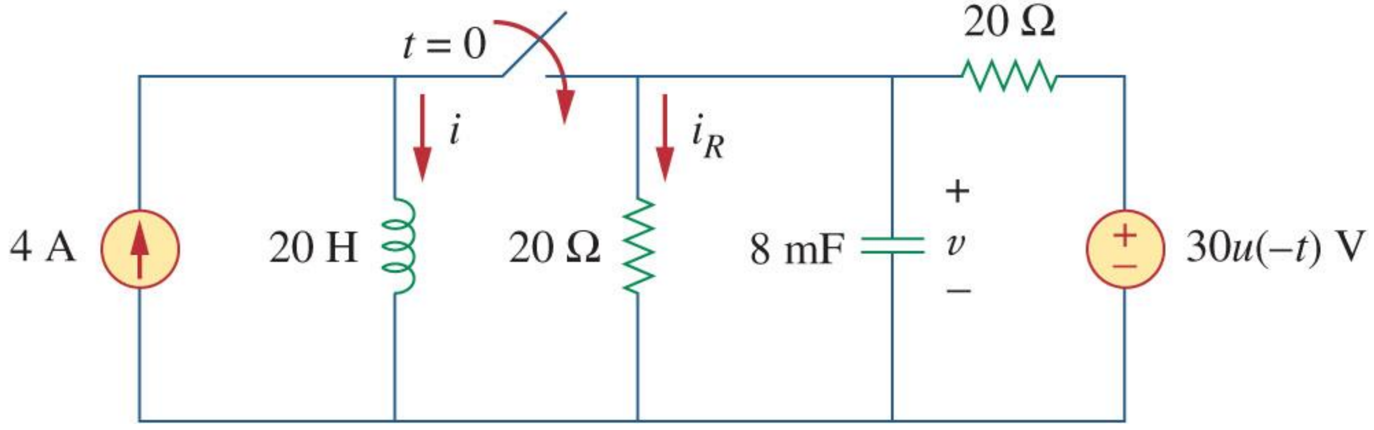
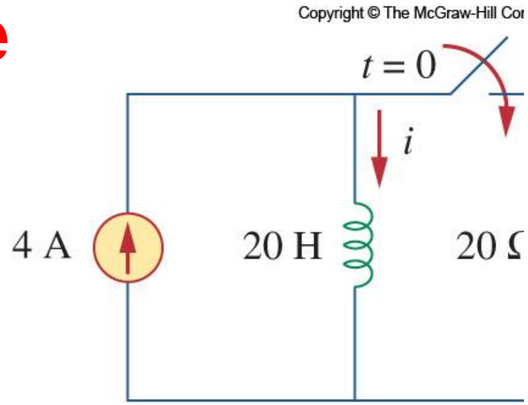
Example

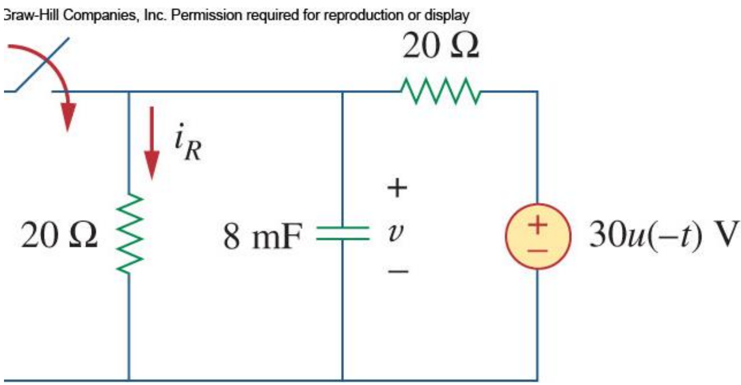
- Find $i(t)$ and $i_R(t)$ for $t > 0$.



