EE150 - Signals and Systems, Fall 2024

Homework Set #1

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Problem 1. (10 points)

(a) Use Euler's formula to write the following expression:

$\sin\theta =$ ______ $\cos\theta =$ ______ (2 points)

(b) Use the Cartesian coordinate system ($z = x + jy$) to represent the following complex numbers:(4 points)

(1) $\sqrt{2}e^{-j\frac{13\pi}{4}}$ (2) $e^{-j\frac{\pi}{2}}$

(c) Use the polar coordinates ($z = re^{j\theta}, -\pi < \theta \le \pi$) to represent the following complex numbers:(4 points)

(1) -3 (2) $\frac{\sqrt{2}+\sqrt{6}j}{2+\sqrt{3}j}$

Solution.

(a) $e^{j\theta} = cos\theta + jsin\theta \quad sin\theta = \frac{e^{j\theta}-e^{-j\theta}}{2j} \quad cos\theta = \frac{e^{j\theta}+e^{-j\theta}}{2}$

(b) $\sqrt{2}e^{-j\frac{13\pi}{4}} = \sqrt{2}(cos\frac{13\pi}{4} - jsin\frac{13\pi}{4}) = -1 + j$

$e^{-j\frac{\pi}{2}} = cos\frac{\pi}{2} - jsin\frac{\pi}{2} = -j$

(c) $-3 = 3e^{j\pi}$

$\frac{\sqrt{2}+\sqrt{6}j}{2+\sqrt{3}j} = \frac{5\sqrt{2}+\sqrt{6}j}{7}$ then $r = \sqrt{(\frac{5\sqrt{2}}{7})^2 + (\frac{\sqrt{6}}{7})^2} = \frac{2\sqrt{14}}{7}$, $\theta = arctan\frac{\frac{\sqrt{6}}{7}}{\frac{5\sqrt{2}}{7}} = arctan\frac{\sqrt{3}}{5}$

so $z = \frac{2\sqrt{14}}{7}e^{jarctan\frac{\sqrt{3}}{5}}$

Problem 2. (20 points) Determine the energy E_∞ and power P_∞ of following signals. Which are finite-energy signals, which are finite-power signals, which are infinite energy and power signals? Write your calculation.

(a) $x_1(t) = t$

(b) $x_2(t) = e^{-\frac{1}{4}t}u(t)$

(c) $x_3[n] = e^{j(\frac{\pi}{4n}+\frac{\pi}{6})}$

Solution.

(a) $E_\infty = \int_{-\infty}^{\infty} |x_1(t)|^2 dt = \int_{-\infty}^{\infty} t^2 dt = \infty$

$P_\infty = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} t^2 dt = lim_{T\to\infty}\frac{1}{3}T^2 = \infty$

Infinite energy and power signal

(b) $E_\infty = \int_0^\infty e^{-\frac{t}{2}} dt = 2$

$P_\infty = \lim_{T\to\infty} \frac{1}{2T}2 = \lim_{T\to\infty}\frac{1}{T} = 0$

Finite energy signal

(c) $E_\infty = \sum_{n=-\infty}^{\infty} |x_3[n]|^2 = \int_{n=-\infty}^{\infty} 1 = \infty$

$P_\infty = \lim_{N\to\infty} \frac{1}{2N+1} \int_{n=-N}^{N} 1 = 1$

Finite power signal

Problem 3. (20 points) We have a signal $x(t)$, the following figures are the parts of $x(t)$ and its odd part $x_o(t)$, for $t \geq 0$ only. Please plot the whole odd part $x_o(t)$, whole even part $x_e(t)$ and whole $x(t)$ for $-\infty < t < \infty$ and write the equation of each function. (Be careful to write the boundary values clearly)

