Computer Graphics I

Lecture 7: Rendering geometries

Xiaopei LIU

School of Information Science and Technology ShanghaiTech University

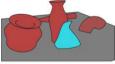
What is rendering?

Rendering (image synthesis)

- The automatic process of generating a photorealistic/nonphotorealistic image from 2D or 3D models by computer programs (renderer)
- The results of displaying such a model can be called a render







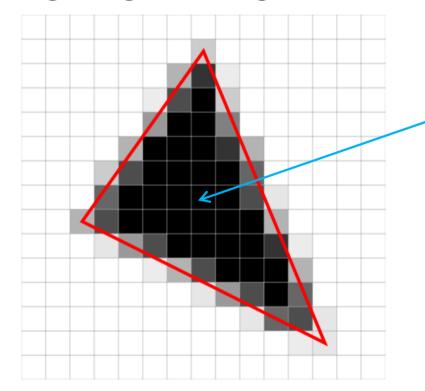






After rasterization

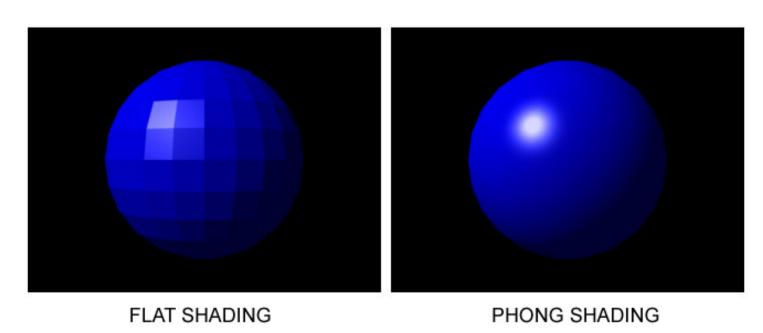
- We have everything represented by pixels
 - The number of pixels determined by the resolution
- Next question: how to determine the pixel color?
 - Lighting+shading & texturing



How shall we determine the color of each pixel?

Shading

- The process of altering the color of an object/surface/polygon in the 3D scene
 - The process to determine the pixel color
 - Narrow sense: particularly refer to lighting with color interpolation



Texturing

• The process of giving details of a surface

- Surface details are hard to model
- Make the surface look more realistic



Light reflection

• Definition:

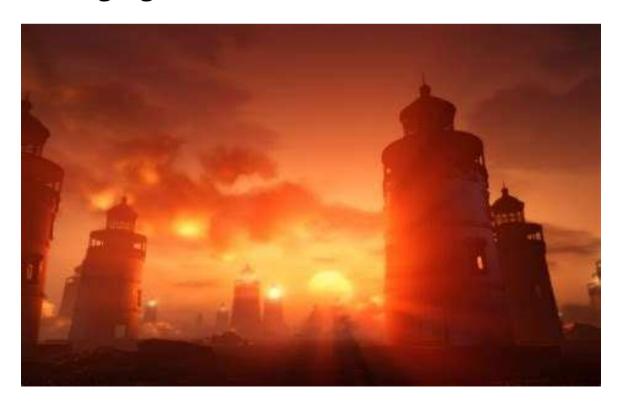
 Reflection is the process by which light incident on a surface interacts with the surface such that it changes direction immediately and leaves off the surface



Graphical lighting

The simulation of light in computer graphics

- Determine the distribution of light energy in space
- A lighting model is used to describe how surfaces respond to incoming light



1. Light sources & material

Light sources

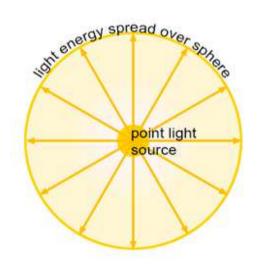
- Where the light comes from
 - The sources in an environment to emit light

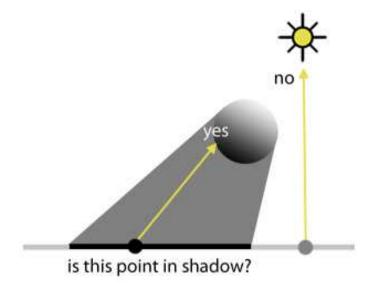




Point lights

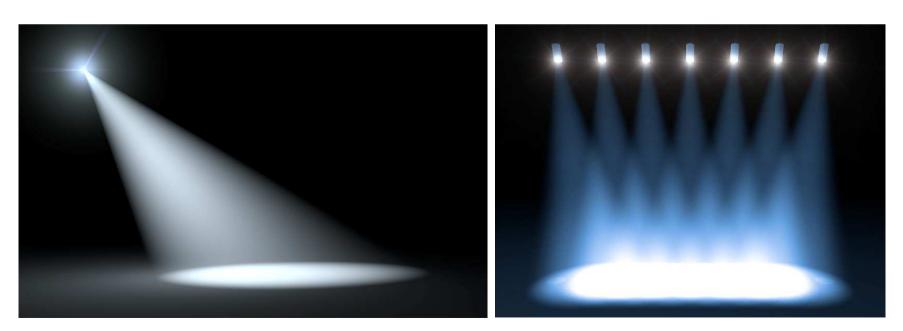
- Light emission from a single point in space
- Possibly angularly varying distribution of out-going light intensity





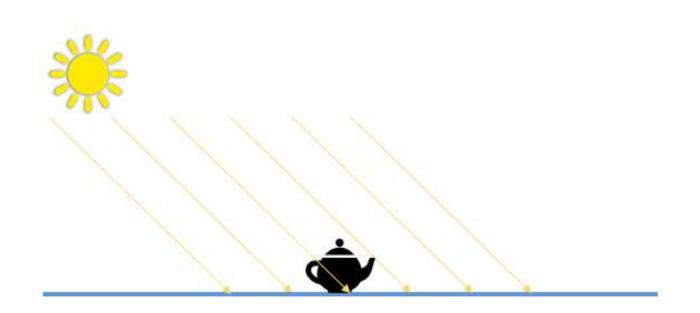
Spot lights

- A variant of point light
- Light emission only within a certain range of angles
- Usually a cone in 3D



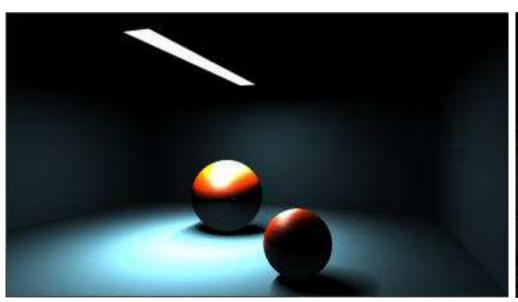
Directional (distant) lights

- A light emission with the same direction at every point in space
- Can be considered as a point light with position at infinity



Area lights

- Light emission from the surface of one or more shapes
- Most commonly existing form of light sources





Infinite area lights

- An infinitely far-away area light source that surrounds the entire scene
- Environment light as one important usage



Directional light rendering



Environment light map



Environment light rendering

What is material?

Material definition in graphics

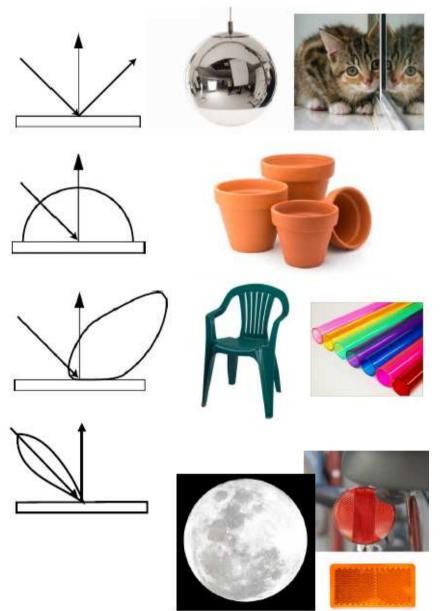
- A surface property used for describing how light is reflected on the surface
- Determine reflected light distribution with respect to surface normal and view direction
- Different physical materials have different reflections



Categories of different materials

- Ideal specular Perfect mirror
- Ideal diffuse
 Uniform reflection in all directions
- Glossy specular
 Majority of light distributed in reflection direction
- Retro-reflective

 Reflects light back toward source



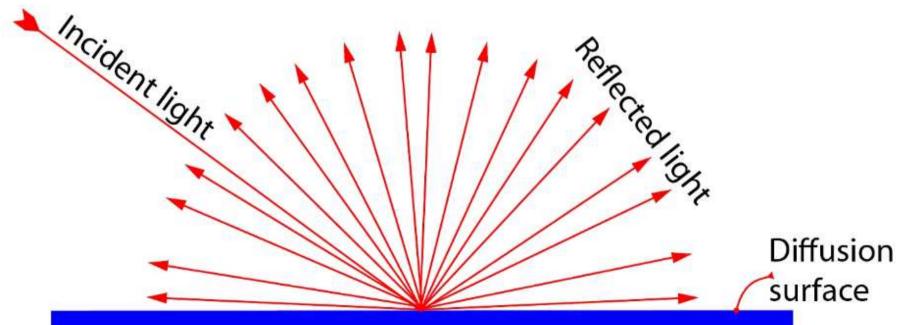
Diagrams illustrate how incoming light energy from given direction is reflected in various directions.

