

# Lecture 14 Image Deblurring

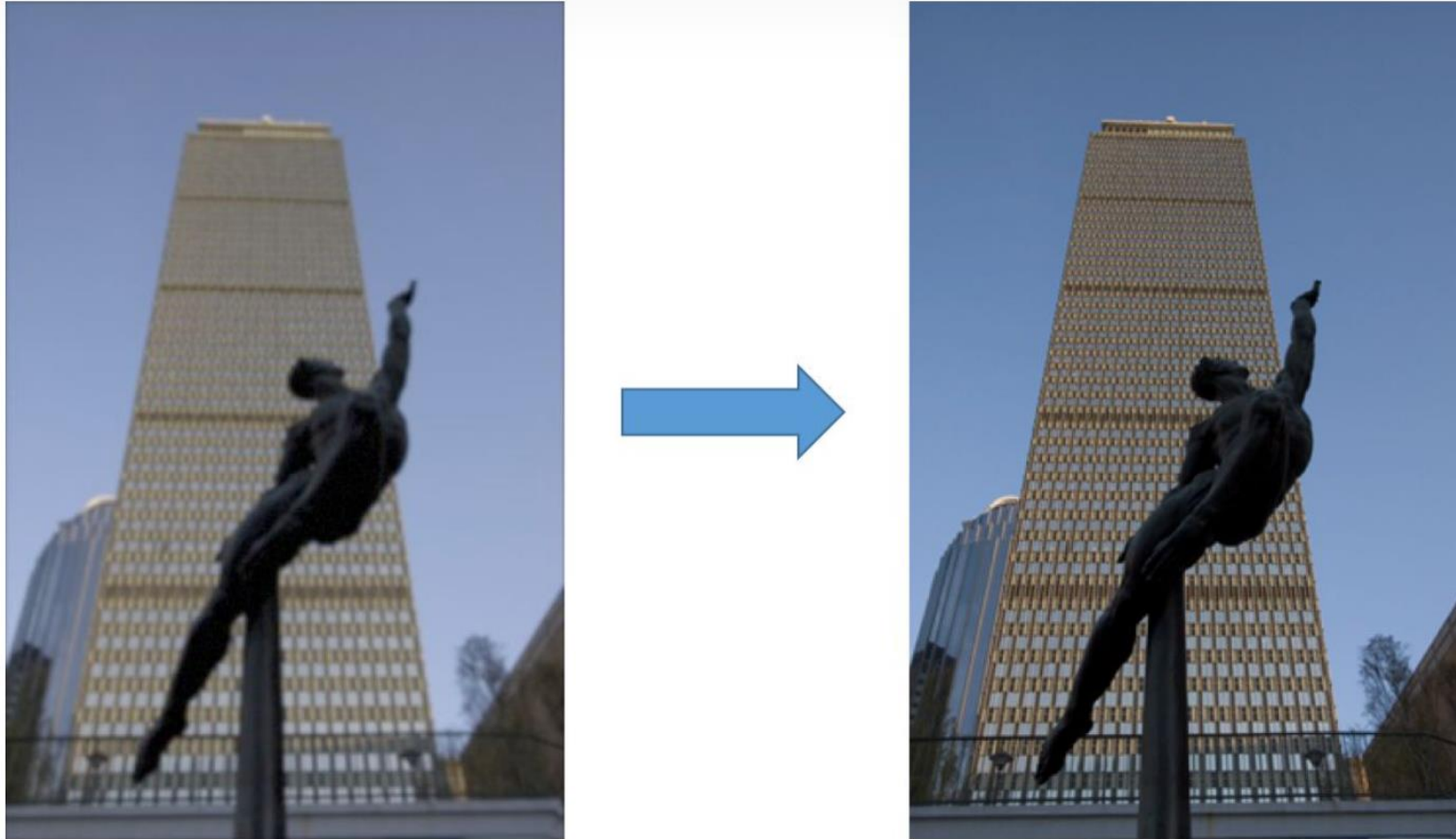
(Introduction and Formulation)

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SIST Building-3 420

# Overview-Deblur



# Overview-Deblur



- original



- optical blur



- motion blur

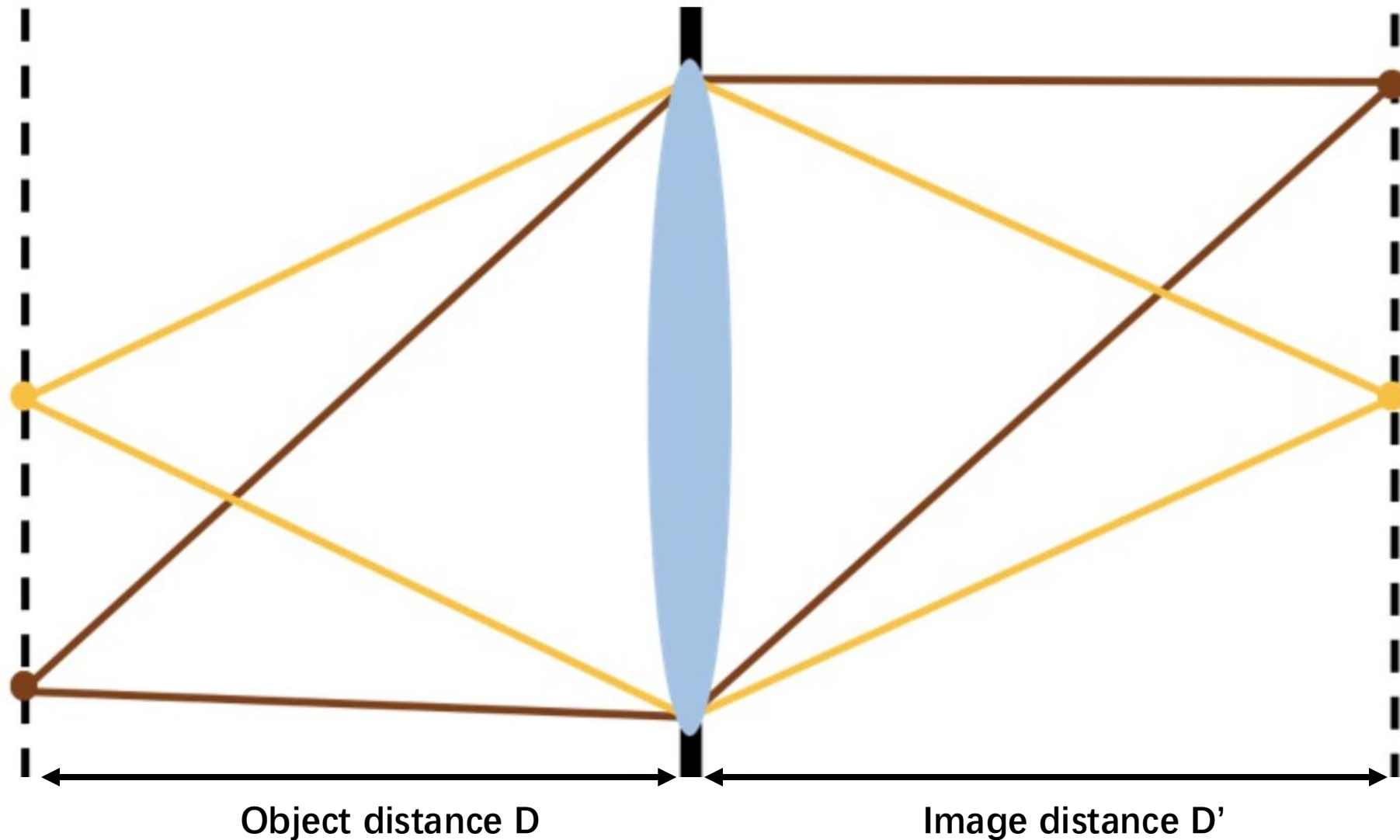


- spatial quantization (discrete pixels)

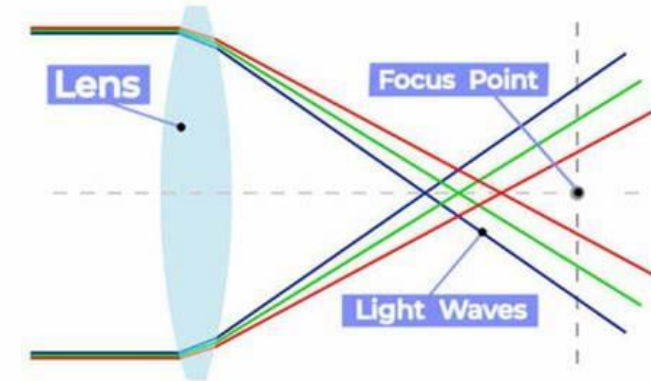
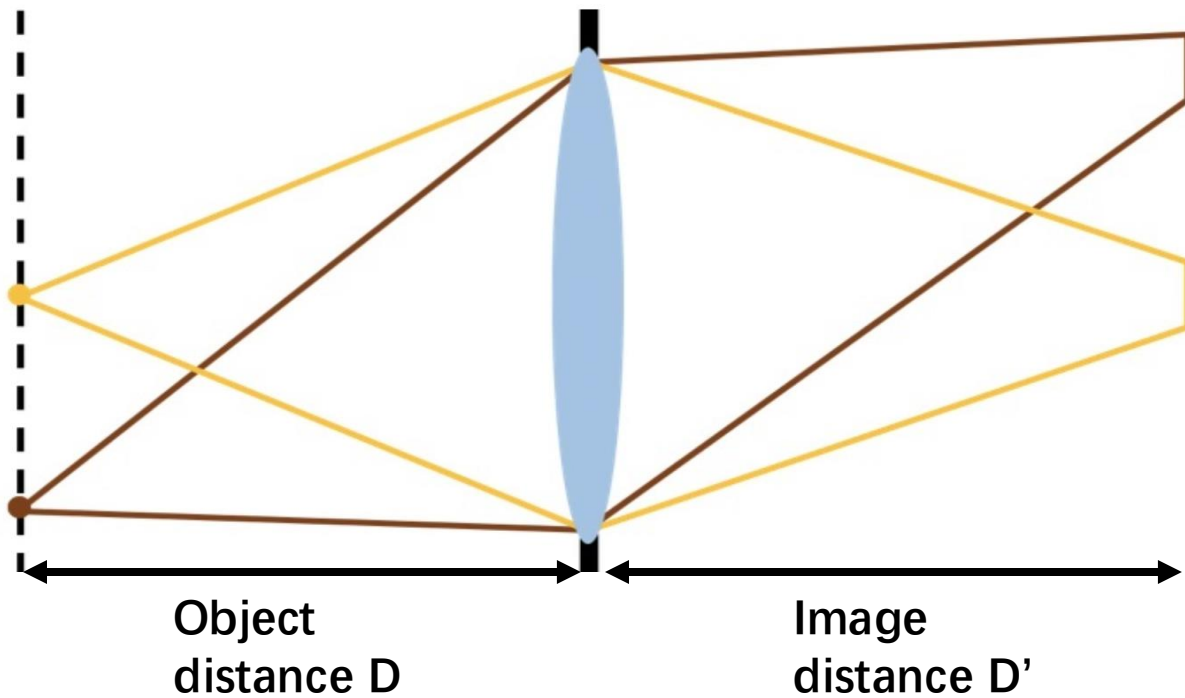


