EE150 Signal and System
Homework 3

Due on 23: 59, April. 14, 2024.

Note:

- Please provide enough calculation process to get full marks.
- Please submit your homework to Blackboard in PDF version.
- It's highly recommended to write every exercise on a single sheet of page.
- Late submissions will have points deducted according to the penalty policy.
- Please use English only to complete the assignment, solutions in Chinese are not allowed.
- Plagiarizer will get zero points.
- The full score of this assignment is 100 points.

Exercise 1. (20pt)

Find the fundamental frequency ω_0 and the Fourier series coefficients a_k for each of the following signals:

(a) $x(t) = \sin(3\pi t) + \cos(4\pi t)$

(b) $x(t) = [1 + \cos(2\pi t)] \sin(10\pi t + \frac{\pi}{6})$

(c) $x(t) = \sum_{m=-\infty}^{\infty} \delta(t - \frac{m}{3}) + \delta(t - \frac{2m}{3})$

(d) $x[n] = 2\sin(\frac{14\pi}{19}n) + \cos(\frac{10\pi}{19}n) + 1$

(e) $x[n] = \sum_{m=-\infty}^{\infty} (-1)^m (\delta[n - 2m] + \delta[n + 3m])$

Exercise 2. (20pt)

Determine the Fourier series representations for the following signals.

(a) Each $x(t)$ illustrated in Figure 1 (a)-(b).

(b) $x(t)$ periodic with period 2 and

$$x(t) = e^{-t}, \quad \text{for} - 1 < t < 1.$$

(c) $x(t)$ periodic with period 4 and

$$x(t) = \begin{cases} \sin \pi t, & 0 \le t \le 2 \\ 0, & 2 < t \le 4. \end{cases}$$

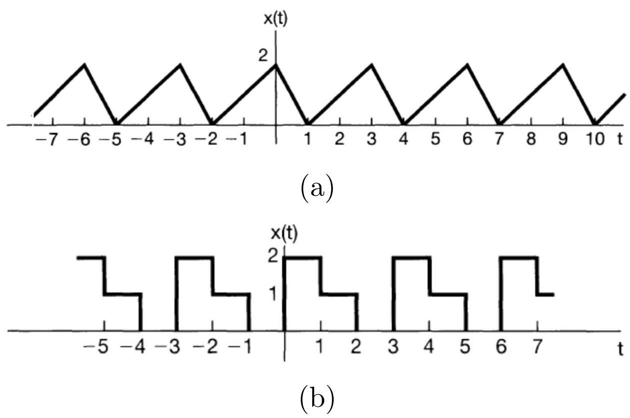

Figure 1: Exercise 2

Exercise 3. (20pt)

Suppose we are given the following information about a signal $x(t)$:

(a) $x(t)$ is a real signal.

(b) $x(t)$ is periodic with period $T = 6$ and has Fourier coefficients a_k.

(c) a_k=0 for $k = 0$ and $k > 2$.

(d) $x(t) = -x(t-3)$.

(e) $\frac{1}{6}\int_{-3}^{3} |x(t)|^2 = \frac{1}{2}$.

(f) a_1 is a positive real number.

Show that $x(t) = A\cos(Bt + C)$, and determine the values of constants A, B, and C.

Exercise 4. (20pt)

Consider the following three discrete-time signals with a fundamental period of 6:

$$x[n] = 1 + \cos(\frac{2\pi}{6}n), \quad y[n] = \sin(\frac{2\pi}{6}n + \frac{\pi}{4}), \quad z[n] = x[n]y[n]$$

(a) Determine the Fourier series coefficients of $x[n]$.

(b) Determine the Fourier series coefficients of $y[n]$.

(c) Use the results of parts (a) and (b), along with the multiplication property of the discrete-time Fourier series, to determine the Fourier series coefficients of $z[n] = x[n]y[n]$.

(d) Determine the Fourier series coefficients of $z[n]$ through direct evaluation, and compare your result with that of part (c).

Exercise 5. (20pt)

(1) Consider a continuous-time ideal lowpass filter $h(t)$ whose frequency response is

$$H(jw) = \begin{cases} 1, & |w| \leq 100 \\ 0, & |w| > 100 \end{cases}$$

When the input to this filter is a signal $x(t)$ with fundamental period $T = \frac{\pi}{6}$ and Fourier series coefficients a_k, it is found that

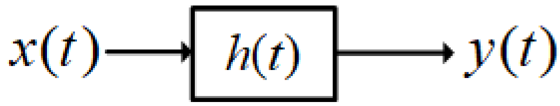

Figure 2: $y(t)$

Where $y(t) = x(t)$, and for what values of k is it guaranteed that $a_k = 0$?

(2) Consider a discrete-time LTI system with impulse response

$$h[n] = \begin{cases} 1, & 0 \leq n \leq 2 \\ -1, & -2 \leq n \leq -1 \\ 0, & \text{otherwise} \end{cases}$$

Given that the input to this system is

$$x[n] = \sum_{k=-\infty}^{\infty} \delta(n - 4k),$$

determine the Fourier series coefficients of the output $y[n]$.