CS150A Database

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Oct. 18, 2024

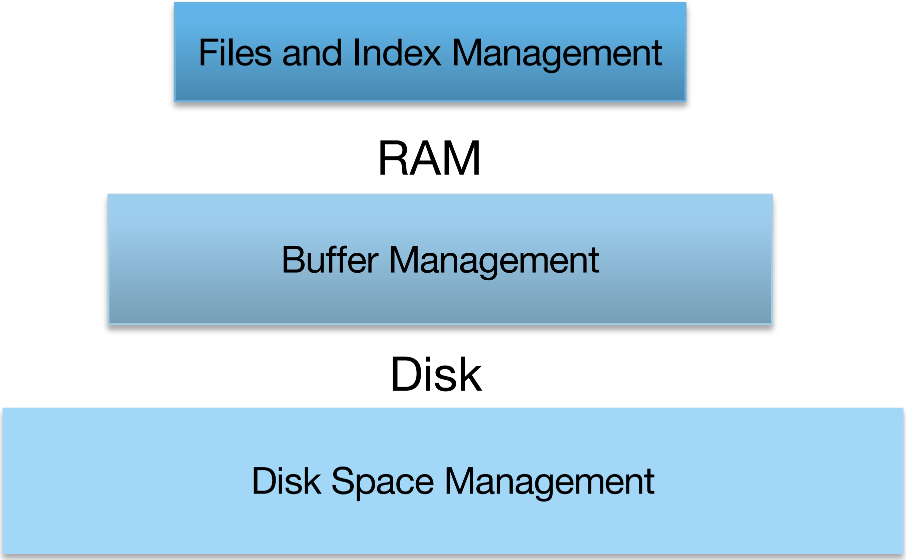
Today:
- Relational Algebra:
    - Basic Operators
    - Compound Operators

Readings:
- Database Management Systems (DBMS), Chapters 4.1-4.2

Acknowledgement: Joe Hellerstein@UCBerkeley's scourse notes

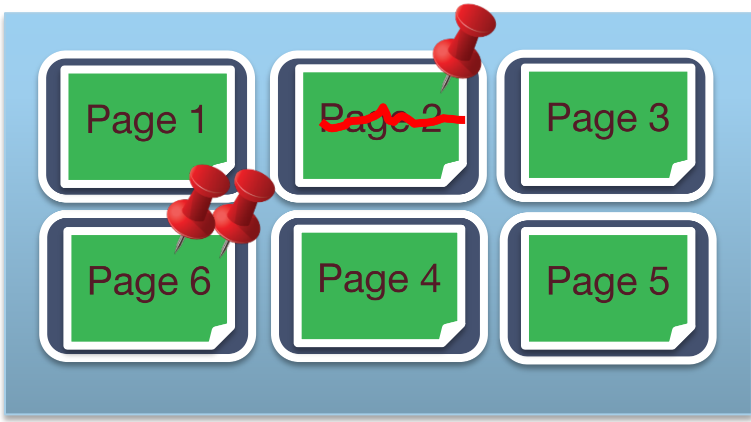
Review

- Buffer Manager provides a level of indirection
  - Connects RAM and Disk
  - Maps disk page Ids to RAM addresses



Review

- Buffer Manager provides a level of indirection
  - Connects RAM and Disk
  - Maps disk page Ids to RAM addresses
- Ensures that each requested page is “pinned” in RAM
  - To be (briefly) manipulated in-memory
  - And then unpinned by the caller!
- Attempts to minimize “cache misses”
  - By replacing pages unlikely to be referenced
  - By prefetching pages likely to be referenced



