Homework 4

Due date: Nov. 14th, 2023

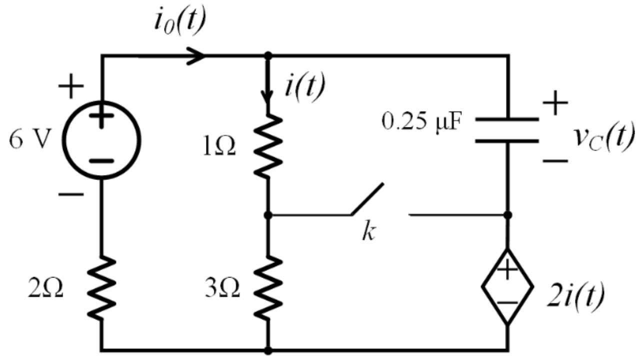
Turn in your hard-copy hand-writing homework to Room 324 #3 SIST

信息学院 3 号楼 324

Rules:

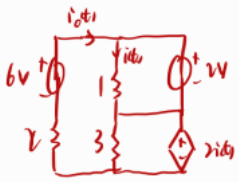
- Work on your own. Discussion is permissible, but extremely similar submissions will be judged as plagiarism.
- Please show all intermediate steps: a correct solution without an explanation will get zero credit.
- Please submit on time. No late submission will be accepted.
- Please prepare your submission in English only. No Chinese submission will be accepted.

1. For the circuit below, the switch has been open for a long time. The switch is closed at t = 0s immediately. Determine $i_0(t)$, $i(t)$ and $v_C(t)$ in the circuit for $t > 0$.



1. $t < 0$. $i = \frac{6}{1+2+3} = 1A$ $u_c(0^-) = (1+3)i - 2i = 2V$

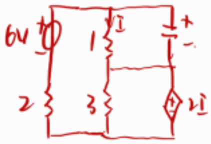
$t = 0^+$ $u_c(0^+) = u_c(0^-) = 2V$



$i(0^+) = \frac{2}{1} = 2A$

$2 + 2i(0^+) + 2i_0(0^+) = 6V \Rightarrow i_0(0^+) = 0A$

$t \to \infty$



$i(\infty) = i_0(\infty)$

$I \times 1 + 2I + 2I = 6V$ $I = 1.2A$

$i(\infty) = i_0(\infty) = 1.2A$

Req:

$$\begin{cases} U = 1 \cdot I'' \\ (I' - I'') \cdot 2 = 2I'' + U \end{cases} \Rightarrow \frac{U}{I'} = 0.4\Omega$$

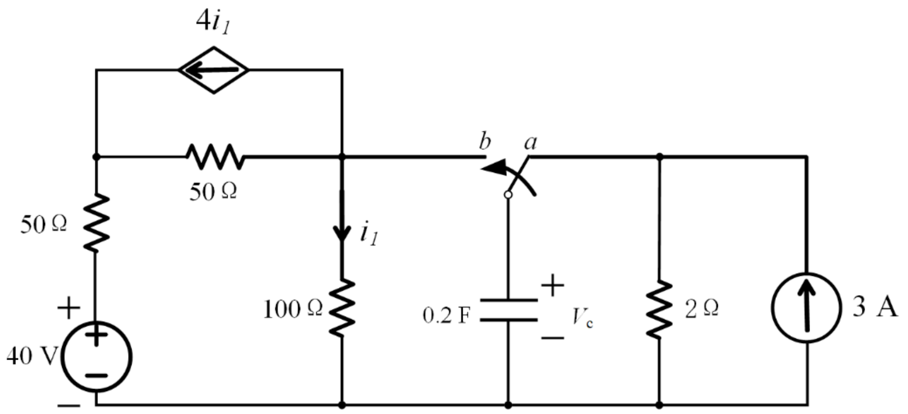
$Req = 0.4\Omega$

$\tau = Req\,C = 0.25\,\mu F \times 0.4\Omega = 10^{-7}s$

$i(t) = 1.2 + 0.8e^{-10^7 t}$ A $(t \geq 0)$

$i_0(t) = 1.2 - 1.2e^{-10^7 t}$ A $(t \geq 0)$

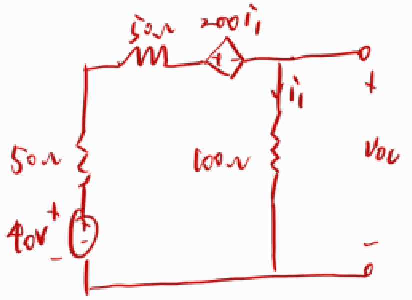
2. When t=0s, the switch changes from node a to node b immediately. Assume that the circuit reaches steady state before t=0. Determine $v_c(t)$, in the circuit for $t > 0$.



