ShanghaiTech University

CS240 Algorithm Design and Analysis

Fall 2024

Problem Set 3

Due: 23:59, Nov. 26, 2024

1. Submit your solutions to the course Gradescope.
2. If you want to submit a handwritten version, scan it clearly.
3. Late homeworks submitted within 24 hours of the due date will be marked down 25%. Homeworks submitted more than 24 hours after the due date will not be accepted unless there is a valid reason, such as a medical or family emergency.
4. You are required to follow ShanghaiTech's academic honesty policies. You are allowed to discuss problems with other students, but you must write up your solutions by yourselves. You are not allowed to copy materials from other students or from online or published resources. Violating academic honesty can result in serious penalties.

Notice: When proving that a problem A is NP-complete, you need to strictly follow the below steps and explicitly state each part.

- Prove that A is in NP.
- Choose an NP-complete problem B. For any instance b of B, construct an instance a of problem A in polynomial time.
- Prove that b has a solution if and only if a has a solution.

Problem 1:

Triple-SAT: Let Triple-SAT denote the following decision problem: given a Boolean formula ϕ, decide whether ϕ has at least three distinct satisfying assignments. Prove that Triple-SAT is NP-complete.

Problem 2:

Given a set $F = \{f_1, f_2, \ldots, f_n\}$ and a system of subsets $\{S_1, S_2, \ldots, S_m\}$, where each $S_i \subseteq F$ satisfies $|S_i| \geq 2$. The objective is to assign each element in F one of two labels, "red" or "blue", so that for each subset S_i, there is at least one element labeled "red" and at least one element labeled "blue".

The task is to prove that determining whether such a labeling scheme exists to satisfy the above condition is an NP-Complete problem.

Problem 3:

Given a finite set of items I, where each item $i \in I$ has a positive integer size $s(i) \in Z^+$, an integer bin capacity B, and an integer K. Is there a way to partition the set I into K disjoint subsets such that the sum of the sizes of items in each subset I_1, I_2, ..., I_i is less than or equal to B? Prove this problem is NP-complete.