CS270-B Advanced Digital Image Processing

Lecture 11 Image Reconstruction

(Compress Sensing)

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Outline

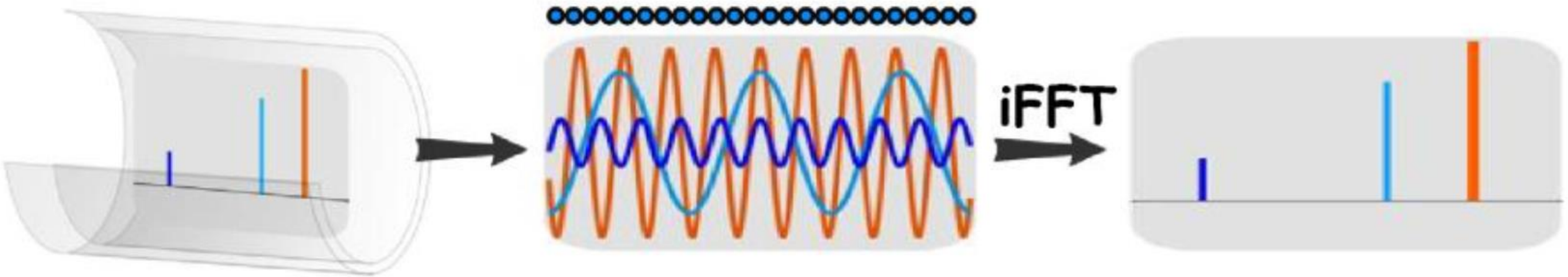
– Intuitive Example of Compress Sensing

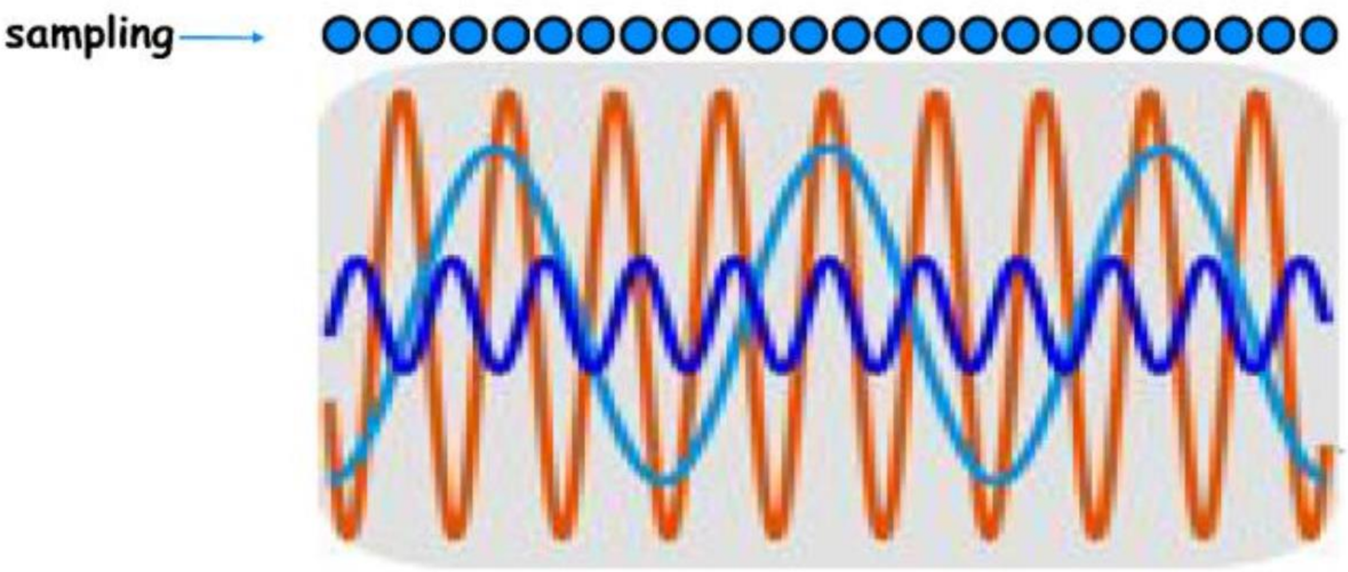
– Compress Sensing

– Restricted Isometry Property

– Example in Image Reconstruction

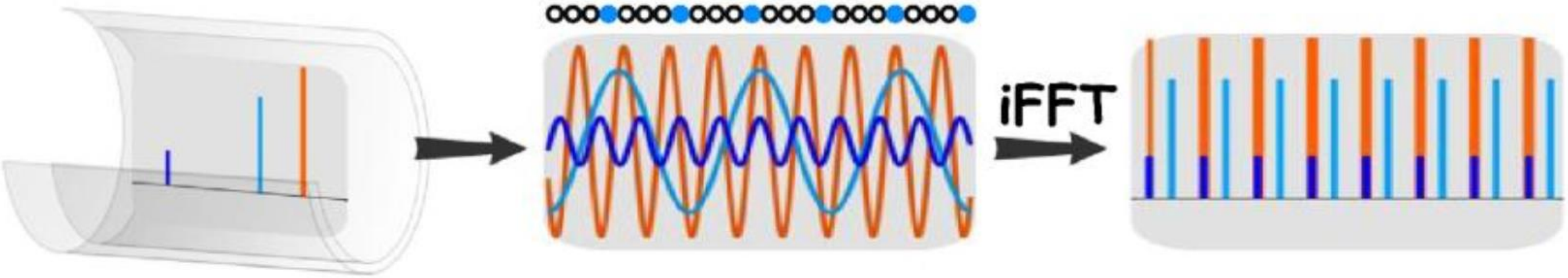
Intuitive Example of Compress Sensing (CS)

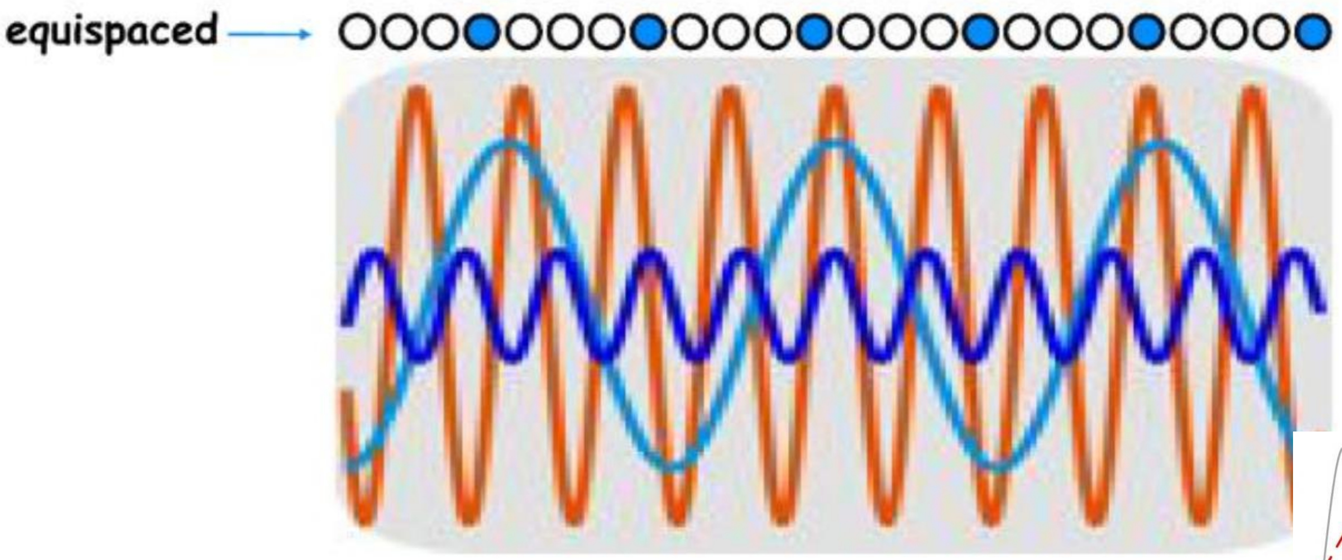


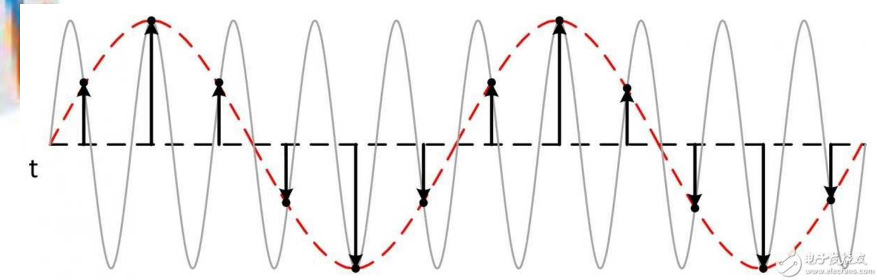


Nyquist

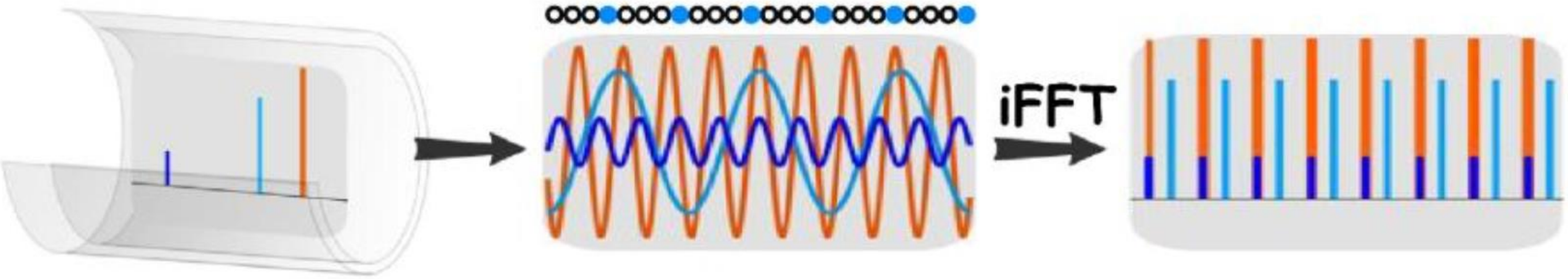
Intuitive Example of Compress Sensing (CS)

