$

where $\alpha 1$ stands for the Lagrangian multiplier. The Kuhn-Tucker conditions are

$$\delta L1/\delta p1 = \delta W1/\delta p1 + \alpha 1\ \delta\pi 1/\delta p1 \leq 0 \quad \text{and } p1(\delta L1/\delta p1)=0$$

$$\delta L1/\delta\alpha 1 = \pi 1(p1,p2) \geq 0 \quad \text{and} \quad \alpha 1(\delta L1/\delta\alpha 1)=0\ .$$

The optimality conditions then become[6]

$$\delta W1/\delta p1 + \alpha 1(\delta\pi 1/\delta p1) = 0 \quad \text{and } p1 > 0 \tag{18-1}$$

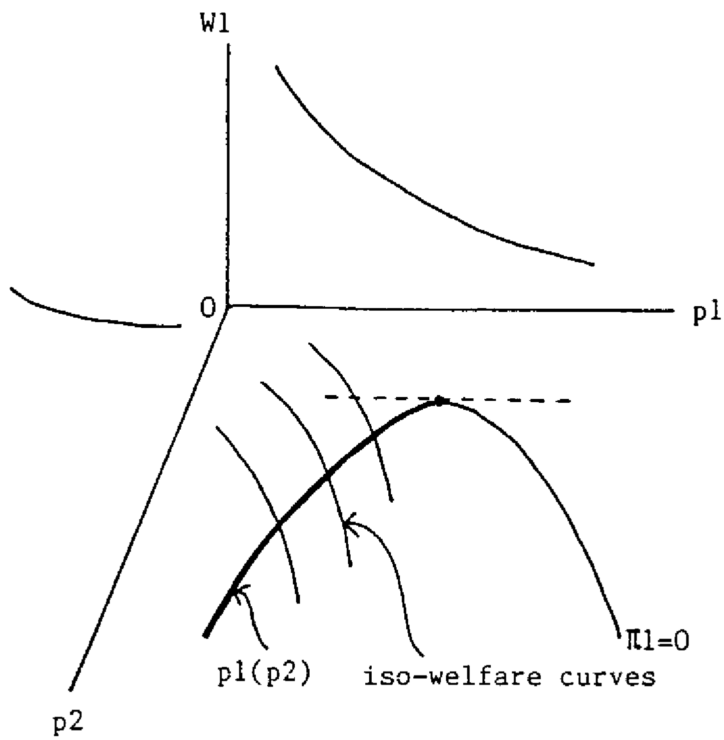
$$\pi 1(p1,p2) = 0 \quad \text{and } \alpha 1 > 0. \tag{18-2}$$

From equation (18-2), we can derive the reaction function of F1, p1(p2), which shows its iso-profit curve for a zero profit. By evaluating equation (18-1) at p1(p2), we reach the solution

$$\alpha 1(p2) = -\ (\delta W1/\delta p1)/(\delta\pi 1/\delta p1).$$

Drawing the iso-profit curve corresponding to a zero profit, we see that the effective reaction curve for firm F1 is the negatively sloped portion of p1(p2). This is represented by the thick line in [Figure 9], since the slope of iso-welfare curve is positive.[7]

[Figure 9] F1's Reaction Curve under Welfare Maximization



2) Firm F2's Behavior

We must analyze firm F2's social welfare maximization behavior simultaneously--due to the price competition between the firms. As in the case of firm F1, we can formulate firm F2's social welfare maximization problem as

$$\max \; W2(p1,p2)$$
$$\text{s.t. } \pi2(p1,p2) \geq 0$$

where W2(p1,p2) represents the social welfare function faced by firm F2, as defined in equation (16), and π2(p1,p2) is its profit function, as defined in equation (12). The Lagrangian equation is then

$$L2(p1,p2,\alpha2) = W2(p1,p2) + \alpha2 \; \pi2(p1,p2).$$

The Kuhn-Tucker conditions are

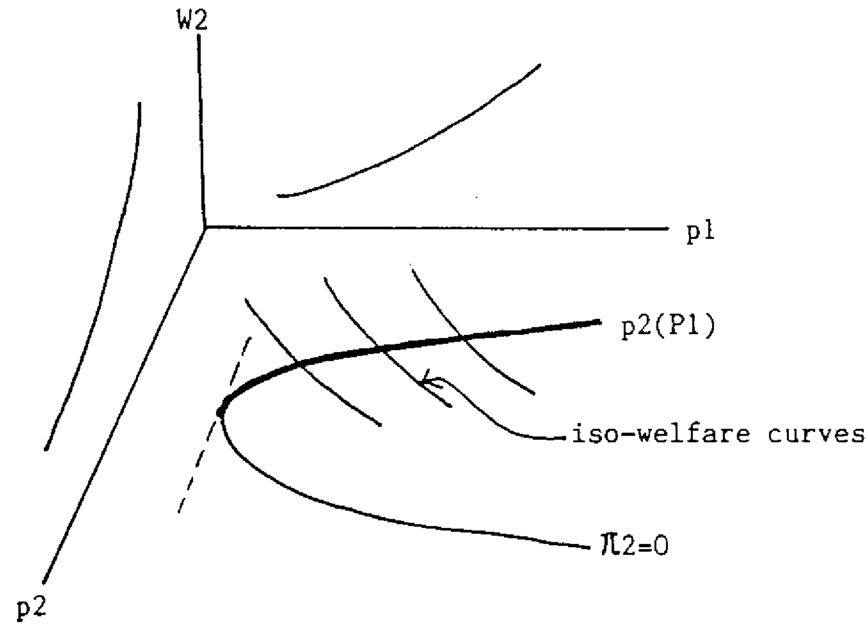
$$\delta L2/\delta p2 = \delta W2/\delta p2 + \alpha2 \; (\delta\pi2/\delta p2) \leq 0 \text{ and } p2(\delta L2/\delta p2)=0$$
$$\delta L2/\delta\alpha2 = \pi2(p1,p2) \geq 0 \text{ and } \alpha2(\delta L2/\delta\alpha2) = 0,$$

and the optimality conditions are

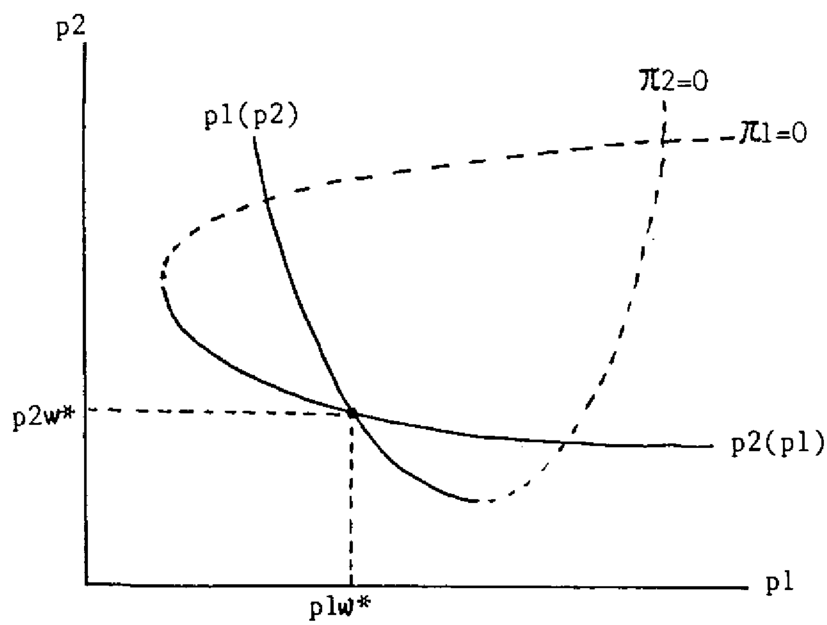
$$\delta W2/\delta p2 + \alpha2(\delta\pi2/\delta p2) = 0 \quad \text{and} \quad p2 > 0 \tag{19-1}$$
$$\pi2(p1,p2) = 0 \text{ and } \alpha2 > 0. \tag{19-2}$$

Solving equation (19-2) results in the reaction curve for firm F2, p2(p1), which simply shows its iso-profit curve for a zero profit level of firm F2. Evaluating equation (19-1) at p2(p1) yields the Lagrangian multiplier as a function of p1, $\alpha2(p1) = -(\delta W2/\delta p2) / (\delta\pi2/\delta p2)$.

[Figure 10] F2's Reaction Curve under Welfare Maximization



[Figure 11] Optimal Prices under Welfare Maximization



[Figure 10] shows the reaction curve and iso-welfare curves for firm F2.

Finally, optimal prices ($p1w^*$,$p2w^*$) for social welfare maximization are obtained by solving the reaction functions for both firms simultaneously. The intersection of these reaction curves shows this solution graphically, see [Figure 11]. Evaluating $\alpha1(p2)=-(\delta W1/\delta p1)/(\delta\pi1/\delta p1)$ and $\alpha2(p1)=-(\delta W2/\delta p2)/(\delta\pi2/\delta p2)$ at ($p1w^*$, $p2w^*$) yields the solutions for $\alpha1^*$ and $\alpha2^*$.

To see the firms' behavioral differences under profit maximization and social welfare maximization, we compare firm F1's optimality condition in equation (13) with that in equation (18).

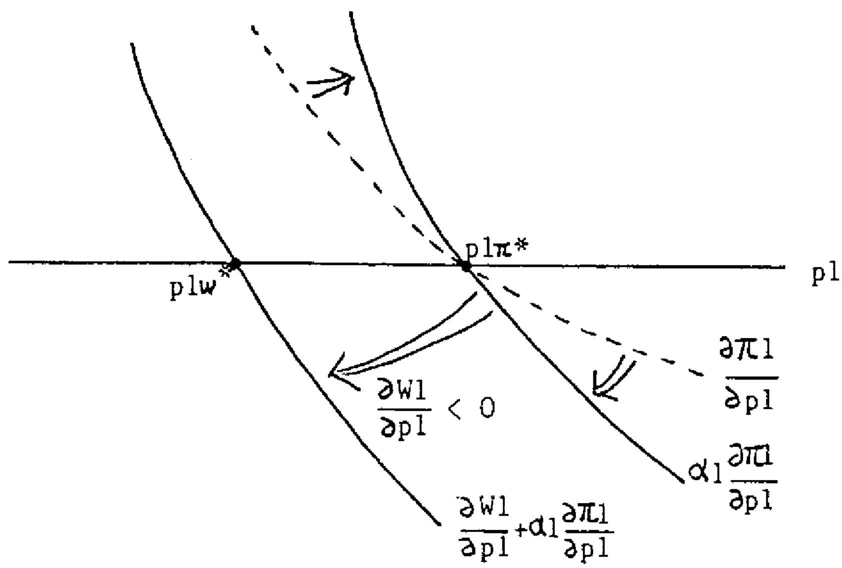
$$\delta\pi1/\delta p1 = 0 \tag{13}$$

$$\delta W1/\delta p1 + \alpha1(\delta\pi1/\delta p1) = 0 \tag{18}$$

Define $p1\pi^*$ as the solution for profit maximizing behavior from equation (13), and $p1w^*$ as the solution for social welfare maximizing behavior from equation (18).

Since $\delta W1/\delta p1<0$, from equation (17-2), we can conclude that $p1w^* < p1\pi^*$, as reflected in [Figure 12]. The profit maximizing price, $p1\pi^*$, is found by solving the condition of $\delta\pi1/\delta p1 = 0$. Multiplying this by $\alpha1>0$ will

[Figure 12] Comparison of Optimal Prices of Profit Maximization versus Welfare Maximization



not affect this price. That is, p1π* is also the solution to $\alpha 1(\delta\pi 1/\delta p1)=0$. Adding $\delta W1/\delta p1$, which is negative, will shift the curve $\alpha 1(\delta\pi 1/\delta p1)$ down. Since p1w* is the solution to $\delta W1/\delta p1 + \alpha 1(\delta\pi 1/\delta p1) = 0$, we can conclude that p1w* is lower than p1π*. Thus, the social welfare maximizing price for firm F1 is always smaller than the profit maximizing price--assuming a fixed price by the rival firm.

Similarly, we compare firm F2's optimality condition in equation (14) with that in equation (19).

$$\delta\pi 2/\delta p2 = 0 \qquad (14)$$

$$\delta W2/\delta p2 + \alpha 2(\delta\pi 2/\delta p2) = 0 \qquad (19)$$

We see that the solution to equation (14), denoted by $p2\pi^*$, is higher than the solution to equation (19), denoted by $p2w^*$. Therefore, the profit maximizing price for firm F2 is always higher than the social welfare maximizing price--assuming a fixed price by the rival firm.

