Lecture 23

CS 131: COMPILERS

Announcements

- HW6: Analysis & Optimizations
  - Alias analysis, constant propagation, dead code elimination, register allocation
  - Due: December 30th

- Final Exam:
  - In class, Jan 2nd
  - Coverage: emphasizes material since the midterm
  - Cheat sheet: one, hand-written, double-sided, letter-sized page of notes

CODE ANALYSIS

CODE ANALYSIS

Live Variable Analysis

- A variable v is ***live*** at a program point if v is defined before the program point and used after it.
- Liveness is defined in terms of where variables are ***defined*** and where variables are ***used***

- ***Liveness analysis***: Compute the live variables between each statement.
  - May be *conservative* (i.e. it may claim a variable is live when it isn't) so because that's a safe approximation
  - To be useful, it should be more *precise* than simple scoping rules.

- Liveness analysis is one example of ***dataflow analysis***
  - Other examples: Available Expressions, Reaching Definitions, Constant-Propagation Analysis, …

Liveness information

- Consider this program:

```
int f(int x) {          <-- x is live
  int a = x + 2;        <-- a and x are live
  int b = a * a;        <-- b and x are live
  int c = b + x;        <-- c is live
  return c;
}
```

- The scopes of a,b,c,x all overlap – they're all in scope at the end of the block.
- But, a, b, c are never live at the same time.
  - So they can share the same stack slot / register

Liveness

- Observation: %uid1 and %uid2 can be assigned to the same register if their values will not be needed at the same time.
  - What does it mean for an %uid to be "needed"?
  - Ans: its contents will be used as a source operand in a later instruction.
- Such a variable is called "***live***"

A variable is ***live*** if its value *might* be used by some future part of the execution path when the program is executed.

Notes:

- the use of the variable might depend on user input or other data not available until the program is run
- even if not, in general, such a property is undecidable

⇒ liveness is a static approximation of the dynamic behavior

- Observe: two variables can share the same register if they are *not* live at the same time.

Control-flow Graphs Revisited

- For the purposes of dataflow analysis, we use the *control-flow graph* (CFG) intermediate form.
- Recall that a basic block is a sequence of instructions such that:
  - There is a distinguished, labeled entry point (no jumps into the middle of a basic block)
  - There is a (possibly empty) sequence of non-control-flow instructions
  - The block ends with a single control-flow instruction (jump, conditional branch, return, etc.)
- A *control flow graph*
  - Nodes are blocks
  - There is an edge from B1 to B2 if the control-flow instruction of B1 might jump to the entry label of B2
  - There are no "dangling" edges – there is a block for every jump target.

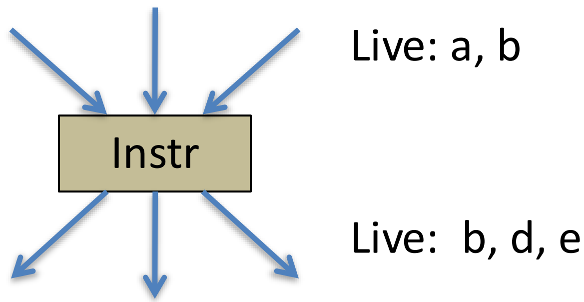
Note: the following slides are intentionally a bit ambiguous about the exact nature of the code in the control flow graphs:

an "imperative" C-like source level
at the x86 assembly level
the LLVM IR level

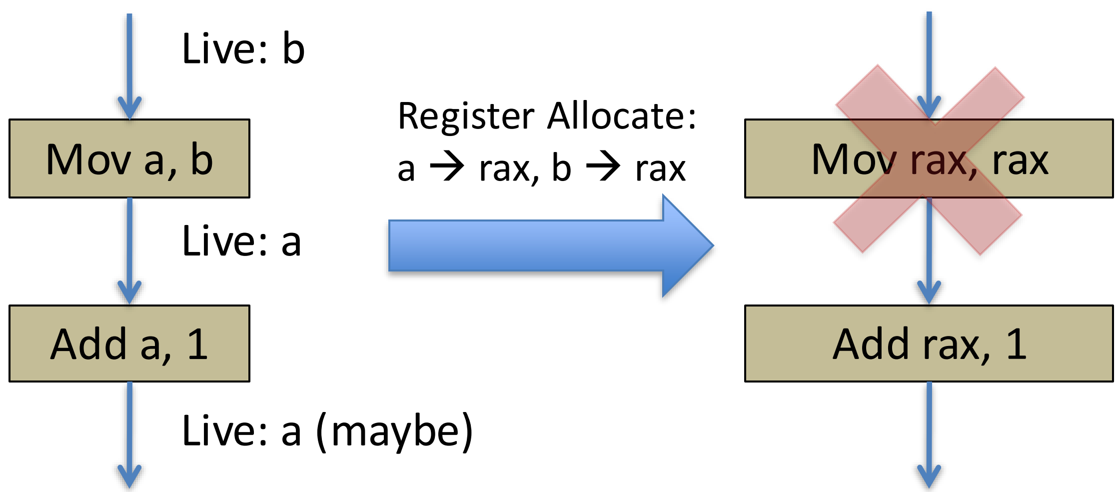
Each setting applies the same general idea, but the exact details will differ.

- e.g., LLVM IR doesn't have "imperative" update of %uid temporaries.
  (The SSA structure of the LLVM IR by design! makes some of these analyses simpler.)

