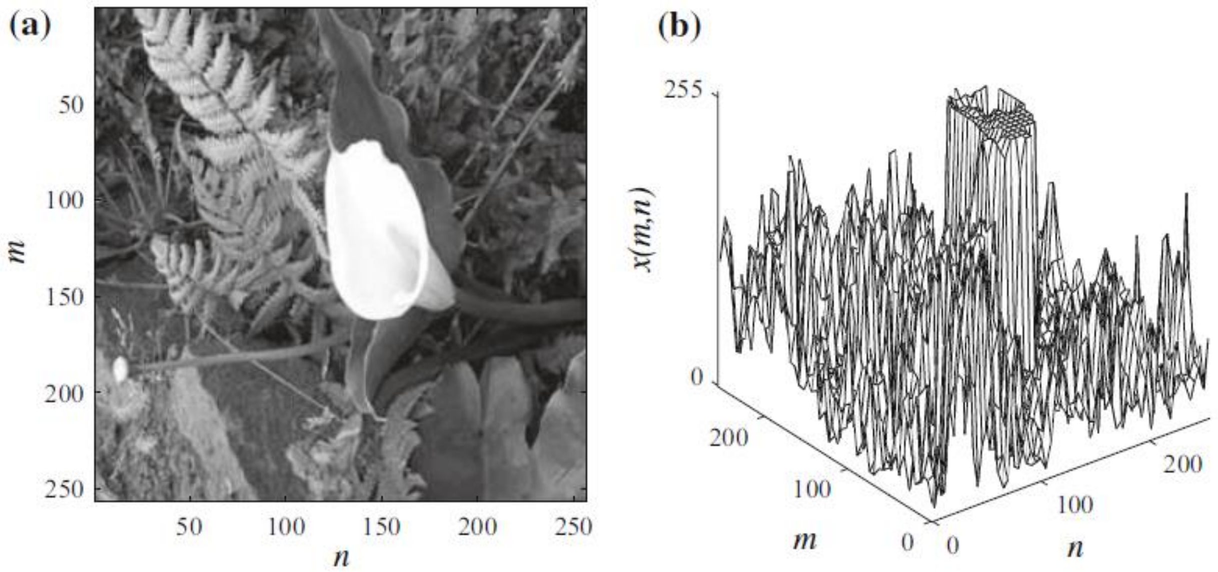
Lecture 2 – Basics of Digital Image

This lecture will cover:

- – Digital Image
- – Image Characteristics
  - Spatial resolution
  - Noise
  - Image artifacts
- – Image Operation
  - Basic operation
  - Spatial Operation
  - Filtering
- – Digital Image Processing

Digital image

- **A visual representation in form of a function *f*(*x*,*y*), where**
  - *f* is related to the intensity or brightness (color) at point
  - (*x*, *y*) are spatial coordinates
  - *x*, *y*, and the amplitude of *f* are finite and discrete quantities



(a) A 256X256 image with 256 gray levels; (b) its amplitude profile

Matrix Representation

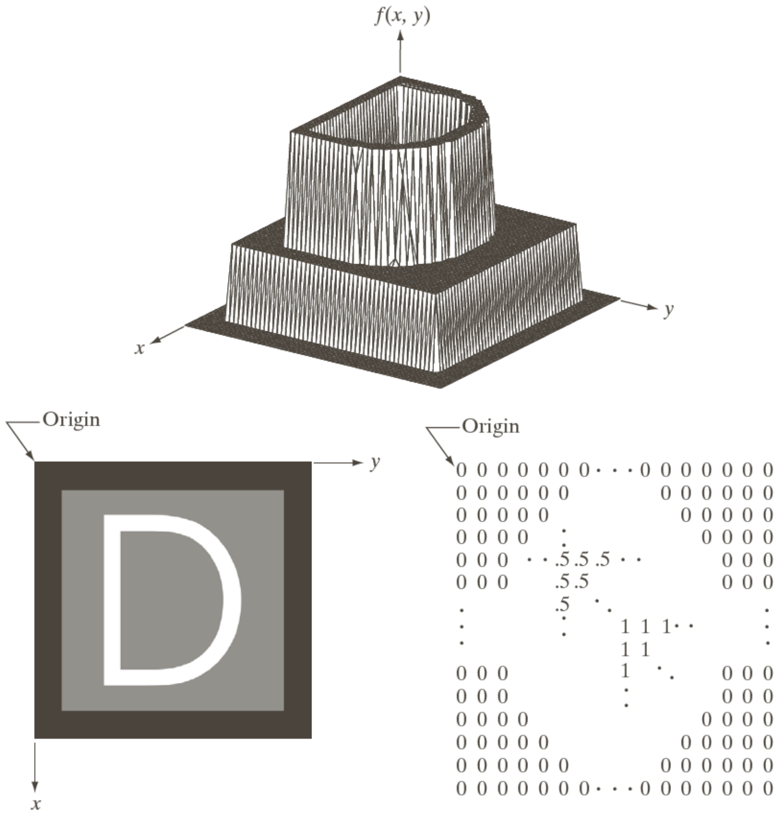
Three basic ways to represent *f*(*x*, *y*)

- Plot of function: *difficult to view and interpret*
- Visual intensity array: *for view*
- numerical array: *for processing and algorithm development*

$$[f(x,y)] = \begin{bmatrix} f(0,0) & f(0,1) & \cdots & f(0,N-1) \\ f(1,0) & f(1,1) & \cdots & f(1,N-1) \\ \vdots & \ddots & \cdots & \vdots \\ f(M-1,0) & f(M-1,1)\cdots & \cdots & f(M-1,N-1) \end{bmatrix}$$

$$A = \begin{bmatrix} a_{0,0} & a_{0,1} & \cdots & a_{0,N-1} \\ a_{1,0} & a_{1,1} & \cdots & a_{1,N-1} \\ \vdots & \ddots & \cdots & \vdots \\ a_{M-1,0} & a_{M-1,1}\cdots & \cdots & a_{M-1,N-1} \end{bmatrix}$$

Intensity level $L = 2^k$, then $b = M \times N \times k$



Discrete Fourier Transform （离散傅里叶变换）

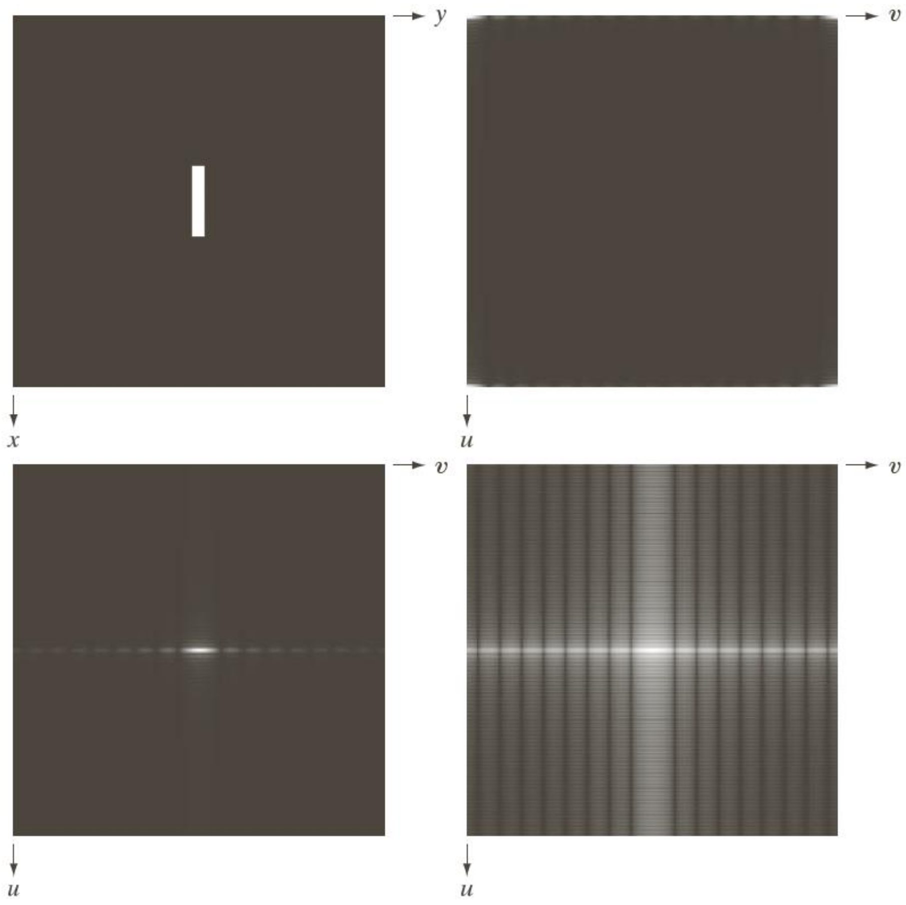
2D Discrete Fourier Transform (DFT)

$$F(u,v) = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y) e^{-j2\pi(\frac{ux}{M}+\frac{vy}{N})}$$

2D Inverse Discrete Fourier Transform (IDFT)

$$f(x,y) = \frac{1}{MN} \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} F(u,v) e^{j2\pi(\frac{ux}{M}+\frac{vy}{N})}$$

- $f(x,y)$: M*N input image
- (x,y): spatial variables
- (μ,ν): frequency variables, defines the continuous frequency domain



a b
c d

FIGURE 4.24
(a) Image. (b) Spectrum showing bright spots in the four corners. (c) Centered spectrum. (d) Result showing increased detail after a log transformation. The zero crossings of the spectrum are closer in the vertical direction because the rectangle in (a) is longer in that direction. The coordinate convention used throughout the book places the origin of the spatial and frequency domains at the top left.