$$x(t) = \begin{cases} 2 & 0 \leq t \leq 1 \\ 0 & t > 1 \end{cases}$$

$$x_o(t) = \begin{cases} t & 0 \leq t \leq 2 \\ 0 & t > 2 \end{cases}$$

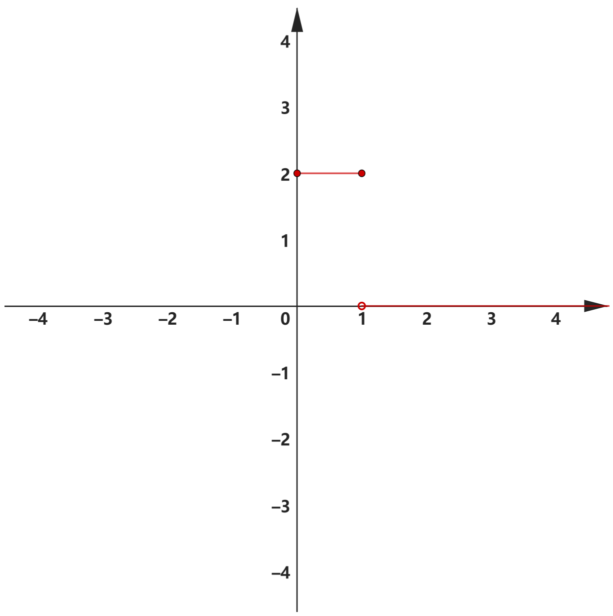

Figure 1: part of $x(t)$

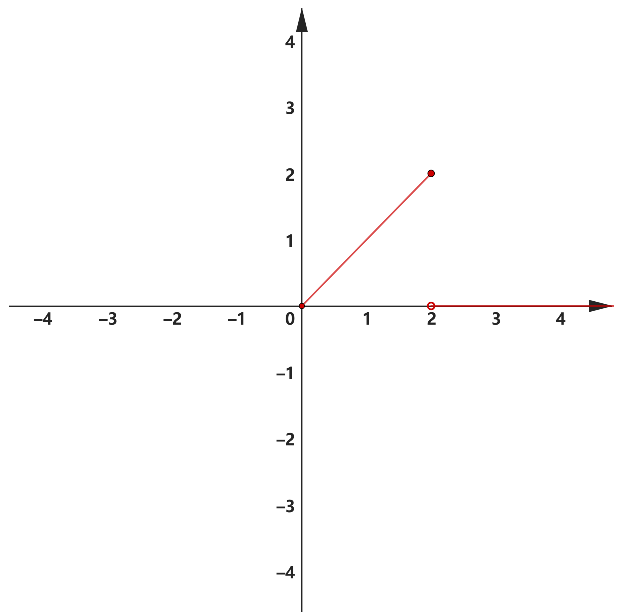

Figure 2: part of $x_o(t)$

Solution.

$$x(t) = \begin{cases} 0 & t < -2 \\ 2t & -2 \leq t < -1 \\ 2t+2 & -1 \leq t < 0 \\ 2 & 0 \leq t \leq 1 \\ 0 & t > 2 \end{cases}$$

$$x_e(t) = \begin{cases} 0 & t < -2 \\ t & -2 \leq t < -1 \\ t+2 & -1 \leq t < 0 \\ -t+2 & 0 \leq t \leq 1 \\ -t & 1 < t \leq 2 \\ 0 & t > 2 \end{cases}$$

$$x_o(t) = \begin{cases} 0 & t < -2 \\ t & -2 \leq t \leq 2 \\ 0 & t > 2 \end{cases}$$

