Lecture 11

- Magnetically Coupled Circuits

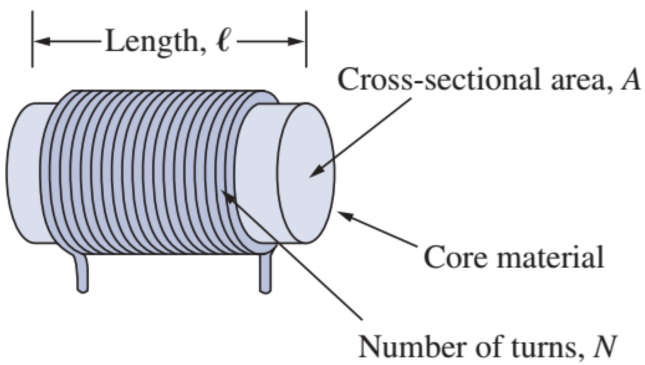
Outline

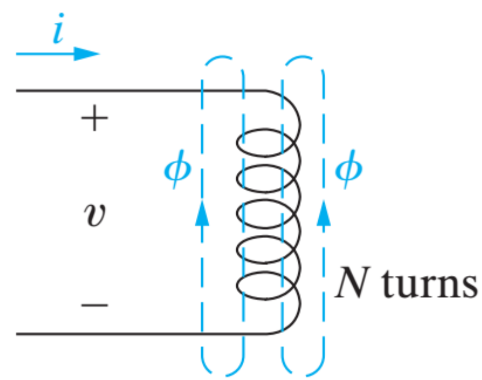
- Mutual inductance
- Transformers

Recall: Self Inductance

- Self inductance:

reaction of the inductor to the change in current ***through itself***.

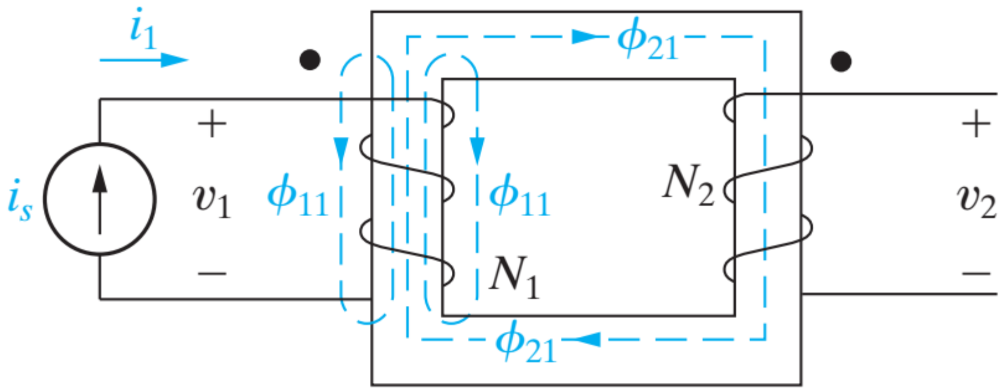




$$v = L\frac{di}{dt}$$

Mutual Inductance

- Mutual inductance: reaction of one inductor to the change in current ***through another inductor***.

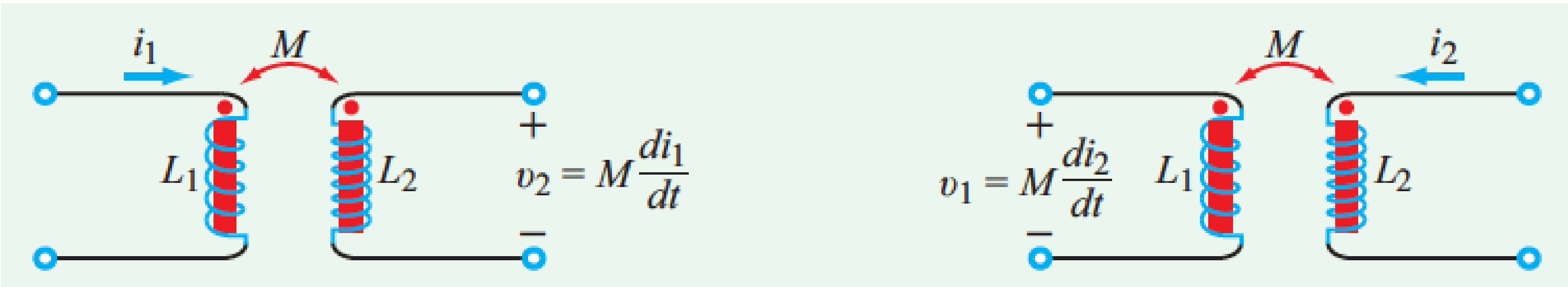


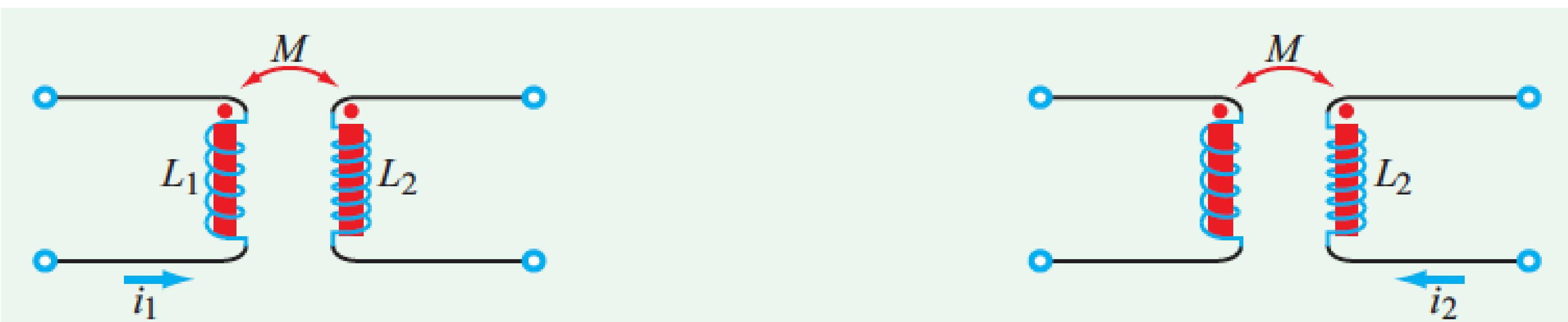
$$v_2 = \frac{d(N_2\phi_{21})}{dt} = N_2\frac{d}{dt}(\mathcal{P}_{21}N_1i_1)$$

