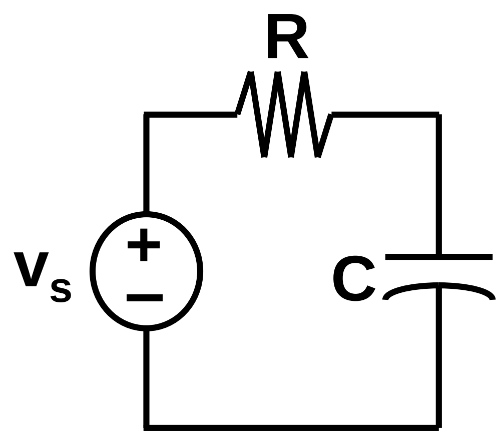
Outline

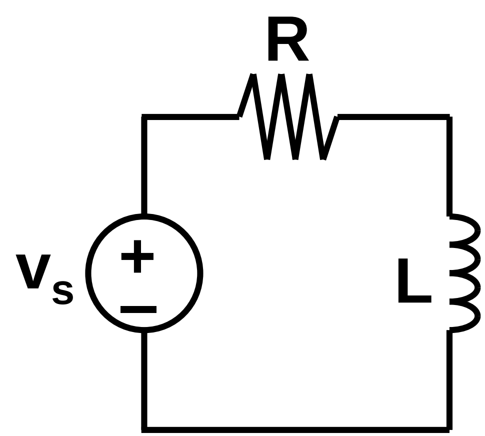
- Capacitors and inductors
- Natural response of RC/RL circuits
- Step response of RC/RL circuits
- Others

RC and RL Circuits

- A circuit that contains only source(s), resistor(s) and <u>a capacitor</u> is called an ***RC circuit***.



- A circuit that contains only source(s), resistor(s) and <u>an inductor</u> is called an ***RL circuit***.

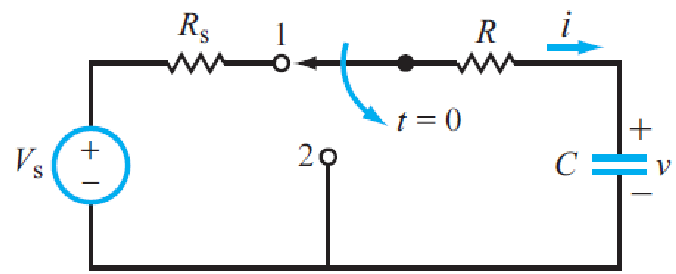


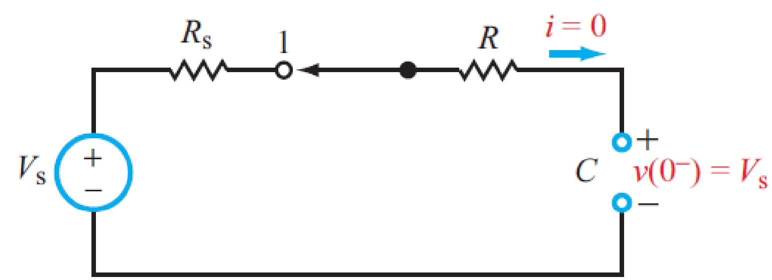
[Source: Berkeley]

Natural Response

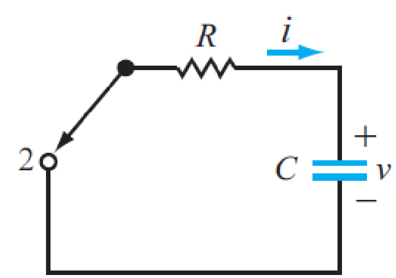
Behavior (*i.e.*, current and voltage) when stored energy in the inductor or capacitor is released to the resistive part of the network (containing <u>no independent sources</u>).

Natural Response of a Charged Capacitor





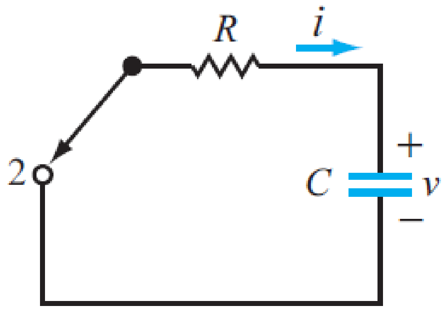
(a) $t = 0^-$ is the instant just before the switch is moved from terminal 1 to terminal 2;



(b) $t = 0$ is the instant just after it was moved, $t = 0$ is synonymous with $t = 0^+$.

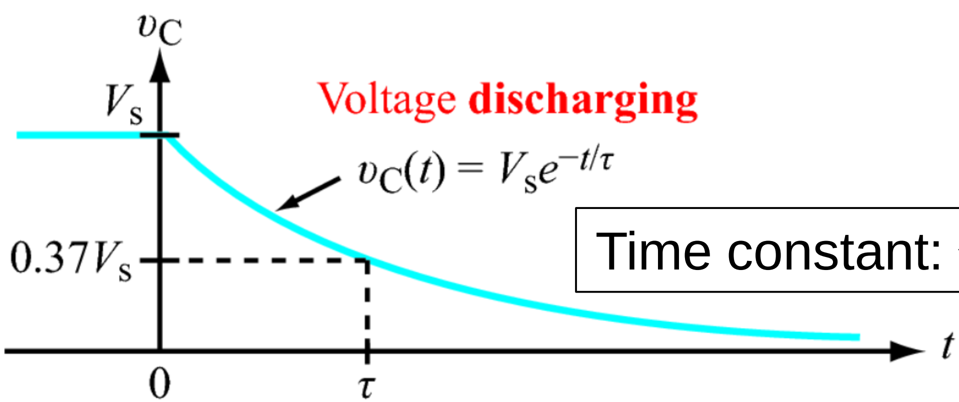
[Source: Berkeley]

