

#### EE150 Signals and Systems

Lecturers: Yijie Mao and Yong Zhou 毛奕婕 周勇

Network Intelligence Center
School of Information Science and Technology
ShanghaiTech University

#### Outline

- Course overview
- Signals and systems introduction
- Classification of signals
- Operation on signals
- Summary

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#### Course Overview

- Course Title: EE 150 Signals and Systems
- Course Level: Undergraduate
- Credit/Contact Hour: 4/64
- Instructor (Week 1-8)

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- Office Hour
  - 14:00-16:00 Tuesday

SIST-2 Room 302I

- Instructor (Week 9-16)

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- Office Hour
  - 4 14:00-16:00 Wednesday

SIST-2 Room 302E

#### Course Overview

- Teaching Assistants (TAs):
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  - ◎ Jinhao Qiu (邱锦灏)
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- Tutorial:
- Blackboard:

https://elearning.shanghaitech.edu.cn:8443 (Slides and text book)

QQ group: 866273449



## Syllabus (Week 1-8, Instructor: Yijie Mao)

Week	Timeline	Chapters	Teaching Contents
1-2	Feb. 27, 29, Mar. 5, 7	Chapter 1: Overview	<ul> <li>1.1 Continuous-time and Discrete-time Signals</li> <li>1.2 Transformation of Independent Variables</li> <li>1.3 Exponential and Sinusoidal Signals</li> <li>1.4 The Unit Impulse and Unit Step Functions</li> <li>1.5 Continuous-Time and Discrete-Time Systems</li> <li>1.6 Basic System Properties</li> </ul>
3-4	Mar. 12, 14, 19, 21	Chapter 2: Linear Time- Invariant Systems	2.1 Discrete-Time LTI Systems 2.2 Continuous-Time LTI Systems 2.3 Properties of Linear Time-Invariant Systems 2.4 Differential and Difference Equations
5-6	Mar. 26, 28, Apr. 2, 4	Chapter 3:  (Fourier Series Representation)  (of Periodic Signals)	<ul> <li>3.1 The Response of LTI Systems to Complex Exponentials</li> <li>3.2 Fourier Series Representation of Continuous-Time</li> <li>Periodic Signals</li> <li>3.3 Convergence of the Fourier Series</li> <li>3.4 Properties of Continuous-Time Fourier Series</li> <li>3.5 Fourier Series Representation of Discrete-Time Periodic</li> <li>Signals</li> <li>3.6 Properties of Discrete-Time Fourier Series</li> <li>3.7 Fourier Series and LTI Systems</li> </ul>
7-8	Apr. 9, 11, 16, 18	Chapter 4: The Continuous-Time Fourier  Transform	4.1 The Continuous-Time Fourier Transform 4.2 The Fourier Transform for Periodic Signals 4.3 Properties of the Continuous-Time Fourier Transform 4.4 The Convolution Property 4.5 The Multiplication Property 4.6 Linear Constant-Coefficient Differential Equations

## Syllabus (Week 9-16, Instructor: Yong Zhou)

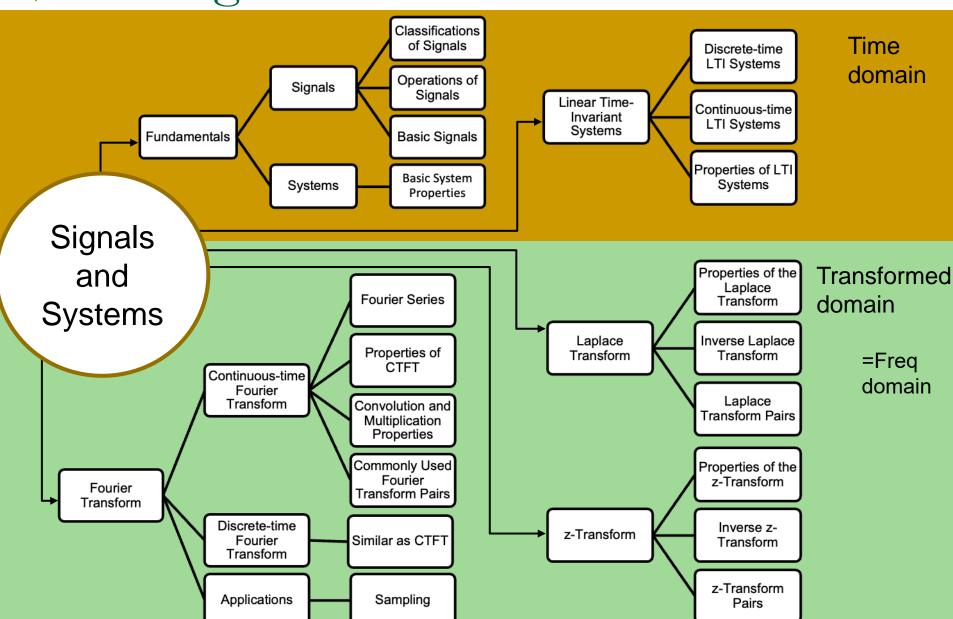
Week	Timeline	Chapters	Teaching Contents
9-10	Apr. 23, 25, 30, May 2	Chapter 5: The Discrete-Time Fourier Transform	<ul> <li>5.1 The Discrete-Time Fourier Transform</li> <li>5.2 The Fourier Transform for Periodic Signals</li> <li>5.3 Properties of the Discrete-Time Fourier Transform</li> <li>5.4 The Convolution Property</li> <li>5.5 The Multiplication Property</li> <li>5.6 Duality</li> <li>5.7 Linear Constant-Coefficient Difference Equations</li> </ul>
11	May 7	Chapter 6: Time and Frequency Characterization of Signals and Systems	<ul> <li>6.1 The Magnitude-Phase Representation of the Frequency Response of LTI Systems</li> <li>6.2 Time-Domain Properties of Ideal Frequency-Selective Filters</li> <li>6.3 Time-Domain and Frequency-Domain Aspects of Nonideal Filters</li> <li>7.1 The Sampling Theorem</li> </ul>
11-12	May 9, 14, 16	Chapter 7: Sampling	<ul> <li>7.2 Reconstruction of a Signal from Its Samples</li> <li>7.3 The Effect of Undersampling: Aliasing</li> <li>7.4 Discrete-Time Processing of Continuous-Time Signals</li> <li>7.5 Sampling of Discrete-Time Signals</li> </ul>
13-14	May 21, 23, 28, 30	Chapter 8: The Laplace Transform	<ul> <li>8.1 The Laplace Transform</li> <li>8.2 The Region of Convergence for Laplace Transforms</li> <li>8.3 The Inverse Laplace Transform</li> <li>8.4 Properties of the Laplace Transform</li> <li>8.5 Some Laplace Transform Pairs</li> <li>8.6 Analysis and Characterization of LTI Systems Using the Laplace Transform</li> <li>8.7 System Function Algebra and Block Diagram</li> <li>Representations</li> </ul>