Spectrum（频谱）

- 2D DFT in polar form: $F(u,v) = |F(u,v)|e^{-j\Phi(u,v)}$, then
  - Fourier spectrum（频谱）: $|F(u,v)| = [R^2(u,v) + I^2(u,v)]^{\frac{1}{2}}$
  - Phase angle（相角）: $\Phi(u,v) = \arctan\frac{I(u,v)}{R(u,v)}$
  - Power spectrum(功率谱): $P(u,v) = |F(u,v)|^2$
  - DC component(直流分量): $F(0,0) = \sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y) = MN\overline{f(x,y)}$
- Convolution theorem (卷积定理)

$f(x,y) \star h(x,y) \Leftrightarrow F(u,v)H(u,v)$ or $f(x,y)\,h(x,y) \Leftrightarrow F(u,v) \star H(u,v)$

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Spatial Resolution

- The spatial resolution of an imaging system is related to
  - the smallest feature that can be visualized
  - the smallest distance between two features such that the features can be individually resolved
- Spatial frequency: $\boldsymbol{\omega} = \frac{\mathbf{1}}{\boldsymbol{\lambda}}$
- The common measures of spatial resolution
  - The line spread function (线扩散函数)
  - The point spread function (点扩散函数)
  - The modulation transfer function (调制传递函数)

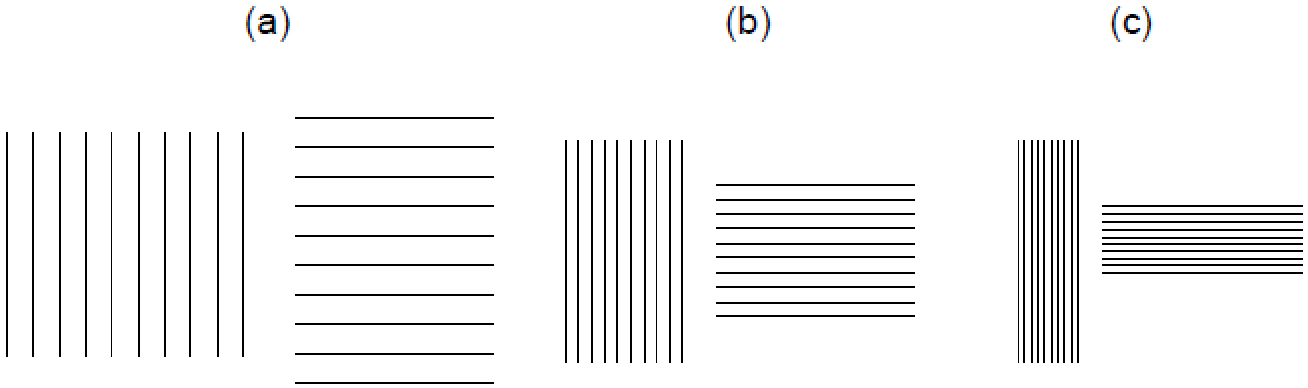


Figure. Grid patterns with increasing spatial frequencies going from (a) to (c)..

The Line Spread Function (LSF)

- The degree of blurring for a line in the image (2D)

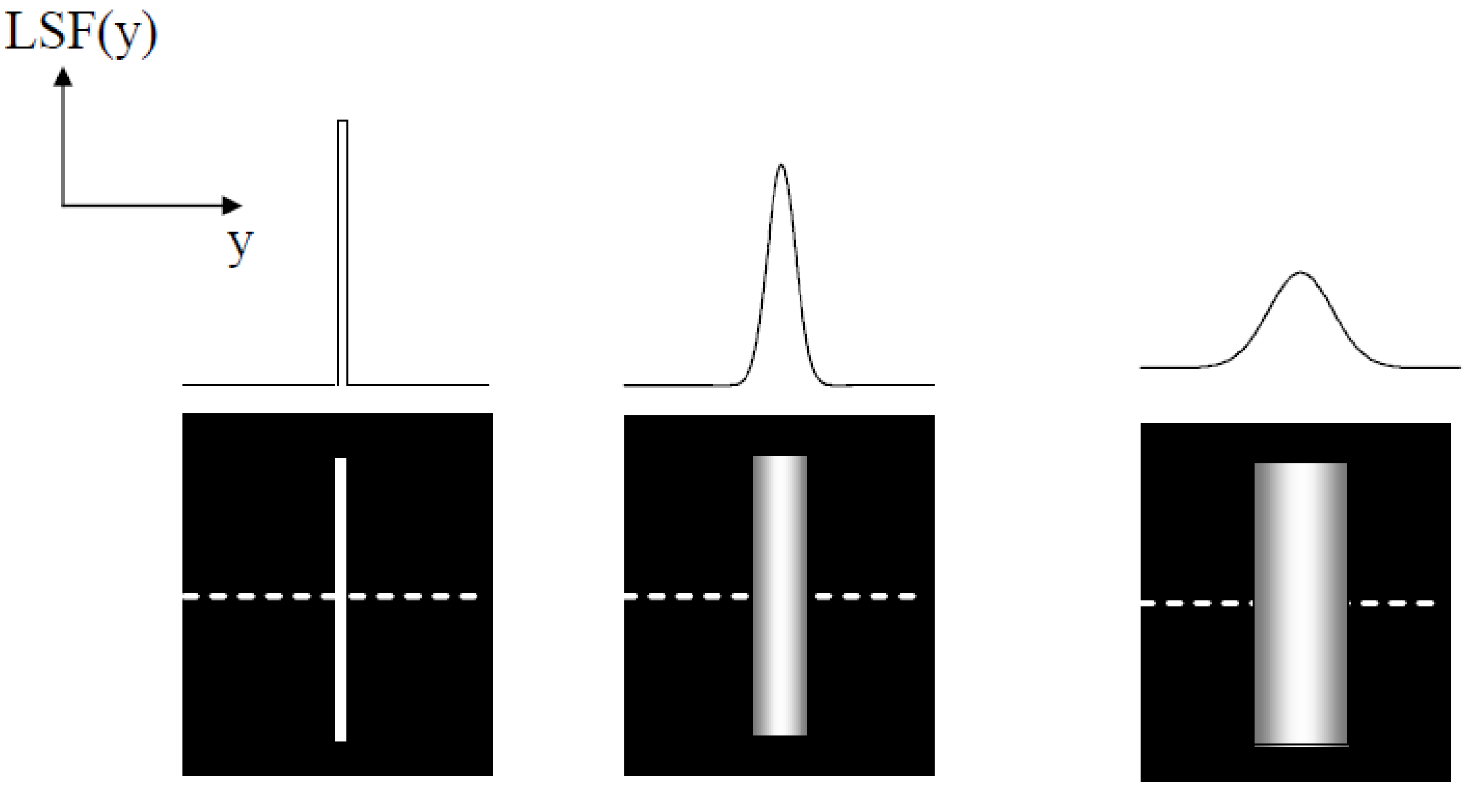


Figure. The concept of the line-spread function. A thin object is imaged using three different imaging systems. The system on the left has the sharpest LSF, as defined by the one-dimensional projection measured along the dotted line and shown above each image. The system in the middle produces a more blurred image, and has a broader LSF, with the system on the right producing the most blurred image with the broadest LSF.

The Line Spread Function (LSF)

$$\mathbf{LSF}(y) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(y - y_0)^2}{2\sigma^2}\right)$$

$$\mathbf{FWHM} = \left(2\sqrt{2\ln 2}\right)\sigma \cong 2.36\sigma$$

where,

LSF : a Gaussian function

y_0 : center of the function

σ : the standard deviation of the distribution

FWHM: full width half maximum (半高宽)

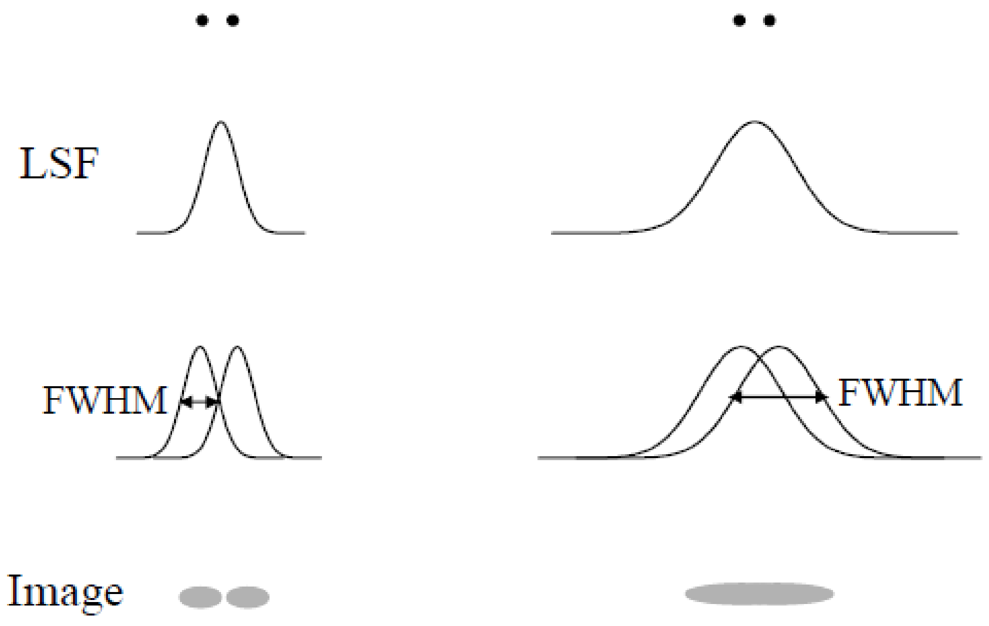


Figure. Imaging results produced by two different systems with a relatively narrow (left) and broad (right) LSF. In the case on the left, two small structures within the body (top) have a separation which is slightly greater than the FWHM of the LSF, and so the resulting image shows the two different structures. In the case on the right, the FWHM of the LSF is greater than the separation of the structures, and so the image appears as one large structure.

The Point Spread Function (PSF)

➢ A full description of the spatial resolution of an imaging system (3D):

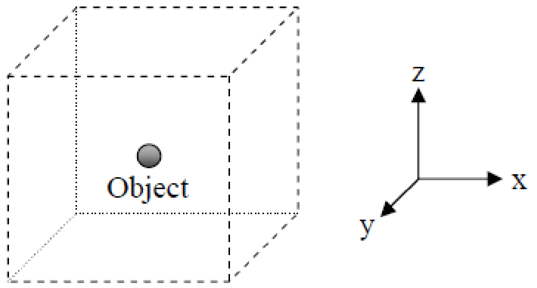
$$\mathbf{I}(x, y, z) = \mathbf{O}(x, y, z) * \mathbf{h}(x, y, z)$$

where,

$\mathbf{h}(x, y, z)$: PSF

$\mathbf{O}$: object

$\mathbf{I}$: image



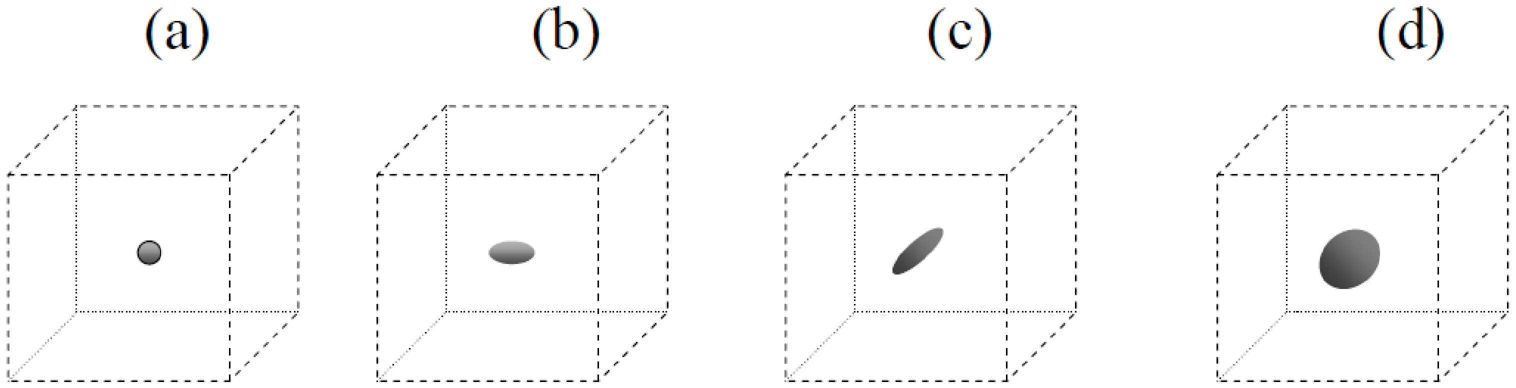


Figure. (top) Small point object being imaged. (a)-(d) Images produced with different point spread functions. (a) A sharp PSF in all three dimensions. (b) A PSF which is significantly broader in x than in y or z. (c) A PSF which is broadest in the y-dimension. (d) A PSF which is broad in all three dimensions.