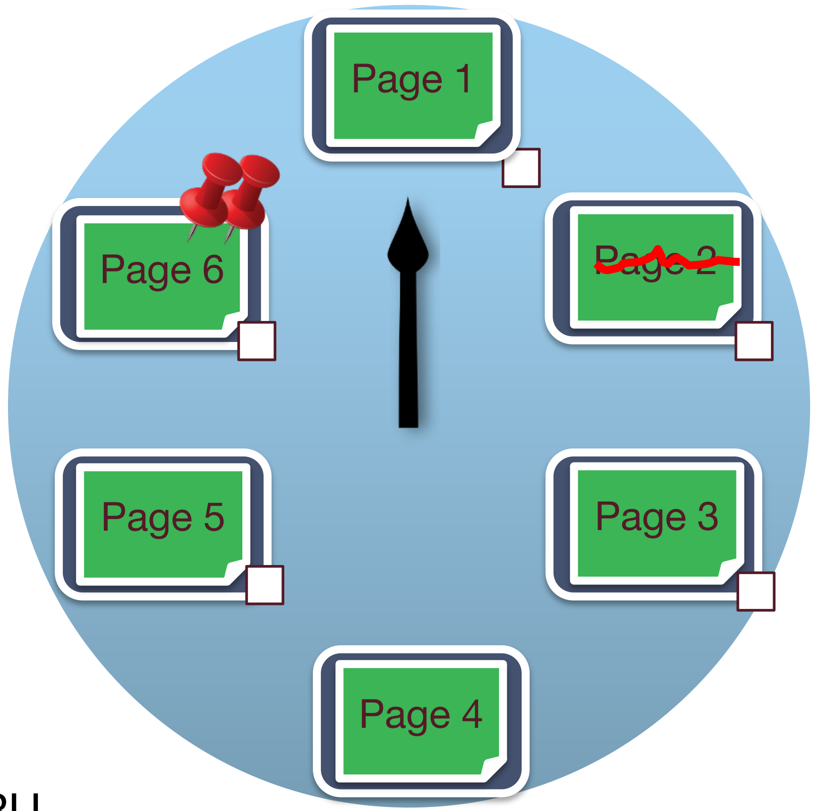
Review

- Pin Counts and Dirty Bits:
  - When do they get set/unset?
  - By what layer of the system?
- LRU, MRU and Clock
  - Be able to run each by hand
  - For Clock:
    - What pages are eligible for replacement
    - When is reference bit set/unset
    - What is the point of the reference bit?
- Sequential flooding
  - And how it behaves for LRU (Clock), MRU

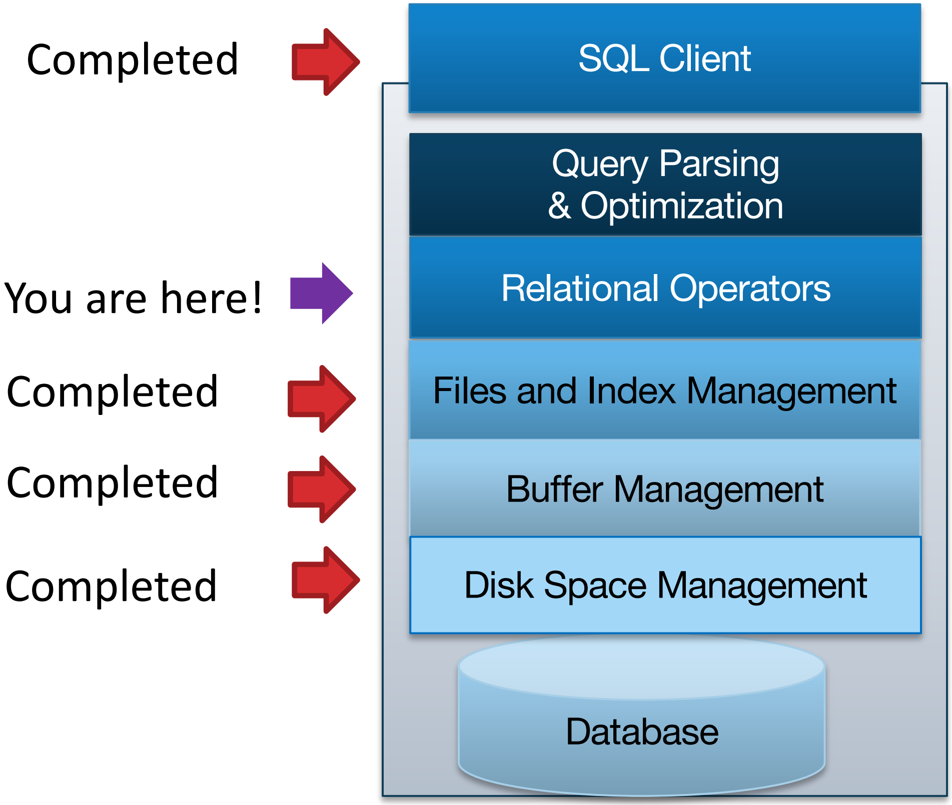
FrameId	PageId	Dirty?	Pin Count
1	1	N	0
2	2	Y	1
3	3	N	0
4	6	N	2
5	4	N	0
6	5	N	0

Review

- Pin Counts and Dirty Bits:
  - When do they get set/unset?
  - By what layer of the system?
- LRU, MRU and Clock
  - Be able to run each by hand
  - For Clock:
    - What pages are eligible for replacement
    - When is reference bit set/unset
    - What is the point of the reference bit?
- Sequential flooding
  - And how it behaves for LRU (Clock), MRU



Architecture of a DBMS: What we've learned



- Finished all the storage issues
- Getting to the parts of query processing

Today:

- *definitions* of the relational operators that we will execute to evaluate queries.
- *implementations* of these operators

An Overview of the Layer Above

SQL Query

SELECT S.name
FROM Reserves R, Sailors S
WHERE R.sid = S.sid
AND R.bid = 100
AND S.rating > 5

Query Parser
& Optimizer

Relational Algebra

$$\pi_{S.name}(\sigma_{bid=100 \wedge rating>5}(\text{Reserves} \bowtie_{R.sid=S.sid} \text{Sailors}))$$

The parser translate the query into a form that the optimizer is going to manipulate

The operator is going to work with operations that look like these, and these are in a language called the relational algebra

An Overview of the Layer Above

SQL Query

```
SELECT S.name
FROM Reserves R, Sailors S
WHERE R.sid = S.sid
AND R.bid = 100
AND S.rating > 5
```

Query Parser & Optimizer

Relational Algebra

$$\pi_{S.name}(\sigma_{bid=100 \wedge rating>5}(\text{Reserves} \bowtie_{R.sid=S.sid} \text{Sailors}))$$

Equivalent to...

