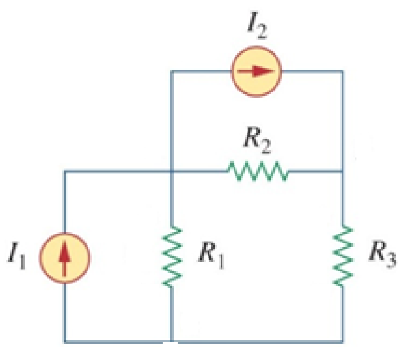
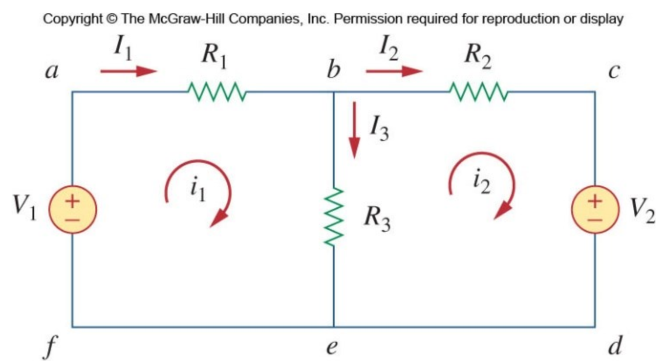
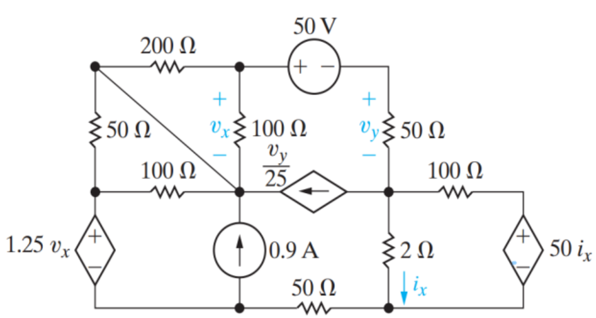
Lecture 5
- RC/RL First-Order Circuits

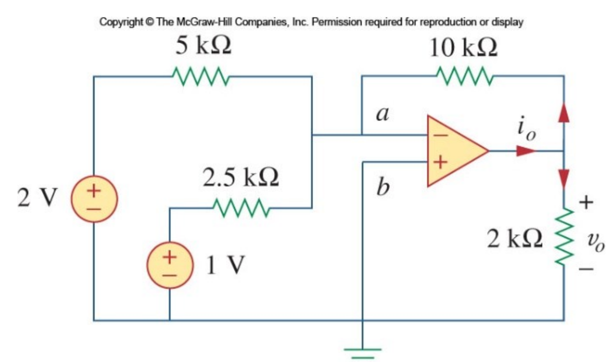
Beginning of Temporal Behavior Analysis of Circuits

- Till now we discussed <u>static</u> analysis of a circuit
  - Responses at a given time depend only on inputs at that time.
  - Circuit responds to input changes infinitely fast.







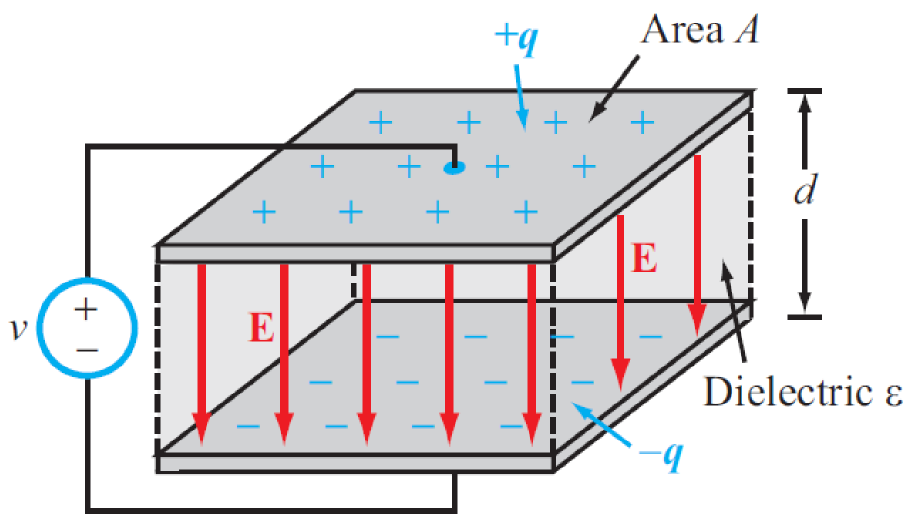


Outline

- Capacitors and inductors
- Natural response of RC/RL circuits
- Step response of RC/RL circuits
- Others

