Steps in Applying the Laplace transform

- **Transform the circuit from the time domain to the Laplace (*s*) domain, including initial conditions.**

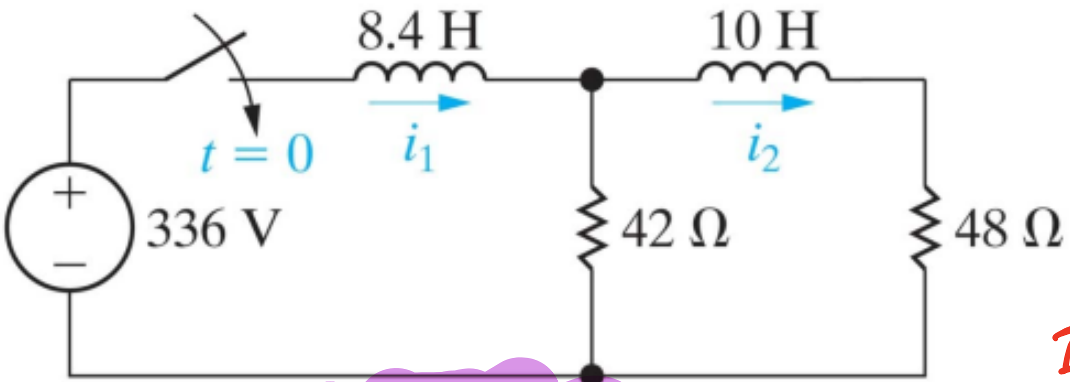
--*The elegance of using the Laplace transform in circuit analysis lies* in (1) transforming the differential equation into an *algebraic* equation; and (2) automatic inclusion of initial conditions in the transformation process, thus providing a complete (**transient and steady-state**) solution.

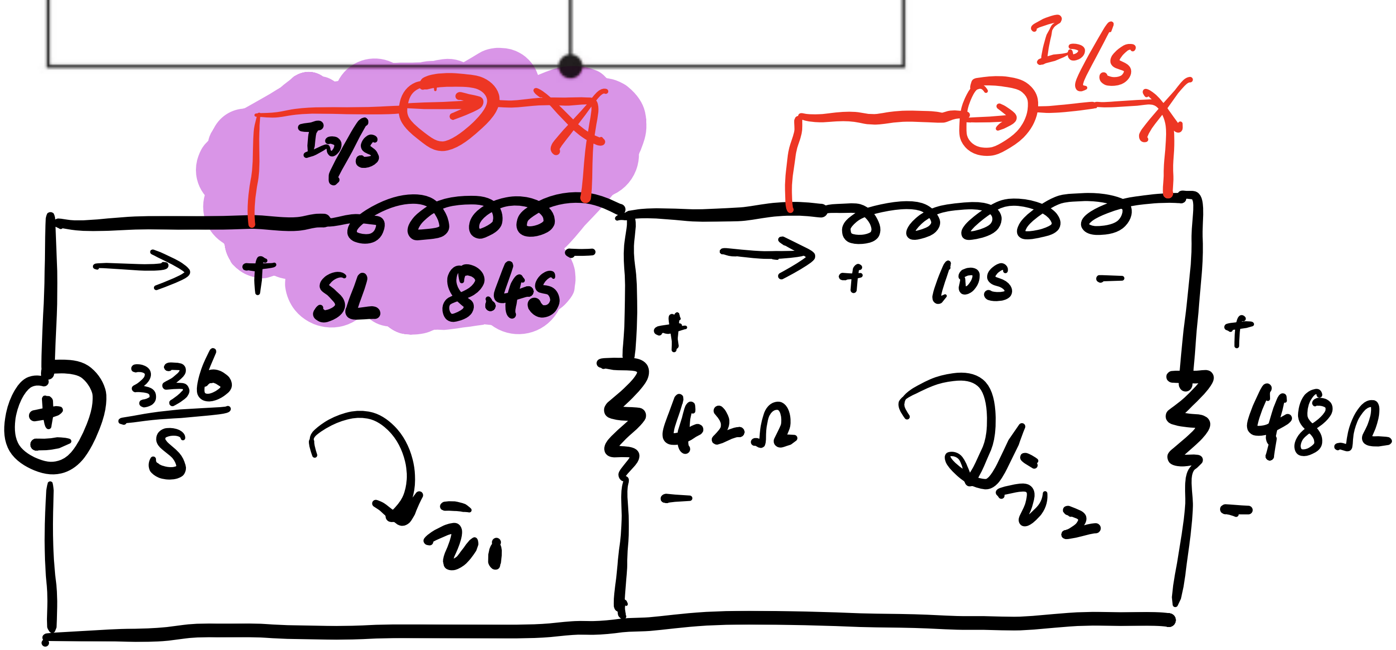
- **Solve the circuit using nodal analysis, mesh analysis, source transformation, superposition, or any other analysis technique with which we are familiar.**

- Take the inverse transform of the solution and thus obtain the solution in the time domain.