## Syllabus (Week 9-16, Instructor: Yong Zhou)

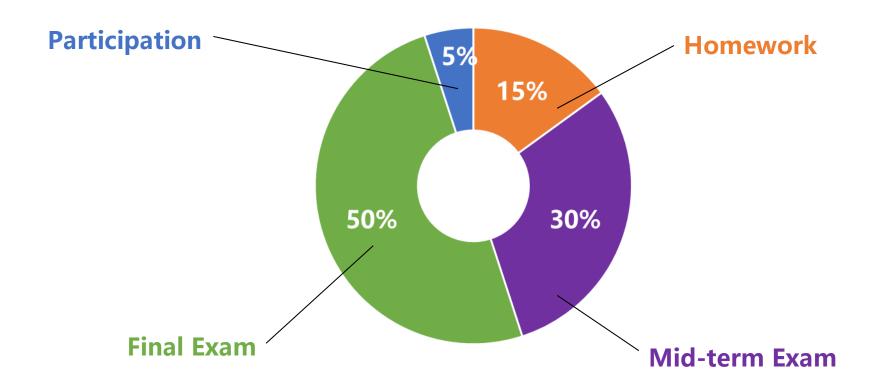
Week	Timeline	Chapters	Teaching Contents
15-16	Jun. 4, 6, 11, 13	Chapter 9: The z-Transform	9.1 The z-Transform 9.2 The Region of Convergence for the z-Transform 9.3 The Inverse z-Transform 9.4 Properties of the z-Transform 9.5 Some Common z-Transform Pairs 9.6 Analysis and Characterization of LTI Systems Using z-Transforms 9.7 System Function Algebra and Block Diagram Representations 9.8 The Unilateral z-Transform

#### Knowledge Framework.

#### 复习用

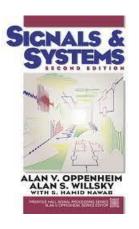


## Grading



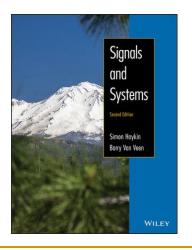
#### References

#### ■ Textbook

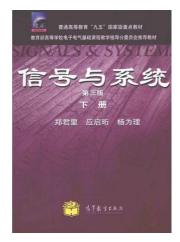












## Ch.1 Overview

# Part I Introduction to Signals and Systems

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- Operation on signals
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#### Introduction to Signals

#### What are Signals?

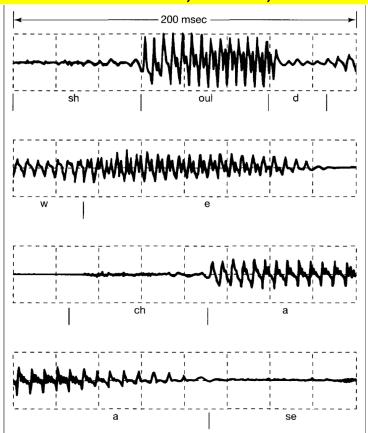
- a function of one or more independent variables (e.g., time and spatial variables);
- typically contains information about the behaviour or nature of some physical phenomena.
- Our world is full of signals, both natural and man-made.
  - Voltage waveform in a circuit.
  - The periodic electrical signals generated by the heart.
  - Stock prices

变量是time

■ Variation in air <u>pressure</u> when we speak. 因变量是pressure

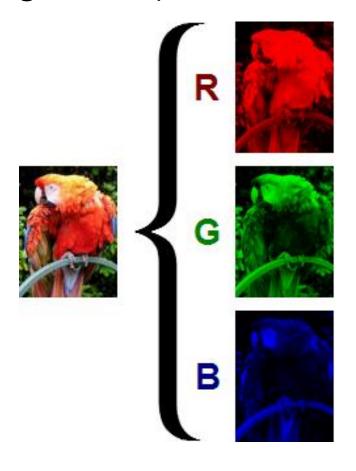
自变量是时间 因变量是intensity?

