Lecture 14
-- Laplace Transform in Circuit Analysis

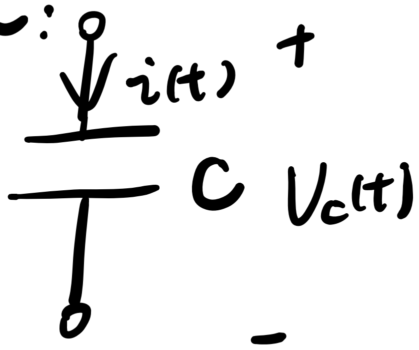
V-I relations of R,L,C

R: $V(t) = R\,i(t)$ $\downarrow\mathcal{L}$ $\downarrow\mathcal{L}$

$\hookrightarrow \mathcal{L}$ $V(s) = R \cdot I(s)$

- R $$U_R(s) = RI_R(s)$$
- C $$V(s) = \frac{1}{sC}I(s) + \frac{V_0}{s}$$ $$I(s) = sCV(s) - CV_0$$
- L $$I(s) = \frac{1}{sL}V(s) + \frac{I_0}{s}$$ $$V(s) = sLI(s) - LI_0$$

C:



$$i(t) = C \cdot \frac{dV_c(t)}{dt}$$

$$I(s) = \mathcal{L}\left[C \cdot \frac{dV_c(t)}{dt}\right] = C\,\mathcal{L}\left[\frac{dV_c(t)}{dt}\right]$$

$$= C \cdot [s \cdot V_c(s) - V_c(0_-)]$$

$$I(s) = C \cdot S \cdot V(s) - CV_0 \qquad ①$$

$$V(s) = \frac{1}{CS}[I(s) + CV_0] = \frac{1}{CS} \cdot I(s) + \frac{V_0}{S} \qquad ②$$

Notes

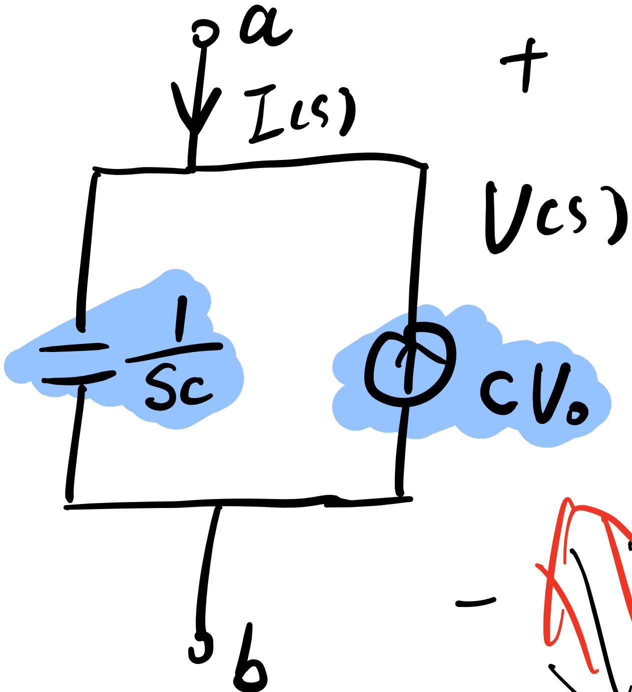
(1) $V_c(t) = V_0 + \frac{1}{C}\int_0^\infty i_c \, dt$

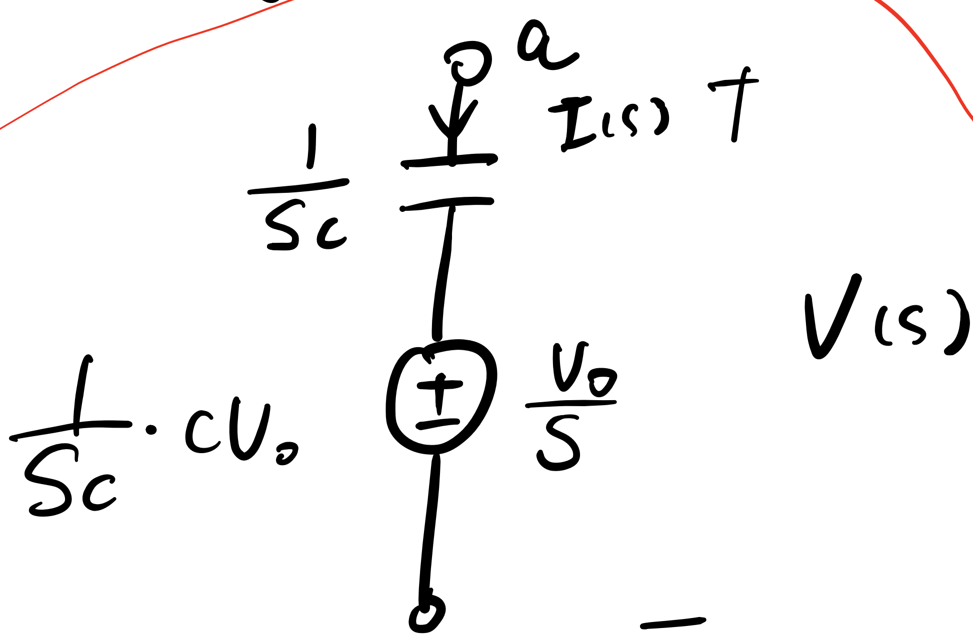
$$V(s) = \frac{V_0}{S} + \frac{1}{C} \cdot \frac{I(s)}{S}$$