Natural Response of a Charged Capacitor

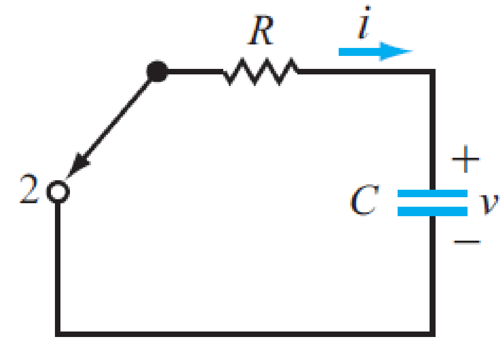


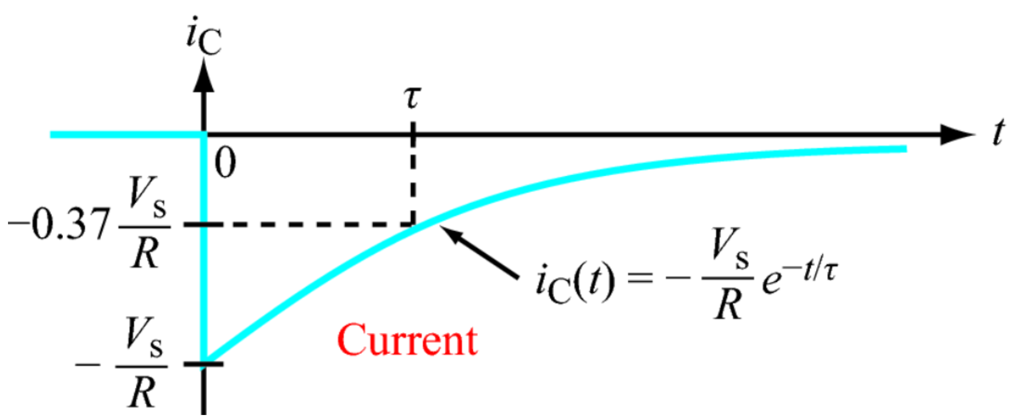
[Source: Berkeley]