The Modulation Transfer Function (MTF)

The Fourier transform of PSF

$$\mathbf{MTF}\left(\boldsymbol{k_x}, \boldsymbol{k_y}, \boldsymbol{k_z}\right) = \mathbf{F}\{\mathbf{PSF}(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z})\}$$

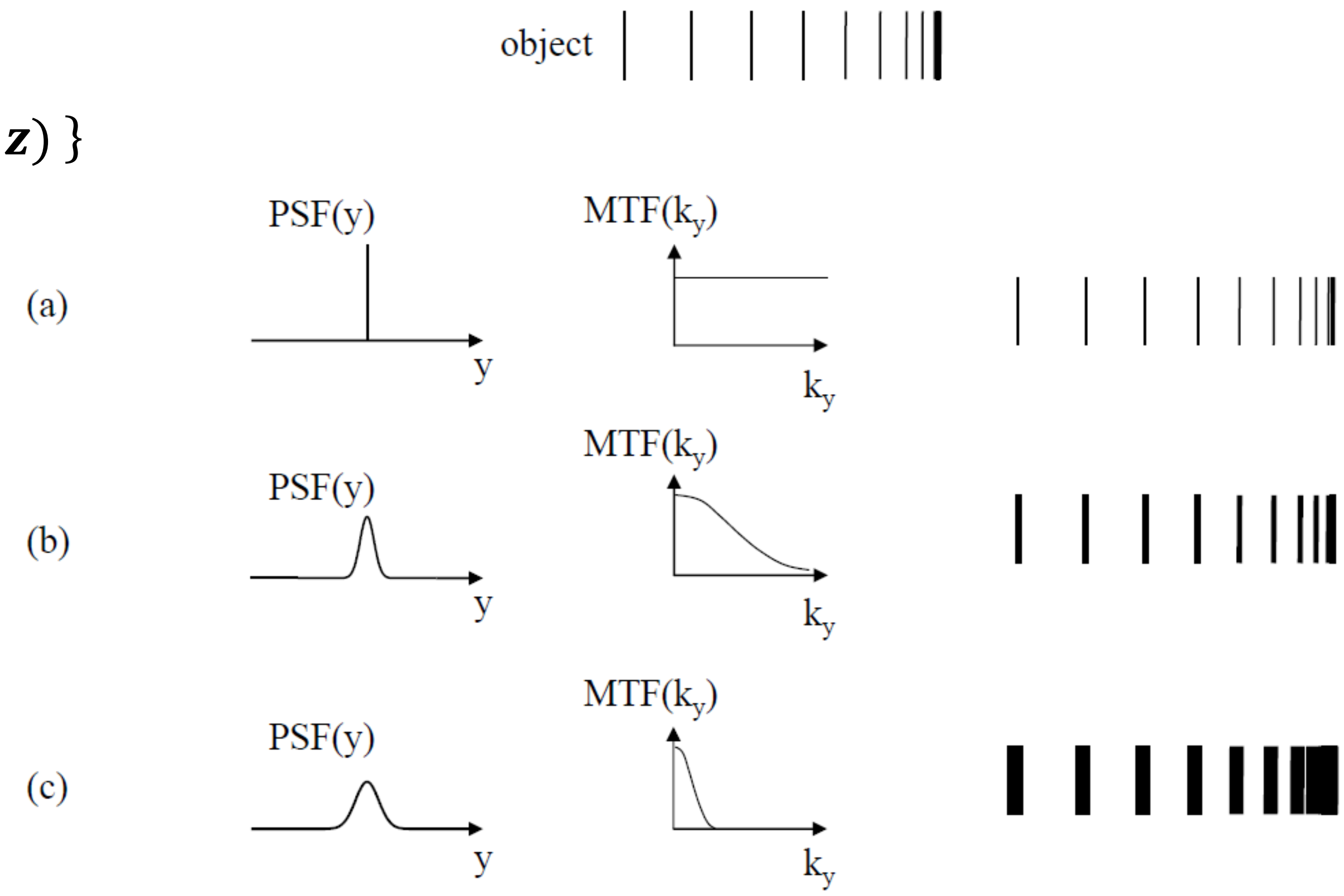


Figure. (top) The object being imaged corresponds to a set of lines with increasing spatial frequency from left-to-right. (a) An ideal PSF and the corresponding MTF produce an image which is a perfect representation of the object. (b) A slightly broader PSF produces an MTF which loses the very high spatial frequency information, and the resulting image is blurred. (c) The broadest PSF corresponds to the narrowest MTF, and the greatest loss of high spatial frequency information.

Resolution of Medical Imaging

TABLE 1-1 THE LIMITING SPATIAL RESOLUTIONS OF VARIOUS MEDICAL IMAGING MODALITIES. THE RESOLUTION LEVELS ACHIEVED IN *TYPICAL* CLINICAL USAGE OF THE MODALITY ARE LISTED

MODALITY	SPATIAL RESOLUTION (mm)	COMMENTS
Screen film radiography	0.08	Limited by focal spot size and detector resolution
Digital radiography	0.17	Limited by size of detector elements and focal spot size
Fluoroscopy	0.125	Limited by detector resolution and focal spot size
Screen film mammography	0.03	Highest resolution modality in radiology, limited by same factors as in screen film radiography
Digital mammography	0.05–0.10	Limited by same factors as digital radiography
Computed tomography	0.3	About ½ mm pixels
Nuclear medicine planar imaging	2.5 (detector face), 5 (10 cm from detector)	Spatial resolution degrades substantially with distance from detector
Single photon emission computed tomography	7	Spatial resolution worst towards the center of cross-sectional image slice
Positron emission tomography	5	Better spatial resolution than the other nuclear imaging modalities
Magnetic resonance imaging	1.0	Resolution can improve at higher magnetic fields
Ultrasound imaging (5 MHz)	0.3	Limited by wavelength of sound

Noise

- Definition: any recorded signal not related to the actual signal that one is trying to measure.
- Properties
  - Mostly random
  - Mean is zero
  - Quantitative measure of noise is conventionally the standard deviation.

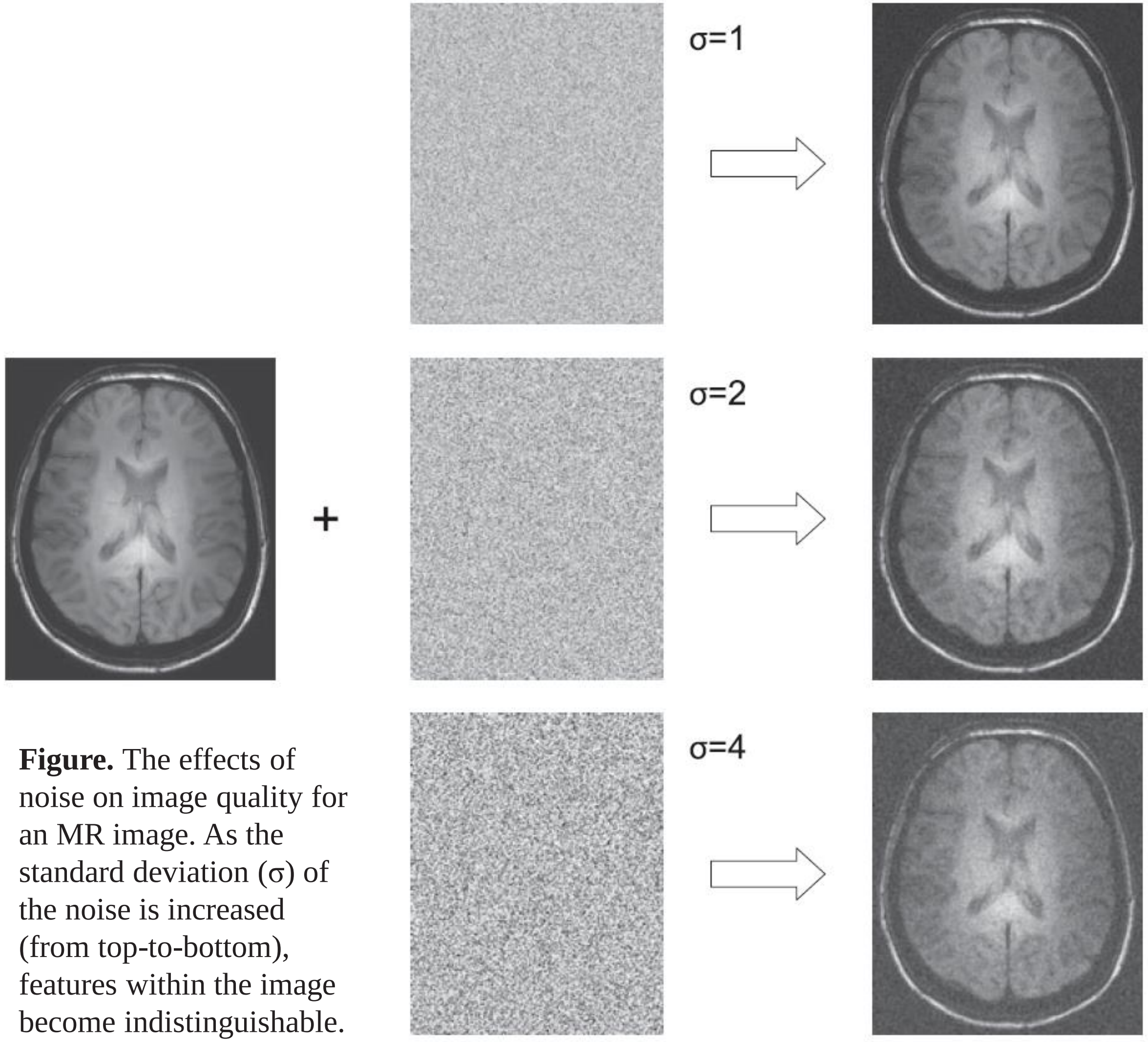


Figure. The effects of noise on image quality for an MR image. As the standard deviation (σ) of the noise is increased (from top-to-bottom), features within the image become indistinguishable.

