Homework 5

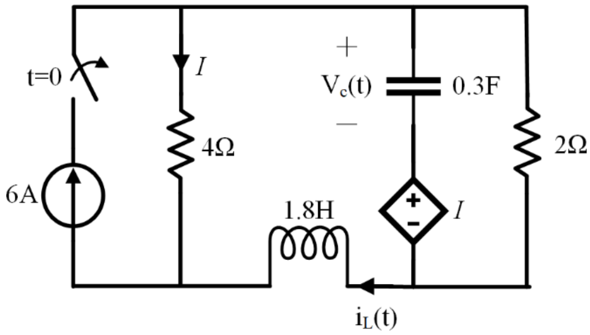
Due date: Dec. 5th, 2023

Turn in your hard-copy hand-writing homework in class

Rules:

- Work on your own. Discussion is permissible, but extremely similar submissions will be judged as plagiarism.
- Please show all intermediate steps: a correct solution without an explanation will get zero credit.
- Please submit on time. No late submission will be accepted.
- Please prepare your submission in English only. No Chinese submission will be accepted.

1. Assume the circuit has reached steady state at $t < 0$, find $v_c(0^+)$, $dv_c(0^+)/dt$, $i_L(0^+)$, $di_L(0^+)/dt$.



$i_L(0^+) = i_L(0^-) = \frac{6\times4}{4+2} = 4A$,

$V_c(0^+) = V_c(0^-) = 2\times4 - 2 = 6V$

when $t > 0$, $I = -4A$, $V_{2\Omega} = -4 + 6 = 2V$, $I_{2\Omega} = \frac{2}{2} = 1A$

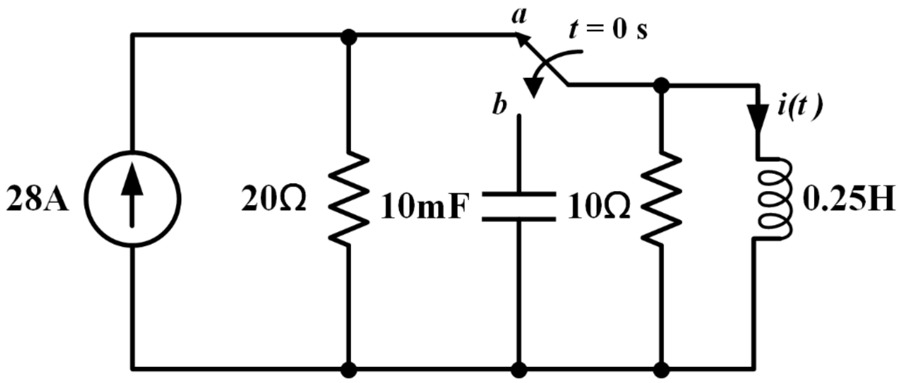
$i_c(0^+) = 4 - 1 = 3A$

$V_{4\Omega} + V_L(0^+) + V_{2\Omega} = 0$, $\therefore$ $V_L(0^+) = -18V$

$$\frac{dv_c(0^+)}{dt} = \frac{i_c(0^+)}{C} = 10$$

$$\frac{di_L(0^+)}{dt} = \frac{V_L(0^+)}{L} = -10$$

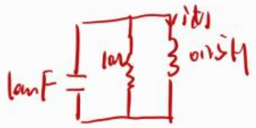
2. When t<0, no energy is stored in the capacitor, the switch has been placed at node ***a*** for a long time. The switch moves from node ***a*** to node ***b*** at $t = 0$ immediately. Determine $i(t)$ for $t \geq 0$.



2.

$t<0$: $i(t) = 28A$ $\quad V_c(t) = 0V$

$t \geq 0$: $i(0^+) = i(0^-) = 28A$ $\quad v_c(0^+) = v_c(0^-) = 0$ $\quad 4'$



$s^2 + 10s + 400 = 0$ $\quad \alpha = \frac{1}{2RC} = 5$ $\quad \omega_0 = \frac{1}{\sqrt{LC}} = 20$