Capacitors

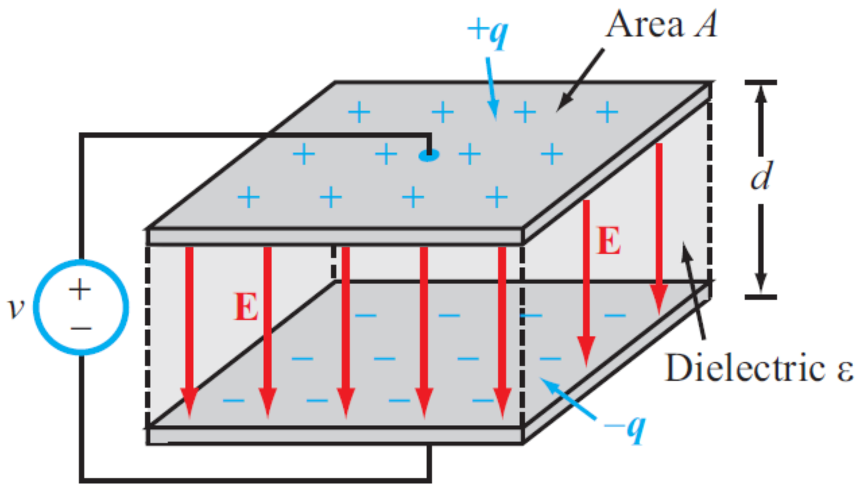
Storage element that stores energy in electric field

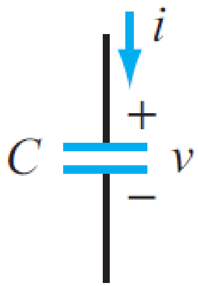


Parallel plate capacitor

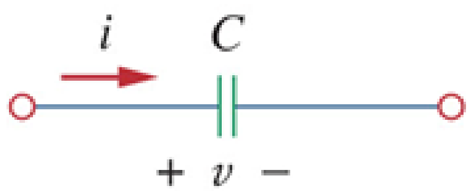
[Source: Berkeley]

V-I Relationship of Capacitors





Stored Energy

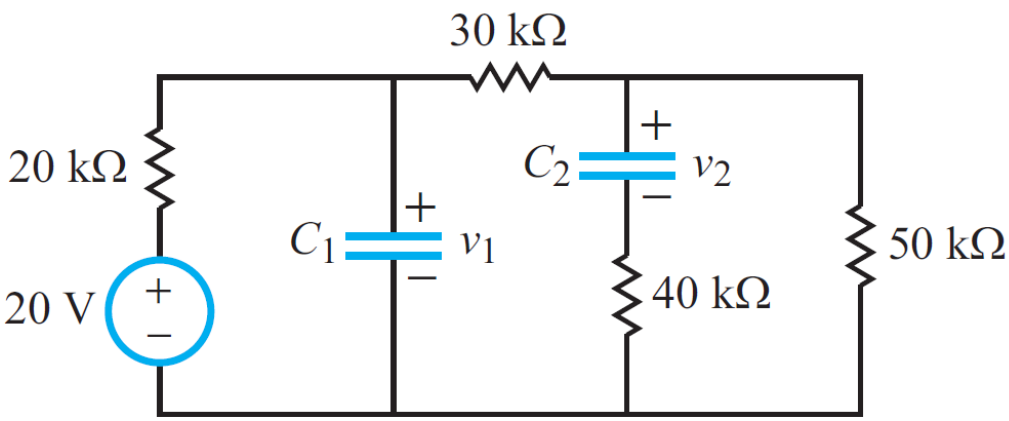


$$i = C\frac{dv}{dt}$$

- The instantaneous power delivered to the capacitor is

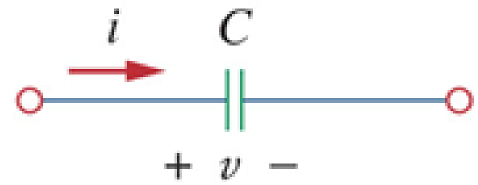
- The energy stored in a capacitor is:

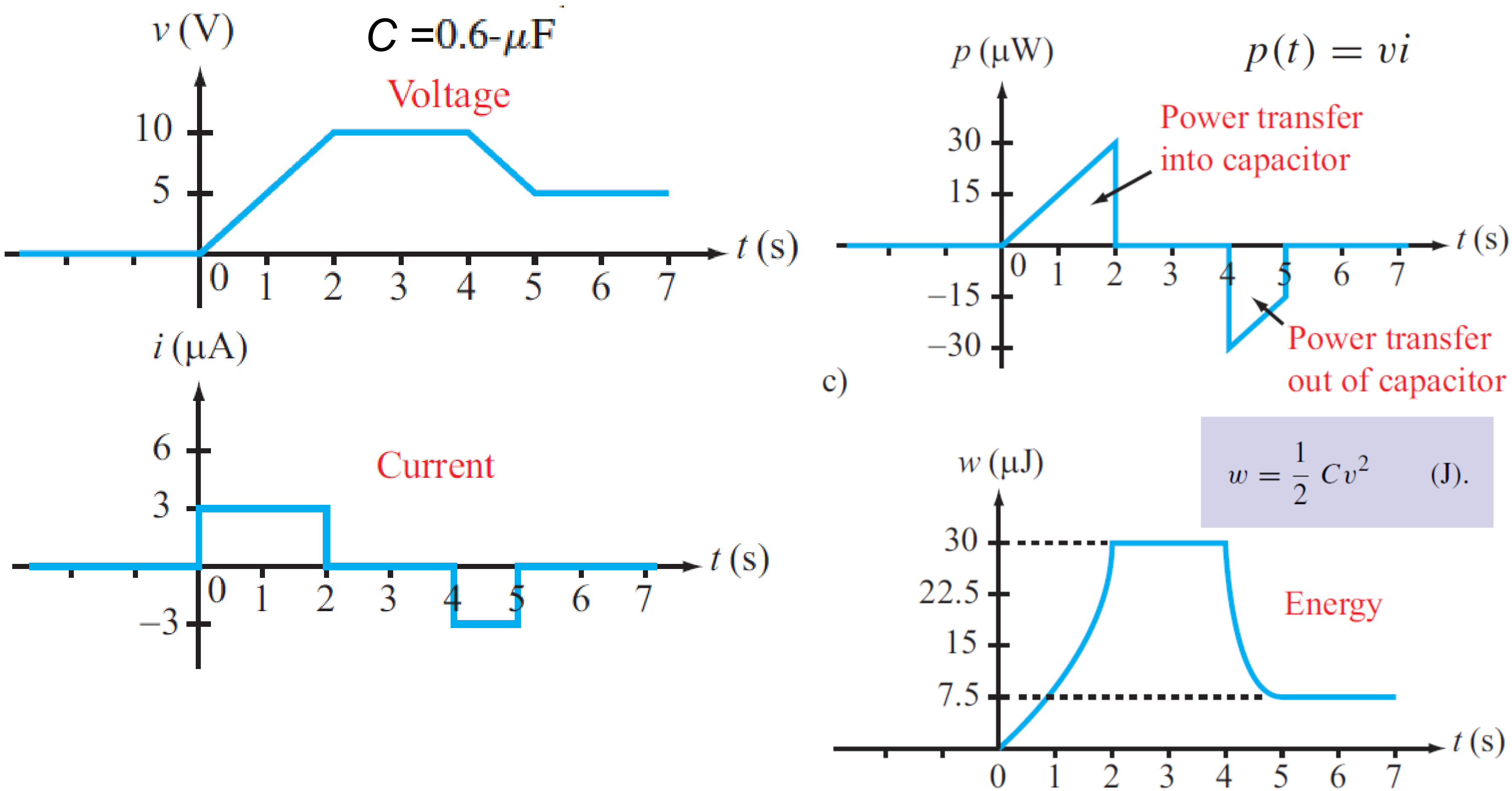
Example-1



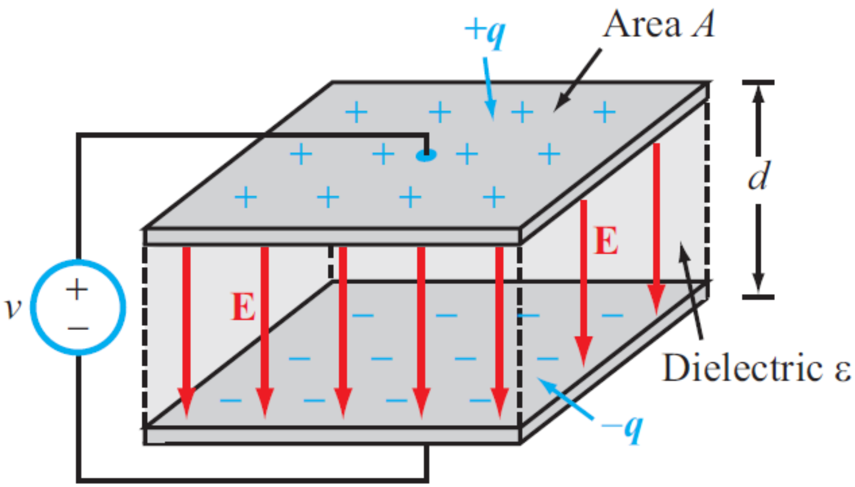
[Source: Berkeley]

