EE150 -Signals and Systems, Fall 2024

Homework Set #7

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Problem 1 (20 pt)

1. Determine the **unilateral Laplace transform** of each of the following signals, and specify the corresponding regions of convergence:

   (a) $x(t) = e^{-2t}u(t+1) + e^{-4t}u(t)$ (b) $x(t) = \delta(t+2) + \delta(t) + e^{-3(t+2)}u(t+2)$

2. Determine the **Laplace transform** and the associated region of convergence for each of the following functions of time:

   (a) $x(t) = te^{-3|t|}$ (b) $x(t) = \delta(2t) + u(2t) + e^{-5t}sin(5t)u(t)$

Problem 2 (10 pt)

Consider a signal $y(t)$ obtained by convolving two signals $x_1(t-3)$ and $x_2(-t+2)$

$$y(t) = x_1(t-3) * x_2(-t+2)$$

where $x_1(t) = e^{-2t}u(t)$ and $x_2(t) = e^{-3t}u(t)$

use properties of the Laplace transform to determine the Laplace transform $Y(s)$ of $y(t)$.

Problem 3 (20 pt)

Consider a continuous-time LTI system for which the input $x(t)$ and output $y(t)$ are related by the differential equation

$$\frac{d^2y(t)}{dt^2} - 2\frac{dy(t)}{dt} - 3y(t) = x(t)$$

Let $X(s)$ and $Y(s)$ denote Laplace transforms of $x(t)$ and $y(t)$, respectively, and let $H(s)$ denote the Laplace transform of $h(t)$, the system impulse response.

(a) Determine $H(s)$ as a ratio of two polynomials in s. Sketch the pole-zero pattern of $H(s)$.

(b) Determine $h(t)$ for each of the following cases:

1. The system is stable.

2. The system is causal.

3. The system is neither stable nor causal

Problem 4 (15 pt)

Suppose we are given the following information about a causal and stable LTI system S with impulse response $h(t)$ and a rational system function $H(s)$:

1.$H(1) = 0.1$.

2. When the input is $u(t)$, the output is absolutely integrable.

3. When the input is $tu(t)$, the output is not absolutely integrable.

4. The signal $\frac{d^2h(t)}{dt^2} + \frac{4dh(t)}{dt} + 5h(t)$is of finite duration.

5.$H(s)$ has exactly one zero at infinity.

Determine $H(s)$ and its region of convergence.

Problem 5 (15 pt)

The system function of a continuous system is

$$H(s) = \frac{2s+4}{s^3+3s^2+5s+3}$$

Try to draw the direct, cascaded and parallel block diagrams respectively

Problem 6 (20 pt)

Consider the system S characterized by the differential equation

$$\frac{d^2y(t)}{dt^2} + 5\frac{dy(t)}{dt} + 6y(t) = x(t)$$

(a) Determine the zero-state response of this system for the input $x(t) = e^{-4t}u(t)$.

(b) Determine the zero-input response of the system for $t > 0$, given that

$$y(0^-) = -2 \quad \frac{dy(t)}{dt}|_{t=0^-} = 1$$

(c) Determine the output of S when the input is $x(t) = e^{-4t}u(t)$ and the initial conditions are the same as those specified in part (b).