Load Balancing and Scheduling

CS121 Parallel Computing

Fall 2024

Load balancing problem

- Goal is to finish a set of tasks as quickly as possible.
  - Requires resources don’t idle, i.e. do similar amounts of work.
- Load balancing decides which tasks to perform on which processors.
- Scheduling decides the order of tasks, which affects fairness, responsiveness, etc.
- Load balancing and scheduling have a vast literature in parallel and distributed computing, operating systems, operations research, etc.
  - Many different models for computation and communication, precedence constraints, heterogeneous systems, etc.
- Many packages available, e.g. Cilk, ADLB, Zoltan, Chombo, ParMetis.
- For most scheduling problems, finding optimal solution is intractable.
  - Goal of load balancing is speed, so load balance algorithm itself needs to be fast.
  - Rely on fast heuristics that work well in practice or have approximate performance guarantees.

Static vs dynamic

- Some applications are static, i.e. the set of tasks in the application, their sizes and communication pattern are known at the start of the execution.
  - Ex Dense and sparse linear algebra, FFT.
  - Load balancing can be done once at beginning of computation.
    - Can afford to spend more time to get higher quality solution.
- For semi-static problems, task information is known periodically at start of some phases.
  - Ex Particle simulations, grid computations.
  - Periodically rebalance when load changes substantially.
    - Requires relatively efficient algorithm.
- For dynamic problems, information is only known at runtime.
  - Ex Search problems.
  - Constantly rebalance on the fly. Need very lightweight methods.
- Can load balance at different granularities.
  - Fine grained task balancing gives best results, but may take too much time and memory.
  - Can group tasks together for coarse grained balancing.

Centralized vs distributed

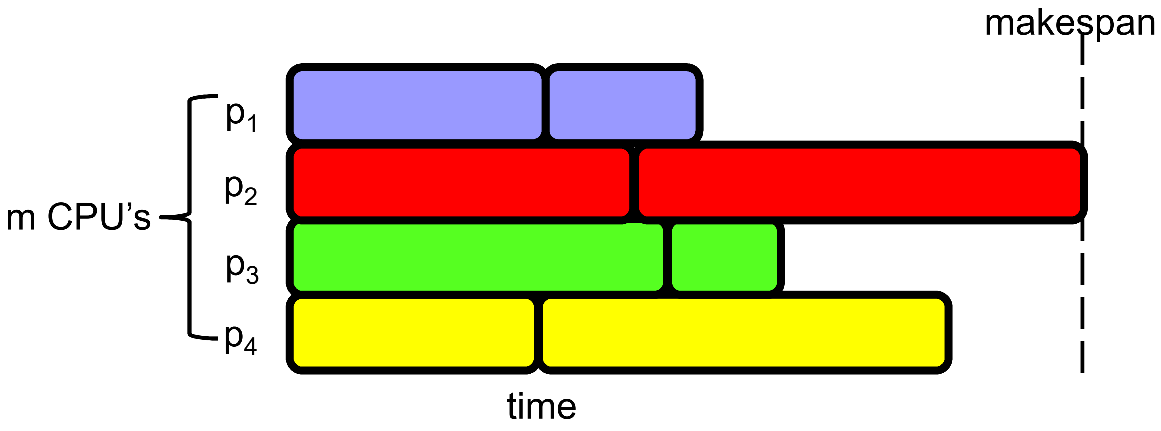
- Centralized load balancing gathers all information at one node.
  - Produces better result since global load info available.
  - Central node becomes performance bottleneck.
- Distributed load balancing lets nodes communicate and make own balancing decisions.
  - More scalable. Can react faster to load changes.
  - Hard to achieve globally optimal result.
  - May be slower than centralized if multiple balancing steps required.
- Hierarchical scheme uses centralized node for coarse grained load balancing, then uses distributed nodes for fine grained balance.
  - Ex First assign groups of tasks to nodes, then divide each group among the processors.

Other issues

- Load estimation tries to predict the load and communication pattern of a task.
  - □ In best case, this can be inferred from the code.
  - □ Otherwise, can profile the task, and assume its future behavior matches the past.
    - Information gathered automatically, without user intervention.
    - Ex Works well for some scientific computations and simulations.
  - □ Alternatively, can build a model for the task behavior.
    - Labor intensive. Must update model if program changes.
- When load changes, can rebalance by migrating tasks from one node to another.
  - □ May be costly, because need to move code and possibly data.

Static load balancing

- Start with a basic model where task sizes are known, there are no precedence constraints between tasks, and ignore communication costs.
  - Even in this model optimal scheduling is NP-hard.
- n independent tasks with different sizes.
  - Tasks can be done in any order.
  - Any task can be done on any processor.
- m processors with the same speed.
  - Each processors can do one task at a time.
- Minimize the makespan, i.e. time when last processor finishes.



Minimizing makespan is NPC

- Show that minimizing makespan on even two processors is NP-complete.
- Decision version of problem is in NP.
- SUBSET-SUM problem: Given a set of numbers S and target t, is there a subset of S summing to t?
  - □ Ex S={1,3,8,9}. For t=9, yes. For t=14, no.
  - □ SUBSET-SUM is NP-complete. Will reduce it to 2 processor makespan scheduling.
- Let (S,t) be an instance of SUBSET-SUM, and let s be sum of all elements in S.
- Make a set of tasks J = S∪{s-2t}, and schedule them on 2 processors.
- Show that SUBSET-SUM reduces to min makespan, i.e. SUBSET-SUM has a solution iff min makespan has a certain solution.

