



Lecture 11

- Magnetically Coupled Circuits



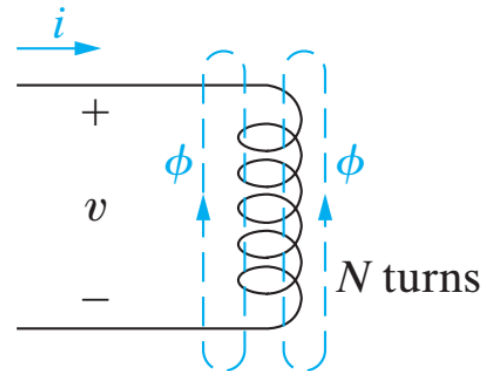
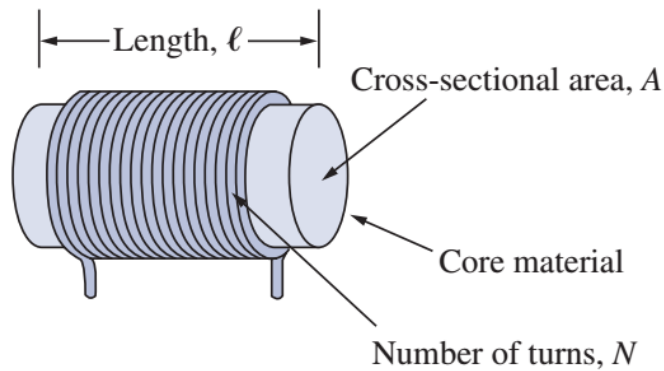
Outline

- Mutual inductance
- Transformers

Recall: Self Inductance

- Self inductance:

Reaction of the inductor to the change in current ***through itself***.

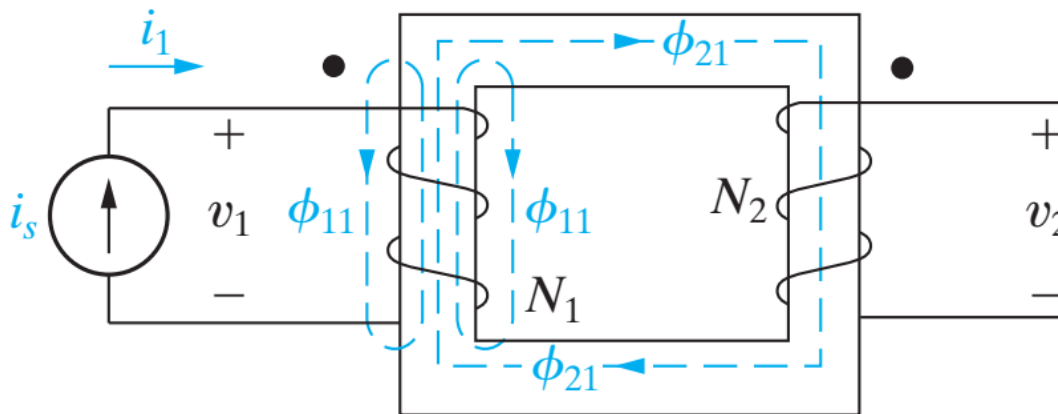


$$v = L \frac{di}{dt}$$



Mutual Inductance

- Mutual inductance: Reaction of one inductor to the change in current ***through another inductor.***

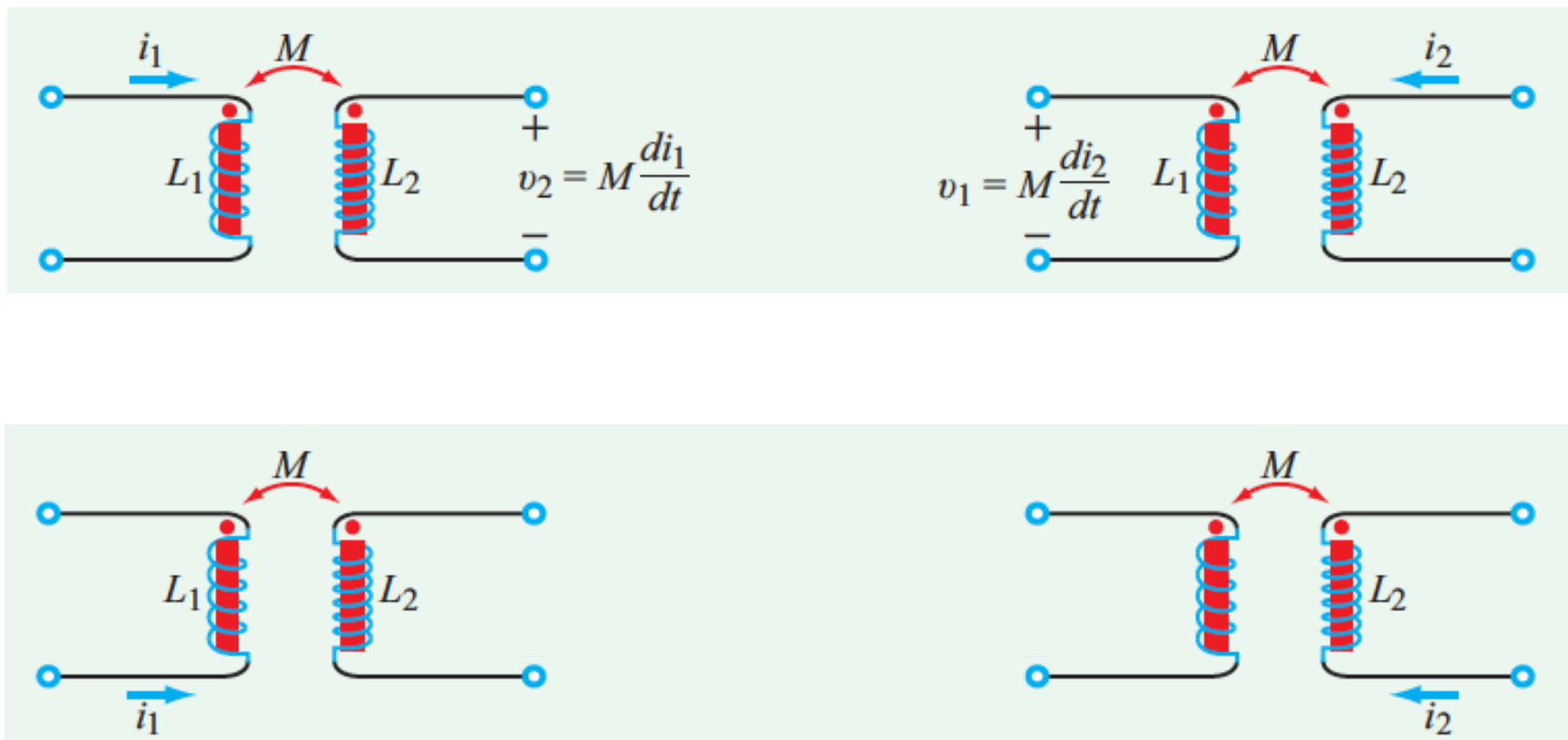


$$\begin{aligned} v_2 &= \frac{d(N_2\phi_{21})}{dt} = N_2 \frac{d}{dt}(\mathcal{P}_{21}N_1i_1) \\ &= N_2N_1\mathcal{P}_{21} \frac{di_1}{dt} \end{aligned}$$

$$= M_{21} \frac{di_1}{dt}$$



Dot Convention: Defines Directions of Windings



If a current enters the dotted terminal of one coil, the reference polarity of mutual voltage in the 2nd coil is the positive at the dotted terminal, negative at the un-dotted terminal.