v1

$t<0$: $V_c(0^-) = 2\times 3 = 6V$

$t=0$: $V_c(0^+) = V_c(0^-) = 6V$

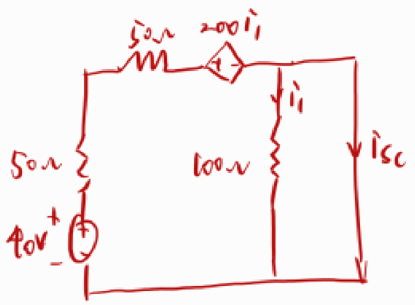


$V_{oc} = 100\Omega \times i_1 = 100 i_1$

$100 i_1 + (50+50) i_1 = 40 - 200 i_1$

$i_1 = 0.1A$

$V_{oc} = 100 \times 0.1A = 10V$



$i_1 = 0 \quad \therefore i_{sc} = \frac{40}{50+50} = 0.4A$

$R_{eq} = \frac{V_{oc}}{i_{sc}} = \frac{10}{0.4} = 25\Omega$

$\tau = R_{eq} \cdot C = 5s$

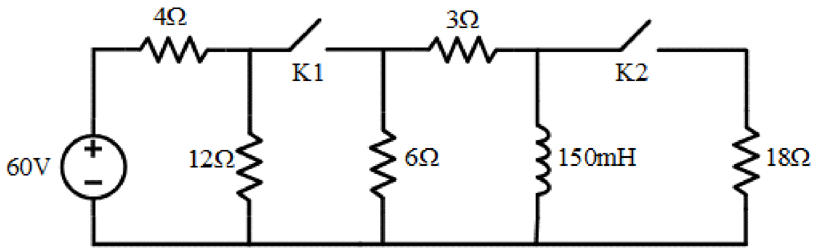
$V_c(\infty) = 10V$

$\therefore V_c(t) = 10 - 4e^{-0.2t}$ V $(t \geq 0)$

3.

a. assuming that K1 and K2 have been closed for a long time, K1 opens at t=0 and K2 opens at t=35ms, calculate the inductance currents for $t > 0$.

b. What is the ratio between energy consumed by the 18Ω resistor and energy stored in the inductor?



a. $I(0) = 6(V)$

当 K_1 断开:

$R_{eq} = (3+6) \,//\, 18 = \frac{9 \times 18}{9+18} = 6\Omega$ 2

$\tau = \frac{L}{R} = \frac{150}{6} \times 10^{-3} = 25ms$ 2

$\therefore i_L(t) = 6e^{-40t} A, \quad 0 \le t \le 35 ms$ 3

b. $i_L(35) = 6e^{-40 \cdot 35 \times 10^{-3}} = 6e^{-1.4} A$ 2

$R_{eq} = 3+6 = 9(\Omega), \quad \tau = \frac{L}{R} = \frac{150}{9} \times 10^{-3} = \frac{50}{3} \times 10^{-3} ms$ 2

$i_L(t) = 1.48e^{-60 \times (t-0.035)} A, \quad t \geq 35 ms$ 4

c. $W(0) = \frac{1}{2} L I(0) = \frac{1}{2} \times 150 \times 10^{-3} \times 36 = 2.7J$ 2

$i_L(t) = 6e^{-40t} A, \quad 0 \le t \le 35ms$

$i_{18\Omega}(t) = 6e^{-40t} \cdot \frac{9}{9+18} = 2e^{-40t}, \quad 0 \le t \le 35ms$ 2

