Machine Learning, 2024 Fall
Quiz 5

1 True/False [30 pts]

1. In a Bayesian Network, two nodes are independent if there is no direct edge connecting them.
   solution: False

2. Bayesian Networks can represent both causal and correlational relationships between variables.
   solution: True

3. D-separation is used to determine conditional independence in Bayesian Networks.
   solution: True

4. Naïve Bayes assumes that all features are conditionally dependent, given the class.
   solution: False

5. The conditional independence in a Bayesian Network can be determined based only on the topology of the graph, without needing the Conditional Probability Tables (CPTs).
   solution: True
6. In a Hidden Markov Model (HMM), the transition model $P(x_t|x_{t-1})$ and the emission model $P(y_t|x_t)$ are vary for each t to ensure flexibility.
   solution: False

2 Conception [20 pts]

1. Do both Bayesian Networks and Markov Networks represent joint distributions over all variables? Please provide your reasons.

solution: Yes, both Bayesian Networks and Markov Networks represent joint distributions over all variables. Bayesian Networks do this by factorizing the joint distribution into a product of conditional probabilities based on the directed acyclic graph (DAG) structure. In contrast, Markov Networks represent the joint distribution as a product of potential functions over cliques in an undirected graph, with a normalization constant to ensure it sums to one.

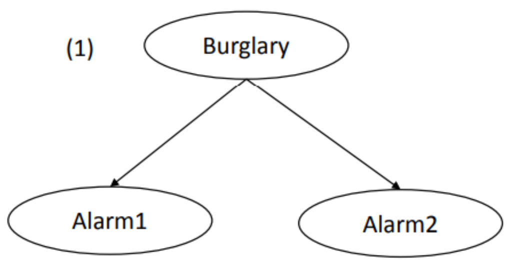
2. Why can Conditional Random Fields (CRF) be used in image segmentation, and what kind of dependencies can CRF model?

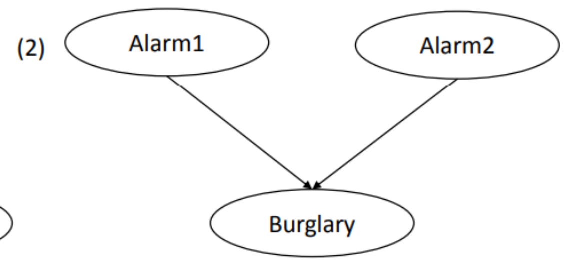
solution: Conditional Random Fields (CRF) can be used in image segmentation because they effectively model spatial dependencies between pixels. The goal of image segmentation is to label pixels such that they form coherent regions, and CRF captures the relationships between neighboring pixels to ensure smoothness and consistency in the segmentation result. CRFs can model the following dependencies: (a) Dependency between neighboring pixels: Neighboring pixels tend to have similar labels, and CRFs capture these local dependencies by connecting adjacent pixels. (b) Global context constraints: CRFs can incorporate global contextual information to influence pixel labeling, ensuring the segmentation is consistent across the image.

3 D-separation [30 pts]

(a) To enhance the security of your house, you installed two different alarm systems from two reputable manufacturers, each using completely independent sensors to detect a burglary. The two Bayesian networks below represent different models of how these alarm systems and a burglary event are related: Alarm1 means that the first alarm system rings, Alarm2 means that the second alarm system rings, and Burglary means that a burglary is in progress.

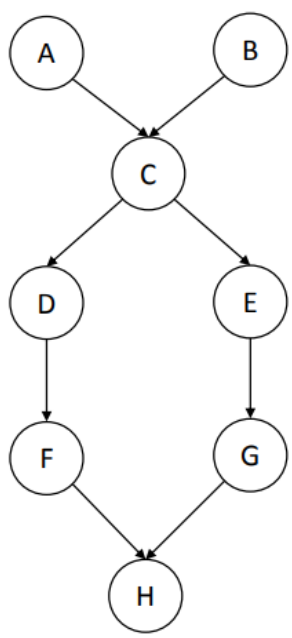
Which of the two Bayesian networks shown below makes incorrect independence assumptions, please give your reasons.





solution: The second one falsely assumes that Alarm1 and Alarm2 are independent if the value of Burglary is unknown. However, if the alarms are working as intended, it should be more likely that Alarm1 rings if Alarm2 rings (that is, they should not be independent).

(b) Consider the following Bayesian network, answer the following questions with explanations.



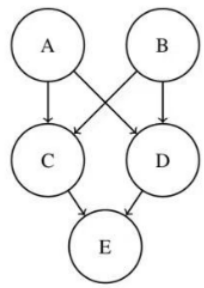
1. Are D and E necessarily independent given evidence about both A and B?
2. Are A and C necessarily independent given evidence about D?
3. Are A and H necessarily independent given evidence about C?

solution:

1. No. The path D-C-E is not blocked.
2. No. They are directly dependent. The path A-C is not blocked.
3. Yes. All paths from A to H are blocked.

4 PGM Example [20 pts]

(a) Write down the joint probability distribution associated with the following Bayes Network. Express the answer in simplest form using the Markov condition.



solution: $P(A, B, C, D, E) = P(A)P(B)P(C|A, B)P(D|A, B)P(E|C, D)$

(b) For the Bayes Network above, assume that:

A	$P(A)$
true	0.7
false	0.3

B	$P(B)$
true	0.6
false	0.4

A	B	$P(C = \text{true} \mid A, B)$	$P(C = \text{false} \mid A, B)$
true	true	0.8	0.2
true	false	0.7	0.3
false	true	0.4	0.6
false	false	0.2	0.8

A	B	$P(D = \text{true} \mid A, B)$	$P(D = \text{false} \mid A, B)$
true	true	0.9	0.1
true	false	0.4	0.6
false	true	0.6	0.4
false	false	0.3	0.7

Calculate the probability $P(C = T, D = F|A = T, B = F)$.

solutions:

$$P(C = T, D = F \mid A = T, B = F) = \frac{P(C = T, D = F, A = T, B = F)}{P(A = T, B = F)}$$

$$= \frac{P(A = T)P(B = F)P(C = T \mid A = T, B = F)P(D = F \mid A = T, B = F)}{P(A = T)P(B = F)}$$

$$= \frac{0.7 \times 0.4 \times 0.7 \times 0.6}{0.7 \times 0.4} = 0.42$$