

## **Homework 3**

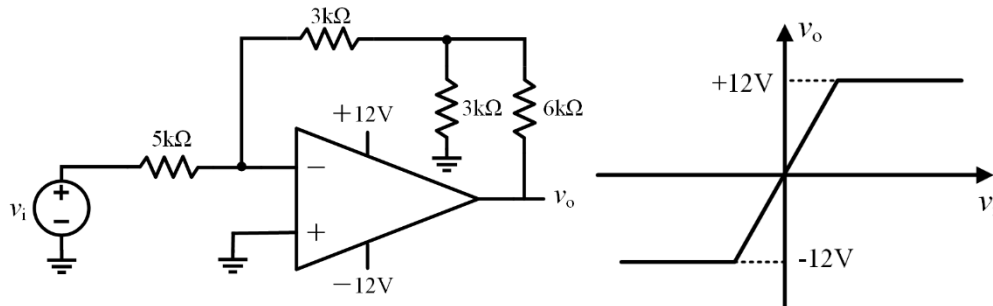
Due date: Nov. 7<sup>th</sup>, 2023

Turn in your hard-copy hand-writing homework in class

Rules:

- Work on your own. Discussion is permissible, but extremely similar submissions will be judged as plagiarism.
- Please show all intermediate steps: a correct solution without an explanation will get zero credit.
- Please submit on time. No late submission will be accepted.
- Please prepare your submission in English only. No Chinese submission will be accepted.

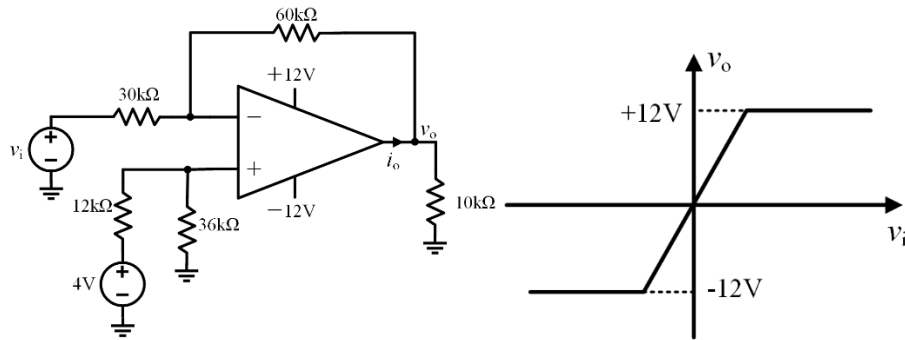
1. The voltage output range of this ideal operational amplifier is  $[-12, 12]$
- assuming the figure shows an ideal operational amplifier, calculate  $v_o/v_i$ .
  - if the operational amplifier is operated in the linear region, determine the range of  $v_i$



$$a. \begin{cases} \frac{v_i}{5} = \frac{0 - v_n}{3} \\ \frac{-v_n}{3} = \frac{v_n - v_o}{6} + \frac{v_n}{3} \end{cases} \Rightarrow \frac{v_o}{v_i} = -3$$

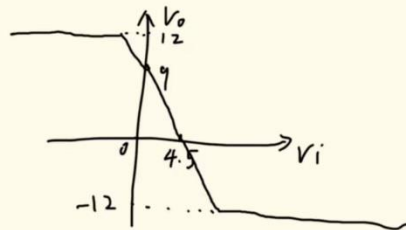
$$b. \because -12 < v_o < 12 \\ \therefore -4 < v_i < 4$$

2. The voltage output range of this ideal operational amplifier is  $[-12, 12]$
- Draw the curve of  $v_o$  changing with  $v_i$  and write down the derivation process.
  - When  $v_i$  is 9 V, find  $i_o$ .



