EE150 - Signals and Systems, Fall 2024

Homework Set #1

Prof. Lin Xu and Prof. Xiran Cai

Problem 1. (10 points)

(a) Use Euler's formula to write the following expression:

$\sin\theta =$ _____ $\cos\theta =$ _____ (2 points)

(b) Use the Cartesian coordinate system ($z = x + jy$) to represent the following complex numbers:(4 points)

(1) $\sqrt{2}e^{-j\frac{13\pi}{4}}$ (2) $e^{-j\frac{\pi}{2}}$

(c) Use the polar coordinates ($z = re^{j\theta}, -\pi < \theta \leq \pi$) to represent the following complex numbers:(4 points)

(1) -3 (2) $\frac{\sqrt{2}+\sqrt{6}j}{2+\sqrt{3}j}$

Problem 2. (20 points) Determine the energy E_∞ and power P_∞ of following signals. Which are finite-energy signals, which are finite-power signals, which are infinite energy and power signals? Write your calculation.

(a) $x_1(t) = t$

(b) $x_2(t) = e^{-\frac{1}{4}t}u(t)$

(c) $x_3[n] = e^{j(\frac{\pi}{4n}+\frac{\pi}{6})}$

Problem 3. (20 points) We have a signal $x(t)$, the following figures are the parts of $x(t)$ and its odd part $x_o(t)$, for $t \geq 0$ only. Please plot the whole odd part $x_o(t)$, whole even part $x_e(t)$ and whole $x(t)$ for $-\infty < t < \infty$ and write the equation of each function. (Be careful to write the boundary values clearly)

$$x(t) = \begin{cases} 2 & 0 \leq t \leq 1 \\ 0 & t > 1 \end{cases}$$

$$x_o(t) = \begin{cases} t & 0 \leq t \leq 2 \\ 0 & t > 2 \end{cases}$$

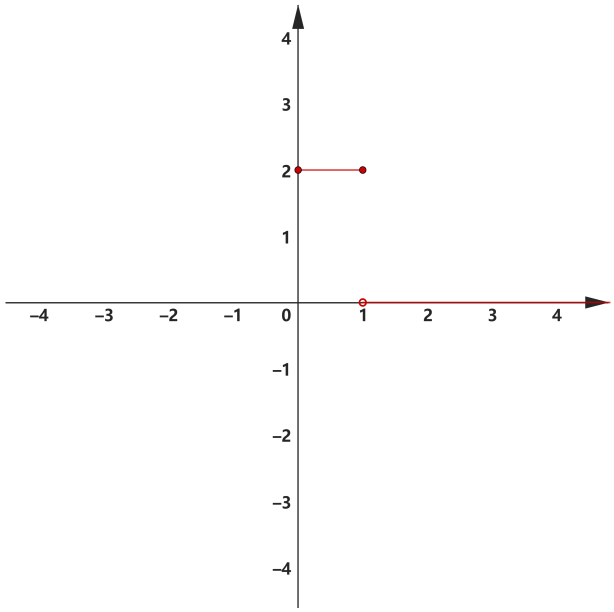

Figure 1: part of $x(t)$

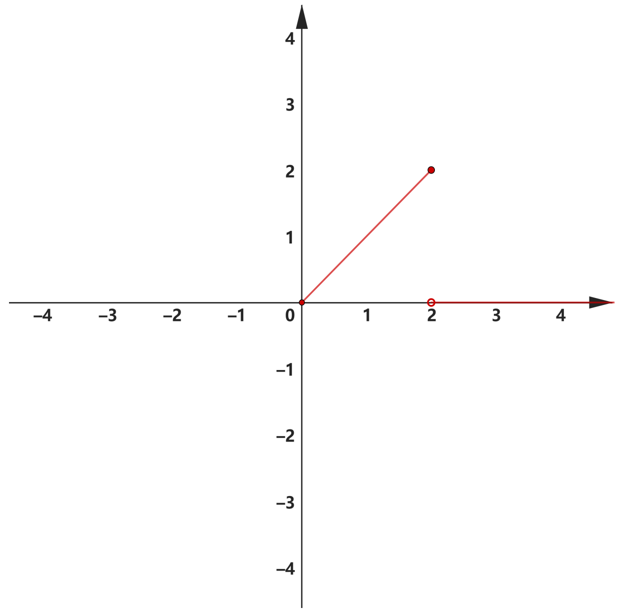

Figure 2: part of $x_o(t)$

Problem 4. (20 points)