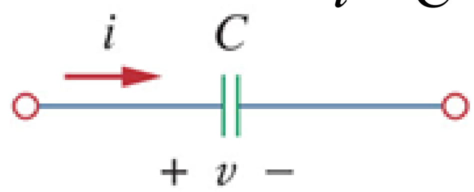
Example-2 Capacitor Response



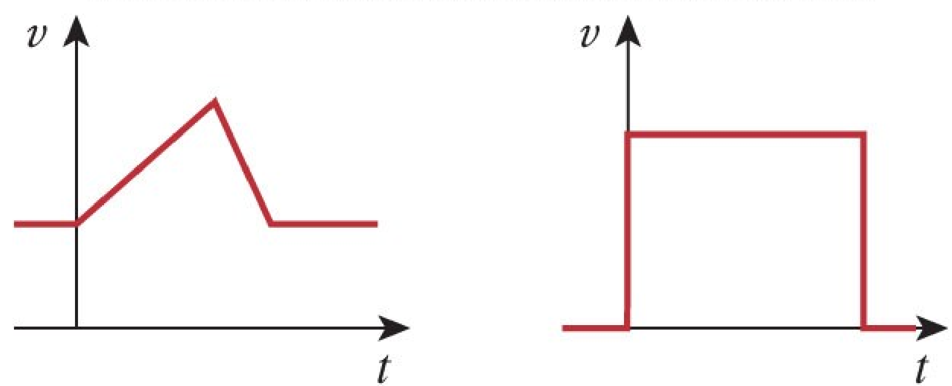


Important Property of Capacitors





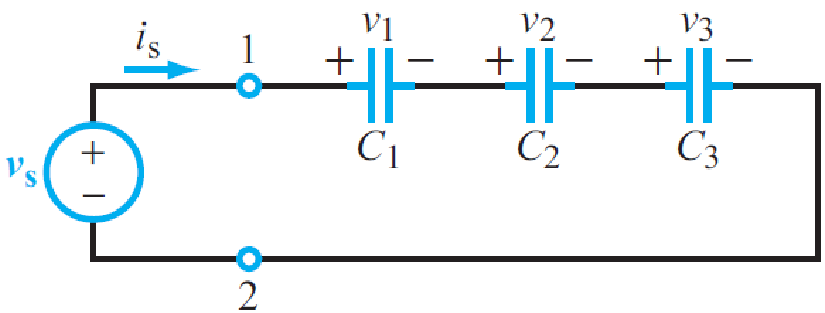
$$i = C\frac{dv}{dt}$$

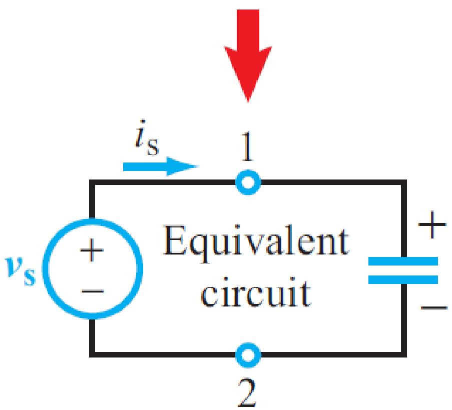


(a) (b)

