1 Myerson's Mechanism

Suppose there are n agents who bid for one single item. Their probability density functions of their valuation distributions are of the Pareto's form and same (i.i.d.):

$$f(x) = \frac{\alpha}{x^{\alpha+1}} \qquad x \geq 1$$

1.1 (1pt)

If $\alpha = 2$ and there are five bidders $\{A, B, C, D, E\}$ with bids $v_A = 20$, $v_B = 18$, $v_C = 16$, $v_D = 14$ and $v_E = 12$. Compute the allocation and payment of Myerson's mechanism.

1.2 (1pt)

If $\alpha = 1/2$, will the mechanism be truthful? Prove your statement.

2 Auction in Social Network

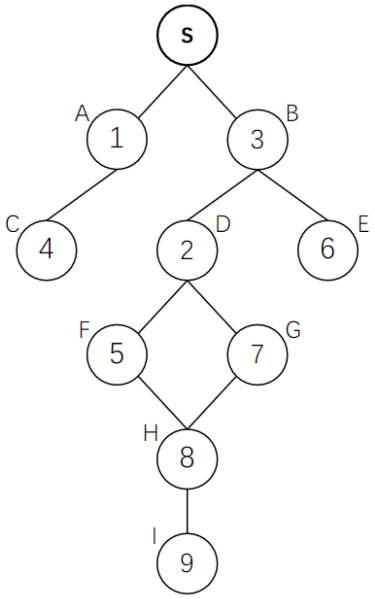
Seller s wants to sell one item in the social network. The other nodes are the buyers. The number in the circle is the buyer's valuation.

2.1 (2pt)

For the extended VCG mechanism in social network, if one buyer does not participate in the auction, the buyers that she invites can not participate either (if D does not participate in the auction, then F, G, H, I can not participate either). Run the extended VCG mechanism in the whole social network rather than just in the first layer. Give the allocation and payment results. Does the extended VCG satisfy IC,IR and WBB? The proof is not needed.
Weakly budget balanced (WBB) means the sum of all buyers' payments (i.e., the revenue of the seller) is non-negative.

2.2 (1pt)

Run IDM in the social network. Give the allocation and payment results.



3 Double Auction (2pt)

Consider a double auction where each seller sells one item and each buyer gets at most one item. All the items are identical. Suppose the sellers' asks are $(s_1 = 3, s_2 = 5, s_3 = 6, s_4 = 13, s_5 = 15, s_6 = 17)$ and the buyers' bids are $(b_1 = 8, b_2 = 10, b_3 = 13, b_4 = 17, b_5 = 19, b_6 = 24)$. Compute each agent's utility the social welfare of the allocations generated by VCG (1pt) and McAfee's mechanisms (1pt) respectively.

4 Advertisement auctions

4.1 VCG allocation (1pt)

Suppose a search engine has two ad slots that it can sell. Slot A has a clickthrough rate of 1000 clicks per hour, slot B has a clickthrough rate of 200 clicks per hour. There are three bidders b_1, b_2, b_3 have values per click of $\{10, 9, 5\}$ respectively. Compute the allocation and the payments for all bidders in VCG.

4.2 GSP allocation (1pt)

Under the same setting as in question 4.1, compute the allocation and payments for all bidders in GSP (without weight).

4.3 The truthfulness of GSP (1pt)

Consider the example of GSP allocation in question 4.2. Is this example truthful? Why? If it is truthful, change one value in the setting above to show that GSP is not truthful in general, and explain.