- additive intensity noise

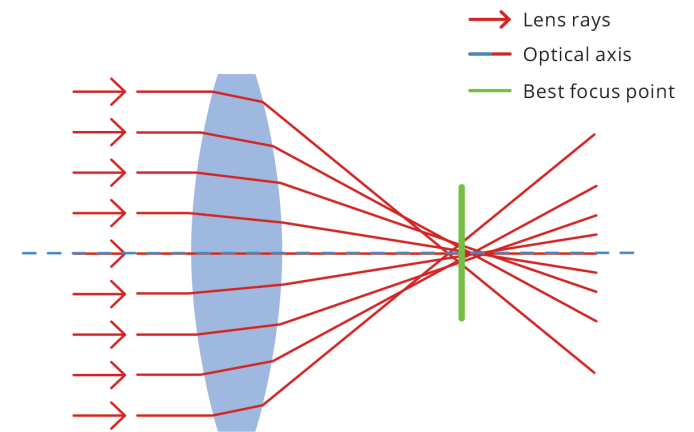
# The properties of the lens are limited



# Optical Blurry

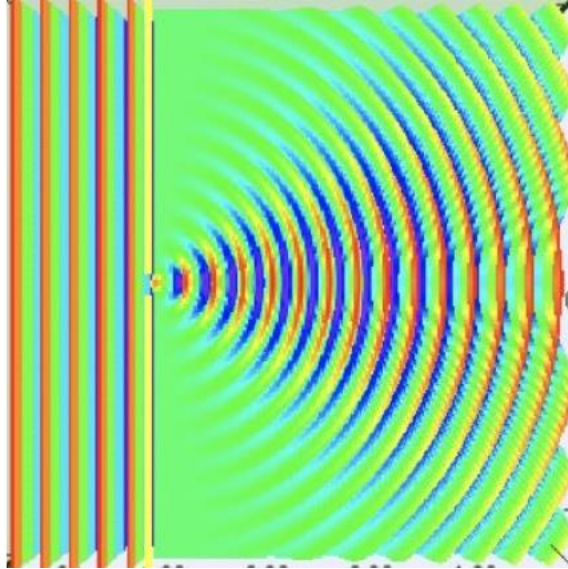


Chromatic aberration

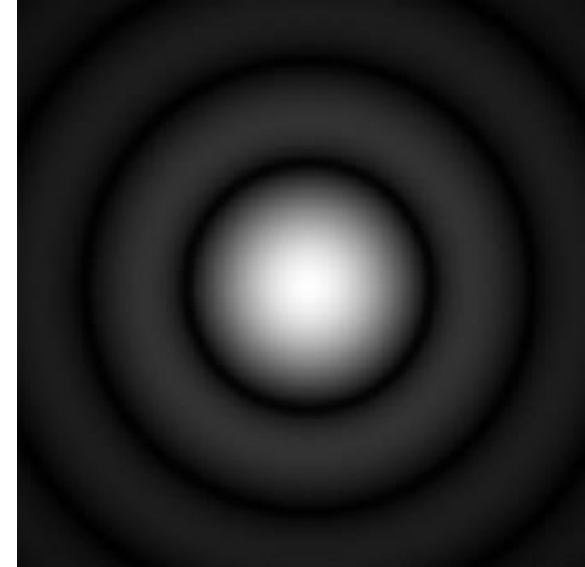


Spherical aberration

# PSF



**Diffraction**, which is caused by the wave nature of light, as long as light passes through the aperture, diffraction occurs.



**PSF**, point spread function caused by the diffraction.

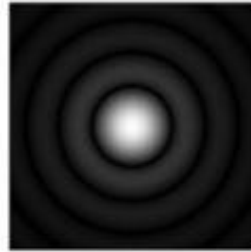
# PSF



Reality

$x$

\*



PSF

$c$

=



Observed image

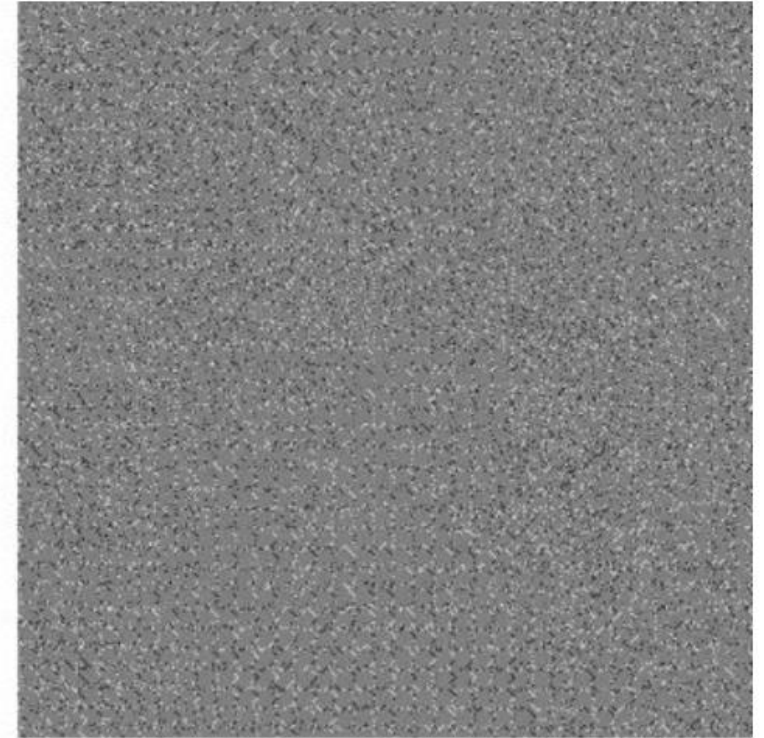
$y$



# Inverse Filter



$$* \left[ \text{Image of a 2D Gaussian kernel} \right]^{-1} =$$





# Wiener Filter

Image domain

$$y = c * x + n$$

Frequency domain

$$Y = CX + N$$

Looking for a function  $H$  that minimize expectation of error:

$$\min_H E[\|X - HY\|^2]$$

$$\min_H E^2[X] - HE[XY] - H^*E[YX] + H^2E^2[Y]$$

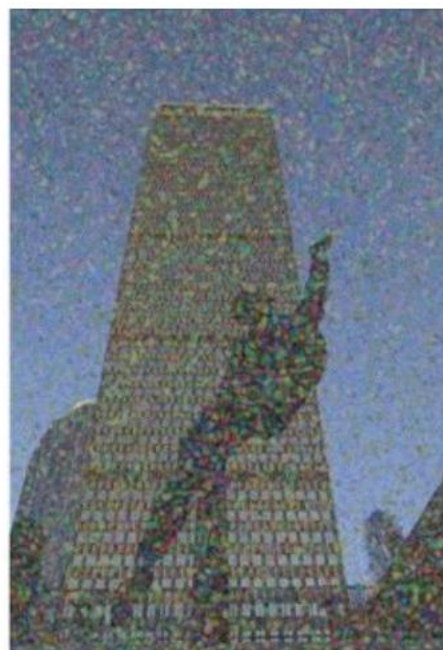
$$H_{opt} = \frac{E[XY]}{E^2[Y]} = \frac{CE^2[X]}{CE^2[X] + E^2[N]}$$

# Deblurring in image domain

Image domain  $\min_x ||y - c * x||^2 + ||\nabla x||^2$



Blurred image  
with noise



w/o regularization



Regularization



GT image

# Overview-Motion Blurring

- The objective is to restore a degraded image to its original form.
- An observed image can often be modelled as:

$$g(x, y) = \iint c(x - x', y - y') f(x', y') dx' dy' + n(x, y)$$

where the integral is a convolution,  $c$  is the point spread function of the imaging system, and  $n$  is additive noise.

- The objective of image restoration in this case is to estimate the original image  $f$  from the observed degraded image  $g$ .

# Maximum a posteriori (MAP) Estimation

# Removing Camera Shake from a Single Photograph

Rob Fergus, Barun Singh, Aaron Hertzmann,  
Sam T. Roweis and William T. Freeman

Massachusetts Institute of Technology  
and  
University of Toronto

# Overview

Original



Our algorithm





# Close-up

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Original



Naïve Sharpening



Our algorithm





# Image formation process



Blurry image

Input to algorithm

Model is approximation

=



Sharp image

Desired output

⊗



Blur  
kernel

Convolution  
operator



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# Why is this hard?

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Simple analogy:

11 is the product of two numbers.

What are they?

