Ch.2 *Linear Time-Invariant Systems*

Lecturer: Yijie Mao

Part IV *Differential or Difference Equations*

Outline

- Differential Equations
- Difference Equations
- Block Diagram Representations

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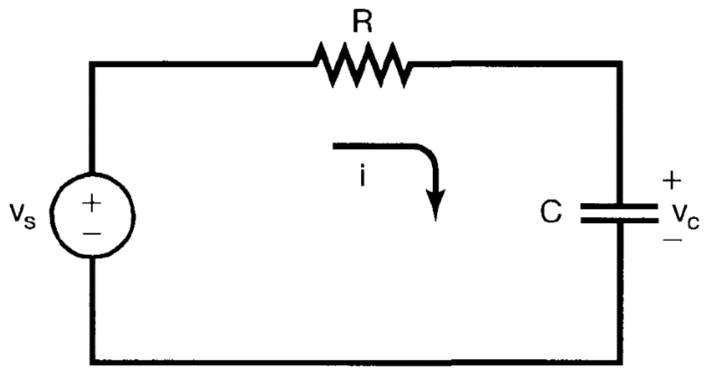
Differential Equation

- First order system



$$\frac{dv_c(t)}{dt} + \frac{1}{RC} v_c(t) = \frac{1}{RC} v_s(t).$$

- In general

$$\frac{dy(t)}{dt} + ay(t) = bx(t)$$

- A differential equation describes a relationship between the input and the output

Differential Equation

- General case: Nth-order linear constant-coefficient differential equation

$$\sum_{k=0}^{N} a_k \frac{d^k y(t)}{dt^k} = \sum_{k=0}^{M} b_k \frac{d^k x(t)}{dt^k}$$

- Particular case $N = 0$, we have

$$a_0 y(t) = \sum_{k=0}^{M} b_k \frac{d^k x(t)}{dt^k}$$

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Difference Equation

- General case: Nth-order linear constant-coefficient difference equation

$$\sum_{k=0}^{N} a_k y[n-k] = \sum_{k=0}^{M} b_k x[n-k]$$

- Recursive solution:

$$y[n] = \frac{1}{a_0}\left\{\sum_{k=0}^{M} b_k x[n-k] - \sum_{k=1}^{N} a_k y[n-k]\right\}$$

Difference Equation

- Recursive solution:

$$y[n] = \frac{1}{a_0}\left\{\sum_{k=0}^{M} b_k x[n-k] - \sum_{k=1}^{N} a_k y[n-k]\right\}$$

  - Particular case *N*=0

$$y[n] = \frac{1}{a_0}\sum_{k=0}^{M} b_k x[n-k]$$ *Non-recursive equation*

$$h[n] = \frac{1}{a_0}\sum_{k=0}^{M} b_k \delta[n-k]$$ *Finite impulse response (FIR) system*

Outline

- Differential Equations
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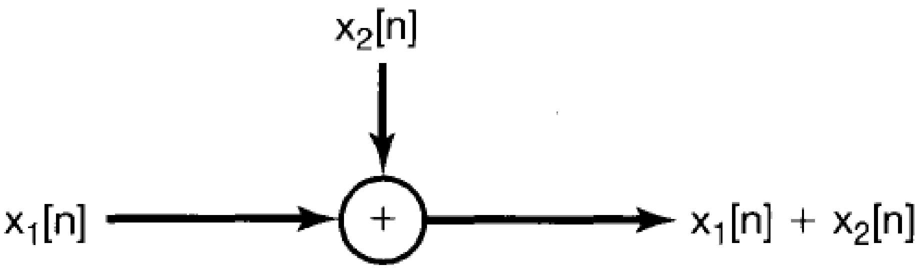
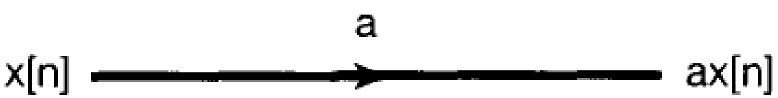
Block Diagram Representations

