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TWO-DIMENSIONAL WATER-SOLID IMPACT

R. Cointe
Ocean Engineering Laboratory
University of California at Santa Barbara
Santa Barbara, California

Present address: Institut Francais du Petrole, Rueil Malmaison, France

ABSTRACT

The general problem of a two-dimensional water-solid impact is considered. The method of matched asymptotic expansions is used to solve the resulting boundary value problem for small penetration depth. The first order solution obtained extends the classical Von Kármán and Wagner theories to non-normal impact and initially curved free surfaces. It accounts for nonlinear features such as the creation of a jet at each water line. The application of the method to the computation of the impact loads exerted by spilling breakers on a marine structure is discussed.

NOMENCLATURE

B	boundary of the structure
c	phase velocity of the spilling breaker
F	impact force per unit length
F_T	total impact force
Fr	Froude number
g	acceleration of gravity
h^*	height of the roller
H	penetration depth (see Figure 3)
ℓ^l (ℓ^r)	x-coordinate of the left (right) water line
ℓ_1	first outer approximation of ℓ^r and $-\ell^l$
L	half wetted width (see Figure 3)
o	much smaller than
O	of order
p	pressure
q	impacting velocity
R_b	radius of curvature of the structure
R_s	radius of curvature of the free surface just prior to impact
S	parametrization of the free surface
t	time (= 0 at the instant of impact)
u	intermediate variable (Eqs. (18) & (19))
V	vertical component of the impacting velocity
x	coordinate along the common tangent at impact
x_s	position of the stagnation point (Eq. (17))
z	coordinate along the common normal at impact
α	angle defining the direction of impact (see Figure 2)
δ	thickness of the jet
Δ	Laplacian (∇^2)
ε	H/L (small parameter)
ε_b	L/R_b
ε_s	L/R_s
ζ_b	height of the crest above the mean water level
η	free surface elevation
η_u	undisturbed free surface
λ	curling factor (Eq. (24))
ξ^l (ξ^r)	z-coordinate of the left (right) water line
ρ	specific mass of the impacting water
ρ_e	specific mass of the water in the roller
ρ_0	specific mass of water
Φ	velocity potential
φ	velocity potential (Eqs. (16))
∇	gradient

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Subscripts

x, z, and t subscripts indicate differentiation

Other Symbols

The following definitions apply to any dimensional variable a :

$\bar{a}$	non-dimensional variable (Eq. (6))
$\tilde{a}$	outer domain variable (Eq. (11))
$\hat{a}$	inner domain variable (Eq. (15))

These symbols are not used on subscripts indicating differentiation.

INTRODUCTION

The problem of water impact has been the subject of numerous experimental and theoretical works in the past sixty years. Von Kármán [1] and Wagner [2] were the first to study it in the early thirties in order to estimate the loads acting on a landing seaplane. Since this period, their method, where potential flow theory is used, has been refined to tackle related problems. Among those, two are relevant to the study of hydrodynamic loadings on ocean structures.

The first one, slamming, concerns the impact of the forebody of ships or floating ocean structures on the sea surface. It may cause sudden and instant loads sufficient to cause serious structural damages. The links between slamming and the water entry problem have been pointed out early. They are both related to rapid changes of the added mass of the submerged part of the structure due to large motions at the water line. The classical Von Kármán's formula [1], as modified by Wagner [2] to take into account the piled-up water, is generally well suited to study this problem, except for very flat structures where air-cushioning may be important or at the very beginning of impact. An extensive review on the subject has been given by Szebehely and Ochi [3]. Recent works include mathematical studies of the flow for the impact of a cylinder [4] or a wedge [5] and extensions of the original works of Von Kármán and Wagner to three-dimensional geometries ([6], [7]).

The second one, green seas impact, concerns the hydrodynamic impact events caused by waves (broken or unbroken) hitting parts of a marine structure. This problem has been studied in the past essentially to compute the loads exerted on coastal structures for which a few measurements exist. If these experiments show that impact forces are greater for breaking waves than for non-breaking waves, the relation between the breaker type and the magnitude of the impact force has still to be clarified ([8], [9], [10], [11], [12]). Wave impact on offshore structures has been studied essentially for small tubular members in the splash zone ([13], [14], [15], [16]) or spherical buoys [17]. However, its importance might be considerable for the stability of a damaged semi-submersible platform in heavy weather [18]. Similarly, wave impingement on decks or superstructures is thought to be the most common source of heavy-weather damage for ships and it has recently been concluded in Norwegian studies of ship loss in the North Sea that a breaking wave is virtually always responsible for capsizing. Despite the considerable importance of this problem, the literature search revealed little.