Example 1

Assuming no initial energy storage, find $i_1(t)$ and $i_2(t)$ for $t > 0$.





$$\frac{336}{S} = 8.4S \cdot i_1 + 42(i_1 - i_2) \quad ①$$

$$42(i_1 - i_2) = 10S \cdot i_2 + 48 i_2 \quad ②$$

By ② $\quad i_1 = \dfrac{90 + 10S}{42} i_2$

$$i_2 = \frac{168}{S^3 + 14S^2 + 24S} = \frac{168}{S(S+2)(S+12)}$$

$$= \frac{k_1}{S} + \frac{k_2}{S+2} + \frac{k_3}{S+12}$$

$$\frac{168}{\cancel{S}(S+2)(S+12)} \cdot \cancel{S} = \frac{k_1}{\cancel{S}} \cdot \cancel{S} + \cancel{\frac{k_2 \cdot S}{S+2}} + \cancel{\frac{k_3 \cdot S}{S+12}}$$

$$k_1 = \frac{168}{(S+2)(S+12)}\bigg|_{S=0} = 7$$

$k_2 = -8.4 \qquad k_3 = 1.4$

$$i_2 = \frac{7}{S} + \frac{-8.4}{S+2} + \frac{1.4}{S+12} \qquad \text{S. Domain}$$

$$\xrightarrow{\mathcal{L}^{-1}} i_2(t) = [7 + (-8.4 \cdot e^{-2t}) + 1.4e^{-12t}]\, u(t) \quad \text{A.}$$

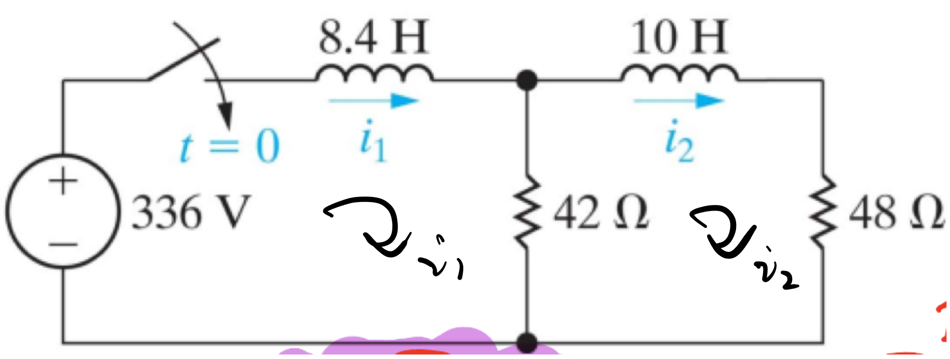
$$i_1 = \frac{90+10s}{42} \cdot \frac{168}{s^3+14s^2+24s}$$

$$= \frac{40s+360}{s^3+14s^2+24s} = \frac{\overset{15}{k_1'}}{s} + \frac{\overset{-14}{k_2'}}{s+2} + \frac{\overset{-1}{k_3'}}{s+12}$$

$\mathcal{L}^{-1}$

$$i_1(t) = (15 - 14e^{-2t} - e^{-12t})\, u(t) \text{ A.}$$

T.D. 2



$$336 = 8.4\frac{di_1}{dt} + 42(i_1 - i_2) \quad ①$$

$$42(i_1 - i_2) = 10\frac{di_2}{dt} + 48 i_2 \quad ②$$

By ② $\quad 42 i_1 = 10\frac{di_2}{dt} + 90 i_2$

$$i_2'' + 14 i_2' + 24 i_2 = 168$$

$$s^2 + 14s + 24 = 0$$

C.E. $(s+2)(s+12) = 0$

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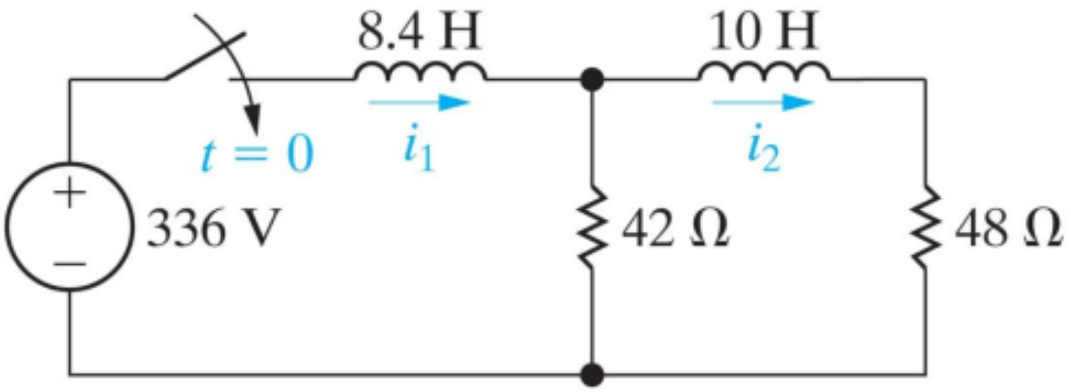
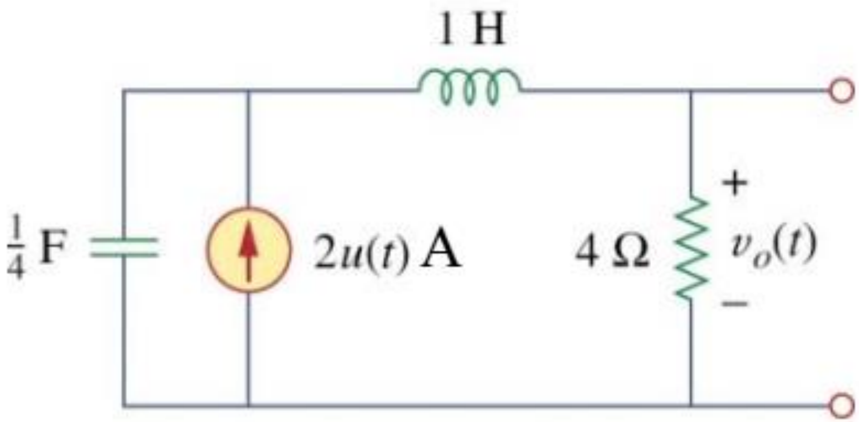
Example 1

