

Causality versus Serial Correlation: an Asymmetric Portmanteau Test

Amedeo Andriollo

Department of Economics
University of Warwick

Motivation

Economic question: how does economic uncertainty impact a target variable (e.g., inflation).

Common practice: 1) identification via structural dynamic model; 2) IR analysis.

↪ The shock series, $\{X_t\}$, needs to be exogenous also w.r.t. **external/omitted variables**, $\{Z_t\}$.

Econometric question: 'Are the shocks, $\{X_t\}$, **exogenous** fluctuations of uncertainty?'

1. The shocks X are exogenous w.r.t. the past of the internal variables (Ramey, 2016).
2. The external/omitted variables Z do not influence the shocks' exogeneity.

↪ Does the past of omitted variables predict the present of the uncertainty shocks?

Testing Exogeneity

The hypotheses of interest are:

$$\mathcal{H}_0 : \mathbb{E}[X_t | \{X_s, Z_s\}_{s < t}] = 0.$$

$$\mathcal{H}_A : \mathbb{E}[X_t | \{X_s\}_{s < t}] = 0, \quad \text{for some } j > 0 : \mathbb{E}[X_t Z'_{t-j}] \neq 0.$$

I consider the class of testing procedures based on **serial cross-correlation**:

- Correlation between lagged/past omitted variables and present shocks.
- Existing tests: 1) To augment (correct parametriz.); 2) Not to augment (mutual independence).

⇒ Conclusions about exogeneity depends on the impact of past shocks to the omitted variables:
the inverse causality \implies **size distortions**.

This Paper

1. Problem: dynamic (linear) model specification testing in the presence of omitted variables.
2. I propose an asymmetric Portmanteau statistic to test exogeneity:
 - Portmanteau statistic: weighted sums of squared sample cross-correlations at all lags.
 - Asymmetric: easy-to-compute correction term that offsets the influence of inverse causality.
 - ↪ Bypass the modeling of the joint dynamics (= robust to misspecification).
3. I establish the asymptotic normality of the corrected test.
4. I test the exogeneity of popular measures for macroeconomic structural shocks.
 - Baker Bloom Davis (2016)'s Economic Policy Uncertainty (EPU) shocks: **not** exogenous.
 - ↪ Reassessing Diercks Hsu Tamoni (2024): EPU shocks as negative supply shocks.

Simple Setting

- $\{X_t, Z_t; t = 1, \dots, T\}$: zero-mean standardized univariate jointly stationary processes.

- The null hypothesis of interest:

$$\mathcal{H}_0 : \mathbb{E}[X_t | \{X_s, Z_s\}_{s < t}] = 0.$$

- The cross-correlation function:

$$\hat{\Gamma}_{XZ}(j) = \frac{1}{T} \sum_{t=j+1}^T X_t Z_{t-j}, \quad \Gamma_{XZ}(j) = \mathbb{E}[X_t Z_{t-j}], \quad j = 1, \dots, T-1.$$

- Benchmark test: one-sided sum of the weighted squared cross-correlations (Hong, 1996):

$$\mathcal{T}_\omega = \sum_{j=1}^{T-1} \omega(j) \left(\hat{\Gamma}_{XZ}(j) \right)^2.$$

$\{\omega(j)\}$ are nonrandom non-negative weights: kernel function of width $M = M(T)$.

Symmetry of the Norm

- “**Squaring**” the cross-correlation induces cross-products at various time indexes.
- The benchmark statistic: $\mathcal{T}_\omega = \mathcal{T}_{1\omega} + \mathcal{T}_{2\omega}$.

$$\text{“Sum of squares”}: \quad \mathcal{T}_{1\omega} = \frac{1}{T^2} \sum_{j=1}^{T-1} \omega(j) \sum_{t=j+1}^T X_t^2 Z_{t-j}^2$$

$$\text{“Sum of cross-products”}: \quad \mathcal{T}_{2\omega} = \frac{1}{T^2} \sum_{j=1}^{T-2} \omega(j) \sum_{s,t=j+1, s \neq t}^T X_t X_s Z_{t-j} Z_{s-j}$$

↪ The second sum treats the two time indexes, s and t , **symmetrically**.

- Asymptotic properties of the benchmark statistic:
 - i) $\mathcal{T}_{1\omega}$ dominates under the alternatives (power of the test);
 - ii) $\mathcal{T}_{2\omega}$ dominates **under the null hypothesis** (size of the test).

↪ Size distortions because of inverse causality?

Inverse Causality in the Variance

Proposition 1

Let $\{X_t, Z_t\}$ be marginally i.i.d. univariate processes with finite fourth moments.

If X is independent of the past of Z , $X_t \perp\!\!\!\perp \{Z_s, s < t\}$, the variance of an element of $\mathcal{T}_{2\omega}$ is:

$$\mathbb{E}[(X_t X_s Z_{t-j} Z_{s-j})^2] = \begin{cases} 1, & s > t - j \\ \mathbb{E}[Z_{t-j}^2 X_s^2 Z_{s-j}^2], & s \leq t - j \end{cases}$$

If X and Z are mutually independent, $X_t \perp\!\!\!\perp Z_s, \forall s, t$, the variance of an element of $\mathcal{T}_{2\omega}$ is:

$$\mathbb{E}[(X_t X_s Z_{t-j} Z_{s-j})^2] = 1$$

↪ The benchmark test (via $\mathcal{T}_{2\omega}$) **incorporates the inverse causality**, unless:

- Mutual independence.
- **A specific ordering of the time indexes is met:** $s > t - j$.