(2) S-domain: No diff. Equation

(3) Pictorial illustration:

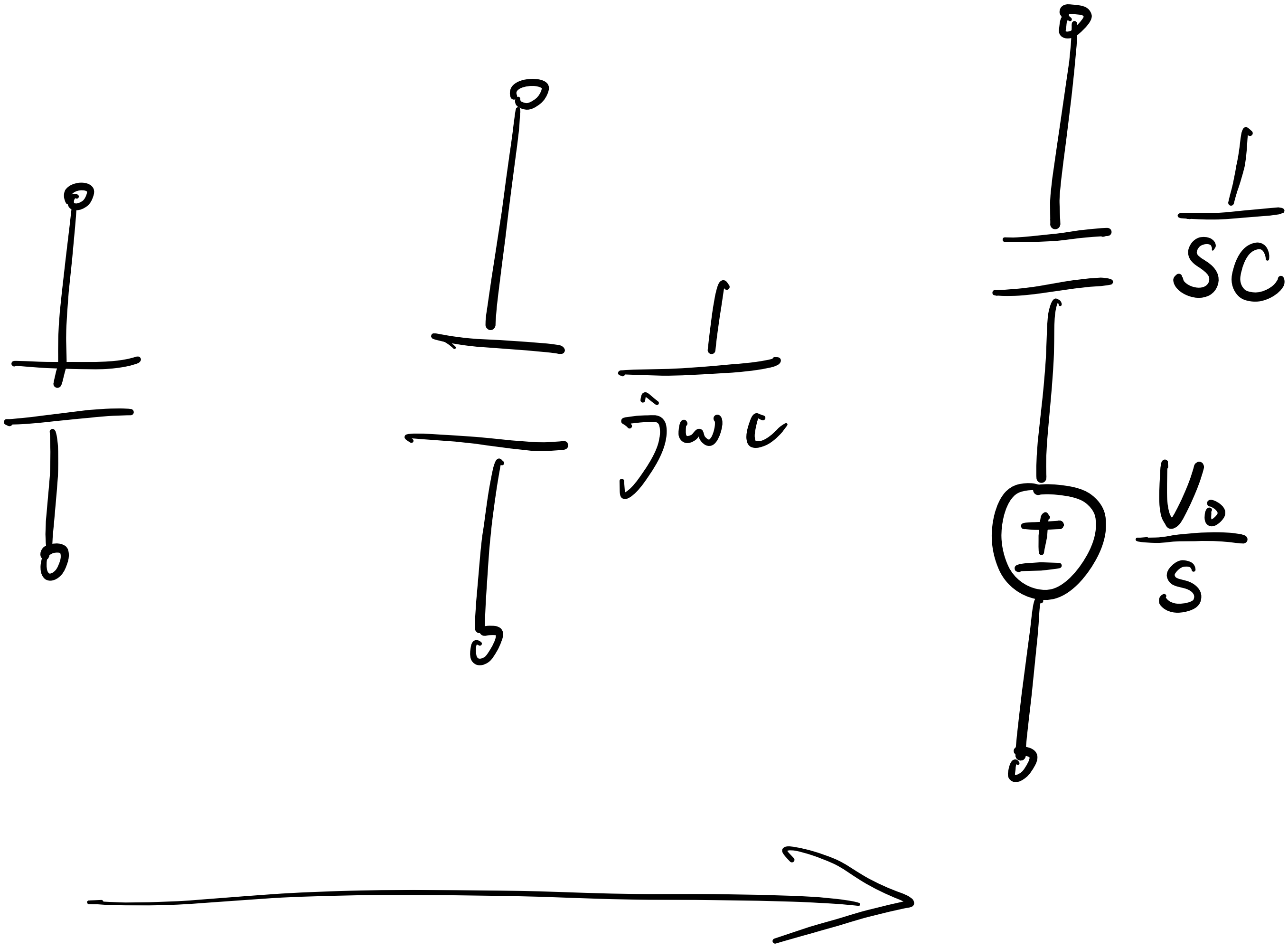
$$I(s) = C\cdot s\cdot V(s) - CV_0 \qquad ①$$