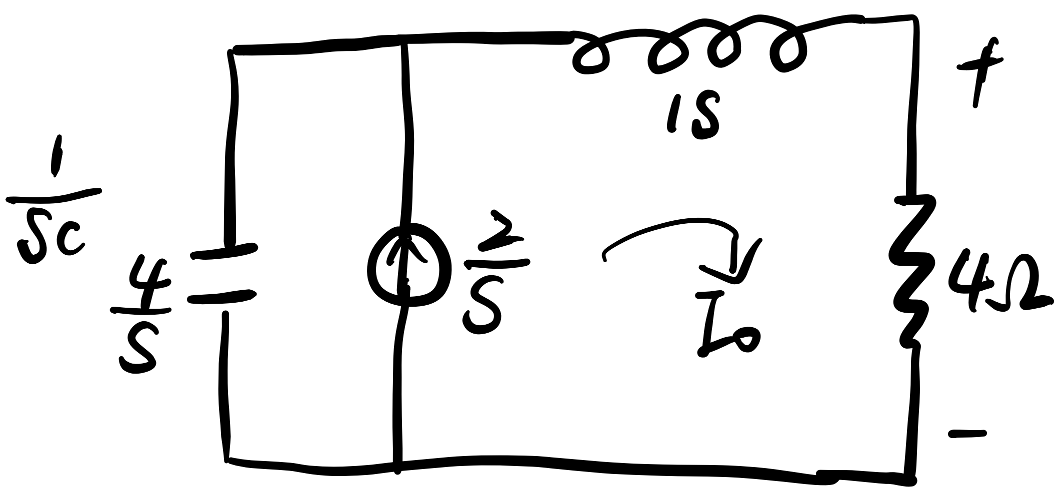
Assuming no initial energy storage, find $i_1(t)$ and $i_2(t)$ for $t > 0$.



Example 2

Determine $v_0(t)$ *for t>0 assuming zero initial conditions:*





$$I_o(s) = \frac{\frac{4}{s}}{\frac{4}{s} + s + 4} \times \frac{2}{s} = \frac{8}{s(s+2)^2}$$

$$V_o(s) = I_o(s) \cdot 4 = \frac{32}{s(s+2)^2}$$

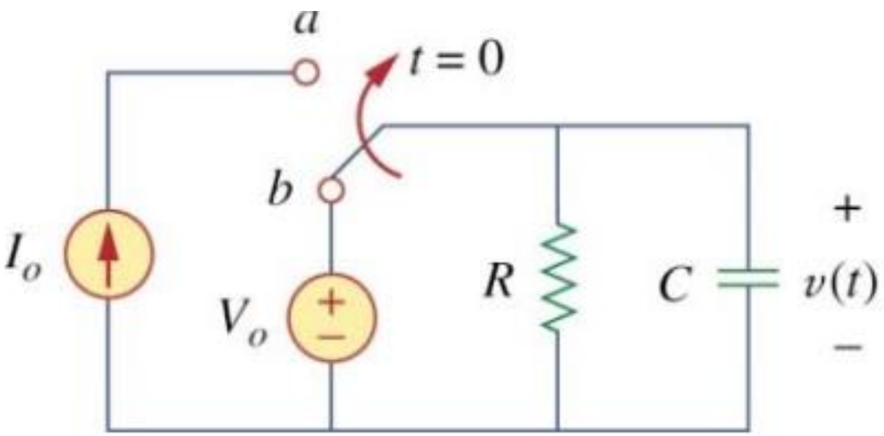
$$= \frac{\overset{8}{A}}{s} + \frac{\overset{-8}{B}}{s+2} + \frac{\overset{-16}{C}}{(s+2)^2}$$

$$B = \left[V_o(s) \cdot (s+2)^2\right]' \Big|_{s=-2} = \left(\frac{32}{s}\right)' = -\frac{32}{s^2} = -8$$

$$V(t) = (\underbrace{8}_{\text{S.S.}} - \underbrace{8 \cdot e^{-2t} - 16t \cdot e^{-2t}}_{\text{T.S.}})\, u(t)$$

Example 3

- The switch has been in position *b* for a long time. It is moved to position *a* at $t = 0$. Determine $v(t)$ for $t > 0$.

