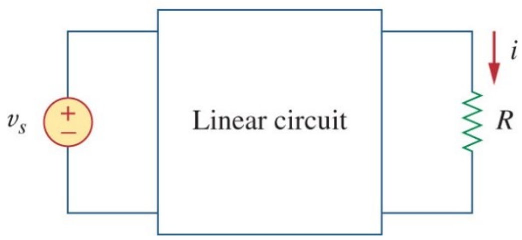
Lecture 3
Circuit Theorems

Outline

- Linearity property
- Superposition
- Thevenin’s theorem
- Source transformation
- Norton’s theorem
- Power transfer

Linear Circuit

- A linear circuit consists of only linear elements (resistors, capacitors and inductors), linear dependent sources, and independent sources.



- In a circuit,
  - Excitation: Sources
  - Response: Voltage or current in the branches

Linearity Property

Excitation x → System → Response $f(x)$

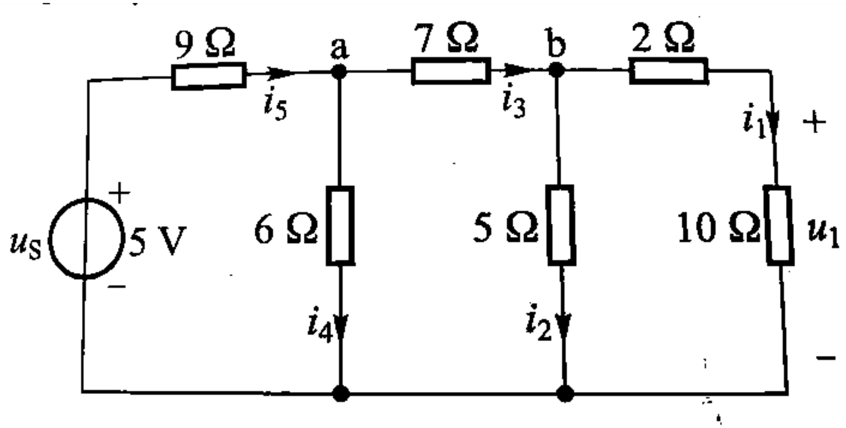
- Linearity is a combination of
  - homogeneity (scaling) property

    Excitation $\alpha \cdot x$ → System → Response $\alpha \cdot f(x)$ α is constant

  - additivity property

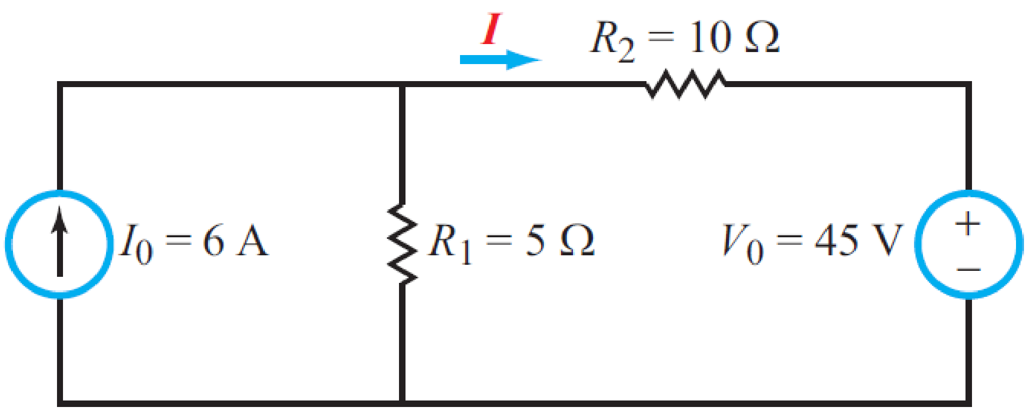
    Excitation $x_1 + x_2$ → System → Response $f(x_1) + f(x_2)$

Example of homogeneity (scaling) property

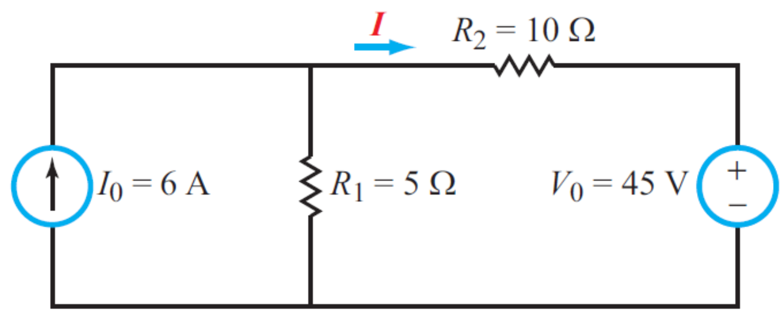


Superposition

- The superposition principle states that the voltage across (or current through) an element in *a linear circuit* is the algebraic sum of the voltages across (or currents through) that element due to each **independent source** acting alone.