Twenty five years ago, Cumberbatch [19] studied the impact of a wedge of water on a flat wall, showing at least the mathematical difficulty of the problem of a mass of water impacting upon a structure. In general, the proper wave kinematics should be taken into account, demanding a proper modelization of the impinging waves. Then, a detailed description of the impact phenomenon should be made in order to predict the loads. Following the development of numerical methods to solve Laplace's equation with full nonlinear free surface boundary conditons, some progress have been accomplished toward the modelisation of nonlinear two-dimensional fluid-solid potential problems (e.g. [20]). Attemps have also been made to study similar problems by solving numerically the Navier-Stokes equations [21]. Even if these techniques increase the understanding of the flow kinematics, the physical nature of the phenomenon remains obscure.

The goal of the present study is to propose a first simple physical and mathematical model for the impact of spilling breakers on a structure. The model for spilling breakers (Tulin & Cointe [22]) is first rapidly described. The application of these results to the impact problem leads to consider the impact of a rigidly moving mass of water on a structure. This problem is solved in two-dimensions for small penetration using the method of matched asymptotic expansions and extending the results of Armand & Cointe [4]. The theoretical results obtained are compared with experiments and empirical formulae used in practice.

THE MODEL FOR STEADY SPILLING BREAKERS

The physical and mathematical modelling of "steady" spilling breakers (as produced by a towed hydrofoil ([23], [24])) has been approched in steps within the last fifteen years. The most important characteristic of a spilling breaker is that it carries along a mass of water, the roller, at its phase velocity, c, see Figure 1. Recently, the roller has been modelled as a low energy recirculating eddy dominated by boundary shear stress (imposed by the wave underneath) and gravity [22], leading to good agreement with experimental data. Behind the roller, a viscous wake extends, as originally pointed out by Banner

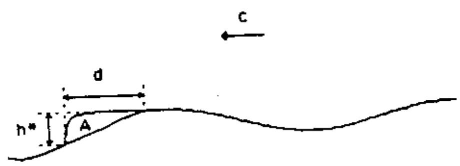

Figure 1 - A steady spilling breaker (schematic)

& Phillips [25] and measured by Duncan [23]. This model shows how the wave is sharply reduced by the roller; the conversion of a steep gravity wave to a weak one by a breaker bearing analogy to the energy loss in a strong shock wave in a compressible fluid.

In connection with the impact problem, it is very important to notice that, within the roller, the velocity of the particles is almost equal to c; but within the wave underneath, this velocity is much smaller than c. The maximum impact force being roughly proportional to the square of the velocity of the impacting particles, this explains why a spilling breaker can induce impact loads which are much greater than those produced by unbroken waves. Due to this difference in velocity, it seems legitimate to assume that impact loads are essentially due to the impact of the roller on the structure. In order to predict these loads, one should first evaluate the geometry of the roller, i.e. the mass of "green-sea" water involved, and then make a model for the impact of the roller. In the case of deep water steady spilling breakers, the model of Tulin & Cointe [22] gives the geometrical properties of the roller (height, length, area, etc...) as a function of the energy which is given to the wave (for a given phase speed c). The problem therefore reduces to the study of the impact of the roller, which has initially a rigid body motion, on a structure.

THE TWO-DIMENSIONAL IMPACT PROBLEM

Formulation of the Problem

In order to be able to obtain an analytical solution giving some insight into the problem, a few simplifying hypothesis are going to be made. In the absence of data relating the geometry of the wave, the wave kinematics and the impact loads, some of these hypothesis cannot be justified without further experiments. Nevertheless they lead to a first reasonable approximation of the problem and it seems fruitful to develop theory at least to a certain point, as such development leads to emphasis on certain aspects of the flows which might later guide experimentation.

The study will be limited to two-dimensional impacts. The fluid will be supposed incompressible and the structure rigid. Viscous effects and air-cushioning will be

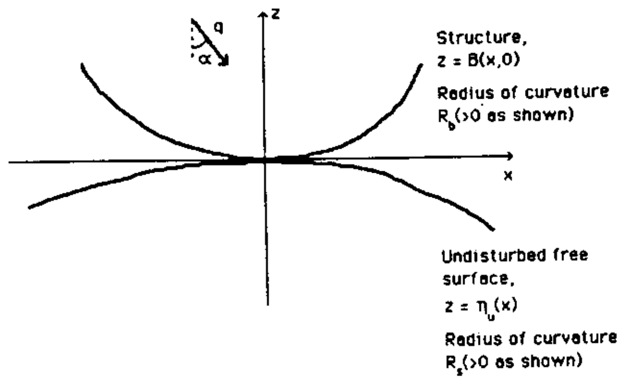

Figure 2 - Geometric definitions just prior to impact ($t=0^-$)

neglected. Finally the surface of the structure and of the fluid prior to impact will be assumed "smooth" in the vicinity of the point of contact. This in particular excludes the case of a wedge, both for the fluid and the structure.

We consider the general case of an impact at constant velocity q making an angle α with the common normal at the instant of impact (see Figure 2). The x-axis of the coordinate system is taken equal to the common tangent at the instant of impact. The z-axis is taken passing through the point attached to the structure which coincided with the point of contact at the instant of impact. In this coordinate system, the structure is moving downward at the velocity $q \cos\alpha \equiv V$ and the fluid leftward at the velocity $q \sin\alpha$ (Figure 3).