Natural Response of RC Circuit



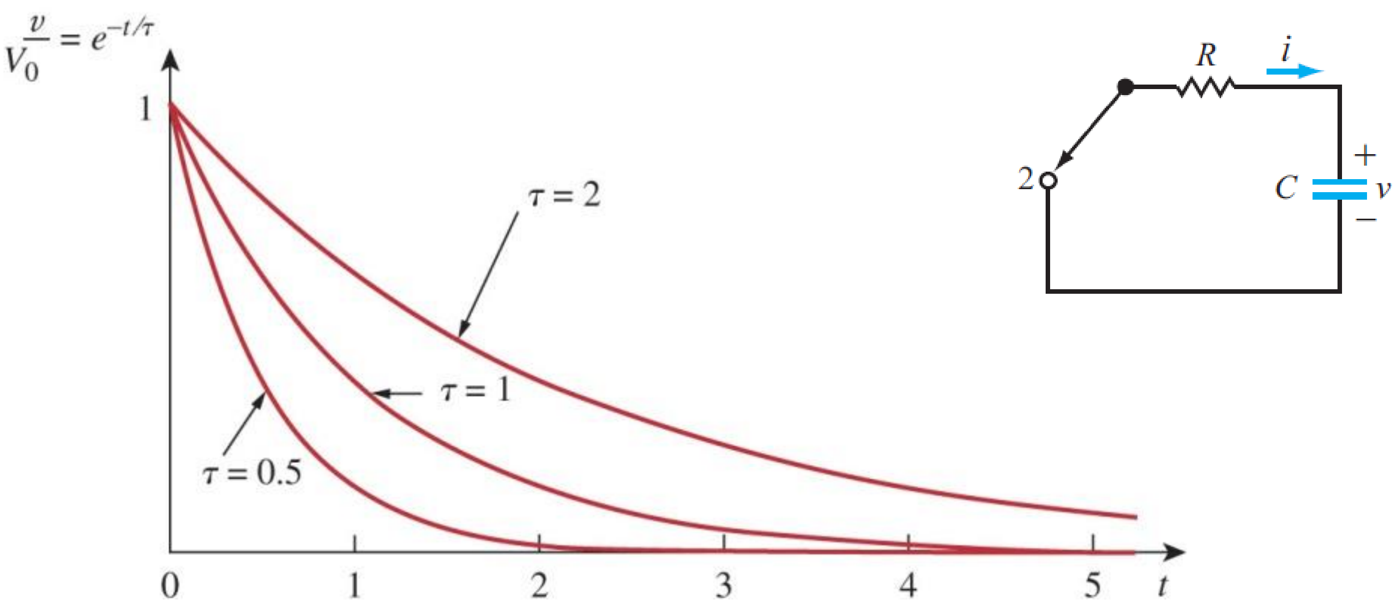
Time constant: $\tau = RC$



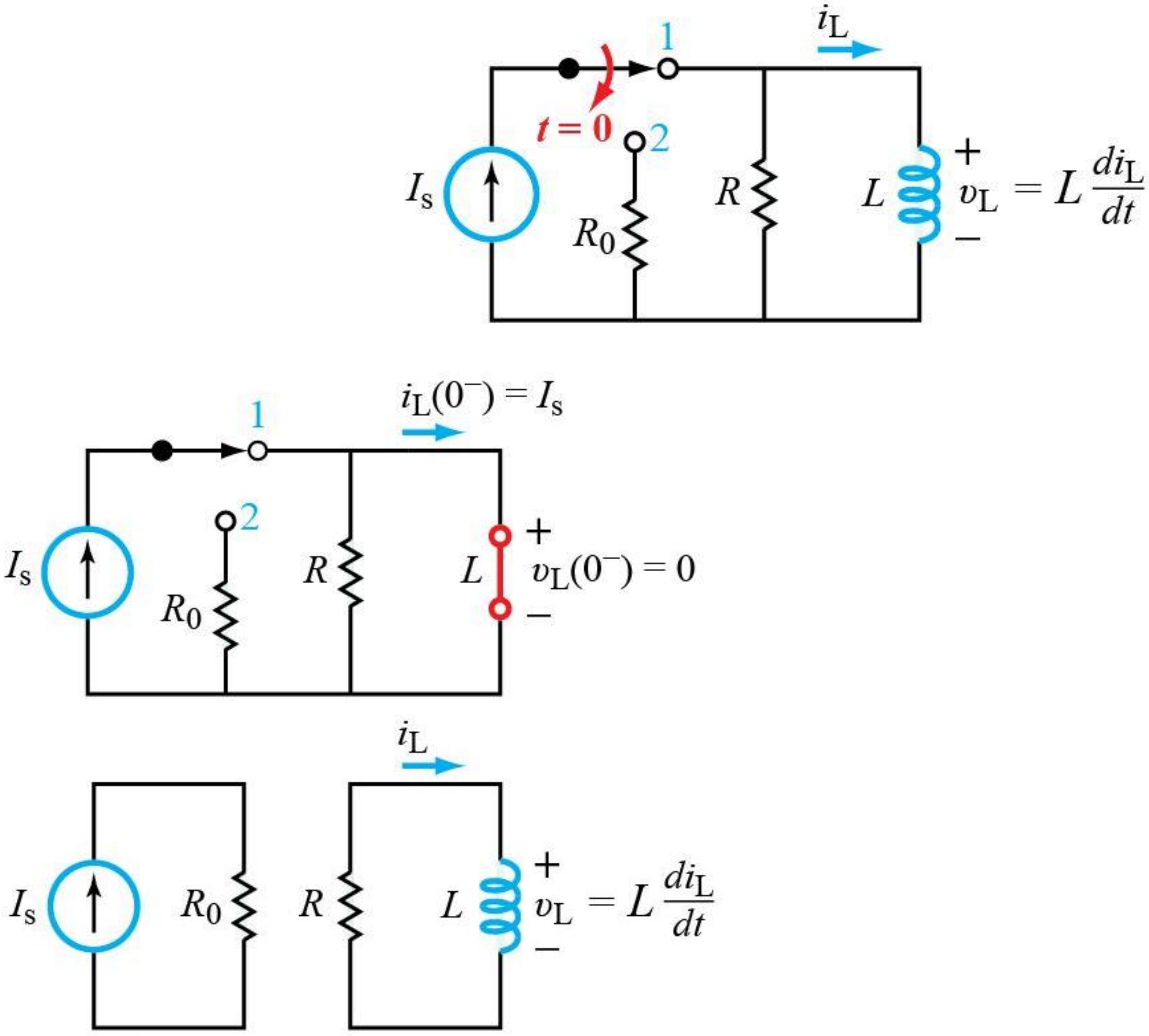


Time Constant τ ($= RC$)

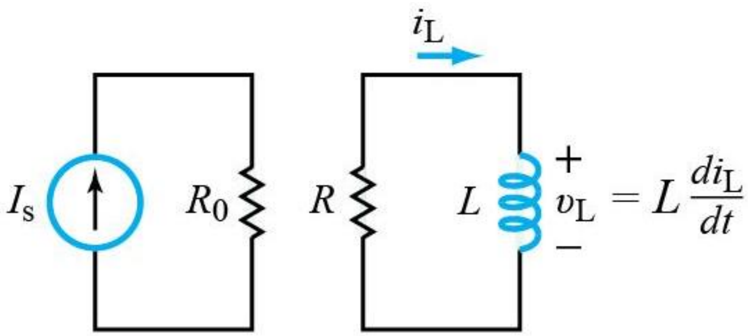
- A circuit with a small time constant has a fast response and vice versa.



