1. Consider a hemisphere represented in the *spherical coordinate system.* The spherical coordinates are defined by two variables:

   (a) $\theta \in [0, \pi/2]$: the polar angle along the z-axis.

   (b) $\phi \in [0, 2\pi)$: the azimuthal angle in the xy-plane.

   You are given two uniform random variables $\xi_1, \xi_2 \in U(0,1)$. Consider the following mappings:

$$\begin{cases} \theta = f(\xi_1), \\ \phi = g(\xi_2), \end{cases}$$

   which transform these uniform random variables into the unit hemisphere's spherical coordinates. If we assume that the BRDF f_r is given as:

$$f_r(\theta_o, \theta_i, \phi_o, \phi_i) = \sin\theta_i,$$

   and we will perform importance sampling, the probability distribution for sampling the directions should conform to the distribution $p(\omega) \propto cos\theta_i \cdot f_r$. Please use *inversion method* to derive the mappings from the uniform random variables (ξ_1, ξ_2) to (θ, ϕ) such that the samples over the solid angle ω follow the distribution $p(\omega)$. Finally, you need to express explicitly the mappings $f(\xi_1)$ and $g(\xi_2)$, and provide the sampled direction (θ, ϕ) and compute the spatial location on the unit hemisphere.

   $p(\omega)d\omega = \sin(\theta)p(\omega)d\theta d\phi, p(\theta,\phi) = c\sin^2(\theta)\cos(\theta)$

   so to calculate $\int_0^{2\pi} d\phi \int_0^{\frac{\pi}{2}} csin^2\theta\cos(\theta)d\theta$ =1, we can get the c as $\frac{3}{2\pi}$

   then we can get $p(\theta) = 3\sin^2\theta cos(\theta)$, P$(\theta) = \sin^3\theta$so $\theta = \arcsin(\xi_1^{\frac{1}{3}})$,$\phi = 2\pi\xi_2$

   The corresponding coordinate is that

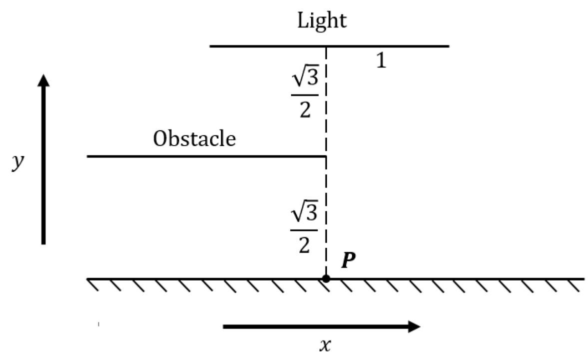
   x=$\sin\theta cos\phi = \xi_1^{\frac{1}{3}} cos(2\pi\xi_2)$,

   $y = \sin\theta\sin(\phi) = \xi_1^{\frac{1}{3}}\sin(2\pi\xi_2)$

   $z = cos(\arcsin\xi_1^{\frac{1}{3}})$

2. Consider in 3D a circular diffuse area light source which emits light along normal direction and is located at the height of $y = \sqrt{3}$ above the ground, see the figure for illustration of a cross-section in 2D below. The light source has a radius of 1 meter, with its center located at the coordinate $(0, \sqrt{3}, 0)$, and its total energy of L. A flat plate obstacle which is infinite along xz-plane is located at $(0, \frac{\sqrt{3}}{2}, 0)$, blocking light in the negative x-direction. To simplify the calculation, we only account for direct illumination and ignore global illumination effects. Additionally, we assume that the ground is diffuse and has a constant BRDF value of c.

   Using the rendering equation, calculate the out-going radiance $L_o(\omega_o, P)$ observed at the point $P(0,0,0)$ in a specified direction ω_o. (Hint: consider evaluating the rendering equation from the light source and convert the integral from solid angle to surface area)



To calculate the radiance L_o ,since the surface is diffuse,it is not related to the outgoing direction.

Then we do calculation by $\int f L_i cos\theta_i dw_i = \int_0^{\pi} \int_0^{\frac{\pi}{6}} f L_i \frac{\cos(\theta_i)\cos(\theta_i)}{|p_1 - p_2|^2} d\theta d\phi = (\frac{\pi}{16} + \frac{9\sqrt{3}}{64}) \times \frac{cL_i\pi}{h^2}$
$L_i = \frac{L}{\pi^2}$,so the ans is $(\frac{\pi}{16} + \frac{9\sqrt{3}}{64}) \times \frac{cL}{3\pi}$