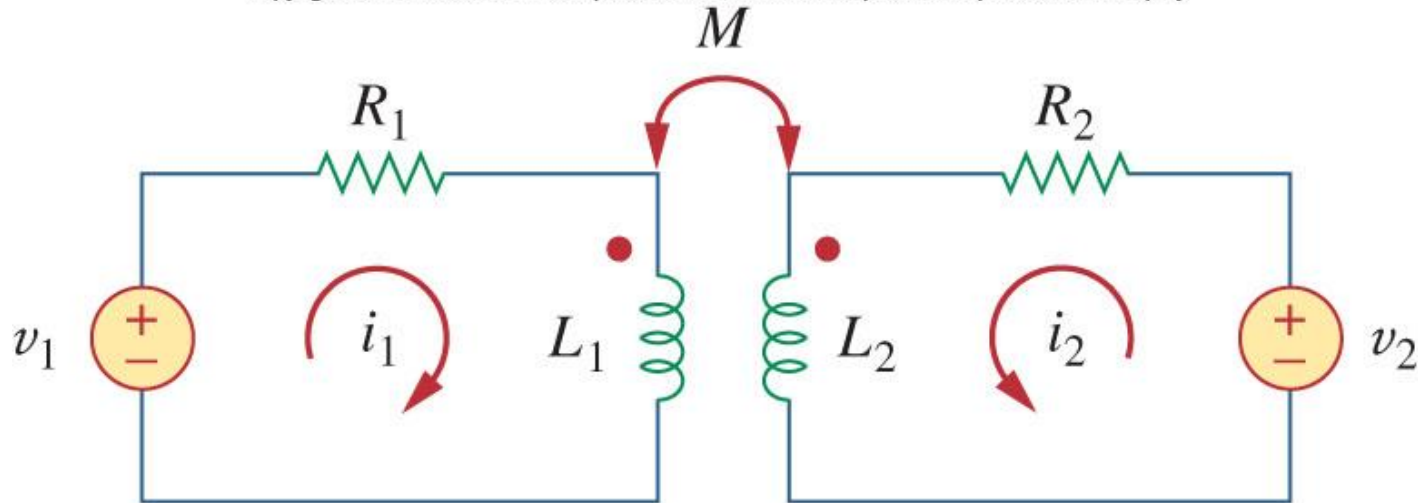




Magnetically Coupled Circuits

- L_1, L_2 : self-inductances
- M : mutual inductance
- Dots: indicating polarity of mutually induced voltages.

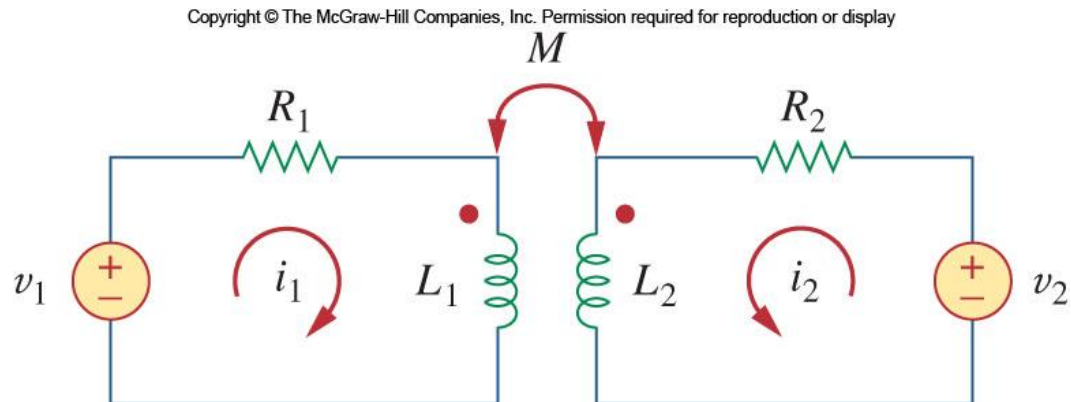
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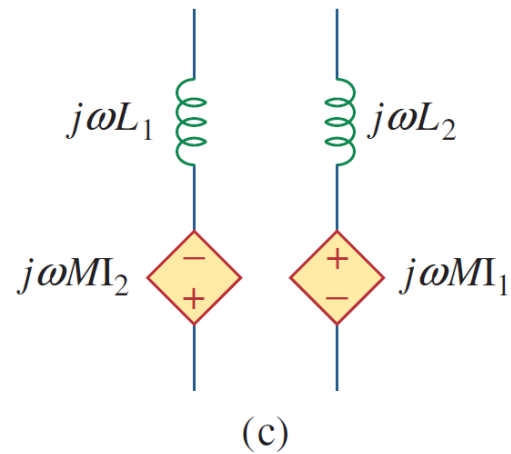
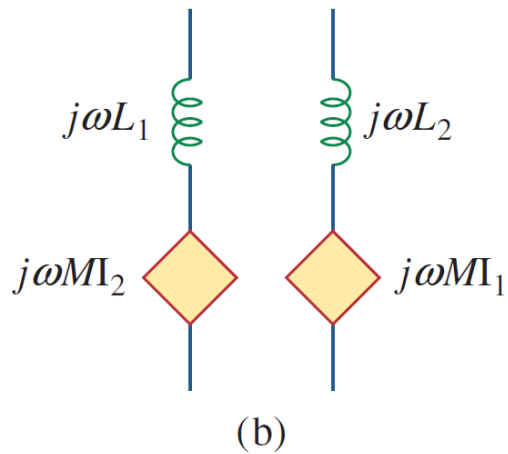
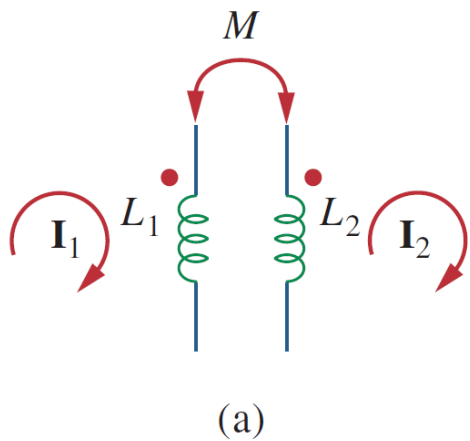
Analysis

