



Lecture 14

-- Laplace Transform in Circuit Analysis



V-I relations of R,L,C

• R
$$U_R(s) = RI_R(s)$$

• C
$$V(s) = \frac{1}{sC} I(s) + \frac{V_0}{s}$$

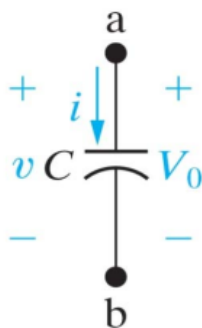
$$I(s) = sCV(s) - CV_0$$

• L
$$I(s) = \frac{1}{sL} V(s) + \frac{I_0}{s}$$

$$V(s) = sLI(s) - LI_0$$

S-domain circuit models for a capacitor

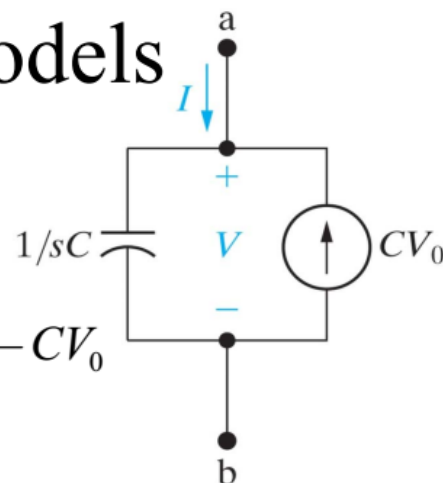
s-Domain Circuit Models



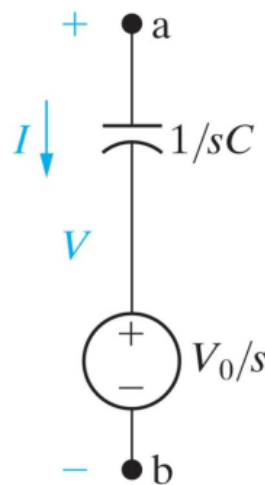
$$i(t) = C \frac{dv(t)}{dt}$$

For a capacitor
(with initial conditions)

$$I(s) = sCV(s) - CV_0$$

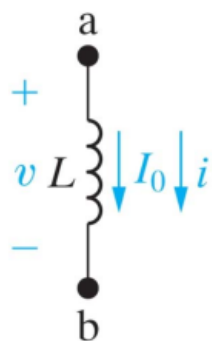


$$V(s) = \frac{1}{sC} I(s) + \frac{V_0}{s}$$



S-domain circuit models for an inductor

s-Domain Circuit Models

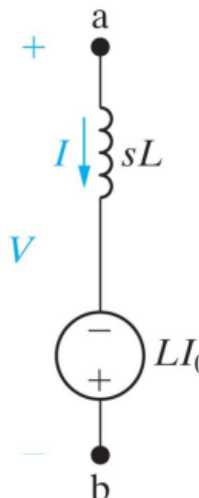


For an inductor
(with initial conditions)

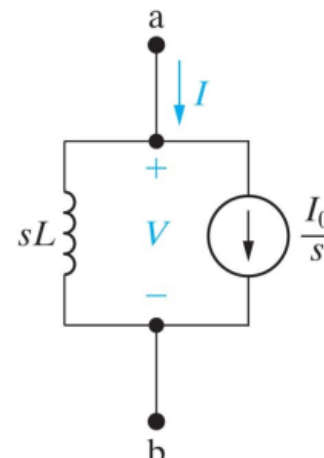
$$v(t) = L \frac{di(t)}{dt}$$



$$V(s) = sLI(s) - LI_0$$



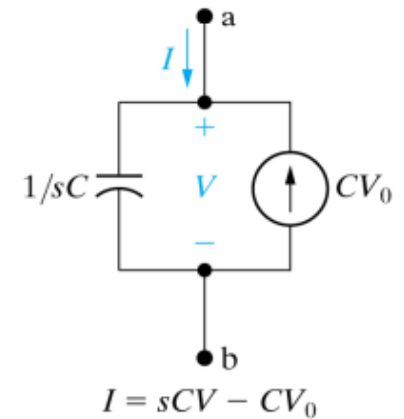
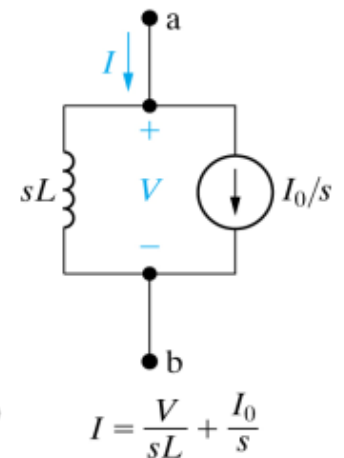
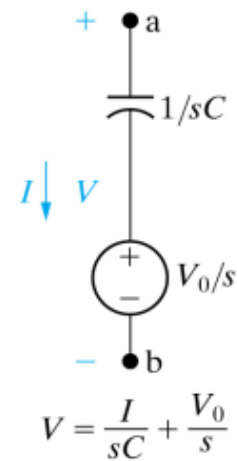
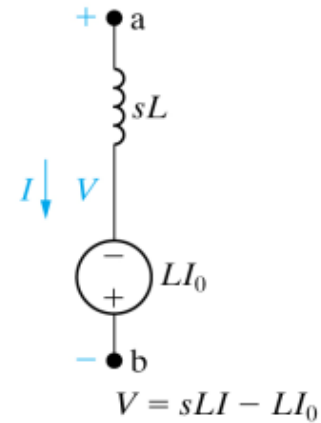
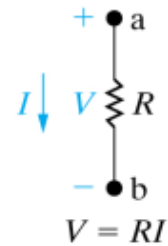
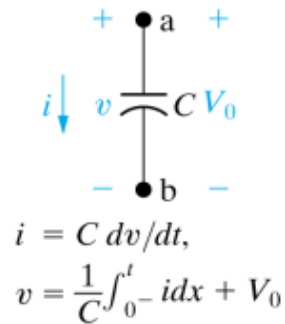
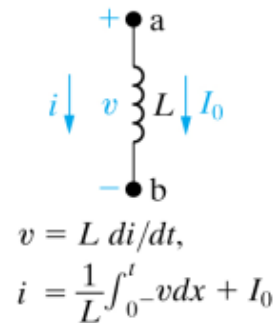
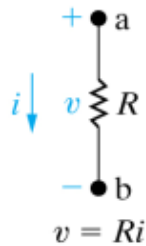
$$I(s) = \frac{1}{sL} V(s) + \frac{I_0}{s}$$





Time domain

s-domain





D.C. sources and Dependent Sources

- The models for dependent sources are easy to develop, drawing from the simple fact that if the Laplace transform of $f(t)$ is $F(s)$, then the Laplace transform of $af(t)$ is $aF(s)$ — the linearity property.

$$\mathcal{L}[av(t)] = aV(s)$$

$$\mathcal{L}[ai(t)] = aI(s)$$



Steps in Applying the Laplace transform

- Transform the circuit from the time domain to the Laplace (s) domain, including initial conditions.
--The elegance of using the Laplace transform in circuit analysis lies in (1) transforming the differential equation into an *algebraic* equation; and (2) automatic inclusion of initial conditions in the transformation process, thus providing a complete (**transient and steady-state**) solution.
- Solve the circuit using nodal analysis, mesh analysis, source transformation, superposition, or any other analysis technique with which we are familiar.
- Take the inverse transform of the solution and thus obtain the solution in the time domain.



Example 1

Assuming no initial energy storage, find $i_1(t)$ and $i_2(t)$ for $t > 0$.