No unique solution:

$$11 = 1 \times 11$$

$$11 = 2 \times 5.5$$

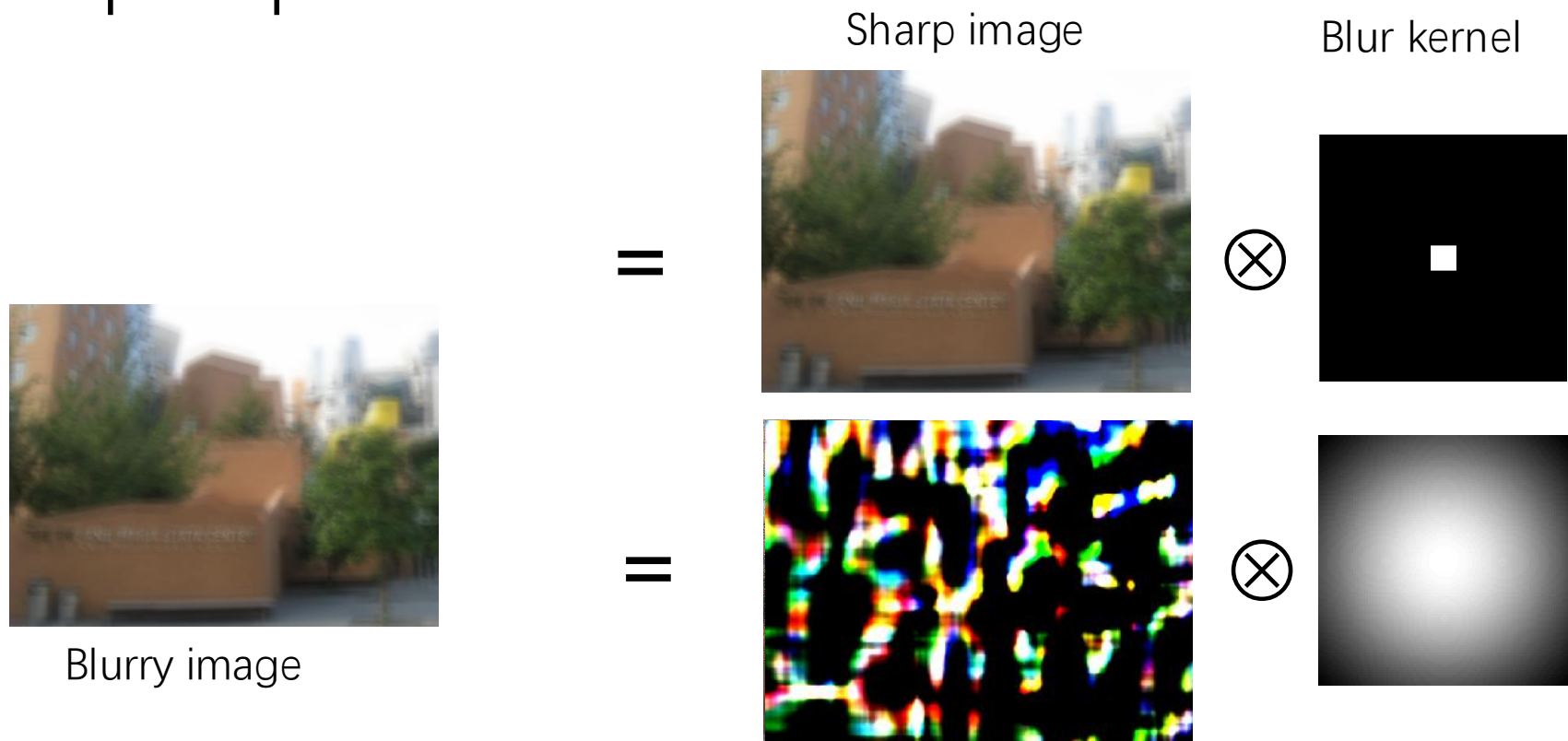
$$11 = 3 \times 3.667$$

etc.....

Need more information !!!!



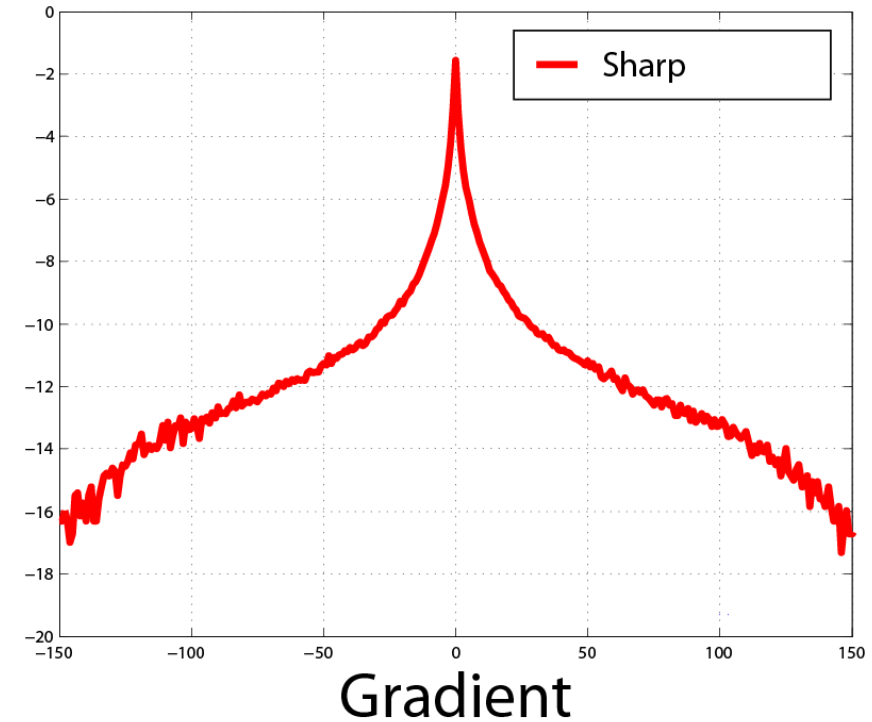
# Multiple possible solutions



# Natural image statistics

Characteristic distribution with heavy tails

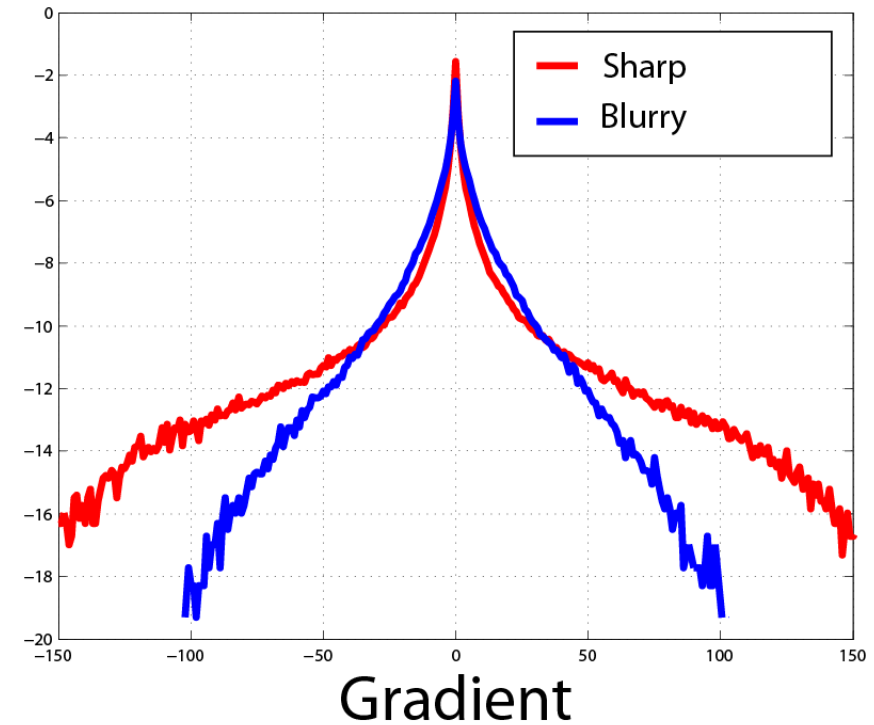
Histogram of image gradients



# Blurry images have different statistics



Histogram of image gradients

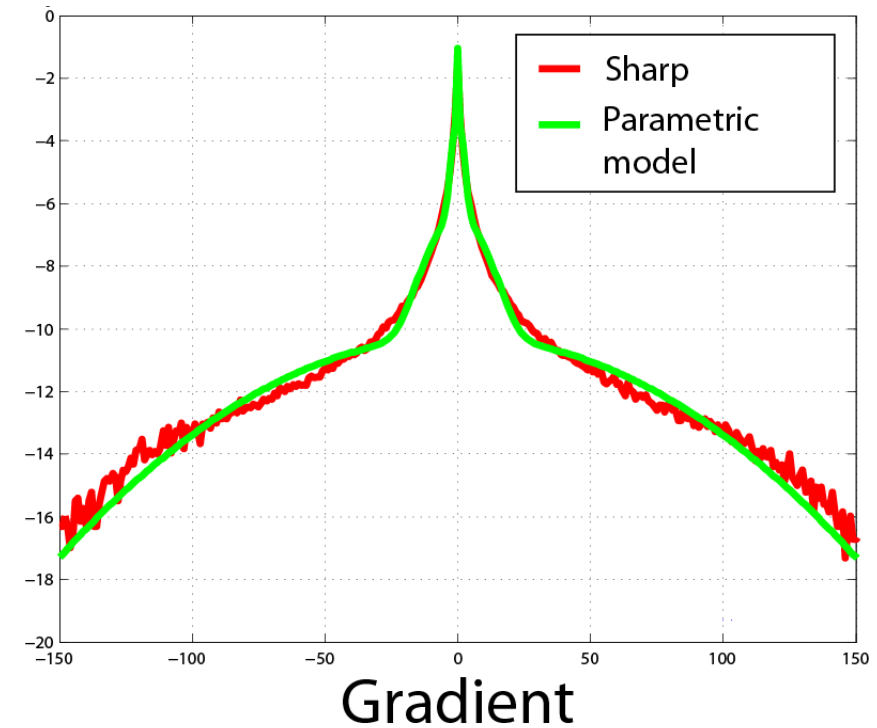




# Parametric distribution



Histogram of image gradients



Use parametric model of sharp image statistics



# Uses of natural image statistics

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- Denoising [Roth and Black 2005]
- Superresolution [Tappen et al. 2005]
- Intrinsic images [Weiss 2001]
- Inpainting [Levin et al. 2003]
- Reflections [Levin and Weiss 2004]
- Video matting [Apostoloff & Fitzgibbon 2005]