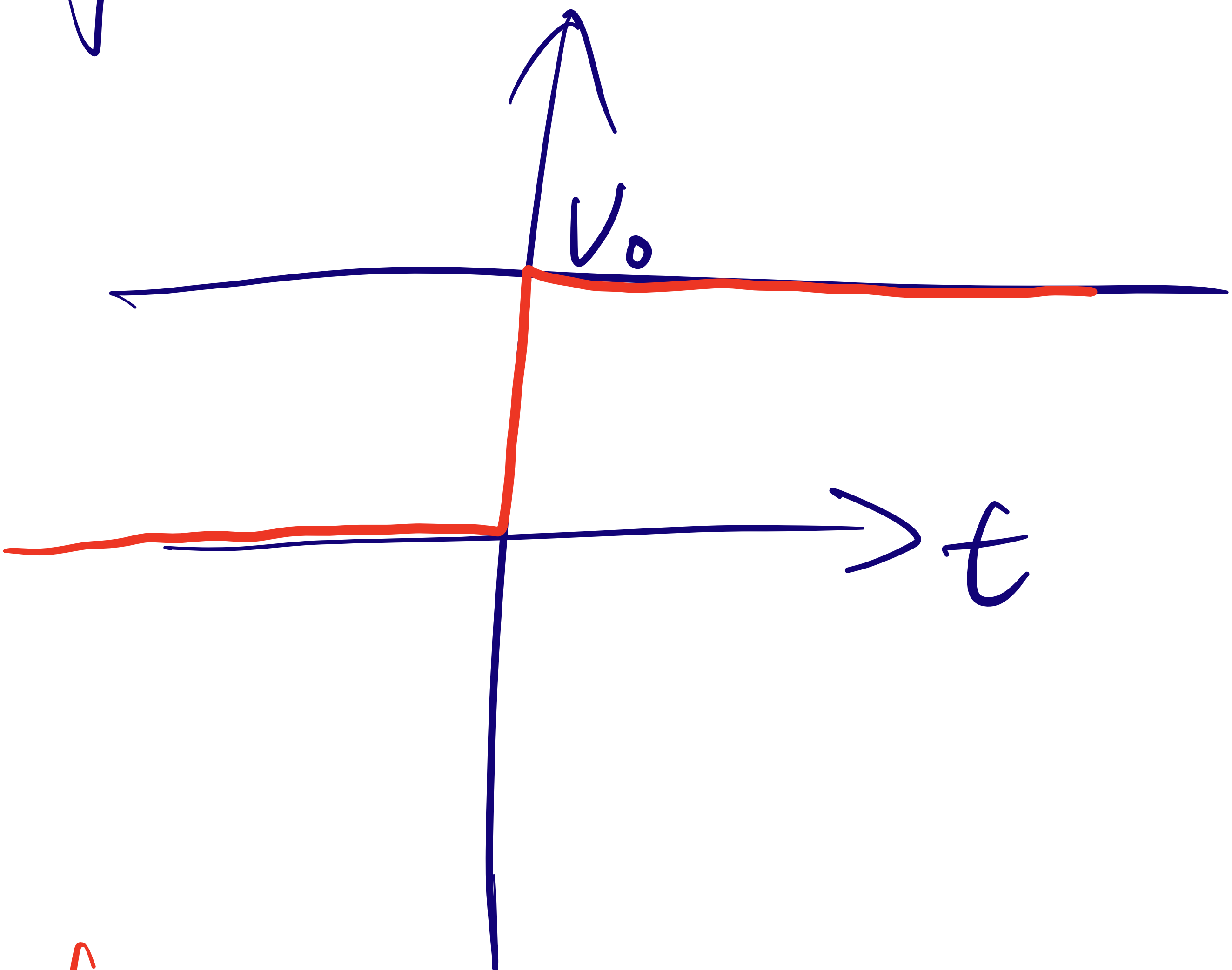


$$V(s) = I(s)\cdot\frac{1}{Sc} + \frac{V_0}{S}$$

T.D. Phasor D. S.D.