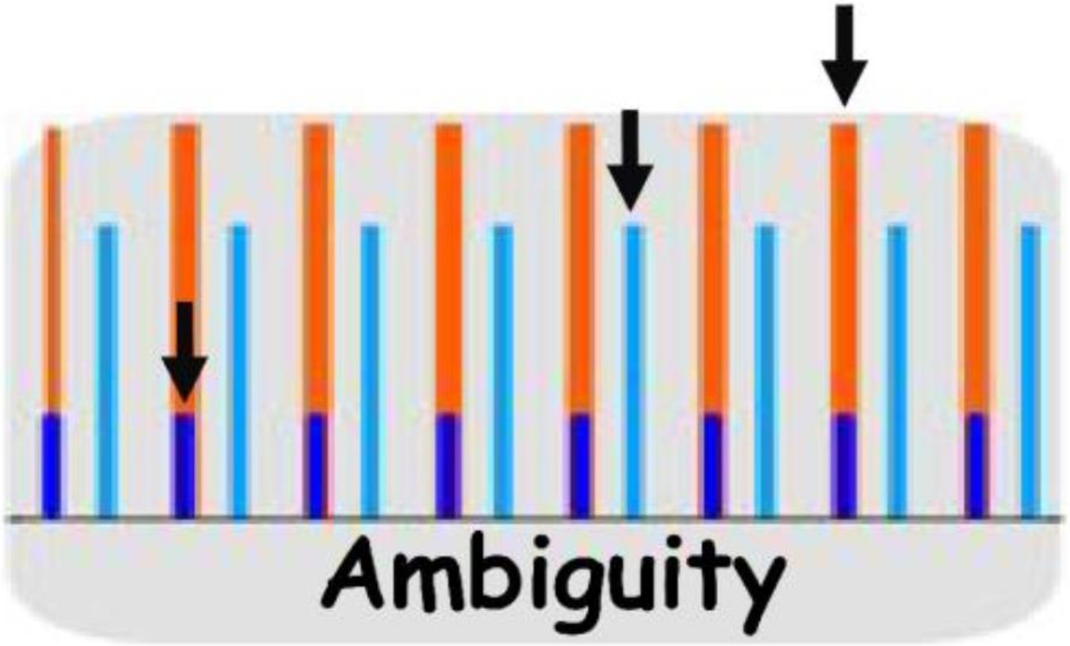






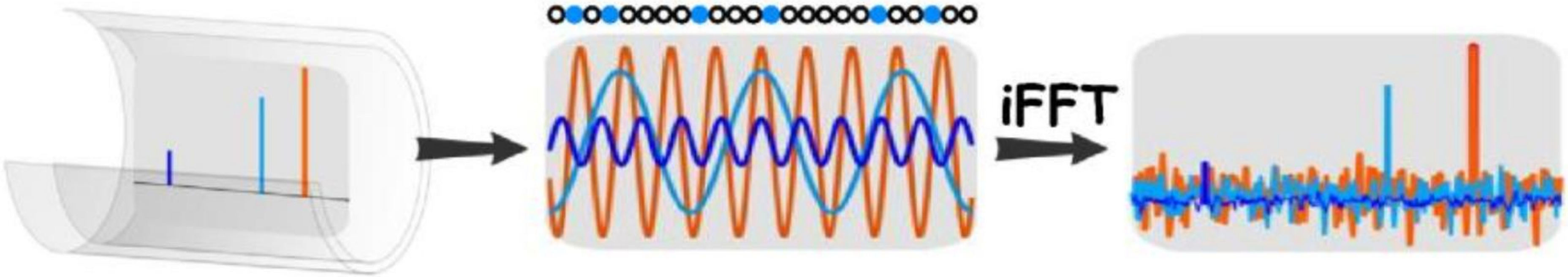
Intuitive Example of Compress Sensing (CS)

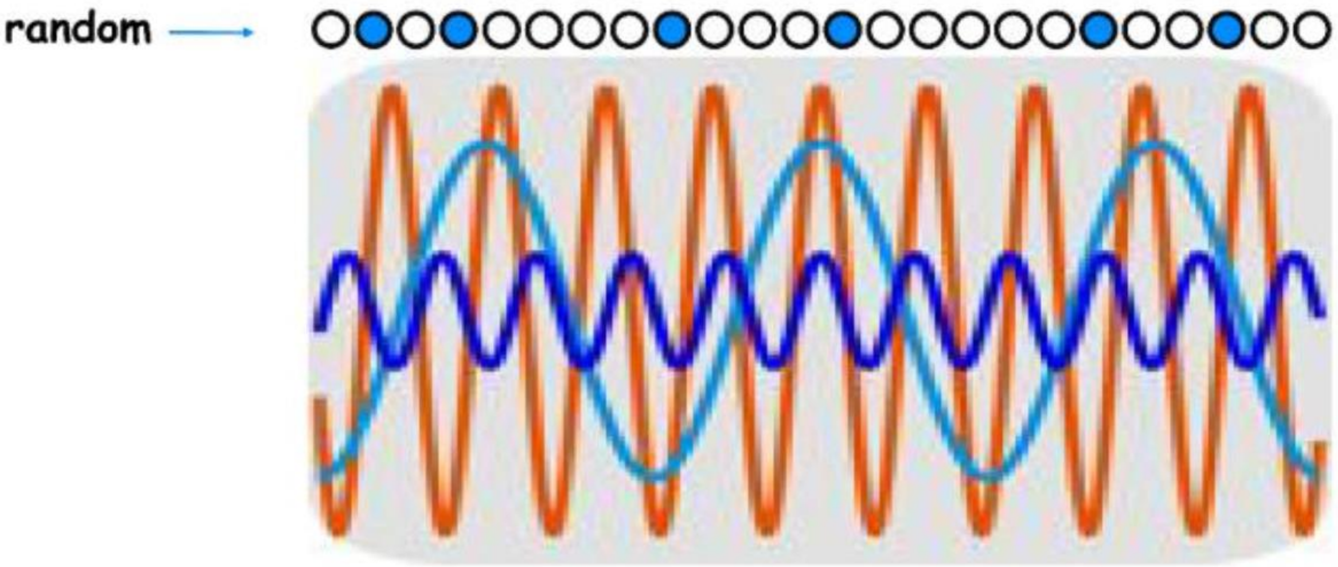




sub-Nyquist

Intuitive Example of Compress Sensing (CS)





