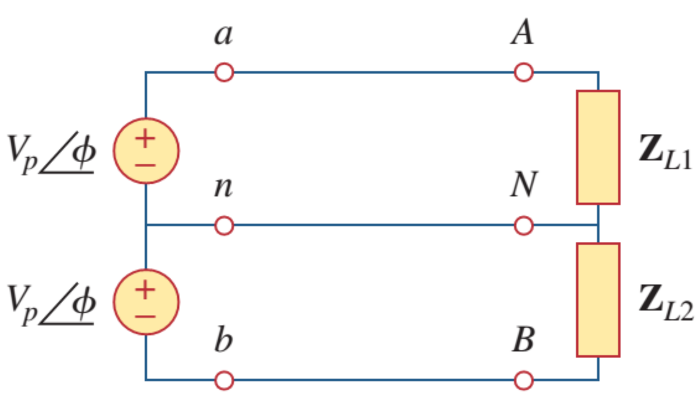
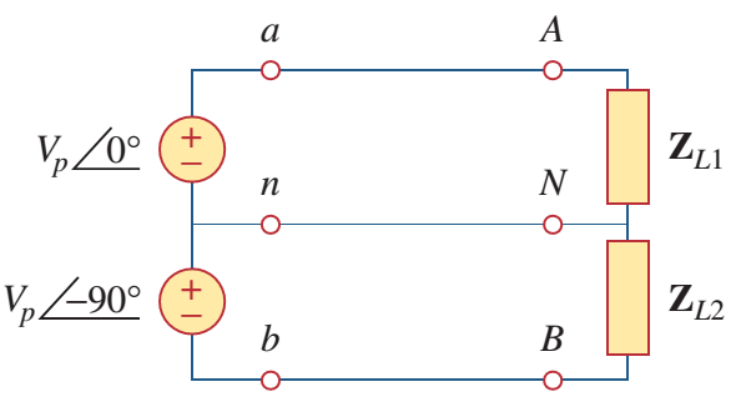
Lecture 10

- Three-Phase Circuits

Single phase vs. Polyphase

- Single-phase power supply
  - For example, two 120V sources with the same phase are connected in series.
  - This allows for appliances to use either 120 or 240V
- Circuits that operate with multiple sources, at the same frequency but *at different phases* are called <u>polyphase</u>.





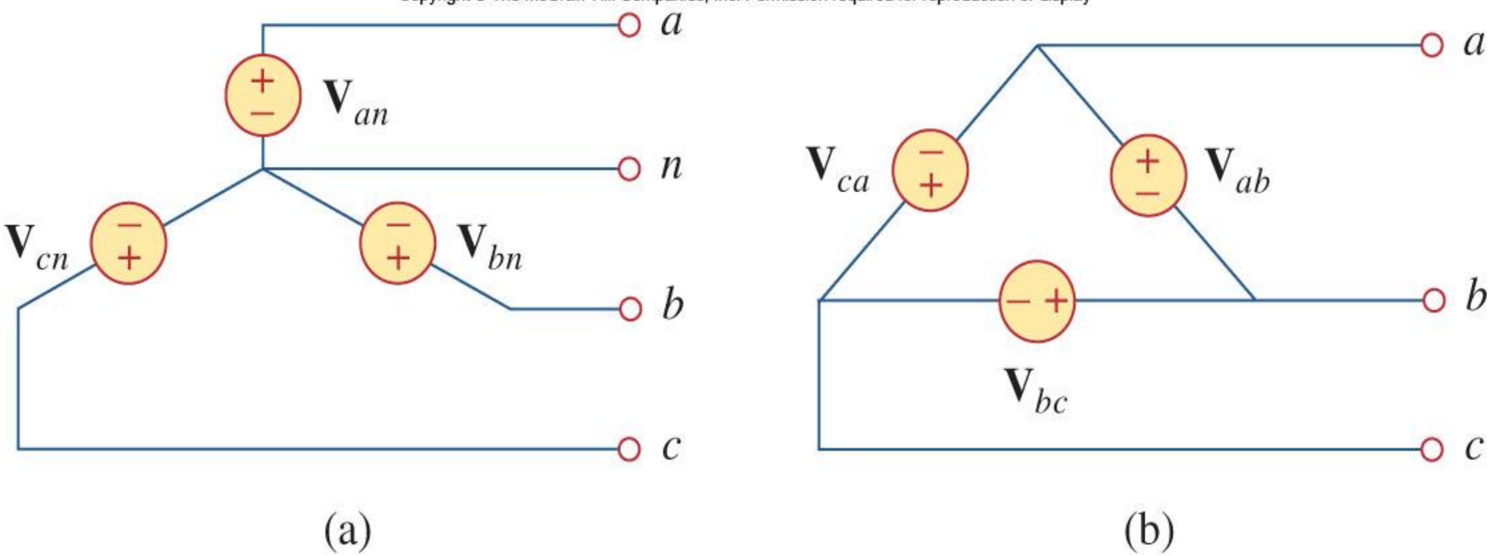
Outline--Three-Phase Circuits

- Balanced Three-Phase System
  - Balanced sources
  - Balanced loads
- Circuit analysis
  - Phase voltage/current
  - Line voltage/current

Balanced Three-Phase Sources

Connecting the Sources

- Three phase voltage sources can be connected by either four or three wire configurations.
  - Four-wire system accomplished using a Y(Wye) connected source.
  - Three-wire configuration accomplished by Delta connected source.



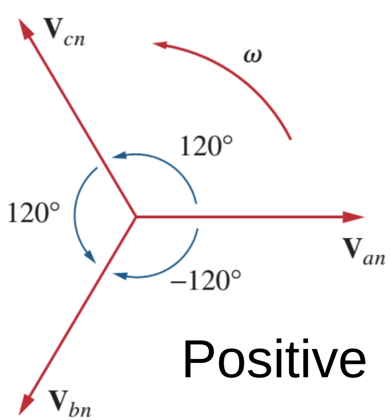
$$V_{an} = V_p \angle 0^\circ$$

$$V_{bn} = V_p \angle -120^\circ$$

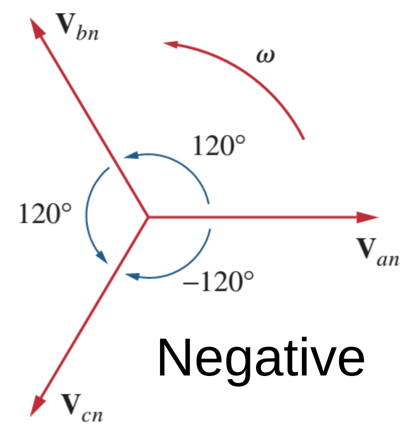
$$V_{cn} = V_p \angle -240^\circ = V_p \angle +120^\circ$$

Balanced Three-Phase Sources

- Balanced phase voltage are equal in magnitude and are out of phase with each other by 120deg
- It's easy to know $\boxed{V_{an}+V_{bn}+V_{cn}=0}$
- Two sequences for the phases:



$$V_{an}=V_p\angle 0^\circ$$
$$V_{bn}=V_p\angle -120^\circ$$
$$V_{cn}=V_p\angle -240^\circ=V_p\angle +120^\circ$$

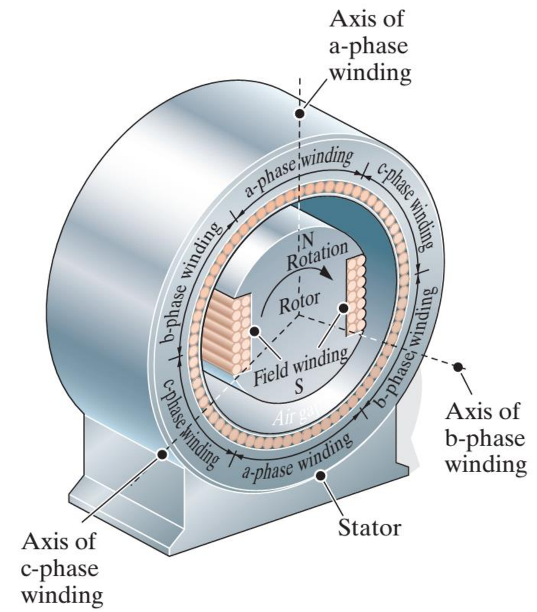


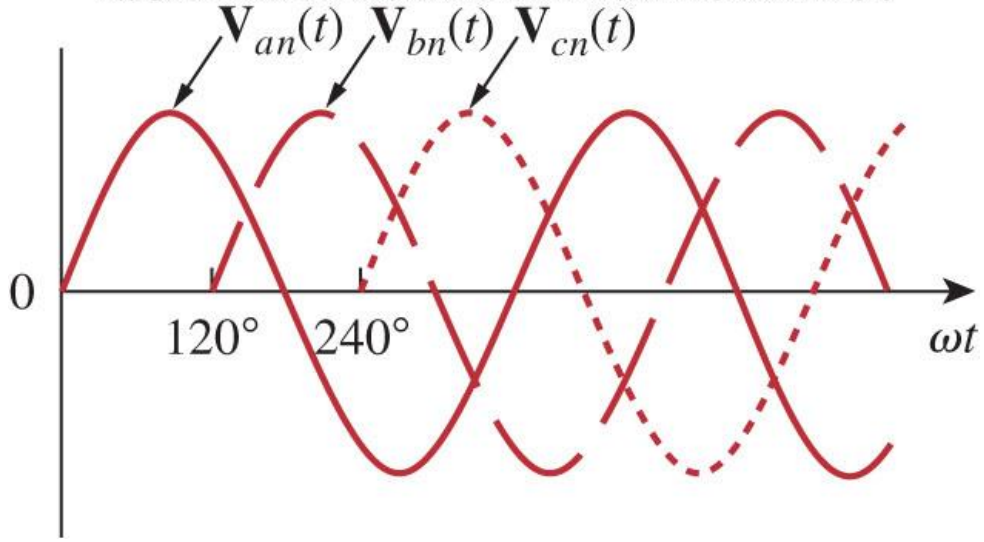
$$V_{an}=V_p\angle 0^\circ$$
$$V_{cn}=V_p\angle -120^\circ$$
$$V_{bn}=V_p\angle -240^\circ=V_p\angle +120^\circ$$