- Find i_1 and i_2 .
 - In time domain
 - In phasor domain





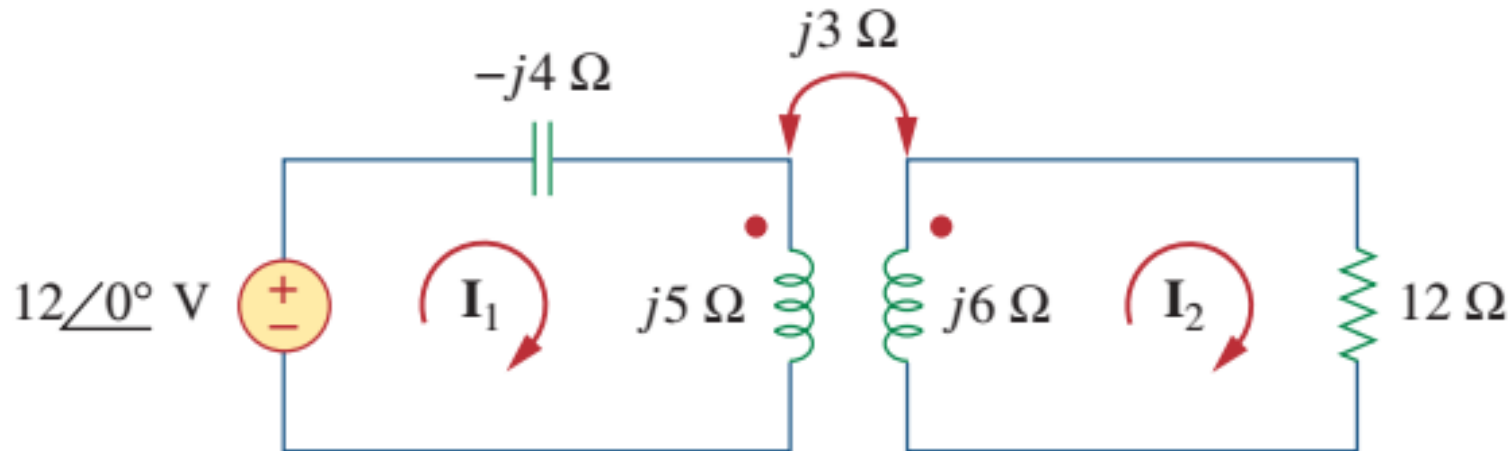
Solve mutually coupled inductors





Example

- Calculate the phasor currents \mathbf{I}_1 , and \mathbf{I}_2
- Calculate the phasor voltages \mathbf{V}_1 , and \mathbf{V}_2 across the inductors



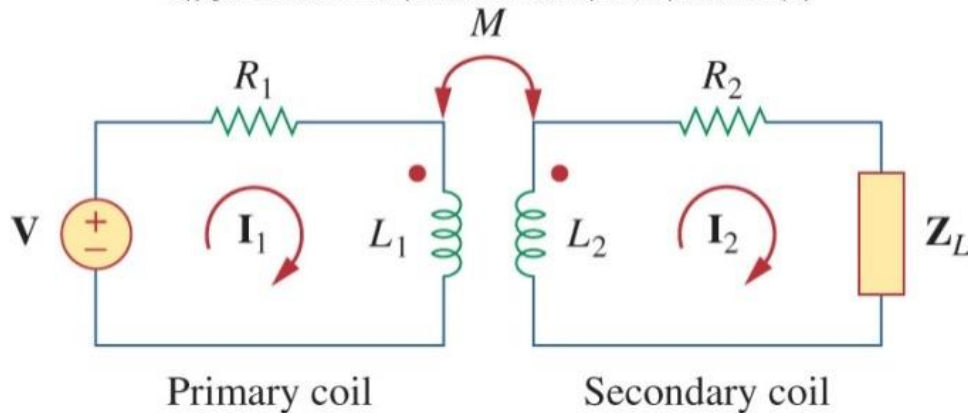




Transformers

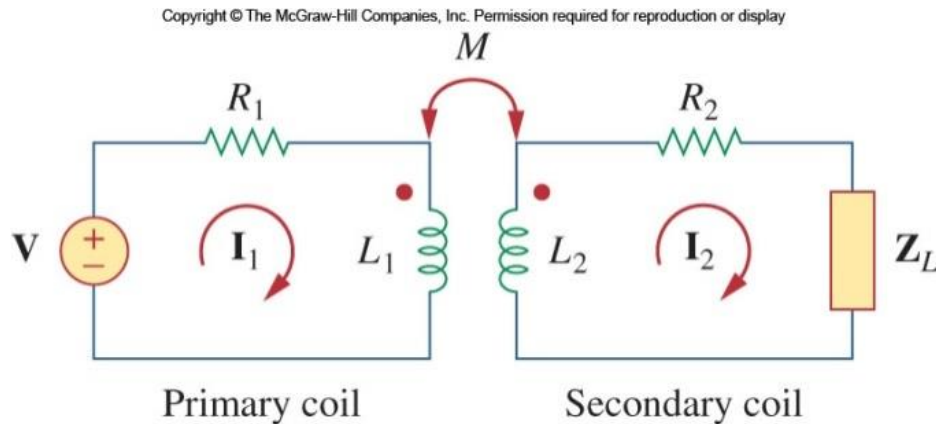
- A transformer is a magnetic device that takes advantage of mutual inductance.

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Transformer Impedance

- An important parameter to know for a transformer is how the input impedance Z_{in} is seen from the source.



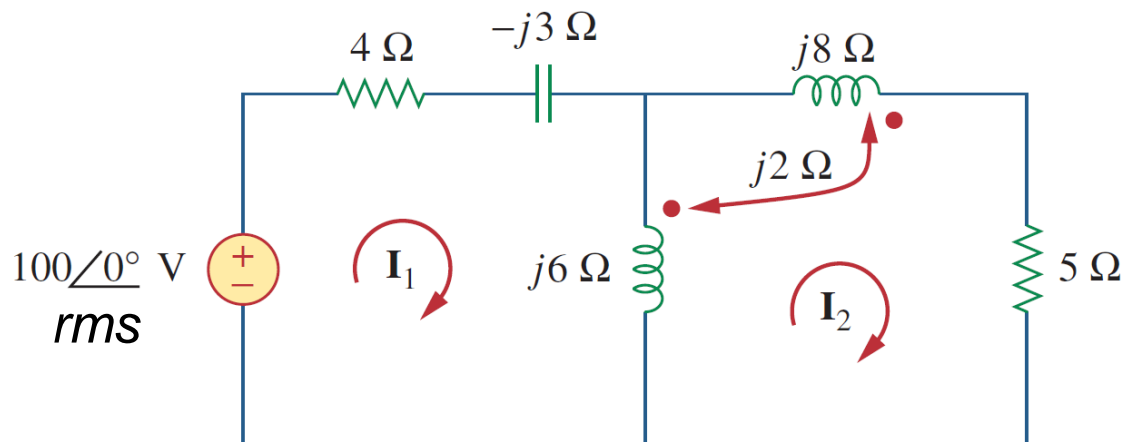
$$Z_{in} = \frac{V}{I_1} = R_1 + j\omega L_1 + \frac{\omega^2 M^2}{R_2 + j\omega L_2 + Z_L}$$