Minimizing makespan is NPC

- Claim If some subset of S sums to t, then min makespan is s-t.
- Proof Say S'⊆S sums to t. Schedule the tasks in S' and task s-2t on processor 1. So processor 1 finishes at time t+s-2t=s-t. Processor 2 does the tasks in S-S', so it finishes at time s-t as well. Since processors finish at same time, the makespan is minimal.
- Claim If the min makespan is s-t, there exists a subset of S that sums to t.
- Proof Suppose WLOG processor 1 does the s-2t task. Since makespan is s-t, the other tasks processor 1 does must have total size s-t-(s-2t)=t.
- So (S,t) is yes instance of SUBSET-SUM iff minimum makespan = s-t, so minimizing makespan is NPC.

List scheduling

- Since scheduling is NPC, it’s unlikely we can find the min makespan in polytime.
- List scheduling (Graham) is a simple greedy algorithm that finds a schedule with makespan at most twice the minimum.
  - Call this a 2-approximation.
- If there are precedence constraints, we can modify list scheduling to allocate a task whenever it’s available, i.e. all its preceding tasks are finished.
  - This still gives a 2-approximation, but we won’t prove it.

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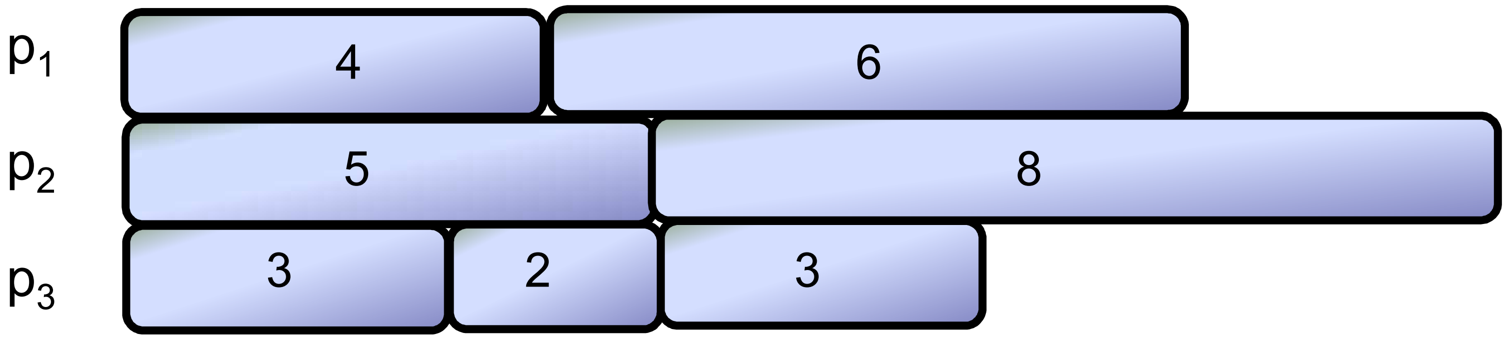
List the tasks in any order.
While there are unfinished tasks.
    If any processor is idle, give it
    the next task in the list.
```

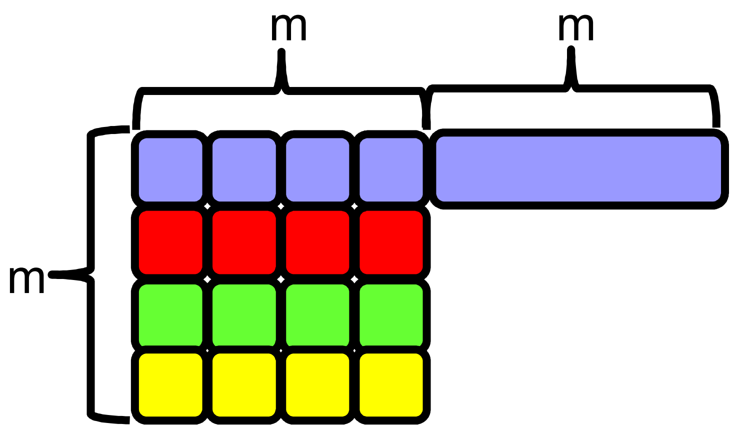
Example

- 3 processors. The tasks have sizes 2, 3, 3, 4, 5, 6, 8.
- List tasks in any order. Say 4, 5, 3, 2, 6, 8, 3.
- All processors finishes by time 13, so makespan = 13.

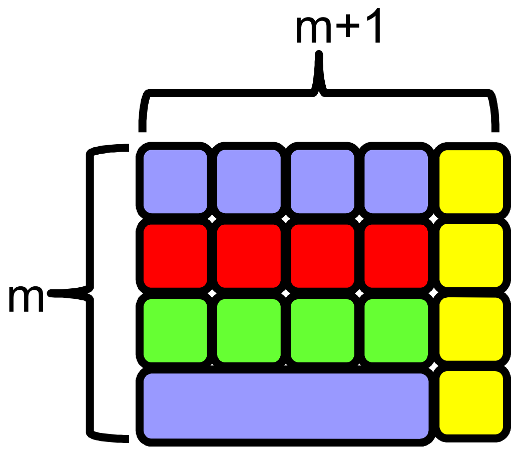


Worst case for LS

- How badly can list scheduling do compared to optimal?
- Say there are m^2 tasks with length 1, and one task with length m.
  - Suppose they're listed in the order 1,1,1,...,1,m.
  - LS has makespan 2m. Optimal makespan is m+1.