$$= N_2N_1\mathcal{P}_{21}\frac{di_1}{dt}$$

$$\boxed{= M_{21}\frac{di_1}{dt}}$$

Dot Convention: Defines Directions of Windings



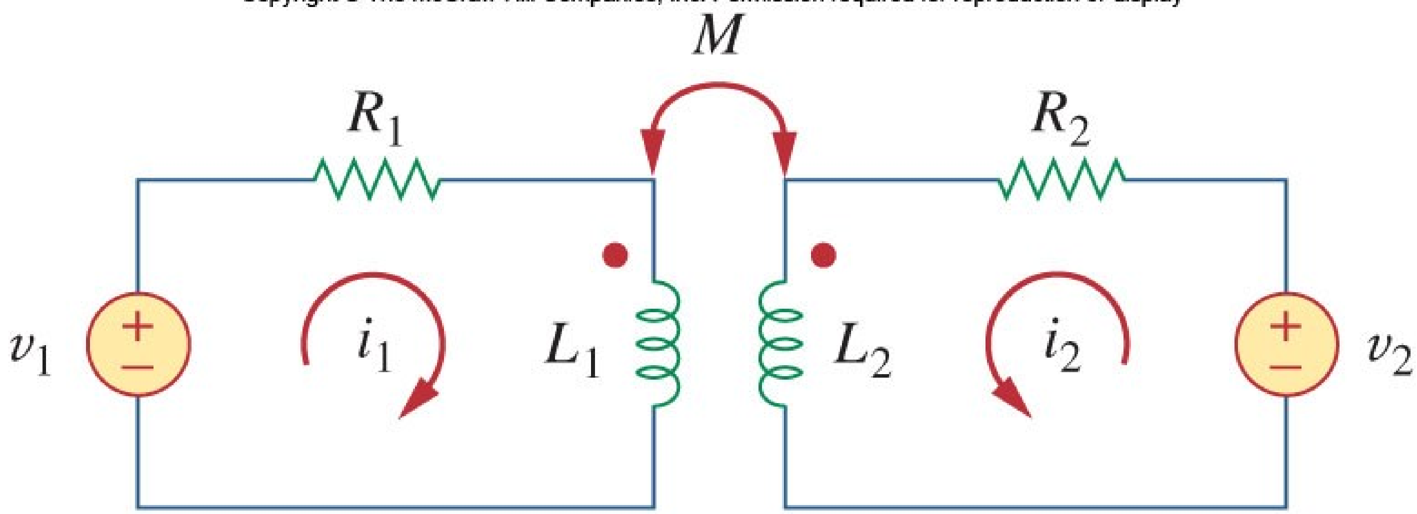


If a current enters the dotted terminal of one coil, the reference polarity of mutual voltage in the 2nd coil is the positive at the dotted terminal, negative at the un-dotted terminal.

[Source: Berkeley]

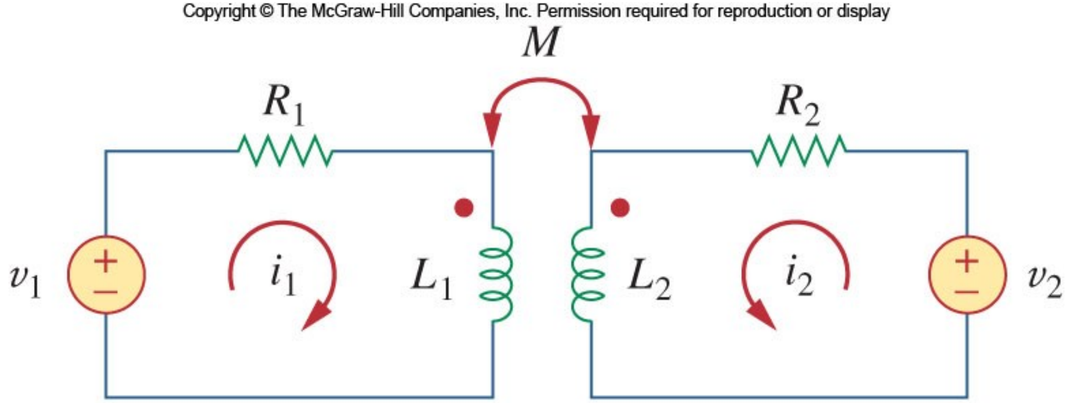
Magnetically Coupled Circuits

- L_1, L_2: self-inductances
- M: mutual inductance
- Dots: indicating polarity of mutually induced voltages.

