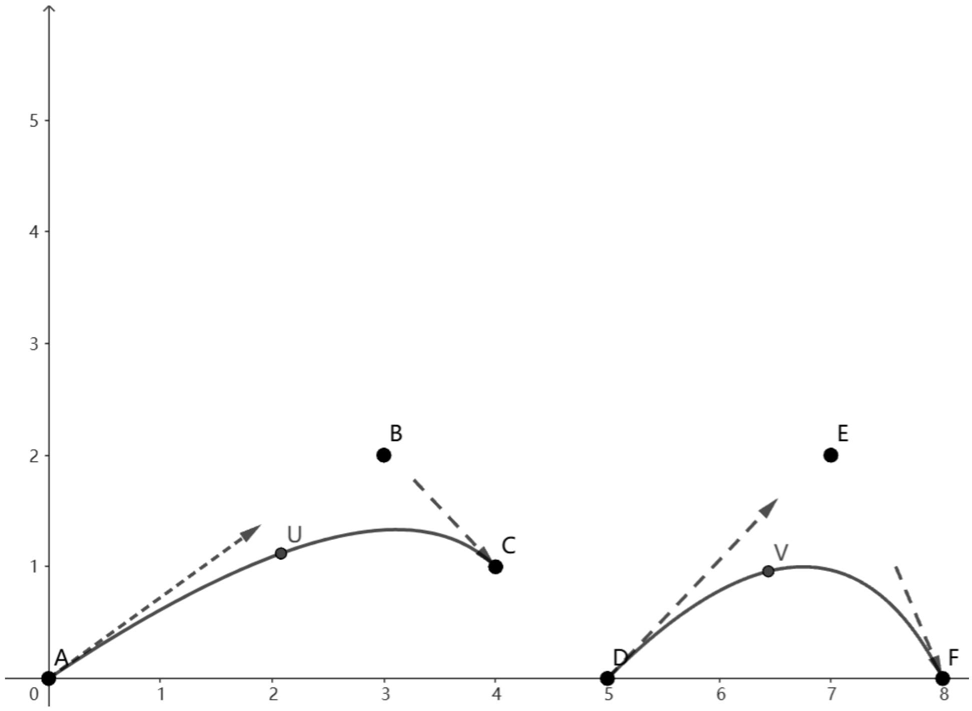
1. Consider two *Bézier curves* with degree $n = 2$ (**three control points**). The curve $U(t)$ is controlled by points A(0,0), B(3,2), C(4,1), and the other curve $V(t)$ is controlled by points D(5,0),E(7,2),F(8,0).

   As you have learned in class, the evaluation of Bézier curve is performed iteratively by the *de Casteljau's algorithm*. We perform the iterative calculation $\mathbf{c}_i^{(n+1)} \leftarrow (1-t)\cdot\mathbf{c}_i^{(n)} + t\cdot\mathbf{c}_i^{(n)}$ where $t \in [0,1]$ is the parameter.



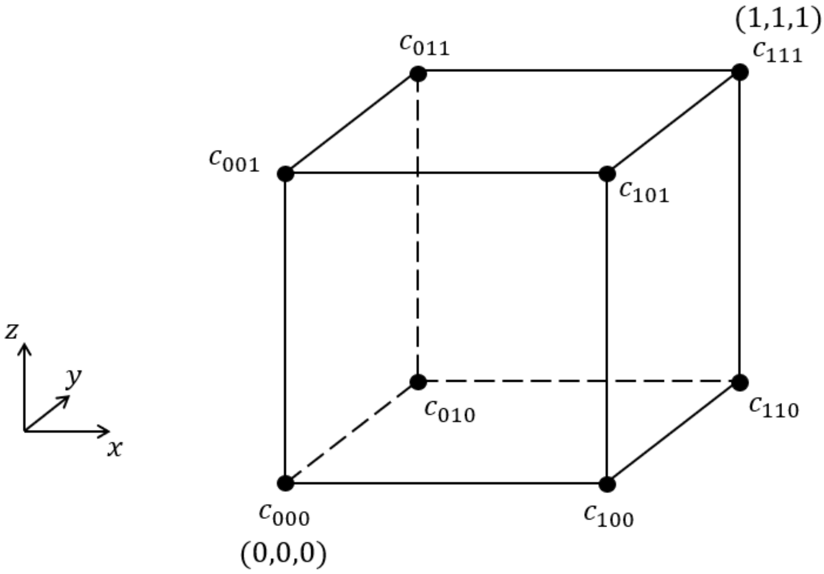
   Now we want to stitch two *Bézier curves* by only applying linear transformations (translation, rotation and scaling) to the control points of $V(t)$ such that the new Bézier curve W(t) satisfy that point D is connected to point C and the curve is not only zero-order but also first-order continuous.
   Please show the detailed linear transformations and explain why the new curve is both zero-order and first-order continuous.
   In order to maintain zero-order and first-order continuous ,we need that
   U(1)=V(0) and U'(1)=V'(0)
   $U(t) = (1-t)^2\mathbf{A} + 2(1-t)t\mathbf{B} + t^2\mathbf{C}$,so $U'(1) = 2\vec{BC}$,$V'(0) = 2\vec{DE}$
   So to connect two curves,

   (a) translate the control points by $\vec{DC}$=(-1,1)
   (b) rotate 90 degrees clockwise about C
   (c) scale the $\vec{DE}$ by 0.5

2. Consider the process of extracting a mesh from an axis-aligned 3D uniform grid to represent the corresponding isosurface using the Marching cubes algorithm. As shown in the figure below, we focus on a single cell/cube. Its 8 vertices/corners are labeled as c_{000}, c_{001}, c_{010}, ..., c_{111}, according to their x, y and z index. We assume the position of c_{000} is $(0, 0, 0)$ and the position of c_{111} is $(1, 1, 1)$.

   Now suppose the values on points c_{100}, c_{010}, c_{110} and c_{001} are 1, and the values on the rest vertices/corners are -1, and we are extracting an iso-surface with an iso-value of 0.5.

   Determine how many triangles will be extracted from this cell/cube, and write down the positions of the three vertices of each triangle.



   Answer: 4 triangles in total.

   1: (0, 0, 0.75), (0.25, 0, 1), (0, 0.25, 1)

   2: (1, 1, 0.25), (1, 0, 0.25), (0, 1, 0.25)

   3/4: two triangles forming the quadrilateral (0.75, 0, 0), (1, 0, 0.25), (0, 1, 0.25), (0, 0.75, 0). Any of the two possibilities is okay.