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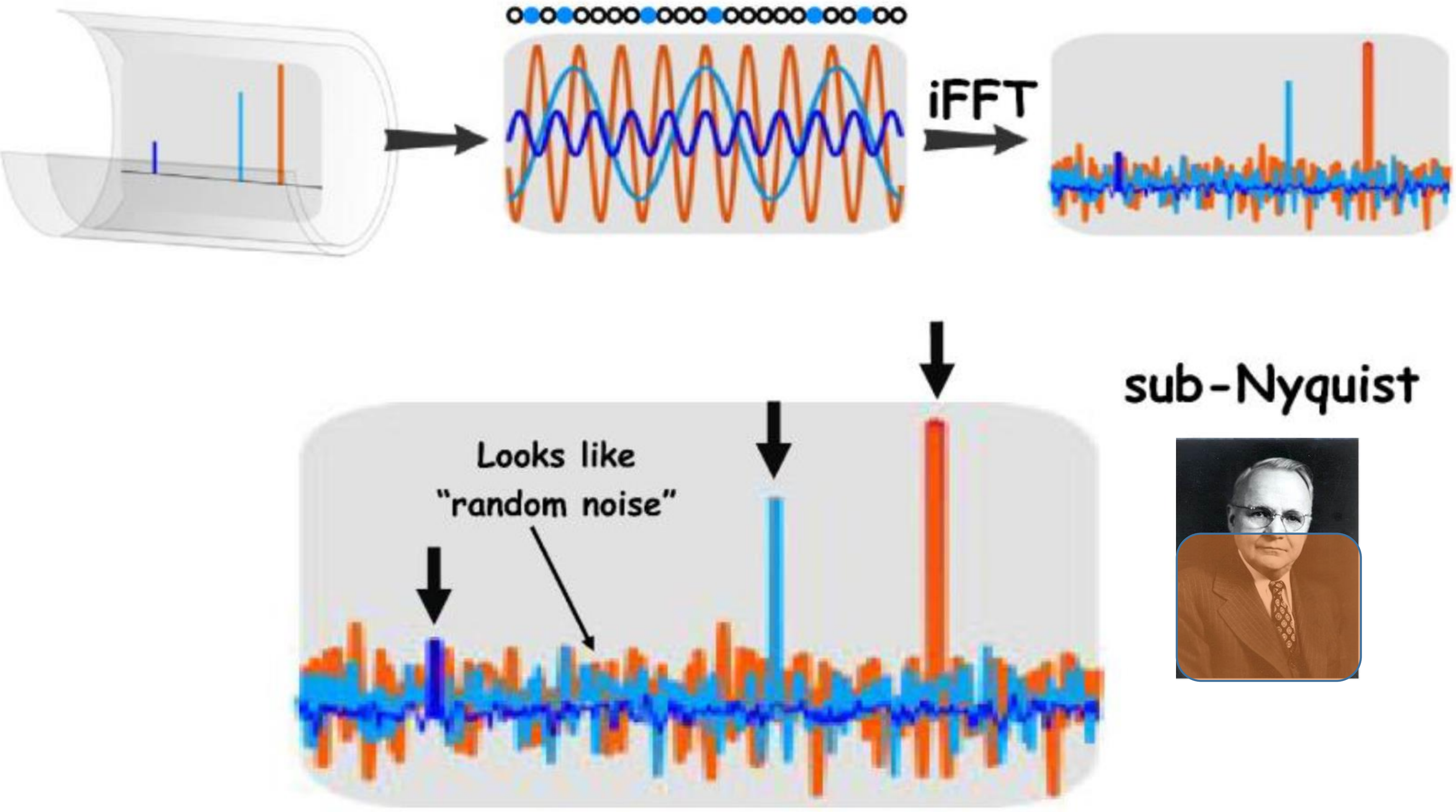
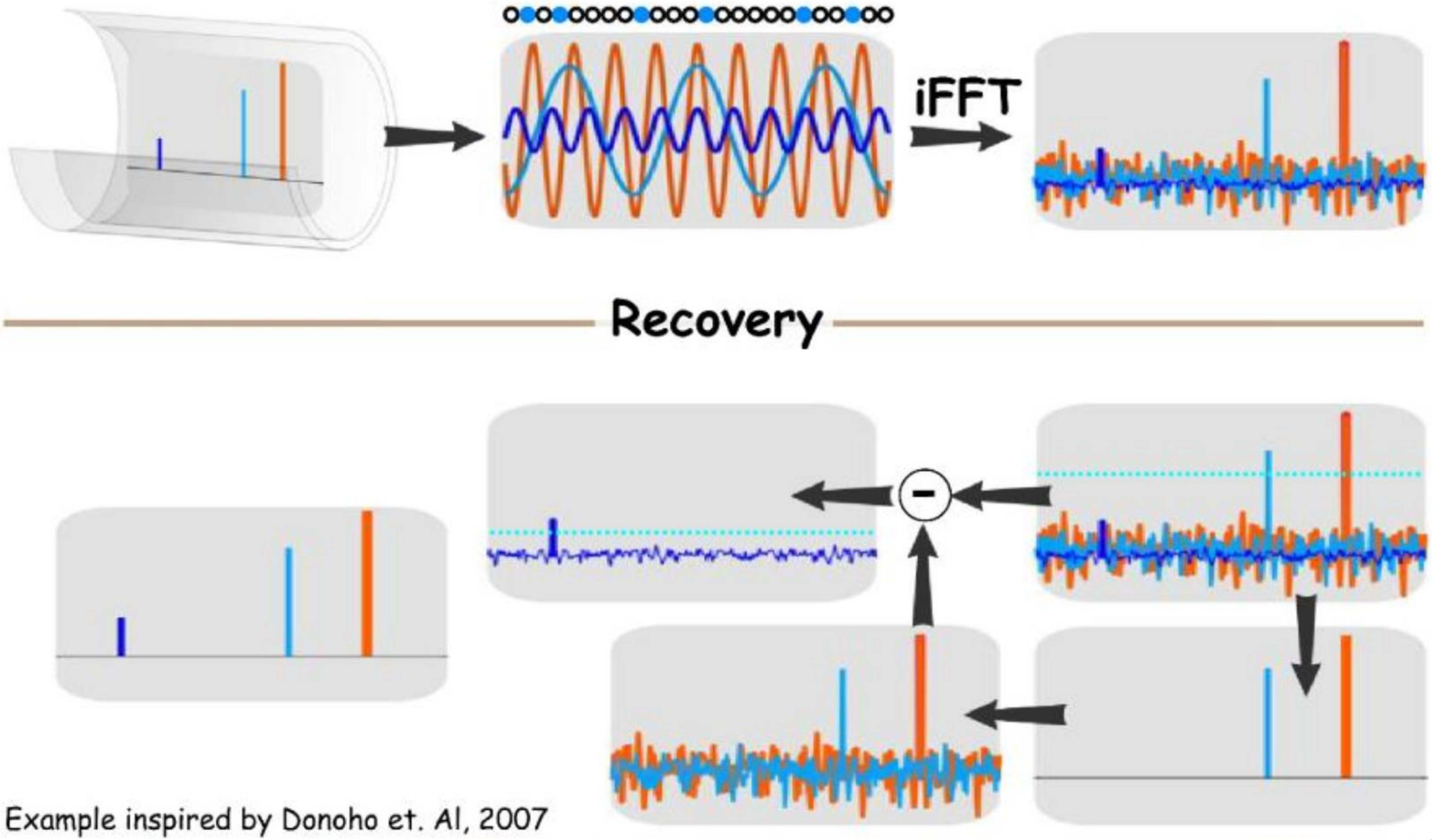


Intuitive Example of Compress Sensing (CS)



Example inspired by Donoho et. Al, 2007

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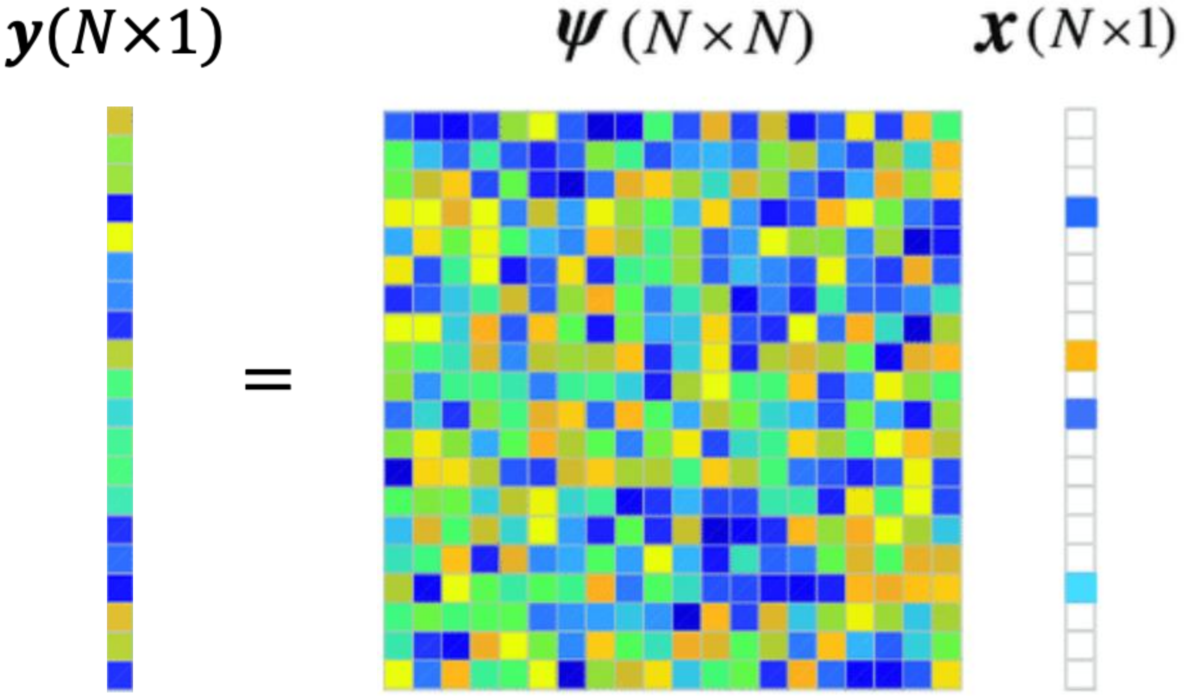
Traditional Sensing

- $x \in \mathcal{R}^N$ is a signal
- Make N linear projections



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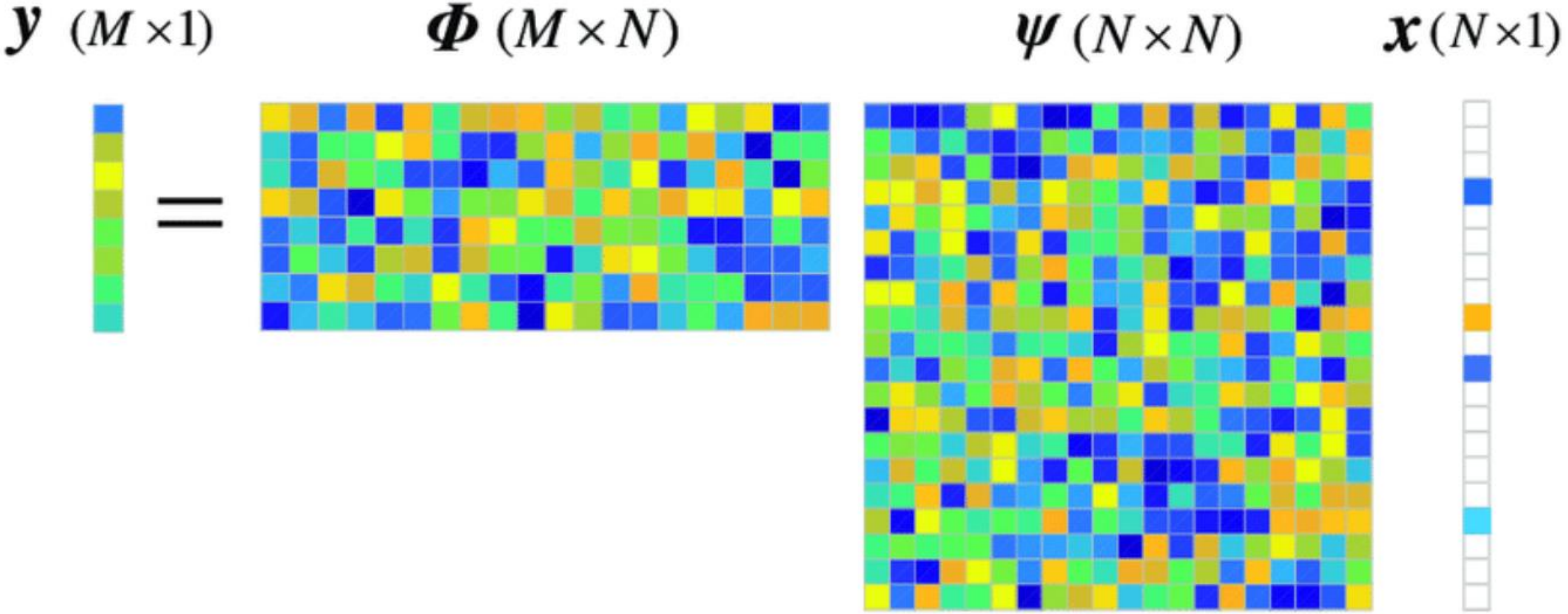
Compress Sensing

- $x \in \mathcal{R}^N$ is a K-sparce signal $(K \ll N)$
- Make $M(K < M \ll N)$ incoherent linear projections



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Restricted Isometry Property (RIP)

Definition 1.1 (Restricted Isometry Constants): Let F be the matrix with the finite collection of vectors $(v_j)_{j\in J} \in \boldsymbol{R}^p$ as columns. For every integer $1 \le S \le |J|$, we define the S-restricted isometry constants δ_S to be the smallest quantity such that F_T obeys

$$(1-\delta_S)\|c\|^2 \le \|F_T c\|^2 \le (1+\delta_S)\|c\|^2 \qquad (1.7)$$

for all subsets $T \subset J$ of cardinality at most S, and all real coefficients $(c_j)_{j\in T}$.

