Profit Maximization in Mechanism Design

> Optimal Auction Design. Myerson 1981
>
> ch 13.2

Vickrey auction with reservation price r

单物品：VCG = second price auction

- Two biders
- $b_1, b_2 \in [0, 1]$ Random/uniform
- $min(b_1, b_2)$
- $Pr(min(b_1, b_2) > x) = (1-x)^2$
- $F(x) = 1 - (1-x)^2$
- $f(x) = F'(x) = 2 - 2x$
- $E(min(b_1, b_2)) = \int_0^1 xf(x)dx = \int_0^1 x(2-2x) = x^2 - 2/3x^3|_0^1 = 1/3$

Reserve Price

- $VCG_r : max(b_1, b_2) \geq r$ -> sell
- $payment = max(r, min(b_1, b_2))$
- $P(pay\ r) = r * (1-r) * 2$
- $E(payment) = r * r * (1-r) * 2 + x^2 - 2/3x^3|_r^1 = r^2 - 4/3r^3 + 1/3$

明显好于VCG 且1/2最优

Virtual Valuation

$\phi_i(v_i) = v_i - \frac{1-F(v_i)}{f(v_i)}$

1. Collect all bids b_i
2. Compute $\phi_i(b_i)$
3. Apply VCG on $\phi_i(b_i)$, get allocation x_i, payment P_i
4. Final allocation is x_i, final payment is $\phi_i^{-1}(P_i)$

$v_i \in [0, 1)$

$\phi_i(b_i) = b_i - (1 - b_i) = 2b_i - 1 \in [-1, 1]$

$2b_i - 1 = 0$

$b_i = 1/2$

> 卖软件：保留价vcg-> fixed price auction

$\frac{1-F(v_i)}{f(v_i)}$ 单调非增能保证truthful