(Logical) Query Plan:

$\pi_{S.name}$

↑

$\sigma_{R.bid=100 \wedge S.rating > 5}$

↑

$\bowtie_{R.sid=S.sid}$

Reserves Sailors

The relational algebra expression can be represented as a tree of operators

A logical query plan is a strategy for executing the query expressed in terms of these operators

These operators are a logical representation of what we want to do and in what order

An Overview of the Layer Above

SQL Query

```
SELECT S.name
FROM Reserves R, Sailors S
WHERE R.sid = S.sid
AND R.bid = 100
AND S.rating > 5
```

Query Parser & Optimizer

Relational Algebra

$\pi_{S.name}(\sigma_{bid=100 \land rating>5}($ Reserves $\bowtie_{R.sid=S.sid}$ Sailors$))$

Equivalent to…

(Logical) Query Plan:

$\pi_{S.name}$

$\sigma_{R.bid=100 \land S.rating > 5}$

$\bowtie_{R.sid=S.sid}$

Reserves   Sailors

But actually will produce…

Optimized (Physical) Query Plan:

$\pi_{S.name}$ — On-the-fly Project Iterator

$\sigma_{S.rating>5}$ — On-the-fly Select Iterator

$\bowtie_{R.sid=S.sid}$ — Indexed Nested Loop Join Iterator

$\sigma_{R.bid=100}$ Sailors — Heap Scan Iterator

Reserves

Operator Code

B+-Tree Indexed Scan Iterator

The optimizer itself is actually going to produce a physical optimized query plan.

Every operator in this tree is going to generate an actual algorithm to pump data up through these edges.

SQL vs Relational Algebra

SQL Query

SELECT S.name
FROM Reserves R, Sailors S
WHERE R.sid = S.sid
AND R.bid = 100
AND S.rating > 5

Query Parser & Optimizer

Relational Algebra

$$\pi_{S.name}(\sigma_{bid=100 \wedge rating>5}(\text{Reserves} \bowtie_{R.sid=S.sid} \text{Sailors}))$$

Equivalent to...

SQL

A **declarative** expression of the query result

Relational Algebra

Operational description of a computation.

Systems execute relational algebra query plan.

R.sid=S.sid

Reserves

Sailors

Operator Code

On-the-fly

SQL (Structured Query Language)

```
SELECT S.name
FROM Reserves R, Sailors S
WHERE R.sid = S.sid
AND R.bid = 100
AND S.rating > 5
```

- Key **System** Features: *Why do we like SQL*
  - Declarative:
    - Say ***what*** you want, not ***how*** to get it
    - Enables system to optimize the ***how***

- Foundation in formal Query Languages
  - **Relational Calculus**

History: Formal Relational QL's

- **Relational Calculus:** (Basis for SQL)
  - Describe the result of computation
  - Based on first order logic：logic in which the predicate of a sentence or statement can only refer to a single subject
  - Tuple Relational Calculus **(TRC)**
    - $\{S \mid S \in \text{Sailors}\ \exists R \in \text{Reserves}\ (R.sid = S.sid \land R.bid = 103)\}$

Are these equivalent?

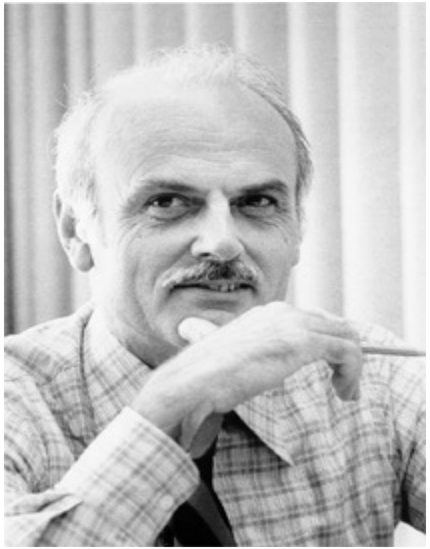
Can we go from one to the other?

