Homework 8

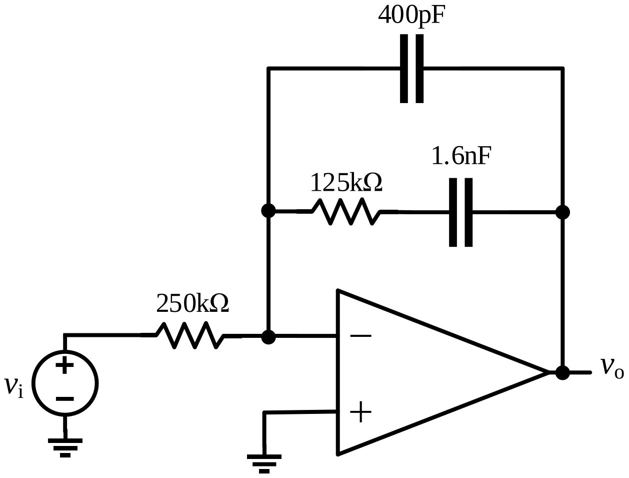
Due date: 11.59 am, Jan. 15th, 2023

Turn in your hard-copy hand-writing homework to Room 324 #3 SIST

Rules:

- Work on your own. Discussion is permissible, but extremely similar submissions will be judged as plagiarism.
- Please show all intermediate steps: a correct solution without an explanation will get zero credit.
- Please submit on time. No late submission will be accepted.
- Please prepare your submission in English only. No Chinese submission will be accepted.

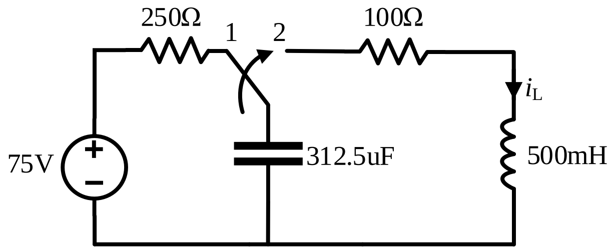
1. Please find the transfer function v_0/v_i.



$$\frac{V_i}{250k} + \frac{V_o}{\frac{1}{s\cdot 400p}} + \frac{V_o}{125k+\frac{1}{s\cdot 1.6n}} = 0 \quad 10$$

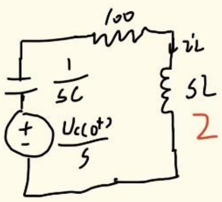
$$H(s) = \frac{V_o}{V_i} = -10000\cdot\frac{s+5000}{s(s+25000)} \quad 5$$

2. For the following circuit, the switch had been at node **1** for a long time before ***t***=0s. When ***t***=0s, the switch was turned to node **2** immediately. If no initial energy was stored for the inductor, please use **Laplace domain method** to find ***i**$_L$*(t) for ***t***>0s



2. $V_c(0^-) = 75\,V$, | $i_L(0^-) = 0$, $V_c(0^+) = V_c(0^-) = 75$ |, $i_L(0^+) = i_L(0^-) = 0$ (15)

when $t>0$:



$$\left(\frac{1}{sC} + sL + 100\right) i_L = \frac{U_C(0^+)}{s} \quad 4$$

$$\therefore\ i_L(s) = \frac{75sC}{(s^2LC + 100sC + 1)s} = \frac{75C}{s^2LC + 100sC + 1}$$

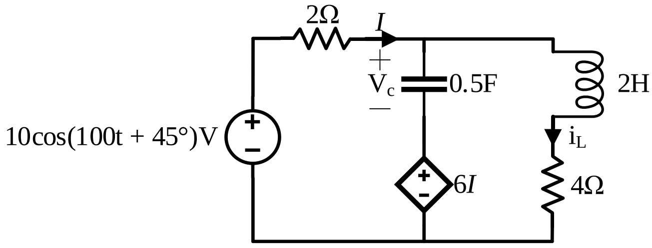
$$= \frac{75C}{(0.00625s+1)(0.025s+1)} \quad 2$$

$$= 25C\left(\frac{4}{0.025s+1} - \frac{1}{0.00625s+1}\right)$$

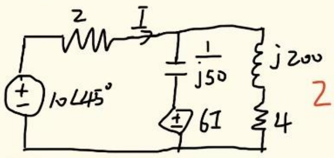
$$= 25C\left(\frac{160}{s+40} - \frac{160}{s+160}\right) \quad 2$$

$$\therefore\ i_L(t) = 1.25\left(e^{-40t} - e^{-160t}\right),\ t>0 \quad 3$$

3. Given no initial energy was stored for the energy storage elements, find steady-state response v_c by **Laplace domain method and phasor domain method**.