- Audio (intensity vs. time)
  - characteristics: volume, tone, timbre



音量音高音色

Picture (brightness)



自变量是RGB的brightness

RGB三个二维信号组成彩色图片(3 channels)

- TV signal (voltage vs. time)
  - modulated picture signal + audio signal



Stock price (index vs. time; \$ vs. time)



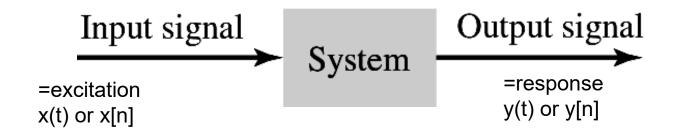
This course focuses on signals involving a single independent variable, i.e., time.

#### Introduction to Systems

What are Systems?

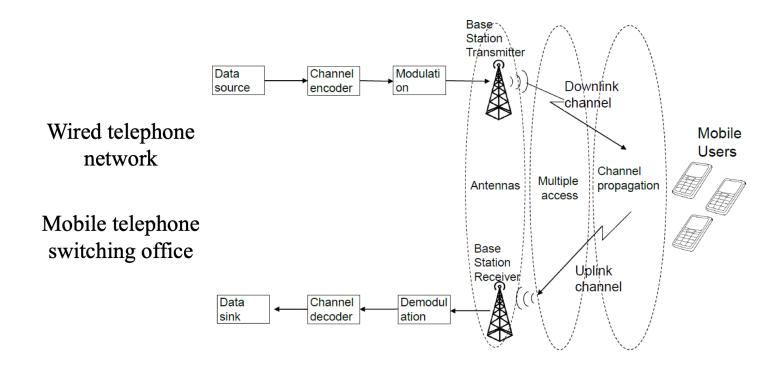
系统==信号生成器/变换器

- A system is a generator of signals or a transformer of signals.
- A system is formally defined as an entity that manipulates one or more signals to accomplish a function, thereby yielding new signals. E.g.,
  - Mobile phone
  - Electronic circuits



## Examples to Systems

#### Cellular Communication Systems



#### Signals and systems introduction

- Typical systems in electrical and electronic engineering
  - Communication systems
  - Control systems 汽车速度控制系统: 脚踩踏板的压力 <-> 加速度
  - Computer systems

#### Signals and systems introduction

- Why study Signals and Systems?
  - Signals and systems are fundamental to all of engineering!
  - Steps involved in engineering are:
    - Model system: Involves writing a mathematical description of input and output signals.
    - Analyze system: Study of the various signals associated with the system.
    - Design system: Requires deciding on a suitable system architecture, as well as finding suitable system parameters.
    - Implement system/test system: Check system, and the input and output signals, to see that the performance is satisfactory.

#### Overview of our course

- This course is about signals and their processing by systems. It involves:
  - Modelling of signals by mathematical functions
  - Modelling of systems by mathematical equations
  - Solution of the equations when excited by the functions
  - Stability of the systems



#### Objectives of our course

- After the course, you will know:
  - System characterization
     how it responds to input signal
     (e.g. human auditory system)
  - System design
    - to process signal in a particular way (e.g. signal restoration, signal identification, image processing)
- The course will serve as the prerequisites for additional coursework in the study of communications, signal processing and control.

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## Classification of Signals 知识要点 信号的分类

oppenheimer的书没有 做具体的总结,但是这 里有

- Methods used for processing a signal or analysing the response of a system to a signal significantly depend on the characteristic attributes of the signal.
- Certain techniques apply to only specific types of signals – hence the need for classification:
  - continuous-time & discrete-time signals
  - even and odd signals
    还有非奇非偶信号
  - periodic and aperiodic signals
  - deterministic and random signals
  - energy and power signals

## Continuous-time and discrete-time signals

- A continuous-time signal if it is defined for all time t, except at some discontinuous point.
- A discrete-time signal is defined only at discrete instants of time.

(b) Representation of x(t) as a

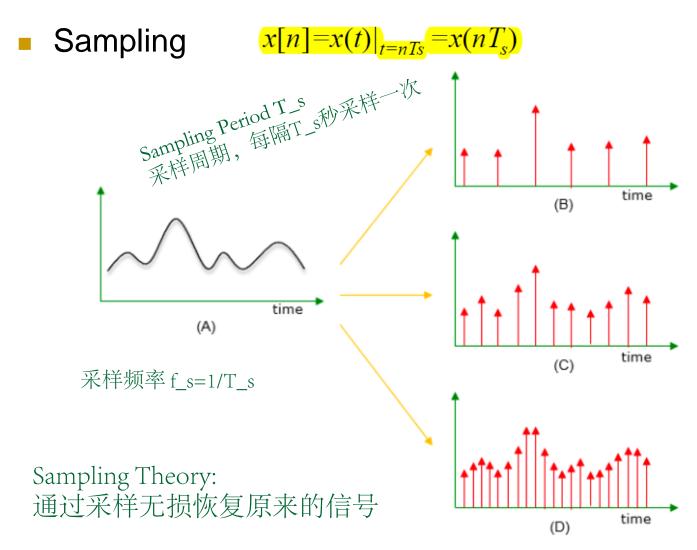
discrete-time signal x[n].