2. Lighting with diffuse & specular reflections

Perfect diffuse reflection

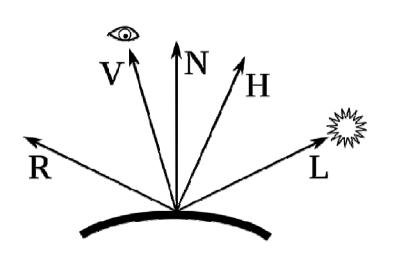
What is a perfect diffuse reflection?

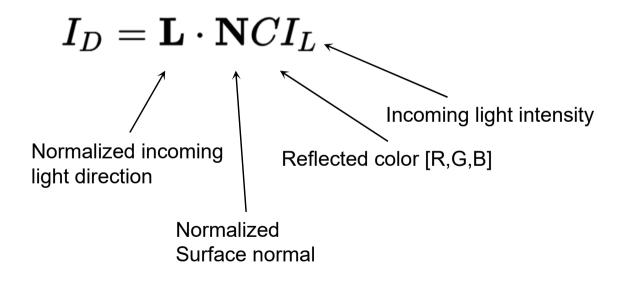
- The reflected light is scattered around uniformly
- Every direction of reflected light has the same intensity
- Occur for rough surfaces



Perfect diffuse reflection

- How to compute perfect diffuse reflection?
 - Lambert's cosine law
 - Independent of the view

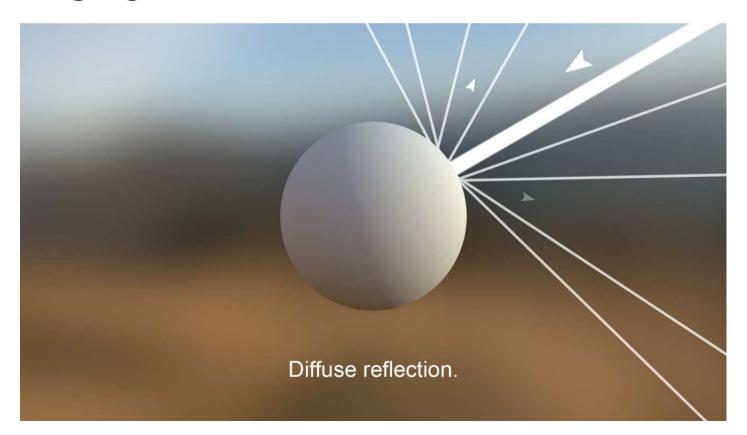




Perfect diffuse reflection

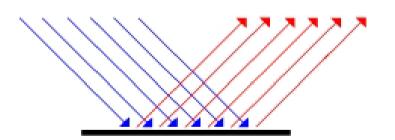
Shading with Lambert's model

- Smooth over the surface
- Light distribution only determined by model geometry
- No highlight area

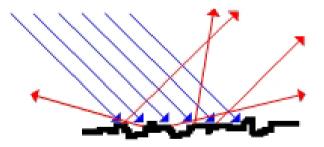


Specular highlight

- An area showing strong reflection
 - The distribution relies on the material property (smooth surface)
 - Depends on the light and view directions



Specular Reflection (smooth surfaces)



Diffuse Reflection (rough surfaces)

Specular highlight

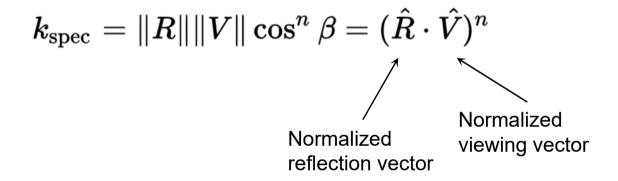
Examples of specular reflection

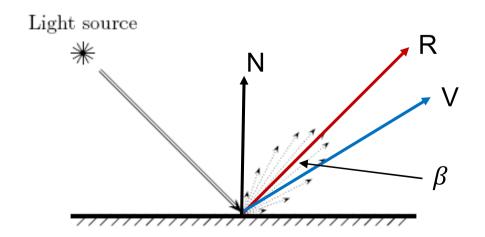
- Environment will have an image on the surface
- Point light sources exhibit speckles on the surface



Phong reflection

- How to model specular light?
 - Phong specular lighting model



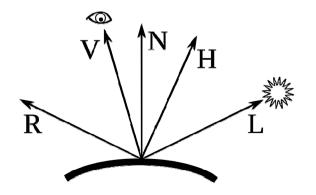


Phong reflection

Phong lighting model

- Constant ambient lighting
- Diffuse lighting
- Specular lighting

$$I_{ ext{p}} = k_{ ext{a}} i_{ ext{a}} + \sum_{m \; \in \; ext{lights}} (k_{ ext{d}} (\hat{L}_m \cdot \hat{N}) i_{m, ext{d}} + k_{ ext{s}} (\hat{R}_m \cdot \hat{V})^lpha i_{m, ext{s}})$$

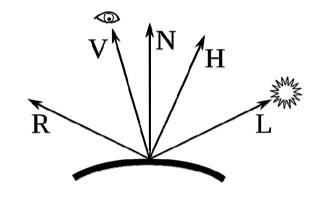


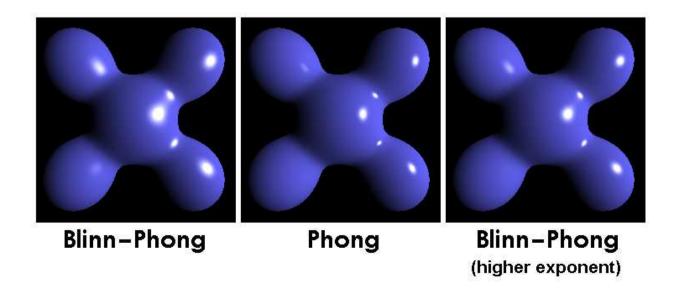
Phong reflection

Approximation

Blinn–Phong lighting model

$$H = rac{L + V}{\|L + V\|} \qquad (N \cdot H)^{lpha'} \stackrel{\mathsf{replace}}{\longrightarrow} (R \cdot V)^{lpha}$$





Lighting in OpenGL

- Determine vertex color based on Phong lighting model
 - Based on point/directional light sources
 - Each vertex in a 3D geometric model will be applied with the Blinn-Phong lighting model
 - The pixel color inside geometric primitive (such as a triangle) is determined by interpolation from projected vertex colors

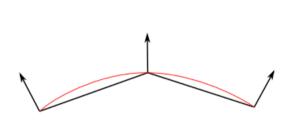


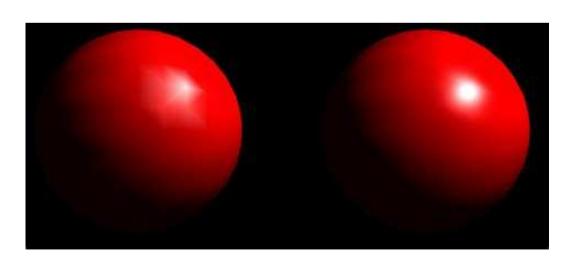


Lighting in OpenGL

Gourand vs. Phong shading

- Interpolate colors vs. interpolate normals
- Gourand shading: colors interpolated within primitives
- Phong shading: normals are interpolated first per pixel,
 then colors are computed based on a lighting model

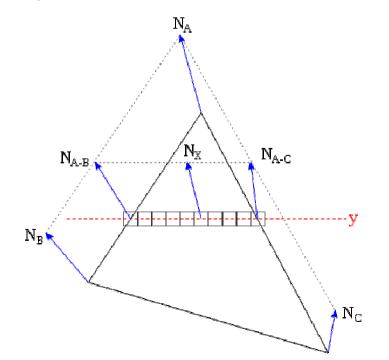




Realizing Phong lighting & shading

Vertex shader

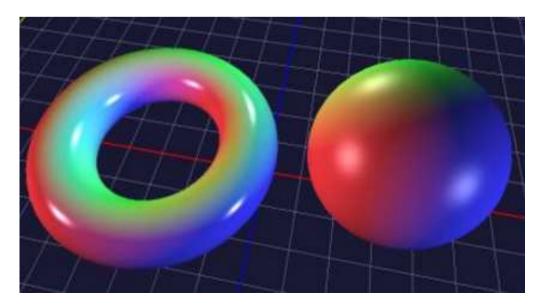
- Variable with automatic interpolation [varying] or [out]
- Such a variable will be interpolated automatically after rasterization and passed to fragment/pixel shader
- Normal interpolation: set normal to such a variable



Realizing Phong lighting & shading

Fragment(pixel) shader

- Receive an interpolated variable, such as normal
- Perform Phong lighting calculation per pixel to determine the pixel color
- Sampling rate: all projected pixels
 - Gourand shading sampling rate: all projected vertices

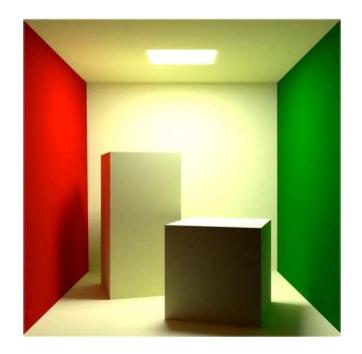


3. Illumination in computer graphics

How we study reflection in graphics?