The equation of the free surface is given in implicit form by

$$S(x,z,t) = 0 \tag{1}$$

and, at t = 0 (corresponding to the instant of impact),

$$S(x,z,0) = z - \eta_u(x) = 0, \tag{2}$$

where $\eta_u(x)$ is the elevation of the free surface just prior to impact. Similarly, the surface of the structure is given by

$$z = B(x,t) \tag{3}$$

Outer Equations

In order to find the outer solution we first assume that the free surface can be parametrized by

$$S(x,z,t) = z - \eta(x,t) = 0. \tag{4}$$

The intersections between the structure and the fluid are given on the left and on the right by

$$x = \ell^l(t), \quad z = \xi^l(t) = \eta(\ell^l(t),t) = B(\ell^l(t),t) \tag{5a}$$

$$x = \ell^r(t), \quad z = \xi^r(t) = \eta(\ell^r(t),t) = B(\ell^r(t),t) \tag{5b}$$

We then define H, the penetration depth, and L, half the wetted width (see Figure 3). The actual relation between H and L, as yet unknown, will be derived later (see Eq. (13)). L will be chosen as length scale, V as velocity scale and H/V as time scale. This means that the outer solution will be expected to be valid in a region of width of order L. Outside of this region, we assume that the flow is undisturbed. This in particular implies that the depth of the fluid has to be at least of order L for the results to apply. The non-dimensional variables are therefore given by

$$\bar\phi = \phi/VL;\ \bar x = x/L;\ \bar z = z/L;\ \bar t = Vt/H;$$
$$\bar\eta = \eta/L;\ \bar B = B/L, \text{ etc...} \tag{6}$$

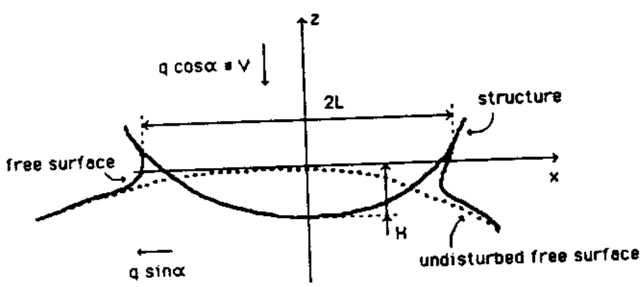

Figure 3 - Geometric definitions after impact (outer domain)

Defining $\varepsilon \equiv H/L$ and $Fr^2 \equiv V^2/gH$, we obtain the non-dimensional equations for the problem :

$$\Delta\bar\phi = 0 \quad \text{in the fluid domain} \tag{7a}$$

$$\bar\eta_t + \varepsilon\,\bar\eta_x\,\bar\phi_x - \varepsilon\,\bar\phi_z = 0$$
$$\bar z/Fr^2 + \bar\phi_t + 1/2\,\varepsilon\,\nabla\bar\phi.\nabla\bar\phi = 0$$
$$\text{for } \bar z = \bar\eta(\bar x,\bar t);\ \bar x \notin [\bar\ell^l, \bar\ell^r] \tag{7b}$$

$$\bar B_t + \varepsilon\,\nabla(\bar B - \bar z)\,.\,\nabla\bar\phi = 0$$
$$\text{for } \bar z = \bar B(\bar x,\bar t);\ \bar x \in [\bar\ell^l, \bar\ell^r] \tag{7c}$$

$$\bar\phi_x \to -\tan\alpha;\ \bar\phi_z \to 0;\ \bar\phi_t \to 0 \quad \text{at infinity} \tag{7d}$$

$$\bar\phi(\bar x,\bar z,0) = -\tan\alpha\ \bar x;\ \bar\eta(\bar x,0) = \bar\eta_u(\bar x) \tag{7e}$$

Perturbation Scheme

In order to simplify these equations, we introduce R_b, the radius of curvature of the structure, and R_s, the radius of curvature of the free surface prior to impact, both taken at the point of contact. The sign convention for these radii is shown in Figure 2. We define $\varepsilon_b \equiv L/R_b$ and $\varepsilon_s \equiv L/R_s$ and we assume that ε_b and ε_s are of the same order of magnitude or smaller than ε. The linearized solution will be found in the limit where ε goes to zero.