 x(t)
 幅值(Magnitude)连续的信号

 Analog Signal
 Sequence 只在这些离散的点上,幅值有定义。 具他点无定义,而不是为0.0 这里对应的幅值也可以是连续的变化的情况-> 这里对应的幅值也可以是连续的变化的情况-> 对幅值连续的连续信号的有限样本点的采样,幅值连续的离散信号。方波,...

#### Continuous-time and discrete-time signals



#### Continuous-time and discrete-time signals

 A discrete-time signal is often derived from a continuoustime signal by sampling it at a uniform rate.

$$x[n] = x(t)/_{t=nT_s} = x(nT_s)$$
  $n = 0,1,2,...,-1,-2,...$  can be less than zero

 $T_s$ : sampling period; n denote an integer

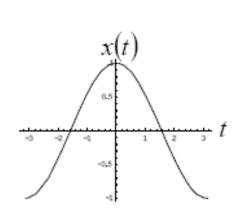
In this lecture, we use t to denote time for a continuous-time signal, and n to denote time for a discrete-time signal.

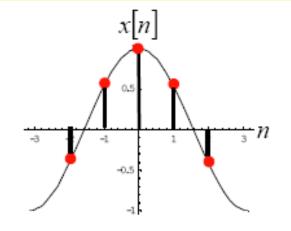
Continuous-time signals: x(t)Parentheses (·)

Discrete-time signals:  $x[n] = x(nT_s), \quad n = 0, \pm 1, \pm 2, \dots$  where  $t = nT_s$ Brackets [·]

## Even and odd signals — 奇偶函数

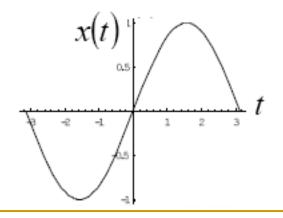
Even signals: x(-t) = x(t), x[-n] = x[n] for all t

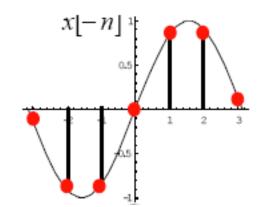




轴对称 **Symmetric** about vertical axis

odd signals: x(-t) = -x(t), x[-n] = -x[n] for all t





原点中心对称 旋转180度不变 **Antisymmetric** about origin

**Problem:** Consider the signal

$$x(t) = \begin{cases} \sin\left(\frac{\pi t}{T}\right), & -T \le t \le T \\ 0, & \text{otherwise} \end{cases}$$

Is the signal x(t) an even or an odd function of time?

sin(omega\*t), 基波周期 T=2pi/omega

**Even-odd decomposition of** x(t): Any signal can be broken into a sum of two signalsone even and one odd.

$$x(t) = x_e(t) + x_o(t)$$

#### 奇偶分解

每个函数都可以分成一个奇函数和偶函数的 加和。

两个分解结果如左下。

where

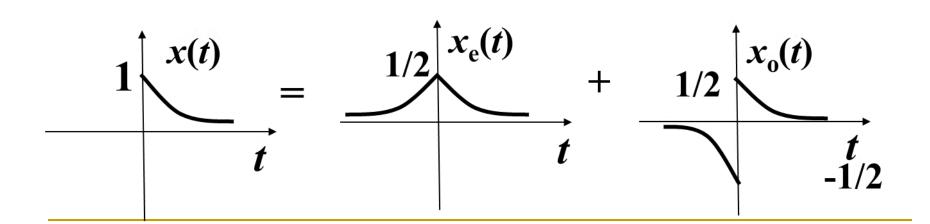
$$x_e(t) = E_v\{x(t)\} = \frac{1}{2}[x(t) + x(-t)]$$

$$x_o(t) = O_d\{x(t)\} = \frac{1}{2}[x(t) - x(-t)]$$

**Even-odd decomposition of** x(t): Any signal can be broken into a sum of two signalsone even and one odd.

$$x_{e}(t) = E_{v}\{x(t)\} = \frac{1}{2}[x(t) + x(-t)]$$

$$x_{o}(t) = O_{d}\{x(t)\} = \frac{1}{2}[x(t) - x(-t)]$$

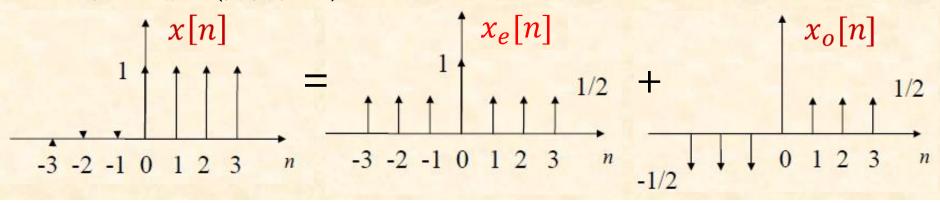


**Even-odd decomposition of** x[n]: Any signal can be broken into a sum of two signalsone even and one odd.

$$x[n] = x_e[n] + x_o[n]$$
  
 $x_e[n] = (x[n] + x[-n])/2$ 

step function 阶跃函数  $x_o[n] = (x[n] - x[-n])/2$  otherwise, f=0

非整数点没有定义(离散情况下)