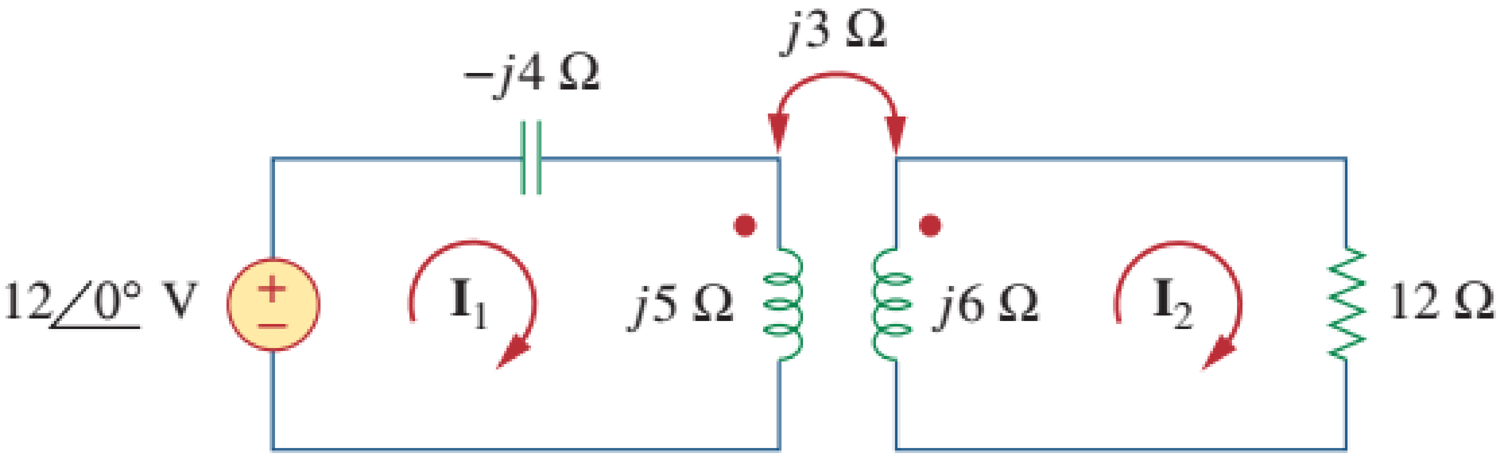


Analysis

- Find i_1 and i_2.
  - In time domain
  - In phasor domain

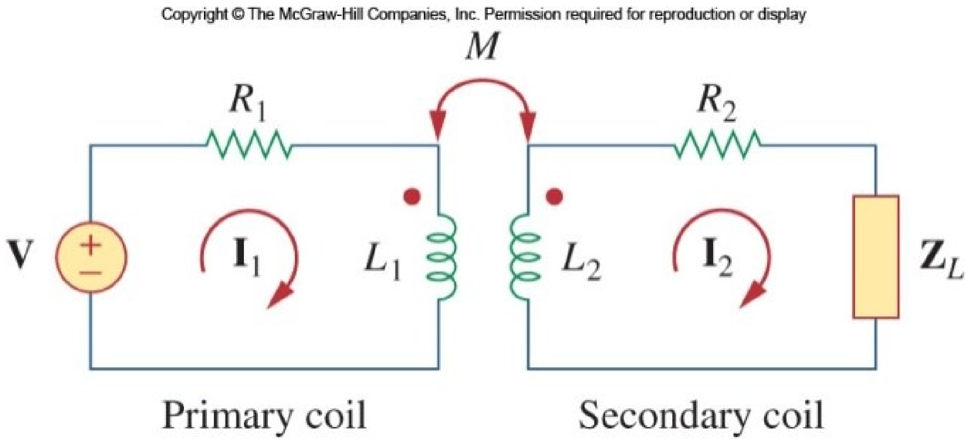


- Calculate the phasor currents $\mathbf{I_1}$, **and** $\mathbf{I_2}$
- Calculate the phasor voltages $\mathbf{V_1}$, **and** $\mathbf{V_2}$ **across the inductors**



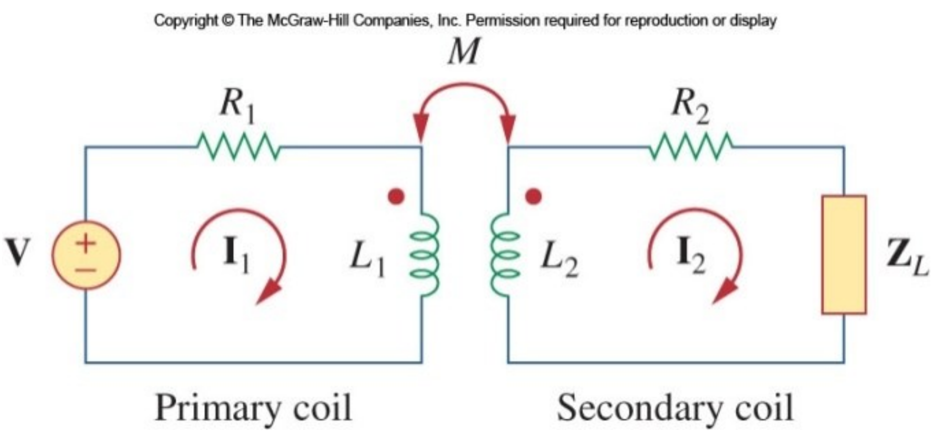
Transformers

- A transformer is a magnetic device that takes advantage of mutual inductance.



Transformer Impedance

- An important parameter to know for a transformer is how the input impedance Z_{in} is seen from the source.

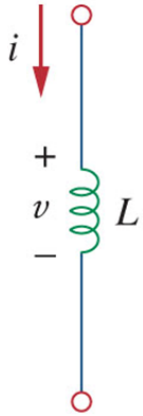


$$\mathbf{Z}_{\text{in}} = \frac{\mathbf{V}}{\mathbf{I}_1} = R_1 + j\omega L_1 + \frac{\omega^2 M^2}{R_2 + j\omega L_2 + \mathbf{Z}_L}$$

Reflected impedance from secondary to primary

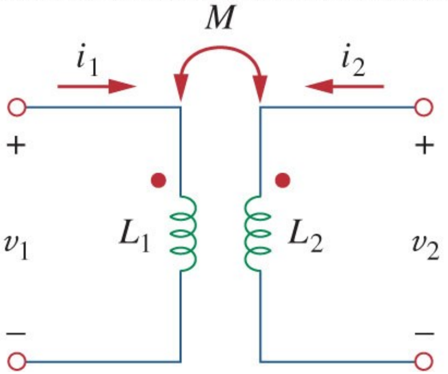
Energy in a Coupled Circuit

- The energy stored in an inductor is



- For coupled inductors, the total energy stored is

$$w = \frac{1}{2} L_1 i_1^{\ 2} + \frac{1}{2} L_2 i_2^{\ 2} \pm M i_1 i_2$$



  - The positive sign is selected when the currents both enter or leave the dotted terminals.