4. Comparison of Autonomous versus Joint Operation of Duopoly Firms

In this section, we study the joint operation of spatial duopoly firms under the objectives of profit maximization and social welfare maximization. Our goal is to analyze the behavioral differences between autonomous operation and joint operation of duopoly by a single authority.

1) Profit Maximizing Firms

When profit maximizing firms are jointly operated, the authority's objective is to maximize profit across all firms.

$$\max \quad \pi(p1,p2) = \pi1(p1,p2) + \pi2(p1,p2)$$

The first order conditions are

$$\delta\pi/\delta p1 = \delta\pi1/\delta p1 + \delta\pi2/\delta p1 = 0 \qquad (20\text{-}1)$$

$$\delta\pi/\delta p2 = \delta\pi1/\delta p1 + \delta\pi2/\delta p2 = 0. \qquad (20\text{-}2)$$

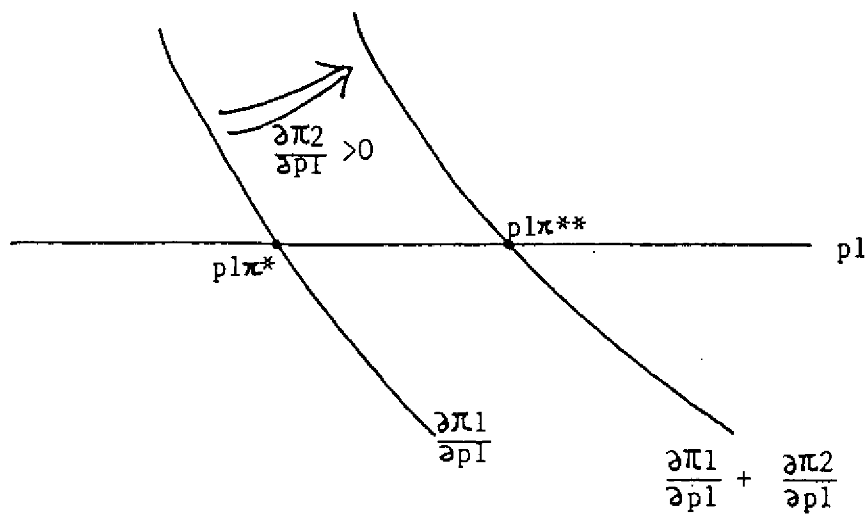
Solving these equations simultaneously, we find the optimal prices $p1\pi^{**}$ and $p2\pi^{**}$. A "**" denotes joint operation, whereas "*" denotes autonomous operation by each firm.

Let us now compare these optimality conditions with those for autonomous operation. From equations (11) and (12), we can derive the following relationships.

$$\begin{aligned}\delta\pi2/\delta p1 &= p2(\delta Q2/\delta p1) - MC2(\delta Q2/\delta p1)\\ &= (p2-MC2)(\delta Q2/\delta p1) > 0 \qquad (21\text{-}1)\end{aligned}$$

$$\begin{aligned}\delta\pi1/\delta p2 &= p1(\delta Q1/\delta p2) - MC1(\delta Q1/\delta p2)\\ &= (p1-MC1)(\delta Q1/\delta p2) > 0 \qquad (21\text{-}2)\end{aligned}$$

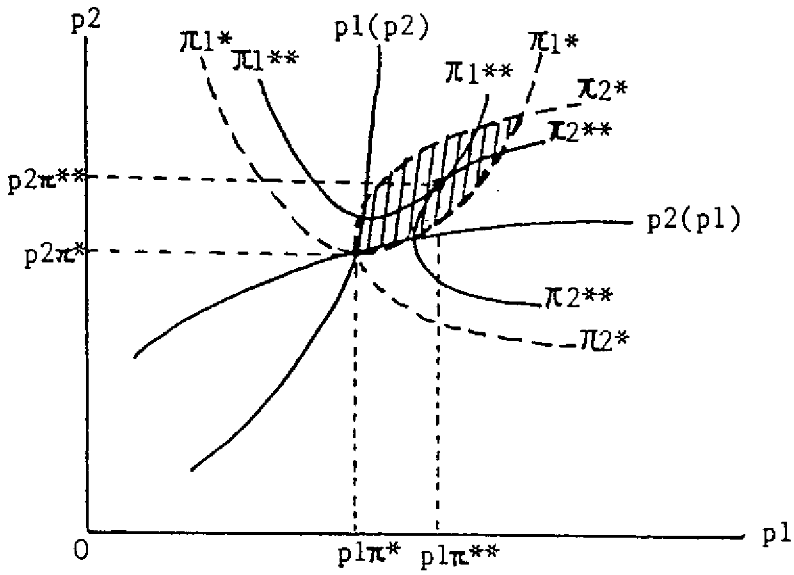
[Figure 13] Comparison of Profit Maximizing Prices Resulting form Autonomous versus Joint Operation



since $\delta Q2/\delta p1 > 0$ and $\delta Q1/\delta p2 > 0$ as shown in equations (10-2) and (10-4).

From equation (20-1), we can conclude that its solution, $p1\pi^{**}$, is greater than the solution to equation (13), $p1\pi^{*}$. This is shown in [Figure 13]. Begin with $p1\pi^{*}$ where $\delta\pi1/\delta p1=0$. Since $\delta\pi2/\delta p1 > 0$, in equation (20-1), adding $\delta\pi2/\delta p1$ to $\delta\pi1/\delta p1$ will shift up the curve $\delta\pi1/\delta p1$ to the $p1\pi^{**}$, where $\delta\pi1/\delta p1 + \delta\pi2/\delta p1 = 0$, which is greater than $p1\pi^{*}$. Thus, the profit maximizing price for firm F1 under joint operation is higher than that from autonomous operation. Similarly, we reach the same conclusion for firm F2. This requires a comparison of

[Figure 14] Optimal Prices under Profit Maximization



equation (20-2) with equation (14), and use of the fact that $\delta\pi 1/\delta p2 > 0$.

These relationships between optimal prices under the different operational regimes can also be explained in [Figure 6]. In [Figure 14], the reaction curves for both firms have been copied from [Figure 6]. The shaded area, bounded by the intersection of the autonomous iso-profit curves which result in optimal prices ($p1\pi^*$,$p2\pi^*$), represents the set of profit increasing price combinations. This set lies to the northeast of ($p1\pi^*$,$p2\pi^*$). Thus, any solution that increases joint profits must be accomplished through higher prices. A movement from ($p1\pi^*$,$p2\pi^*$) toward any point within this shaded area yields a higher profit to both firms. The authority will then choose the price combination that maximizes joint profits.

2) Social Welfare Maximizing Firms

Let us now consider the case of joint operation of the duopoly firms for social welfare maximization. An authority seeks to maximize aggregate social welfare over the market range [0,1]. This is simply the summation of the social welfare functions faced by each firm under the constraint of non-negative profit.

$$\max \quad W(p1,p2) = W1(p1,p2) + W2(p1,p2)$$
$$\text{s.t.} \quad \pi(p1,p2) = \pi1(p1,p2) + \pi2(p1,p2) \geq 0.$$

The Lagrangian function for this problem is

$$L(p1,p2,\alpha)=W1(p1,p2)+W2(p1,p2) + \alpha[\pi1(p1,p2)+\pi2(p1,p2)]$$

and the Kuhn-Tucker conditions are

$$\delta L/\delta p1 = \delta W1/\delta p1 + \delta W2/\delta p1 + \alpha\ [\delta\pi1/\delta p1 + \delta\pi2/\delta p1] \leq 0$$
$$\text{and} \quad p1(\delta L/\delta p1) = 0$$

$$\delta L/\delta p2 = \delta W1/\delta p2 + \delta W2/\delta p2 + \alpha\ [\delta\pi 1/\delta p2 + \delta\pi 2/\delta p2] \le 0$$
$$\text{and} \quad p2(\delta L/\delta p2) = 0$$
$$\delta L/\delta\alpha = \pi 1(p1,p2) + \pi 2(p1,p2) \ge 0 \quad \text{and } \alpha(\delta L/\delta\alpha) = 0.$$

From these conditions we derive the optimality conditions as follows.[8]

$$\delta W1/\delta p1 + \delta W2/\delta p1 + \alpha\ [\delta\pi 1/\delta p1 + \delta\pi 2/\delta p1] = 0$$
$$\text{and} \quad p1>0 \tag{22-1}$$
$$\delta W1/\delta p2 + \delta W2/\delta p2 + \alpha\ [\delta\pi 1/\delta p2 + \delta\pi 2/\delta p2] = 0$$
$$\text{and } p2 > 0 \tag{22-2}$$
$$\pi 1(p1,p2) + \pi 2(p1,p2) = 0 \qquad \text{and} \quad \alpha > 0 \tag{22-3}$$

From equation (22-3) we find F1's price reaction function p1(p2), and substitute this into equations (22-1) and (22-2). This yields

$$\alpha(p2) = -\ [\delta W1/\delta p1+\delta W2/\delta p1]\ /\ [\delta\pi 1/\delta p1+\delta\pi 2/\delta p1]$$
$$= -\ [\delta W1/\delta p2+\delta W2/\delta p2]\ /\ [\delta\pi 1/\delta p2+\delta\pi 2/\delta p2]. \tag{23}$$

Solving the last two terms of this equation, we find the social welfare maximizing prices for both firms under joint operation, (p1w**, p2w**). We then substitute this result back into equation (23) to solve for the value of the multiplier $\alpha^{**} = \alpha(p2w^{**})$.

We must compare the optimal prices under joint operation (p1w**,p2w**), derived from equation (22), with the results of the previous cases of autonomous operation (p1w*,p2w*), derived from equations (18) and (19). The latter are reproduced as

$$\delta W1/\delta p1 + \alpha 1\ (\delta\pi 1/\delta p1) = 0 \quad \text{and} \quad \pi 1 = 0 \tag{18}$$

$$\delta W2/\delta p2 + \alpha 2\ (\delta\pi 2/\delta p2) = 0 \quad \text{and} \quad \pi 2 = 0. \tag{19}$$

A comparison of multipliers ($\alpha 1$, $\alpha 2$, α), which represent the negative effect of a unit change in the profit constraint on the welfare level, is considered in the following proposition.

<u>Proposition 1</u>. $\alpha 1(p1,p2) > \alpha(p1,p2)$ if $\delta\pi 1/\delta p1 > 0$, and $\alpha 2(p1,p2) > \alpha(p1,p2)$ if $\delta\pi 2/\delta p2 > 0$.

<Proof> From equations (18-1), (19-1), (22-1), and (22-2),

$$\alpha 1(p1,p2) = -\ (\delta W1/\delta p1)/(\delta\pi 1/\delta p1) \qquad \langle 1\rangle$$

$$\alpha 2(p1,p2) = -\ (\delta W2/\delta p2)/(\delta\pi 2/\delta p2) \qquad \langle 2\rangle$$

$$\begin{aligned}\alpha(p1,p2) &= -\ (\delta W1/\delta p1+\delta W2/\delta p1)\ /\ (\delta\pi 1/\delta p1+\delta\pi 2/\delta p1)\\ &= -\ (\delta W1/\delta p2+\delta W2/\delta p2)\ /\ (\delta\pi 1/\delta p2+\delta\pi 2/\delta p2).\end{aligned} \qquad \langle 3\rangle$$

We find $\delta W1/\delta p1 = -\alpha 1(\delta\pi 1/\delta p1)$ and $\delta W2/\delta p2 = -\alpha 2(\delta\pi 2/\delta p2)$ from equations <1> and <2>. Substituting these relations into <3> yields

$$\alpha = [\alpha 1(\delta\pi 1/\delta p1)-\delta W2/\delta p1] \;/\; [\delta\pi 1/\delta p1+\delta\pi 2/\delta p1]$$
$$= [\alpha 2(\delta\pi 2/\delta p2)-\delta W1/\delta p2] \;/\; [\delta\pi 1/\delta p2+\delta\pi 2/\delta p2].$$