Applying Superposition



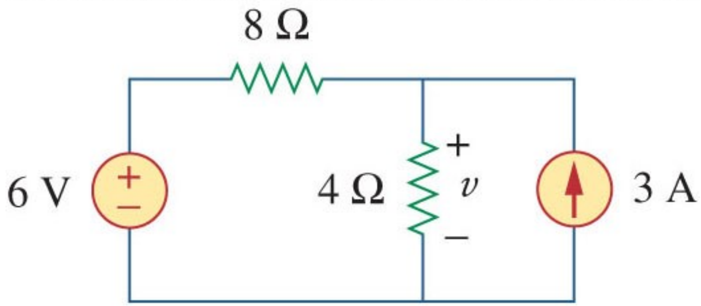
- The steps are:
  1. Turn off all other **independent** sources except for the source of interest. Find the output (voltage or current) due to that active source.
     - "Turn off" means to replace **independent** voltage source by short circuit (0 **V**), **independent** current source by open circuit (0 **A**).
  2. Repeat step 1 for **each independent** source.
  3. Find the total contribution by adding algebraically ***all*** the contributions due to the **independent** sources.

Note that

1) Using superposition means applying one independent source at a time.
2) Dependent sources are left alone.

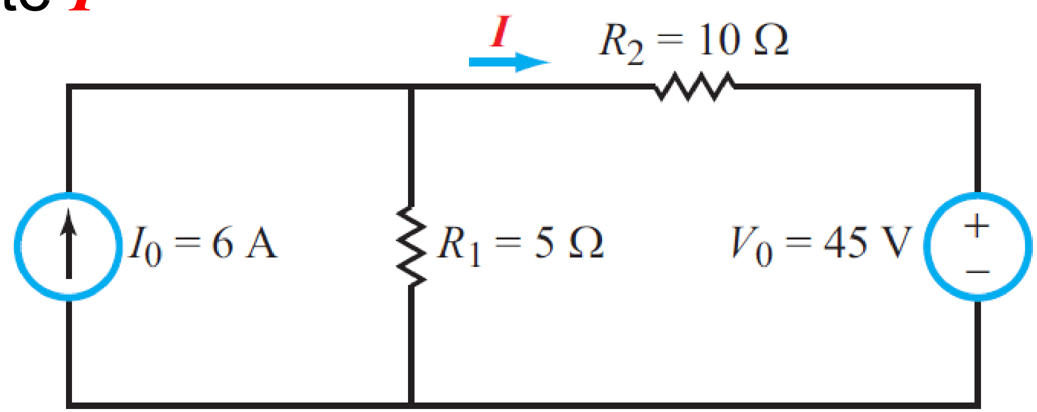
Open Circuit and Short Circuit

- Turn off an independent voltage source means
  - v=0
  - Replace by wire
  - Short circuit
- Turn off an independent current source means
  - i=0
  - Cut off the branch
  - Open circuit