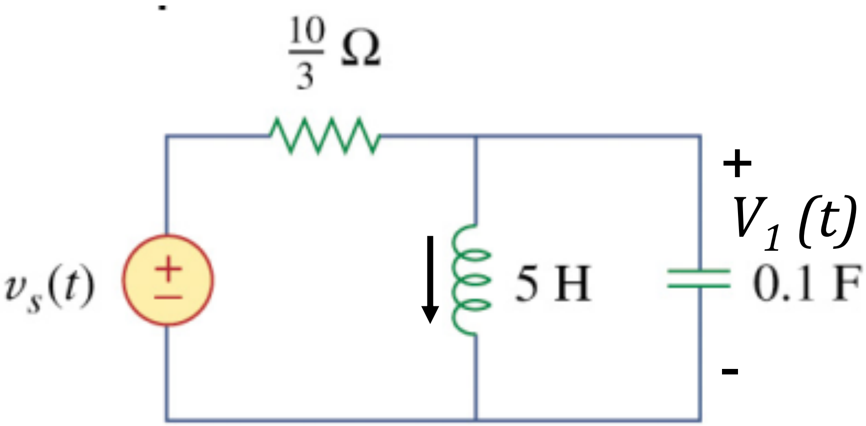


Example 4

- Find (1) the voltage across the capacitor

  (2) current through the inductor

  assuming that $v_s(t) = 10u(t)$ V, and assume that at $t = 0$, *-1 A* flows through the inductor and +5 V is across the capacitor.



Example 4

- Find (1) the voltage across the capacitor

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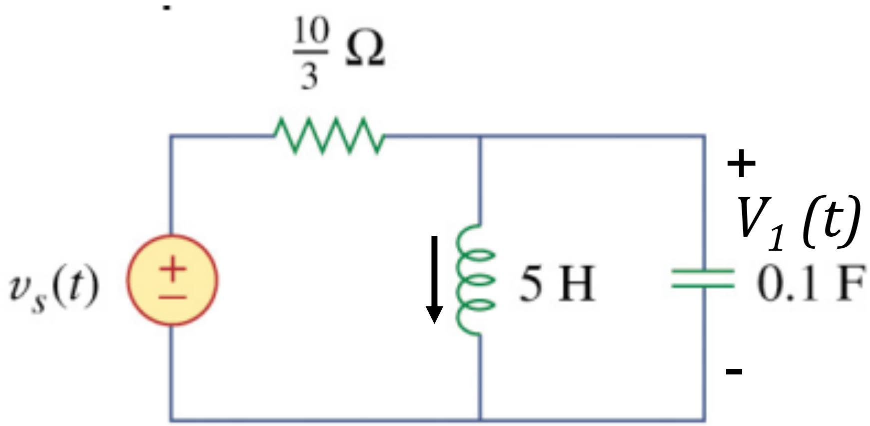
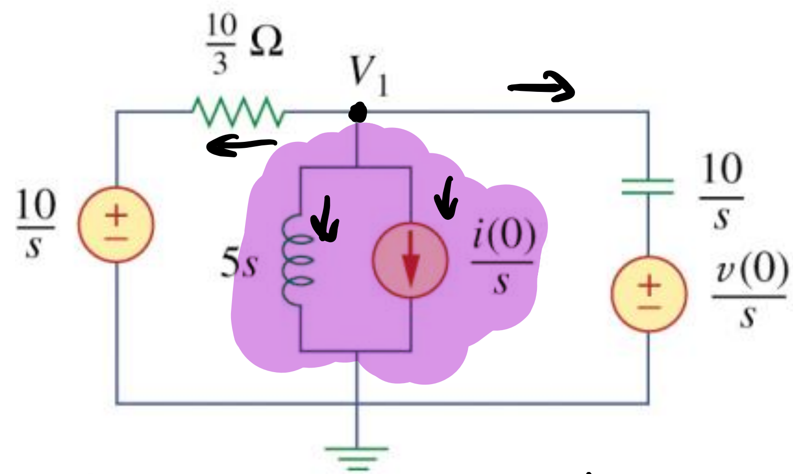
  assuming that $v_s(t) = 10u(t)$ V, and assume that at $t = 0$, *-1 A* flows through the inductor and +5 V is across the capacitor.





$$\frac{V_1 - \frac{10}{s}}{\frac{10}{3}} + \frac{V_1 - 0}{5s} + \frac{-1}{s} + \frac{V_1 - \frac{5}{s}}{10/s} = 0$$

$$V_1 = \frac{40+5s}{(s+1)(s+2)} = \frac{35}{s+1} - \frac{30}{s+2}$$

$$V_1(t) = (35 \cdot e^{-t} - 30 e^{-2t}) u(t) \quad V$$

$$i_L(s) = \frac{V_1}{5s} + \frac{-1}{s}$$

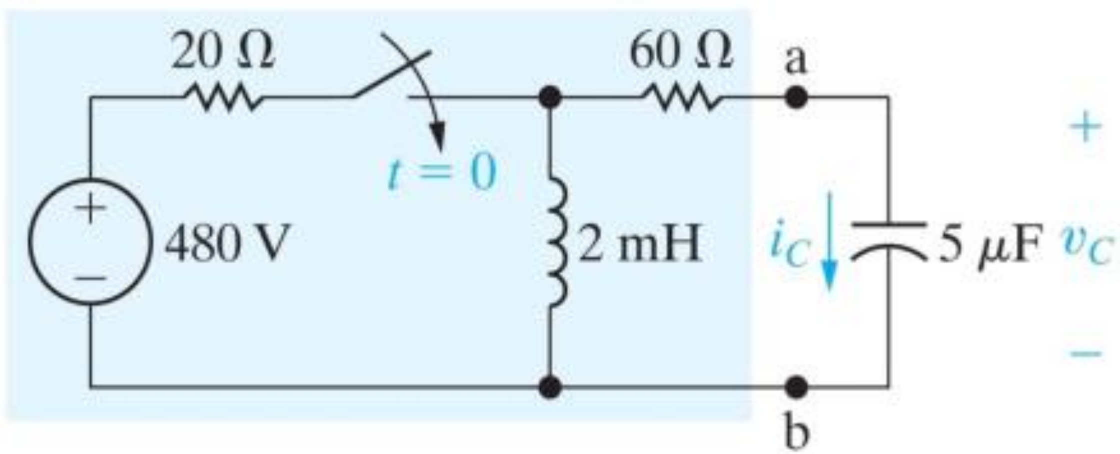
$$= \frac{40 + 5s}{5s(s+1)(s+2)} - \frac{1}{s}$$