(a) Determine which of the following signals is periodic. If a signal is periodic, write its fundamental period.(10 points)

(1) $x[n] = \mathrm{e}^{j(2\pi n/5)}$

(2) $x(t) = \frac{\sqrt{3}}{2}\tan(4\pi t + \frac{\pi}{4} + \frac{\sqrt{5}}{2})$

(b) Find the fundamental period of discrete signal $x_1[n] = e^{j\pi n}$ and $x_2[n] = e^{j\frac{2\pi}{3}n}$ (2 points), then answer following questions.

(1) What's the fundamental period of $x_1[n] + x_2[n]$? (4 points)

(2) What's the fundamental period of $x_1[n] \cdot x_2[n]$? (4 points)

Problem 5.(5 points $\times$ 4) For each of the following systems, determine whether the system is (1) stable, (2) causal, (3) linear, (4) time invariant, and (5) memoryless:

1) $y[n] = \sum_{k=-\infty}^{n} x[k]$,

2) $y[n] = x[Mn]$, for $M > 1$

3) $y(t) = x(-t)$,

4) $y(t) = x(\frac{t}{3})$.

(Notice: You need to give your reasons.)

Problem 6. (10 points) We have a signal $x(t)$.

(a) Express x(t) in terms of the unit step function, then calculate the $x'(t)$. (4 points)

(b) Plot the $x(-t+2)$ and $x(\frac{2}{3}t+1)$(6 points)

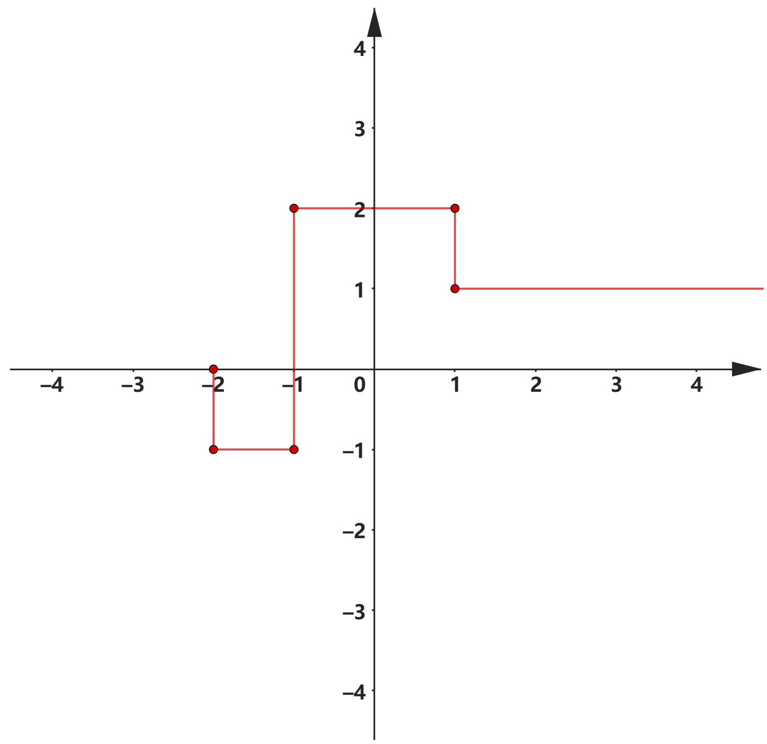

Figure 3: $x(t)$