$$a. \quad \frac{v_i - 3}{30} = \frac{3 - v_o}{60} \quad \therefore \quad 2v_i - 9 = -v_o$$

$$v_o = 9 - 2v_i$$



$$b. \quad v_i = 9\text{V}$$

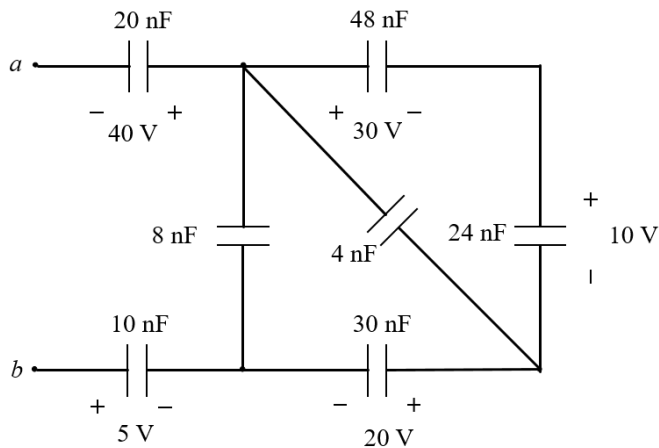
$$v_o = -9\text{V}$$

$$i_o = \frac{-9}{10\text{k}} + \frac{-9 - 3}{60\text{k}}$$

$$= -11 \times 10^{-4}$$

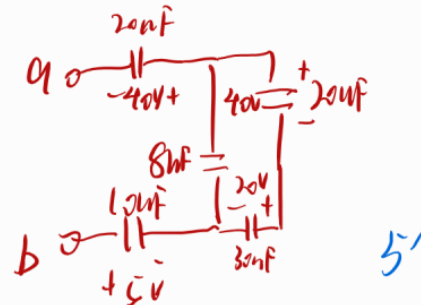
$$= -1.1 \times 10^{-3} \text{A}$$

3. The capacitance and associated voltage for each capacitor is given. Find the equivalent capacitance  $C_{ab}$  and the voltage  $v_{ab}$  for the circuit below.



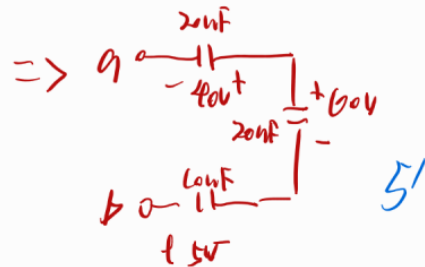
$$3. \quad \frac{1}{C_1} = \frac{1}{48} + \frac{1}{24} = \frac{1}{16} \quad C_1 = 16 \text{ nF}$$

$$C_2 = 4 + 16 = 20 \text{ nF}$$

 $\Rightarrow$ 


$$\frac{1}{C_3} = \frac{1}{30} + \frac{1}{20} = \frac{1}{12} \quad C_3 = 12 \text{ nF}$$

$$C_4 = 8 + 12 = 20 \text{ nF}$$



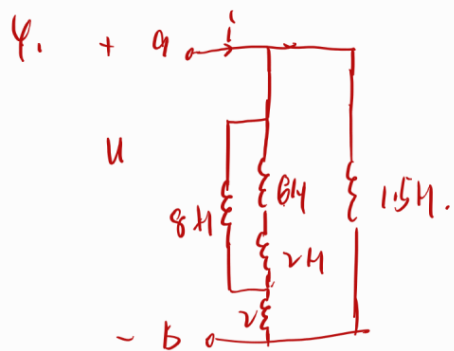
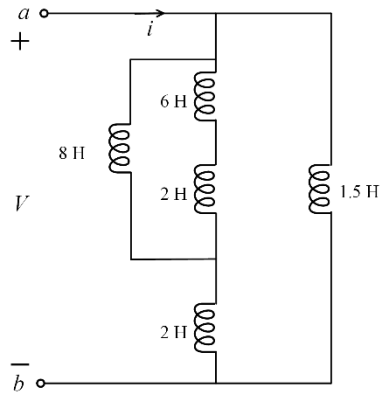
$$\frac{1}{C_5} = \frac{1}{20} + \frac{1}{20} + \frac{1}{10} = \frac{1}{5} \quad C_5 = 5 \text{ nF}$$

$$V_{ab} = 60 - 40 - 5 = 15 \text{ V} \quad 5'$$

4. For the circuit below,  $i(0)=0$  A,  $v = 6e^{-2t}$  V. The initial energy of all inductance are all 0.

a. Calculate the equivalent inductance  $L_{ab}$ .

b. Find the  $i(t)$ .