Coupling Coefficient k

- The system cannot have negative energy

$$\frac{1}{2}L_1 i_1^{\ 2} + \frac{1}{2}L_2 i_2^{\ 2} - M i_1 i_2 \geq 0 \quad \Rightarrow \quad \boxed{M \leq \sqrt{L_1 L_2}}$$

- Define a parameter describes how closely M approaches upper limit.

$$\boxed{k = \frac{M}{\sqrt{L_1 L_2}}}$$

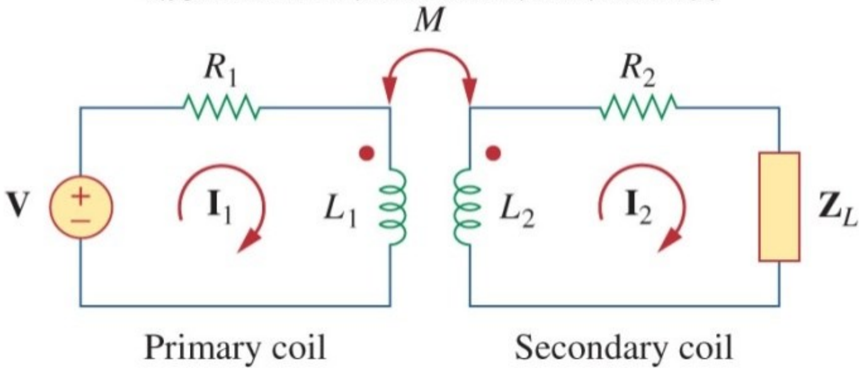
  - Coupling coefficient, $0 \leq k \leq 1$.

Ideal Transformers

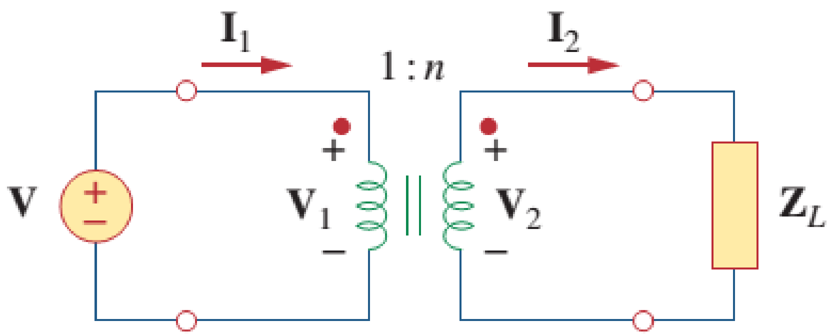
- The ideal transformer has:
  - Coils with very large reactance

    $(L_1, L_2, M \rightarrow \infty)$

  - Coupling coefficient k=1.
  - Primary and secondary coils are lossless, $R_1 = R_2 = 0$.



Ideal Transformers

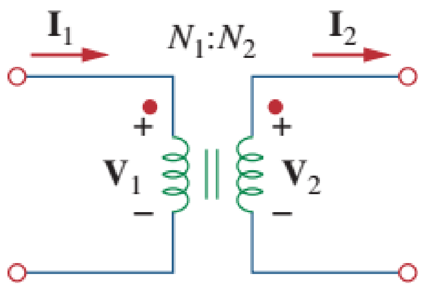


- The voltage is related as:

$$\frac{\mathbf{V_2}}{\mathbf{V_1}} = \frac{N_2}{N_1} = n$$

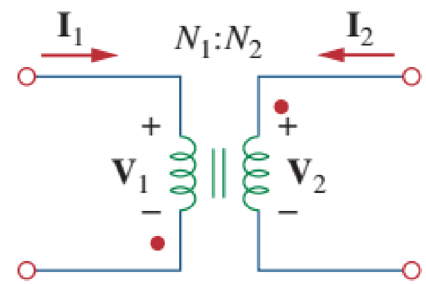
- The current is related as:

1. If $\mathbf{V}_1$ and $\mathbf{V}_2$ are *both* positive or both negative at the dotted terminals, use $+n$ in Eq. (13.52). Otherwise, use $-n$.
2. If $\mathbf{I}_1$ and $\mathbf{I}_2$ *both* enter into or both leave the dotted terminals, use $-n$ in Eq. (13.55). Otherwise, use $+n$.



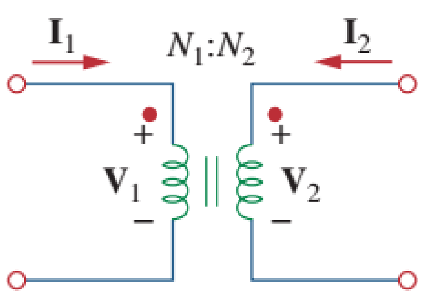
$$\frac{\mathbf{V}_2}{\mathbf{V}_1} = \frac{N_2}{N_1} \qquad \frac{\mathbf{I}_2}{\mathbf{I}_1} = \frac{N_1}{N_2}$$

(a)



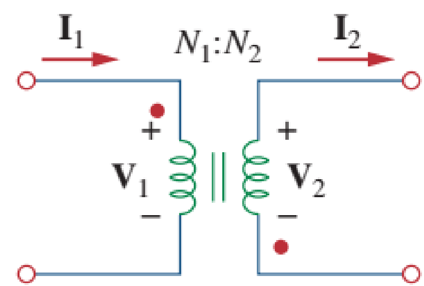
$$\frac{\mathbf{V}_2}{\mathbf{V}_1} = -\frac{N_2}{N_1} \qquad \frac{\mathbf{I}_2}{\mathbf{I}_1} = \frac{N_1}{N_2}$$

(c)



$$\frac{\mathbf{V}_2}{\mathbf{V}_1} = \frac{N_2}{N_1} \qquad \frac{\mathbf{I}_2}{\mathbf{I}_1} = -\frac{N_1}{N_2}$$