3. ① $\dot{V} = 10\angle 45°$ 1

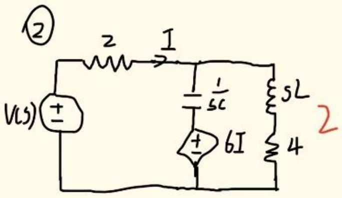


$$\begin{cases} \dot{V}_c + 6\dot{I} + 2\dot{I} = 10\angle 45° \\ (\dot{I} - \dot{V}_c\, j50)(j200+4) + 2\dot{I} = 10\angle 45° \end{cases}$$ 2 2 (20)

$\dot{V}_c = 0.018 - 0.017j \approx 0.025\angle -45°$ 1

$\therefore V_c(t) = 0.025\cos(100t - 45°)$

②



$$\begin{cases} V_c(s) + 6I + 2I = V(s) \\ (I - V_c(s)sC)(sL+4) + 2I = V(s) \end{cases}$$ 2 2

$V(s) = 10e^{0.45s}\frac{s}{s^2+100^2}$ 1

$H(s) = \frac{V_c(s)}{V(s)} = \frac{s-1}{4s^2+9s+3}$ 1

$V_c(s) = H(s)V(s) = 10e^{0.45s}\cdot\frac{s(s-1)}{(s^2+100^2)(4s^2+9s+3)}$ 1

$V_c(s) = 10e^{0.45s}\cdot\frac{s(s-1)}{(s^2+100^2)(4s^2+9s+3)}$

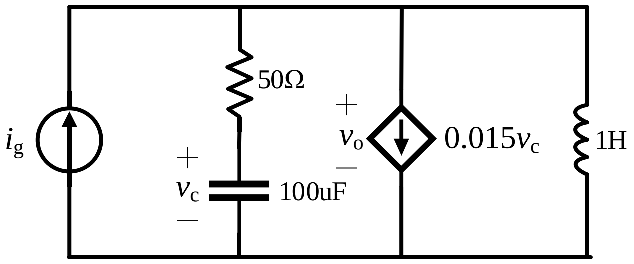
$= \left(\frac{-4\times(129997s+29991)}{1600570009\times(4s^2+9s+3)} + \frac{129997s+399880000}{1600570009\times(s^2+100^2)}\right)10e^{0.45s}$ 1

steady-state response $V_c(s) = \left(\frac{8.12\times10^{-5}s}{s^2+100^2} + \frac{2.50\times10^{-3}\times100}{s^2+100^2}\right)10e^{0.45s}$ 1

$= 8.12\times10^{-4}\cos(100t+45) + 2.5\times10^{-2}\sin(100t+45)$

$\approx 0.025\cos(100t-45°)$ 1

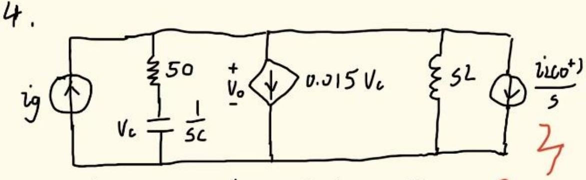
4. When $t=0$, the current through the inductor is 5mA and no initial energy is stored for the capacitor. If i_g=20u(t)mA, find $v_o(t)$ for $t>0$ by **Laplace domain method.**



4.

$i_g(s) = \frac{0.02}{s}$ 2 (15)

$i_L(0^+) = i_L(0^-) = 5mA$ 1



3

$V_o(s) = V_c(s) + 50sC\,V_c(s)$ 3

$V_c(s)sC + \frac{V_o(s)}{sL} + 0.015\,V_c(s) + \frac{0.005}{s} = \frac{0.02}{s}$ 3

$$V_o(s) = \frac{7.5\times10^{-5}s + 0.015}{0.0001s^2 + 0.02s + 1} = \frac{0.75}{s+100} + \frac{75}{(s+100)^2}$$ 2

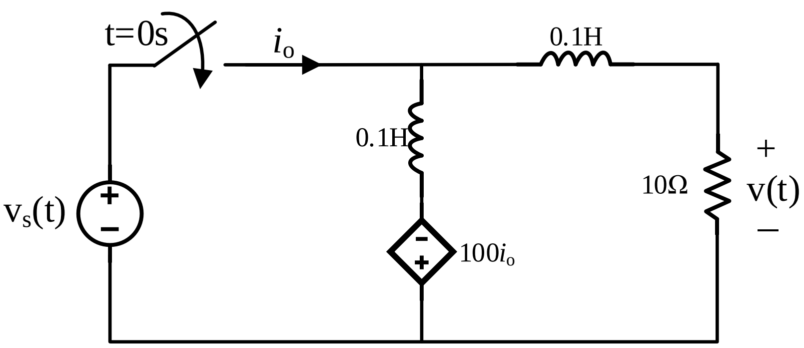
$\therefore\ V_o(t) = 0.75e^{-100t} + 75te^{-100t}.\quad t>0$ 1

来自华为笔记

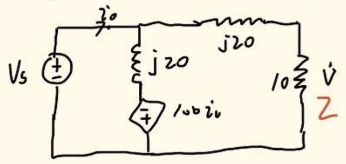
5. For the following circuit, $v_s(t)=10\cos(200t)$ V, and the switch closed immediately at **t**=0s. There is no energy stored for the inductors before **t**=0s. Please

(a)Use **phasor method** to find the **steady-state** for the voltage of ***v*(t)**.