The Correction Term

- My approach: a correction that **differences out** the terms accounting for the inverse causality.
 \hookrightarrow Remove the terms associated to X happening before Z (when: $s \leq t - j$).
- The **corrected version** of the test statistic, \mathcal{T}_ω^C :

$$\begin{aligned}\mathcal{T}_\omega^C &= \mathcal{T}_\omega - \left(\frac{1}{T^2} \sum_{j=1}^{T-2} \omega(j) \sum_{s, t=j+1, s \leq t-j}^T X_t X_s Y_{t-j} Y_{s-j} \right) \\ &= \mathcal{T}_\omega - \underbrace{\mathcal{C}_\omega}_{\text{Inverse Causality}} = \mathcal{T}_{1\omega} + \underbrace{\mathcal{T}_{2\omega}^C}_{\mathcal{T}_{2\omega} - \mathcal{C}_\omega}\end{aligned}$$

Under the Null

- $\mathcal{I}(t)$: the information set up to period t of the joint time series $\{X_s, Z_s; s < t\}$.
- **Assumption 1** (Weighting function): let $\{\omega(j)\}$ be a function of some sequence of integers $M = M(T)$ for a square-integrable kernel $k(\cdot) : \mathbb{R} \rightarrow [-1, 1]$, s.t.: $\omega(j) = k^2(j/M)$, $k(0) = 1$.

Theorem 1 (Size)

Suppose $\{X_t\}$ is such that:

$$\mathbb{E}[X_t^2 | \mathcal{I}(t-1)] = \mathbb{E}[X_t^2], \quad \mathbb{E}[X_t^4 | \mathcal{I}(t-1)] = \mathbb{E}[X_t^4].$$

Suppose the time series $\{Z_t\}$ is fourth-order stationary with finite eighth-order moments, the joint process $\{X_t, Z_t\}$ is strictly stationary, and Assumption 1 holds with $\frac{M^2}{T} \rightarrow 0$, as $T, M \rightarrow \infty$.

Under the null hypothesis of interest \mathcal{H}_0 , we have:

$$\frac{T \cdot \mathcal{T}_\omega^c - \mu_\omega}{\sqrt{D_\omega^{(Hete)}}} \xrightarrow{d} \mathcal{N}(0, 1).$$

Under the Null (Continued)

- 1) Restrictions on the conditional moments: “*isolating effects rather than causes*”.

Cond. homosk.: **necessary to isolate the mean** (norm); cond. homokur.: standard CLT.

↪ Typically implied by mutual independence.

- 2) Under additional conditions on the joint process, the result holds with $\frac{M}{T} \rightarrow 0$, as $T, M \rightarrow \infty$.

- 3) Theorem 2: X as the residual/innovation from the (causal) parametric models:

$$W_t = \mu_X(\theta_0, \{X_s, s < t\}) + X_t,$$

with \sqrt{T} -consistent estimator of $\{\theta^0\}$ plus additional conditions, the (plug-in) result holds.

The center and the scale of the corrected statistic are:

$$\mu_\omega = \sum_{j=1}^{T-1} \left(1 - \frac{j}{T}\right) \omega(j), \quad D_\omega^{(Hete)} = \frac{1}{T^2} \sum_{j=1}^{T-1} \omega^2(j) \sum_{s,t=j+1, s>t-j}^T \mathbb{E}[Z_t^2 Z_s^2].$$

Are the EPU Shocks Exogenous?

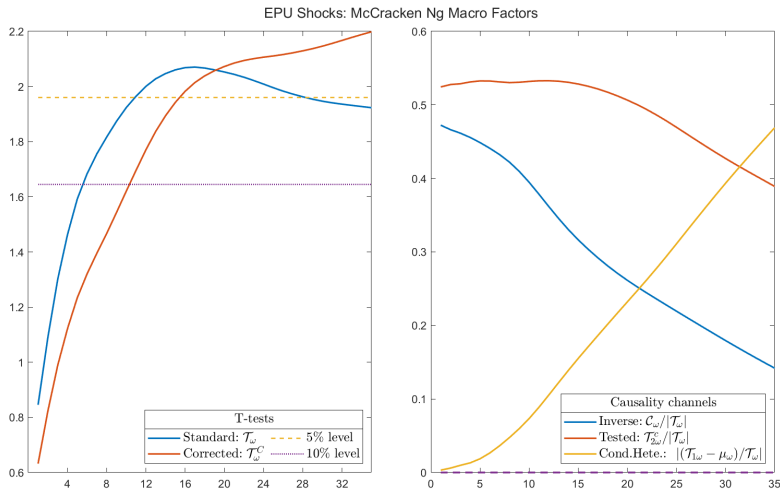
- **Economic Policy Uncertainty (EPU)** index: 1) the monthly frequency of newspaper articles containing terms related to: uncertainty, the economy, and policy; 2) other indicators (tax code expiration, forecaster disagreement).
- Following Baker Bloom Davis (2016), the uncertainty shock series:
 - i) Fit a VAR(3) to monthly data from Jan. 1985 to Dec. 2019.
 - ii) Identification: Cholesky decomposition with the following ordering:
 - 1) EPU index, 2) log of the S&P500 index, 3) fed funds rate, 4) log employment, and 5) log IP.