$\therefore \alpha^2 < \omega_0^2$

$\omega_d = \sqrt{\omega_0^2 - \alpha^2} = 19.36$

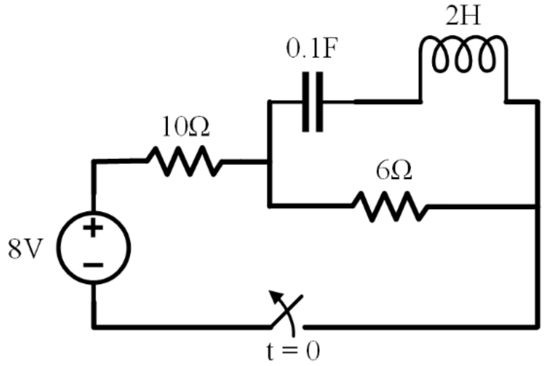
$i(t) = e^{-5t}(B_1 \cos 19.36t + B_2 \sin 19.36t)$ $\quad 6'$

$$\begin{cases} \text{when } t=0^+ \quad i(0) = 28\,A = B_1 \\ \frac{di_L(0^+)}{dt} = \frac{V_0}{L} = 0 = -5B_1 + 19.36B_2 = 0 \end{cases} \Rightarrow \begin{cases} B_1 = 28 \\ B_2 = 7.23 \end{cases}$$

$5'$

$\therefore i(t) = e^{-5t}(28\cos 19.36t + 7.23 \sin 19.36t)\ A.$

3. Assume the circuit has reached steady state at $t < 0$, calculate the current of a 6ohm resistor for $t > 0.6$.



3.

$$\begin{cases} i = C\dfrac{dV_c}{dt} \\ V_L = L\dfrac{di}{dt} \\ 6i + V_L + V_c = 0 \end{cases} \quad 3 \quad \Rightarrow$$

$$i = C\frac{d(-V_L)}{dt} + C\frac{d(-6i)}{dt} = -C\frac{dV_L}{dt} - 6C\frac{di}{dt}$$

$$LC\frac{d^2i}{dt^2} + 6C\frac{di}{dt} + i = 0 \quad 2$$

$$\frac{d^2i}{dt^2} + 3\frac{di}{dt} + 5i = 0$$

$$\therefore\ i = e^{-1.5t}(A_1\cos 1.66t + A_2\sin 1.66t) \quad 2$$

$t<0$

$i(0^-) = 0A$

$V_C(0^-) = 3V$ 2

$t>0$. $i_L(0^+) = i_L(0^-) = 0A$

$V_C(0^+) = V_C(0^-) = 3V$

$V_C(0^+) + V_L(0^+) = 0$

$\therefore\ V_L(0^+) = -3V$

$$\frac{V_L}{L} = \frac{di}{dt} = -1.5e^{-1.5t}(A_1\cos 1.66t + A_2\sin 1.66t) + e^{-1.5t}(-1.66A_1\sin 1.66t + 1.66A_2\cos 1.66t) \quad 2$$

代入边界条件:

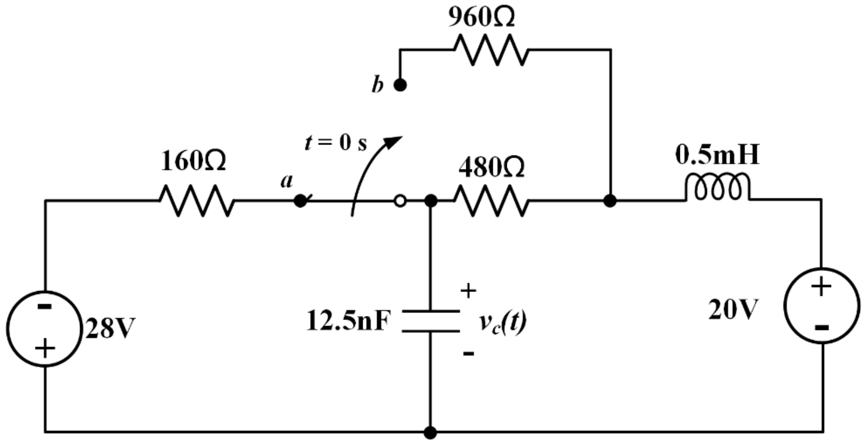
$$\begin{cases} A_1 = 0 \\ 1.66A_2 = -3/2 \end{cases} \quad \therefore A_2 = -0.9 \quad 4$$

$i = -0.9e^{-1.5t}\sin 1.66t,\ t>0$

$i = -0.9e^{-1.5(t-0.6)}\sin 1.66t,\ t>0.6$

来自华为笔记

4. For the following circuit, the switch has been placed at node ***a*** for a long time. At ***t***= 0 s, the switch is switched from ***a*** to ***b*** immediately. Please find the voltage on the capacitor ***$v_c(t)$*** for ***t*** ≥ 0s.



