Homework 6

Due date: Dec. 14th, 2023

Turn in your hard-copy hand-writing homework in class

Rules:

- Work on your own. Discussion is permissible, but extremely similar submissions will be judged as plagiarism.
- Please show all intermediate steps: a correct solution without an explanation will get zero credit.
- Please submit on time. No late submission will be accepted.
- Please prepare your submission in English only. No Chinese submission will be accepted.

1. Given that the voltage and current of two-terminal elements adopt the passive sign convention, the instantaneous values are expressed as:

(1) $\boldsymbol{v(t)} = 15\cos(400t + 30°)\ V$, $\boldsymbol{i(t)} = 3\sin(400t + 30°)\ A$;

(2) $\boldsymbol{v(t)} = 8\sin(500t + 50°)\ V$, $\boldsymbol{i(t)} = 2\sin(500t + 140°)\ A$;

(3) $\boldsymbol{v(t)} = 8\cos(250t + 60°)\ V$, $\boldsymbol{i(t)} = 5\sin(250t + 150°)\ A$;

(a) Transform the three voltage & current pairs into phasors.

(b) Try to determine whether the element is a resistor, inductor or capacitor, and determine its value (R=?,C=?, L=?) for **(1), (2), and (3), respectively**.

1.

(a) (1) $\dot{V} = 15\angle 30°$, $\dot{I} = 3\angle -60°$, at $\omega = 400$ rad/s 2'

(2) $\dot{V} = 8\angle -40°$, $\dot{I} = 2\angle 50°$, at $\omega = 500$ rad/s 2'

(3) $\dot{V} = 8\angle 60°$, $\dot{I} = 5\angle 60°$, at $\omega = 250$ rad/s 2'

(b) ① $Z_{(1)} = \frac{\dot{V}}{\dot{I}} = 5\angle 90°$ (inductor)

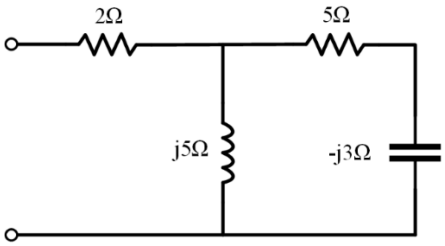
$j\omega L = 5j \Rightarrow L = \frac{5}{400}\ H = 0.0125\ H$ 3'

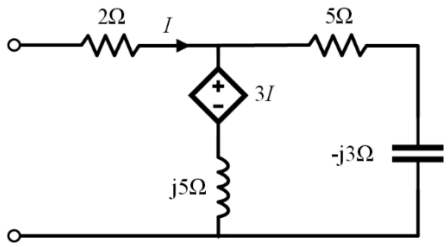
② $Z_{(2)} = \frac{\dot{V}}{\dot{I}} = \frac{8\angle -40°}{2\angle 50°} = 4\angle -90°$ (capacitor)

$\frac{-j}{\omega L} = -4j \Rightarrow C = \frac{1}{4\times 500}\ F = 0.5\ mF$ 3'

③ $Z_{(3)} = \frac{\dot{V}}{\dot{I}} = \frac{8\angle 60°}{5\angle 60°} = 1.6\ \Omega$ (resistor) 3'

2. Finding the equivalent impedance and admittance of these **two** circuits.





2. ① $Req = 2 + j5 // (5 - j3)$

$= 2 + \frac{j25 + 15}{5 + j2}$

$= 2 + \frac{j125 + 75 + 50 - j30}{29}$

$= \frac{125}{29} + 2 + j\frac{95}{29} = 6.31 + j3.28 = 7.11\angle 27.43°\,\Omega$ 5

② $\begin{cases} \frac{i(5-j3) - 3I}{j5} + i = I \\ V = 2I + i(5-j3) \end{cases}$ $\Rightarrow Req = \frac{V}{I} = 8.28 + j0.69 = 8.31\angle 4.76°\,\Omega$

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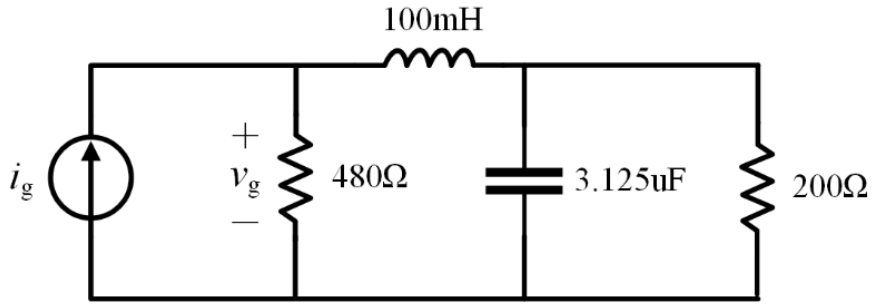
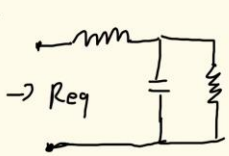
3. **ig = 60cos(ωt)** mA, if ig is in phase with vg, calculate the frequency of the current source and find the expression for **vg**



3.