Liveness is Associated with *Edges*



- This is useful so that the same register can be used for different temporaries in the same statement.
- Example: a = b + 1
- Compiles to:



Uses and Definitions

- Every instruction/statement ***uses*** some set of variables
  - i.e. reads from them
- Every instruction/statement ***defines*** some set of variables
  - i.e. writes to them

- For a node/statement s define:
  - **use[s]** : set of variables used by s
  - **def[s]** : set of variables defined by s

- General Examples:

s: a = b + c	use[s] = {b,c}	def[s] = {a}
s: a = a + 1	use[s] = {a}	def[s] = {a}

Liveness, Formally

- A variable v is ***live*** on edge e if:
  There is
  - a node n in the CFG such that use[n] contains v, *and*
  - a directed path from e to n such that for every statement s’ on the path, def[s’] does not contain v

- The first clause says that v will be used on some path starting from edge e.
- The second clause says that v won’t be redefined on that path before the use.

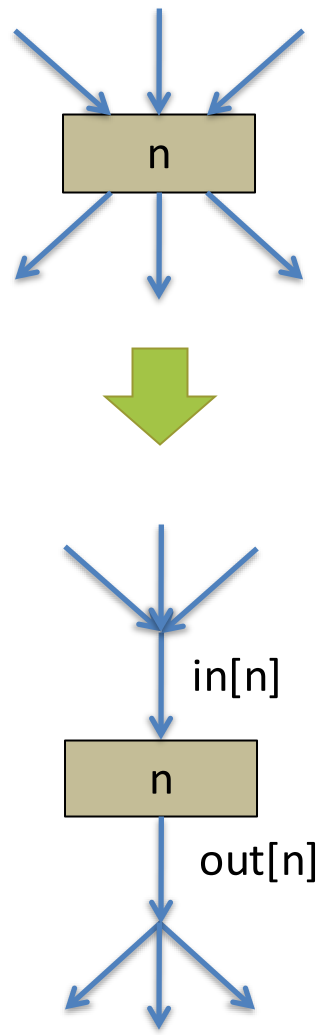
- Questions:
  - How to compute this efficiently?
  - How to use this information (e.g. for register allocation)?
  - How does the choice of IR affect this?
    (e.g. LLVM IR uses SSA, so it doesn’t allow redefinition ⇒ simplify liveness analysis)

Dataflow Analysis

- *Idea*: compute liveness information for all variables simultaneously.
  - Keep track of sets of information about each node
- Approach: define *equations* that must be satisfied by any liveness determination.
  - Equations based on “obvious” constraints.
- Solve the equations by iteratively converging on a solution.
  - Start with a “rough” approximation to the answer
  - Refine the answer at each iteration
  - Keep going until no more refinement is possible: a ***fixpoint*** has been reached
- This is an instance of a general framework for computing program properties: dataflow analysis

Dataflow Value Sets for Liveness

- Nodes are program statements, so:
- **use[n]** : set of variables used by n
- **def[n]** : set of variables defined by n
- **in[n]** : set of variables live on entry to n
- **out[n]** : set of variables live on exit from n



- Associate in[n] and out[n] with the "collected" information about incoming/outgoing edges

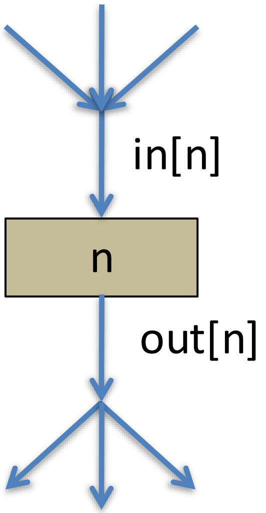
- For Liveness: what constraints are there among these sets?
- Clearly:

$$in[n] \supseteq use[n]$$

- What other constraints?

Other Dataflow Constraints

- We have: $in[n] \supseteq use[n]$
  - "A variable must be live on entry to n if it is used by n"
- Also: $in[n] \supseteq out[n] - def[n]$
  - "If a variable is live on exit from n, and n doesn't define it, it is live on entry to n"
  - Note: here '-' means "set difference"
- And: $out[n] \supseteq in[n']$ if $n' \in succ[n]$
  - "If a variable is live on entry to a successor node of n, it must be live on exit from n."



Iterative Dataflow Analysis

- Find a solution to those constraints by starting from a rough guess.
  - Start with: $in[n] = \emptyset$ and $out[n] = \emptyset$
- The guesses don't satisfy the constraints:
  - $in[n] \supseteq use[n]$
  - $in[n] \supseteq out[n] - def[n]$
  - $out[n] \supseteq in[n']$ if $n' \in succ[n]$

- Idea: iteratively re-compute in[n] and out[n] where forced to by the constraints.
  - Each iteration will add variables to the sets in[n] and out[n]
    (i.e. the live variable sets will increase monotonically)
- We stop when in[n] and out[n] satisfy these equations:
  (which are derived from the constraints above)
  - $in[n] = use[n] \cup (out[n] - def[n])$
  - $out[n] = \bigcup_{n' \in succ[n]} in[n']$