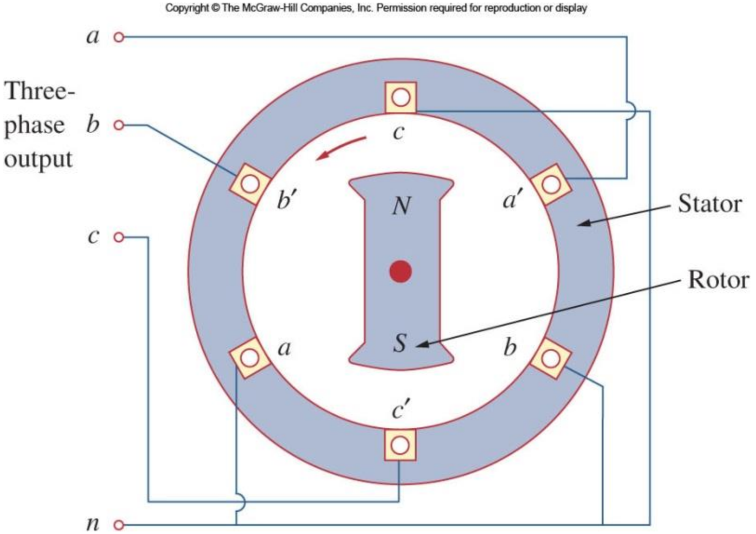
Three-Phase Sources

- Three phase voltages are typically produced by a three-phase AC generator.

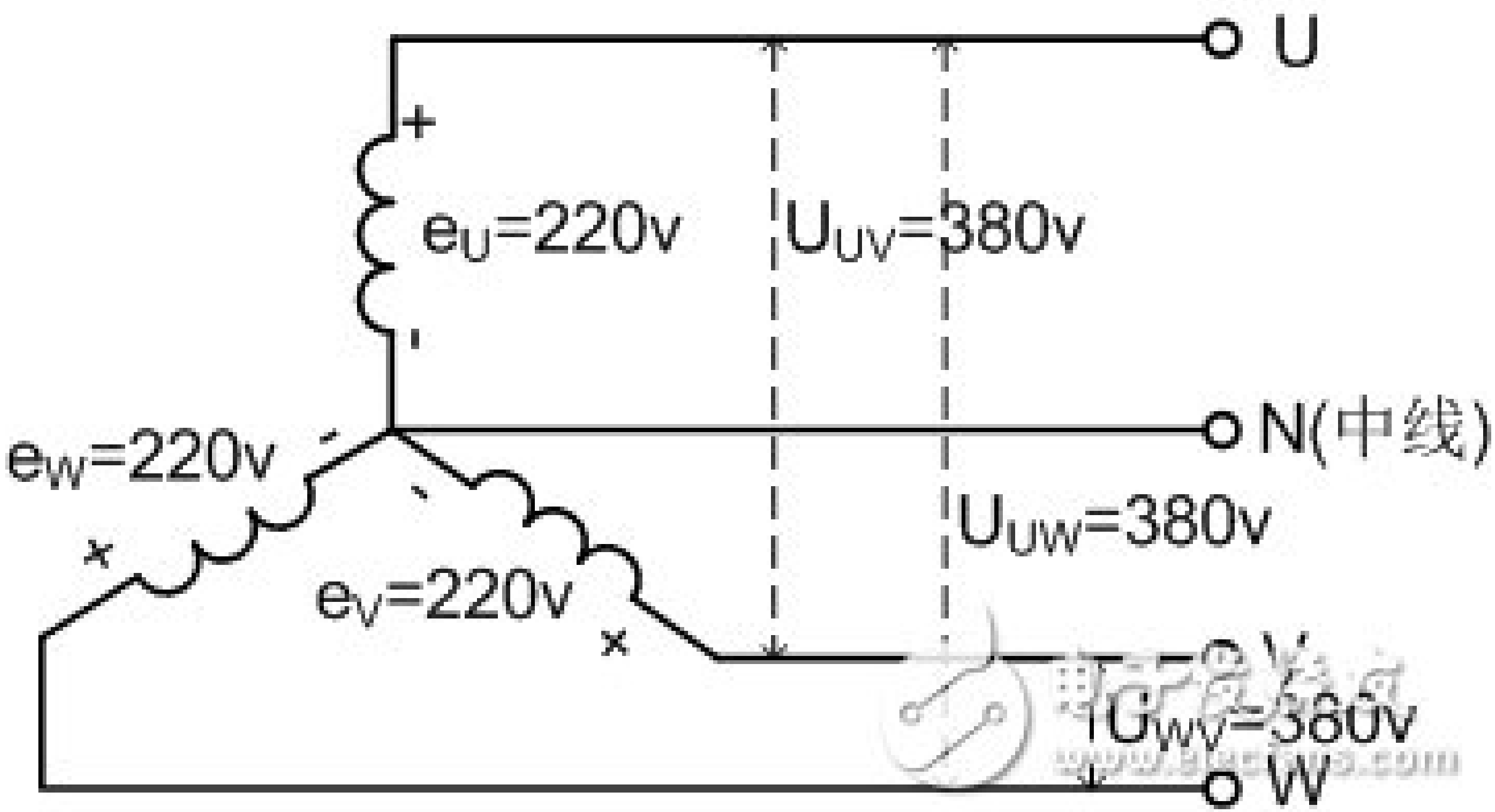
- The output voltages look like below.







Three-phase four-wire system in China

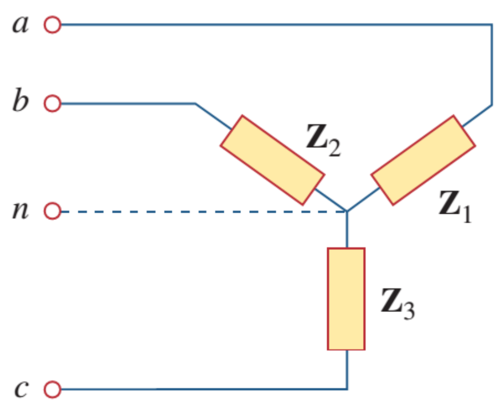


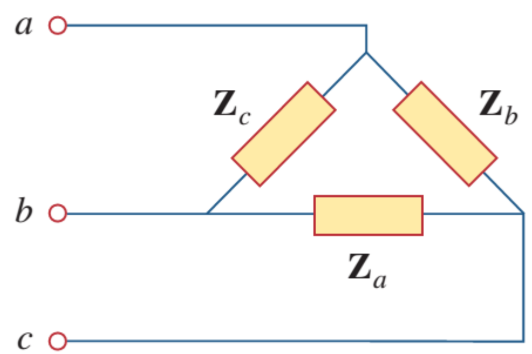
Balanced Loads

- A <u>balanced</u> load means the same impedance for each load.

-- Impedance are equal in magnitude and in phase

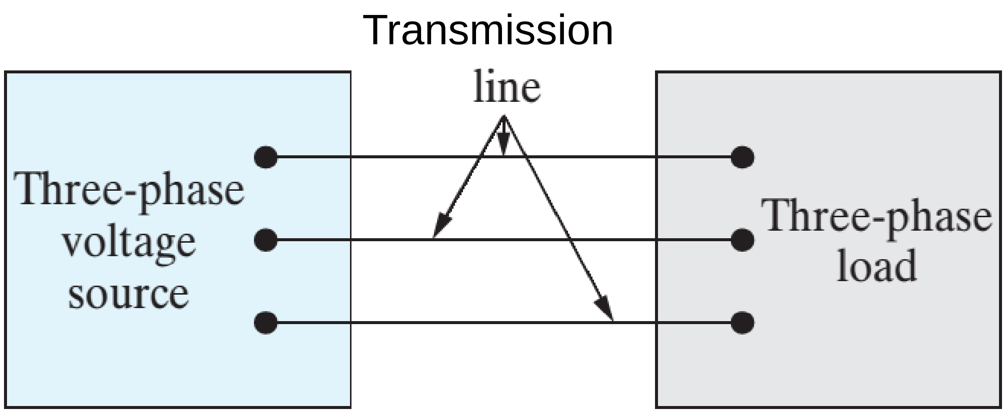
- They may also be connected in either Delta or wye
  - For a balanced wye connected load: $Z_1 = Z_2 = Z_3 = Z_Y$
  - For a balanced delta connected load: $Z_a = Z_b = Z_c = Z_\Delta$





- The load impedance per phase for the above configurations can be interchanged.