$Req = j0.1w + 200 \,//\, -j\frac{1}{w}\cdot 32\times10^4$ 2

$= j0.1w + \dfrac{-j\frac{1}{w}\cdot 64\times10^6}{200 - j\frac{1}{w}32\times10^4}$

$= j0.1w + \dfrac{-j\frac{1}{w}32\times10^4 + \frac{1}{w^2}512\times10^6}{1+\frac{1}{w^2}16^2\times10^4}$

$= j\left(0.1w - \dfrac{\frac{1}{w}32\times10^4}{1+\frac{1}{w^2}16^2\times10^4}\right) + \dfrac{512\times10^6}{w^2+256\times10^4}$ 4

$\therefore 0.1w = \dfrac{\frac{1}{w}32\times10^4}{1+\frac{1}{w^2}16^2\times10^4} \Rightarrow w = 800$ 4

$Req = 160\Omega$ 2

$Vg = 2g \times (480 \,//\, 160) = 7.2\cos(800t)V$ 3

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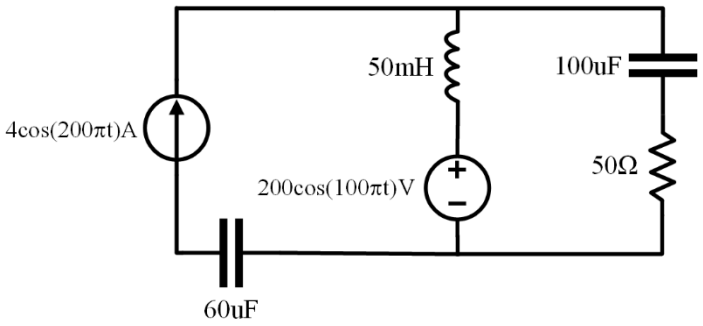
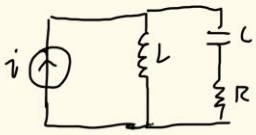
4. Finding the Time Domain Voltage Expression for Resistance.



4.

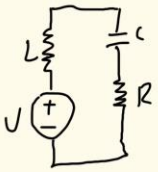


$\dot{V}_R = \dot{I} \cdot \frac{j\omega L}{j\omega L - j\frac{1}{\omega C} + R}$ 2, $\omega = 200\pi$, $I = 4\angle 0$ 2

$V_R = \dot{V}_R \cdot R = \dot{I} \cdot \frac{j\omega LR}{j\omega L - j\frac{1}{\omega C} + R}$ 2

$= 4 \cdot \frac{j500\pi}{j10\pi - j\frac{50}{\pi} + 50} = 120.03\angle 72.78°$ 2

$\therefore V_R = 120.03\cos(200\pi t + 72.78)$ 2



$V_R' = V \cdot \frac{R}{j\omega L - j\frac{1}{\omega C} + R}$ 4, $\omega = 100\pi$, $V = 200\angle 0$ 2

$\therefore V_R' = \frac{200 \times 50}{j5\pi - j\frac{100}{\pi} + 50} = 190.35\angle 17.87°$ 2

$\therefore V_{R\,total} = V_R + V_R' = 120.03\cos(200\pi t + 72.78)$ 2
$+ 190.35\cos(100\pi t + 17.87)$

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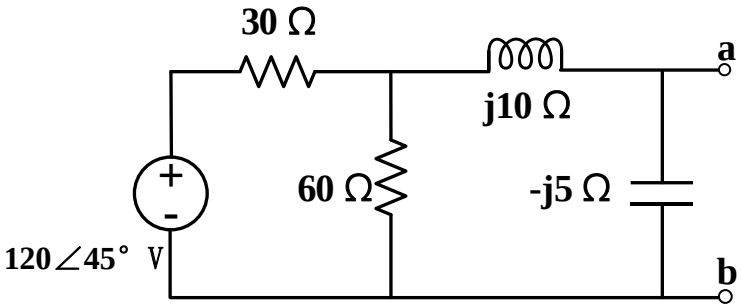
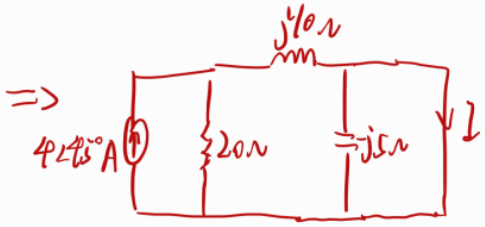
5. For the circuit below, please find the Thevenin equivalent circuit with respect to node **a** and node **b**.