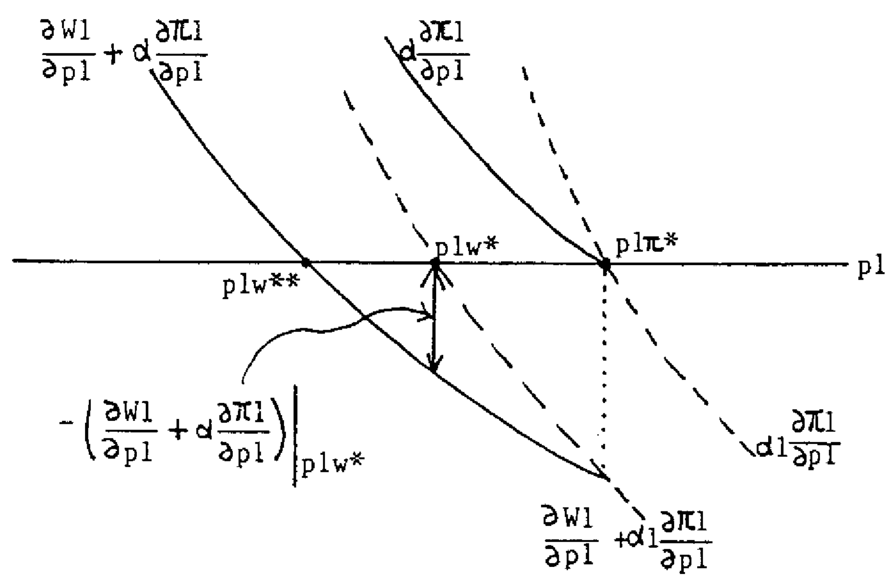
Define $\alpha'=[\alpha 1(\delta\pi 1/\delta p1)-\delta W2/\delta p1]/(\delta\pi 1/\delta p1)$, then $\alpha'>\alpha$ since $\delta\pi 2/\delta p1>0$. Therefore, $\alpha'=\alpha 1-(\delta W2/\delta p1)/(\delta\pi 1/\delta p1) > \alpha$. Given that $\delta W2/\delta p1 > 0$, $\alpha 1>\alpha$ if $\delta\pi 1/\delta p1>0$. Similarly if we define $\alpha''=[\alpha 2(\delta\pi 2/\delta p2)-\delta W1/\delta p2]/(\delta\pi 2/\delta p2)$, then $\alpha''>\alpha$ since $\delta\pi 1/\delta p2>0$. Thus, $\alpha'' = \alpha 2-(\delta W1/\delta p2)/(\delta\pi 2/\delta p2)>\alpha$. Finally, since $\delta W1/\delta p2>0$, we get $\alpha 2>\alpha$ if $\delta\pi 2/\delta p2>0$. (*)

From equations (18) and (19), we know that $\delta\pi 1/\delta p1>0$ and $\delta\pi 2/\delta p2>0$ at equilibrium prices $(p1w^*,p2w^*)$, because $\delta W1/\delta p1<0$, $\delta W2/\delta p2<0$, $\alpha 1>0$, and $\alpha 2>0$, as shown in equations (17-1), (17-3), (18-2), and (19-2). Thus, we conclude that $\alpha 1(p1w^*, p2w^*) > \alpha(p1w^*, p2w^*)$ and $\alpha 2(p1w^*, p2w^*) > \alpha(p1w^*, p2w^*)$. The question then becomes, under what conditions are the optimal prices for social welfare maximization under joint operation higher (lower) than those under autonomous operation?

<u>Proposition 2</u>. $p1w^{**} < p1w^{*}$ if $-\delta W/\delta p1 > \alpha(\delta\pi/\delta p1)$ at $p1w^{*}$, and $p1w^{**} > p1w^{*}$ if $-\delta W/\delta p1 < \alpha(\delta\pi/\delta p1)$ at $p1w^{*}$. Similarly $p2w^{**} < p2w^{*}$ if $-\delta W/\delta p2 > \alpha(\delta\pi/\delta p2)$ at $p2w^{*}$, and $p2w^{**} > p2w^{*}$ if $-\delta W/\delta p2 < \alpha(\delta\pi/\delta p2)$ at $p2w^{*}$.

<Proof> The curves $\alpha1(\delta\pi1/\delta p1)$ and $\delta W1/\delta p1 + \alpha1(\delta\pi1/\delta p1)$ are drawn as dotted lines in [Figure 15] which is copied from [Figure 12]. We then draw $\alpha(\delta\pi1/\delta p1)$ below the curve $\alpha1(\delta\pi1/\delta p1)$ in the range of $\delta\pi1/\delta p1>0$, since $\alpha1 > \alpha$ when $\delta\pi1/\delta p1 > 0$ from proposition 1. The $\delta W1/\delta p1 + \alpha(\delta\pi1/\delta p1)$ curve is below the curve $\alpha(\delta\pi1/\delta p1)$ because

[Figure 15] Comparison of Welfare Maximizing Prices Resulting from Autonomous versus Joint Operation

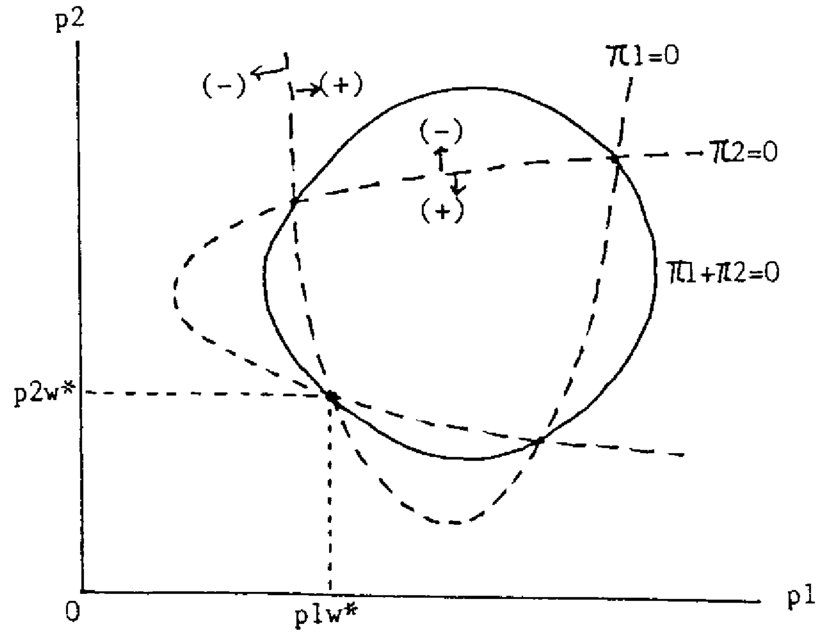


$\delta W1/\delta p1<0$. In order to find the location of $p1w^{**}$, we need to add $\delta W2/\delta p1 + \alpha(\delta\pi 2/\delta p1) > 0$ to $\delta W1/\delta p1 + \alpha(\delta\pi 1/\delta p1)$, since the equilibrium condition is $(\delta W1/\delta p1 + \delta W2/\delta p1) + \alpha (\delta\pi 1/\delta p1 + \delta\pi 2/\delta p1) = 0$. Thus, the location of the social welfare maximizing price under joint operation, $p1w^{**}$, depends on the magnitude of $-[(\delta W1/\delta p1) + \alpha(\delta\pi 2/\delta p1)]$ and $[(\delta W2/\delta p1) + \alpha(\delta\pi 2/\delta p1)]$, evaluated at $p1w^*$. If $-[\delta W1/\delta p1+\alpha(\delta\pi 1/\delta p1)] > [\delta W2/\delta p1+\alpha(\delta\pi 2/\delta p1)]$--if the equilibrium condition for social welfare maximization under joint operation evaluated at $p1w^*$ is $(\delta W1/\delta p1 + \delta W2/\delta p1) + \alpha(\delta\pi 1/\delta p1 + \delta\pi 2/\delta p1) < 0$--then $p1w^{**} < p1w^*$. Alternatively, if $-[\delta W1/\delta p1 + \alpha(\delta\pi 1/\delta p1)] < [\delta W2/\delta p1 + \alpha(\delta\pi 2/\delta p1)]$ at $p1w^*$, that is, if $(\delta W1/\delta p1 + \delta W2/\delta p1) + \alpha(\delta\pi 1/\delta p1 + \delta\pi 2/\delta p1) > 0$, then $p1w^{**} > p1w^*$. (*)

<u>Proposition 3</u>. If $p1w^{**} < p1w^*$, then $p2w^{**} > p2w^*$. If $p1w^{**} > p1w^*$, then $p2w^{**} < p2w^*$. If $p1w^{**} = p1w^*$, then $p2w^{**} = p2w^*$.

<Proof> From a zero iso-profit curve for each firm, we draw the profit constraint $\pi 1+\pi 2=0$ for the joint operation case, as shown in [Figure 16]. The slope of the zero iso-profit curve can be derived through total derivation of the profit constraint $\pi 1(p1,p2) + \pi 2(p1,p2) = 0$: $(\delta\pi 1/\delta p1+\delta\pi 2/\delta p1)\, dp1 + (\delta\pi 1/\delta p2+\delta\pi 2/\delta p2)\, dp2 = 0$.

[Figure 16] Zero Iso-profit Curve



Thus, $dp2/dp1 = -[\delta\pi1/\delta p1 + \delta\pi2/\delta p1]/[\delta\pi1/\delta p2+\delta\pi2/\delta p2]$ can be positive, negative, or zero.

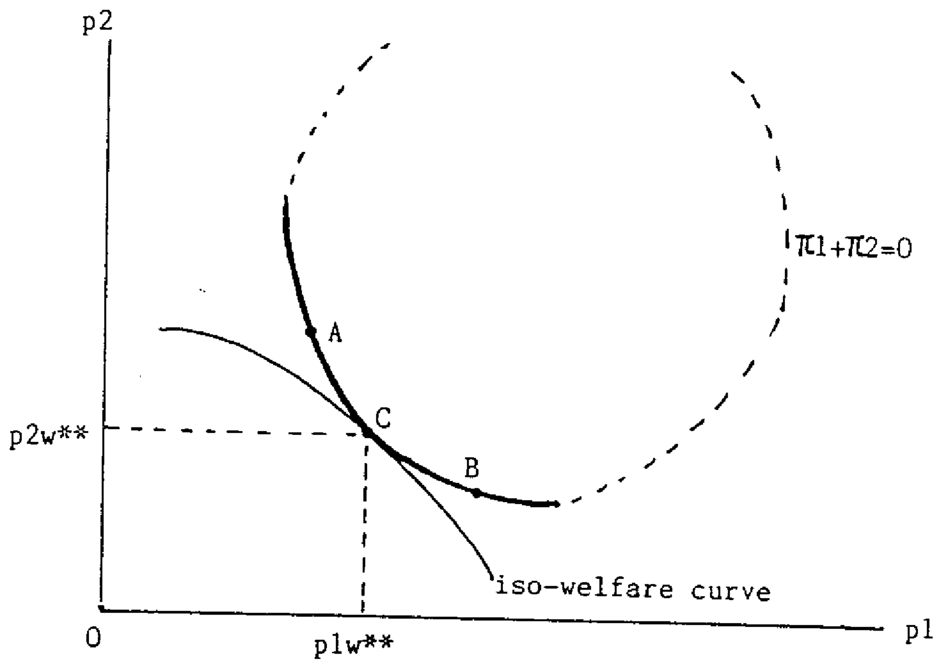
We now derive the iso-welfare curve from the total derivation of constant welfare level, $W1(p1,p2) + W2(p1,p2) = Wc$:

$$[\delta W1/\delta p1 + \delta W2/\delta p1]\, dp1 + [\delta W1/\delta p2 + \delta W2/\delta p2]\, dp2 = 0.$$

Then the slope of iso-welfare curve is negative,

$$dp2/dp1 = -[\delta W1/\delta p1+\delta W2/\delta p1] / [\delta W1/\delta p2+\delta W2/\delta p2] < 0$$

[Figure 17] Iso-welfare curve and Zero Profit Constraint



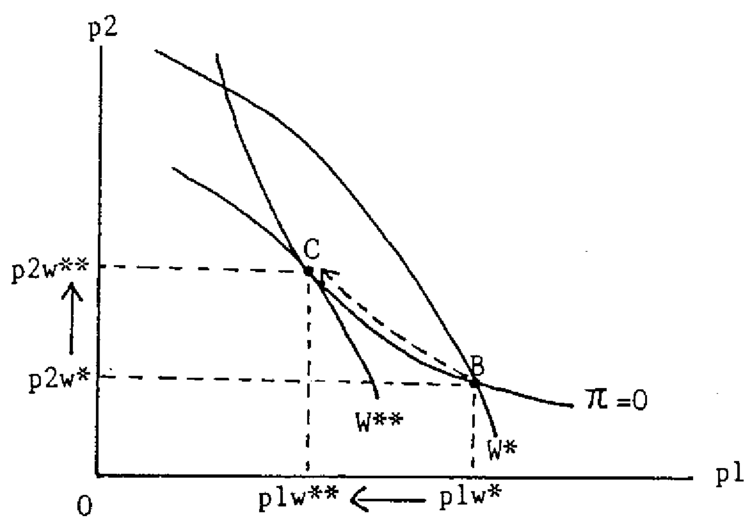
since $\delta W1/\delta p1<0$, $\delta W2/\delta p1>0$, $\delta W1/\delta p2>0$, and $\delta W2/\delta p2<0$ from equations (17). Optimal prices are obtained at point C in [Figure 17], where the welfare level is maximized on the zero iso-profit curve. Since social welfare increases as both firms' prices fall, the effective zero profit constraint is the bottom left, negatively sloped portion of the circular constraint.