We will assume that he boundary of the structure is given by

$$\bar B = -\varepsilon\,\bar t + \varepsilon_b/2\,\bar x^2 + o(\varepsilon) \tag{8}$$

and the undisturbed free surface by

$$\bar\eta_u = -\varepsilon_s/2\,(\bar x - \varepsilon\tan\alpha\,\bar t)^2 + o(\varepsilon) \tag{9}$$

As long as $\tan\alpha \ll 1/\varepsilon$, the horizontal motion of the free surface can be neglected at first order in Eq. (9). The assumptions made in order to linearize the problem are therefore :

$$0 \le \varepsilon \ll 1;\ |\varepsilon_b|/\varepsilon \le O(1);\ |\varepsilon_s|/\varepsilon \le O(1);$$
$$Fr \gg 1;\ \tan\alpha \ll 1/\varepsilon \tag{10}$$

Linearized Outer Solution

In order to find the linearized solution of the problem, a change of variable has to be made to fix the position of the water lines and to allow for a self-similar solution. The new variables are :

$$\tilde x = (2\bar x - \bar\ell^r - \bar\ell^l)\,/\,(\bar\ell^r - \bar\ell^l) \tag{11}$$
$$\tilde z = (2\bar z - \bar\xi^r - \bar\xi^l - (\bar\xi^r - \bar\xi^l)(2\bar x - \bar\ell^r - \bar\ell^l)/(\bar\ell^r - \bar\ell^l))\,/\,(\bar\ell^r - \bar\ell^l)$$
$$\tilde\eta = 2\,(2\bar\eta - \bar\xi^r - \bar\xi^l - (\bar\xi^r - \bar\xi^l)(2\bar x - \bar\ell^r - \bar\ell^l)/(\bar\ell^r - \bar\ell^l))\,/\,(\bar\ell^r - \bar\ell^l)^2$$
$$\tilde t = \bar t\,;\ \tilde\phi = 2\bar\phi\,/\,(\bar\ell^r - \bar\ell^l)$$

The square in the right hand term of (11c) is introduced in order to get a self-similar solution at first order, i.e. a solution independent of time in the ~ variables. A similar change of variable, but in the symmetric case, was introduced by Armand and Cointe [4]. The first order self-similar solution is found here following the same method which is a regular perturbation scheme applied on the ~ equations. It is given by the unbounded flow without circulation around a flat plate of width $\bar\ell^r - \bar\ell^l$ and equal to the undisturbed flow at infinity. The positions of the water lines and the elevation of the free surface are given at first order by :

$$\bar{\ell}^r \approx \bar{\ell}_1 = (4\,\varepsilon\,\bar{t}\,/\,(\varepsilon_b+\varepsilon_s))^{1/2}\ ;\ \bar{\ell}^l \approx -\,\bar{\ell}_1 \qquad (12)$$

$$\bar{\eta} \approx 2\,\varepsilon\varepsilon_b\,\bar{x}^2/(\varepsilon_b+\varepsilon_s)\ -\ 2\,\varepsilon\,|\bar{x}|(\bar{x}^2-\bar{\ell}_1{}^2)^{1/2} - \varepsilon\,\bar{\ell}_1{}^2$$

Since L has to scale half the wetted width, we choose

$$\bar{\ell}_1 = 1 \text{ for } \bar{t} = 1 \text{ or } \varepsilon = (\varepsilon_b+\varepsilon_s)\,/\,4 \qquad (13)$$

which explicits the relation between the penetration depth, H, and half the wetted width, L. Neglecting the piled-up water, we would obtain, at this order, $\varepsilon \approx (\varepsilon_b+\varepsilon_s)/2$. The effect of the splash is therefore to double the wetted depth, as in the case of the impact of a circular cylinder on a flat free surface. ε has to be positive and Eq. (10) is satisfied except when $\varepsilon_b \approx -\varepsilon_s$, i.e. when the structure and the free surface are almost superimposable.

From Eqs. (8), (9) and (12b), we get at first order

$$\int_{body} (\bar{\eta}_u - \bar{B})\,d\bar{x} = \int_{free\ surface} (\bar{\eta} - \bar{\eta}_u)\,d\bar{x} =$$
$$= (\varepsilon_b + \varepsilon_s)\,\bar{\ell}_1{}^3\,/\,12 \qquad (14)$$

This means that the volume of water above the undisturbed free surface is exactly equal to the volume of water displaced by the impacting structure. In other words, the present solution conservs the mass of water.

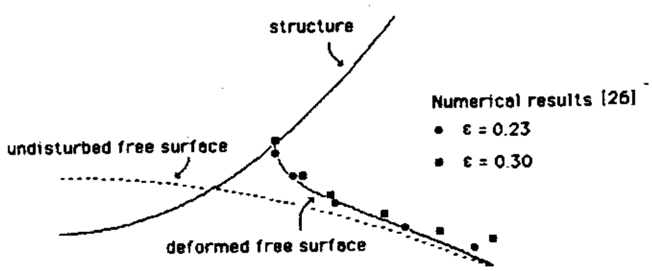

Figure 4 - Outer domain free surface elevation (R_b = 0.216 m, R_s = 0.788 m, V = 5.15 m/s)

