Homework 1

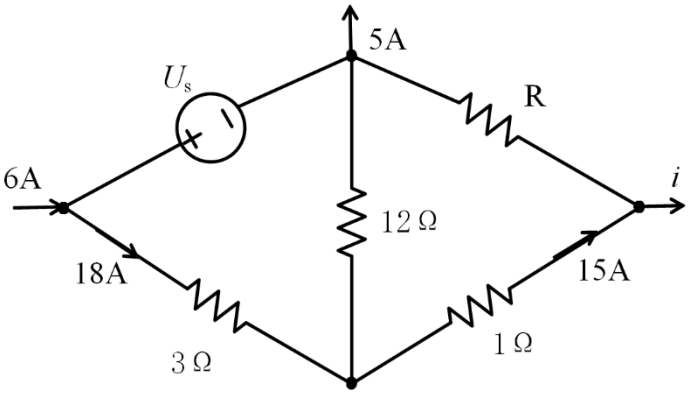
Due date: Oct. 19th, 2023

Turn in your hard-copy hand-writing homework in class

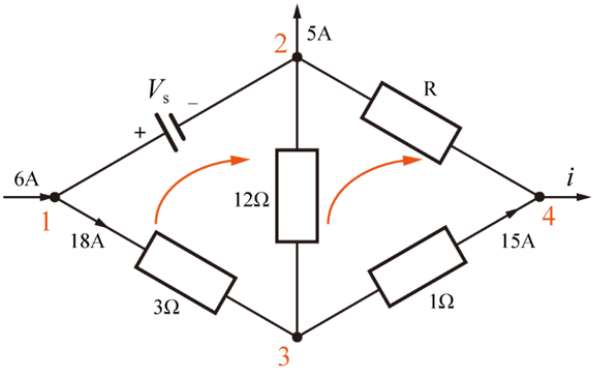
Rules:

- Work on your own. Discussion is permissible, but extremely similar submissions will be judged as plagiarism.
- Please show all intermediate steps: a correct solution without an explanation will get zero credit.
- Please submit on time. No late submission will be accepted.
- Please prepare your submission in English only. No Chinese submission will be accepted.

1. (a) Use Kirchhoff's law to find U_s and i. 10'
   (b) Calculate resistance R. 5'



Answer:



As shown in the figure, list the current equations for each node and define the current entering the node as positive and the current leaving the node as negative:

node 1: 6A − 18A + i_{vs} = 0

node 2: − i_{vs} − 5A + i_R + $i_{12\Omega}$ = 0

node 3: 18A − $i_{12\Omega}$ − 15A = 0

node 4: − i + 15A − i_R = 0

So: i_{vs} = 12A, i_R = 14A, $i_{12\Omega}$ = − 3A, i = 1A

List the circuit voltage equation in the reference direction of the figure:

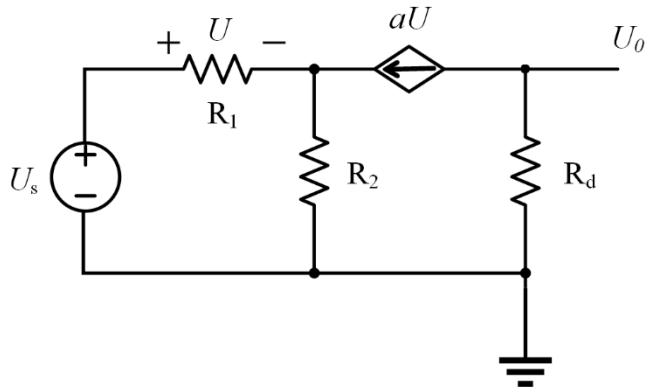
$V_s - V_{12\Omega} - V_{3\Omega} = 0$ ..... $V_{12\Omega} - V_R - V_{1\Omega} = 0$

$V_{12\Omega} = 3A \times 12\Omega = 36V$ ..... $V_{3\Omega} = 18A \times 3\Omega = 54V$