Signal-to-noise ratio (SNR)

➢ **Signal-to-noise ratio (SNR，信噪比)**

$$\mathrm{SNR} = \sqrt{\frac{P_S}{P_n}} = \sqrt{\frac{E(S^2)}{\sigma^2}} = \frac{\sqrt{E(S^2)}}{\sigma}$$

➢ **Increase SNR by factor of $\sqrt{N}$**

- Repeating scan numbers (Averaging)
- Increasing scan time (Higher energy)

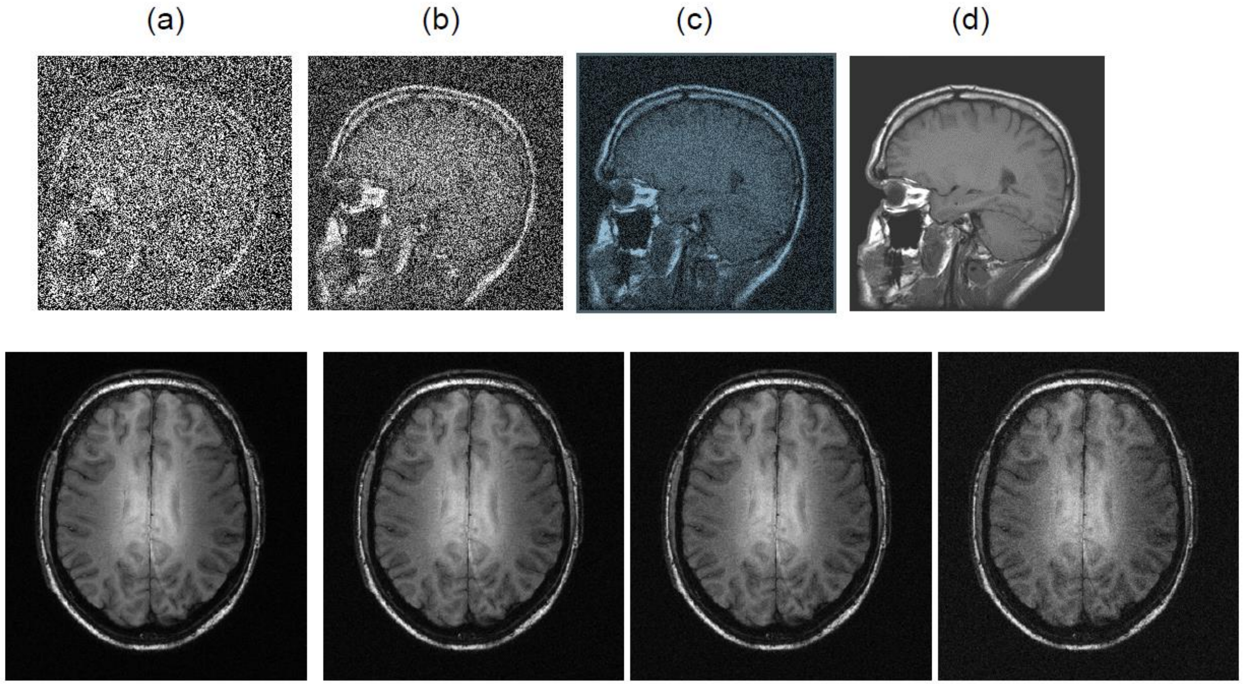


Figure. Signal averaging to improve the image SNR. (a) MR image acquired in a single scan, (b) two identical scans averaged together, (c) four scans, and (d) sixteen scans.

Contrast-to-noise ratio (CNR)

- **Contrast-to-noise ratio (CNR，对比噪声比)**

$$\mathrm{CNR}_{AB} = \frac{C_{AB}}{\sigma_n} = \frac{|S_A - S_B|}{\sigma_n} = |\mathrm{SNR}_A - \mathrm{SNR}_B|$$

Where

CNR_{AB}: the contrast between tissue A and tissue B

S_A, S_B: the signals from tissue A and B

σ_n : the standard deviation of noise

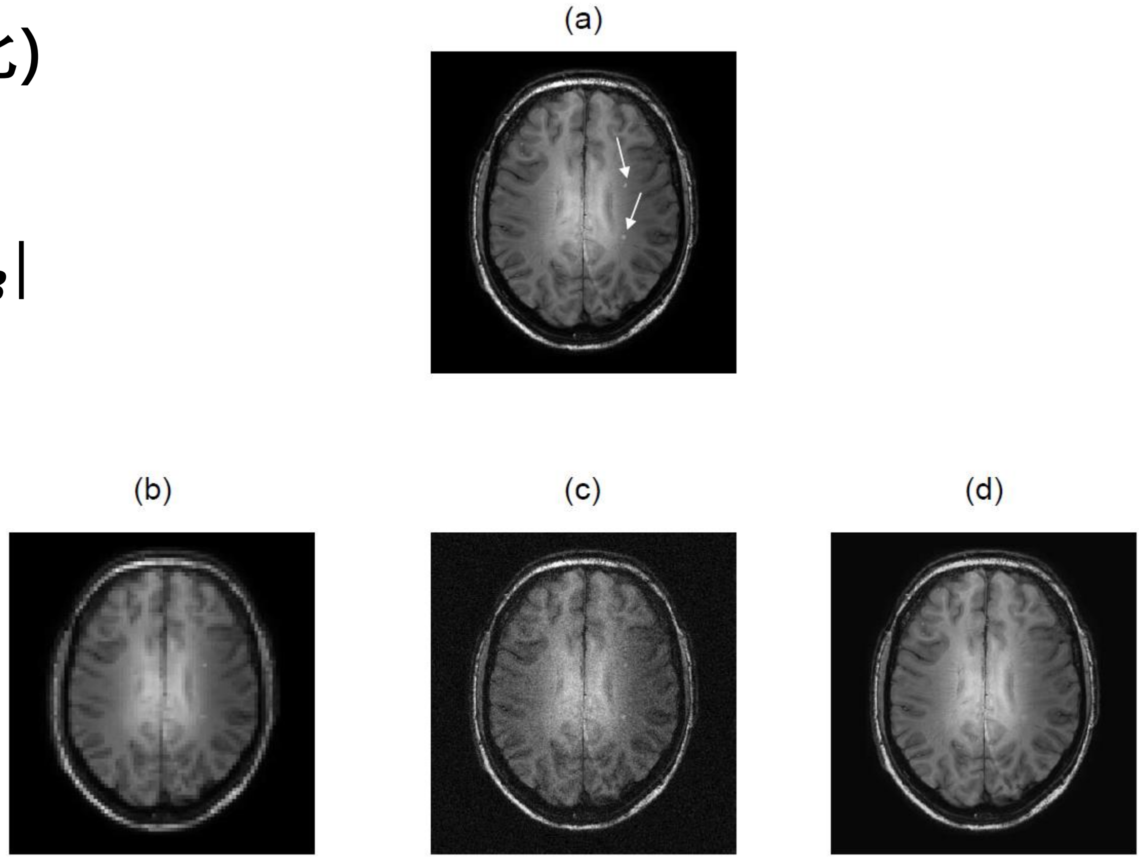


Figure. (a) MR image showing two small white-matter lesions indicated by the arrows. Corresponding images acquired with (b) four times poorer spatial resolution, (c) four times lower SNR, and (d) a reduced CNR between the lesions and the surrounding healthy tissue. The arrows point to lesions that can be detected.

Image Artifacts (伪影)

Signals in an image

① caused by a phenomenon related to the imaging process
② distorting the image or introduces an apparent features without physical counterpart

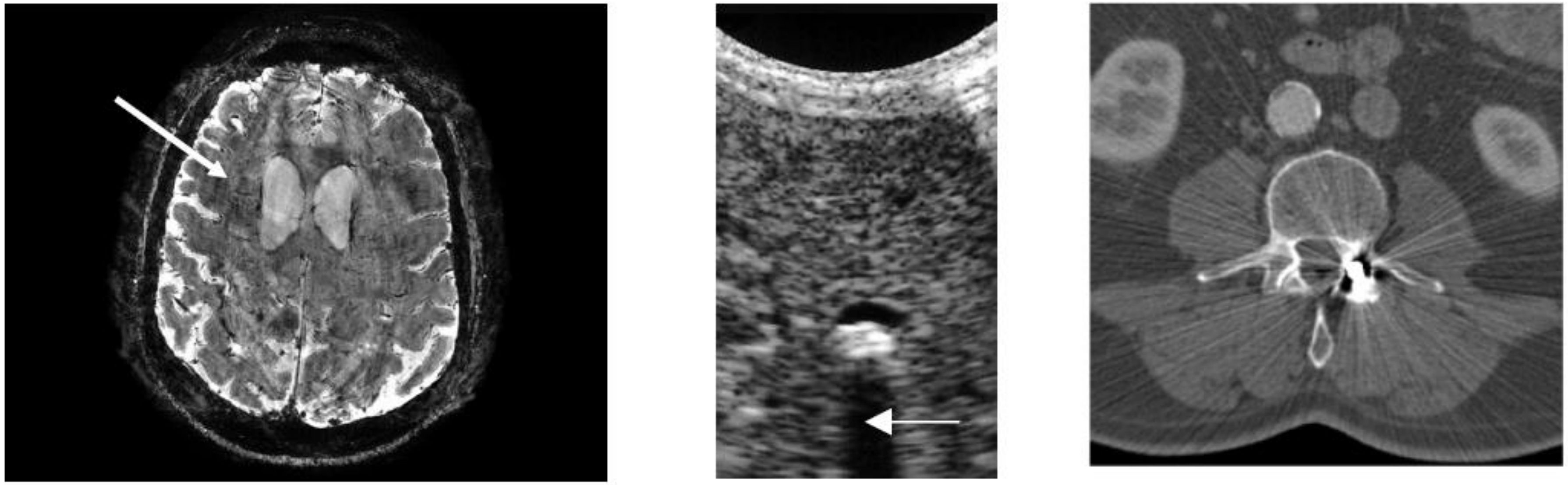


Figure. Examples of image artifacts. (a) Motion in MRI causes extra lines to appear in the image (arrowed), (b) acoustic shadowing in ultrasound produces a black hole in the image (arrowed), and (c) a metal implant causes 'streaking artifacts' in a CT image.