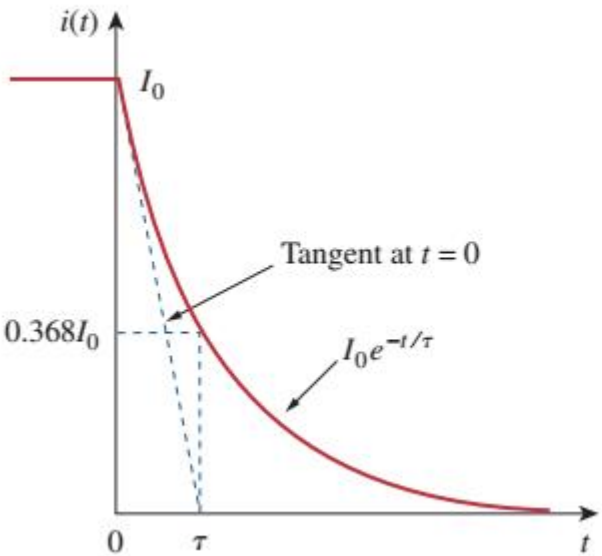
Natural Response of the RL Circuit



Natural Response of the RL Circuit

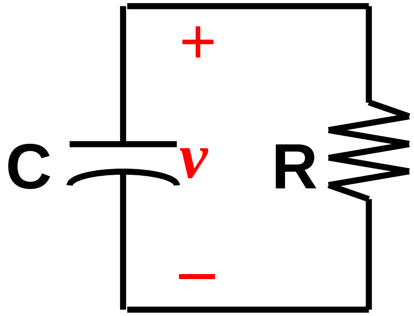


Natural Response of the RL Circuit



Natural Response Summary

RC Circuit



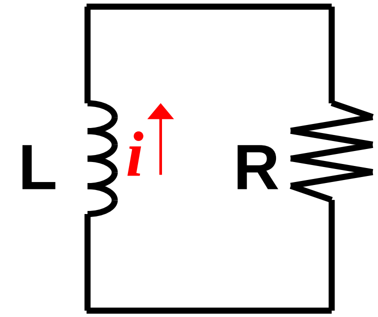
- Capacitor voltage cannot change instantaneously

$$v(0^-) = v(0^+)$$

$$v(t) = v(0)e^{-t/\tau}$$

- time constant $\tau = RC$

RL Circuit



- Inductor current cannot change instantaneously

$$i(0^-) = i(0^+)$$

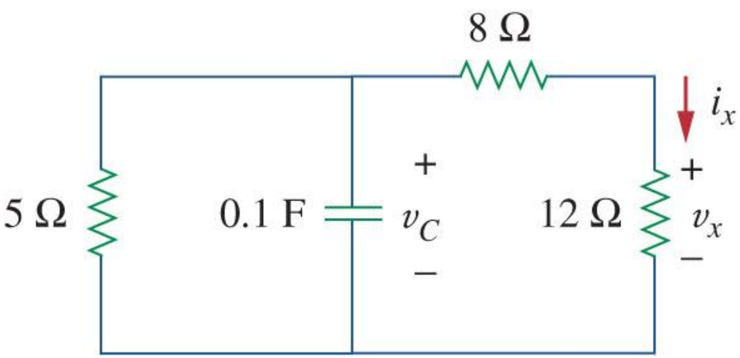
$$i(t) = i(0)e^{-t/\tau}$$

- time constant $\tau = \frac{L}{R}$

[Source: Berkeley]

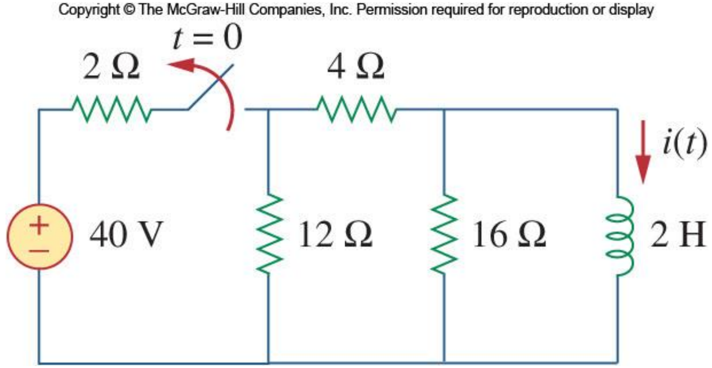
Example

- In the circuit below, let $v_C(t=0) = 15\text{V}$. Find v_C, v_x, and i_x for $t > 0$.

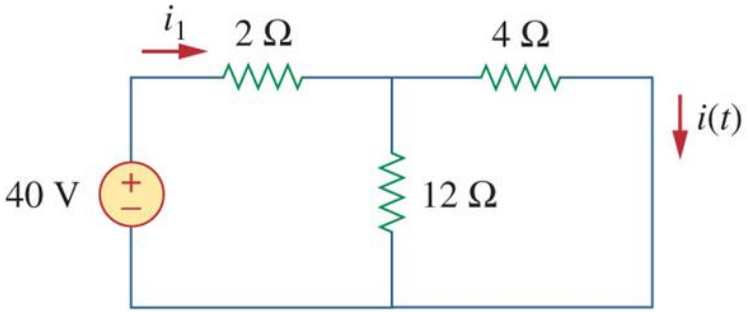


Example

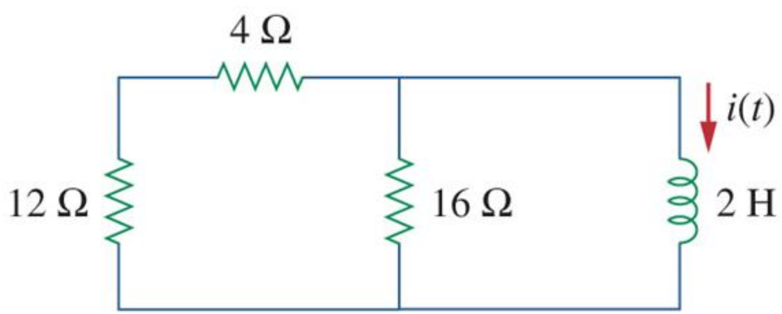
- The switch in the circuit below has been closed for a long time. At $t = 0$, the switch is opened. Calculate $i(t)$ for $t > 0$.