Example

$t < 0$

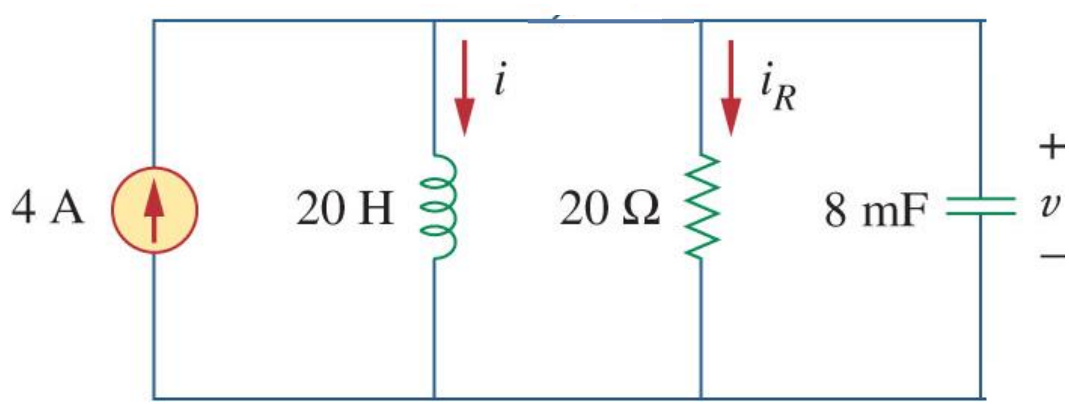




Initial values : $i(0^+) = 4A$

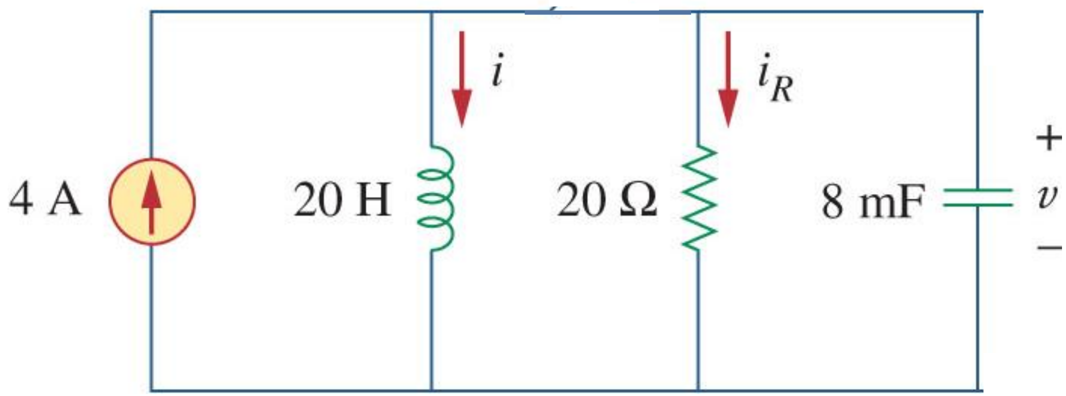
$$v(0^+) = \frac{20}{20+20} \times 30V = 15V$$

$t = 0^+$



$$v(0^+) = 15V = L\frac{di(0^+)}{dt}$$

$t = 0^+$



For $t > 0$, $\alpha = \frac{1}{2RC} = \frac{1}{2\times10\times8\times10^{-3}} = 6.25, \omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{20\times8\times10^{-3}}} = 2.5$