→ EPU uncertainty shocks: $\{X_t\}$.

- Following Forni and Gambetti (2014), the omitted variables:
The first 8 **principal components** of a dataset that summarizes the relevant macroeconomic/financial information, McCracken and Ng (2016)'s FRED-MD.

→ McCracken and Ng (2016) macro factors: $\{Z_t\}$.

Are the EPU Shocks Exogenous? (Continued)



Left: Orange: Corrected, Blue: Benchmark/Standard. **Right:** Orange: Tested, Blue: Inverse, Yellow: Conditional Hete.

Reassessing Diercks Hsu Tamoni (2024): Economic Effects of Endogeneity

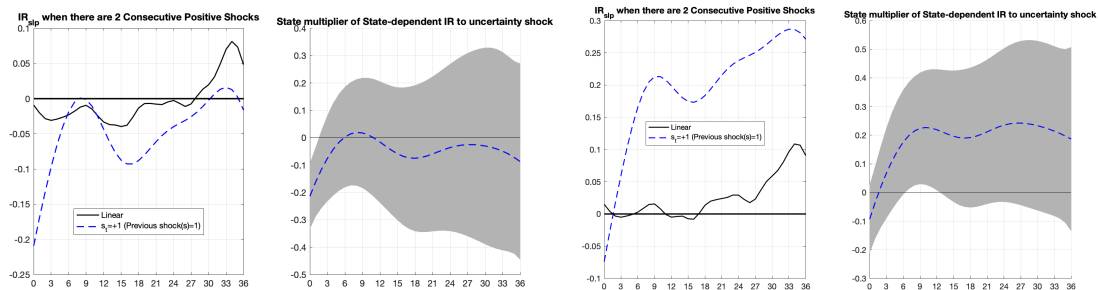


Figure: Response of inflation (PCE index) to consecutive positive EPU uncertainty shocks:

LEFT: Diercks Hsu Tamoni (2024). RIGHT: Adding two lags of McCracken and Ng (2016)'s macro factors.

LEFT PANELS: the empirical state-dependent impulse responses to two consecutive positive uncertainty shocks (dashed blue line) and contrast it to the response to a single shock (solid black line). RIGHT PANELS: the incremental effect of the second shock, with 90% confidence intervals (shaded area). In both panels, on the y-axes, the level of impulse responses; on the x-axes, the horizons, h .

Testing Shock Exogeneity: Summary

[Visualize](#)

		Exogenous	Tested	Cond. Heter.	Omitted factors
Uncertainty	Baker Bloom Davis (QJE 16)	?	★		Macro (McK NG)
				★	Finance (GX)
	Jurado Ludvisong Ng (AER 15)	?	★		Macro (McK NG)
			★		Finance (GX)
	Berger Dew-Becker Giglio (RES 20)	?	★	★	Finance (LP) Finance (GX)
Monetary	Aruoba Drechsel (wp 23)	✓			
	Bu Rogers Wu (JME 21)	✓			
	Miranda-Agrippino Ricco (AEJ 21)	✓			
	Bauer Swanson (NBER 23)	?		★	Finance (GX, LP)
		?	★	★	Macro (McK NG, RZ)
	Jarociński Karadi (AEJ 20)	?		★	Finance (GX, LP)
Carbon/Oil	Känzig (AER 21; wp 23)	✓			

Conclusions

- I offer a strategy to test exogeneity of structural shocks based on serial cross-correlations.
- **Contributions:**
 - New insights about a class of tests (dynamic linear model specification testing).
 - Theory: a correction term to impose directionality to the Portmanteau statistics.
↪ It offsets size distortion due to inverse causality.
 - Empirics: testing the exogeneity of popular measure of macroeconomic shocks.
- **Bottom lines:**
 - The squares incorporate inverse causality.
 - EPU shocks are not exogenous: behave as a negative (superadditive) supply shock.

Thank you for your attention!!

Simulations

- The process X is defined as:

$$X_t = \epsilon_x, \quad \epsilon_x \sim i.i.d.(0, 1)$$

→ The null hypothesis of interest holds true.

- For the process Z , I consider three families of DGPs:

a) DGP 1A: LINEAR-IN-MEAN

$$Z_t = \alpha Z_{t-1} + \beta X_{t-1} + \epsilon_z, \quad \epsilon_z \sim i.i.d.(0, 1)$$

b) DGP 2A: SQUARED-IN-MEAN (in the paper);

c) DGP 3A: SQUARED-IN-VARIANCE (in the paper).

with: $\alpha = \{0.2, 0.3, 0.4, 0.5, 0.6, 0.7\}$, $\beta = \{0, 0.2, 0.4, 0.6, 1, 2\}$.

