1. $x[n] = \left(\frac{2}{3}\right)^{\frac{1}{2}n} u[n]$

$$E_\infty = \sum_{n=-\infty}^{\infty} \left(\frac{2}{3}\right)^n u[n]$$

$$= \sum_{n=0}^{\infty} \left(\frac{2}{3}\right)^n = \frac{1}{1-\frac{2}{3}} = 3$$

$$P_\infty = 0$$

2. Euler $\quad e^{j\theta} = \cos\theta + j\sin\theta$

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \cdots$$

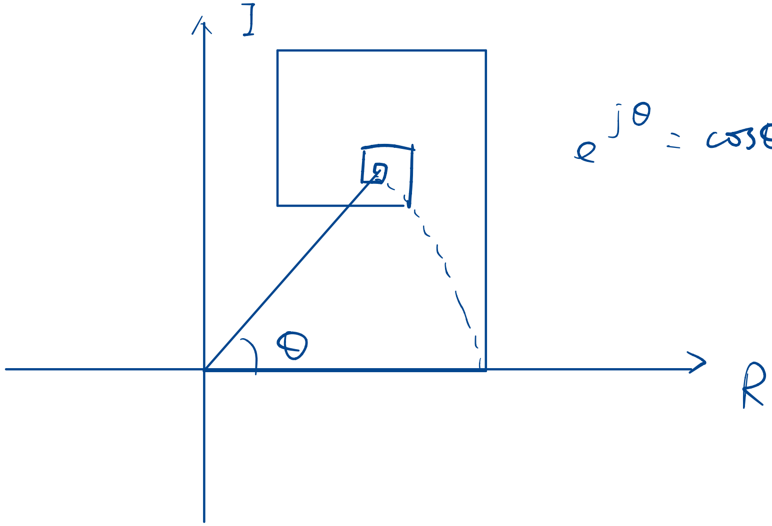
$$e^{j\theta} = 1 + \frac{j\theta}{1!} + \frac{(j\theta)^2}{2!} + \cdots$$

$$= 1 + \frac{-\theta^2}{2!} + \frac{\theta^4}{4!} + \frac{-\theta^6}{6!} + \cdots$$

$$+ j\frac{\theta}{1!} + j\frac{-\theta^3}{3!} + j\frac{\theta^5}{5!} + \cdots$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n \theta^{2n}}{(2n)!} + j\sum_{n=0}^{\infty} \frac{(-1)^n \theta^{2n+1}}{(2n+1)!}$$

$$= \cos\theta + j\sin\theta$$



$e^{j\theta} = \cos\theta + j\sin\theta$

3. $x(t) = ce^{at}$

① c, a are real

② c, a are not real

let $c = |c|e^{j\phi}$ $a = r + j\omega_0$

1° if $r = 0$, $\omega_0 \neq 0$: periodic $T_0 = \frac{2\pi}{\omega_0}$

2° if $r \neq 0$, aperiodic

4. $x[n] = c\alpha^n$

$c = a + jb = |c|e^{j\theta}$

$\alpha = \beta + j\gamma = |\alpha|e^{j\phi_2} = e^{\beta}$

① c, α are both real

② c is real, β is imaginary.

periodic iff $x[n+N] = x[n] \quad \forall n$

$\Rightarrow e^{j\omega_0(n+N)} = e^{j\omega_0 n}$

$\Rightarrow \quad \omega_0 = m\frac{2\pi}{N}$

③ Both c and α are complex

$x[n] = c\alpha^n$

$= |c|e^{j\theta} \cdot (|\alpha|e^{j\omega_0 n})^n$

$= |c| \cdot |\alpha|^n \cos(\omega_0 n + \theta) + j|c| \cdot |\alpha|^n \sin(\omega_0 n + \theta)$

5. $x(t) = \cos\left(\frac{\pi}{3} t\right)$

$$E_\infty = \int_{-\infty}^{\infty} \left| \cos^2\left(\frac{\pi}{3} t\right) \right| dt = \infty$$

$$P_\infty = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} \left| \cos^2\left(\frac{\pi}{3} t\right) \right| dt$$

$$= \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} \frac{1 + \cos \frac{2}{3}\pi t}{2} \, dt = \frac{1}{2}$$

Note: $\int_{-\infty}^{\infty} \cos at \, dt = 0$ <u>in this course</u>.

in fact, it is divergent.

6. $x[n] = \cos \frac{2}{3}\pi n$

$$E_\infty = \infty$$

$$P_\infty = \frac{1}{2}$$

7. $x(t) = e^{j5t} + e^{j\frac{7}{3}t}$

$$T_{01} = \frac{2\pi}{5} \qquad T_{02} = \frac{2\pi}{7/3} = \frac{6}{7}\pi$$

$$6\pi = \frac{2}{5}\pi \times 15 = \frac{6}{7}\pi \times 7 \quad \Rightarrow T = 6\pi$$

8. $x[n] = 2\cos\left(\frac{\pi}{4}n\right) + \sin\left(\frac{\pi}{8}n\right) - 2\cos\left(\frac{\pi}{2}n + \frac{\pi}{6}\right)$

$$N_1 = \frac{2\pi}{\pi/4} = 8 \qquad N_2 = \frac{2\pi}{\pi/8} = 16 \qquad N_3 = 4$$

$$\Rightarrow N = 16$$

9. $x[n] = \cos\left(\frac{\pi}{8}n^2\right)$

$$x[n+N] = \cos\left[\frac{\pi}{8}(n+N)^2\right]$$

$$\Rightarrow \frac{\pi}{8}(n+N)^2 = \frac{\pi}{8}n^2 + \frac{\pi}{8}\cdot 2N\cdot n + \frac{\pi}{8}N^2$$

$$= \frac{\pi}{8}n^2 + 2\pi\left(\frac{1}{8}N\cdot n + \frac{1}{16}N^2\right)$$

$$N_{min} = 8 \Rightarrow = \frac{\pi}{8}n^2 + 2\pi(n+4)$$