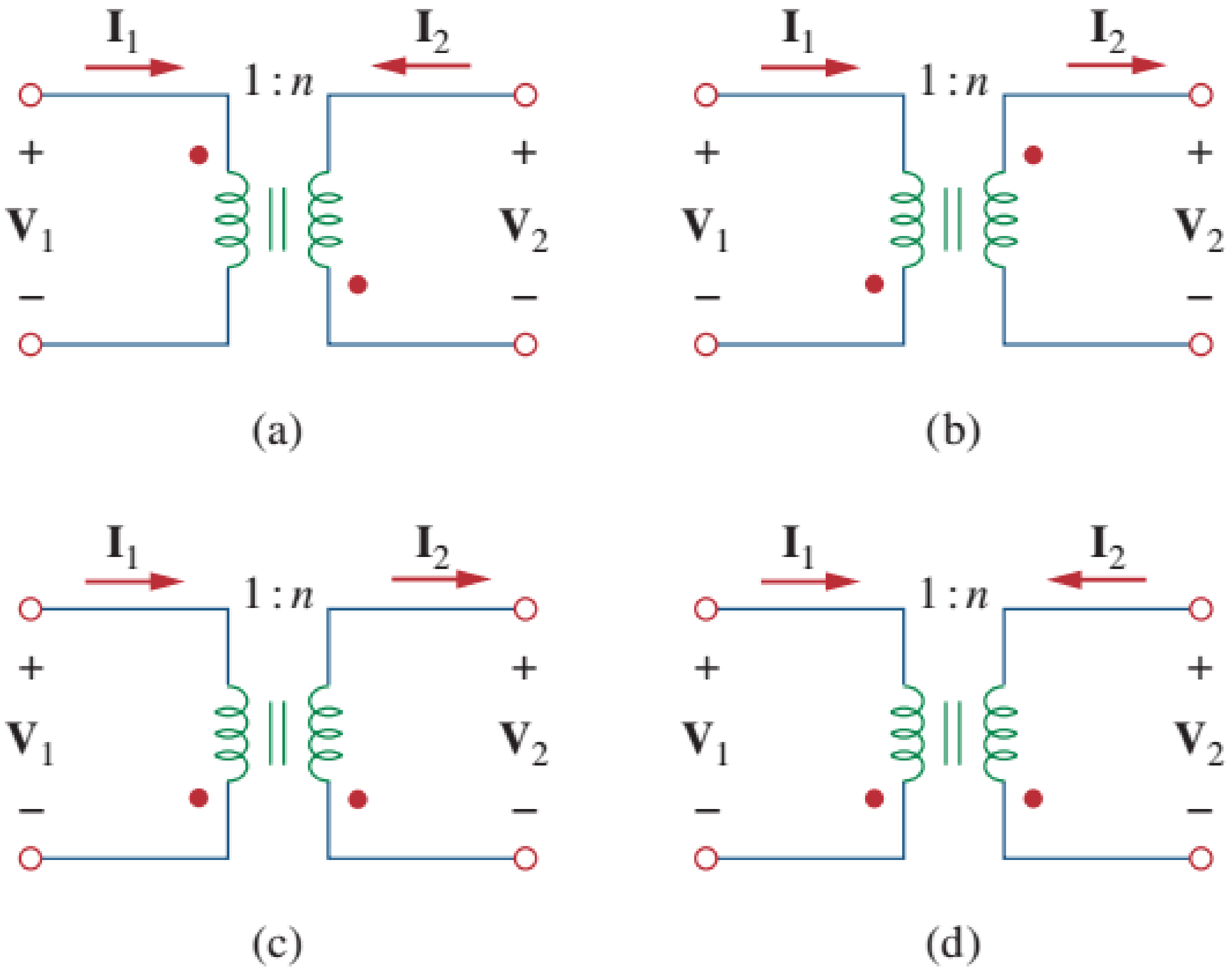
(b)



$$\frac{\mathbf{V}_2}{\mathbf{V}_1} = -\frac{N_2}{N_1} \qquad \frac{\mathbf{I}_2}{\mathbf{I}_1} = -\frac{N_1}{N_2}$$

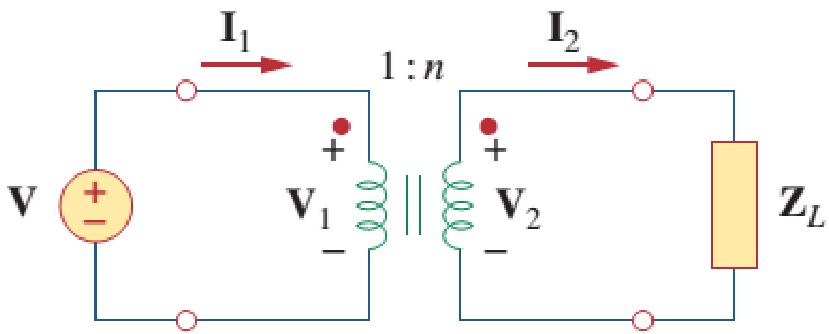
(d)

Practice



Ideal Transformers

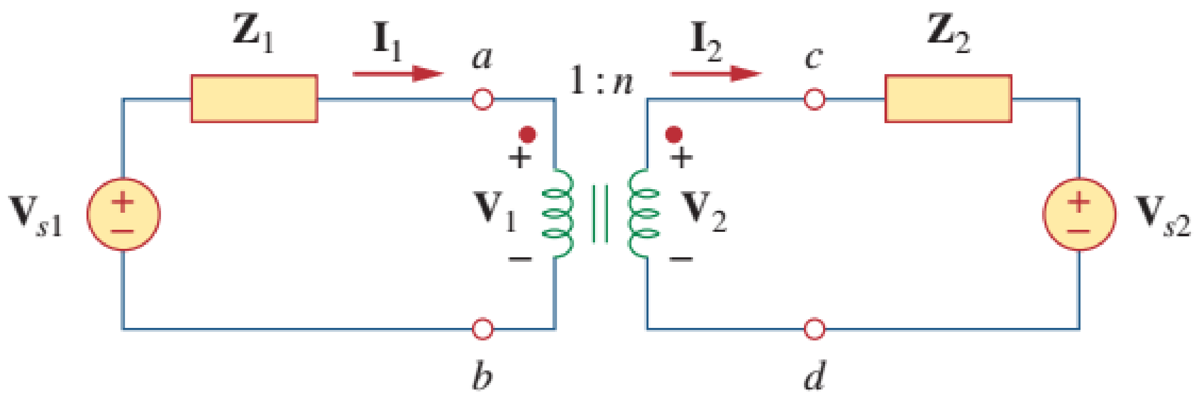
$$\frac{\mathbf{V_2}}{\mathbf{V_1}} = \frac{N_2}{N_1} = n$$