- The noise are generated by a multivariate t-distribution: $(\epsilon_x, \epsilon_z) \sim t_6(0, I_2)$.
(Brunnermeier Palia Sastry Sims, 2021)

Simulations: Linear-in-mean

Table: Rejection frequencies for DGP1A: This table presents the rejection frequencies of two testing procedure, corrected (\mathcal{T}_ω^c) and benchmark (\mathcal{T}_ω), when the time series are generated by DGP1A; sample size, $T = 700$; 700 iterations; the weighting function is the Bartlett kernel; the smoothing parameter range is: $M = \{12, 30\}$; nominal significance level is 5%.

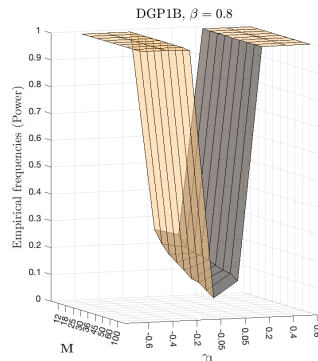
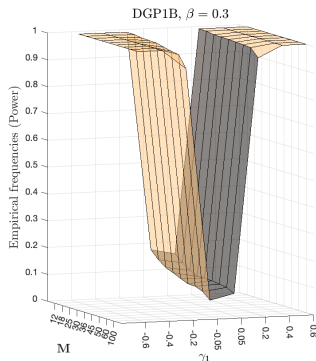
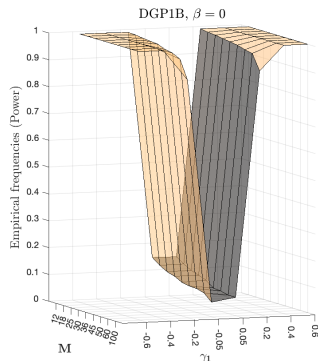
$$M = 12 \quad \approx 2(10T)^{1/5} \quad \approx 2 \ln T$$

	Corrected						Benchmark					
	$\beta = 0$	$\beta = 0.2$	$\beta = 0.4$	$\beta = 0.6$	$\beta = 1$	$\beta = 2$	$\beta = 0$	$\beta = 0.2$	$\beta = 0.4$	$\beta = 0.6$	$\beta = 1$	$\beta = 2$
$\alpha = 0.2$	0.026	0.015	0.01	0.015	0.023	0.019	0.053	0.048	0.039	0.05	0.049	0.046
$\alpha = 0.3$	0.019	0.018	0.019	0.033	0.022	0.025	0.039	0.043	0.068	0.053	0.04	0.05
$\alpha = 0.4$	0.03	0.03	0.019	0.033	0.032	0.019	0.065	0.058	0.056	0.062	0.062	0.045
$\alpha = 0.5$	0.048	0.04	0.023	0.043	0.035	0.036	0.08	0.075	0.055	0.078	0.073	0.063
$\alpha = 0.6$	0.045	0.04	0.052	0.039	0.042	0.038	0.076	0.075	0.089	0.076	0.073	0.083
$\alpha = 0.7$	0.053	0.059	0.055	0.058	0.055	0.063	0.073	0.09	0.09	0.079	0.089	0.1

$$M = 30 \quad \approx 5(10T)^{1/5} \quad \approx \sqrt{T}$$

	Corrected						Benchmark					
	$\beta = 0$	$\beta = 0.2$	$\beta = 0.4$	$\beta = 0.6$	$\beta = 1$	$\beta = 2$	$\beta = 0$	$\beta = 0.2$	$\beta = 0.4$	$\beta = 0.6$	$\beta = 1$	$\beta = 2$
$\alpha = 0.2$	0.016	0.015	0.009	0.009	0.023	0.016	0.055	0.045	0.033	0.045	0.053	0.053
$\alpha = 0.3$	0.02	0.018	0.019	0.028	0.023	0.019	0.052	0.056	0.059	0.063	0.053	0.056
$\alpha = 0.4$	0.028	0.029	0.019	0.039	0.025	0.018	0.072	0.072	0.056	0.075	0.066	0.053
$\alpha = 0.5$	0.04	0.038	0.028	0.043	0.043	0.023	0.085	0.088	0.058	0.093	0.093	0.068
$\alpha = 0.6$	0.048	0.039	0.056	0.052	0.045	0.045	0.095	0.089	0.103	0.108	0.098	0.096
$\alpha = 0.7$	0.053	0.078	0.065	0.068	0.07	0.069	0.13	0.129	0.12	0.123	0.132	0.128

Simulation: Under the alternatives



DGP: $X_t = \gamma_1 Z_{t-1} + \epsilon_x$, $Z_t = 0.4Z_{t-1} + \beta X_{t-1} + \epsilon_z$, $(\epsilon_x, \epsilon_z)' \sim \mathcal{N}(0, I_2)$.
 $\gamma_1 = \{-0.6, -0.4, -0.2, -0.05, 0.05, 0.2, 0.4, 0.6\}$, and $\beta = \{0, 0.3, 0.8\}$.

Guidelines for the practitioner

Two key guidelines for the practitioner about my proposed testing procedure:

1. When to use?

Use the corrected test statistic when the omitted variables Z have some **temporal dependence**.