4. $t<0$: $V_C(0^-) = 20 - (20+28) \times \frac{480}{160+480} = -16V$ $i_L(0^-) = \frac{20+28}{160+480} = \frac{3}{40} A$ 4'

$t>0$: $960\Omega \parallel 480\Omega = 320\Omega$

$\alpha = \frac{R}{2L} = 3.2\times10^5$ $w_0 = \frac{1}{\sqrt{LC}} = 4\times10^5$

$\alpha^2 < w_0^2$

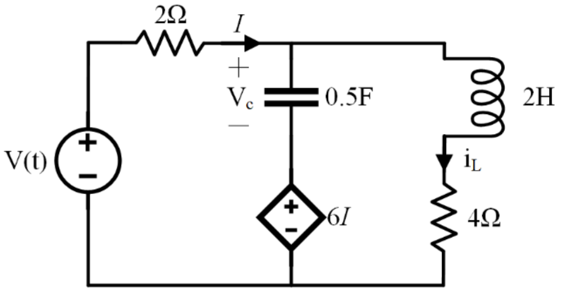
$S_{1,2} = -\alpha \pm \sqrt{\alpha^2 - w_0^2} = -3.2\times10^5 \pm 2.4\times10^5 j$ 6'

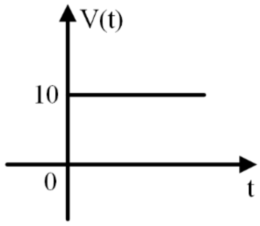
$V(t) = 20 + (B_1 \cos 2.4\times10^5 t + B_2 \sin 2.4\times10^5 t) e^{-3.2\times10^5 t}$

$$\begin{cases} V(0^+) = -16 = 20 + B_1 \\ \frac{dV_c(0^+)}{dt} = 6\times10^6 = 2.4\times10^5 B_2 - 3.2\times10^5 B_1 \end{cases} \Rightarrow \begin{cases} B_1 = -36 \\ B_2 = -23 \end{cases}$$

$\therefore V(t) = 20 + (-36 \cos 2.4\times10^5 t - 23 \sin 2.4\times10^5 t) e^{-3.2\times10^5 t}$ 5'

5. Assume $\boldsymbol{V_c(0^+)} = 4\text{V}$, $\boldsymbol{i_L(0^+)} = 0\text{A}$. find $\boldsymbol{V_c(t)}$ and $\boldsymbol{i_L(t)}$.





5. $\begin{cases} i_L = I - C\dfrac{dV_c}{dt} \\ V_c + 6I = 4i_L + L\dfrac{di_L}{dt} \\ 2I + V_c + 6I = 10 \end{cases}$ ⇒ 3

$LC\dfrac{d^2V_c}{dt^2} + \left(4C + \dfrac{L}{8}\right)\dfrac{dV_c}{dt} + \dfrac{3}{4}V_c + \dfrac{5}{2} = 0$

$\dfrac{d^2V_c}{dt^2} + 2.25\dfrac{dV_c}{dt} + \dfrac{3}{4}V_c = -\dfrac{5}{2}$ 2

$V_c = C_1 e^{-0.41t} + C_2 e^{-1.84t} - \dfrac{10}{3}$ 2

$i_c = (-0.41C_1 e^{-0.41t} - 1.84C_2 e^{-1.84t})\cdot\dfrac{1}{2}$ 2

$V_c(0^+) = 4$

$i_L(0^+) = 0$ 2

$i_c(0^+) = \dfrac{3}{4}$

代入边界条件：

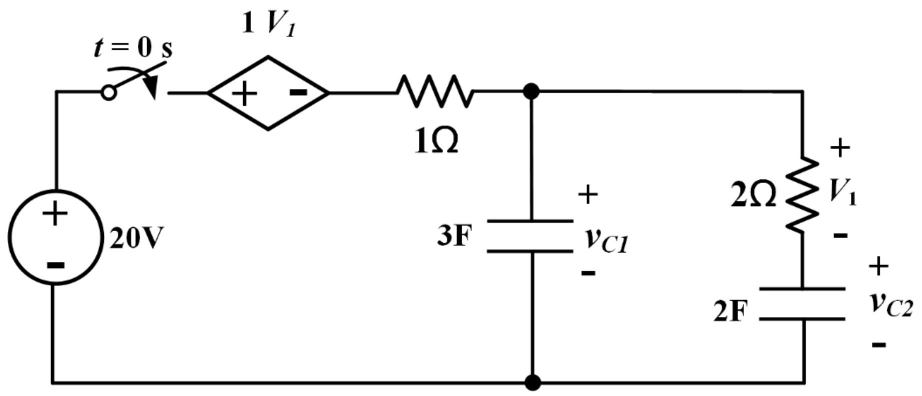
$\begin{cases} C_1 + C_2 - \dfrac{10}{3} = 4 \\ -0.41C_1 - 1.84C_2 = \dfrac{3}{2} \end{cases} \Rightarrow \begin{cases} C_1 = 10.48 \\ C_2 = -3.15 \end{cases}$ 2

