Problem 1. (**Cholesky Decomposition**)

Let $\mathbf{u} \in \mathbb{R}^n$, $\tau > 0$, and $\mathbf{A} = \mathbf{I}_n + \tau \mathbf{u}\mathbf{u}^T = \mathbf{G}\mathbf{G}^T$ be the Cholesky decomposition of $\mathbf{A}$. Also let $\mathbf{d}$ be an n-dimensional column vector with $\mathbf{d} = \mathrm{diag}(\mathbf{G})$, i.e., $d(i) = \mathbf{G}(i,i)$ for $i = 1, 2, \ldots n$.

Algorithm 1 Diagonalize of the Cholesky factor

1: **Input:** $\mathbf{u} \in \mathbb{R}^n, \tau > 0$.

2: *Complete the algorithm here...*

3: **Output:** An n-dimensional column vector $\mathbf{d}$ such that $\mathbf{d} = \mathrm{diag}(\mathbf{G})$.

1) Determine the vector $\mathbf{d}$ and complete Algorithm 1 while satisfying the following two conditions, respectively.
   (a) Obtain the Cholesky decomposition of $\mathbf{A}$ with $\mathcal{O}(\frac{n^3}{3})$ computational complexity. (**Hint**: you may use the LDL decomposition to obtain the Cholesky decomposition of $\mathbf{A}$)
   (b) Determine the vector $\mathbf{d}$ with $\mathcal{O}(n)$ computational complexity. (**Hint**: develop recipes for $d_1 \in \mathbb{R}, \mathbf{v} \in \mathbb{R}^{n-1}$, and the lower triangular $\mathbf{G}_1^{(n-1)\times(n-1)}$ in
   $$\mathbf{G}\mathbf{G}^T = \begin{bmatrix} d_1 & \mathbf{0} \\ \mathbf{v} & \mathbf{G}_1 \end{bmatrix} \begin{bmatrix} d_1 & \mathbf{v}^T \\ \mathbf{0} & \mathbf{G}_1^T \end{bmatrix} = \mathbf{I}_n + \tau \mathbf{u}\mathbf{u}^T.)$$
2) Let $\tau = 2024$ and $\mathbf{u} = [1, 2, 3, 4]^T$. Calculate the n-dimensional column vector $\mathbf{d}$ in $\mathbf{Matlab}$.