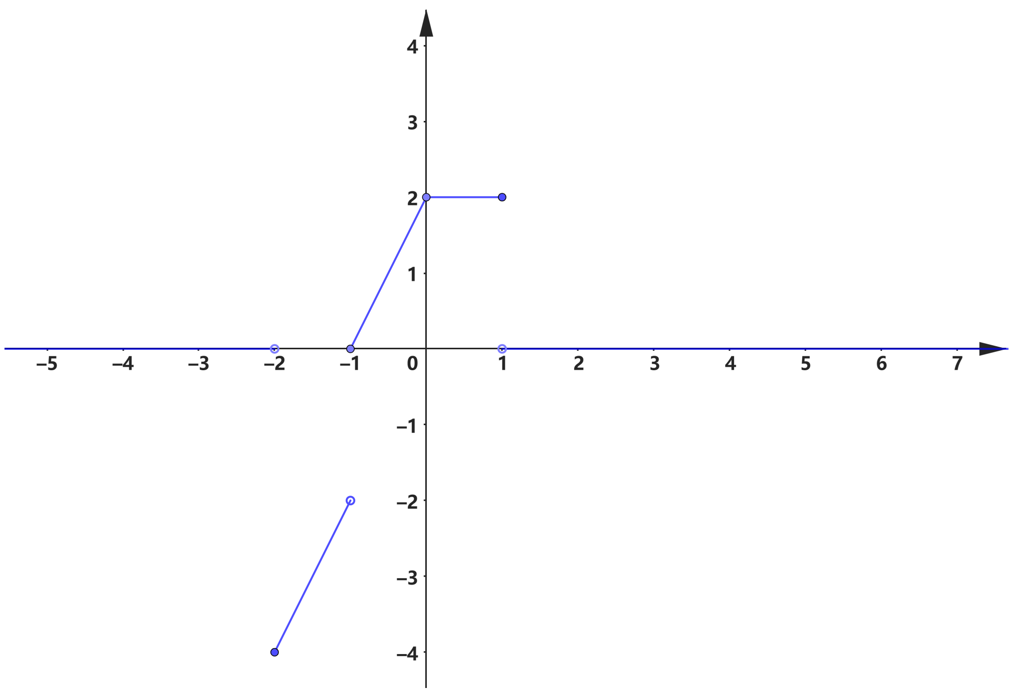

Figure 3: $x(t)$

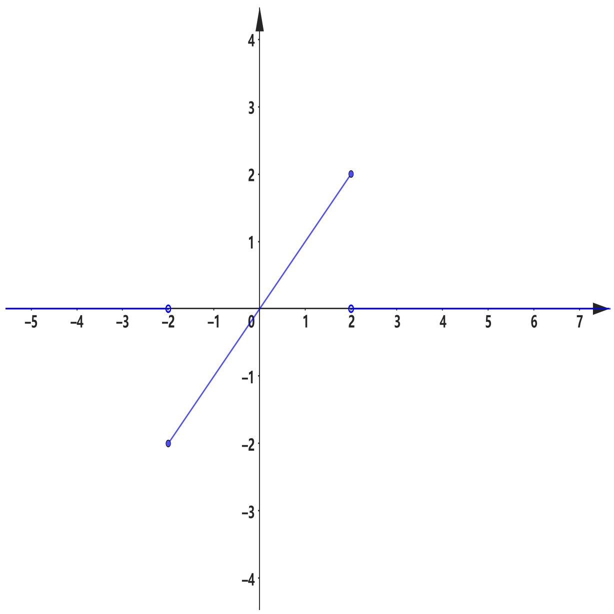

Figure 4: $x_o(t)$

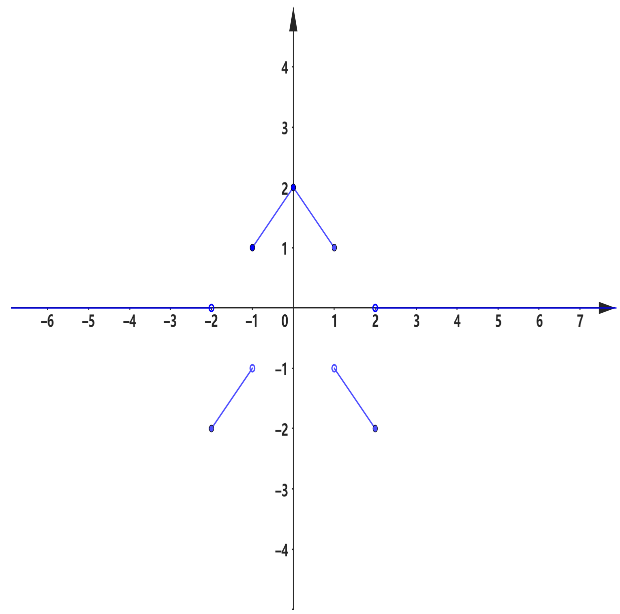

Figure 5: $x_e(t)$

Problem 4. (20 points)

(a) Determine which of the following signals is periodic. If a signal is periodic, write its fundamental period.(10 points)

(1) $x[n] = \mathrm{e}^{j(2\pi n/5)}$

(2) $x(t) = \frac{\sqrt{3}}{2}\tan(4\pi t + \frac{\pi}{4} + \frac{\sqrt{5}}{2})$

(b) Find the fundamental period of discrete signal $x_1[n] = e^{j\pi n}$ and $x_2[n] = e^{j\frac{2\pi}{3}n}$ (2 points), then answer following questions.