  - makespan(LS) / makespan(opt) = 2m/(m+1) $\approx$ 2.
- This is worst possible case for list scheduling.
- Thm Suppose the optimal makespan is M*, and LS produces a schedule with makespan M. Then M$\leq$ 2M*.



makespan(LS) = 2m



makespan(opt) = m+1

LPT scheduling

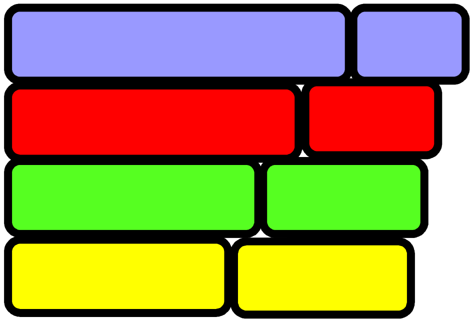
- Worst case for LS occurred when longest job was scheduled last.
  - Large jobs are “dangerous” at end.
- Let’s try to schedule longest jobs first.
- Longest processing time (LPT) schedule is just like list scheduling, except it first sorts tasks by nonincreasing order of size.
- Ex For three processors and tasks with sizes 2, 3, 3, 4, 5, 6, 8, LPT first sorts the jobs as 8,6,5,4,3,3,2. Then it assigns p_1 tasks 8,3, p_2 tasks 6,3, p_3 tasks 5,4,2, for a makespan of 11.
- LPT has an approximation ratio of 4/3.
- LS still has two advantages.
  - It can schedule tasks dynamically, as they’re generated on the fly.
  - It doesn’t need to know the sizes of the tasks.

LPT is a 4/3-approximation

- Thm Suppose the optimal makespan is M*, and LPT produces a schedule with makespan M. Then M $\leq$ 4/3 M*.
- Let X be the last job to finish. Assume it starts at time T and has size t.
- Assume WLOG that X is the last job to start.
  - If not, then say Y starts after T.
  - Y finishes before T+t. So we can remove Y without increasing the makespan.
- Cor 1 X is the smallest job.
  - X is the last job to start, so due to LPT scheduling it's the smallest.

LPT is a 4/3-approximation

- Claim 1 LPT's makespan = $T+t \leq M^* + t$.
  - No processor is idle up to time T.
  - So total work from all jobs is so $W \geq T^*p$, and $M^* \geq W / p = T$.
- Case 1 $t \leq M^*/3$.
  - Then LPT's makespan $\leq M^* + t \leq M^* + M^*/3 = 4/3\ M^*$.
- Case 2 $t > M^*/3$.
  - Since X is the smallest task, all tasks have size $> M^*/3$.
  - So the optimal schedule has at most 2 tasks per processor. So $n \leq 2m$.