- **Relational Algebra:**
  - Algebra on sets
  - It comes from a different branch of mathematics
  - It is an algebra, not a logic.
  - Operational description of transformations

Codd's Theorem

- Established equivalence in expressivity between :
  - Relational Calculus
  - Relational Algebra

- Why an important result?
  - Connects declarative representation of queries with operational description
  - Constructive: we can compile SQL into relational algebra



Edgar F. "Ted" Codd
(1923 - 2003)
Turing Award 1981

Relational Algebra Preliminaries

- Algebra of operators on relation instances
- Can cascade these operators in nested expressions like the ones here

$$\pi_{S.name}(\sigma_{R.bid=100 \wedge S.rating>5}(R \bowtie_{R.sid=S.sid} S))$$

- Closed: result is also a relation instance
  - Enables rich composition!
- Typed: input schema determines output
  - Can statically check whether queries are legal.

Relational Algebra and Sets

- Pure relational algebra has set semantics
  - No duplicate tuples in a relation instance
  - vs. SQL, which has multiset (bag) semantics

Relational Algebra Operators: Unary

- Unary Operators: on **single relation**
- **Projection** (π): Retains only desired columns (vertical)
- **Selection** (σ): Selects a subset of rows (horizontal)
- **Renaming** (ρ): Rename attributes and relations.

Relational Algebra Operators: Binary

- <u>Binary Operators</u>: on **pairs of relations**
- **Union** ($\cup$): Tuples in r1 or in r2.
- **Set-difference** ($-$): Tuples in r1, but not in r2.
- **Cross-product** ($\times$): Allows us to combine two relations.

Relational Algebra Operators: Compound

- Compound Operators: common "*macros*" for the above
- **Intersection** ($\cap$): Tuples in r1 and in r2.
- **Joins** ($\bowtie_{\theta}$, $\bowtie$): Combine relations that satisfy predicates

Projection (π)

- Corresponds to the SELECT list in SQL
- Schema determined by schema of attribute list
  - Names and types correspond to input attributes
- *Selects a subset of columns (vertical)*

$$\pi_{\underbrace{\text{sname,age}}_{\text{List of Attributes}}}(S2)$$

Relational *Instance* **S2**

sid	sname	rating	age
28	yuppy	9	35.0
31	lubber	8	55.5
44	guppy	5	35.0
58	rusty	10	35.0

sname	age
yuppy	35.0
lubber	55.5
guppy	35.0
rusty	35.0

Projection (π), cont.

- Set semantics → results in fewer rows
  - Real systems don't automatically remove duplicates
    - Why? (Semantics and Performance reasons)
  - *Selects a subset of columns (vertical)*

$$\pi_{age}(S2)$$

Relational *Instance* **S2**

sid	sname	rating	age
28	yuppy	9	35.0
31	lubber	8	55.5
44	guppy	5	35.0
58	rusty	10	35.0

→

Multiset

age
35.0
55.5
35.0
35.0

→

Set

age
35.0
55.5

Selection(σ)

- Corresponds to the WHERE clause in SQL
- Output schema same as input
- Duplicate Elimination? Not needed.
- *Selects a subset of rows (horizontal)*

$$\sigma_{\underbrace{rating>8}_{\text{Selection Condition (Boolean Expression)}}}(S2)$$

Relational *Instance* **S2**

sid	sname	rating	age
28	yuppy	9	35.0
~~31~~	~~lubber~~	~~8~~	~~55.5~~
~~44~~	~~guppy~~	~~5~~	~~35.0~~
58	rusty	10	35.0

sid	sname	rating	age
28	yuppy	9	35.0
58	rusty	10	35.0

Composing Select and Project

- Names of sailors with rating > 8: $\pi_{sname}(\sigma_{rating>8}(S2))$

sid	sname	rating	age
28	yuppy	9	35.0
31	lubber	8	55.5
44	guppy	5	35.0
58	rusty	10	35.0