The solution for the free surface elevation (Eq. (12b)) is plotted in Figure 4 together with numerical results obtained by Nichols and Hirt [26] using the SOLA-SURF code. Nichols and Hirt calculated the deformation after impact of a fluid layer which had initially a cosine shape. As stated before, our computation is supposed to be valid in this case as long as the ratio of the wetted width to the thickness of the layer is of order 1 or smaller. Since the thickness of the layer was equal in this case to the cylinder diameter, this ratio is of order ε. The agreement between the two computations is excellent. The relative discrepancy far from the water line comes from our approximation of the cosine by a parabola (see Eq. (9)). Near the water line, Nichols and Hirt had to apply a pressure before the actual contact between the fluid and the structure in order to avoid a singularity at this point. A similar problem appears in our outer solution at the water line. A careful look at the equations near this singularity will show that it leads to the formation of a small jet (absent from Nichols and Hirt computations due to their smoothing procedure).

Nonlinear Inner Solution and Matching

The outer solution being given, at first order, by the unbounded flow around a flat plate, it is singular in the vicinity of the water lines. The quadratic term in the pressure becomes as important as the linear term in a domain of width ε^2 (as shown by Eq. (21)). Since the problem is, at first order, symmetric, we only define the inner variables for the right water line :

$$\hat{x} = (\tilde{x} - 1)\,/\,\varepsilon^2;\ \hat{z} = \tilde{z}\,/\,\varepsilon^2;\ \hat{t} = \bar{t} \qquad (15a)$$

$$\hat{\Phi} = \tilde{\Phi}\,/\,\varepsilon;\ \hat{S} = S\,/\,(\varepsilon^2 L\,\bar{\ell}) \qquad (15b)$$

With the change of variable $\hat{\varphi} = \hat{\Phi} - d\bar{\ell}_1/d\bar{t}\ \hat{x}$, the inner problem is given at lowest order by :

$$\Delta\hat{\varphi} = 0 \qquad (16a)$$

$$\nabla\hat{\varphi}\,.\,\nabla\hat{S} = 0$$
$$\nabla\hat{\varphi}\,.\,\nabla\hat{\varphi} = (d\bar{\ell}_1/d\bar{t})^2 \quad \text{for } \hat{S}(\hat{x},\hat{z},\hat{t}) = 0 \qquad (16b)$$

$$\hat{\varphi}_z = 0 \quad \text{for } \hat{z} = 0 \qquad (16c)$$

This $\hat{\varphi}$-problem, which is a classical jet problem, has been obtained and solved by Armand and Cointe [4] for the impact on an initially flat free surface. It actually corresponds to Wagner's inner solution [2] obtained in 1932, but it differs from the solution derived recently by Watanabe ([5], [7]). However Watanabe's solution, given by the flow below a planning plate, is only an approximation of the inner boundary value problem (it has a free surface downstream).

The solution is given in terms of two unknowns : the thickness of the jet and the position of the stagnation point which are found by matching of the free surfaces (or potentials) and of the loci of the maximum pressures, respectively. This gives :

$$\hat{\delta} = \pi\,\hat{t}\,/\,2 = \pi\,/\,(8\,(d\bar{\ell}_1/d\bar{t})^2)\ ;$$
$$\hat{x}_s = -\,4\,\hat{\delta}\,/\,\pi = -\,2\,\hat{t} \qquad (17)$$

The free surface is given in the inner domain by

$$\hat{x} = \hat{t} / 2 \, (u^2 - 2 \log u + 1) \; ; \; u \in \mathbb{R}^+ \tag{18a}$$

$$\hat{z} = - \hat{t} \, (2 u + \pi/2) \tag{18b}$$

and the pressure by

$$\hat{x} = \hat{t} / 2 \, (- u^2 - 4 u - 2 \log u + 1) \; ; \; u \in \mathbb{R}^+ \tag{19a}$$

$$\hat{p} = p H^2 / (\rho V^2 L^2) = (1 - (u-1)^2/(u+1)^2) / 8 \hat{t} \tag{19b}$$

The pressure in the outer domain is given by

$$\bar{p} = p H^2 / (\rho V^2 L^2) =$$

$$= \varepsilon \, d\bar{\ell}_1/d\bar{t} \, / \, (1-\bar{x}^2)^{1/2} - \varepsilon^2 / (2 \, (1-\bar{x}^2)) \tag{20}$$

The quadratic term in the pressure has been retained because of its importance in the vicinity of the water line. Actually, this expression can be expanded in the inner region to give

$$\bar{p} \approx 2 \, (d\bar{\ell}_1/d\bar{t})^2 \, \{ + (- \hat{\delta} / \pi \hat{x})^{1/2} + \hat{\delta} / \pi \hat{x} \} \tag{21}$$