  - If $1 \leq n \leq m$, then LPT and optimal schedule both put one task per processor.
  - If $m < n \leq 2m$, then optimal schedule is to put tasks in nonincreasing order on processors 1,...,m, then on m,...,1.
    - LPT also schedules tasks this way, so it's optimal.

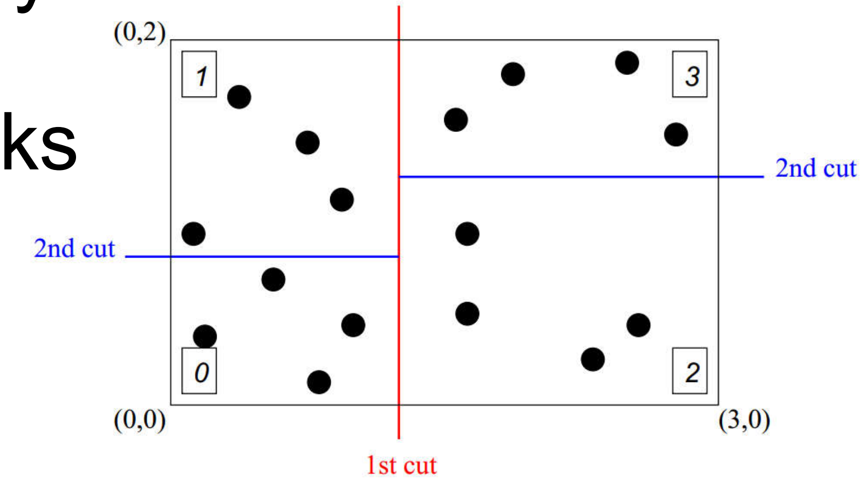


Geometric load balancing

- In many parallel applications, tasks have geometric coordinates, and nearby tasks communicate with each other.
  - Ex In a particle simulation, nearby particles interact.
  - Assume a static or semi-static setting, where tasks have same size.
- We want to load balance and also minimize communication.
  - Want to place nearby tasks on same processor.
  - Still assume the task sizes are known.
- Represent each task by a point at some coordinates.
- Partition the points into m sets. Assign tasks in each set to one processor.

Recursive bisection

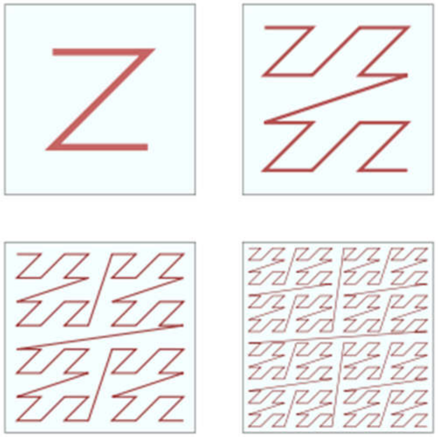
- First partition tasks evenly in the x direction.
- In each half, partition tasks evenly in the y direction.
- In each quarter, partition tasks evenly in the x direction. Etc.
- This might lead to partitions with large aspect ratios, causing many communicating tasks to lie in different partitions.

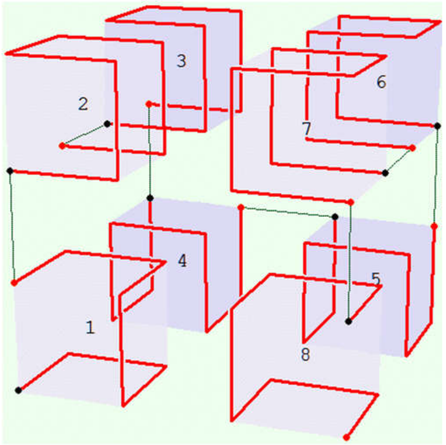


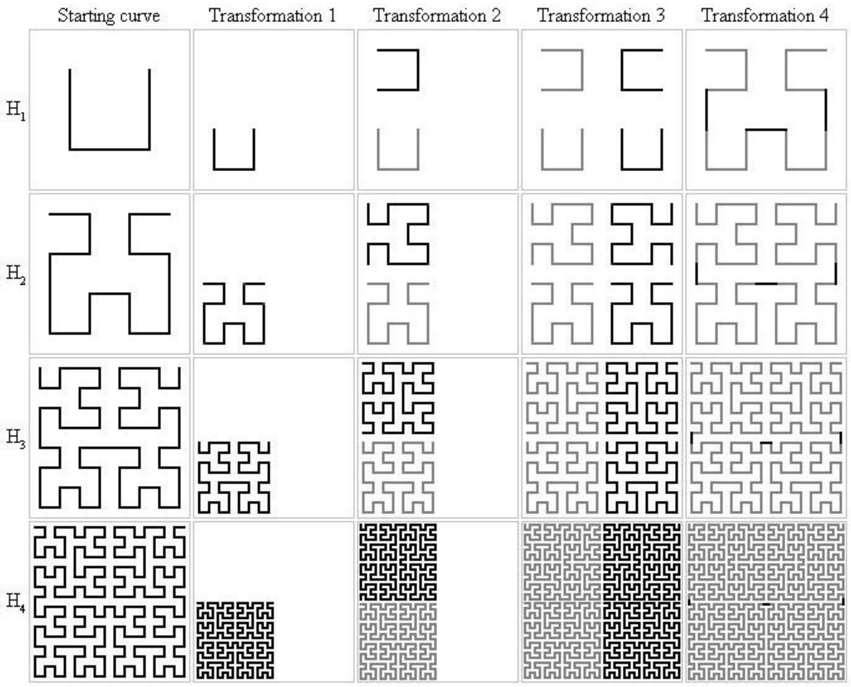
Source: New Challenges in Dynamic Load Balancing, Devine et al.

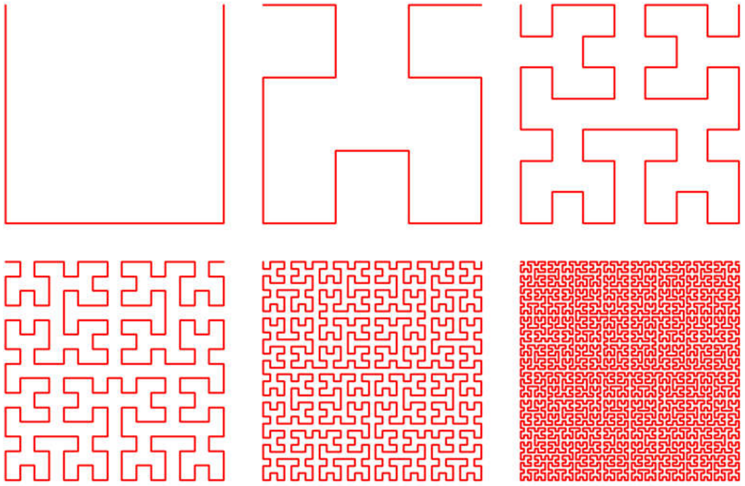
Space filling curve

- A space filling curve (SFC) is a 1-D curve that passes through all the points in a discrete / continuous space.
- SFC's have good locality properties, i.e. nearby points in the SFC are nearby in space, and usually vice versa.
- Many types of SFC's, e.g. Morton (Z-order) and Hilbert curves.
- SFC's can be generalized to higher dimensions.



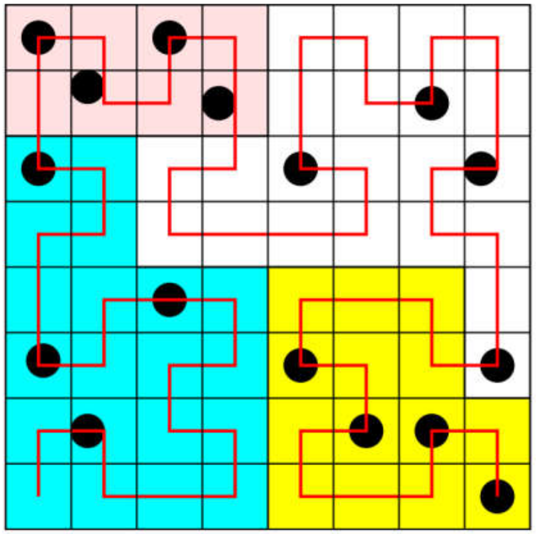






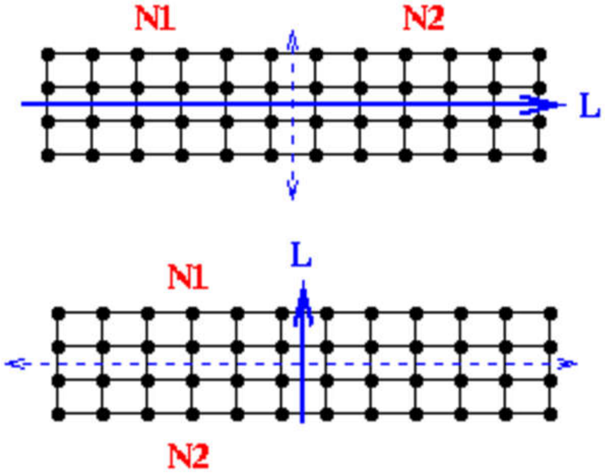