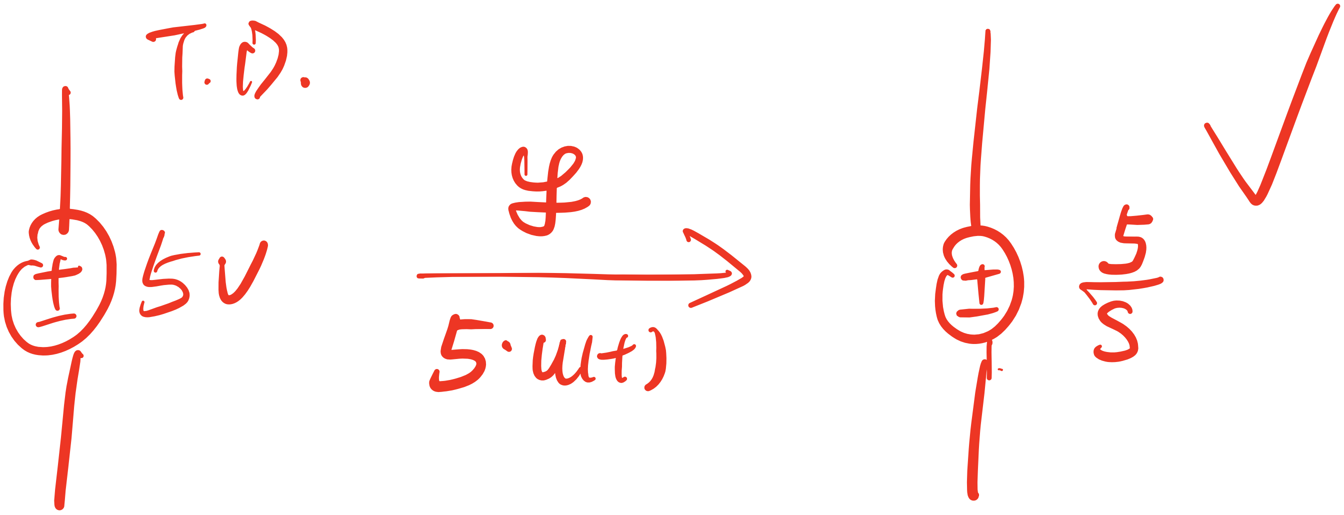


$f(t) = V_0$

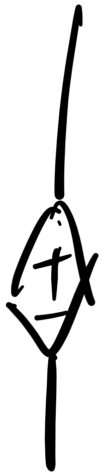


$f(t) = V_0 \cdot u(t)$

T.D.



T. D. $\xrightarrow{\mathcal{L}}$ S. domain



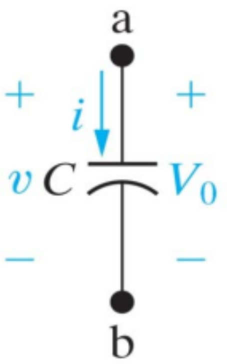
$5 i_x(t)$

$\underline{5 i(s)}$

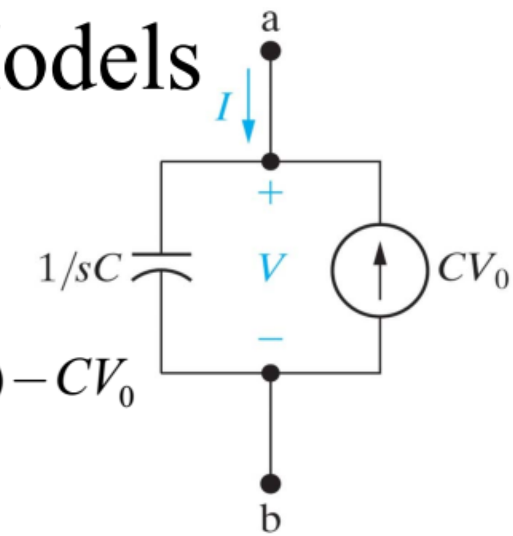
S-domain circuit models for a capacitor

s-Domain Circuit Models

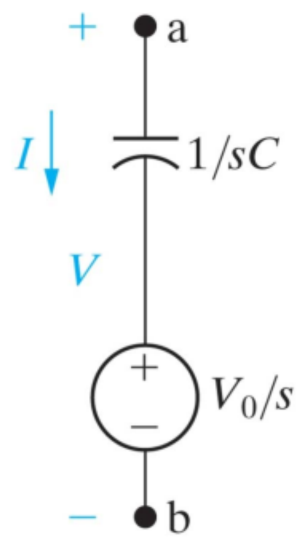
For a capacitor
(with initial conditions)