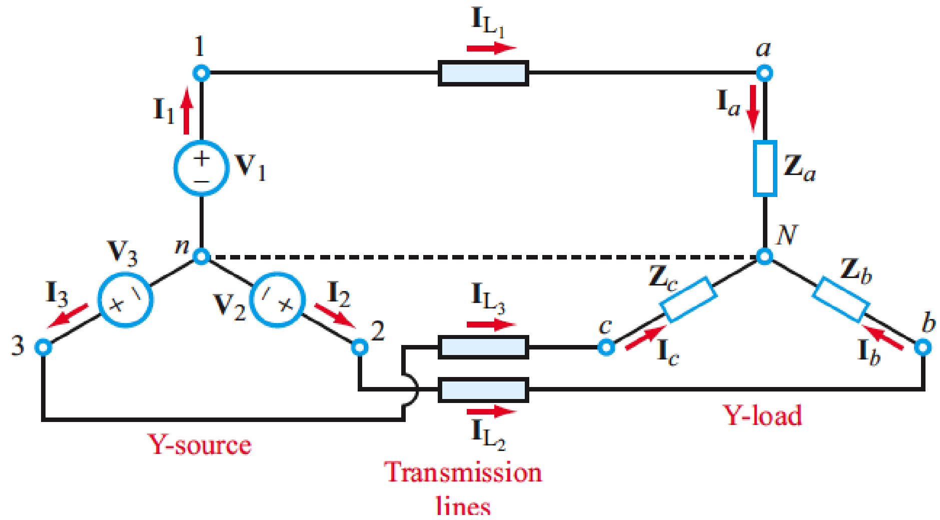
Source-Load configurations



Source	Load
Y	Y
Y	Δ
Δ	Y
Δ	Δ

Source-Load Configurations

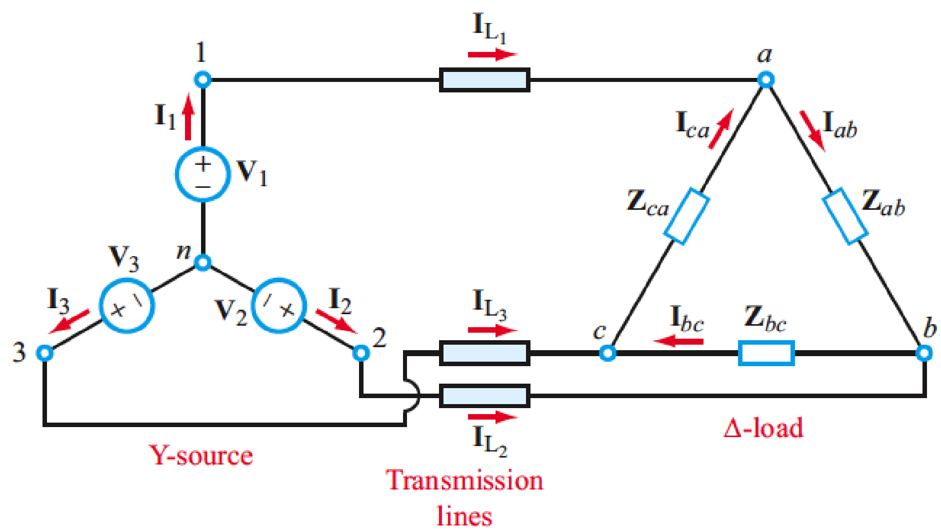
Y-Y



Load Phase Currents
I_a, I_b, I_c
(same as line currents I_{L_1}, I_{L_2}, and I_{L_3})

Load Phase Voltages
V_{aN}, V_{bN}, V_{cN}

Y-Delta

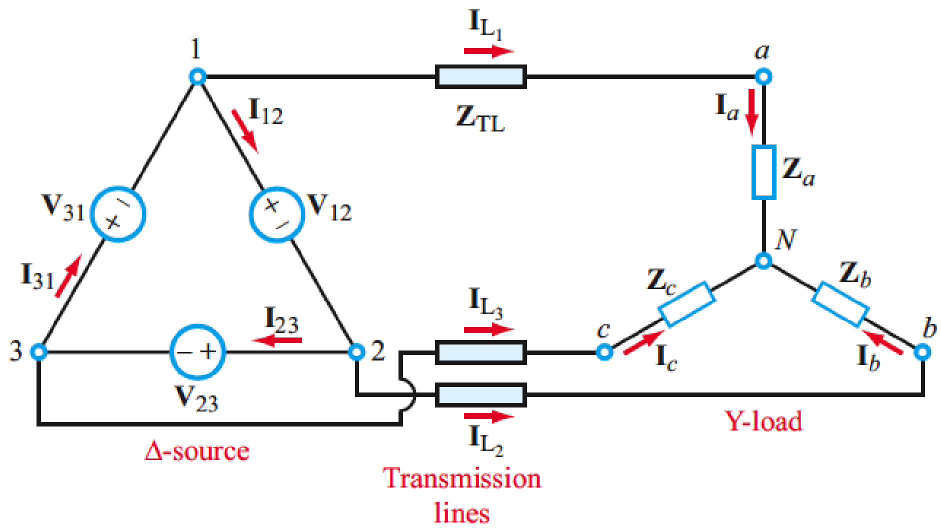


Load Phase Currents
I_{ab}, I_{bc}, I_{ca}

Load Phase Voltages
V_{ab}, V_{bc}, V_{ca}

Source-Load Configurations

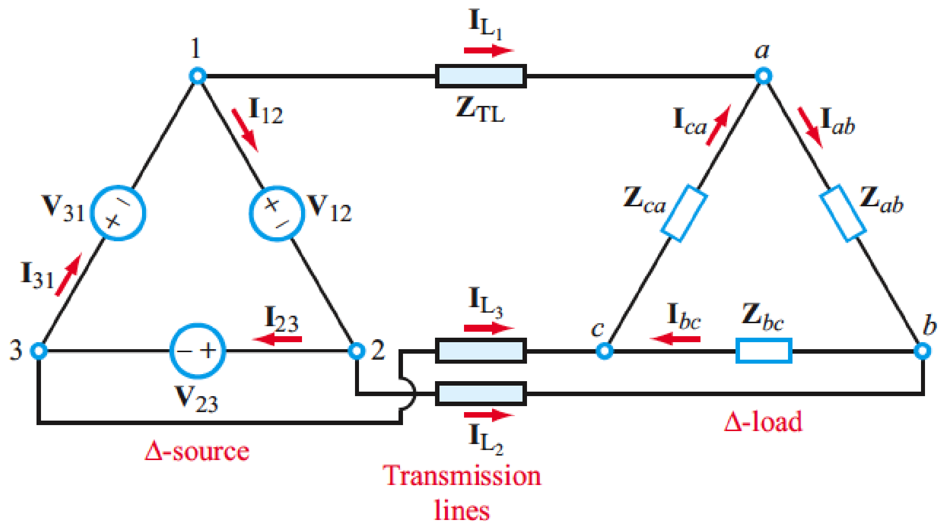
Delta-Y



Load Phase Currents
$\mathbf{I}_a$, $\mathbf{I}_b$, $\mathbf{I}_c$
(same as line currents
$\mathbf{I}_{L_1}$, $\mathbf{I}_{L_2}$, and $\mathbf{I}_{L_3}$)

Load Phase Voltages
$\mathbf{V}_{aN}$, $\mathbf{V}_{bN}$, $\mathbf{V}_{cN}$

Delta-Delta



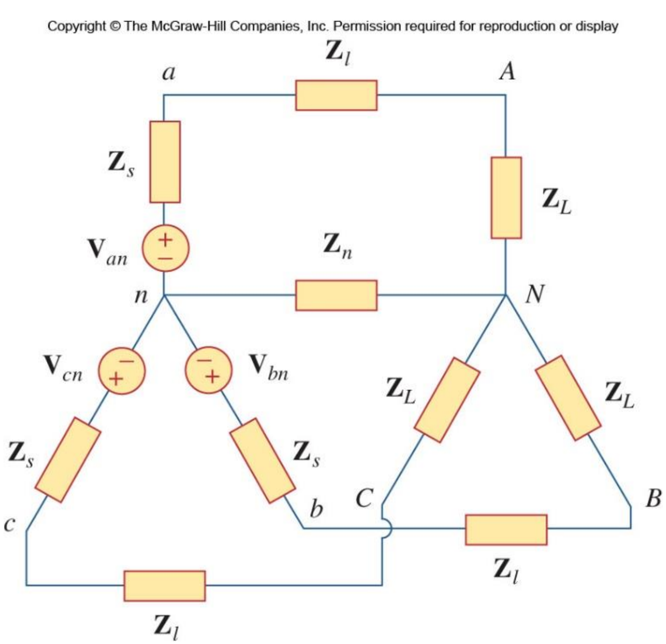
Load Phase Currents
$\mathbf{I}_{ab}$, $\mathbf{I}_{bc}$, $\mathbf{I}_{ca}$

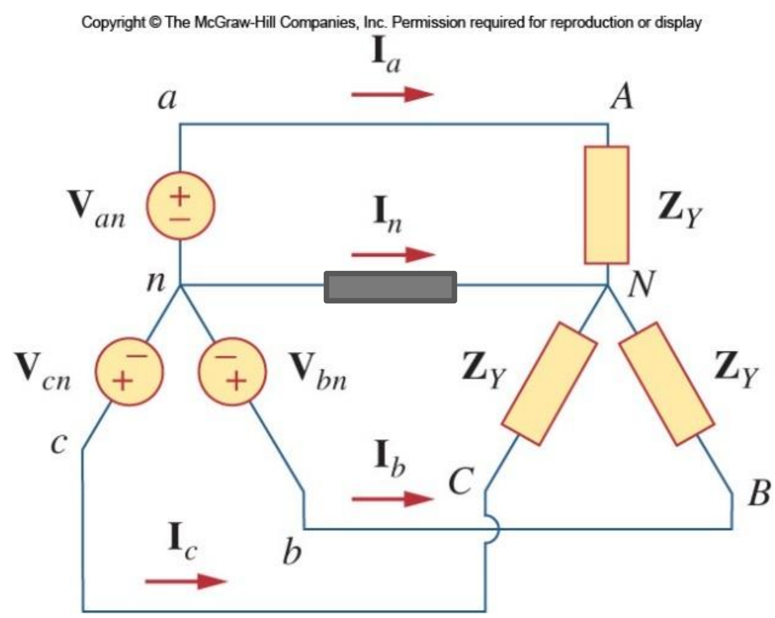
Load Phase Voltages
$\mathbf{V}_{ab}$, $\mathbf{V}_{bc}$, $\mathbf{V}_{ca}$
(same as source voltages
if $\mathbf{Z}_{TL}$ is negligible)