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  - **Filtering**
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Array and Matrix Operation

Consider two 2 x 2 image

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \text{ and } \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$$

- **Array product**

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} a_{11}b_{11} & a_{12}b_{12} \\ a_{21}b_{21} & a_{22}b_{22} \end{bmatrix}$$

- **Matrix product**

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} \end{bmatrix}$$

Vector and Matrix Operation

- **Multispectral image processing**

A pixel in a n-dimensional space can be expressed as a column vector $Z = [z_1, z_2 \dots z_n]^T$, then a vector norm between two pixels Z and A

$$\|Z - A\| = [(Z - A)^T (Z - A)]^{\frac{1}{2}}$$
$$= [(z_1 - a_1)^2 + (z_2 - a_2)^2 + \cdots + (zn - an)^2]^{\frac{1}{2}}$$

- **Linear transformations**

$$g = Hf + n$$

Linear and Nonlinear Operation

An operator

$$H[f(x,y)] = g(x,y)$$

is linear if

$$\begin{aligned} H\left[a_i f_i(x,y) + a_j f_j(x,y)\right] &= a_i H[f_i(x,y)] + a_j H\left[f_j(x,y)\right] \\ &= a_i g_i(x,y) + a_j g_j(x,y) \end{aligned}$$

- Additivity (相加性)
- Homogeneity (同质性)

Set Operation (Coordinates)

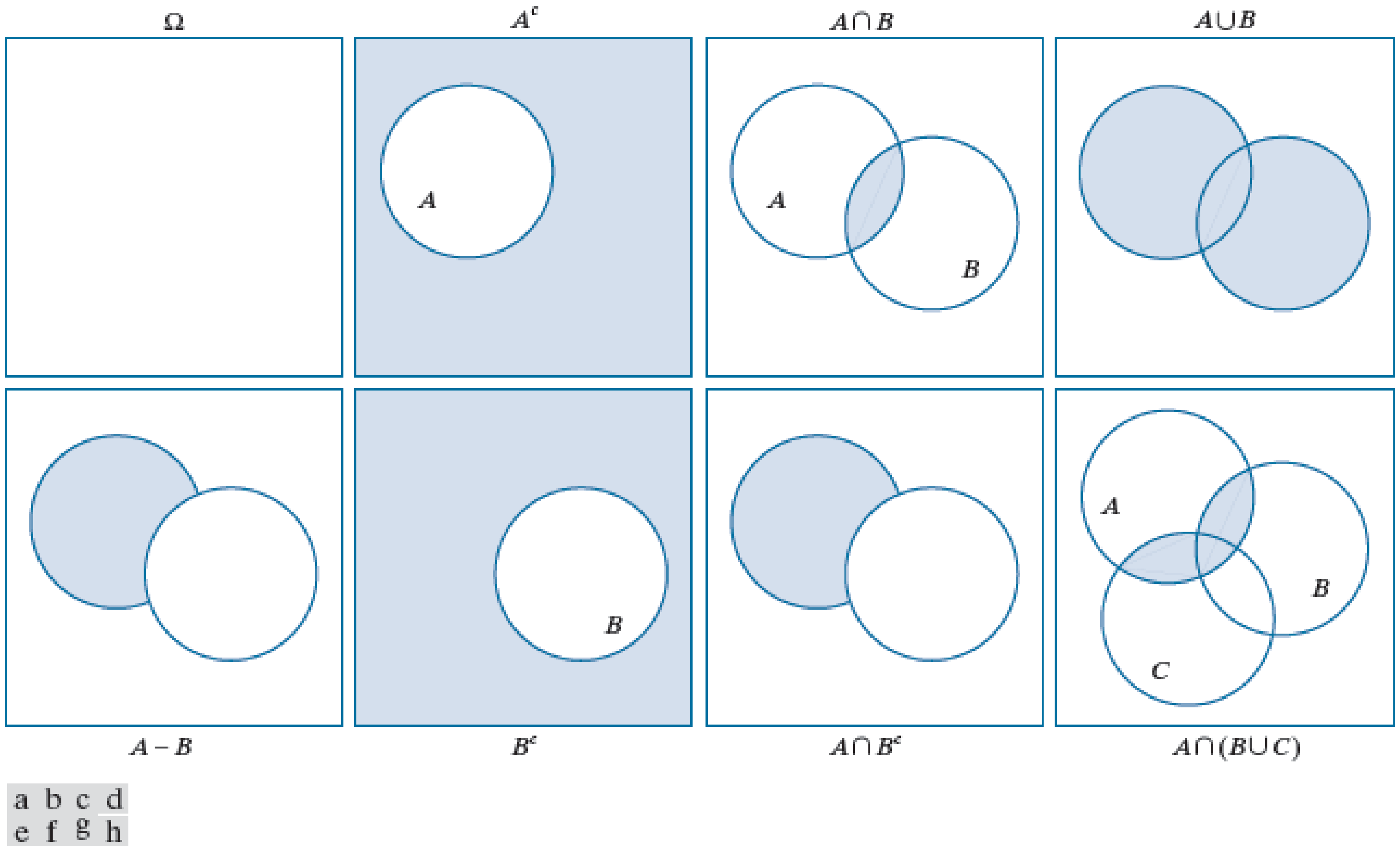


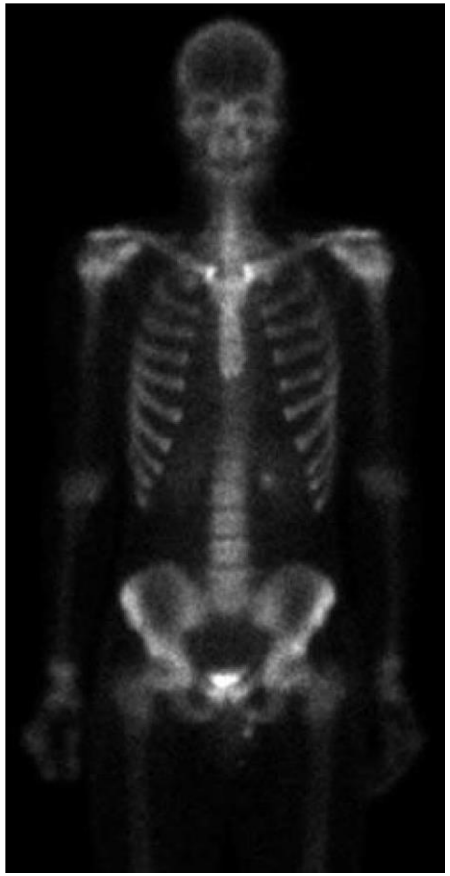
FIGURE 2.35 Venn diagrams corresponding to some of the set operations in Table 2.1. The results of the operations, such as A^c, are shown shaded. Figures (e) and (g) are the same, proving via Venn diagrams that $A - B = A \cap B^c$ [see Eq. (2-40)].

Set Operation (Intensity)

a b c

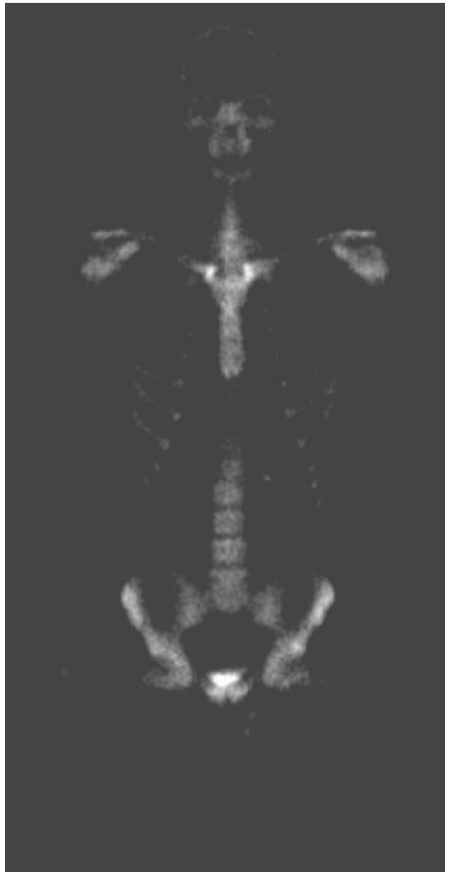
FIGURE 2.36 Set operations involving grayscale images. (a) Original image. (b) Image negative obtained using grayscale set complementation. (c) The union of image (a) and a constant image. (Original image courtesy of G.E. Medical Systems.)