$$= \frac{3}{s} - \frac{7}{s+1} + \frac{3}{s+2}$$

$$i_L(t) = (3 - 7 \cdot e^{-t} + 3 \cdot e^{-2t}) u(t) \quad A.$$

Example 5

- Use Thevenin's equivalent circuit w.r.t. terminals *a*-*b* to find current $i_C(t)$ for t>0.

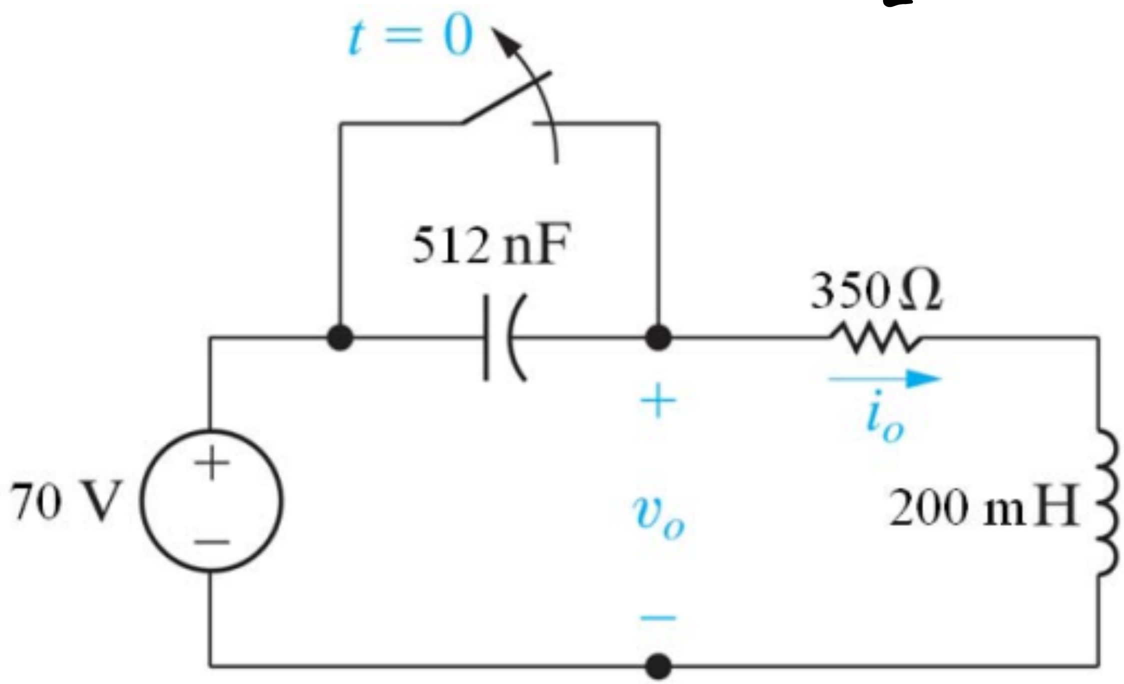


Example 6

- Find $v_o(t)$ for $t>0$

$V_c(0) = V_0 = 0\ V$

$i_L(0) = I_0 = 0.2A$



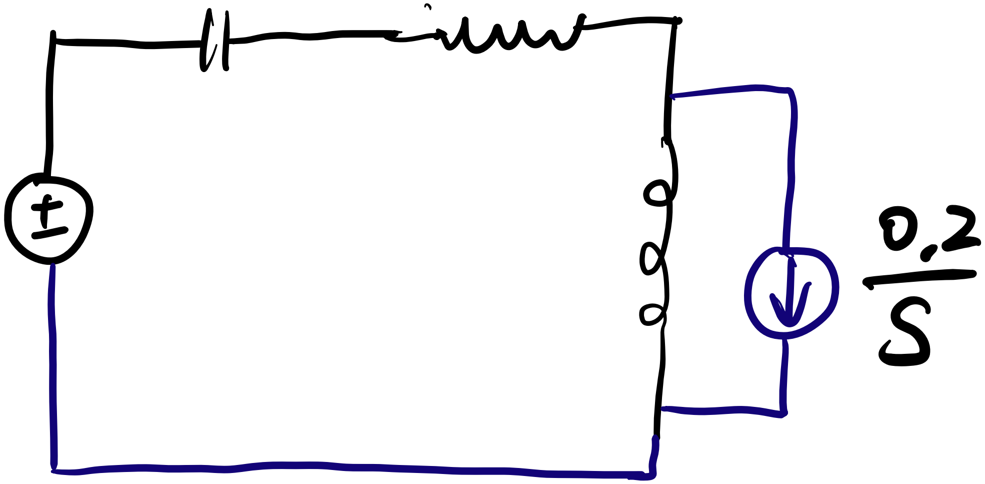
$\alpha = \frac{R}{2L}$

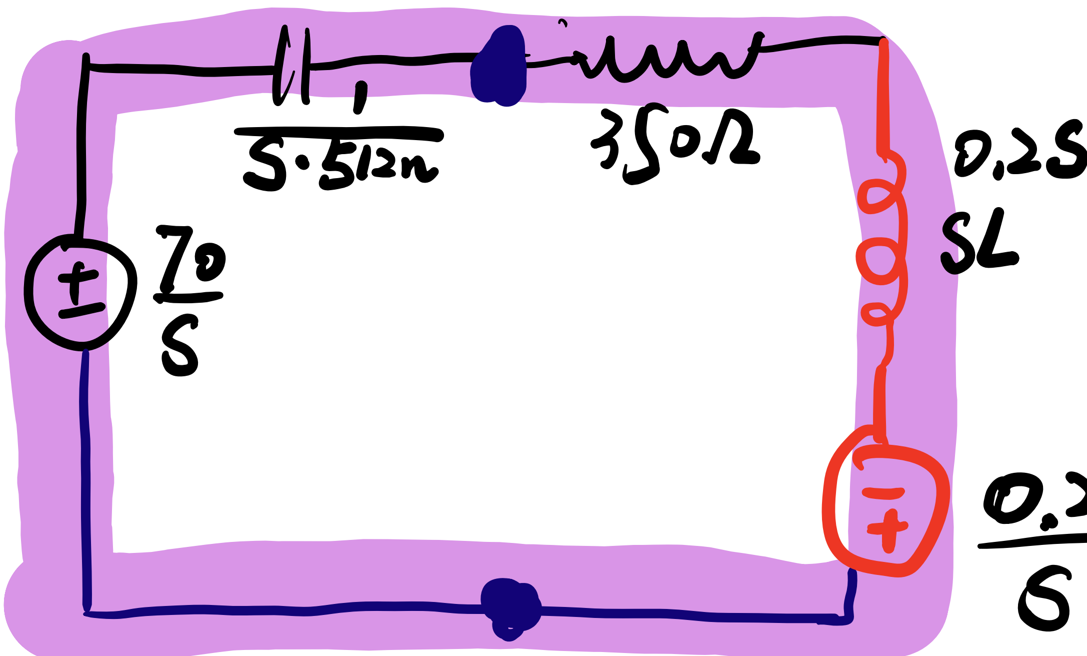
875

$\omega_0 = \frac{1}{\sqrt{LC}}$

$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$

$= -875 \pm 3000j$





$$\frac{0.2}{S} \times 0.2S = 0.04$$

$s-(\alpha+j\omega)$