$\sigma_{rating>8}$

sid	sname	rating	age
28	yuppy	9	35.0
58	rusty	10	35.0

π_{sname}

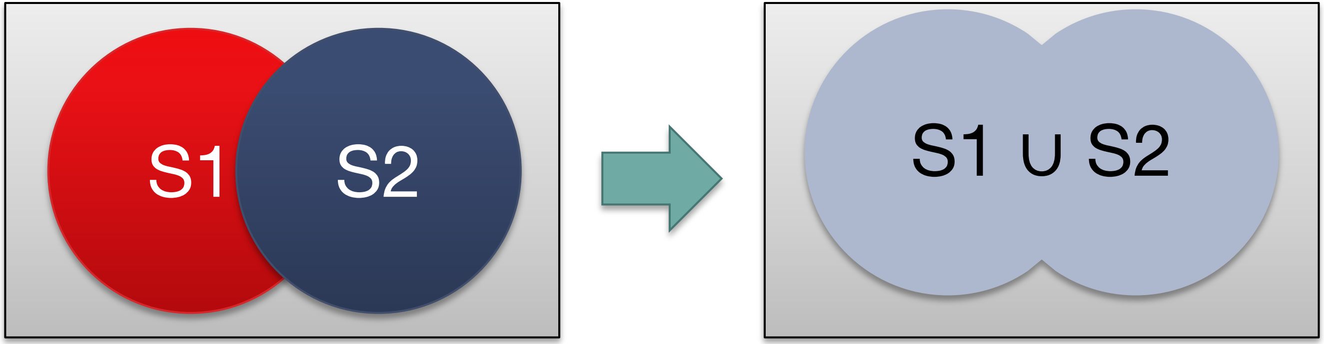
sname
yuppy
rusty

- What about: $\sigma_{rating>8}(\pi_{sname}(S2))$
    - *Invalid types. Input to $\sigma_{rating>8}$ does not contain rating.*

Union (∪)

- Two input relations, must be *compatible*:
  - Same number of fields
  - Fields in corresponding positions have same **type**
- SQL Expression: UNION

S1 ∪ S2



Union (∪) VS Union ALL

- Duplicate elimination in practice?
  - SQL's UNION vs UNION ALL

S1 ∪ S2

sid	sname	rating	age
22	dustin	7	45
28	yuppy	9	35.0
31	lubber	8	55.5
44	guppy	5	35.0
58	rusty	10	35.0

Relational *Instance* **S1**

sid	sname	rating	age
22	dustin	7	45.0
31	lubber	8	55.5
58	rusty	10	35.0

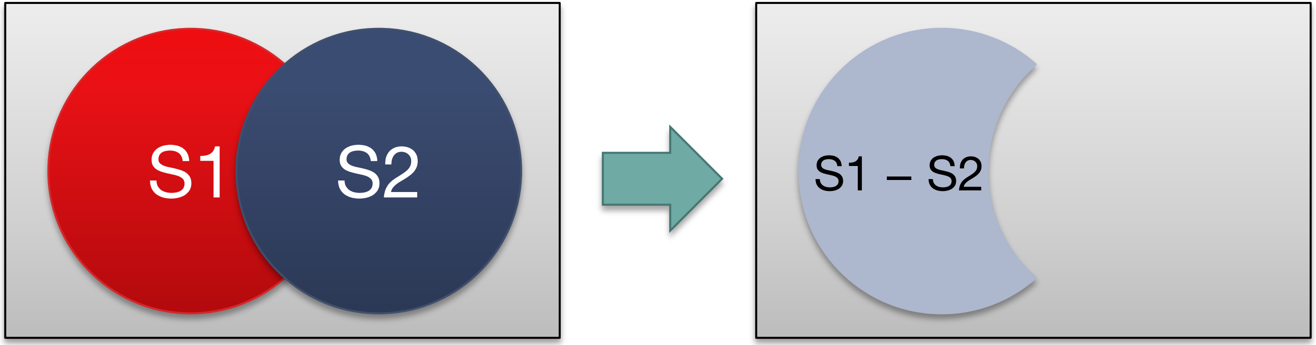
Relational *Instance* **S2**

sid	sname	rating	age
28	yuppy	9	35.0
31	lubber	8	55.5
44	guppy	5	35.0
58	rusty	10	35.0

Set Difference (–)

- Same as with union, both input relations must be *compatible*.
- SQL Expression: EXCEPT

S1 – S2



Set Difference (–), cont.

- Duplicate elimination?
  - Not required
- EXCEPT vs EXCEPT ALL

Relational *Instance* **S1**

sid	sname	rating	age
22	dustin	7	45.0
31	lubber	8	55.5
58	rusty	10	35.0

Relational *Instance* **S2**

sid	sname	rating	age
28	yuppy	9	35.0
31	lubber	8	55.5
44	guppy	5	35.0
58	rusty	10	35.0

S1 – S2

sid	sname	rating	age
22	dustin	7	45

S2 – S1

sid	sname	rating	age
28	yuppy	9	35.0
44	guppy	5	35.0

Cross-Product (×)

- **R1 × S1:** *Each row of* ***R1*** *paired with each row of* ***S1***

R1:

sid	bid	day
22	101	10/10/96
58	103	11/12/96

×

S1:

sid	sname	rating	age
22	dustin	7	45.0
31	lubber	8	55.5
58	rusty	10	35.0

=

sid	bid	day	sid	sname	rating	age
22	101	10/10/96	22	dustin	7	45.0
22	101	10/10/96	31	lubber	8	55.5
22	101	10/10/96	58	rusty	10	35.0
58	103	11/12/96	22	dustin	7	45.0
58	103	11/12/96	31	lubber	8	55.5
58	103	11/12/96	58	rusty	10	35.0

- ***How many rows in result? |R1|*|R2|***
- ***Schema compatability? Not needed.***
- ***Duplicates? None generated.***

Renaming (ρ)