A more systematic approach

- More general description about how light is reflected over surface material
- More generally, how light is distributed over space at a certain time instance



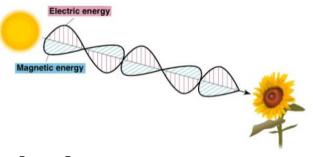


Radiometry and photometry

Radiometric techniques in optics

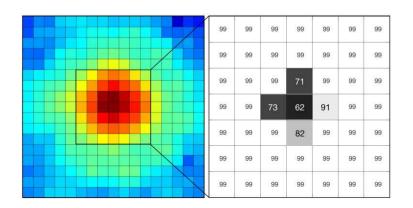
Characterize the distribution of the radiation's power in

space



Photometric techniques

 The measurement of light, in terms of its perceived brightness to the human eye



Spectral power distribution of light

The radiometric description

- Flux

• The rate of power transfer over a surface area

Intensity

• Power transferred per unit area

- Irradiance

Power received by a surface per unit area

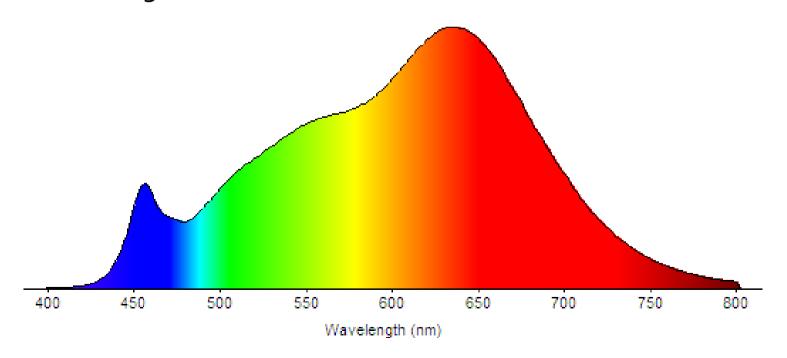
- Radiance

 Power emitted, reflected, transmitted or received by a surface per unit solid angle per unit projected area

Spectral power distribution of light

What is SPD of light?

- Spectral power distribution (SPD)
 - A distribution function w.r.t wavelength, given an illumination
 - Describe the amount of light (power) at each (continuous) wavelength



Spectral power distribution of light

- How can we represent SPD?
 - Recall basis function

$$u(x) = \sum_{i=1}^{n} c_i \varphi_i$$

- Find good basis functions
 - Project onto the low-dimensional space with coefficients c_i
 - Many different basis functions have been investigated in computer graphics
 - Trade-offs in complexity for coefficient computation
 - Different representability and accuracy for approximating SPD

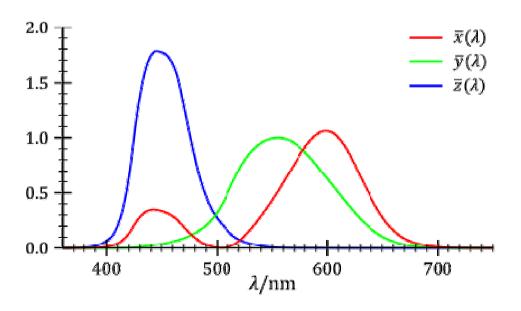
Color

Visual perception property for human eyes

- Derive from spectrum of light interacting in the eye with the spectral sensitivities of the light receptors
- Color can be described by the coefficients of basis functions that span the SPD
- Color space: colors can be identified numerically by their coordinates (coefficients)

Tristimulus theory of color perception

- All visible SPDs can be accurately represented for human observers with three values
- Given an SPD S(λ), these values can be computed from the matching functions by integration



What is the matching function?

- The basis functions that are used to synthesize different power spectral distribution of light
- The matching function can be produced by the imaging hardware to generate emitted light per pixel
- The light for each pixel is finally created by combining different matching functions with corresponding coefficients

XYZ coefficients – how to compute?

$$egin{align} X &= \int_{380}^{780} L_{ ext{e},\Omega,\lambda}(\lambda)\, \overline{x}(\lambda)\, d\lambda, \ Y &= \int_{380}^{780} L_{ ext{e},\Omega,\lambda}(\lambda)\, \overline{y}(\lambda)\, d\lambda, \ Z &= \int_{380}^{780} L_{ ext{e},\Omega,\lambda}(\lambda)\, \overline{z}(\lambda)\, d\lambda. \ \end{align}$$

Inner product: projection onto basis functions

Note:

- Metamer: SPDs with remarkably different distributions may have very similar x,y,z values
- The y component of XYZ color is closely related to luminance (perceived brightness of color)

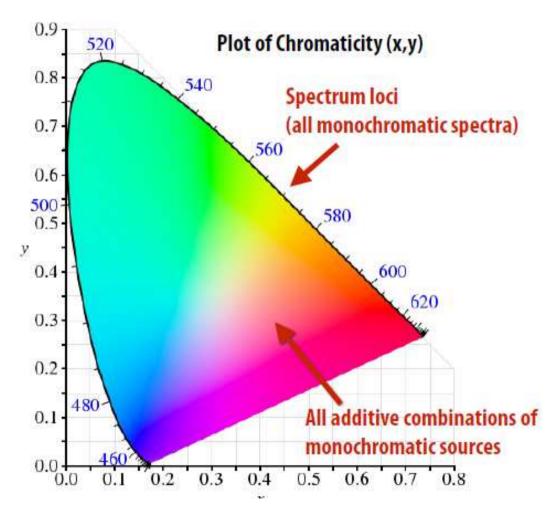
CIE xy chromaticity diagram – specify the color

Calculating x-y

$$x=rac{X}{X+Y+Z}$$
 $y=rac{Y}{X+Y+Z}$ $z=rac{Z}{X+Y+Z}=1-x-y$

Calculating XYZ

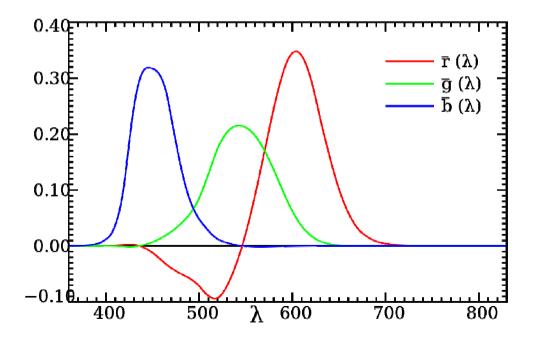
$$X = rac{Y}{y} x \ Z = rac{Y}{y} (1 - x - y)$$



RGB color

Different set of matching functions

$$\int_0^\infty ar r(\lambda)\,d\lambda = \int_0^\infty ar g(\lambda)\,d\lambda = \int_0^\infty ar b(\lambda)\,d\lambda$$



$$egin{aligned} R &= \int_0^\infty S(\lambda)\, ar r(\lambda)\, d\lambda \ G &= \int_0^\infty S(\lambda)\, ar g(\lambda)\, d\lambda \ B &= \int_0^\infty S(\lambda)\, ar b(\lambda)\, d\lambda \end{aligned}$$

XYZ-RGB conversions

Conversion between XYZ and RGB colors

$$\begin{bmatrix} R \\ G \\ B \end{bmatrix} = \begin{bmatrix} 0.418,47 & -0.158,66 & -0.082,835 \\ -0.091,169 & 0.252,43 & 0.015,708 \\ 0.000,920,90 & -0.002,549,8 & 0.178,60 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \frac{1}{0.176,97} \begin{bmatrix} 0.490,00 & 0.310,00 & 0.200,00 \\ 0.176,97 & 0.812,40 & 0.010,630 \\ 0.000,0 & 0.010,000 & 0.990,00 \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

Radiative transfer

- Phenomenological study of transfer of radiant energy
- Based on radiometric principles at geometric optics level (macroscopic light property)

Assumption of light

- Linearity
 - The combined effect of two inputs equals the sum of effect of each individual input
- Energy conservation
 - When light scatters from a surface or participating media, no more additional energy produced

Assumption of light

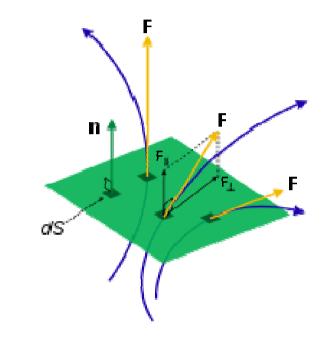
- No florescence or phosphorescence
 - The behavior of light at one wavelength is independent of other wavelengths
- Steady state
 - Light in the environment is assumed to have reached equilibrium

