EE150 Signal and System
Homework 4

Due on 23: 59, April 30, 2024.

Note:

- Please provide enough calculation process to get full marks.
- Please submit your homework to Blackboard in PDF version.
- It's highly recommended to write every exercise on a single sheet of page.
- Late submissions will have points deducted according to the penalty policy.
- Please use English only to complete the assignment, solutions in Chinese are not allowed.
- Plagiarizer will get zero points.
- The full score of this assignment is 100 points.

Exercise 1. (20pt)

Find the continuous-time Fourier transform representations for the following signals:

(a) $x(t) = 2\cos(\pi t) + \sin(2\pi t)$

(b) $x(t) = \begin{cases} 1 + \cos(\pi t), & |t| \leq 1 \\ 0, & |t| > 1 \end{cases}$

(c) $x(t) = \sum_{k=0}^{\infty} a^k \delta(t - kT), \quad |a| < 1$

(d) $x(t) = [te^{-2t}\sin(4t)]u(t)$

(e) $x(t) = \begin{cases} 1 - t^2, & 0 < t < 1 \\ 0, & \text{otherwise} \end{cases}$

(f) $x(t)$ as shown in Figure 1.

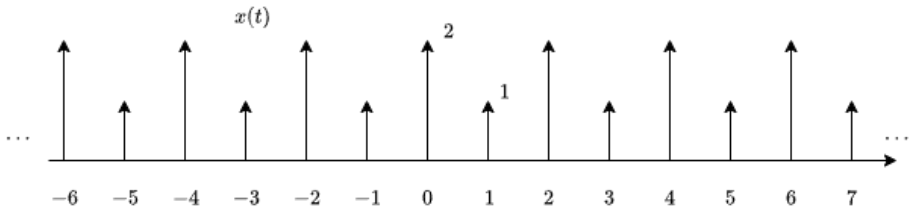

Figure 1: The graph of signal $x(t)$ in (f)

Exercise 2. (20pt)

Consider the signal

$$x_0(t) = \begin{cases} e^{-t}, & 0 \le t \le 1 \\ 0, & \text{otherwise} \end{cases}$$

Determine the Fourier transform of each of the signals shown in Figure 2. You should be able to do this by explicitly evaluating only the transform of $x_0(t)$ and then using properties of the Fourier transform.

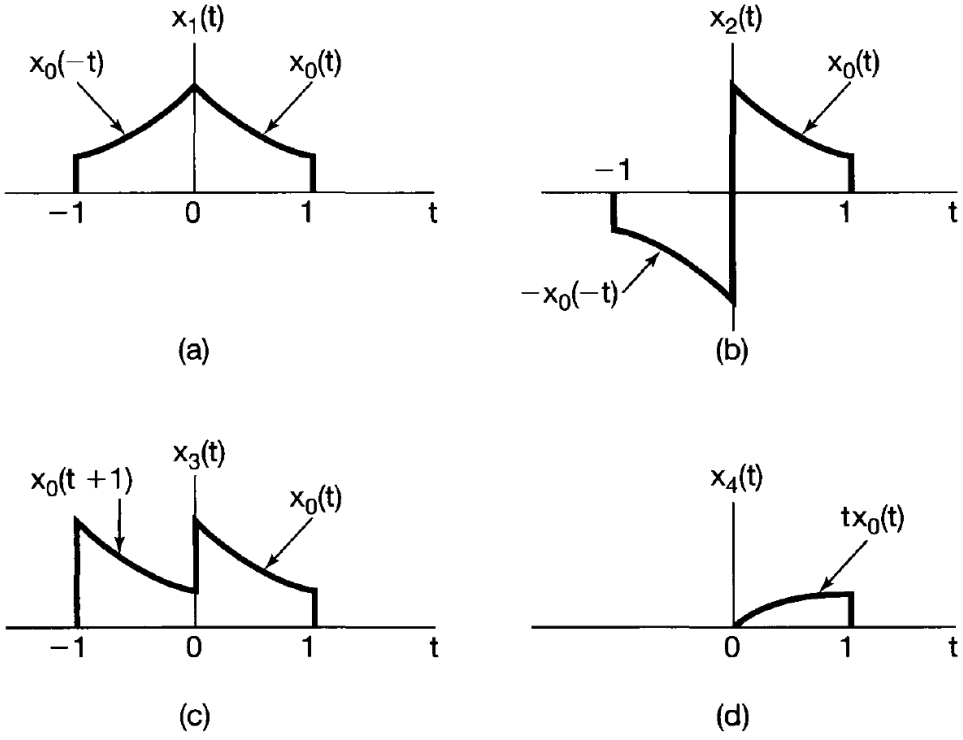

Figure 2

Exercise 3. (20pt)

An LTI system has the impulse response $h(t) = 2\frac{\sin(2\pi t)}{\pi t}\cos(7\pi t)$.
Use the FT to determine the system output for the following inputs, x(t).