$i_L(t) = \dfrac{10 - V_c}{8} - C\dfrac{dV_c}{dt}$

$= 0.84e^{-0.41t} - 2.5e^{-1.84t} + 1.67$ 2

来自华为笔记

6. For the following circuit, the switch closes at t = 0s immediately. Please find the voltage on the capacitors $v_{C1}(t)$ and $v_{C2}(t)$ for t > 0s, respectively. Note that the switch has been open for a long time before t = 0s.



6. $t<0$: $V_{C1}(0^-) = V_{C2}(0^+) = 0$ 2'

$t>0$: $V_{C2} = V_{C1} - V_1 = V_{C1} - 2\times 2\frac{dV_{C2}(t)}{dt} \Rightarrow V_{C1} = V_{C2} + 4\frac{V_{C2}(t)}{dt}$

$\Rightarrow \frac{dV_{C1}(t)}{dt} = \frac{dV_{C2}(t)}{dt} + 4\frac{d^2V_{C2}(t)}{dt^2}$ 3'

$I_1 = 3\frac{dV_{C1}(t)}{dt} + 2\frac{dV_{C2}(t)}{dt} = 12\frac{d^2V_{C2}(t)}{dt} + 5\frac{dV_{C2}(t)}{dt}$

$20 = V_1 + I_1 + V_{C1}$

$\Rightarrow$

$12\frac{d^2V_{C2}(t)}{dt^2} + 13\frac{dV_{C2}(t)}{dt} + V_{C2}(t) = 20$

$\frac{d^2V_{C2}(t)}{dt^2} + \frac{13}{12}\frac{dV_{C2}(t)}{dt} + V_{C2}(t) = \frac{5}{3}$ 5'

$\alpha = \frac{13}{24}$ $\quad \omega_0 = \sqrt{\frac{1}{12}}$ $\quad \because \alpha^2 > \omega_0^2$ $\qquad S_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2} = -\frac{1}{12}$

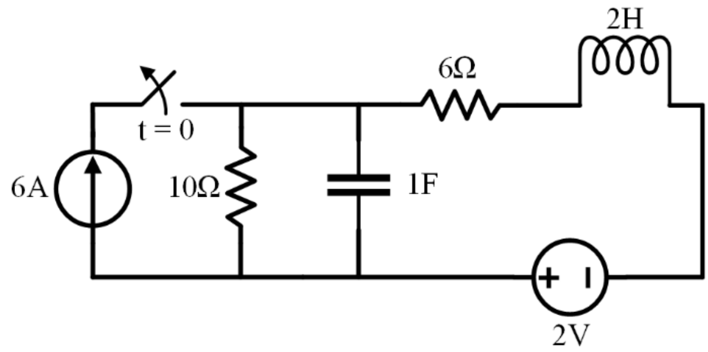
$V_{C2}(t) = A_1 e^{-\frac{1}{12}t} + A_2 e^{-t} + 20$ $\qquad S_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2} = -1$ 4'

$\begin{cases} V_{C2}(0^+) = 0 = A_1 + A_2 + 20 \\ \frac{dV_{C2}(0^+)}{dt} = 0 = -\frac{1}{12}A_1 - A_2 \end{cases} \Rightarrow \begin{cases} A_1 = -21.81 \\ A_2 = 1.81 \end{cases}$ 2'

$V_{C2}(t) = 20 - 21.81e^{-\frac{1}{12}t} + 1.81e^{-t}$ (V) 4'

$V_{C1} = 4V'_{C2}(t) + V_{C2}(t) = 20 - 14.57e^{-\frac{1}{12}t} - 5.43e^{-t}$ (V)

7. Assume the circuit has reached steady state at $t < 0$, calculate the current of a 6ohm resistor for $t > 0$ for $t > 0$.