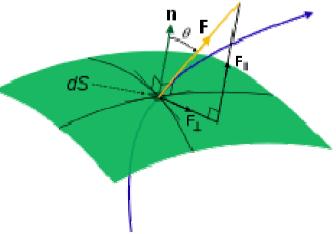
What has been lost

- Diffraction and interference
- Polarization

Radiant flux

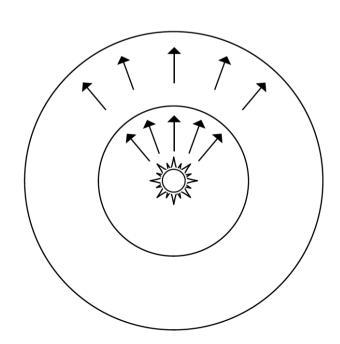
- Also known as power
- Total amount of energy passing through a surface or region of space per unit time
- It is computed following a vector field carrying energy





- Irradiance and radiant exitance
 - Irradiance (*E*)
 - Area density of flux arriving at a surface
 - Radiance exitance (*M*):
 - Area density of flux leaving a surface
 - For a sphere

$$E = \frac{\Phi}{4\pi r^2}$$



- Understanding Lambert's law from irradiance equation
 - Consider a light source with area A and flux Φ which is illuminating the surface
 - Perpendicular case

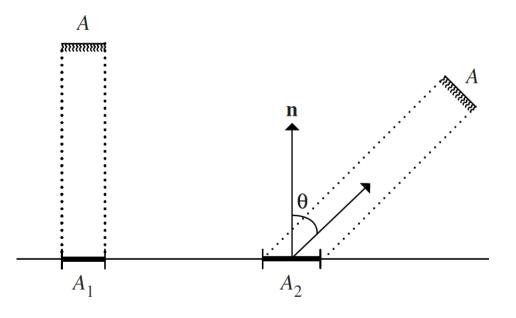
$$E_1 = \frac{\Phi}{A}$$

Non-perpendicular case

$$E_2 = \frac{\Phi \cos \theta}{A}$$

In general

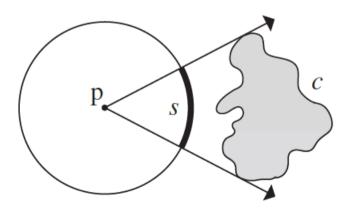
$$E = \frac{\mathrm{d}\Phi}{\mathrm{d}A}$$



Solid angle

• Solid angle ω

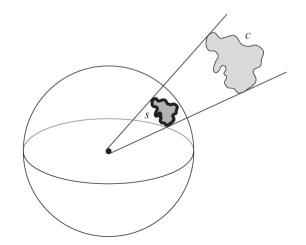
- An extension of 2D angles on a sphere
- 2D angle case
 - Projected arc on unit circle subtended by an object, measured in radians



Solid angle

• Solid angle ω

- An extension of 2D angles on a sphere
- 3D angle case
 - Projected area on unit sphere subtended by an object, measured in steradians (square of radians)
 - The entire sphere has a solid angle of 4π , and a hemisphere subtends 2π



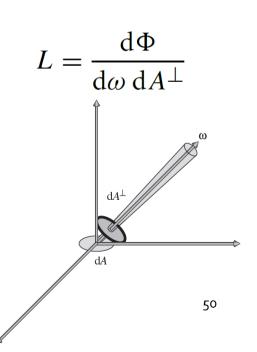
Intensity and radiance

• Intensity I

- Flux density per solid angle $I = \frac{\mathrm{d}\Phi}{\mathrm{d}\omega}$
- Intensity describes directional distribution of light
 - Only meaningful for point light sources

Radiance L

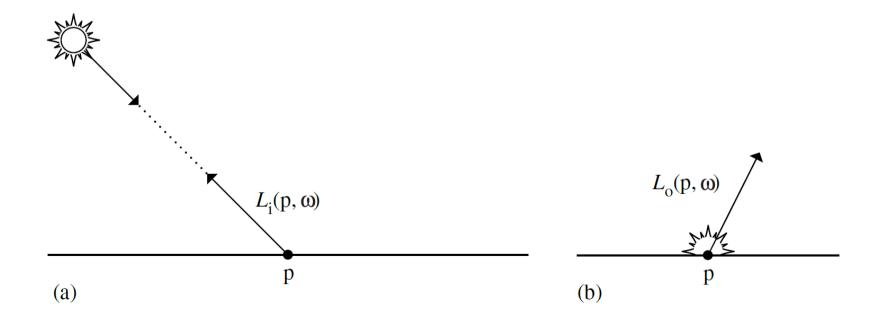
- Flux density per unit area per solid angle
 - Most frequently used
 - Most fundamental of all radiometric quantities
 - All other quantities can be computed using radiance
 - It remains constant along rays in empty space



Incident/exitant radiance functions

It is useful to make a distinction

- Radiance arriving at a point L_i
- Radiance leaving at a point L_{\circ}
- In general $L_{\rm i}({\rm p},\omega) \neq L_{\rm o}({\rm p},\omega)$
- If there is no surface $L_o(p, \omega) = L_i(p, -\omega)$



Photometry

Photometry

- The study of visible electromagnetic radiation in terms of human perception
- Radiometric quantity can be converted into photometric quantity
- By integrating against spectral response curve $V(\lambda)$
- $V(\lambda)$ describes the relative sensitivity of human eye to various wavelengths

Luminance

Luminance

- Measure how bright a spectral power distribution appears to a human observer
- Computing the luminance

$$Y = \int_{\lambda} L(\lambda) \ V(\lambda) \ d\lambda$$

- Luminance and the spectral response curve V (λ) are closely related to the XYZ representation of color
 - CIE Y(λ) tristimulus curve was chosen to be proportional to V(λ)

$$Y = 683 \int_{\lambda} L(\lambda) Y(\lambda) d\lambda$$

Luminance

• Typical luminance values

Condition	Luminance (cd/m ² , or nits)
Sun at horizon	600,000
60-watt light bulb	120,000
Clear sky	8,000
Typical office	100–1000
Typical computer display	1–100
Street lighting	1–10
Cloudy moonlight	0.25

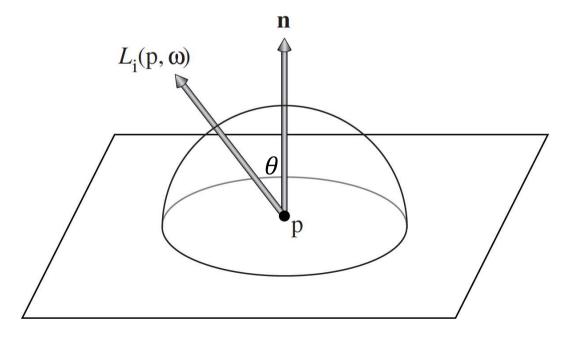
Radiometric integrals

Computation of irradiance E

– Irradiance at a point p with surface normal ${\bf n}$ due to radiance over a set of directions Ω is

$$E(\mathbf{p}, \mathbf{n}) = \int_{\Omega} L_{i}(\mathbf{p}, \omega) |\cos \theta| d\omega$$

$$L = \frac{\mathrm{d}\Phi}{\mathrm{d}\omega \,\mathrm{d}A^{\perp}}$$
$$E = \frac{\mathrm{d}\Phi}{\mathrm{d}A}$$

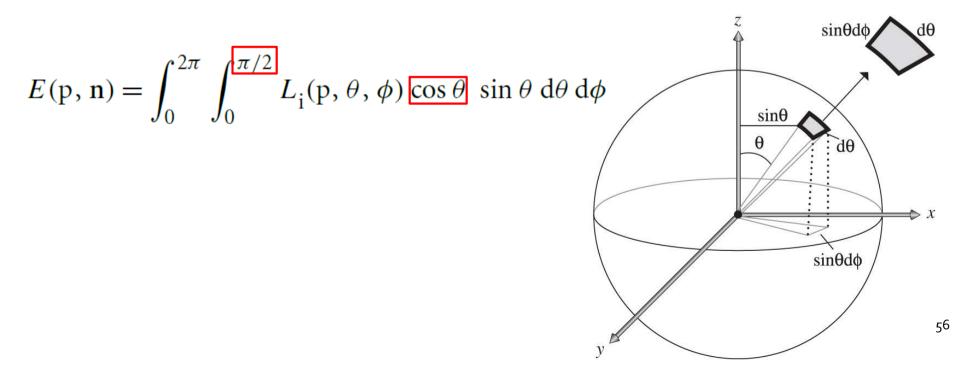


Radiometric integrals

- Computation of irradiance E
 - Integral over spherical coordinates

$$d\omega = \sin\theta \ d\theta \ d\phi$$

Computing irradiance in spherical coordinates



Radiometric integrals

Integral over area

- Turn integrals over directions into integrals over areas
- Irradiance will be much easier to compute over area
- Differential area is related to differential solid angle:

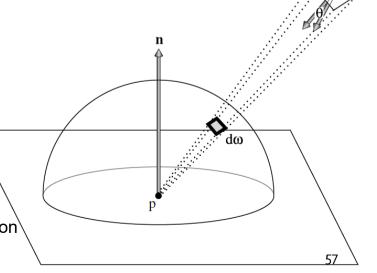
$$d\omega = \frac{dA \cos \theta}{r^2}$$
Distance to p