If the optimal prices for social welfare maximization under autonomous operation, (p1w*,p2w*), are obtained at point A in [Figure 17], then p1w** > p1w* and p2w** < p2w*. If the prices (p1w*,p2w*) are obtained at point B, then p1w** < p1w* and p2w** > p2w*. And if the prices (p1w*,p2w*) are obtained at point C, then p1w**=p1w* and p2w**=p2w*. (*)

<u>Proposition 4</u>. From propositions 2 and 3, we can derive the following conditions for the relationship between the social welfare maximizing prices under autonomous operation and these prices under joint operation.

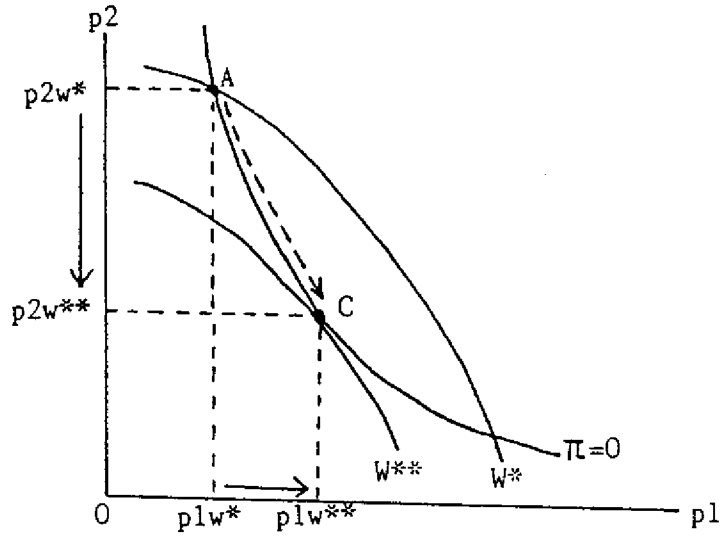
1) p1w** < p1w* and p2w** > p2w*:

if $-(\delta W/\delta p1) > \alpha\,(\delta\pi/\delta p1)$ and $-(\delta W/\delta p2) < \alpha(\delta\pi/\delta p2)$ at (p1w*, p2w*). (Point B case)



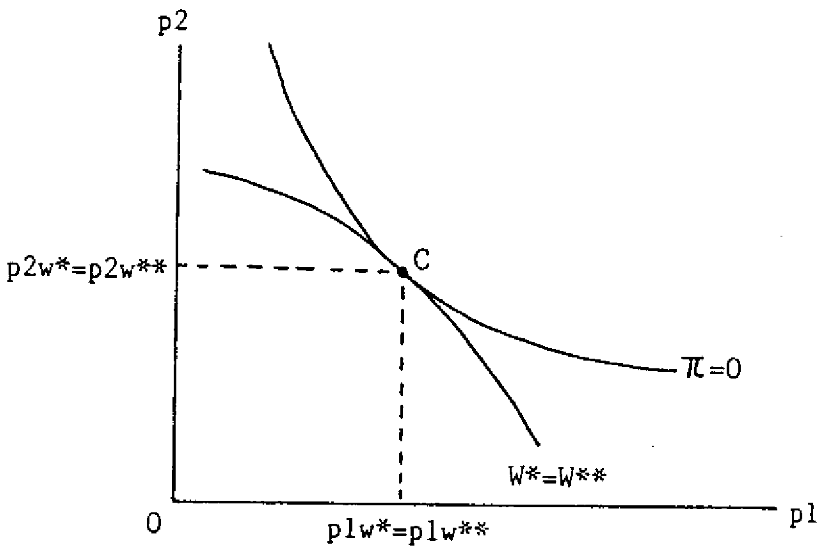
2) p1w** > p1w* and p2w** < p2w*:

if $-(\delta W/\delta p1) < \alpha(\delta\pi/\delta p1)$ and $-(\delta W/\delta p2) > \alpha(\delta\pi/\delta p2)$ at (p1w*,p2w*).(Point A case)



3) p1w** = p1w* and p2w** = p2w*:

if $-(\delta W/\delta p1) = \alpha(\delta\pi/\delta p1)$ and $-(\delta W/\delta p2) = \alpha(\delta\pi/\delta p2)$ at (p1w*, p2w*). (Point C case)



In a general framework, we have analyzed the spatial duopoly firms' optimizing behavior that corresponds to profit maximization and welfare maximization under both autonomous operation and joint operation. Explicit results for the optimal prices and objective values could not be shown, due to the general assumptions of the model, but we did make a behavioral comparison of spatial duopoly firms under the presence of price competition.

A summary table based on the results of the previous analyses is provided by [Table 1]. The superscript "*" represents optimal results for autonomous operation, while superscript "**" represents the optimal results for joint operation. Subscript "π" denotes the profit maximizing objective, while subscript "w" denotes the social welfare maximizing objective. Subscripts (1,2) refer to firms. For example, $p^*_{1\pi}$ denotes the optimal price for firm F1 resulting from profit maximization behavior under autonomous operation.

As we can see from [Table 1], the price level resulting from profit maximization behavior is highest when there is joint operation by one authority. The price level resulting from profit maximization is always higher than that from social welfare maximization, because a higher price yields less indirect utility to the consumers at each location. We found that if $p1w^*$ exceeds $p1w^{**}$, then $p2w^*$ is less than $p2w^{**}$, and if $p1w^*$ is less than $p1w^{**}$, then $p2w^*$ exceeds $p2w^{**}$, as shown in proposition 3. The profit level is constrained at zero when the firm's goal is to maximize social welfare, and maximum profit from joint operation exceeds those from autonomous operation. As expected, the welfare level is lowest when the firms pursue profit maximization under joint operation.

[Table 1] Summary

Objective		max Π		max W s.t. $\Pi \geq 0$	
Operation		Autonomous	Joint	Autonomous	Joint
Optimality Conditions		$\frac{\partial \Pi i}{\partial pi} = 0$ $i = 1, 2$	$\frac{\partial \Pi i}{\partial pi} + \frac{\partial \Pi j}{\partial pi} = 0$ $i, j = 1, 2$ $i \neq j$	$\begin{cases} \frac{\partial Wi}{\partial pi} + \alpha i \frac{\partial \Pi i}{\partial pi} = 0 \\ \Pi i = 0 \end{cases}$ $i = 1, 2$	$\begin{cases} \left(\frac{\partial Wi}{\partial pi} + \frac{\partial Wj}{\partial pi}\right) + \alpha\left(\frac{\partial \Pi i}{\partial pi} + \frac{\partial \Pi j}{\partial pi}\right) = 0 \\ \Pi = 0 \end{cases}$
Notation	Price	$p_\pi{}^*$	$p_\pi{}^{**}$	$p_w{}^*$	$p_w{}^{**}$
	Profit	$\Pi{}^*$	$\Pi{}^{**}$	$\Pi_w^* = 0$	$\Pi_w{}^{**} = 0$
	Welfare	$W_\pi{}^*$	$W_\pi{}^{**}$	W^*	W^{**}
Relationship	Price	$p_w{}^* < p_\pi{}^* < p_\pi{}^{**}$, $p1w^* \lesseqgtr p1w^{**}$ and $p2w^* \gtreqless p2w^{**}$			
	Profit	$\Pi^{**} > \Pi^* > \Pi_w{}^* = 0$			
	Welfare	$W_\pi{}^{**} < W_\pi{}^* < W^* < W^{**}$			

NOTES

1) This model shows the same situation as in Hotelling's model, except for the general assumptions on the population density function, demand function, and cost function. He used a uniform population density, perfectly inelastic unit demand, and zero production costs.

2) Since indirect utility function is

$$V(p',y) = U\,[\,q(p',y),\ \bar{q}(p',y)\,] + \sigma(p',y)[y-p'q(p',y)-\bar{q}(p',y)],$$

$$\delta V/\delta p' = (\delta U/\delta q)(\delta q/\delta p') + (\delta U/\delta \bar{q})(\delta \bar{q}/\delta p') + (\delta\sigma/\delta p')(y-p'q-\bar{q}) + \sigma[-p'(\delta q/\delta p')-q-\delta\bar{q}/\delta p')]$$

$$\delta V/\delta y = (\delta U/\delta q)(\delta q/\delta y) + (\delta U/\delta \bar{q})(\delta \bar{q}/\delta y) + (\delta\sigma/\delta y)(y-p'q-\bar{q}) + \sigma[1-p'(\delta q/\delta y)-(\delta\bar{q}/\delta y)].$$

From equations (1) and (2) we have $\delta U/\delta q=\sigma p'$ and $\delta U/\delta\bar{q}=\sigma$. Thus,

$$\delta V/\delta p' = \sigma p'(\delta q/\delta p') + \sigma(\delta\bar{q}/\delta p') + (\delta\sigma/\delta p')(y-p'q-\bar{q}) -\sigma p'(\delta q/\delta p')-\sigma q-\sigma(\delta\bar{q}/\delta p') = -\sigma q < 0\ ,$$

$$\delta V/\delta y = \sigma p'(\delta q/\delta y) + \sigma(\delta\bar{q}/\delta y) + (\delta\sigma/\delta y)(y-p'q-\bar{q}) + \sigma - \sigma p'(\delta q/\delta y) - \sigma(\delta\bar{q}/\delta y) = \sigma > 0$$

since $y-p'q-\bar{q} = 0$ from budget constraint.

3) From $\delta V/\delta p'= -\sigma q < 0$ and $p'= p+cd$,

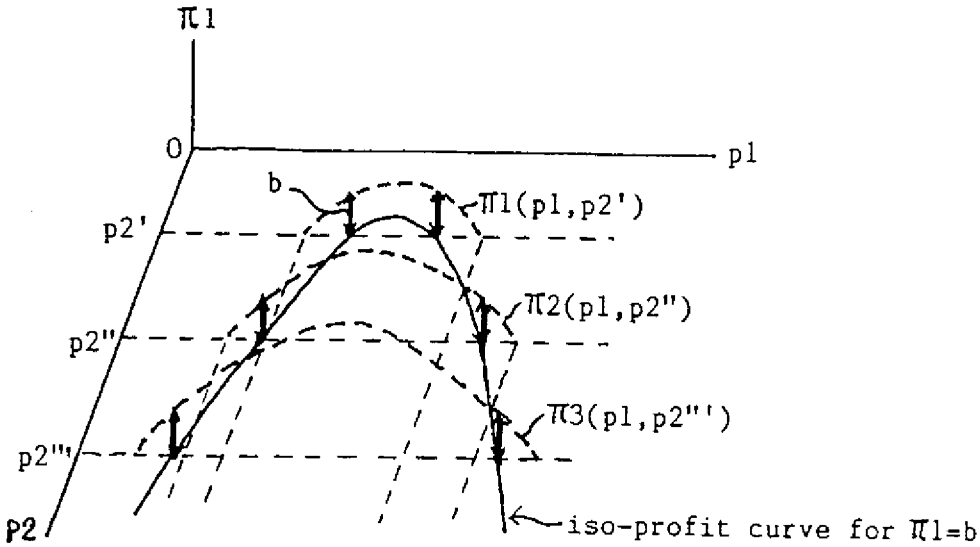
$$\delta V/\delta p = (\delta V/\delta p')\ (\delta p'/\delta p) = (-\sigma q)\ 1 = -\sigma q < 0$$

$$\delta V/\delta c = (\delta V/\delta p')\ (\delta p'/\delta c) = (-\sigma q)\ d = -\sigma qd < 0$$

$$\delta V/\delta d = (\delta V/\delta p')\ (\delta p'/\delta d) = (-\sigma q)\ c = -\sigma qc < 0$$

4) Since Hotelling's model assumed that the population density is uniformly distributed and that quantity demanded for all consumers residing at any location is the same, the demand faced by each firm is just the length of the market covered by that firm.