- *Renames relations and their attributes:*
- Note that relational algebra doesn't require names.
  - We could just use positional arguments.

$$\rho(\underbrace{\text{Temp1}}_{\text{Output Relation Name}}(\underbrace{1 \rightarrow \text{sid1},\ 4 \rightarrow \text{sid2}}_{\text{Renaming List: position} \rightarrow \text{New Name}}),\ \underbrace{R1 \times S1}_{\text{Input Relation}})$$

R1 × S1

sid	bid	day	sid	sname	rating	age
22	101	10/10/96	22	dustin	7	45.0
22	101	10/10/96	31	lubber	8	55.5
22	101	10/10/96	58	rusty	10	35.0
58	103	11/12/96	22	dustin	7	45.0
58	103	11/12/96	31	lubber	8	55.5
58	103	11/12/96	58	rusty	10	35.0

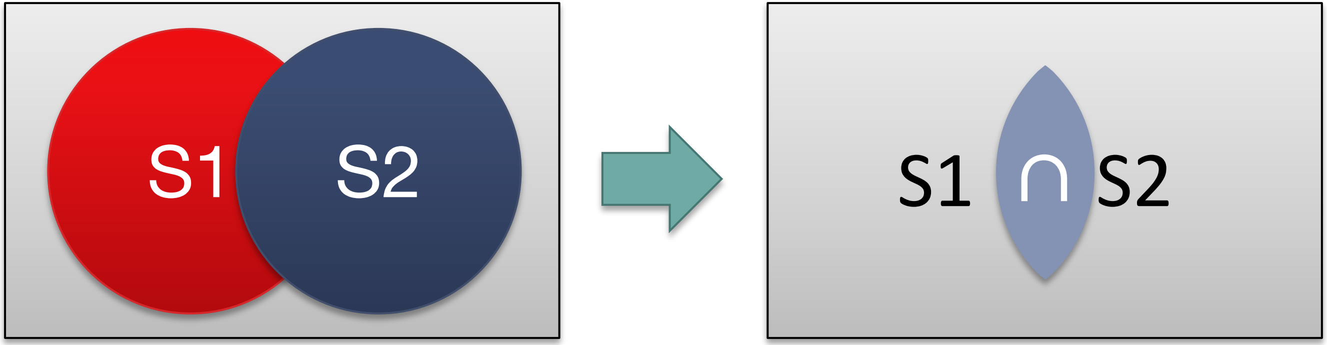
Temp1

sid1	bid	day	sid2	sname	rating	age
22	101	10/10/96	22	dustin	7	45.0
22	101	10/10/96	31	lubber	8	55.5
22	101	10/10/96	58	rusty	10	35.0
58	103	11/12/96	22	dustin	7	45.0
58	103	11/12/96	31	lubber	8	55.5
58	103	11/12/96	58	rusty	10	35.0

Compound Operator: Intersection

- Same as with union, both input relations must be *compatible*.
- SQL Expression: INTERSECT

S1 ∩ S2



Intersection (∩)

- Equivalent to:
  S1 — (S1 — S2)

S1 ∩ S2

sid	sname	rating	age
31	lubber	8	55.5
58	rusty	10	35.0

Relational *Instance* **S1**

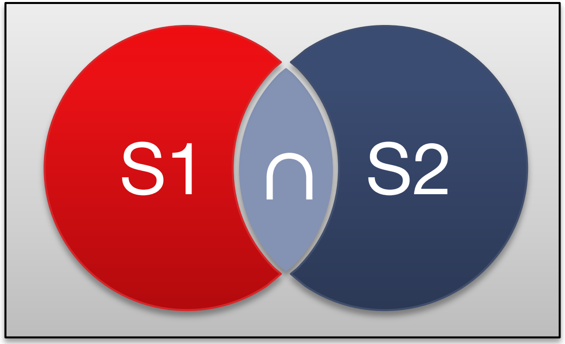
sid	sname	rating	age
22	dustin	7	45.0
31	lubber	8	55.5
58	rusty	10	35.0

Relational *Instance* **S2**

sid	sname	rating	age
28	yuppy	9	35.0
31	lubber	8	55.5
44	guppy	5	35.0
58	rusty	10	35.0