Space filling curve partitioning

- We can use an SFC to map nodes onto a 1D line.
- Then we partition nodes along the line evenly.
- Given a node, there are efficient algorithms to determine which partition it lies in.
- Given a box, can also efficiently enumerate all nodes lying inside the box.
- These operations are useful for particle simulations and collision detection.

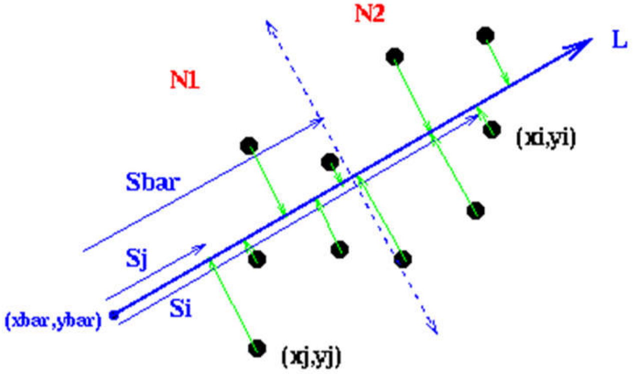


Inertial partitioning

- The top partitioning (across the dotted line) is better than the bottom one, because it cuts fewer communicating pairs of nodes.
- Intuitively, we want to find a line L that minimizes the moment of inertia of the nodes rotating around L.
  - □ A closed form solution exists for the optimal L.
- Once we have L, project all the nodes onto L, then find the median.
  - □ Partition nodes based on which side of the median their projection lies.
- This produces two partitions. For m partitions, apply the partitioning recursively.



Inertial Partitioning in 2D



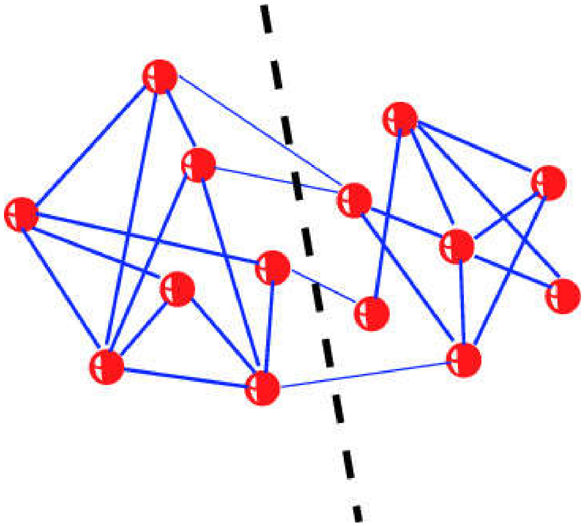
Source: https://people.eecs.berkeley.edu/~demmel/cs267/lecture18/lecture18.html

Graph based partitioning

- In geometric partitioning, the communicating tasks were defined implicitly based on proximity.
- In graph based partitioning, we're given an explicit graph showing the pairs of communicating nodes.
    - Graph nodes can be weighted based on size of the task.
    - Graph edges can be weighted based on amount of communication.
- Want to partition nodes of graph in two parts, and map each part onto a processor.
    - If we have more than two processors, do the partitioning recursively.
    - Want each part to have approximately same number / weight of nodes, for load balance.
    - Want to minimize number of edges cut (i.e. crossing between partitions), because these represent communication between processors.

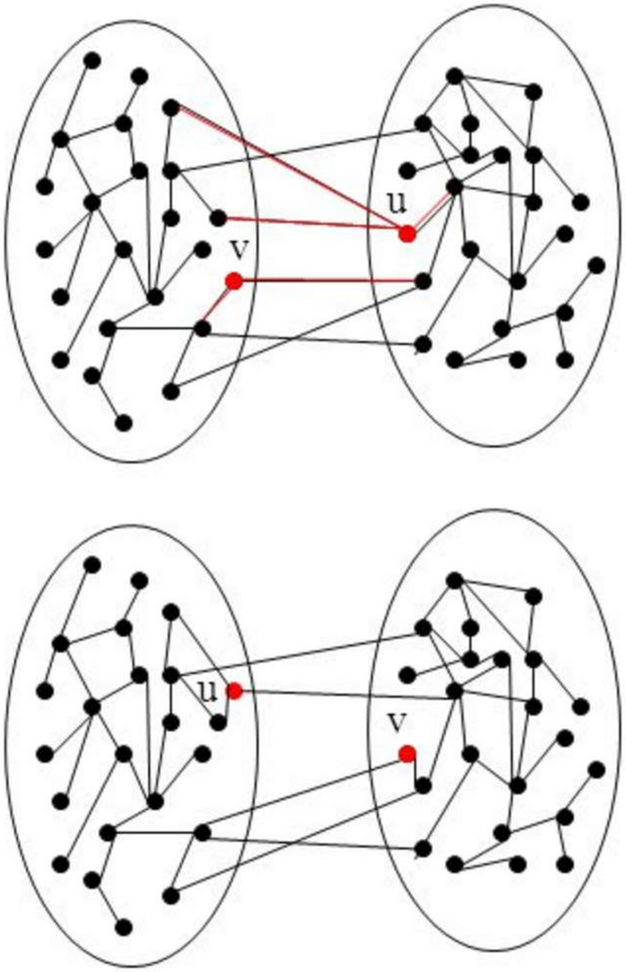
