1. (12') Find a state space description of the following expression.

(1) $\dddot{y} + 2\ddot{y} + 6\dot{y} + 3y = 5u$

(2) $\dddot{y} + 8\ddot{y} + 5\dot{y} + 13y = 4\dot{u} + 7u$

(3) $\frac{Y(s)}{U(s)} = \frac{3(s+5)}{(s+3)^2(s+1)}$

2. (8') Calculate the transfer function matrix $G(s)$ for the following state-space representation:

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -3 & -1 & -2 \end{bmatrix} x + \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix} u$$

$$y = [1 \quad 1 \quad 1]x$$

3. (12') Try to determine the observability of the following system:

(1) $\dot{x} = \begin{bmatrix} -1 & -2 & -2 \\ 0 & -1 & 1 \\ 1 & 0 & -1 \end{bmatrix} x + \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} u, y = [1 \quad 1 \quad 0]x$

(2) $\dot{x} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 3 & 1 \end{bmatrix} x, y = [1 \quad 1 \quad 1]x$

(3) $\dot{x} = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -3 \end{bmatrix} x, y = [0 \quad 1 \quad 1]x$

4. (12') Determine the state controllability of the following system.

(1) $\dot{x} = \begin{bmatrix} -2 & 2 & -1 \\ 0 & -2 & 0 \\ 1 & -4 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$

(2) $\dot{x} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} u$

(3) $\dot{x} = \begin{bmatrix} 0 & 4 & 3 \\ 0 & 20 & 21 \\ 0 & -25 & -20 \end{bmatrix} x + \begin{bmatrix} -1 \\ 3 \\ 0 \end{bmatrix} u$

5. (16') Given the following continuous-time time-invariant controlled system:

$$\dot{x} = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} u$$

Determine a state feedback matrix k such that $(A - Bk)$ is similar to:

$$F = \begin{bmatrix} -3 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

6. (16’) Given the following continuous-time linear time-invariant system:

$$\dot{x} = \begin{bmatrix} -1 & -2 & -2 \\ 0 & -1 & 1 \\ 1 & 0 & -1 \end{bmatrix} x + \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}, y = \begin{bmatrix} 1 & 1 & 0 \end{bmatrix} x$$

Design a full-order state observer with eigenvalues -3, -3, -4.

7. (20’+4’=24’) Given that the transfer function of the controlled system is: $G(s) = \frac{1}{s(s+6)}$,

   design a Full-state observer-based feedback controller such that the closed-loop poles are located at $-4 \pm j6$, and the poles of the observer are at -10, -10 and draw the final system block diagram.