(1) What's the fundamental period of $x_1[n] + x_2[n]$? (4 points)

(2) What's the fundamental period of $x_1[n] \cdot x_2[n]$? (4 points)

Solution.

(a)

(1) periodic with period 5:

$$e^{j(\frac{2\pi}{5}n) = e^{j(\frac{2\pi}{5})(n+N)}} = e^{j(\frac{2\pi}{5}n + 2\pi k)}$$

then let $2\pi k = \frac{2\pi}{5}N$ which k and N are integers. So $k = 1$, $N = 5$, period is 5.

(2) periodic with period is $T = \frac{\pi}{|\omega|} = \frac{1}{4}$.

(b) Fundamental period of $x_1[n]$ and $x_2[n]$ are $N_1 = 2$ and $N_2 = 3$

(1) $N = LCM(N_1, N_2) = 6$, LCM is least common multiple.

(2) $N = LCM(N_1, N_2) = 6$

$$lg(x_1[n] \cdot x_2[n]) = lgx_1[n] + lgx_2[n]$$

because use lg function will not change the fundamental period, so the fundamental period of $lgx_1[n]$ and $lgx_2[n]$ are still N_1 and N_2. Then the fundamental period of $lgx_1[n] + lgx_2[n]$ is $LCM(N_1, N_2)$ which means the fundamental period of $x_1[n] \cdot x_2[n]$ is $LCM(N_1, N_2)$

Problem 5.(5 points $\times$ 4) For each of the following systems, determine whether the system is (1) stable, (2) causal, (3) linear, (4) time invariant, and (5) memoryless:

1) $y[n] = \sum_{k=-\infty}^{n} x[k]$,
2) $y[n] = x[Mn]$, for $M > 1$
3) $y(t) = x(-t)$,
4) $y(t) = x(\frac{t}{3})$.

(Notice: You need to give your reasons.)

Solution.

(1) not stable, causal, linear, time invariant, not memoryless

(2) stable, not causal, linear, not time invariant, not memoryless

(3) stable, not causal, linear, not time invariant, not memoryless

(4) stable, not causal, linear, not time invariant, not memoryless

Problem 6. (10 points) We have a signal $x(t)$.

(a) Express x(t) in terms of the unit step function, then calculate the $x'(t)$. (4 points)

(b) Plot the $x(-t+2)$ and $x(\frac{2}{3}t+1)$(6 points)

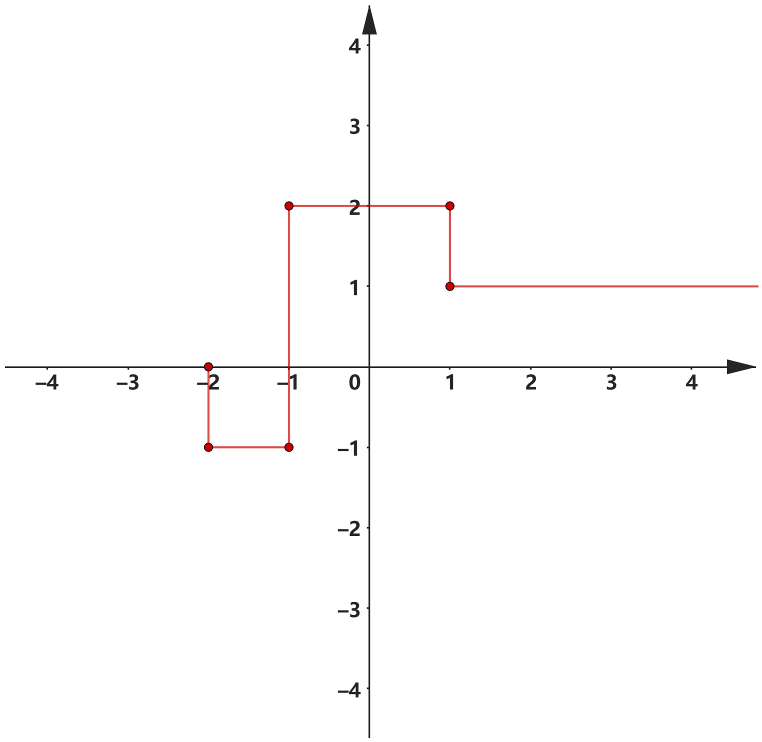

Figure 6: $x(t)$

Solution.