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Restricted Isometry Property (RIP)

陶哲轩和Candès于2005年给出了更为准确的要求：观测矩阵Φ应满足约束等距性条件(Restricted Isometry Property，简称RIP)：

即对于任意和常数，有:

$$(1-\delta_k)\|c\|_2^2 \le \|\phi c\|_2^2 \le (1+\delta_k)\|c\|_2^2$$

Restricted Isometry Property (RIP)

Baraniuk证明：

RIP的等价条件是观测矩阵和稀疏表示基不相关（incoherent）

$$y = \Phi \Psi s$$

$$\Phi \xleftrightarrow{\text{不相关}} \Psi$$

相关性的定义：

$$\mu(\Phi, \Psi) = \sqrt{n} \cdot \max_{1 \le k, j \le n} |\langle \varphi_k, \psi_j \rangle|.$$

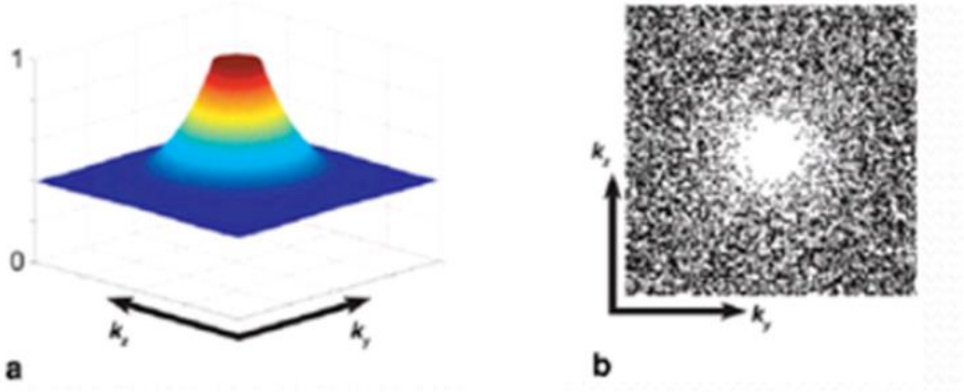
μ的范围： $\mu(\Phi, \Psi) \in [1, \sqrt{n}]$

μ越小，Φ和Ψ越不相关

Restricted Isometry Property (RIP)

陶哲轩和Candès证明：

独立同分布的高斯随机测量矩阵可以成为普适的压缩感知测量矩阵。



CS recovery

- Given $y = \Phi x$ } Under-determined
  find x
- But there' s hope, x is sparse.

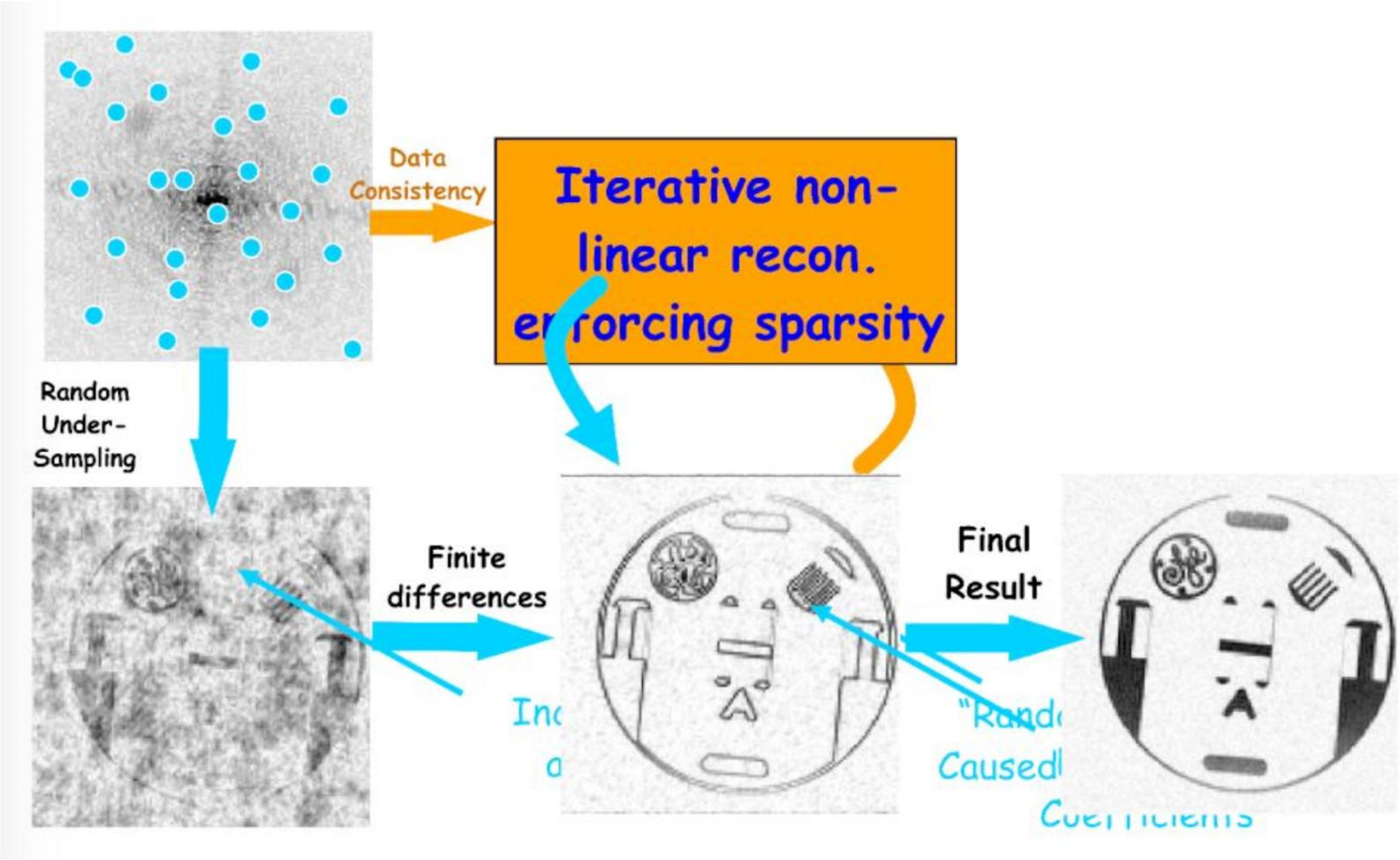
Enforce Sparsity

$$\min_x \|x\|_{L1}, s.t. \|\Phi\Psi x - y\| < \varepsilon$$

Enforce Data Consistency

Need $M \approx Klog(N) \ll N$
Solved by linear-programming

CS recovery



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Not a good idea for CT， why?

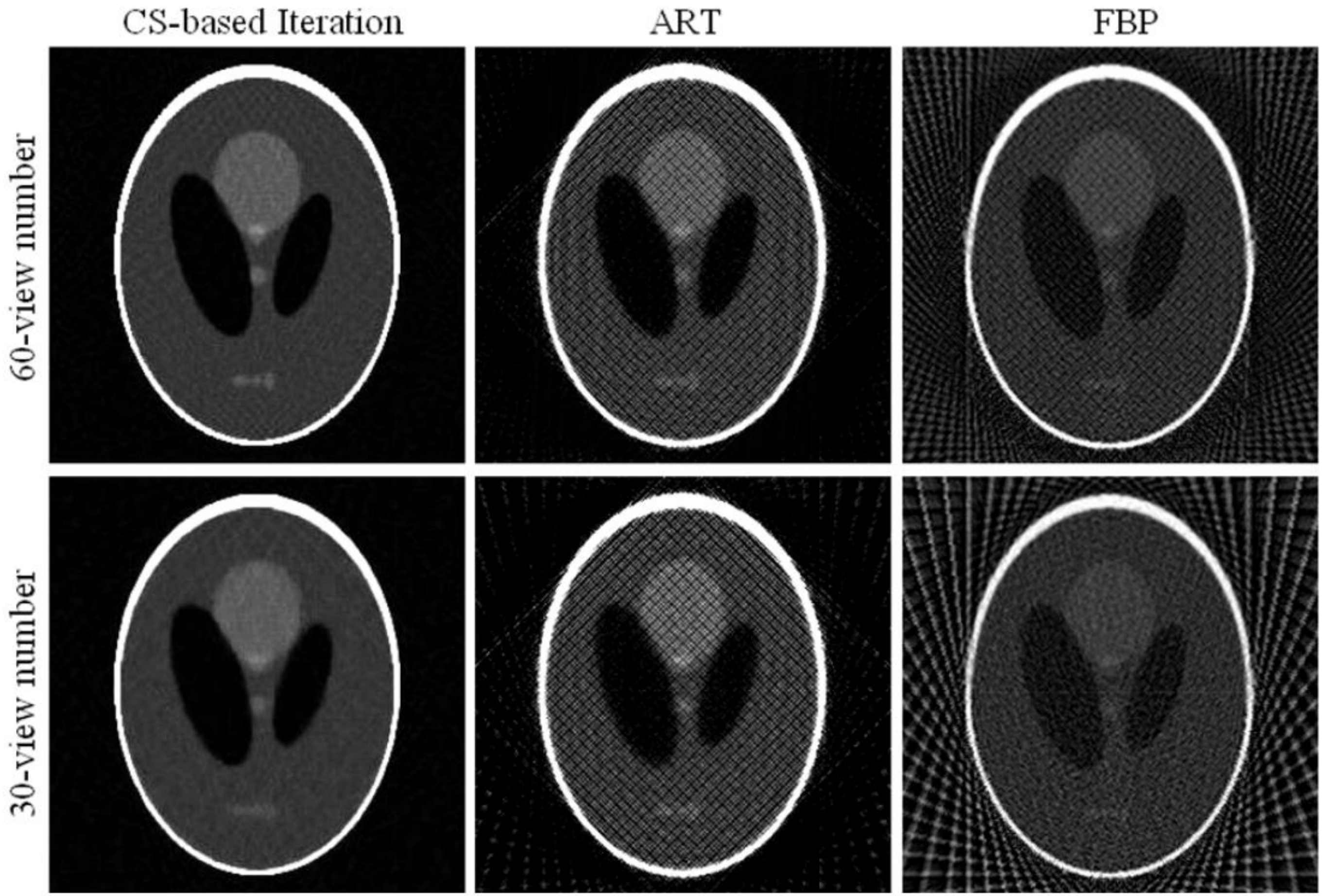


Figure 7 Shepp-Logan phantom images reconstructed from 60 and 30 view numbers using CS-based iterative algorithm (column 1), ART (column 2), and FBP (column 3).