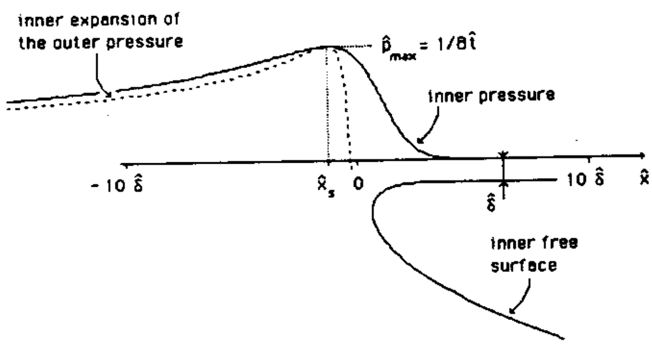

Figure 5 - Inner solution : pressure and free surface elevation

Eqs. (18), (19) and (21) are shown in Figure 5 which actually corresponds to Wagner's Figure 9 [2]. The only difference comes from the position of the stagnation point which is chosen here in order to have the maximum inner and outer pressures reached at the same point. Because of this choice, a uniformly valid expression for the pressure is Eq. (20) for $\hat{x} \leq \hat{x}_s$ and Eqs. (19) for $\hat{x} \geq \hat{x}_s$.

An example of the composite solution is given in Figure 6 where the deformation of a cylinder of water impacting a flat wall is shown. This is somehow similar to Cumberbatch [19] computations for the wedge. Since a uniformly valid approximation is now avalaible, it is possible to find the impact force exerted on the structure at the beginning of impact.

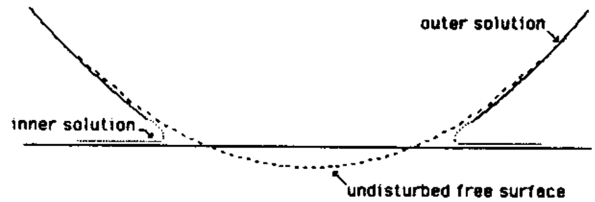

Figure 6 - Impact of a cylinder of water on a flat wall ($L/R_s = 0.5$)

First Order Impact Force

The first order impact force is given by integration of the pressure over the wetted part of the structure. A priori, both the inner and outer pressures should be considered, but at this order only the linear outer pressure gives a non-zero contribution. The impact force is therefore given by

$$F = \pi/2 \, \rho \, V^2 L^2 / H \approx 2\pi \, \rho \, V^2 R_b R_s / (R_b + R_s) \tag{22}$$

In the case where the free surface is initially flat, this leads to the classical Wagner's result [2] which is just twice Von Kármán's result [1]. The impact force is predicted, at this order, to be constant. The rapid decrease in time could be predicted going to next order since the small parameter ε varies like the square root of time. Actually, the integration of the uniformly valid expression for the pressure would lead to this behavior, but the coefficient would be incorrect since it should also include the second order outer problem (see [4] for the case of the impact of a cylinder on a flat free surface). Considering the other approximations made here to study the impact of spilling breakers, it did not seem necessary to go to the next order solution. This should be done numerically in a more refined model. The value given by Eq. (18) should therefore only be considered as a maximum for the impact force. Moreover, due to air-cushioning, the observed maximum impact force can be smaller than the one predicted here.

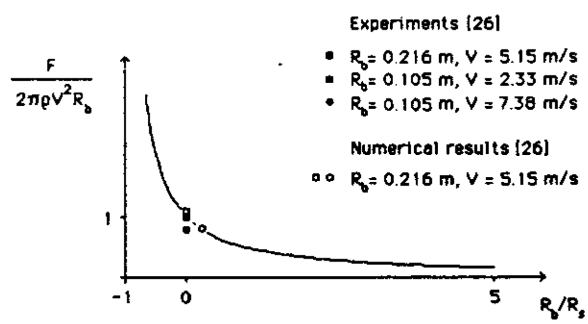

Figure 7 - Effect of the curvature of the free surface on the maximum impact force

The value for the maximum impact force exerted on a structure of radius of curvature R_b has been plotted in Figure 7 as a function of the curvature of the free surface. Some data obtained in experiments and numerical computations performed for the Electric Power Research Institute [26] for an initially flat free surface are also plotted as well as the result corresponding to Nichols and Hirt computation shown in Figure 4 for a cosine shaped free surface. Keeping in mind that the curve shown is only an upper bound, the agreement with these data is excellent. The effect of a positive curvature of the free surface is to reduce the maximum impact force. This might be related to the observation made by Dalton and Nash [14] that waves of longer periods induce larger slam coefficients.

THE IMPACT OF SPILLING BREAKERS

The general results obtained for the two-dimensional impact problem can be applied to the impact of spilling breakers once the geometry of the roller is known. In the absence of experiments relating the geometry of the roller, the wave kinematics and the impact loads exerted, we will only study the impact of spilling breakers on a pile. Even though this problem can only be treated here doing some crude approximations, it should illustrate how similar computations could be made in cases where experiments and theory could be compared more accurately.