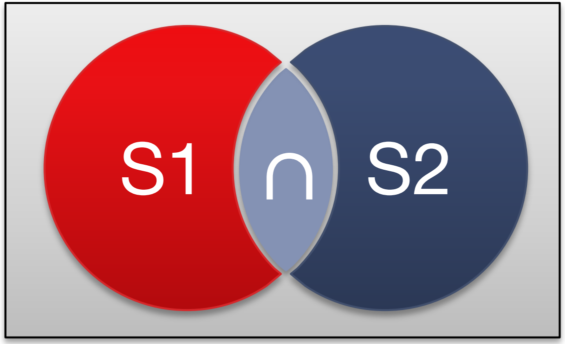
Intersection (∩), Pt 2

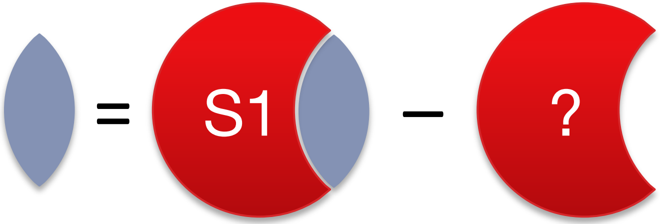
- $S1 \cap S2 = ?$



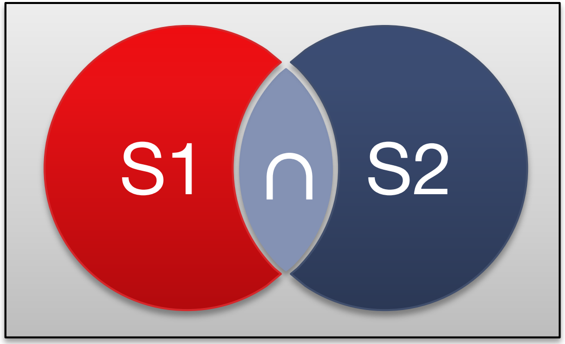
Intersection (∩), Pt 3

- $S1 \cap S2 = S1 - ?$

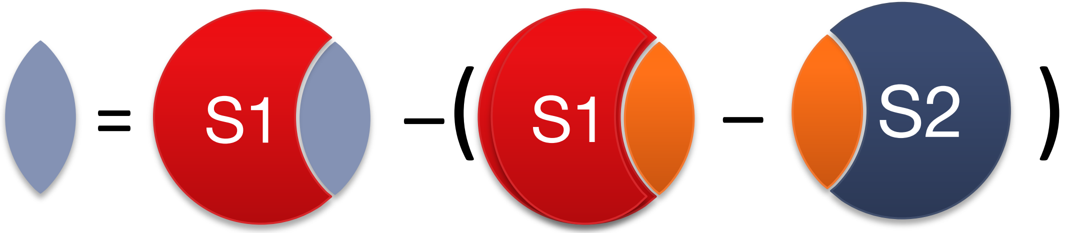




Intersection (∩), Pt 4



- $S1 \cap S2 = S1 - (S1 - S2)$



- Is intersection monotonic?

$R_1 \subseteq R_2 \quad \Rightarrow \quad S \cap R_1 \subseteq S \cap R_2$

Compound Operator: Join

- Joins are compound operators (like intersection):
  - Generally, σ_θ(R × S)
- Hierarchy of common kinds:
  - **Theta Join** ($\bowtie_\theta$): join on logical expression θ
    - **Equi-Join:** theta join with theta being a conjunction of equalities
      - **Natural Join** ($\bowtie$): equi-join on all matching column names

Note: we will need to learn a good join algorithm.

Avoid cross-product if we can!!

Theta Join ($\bowtie_\theta$) Example

- **R1** $\bowtie_{\text{sid=sid}}$ **S1**

R1:

sid	bid	day
22	101	10/10/96
58	103	11/12/96

$\bowtie_{\text{sid=sid}}$

S1:

sid	sname	rating	age
22	dustin	7	45.0
31	lubber	8	55.5
58	rusty	10	35.0

=

sid	bid	day	sid	sname	rating	age
22	101	10/10/96	22	dustin	7	45.0
58	103	11/12/96	58	rusty	10	35.0

- Note that output needs a rename operator!

Another Theta Join ($\bowtie_\theta$) Example

- $\mathbf{R} \bowtie_\theta \mathbf{S} = \sigma_\theta(\mathbf{R} \times \mathbf{S})$
- **Example:** *More senior sailors for each sailor.*
- S1 $\bowtie_{f4 < f8}$ S1

S1:

f1	f2	f3	f4
22	dustin	7	45.0
31	lubber	8	55.5
58	rusty	10	35.0

S1				S1			
f1	f2	f3	f4	f5	f6	f7	f8
22	dustin	7	45.0	22	dustin	7	45.0
22	dustin	7	45.0	31	lubber	8	55.5
22	dustin	7	45.0	58	rusty	10	35.0
31	lubber	8	55.5	22	dustin	7	45.0
31	lubber	8	55.5	31	lubber	8	55.5
31	lubber	8	55.5	58	rusty	10	35.0
58	rusty	10	35.0	22	dustin	7	45.0
58	rusty	10	35.0	31	lubber	8	55.5
58	rusty	10	35.0	58	rusty	10	35.0

Another Theta Join ($\bowtie\theta$), Pt 2

- **R $\bowtie_\theta$ S = σ_θ(R × S)**
- **Example:** *More senior sailors for each sailor.*
- S1 $\bowtie_{\text{age} < \text{age2}}$ S1

