Numerical Optimization, Fall 2024
Homework 3

Due 23:59 (CST), Oct. 31, 2024

Problem 1

Prove the dual of the dual of the primal problem is itself. [20pts]

Problem 2

Write the optimality conditions for the following linear programming problem. [15pts]

$$\begin{aligned} \min \quad & x_1 + 2x_2 \\ \text{s.t.} \quad & x_1 + x_2 \geq 1, \\ & 2x_1 + x_2 \geq 2, \\ & x_1, x_2 \geq 0. \end{aligned}$$

Problem 3

Write the dual problem for the following linear programming problem. [15pts]

$$\begin{aligned} \min \quad & 10x_1 + 15x_2 \\ \text{s.t.} \quad & 2x_1 + x_2 \geq 3, \\ & x_1 + 3x_2 \geq 5, \\ & x_1, x_2 \geq 0. \end{aligned}$$

Problem 4

Give an example where neither the primal problem nor the dual problem is feasible. [20pts]

Problem 5

(1) Prove that one and only one of $(\boldsymbol{Ax} \leq \mathbf{0}, \boldsymbol{c^T x} > 0)$ or $(\boldsymbol{A}^T\boldsymbol{y} = \boldsymbol{c}, \boldsymbol{y} \geq \mathbf{0})$ is solvable, where $\boldsymbol{A} \in \mathbb{R}^{m\times n}, \boldsymbol{c} \in \mathbb{R}^n$. [15pts]

(2) Prove that one and only one of $(\boldsymbol{By} + \boldsymbol{Cw} = \boldsymbol{g}, \boldsymbol{y} > \mathbf{0})$ or $(\boldsymbol{g}^T\boldsymbol{d} < 0, \boldsymbol{B}^T\boldsymbol{d} \geq \mathbf{0}, \boldsymbol{C}^T\boldsymbol{d} = 0)$ is solvable, where $\boldsymbol{B} \in \mathbb{R}^{n\times m}, \boldsymbol{C} \in \mathbb{R}^{n\times p}, \boldsymbol{g} \in \mathbb{R}^n$. [15pts]