Balanced Y-Y connection

- The load impedance Z_Y will be assumed to be balanced.
  - This can be the source Z_s, line Z_l and load Z_L together.

$$\mathbf{Z}_Y = \mathbf{Z}_s + \mathbf{Z}_\ell + \mathbf{Z}_L$$



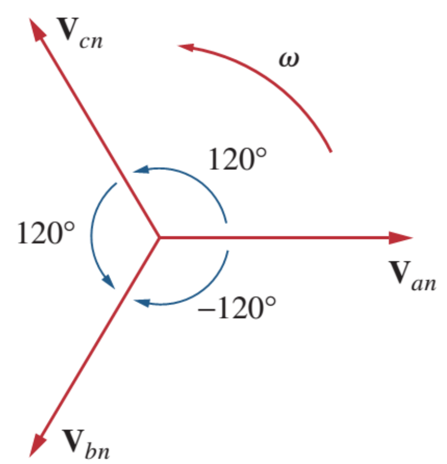


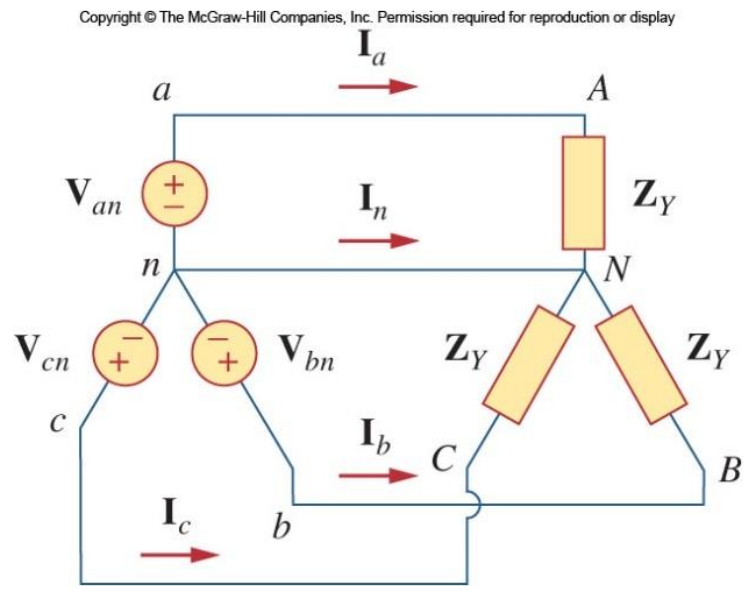
Phase Voltage & Line-to-Line Voltage

- Use the positive sequence:

Phase Voltage $V_{an} = V_p \angle 0^\circ$

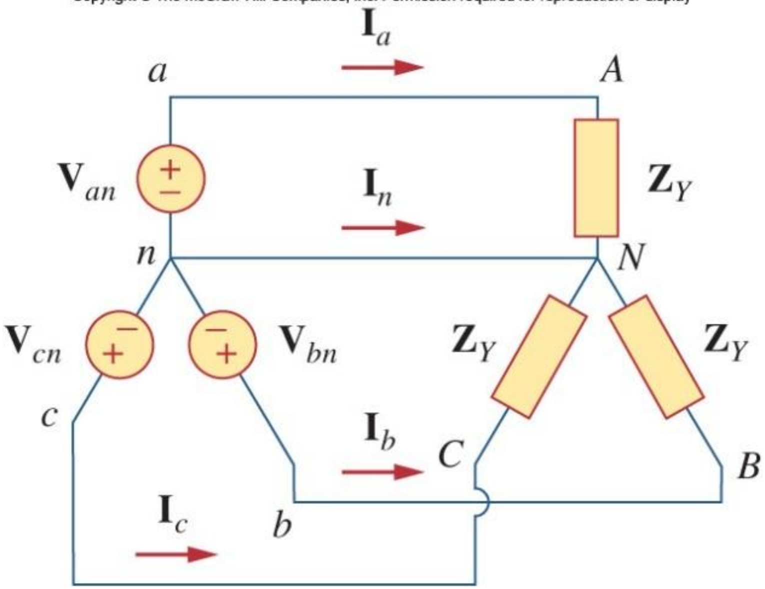
$$V_{bn} = V_p \angle -120^\circ \quad V_{cn} = V_p \angle +120^\circ$$





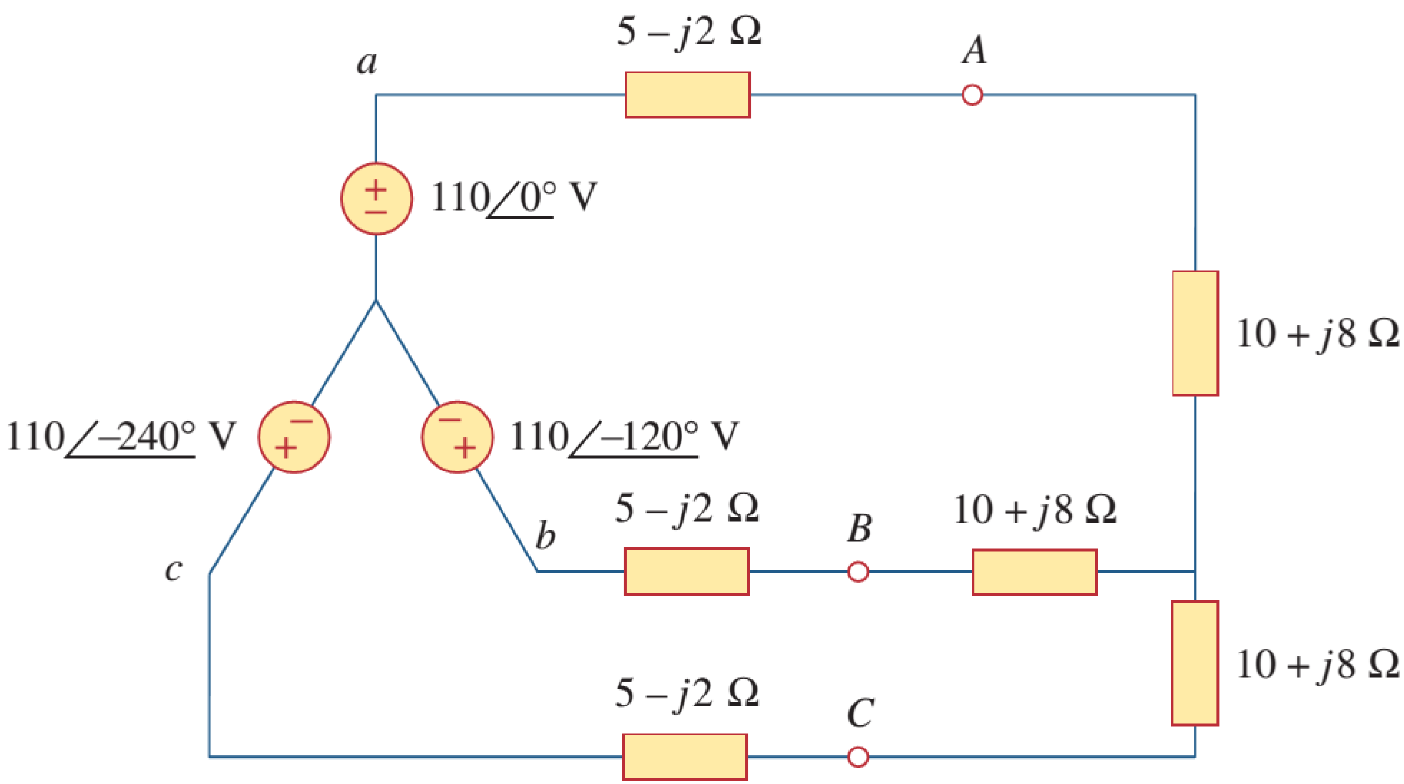
- ***The line to line voltages (or just line voltages in short):***

Line Currents



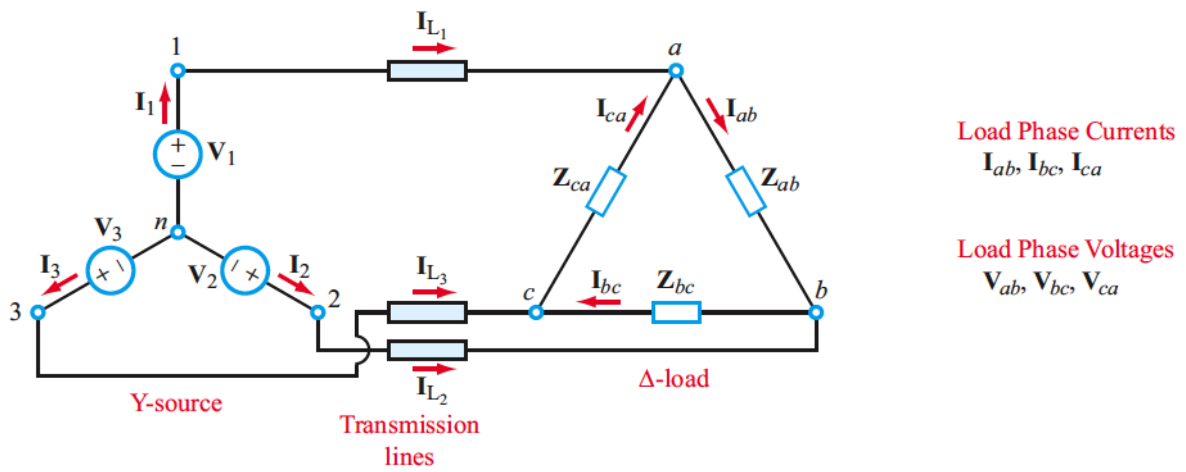
Example

- Calculate the line currents.