$$V_0(s)=\frac{70s-268{,}125}{s^2+1750s+9{,}765{,}625}=\frac{K_1}{(s+875-j3000)}+\frac{K_2}{(s+875+j3000)}$$

$$K_1=\left.\frac{70s-268{,}125}{(s+875+j3000)}\right|_{s=-875+j3000}=\frac{70(-875+j3000)-268{,}125}{[(-875+j3000)+875+j3000]}=65.1\angle 57.48^\circ$$

$$K_2=\left.\frac{70s-268{,}125}{(s+875-j3000)}\right|_{s=-875-j3000}=\frac{70(-875-j3000)-268{,}125}{[(-875-j3000)+875+-j3000]}=65.1\angle-57.48^\circ$$

$$V_0(s)=\frac{65.1\angle 57.48^\circ}{(s+875-j3000)}+\frac{65.1\angle-57.48^\circ}{(s+875+j3000)}$$

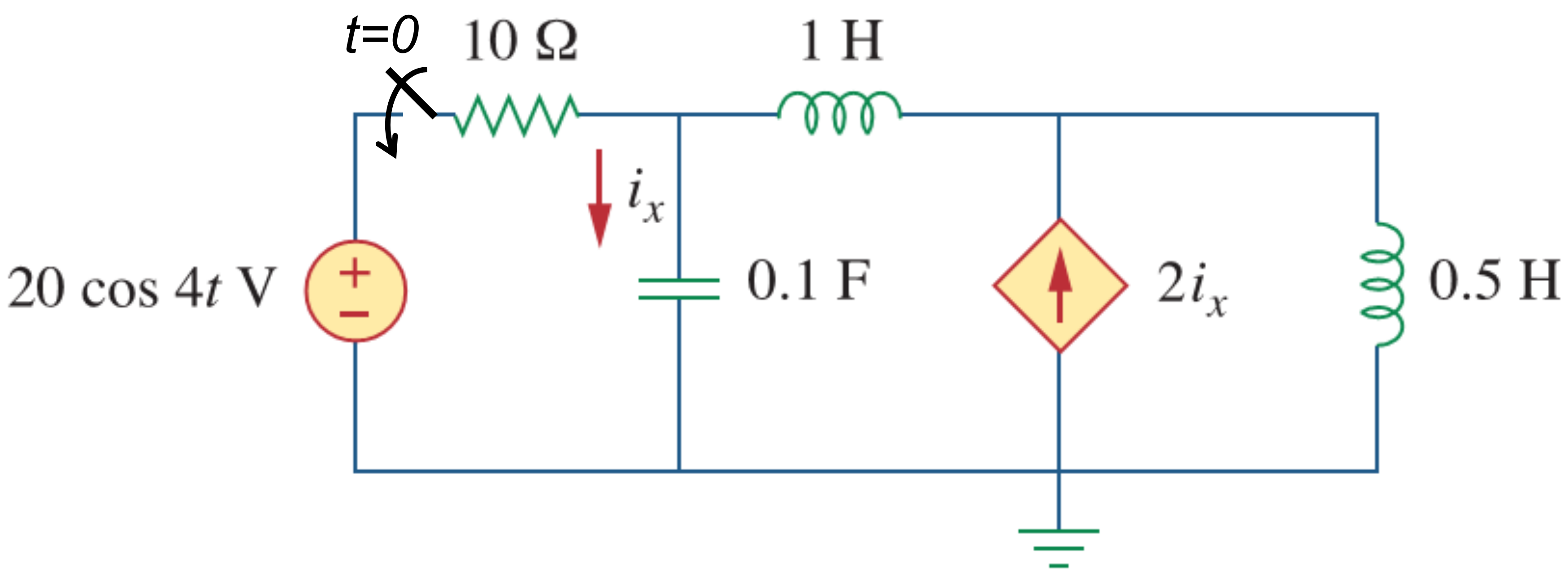
$$v_0(t)=2(65.1)e^{-875t}\cos(3000t+57.48^\circ)=\underbrace{130.2e^{-875t}\cos(3000t+57.48^\circ)u(t)\text{ V}}$$