Corruption process assumed known



# Existing work on image deblurring

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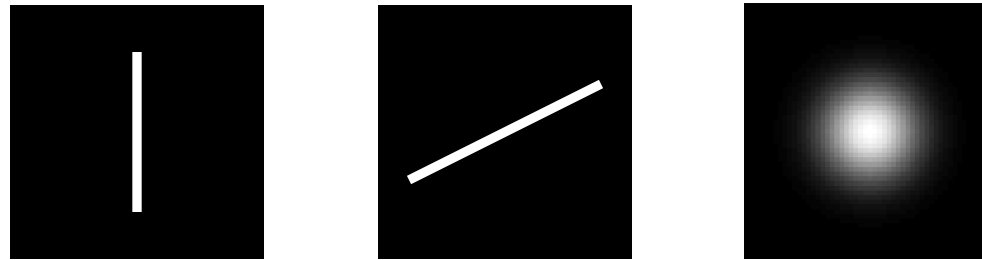
Software algorithms:

- Extensive literature in signal processing community

Mainly Fourier and/or Wavelet based

Strong assumptions about blur

→ not true for camera shake



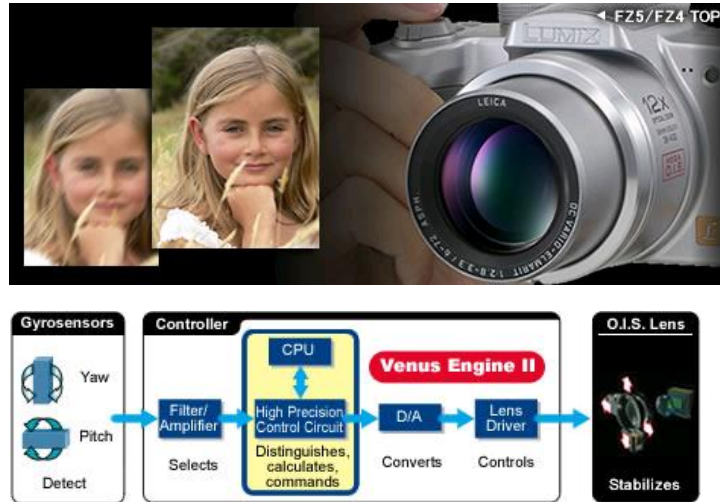
Assumed forms of blur kernels

Image constraints are frequency-domain power-laws

# Existing work on image deblurring

## Hardware approaches

Image stabilizers



Dual cameras



Ben-Ezra and  
Nayar 2004

Coded shutter



Raskar et al.  
SIGGRAPH 2006

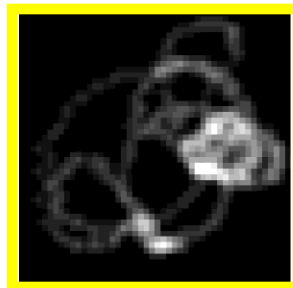
Our approach can be combined with these hardware methods

# Three sources of information

## 1. Reconstruction constraint:



Estimated sharp image

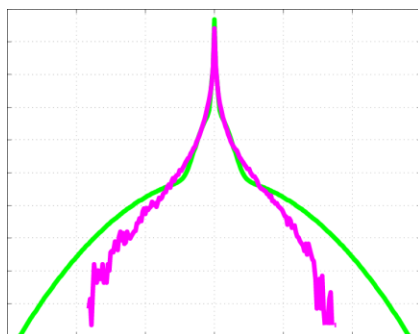


Estimated  
blur kernel



Input blurry image

## 2. Image prior:



Distribution of  
gradients

## 3. Blur prior:



Positive  
&  
Sparse



# How do we use this information?

Obvious thing to do:

- Combine 3 terms into an objective function
- Run conjugate gradient descent
- This is Maximum a-Posteriori (MAP)

$$\max_{K, X} p(K, X | Y)$$

Kernel

Latent clean image

Observed image

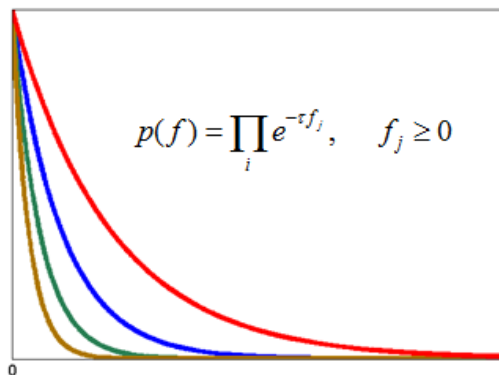
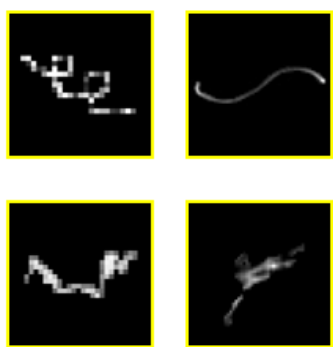


# How do we use this information?

Since these statistics are based on the image gradients rather than the intensities, we perform the optimization in the gradient domain

$$\nabla Y = \nabla X \otimes K$$

Given the measured image gradients  $\nabla Y$ , we can write the posterior distribution over the unknowns with Bayes' Rule:



$$\begin{aligned} p(K, \nabla X | \nabla Y) &\propto p(\nabla Y | K, \nabla X) p(\nabla X) p(K) \\ &= \prod_i N(\nabla Y(i) | (K \otimes \nabla X(i)), \sigma^2) \\ &= \prod_i \sum_{c=1}^C N(\nabla X | 0, v_c) \prod_j \sum_{d=1}^D \pi_d E(K_j | \lambda_d) \end{aligned}$$



# Loss Function

- The variational algorithm minimizes a cost function representing the distance between the approximating distribution and the true posterior, measured as:

$$KL(q(K, \nabla X, \sigma^2) || p(K, \nabla X | \nabla Y))$$

- The independence assumptions in the variational posterior allows the cost function  $\mathcal{C}_{KL}$  to be factored:

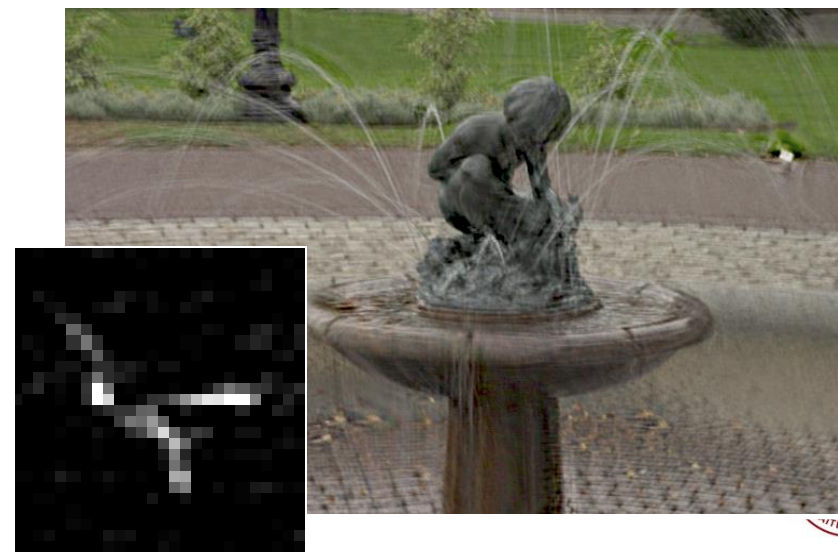
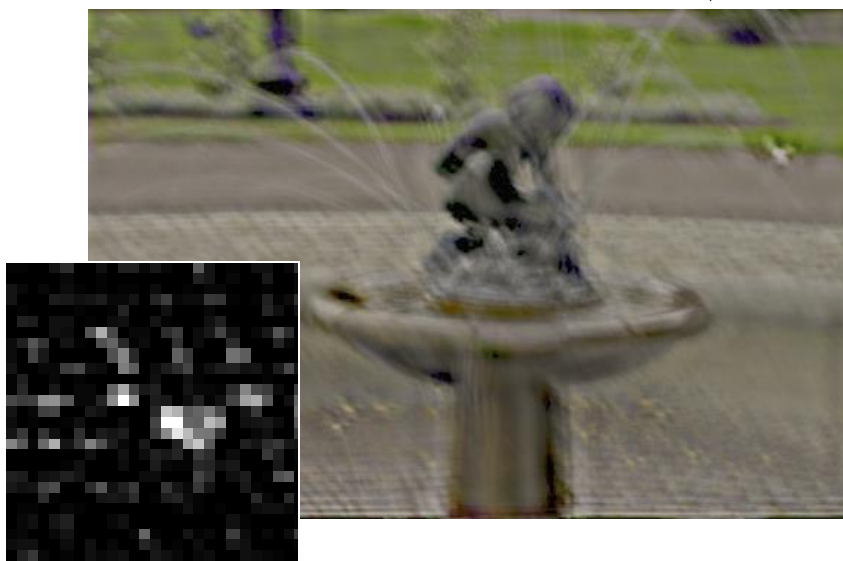
$$\left\langle \log \frac{q(\nabla X)}{p(\nabla X)} \right\rangle_{q(\nabla X)} + \left\langle \log \frac{q(K)}{p(K)} \right\rangle_{q(K)} + \left\langle \log \frac{q(\sigma^{-2})}{p(\sigma^2)} \right\rangle_{q(\sigma^{-2})}$$