$$i(t) = C\frac{dv(t)}{dt} \Rightarrow I(s) = sCV(s) - CV_0$$



$$V(s) = \frac{1}{sC}I(s) + \frac{V_0}{s}$$



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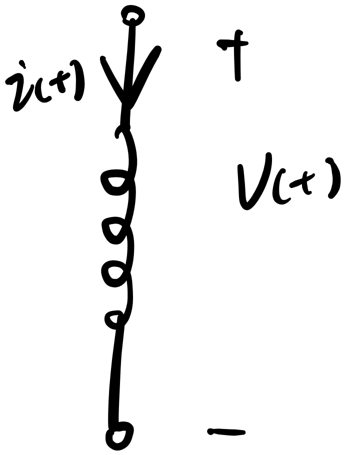
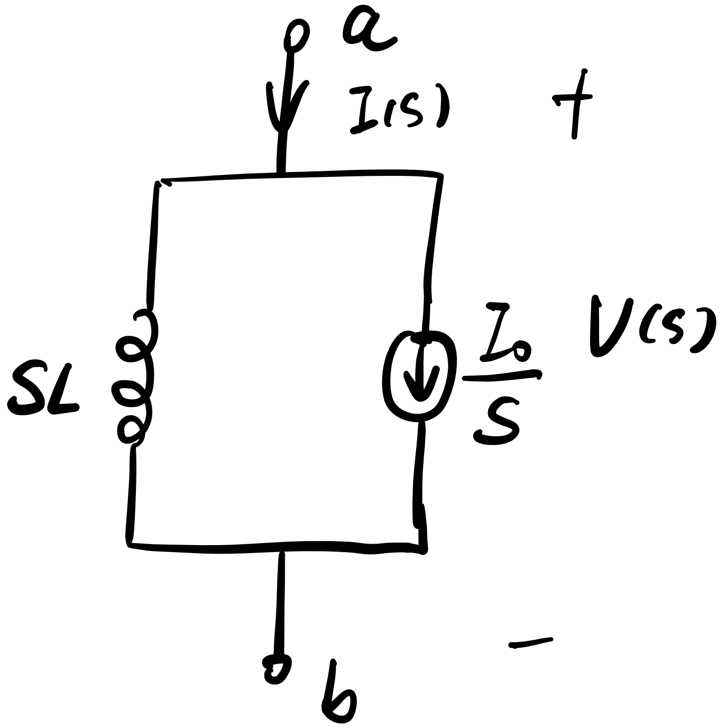


$$V(t) = L\frac{di(t)}{dt}$$

$$V(s) = L[sI(s) - i(0_-)]$$

$$= LSI(s) - LI_0 \qquad ③$$

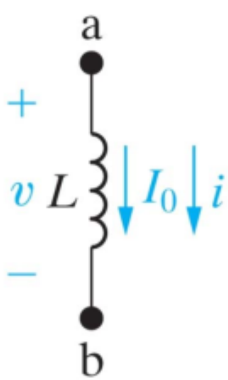
$$I(s) = \frac{1}{Ls}V(s) + \frac{I_0}{s} \qquad ④$$



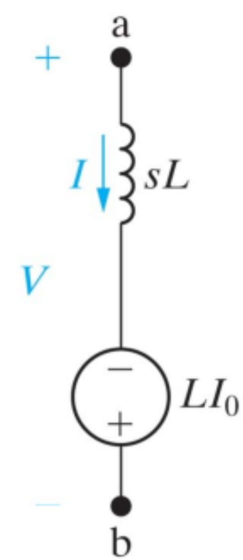
S-domain circuit models for an inductor

s-Domain Circuit Models

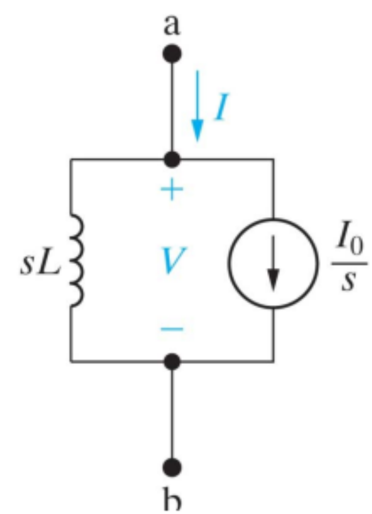
For an inductor
(with initial conditions)