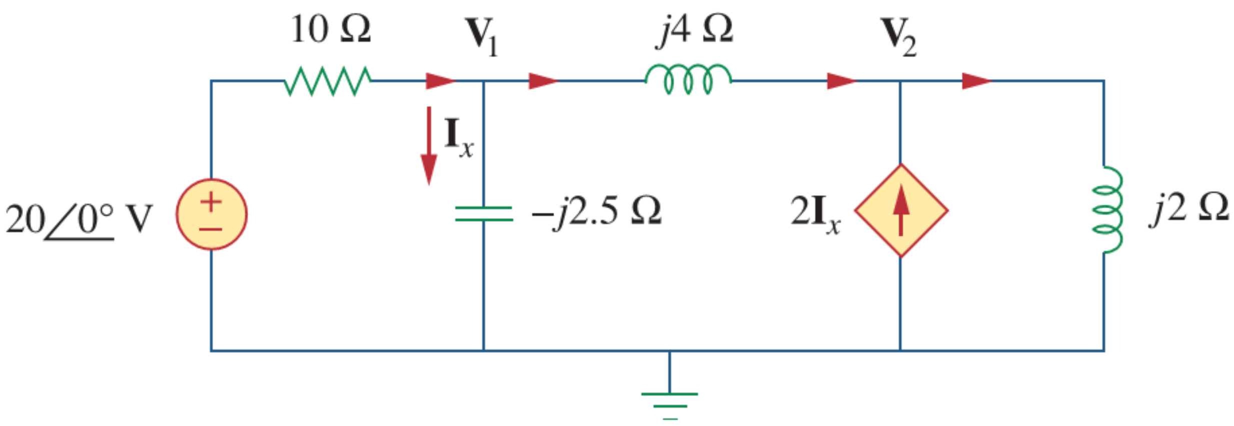
$|K_1|$ α ω φ_K

Example 7

- Example---Find i_x(s.s.) assuming no initial energy stored

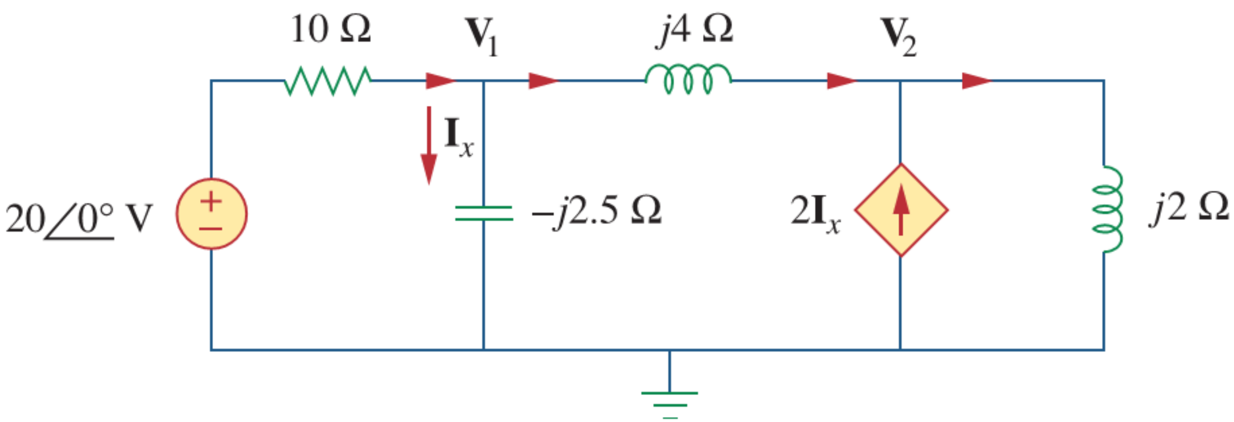
Using phasor method and Laplace transform method





$$\frac{20 - \mathbf{V}_1}{10} = \frac{\mathbf{V}_1}{-j2.5} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{j4}$$