$$s_{1,2} = -6.25 \pm 5.7282$$

$$i(t) = I_{s.s.} + A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

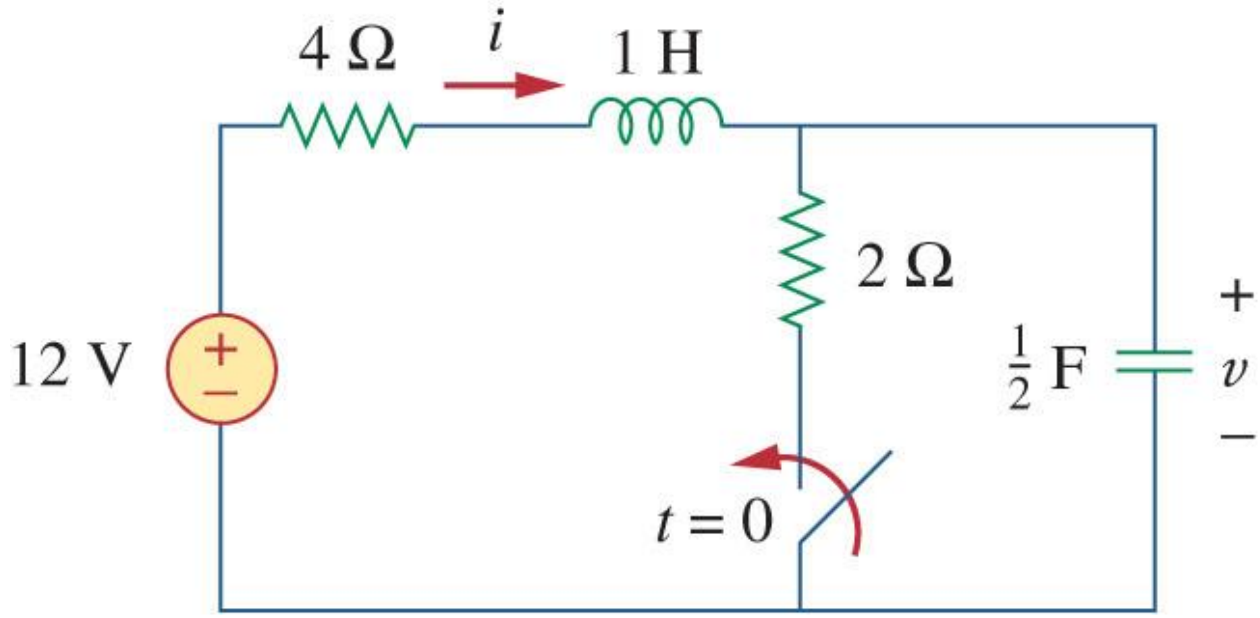
Apply

$$i(0^+) = 4A$$

$$\frac{di(0^+)}{dt}$$

General Second-Order Circuits

- An example



General Second-Order Circuits

- The principles of solving the series/parallel forms of RLC circuits can **be applied/extended** to **general** **second-order circuits**, by taking the following six steps:

1. First determine the initial conditions, $x(0)$ *and* $dx(0)/dt$.

2. Applying KVL and KCL, to find a general second-order differential equation about $x(t)$.

Then solve the equation:

3. **Depending on the roots of C.E. , the form of general solution** $x_{g.s.}(t)$ **(3 cases) of homogeneous equation can be determined.**

4. We obtain the **particular solution** by observation/calculation, **specially** for a DC/step response $x_{p.s.}(t)=x(\infty)$

5. The complete response = general solution + particular solution.

$$x(t)=x_{P.S.}(t)+x_{G.S.}(t)$$

6. Using the initial conditions to determine the constants of $x(t)$.

$x(t)$ = unknown variable (voltage or current)

Differential equation: $x'' + ax' + bx = c$

Initial conditions: $x(0)$ and $x'(0)$

Final condition: $x(\infty) = \dfrac{c}{b}$

$$\alpha = \frac{a}{2} \qquad \omega_0 = \sqrt{b}$$

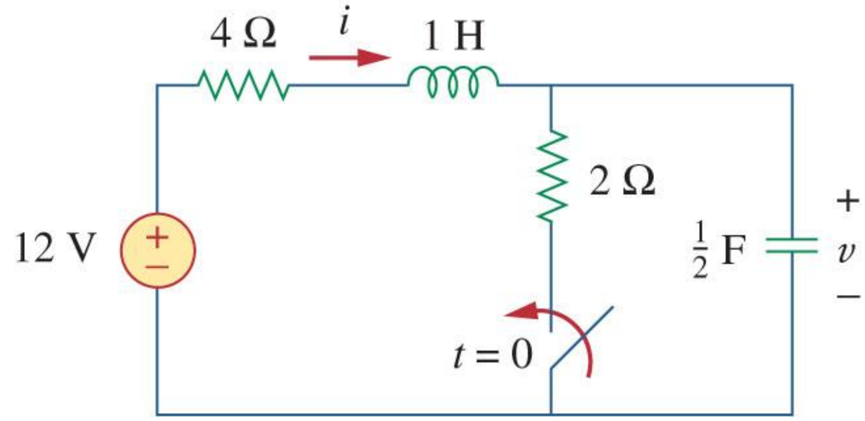
$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2} \qquad s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$$