When $t < 0$



When $t > 0$



Outline

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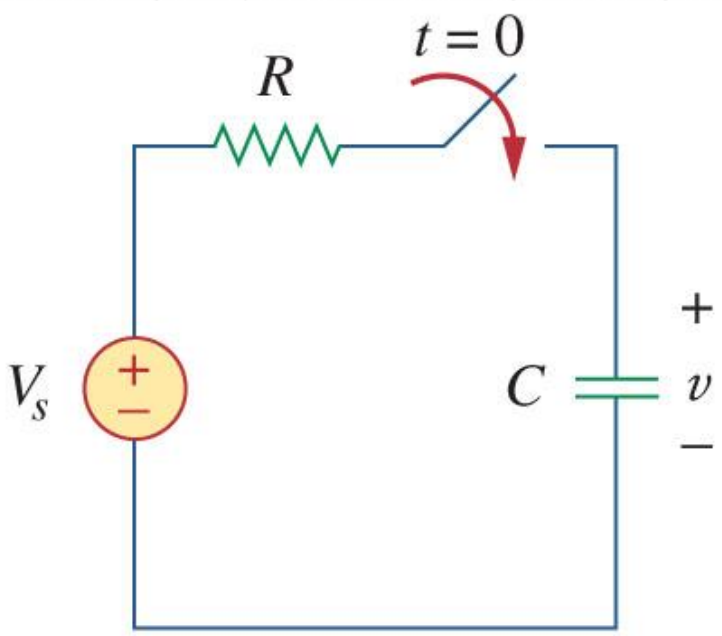
Step Response of RC Circuit

- When a DC source is suddenly applied to a RC circuit, the circuit response is known as the step response.

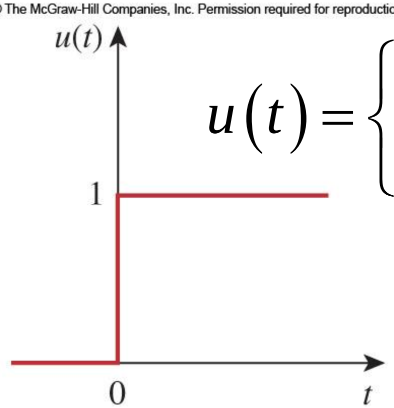


$$v(0^-) = v(0^+) = v_0$$

The unit step function *u(t)*

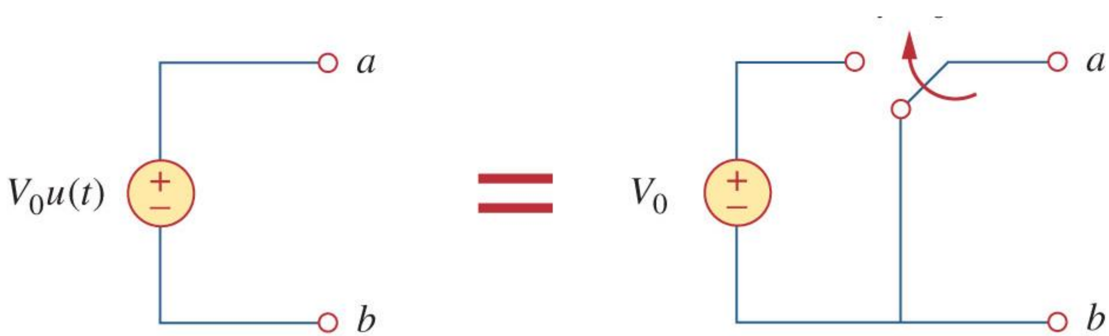
- A step function is one that maintains a constant value before a certain time and then changes to another constant afterwards.

switching time may be shifted to $t = t_0$ by



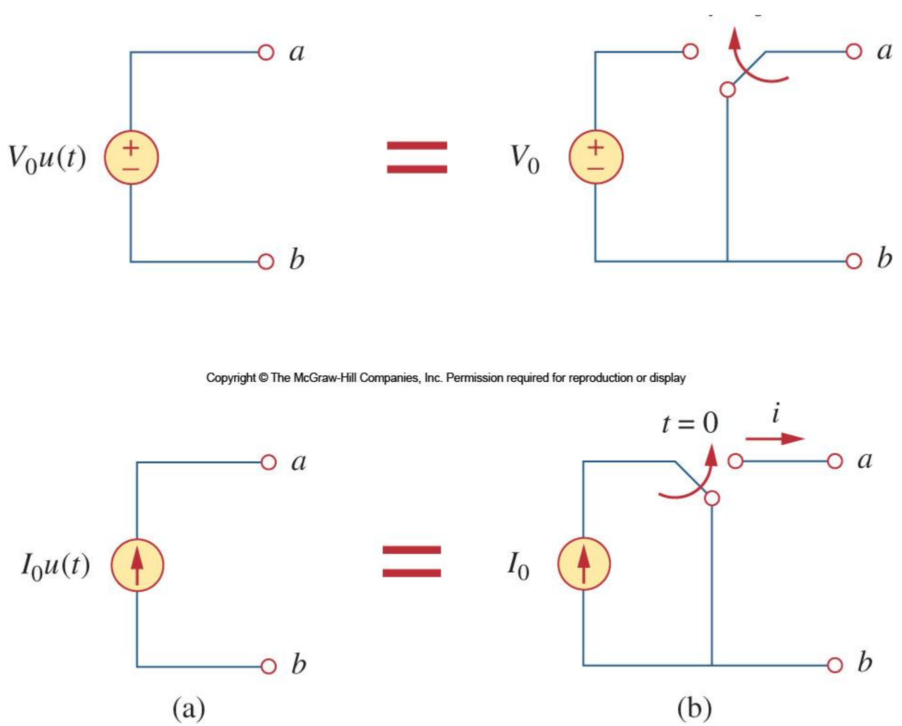
$$u(t) = \begin{cases} 0, & t < 0 \\ 1, & t > 0 \end{cases}$$