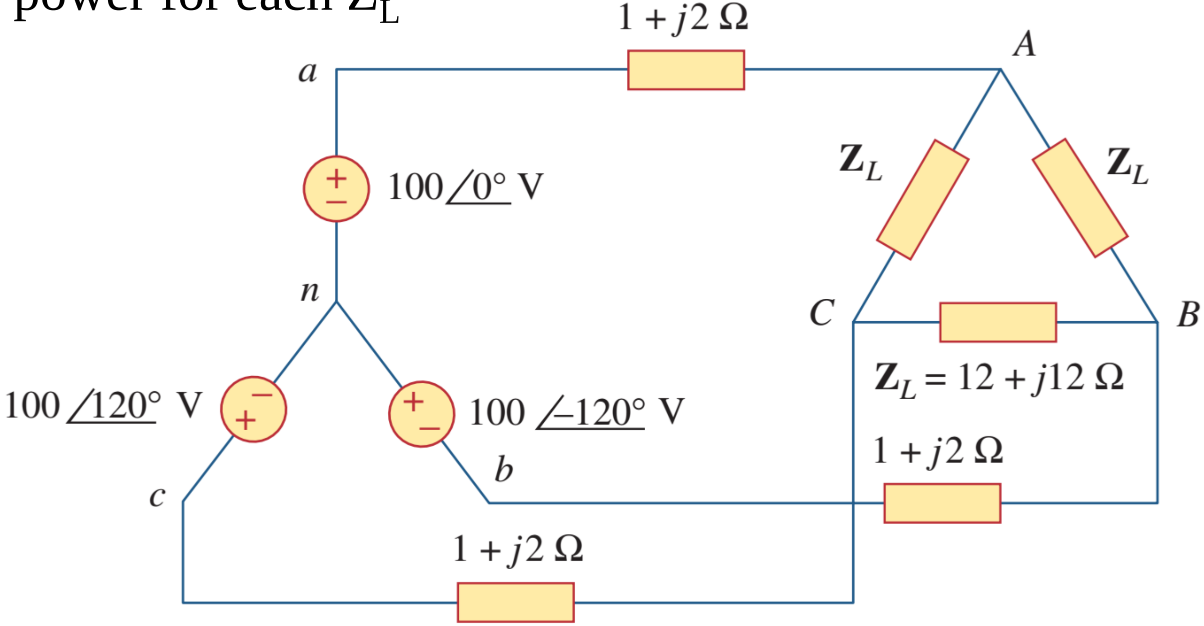


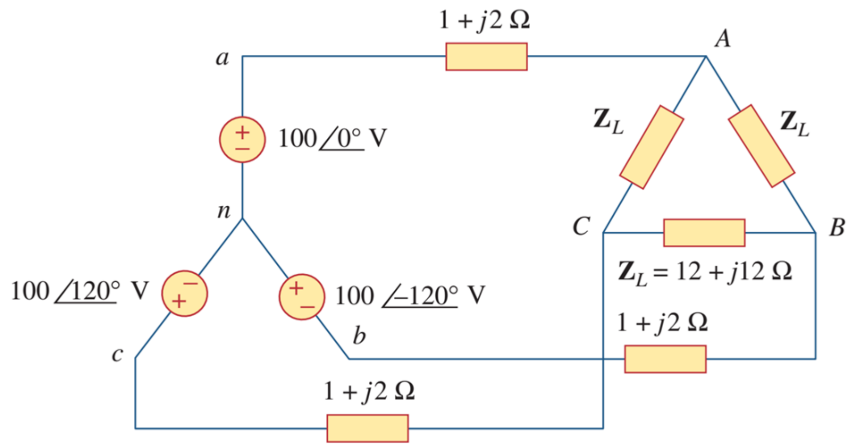
Wye-Δ

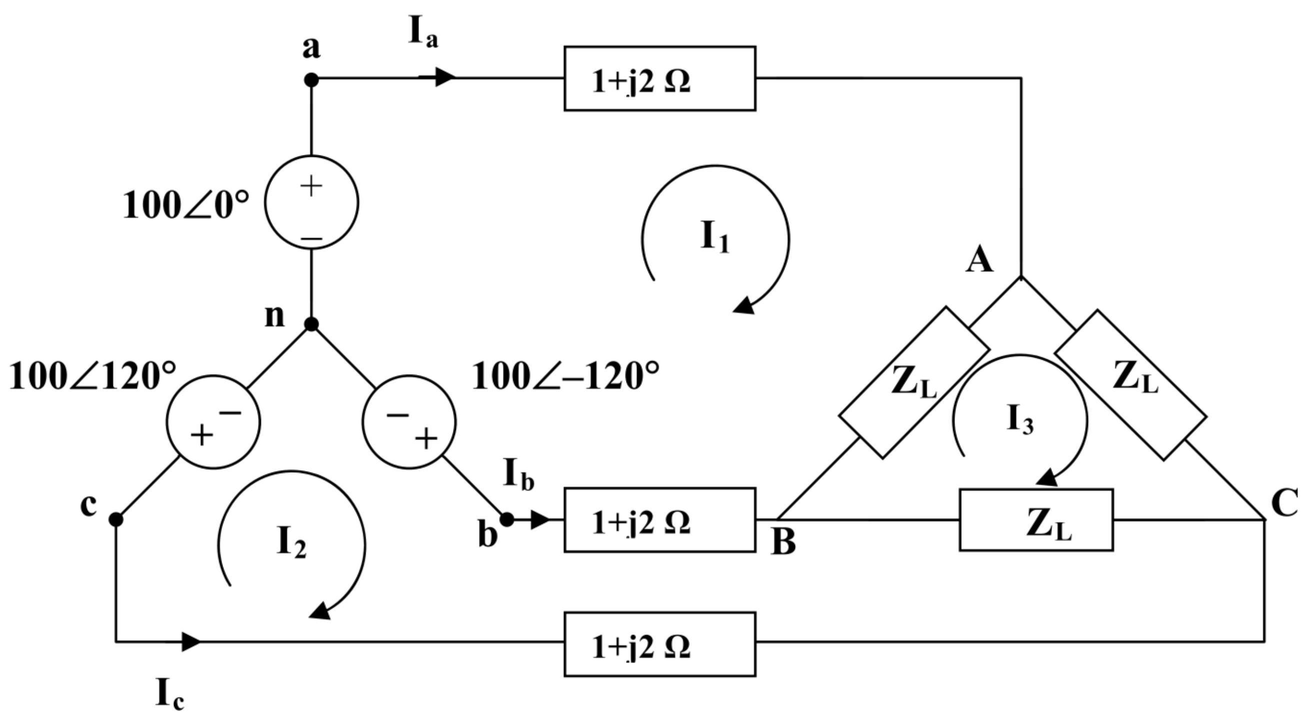
Y-Delta



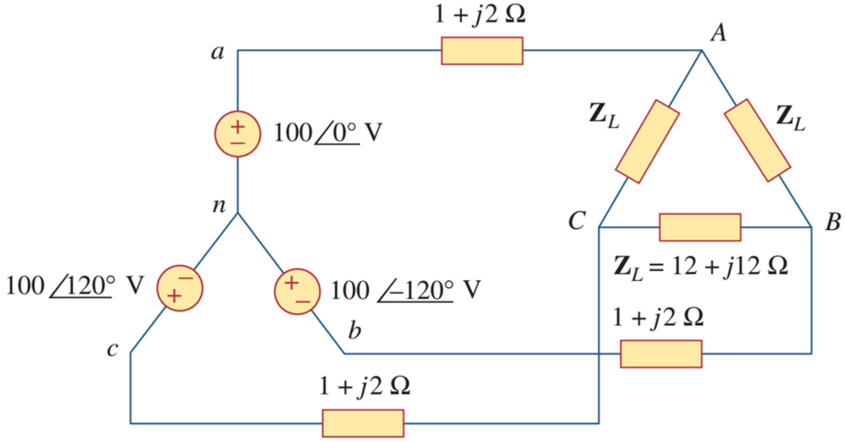
Find complex power for each Z_L




1 + j2 Ω
a
A
100∠0° V
Z_L
Z_L
n
C
B
100∠120° V
100∠−120° V
Z_L = 12 + j12 Ω
b
1 + j2 Ω
c
1 + j2 Ω