Overdamped Response $\alpha > \omega_0$

$$x(t) = [A_1 e^{s_1 t} + A_2 e^{s_2 t} + x(\infty)]$$

General RLC Circuits

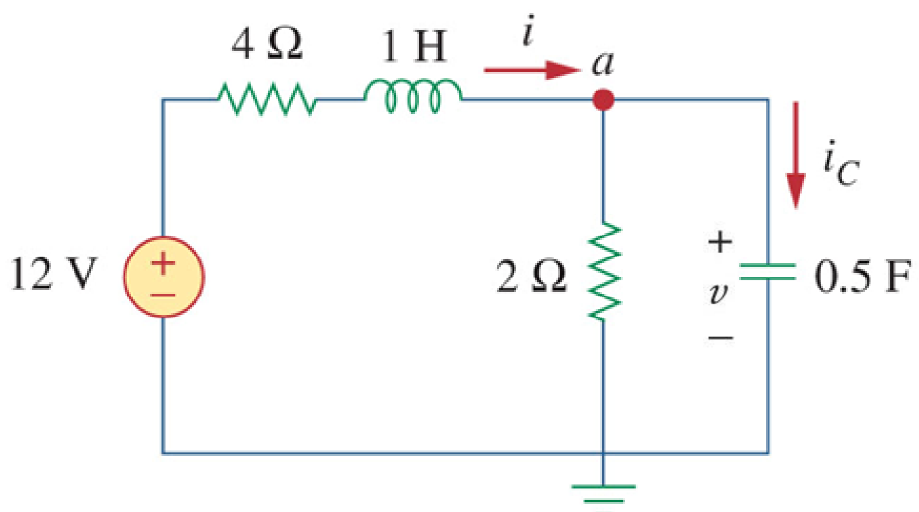
- Find the complete response $v(t)$ for $t > 0$ in the circuit.