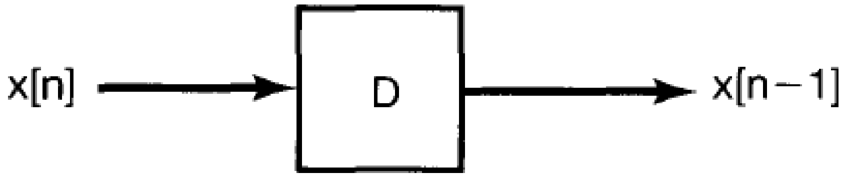
- Basic elements: discrete-time

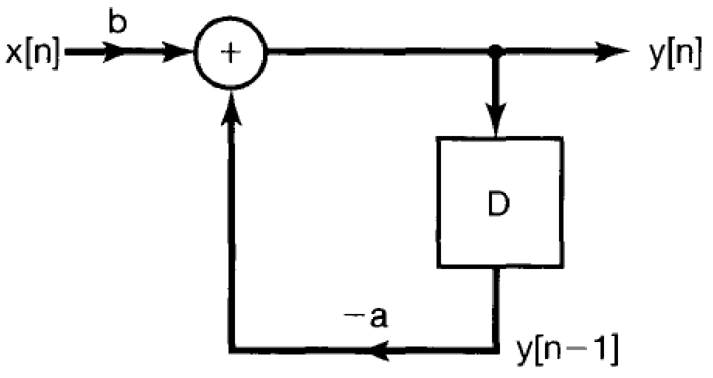


$$y[n] + ay[n-1] = bx[n]$$

$$y[n] = -ay[n-1] + bx[n]$$

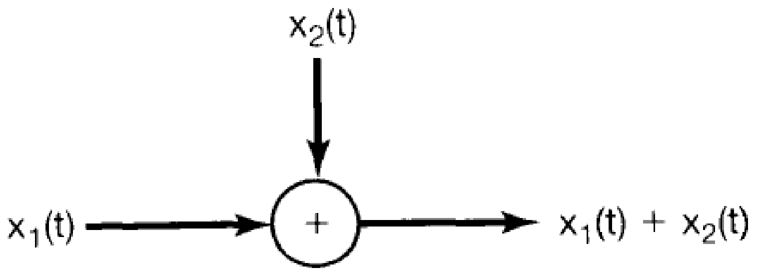


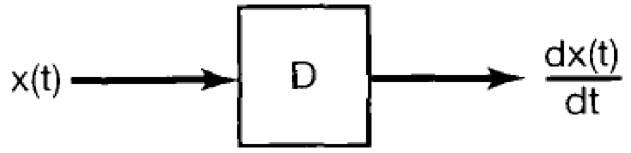




Block Diagram Representations

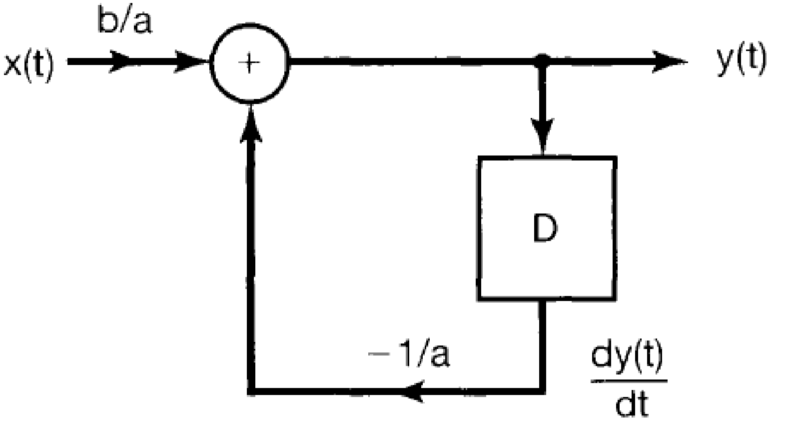
- Basic elements: continuous-time





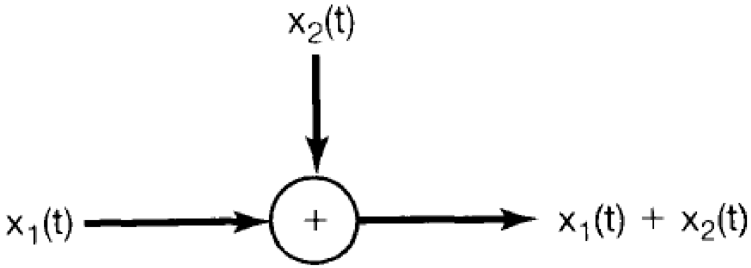
$$\frac{dy(t)}{dt} + ay(t) = bx(t)$$

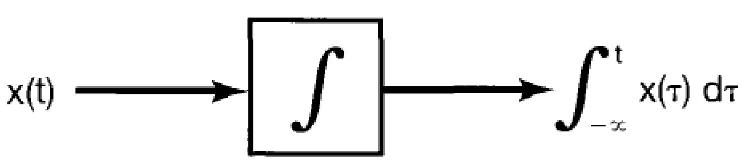
$$y(t) = -\frac{1}{a}\frac{dy(t)}{dt} + \frac{b}{a}x(t)$$



Block Diagram Representations

- Basic elements: continuous-time

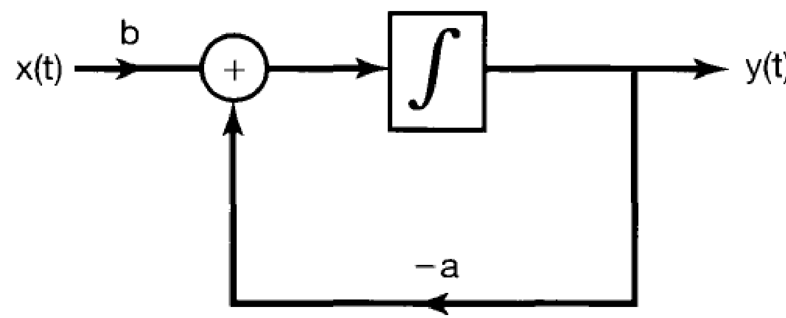




$$\frac{dy(t)}{dt} + ay(t) = bx(t)$$

$$\frac{dy(t)}{dt} = -ay(t) + bx(t)$$

$$y(t) = \int_{-\infty}^{t} [bx(\tau) - ay(\tau)]d\tau$$



Summary

- Differential Equations
- Difference Equations
- Block Diagram Representations

- Reference in textbook:
  - 2.4