$$v(t) = L\frac{di(t)}{dt} \Rightarrow V(s) = sLI(s) - LI_0$$



$$I(s) = \frac{1}{sL}V(s) + \frac{I_0}{s}$$



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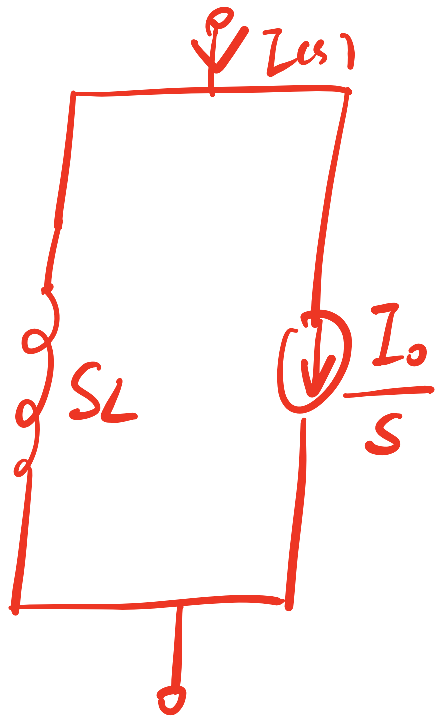
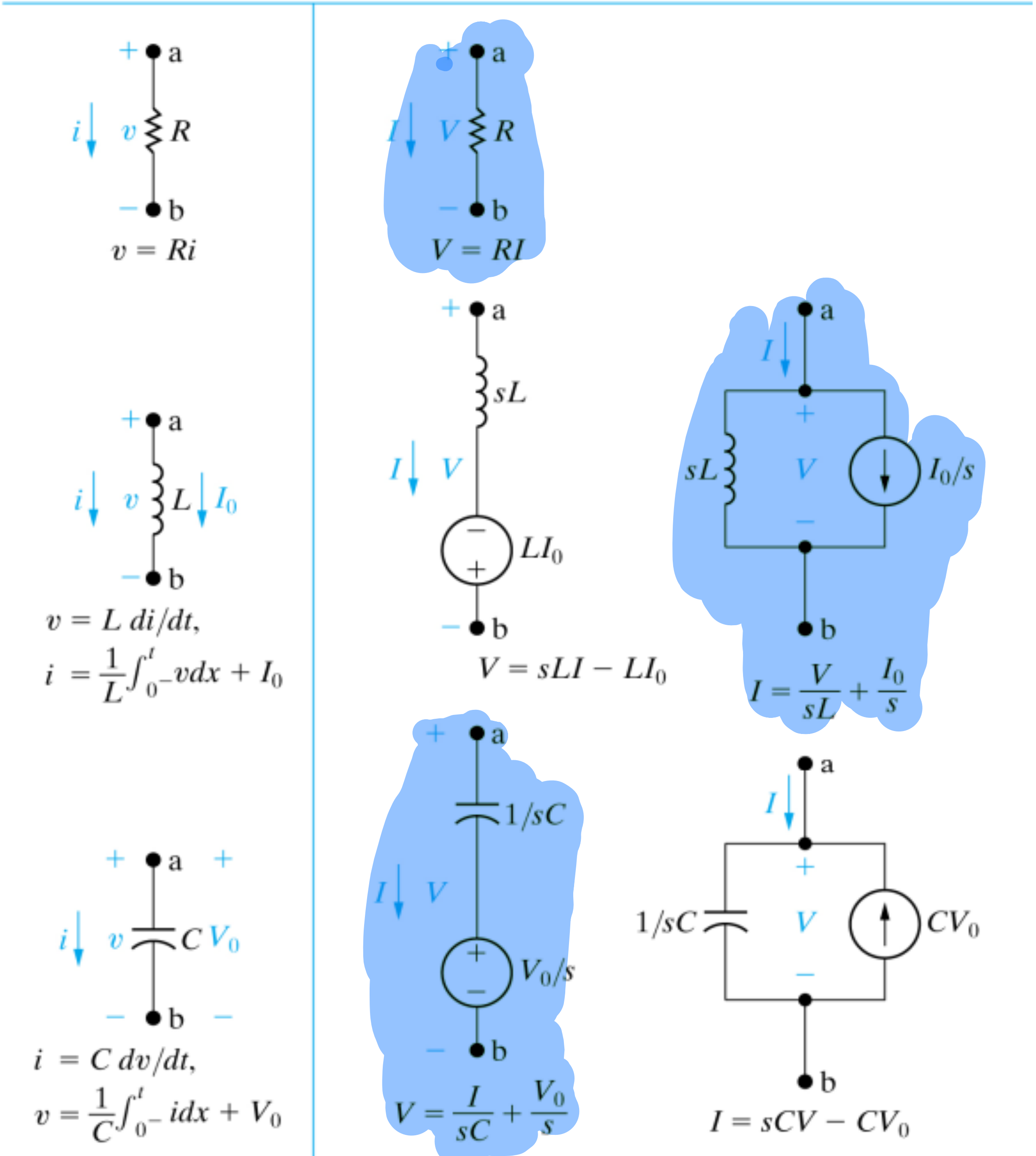