1. Initial conditions

$$v(0^+) = v(0^-) = 12V,\ i(0^+) = i(0^-) = 0$$

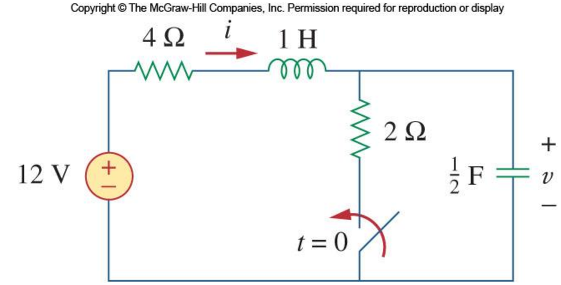
$t = 0$

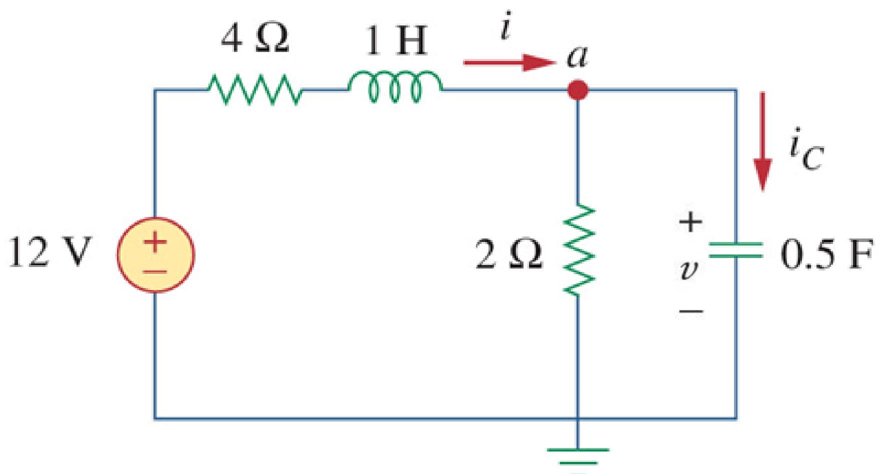


$$\frac{dv(0^+)}{dt} = \frac{i_C(0^+)}{C} = \frac{-12/2}{0.5} = -12\ V/s$$

General RLC Circuits

- Find the complete response $v(t)$ for $t > 0$





2. To find a general second-order differential equation about $x(t)$

$$\frac{d^2v}{dt^2} + 5\frac{dv}{dt} + 6v = 24$$

3. General Solution:

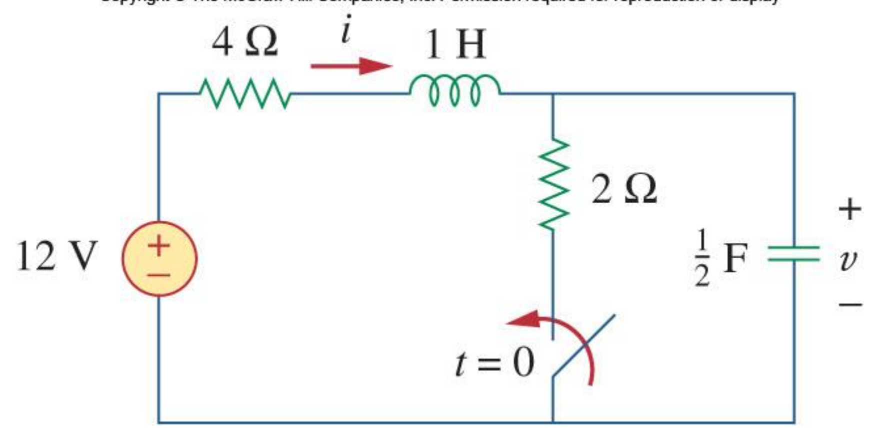
⇨ General Solution $v_t(t) = A_1e^{-2t} + A_2e^{-3t}$

4. Particular Solution : Steady-state response $v_{ss}(t) = 4V$

5. Put together : $v(t) = 4 + \mathrm{A}_1e^{-2t} - \mathrm{A}_2e^{-3t}$

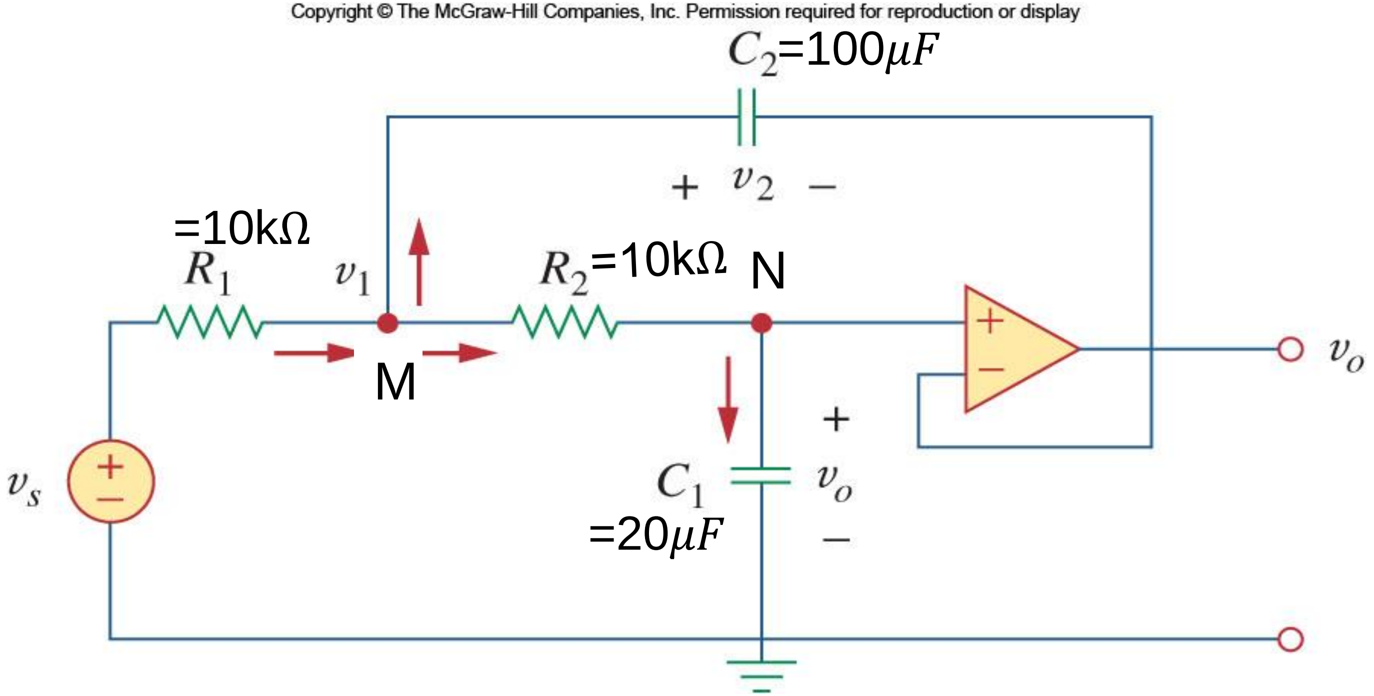
6. Using initial conditions to determine A_1 , A_2

