EE150-Signals and System, Fall 2024
Homework 8

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1. (15 pts) Given the following discrete time signal

$$x[n] = -(\frac{1}{3})^n \, u[-n-1]$$

$$x[n] = (\frac{1}{2})^n \{u[n] - u[n-5]\}$$

$$x[n] = 11(\frac{1}{3})^n \cos\left[\frac{\pi}{3}n + \frac{\pi}{4}\right] u[n]$$

Use the definition of the Z-transform to find their Z-transform expression and give the ROC (5 points each)

2. (15 pts) The expression of Z-transform $X(z)$ of the discrete time signal $x[n]$ is shown below, please discuss all possibilities of region of convergence and find the corresponding $x[n]$. What will the region of convergence be like if the Fourier transform of the $x[n]$ converges.

$$X(z) = \frac{8 - 13z^{-1}}{2 - 5z^{-1} + 2z^{-2}}$$

3. **(10 pts)** Given the equation below, determine the Z transform or inverse Z transform using the properties of the Z transform

(a) $X(z) = \dfrac{\frac{2}{3}z^{-1}}{(1-\frac{1}{3}z^{-1})^2}$, $|z| > \dfrac{1}{3}$

(b) $y[n] = \{((\frac{1}{2})^n u[n]) * ((3)^n u[-n])\}$

Note: Sign "∗" represents the convolution operator.

4. (20 pts) Consider a LTI system whose zero-pole plot is shown below and the system function $H(z)$ is rational, with the impulse response function defined as $h[n]$:
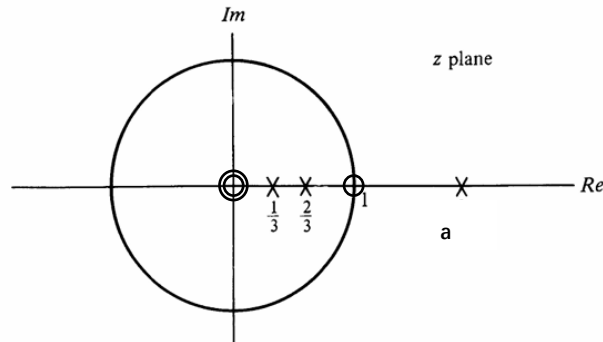
a. Determine whether the following statements are true or false and explain your judgement.
   a) If the system is causal, then it is stable.
   b) $H(e^{j\omega})$ could not be zero for any ω
   c) $h[n]$ is finite duration
   d) If the system is stable, then $F\{(\frac{5}{4})^n h[n]\}$ converges

b. Find out H(z) if we know:
   a) $H(z)$ has exactly three poles and three zeros.
   b) $H(z)$'s region of convergence is $|z| > 2$
   c) $h[0] = 3$

5. (30 pts) For a causal LTI system characterized by the difference equation:
$$y[n-2] - 7y[n-1] + 10y[n] = 10x[n]$$
a). Find the transfer function of the system.

b). Sketch the zero-pole plot of this system and find its region of convergence. Is this system stable? Why?

c). Find the unit impulse response of the system.

d). Sketch the block diagram of the system in **parallel**, **cascade**, and **direct form.**

6. (10 pts) The system equation is characterized as:
$$y[n] - y[n-1] - 2y[n-2] = x[n] + 2x[n-2]$$
Given the initial condition of the system $y[-1] = \beta$ and $y[-2] = \gamma$,and the input signal $x[n] = \alpha u[n]$, find the zero-input response and zero-state response of the system.