1. Given that the voltage and current of two-terminal elements adopt the passive sign convention, the instantaneous values are expressed as:

(1) $v(t) = 15\cos(400t + 30°)\ V$, $i(t) = 3\sin(400t + 30°)\ A$;

(2) $v(t) = 8\sin(500t + 50°)\ V$, $i(t) = 2\sin(500t + 140°)\ A$;

(3) $v(t) = 8\cos(250t + 60°)\ V$, $i(t) = 5\sin(250t + 150°)\ A$;

(a) Transform the three voltage & current pairs into phasors.

(b) Try to determine whether the element is a resistor, inductor or capacitor, and determine its value (R=?,C=?, L=?) for (1), (2), and (3), respectively.

a) (1) $\dot{V} = 15\angle 30°\ (V)$

$i(t) = 3\cos(400t - 60°)\ (A)$

$\Rightarrow \dot{I} = 3\angle -60°\ (A)$

(2) $v(t) = 8\cos(500t - 40°)$

$\Rightarrow \dot{V} = 8\angle -40°\ (V)$

$i(t) = 2\cos(500t + 50°)\ (A)$

$\Rightarrow \dot{I} = 2\angle 50°\ (A)$

(3) $\dot{V} = 8\angle 60°\ (V)$

$i(t) = 5\cos(250t + 60°)\ (A)$

$\Rightarrow \dot{I} = 5\angle 60°\ (A)$

b) (1) $30 - (-60) = 90°$

$\Rightarrow$ voltage leads current by $90°$

$\Rightarrow$ inductor

$$L = \frac{\dot{V}}{j\omega \dot{I}} = \frac{15\angle 30°}{j400(3\angle -60°)} = 12.5\,mH$$

(2) $-40 - 50 = -90°$

$\Rightarrow$ voltage lags current by $90°$

$\Rightarrow$ capacitor

$$C = \frac{\dot{I}}{j\omega \dot{V}} = \frac{2\angle 50°}{j500(8\angle -40°)} = 0.5\,mF$$

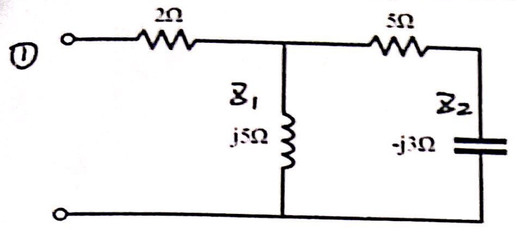
(3) $60 - 60 = 0$

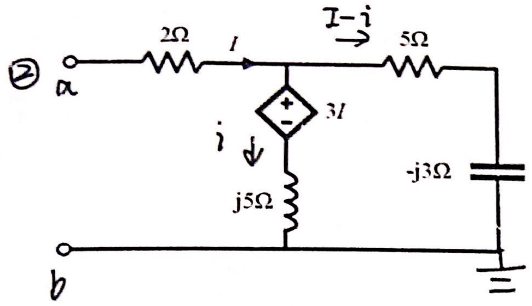
$\Rightarrow$ voltage and current in same phase

$\Rightarrow$ resistor

$$R = \frac{\dot{V}}{\dot{I}} = \frac{8\angle 60°}{5\angle 60°} = 1.6\Omega$$

2. Find the equivalent impedance for the following two circuits.





① $Z_{eq} = 2 + j5 // (5-j3) = 2 + \dfrac{j5(5-j3)}{5+j2} = 7.11\angle 27.43^\circ$ (Ω)

② $3\tilde{I} + j5\tilde{i} = 5(\tilde{I}-\tilde{i}) - j3(\tilde{I}-\tilde{i})$

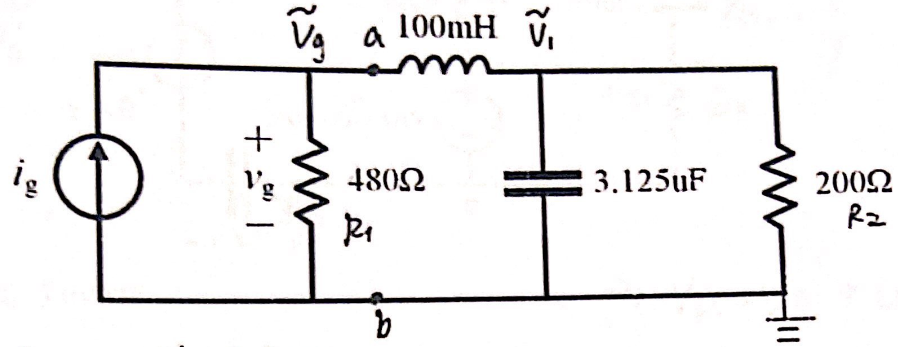
$\Rightarrow \tilde{i} = \dfrac{-2\tilde{I}+j3\tilde{I}}{-5+3j-5j} = \dfrac{j3-2}{-5-2j}\tilde{I}$

$\dot{V}_{ab} = 5\tilde{I} + j5\tilde{i}$

$= (8.30\angle 4.76)\tilde{I}$ (A)

$\Rightarrow Z_{eq} = \dfrac{\dot{V}_{ab}}{\tilde{I}} = 8.30\angle 4.76$ (Ω)

3. Given $i_g = 60\cos(\omega t)$ mA.
If i_g is in phase with v_g, calculate the angular frequency of the current source and find the expression for v_g.