(A) Original image



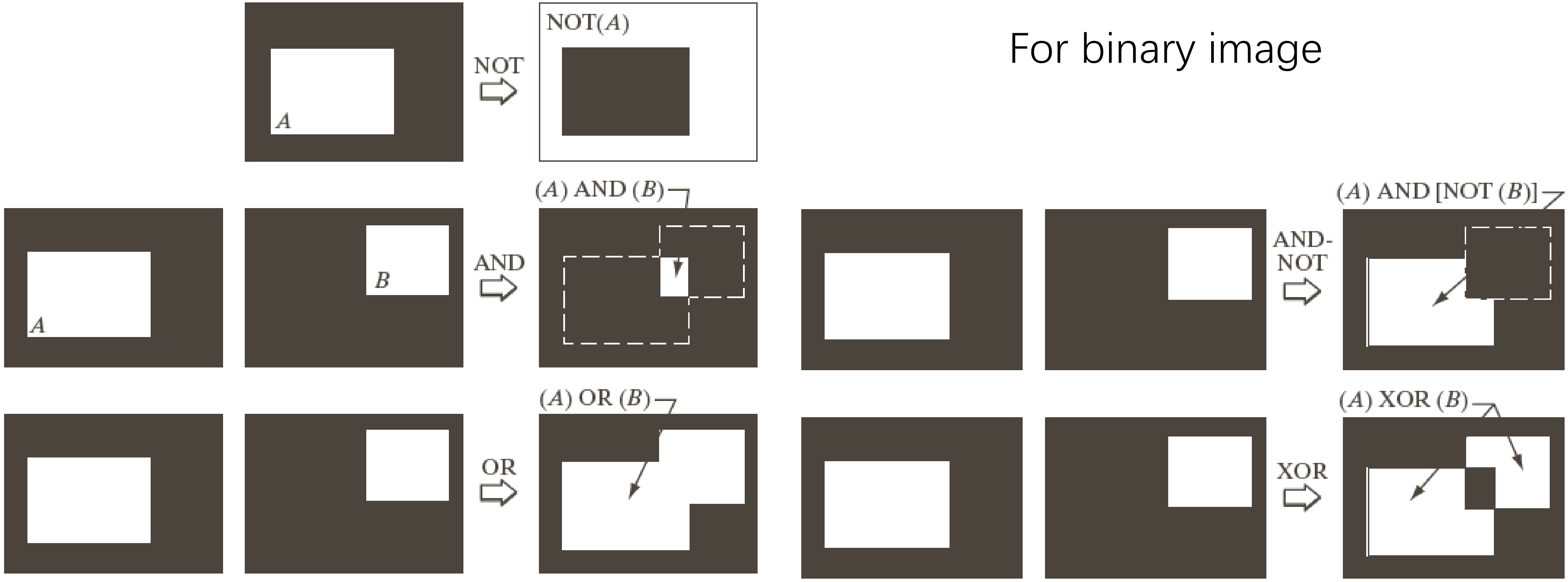
(B) Complement image
$Z = 255 - A$

(C) Union: $A \cup 3\bar{Z} = \{\max(a, 3\bar{Z}) \,|\, a \in A\}$



Logical Operation

For binary image



Arithmetic Operation

- **Addition**

$$s(x, y) = f(x, y) + g(x, y)$$

- **Subtraction**

$$d(x, y) = f(x, y) - g(x, y)$$

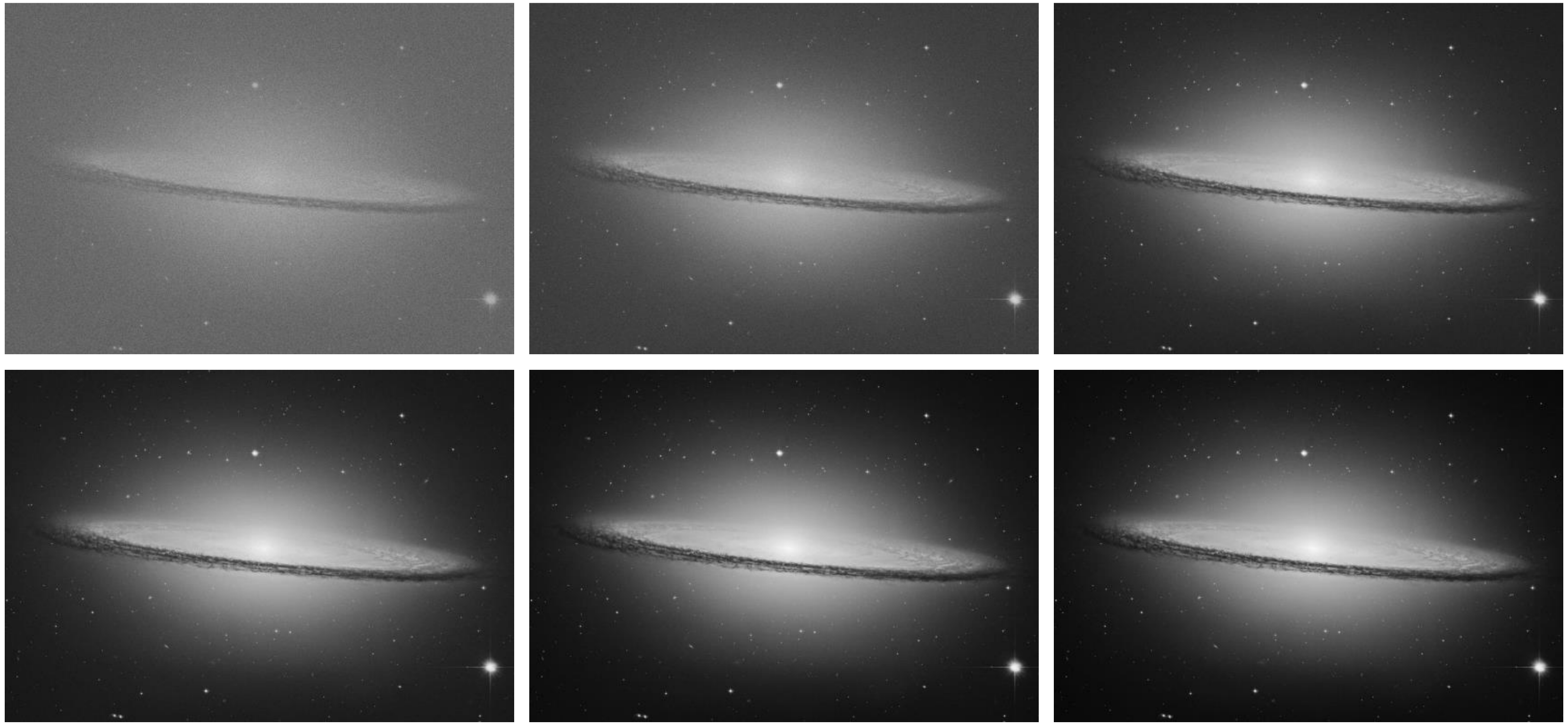
- **Multiplication**

$$p(x, y) = f(x, y) \times g(x, y)$$

- **Division**

$$v(x, y) = f(x, y) \div g(x, y)$$

Image Addition



a b c
d e f

FIGURE 2.29 (a) Sample noisy image of the Sombrero Galaxy. (b)-(f) Result of averaging 10, 50, 100, 500, and 1,000 noisy images, respectively. All images are of size 1548×2238 pixels, and all were scaled so that their intensities would span the full [0, 255] intensity scale. (Discovered in 1767, the Sombrero Galaxy is 28 light years from Earth. Original image courtesy of NASA.)

Image Addition

If $f(x, y)+g(x, y) > L_{max}$, $s(x, y)$ can be calculated as

- **Average**

$$s(x,y) = \frac{f(x,y) + g(x,y)}{2}$$

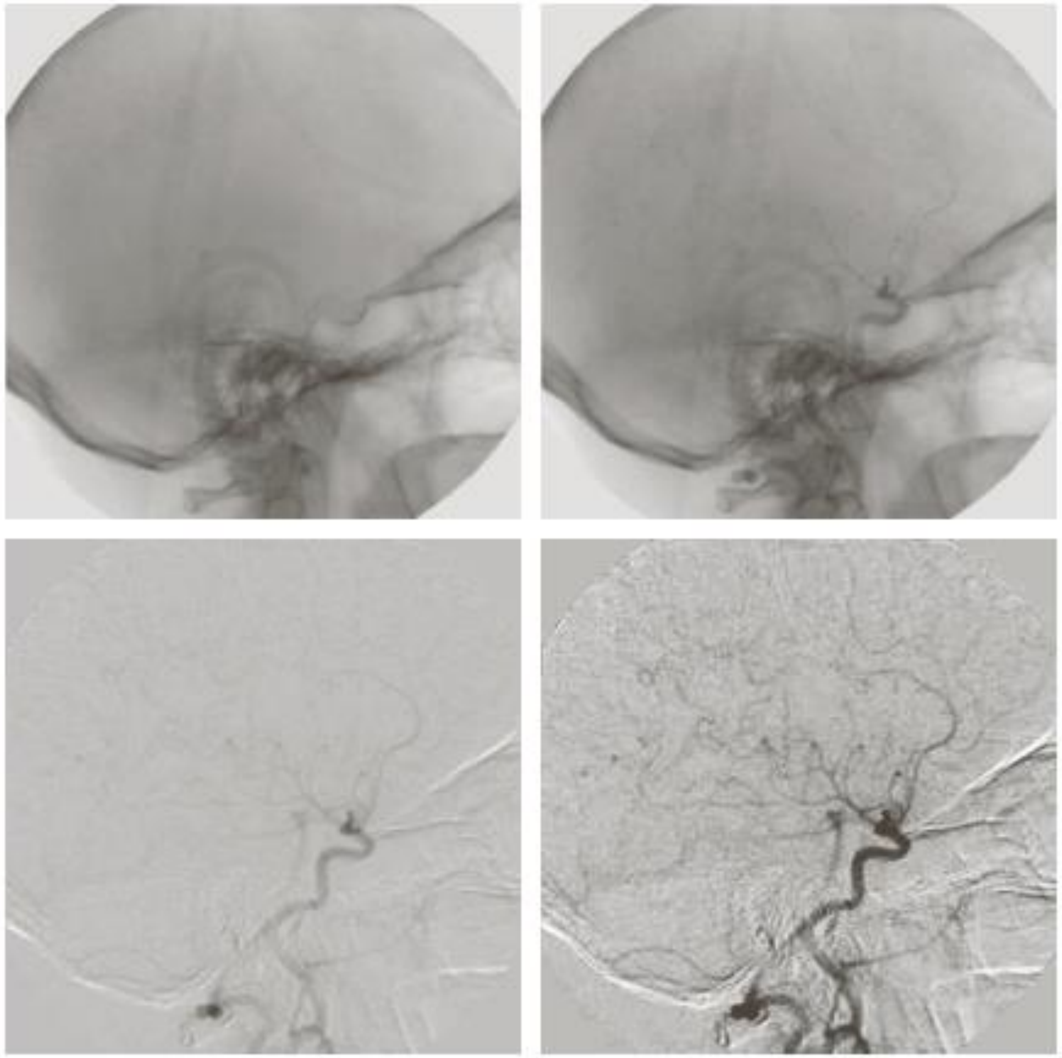
- **Scale**

$$s(x,y) = f(x,y) + g(x,y): \quad \{\min[s(x,y)], \max[s(x,y)]\} = \{0, L_{\max}\}$$

- **Max intensity value**

If $s(x,y) = f(x,y) + g(x,y) > L_{\max},\qquad s(x,y) = L_{\max}$

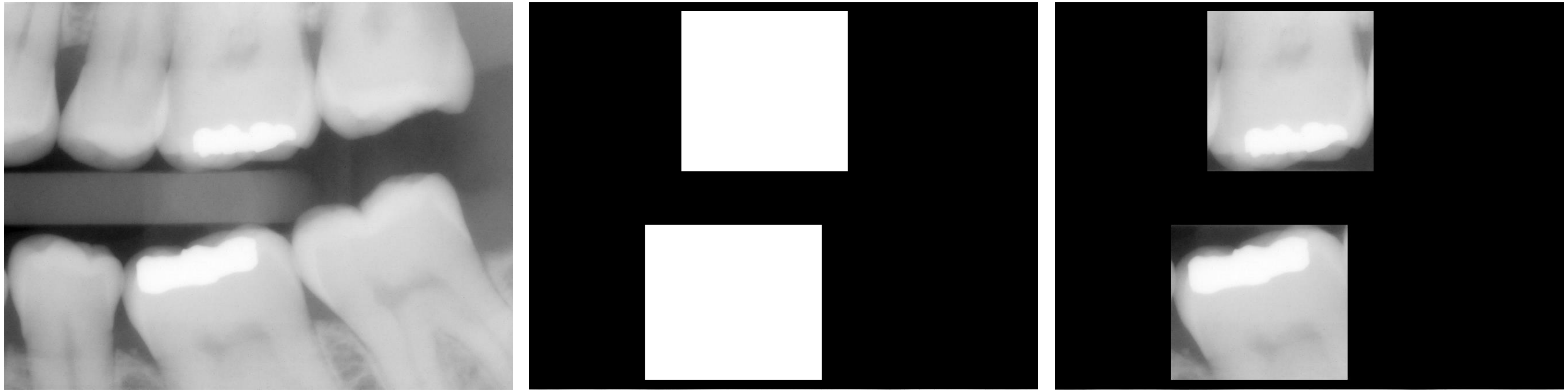
Image Subtraction



a b
c d

FIGURE 2.32 Digital subtraction angiography. (a) Mask image. (b) A live image. (c) Difference between (a) and (b). (d) Enhanced difference image. (Figures (a) and (b) courtesy of the Image Sciences Institute, University Medical Center, Utrecht, The Netherlands.)