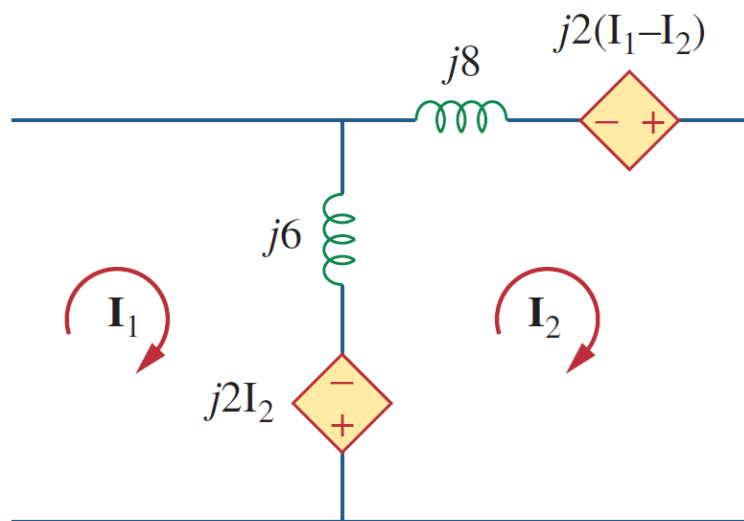
Reflected impedance from secondary to primary



Example

Find (1) mesh currents &
(2) complex power absorbed by the voltage source

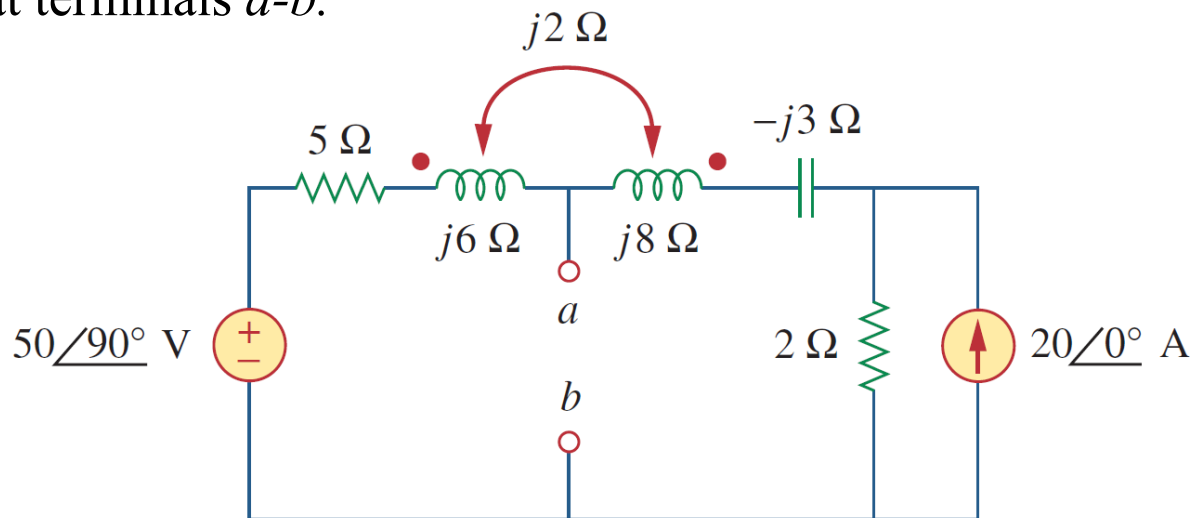






Practice

Obtain the Thevenin equivalent circuit for the circuit at terminals a - b .

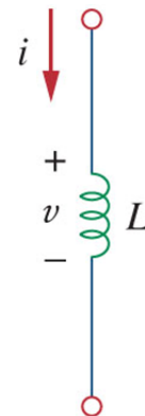


Energy in a Coupled Circuit

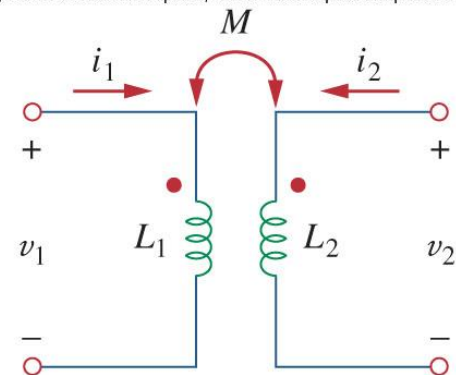
- The energy stored in an inductor is
- For coupled inductors, the total energy stored is

$$w = \frac{1}{2} L_1 i_1^2 + \frac{1}{2} L_2 i_2^2 \pm M i_1 i_2$$

- The positive sign is selected when the currents both enter or leave the dotted terminals.



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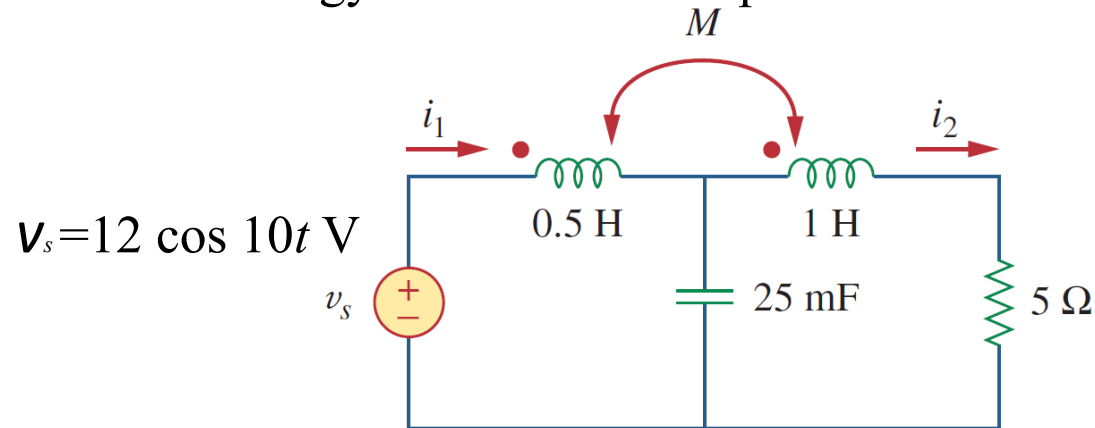


Practice

If $M = 0.2$ H

Find i_1 i_2 .

Calculate the energy stored in the coupled coils at $t = 15$ ms.





Coupling Coefficient k

- The system cannot have negative energy

$$\frac{1}{2}L_1 i_1^2 + \frac{1}{2}L_2 i_2^2 - M i_1 i_2 \geq 0 \quad \Rightarrow \quad M \leq \sqrt{L_1 L_2}$$

- Define a parameter describes how closely M approaches upper limit.