For steady deep water spilling breaker, the model of Tulin and Cointe [22] predicts that the height of the roller grows as the square root of the distance from its toe in a narrow zone near the toe. After this short growth region, the top of the roller stays flat. An idealization of this shape is a triangle of height h^*. If we consider the impact of a spilling breaker on a vertical circular cylinder of radius R_b (see Figure 8), we therefore predict in a first (and rather crude) approximation that the maximum impact force is

$$F_T \approx 2\pi \rho_e c^2 R_b h^* \qquad (23)$$

where ρ_e is the specific mass of the water in the roller. This equation is qualitatively equivalent to the one used in practice by coastal engineers and introducing the curling factor, λ ([8], [10], [12]):

$$F_T = \pi \rho_0 c^2 R_b \lambda \zeta_b \qquad (24)$$

where ζ_b is the height of the crest above the mean water level. Both formulae are in quantitative agreement if the curling factor, which is generally found empirically, is given by

$$\lambda = 2 (\rho_e / \rho_0) (h^* / \zeta_b) \qquad (25)$$

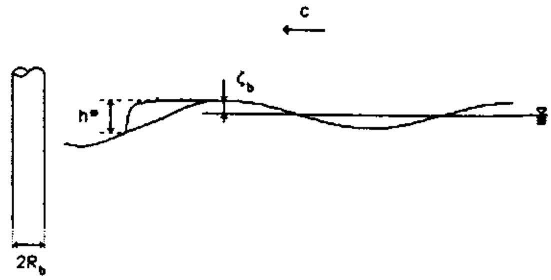

Figure 8 - Impact of a spilling breaker on a vertical pile (schematic)

Using the model of Tulin and Cointe [22] we get for steady deep water spilling breakers

$$\lambda = 2 (\rho_e / \rho_0) (1 / (c^2/2gh^* - 1)) \qquad (26)$$

The value of gh^*/c^2 being of order 0.3 and ρ_e/ρ_0 of order 0.5 [23], we obtain a value for λ around 1.5. Typical values for breakers on a beach range between 0.1 and 0.5, the lowest values corresponding to spilling breakers ([10], [12]). The value obtained here is therefore much higher. However, the geometry of a steady deep water spilling breaker can be very different from the geometry of an unsteady shallow water spilling breaker. In particular, the height above the mean water level is very small for a steady deep water spilling breaker, explaining the relatively high value of λ obtained. Before any conclusion on the validity of the model can be drawn (and other effects such as the compressiblity of the air-water mixture composing the roller and air-cushioning are considered), it seems therefore necessary to perform experiments pertaining to these computations and/or evaluate h^* / ζ_b for a spilling breaker on a beach.

CONCLUSION

A rational method was derived to deal with general two-dimensional fluid-solid impact problems. The basic assumptions made are that the fluid is incompressible, the structure rigid and any air-cushioning absent. This method yields analytical expressions for the pressure distribution and the deformation of the free surface at the beginning of impact (first order solution) within a wide range of impact conditions (given by Eqs. (10)). Realistic nonlinear features such as the presence of a jet at each water line are predicted. Limiting cases include the impact of a rigid cylinder on a flat free surface [4] for which this method agrees with Wagner [2] results and the impact of a cylinder of water on a plane wall.

The main conclusions of this study are :

1/ At first order, only the normal component of the impacting velocity is important, as that has been observed experimentally for the impact of a sphere [6].

2/ The wetting correction, introduced by Wagner

[2], is extremely important and double the effective penetration depth.

3/ The maximum impact pressure is always reached in the vicinity of the water line. It is equal to 1/2 ρ $(d\ell/dt)^2$, where ℓ is half the wetted width of the structure.

4/ The effect of an initial positive curvature of the free surface is to reduce the maximum impact force. For a radius of curvature of the free surface 4 times greater than the radius of curvature of the structure, the reduction reaches 20 % and the present results agree well with numerical computations [26]. Similarly, the effect of an initial negative curvature of the free surface is to increase the maximum impact force which blows up when the structure and the fluid become superimposable.

Finally, it has been shown how these results can be used to provide a first reasonable approximation for the impact loads exerted by spilling breakers on marine structures. The model derived by Tulin and Cointe [22] for steady deep water spilling breakers has been used to get a first quantitative estimate of these loads for a vertical pile. However, even if empirical formulae used by coastal engineers are qualitatively similar to the one derived here, the absence of data pertaining to deep water spilling breakers does not allow the validation of the model. In particular, the importance of air-cushioning and/or the compressiblity of the air water mixture in the roller cannot be estimated. The need for carefully controled experiments concerning the geometry of the roller and the classification of the effects which might be important is apparent. It is hoped that they might be guided by the present model.

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REFERENCES

[1] Von Kármán, T., "The Impact of Seaplane Floats during Landing," NACA TN 321, October 1929.

[2] Wagner, H., "Über Stoß und Gleitvorgänge an der Oberfläche von Flüssigkeiten," ZAMM, Vol. 12, 1932, pp. 193-215.