S1:

<u>f1</u>	f2	f3	f4
22	dustin	7	45.0
31	lubber	8	55.5
58	rusty	10	35.0

S1				S1			
f1	f2	f3	f4	f5	f6	f7	f8
~~22~~	~~dustin~~	~~7~~	~~45.0~~	~~22~~	~~dustin~~	~~7~~	~~45.0~~
22	dustin	7	45.0	31	lubber	8	55.5
~~22~~	~~dustin~~	~~7~~	~~45.0~~	~~58~~	~~rusty~~	~~10~~	~~35.0~~
~~31~~	~~lubber~~	~~8~~	~~55.5~~	~~22~~	~~dustin~~	~~7~~	~~45.0~~
~~31~~	~~lubber~~	~~8~~	~~55.5~~	~~31~~	~~lubber~~	~~8~~	~~55.5~~
~~31~~	~~lubber~~	~~8~~	~~55.5~~	~~58~~	~~rusty~~	~~10~~	~~35.0~~
58	rusty	10	35.0	22	dustin	7	45.0
58	rusty	10	35.0	31	lubber	8	55.5
~~58~~	~~rusty~~	~~10~~	~~35.0~~	~~58~~	~~rusty~~	~~10~~	~~35.0~~

Another Theta Join ($\bowtie_\theta$), Pt 3

- **R $\bowtie_\theta$ S = σ_θ(R × S)**
- **Example:** *More senior sailors for each sailor.*
- S1 $\bowtie_{f4 < f8}$ S1

S1:

sid	sname	rating	age
22	dustin	7	45.0
31	lubber	8	55.5
58	rusty	10	35.0

S1				S1			
sid	sname	rating	age	sid	sname	rating	age2
22	dustin	7	45.0	31	lubber	8	55.5
58	rusty	10	35.0	22	dustin	7	45.0
58	rusty	10	35.0	31	lubber	8	55.5

- Result *schema* same as that of cross-product.
- Special Case:
  - **Equi-Join:** *theta join with AND of = predicates*
    - *Special special case **Natural Join** …*

Natural Join (⋈)

- Special case of equi-join in which equalities are specified for all matching fields and duplicate fields are projected away

$$\mathbf{R \bowtie S} = \pi_{\text{unique fld.}}\ \sigma_{\text{eq. matching fld.}}(\mathbf{R} \times \mathbf{S})$$

- Compute R × S
- Select rows where fields appearing in both relations have equal values
- Project onto the set of all unique fields.

Natural Join (⋈) Pt 2

- $\mathbf{R} \bowtie \mathbf{S} = \pi_{\text{unique fld.}}\, \sigma_{\text{eq. matching fld.}}(\mathbf{R} \times \mathbf{S})$

R1 ⋈ S1

sid	bid	day	sid	sname	rating	age
22	101	10/10/96	22	dustin	7	45.0
~~22~~	~~101~~	~~10/10/96~~	~~31~~	~~lubber~~	~~8~~	~~55.5~~
~~22~~	~~101~~	~~10/10/96~~	~~58~~	~~rusty~~	~~10~~	~~35.0~~
~~58~~	~~103~~	~~11/12/96~~	~~22~~	~~dustin~~	~~7~~	~~45.0~~
~~58~~	~~103~~	~~11/12/96~~	~~31~~	~~lubber~~	~~8~~	~~55.5~~
58	103	11/12/96	58	rusty	10	35.0

R1:

sid	bid	day
22	101	10/10/96
58	103	11/12/96

S1:

sid	sname	rating	age
22	dustin	7	45.0
31	lubber	8	55.5
58	rusty	10	35.0

Natural Join (⋈), Pt 3

- $\mathbf{R} \bowtie \mathbf{S} = \pi_{\text{unique fld.}}\, \sigma_{\text{eq. matching fld.}}(\mathbf{R} \times \mathbf{S})$

R1:

sid	bid	day
22	101	10/10/96
58	103	11/12/96

sid	sname	rating	age
22	dustin	7	45.0
31	lubber	8	55.5
58	rusty	10	35.0

R1 ⋈ S1

sid	bid	day	sname	rating	age
22	101	10/10/96	dustin	7	45.0
58	103	11/12/96	rusty	10	35.0

- Commonly used for foreign key joins (as above).

Extended Relational Algebra

- Group By / Aggregation Operator (γ):
  - $\gamma_{\text{age, AVG(rating)}}$(Sailors)
  - With selection (HAVING clause):
    - $\gamma_{\text{age, AVG(rating), COUNT(*)>2}}$(Sailors)

Summary

- Relational Algebra: a small set of operators mapping relations to relations
  - Operational, in the sense that you specify the explicit order of operations
  - A closed set of operators! Mix and match.
- Basic ops include: σ, π, $\times$, $\cup$, $-$
- Important compound ops: $\cap$, $\bowtie$