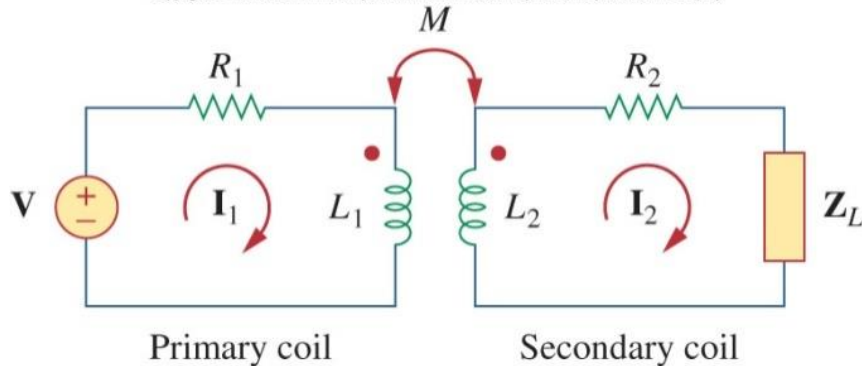
$$k = \frac{M}{\sqrt{L_1 L_2}}$$

- Coupling coefficient, $0 \leq k \leq 1$.

Ideal Transformers

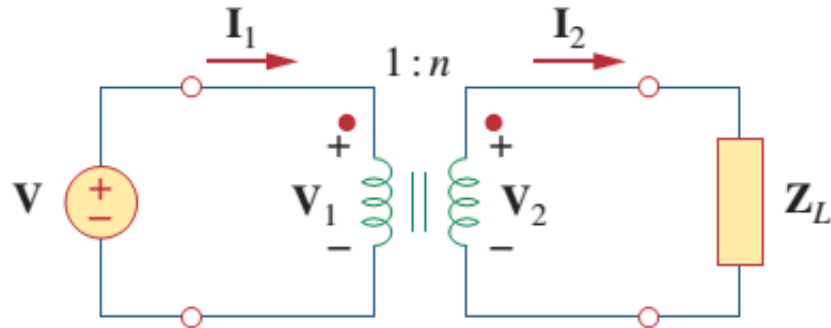
- The ideal transformer has:
 - Coils with very large reactance
($L_1, L_2, M \rightarrow \infty$)
 - Coupling coefficient $k=1$.
 - Primary and secondary coils are lossless, $R_1 = R_2 = 0$.

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Ideal Transformers



- The voltage is related as:

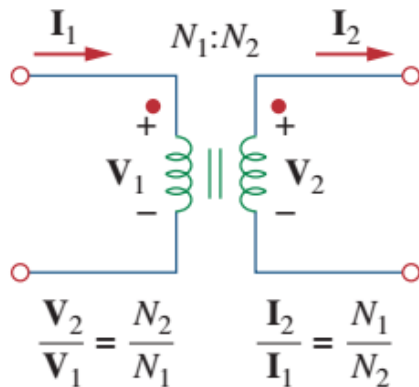
$$\frac{V_2}{V_1} = \frac{N_2}{N_1} = n \quad (13.52)$$

- The current is related as:

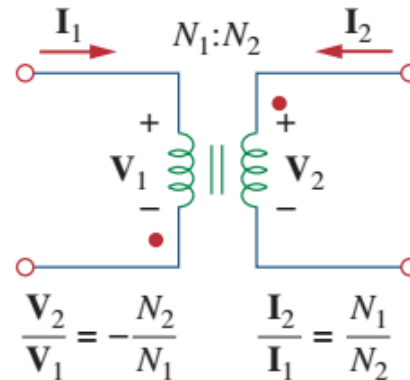
$$\frac{I_2}{I_1} = \frac{N_1}{N_2} = \frac{1}{n} \quad (13.55)$$



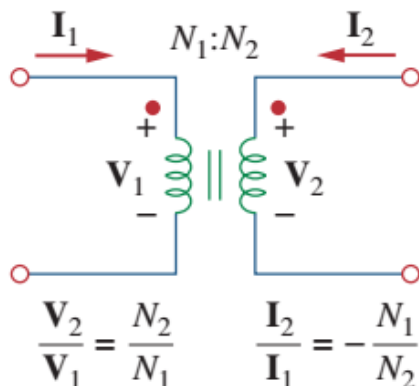
1. If V_1 and V_2 are *both* positive or both negative at the dotted terminals, use $+n$ in Eq. (13.52). Otherwise, use $-n$.
2. If I_1 and I_2 *both* enter into or both leave the dotted terminals, use $-n$ in Eq. (13.55). Otherwise, use $+n$.



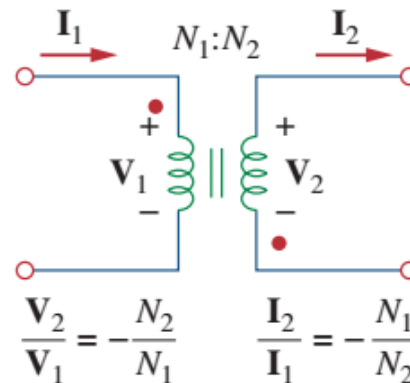
(a)



(c)



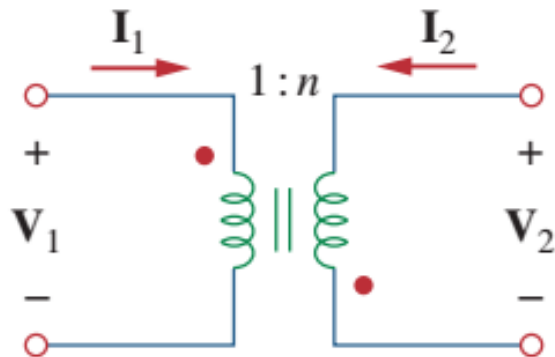
(b)



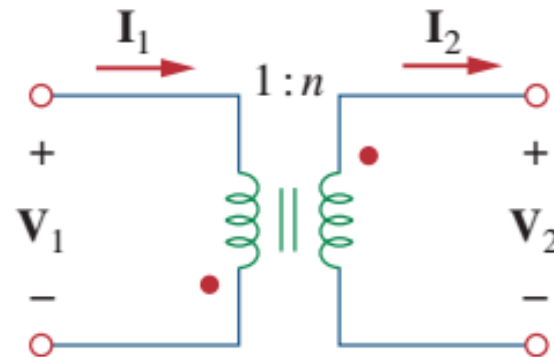
(d)