$j\omega \to s$

Time domain	s-domain



D.C. sources and Dependent Sources

- The models for dependent sources are easy to develop, drawing from the simple fact that if the Laplace transform of *f(t)* is *F(s)*, then the Laplace transform of *af*(*t*) is *aF*(*s*) — the linearity property.

$$\mathcal{L}[av(t)] = aV(s)$$

$$\mathcal{L}[ai(t)] = aI(s)$$

Steps in Applying the Laplace transform

- **Transform the circuit from the time domain to the Laplace (*s*) domain, including initial conditions.**

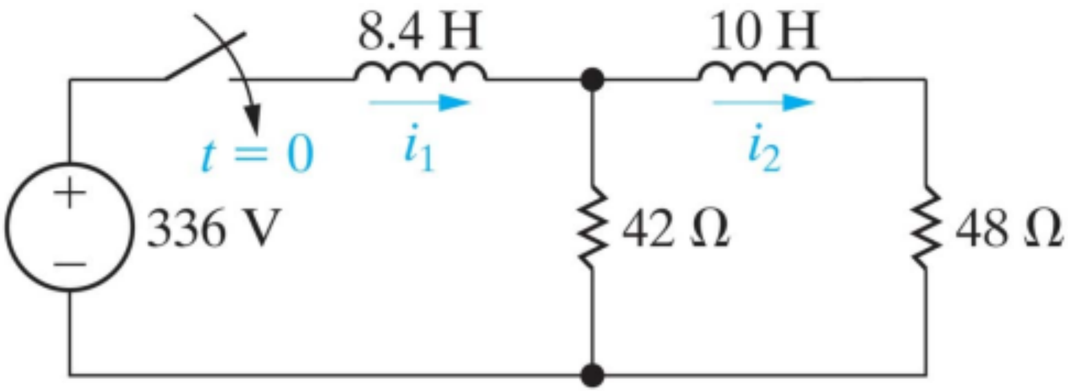
--*The elegance of using the Laplace transform in circuit analysis lies* in (1) transforming the differential equation into an *algebraic* equation; and (2) automatic inclusion of initial conditions in the transformation process, thus providing a complete (**transient and steady-state**) solution.

- **Solve the circuit using nodal analysis, mesh analysis, source transformation, superposition, or any other analysis technique with which we are familiar.**

- Take the inverse transform of the solution and thus obtain the solution in the time domain.

Example 1

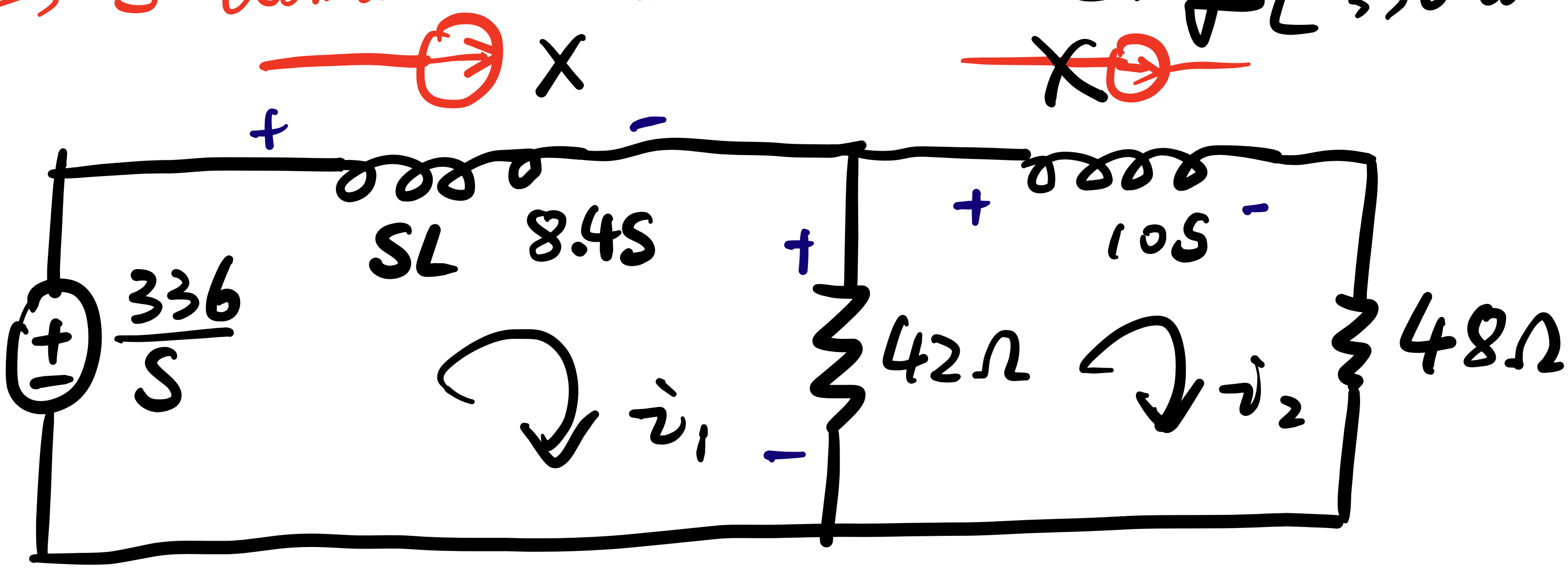
Assuming no initial energy storage, find $i_1(t)$ and $i_2(t)$ for $t > 0$.