$$u(t - t_0) = \begin{cases} 0, & t < t_0 \\ 1, & t > t_0 \end{cases}$$

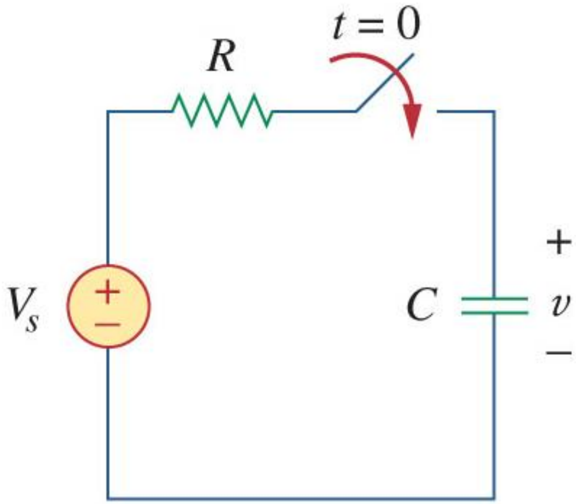


Equivalent Circuit of Unit Step

- The unit step function has an equivalent circuit to represent when it is used to switch on a source.

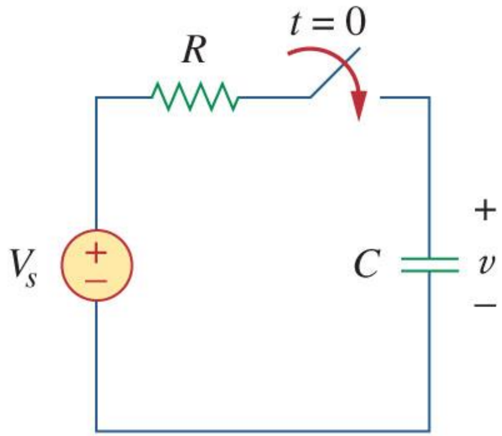


Step Response of the RC Circuit



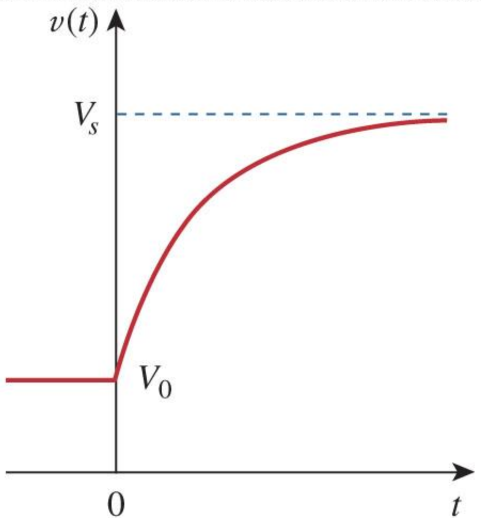
$$v(0^-) = v(0^+) = V_0$$

Step Response of the RC Circuit



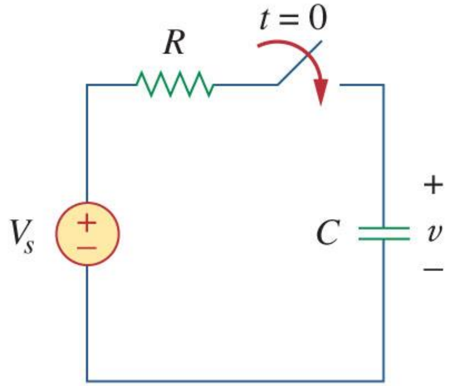
$$v(0^-) = v(0^+) = V_0$$

$$v(t) = \begin{cases} V_0, & t < 0 \\ V_s + (V_0 - V_s)e^{-t/\tau} & t > 0 \end{cases}$$



- This is known as the complete response, or total response.

Complete response



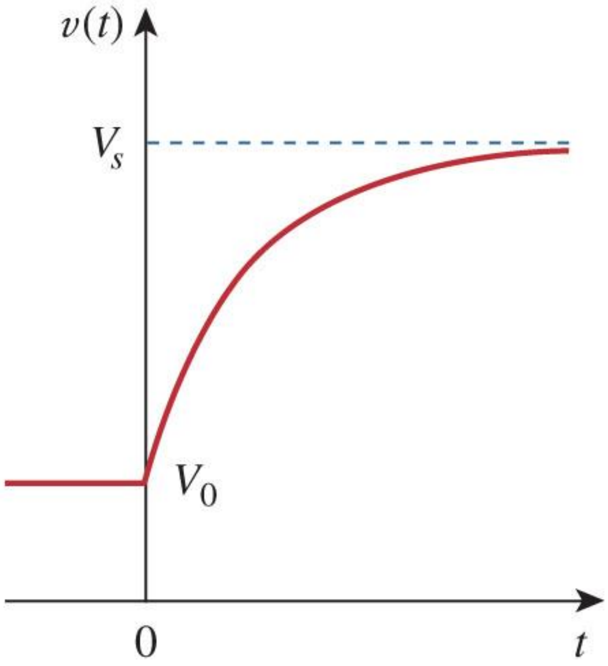
$$v(0^-) = v(0^+) = V_0$$

- The complete response

$$v(t) = \begin{cases} V_0, & t<0 \\ V_s + (V_0 - V_s)e^{-t/\tau} & t>0 \end{cases}$$