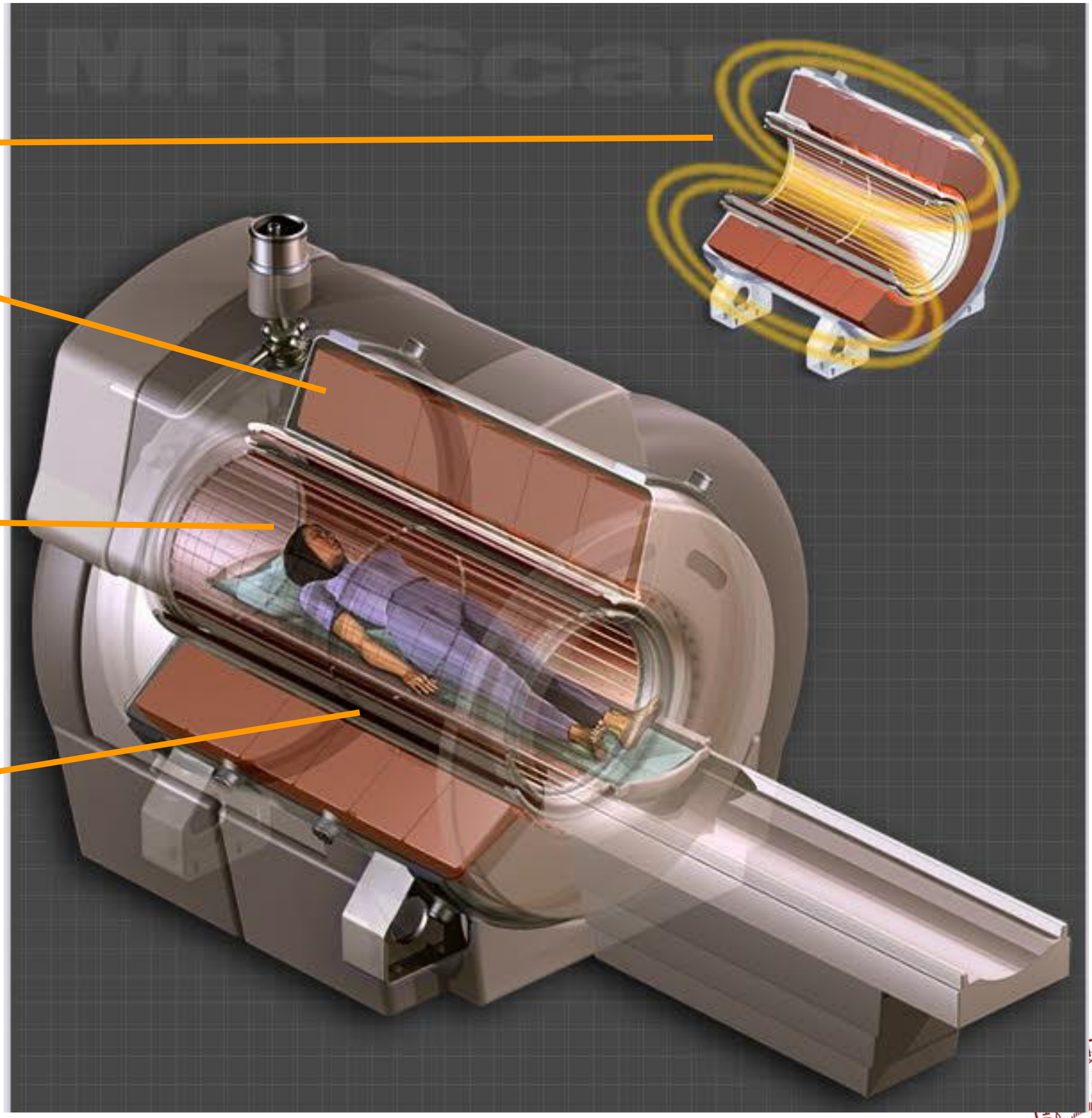
Magnetic Resonance Imaging (MRI): Fourier Encoding
A natural CS hardware