7. $\begin{cases} V_c = (i_L - C\dfrac{dV_c}{dt}) \times 10 \\ 2 + V_L + 6i_L + V_c = 0 \\ \dfrac{dV_L}{dt} + 6\dfrac{di_L}{dt} + \dfrac{dV_c}{dt} = 0 \\ V_L = L\dfrac{di_L}{dt} \end{cases}$ 1 1 1

$\Rightarrow \frac{1}{10}(-2 - V_L - 6i_L) = i_L + C\frac{dV_L}{dt} + 6C\frac{di_L}{dt}$

$10CL\frac{d^2 i_L}{dt^2} + (60C + L)\frac{di_L}{dt} + 16i_L + 2 = 0$

$20\frac{d^2 i_L}{dt^2} + 62\frac{di_L}{dt} + 16 i_L = -2$ 4

$i_L = C_1 e^{-0.28t} + C_2 e^{-2.82t} - \frac{1}{8}$ 2

$V_L = L\frac{di_L}{dt} = -0.56C_1 e^{-0.28t} - 5.64C_2 e^{-2.82t}$ 2

$i_L(0^-) = -\frac{6\times 10}{10+6} - \frac{2}{10+6} = -3.875A$ 1

$V_c(0^-) = -\frac{2\times 10}{10+6} + 6\times(10//6) = 21.25V$ 1

$\therefore V_c(0^+) = V_c(0^-),\ i_L(0^+) = i_L(0^-)$

$V_L(0^+) = 2 + V_c(0^+) + 6\times i_L(0^+) = 0$ 1

代入边界条件:

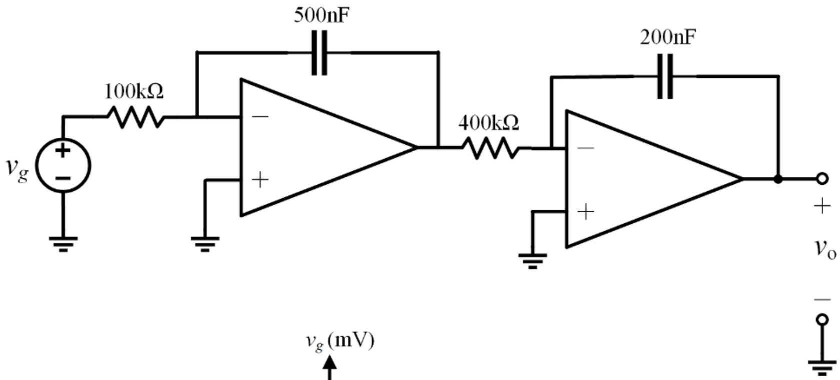
$\begin{cases} C_1 + C_2 - \frac{1}{8} = -3.875 \\ -0.56C_1 - 5.64C_2 = 0 \end{cases}$ 2

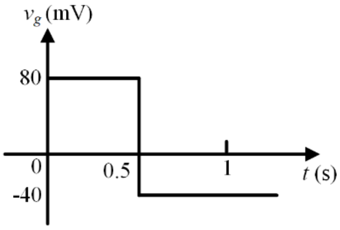
$\therefore \begin{cases} C_1 = -4.16 \\ C_2 = 0.41 \end{cases}$ 2

$i_R = i_L = -4.16e^{-0.28t} + 0.41e^{-2.82t} - \frac{1}{8}$ 2

来自华为笔记

8. The waveform of voltage source $\boldsymbol{v}_g$ as shown and the initial value of the capacitance are 0, find $\boldsymbol{v}_0(\boldsymbol{t})$ for $\boldsymbol{t} \geq 0$.





$$8.\ \begin{cases} \dfrac{V_g}{100k} = C_1 \dfrac{dV_{c1}}{dt} & 2 \\ -\dfrac{V_{c1}}{400k} = C_2 \dfrac{dV_{c2}}{dt} & 2 \Rightarrow \\ V_o = -V_{c2} & 2 \end{cases}$$

$V_{c1} = \dfrac{V_g}{100k \times C_1} t + A_1 = 20 V_g t + A_1$ 2

$V_{c2} = -125 V_g t^2 - 12.5 A_1 t + A_2$ 2

$\because V_{c1}(0^+) = V_{c2}(0^+) = 0$ 1

$\therefore A_1 = A_2 = 0,\ \therefore V_o = -V_{c2} = 10t^2,\ 0 < t < 0.5$ 2

$\because V_{c1}(0.5^-) = 0.8V,\ V_{c2}(0.5^-) = -2.5V$ 2

$V_{c1}(0.5^+) = V_{c1}(0.5^-),\ V_{c2}(0.5^+) = V_{c2}(0.5^-)$ 1

代入得 $\begin{cases} -0.4 + A_1 = 0.8 \\ 1.25 - 6.25A_1 + A_2 = -2.5 \end{cases}$

$\begin{cases} A_1 = 1.2 \\ A_2 = 3.75 \end{cases}$ 2

$\therefore V_o = -5t^2 + 15t - 3.75,\ t > 0.5$

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