Graph based partitioning

- Computing optimal partitioning is NP-complete, so we have to settle for heuristics.
  - □ Local search (e.g. Kernighan-Lin).
  - □ Spectral.
  - □ Multilevel.



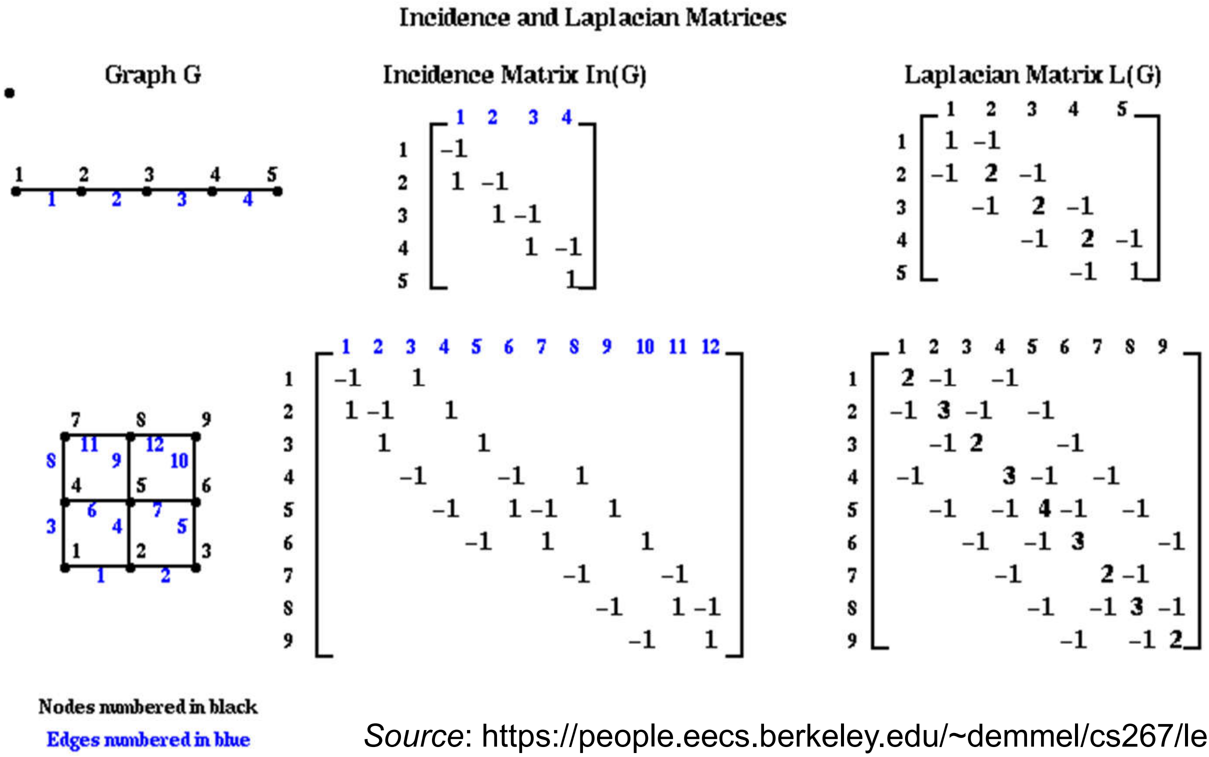
Kernighan-Lin partitioning

- Greedily improve a partition by swapping nodes until no improvement possible.
- Start with an arbitrary partition A, B, each with half the nodes.
- For i=1, ..., n/2 iterations.
  - Choose $a_i \in A$ and $b_i \in B$ s.t. a_i and b_i have never been swapped before, and swapping a_i and b_i results in smallest cut.
  - Let C_i be the cost of the partition after swapping a_i and b_i.
- Choose the C_i with the lowest cost.
- If C_i's cost is lower than cost of (A, B), restart the algorithm with partition C_i.
- Otherwise return C_i.
- In practice KL is fast and returns reasonably good partitions.



Graph Laplacian

- Given an undirected graph G, define the Laplacian matrix L(G)
  - For each edge (i,j) in G, set entry (i,j) to -1 in L(G).
  - For each node i, set entry (i,i) of L(G) to d, where d is the degree of i.
  - Set all other entries to 0.
- Similar to adjacency matrix, but has interesting spectral properties.



Incidence and Laplacian Matrices

Graph G

Incidence Matrix In(G)

$$\begin{array}{c|cccc}
 & 1 & 2 & 3 & 4 \\
\hline
1 & -1 & & & \\
2 & 1 & -1 & & \\
3 & & 1 & -1 & \\
4 & & & 1 & -1 \\
5 & & & & 1
\end{array}$$

Laplacian Matrix L(G)

$$\begin{array}{c|ccccc}
 & 1 & 2 & 3 & 4 & 5 \\
\hline
1 & 1 & -1 & & & \\
2 & -1 & 2 & -1 & & \\
3 & & -1 & 2 & -1 & \\
4 & & & -1 & 2 & -1 \\
5 & & & & -1 & 1
\end{array}$$

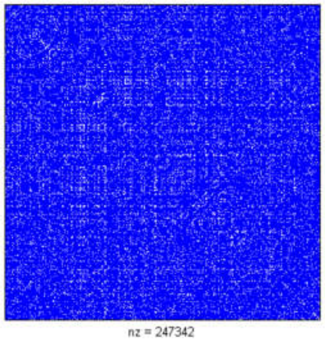
$$\begin{array}{c|cccccccccccc}
 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 \\
\hline
1 & -1 & & 1 & & & & & & & & & \\
2 & 1 & -1 & & 1 & & & & & & & & \\
3 & & 1 & & & 1 & & & & & & & \\
4 & & & -1 & & & -1 & & 1 & & & & \\