(a)

$$x(t) = -u(t+2) + 3u(t+1) - u(t-1)$$
$$x'(t) = -\delta(t+2) + 3\delta(t+1) - \delta(t-1)$$

(b)

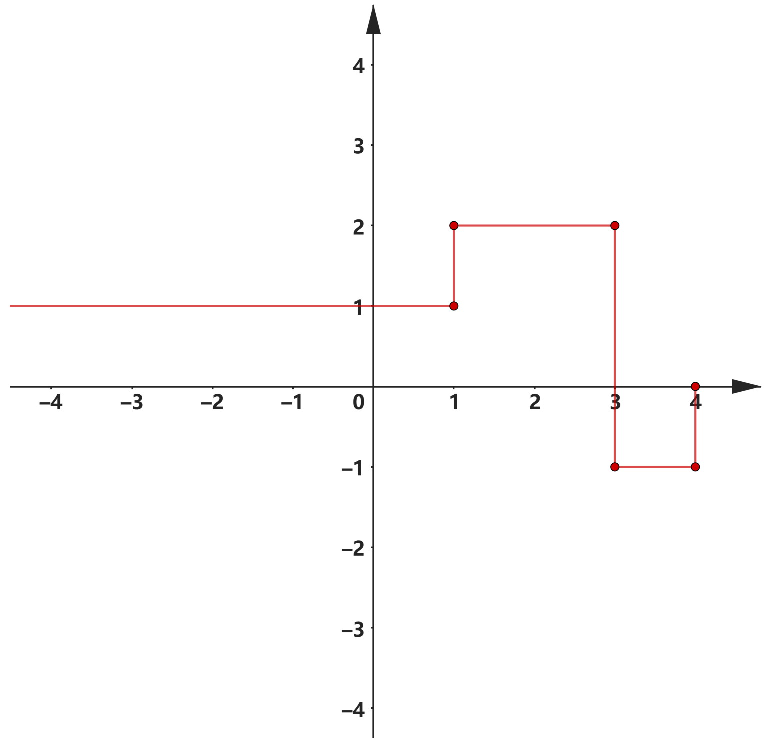

Figure 7: $x(-t+2)$

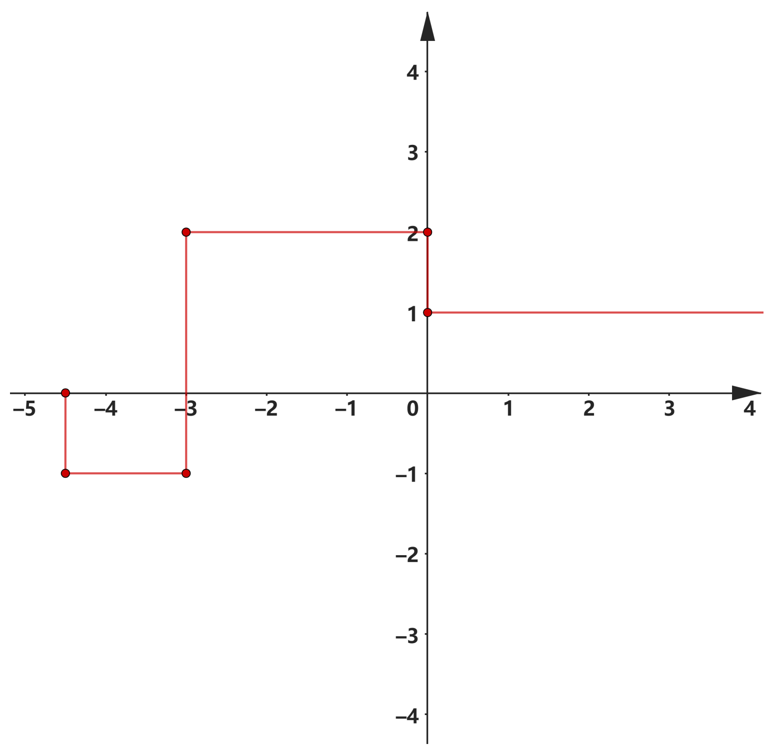

Figure 8: $x(\frac{2}{3}t+1)$