# Results from MAP estimation

Input blurry image



Maximum a-Posteriori (MAP)      Our method: Variational Bayes





# Variational Bayesian method

$$P(\text{Image}, \text{Mask}) > P(\text{Image}, \text{Mask})$$

$$p(K, \nabla X | \nabla Y)$$



$$\#Y < \#X + \#K$$



# Variational Bayesian method

$$\operatorname{argmax}_{\{K,X\}} p(K, X|Y) \rightarrow \operatorname{argmax}_{\{K\}} p(K|Y)$$

$$\#Y \gg \#K$$

$$p(K|Y) = \int_{\mathbf{X}} p(K, \mathbf{X}|Y) d\mathbf{X}$$



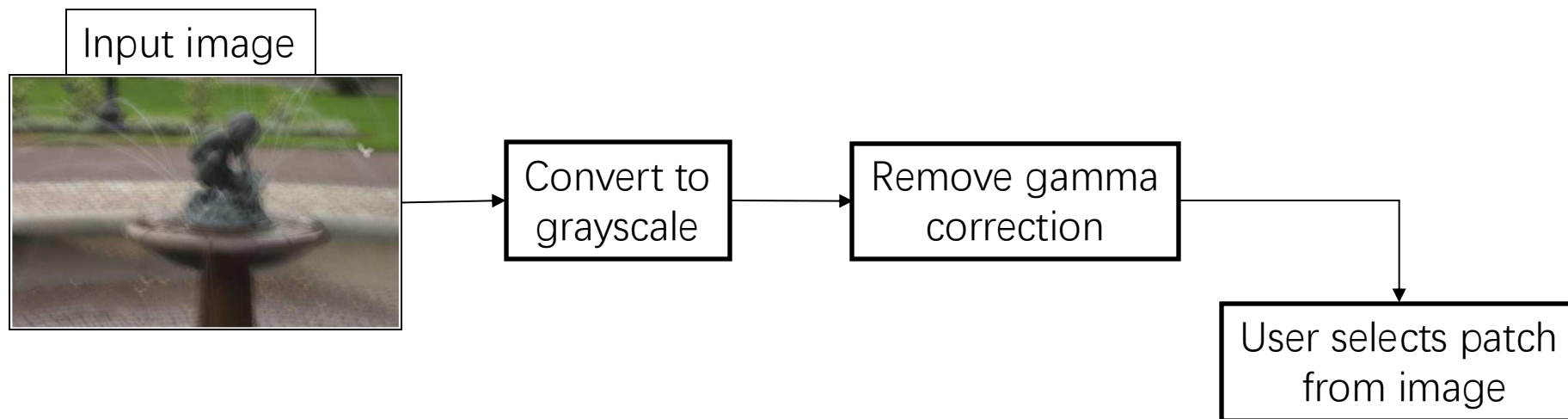
# Overview of algorithm

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1. Pre-processing
2. Kernel estimation
  - Multi-scale approach
3. Image reconstruction
  - Standard non-blind deconvolution routine



# Preprocessing



Bayesian inference  
too slow to run on  
whole image

Infer kernel  
from this patch



# Initialization

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Input image

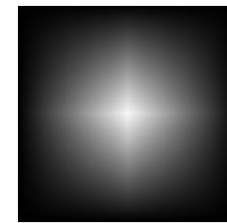


Convert to  
grayscale

Remove gamma  
correction

User selects patch  
from image

Initialize 3x3  
blur kernel



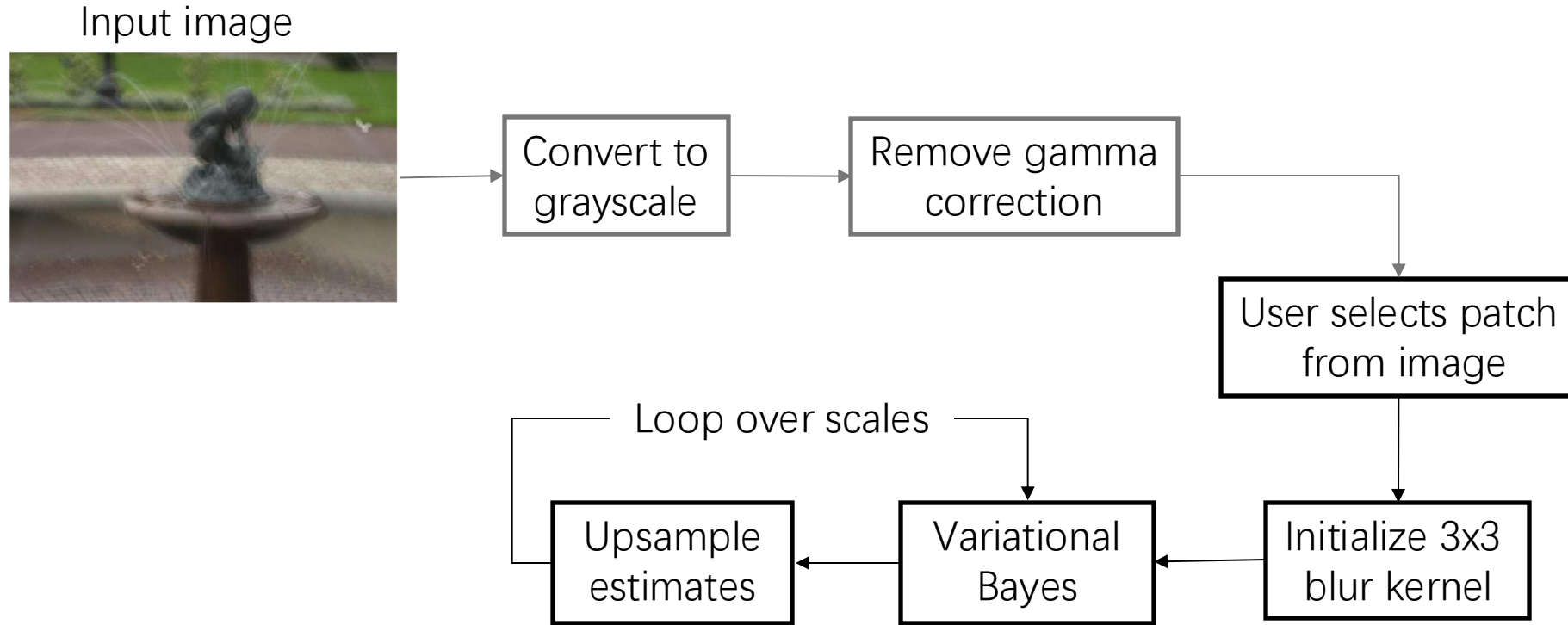
Blurry patch

Initial image estimate

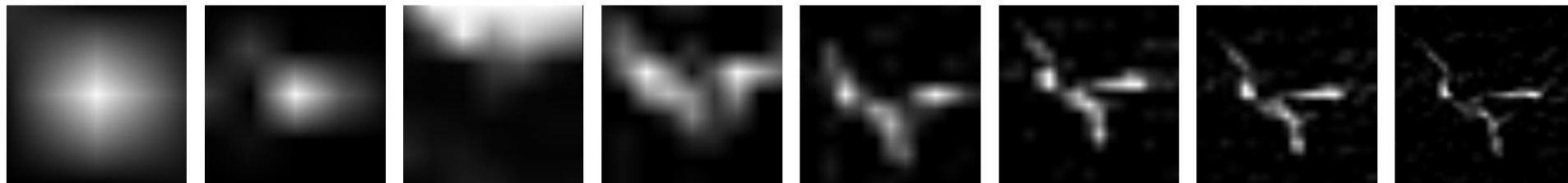
Initial blur kernel



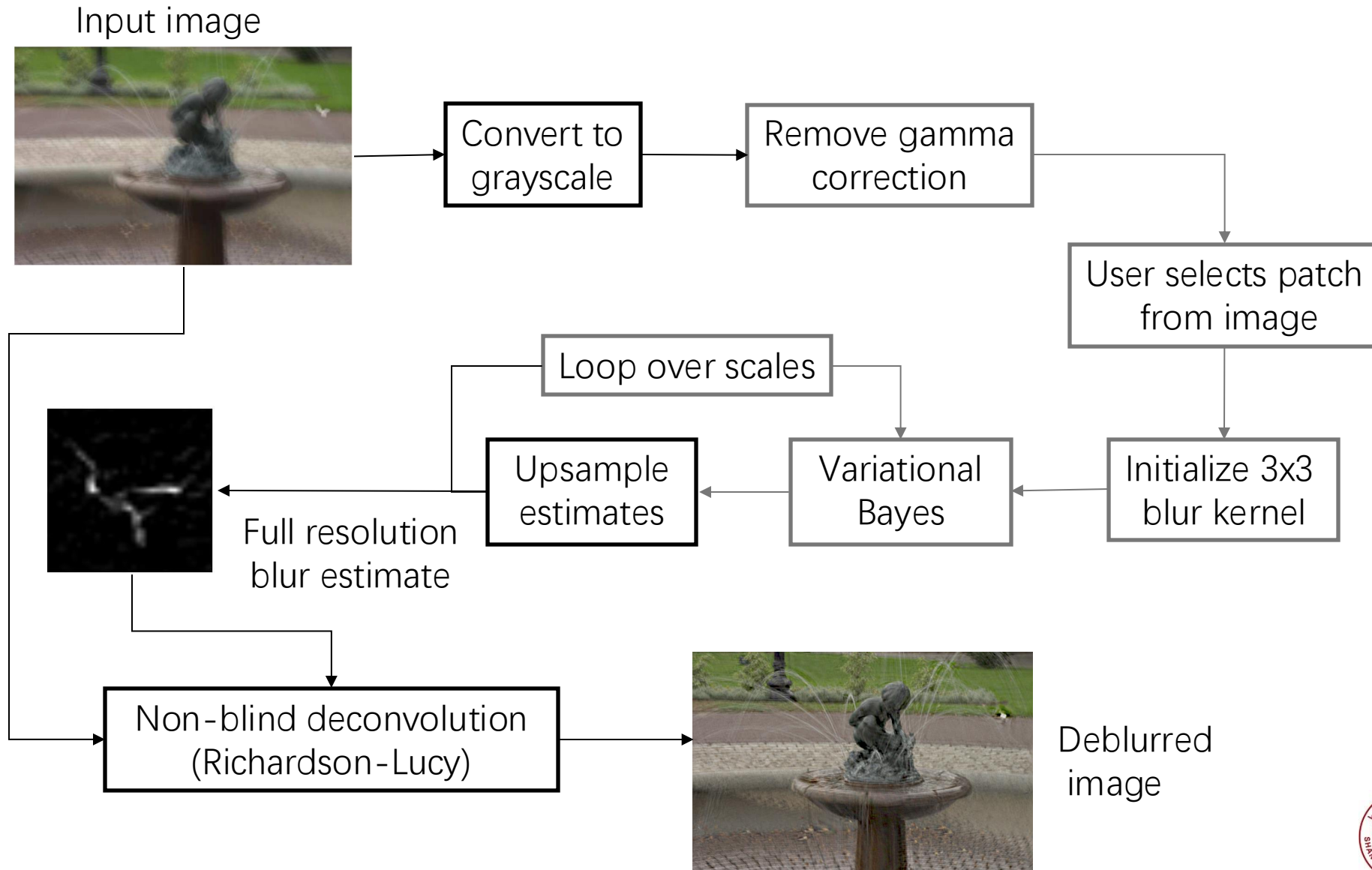
# Inferring the kernel: multiscale method



