# Homework 5

**Problem 1 (15 points)**

Compute the Fourier transform of each of the following signals:

$(a)\quad x[n] = \left(\frac{2}{3}\right)^{-n} u[-n]$

$(b)\quad x[n] = \sin\left(\frac{\pi}{6}n\right)\cos\left(\frac{\pi}{6}n\right)$

$(c)\quad x[n] = \begin{cases} 1,\ n = 8k-1,\ 8k,\ 8k+1\ (k \epsilon Z) \\ 0,\ others \end{cases}$

**Problem 2 (15 points)**

Compute the inverse Discrete-Time Fourier transform of $X(e^{j\omega})$ of each of the following signals:

$$(a)\ X_1(e^{jw}) = \sum_{k=-\infty}^{\infty} \left\{ 2\pi\delta(w-2\pi k) - \pi\delta\left(w-\frac{\pi}{3}-2\pi k\right) - \pi\delta\left(w+\frac{\pi}{3}-2\pi k\right) \right\}$$

$$(b)\ X_2(e^{jw}) = \frac{1}{(1-ae^{-jw})^2}\ ,\quad |a|<1$$

$$(c)\ X_3(e^{jw}) = \frac{1-\frac{1}{729}e^{-j6w}}{1-\frac{1}{3}e^{-jw}},\quad hint:\ \frac{1}{729} = \frac{1}{3^6}$$

**Problem 3 (20 points)**

Let $X(e^{jw})$ denote the Fourier transform of the signal $x[n]$ depicted in Figure below. Perform the following calculations without explicitly evaluating $X(e^{jw})$:

**(a)** Evaluate $X(e^{i0})$. **(2 points)**

**(b)** Evaluate $\int_{-\pi}^{\pi} X(e^{jw})\, dw$. **(2 points)**

**(c)** Find $X(e^{-j\pi})$ **(2 points)**

**(d)** Determine and sketch the signal whose Fourier transform is $\mathrm{Re}\{X(e^{jw})\}$ **(3 points)**

**(e)** If a signal whose Fourier transform is $(1-e^{-2jw})X(e^{jw})$, draw its figure please. **(4 points)**

**(f)** Evaluate:

$(i)\ \int_{\pi}^{-\pi} |X(e^{jw})|^2 dw$

$(ii)\ \int_{\pi}^{-\pi} \left|\frac{dX(e^{jw})}{dw}\right|^2 dw$

**(i) 3 points** **(ii) 4 points**

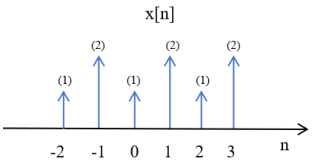



Figure of Problem 3

**Problem 4 (15 points)**

**Simple calculation，it is known that** $x[n]=\left(\frac{1}{2}\right)^n u[n-4]$.

**(a)** Determine $X(e^{jw})$. **(5 points)**

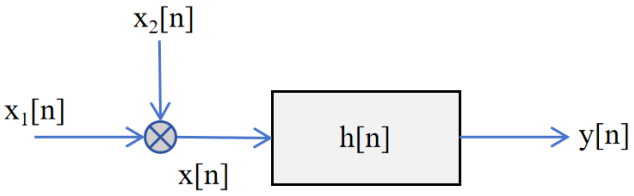
(b) If $y[n]=\sum_{k=-\infty}^{n-2} x[k]$, determine $Y(e^{jw})$. Note: use the answer in (a) to find the final expression of $Y(e^{jw})$. **(10 points)**

**Problem 5 (15 points)**

Given $x[n]=x_1[n]\cdot x_2[n]$, $x_1[n] = \dfrac{\sin\left(\frac{\pi n}{4}\right)}{\pi n}$, $x_2[n] = \cos\left(\dfrac{3\pi n}{4}\right)$.

(a) Draw the spectrum diagram of $X(e^{jw})$. **(9 points)**

(b) Given a discrete-time LTI system (see the system block diagram below) whose unit impulse response is $h[n]=\dfrac{\sin\left(\frac{3\pi}{4}n\right)}{\pi n}$, draw the spectrum diagram of $Y(e^{jw})$ in one period. **(6 points)**




System block figure of Problem 5

**Problem 6 (20 points)**

We are given a discrete-time, liner, time-invariant, causal system with input denoted by $x[n]$ and output denoted by $y[n]$. This system is specified by the following difference equations, involving an intermediate signal $w(n)$:

$$y[n]+\frac{1}{4}y[n-1]+w[n]+\frac{1}{2}w[n-1]=\frac{2}{3}x[n]$$

$$y[n]-\frac{5}{4}y[n-1]+2w[n]-2w[n-1]=-\frac{5}{3}x[n]$$

(a) Find a difference equation relating $y[n]$ and $x[n]$, directly (without using $w[n]$) for the system. **(8points)**

(b) Calculate $h[n]$ and $H(jw)$. **(4 points)**

(c) If the input $x[n] = \left(\frac{1}{3}\right)^n u(n)$, find $y[n]$. **(8 points)**