2. What M to choose?

Given the trade-off between size and power with respect to M (number of cross-correlations),

Prioritize a 'large' smoothing parameter, proportional to the **parametric rate** \sqrt{T} .

(see the bandwidth rule in Hong and Lee (2005))

↪ To avoid the problem of under-sized/low power.

Time series and EPU shocks

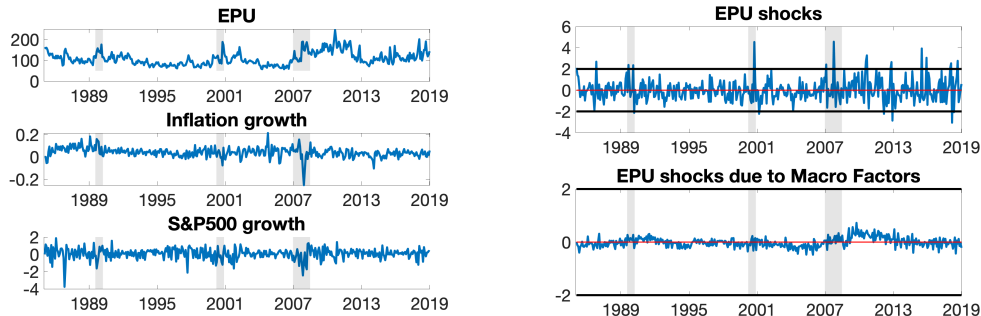


Figure: Time series and shocks:

Parallel to Figure 1 in Diercks et al. (2024), the left panel displays the time series of EPU, together with the time series associated to inflation and stock market in percentage growth (i.e., $(\text{current}/\text{previous} - 1) \times 100$). The right panel displays the the estimated EPU shock series and its part that correlates with the past of the macroeconomic factors. The shaded areas represent NBER (National Bureau of Economic Research) recessions.

EPU shocks: Restoring Exogeneity

- Baker Bloom Davis (2016)'s shock series is not exogenous (fundamental):

Causal inference about uncertainty can benefit by including the macro factors.

↪ What are the economic consequences of the endogeneity?

- Revisit Diercks Hsu Tamoni (2024): '*Are the effects of uncertainty shocks superadditive?*'.
- **Superadditivity**: the effect of positive shock followed by a positive shock in the previous period is amplified (state multiplier).

$$y_{t+h} = \text{const.} + (\beta_{0,h} + \underbrace{\beta_{1,h}}_{\text{state multiplier}} \mathbf{1}\{\epsilon_{t-1}^{EPU} > 0\})\epsilon_t^{EPU} + \text{controls} + u_{t+h}$$

↪ +2 lags of Macro factors

Linear effect of EPU shocks: Inflation

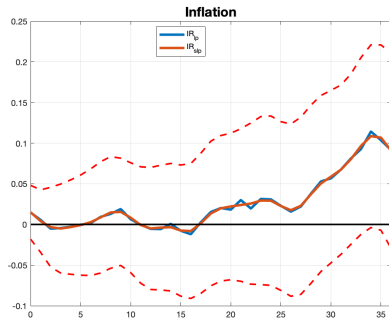
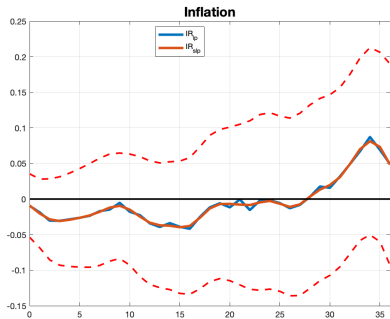


Figure: Linear response of price level to EPU uncertainty shocks::

LEFT: Diercks Hsu Tamoni (2024). RIGHT: Adding two lags of McCracken and Ng (2016)'s macro factors.

The panels show the empirical unconditional impulse responses, i.e. $\{\beta_{0,h}\}_{h=1,\dots,H}$. On the y-axes, the level of impulse responses; on the x-axes, the horizons, h ; solid blue line represents the standard LPs and red solid line represents the Smoothed LPs; dashed red line stands for the 90% confidence intervals.

Superadditivity of EPU shocks: Ind. Production

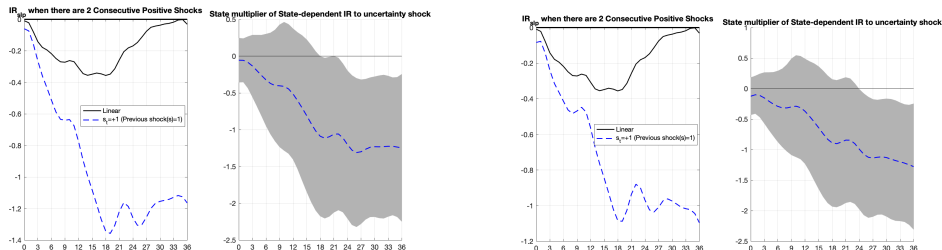


Figure: Response of industrial production to consecutive positive EPU uncertainty shocks:

LEFT: Diercks Hsu Tamoni (2024). RIGHT: Adding two lags of McCracken and Ng (2016)'s macro factors.