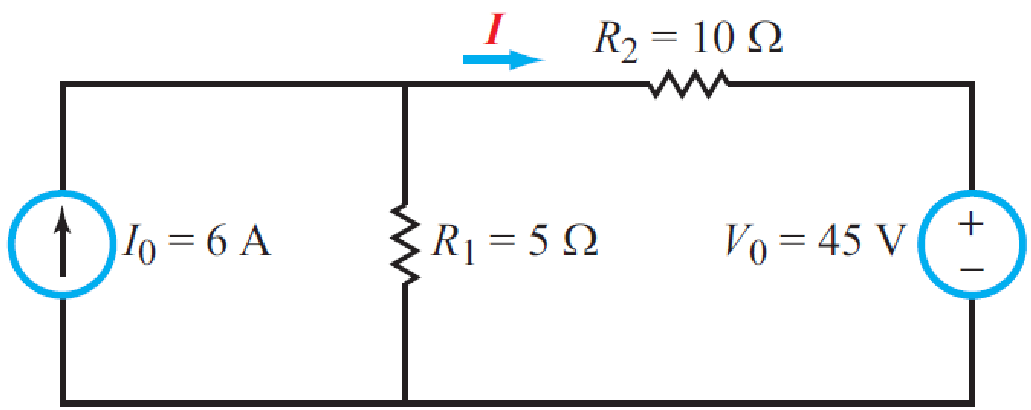
Example: Superposition

Calculate I



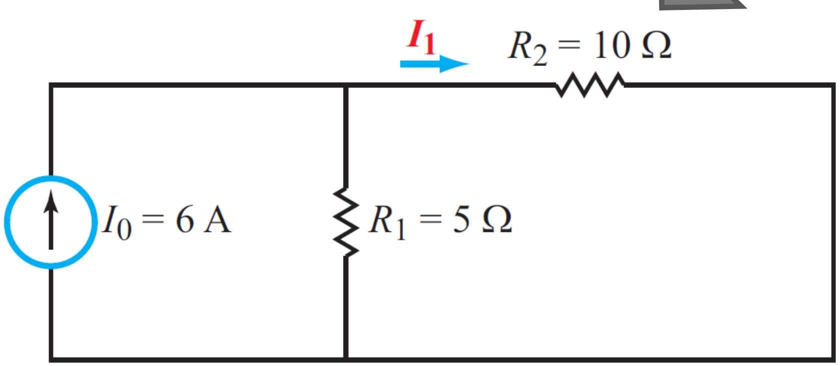
Example: Superposition

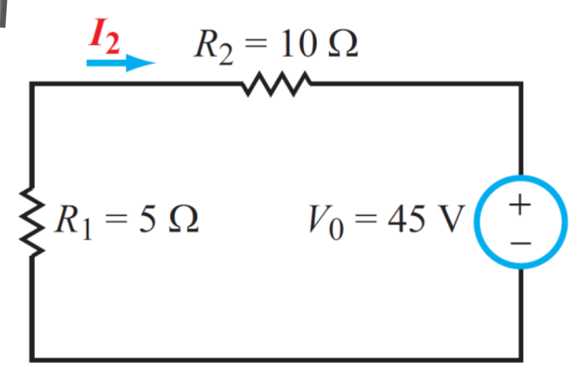
Calculate I



Contribution from I_0 alone

Contribution from V_0 alone





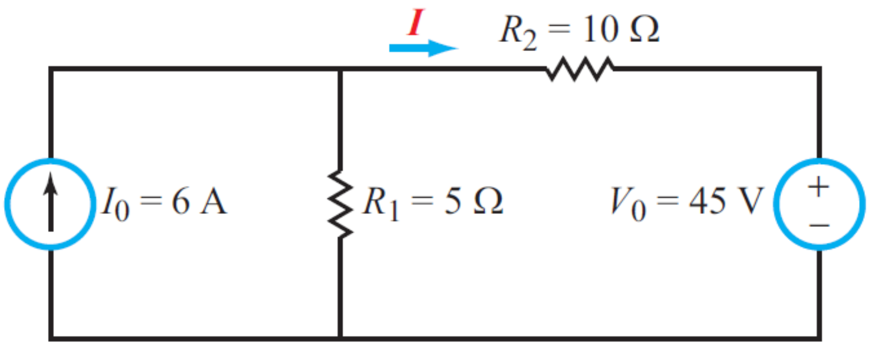
$I_1 = 2$ A

$I_2 = -3$ A

$$\boxed{I = I_1 + I_2 = 2 - 3 = -1 \text{ A}}$$

Why Superposition?

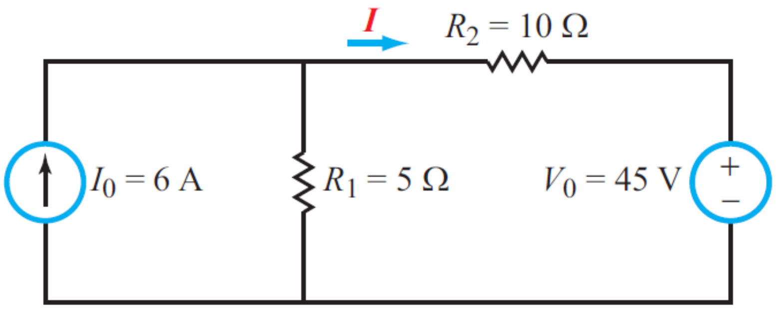
- It is useful to evaluate the sensitivity of a response to specific sources in the circuit.
- Because it entails solving a circuit multiple times, this source-superposition method may not be attractive.