$f(t) = 336V$

↳ $\mathcal{L}[336 \cdot u(t)]$

↳ S-domain circuit:



Mesh:

$$\frac{336}{s} = 8.4s \cdot i_1 + 42(i_1 - i_2) \quad ①$$

$$42(i_1 - i_2) = 10.5 \cdot (i_2) + 48 i_2 \quad ②$$

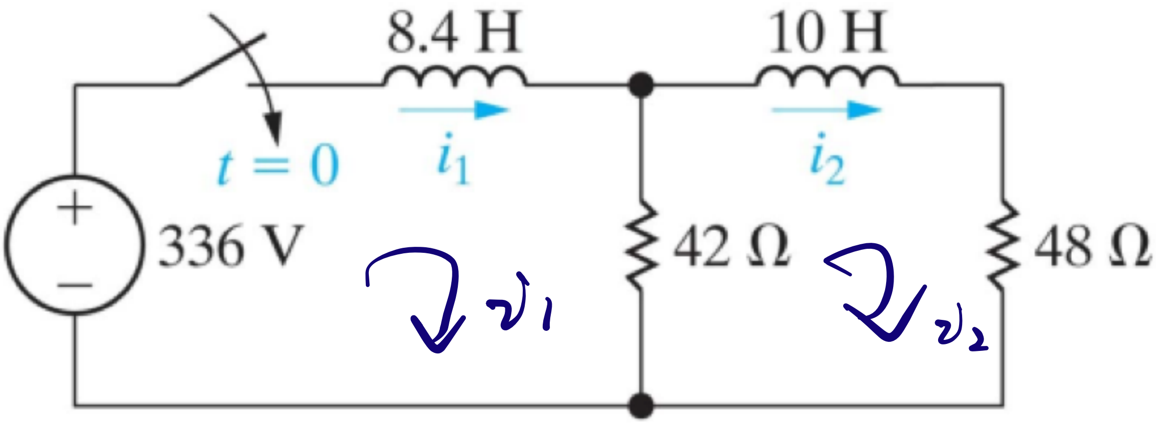
由 ② $\quad i_1 = \dfrac{90 + 10.5}{42} \cdot i_2$

$$\Rightarrow i_2 = \frac{168}{s^3 + 14s^2 + 24s} = \frac{168}{s(s+2)(s+12)}$$

$$= \frac{k_1}{s}^{\,7} + \frac{k_2}{s+2}^{\,-8.4} + \frac{k_3}{s+12}^{\,1.4}$$

$$\mathcal{L}^{-1} \quad i_2(t) = [7 - 8.4e^{-2t} + 1.4e^{-12t}]u(t)$$

$$i_1 = (15 - 14e^{-2t} - e^{-12t})u(t)$$



$$336 = 8.4\frac{di_1}{dt} + 42(i_1 - i_2) \qquad ①$$

$$42(i_1 - i_2) = 10\frac{di_2}{dt} + 48i_2 \qquad ②$$

$$\rightarrow i_2'' + 14i_2' + 24i_2 = 168$$

$$s^2 + 14s + 24 = 0$$

$(s+2)(s+12)=0$