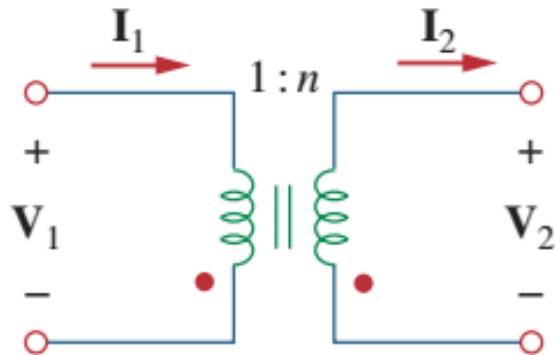
Practice



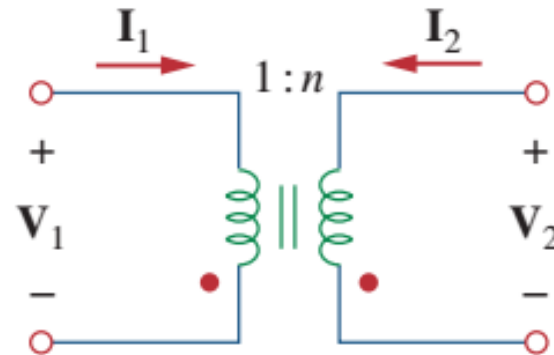
(a)



(b)



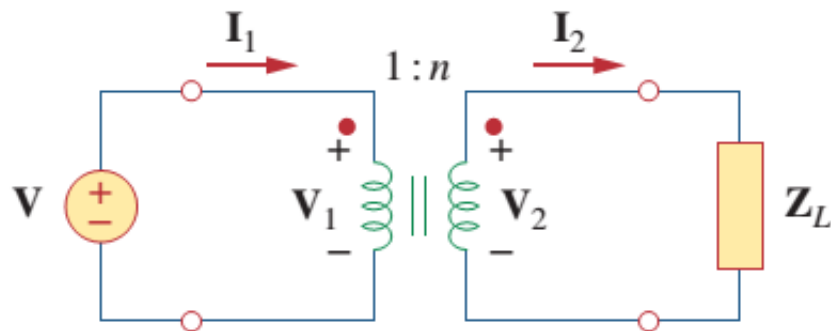
(c)



(d)



Ideal Transformers



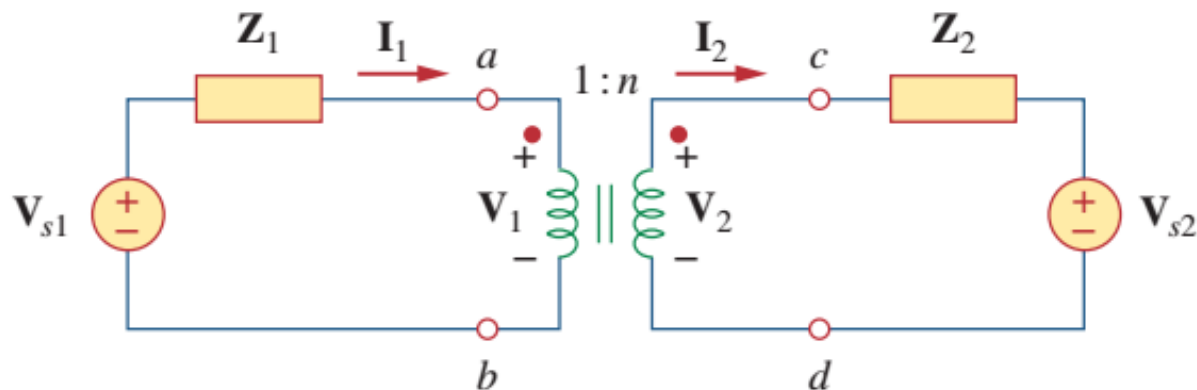
$$\frac{V_2}{V_1} = \frac{N_2}{N_1} = n$$

- Reflected impedance

$$Z_{\text{in}} = \frac{V_1}{I_1} =$$



Ideal Transformers

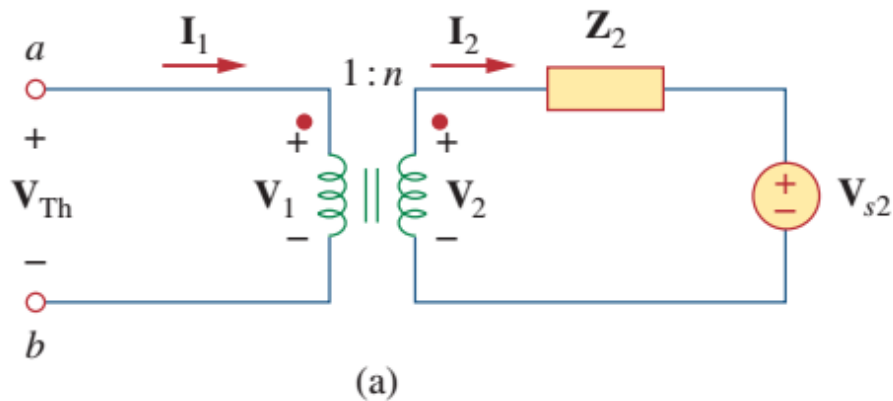
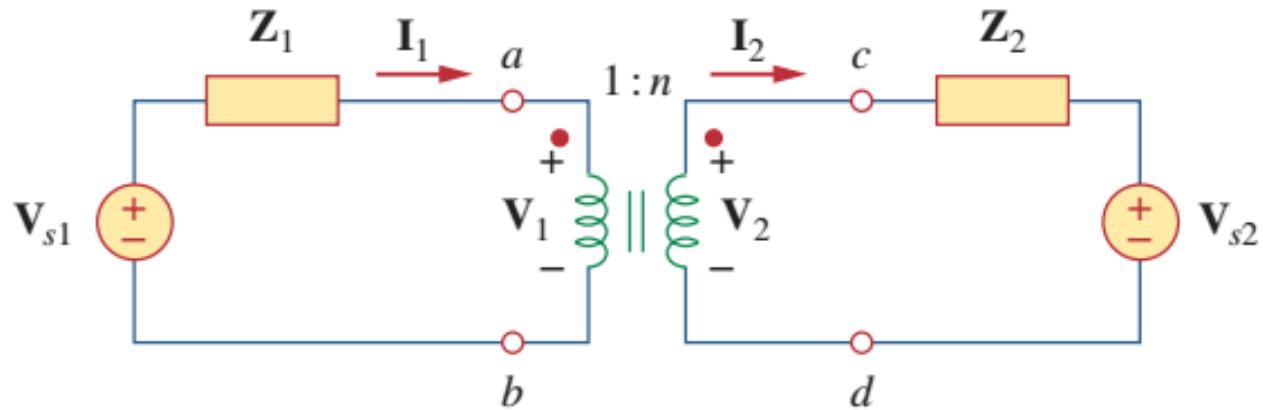