$$I = aI_0 + bV_0$$

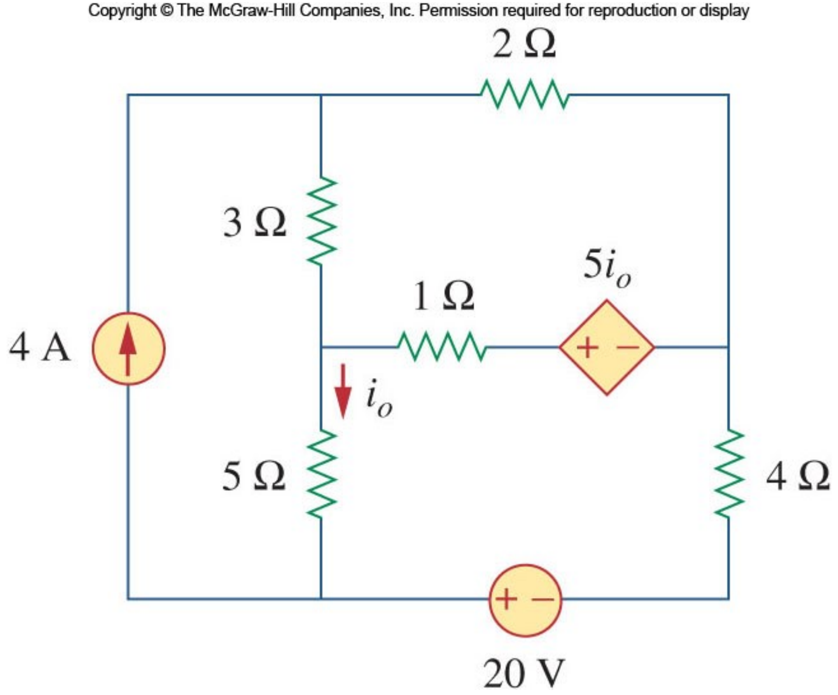
How about Power absorbed by R_2

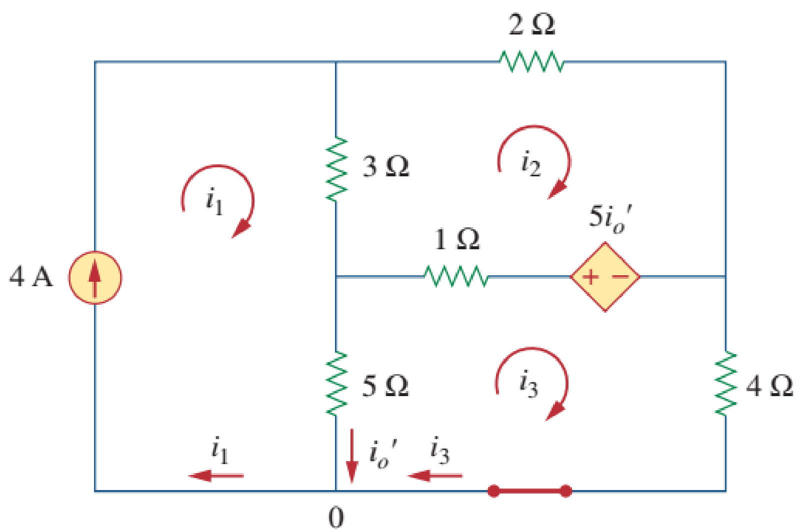
- Power due to $I_0, P_1 =?$
- Power due to $V_0, P_2 =?$
- Power due to both V_0 and $I_0, P =?$

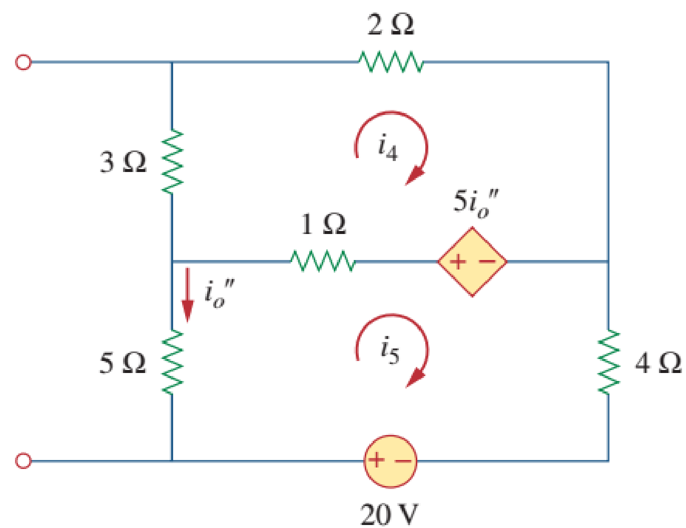


Practice 1

- Find i_0 in the circuit shown below.




2 Ω
3 Ω
i_1
i_2
$5i_o'$
1 Ω
4 A
5 Ω
i_3
4 Ω
i_1
i_o'
i_3
0


2 Ω
i_4
3 Ω
$5i_o''$
1 Ω
i_o''
i_5
5 Ω
4 Ω
20 V

Practice 2

- Express node voltage e_1 as a function of two voltage sources V_1, V_2 and one current source I.