1 Myerson's Mechanism

Suppose there are n agents who bid for one single item. Their probability density functions of their valuation distributions are of the Pareto's form and same (i.i.d.):

$$f(x) = \frac{\alpha}{x^{\alpha+1}} \qquad x \geq 1$$

1.1 (1pt)

If $\alpha = 2$ and there are five bidders $\{A, B, C, D, E\}$ with bids $v_A = 20$, $v_B = 18$, $v_C = 16$, $v_D = 14$ and $v_E = 12$. Compute the allocation and payment of Myerson's mechanism.

$$F(x) = 1 - \frac{1}{x^\alpha}$$

When $\alpha = 2$,

$$\phi(v) = \frac{1 - \frac{1}{x^\alpha}}{f(x)} = v/2$$

The allocation and payment will be same as VCG.
Then A is the winner and she should pay 18.

1.2 (1pt)

If $\alpha = 1/2$, will the mechanism be truthful? Prove your statement.

When $\alpha = 1/2$, $\phi(v) = -v$, which is not monotone increasing. Hence the mechanism is not truthful.

2 Auction in Social Network

Seller s wants to sell one item in the social network. The other nodes are the buyers. The number in the circle is the buyer's valuation.

2.1 (2pt)

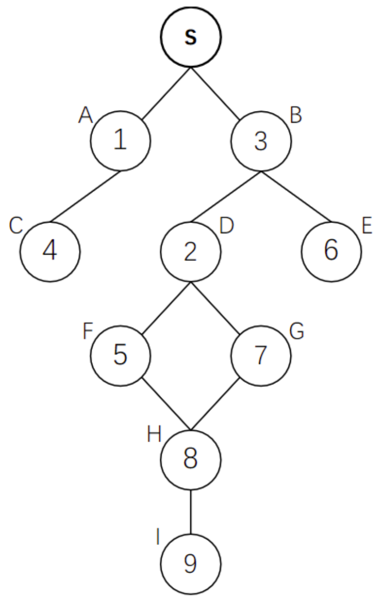
For the extended VCG mechanism in social network, if one buyer does not participate in the auction, the buyers that she invites can not participate either (if D does not participate in the auction, then F, G, H, I can not participate either). Run the extended VCG mechanism in the whole social network rather than just in the first layer. Give the allocation and payment results. Does the extended VCG satisfy IC,IR and WBB? The proof is not needed.
Weakly budget balanced (WBB) means the sum of all buyers' payments (i.e., the revenue of the seller) is non-negative.

Allocation: node I gets the item
Payment: $p_B = 4 - 9 = -5$, $p_D = 6 - 9 = -3$, $p_H = 7 - 9 = -2$, $p_I = 8 - 0 = 8$. The other buyers are 0.
The revenue of the seller is $-5 - 3 - 2 + 8 = -2$.
It satisfies IC, IR but does not satisfy WBB.

2.2 (1pt)

Run IDM in the social network. Give the allocation and payment results.

Allocation: node H gets the item
Payment: $p_B = 4 - 6 = -2$, $p_D = 6 - 7 = -1$, $p_H = 7$. The other buyers are 0.
The revenue of the seller is 4.



3 Double Auction (2pt)

Consider a double auction where each seller sells one item and each buyer gets at most one item. All the items are identical. Suppose the sellers' asks are $(s_1 = 3, s_2 = 5, s_3 = 6, s_4 = 13, s_5 = 15, s_6 = 17)$ and the buyers' bids are $(b_1 = 8, b_2 = 10, b_3 = 13, b_4 = 17, b_5 = 19, b_6 = 24)$. Compute each agent's utility the social welfare of the allocations generated by VCG (1pt) and McAfee's mechanisms (1pt) respectively.