Use multi-scale approach to avoid local minima:



# Image Reconstruction





# Results on real images

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Submitted by people from their own photo collections

Type of camera unknown

Output does contain artifacts

- Increased noise
- Ringing

Compares well to existing methods

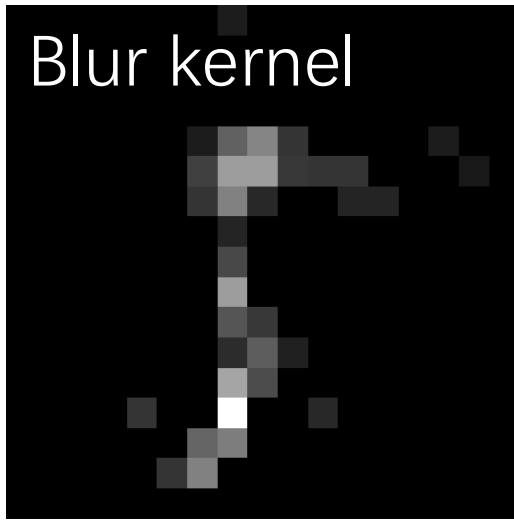


Original photograph



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Blur kernel



Our output





Matlab's deconvblind



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# Close-up of garland

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Original



Matlab's  
deconvblind



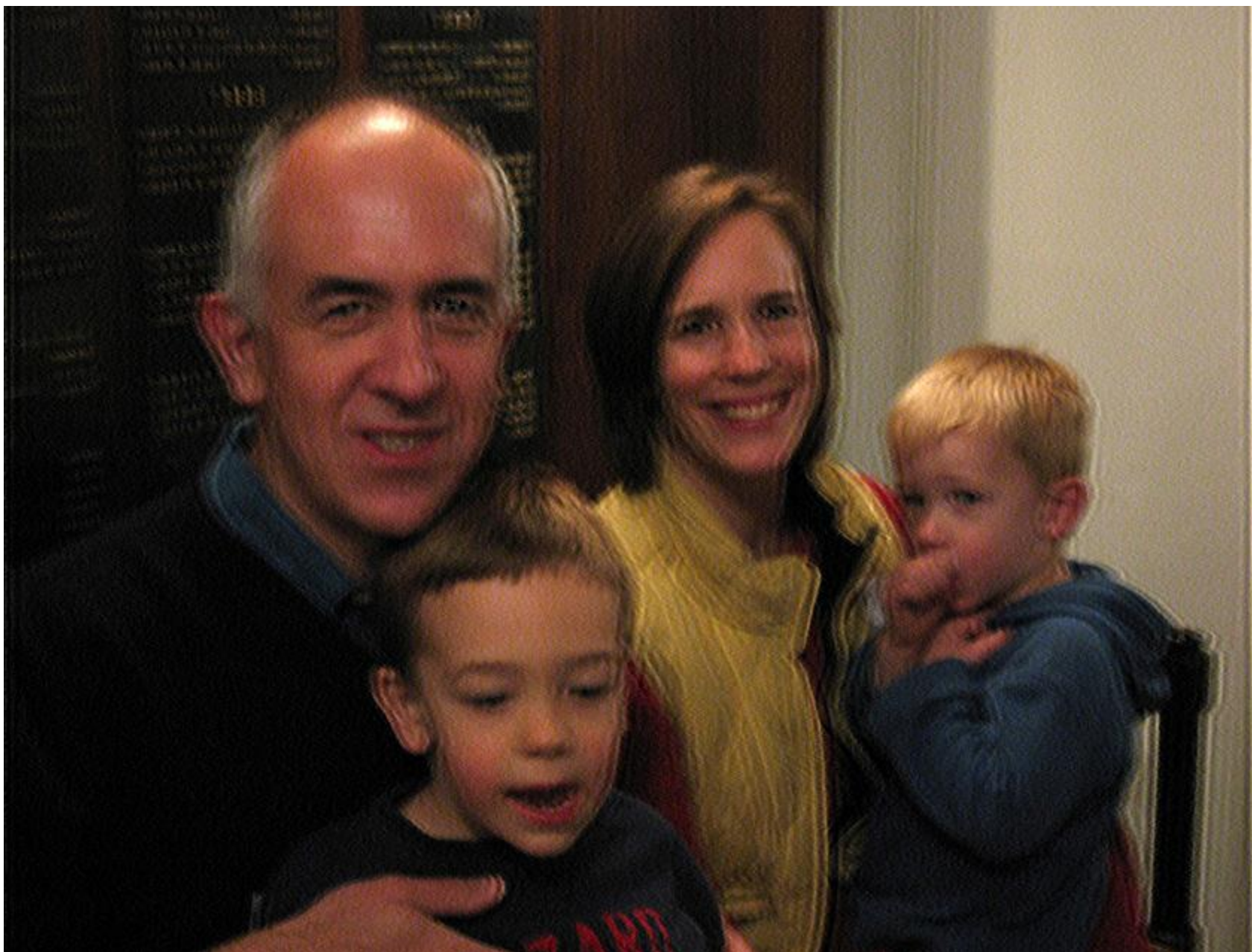
Our output



Original photograph



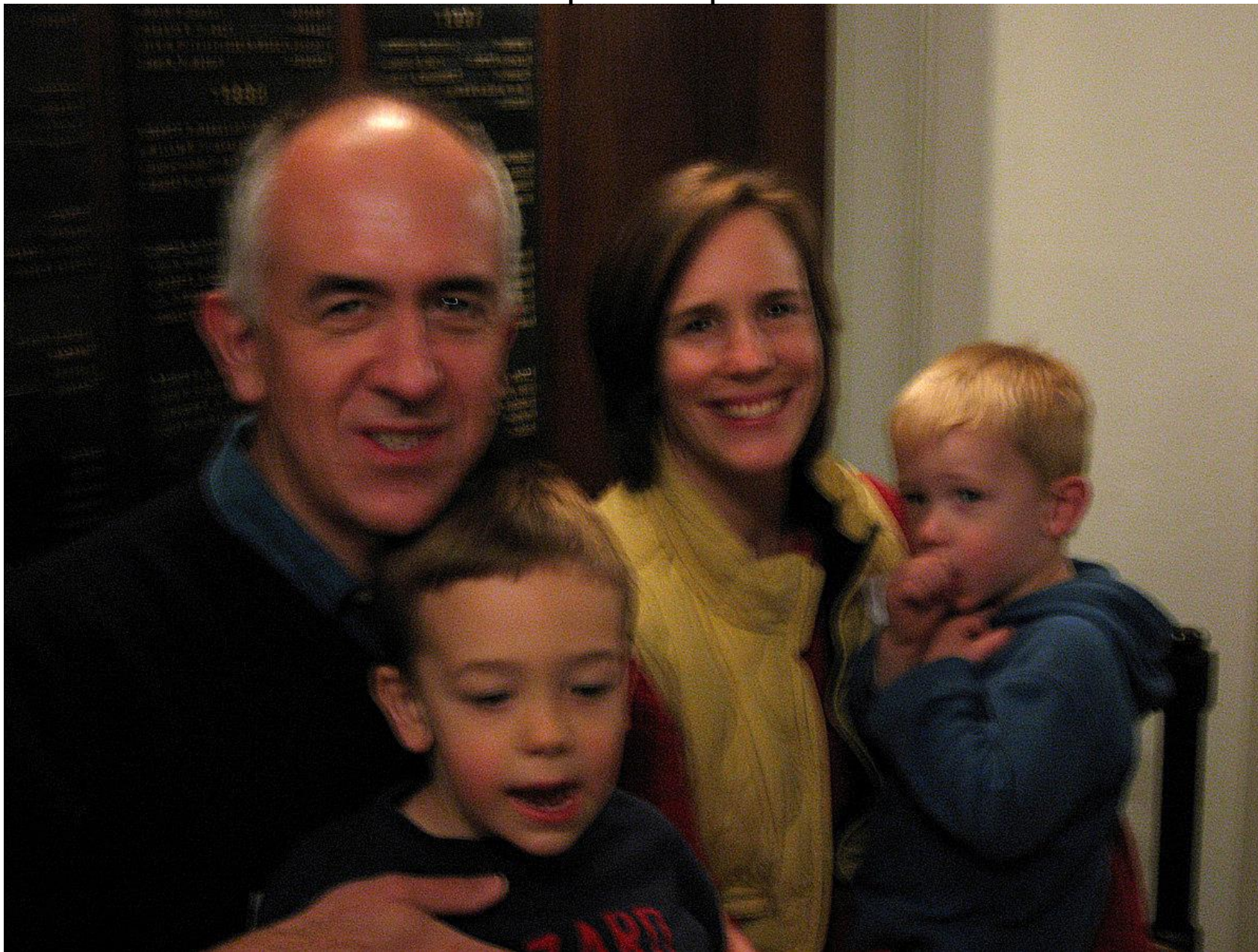




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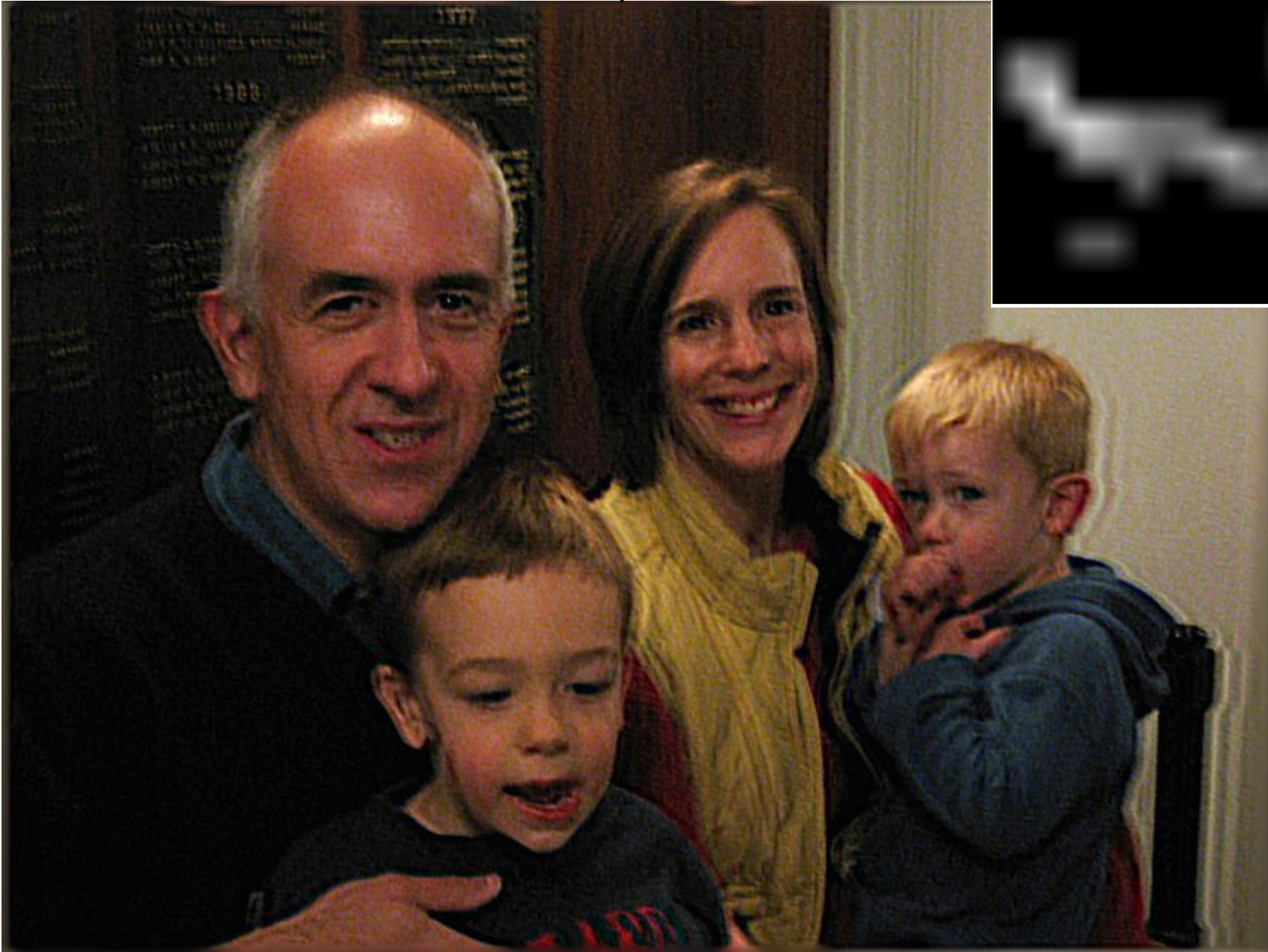


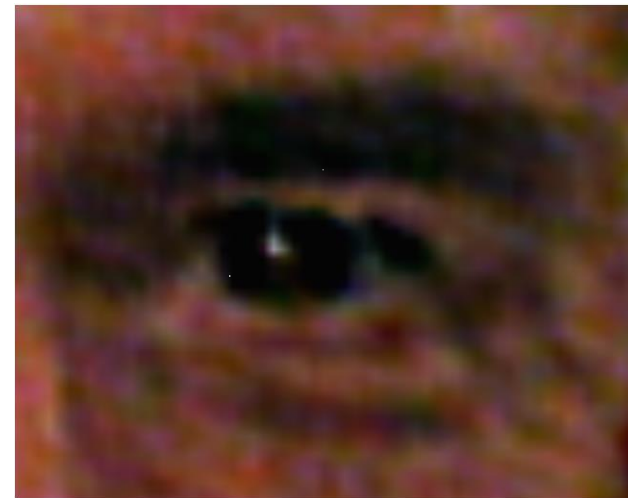
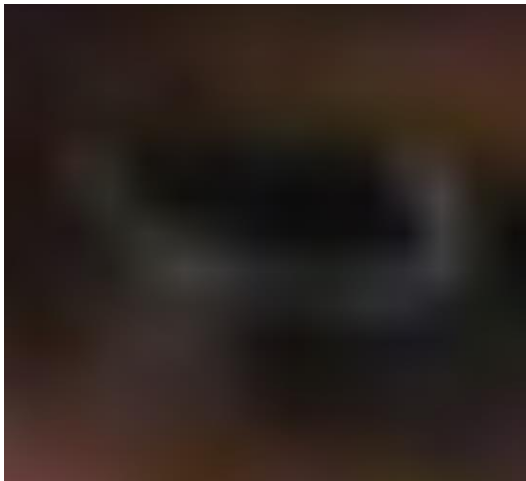
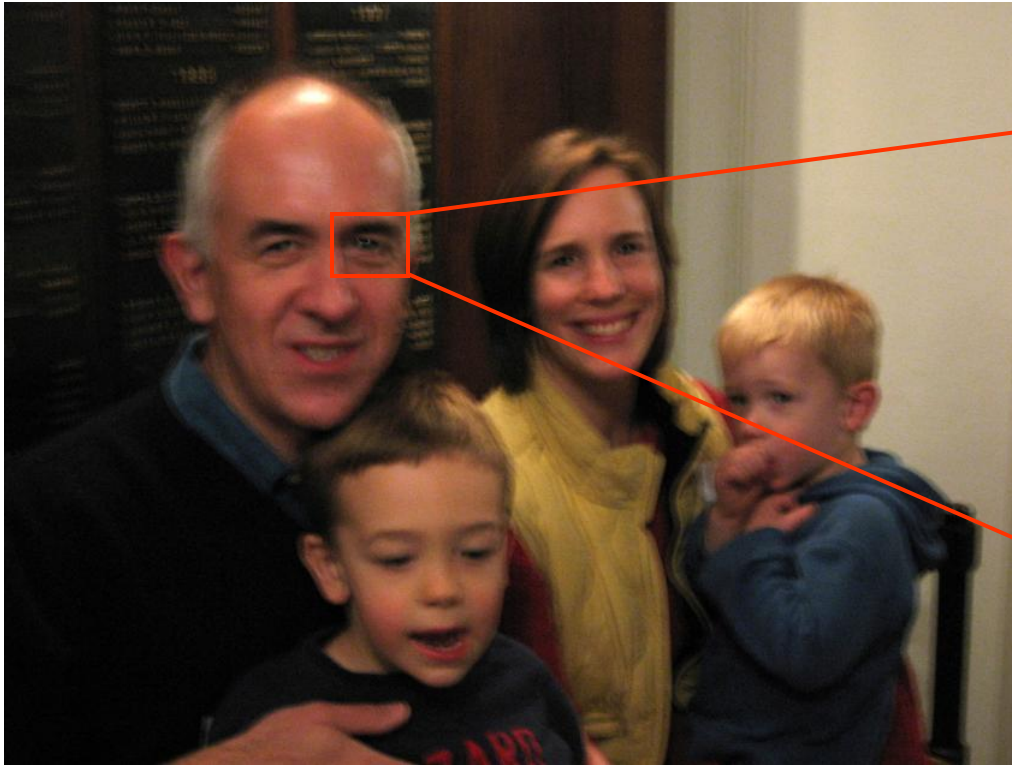
Photoshop sharpen more