- Use mesh analysis method to find the complete response $i(t)$ for $t > 0$ in the circuit.



Example of 2nd-order op-amp circuits

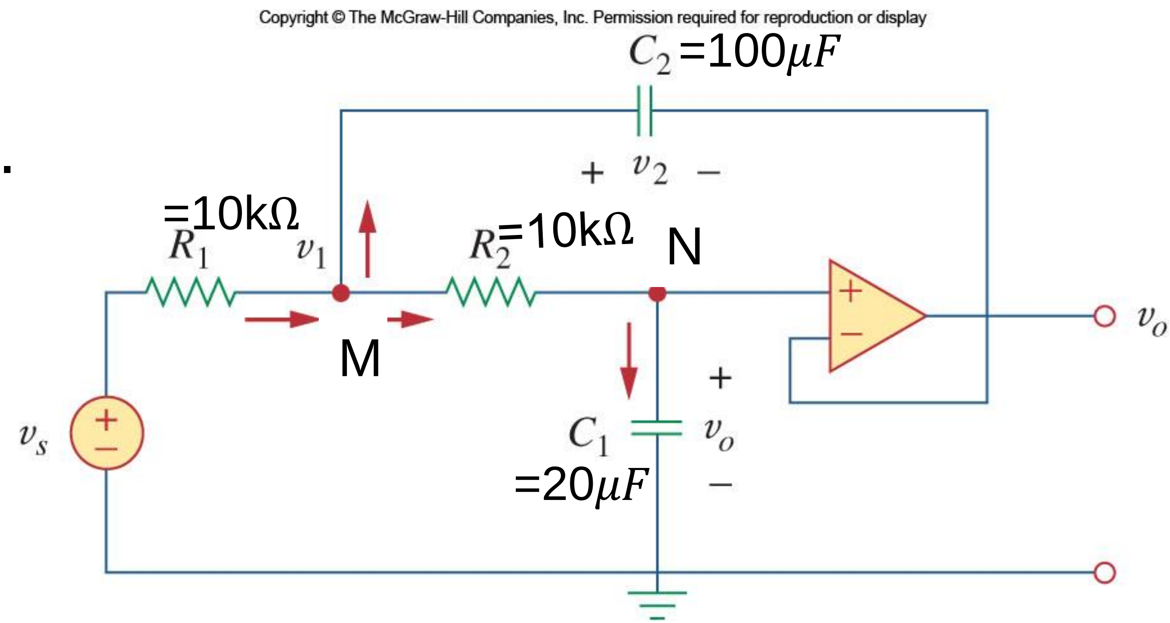
- Find v_o for $t > 0$
  when $v_s = 10u(t)mV$.



Initial conditions: $v_o(0^+) = 0$, $C_1 \frac{dv_o(0^+)}{dt} = \frac{v_1(0^+)-v_o(0^+)}{R_2} = \frac{v_2(0^+)}{R_2} = 0$

Example of 2nd-order op-amp circuits

- Find v_o for $t > 0$ when $v_s = 10u(t)mV$.



KCL at node M:

$$\frac{v_s - v_1}{R_1} = C_2\frac{dv_2}{dt} + \frac{v_1 - v_o}{R_2}$$

KCL at node N:

$$C_1\frac{dv_o}{dt} = \frac{v_1 - v_o}{R_2} \qquad and\ we\ have\ v_1 - v_2 = v_o$$

$$\Rightarrow \quad \frac{d^2v_o}{dt^2} + \left(\frac{1}{R_1C_2} + \frac{1}{R_2C_2}\right)\frac{dv_o}{dt} + \frac{v_o}{R_1R_2C_1C_2} = \frac{v_s}{R_1R_2C_1C_2}$$