Quiz6

1 (35pt) True of False

- In the Posterior Belief Query with filtering, it is necessary to know future observations in order to calculate the current hidden variable.
- In enumeration inference with n Boolean variables, the space complexity is $O(2^n)$.
- Inference in a polytree can be non-linear if the inference is not conducted in an optimal traversal order.
- In variable elimination, every factor computed at each step through the Distributive Law represents the probability or joint probability associated with the corresponding variables.
- In variable elimination, the factor $f(a)$ is a value when its corresponding variable A is irrelevant to the query variable.
- Enumeration and Variable Elimination both belong to the Exact Inference.
- In variable elimination, different elimination orders result in different numbers of elimination steps and computational complexities.

FFTFTTF

Explanation:

1. See slide 6, page 64. Posterior Belief Query with filtering is only relevant to the current observation.

2. See slide 6, page 85. The space complexity is $O(n)$ because they only need n CPTs to store variables.

3. The theorem in slide 7, page 8 is not always true. Remember that size of the largest clique in the induced graph is an indicator for the complexity of variable elimination. (See slide 7, page 57). So if we do not select the optimal traversal order, the size of the largest clique may be very large, leading to a non-linear inference complexity.

4. See slide 7, page 22. The factor at each step is just a number. That's why we need the normalization term α.

5. Variable A and the query variable is independent to each other, which means A does not affect the posterior belief of the query variable. Therefore, $f(a)$ is a constant value.
6. See slide 6, page 91.
7. See slide 7, page 35. Different elimination orders result in different computation size at each step. But the number of elimination steps remains fixed because the variables need to be eliminated is determined by the graph.

2 (30pt) Queries in Inference

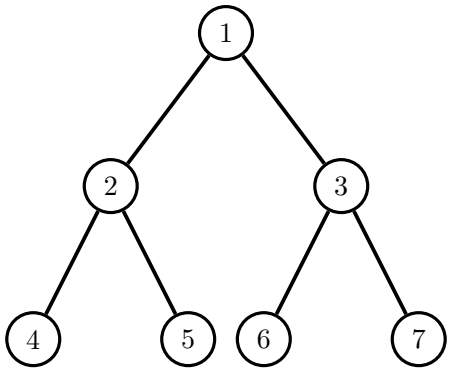
(20pt) (1) Write down the expressions for the following queries and describe their practical meaning briefly. **Note:** You will receive a few points if you only provide the expressions of the queries.

- Likelihood query:
- Maximum likelihood query:
- Posterior belief query:
- Maximum a posterior query:
- Most probable explanation query:

see slide 6, page 61

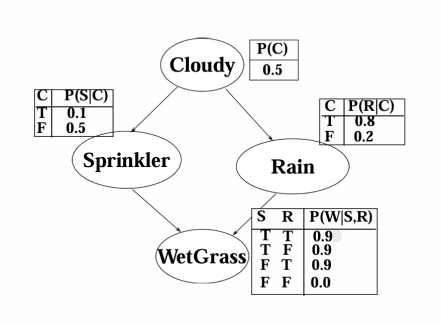
(10pt) (2) Consider a Bayesian Network with a binary tree structure. Assume there are m variables, each of which can take k possible values and the height of the tree is h. What is the computational complexity of inferring the query $P(X|E = e)$? by using **enumeration** and **variable elimination** methods? Write down the answers and give an explanation.

enumeration: $O(k^m)$
Reason: Enumeration needs to consider all possible values to all variables.
variable elimination: $O(k^2 \cdot m)$
Reason: The structure of the moralized graph of the Bayesian Network is shown below. If the elimination order start from the leaf nodes, the size of the largest clique is always 2, which means the maximum factor size during variable elimination is k^2. Since there are m variables, the complexity of variable elimination is $O(k^2 \cdot m)$.



3 (35pt) Enumeration & Variable Elimination

Consider the Water-Sprinkler example discussed in class, where each value in the CPT represents the probability of the current variable being True.

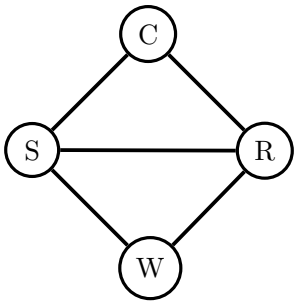


(1) (10pt) What is the probability that the grass is wet given that the weather is cloudy, namely P(W=True|C=True)? Use enumeration method to calculate.

$$
\begin{aligned}
&P(W=T|C=T) \\
&= \frac{P(W=T, C=T)}{P(C=T)} \\
&= \frac{\sum_{s,r} P(W=T, C=T, S=s, R=r)}{P(C=T)} \\
&= \frac{0.5*0.1*0.8*0.9 + 0.5*0.1*0.2*0.9 + 0.5*0.9*0.8*0.9}{0.5} \\
&= 0.738
\end{aligned}
$$

(2) (10pt) Draw the moralized graph of the given Bayesian Network.

Moralized graph:

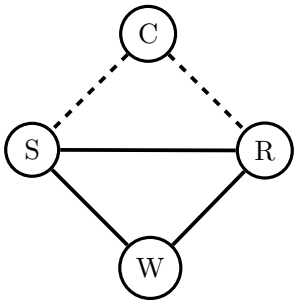


(3) (15pt) If we want to infer $P(W)$ and make the inference most efficient. How can we choose the elimination order? Write down the corresponding factor and draw the induced graph.

The most efficient elimination order is C, S, R or C, R, S

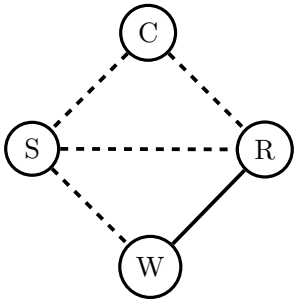
1. elimination C:

factor: $f_c(s, r)$



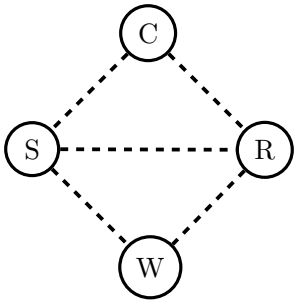
2. elimination S:

factor: $f_s(r, w)$



3. elimination R:

factor: $f_r(w)$



Tips: If we choose the elimination order starting from S or R, the induced graphs must include an edge between C and W, which increases the computational cost of inference.