LEFT PANELS: the empirical state-dependent impulse responses (estimated with LPs as in Diercks et al. (2024)) to two consecutive positive uncertainty shocks (dashed blue line) and contrast it to the response to a single shock (solid black line). RIGHT PANELS: the incremental effect of the second shock, i.e. $\{\beta_{1,h}\}_{h=1,\dots,H}$, with 90% confidence intervals (shaded area). In both panels, on the y-axes, the level of impulse responses; on the x-axes, the horizons, h .

Superadditivity of EPU shocks: Short Rates

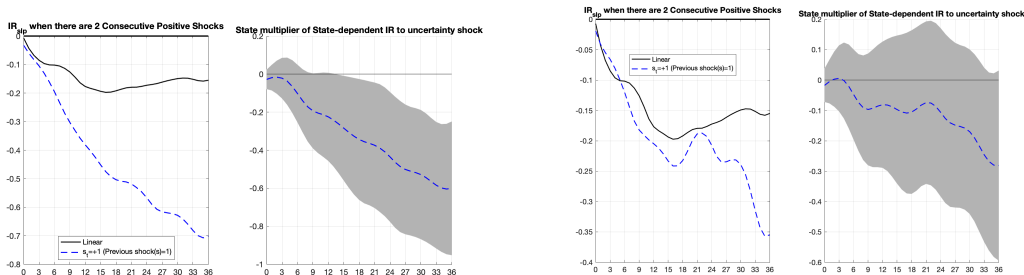


Figure: Response of short rate to consecutive positive EPU uncertainty shocks:

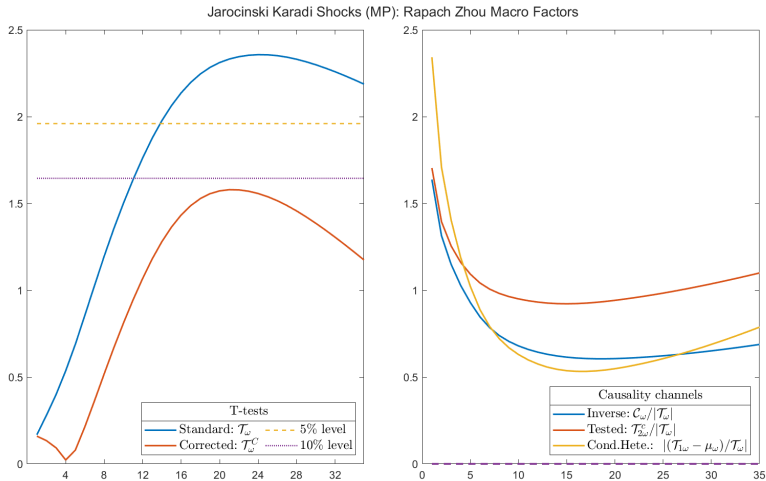
LEFT: Diercks Hsu Tamoni (2024). RIGHT: Adding two lags of McCracken and Ng (2016)'s macro factors.

LEFT PANELS: the empirical state-dependent impulse responses (estimated with LPs as in Diercks et al. (2024)) to two consecutive positive uncertainty shocks (dashed blue line) and contrast it to the response to a single shock (solid black line). RIGHT PANELS: the incremental effect of the second shock, i.e. $\{\beta_{1,h}\}_{h=1,\dots,H}$, with 90% confidence intervals (shaded area). In both panels, on the y-axes, the level of impulse responses; on the x-axes, the horizons, h .

List of tested popular macroeconomic shocks

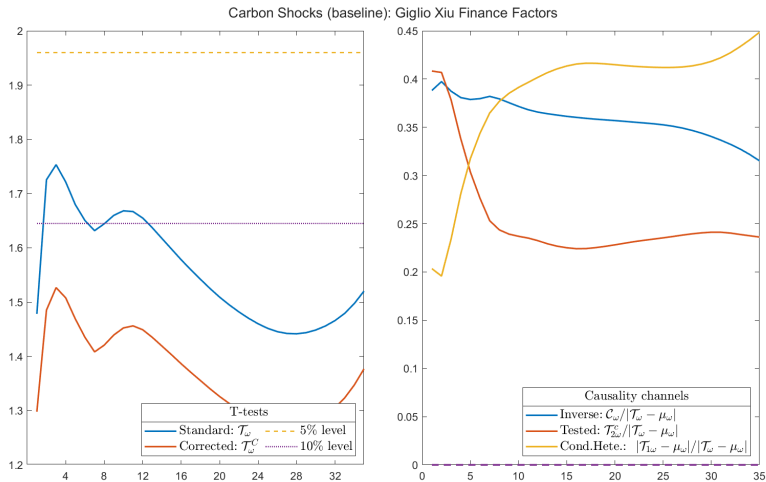
1. Jarociński and Karadi (2020);
2. Känzig (2023);
3. Jurado et al. (2015);
4. Berger et al. (2020);
5. Bauer and Swanson (2023).
6. Not here: Aruoba and Drechsel (2024); Bu et al. (2021); Miranda-Agrippino and Ricco (2021); Känzig (2021).