5) Derivation of iso-profit curve.



6) From Kuhn-Tucker conditions, we have $\delta W1/\delta p1 + \alpha 1\ \delta\pi 1/\delta p1 = 0$ since $p1>0$. We know $\delta W1/\delta p1 < 0$, thus $\alpha 1\ \delta\pi 1/\delta p1 > 0$. Since $\alpha 1 \geq 0$, $\alpha 1$ cannot be zero and must be positive, thus $\delta\pi 1/\delta p1>0$ at equilibrium. And from $\alpha 1\ \delta L1/\delta\alpha 1 = 0$, we have $\delta L1/\alpha 1=0$ since $\alpha 1 > 0$. Therefore, $\pi 1(p1,p2)=0$.

7) $W1(p1,p2) = Wo$(constant)
The total derivation yields
$(\delta W1/\delta p1)dp1 + (\delta W1/\delta p2)dp2 = 0$. Thus,
$dp2/dp1 = - (\delta W1/\delta p1)/(\delta W1/\delta p2) = - (-)/(+) > 0$.

8) From Kuhn-Tucker conditions, $p1(\delta L/\delta p1)=0$ and $p2(\delta L/\delta p2)=0$, we have $\delta L/\delta p1=0$ since $p1>0$ and $\delta L/\delta p2=0$ since $p2>0$. We know that $\delta W1/\delta p1+\delta W2/\delta p1<0$ and $\delta W1/\delta p2+\delta W2/\delta p2<0$ from equation (17). Thus, $\alpha(\delta\pi 1/\delta p1+\delta\pi 2/\delta p1)>0$ must hold to get $\delta L/\delta p1=0$, and $\alpha(\delta\pi 1/\delta p2+\delta\pi 2/\delta p2)>0$ must hold to get $\delta L/\delta p2=0$. Since $\alpha\geq 0$, α must be positive. Therefore, $\delta L/\delta\alpha=0$ from $\alpha(\delta L/\delta\alpha)=0$. The Kuhn-Tucker conditions then become $\delta L/\delta p1=0$ and $p1>0$, $\delta L/\delta p2=0$ and $p2>0$, and $\delta L/\delta\alpha=0$ and $\alpha>0$.

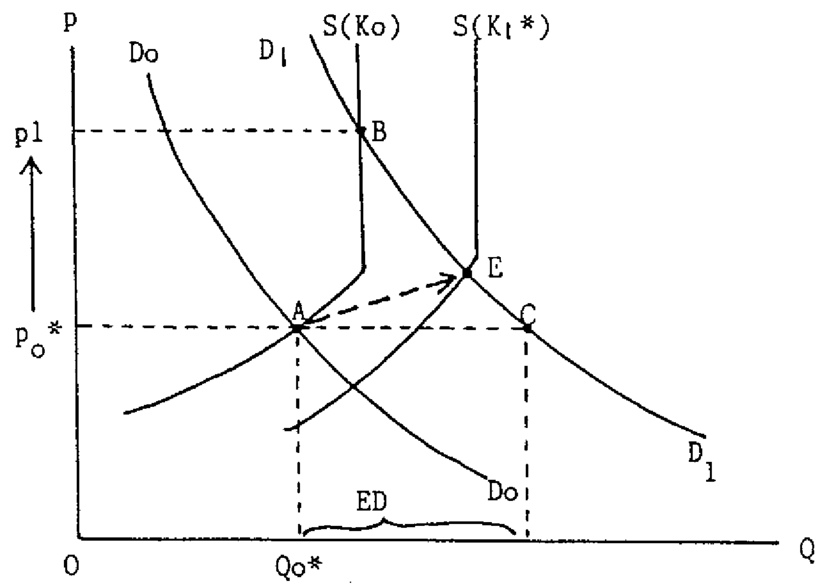
IV. CAPACITY EXPANSION PLAN IN SPATIAL DUOPOLY

In the previous chapter, we have analyzed the optimal pricing policies of spatial duopoly firms according to various objectives in a certain time period. Based upon that theoretical framework, we now extend the model to include the capital investment problem in this chapter and to consider the location decisions of spatial duopoly firms in the next chapter.

In this chapter, we attempt to relate the static pricing policies to the dynamic investment plan under the presence of spatial competition between duopoly firms. We analyze the optimal combination of price and capital stock of spatial duopoly which has fixed location with entry barriers, in the long-term respective. Entry barriers are mainly caused by large amount of fixed cost, economies of scale, sunk cost, or regulations in the industry, which also support the assumption of their fixed location.

We were not concerned with the capital stock level in the previous chapter because it is assumed to be fixed in a short-run analysis. When the industry expects demand to increase, firms encounter the strategic problem of choosing the optimal mix between pricing policy and capacity expansion or capital investment policy. We

[Figure 18] Strategic Mix of Pricing and Investment



firms with optimal pricing policies via the calculus of variations.

With a given initial level of capital stock Ko, the firm will charge optimal price po*, in order to maximize its objective function. This is shown at point A in [Figure 1]. An increased demand, from Do to D1, results in higher revenue (as both price and quantity rise) if the capital stock is fixed (point B), and results in excess demand if the price is fixed in the short-run (point C). However, the increased revenue will be reduced by the actions of the rival firm. To maximize its long-run objective in the face of rival behavior, the firm must determine a new optimal pricing-investment combination. To accomplish this, the firms' profit functions, defined in equations (11) and (12) must be modified to include investment cost with the production and capital maintenance costs. Hence:

$$\pi_{1t} = P_{1t}\ Q_{1t}(P_{1t}, P_{2t}) - C_{1t}[Q_{1t}(P_{1t}, P_{2t})] - G_t(K'_{1t})$$

$$\pi_{2t} = P_{2t}\ Q_{2t}(P_{1t}, P_{1t}) - C_{2t}[Q_{2t}(P_{1t}, P_{2t})] - G_t(K'_{2t})$$

where (π_{1t}, π_{2t}) denote profit functions for firm F1 and firm F2, respectively, at time t. (P_{1t}, P_{2t}) are the prices charged by each firm at time t. (Q_{1t}, Q_{2t}) are the demand functions faced by firms. (C_{1t}, C_{2t}) are production and maintenance costs. G_t is investment cost, and is identical for each firm. (K'_{1t}, K'_{2t}) are changes in the capital level, defined as investment at time t. We constrain the demand functions, derived in the previous chapter, must meet the supply function (defined as the production function of each firm depending on the capital level) in order to clear the market. For simplicity, capital is assumed to be the only input in this industry.

$$Q_{1t}(P_{1t}, P_{2t}) = F_{1t}(K_{1t}) \tag{1-1}$$

where $\varepsilon_{1t} = (\delta p_{1t}/\delta Q_{1t})\ F_{1t}(K_{1t})/p_{1t}$ is the price elasticity of demand faced by firm F1 at time t, MPP_k is marginal physical product of capital, and MR is marginal revenue for firm F1. Thus equation (5) can be reduced to

$$MR(K_{1t}, p_{2t})\ MPP_k(K_{1t}) = \delta C_{1t}/\delta K_{1t} + r(\delta G_t/\delta K'_{1t}). \qquad (8)$$

Since the above condition is a function of its own capital stock and rival's price, solving the differential equation (8) with the boundary condition $f_{1k'} = 0$ at t=T and an initial value of K_{10} at t=0, we find the optimal level of of capital stock for firm F1 as a function of its rival's price, $K_{1t}(p_{2t})$. We call this solution as capital reaction function of firm F1.

2) Firm F2's Behavior

Similarly, profit maximizing firm F2's dynamic objective can be formulated as

$$\max \int_0^T e^{-rt}\ \pi_{1t}\ dt$$

s.t. K_{20} is given, K_{2T} is free.

We define

$$f_2(t,K_2,K_2') = e^{-rt}\ \pi_{2t}$$

$$= e^{-rt}\{p_{2t}[F_{2t}(K_{2t}),p_{1t}]F_{2t}(K_{2t})-C_{2t}(K_{2t})-G_t(K_{2t}')\}$$

using equation (4), then calculate the following in order to employ the calculus of variation method.

$$f_{2k} = \delta f_2/\delta K_{2t} = e^{-rt}\{(\delta p_{2t}/\delta Q_{2t})(\delta F_{2t}/\delta K_{2t})F_{2t}(K_{2t})$$

$$+ p_{2t}[F_{2t}(K_{2t}),p_{1t}]\delta F_{2t}/\delta K_{2t}-\delta C_{2t}/\delta K_{2t}\} \qquad (9)$$

$$f_{2k'} = \delta f_2/\delta K_{2t}' = e^{-rt}\ \delta G_t/\delta K_{2t}'$$

$$df_{2k'}/dt = r\ e^{-rt}(\delta G_t/\delta K_{2t}') \qquad (10)$$

According to Euler's equation $f_{2k} - df_{2k'}/dt = 0$, equation (9) must be equal to equation (10). Thus,

$$e^{-rt}\ \{(\delta p_{2t}/\delta Q_{2t})(\delta F_{2t}/\delta K_{2t})\ F_{2t}(K_{2t})$$

$$+ p_{2t}[F_{2t}(K_{2t}),p_{1t}]\ \delta F_{2t}/\delta K_{2t} - \delta C_{2t}/\delta K_{2t}\}$$

$$= r\ e^{-rt}\ \delta G_t/\delta K_{2t}'$$

Rearranging the above equation leads to

$$MR(K_{2t},p_{1t})MPP_k(K_{2t}) = \delta C_{2t}/\delta K_{2t} + r(\delta G_t/\delta K'_{2t}). \qquad (11)$$

Solving this differential equation with an initial condition of K_{20} and boundary condition $f_{2k'} = 0$ at time $t=T$, we find the optimal level of capital stock for firm F2 as a function of its rival's price, $K_{2t}(p_{1t})$, which is called as capital reaction function of firm F2.

Finally we consider both firms' behavior, equations (8) and (11), simultaneously, to find the optimal solution for price and capital stock level for spatial duopoly firms. That is, price reaction for each firm, $p_{1t}(p_{2t})$ and $p_{2t}(p_{1t})$, can be derived by substituting each firm's capital reaction function, $K_{1t}(p_{2t})$ and $K_{2t}(p_{1t})$, into the inverse demand functions, equations (2-1) and (2-2), respectively. Optimal prices (p^*_{1t}, p^*_{2t}) are obtained by solving the price reaction functions simultaneously. The optimal level of capital stock for each firm (K^*_{1t}, K^*_{2t}) are obtained by substituting the optimal prices into $K_{1t}(p^*_{2t})$ and $K_{2t}(p^*_{1t})$.

2. Numerical Application

Due to our use of general functional forms in the model, we cannot derive many implications about the investment plan for a spatial duopoly without specific information on demand, production, and cost functions. Thus, we now consider specific functional forms in a simplified numerical application of our model.

1) Assumptions on Demand Side

Let us consider a linear country scaled [0,1] with a uniform distribution of population, $f(x)=1$ for x in [0,1]. There are two firms, F1 and F2, located at k1=0.4 and k2=0.8 within [0,1]. Each consumer purchases only one unit of the product, at the lowest delivered price $q(p')=1$, where transportation cost $c=1$. Then the market boundary $k1+L^*$, which depends on p_1 and p_2, can be found by using equation (7) from chapter three.

$$k_1+L^* = k_1 + [p_2-p_1+c(k_2-k_1)]/2c$$

$$= 0.6 + p_2/2 - p_1/2 \qquad (12)$$

The market demand faced by each firm becomes

$$Q_1(p_1,p_2) = \int_0^{k1+L^*} q(p')\ f(x)\ dx$$

$$= 0.6 + p_2/2 - p_1/2 \qquad (13)$$

$$Q_2(p_1,p_2) = \int_{k1+L^*}^{1} q(p')\ f(x)\ dx$$

$$= 0.4 + p_1/2 - p_2/2 \qquad (14)$$

As expected in chapter three, $\delta Q1/\delta p1 < 0$, $\delta Q1/\delta p2 > 0$, $\delta Q2/\delta p1 < 0$, and $\delta Q2/\delta p2 < 0$. In other words, an increase in the relative price of a firm reduces the market demand for that firm. From these market demand functions, we can derive the inverse demand function for each firm:

$$p_1 = 1.2 + p_2 - 2Q_1 \qquad (15)$$

$$p_2 = 0.8 + p_1 - 2Q_2 \qquad (16)$$

2) Assumptions on Supply Side

For simplicity, production and maintenance cost C, and investment cost G, are assumed to be identical for both firms and to be constant over time. C is linear in input K, while G is linear in the investment level K'. Capital is the only input, and the technology is identical in both firms with a functional form of $F(K) = K^{\alpha}$.