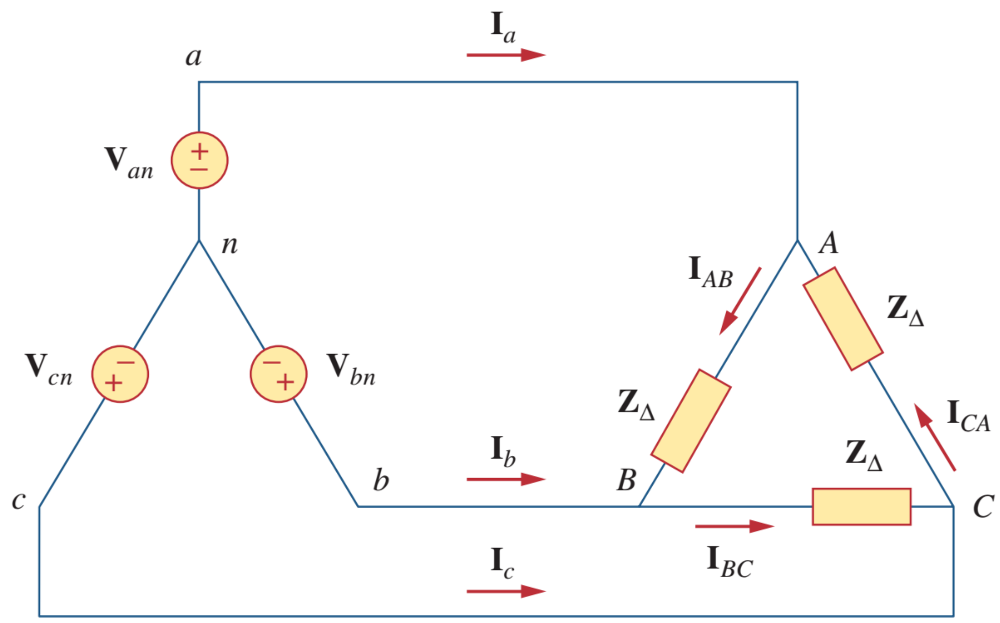

I_a
a
1+j2 Ω
100∠0°
I_1
A
n
100∠120°
100∠−120°
Z_L
Z_L
I_3
I_b
c
I_2
b
1+j2 Ω
B
Z_L
C
1+j2 Ω
I_c

Wye-Δ ➔ Wye-Wye


1 + j2 Ω
a
A
100 /0° V
Z_L
Z_L
n
C
B
100 /120° V
100 /−120° V
Z_L = 12 + j12 Ω
1 + j2 Ω
b
c
1 + j2 Ω

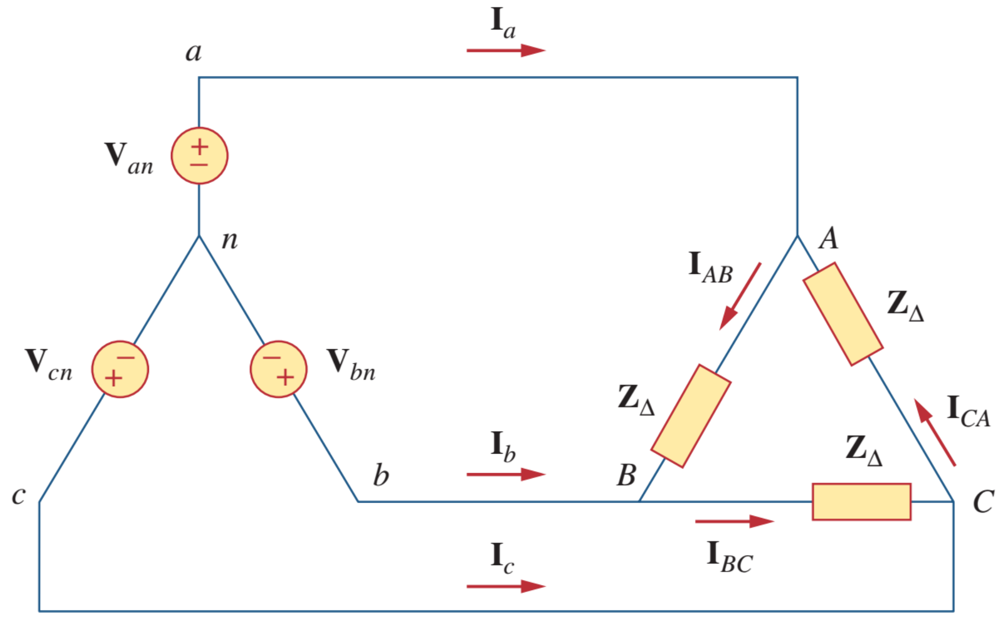
Wye-Δ

Find load phase current and line current



Wye-Δ

Find load phase current and line current



Assume positive sequence:

$$\mathbf{V}_{an} = V_p\angle 0^\circ$$

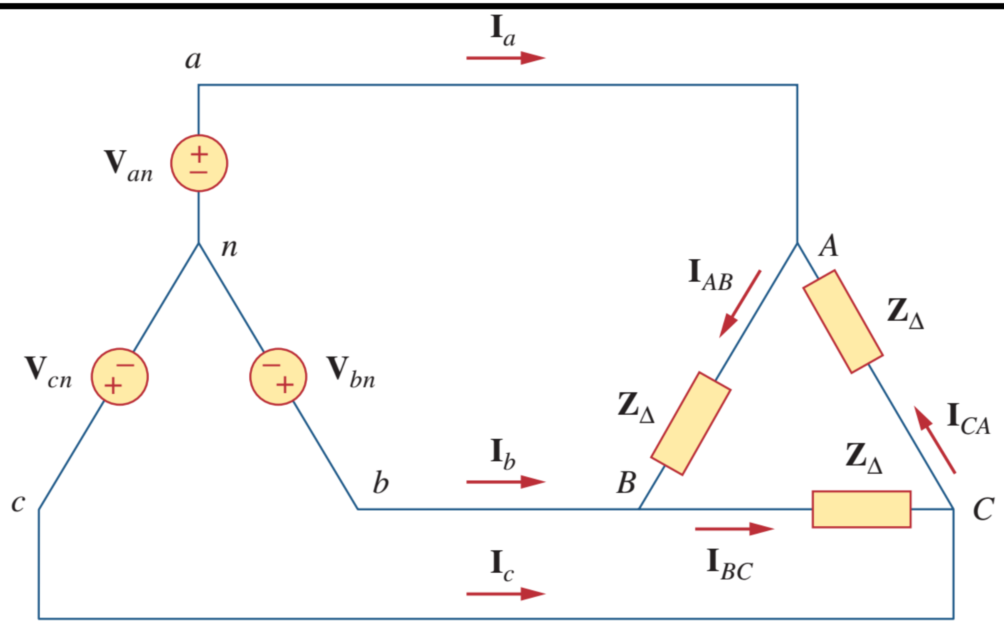
$$\mathbf{V}_{bn} = V_p\angle -120^\circ, \qquad \mathbf{V}_{cn} = V_p\angle +120^\circ$$

$$\mathbf{V}_{ab} = \sqrt{3}V_p\angle 30^\circ = \mathbf{V}_{AB}, \qquad \mathbf{V}_{bc} = \sqrt{3}V_p\angle -90^\circ = \mathbf{V}_{BC}$$

$$\mathbf{V}_{ca} = \sqrt{3}V_p\angle -150^\circ = \mathbf{V}_{CA}$$

$$\mathbf{I}_{AB} = \frac{\mathbf{V}_{AB}}{\mathbf{Z}_\Delta}, \qquad \mathbf{I}_{BC} = \frac{\mathbf{V}_{BC}}{\mathbf{Z}_\Delta}, \qquad \mathbf{I}_{CA} = \frac{\mathbf{V}_{CA}}{\mathbf{Z}_\Delta}$$

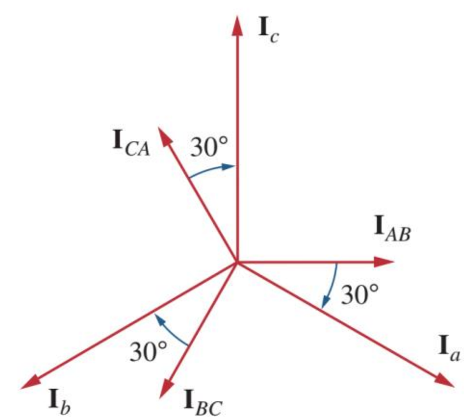
Wye-Δ



$$\mathbf{I}_a = \mathbf{I}_{AB} - \mathbf{I}_{CA}, \qquad \mathbf{I}_b = \mathbf{I}_{BC} - \mathbf{I}_{AB}, \qquad \mathbf{I}_c = \mathbf{I}_{CA} - \mathbf{I}_{BC}$$

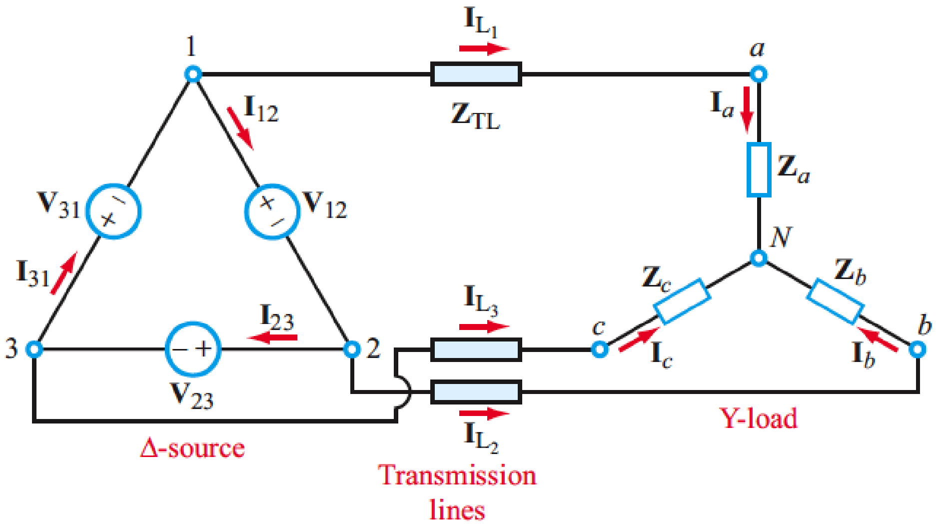
Since $\mathbf{I}_{CA} = \mathbf{I}_{AB}\underline{/-240^\circ}$,

$$\begin{aligned}\mathbf{I}_a = \mathbf{I}_{AB} - \mathbf{I}_{CA} &= \mathbf{I}_{AB}(1 - 1\underline{/-240^\circ}) \\ &= \mathbf{I}_{AB}(1 + 0.5 - j0.866) = \mathbf{I}_{AB}\sqrt{3}\underline{/-30^\circ}\end{aligned}$$