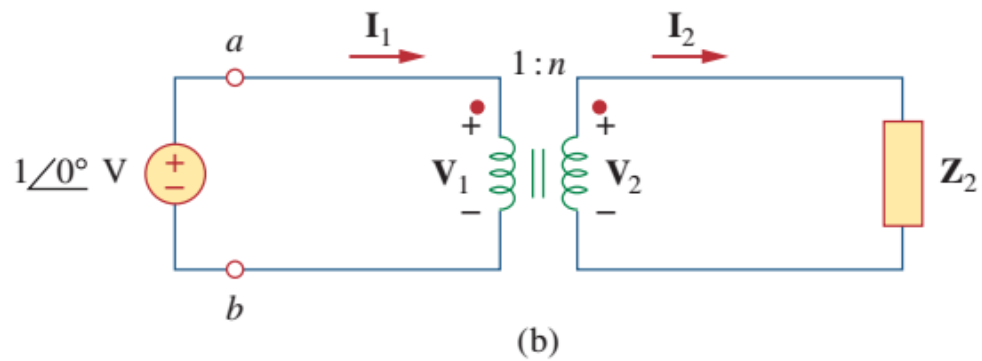
$$\frac{V_2}{V_1} = \frac{N_2}{N_1} = n$$

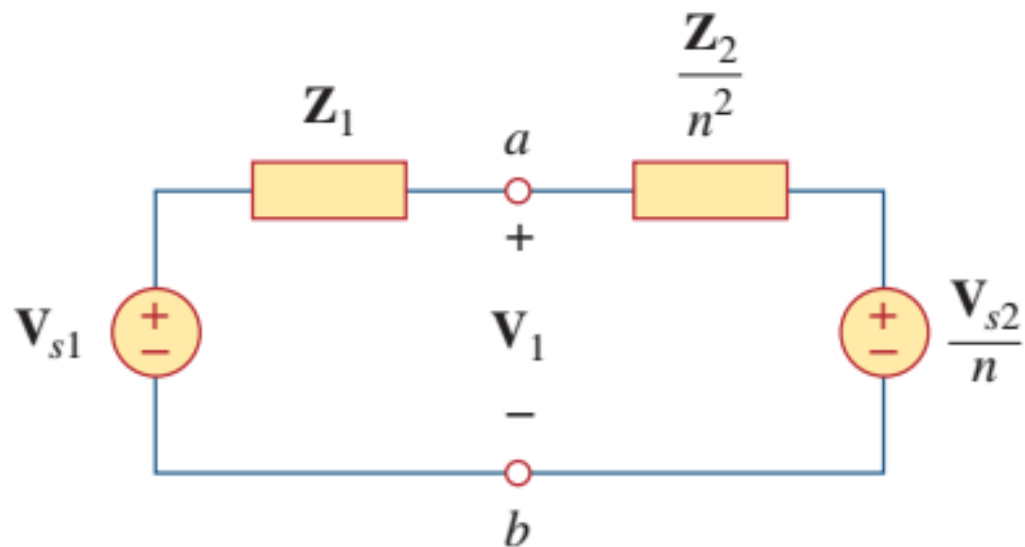
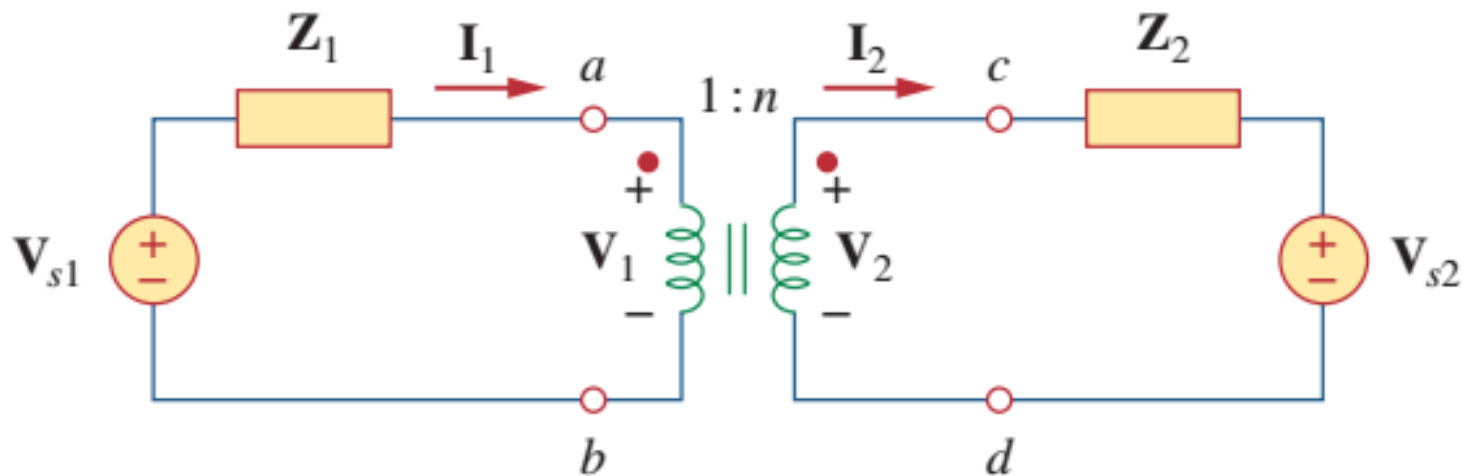


Ideal Transformers

- Reflected impedance and source



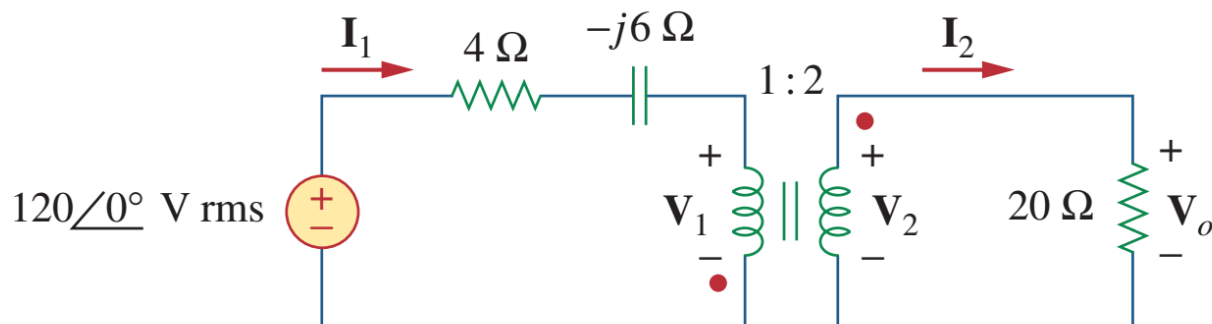


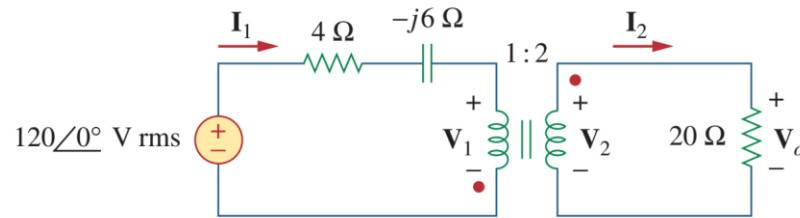




Example

For the ideal transformer circuit of Fig. 13.37, find: (a) the source current \mathbf{I}_1 , (b) the output voltage \mathbf{V}_o , and (c) the complex power supplied by the source.