5 & & & & -1 & & 1 & -1 & & 1 & & & \\
6 & & & & & -1 & & 1 & & & 1 & & \\
7 & & & & & & & & -1 & & & -1 & \\
8 & & & & & & & & & -1 & & 1 & -1 \\
9 & & & & & & & & & & -1 & & 1
\end{array}$$

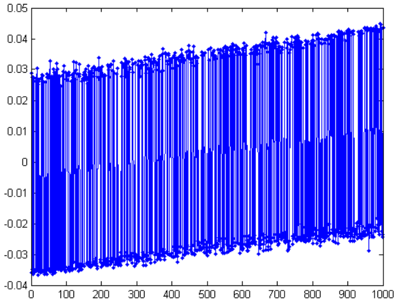
$$\begin{array}{c|ccccccccc}
 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\
\hline
1 & 2 & -1 & & -1 & & & & & \\
2 & -1 & 3 & -1 & & -1 & & & & \\
3 & & -1 & 2 & & & -1 & & & \\
4 & -1 & & & 3 & -1 & & -1 & & \\
5 & & -1 & & -1 & 4 & -1 & & -1 & \\
6 & & & -1 & & -1 & 3 & & & -1 \\
7 & & & & -1 & & & 2 & -1 & \\
8 & & & & & -1 & & -1 & 3 & -1 \\
9 & & & & & & -1 & & -1 & 2
\end{array}$$

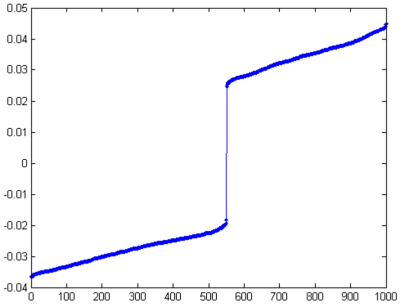
Source: https://people.eecs.berkeley.edu/~demmel/cs267/lecture18/lecture18.html

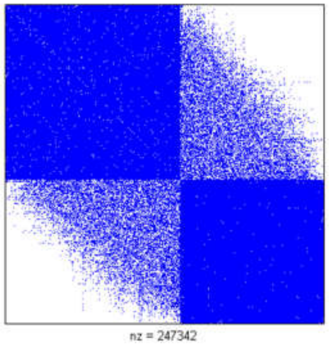
Spectral partitioning

- Fact L(G) is positive semidefinite, and all the eigenvalues of L(G) are real and nonnegative.
- Let $(x_1, x_2, ..., x_n)$ be the eigenvector of L(G) corresponding to the second smallest eigenvalue.
- Partition G as $A = \{i : x_i \leq 0\}$ and $B = \{i : x_i > 0\}$.
- Usually produces better partitions than Kernighan-Lin.
- But finding second eigenvector is quite expensive.
  - □ Suffices to find approximate eigenvector. But this is still costly.







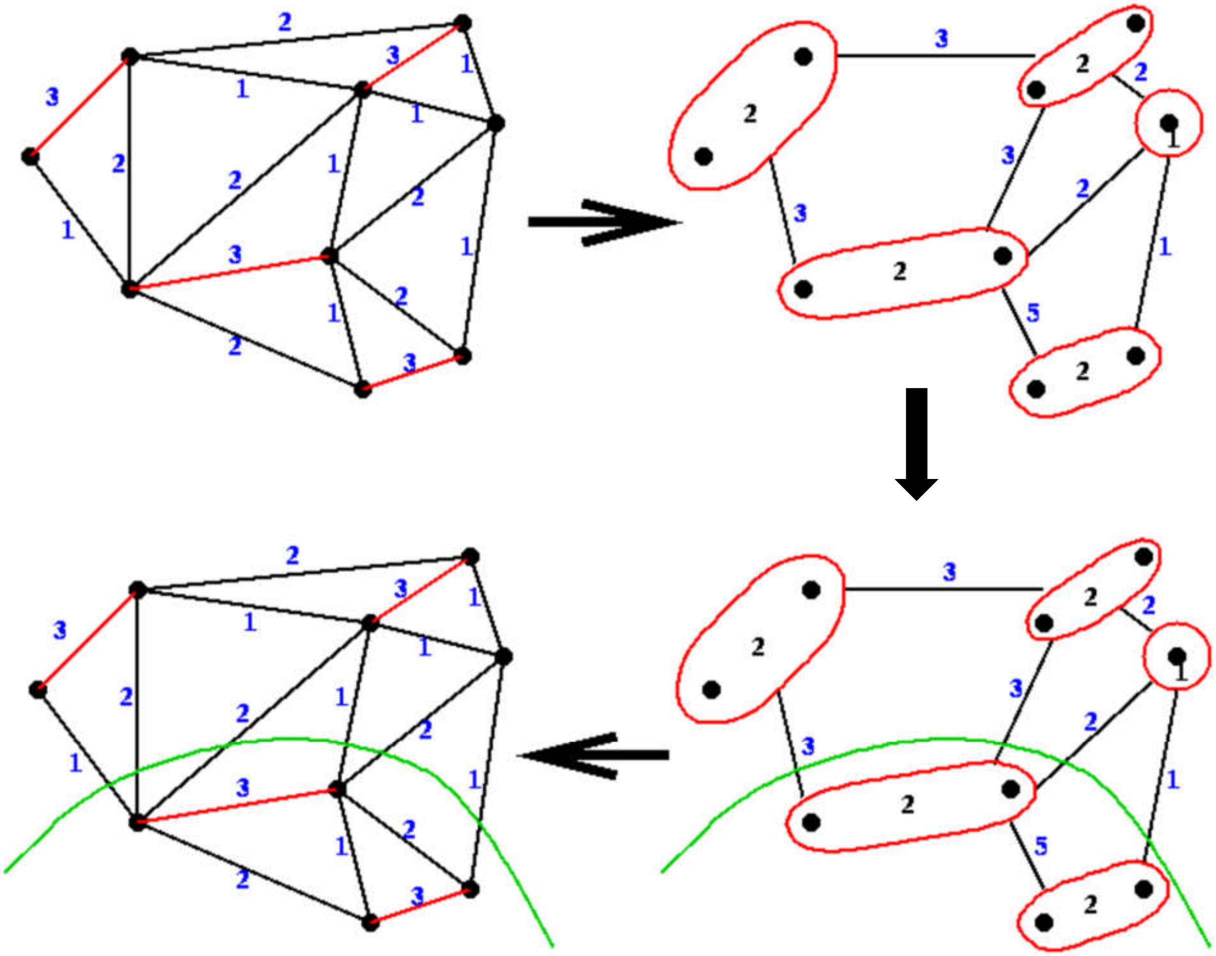


Source: https://www.cs.purdue.edu/homes/dgleich/demos/matlab/spectral/spectral.html

Multilevel partitioning

- Spectral, and even KL partitioning are too slow on very large graphs.
- To speed them up we run them on coarsened versions of the task graph.
- In fact, we coarsen the graph several times, until the number of nodes is small. Then we partition the coarse graph. Finally, we recover a partition on the fine graph using the coarse partition.
  - During the recovery, we can refine the coarse partitioning, by e.g. using it as the starting guess for Kernighan-Lin.