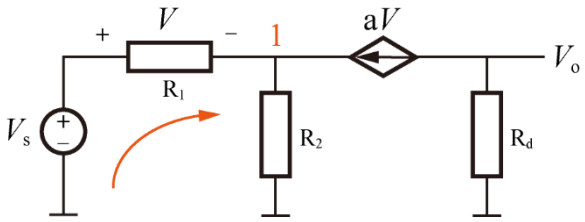
$V_{1\Omega} = 15A \times 1\Omega = 15V$ → → $V_R = 14A \times R = 14R$

So: $V_s = 90V$ R = 21/14Ω = 1.5Ω

2. Use Kirchhoff's law to calculate U_o/U_s in terms of a, R_1, R_2, and R_d. Except for U, all other variables are known quantities. 10'



Answer



Kirchhoff's current law

node 1 $i_{R1} + aV - i_{R2} = 0$

Kirchhoff's voltage Law

$-V_s + V + V_{R2} = 0$

Ohm's law

$V_{R2} = i_{R2} \times R_2$

$V = i_{R1} \times R_1$

So $V_s = (1 + R_2/R_1 + aR_2)V$ $V_o = -aVR_d$

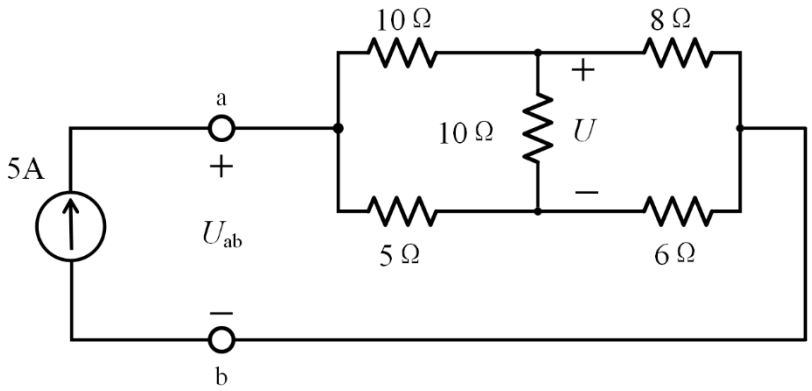
$V_o/V_s = (-aR_d)/(1 + R_2/R_1 + aR_2)$

The dimension of a is S.

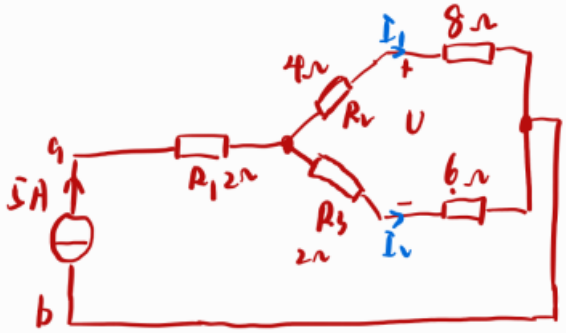
3. For the circuit below, using Δ-Y conversion

(a) Calculate the U. 10'

(b) Calculate the U_{ab}. 5'



transfer Δ → Y :



$R_1 = R_3 = \frac{5\times10}{10+10+5}\Omega = 2\Omega$, $R_V = \frac{10\times10}{10+10+5}\Omega = 4\Omega$

$I_1 = \frac{16}{8+4+2+6}\times5A = 2A$ $I_V = \frac{8+4}{8+4+2+6}\times5A = 3A$

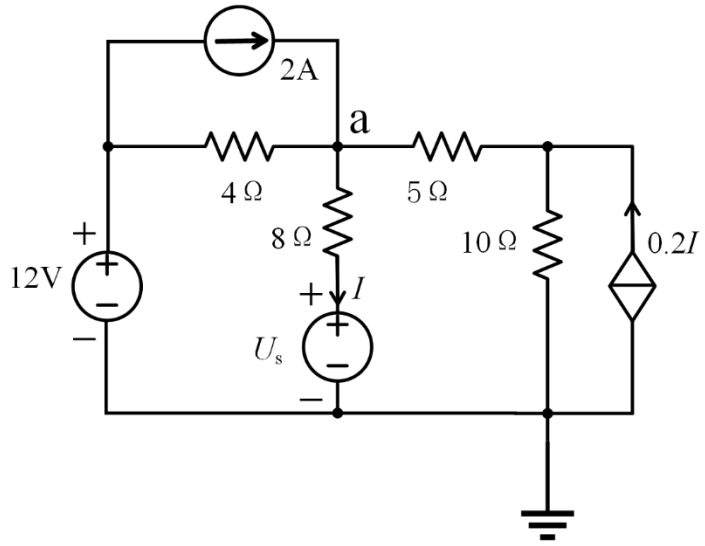
$\therefore U = 8I_1 - 6I_V = 16 - 18 = -2V$