The market equilibrium conditions are,

$$Q_{1t}(p_{1t}, p_{2t}) = F_{1t}(K_{1t}) = K_{1t}^{\alpha}$$

$$Q_{2t}(p_{1t}, p_{2t}) = F_{2t}(K_{2t}) = K_{2t}^{\alpha}$$

The profit functions for each firm at time t are

$$\pi_{1t} = p_{1t}\, Q_{1t}(p_{1t}, p_{2t}) - C\, K_{1t} - G\, K'_{1t}$$

$$= (1.2 + p_{2t} - 2\, K_{1t}^{\alpha})\, K_{1t}^{\alpha} - C\, K_{1t} - G\, K'_{1t}$$

$$\pi_{2t} = p_{2t}\, Q_{2t}(p_{1t}, p_{2t}) - C\, K_{2t} - G\, K'_{2t}$$

$$= (0.8 + p_{1t} - 2\, K_{2t}^{\alpha})\, K_{2t}^{\alpha} - C\, K_{2t} - G\, K'_{2t}$$

3) Firms' Behavior

Under the specific assumptions above, we can formulate firm F1's objective function as

$$\max \int_{\emptyset}^{T} e^{-rt}\, \pi_{1t}\, dt$$

s.t. $K_{1\emptyset}$ is given, K_{1T} is free.

Letting $f_1(t, K_1, K_1'; p_1) = e^{-rt}\, \pi_{1t}$

$$= e^{-rt}[(1.2+p_{2t}-2K_{1t}^{\alpha})\, K_{1t}^{\alpha} - CK_{1t}-GK_{1t}']$$

we calculate the followings in order to employ the calculus of variation method:

$$f_{1k} = \delta f_1/\delta K_{1t} = e^{-rt}[(1.2 + p_{2t})\alpha K_{1t}^{\alpha-1} - 4\alpha K_{1t}^{2\alpha-1} - C] \quad (17)$$

$$f_{1k'} = \delta f_1/\delta K_{1t}' = e^{-rt}(-G)$$

$$df_{1k'}/dt = r\, G\, e^{-rt} \quad (18)$$

From Euler's equation $f_{1k} = df_{1k'}/dt$, we find

$$(1.2 + p_2)\, \alpha K_{1t}^{\alpha-1} - 4\alpha K_{1t}^{2\alpha-1} = H \quad (19)$$

where $H = C + r\, G$ denotes some constant value. The optimality condition (19) is then reduced to a static problem from a dynamic differential equation problem due to the simplified assumption of this example.

The optimality condition for firm F2 can be derived similarly. Firm F2's objective function is

$$\max \int_{0}^{T} e^{-rt} \pi_{2t} \, dt$$

s.t. K_{20} is given, K_{2T} is free,

letting $f_2(t,K_2,K_2';p1) = e^{-rt} \pi_{2t}$ and following the same procedure as in the case for firm F1, we can find the optimality condition for firm F2 as followings:

$$(0.8 + p_1) \alpha K_{2t}^{\alpha-1} - 4\alpha K_{2t}^{2\alpha-1} = H \tag{20}$$

where $H = C + r\,G$ shows some constant value. Equation (20) also reduces to a static problem. By solving equations (19) and (20) we find the capital reactions as functions of the rival's price, $K_1(p_2)$ and $K_2(p_1)$. For simplicity let $\alpha=1$, i.e. there are constant returns to scale.[5] At any time period

$$K_1 = 0.3 + p_2/4 - H/4 \tag{19'}$$

$$K_2 = 0.2 + p_1/4 - H/4 \; . \tag{20'}$$

$$Q_{2t}(p_{1t},p_{2t}) = F_{2t}(K_{2t}) \tag{1-2}$$

Using the market equilibrium conditions (1-1) and (1-2), we can derive the inverse demand function for each firm as a function of quantity and rival's price.

$$p_{1t} = p_{1t}[F_{1t}(K_{1t}),\ p_{2t}] \tag{2-1}$$

$$p_{2t} = p_{2t}[F_{2t}(K_{2t}),\ p_{1t}] \tag{2-2}$$

Substituting these inverse demand functions into the profit functions yields

$$\pi_{1t} = p_{1t}[F_{1t}(K_{1t}),p_{2t}]\ F_{1t}(K_{1t}) - C_{1t}(K_{1t}) - G_t(K_{1t}') \tag{3}$$ [1]

$$\pi_{2t} = p_{2t}[F_{2t}(K_{2t}),p_{1t}]\ F_{2t}(K_{2t}) - C_{2t}(K_{2t}) - G_t(K_{2t}'). \tag{4}$$

Each is a function of its own level of capital stock and its rival's price at time t. We now consider each firm's behavior.

1. Firms' Behavior

1). Firm F1's Behavior

When we consider the investment plan over a time period [Ø,T], firms need to consider the integrated profit level over the period, since investment is defined by the change of capital stock over time. The firm's objective will be to maximize the present value of cumulative profits over period [Ø,T].

$$\max \int_{0}^{T} e^{-rt} \pi_{1t}\, dt \quad 2)$$

$$\text{s.t. } K_{10} \text{ is given, } K_{1T} \text{ is free}$$

where r represents the discount rate. Using equation (3), we define

$$f_1(t,K_1,K_1') = e^{-rt} \pi_{1t}$$

$$= e^{-rt}\{p_{1t}[F_{1t}(K_{1t}),p_{2t}]F_{1t}(K_{1t})-C_{1t}(K_{1t})-G_t(K_{1t}')\}$$

In order to apply the calculus of variations, we need to calculate the following:

$$f_{1k} = \delta f_1/\delta K_{1t} = e^{-rt}\{(\delta p_{1t}/\delta Q_{1t})(\delta F_{1t}/\delta K_{1t})F_{1t}(K_{1t})$$

$$+ p_{1t}[F_{1t}(K_{1t}), p_{2t}]\delta F_{1t}/\delta K_{1t} - \delta C_{1t}/\delta K_{1t}\} \quad (5)$$

$$f_{1k'} = \delta f_1/\delta K'_{1t} = e^{-rt}\ \delta G_t/\delta K'_{1t}$$

$$df_{1k'}/dt = r\ e^{-rt}(\delta G_t/\delta K'_{1t}) \quad (6)$$

From Euler's equation, $f_{1k} - df_{1k'}/dt = 0$,[3] equation (5) must be equal to equation (6). Thus

$$e^{-rt}\ \{(\delta p_{1t}/\delta Q_{1t})(\delta F_{1t}/\delta K_{1t})\ F_{1t}(K_{1t})$$

$$+p_{1t}[F_{1t}(K_{1t}), p_{2t}]\ \delta F_{1t}/\delta K_{1t} - \delta C_{1t}/\delta K_{1t}\}$$

$$= r\ e^{-rt}\ \delta G_t/\delta K'_{1t}$$

Rearranging the above equation leads to

$$(\delta F_{1t}/\delta K_{1t})p_{1t}[F_{1t}(K_{1t}), p_{2t}]\ [1+\delta p_{1t}/\delta Q_{1t}\ F_{1t}(K_{1t})/p_{1t}]$$

$$= \delta C_{1t}/\delta K_{1t} + r\ \delta G_t/\delta K'_{1t} \quad (7)$$

LHS(left hand side) of equation (7) becomes

$$(\delta F_{1t}/\delta K_{1t})\ p_{1t}(1+1/\varepsilon_{1t}) = MPP_k(K_{1t})\ MR(K_{1t}, p_{2t})$$

Substituting these equations into the inverse demand functions, equations (15) and (16), and using production function $Q = K^{\alpha}$, we find the price reaction function for each firm.

$$p_1 = 0.6 + p_2/2 + H/2 \tag{21}$$

$$p_2 = 0.4 + p_1/2 + H/2 \tag{22}$$

Solving the price reaction functions simultaneously yields the optimal price for each firm,

$$p_1^* = 16/15 + H \,, \quad \text{and} \quad p_2^* = 14/15 + H \,.$$

The optimal level of capital stock for each firm is calculated to be $K_1^* = 8/15$ and $K_2^* = 7/15$ by substituting the optimal prices into equations (19)' and (20)'. Clearly, the optimal price and level of capital for firm F1 are higher than those for firm F2. This is due to F1's fixed locational advantage which gave the firm more market power. The market boundary can be found from equation (12) to be $k1+L^* = 0.533$. The boundary resulting from spatial competition is less than the midpoint (0.6) between the two firms' location--the generally expected result. This is mainly caused by the spatial competition between the duopolists and by asymmetric location of the two firms.

We found the optimal capital stock level and price for one time period by reducing the dynamic optimality conditions to static conditions. Therefore, we can find the optimal solutions at every time period for any increased demand using the static optimality conditions for that period. For any exogenous demand expansion path, the corresponding optimal path for capital and prices can be found.

Assume that the above analysis is for an initial period of time with the solutions $p_{10} = 16/15 + H$, $p_{20} = 14/15 + H$, $K_{10} = 8/15$, and $K_{20} = 7/15$; and with demand functions $Q_{10}(p_1, p_2) = 0.6 + p_2/2 - p_1/2$ and $Q_{20}(p_1, p_2) = 0.4 + p_1/2 - p_2/2$ as shown in equations (12) and (13). Let the dynamic path of demand for each firm be given by

$$Q_{1t}(p_1, p_2) = Q_{10}(p_1, p_2) + \mu_{1t} = (0.6 + p_2/2 - p_1/2) + \mu_{1t}$$

$$Q_{2t}(p_1, p_2) = Q_{20}(p_1, p_2) + \mu_{2t} = (0.4 + p_1/2 - p_2/2) + \mu_{2t}$$

From these demand functions we can derive inverse demand functions.

$$p_1 = 1.2 + p_2 + 2\mu_{1t} - 2Q_{1t} \tag{23}$$

$$p_2 = 0.8 + p_1 + 2\mu_{2t} - 2Q_{2t} \tag{24}$$

The profit functions are then

$$\pi_{1t} = (1.2 + p_2 + 2\mu_{1t} - 2K_{1t}^{\alpha})K_{1t}^{\alpha} - C\,K_{1t} - G\,K_{1t}'$$

$$\pi_{2t} = (0.8 + p_1 + 2\mu_{2t} - 2K_{2t}^{\alpha})K_{2t}^{\alpha} - C\,K_{2t} - G\,K_{2t}'.$$

In every time period, the optimality condition $MR\ MPP_k = H$ must be satisfied. Thus,

$$(1.2 + p_2 + 2\mu_{1t})\alpha K_{1t}^{\alpha-1} - 4\alpha K_{1t}^{2\alpha-1} = H \tag{25}$$

$$(0.8 + p_1 + 2\mu_{2t})\alpha K_{2t}^{\alpha-1} - 4\alpha K_{2t}^{2\alpha-1} = H \tag{26}$$

When we assume that $\alpha=1$ for simplicity, then equations (25) and (26) become

$$K_{1t} = 0.3 + p_2/4 - H/4 + \mu_{1t}/2 \tag*{(25)'}$$

$$K_{2t} = 0.2 + p_1/4 - H/4 + \mu_{2t}/2 \tag*{(26)'}$$

Substituting these relations into the inverse demand functions yields price reaction functions.

$$p_1 = 0.6 + p_2/2 + H/2 + \mu_{1t} \tag{23)'$$

$$p_2 = 0.4 + p_1/2 + H/2 + \mu_{2t}. \tag{24)'$$

Solving equations (23)' and (24)' simultaneously yields a series of optimal prices for both firms.

$$p_{1t} = (16/15 + H) + 4/3\ \mu_{1t} + 2/3\ \mu_{2t}$$

$$= p_{10} + 4/3\ \mu_{1t} + 2/3\ \mu_{2t} \tag{27}$$

$$p_{2t} = (14/15 + H) + 2/3\ \mu_{1t} + 4/3\ \mu_{2t}$$

$$= p_{20} + 2/3\ \mu_{1t} + 4/3\ \mu_{2t} \tag{28}$$

The optimal capital level for each firm is obtained by substituting equations (27) and (28) into equations (25)' and (26)'.

$$K^*_{1t} = 8/15 + 2\ \mu_{1t}/3 + \mu_{2t}/3 = K^*_{10} + 2\ \mu_{1t}/3 + \mu_{2t}/3$$

$$K^*_{2t} = 7/15 + \mu_{1t}/3 + 2\ \mu_{2t}/3 = K^*_{20} + \mu_{1t}/3 + 2\ \mu_{2t}/3$$

Once the optimal path for capital has been found, the optimal investment level for each period is determined by comparing the expected capital level with the existing level--since investment is defined as the change in capital stock over time.