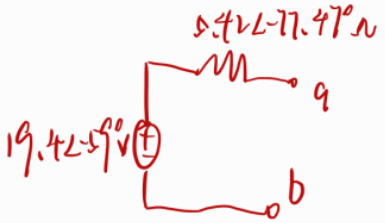


5. $Z_{TH} = -j5\Omega \,//\, (j10\Omega + 30\Omega // 60\Omega) = 5.42\angle -77.47°\,\Omega$ 4'



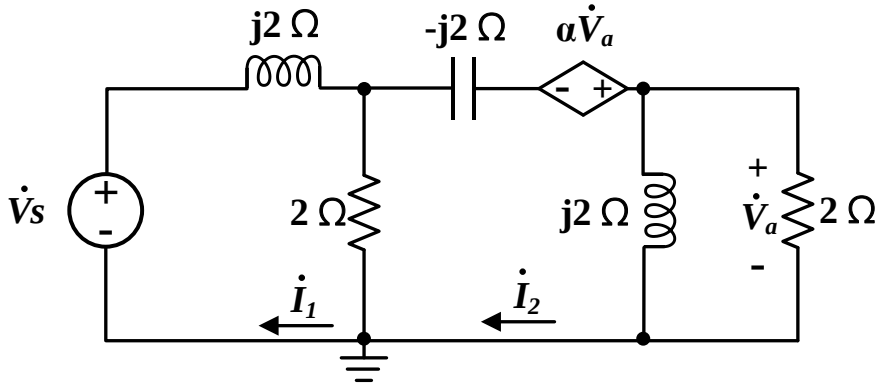
$I = \frac{20}{20+j10}(4\angle 45°) = 3.578\angle 18.43°\,A$ 4'

$V_{th} = Z_{th} I = 19.4\angle -59°\,V$ 4'

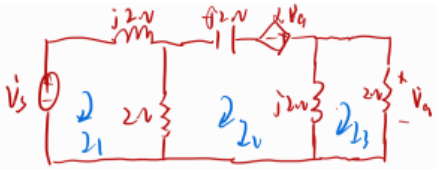


5'

6. Use **nodal AND mesh** analyze method to find $\dot{V}_a$, $\dot{I}_1$ and $\dot{I}_2$ for the circuit below, assuming that $\dot{V}_S = 10\angle 0°\ V$, $\alpha = 0.5$.



6.



Mesh:

$$\begin{cases} j2\cdot \dot{I}_1 + (\dot{I}_1 - \dot{I}_2)\cdot 2 = \dot{V}_S \\ (\dot{I}_2 - \dot{I}_1)\cdot 2 - j2\dot{I}_2 - \alpha\dot{V}_a + j2(\dot{I}_2 - \dot{I}_3) = 0 \\ j2(\dot{I}_3 - \dot{I}_2) + \dot{I}_3\cdot 2 = 0 \\ \dot{V}_a = 2\cdot \dot{I}_3 \end{cases} \quad 12'$$

$$\Rightarrow \begin{cases} \dot{I}_1 = 6.25\angle -5.53°\ A \\ \dot{I}_2 = 4.47\angle -6.57°\ A \\ \dot{I}_3 = 3.16\angle 18.43°\ A \end{cases} \quad 6'$$

$$\dot{V}_a = 2\dot{I}_3 = 6.32\angle 18.43°\ V \quad 2'$$

Nodal:

$$\begin{cases} \dfrac{\dot{V}_S - \dot{V}_1}{j2} + \dfrac{0 - \dot{V}_1}{2} + \dfrac{\dot{V}_2 - \alpha\dot{V}_a - \dot{V}_1}{-j2} = 0 \\ \dfrac{\dot{V}_2 - \alpha\dot{V}_a - \dot{V}_1}{-j2} + \dfrac{\dot{V}_2}{j2} + \dfrac{\dot{V}_2}{2} = 0 \\ \dot{V}_2 = \dot{V}_a \end{cases} \quad \ldots$$