Irradiance integral w.r.t area

$$E(\mathbf{p}, \mathbf{n}) = \int_{A} L \cos \theta_{i} \frac{\cos \theta_{o} dA}{r^{2}}$$

Incoming angle receiving light

Outgoing angle of light source \w.r.t the current incoming direction



Surface reflection

Surface scatter

- When light is incident on a surface, the surface scatters light
- Some light is reflected into the environment

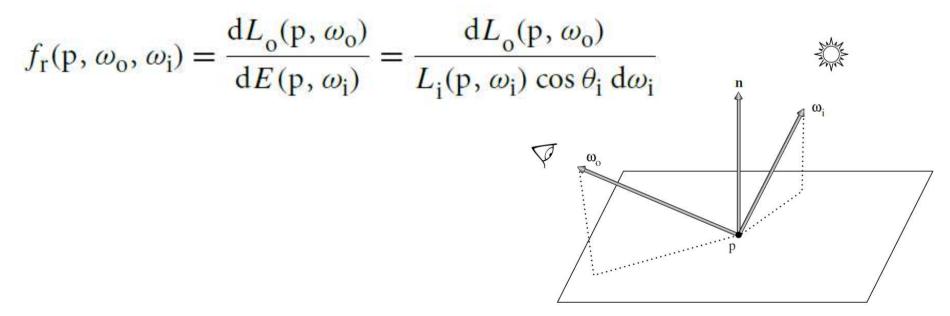
How to model reflection

- The spectral distribution of reflected light (color)
- Directional distribution
- Two abstractions to describe light reflection
 - BRDF
 - BSSRDF

Bidirectional reflectance distribution function

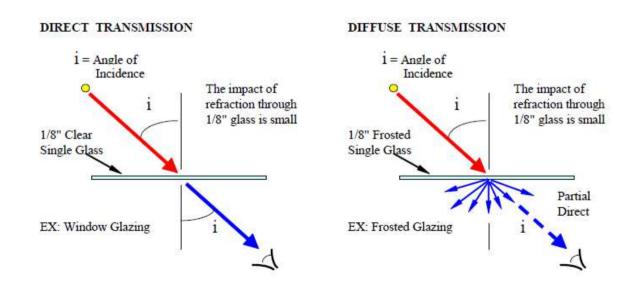
Bidirectional reflectance distribution function (BRDF)

- A formalism for describing reflection from a surface
- How much radiance is leaving the surface as a result of incident radiance



Bidirectional transmittance distribution function

- Bidirectional transmittance distribution function (BTDF)
 - Describe the distribution of transmitted light
 - Defined similarly as BRDF
 - Usually denoted by $f_t(p, \omega_0, \omega_i)$
 - $-\omega_i$ and ω_o are opposite hemispheres around point p



Rendering equation

The fundamental rendering equation

 Describe how an incident distribution of light at a point is transformed into an outgoing distribution

$$L_{o}(\mathbf{p}, \omega_{o}) = \int_{\mathbb{S}^{2}} f(\mathbf{p}, \omega_{o}, \omega_{i}) L_{i}(\mathbf{p}, \omega_{i}) |\cos \theta_{i}| d\omega_{i}$$

- When S² (the entire sphere) is the domain, it is often called the <u>scattering equation</u>
- When upper hemisphere is the domain, it is often called the <u>reflection equation</u>

Bidirectional scattering-surface reflectance distribution function

Subsurface light transport

- Light enters the material at one point and may exit quite far away
 - Wax, candle, marble, skin, many biological tissues, etc.
- Bidirectional scattering-surface reflectance distribution function (BSSRDF)
 - A formalism for describing scattering processes
 - Describe the ratio of the exitant and incident differential radiance at different positions and solid angles

$$S(p_o, \omega_o, p_i, \omega_i) = \frac{dL_o(p_o, \omega_o)}{dE(p_i, \omega_i)}$$

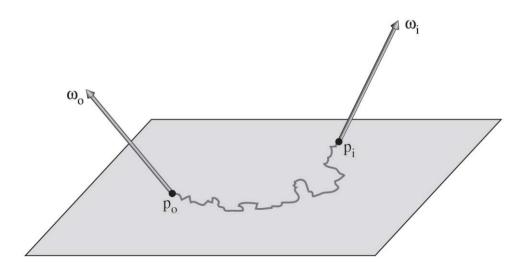
Bidirectional scattering-surface reflectance distribution function

Generalization of scattering equation

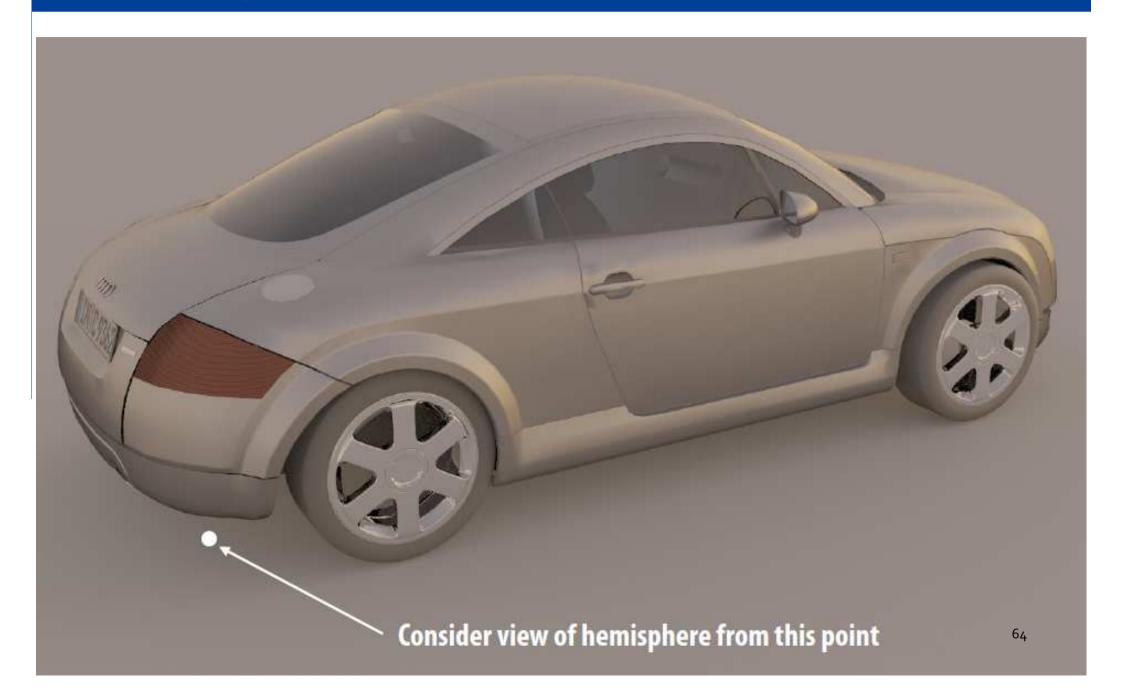
Turn fundamental rendering equation into more complex integral equation

$$L_{o}(p_{o}, \omega_{o}) = \int_{A} \int_{\mathcal{H}^{2}(\mathbf{n})} S(p_{o}, \omega_{o}, p_{i}, \omega_{i}) L_{i}(p_{i}, \omega_{i}) |\cos \theta_{i}| d\omega_{i} dA$$



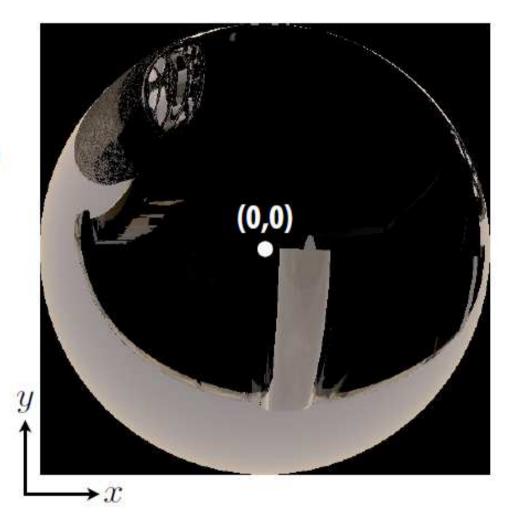


Hemispherical incident radiance

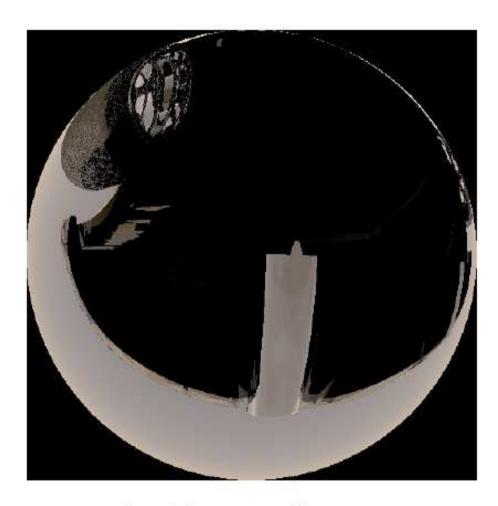


Hemispherical incident radiance

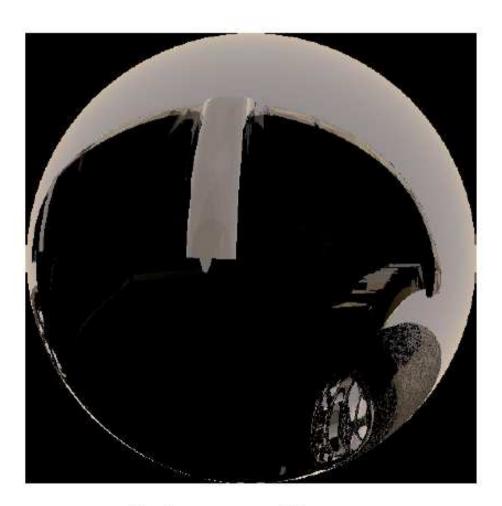
At any point on any surface in the scene, there's an incident radiance field that gives the directional distribution of illumination at the point