From the solution values for prices and capital stocks at an arbitrary time t, we observe that prices and capacities for both firms tend to increase as demands increase. A comparison of prices and capital stocks between the two firms depends upon the relative increase in their market demands. If $\mu_{1t} > \mu_{2t}$, then the price increase for firm F1 is greater than the price increase for firm F2. This will shrink the market boundary of firm F1, since $k1+L^* = 0.6 + p_2/2 - p_1/2 = 8/15 - (\mu_{1t} - \mu_{2t})/3$. This results can be varied according to the assumptions on demand and supply. We have used the simplest case to avoid computational complexity. We can modify the objective function to imply a social welfare maximizing goal, vary the population density, the given location of firms, etc. The resulting complexities require the use of a computer.

This chapter analyzed the optimal combination of price and capital stock of spatial duopoly which has the fixed location with entry barriers. Entry barriers like large amount of fixed cost, economies of scale, sunk cost, or regulations are easily found in such industries as seaports, utility plant, airports, etc. In other words, this analysis can be applied to consider the problems of these industries. For example, urban seaport authorities often tend to neglect spatial price competition between neighboring ports and to expand the port capacity excessively without considering the combination with price adjustment. Urban seaport authorities could employ this analysis in order to solve the problem whether to raise the service price of port use or to expand the port capacity to meet the expected demand increase.

NOTES

1) We define $C_{1t}(Q_{1t}) = C_{1t}[F_{1t}(K_{1t})] = C_{1t}(K_{1t})$ and $C_{2t}(Q_{2t}) = C_{2t}[F_{2t}(K_{2t})] = C_{2t}(K_{2t})$.

2) A dollars at time Ø become $A(1+r)^t$ at time t with annual interest rate r. If the interest were compounded m times a year, then A dollars at time Ø becomes $A(1+r/m)^{mt}$ at time t. Continuous compounding amounts to letting $m \to \infty$. Since $\lim_{m\to\infty} (1+r/m)^{mt} = e^{rt}$, it follows that A dollars invested at annual rate r, continuously compounded, grow to $e^{rt}A$ dollars in t years. Inversely, present value of B dollars in t years is $e^{-rt}B$.

3) For the derivation of Euler's equation, refer to Kamien, M.I. and N.L.Schwartz [1981], Dynamic Optimization: The Calculus of Variations and Optimal Control in Economics and Management, North Holland, pp.14-16.

4) For the derivation of the boundary condition, see Kamien and Schwartz [1981],pp.48-49.

5) We could vary the value of α and use a computer in order to see the implication of different scale economies on the capital investment problem.

V. LOCATIONAL CONSIDERATIONS

We have assumed fixed location of firms in the previous chapter. Now, we try to analyze the locational behavior of duopoly firms pursuing profit maximization objective as an another extension of the basic model in chapter three.

If we keep the assumptions used in the above, locational considerations are raised when the two firms are planned to located simultaneously by a single authority or when the second firm try to locate under the situation that the first firm already exists with its fixed location. If we loose the assumption of entry barriers, new firm planning to enter the industry will also consider the locational choice.

In this chapter we try to figure out the profit maximizing location and corresponding optimal prices under spatial competition and consider the comparative statics of optimal prices with respect to firms' location.

1. Firms' Behavior

From profit functions for both firms as shown in equations (11) and (12) in chapter three, we obtained the optimal prices as functions of fixed locations k1 and k2 by solving first order conditions of maximizing profits. Substituting these solutions p1(k1,k2) and p2(k1,k2) into profit functions, we get profit functions dependent on fixed location of firms.

$$\pi 1(k1,k2) = p1(k1,k2)\ Q1[p1(k1,k2),p2(k1,k2)] - C1\{Q1[p1(k1,k2),p2(k1,k2)]\}$$

$$\pi 2(k1,k2) = p2(k1,k2)\ Q2[p1(k1,k2),p2(k1,k2)] - C2\{Q2[p1(k1,k2),p2(k1,k2)]\}$$

In order to see the optimal location of firms we need to take partial derivation of profit functions with respect to locations k1 and k2.

$$\delta\pi 1/\delta k1 = (\delta p1/\delta k1)\ Q1[p1(k1,k2),p2(k1,k2)] + (p1-MC1)\ [(\delta Q1/\delta p1)(\delta p1/\delta k1)+(\delta Q1/\delta p2)(\delta p2/\delta k1)] = 0 \qquad (1)$$

$$\delta\pi 2/\delta k2 = (\delta p2/\delta k2)\ Q2[p1(k1,k2),p2(k1,k2)] + (p2-MC2)\ [(\delta Q2/\delta p1)(\delta p1/\delta k2)+(\delta Q2/\delta p2)(\delta p2/\delta k2)] = 0 \qquad (2)$$

Solving equations (1) and (2) simultaneously, we can get optimal location (k1*, k2*).

We cannot derive any implication from the above analysis in a general framework because we cannot solve for the optimal locations explicitly. Thus we take a simple explicit example of the locational consideration problem.

2. Numerical Application

Let us consider the simplest case like Hotelling's assumptions such as extremely inelastic demand $q(p')=1$, unit transportation cost $c=1$, zero production cost $C=0$ which is identical to both firms, and uniform population density $f(x)=1$ for x in $[0,1]$.

Then market demands faced by two firms are

$$Q1(p1,p2) = \int_{0}^{k1=L*} q(p')\ f(x)\ dx = (p2-p1)/2 + (k1+k2)/2$$

$$Q2(p1,p2) = \int_{k1+L*}^{1} q(p')f(x)dx = 1-(p2-p1)/2 - (k1+k2)/2$$

First order conditions for profit maximizing firms are

$$\delta\pi 1/\delta p1 = Q1 + p1(\delta Q1/\delta p1) = p2/2 - p1 + (k1+k2)/2 = 0$$
$$\delta\pi 2/\delta p2 = Q2 + p2(\delta Q2/\delta p2) = 1-p2 + p1/2 - (k1+k2)/2 = 0$$

where profit functions are $\pi 1(p1,p2) = p1\ Q1(p1,p2)$ and $\pi 2(p1,p) = p2\ Q2(p1,p2)$. Solving the above conditions simultaneously, we can obtain optimal prices as

$$p1 = 2/3 + (k1+k2)/3 \tag{1}$$
$$p2 = 4/3 - (k1+k2)/3 \tag{2}$$

which is the same result as Hotelling's.

From the optimal prices dependent upon fixed locations of firms, we can derive the following:

$$\delta p1/\delta k1 = 1/3 > 0, \qquad \delta p1/\delta k2 = 1/3 > 0$$
$$\delta p2/\delta k1 = -1/3 < 0, \qquad \delta p2/\delta k2 = -1/3 < 0$$

These relations show that the optimal price tends to increase when the firm moves toward the other firm which results decrease in the other firm's optimal price under the assumption that rival firm's location is fixed and $k1 < k2$. Moving toward the rival firm gives the firm more market power which results higher optimal price of the firm which may be due to the assumption of extremely inelastic demand.

Let us consider the profit level by plugging the above optimal prices (1) and (2) into profit functions.

$$\pi 1(k1,k2) = (k1+k2+2)^2/18$$

$$\pi 2(k1,k2) = (4-k1-k2)^2/18$$

As we can see from the partial derivation of profit functions with respect to its own location to get the optimal location

$$\delta\pi 1/\delta k1 = (k1+k2+2)/9 > 0$$

$$\delta\pi 2/\delta k2 = -(4-k1-k2)/9 < 0 ,$$

firms tend to move toward rival firms to get higher profits through higher prices.

The above analysis is exactly the same version of spatial model as Hotelling's which is regarded as the simplest one to calculate the results. We could try other assumptions about population distribution function to obtained the implication of population density for the location of firms. However, more general explicit function results so complicated equational forms of optimal conditions that we cannot get the solution without computer use. We can employ computer simulation method in order to solve this problem.

3. Comparative Statics

Since we could not derive any implication on locational problem in general version, comparative statics will be analyzed in this section, through which we can derive some implications on optimal price with respect to location change even if explicit solution cannot be obtained. Define Φ as first order conditions of firm to look for profit maximizing price as shown in equation (13) and (14) of chapter three.

$$\Phi1(p1,p2;k1,k2) = 0$$
$$\Phi2(p1,p2;k1,k2) = 0$$

Solving these conditions to get $p1(k1,k2)$ and $p2(k1,k2)$ and return these solution to the above equations to get

$$\Phi1[p1(k1,k2),p2(k1,k2);k1,k2] = 0$$
$$\Phi2[p1(k1,k2),p2(k1,k2);k1,k2] = 0$$

Rewriting the above as $\Phi(X;\alpha) = 0$ where $\Phi=(\Phi1,\Phi2)'$, $X=(p1,p2)$, and $\alpha=(k1,k2)$. Then we can derive a matrix of comparative static $(\delta X/\delta\alpha)$ from total derivation of $\Phi(X;\alpha) = 0$.

$$(\delta X/\delta\alpha) = - (\Phi x)^{-1} \quad \Phi\alpha \tag{1}$$

where $(\delta X/\delta\alpha) = \begin{pmatrix} \delta p1/\delta k1 & \delta p1/\delta k2 \\ \delta p2/\delta k1 & \delta p2/\delta k2 \end{pmatrix}$

$$\Phi x = \begin{pmatrix} \delta\Phi 1/\delta p1 & \delta\Phi 1/\delta p2 \\ \delta\Phi 2/\delta p1 & \delta\Phi 2/\delta p2 \end{pmatrix}$$

$$\Phi\alpha = \begin{pmatrix} \delta\Phi 1/\delta k1 & \delta\Phi 1/\delta k2 \\ \delta\Phi 2/\delta k1 & \delta\Phi 2/\delta k2 \end{pmatrix} .$$

Since $\Phi x^{-1} = (1/|\Phi x|) \begin{pmatrix} \delta\Phi 2/\delta p2 & -\delta\Phi 1/\delta p2 \\ -\delta\Phi 2/\delta p1 & \delta\Phi 1/\delta p1 \end{pmatrix}$

we have

$$\delta p1/\delta k1 = (1/|\Phi x|)\ [(\delta\Phi 2/\delta p2)(\Phi 1/\delta k1) - (\delta\Phi 1/\delta p2)(\delta\Phi 2/\delta k1)] \tag{2}$$

$$\delta p1/\delta k2 = (1/|\Phi x|)\ [(\delta\Phi 2/\delta p2)(\delta\Phi 1/\delta k2) - (\delta\Phi 1/\delta p2)(\delta\Phi 2/\delta k2)] \tag{3}$$

$$\delta p2/\delta k1 = (1/|\Phi x|)\ [(\delta\Phi 1/\delta p1)(\delta\Phi 2/\delta k1) - (\delta\Phi 2/\delta p1)(\delta\Phi 1/\delta k1)] \tag{4}$$

$$\delta p2/\delta k2 = (1/|\Phi x|)\ [(\delta\Phi 1/\delta p1)(\delta\Phi 2/\delta k2) - (\delta\Phi 2/\delta p1)(\delta\Phi 1/\delta k2)] \tag{5}$$

where $|\Phi x| = (\delta\Phi 1/\delta p1)(\delta\Phi 2/\delta p2)-(\delta\Phi 1/\delta p2)(\delta\Phi 2/\delta p1)$.