Basic Components of MRI System

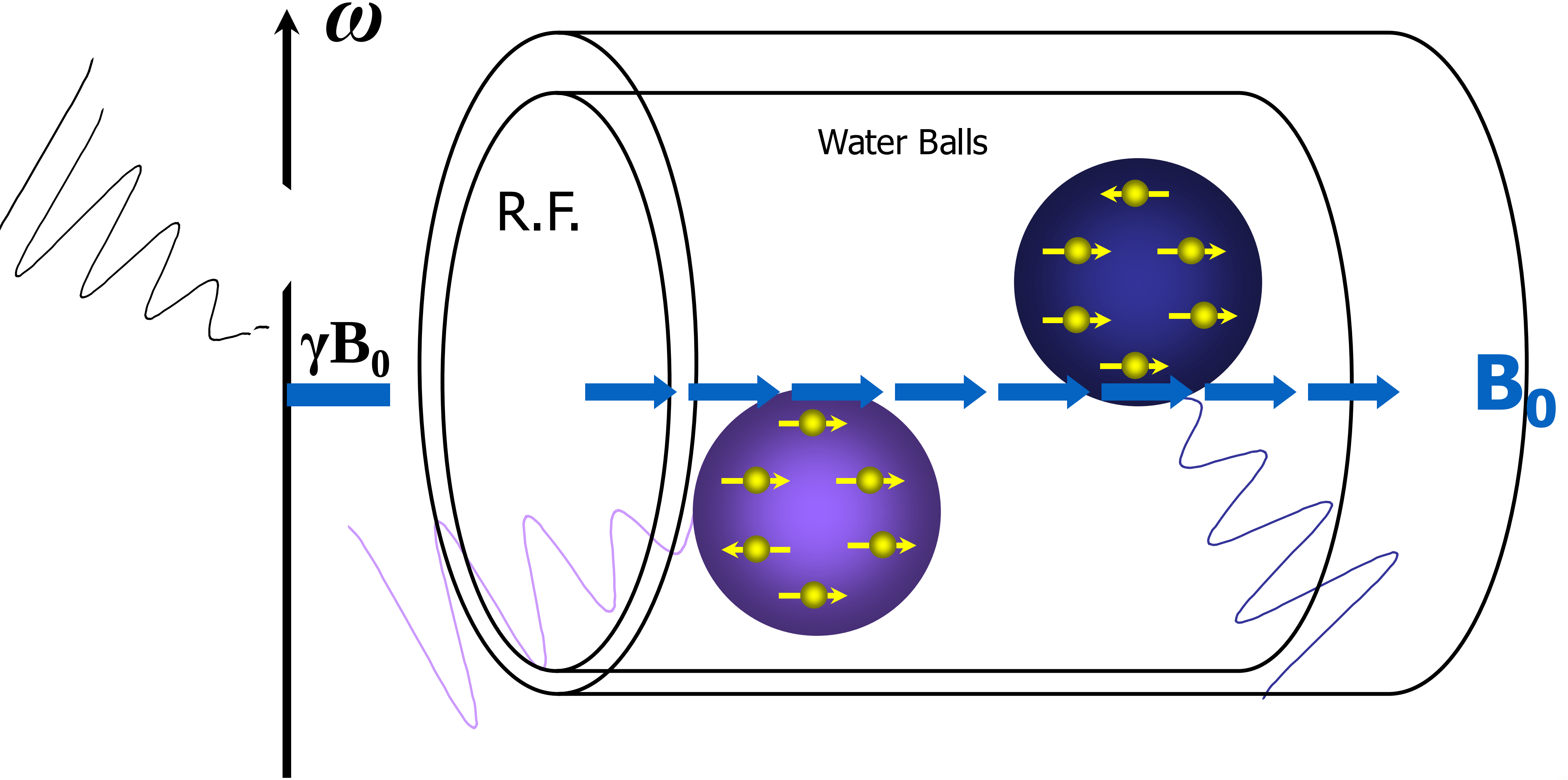
B_0 Field

RF Coil: B_1

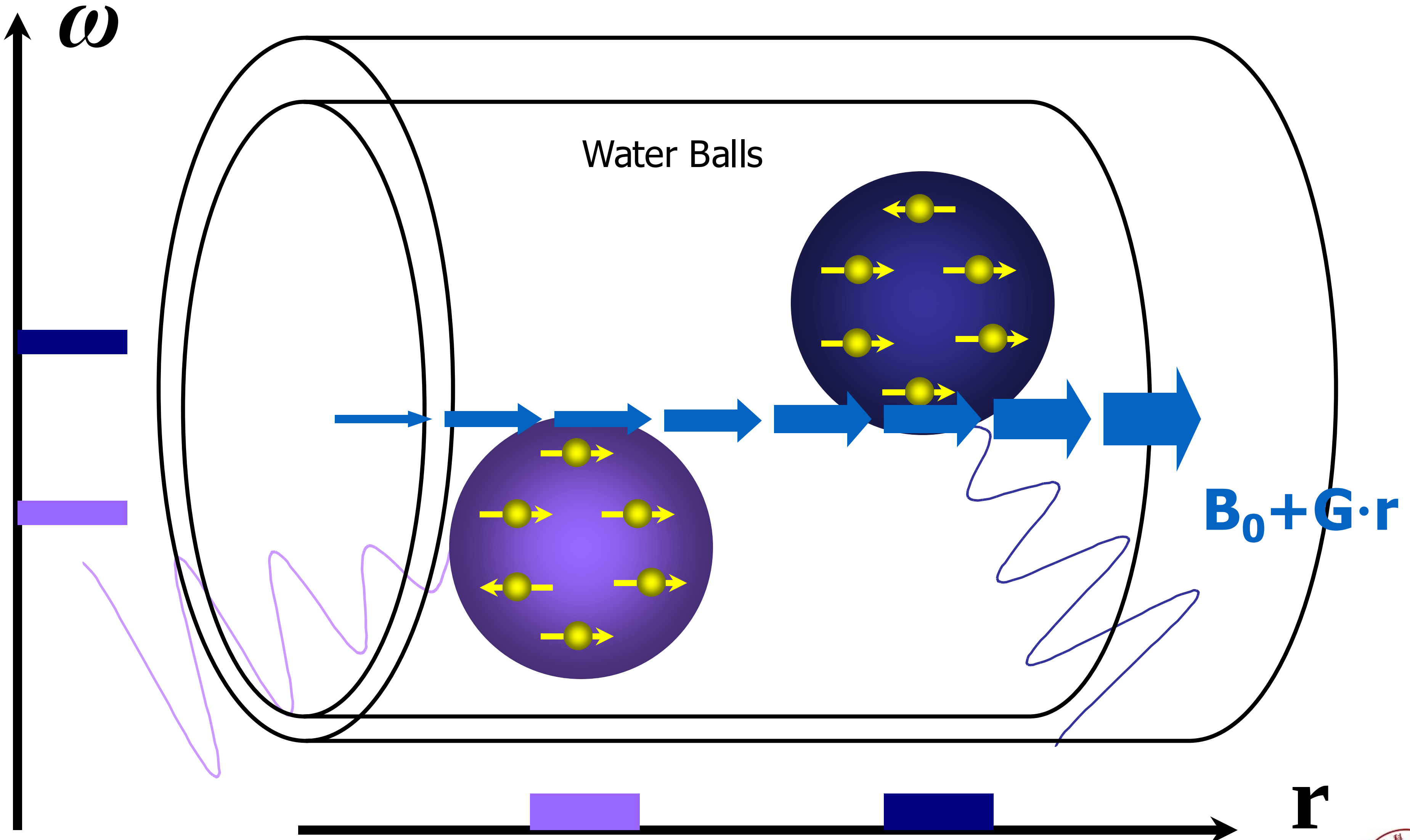
Gradient Coil: G



Magnetic Resonance Phenomenon



“Big” Idea: Magnetic Field Gradient



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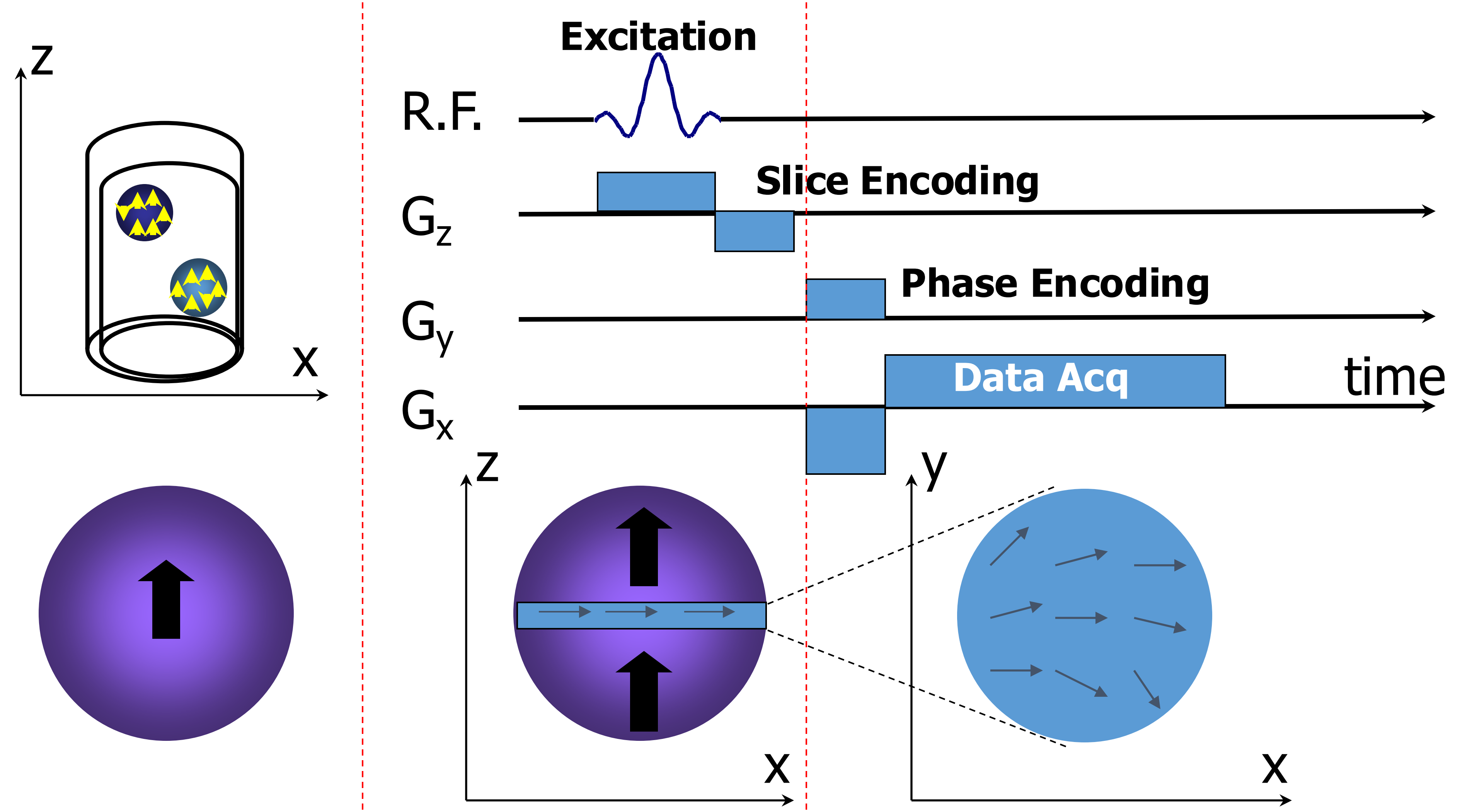
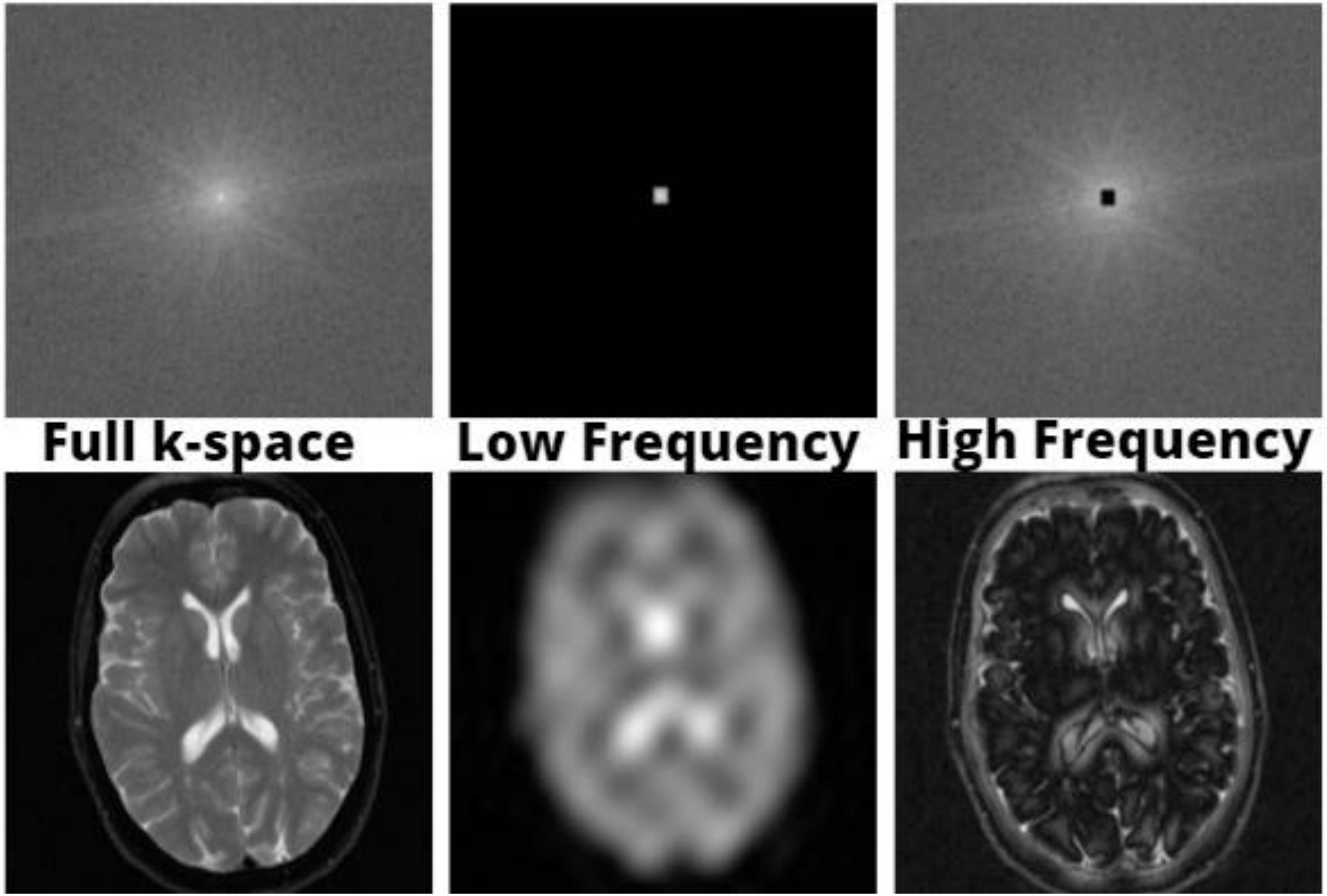
Pulse Sequence: Excitation, Encoding & Acquisition