#### Even-odd decomposition of x(t):

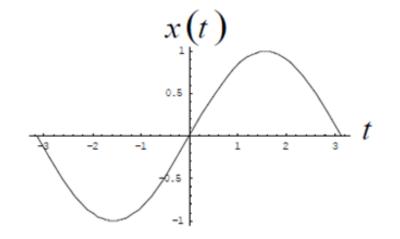
**Problem:** Find the even and odd components of the signal

$$x(t) = e^{-2t} \cos t$$

$$x(t)=e^{-2t}\cos t$$
  $x(-t)=e^{2t}\cos t$   $x_o(t)=rac{1}{2}(e^{-2t}-e^{2t})\cos t$   $\sinh(2t)\cos(t)$   $x_e(t)=rac{1}{2}(e^{-2t}+e^{2t})\cos t$   $\cosh(2t)\cos(t)$ 

# Even and odd signals

#### PRODUCT rule



$$s = \int_{-T}^{T} x(t)dt = 0$$
, always if  $x(t)$  is odd.

$$s = \int_{-T}^{T} x(t)dt = 2\int_{0}^{T} x(t)dt$$
, for  $x(t)$  is even.

#### Continuous-Time Case

Periodic signals:

$$x(t) = x(t+T)$$
  $\forall t$ , where T is a positive constant.  
 $T = T_0, 2T_0, 3T_0, \dots$ 

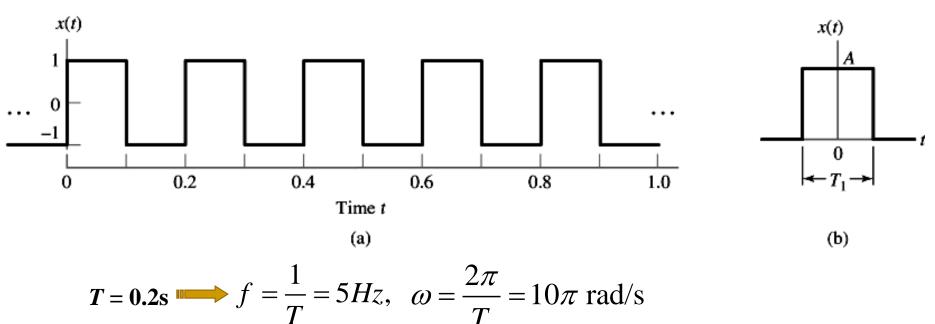
- Fundamental period (smallest positive value of T):  $T = T_0$
- Fundamental frequency:  $\omega_0 = \frac{2\pi}{T_0}$  measured in radians per second.
- $\Box$  Aperiodic signals: x(t) where  $T_0$  does not exist.

#### Continuous-Time Case

Example of periodic and nonperiodic signals.

(a) Square wave with amplitude A = 1 and period T = 0.2s.

(b) Rectangular pulse of amplitude A and duration  $T_1$ .

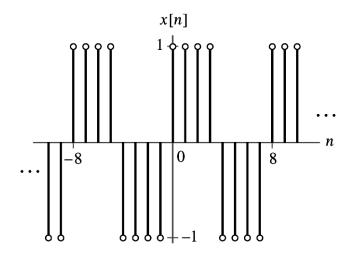


#### Discrete-Time Case

- □ Periodic signals: x[n] = x[n+N] for integer n
  - Fundamental period: The smallest positive integer value of N for which the periodicity holds
  - Fundamental angular frequency:  $\Omega = \frac{2\pi}{N}$ , measured in radians, or radians/cycle.

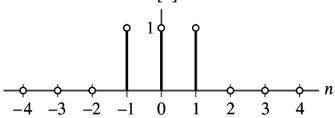
Discrete-time square wave alternative between –1 and +1:

$$N = 8$$
  $\Omega = \frac{2\pi}{8} = \frac{\pi}{4}$  radians.



#### Discrete-Time Case

Aperiodic discrete-time signal consisting of three nonzero samples:

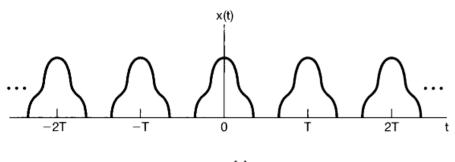


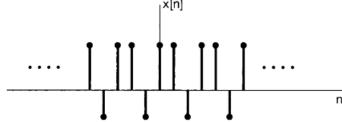
#### Notes

- A sequence obtained by uniform sampling of a periodic continuous-time signal may not be periodic.
- □ The sum of two continuous-time periodic signals may not be periodic.
- The sum of two periodic sequences is always periodic.