$$L_1 = 24 \frac{8 \times 8}{8 + 8} = 6 \text{ H} \quad 5'$$

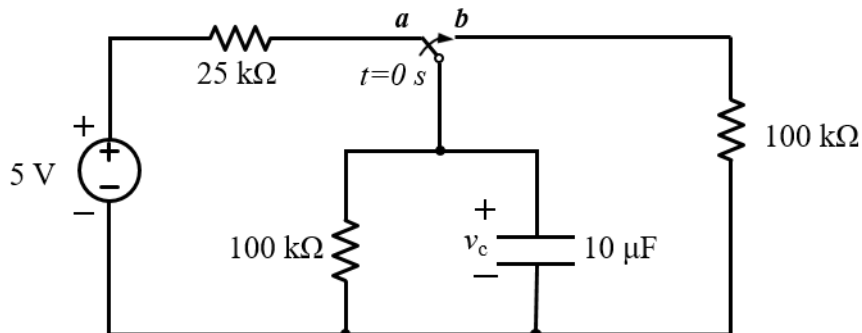
$$L_{ab} = \frac{6 \times 1.5}{6 + 1.5} = 1.2 \text{ H}$$

$$i(0) = 0 \text{ A} \quad 5'$$

$$i(t) = i(0) + \frac{1}{L} \int_0^t v(\xi) d\xi = 0 + \frac{1}{1.2} \int_0^t 6e^{-2\xi} d\xi$$

$$= -2.5(e^{-2t} - 1) \text{ A} \quad 5'$$

5. For the circuit below, When  $t = 0$ s, the switch changes from node  $a$  to node  $b$  immediately. Assume that the circuit reaches steady state before  $t = 0$ . Determine the expression for  $v_c(t)$  and  $i_c(t)$  when  $t \geq 0$ s.



5.  $V_{c(0^-)} = \frac{5}{100+100} \times 100 \text{ V} = 4 \text{ V}$      $V_{c(0^+)} = V_{c(0^-)} = 4 \text{ V}$  '5

When  $t > 0$ ,  $R_{eq} = \frac{100 \times 100}{100+100} \text{ k}\Omega = 50 \text{ k}\Omega$  '5

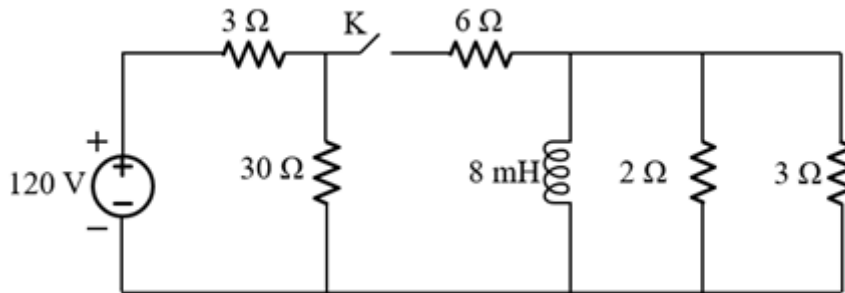
$\tau = RC = 50 \times 10^3 \times 10 \times 10^{-6} \text{ s} = \frac{1}{2} \text{ s}$

$V_c(t) = V_{c(0^+)} e^{-\frac{t}{\tau}} = 4e^{-2t} \text{ V}$

$i_c(t) = \frac{V_c(t)}{50} = 0.08e^{-2t} \text{ mA}$  '10

6.

- a. assuming that K have been closed for a long time, K opens at  $t=0$ , calculate the inductance currents for  $t > 0$ .
- b. What is the percentage of energy stored in the inductor consumed on the resistor  $2\ \Omega$ .



$$(a) \quad i = \frac{120}{3 + (30 \parallel 6)} \cdot \frac{30}{30 + 6} = 12.5 \text{ A} \quad i_L = -i = -12.5 \text{ A}$$

$$J = \frac{L}{R} = \frac{8 \times 10^{-3}}{2 \parallel 3} \quad i_L(t) = -12.5 e^{-\frac{t}{J}} = -12.5 e^{-150t}$$

$$(b) \quad W = \frac{1}{2} L i^2 = \frac{1}{2} \times \left(\frac{80}{4}\right)^2 \times 8 \times 10^{-3} = 625 \text{ mJ}$$

$$p_L(t) = \left( \frac{i_L(t) R_3}{R_2 + R_3} \right)^2 \cdot R_2 = 112.5 e^{-300t}$$

$$W = \int_0^{\infty} p_L(t) dt = \frac{112.5}{300} = 375 \text{ mJ}$$

$$\eta = \frac{375}{625} \times 100 = 60\%$$