$\mathbf{M}_z(\mathbf{r})$ $\quad m(x, y)$ $\quad s(t) = \iint m(x, y)\, e^{i\varphi(x,y,t)}\, dxdy$

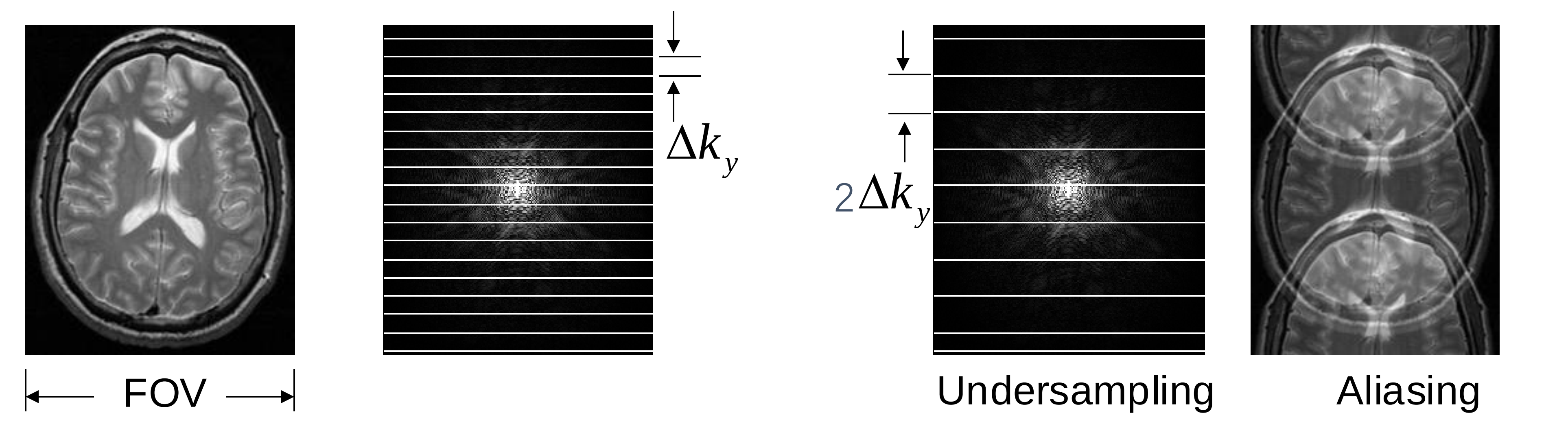
https://mriquestions.com/what-is-phase-encoding.html