### **Even-odd decomposition of** x(t):

**Problem:** What is the fundamental period of a constant function?



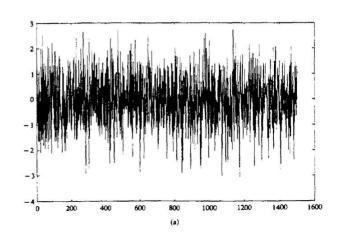


$$x(t) = \begin{cases} \cos(t) & \text{if } t < 0 \\ \sin(t) & \text{if } t \ge 0 \end{cases} \xrightarrow{-6\pi} \int_{-4\pi}^{1} \int_{-2\pi}^{1} \int_{0}^{1} \int_{0}^{1}$$

# Deterministic signals and random signals

- Deterministic signals are those signals whose values are completely specified for any given time. Thus, a deterministic signal can be modeled by a known function of time.
- Random signals are those signals that take random values at any given time and must be characterized statistically.
- Random signals will not be discussed in this course.

#### Random signals



 $\Box v(t)$  and i(t) are voltage and current across a resistor R, the instantaneous power is

$$p(t) = v(t)i(t) = \frac{1}{R}v^{2}(t) = Ri^{2}(t)$$

 $\Box$  The total energy over the time interval  $t_1 \le t \le t_2$  is

$$E_R = \int_{t_1}^{t_2} p(t) dt = \int_{t_1}^{t_2} \frac{1}{R} v^2(t) dt$$

 $\Box$  The average power over the time interval  $t_1 \le t \le t_2$  is

$$P_R = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} p(t) \, dt = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} \frac{1}{R} v^2(t) \, dt$$

 $\Box$  If R = 1  $\Omega$  and x(t) represents a current or a voltage, then

$$p(t) = x^2(t)$$

- Over finite time interval  $t_1 \le t \le t_2$  or  $n_1 \le n \le n_2$
- Continuous-time signal x(t)
  - Total energy:

$$E = \int_{t_1}^{t_2} |x(t)|^2 dt,$$

Time-averaged/average power

$$P = \frac{E}{t_2 - t_1}$$

- Discrete-time signal x[n]
  - Total energy:

$$E = \sum_{n=n_1}^{n_2} |x[n]|^2,$$

Average power

$$P = \frac{E}{n_2 - n_1 + 1}$$

- Over infinite time interval  $-\infty \le t \le \infty$  or  $-\infty \le n \le \infty$
- Continuous-time signal x(t)
  - Total energy:

$$E_{\infty} \triangleq \lim_{T \to \infty} \int_{-T}^{T} |x(t)|^2 dt$$
$$= \int_{-\infty}^{\infty} |x(t)|^2 dt$$

Time-averaged/average power

$$P_{\infty} \triangleq \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt$$

- Discrete-time signal x[n]
- **Total energy:**  $E_{\infty} \triangleq \lim_{N \to \infty} \sum |x[n]|^2$  $= \sum_{n=1}^{\infty} |x[n]|^2$

average power

$$P_{\infty} \triangleq \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt \qquad P_{\infty} \triangleq \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x[n]|^2$$

### Finite-energy signal

If and only if the total energy of the signal satisfies the condition

$$E_{\infty} < \infty$$

$$P_{\infty} \triangleq \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt = 0$$

$$P_{\infty} \triangleq \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x[n]|^2 = 0$$

- Finite-power signal  $P_{\infty} < \infty$ ,  $E_{\infty} = \infty$
- Infinite energy & power signal  $P_{\infty} \rightarrow \infty$ ,  $E_{\infty} \rightarrow \infty$

Finite-energy signal

(1) 
$$x(t) = \begin{cases} 0, & t < 0 \\ 1, & 0 \le t \le 1 \\ 0, & t > 1 \end{cases}$$
  $E_{\infty} < \infty, P_{\infty} = 0$ 

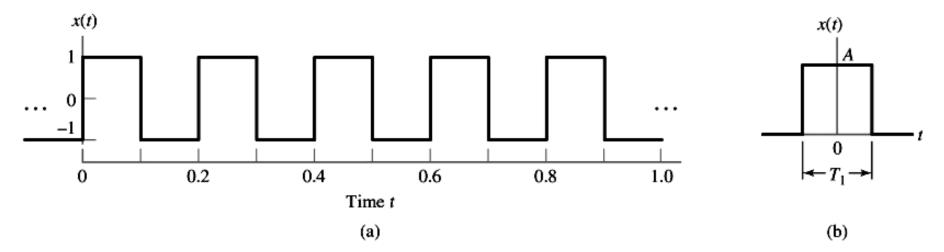
Finite-power signal

(2) 
$$x[n] = 4$$
  $P_{\infty} < \infty$ ,  $E_{\infty} = \infty$ 

Infinite energy & power signal

(3) 
$$x(t) = t$$
  $P_{\infty} \to \infty$ ,  $E_{\infty} \to \infty$ 

#### **Problem:**



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# Operation on Signals

- An issue of fundamental importance in the signals and systems is the use of systems to process or manipulate signals. This issue usually involves a combination of some basic operations in signals.
  - Three transformation in amplitude
    - Amplitude scaling
    - Addition
    - Multiplication
  - Three transformations in time domain
    - Time Scaling
    - Time Reversal
    - Time Shifting