(a) $x(t) = \cos(2\pi t) + \sin(6\pi t)$

(b) $x(t) = \sum_{m=-\infty}^{\infty}(-1)^m \delta(t-m)$

(c) $x(t)$ as depicted in Figure 3

(d) $x(t)$ as depicted in Figure 4

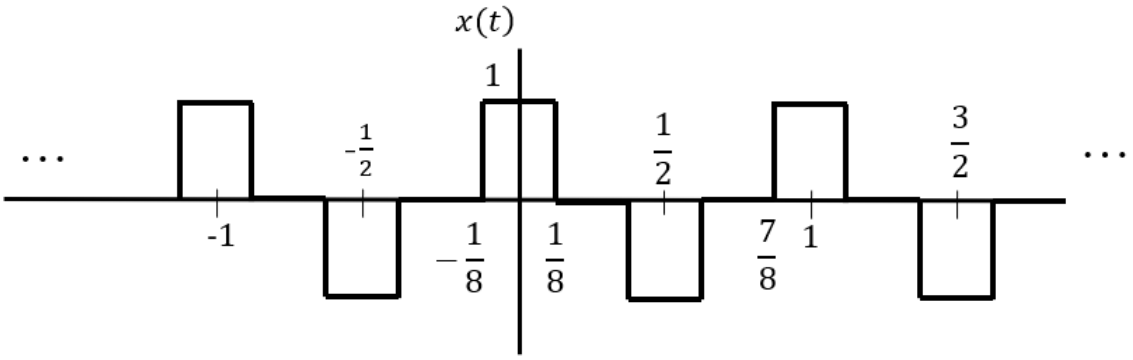

Figure 3

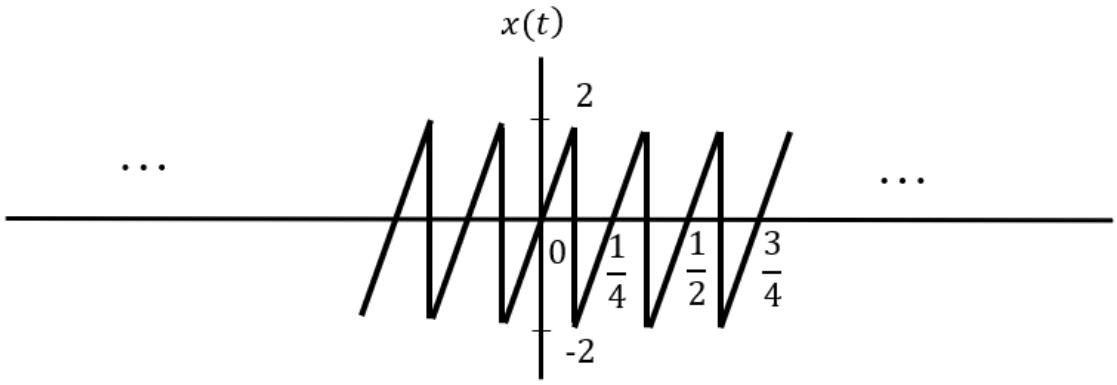

Figure 4

Exercise 4. (20pt)

The input and the output of a stable and casual LTI system are related by the differential equation

$$\frac{d^2y(t)}{dt^2} + 6\frac{dy(t)}{dt} + 8y(t) = 2x(t)$$

(a) Find the impulse response of this system.

(b) What is the response of this system if $x(t) = te^{-2t}u(t)$.

(c) Repeat part (a) for the stable and causal LTI system described by the equation

$$\frac{d^2y(t)}{dt^2} + \sqrt{2}\frac{dy(t)}{dt} + y(t) = 2\frac{d^2x(t)}{dt^2} - 2x(t)$$

Exercise 5. (20pt)

A casual and stable LTI system S has the frequency response:

$$H(jw) = \frac{jw + 4}{6 - w^2 + 5jw}$$

(a) Determine a differential equation relating the input $x(t)$ and output $y(t)$ of S.

(b) Determine the impulse response $h(t)$ of S.

(c) What is the output of S when the input is:

$$x(t) = e^{-4t}u(t) - te^{-4t}u(t)$$