Our output





# Original photograph





Our output



Blur kernel



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# Original photograph



Our output



# Matlab's deconvblind



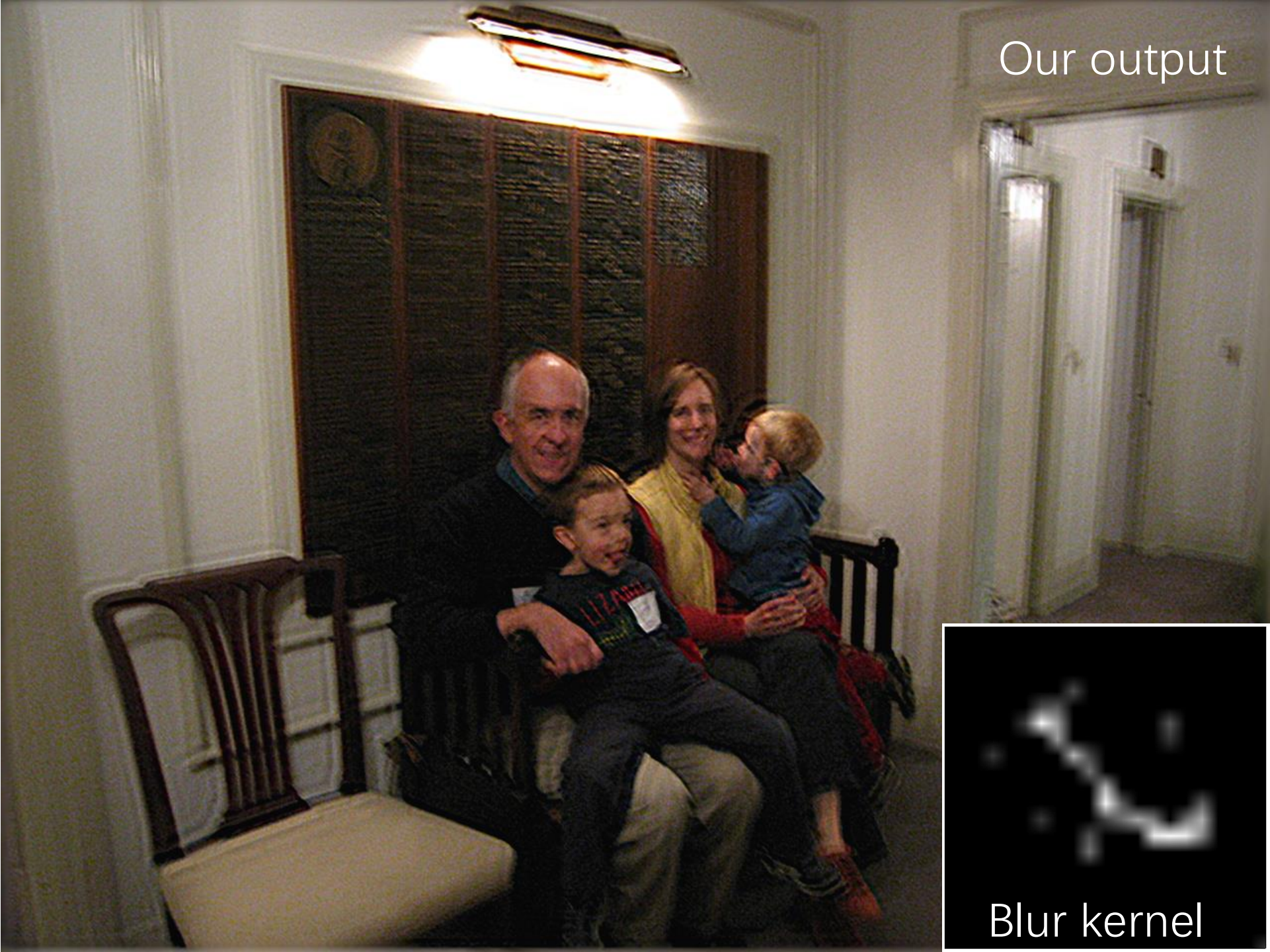


Original photograph





Our output



Blur kernel

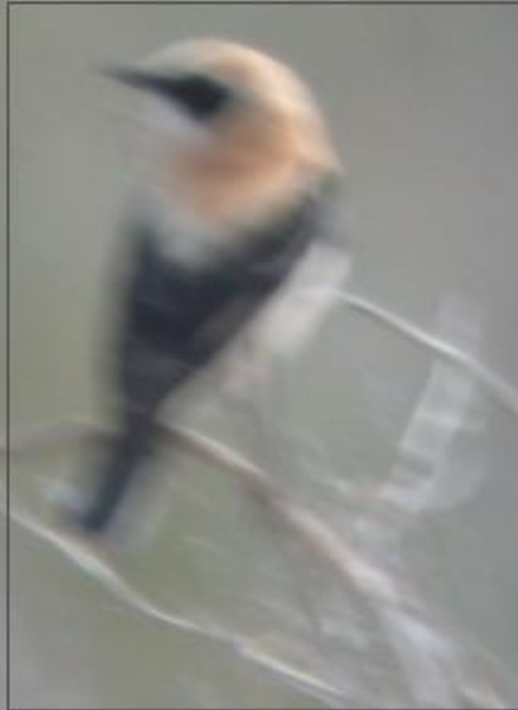
# Close-up of child

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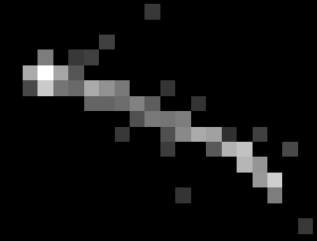




Original photograph



Our output



Blur kernel

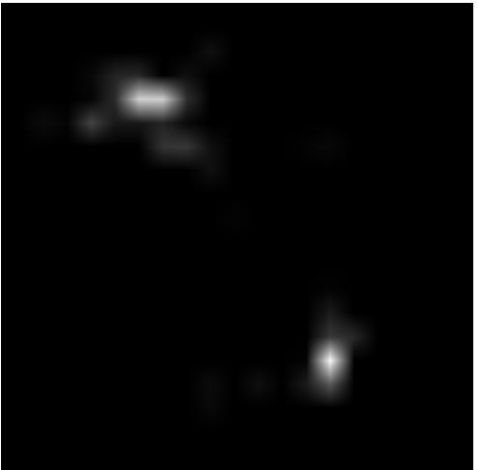






Original photograph

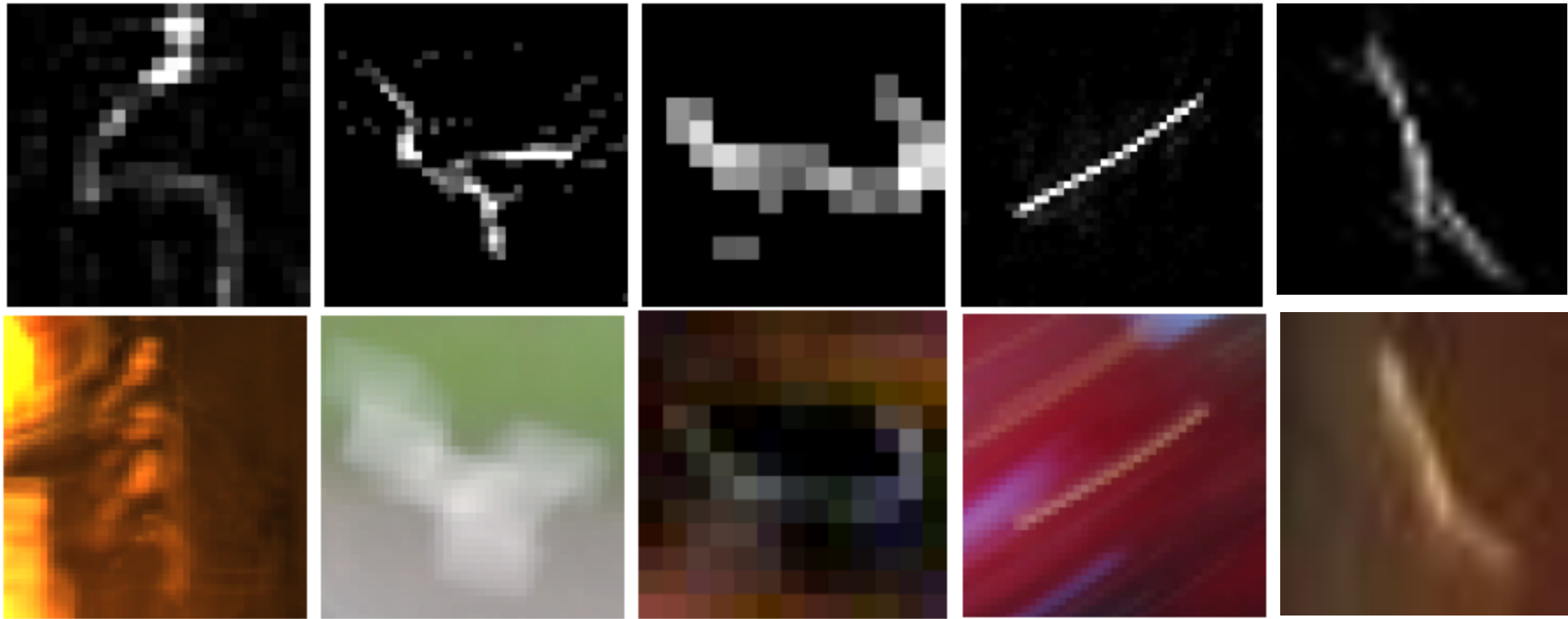




Our output



# Image artifacts & estimated kernels



Note: blur kernels were inferred from large image patches,  
NOT the image patterns shown





# Summary

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Method for removing camera shake  
from real photographs

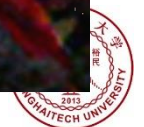
First method that can handle  
complicated blur kernels

Uses natural image statistics

Non-blind deconvolution  
currently simplistic

Things we have yet to model:

- Correlations in colors, scales, kernel continuity
- JPEG noise, saturation, object motion



# A Neural Approach to Blind Motion Deblurring

[1] Chakrabarti, A. (2016). A neural approach to blind motion deblurring. In Computer Vision–ECCV 2016: 14th European Conference, Amsterdam, The Netherlands, October 11–14, 2016, Proceedings, Part III 14 (pp. 221–235). Springer International Publishing.

# Learning a Convolutional Neural Network for Non-uniform Motion Blur Removal

[1] Jian Sun, Wenfei Cao, Zongben Xu, Jean Ponce,, Learning a Convolutional Neural Network for Non-uniform Motion Blur Removal, CVPR, 2015.

# Scale-recurrent Network for Deep Image Deblurring

[1] Xin Tao, Hongyun Gao, Xiaoyong Shen, Jue Wang, Jiaya Jia Scale-recurrent Network for Deep Image Deblurring, CVPR 2018



# Uncertainty-Aware Unsupervised Image Deblurring with Deep Residual Prior

[1] Xiaole Tang, Xile Zhao, Jun Liu, Jianli Wang, Yuchun Miao, Tiejong Zeng, Uncertainty-Aware Unsupervised Image Deblurring with Deep Residual Prior, CVPR 2023

# Self-supervised Non-uniform Kernel Estimation with Flow-based Motion Prior for Blind Image Deblurring

[1] Z. Fang, F. Wu, W. Dong, X. Li, J. Wu and G. Shi, "Self-supervised Non-uniform Kernel Estimation with Flow-based Motion Prior for Blind Image Deblurring," 2023 IEEE/CVF Conference on Computer Vision and Pattern Recognition (CVPR), Vancouver, BC, Canada, 2023, pp. 18105-18114, doi: 10.1109/CVPR52729.2023.01736.

# ID-Blau: Image Deblurring by Implicit Diffusion-based reBLurring AUgmentation

[1] Jia-Hao Wu, Fu-Jen Tsai, Yan-Tsung Peng, Chung-Chi Tsai, Chia-Wen Lin, Yen-Yu Lin ID-Blau: Image Deblurring by Implicit Diffusion-based reBLurring AUgmentation, CVPR 2024



# Joint coil sensitivity and motion correction in parallel MRI with a self-calibrating score-based diffusion model

[1] Lixuan Chen, Xuanyu Tian, Jiangjie Wu, Guoyan Lao, Yuyao Zhang, Hongjiang Wei. "Joint Coil Sensitivity and Motion Correction in Parallel MRI with a Self-Calibrating Score-Based Diffusion Model." Medical Image Analysis, 2025 Feb 21; 102: 103502.