can be written as:

$$\boxed{v = v_n + v_f}$$



$$\text{Complete response} = \underbrace{\text{natural response}}_{\text{stored energy}} + \underbrace{\text{forced response}}_{\text{independent source}}$$

or

$$v = v_n + v_f$$

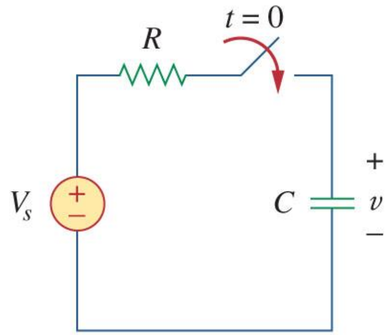
where

$$v_n = V_o e^{-t/\tau}$$

and

$$v_f = V_s(1 - e^{-t/\tau})$$

Another Perspective

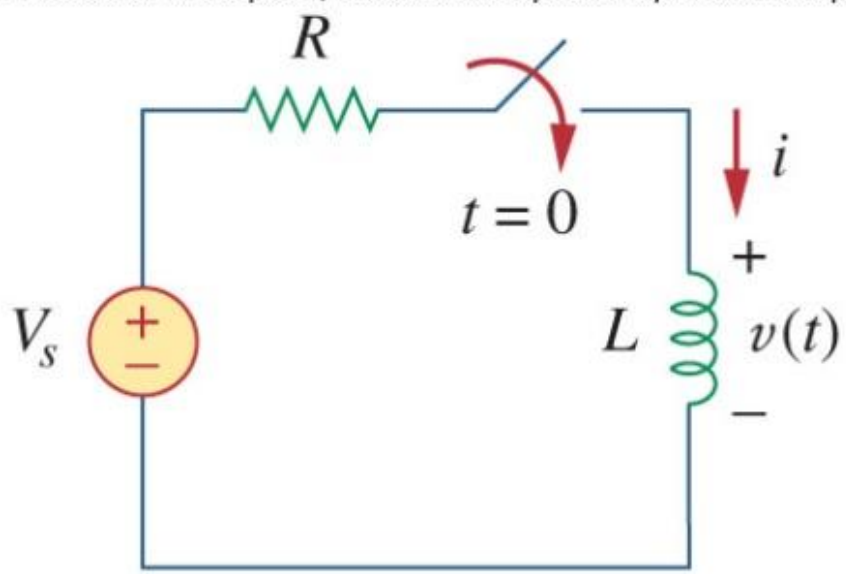


$$v(t) = \begin{cases} V_0, & t < 0 \\ V_s + (V_0 - V_s)e^{-t/\tau} & t > 0 \end{cases}$$

- Another way to look at the response is to break it up into the transient response and the steady state response:

$$v(t) = \underbrace{v(\infty)}_{\text{steady } v_{ss}} + \underbrace{[v(0) - v(\infty)]e^{-t/\tau}}_{\text{transient } v_t}$$

Step Response of the RL Circuit



$$i\left(0^{+}\right)=i\left(0^{-}\right)=I_0$$

$$i(t)=\frac{V_s}{R}+\left(I_0-\frac{V_s}{R}\right)e^{-t/\tau}$$

- Again one may break the response up into the transient response and the steady state response:

$$i(t)=\underbrace{i(\infty)}_{\text{steady } i_{ss}}+\underbrace{[i(0)-i(\infty)]e^{-t/\tau}}_{\text{transient } i_t}$$

General Procedure of Finding RC/RL Response with D.C. sources

1. **Identify the variable of interest**
   - For RL circuits, it is usually the inductor current $i_L(t)$.
   - For RC circuits, it is usually the capacitor voltage $v_c(t)$.

2. **Determine the initial value of the variable at T_0**
   - Recall that $i_L(t)$ and $v_c(t)$ are continuous variables:

$$i_L(T_0^+) = i_L(T_0^-) \quad \text{and} \quad v_c(T_0^+) = v_c(T_0^-)$$

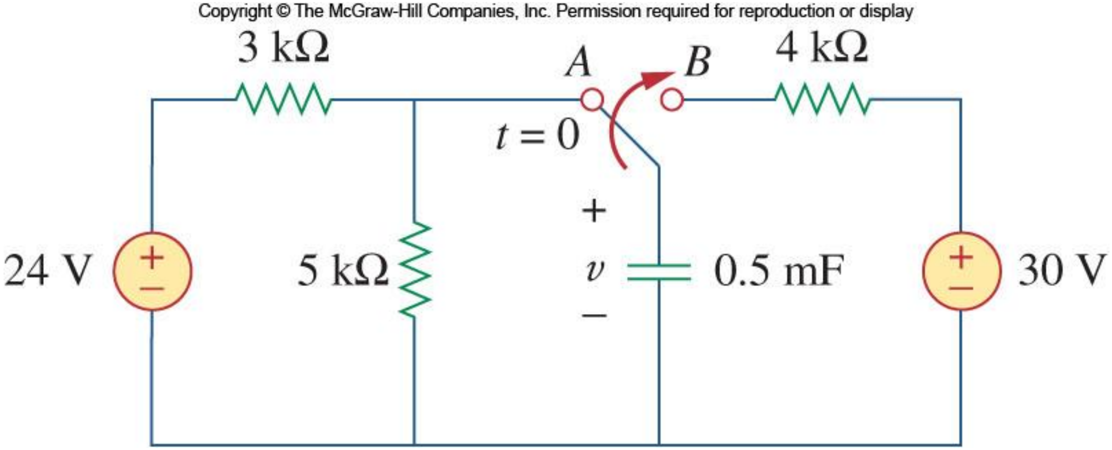
3. **Determine the final value of the variable** (as $t \rightarrow \infty$)

If needed, recall that an inductor behaves like a short circuit & that a capacitor behaves like an open circuit in steady state (e.g., $t \rightarrow \infty$).

4. **Calculate the time constant for the circuit**
   - $\tau = CR$ for an RC circuit where R is the Thévenin equivalent resistance "seen" by the capacitor.
   - $\tau = L/R$ for an RL circuit, where R is the Thévenin equivalent resistance "seen" by the inductor.

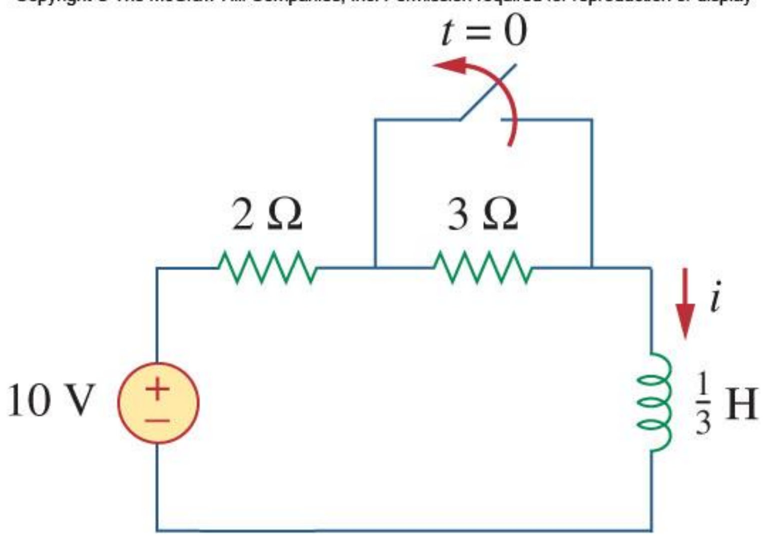
Example

- The switch has been in position A for a long time. At $t = 0$, the switch moves to B. Find $v(t)$.



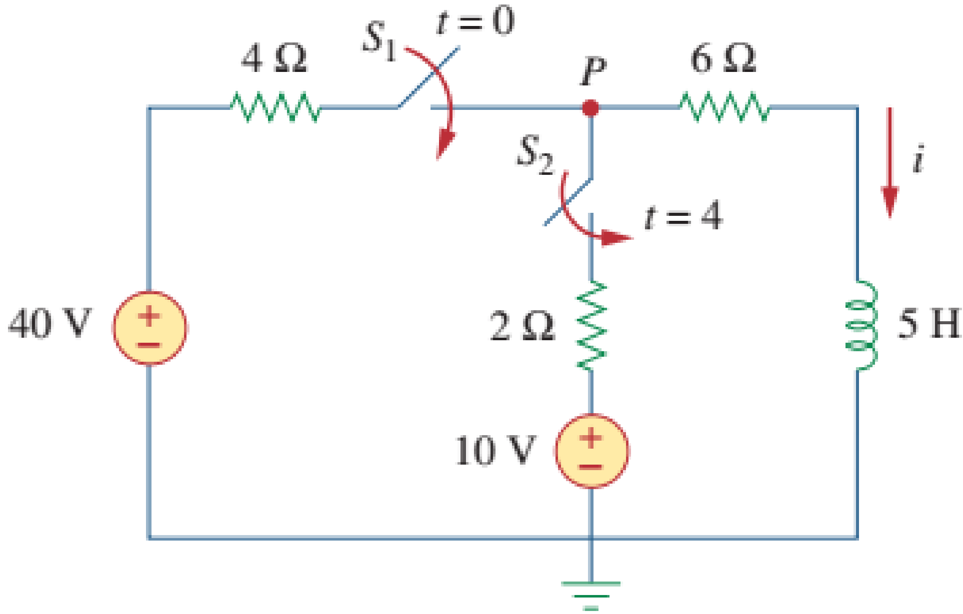
Example

- Find $i(t)$ in the circuit for $t > 0$. Assume that the switch has been closed for a long time.



Sequential switch

At $t = 0$, switch 1 in Fig. 7.53 is closed, and switch 2 is closed 4 s later. Find $i(t)$ for $t > 0$. Calculate i for $t = 2$ s and $t = 5$ s.



We need to consider the three time intervals $t \le 0$, $0 \le t \le 4$, and $t \ge 4$ separately. For $t < 0$, switches S_1 and S_2 are open so that $i = 0$. Since the inductor current cannot change instantly,

$$i(0^-) = i(0) = i(0^+) = 0$$