$R_{ab} = 2 + \frac{12\times8}{12+8} = 6.8\Omega$

$\therefore U_{ab} = 5R_{ab} = 34V$

4. The node voltage U_a equals 15V, using nodal analysis method to obtain U_s and I.

10’

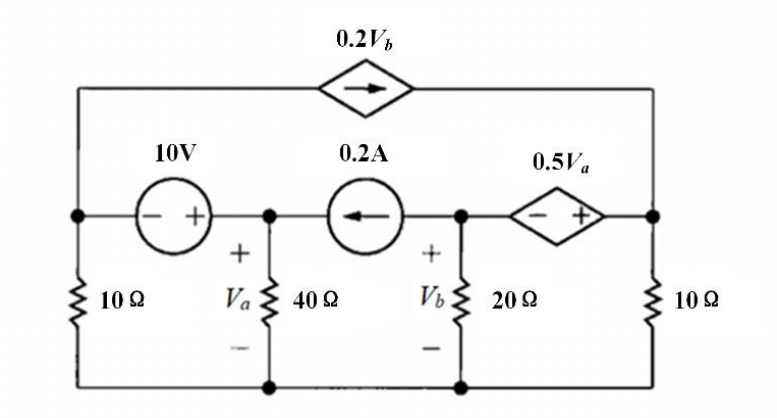


$$\begin{cases} -\frac{12}{4}+\left(\frac{1}{4}+\frac{1}{8}+\frac{1}{5}\right)U_b-\frac{1}{5}U_c=2+\frac{U_s}{8} \\ -\frac{1}{5}U_b+\left(\frac{1}{5}+\frac{1}{10}\right)U_c=0.2\times\frac{U_b-U_s}{8} \end{cases}$$, when $U_b=13\,V$.

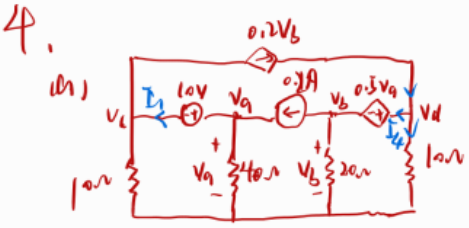
$$\begin{cases} 8.625-0.2U_c=5+0.125U_s \\ -5+0.3U_c=0.375-0.075U_s \end{cases} \Rightarrow U_s=\frac{165}{13}\,V=12.69\,V$$

$$I=\frac{12-U_s}{8}=0.788=\frac{12}{52}\,A$$

5. For the circuit below,
   (a) apply nodal analysis method to find V_a, V_b. 10'
   (b) find the power delivered by each source (2 voltage sources and 2 current sources). 10'



4.



$$\begin{cases} \frac{V_a - 10}{10} + 0.2V_b + \frac{V_a}{40} = 0.2 \\ 0.1 + \frac{V_b}{20} + \frac{V_b + 0.5V_a}{10} = 0.2V_b \end{cases} \Rightarrow \begin{cases} V_a = \frac{46}{13} = 1.\dot{5}3\ A \\ V_b = \frac{68}{13} = 5.\dot{2}3\ A \end{cases}$$

(b)

for $0.2V_b$: $V_c = V_a + 10 = \frac{-114}{13}$ $V_d = V_b + 0.5V_a = \frac{76}{13}$ V