Ideal specular reflection

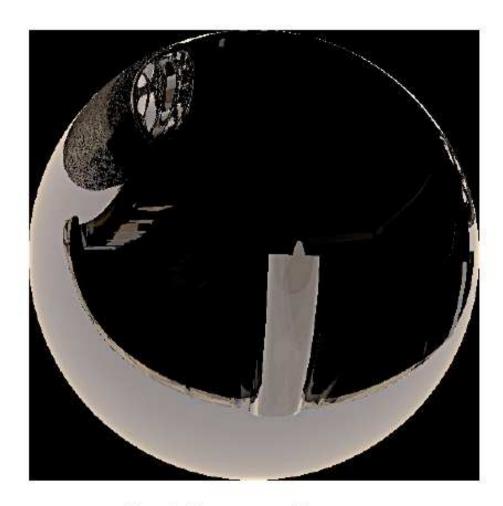


Incident radiance

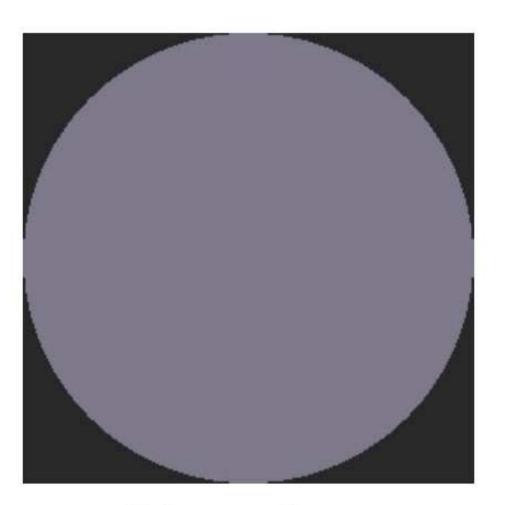


Exitant radiance

Diffuse reflection

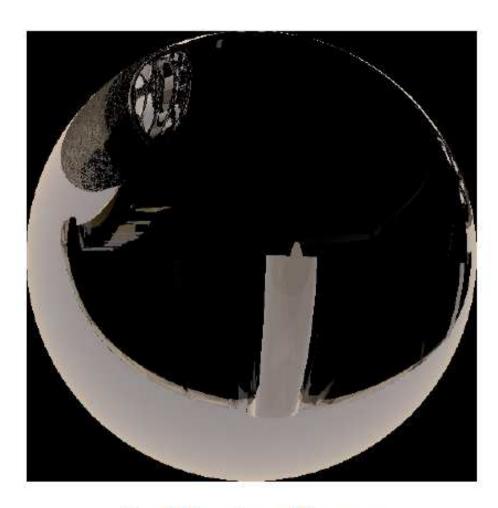


Incident radiance

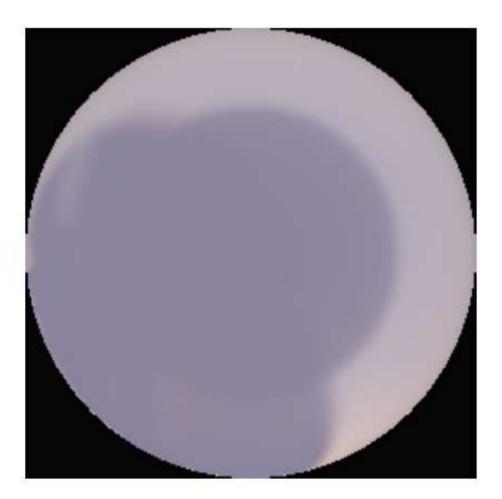


Exitant radiance

Plastic

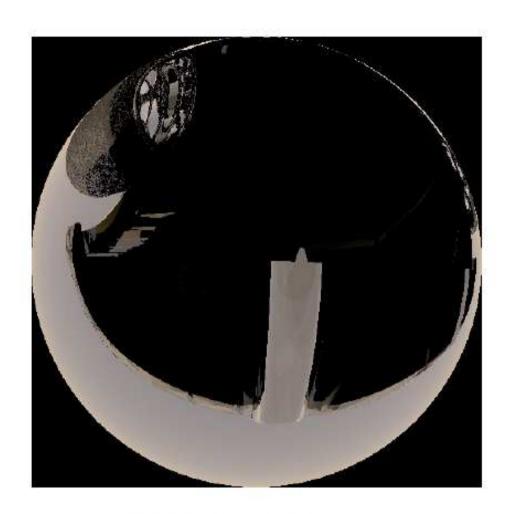


Incident radiance

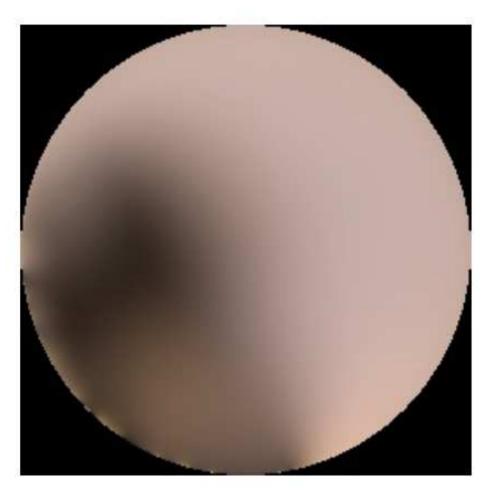


Exitant radiance

Copper



Incident radiance

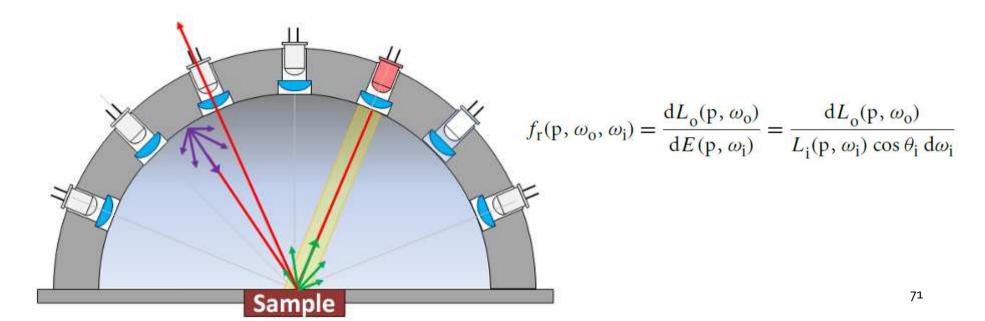


Exitant radiance

3. Constructing general BRDFs

Sources of surface reflection model

- Surface reflection models come from a number of sources
 - Measured data
 - Real-world material surface reflection is measured in laboratories
 - May be used in tabular form with interpolation
 - Or compute coefficient for basis function to fit



Sources of surface reflection model

Surface reflection models come from a number of sources

- Phenomenological models
 - Equations attempting to describe the qualitative properties of real-world materials, which is particularly easy to use
 - Many reflection functions used in computer graphics belong to this type, e.g., Phong reflection model

Simulation

- Simulate optical phenomena based on the low-level information
- Composition of different materials, microgeometry, etc.
- Generate reflection data

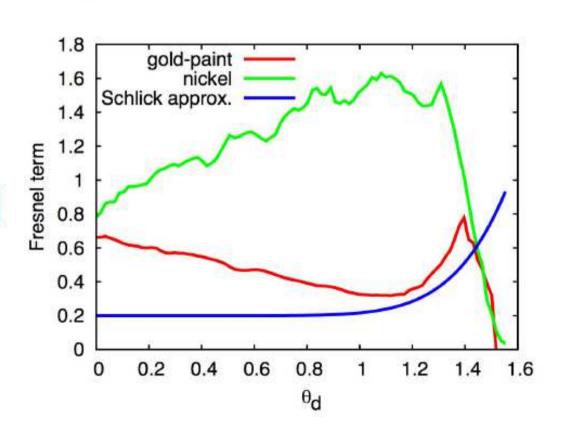
Sources of surface reflection model

- Surface reflection models come from a number of sources
 - Wave optics
 - Treating reflection as a wave and computing the solution of Maxwell's equations
 - Tend to be computationally intensive, but in practice not appreciably more accurate than geometric optics model
 - Geometric optics
 - More tractable solution
 - Ignore complex wave effects like polarization