Capacitors in Series

Combining In-Series Capacitors

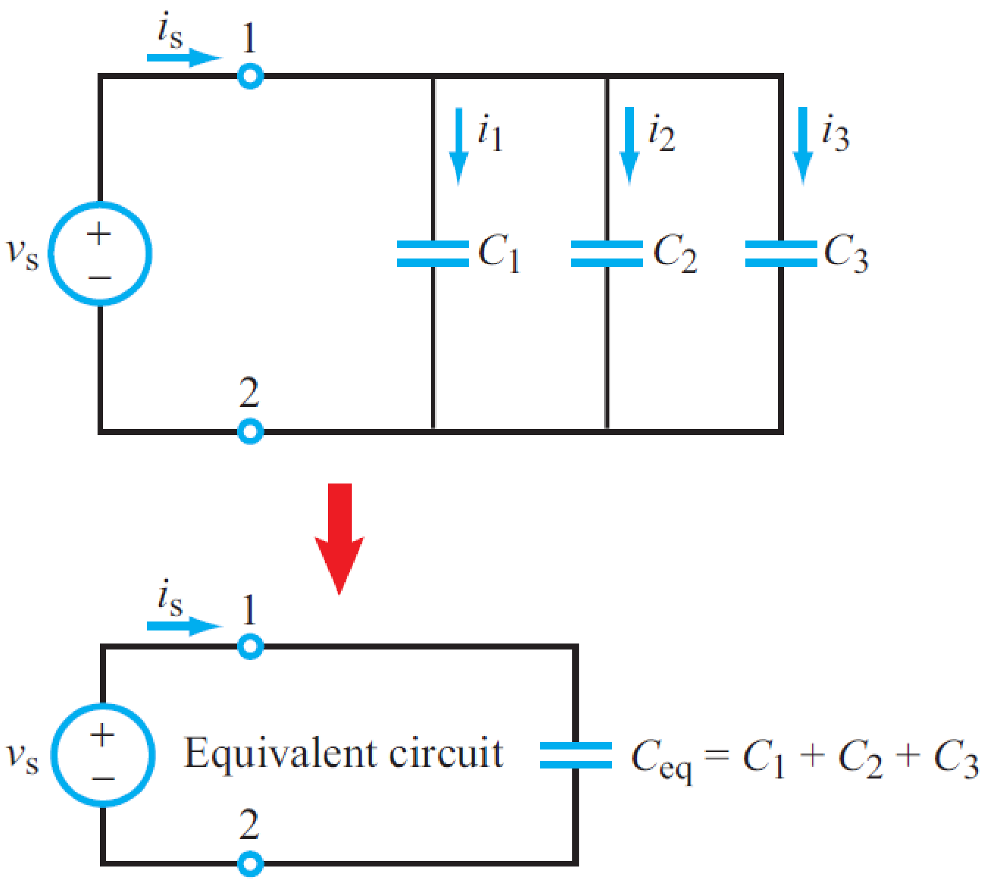




$$C_{eq} = \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)^{-1}$$

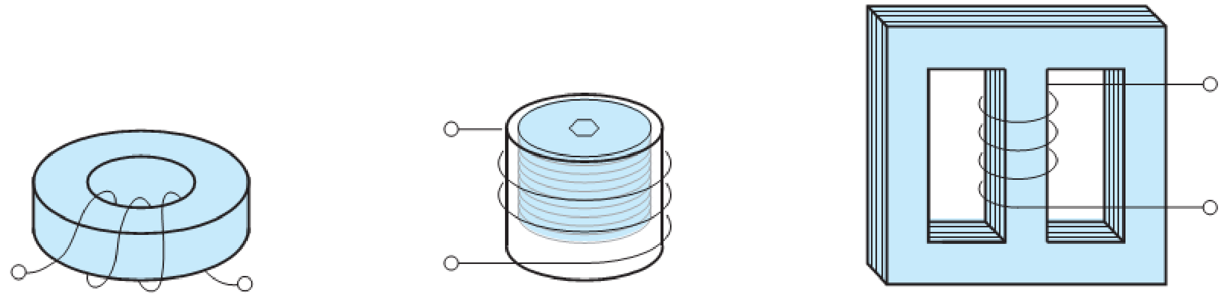
[Source: Berkeley]

Capacitors in Parallel



Inductors

- A storage element that stores energy in magnetic field.
  - They have applications in power supplies, transformers, radios, TVs, radars, and electric motors.
- Any conductor has inductance, but the effect is typically enhanced by coiling the wire up.

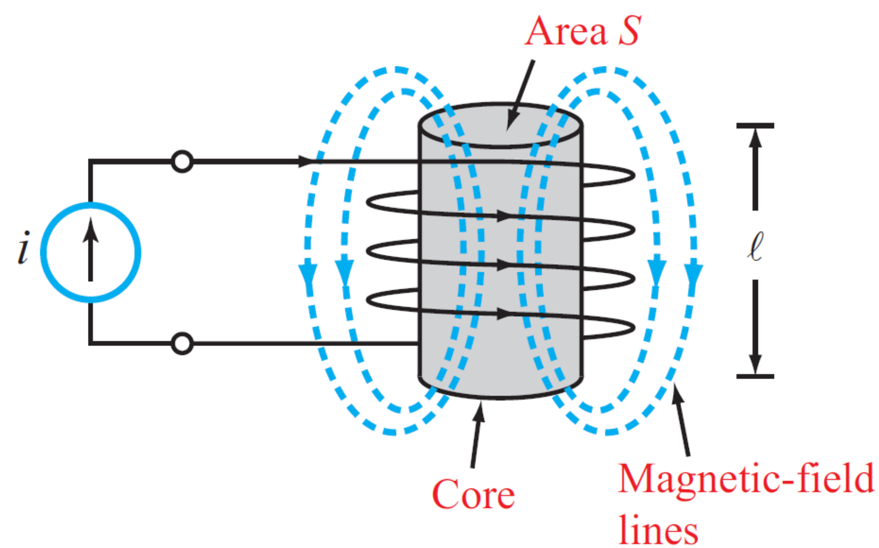


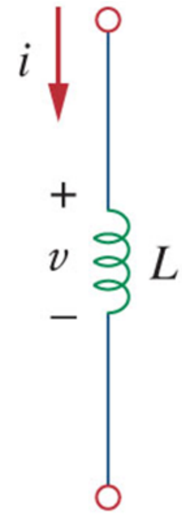
(a) Toroidal inductor

(b) Coil with an iron-oxide slug that can be screwed in or out to adjust the inductance