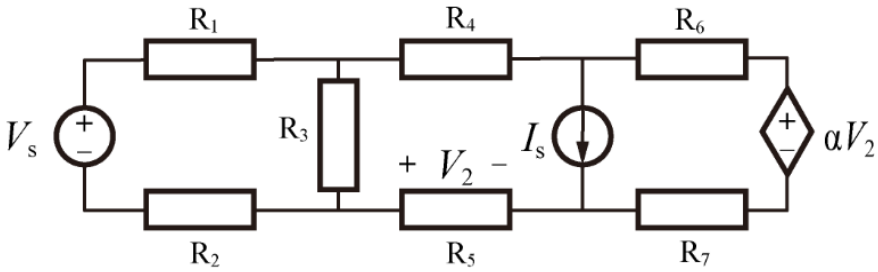
$P_{0.2V_b} = 0.2 \times \frac{68}{13} \times (\frac{-114}{13} - \frac{76}{13}) = -\frac{3584}{169}$ W, delivered $\frac{3584}{169}$ W

for 10V: $P_{10V} = (10 \times) I_1 = \frac{22}{13}$ W, delivered $-\frac{22}{13}$ W

for 0.2A: $P_{0.2A} = 0.2 \times (\frac{68}{13} - \frac{46}{13}) = 0.8$ W, delivered -0.8 W

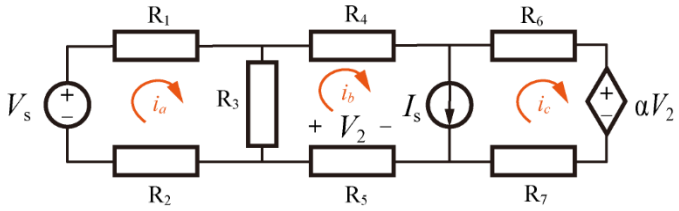
for $0.5V_a$: $P_{0.5V_a} = 0.5 V_a (\frac{V_d}{10} - 0.2V_b) = \frac{48}{169}$ W, delivered $\frac{48}{169}$ W

6. Use mesh current analysis method, calculate the absorbed power of current source and voltage source, given $R_1 = R_2 = R_3 = R_4 = R_5 = R_6 = R_7 = 1\Omega$, $\alpha = 4$, $I_s = 1A$, $V_s = 1V$ 20’



Answer:

Define the voltage of the current source as V:



$-V_s + i_a R_1 + (i_a - i_b)R_3 + i_a R_2 = 0$

$(i_b - i_a)R_3 + i_b R_4 - V + i_b R_5 = 0$

$V + i_c R_6 + \alpha V_2 + i_c R_7 = 0$

Additional equations:

$V_2 = -i_b R_5$

$i_b = i_c + I_s$

So: $i_a = 1.5A$, $i_b = 3.5A$, $i_c = 2.5A$, $V = -9V$

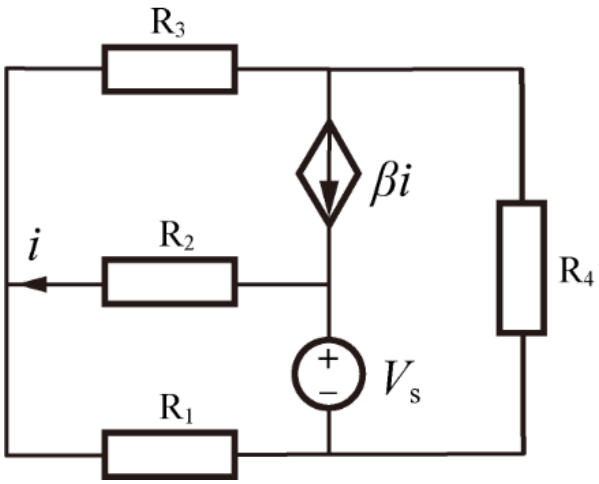
$P_{vs} = i_a \times V_s = -1.5w$

$P_{Is} = I_s \times V = -9w$

$P_{\alpha V} = -35w$

6. $R_1 = R_2 = R_3 = R_4 = 10\Omega$, $\beta = 2$, use the mesh current method to find V_s/i

10’



Answer:

Define the voltage of the controlled current source as V:

$i_a R_3 + V + (i_a - i_b)R_2 = 0$

$(i_b - i_a)R_2 + V_s + i_b R_1 = 0$

$-V_s - V + i_c R_4 = 0$

Additional equations:

$i_a - i_b = i$

$i_a - i_c = \beta i$

eliminate the V, i_a, i_b, i_c, to get: $V_s/i = \frac{(R_1+R_2)(R_3+R_4)-(\beta R_4-R_2)R_1}{R_1+R_3+R_4} = 10$