$\tilde{i}_g = 60\angle 0^\circ = 0.06\angle 0^\circ\ (A)$

$$Z_{eq}' = (Z_L + Z_C // Z_{R_2}) = 0.1j\omega + \frac{1}{+3.125\times10^{-6}\omega j + \frac{1}{200}}$$

$$= 0.1j\omega + \frac{200}{+6.25\times10^{-4}\omega j + 1}$$

$$= 0.1j\omega + \frac{200(1 - 6.25\times10^{-4}\omega j)}{1 + 6.25^2\times10^{-8}\omega^2}$$

$\because$ in phase $\therefore$ $Im(Z_{eq}) = 0$

$$0.1\omega + \frac{-0.125\omega}{1 + 6.25^2\times10^{-8}\omega^2} = 0 \Rightarrow \omega = 800 \text{ rad/s}$$

$Z_L = j\omega L = 0.1j\omega = 80j\ \Omega$

$Z_C = \frac{1}{j\omega C} = \frac{-3.2\times10^5 j}{\omega} = -400j\ \Omega$

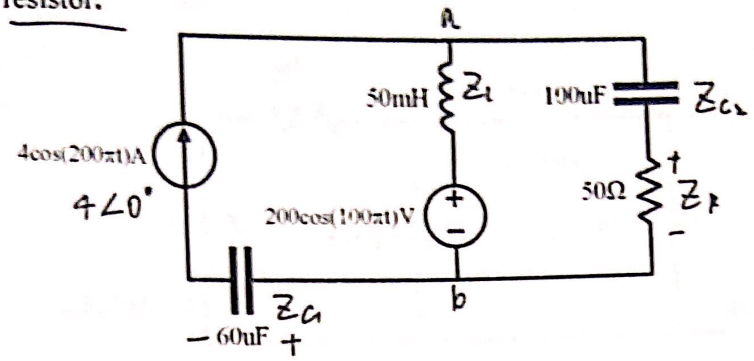
$Z_{ab(right)} = Z_L + Z_C // Z_{R_2} = 160\ \Omega$

$\Rightarrow \tilde{V}_g = \tilde{i}_g \times 480 // Z_{ab(right)} = 7200\angle 0^\circ (V)$

$\Rightarrow V_g = 7200\cos(800t)$ (mV)

$= 7.2\cos(800t)$ (V)

4. Use superposition method to find the Time Domain voltage expression for the resistor.



① current source

$Z_{eq(ab)} = Z_L \parallel (Z_{C_2} + Z_R)$

$Z_{C_1} = \frac{1}{j\omega C_1} = j-26.53\,\Omega$

$Z_L = j\omega L = j31.42\,\Omega$

$Z_{C_2} = \frac{1}{j\omega C_2} = j-15.92\,\Omega$

$Z_R = 50\,\Omega$

$\Rightarrow Z_{eq(ab)} = 18.01 + j25.84\,(\Omega)$

$\tilde{V}_{R_1} = \tilde{I} \cdot Z_{eq(ab)} \frac{Z_R}{Z_{C_2} + Z_R} = 120.05\angle 72.79°\,(V)$

$\Rightarrow V_{R_1} = 120.05\cos(200\pi t + 72.79°)\,(V)$

② voltage source

$Z_L' = j\omega' L = 15.71\angle 90°\,\Omega$

$Z_{C_2}' = \frac{1}{j\omega' C_2} = 31.83\angle -90°\,\Omega$

$Z_R = 50\,\Omega$

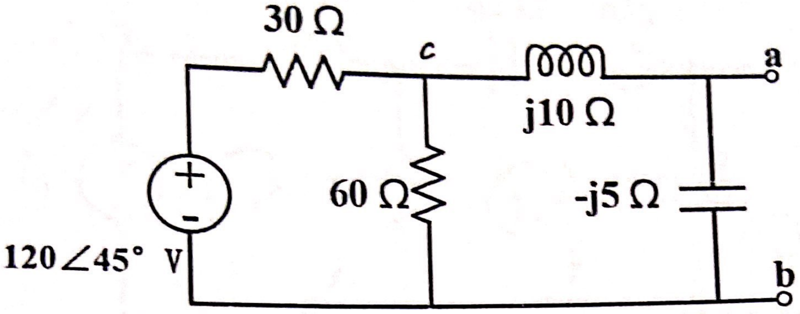
$\Rightarrow \tilde{V}_{R_2} = 200\angle 0 \times \frac{Z_R}{Z_L' + Z_{C_2}' + Z_R} = 190.35\angle 17.87°\,(V)$