- Reflected impedance

$$\mathbf{Z}_{\text{in}} = \frac{\mathbf{V}_1}{\mathbf{I}_1} =$$

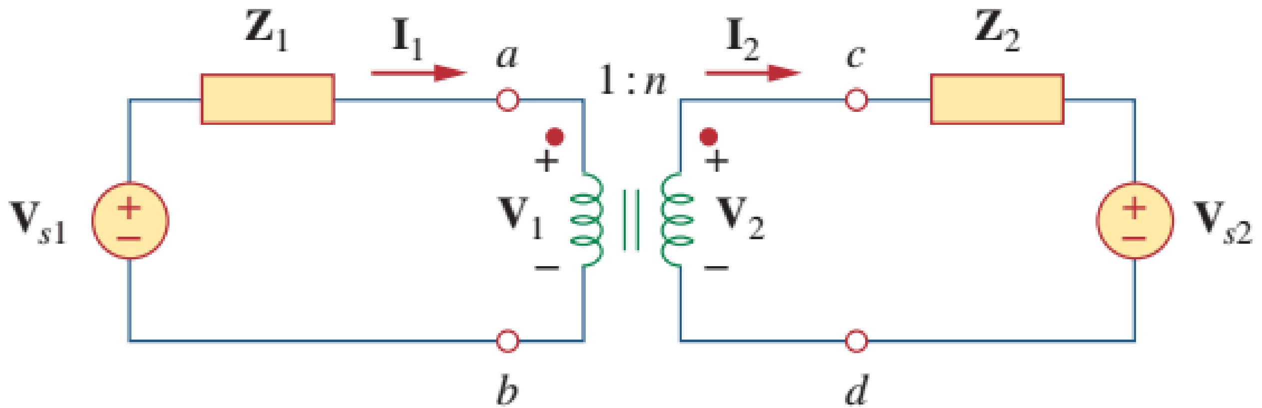
Ideal Transformers

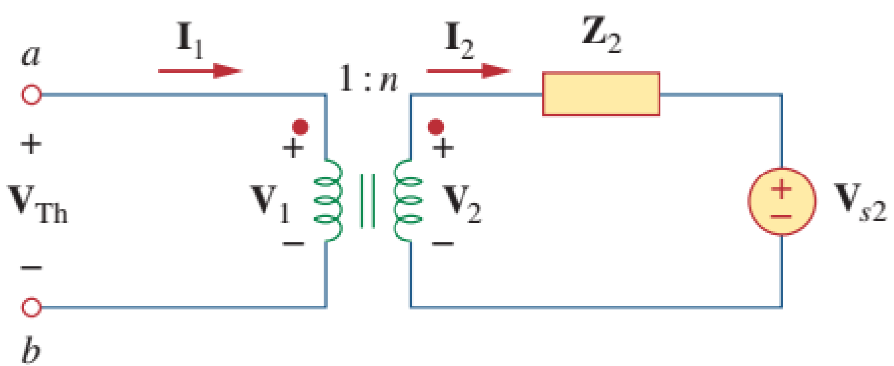


$$\frac{\mathbf{V}_2}{\mathbf{V}_1} = \frac{N_2}{N_1} = n$$

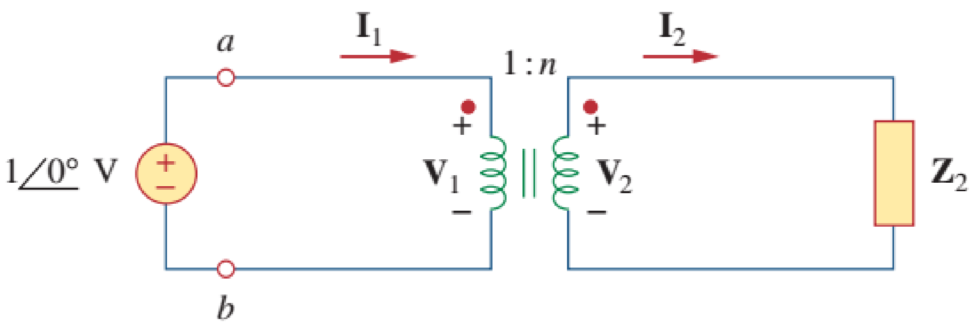
Ideal Transformers

- Reflected impedance and source

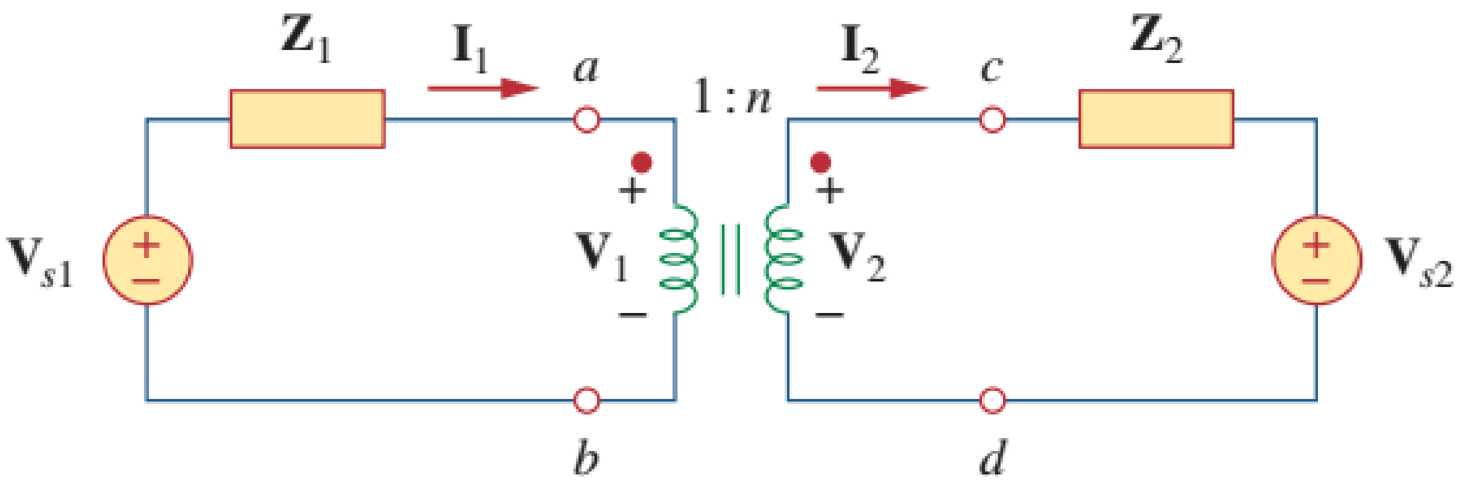


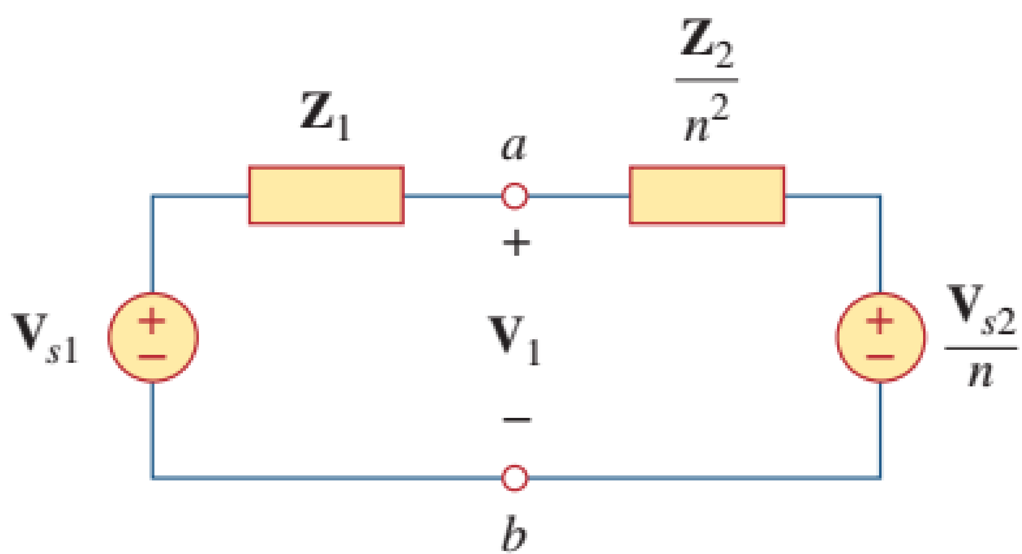


(a)


$\mathbf{I}_1$
$\mathbf{I}_2$
a
$1 : n$
$1\angle 0^\circ$ V
$\mathbf{V}_1$
$\mathbf{V}_2$
$\mathbf{Z}_2$
b

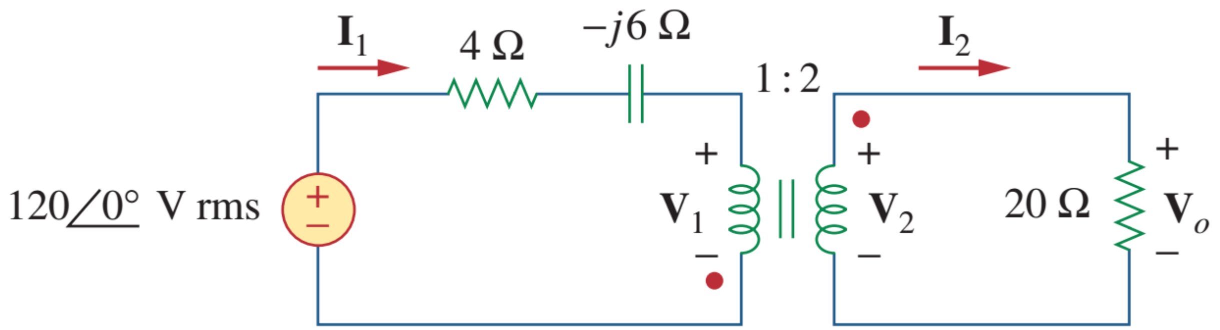
(b)

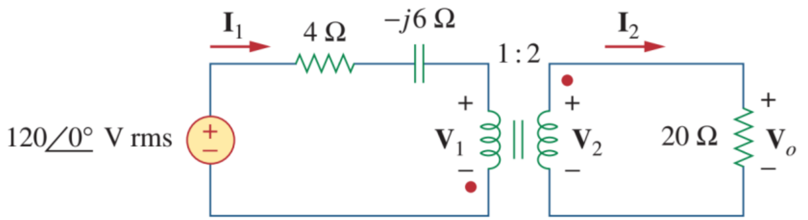

Z1
I1
a
1 : n
I2
c
Z2
Vs1
V1
V2
Vs2
b
d


Z2
n2
Z1
a
Vs1
V1
Vs2
n
b

Example

For the ideal transformer circuit of Fig. 13.37, find: (a) the source current $\mathbf{I}_1$, (b) the output voltage $\mathbf{V}_o$, and (c) the complex power supplied by the source.





Solution:

(a) The 20-Ω impedance can be reflected to the primary side and we get

$$\mathbf{Z}_R = \frac{20}{n^2} = \frac{20}{4} = 5\ \Omega$$

Thus,

$$\mathbf{Z}_{\text{in}} = 4 - j6 + \mathbf{Z}_R = 9 - j6 = 10.82\underline{/-33.69^\circ}\ \Omega$$

$$\mathbf{I}_1 = \frac{120\underline{/0^\circ}}{\mathbf{Z}_{\text{in}}} = \frac{120\underline{/0^\circ}}{10.82\underline{/-33.69^\circ}} = 11.09\underline{/33.69^\circ}\ \text{A}$$

(b) Since both $\mathbf{I}_1$ and $\mathbf{I}_2$ leave the dotted terminals,

$$\mathbf{I}_2 = -\frac{1}{n}\mathbf{I}_1 = -5.545\underline{/33.69^\circ}\ \text{A}$$