(c) Inductor with a laminated iron core

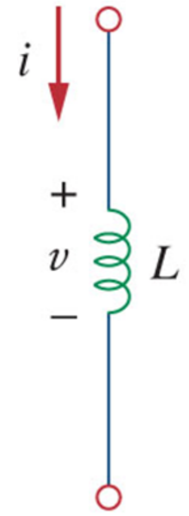
V-I Relationship of Inductors





$$v = L\frac{di}{dt}$$

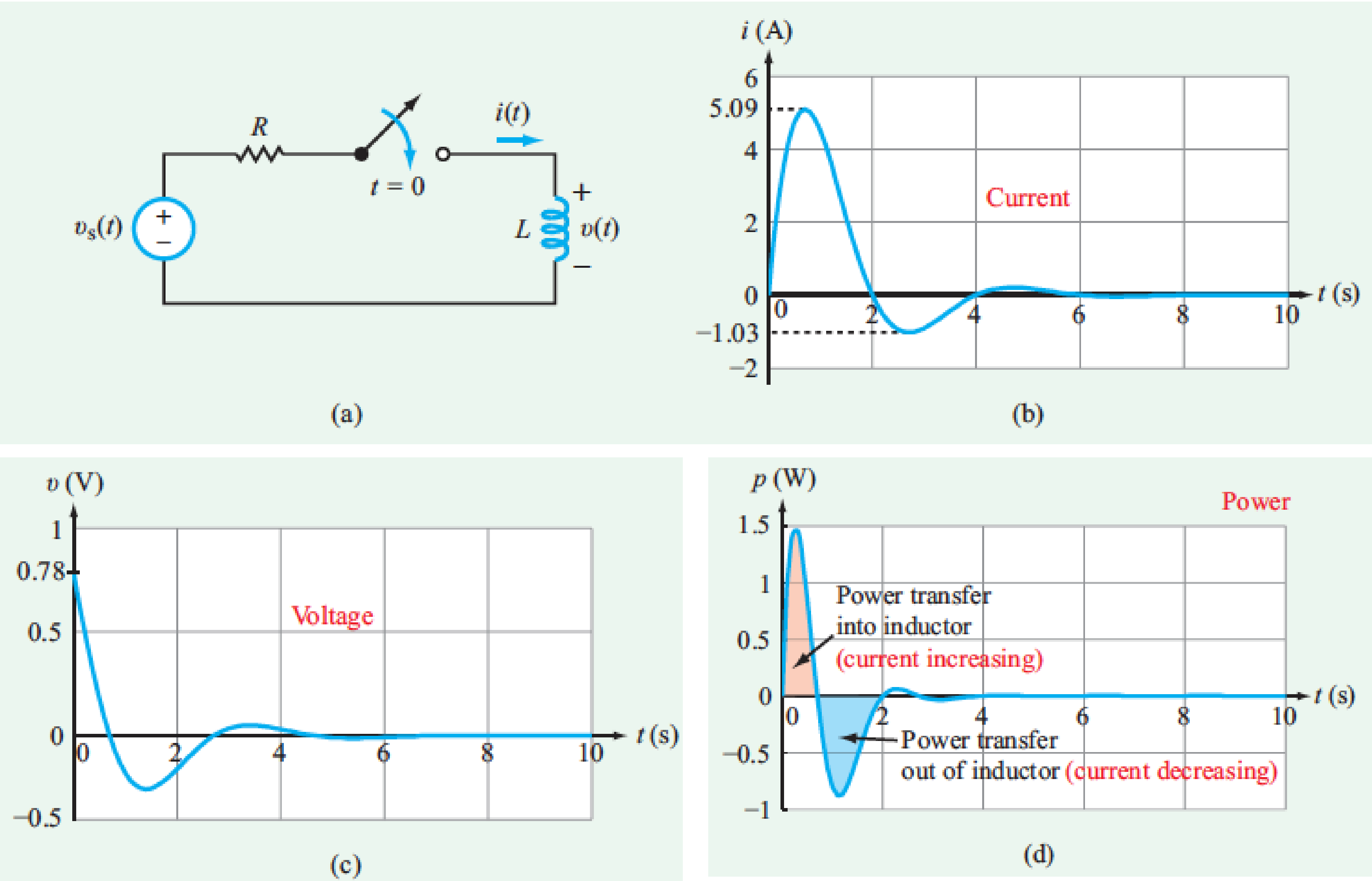
Energy Stored in an Inductor



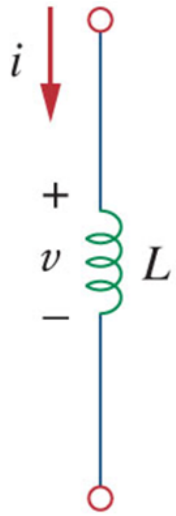
$$v = L\frac{di}{dt}$$

- The power delivered to the inductor is:
- The energy stored is:

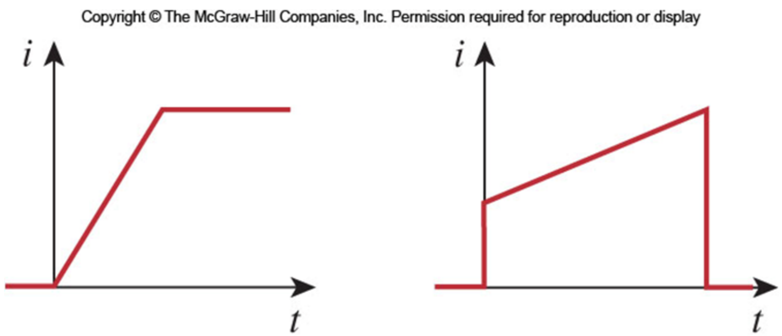
Inductor Response



Important Property of Inductors

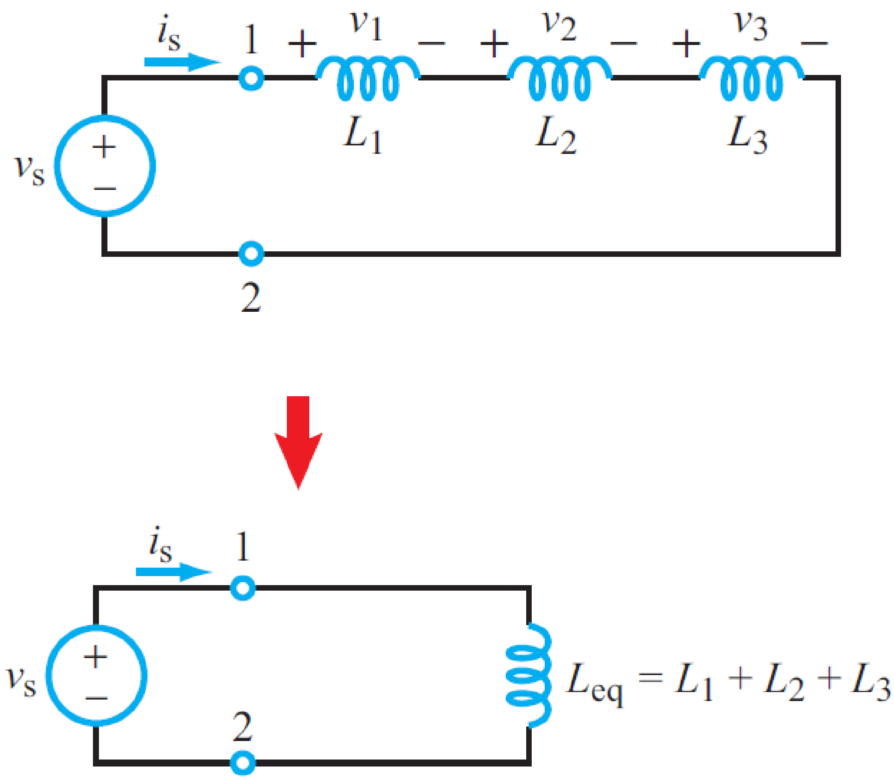


$$v = L\frac{di}{dt}$$

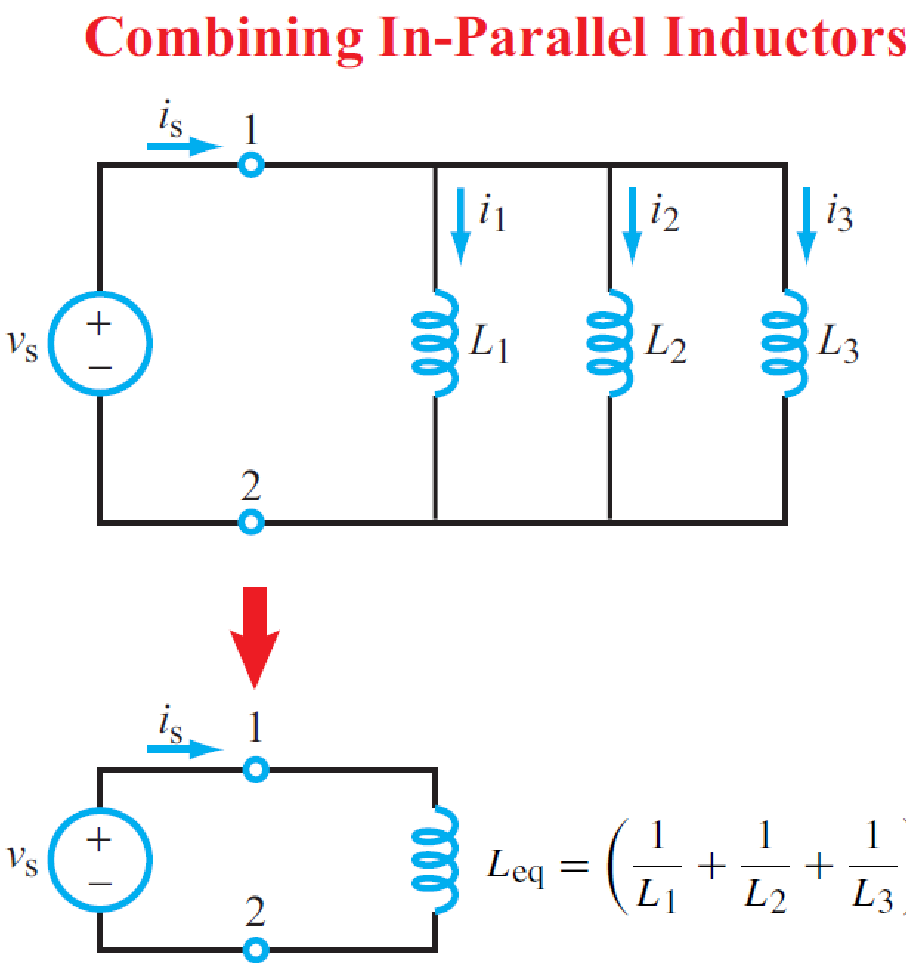


[Source: Berkeley]

Inductors in Series



Inductors in Parallel



$$L_{eq} = \left(\frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}\right)^{-1}$$

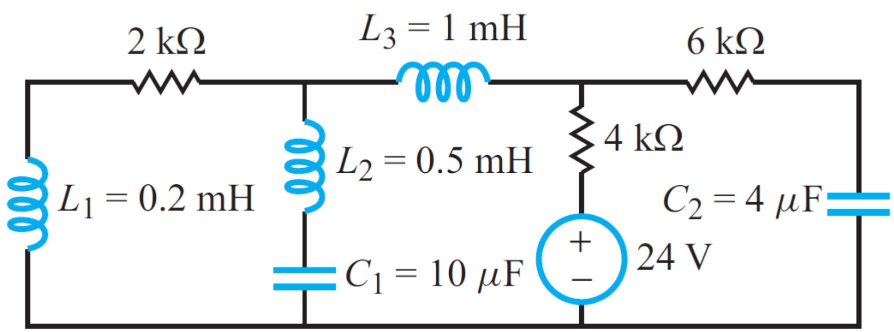
Summary of Resistors, Capacitors and Inductors

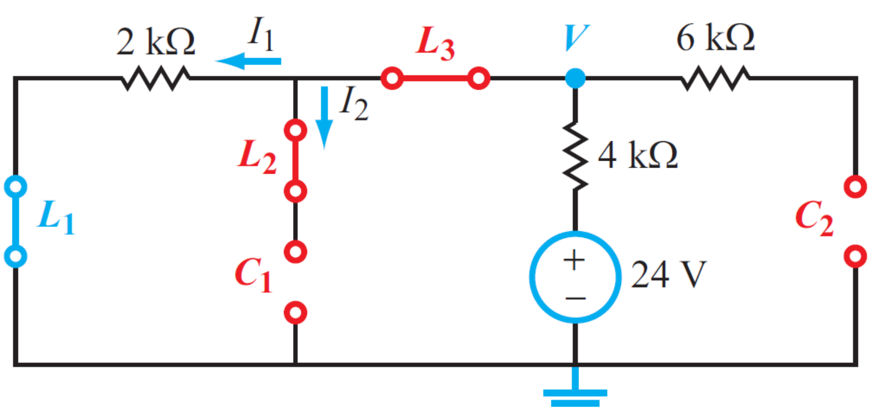
Table 5-4: Basic properties of R, L, and C.

Property	R	L	C
i–υ relation	$i = \frac{\upsilon}{R}$	$i = \frac{1}{L}\int_{t_0}^{t} \upsilon\, dt' + i(t_0)$	$i = C\frac{d\upsilon}{dt}$
υ-i relation	$\upsilon = iR$	$\upsilon = L\frac{di}{dt}$	$\upsilon = \frac{1}{C}\int_{t_0}^{t} i\, dt' + \upsilon(t_0)$
p (power transfer in)	$p = i^2R$	$p = Li\frac{di}{dt}$	$p = C\upsilon\frac{d\upsilon}{dt}$
w (stored energy)	0	$w = \frac{1}{2}Li^2$	$w = \frac{1}{2}C\upsilon^2$
Series combination	$R_{eq} = R_1 + R_2$	$L_{eq} = L_1 + L_2$	$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$
Parallel combination	$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$	$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2}$	$C_{eq} = C_1 + C_2$
dc behavior	no change	short circuit	open circuit
Can υ change instantaneously?	yes	yes	no
Can i change instantaneously?	yes	no	yes

[Source: Berkeley]

Example


2 kΩ
$L_3 = 1$ mH
6 kΩ
4 kΩ
$L_2 = 0.5$ mH
$L_1 = 0.2$ mH
$C_2 = 4\ \mu$F
+
−
24 V
$C_1 = 10\ \mu$F


2 kΩ
I_1
L_3
V
6 kΩ
I_2
L_2
4 kΩ
L_1
C_2
C_1
+
−
24 V