[3] Szebehely, V.G., and Ochi, M.K., "Hydrodynamic Impact and Water Entry," Applied Mechanics Survey, H.N. Abramson et al. Eds., Spartan, Washington, D.C., 1966, pp. 951-957.

[4] Armand, J.-L., and Cointe, R., "Hydrodynamic Impact Analysis of a Cylinder," Proceedings, 5th OMAE Symposium, Tokyo, 1986, Vol. 1, pp. 250-256 (also to appear in the ASME J. Offshore Mechanics and Arctic Engineering, 1987).

[5] Watanabe, I., "Analytical Expression of Hydrodynamic Impact Pressure by Matched Asymptotic Expansion Technique," Trans. West-Japan Soc. Naval Arch., Vol. 71, 1986, pp. 77-85.

[6] Troesh, A.W., and Kang, C.G., "Hydrodynamic Impact Loads on Three-Dimensional Bodies," Proceedings, 16th Symposium Naval Hydrodynamics, Berkeley, Ca., July 1986.

[7] Watanabe, I., "Theoretical Investigation on the Wave Impact Loads on Ships," Proceedings, 16th Symposium Naval Hydrodynamics, Berkeley, Ca., July 1986.

[8] Goda, Y., et al., "Study of Impulsive Breaking Wave Force on Piles," Report of Port and Harbour Research Institute, Vol. 5, April 1966 (in Japanese).

[9] Miller, R.L., et al., "Field Measurements of Impact Pressures in Surf," Proceedings, 14th Coastal Eng. Conf., Copenhagen, Denmark, 1974, pp. 1761-1777.

[10] Wiegel, R.L., "Forces Induced by Breakers on Piles," Proceedings, 18th Coastal Eng. Conf., 1982, pp. 1699-1715.

[11] Ochi, M.K., and Tsai, C.-H., "Prediction of Impact Pressure Induced by Breaking Waves on Vertical Cylinders in Random Seas," Applied Ocean Research, Vol. 6, 1984, pp. 157-165.

[12] Tanimoto, K., et al., "Impact Force of Breaking Waves on an Inclined Pile," Proceedings, 5th OMAE Symposium, Tokyo, 1986, Vol. 1, pp. 235-241.

[13] Kaplan, P., and Silbert, M., "Impact Forces on Platform Horizontal Members in the Splash Zone," Proceedings, 8th Offshore Technology Conference, OTC Paper 2498, Houston, Texas, 1976.

[14] Dalton, C., and Nash, J., "Wave Slam on Horizontal Members of an offshore Platform," Proceedings, 8th Offshore Technology Conference, OTC Paper 2500, Houston, Texas, 1976.

[15] Faltinsen, O., et al., "Water Impact Loads and Dynamic Responses of Horizontal Circular Cylinders in Offshore Platforms," Proceedings, 9th Offshore Technology Conference, OTC Paper 27-i, Houston, Texas, 1977.

[16] Sarpkaya, T., "Wave Impact Loads on Cylinders," Proceedings, 10th Offshore Technology Conference, OTC Paper 3065, Houston, Texas, 1978.

[17] Miloh, T., "Wave Slam on a Sphere Penetrating a Free Surface," J. Eng. Mathematics, Vol. 15, 1981, pp. 221-240.

[18] Numata, E., et al., "Assessment of Stability Requirements for Semisubmersible Units," Proceedings, Annual Meeting of the SNAME, N.Y., 1976.

[19] Cumberbatch, E., "The Impact of a Water Wedge on a Wall," J. Fluid Mech., Vol. 7, 1960, pp. 353-374.

[20] Vinje, T., and Maogang, X., "A Numerical Approach to Nonlinear Ship Motion," Proceedings, 14th Symposium Naval Hydrodynamics, Washington, D.C., 1982.

[21] Miyata, H., et al., "Nonlinear Wave-Induced Forces on a Shallowly Submerged Body," Proceedings, 5th OMAE Symposium, Tokyo, 1986, Vol. 1, pp. 218-224.

[22] Tulin, M.P., and Cointe, R., "On Spilling Breakers," Proceedings, 16th Symposium Naval Hydrodynamics, Berkeley, Ca., July 1986.

[23] Duncan, J.H., "An Experimental Investigation of Breaking Waves Produced by a Towed Hydrofoil," Proc. R. Soc. London, Vol. A 377, 1981, pp. 331-348.

[24] Duncan, J.H., "The Breaking and Non-Breaking Resistance of a Two-Dimensional Hydrofoil," J. Fluid Mech., Vol. 126, 1983, pp. 507-520.

[25] Banner, M.L., and Phillips, O.M., "On the Incipient Breaking of Small Scale Waves," J. Fluid Mech., Vol. 65, 1974, pp. 647-656.

[26] Nichols, B.D., and Hirt, C.W., "Hydrodynamic Impact Analysis," EPRI NP-824, Electric Power Research Institute, Palo Alto, Ca., June 1978.