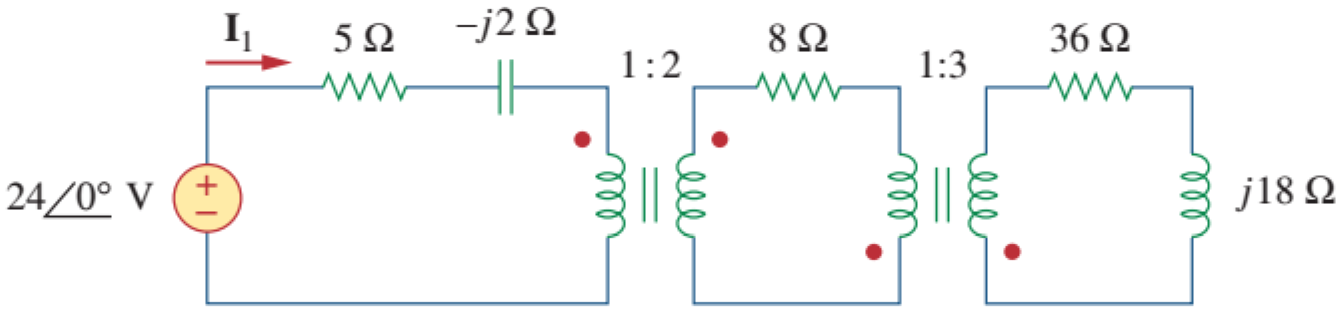
$$\mathbf{V}_o = 20\mathbf{I}_2 = 110.9\underline{/213.69^\circ}\ \text{V}$$

(c) The complex power supplied is

$$\mathbf{S} = \mathbf{V}_s\mathbf{I}_1^* = (120\underline{/0^\circ})(11.09\underline{/-33.69^\circ}) = 1{,}330.8\underline{/-33.69^\circ}\ \text{VA}$$

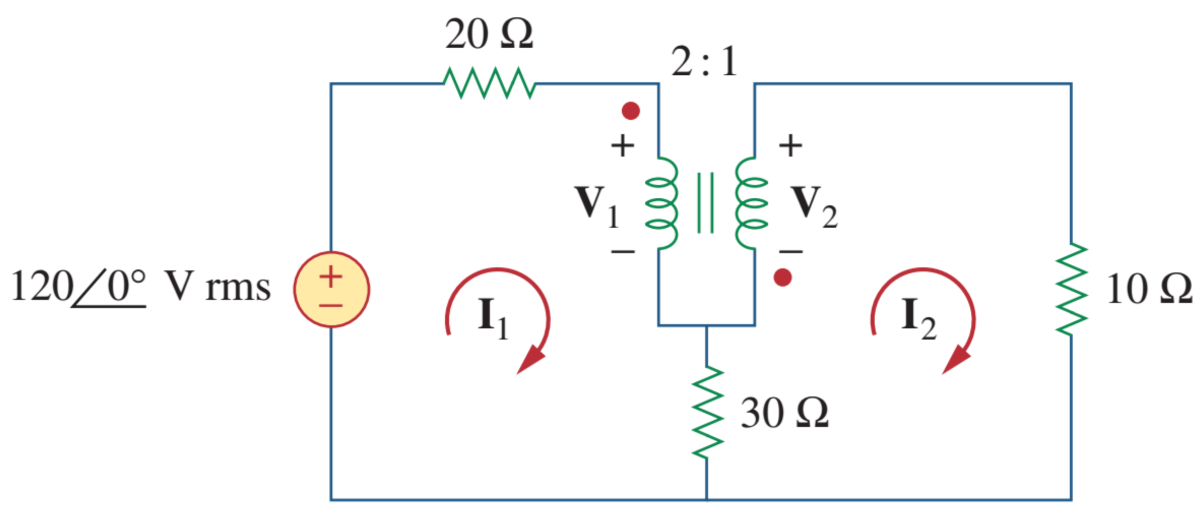
Practice

- Find reflected impedance and $\mathbf{I}_1$



Example

Calculate the power supplied to the 10-Ω resistor in the ideal transformer circuit of Fig. 13.39.



$$-120 + (20 + 30)\mathbf{I}_1 - 30\mathbf{I}_2 + \mathbf{V}_1 = 0$$

or

$$50\mathbf{I}_1 - 30\mathbf{I}_2 + \mathbf{V}_1 = 120 \tag{13.9.1}$$

For mesh 2,

$$-\mathbf{V}_2 + (10 + 30)\mathbf{I}_2 - 30\mathbf{I}_1 = 0$$

or

$$-30\mathbf{I}_1 + 40\mathbf{I}_2 - \mathbf{V}_2 = 0 \tag{13.9.2}$$

At the transformer terminals,

$$\mathbf{V}_2 = -\frac{1}{2}\mathbf{V}_1 \tag{13.9.3}$$

$$\mathbf{I}_2 = -2\mathbf{I}_1 \tag{13.9.4}$$

(Note that $n = 1/2$.) We now have four equations and four unknowns, but our goal is to get $\mathbf{I}_2$. So we substitute for $\mathbf{V}_1$ and $\mathbf{I}_1$ in terms of $\mathbf{V}_2$ and $\mathbf{I}_2$ in Eqs. (13.9.1) and (13.9.2). Equation (13.9.1) becomes

$$-55\mathbf{I}_2 - 2\mathbf{V}_2 = 120 \tag{13.9.5}$$

and Eq. (13.9.2) becomes

$$15\mathbf{I}_2 + 40\mathbf{I}_2 - \mathbf{V}_2 = 0 \quad \Rightarrow \quad \mathbf{V}_2 = 55\mathbf{I}_2 \tag{13.9.6}$$

Substituting Eq. (13.9.6) in Eq. (13.9.5),

$$-165\mathbf{I}_2 = 120 \quad \Rightarrow \quad \mathbf{I}_2 = -\frac{120}{165} = -0.7272 \text{ A}$$

The power absorbed by the 10-Ω resistor is

$$P = (-0.7272)^2(10) = 5.3 \text{ W}$$