- Multilevel schemes achieve good quality and speed in practice.

Multilevel partitioning



- One way to coarsen a graph is based on matchings.
- First, find a maximal weight matching greedily.
- Collapse matched nodes.
- Merge edges connected to matched nodes.

- After partitioning the coarse graph, expand the merged edges to recover partition in the original graph.
- Can refine the partition using e.g. Kernighan-Lin.

Source: https://people.eecs.berkeley.edu/~demmel/cs267/lecture18/lecture18.html

Dynamic load balancing

- In some applications tasks are created by processes dynamically.
  - Ex Search algorithms. Recursive algorithms.
- Ideally do distributed load balancing, since tasks are created by distributed processes.
- One method is diffusion (aka push).
  - If a process has too many tasks, it sends some to its neighbors. If a neighbor becomes overloaded, it does the same thing.
  - Eventually load spreads out and equalizes.
  - But might take a long time and cause lots of communication.

Dynamic load balancing

- Another technique is work stealing (aka pull).
  - Processes without work steal work from processes with work.
- In work stealing, each process maintains a double-ended queue (deque).
- Process performs task on the top of the deque.
- If process creates a task, it pushes it onto top of the deque.
- If the process's deque is empty, it needs to load balance.
  - It picks a random other process and steals a task from the bottom of that process's deque.
  - This minimizes (but doesn't completely avoid) contention between the two processes, since one takes tasks from top and one from bottom.
- Work stealing doesn't incur any overhead when processes have tasks.
- Overhead when stealing is also borne by idle processes.
  - In contrast, for work pushing busy processes incur overhead for load balancing.
- Work stealing is used in the Cilk parallel runtime.