We can sort the sellers' asks and the buyers' bids.
By VCG mechanism, the front four pairs will make deals, so $u_{s_1} = 13-3 = 10, u_{s_2} = 13-5 = 8, u_{s_3} = 13-6 = 7, u_{s_4} = 13-13 = 0, u_{s_5} = 0, u_{s_6} = 0$
and $u_{b_1} = 0, u_{b_2} = 0, u_{b_3} = 13-13 = 0, u_{b_4} = 17-13 = 4, u_{s_5} = 19-13 = 6, u_{s_6} = 24-13 = 11$
social welfare is:

$$SW_{VCG} = 24 + 19 + 17 + 13 + 15 + 17 = 105$$

By McAfee's mechanism, only the front three pairs will make deals, so we will get $p_b = p_s = 13$
$u_{s_1} = 13-2 = 11, u_{s_2} = 13-3 = 10, u_{s_3} = 13-5 = 8, u_{s_4} = 0, u_{s_5} = 0, u_{s_6} = 0$
and $u_{b_1} = 0, u_{b_2} = 0, u_{b_3} = 0, u_{b_4} = 17-13 = 4, u_{s_5} = 19-13 = 6, u_{s_6} = 20-12 = 7$
the social welfare is:

$$SW_{McAfee} = 24 + 19 + 17 + 13 + 15 + 17 = 105$$

4 Advertisement auctions

4.1 VCG allocation (1pt)

Suppose a search engine has two ad slots that it can sell. Slot A has a clickthrough rate of 1000 clicks per hour, slot B has a clickthrough rate of 200 clicks per hour. There are three bidders b_1, b_2, b_3 have values per click of $\{10, 9, 5\}$ respectively. Compute the allocation and the payments for all bidders in VCG.

Under VCG, bidder b_1 and b_2 win slot A and B respectively. The payment for bidder b_1 is the social welfare loss by moving b_2 from slot A to slot B and forcing b_3 out of slot B. The payment for bidder b_2 is the social welfare loss by forcing b_3 out of slot B:

$$p_{b_1} = (9 * 1000 + 5 * 200) - (9 * 200) = 8200$$
$$p_{b_2} = (10 * 1000 + 5 * 200) - (10 * 1000) = 1000$$

4.2 GSP allocation (1pt)

Under the same setting as in question 4.1, compute the allocation and payments for all bidders in GSP (without weight).

Under GSP, bidder b_1 and b_2 win slot A and B respectively. The payments for two slots are 9 and 5 per click. The total payments of bidders b_1, b_2 are:

$$p_{b_1} = 9 * 1000 = 9000$$
$$p_{b_2} = 5 * 200 = 1000$$

4.3 The truthfulness of GSP (1pt)

Consider the example of GSP allocation in question 4.2. Is this example truthful? Why? If it is truthful, change one value in the setting above to show that GSP is not truthful in general, and explain. Under VCG, bidder b_1

and b_2 win slot A and B respectively. The payment for bidder b_1 is the social welfare loss by moving b_2 from slot A to slot B and forcing b_3 out of slot B. The payment for bidder b_2 is the social welfare loss by forcing b_3 out of slot B:

$$p_{b_1} = (9 * 500 + 5 * 200) - (9 * 200) = 3700$$
$$p_{b_2} = (10 * 500 + 5 * 200) - (10 * 500) = 1000$$

Under GSP, bidder b_1 and b_2 win slot A and B respectively. The payments for two slots are 9 and 5 per click. The total payments of bidders b_1, b_2 are:

$$p_{b_1} = 9 * 500 = 4500$$
$$p_{b_2} = 5 * 200 = 1000$$

4.4 The truthfulness of GSP (1pt)

Use the example of question 4.1 to show that GSP is not truthful.

Bidders' valuation per click are $\{10, 9, 5\}$. Slots' CTRs are $\{500, 200\}$ clicks per hour respectively. If everyone bids truthfully, the revenue for b_1 is:

$$r_1 = (10 - 9) * 1000 = 1000$$

If bidder b_1 bids lower than 9 and higher than 5, he will get a revenue of:

$$r_1' = (10 - 5) * 200 = 1000$$

If bidder b_1 bids higher than 10 it will not affect his revenue, if he bids lower than 5, his revenue will be even lower. So this example of GSP is truthful.
Change one value: anything that makes $r_1 < r_1'$ is ok.