### Transformation in Amplitude

Amplitude scaling: y(t) = cx(t)

$$y(t) = cx(t)$$

c: scaling factor

$$y[n] = cx[n]$$

- Performed by amplifier or resistor

**Addition:** 
$$y(t) = x_1(t) + x_2(t)$$

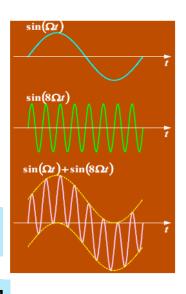
$$y[n] = x_1[n] + x_2[n]$$

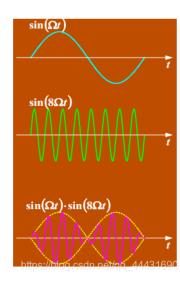
- E.g. audio mixer
- **Multiplication:**  $y(t) = x_1(t)x_2(t)$

$$y(t) = x_1(t)x_2(t)$$

$$y[n] = x_1[n]x_2[n]$$

□ E.g. AM radio signal

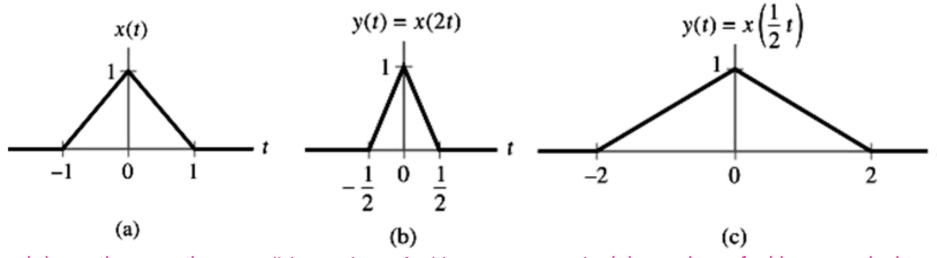




### Time Scaling

#### **Continuous-Time Case**

$$y(t) = x(at), \quad a > 0$$
  $\longrightarrow$  
$$\begin{cases} a > 1 \Rightarrow \text{compressed} \\ 0 < a < 1 \Rightarrow \text{expanded} \end{cases}$$



- (a) continuous-time signal x(t)
- by a factor of 2
- (b) version of x(t) compressed (c) version of x(t) expanded by a factor of 2.

### Time Scaling

#### Discrete-Time Case

$$y[n] = x[kn], \quad k > 0$$

k = integer Some values lost!

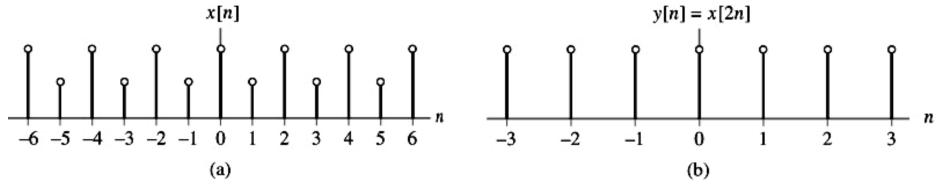


Figure 1.21

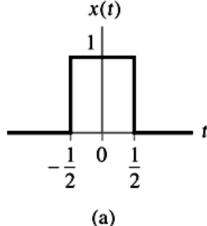
Effect of time scaling on a discrete-time signal: (a) discrete-time signal x[n] and (b) version of x[n] compressed by a factor of 2, with some values of the original x[n] lost as a result of the compression.

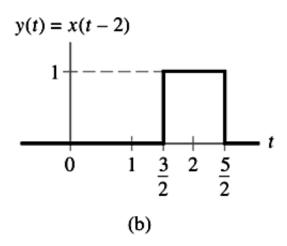
### Time Shifting

#### Continuous-Time Case

$$y(t) = x(t - t_0)$$
 $t_0 > 0 \Rightarrow \text{shift toward right}$ 
 $t_0 < 0 \Rightarrow \text{shift toward left}$ 

**Example** y(t) = x (t - 2)





#### Discrete-Time Case

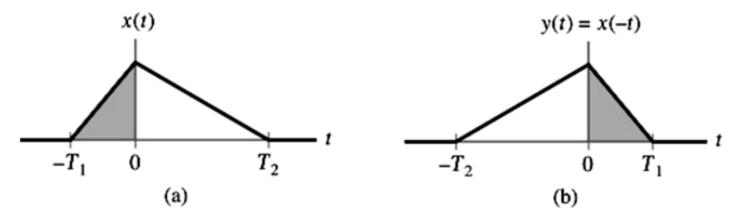
y[n] = x[n-m] where m is a positive or negative integer

### Time Reversal

$$y(t) = x(-t)$$
  $y(t)$  represents a reversed version of  $x(t)$  about  $t = 0$ .