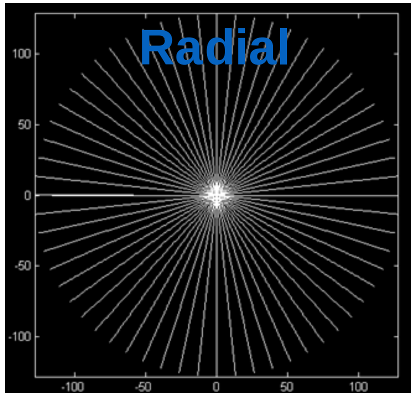
K-Space

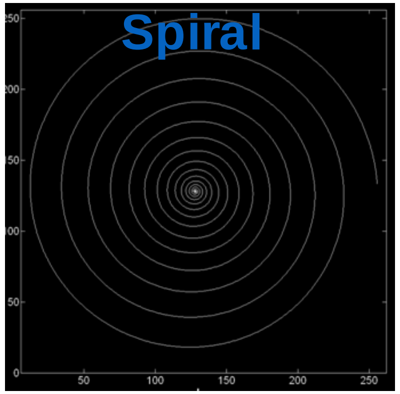


K-Space Sampling

Nyquist rate: f = 2*Bw → $\Delta k_x = 1/FOV_x, \Delta k_y = 1/FOV_y$

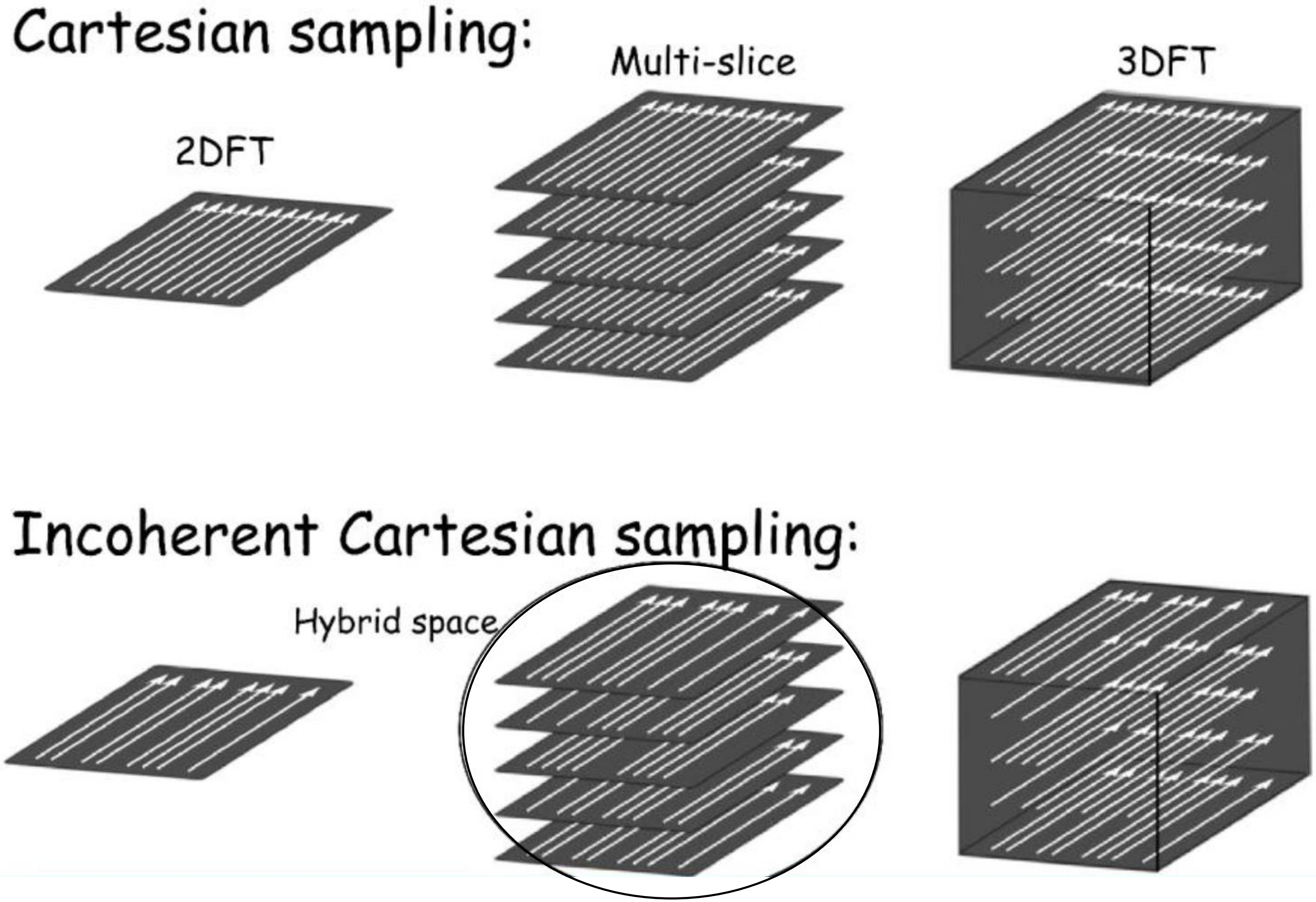




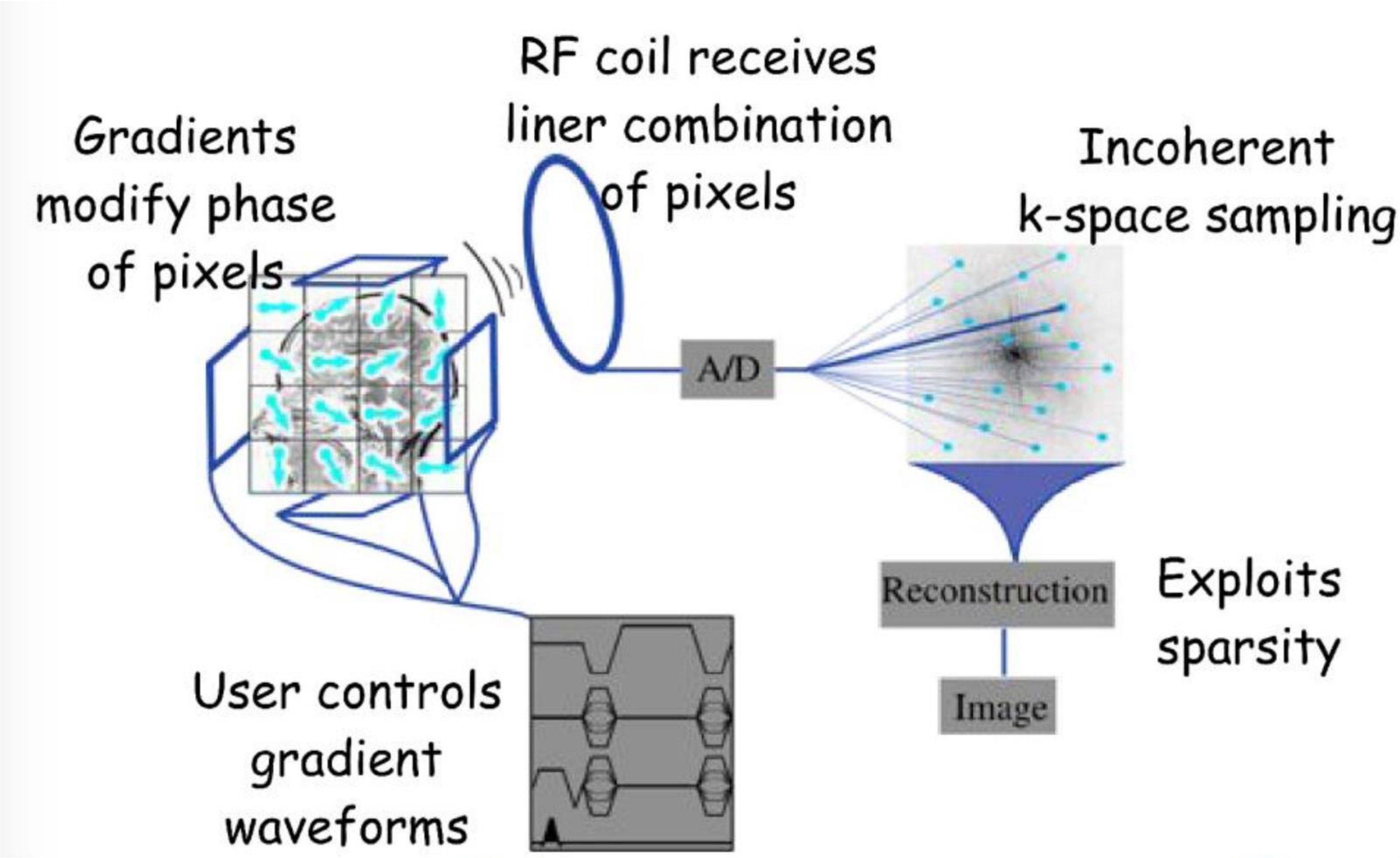


- Non-Cartesian Sampling
  - Design of gradient waveform
  - Image recon needs gridding

MRI – A Natural CS Hardware



Incoherent Sampling



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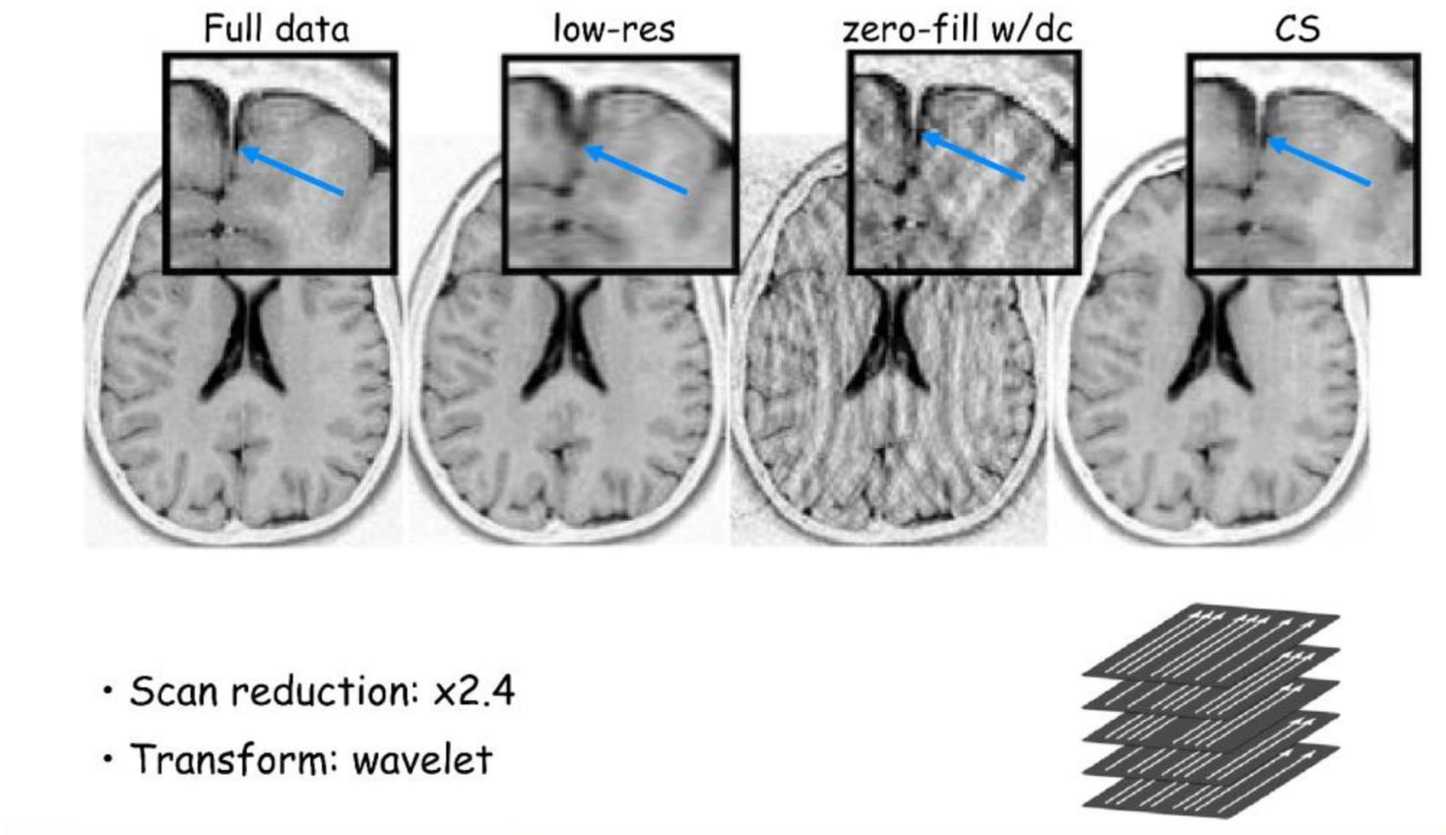
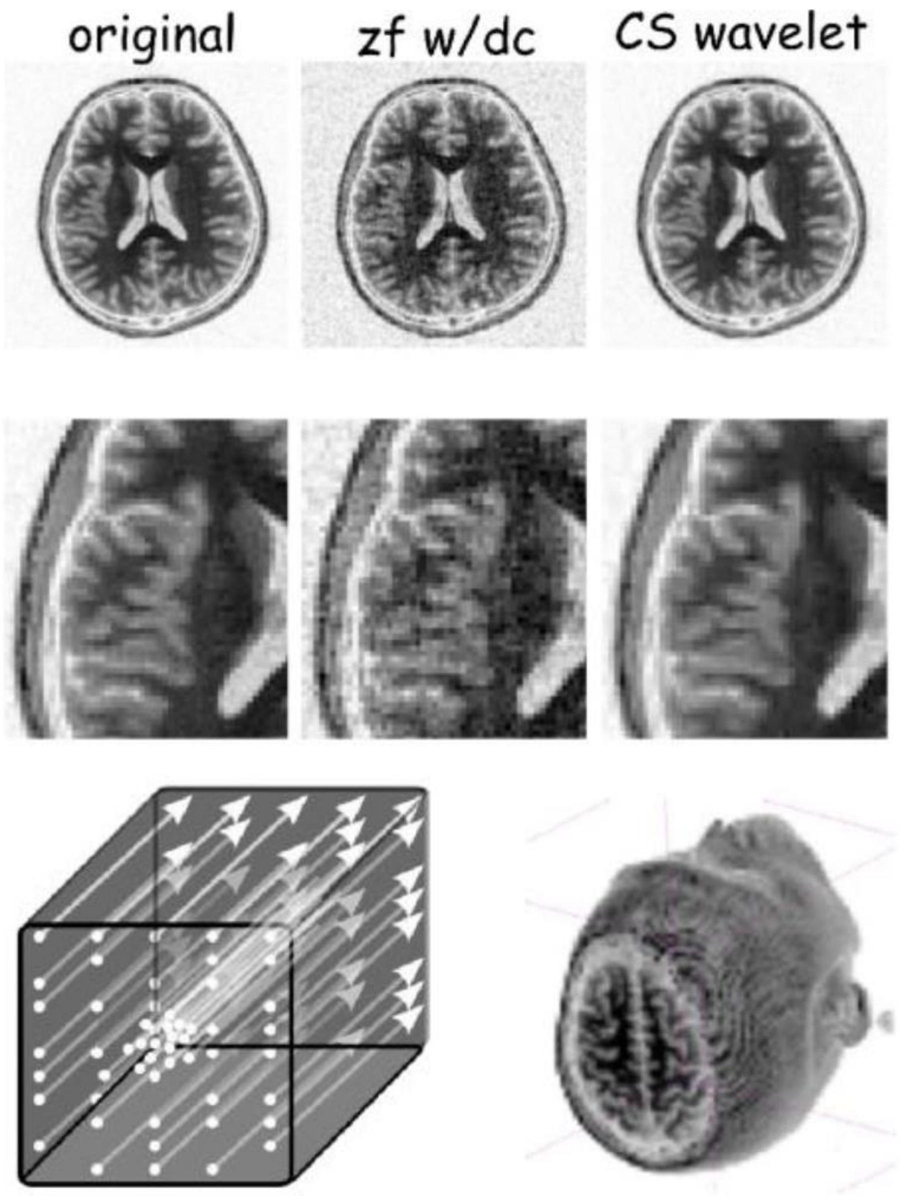
- Scan time reduction: 2.4 times
- Transform: Wavelet





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Resources

- SparseMRI V0.2: matlab code, examples
  http://www.stanford.edu/~mlustig/SparseMRI.html

- Rice University CS page: papers, tutorials, codes, ….
  http://www.dsp.ece.rice.edu/cs/

- IEEE Signal Processing Magazine, special issue on compressive sampling 2008;25(2)

- Blog:
  http://nuit-blanche.blogspot.com/