Complete Liveness Analysis Algorithm

for all n, in[n] := Ø, out[n] := Ø

repeat until no change in 'in' and 'out'

    for all n

        $out[n] := \bigcup_{n' \in succ[n]} in[n']$

        $in[n] := use[n] \cup (out[n] - def[n])$

    end

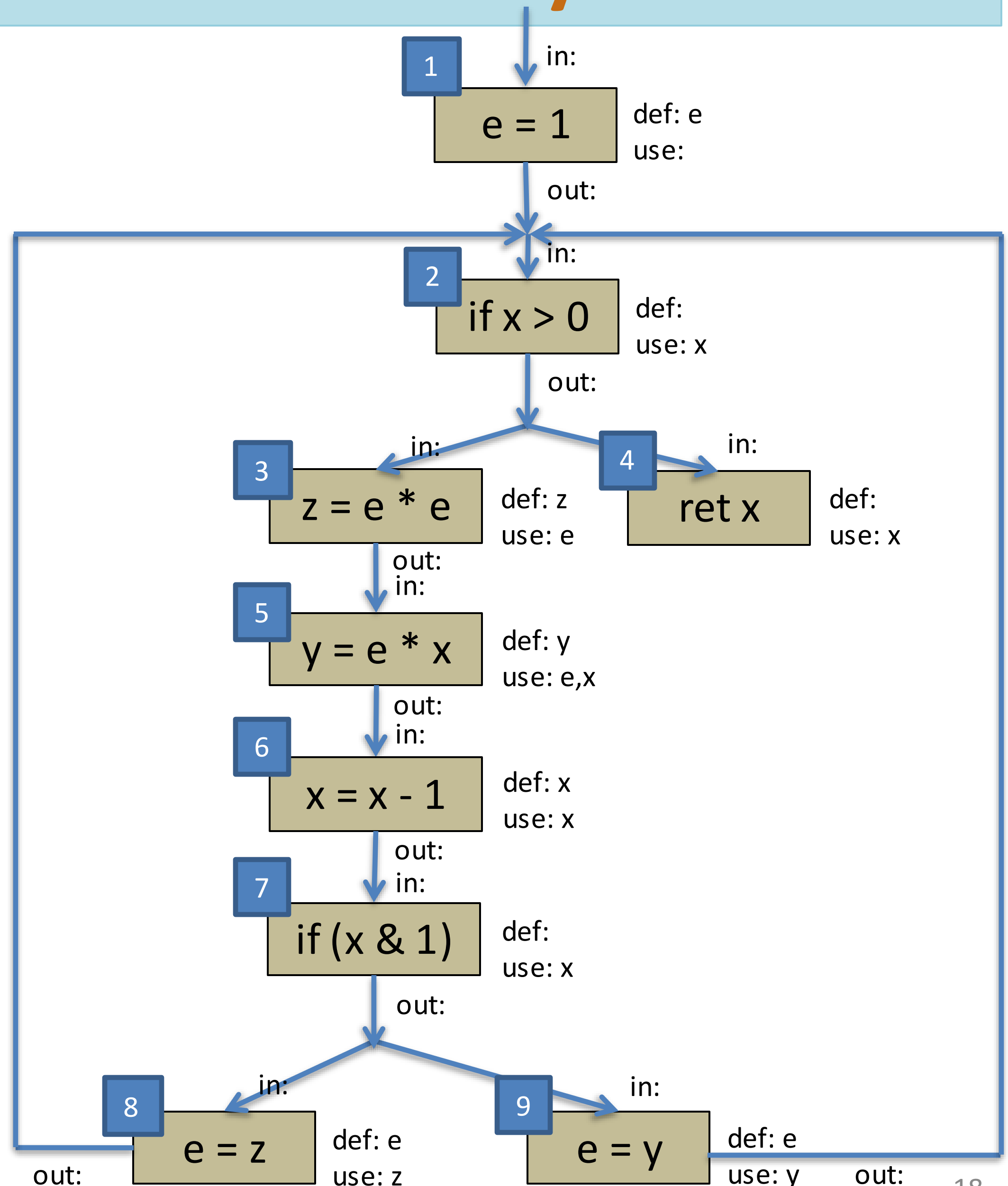
end

- Finds a *fixpoint* of the in and out equations.
  - The algorithm is guaranteed to terminate… Why?
- Why do we start with Ø?

Example Liveness Analysis

- Example flow graph:

```
e = 1;
while(x>0) {
  z = e * e;
  y = e * x;
  x = x – 1;
  if (x & 1) {
    e = z;
  } else {
    e = y;
  }
}
return x;
```



Example Liveness Analysis

Each iteration update:

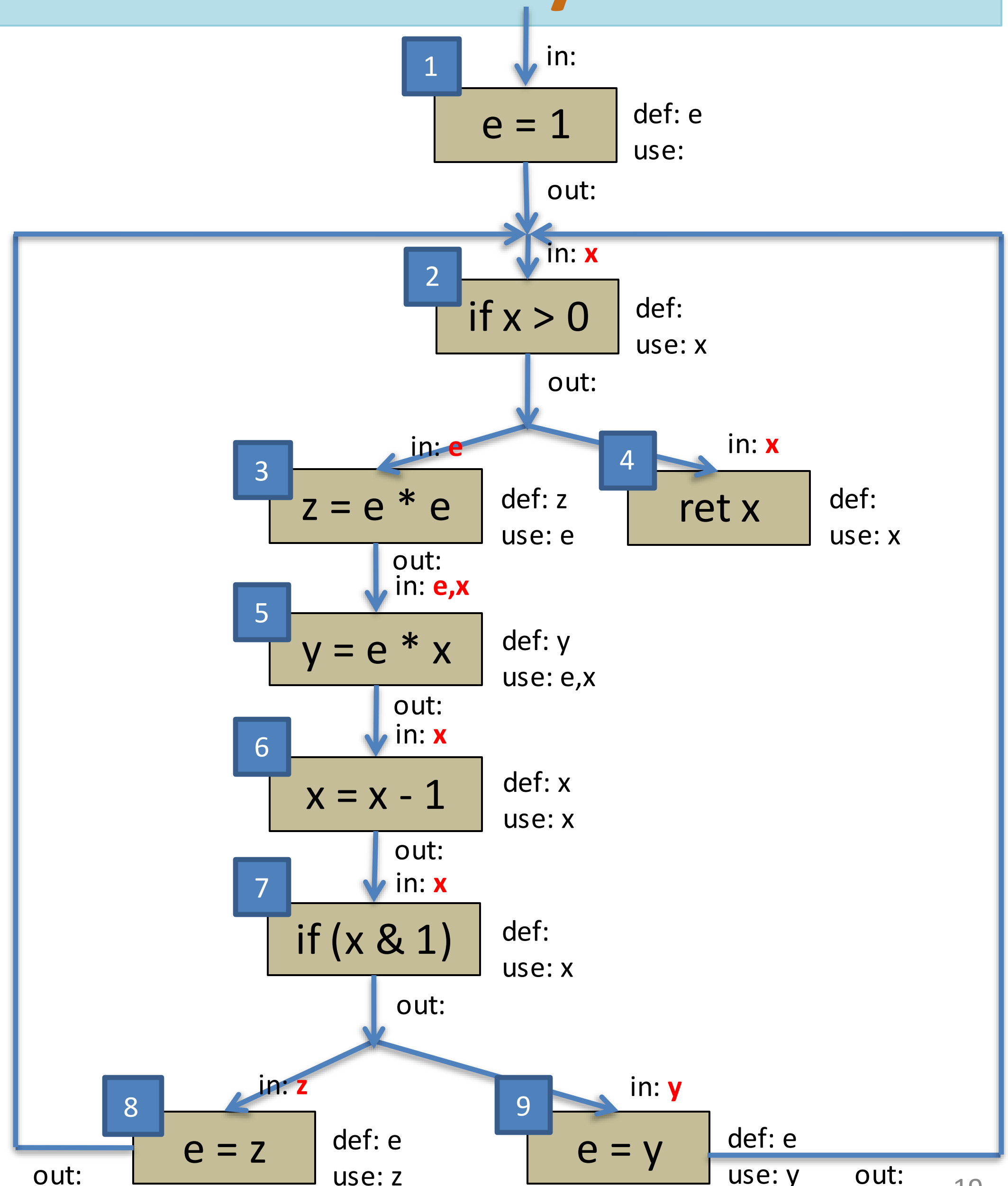
out[n] := $\cup_{n' \in succ[n]}$ in[n']

in[n] := use[n] ∪ (out[n] - def[n])

- Iteration 1:

in[2] = x
in[3] = e
in[4] = x
in[5] = e,x
in[6] = x
in[7] = x
in[8] = z
in[9] = y

(showing only updates that make a change)



Example Liveness Analysis

Each iteration update:

out[n] := $\cup_{n' \in succ[n]}$ in[n']

in[n] := use[n] ∪ (out[n] - def[n])

- Iteration 2:

out[1]= x

in[1] = x

out[2] = e,x

in[2] = e,x

out[3] = e,x

in[3] = e,x

out[5] = x

out[6] = x

out[7] = z,y

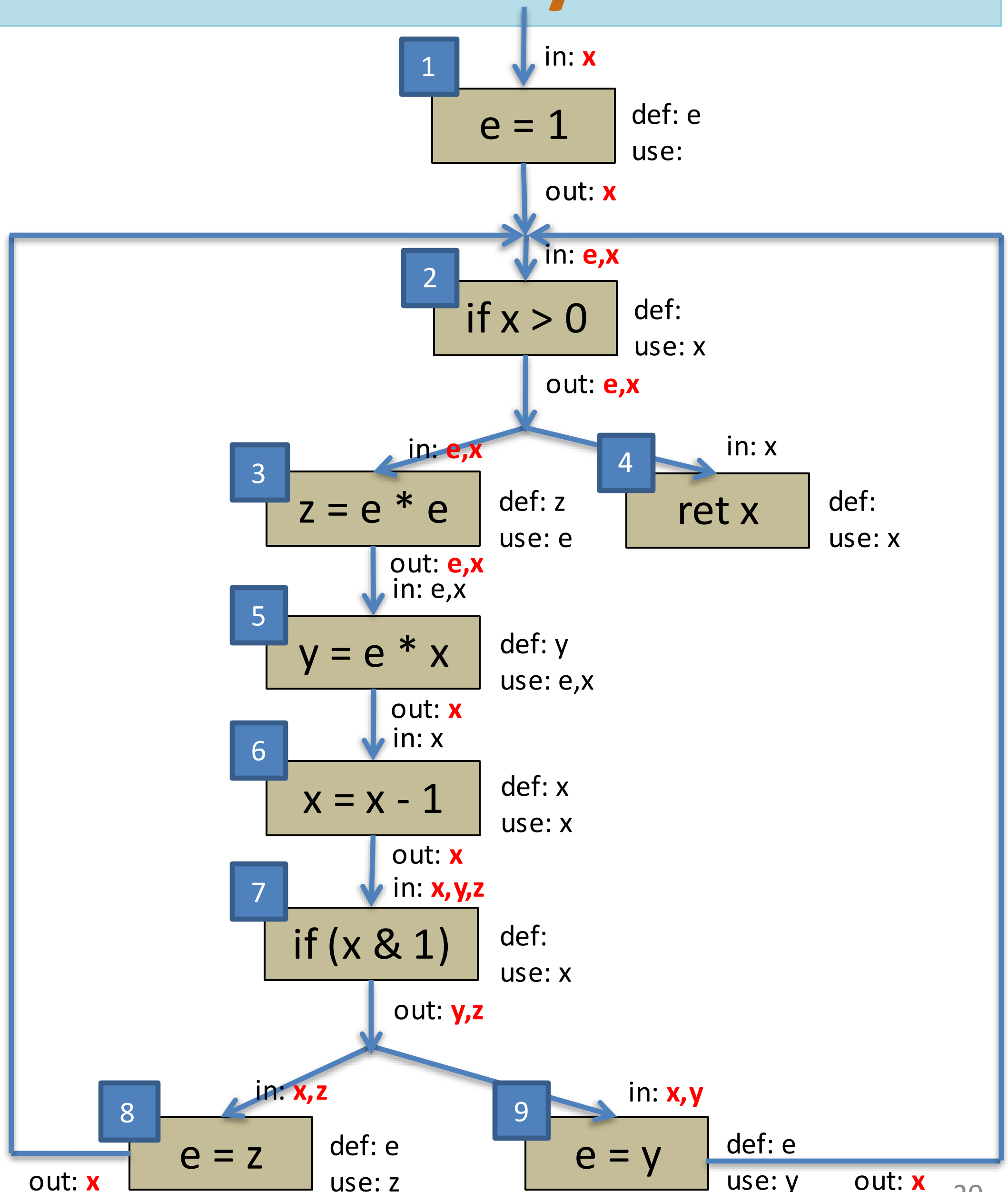
in[7] = x,z,y

out[8] = x

in[8] = x,z

out[9] = x

in[9] = x,y



Example Liveness Analysis

Each iteration update:

out[n] := $\cup_{n' \in succ[n]}$ in[n']

in[n] := use[n] ∪ (out[n] - def[n])

- Iteration 3:

out[1]= e,x

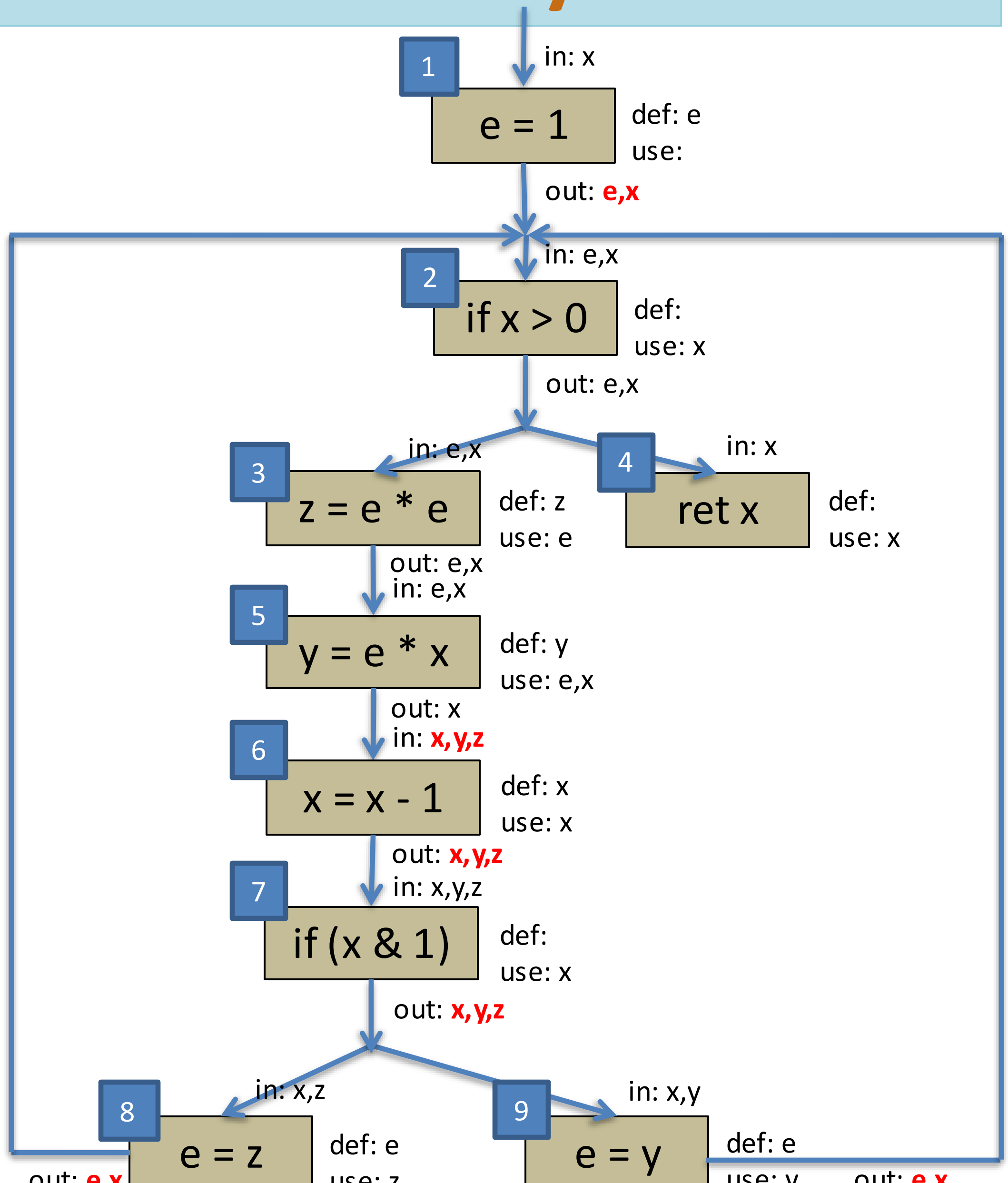
out[6]= x,y,z

in[6]= x,y,z

out[7]= x,y,z

out[8]= e,x

out[9]= e,x



Example Liveness Analysis

Each iteration update:

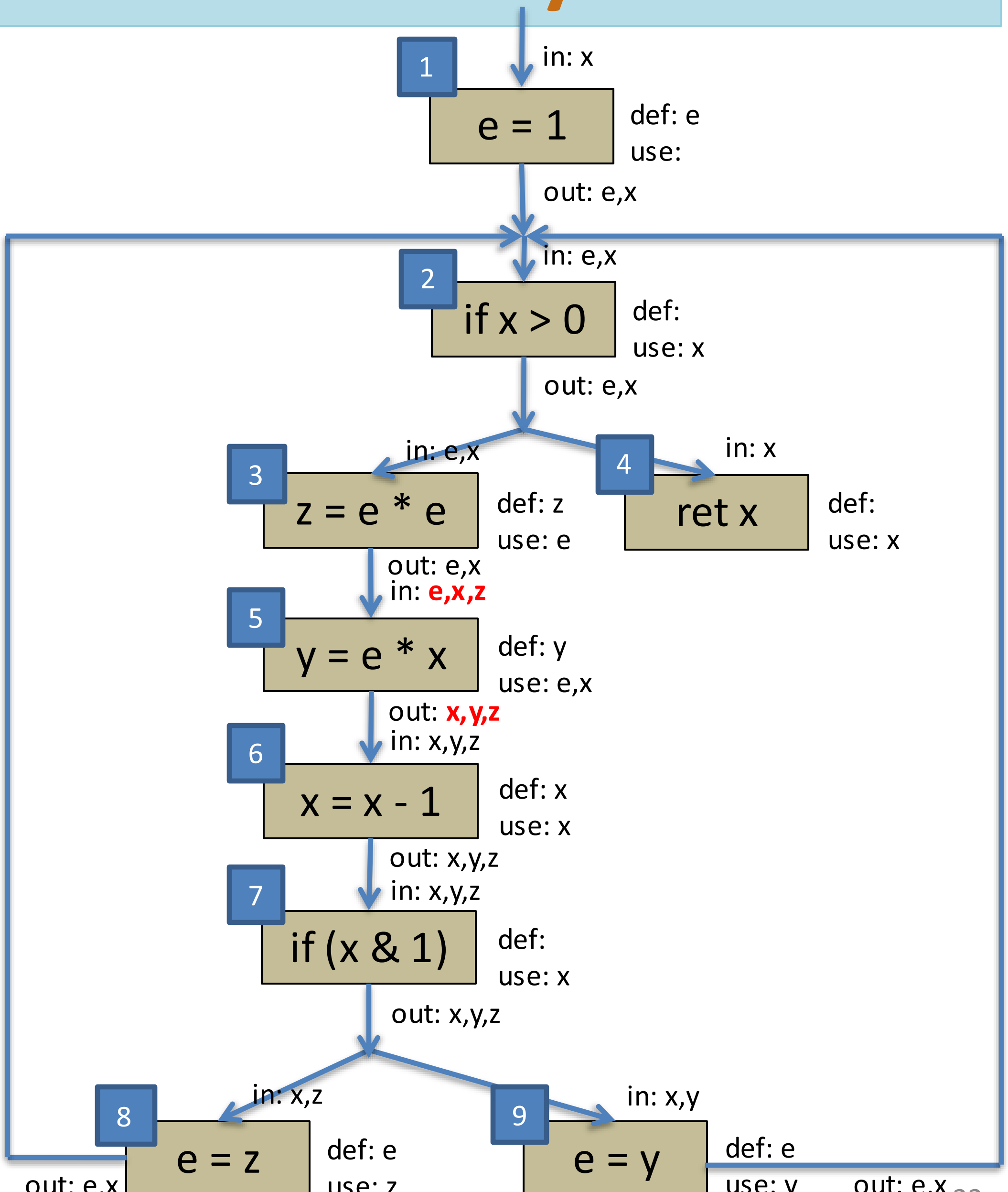
out[n] := $\cup_{n' \in succ[n]}$ in[n']

in[n] := use[n] ∪ (out[n] - def[n])

- Iteration 4:

out[5]= x,y,z

in[5]= e,x,z



Example Liveness Analysis

Each iteration update:

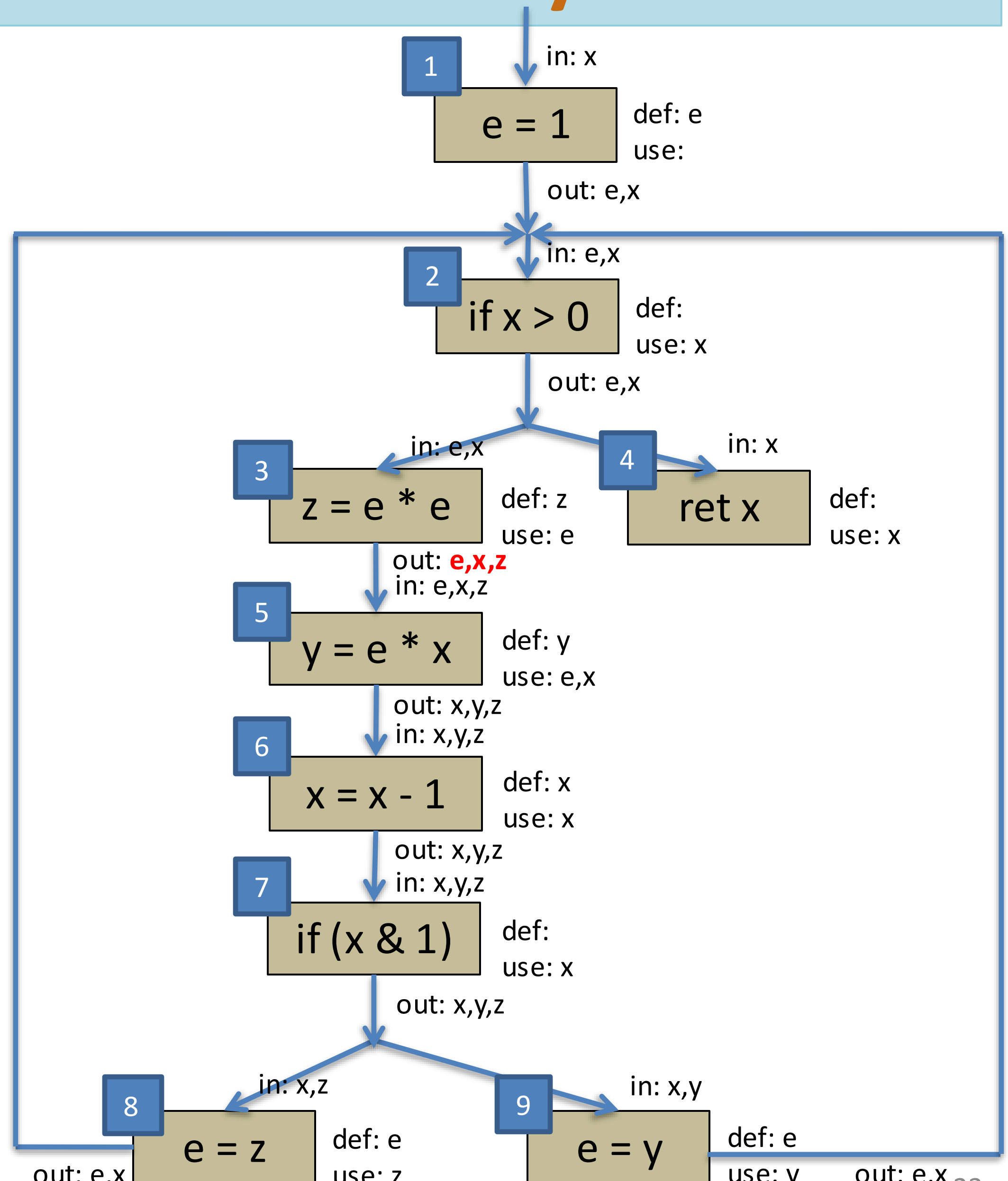
out[n] := $\cup_{n' \in succ[n]}$in[n']

in[n] := use[n] ∪ (out[n] - def[n])

- Iteration 5:

out[3]= e,x,z

Done!



Improving the Algorithm

- Can we do better?

- Observe: the only way information propagates from one node to another is using: $out[n] := \bigcup_{n' \in succ[n]} in[n']$
  - This is the only rule that involves more than one node

- If a node's successors haven't changed, then the node itself won't change.

- Idea for an improved version of the algorithm:
  - Keep track of which node's successors have changed

A Worklist Algorithm

- Use a FIFO queue of nodes that might need to be updated.

```
for all n, in[n] := Ø, out[n] := Ø
w = new queue with all nodes
repeat until w is empty
    let n = w.pop()                          // pull a node off the queue
     old_in = in[n]                          // remember old in[n]

     out[n] := ∪_{n'∈succ[n]} in[n']
     in[n] := use[n] ∪ (out[n] - def[n])
     if (old_in != in[n]),                   // if in[n] has changed
        for all m in pred[n], w.push(m)      // add to worklist
end
```

OTHER DATAFLOW ANALYSES

Generalizing Dataflow Analyses

- The kind of iterative constraint solving used for liveness analysis applies to other kinds of analyses as well.
  - Reaching definitions analysis
  - Available expressions analysis
  - Alias Analysis
  - Constant Propagation
  - These analyses follow the same 3-step approach as for liveness.

- To see these as an instance of the same kind of algorithm, the next few examples to work over a canonical intermediate instruction representation called *quadruples*
  - Allows easy definition of def[n] and use[n]
  - A slightly "looser" variant of LLVM's IR that doesn't require the "static single assignment" – i.e. it has *mutable* local variables
  - We will use LLVM-IR-like syntax

Def / Use for SSA

- Instructions n:def[n] use[n] description

Instruction	def[n]	use[n]	description
a = op b c	{a}	{b,c}	arithmetic
a = load b	{a}	{b}	load
store a, b	Ø	{a,b}	store
a = alloca t	{a}	Ø	alloca
a = bitcast b to u	{a}	{b}	bitcast
a = gep b [c,d, …]	{a}	{b,c,d,…}	getelementptr
a = f(b_1,…,b_n)	{a}	{b_1,…,b_n}	call w/return
f(b_1,…,b_n)	Ø	{b_1,…,b_n}	void call (no return)

- Terminators

Instruction	def[n]	use[n]	description
br L	Ø	Ø	jump
br a L1 L2	Ø	{a}	conditional branch
return a	Ø	{a}	return