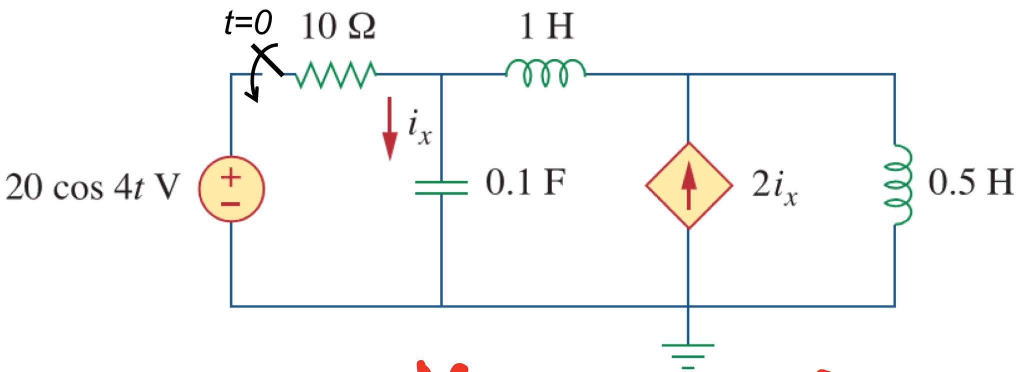
$$2\mathbf{I}_x + \frac{\mathbf{V}_1 - \mathbf{V}_2}{j4} = \frac{\mathbf{V}_2}{j2}$$

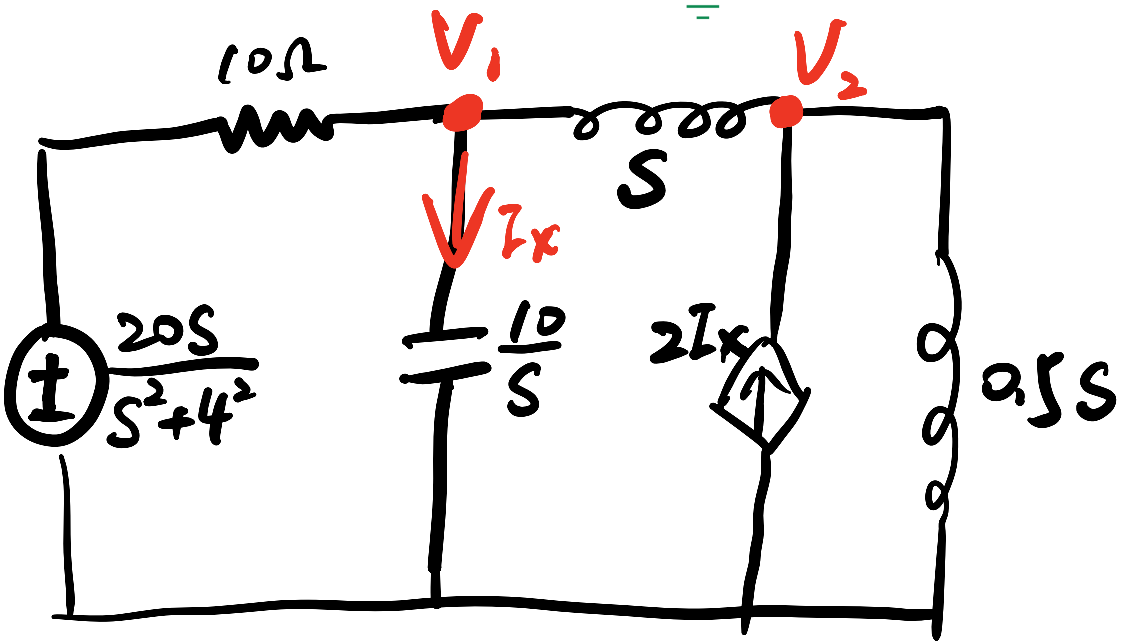


$$\frac{20 - \mathbf{V}_1}{10} = \frac{\mathbf{V}_1}{-j2.5} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{j4}$$

$$2\mathbf{I}_x + \frac{\mathbf{V}_1 - \mathbf{V}_2}{j4} = \frac{\mathbf{V}_2}{j2}$$

$$i_x = 7.59 \cos(4t + 108.4^\circ) \text{ A}$$





$$I_x = \frac{6S^3}{(S^2+4^2)(S^2+3S+20)}$$

$$I_x = \frac{k_1}{s-4j} + \frac{k_1^*}{s-(-4j)} + \frac{k_2}{s-(-1.5+4.2j)} + \frac{k_2^*}{s-(-1.5-4.2j)}$$

$\alpha_1 = 0 \quad \omega_1 = 4$

$\alpha_2 = -1.5 \quad \omega_2 = 4.2$

$k_1 = 3.79\angle 108.43^\circ$

$k_2 = 5.0\angle -146.2^\circ$

$$i_x(t) = \left[2|k_1| e^{\alpha_1 t}\cos(\omega_1 t + \varphi_{k_1}) + 2|k_2| e^{\alpha_2 t}\cos(\omega_2 t + \varphi_{k_2})\right]u(t)$$

$$= \left[7.58\cos(4t+108.43^\circ) + 10e^{-1.5t}\cos(4.2t-146.2^\circ)\right]u(t)$$