Solution:

(a) The 20-Ω impedance can be reflected to the primary side and we get

$$\mathbf{Z}_R = \frac{20}{n^2} = \frac{20}{4} = 5 \, \Omega$$

Thus,

$$\begin{aligned}\mathbf{Z}_{\text{in}} &= 4 - j6 + \mathbf{Z}_R = 9 - j6 = 10.82 \angle -33.69^\circ \, \Omega \\ \mathbf{I}_1 &= \frac{120 \angle 0^\circ}{\mathbf{Z}_{\text{in}}} = \frac{120 \angle 0^\circ}{10.82 \angle -33.69^\circ} = 11.09 \angle 33.69^\circ \, \text{A}\end{aligned}$$

(b) Since both \mathbf{I}_1 and \mathbf{I}_2 leave the dotted terminals,

$$\begin{aligned}\mathbf{I}_2 &= -\frac{1}{n} \mathbf{I}_1 = -5.545 \angle 33.69^\circ \, \text{A} \\ \mathbf{V}_o &= 20 \mathbf{I}_2 = 110.9 \angle 213.69^\circ \, \text{V}\end{aligned}$$

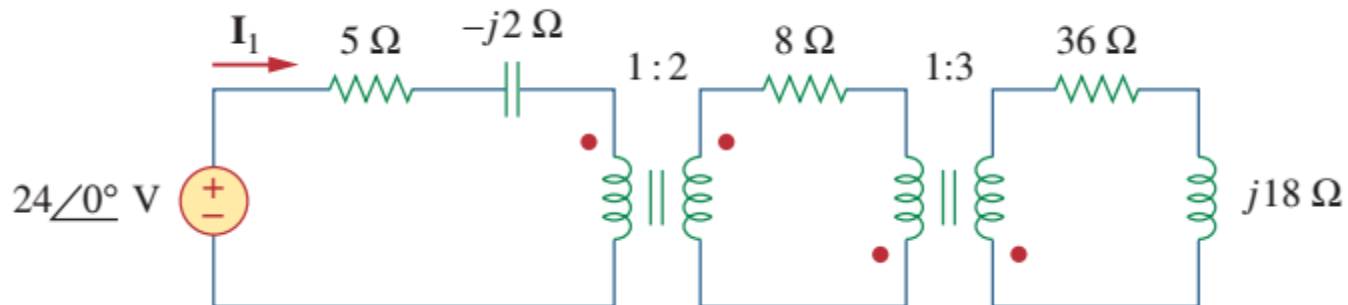
(c) The complex power supplied is

$$\mathbf{S} = \mathbf{V}_s \mathbf{I}_1^* = (120 \angle 0^\circ)(11.09 \angle -33.69^\circ) = 1,330.8 \angle -33.69^\circ \, \text{VA}$$



Practice

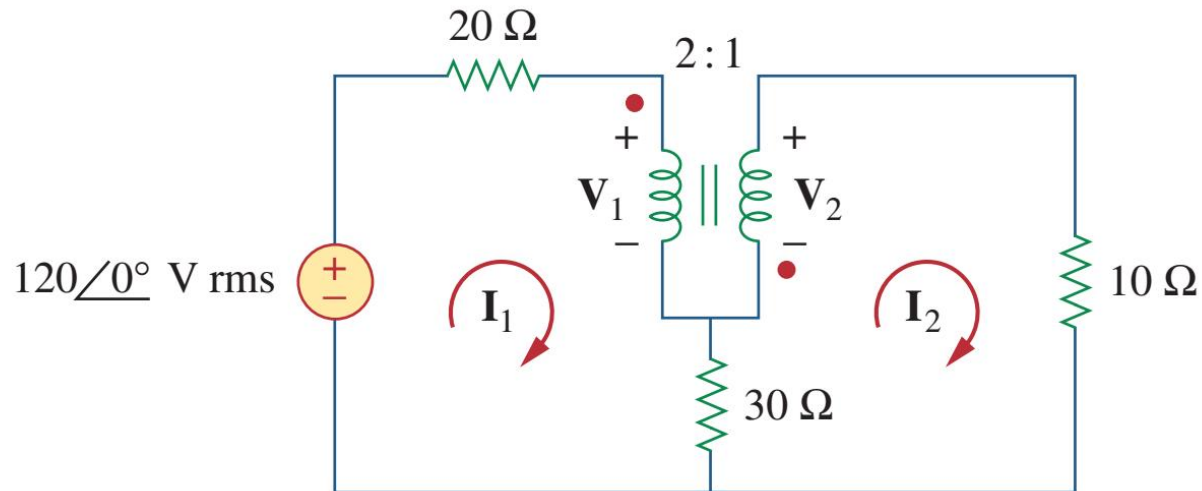
- Find reflected impedance and I_1





Example

Calculate the power supplied to the $10\text{-}\Omega$ resistor in the ideal transformer circuit of Fig. 13.39.





$$-120 + (20 + 30)\mathbf{I}_1 - 30\mathbf{I}_2 + \mathbf{V}_1 = 0$$

or

$$50\mathbf{I}_1 - 30\mathbf{I}_2 + \mathbf{V}_1 = 120 \quad (13.9.1)$$

For mesh 2,

$$-\mathbf{V}_2 + (10 + 30)\mathbf{I}_2 - 30\mathbf{I}_1 = 0$$

or

$$-30\mathbf{I}_1 + 40\mathbf{I}_2 - \mathbf{V}_2 = 0 \quad (13.9.2)$$

At the transformer terminals,

$$\mathbf{V}_2 = -\frac{1}{2}\mathbf{V}_1 \quad (13.9.3)$$

$$\mathbf{I}_2 = -2\mathbf{I}_1 \quad (13.9.4)$$

(Note that $n = 1/2$.) We now have four equations and four unknowns, but our goal is to get \mathbf{I}_2 . So we substitute for \mathbf{V}_1 and \mathbf{I}_1 in terms of \mathbf{V}_2 and \mathbf{I}_2 in Eqs. (13.9.1) and (13.9.2). Equation (13.9.1) becomes

$$-55\mathbf{I}_2 - 2\mathbf{V}_2 = 120 \quad (13.9.5)$$

and Eq. (13.9.2) becomes

$$15\mathbf{I}_2 + 40\mathbf{I}_2 - \mathbf{V}_2 = 0 \quad \Rightarrow \quad \mathbf{V}_2 = 55\mathbf{I}_2 \quad (13.9.6)$$

Substituting Eq. (13.9.6) in Eq. (13.9.5),

$$-165\mathbf{I}_2 = 120 \quad \Rightarrow \quad \mathbf{I}_2 = -\frac{120}{165} = -0.7272 \text{ A}$$

The power absorbed by the $10\text{-}\Omega$ resistor is

$$P = (-0.7272)^2(10) = 5.3 \text{ W}$$