Are Jarociński and Karadi (2020)'s exogenous?



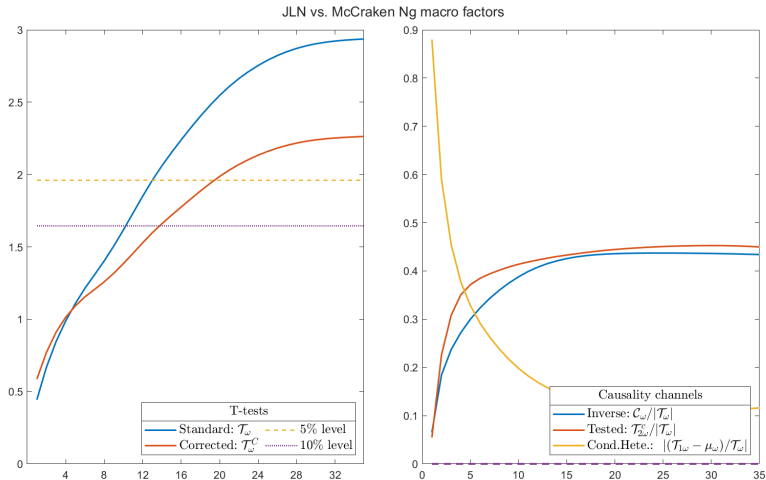
Left: Orange: Corrected, Blue: Benchmark/Standard. **Right:** Orange: Tested, Blue: Inverse, Yellow: Conditional Hete.

Are Känzig (2023)'s shocks exogenous?



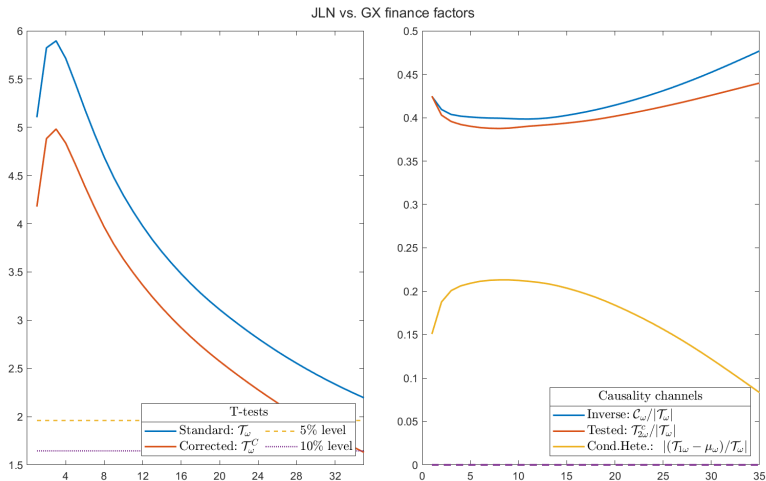
Left: Orange: Corrected, Blue: Benchmark/Standard. **Right:** Orange: Tested, Blue: Inverse, Yellow: Conditional Hete.

Are JLN (2015)'s uncertainty shocks exogenous?



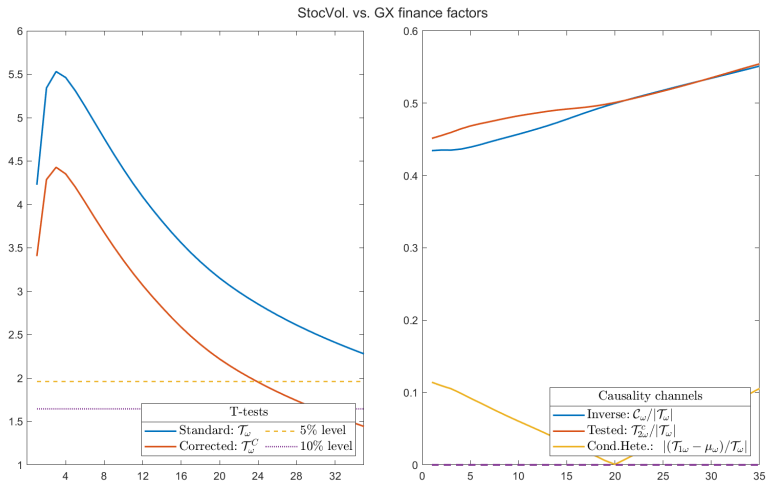
Left: Orange: Corrected, Blue: Benchmark/Standard. **Right:** Orange: Tested, Blue: Inverse, Yellow: Conditional Hete.

Are JLN (2015)'s uncertainty shocks exogenous?