Image Multiplication



a b c

FIGURE 2.34 (a) Digital dental X-ray image. (b) ROI mask for isolating teeth with fillings (white corresponds to 1 and black corresponds to 0). (c) Product of (a) and (b).

Image Division

$g(x, y) = f(x, y)\,h(x, y)$

$h(x, y)$

$f(x, y)$

$$f(x, y) = g(x, y) / h(x, y)$$

Spatial Operation

Performed directly on the pixels of the image

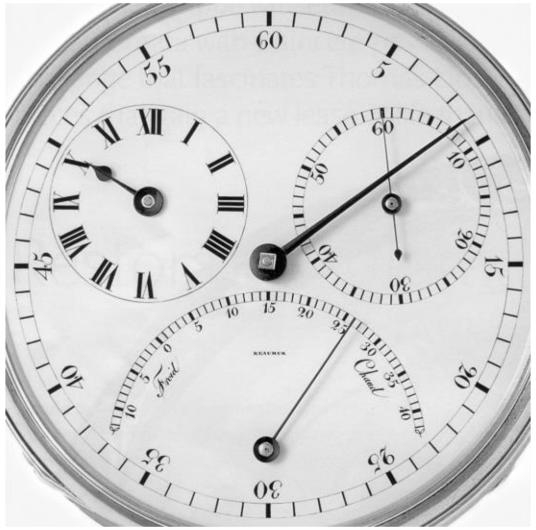
- Single-pixel operations
- Neighborhood operations
- Image geometry

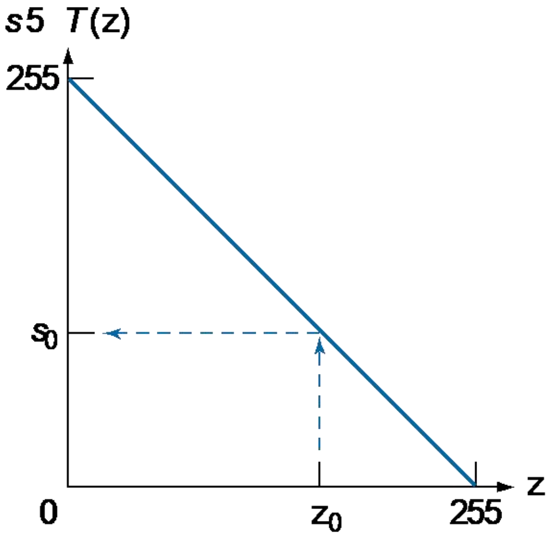
    Scale, Rotate, Translate, Mirror, Transpose, Shear, etc.

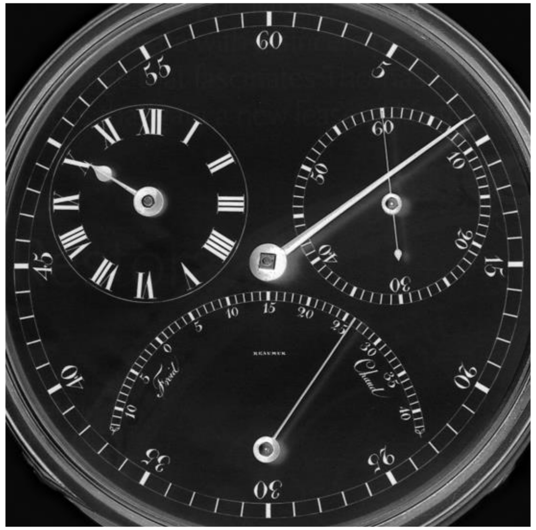
- Interpolation

Single-pixel Operation

$$S = T(z)$$







a b c

Figure 2.38 (a) An 8-bit image. (b) Intensity transformation function used to obtain the digital equivalent of a "photographic" negative of an 8-bit image. The arrows show transformation of an arbitrary input intensity value z_0 into its corresponding output value s_0. (c) Negative of (a), obtained using the transformation function in (b).

Neighborhood operation

S_{xy} is a region with center (x, y), $g(x, y) = \frac{1}{mn}\sum_{(r,c)\in S_{xy}} f(r, c)$

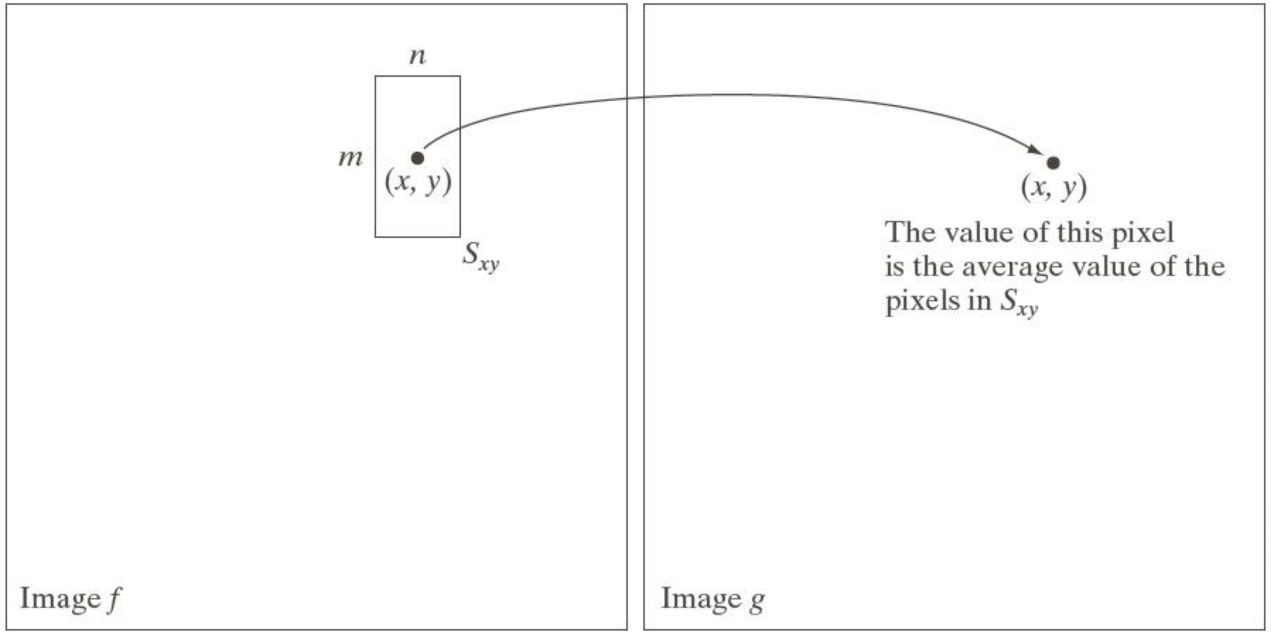


Image geometry

- **Modify spatial relationship between pixels –** *rubber-sheet*
  - Forward mapping (前向映射): $(x\ y) = T(v\ w)$
  - Inverse mapping (反向映射): $(v\ w) = T^{-1}(x\ y)$
- **Affine transform (仿射变换)**

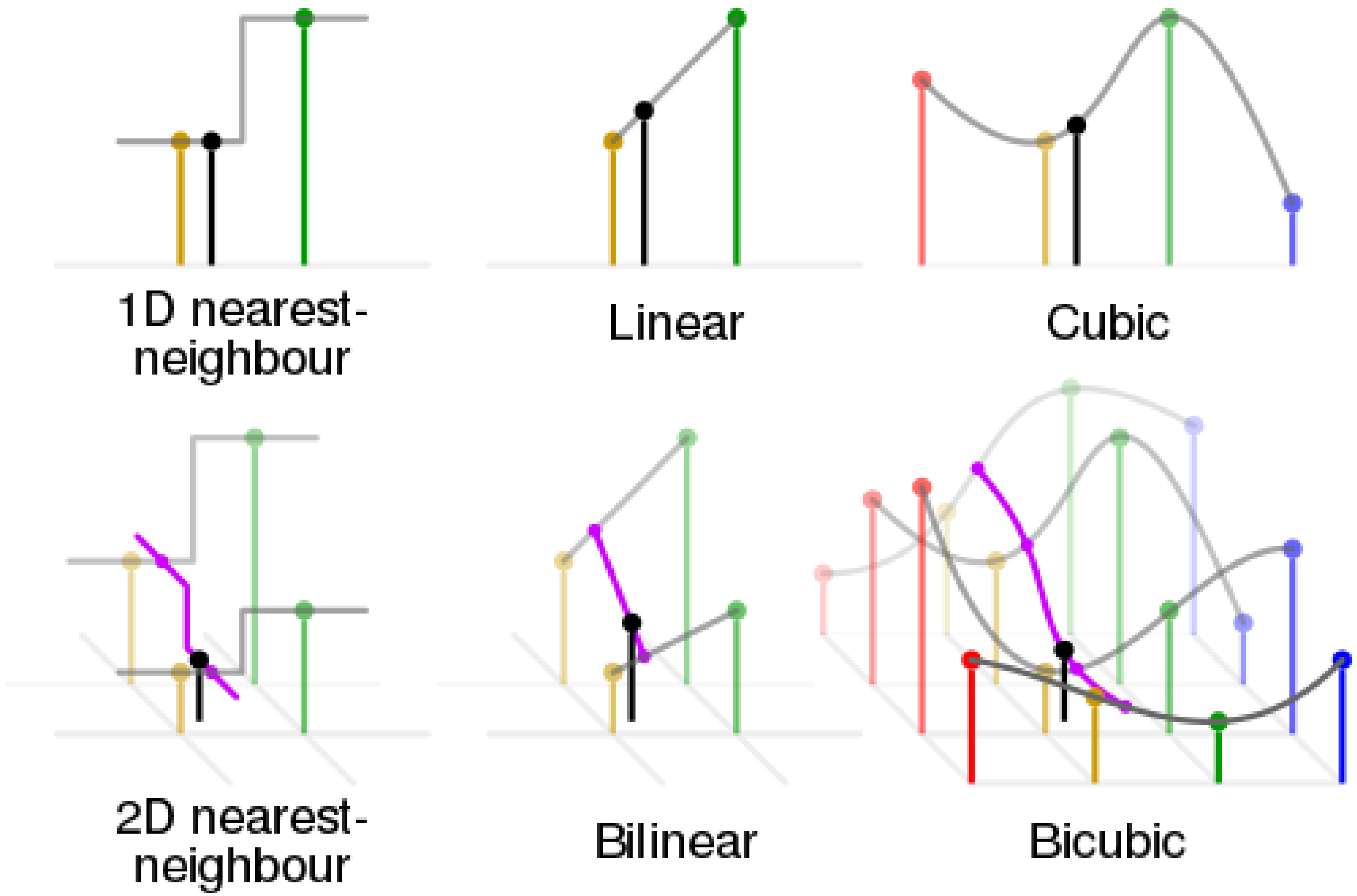
$$[x\ y\ 1] = [v\ w\ 1]T = [v\ w\ 1]\begin{bmatrix} t_1 & t_4 & 0 \\ t_2 & t_5 & 0 \\ t_3 & t_6 & 1 \end{bmatrix}$$