REACHING DEFINITIONS

Reaching Definition Analysis

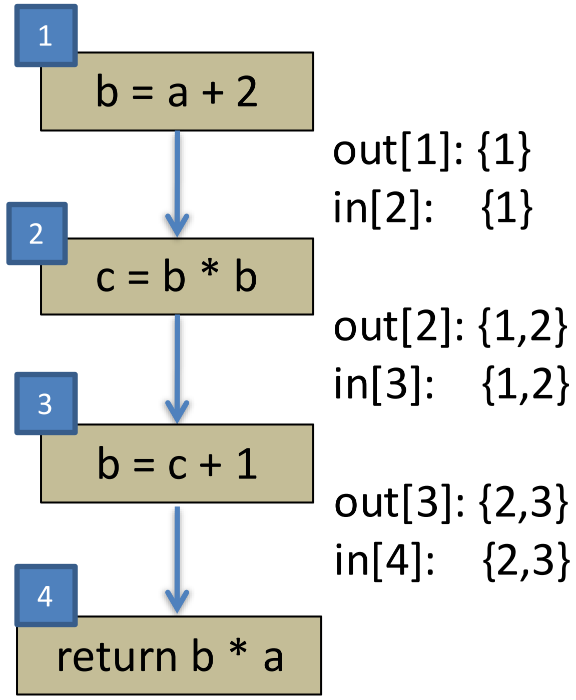
- Question: what uses in a program does a given variable definition reach?

- This analysis is used for constant propagation & copy prop.
  - If only one definition reaches a particular use, can replace use by the definition (for constant propagation).
  - Copy propagation additionally requires that the copied value still has its same value – computed using an *available expressions* analysis (next)

- Input: Quadruple CFG
- Output: in[n] (resp. out[n]) is the set of nodes defining some variable such that the definition may reach the beginning (resp. end) of node n

Example of Reaching Definitions

- Results of computing reaching definitions on this simple CFG:



Reaching Definitions Step 1

- Define the sets of interest for the analysis
- Let defs[a] be the set of *nodes* that define the variable a
- Define gen[n] and kill[n] as follows:
- Quadruple forms n:

Quadruple forms n	gen[n]	kill[n]
a = b op c	{n}	defs[a] - {n}
a = load b	{n}	defs[a] - {n}
store b, a	Ø	Ø
$a = f(b_1,\ldots,b_n)$	{n}	defs[a] - {n}
$f(b_1,\ldots,b_n)$	Ø	Ø
br L	Ø	Ø
br a L1 L2	Ø	Ø
return a	Ø	Ø

Reaching Definitions Step 2

- Define the constraints that a reaching definitions solution must satisfy.
- $out[n] \supseteq gen[n]$
  "The definitions that reach the end of a node at least include the definitions generated by the node"
- $in[n] \supseteq out[n']$ if n' is in $pred[n]$
  "The definitions that reach the beginning of a node include those that reach the exit of *any* predecessor"
- $out[n] \cup kill[n] \supseteq in[n]$
  "The definitions that come in to a node either reach the end of the node or are killed by it."
  - Equivalently: $out[n] \supseteq in[n] - kill[n]$

Reaching Definitions Step 3

- Convert constraints to iterated update equations:
- $in[n] := \bigcup_{n' \in pred[n]} out[n']$
- $out[n] := gen[n] \cup (in[n] - kill[n])$

- Algorithm: initialize in[n] and out[n] to Ø
  - Iterate the update equations until a fixed point is reached

- The algorithm terminates because in[n] and out[n] increase only *monotonically*
  - At most to a maximum set that includes all variables in the program
- The algorithm is precise because it finds the *smallest* sets that satisfy the constraints.

AVAILABLE EXPRESSIONS

Available Expressions

- Idea: want to perform common subexpression elimination:
  - a = x + 1 ... b = x + 1 → a = x + 1 b = a
- This transformation is safe if x+1 means computes the same value at both places (i.e. x hasn't been assigned).
  - "x+1" is an *available expression*
- Dataflow values:
  - in[n] = set of nodes whose values are available on entry to n
  - out[n] = set of nodes whose values are available on exit of n

Available Expressions Step 1

- Define the sets of values
- Define gen[n] and kill[n] as follows:
- Quadruple forms n:

Quadruple forms n	gen[n]	kill[n]
a = b op c	{n} - kill[n]	uses[a]
a = load b	{n} - kill[n]	uses[a]
store b, a	Ø	uses[[x]] (for all x that may equal a)
br L	Ø	Ø
br a L1 L2	Ø	Ø
a = f(b_1,…,b_n)	Ø	uses[a]∪ uses[[x]] (for all x)
f(b_1,…,b_n)	Ø	uses[[x]] (for all x)
return a	Ø	Ø

Note the need for "may alias" information…

Note that functions are assumed to be impure…

Available Expressions Step 2

- Define the constraints that an available expressions solution must satisfy.
- $out[n] \supseteq gen[n]$
  "The expressions made available by n that reach the end of the node"
- $in[n] \subseteq out[n']$ if n' is in pred[n]
  "The expressions available at the beginning of a node include those that reach the exit of *every* predecessor"
- $out[n] \cup kill[n] \supseteq in[n]$
  "The expressions available on entry either reach the end of the node or are killed by it."
  - Equivalently: $out[n] \supseteq in[n] - kill[n]$

Note similarities and differences with constraints for "reaching definitions".

Available Expressions Step 3

- Convert constraints to iterated update equations:
- $in[n] := \bigcap_{n' \in pred[n]} out[n']$
- $out[n] := gen[n] \cup (in[n] - kill[n])$
- Algorithm: initialize in[n] and out[n] to {set of all nodes}
  – Iterate the update equations until a fixed point is reached
- The algorithm terminates because in[n] and out[n] *decrease* only *monotonically*
  – At most to a minimum of the empty set
- The algorithm is precise because it finds the *largest* sets that satisfy the constraints.