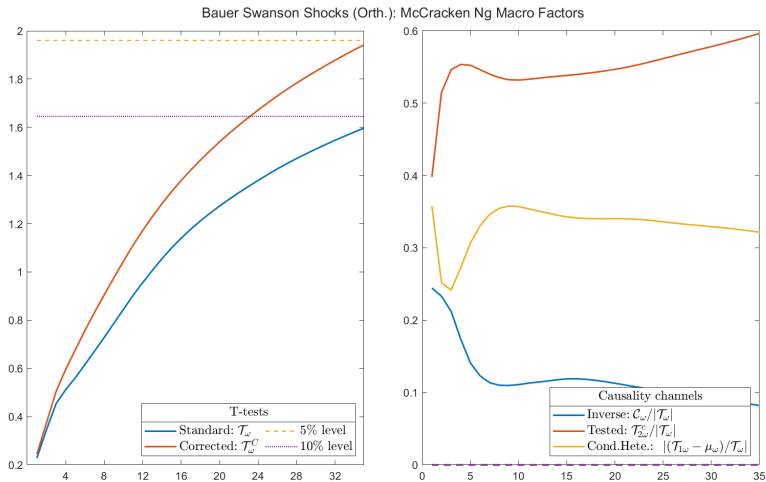
Left: Orange: Corrected, Blue: Benchmark/Standard. **Right:** Orange: Tested, Blue: Inverse, Yellow: Conditional Hete.

Are BDG (2020)'s uncertainty shocks exogenous?



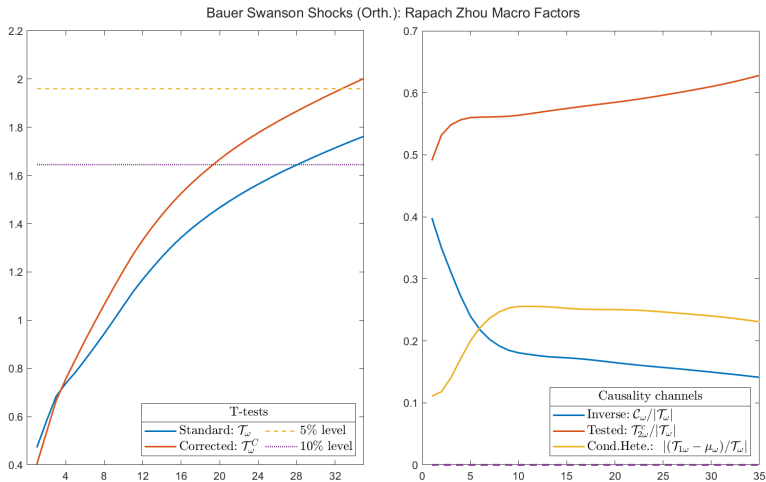
Left: Orange: Corrected, Blue: Benchmark/Standard. Right: Orange: Tested, Blue: Inverse, Yellow: Conditional Hete.

Are BS (2023)'s monetary shocks exogenous?



Left: Orange: Corrected, Blue: Benchmark/Standard. **Right:** Orange: Tested, Blue: Inverse, Yellow: Conditional Hete.

Are BS (2023)'s monetary shocks exogenous?



Left: Orange: Corrected, Blue: Benchmark/Standard. **Right:** Orange: Tested, Blue: Inverse, Yellow: Conditional Hete.

Bibliography I

- Aruoba, S. B. and Drechsel, T. (2024). Identifying monetary policy shocks: A natural language approach. Technical report, National Bureau of Economic Research.
- Bauer, M. D. and Swanson, E. T. (2023). A reassessment of monetary policy surprises and high-frequency identification. *NBER Macroeconomics Annual*, 37(1):87–155.
- Berger, D., Dew-Becker, I., and Giglio, S. (2020). Uncertainty shocks as second-moment news shocks. *The Review of Economic Studies*, 87(1):40–76.
- Bu, C., Rogers, J., and Wu, W. (2021). A unified measure of fed monetary policy shocks. *Journal of Monetary Economics*, 118:331–349.
- Diercks, A. M., Hsu, A., and Tamoni, A. (2024). When it rains it pours: Cascading uncertainty shocks. *Journal of Political Economy*, 132(2):694–720.
- Forni, M. and Gambetti, L. (2014). Sufficient information in structural vars. *Journal of Monetary Economics*, 66:124–136.
- Hong, Y. (1996). Testing for independence between two covariance stationary time series. *Biometrika*, 83(3):615–625.
- Hong, Y. and Lee, Y.-J. (2005). Generalized spectral tests for conditional mean models in time series with conditional heteroscedasticity of unknown form. *The Review of Economic Studies*, 72(2):499–541.
- Jarociński, M. and Karadi, P. (2020). Deconstructing monetary policy surprises—the role of information shocks. *American Economic Journal: Macroeconomics*, 12(2):1–43.
- Jurado, K., Ludvigson, S. C., and Ng, S. (2015). Measuring uncertainty. *American Economic Review*, 105(3):1177–1216.
- Känzig, D. R. (2021). The macroeconomic effects of oil supply news: Evidence from opec announcements. *American Economic Review*, 111(4):1092–1125.
- Känzig, D. R. (2023). The unequal economic consequences of carbon pricing. Technical report, National Bureau of Economic Research.
- McCracken, M. W. and Ng, S. (2016). Fred-md: A monthly database for macroeconomic research. *Journal of Business & Economic Statistics*, 34(4):574–589.
- Miranda-Agrippino, S. and Ricco, G. (2021). The transmission of monetary policy shocks. *American Economic Journal: Macroeconomics*, 13(3):74–107.
- Ramey, V. A. (2016). Macroeconomic shocks and their propagation. *Handbook of macroeconomics*, 2:71–162.