or

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = T\begin{bmatrix} v \\ w \\ 1 \end{bmatrix} = \begin{bmatrix} t_1 & t_2 & t_3 \\ t_4 & t_5 & t_6 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} v \\ w \\ 1 \end{bmatrix}$$

Image Interpolation (插值)

- ➢ Use known data to estimate values at unknown locations
- ➢ A resampling method
- ➢ Intensity interpolation



Interpolation

a b c
d e f

Image interpolation:

interpolate the image from 72dpi to 1250 dpi using (a)nearest neighbor method, (b) Bilinear method, (c) Bicubic method;

interpolate the image from 150dpi to 1250 dpi using (a)nearest neighbor method, (b) Bilinear method, (c) Bicubic method;

Image Filtering

- **High-pass**
  - Small objects and edges
  - Sharpening
  - Improving spatial resolution
  - More noise
  - SNR decrease
- **Low-pass**
  - Smoothing
  - Little effect on spatial resolution
  - Attenuating noise
  - SNR increase

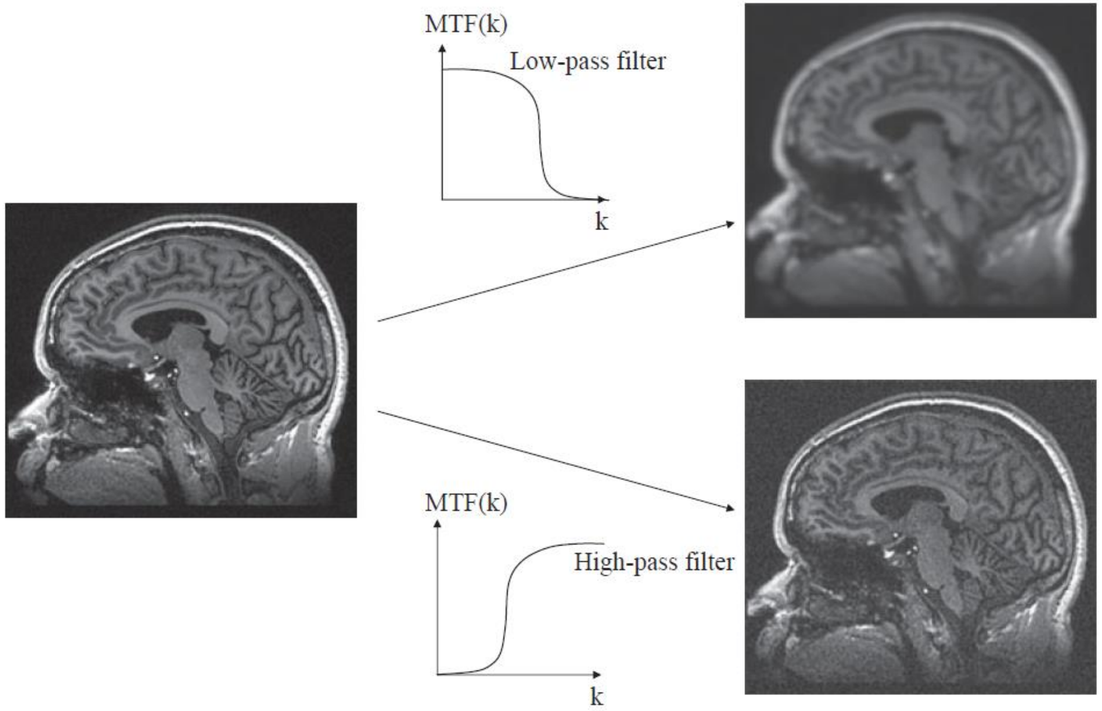


Image Filtering

original image

1	5	3	5	4	6
4	3	32	5	6	9
6	10	4	8	8	7

$*$

filter

1/12	1/12	1/12
1/12	4/12	1/12
1/12	1/12	1/12

$=$

filtered image

1	5	3	5	4	6
4	a	b	c	d	9
6	10	4	8	8	7

$$
\left\{
\begin{array}{l}
a=(1)(1/12)+(5)(1/12)+(3)(1/12)+(4)(1/12)+(3)(4/12)+(32)(1/12)+(6)(1/12)+(10)(1/12)+(4)(1/12)=6.4 \\
b=(5)(1/12)+(3)(1/12)+(5)(1/12)+(3)(1/12)+(32)(4/12)+(5)(1/12)+(10)(1/12)+(4)(1/12)+(8)(1/12)=14.3 \\
c=(3)(1/12)+(5)(1/12)+(4)(1/12)+(32)(1/12)+(5)(4/12)+(6)(1/12)+(4)(1/12)+(8)(1/12)+(8)(1/12)=7.5 \\
d=(5)(1/12)+(4)(1/12)+(6)(1/12)+(5)(1/12)+(6)(4/12)+(9)(1/12)+(8)(1/12)+(8)(1/12)+(7)(1/12)=6.3
\end{array}
\right\}
$$

1	5	3	5	4	6
4	6.4	14.3	7.5	6	9
6	10	4	8	8	7

filtered image

Lecture 2 – Basics of Digital Image

This lecture will cover:

- – Digital Image
- – Image Characteristics
    - Spatial resolution
    - Noise
    - Image artifacts
- – Image Operation
    - Basic operation
    - Spatial Operation
    - Filtering
- – **Digital Image Processing**

Image Enhancement

- **Spatial domain: direct manipulation of pixels**
  - Intensity Transformation: Linear transformation, Log Transformation, Power-law (gamma) Transformation
  - Histogram:
    - ✓ Global histogram processing: Equalization, Matching, Exact Matching
    - ✓ Local histogram processing
    - ✓ Histogram Statistics for Image Enhancement
  - Spatial filtering : convolution and correlation
    - ✓ Smoothing: Box and Gaussian (Linear), Order-statistic filter (Nonlinear)
    - ✓ Sharpening: Laplacian, gradient, Unsharp Masking
- **Frequency domain : transform and inverse transform image**
  - 2D Image Transform: sampling theorem, DFT, spectrum and phase angle
  - Frequency Domain Filtering: Ideal, Butterworth, Gaussian Filter
    - ✓ Lowpass Filtering
    - ✓ Highpass Filtering : Laplacian, High Frequency Emphasis Filter, Homomorphic Filtering
    - ✓ Selective Filtering : Bandreject and Bandpass Filters, Notch Filter

Image Restoration

- **Model of Image Degradation Process**
- **Noise Reduction**
  - Noise Models: properties, Probability Density Function (PDF), estimation of noise parameter
  - Spatial Filtering
    - ✓ Mean Filters: arithmetic, geometric, harmonic, Contraharmonic
    - ✓ Order-statistic Filters: median, max and min, midpoint, alpha-trimmed mean filter
    - ✓ Adaptive Filters: adaptive local noise reduction filter, adaptive median filter
  - Frequency Domain Filtering
- **Image degradation and restoration**
  - Degradation Function
  - Restoration Filtering
    - ✓ Inverse Filtering
    - ✓ Wiener Filtering
    - ✓ Constrained Least Squares Filtering
    - ✓ Geometric Mean Filtering

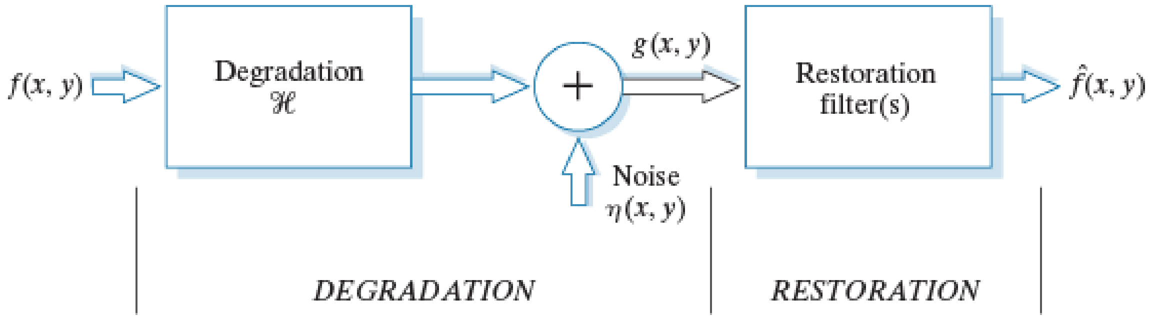


FIGURE 5.1 A model of the image degradation/restoration process.

Image Segmentation

- **Traditional segmentation method**
  - Based on Discontinuity (Edge-based segmentation):
    - ✓ Point, line, edge detection: Sobel, LoG, Canny
    - ✓ Edge Linking: local or region processing, global (Hough transform)
  - Based on Similarity
    - ✓ Thresholding: Global (Otsu's method) and Variable
    - ✓ Region-based segmentation: Region Growing, Region Splitting and Merging, Region Clustering, Superpixels, Graph Cuts
- **Morphological Image Processing**
  - Morphological operation: erosion and dilation, opening and closing, HMT
  - Morphological algorithms: basic algorithms, morphological reconstruction
  - Morphological Watersheds
- **Active Contours**
  - Snakes: explicit (parametric) representation of segmentation curves
  - Level Sets: implicit representation of curves