Source-Load Configurations

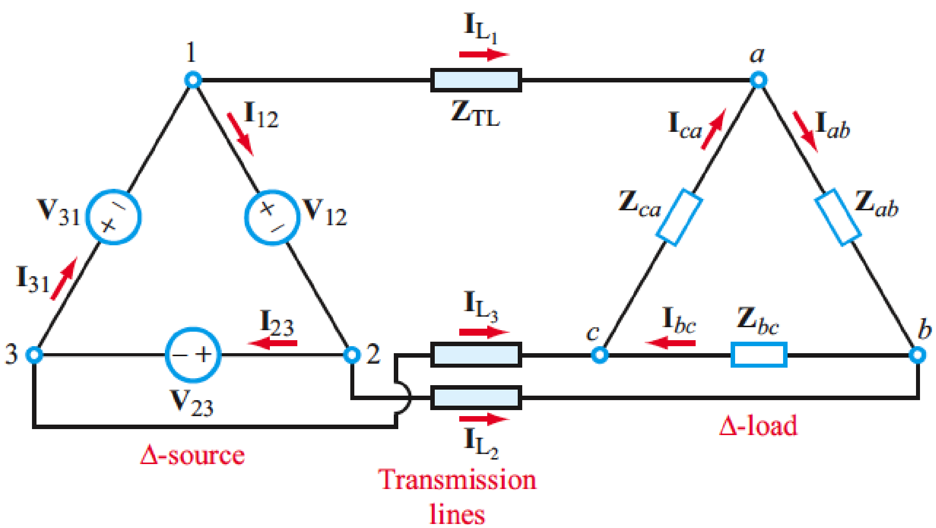
Delta-Y



Load Phase Currents
I_a, I_b, I_c
(same as line currents I_{L_1}, I_{L_2}, and I_{L_3})

Load Phase Voltages
V_{aN}, V_{bN}, V_{cN}

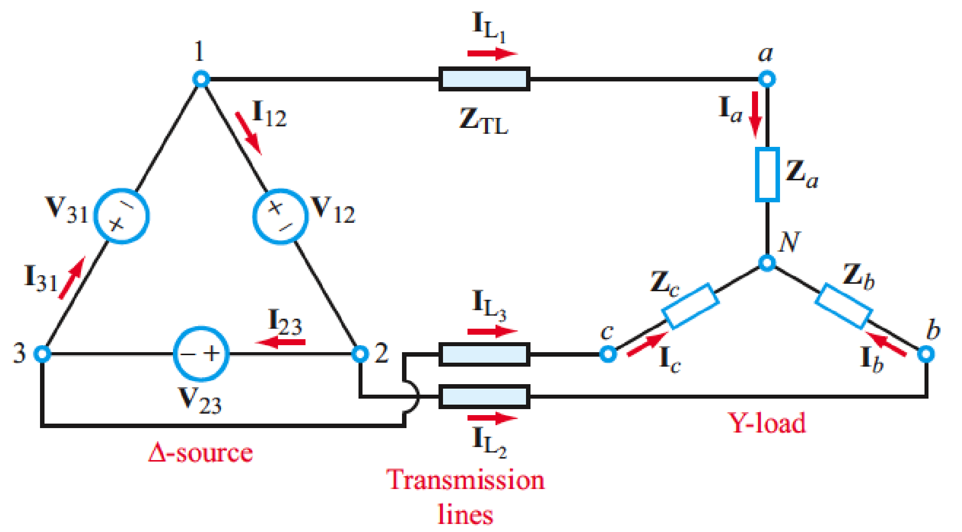
Delta-Delta



Load Phase Currents
I_{ab}, I_{bc}, I_{ca}

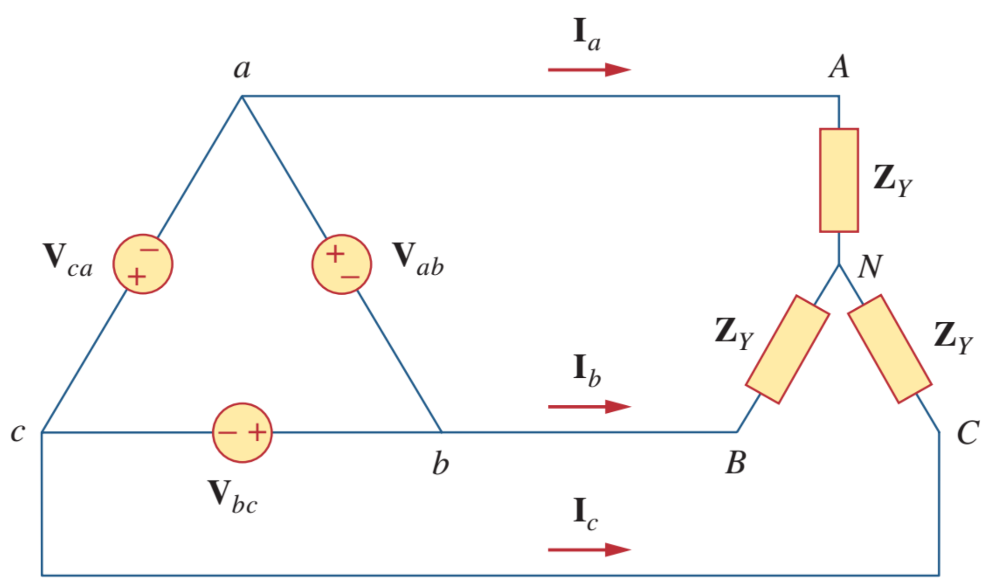
Load Phase Voltages
V_{ab}, V_{bc}, V_{ca}
(same as source voltages if Z_{TL} is negligible)

Δ-Wye

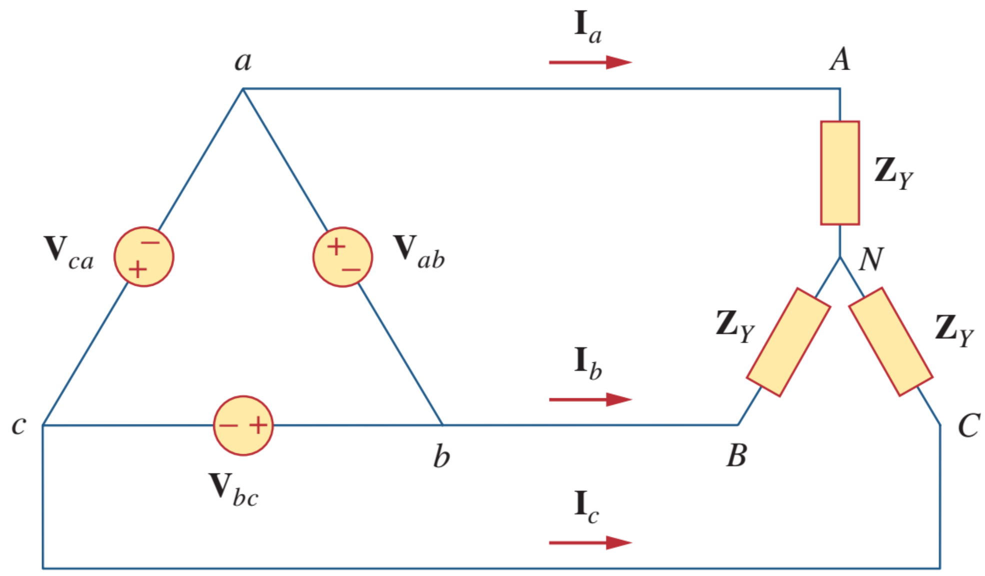


Load Phase Currents
I_a, I_b, I_c
(same as line currents I_{L_1}, I_{L_2}, and I_{L_3})

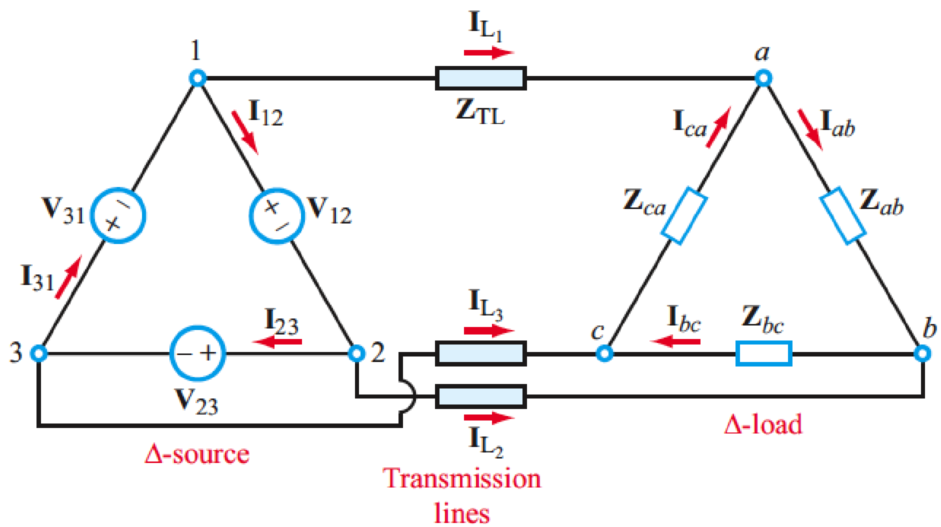
Load Phase Voltages
V_{aN}, V_{bN}, V_{cN}