(b)Use **Laplace domain method** to find **complete response** of ***v*(t)** for ***t*>0** and compare the results from (a).



5. (a) $\dot{V}_s = 10\angle 0^\circ$, $w = 200$ 1 (15)



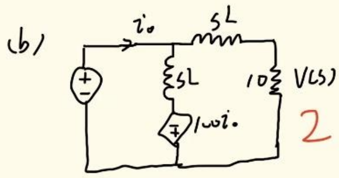
2

$\frac{\dot{V}}{10}(10+j20) = \dot{V}_s$ 2

$\therefore \dot{V} = \frac{10}{1+j2} = 2-4j = 4.47\angle -63.4^\circ$ 1

$\therefore v(t) = 4.47\cos(200t - 63.4^\circ),\ t>0$ 1

(b)



2

$\begin{cases} \frac{V(s)}{10}(sL+10) = V_s(s) \\ V_s(s) = \frac{10s}{s^2+200^2} \end{cases}$ 2, 1

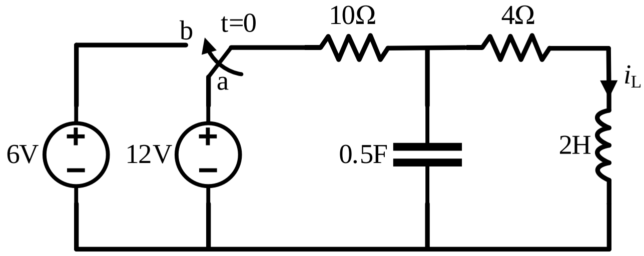
$\therefore V(s) = \frac{100s}{s^2+200^2}\cdot\frac{10}{s+100} = \frac{2(s+400)}{s^2+200^2} - \frac{2}{s+100}$ 2

$\therefore v(t) = 2\cos(200t) + 4\sin(200t) - 2e^{-100t}$

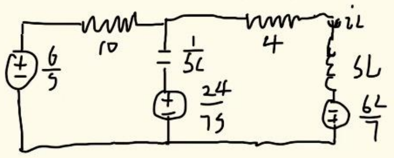
$= 4.47\cos(200t - 63.4^\circ) - 2e^{-100t},\ t>0$ 1

来自华为笔记

6. For the following circuit, the switch had been at node ***a*** for a long time before ***t***=0s. When ***t***=0s, the switch was turned to node ***b*** immediately. Please use (a) **Laplace domain method and (b) time domain method** to find $i_L(t)$ for t>0s.



6.(a) $V_C(0^-) = V_C(0^+) = \frac{12}{10+4}\times 4 = \frac{24}{7}V$ 1 $\quad i_L(0^-) = i_L(0^+) = \frac{12}{10+4} = \frac{6}{7}A$ 1 $\quad$ (20)



$$\begin{cases} i_L(s)(sL+4) - \frac{6L}{7} = V_C(s) + \frac{24}{7s} & 2 \\ \left[V_C(s)sC + i_L(s)\right]\cdot 10 + V_C(s) + \frac{24}{7s} = \frac{6}{s} & 2 \end{cases}$$

$$\therefore\ i_L(s) = \frac{30s^2+66s+21}{35s^3+77s^2+49s} \quad 2$$

$$= 0.43\frac{s+1.1}{(s+1.1)^2+0.44^2} + 1.06\frac{0.44}{(s+1.1)^2+0.44^2} + \frac{0.43}{s} \quad 2$$

$$\therefore\ i_L(t) = e^{-1.1t}(0.43\cos 0.44t + 1.06\sin 0.44t) + 0.43,\quad t>0 \quad 1$$

(b) $$\begin{cases} (C\frac{dV_C}{dt} + i_L)\cdot 10 + 4i_L + L\frac{di_L}{dt} = 6 & 2 \\ V_C = 4i_L + L\frac{di_L}{dt} & 1 \end{cases}$$

$$\Rightarrow 10\frac{d^2 i_L}{dt^2} + 22\frac{di_L}{dt} + 14 i_L = 6 \quad 1$$

$x_1 = -1.1-0.44j$, $x_2 = -1.1+0.44j$ 1

齐次解：

$$\bar{i}_L(t) = e^{-1.1t}(C_1\cos 0.44t + C_2\sin 0.44t) \quad 1$$

特解： $i_L^*(t) = 0.43$

$$\therefore\ i_L(t) = e^{-1.1t}(C_1\cos 0.44t + C_2\sin 0.44t) + 0.43 \quad 1$$

代入 $$\begin{cases} i_L(0^+) = \frac{6}{7} \\ V_C(0^+) = \frac{24}{7} \end{cases} \Rightarrow \begin{cases} C_1 + 0.43 = \frac{6}{7} \\ (C_1+0.43)\times 4 + 2(-1.1C_1 + 0.44C_2) = \frac{24}{7} \end{cases}$$

$$\therefore \begin{cases} C_1 = 0.43 \\ C_2 = 1.06 \end{cases} \quad 2 \qquad \therefore\ i_L(t)\ 同上$$