$P_{18\Omega}(t) = i^2 R = 4 \cdot e^{-80t} \cdot 18 = 72e^{-80t}$ 2

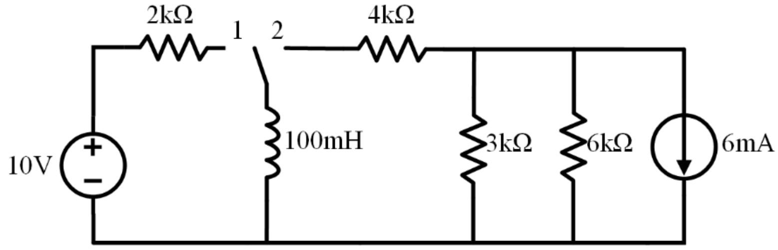
$W_{18\Omega} = \int_0^{35ms} P_{18\Omega}(t)\, dt = \frac{72}{-80} e^{-80t} \Big|_0^{35ms} \approx 0.85J$ 2

$\therefore \eta = \frac{W_{18\Omega}}{W(0)} = \frac{0.85}{2.7} \times 100 = 31.31\%$ 2

4.

a. The switch has been placed at terminal 2 for a long time. When $t=0$, the switch is placed at terminal 1. Calculate the inductance current at $t=0.6ms$.

b. If at t=0.6ms, the switch was placed back to terminal 2. Calculate the inductance current for $t > 0.6ms$.



a. $I(0) = \frac{6m \cdot 2k}{4k + 2k} = 2m(A)$, $R_{eq} = 2k\Omega$

$I(\infty) = -\frac{10}{2k} = -5m(A)$, $\tau = \frac{L}{R_{eq}} = \frac{100m}{2k} = 5\times10^{-6}s$

$i(t) = I(\infty) + [I(0) - I(\infty)]e^{-\frac{t}{\tau}} = -5 + 7e^{-2\times10^{4}t}$ mA, $t>0$

$i(0.6\times10^{-3}) = -5$ mA

b. $I(0) = -5mA$, $I(\infty) = 2mA$, $R_{eq} = 6k\Omega$, $\tau = \frac{L}{R_{eq}} = \frac{100m}{6k}$

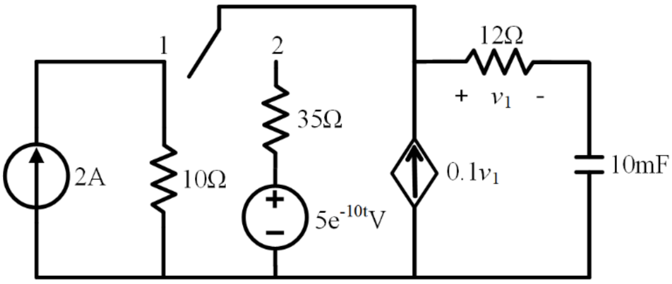
$i(t) = 2 - 7e^{-6\times10^{4}(t - 6\times10^{-4})}$ mA, $t > 0.6ms$

来自华为笔记

5.

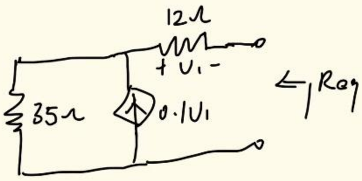
When t<0, the switch is set to terminal 1 and the circuit reaches steady state.

When t>0, the switch is set to terminal 2. Calculate the capacitance voltage for *t>0*.



when switch is set to 1. $U_c(0) = 20V$

when switch is set to 2.



$(i - 12i\times 0.1)35 + 12i = U$

$R_{eq} = \frac{U}{i} = 5\Omega$, $\tau = \frac{1}{20}$

$(12\cdot C\frac{dU_c}{dt} + U_c) + (C\frac{dU_c}{dt} - 1.2C\frac{dU_c}{dt})35 = 5e^{-10t}$

$5C\frac{dU_c}{dt} + U_c = 5e^{-10t}$

$\frac{1}{20}\frac{dU_c}{dt} + U_c = 5e^{-10t}$

设$U_c^* = Ae^{-10t}$，代入解得 $A = 10$

通解 $U_c' = Be^{-\frac{t}{\tau}} = Be^{-20t}$

$\therefore U_c = 10e^{-10t} + Be^{-20t}$. 代 $U_c(0) = 20$得

$U_c = 10e^{-10t} + 10e^{-20t}$, $t>0$