Delta-Y


$\mathbf{I}_a$
a
A
$\mathbf{Z}_Y$
$\mathbf{V}_{ca}$
$\mathbf{V}_{ab}$
N
$\mathbf{Z}_Y$
$\mathbf{Z}_Y$
$\mathbf{I}_b$
c
C
b
B
$\mathbf{V}_{bc}$
$\mathbf{I}_c$

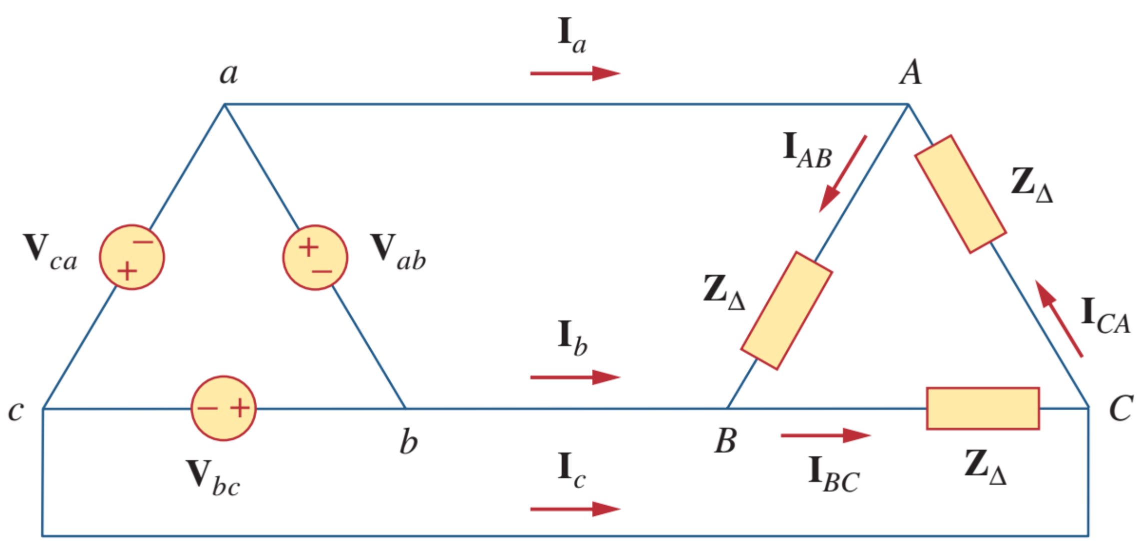
Delta-Delta

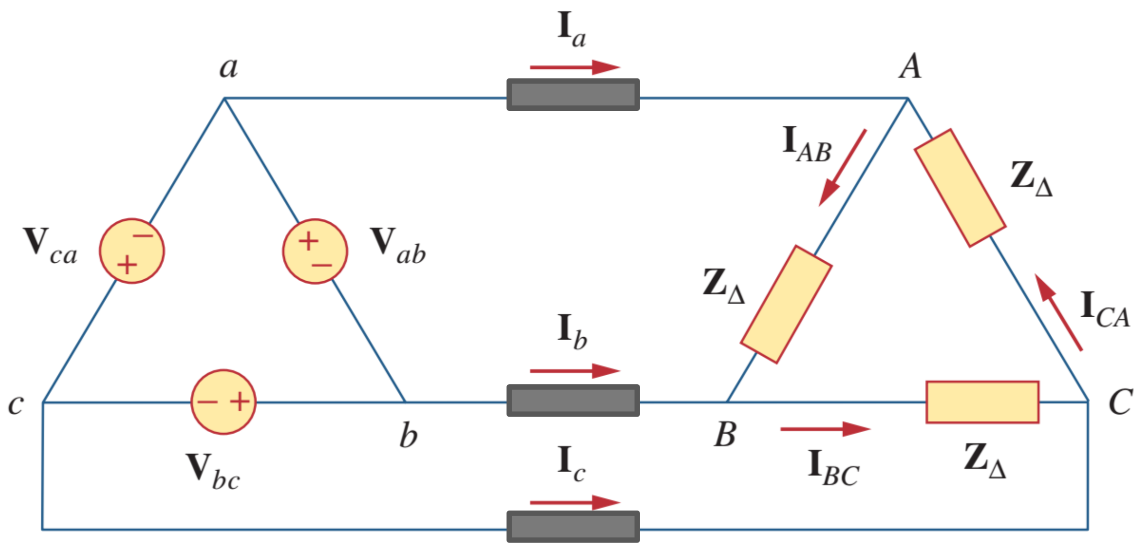


Load Phase Currents
$\mathbf{I}_{ab}, \mathbf{I}_{bc}, \mathbf{I}_{ca}$

Load Phase Voltages
$\mathbf{V}_{ab}, \mathbf{V}_{bc}, \mathbf{V}_{ca}$
(same as source voltages if $\mathbf{Z}_{TL}$ is negligible)

- Δ-Δ




$\mathbf{I}_a$
a
A
$\mathbf{I}_{AB}$
$\mathbf{Z}_\Delta$
$\mathbf{V}_{ca}$
$\mathbf{V}_{ab}$
$\mathbf{Z}_\Delta$
$\mathbf{I}_b$
$\mathbf{I}_{CA}$
c
b
B
C
$\mathbf{V}_{bc}$
$\mathbf{I}_c$
$\mathbf{I}_{BC}$
$\mathbf{Z}_\Delta$

2. (16 points). Given the circuit in Fig.2 is in steady state, use phasor domain method to find $i_0(t)$; and $i_1(t)$.

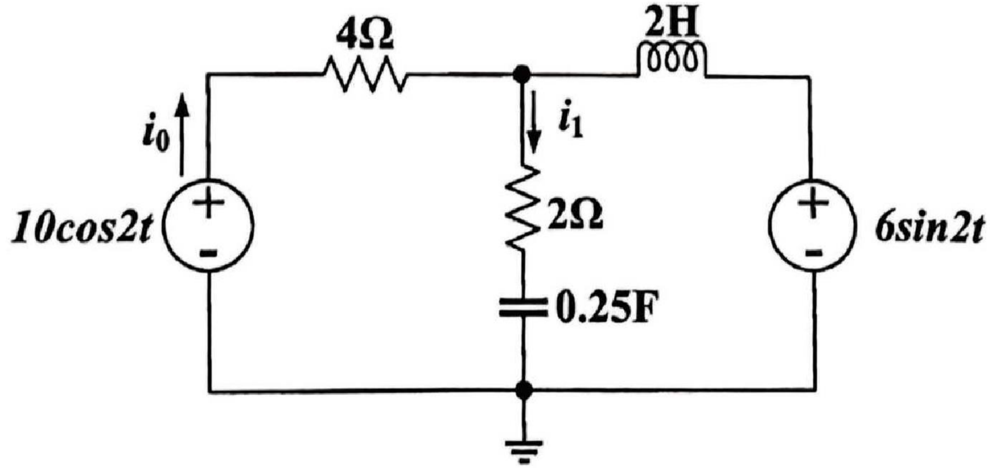


Fig.2

3. (16 points). As shown in Fig.3, the balanced three-phase circuit holds positive a→b→c sequence. Calculate I_a, I_b, I_c and the total complex power delivered to the whole load.

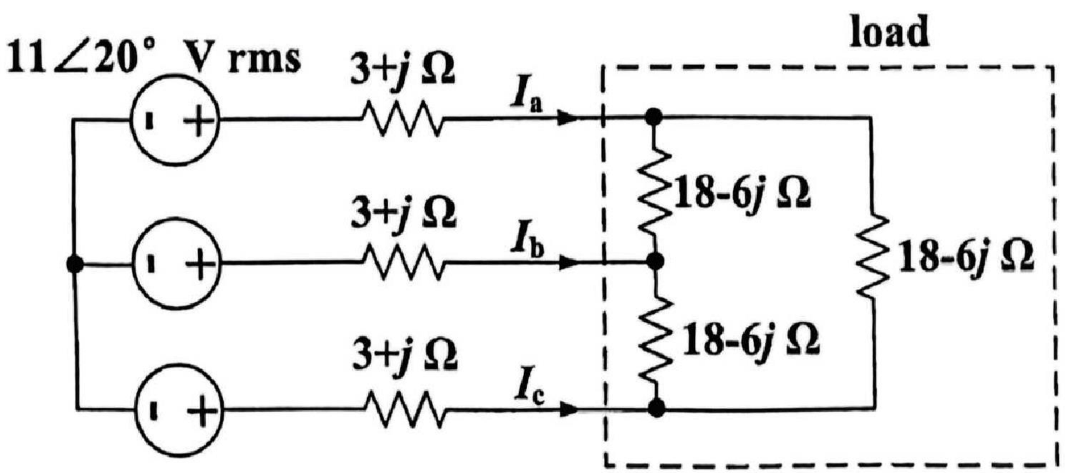


Fig.3