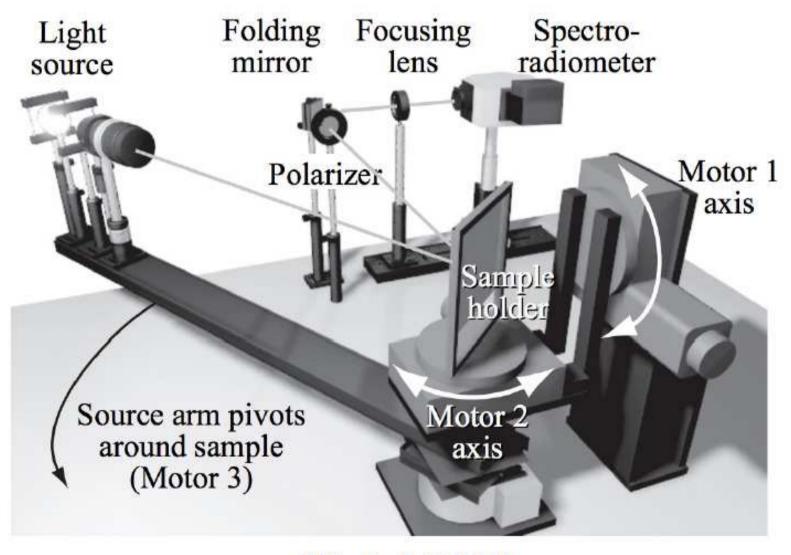
Measuring BRDFs: motivation

- Avoid need to develop / derive models
 - Automatically includes all of the scattering effects present
- Can accurately render with real-world materials
 - Useful for product design, special effects, ...
- Theory vs. practice:

[Bagher et al. 2012]

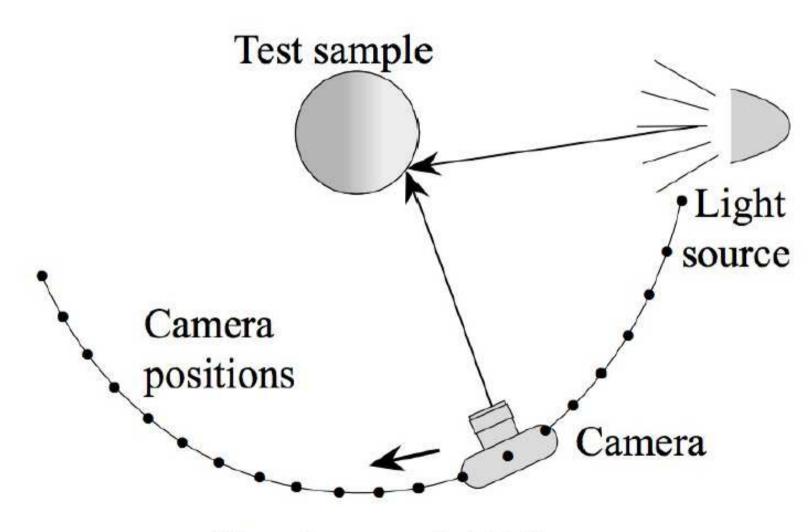


Measuring BRDFs: gonioreflectometer



[Li et al. 2005]

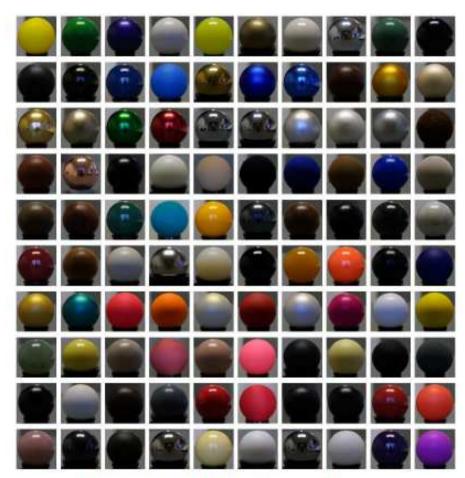
Image-based BRDF measurement



[Marschner et al. 1999]

Tabular representation

- Store regularly-spaced samples in $(\theta_i, \theta_o, |\phi_i \phi_o|)$
 - Better: reparameterize angles to better match specularities
- Generally need to resample measured values to table
- Very high storage requirements



MERL BRDF Database [Matusik et al. 2004] 90*90*180 measurements

4. Light transmission

Transmission

In addition to reflecting off surface, light may be transmitted through surface.

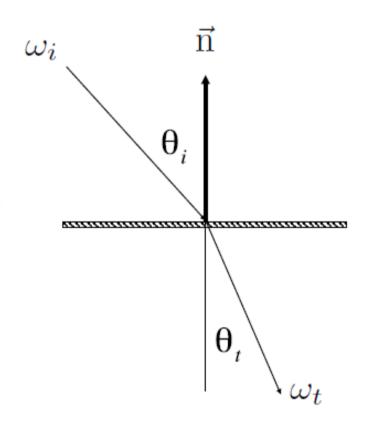
Light refracts when it enters a new medium.

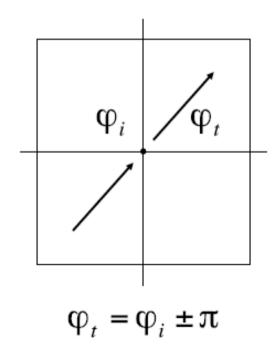




Snell's Law

Transmitted angle depends on index of refraction of medium incident ray is in and index of refraction of medium light is entering.

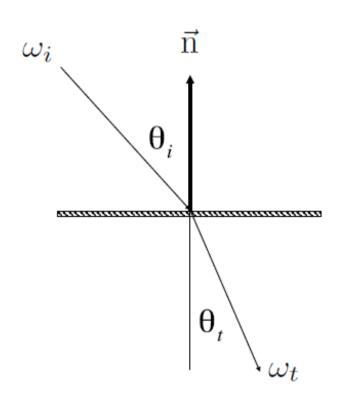




Medium	η *
Vacuum	1.0
Air (sea level)	1.00029
Water (20°C)	1.333
Glass	1.5-1.6
Diamond	2.42

* index of refraction is wavelength dependent (these are averages)

Law of refraction



$$\eta_i \sin \theta_i = \eta_t \sin \theta_t$$

$$\cos \theta_t = \sqrt{1 - \sin^2 \theta_t}$$

$$= \sqrt{1 - \left(\frac{\eta_i}{\eta_t}\right)^2 \sin^2 \theta_i}$$

$$= \sqrt{1 - \left(\frac{\eta_i}{\eta_t}\right)^2 (1 - \cos^2 \theta_i)}$$

Total internal reflection:

When light is moving from a more optically dense medium to a less optically dense medium: $\frac{\eta_i}{}>1$

$$1 - \left(\frac{\eta_i}{\eta_t}\right)^2 \left(1 - \cos^2 \theta_i\right) < 0$$

Fresnel reflection

Reflectance depends on angle of incidence and polarization of light



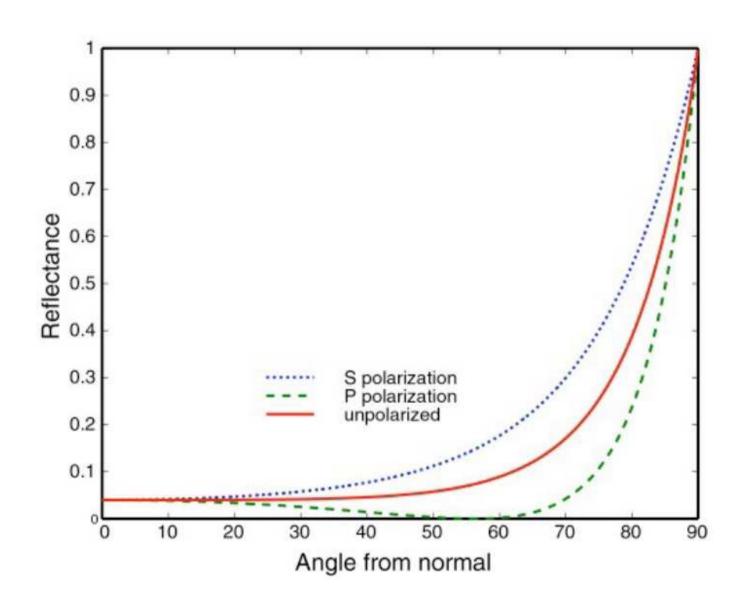




This example: reflectance increases with grazing angle

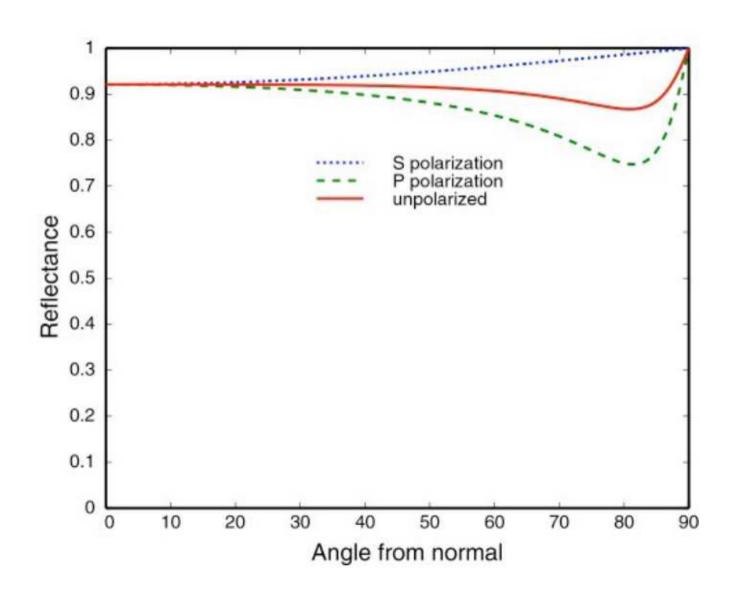
Fresnel reflection

Fresnel reflection (dielectric, $\eta = 1.5$)