$\Rightarrow V_{R_2} = 190.35\cos(100\pi t + 17.87°)$

$\Rightarrow V_R = V_{R_1} + V_{R_2}$

$= 120.05\cos(200\pi t + 72.79°) + 190.35\cos(100\pi t + 17.87°)$

5. For the circuit below, please find the Thevenin equivalent circuit with respect to node **a** and node **b**.



$$Z_{cb} = 60 \,//\, (j10 - j5)$$

$$= \frac{60 \times j5}{60 + j5} = 4.98\angle 85.24^\circ\ \Omega$$

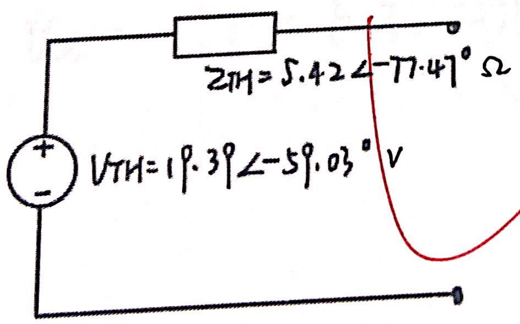
$$\Rightarrow V_{TH} = V_{oc}$$

$$= 120\angle 45^\circ \times \frac{4.98\angle 85.24^\circ}{30 + 4.98\angle 85.24^\circ} \times \frac{-j5}{j10 - j5} = 19.39\angle -59.03^\circ\ (V)$$

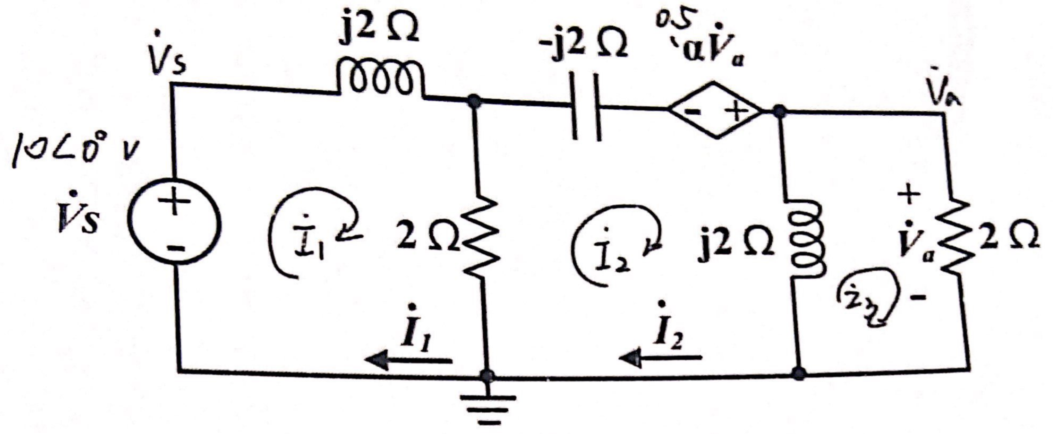
$$Z_{TH} = (30 // 60 + j10) \,//\, (-j5)$$

$$= 5.42\angle -77.47^\circ\ (\Omega)$$

$\Rightarrow$ Thevenin equivalent:



6. Use **nodal or mesh** methods to find $\dot{V}_a$, $\dot{I}_1$and $\dot{I}_2$for the circuit below, assuming that $\dot{V}_S$= $10\angle 0°\ V$, $\alpha = 0.5$.



mesh analysis:

$$\begin{cases} -\dot{V}_s + j2\dot{I}_1 + 2(\dot{I}_1 - \dot{I}_2) = 0 \\ 2(\dot{I}_2 - \dot{I}_1) - j2\dot{I}_2 - \alpha\dot{V}_a + j2(\dot{I}_2 - \dot{I}_3) = 0 \\ j2(\dot{I}_3 - \dot{I}_2) + 2\dot{I}_3 = 0 \\ \dot{V}_a = 2\dot{I}_3 \end{cases}$$

$$\Rightarrow \begin{cases} \dot{I}_1 = 6.52\angle -57.53° \text{ (A)} \\ \dot{I}_2 = 4.47\angle -26.57° \text{ (A)} \end{cases}$$

$$\dot{I}_3 = \frac{j2\dot{I}_2}{j2+2} = 3.16\angle 18.43° \text{ (A)}$$

$$\Rightarrow \dot{V}_a = 2\dot{I}_3 = 6.32\angle 18.43° \text{ (V)}$$