Homework 4

Due: 2025-05-11
Release: 2025-04-30

1 Facility Location

Now we consider the case where there are two facilities and each facility can only serve a limited number of agents. There are n agents. Assume the first facility can serve c_1 agents and the second can serve c_2 agents.

1.1 (1.5pt)

When $n = 2c_1 = 2c_2$, mechanism M_1 places one facility at the $c_1 th$ agent from left. serving the leftmost c_1 agents, and the other facility at the $c_1 + 1th$ agent, serving the rightmost c_2 agents. Is M_1 truthful? Give the prove or find a counterexample.

1.2 (2pt)

Calculate the approximation ratio of M_1 for maximum cost and the total cost separately. (You can rescale the problem so the leftmost agent is at 0, and the rightmost agent at 1.)

2 Cost Sharing

Consider the 3-player game (N, c) given by the following cost function:

$$c\{1\} = 4; \quad c\{2\} = 6; \quad c\{3\} = 8 \quad c\{1,2\} = 8; \quad c\{1,3\} = 11; \quad c\{2,3\} = 14 \quad c\{1,2,3\} = 15$$

2.1 (1pt)

Is $(3, 5, 7)$ in the core of the game? Give the proof.

2.2 (2pt)

If $c\{1,2,3\} = 17$ (other costs remain unchanged), give a vector that is in the core of the game or prove that the core is empty.

3 Core

Consider the following game. $N = \{A, B, C\}$, the valuation function is as below:

$$v(A) = 2, v(B) = 3, v(C) = 2$$

$$v(AB) = 8, v(AC) = 10, v(BC) = 8, v(ABC) = 12$$

3.1 (2pt)

Find the Shapley value of this game. Is the Shapley value in the core?

3.2 (1.5pt)

Find the core of this game. If there isn't a core, explain.