We can calculate Φx as following:

$$\delta\Phi 1/\delta p1 = 2(\delta Q1/\delta p1) + (p1-\delta C1/\delta Q1)\ \delta^2 Q1/\delta p1^2 \quad (6)$$

$$\delta\Phi 1/\delta p2 = \delta Q1/\delta p2 + (p1-\delta C1/\delta Q1)\ \delta^2 Q1/\delta p1\delta p2 \quad (7)$$

$$\delta\Phi 2/\delta p1 = \delta Q2/\delta p1 + (p2-\delta C2/\delta Q2)\ \delta^2 Q2/\delta p2\delta p1 \quad (8)$$

$$\delta\Phi 2/\delta p2 = 2(\delta Q2/\delta p2) + (p2-\delta C2/\delta Q2)\ \delta^2 Q2/\delta p2^2 \quad (9)$$

We know that $\delta Q1/\delta p1<0$, $\delta Q1/\delta p2>0$, $\delta Q2/\delta p1>0$, and $\delta Q2/\delta p2<0$ from equations (10-1)-(10-4) in chapter three. We need to know $\delta^2 Q1/\delta p1^2$, $\delta^2 Q1/\delta p1\delta p2$, $\delta^2 Q2/\delta p2\delta p1$, and $\delta^2 Q2/\delta p2^2$ in order to see the sign of the above equations (6)-(9) which are used to determine the sign of equation (2)-(5).

$$\delta^2 Q1/\delta p1^2 = \int_0^{k1+L^*} (\delta^2 q/\delta p1^2)\ f(x)dx - (3/4c)\ q'\ f$$

$$+ (1/4c^2)\ q\ f' \geq 0 \quad (10)$$

It will be positive when we assume $\delta^2 q/\delta p1^2 \geq 0$ and $f'(k1+L^*) > 0$ since $q'<0$. $q'=\delta q/\delta p'$, q, $f'=\delta f/\delta x$, and f are the ones evaluated at boundary.

$$\delta^2 Q1/\delta p1\delta p2 = (1/4c)\ q'f - (1/4c^2)\ q\ f' < 0 \quad (11)$$

$$\delta^2 Q2/\delta p2^2 = \int_{k1+L*}^{1} (\delta^2 q/\delta p2^2)\ f(x)\ dx$$

$$- (1/4c^2)\ q\ f' \quad (12)$$

$$\delta^2 Q2/\delta p2\delta p1 = (1/4c)q'f + (1/4c^2)\ q\ f' \quad (13)$$

$$\delta^2 Q1/\delta p2^2 = \delta^2 Q2/\delta p1\delta p2 \quad (14)$$

$$\delta^2 Q2/\delta p1^2 = \delta^2 Q1/\delta p1\delta p2 \quad (15)$$

By substituting the above equations (10)-(13) and equations (10-1)-(10-4) in chapter three into equations (6)-(9), we obtain the following:

$$\delta\Phi 1/\delta p1 = 2\int_{0}^{k1+L*} q'f\ dx - (1/c)qf + (p1-MC1)\ [-(3/4c)q'f$$

$$+ (1/4c^2)qf' + \int_{0}^{k1+L*} q''f\ dx\ ] \quad (6)'$$

$$\delta\Phi 1/\delta p2 = (1/2c)qf + (p1-MC1)[(1/4c)\ q'f - (1/4c^2)qf'] \quad (7)'$$

$$\delta\Phi 2/\delta p1 = (1/2c)qf + (p2-MC2)[(1/4c)q'f + (1/4c^2)qf'] \quad (8)'$$

$$\delta\Phi 2/\delta p2 = 2\int_{k1+L*}^{1} q'f\ dx - (1/c)qf + (p2-MC2)\ [-(3/4c)q'f$$

$$- (1/4c^2)qf' + \int_{k1+L*}^{1} q''f\ dx\ ] \quad (9)'$$

From $|\Phi x| = (\delta\Phi 1/\delta p1)(\delta\Phi 2/\delta p2)-(\delta\Phi 1/\delta p2)(\delta\Phi 2/\delta p1)$, it's very difficult to find the sign of $|\Phi x|$ using the above equations, due to extremely complicated functional form. We could find the sign of $|\Phi x|$ with very simplified assumptions.

Let us turn to find $\delta\Phi 1/\delta k1$, $\delta\Phi 1/\delta k2$, $\delta\Phi 2/\delta k1$, and $\delta\Phi 2/\delta k2$ using equations (8)-(10) and (13)-(14) in chapter three, they are:

$$\delta\Phi 1/\delta k1 = (1/2)qf + c\int_0^{k1} q'f\,dx - c\int_{k1}^{k1+L*} q'f\,dx$$
$$+ (p1-MC1)\,[(3/4)q'f - (1/4c)\,q\,f'$$
$$+ c\int_0^{k} q''\,f\,dx - c\int_{k1}^{k1+L*} q''\,f\,dx\,] \tag{17}$$

$$\delta\Phi 1/\delta k2 = (1/2)\,q\,f + (p1-MC1)\,[(1/4)q'f - (1/4c)qf'] \tag{18}$$

$$\delta\Phi 2/\delta k1 = -(1/2)qf - (p2-MC2)\,[(1/4)q'f - (1/4c)\,qf'] \tag{19}$$

$$\delta\Phi 2/\delta k2 = -(1/2)qf + c\int_{k1+L*}^{k2} q'f\,dx - c\int_{k2}^{1} q'f\,dx$$
$$+ (p2-MC2)\,[-(3/4)\,q'f - (1/4c)\,q\,f'$$
$$+ c\int_{k1+L*}^{k2} q''f\,dx - c\int_{k2}^{1} q''f\,dx\,] \tag{20}$$

Substituting equations (17)-(20) and (6)'-(9)' along with Φx into equations (2)-(5), we can figure out the sign of these equations, theoretically, which are used to interpret the comparative statics.

Unfortunately, however, we cannot find the sign of the comparative statics due to many complexities of general implicit functional forms, even though generalization results in consistent implications on pricing policies for spatial duopoly firms pursuing various objectives with different operation regimes. As a result, general assumptions fail to give some implications on the locational context since location choice is much affected by such specific situations faced by each firm as population distribution and cost functions. Under several specific assumptions and with the help of a computer, we can find the sign of these equations.

As we can see from the equations, population distribution seems to have more effect on the determination of their signs than cost functions since we know that price is greater than marginal cost in the spatial context. Thus we can guess that the slope of population distribution around the market boundary has a critical role in determining the sign of the equations. This problem can be solved by the computer simulation method with various assumptions on population distribution.

NOTE

1) Solving $\Phi(X,\alpha) = \emptyset$, we have a solution $X(\alpha)$. Substituting $X(\alpha)$ back into $\Phi(X,\alpha)=\emptyset$ yields $\Phi[X(\alpha),\alpha] = \emptyset$. Total derivation of this function with respect to α is:
$\Phi x(\delta X/\delta\alpha) + \Phi\alpha = \emptyset$. Thus $(\delta X/\delta\alpha) = - (\Phi x)^{-1} \Phi\alpha$.

VI. A SUMMARY AND CONCLUSIONS

Due to transportation (delivery) cost, the price that consumers pay for a homogeneous product may vary by location), even if factory prices are identical. Thus, the concept of perfect competition in classical microeconomics will not exist in a spatial context.

Spatial concerns have been included in many papers. The Hotelling-Smithies type of models are considered to be reasonable since those models assume that the rival firms' behavior will be Cournot-type reaction. The other groups of models do not sufficiently consider the behavior of rival firms.

There still does not exist an acceptable generalized spatial competition model. Capozza and Van Order tried to achieve this through differences in conjectural variations; however, they failed to completely generalize because they maintained the assumptions of the specific models except for non-zero identical cost functions and linear demand curves. Given that results from explicit models can vary depending upon the assumptions used, we need a generalized model in order to get consistent results.

This study tried to analyze a generalized version of spatial competition for duopoly firms with entry barriers. Imperfect competition is more common in the real world due to such entry barriers as large fixed costs, scale economies, sunk cost, regulations, etc. We assumed heterogenous cost functions which were implicitly given, an arbitrary population distribution function, and an endogenous demand which was derived by analyzing consumer behavior. We analyzed the spatial price competition between firms with fixed locations by assuming Cournot-Nash behavior, considered welfare implications and profit maximizing behavior, and compared the operational efficiency of autonomous operation by individual firms versus joint operation by a single authority.

From the analysis of profit maximizing behavior by duopoly firms, we found that the optimal price level under joint operation by a single authority is higher than that from an autonomous operation, and that the maximum profit level achieved is greater for joint operation than for autonomous operation. If firms act to maximize social welfare, then the resulting optimal price is lower than if profit maximization is sought--regardless of the operational type. The profit level achieved from social welfare maximizing behavior is bounded at zero; thus, it results in less profit than does profit maximizing behavior.

There is an inverse relationship between the two firms' prices if they are operated by a single authority whose objective is to maximize social welfare. If a firm's price is raised in joint operation, relative to its price in autonomous operation ($p1w^{**} > p1w^{*}$), then the price of the other firm is decreased ($p2w^{**} < p2w^{*}$). Whether the price of a firm will be increased or decreased from the autonomous level depends on the relative change in profit, measured in welfare terms, versus the change in social welfare, evaluated at the optimal price for social welfare maximization that exists under autonomous operation. In other words, starting from the equilibrium price that maximizes social welfare under autonomous authority, if increasing a firm's price results in a welfare decrease that exceeds a profit increase (measured in welfare terms), then increasing the price of that firm will increase the social welfare under a single authority. Due to the zero profit constraint of the social welfare maximization problem, the price of the rival must then be decreased. This relationship was shown in propositions 3 and 4 in chapter three.

Social welfare is lowest when the two firms pursue profit maximization and are jointly operated by a single authority. Therefore, operational efficiency is maximized under joint operation, regardless of the operational type, because it can avoid price competition. However, when the firms are pursuing profit maximization, joint operation is not desirable because increases in the firms' benefits result from reductions in consumers' satisfaction.

The investment problem for spatial duopolists that expect demand increases was analyzed as an extension of the basic model. Firms must choose a combination of price increases and capacity expansion in order to satisfy the increased demand. The solution to this problem under price competition between spatial duopoly firms was given in chapter four. The calculus of variations led to the optimal combination of price and capital stock level. The optimal investment plan could then be established by identifying the corresponding changes in the planned capital. When we assumed that the cost functions are linear with respect to their capital stock or investment level, then the optimality condition simplified from a dynamic differential equation system to a static equation. This made the problem easier to solve. We then used a simple numerical example to show an explicit result and to derive its implications. We assumed a uniform population distribution, perfectly inelastic individual demand, unitary transportation cost, two firms located at 0.4 and 0.8 within a linear market range [0,1], identical maintenance, production, and investment costs for each firm, and a constant-returns-to-scale technology that depended only on the capital input.

Several conclusions were derived. The market boundary [(p2-p1)/2 + (k1+k2)/2] was not at the mid-point between the firms' locations--this result was intuitively unexpected. If firms are located unsymmetrically, as in our example (k1=0.4, k2=0.8), then p1 exceeds p2 due to the larger market power of firm F1 that resulted from its locational advantage. Price inequality causes the market boundary to deviate from the mid-point toward the firm with the higher price. If firms are located symmetrically in a linear space, then the price for both firms will be equal. This is due to the assumptions of uniform distribution of population, perfectly inelastic individual demand, and identical costs. This also results in the market boundary remaining at the mid-point. We considered a series of demand changes, and derived the optimal paths for the capital level and prices. The capital stock level tended to increase as demand increased, and price increases depended upon demand increases for both firms, as shown in equations (27) and (28) in chapter four.

The locational behavior of spatial duopoly firms was analyzed as a second extension of the basic model--previously, firms' locations were assumed to be fixed in advance. Due to complexities involved with generalizing the assumptions and with using implicit functional forms, we could not derive the locational implications of this model. Comparative static analysis also failed to provide any locational implications unless explicit functional forms for the demand and cost functions were used.

This study can be differentiated from the previous studies on spatial models in many respects. We assumed: i) heterogeneous cost functions, implicitly given, while the previous studies usually assumed identical zero cost or linear cost functions, ii) an arbitrary population distribution, while the others used uniform population distribution, iii) a general form for the individual demand function, derived from an analysis of consumer behavior, while others used a perfectly inelastic demand or a linear demand function. In addition to these more generalized assumptions, we also considered: iv) spatial price competition between firms within a general equilibrium framework, while the others either completely ignored the spatial price competition or considered it within a partial equilibrium framework, v) the welfare implications for consumers by analyzing firms' social welfare maximizing behavior and their profit maximizing behavior, vi) the relative efficiency between autonomous operation by each firm and joint operation.

From the generalized assumptions of this model, we found consistent implications for a spatial duopoly when there are entry barriers and spatial competition between firms. These results are robust to any kind of demand function, population distribution, and cost function. Due to the complexities of realistic numerical examples, simplifying assumptions and computer use are necessary if one is to attempt an empirical study. Computer simulations based upon various population distributions and firm locations could provide many implications for the locational problem. Moreover, computer simulations based upon production functions with various scale economies could provide additional implications for the capital investment path.

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