- $\Box$  An even signal is the same as its reversed version: x(-t) = x(t)
- □ An odd signal is the negative of its reversed version: x(-t) = -x(t)

Example Consider the triangular pulse x(t) shown in (a). Find the reversed version of x(t) about the amplitude axis (i.e., the origin).



$$x(t) = 0$$
 for  $t < -T_1$  and  $t > T_2$   $\longrightarrow$   $y(t) = 0$  for  $t > T_1$  and  $t < -T_2$ 

### **General:** Let $x(t) \rightarrow x(\alpha t + \beta)$

- $\triangleright$  if  $|\alpha| > 1$ , compressed
- $\triangleright$  if  $|\alpha| < 1$ , stretched
- $\triangleright$  if  $\alpha < 0$ , reversed
- $\triangleright$  if  $\beta \neq 0$ , shifted

### Recommended operation order

1st step: time shifting 
$$x(t) \rightarrow x(t + \beta)$$

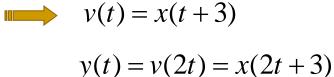
2nd step: time scaling 
$$\chi(t+\beta) \rightarrow \chi(|a|t+\beta)$$

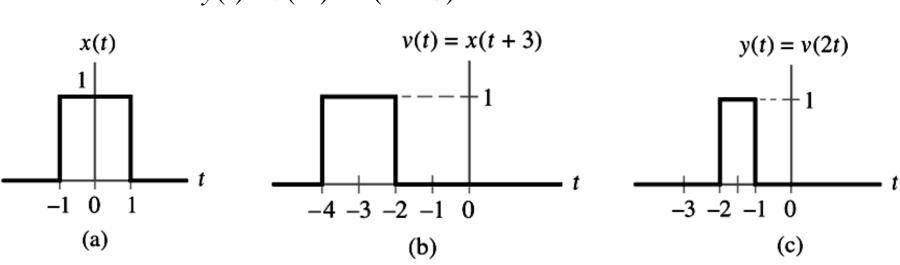
3rd step: time reverse 
$$\chi(|a|t+\beta) \rightarrow \chi(at+\beta)$$
 if  $a < 0$ 

### **Example.** Consider the rectangular pulse x(t) in (a). Find

$$y(t) = x(2t+3).$$

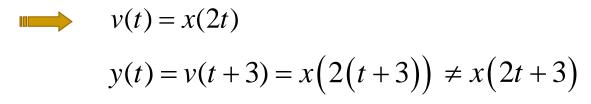
#### <Sol.> Case 1: Shifting first, then scaling

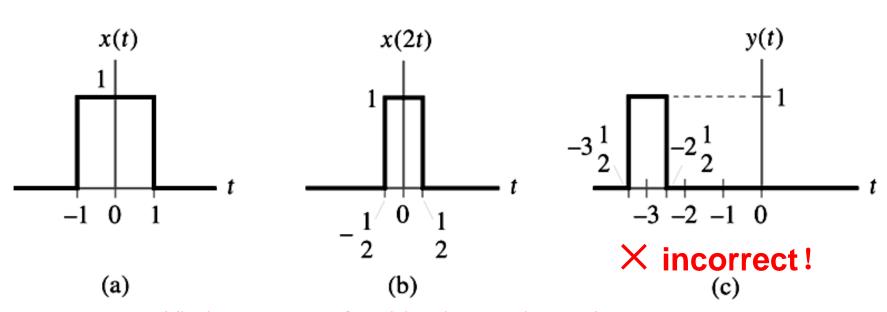




The proper order of time scaling and time shifting operations

#### Case 2: Scaling first, then shifting





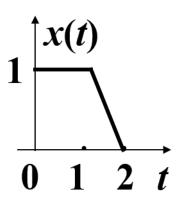
The incorrect way of applying the precedence rule.

#### **Problem.** Given the signal x(t) below, determine and sketch:

$$\geq x(t+1)$$

$$\geq x(-t+1)$$

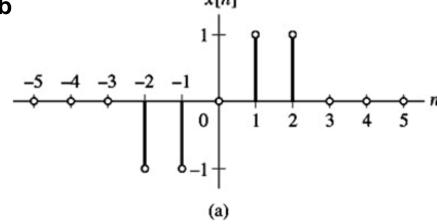
$$> x(\frac{3t}{2}+1)$$

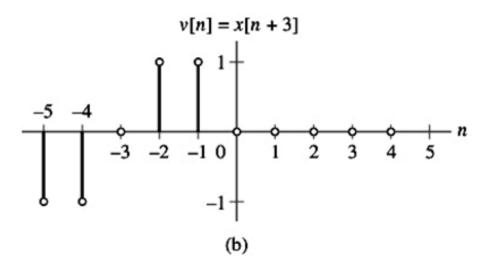


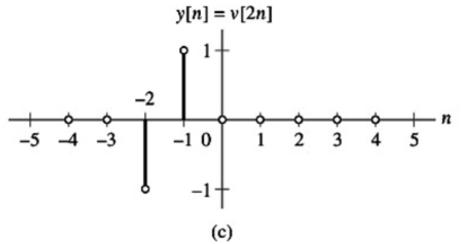
#### Example A discrete-time signal is defined b

$$x[n] = \begin{cases} 1, & n = 1, 2 \\ -1, & n = -1, -2 \\ 0, & n = 0 \text{ and } |n| > 2 \end{cases}$$

Find y[n] = x[2n + 3].







# Summary

- Signals and systems introduction
- Overview of our course
- Classification of signals
- Operation on signals
- Reference in textbook:
  - **1.1,1.2**