Fresnel reflection

Fresnel reflectance (conductor)



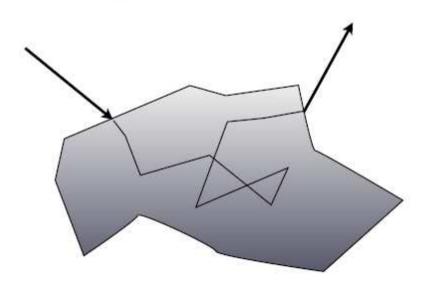
Translucent materials





Subsurface scattering

- Visual characteristics of many surfaces caused by light entering at different points than it exits
 - Violates a fundamental assumption of the BRDF



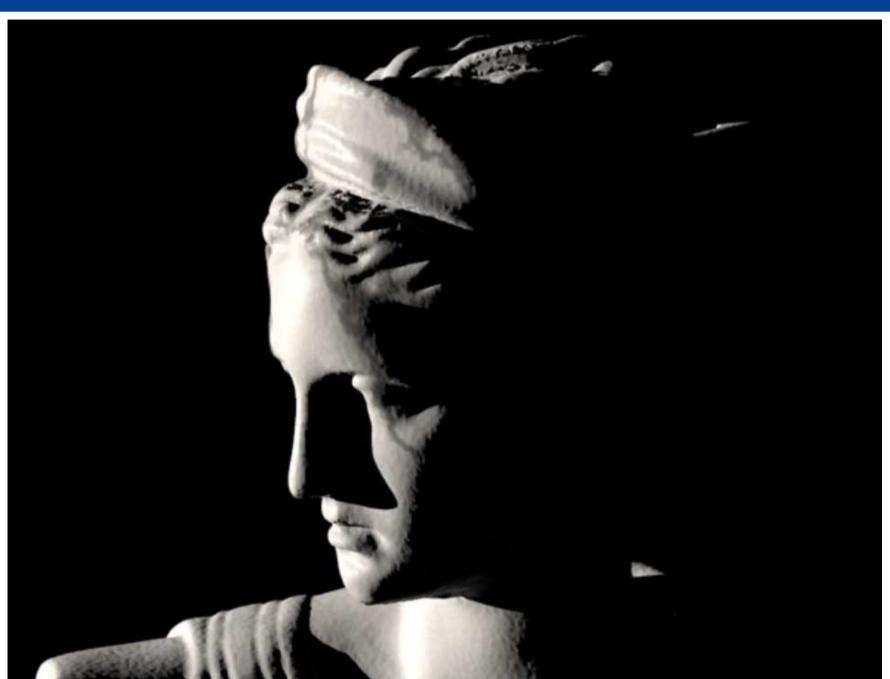


[Jensen et al 2001]

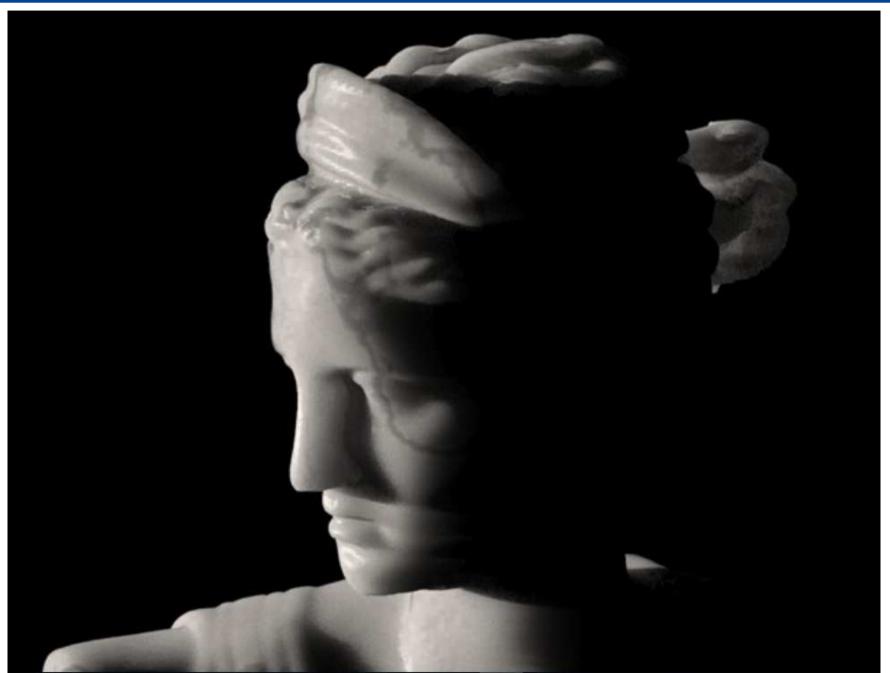


[Donner et al 2008]

BRDF only



BSSRDF



5. Shadow Rendering in OpenGL

Shadow

What is shadow

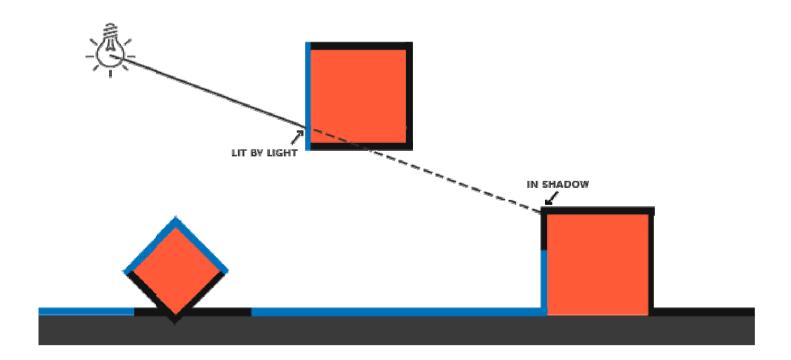
- A result of the absence of light due to occlusion
- when a light source's light rays do not hit an object because it gets occluded by some other object the object is in shadow





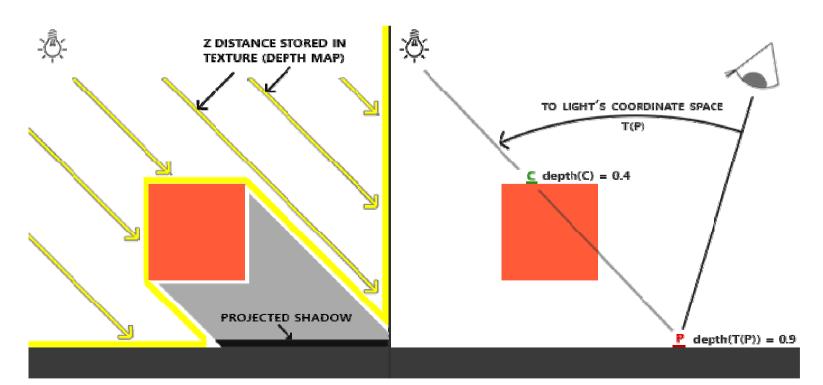
• Basic idea

- We render the scene from the light's point of view
- Everything we see from the light's perspective is lit and everything we can't see must be in shadow



• Basic idea

– What if we were to render the scene from the light's perspective and store the resulting depth values in a texture?



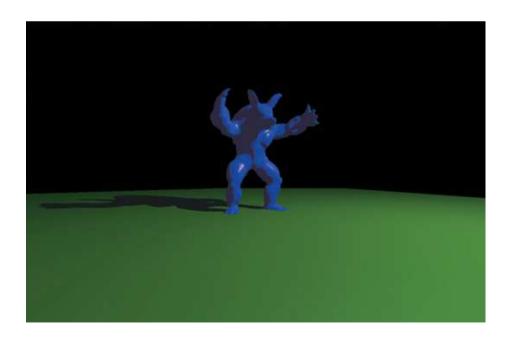
First pass

- View the scene from the shadow-casting light source
- Render scene's depth from the point of view of the light into a depth buffer
- Points visible to the light will be rendered



Second pass

- Project the surface coordinates into the light's reference frame
- Compare their depths to that in shadow map



Next lecture: Geometric parameterization & texturing