EE150 -Signals and Systems, Fall 2024

Homework Set #3

Prof. Lin Xu and Prof. Xiran Cai

Problem 1 (10 pt)

The impulse response of a LTI system is:

$$h(t) = e^{-3t}u(t)$$

(a) Find the system function H(s)

(b) Find the output signal of the system if the input signal is

$$x(t) = 5\sin(3t) + 3\cos(5t)$$

Problem 2 (15 pt)

(a) A continuous-time periodic signal x(t) is real valued and has a fundamental period T=6. The nonzero Fourier series coefficients for x(t) are

$$a_1 = a_{-1}^* = 3j \quad , \quad a_3 = a_{-3} = 5$$

Express x(t) in the form:

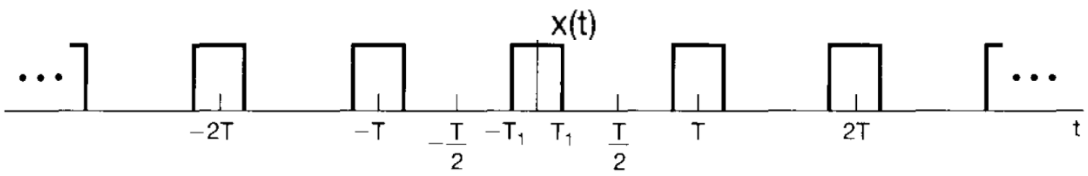
$$x(t) = \sum_{k=0}^{\infty} A_k \cos(w_k t + \emptyset_k)$$

(b) Find the Fourier series coefficients for the following signal:

$$x(t) = 4 + 2\cos\left(\frac{2\pi}{3}t\right) + 4\sin\left(\frac{4\pi}{5}t\right) + 6\cos\left(\frac{8\pi}{15}t\right)$$

(Hint: Calculate the basic signal period first)

(c) $T_1 = 1, T = 4$, The magnitude of x(t) is 1. Find the Fourier series coefficients of x(t).



Problem 3 (15 pt)

Suppose that we are given the following information about a signal x[n]:

(a) x[n] is a real and odd signal.

(b) x[n] has period N = 10 and Fourier series coefficients a_k

(c) $a_{21} = 5j$

(d) $\frac{1}{10}\sum_{n=0}^{9}|x[n]|^2 = 50$

Show that $x[n] = A\sin(Bn + C)$, and specify numerical values for the constants A, B, and C.

Problem 4 (10 pt)

Let x(t) be a periodic signal with fundamental period T and Fourier series coefficients a_k. Derive the Fourier series coefhcients of each of the following signals in terms of a_k:

(a) Re{x(t)}

(b) x(2t-1) [for this part, first determine the period of x(2t-1)]

Problem 5 (10 pt)

(a) Let $x_1(t)$ be a continuous-time periodic signal with fundamental frequency w_1, and Fourier series coefficients a_k. Given that

$$x_2(t) = x_1(2 - t) + x_1(t - 3)$$

how is the fundamental frequency w_2 of $x_2(t)$ related to w_1? Also, find a relationship between the Fourier series coefficients b_k of $x_2(t)$ and the coefficients a_k.

(b) Each of the two sequences $x_1[n]$ and $x_2[n]$ has a period N = 4, and the corresponding Fourier series coefficients are specified as $x_1[n] \overset{FS}{\leftrightarrow} a_k,\ x_2[n] \overset{FS}{\leftrightarrow} b_k$

Where

$a_0 = 1, a_1 = 2, a_2 = 3, a_3 = 4$, $b_0 = 4, b_1 = 3, b_2 = 2, b_3 = 1$

Determine the Fourier series coefficients c_k for the signal $g[n] = x_1[n]x_2[n]$.

Problem 6 (20 pt)

Let

$$x(t) = \begin{cases} -2t & 0 \le t \le 1 \\ 2t - 4 & 1 \le t \le 2 \end{cases}$$

be a periodic signal with fundamental period T = 2 and Fourier series coefficients a_k.

(a) Determine the value of a_0.

(b) Determine the Fourier series representation of d x(t)/dt .

(c) Use the result of part (b) and the differentiation property of the continuous-time Fourier series to help determine the Fourier series coefficients of x(t).

Problem 7 (20 pt)

Consider a causal discrete-time LTI system :

$$h[n] = \left(\frac{1}{2}\right)^n u(n)$$

Find the Fourier series representation of the output y[n] for each of the following inputs:

(a) $x[n] = \cos(\frac{5\pi}{6}n)$

(b) $x[n] = \sin\left(\frac{\pi}{3}n\right) + \cos\left(\frac{4\pi}{3}n\right) + \sin\ (\frac{4\pi}{3}n)$