Lecture 14

-- Laplace Transform in Circuit Analysis

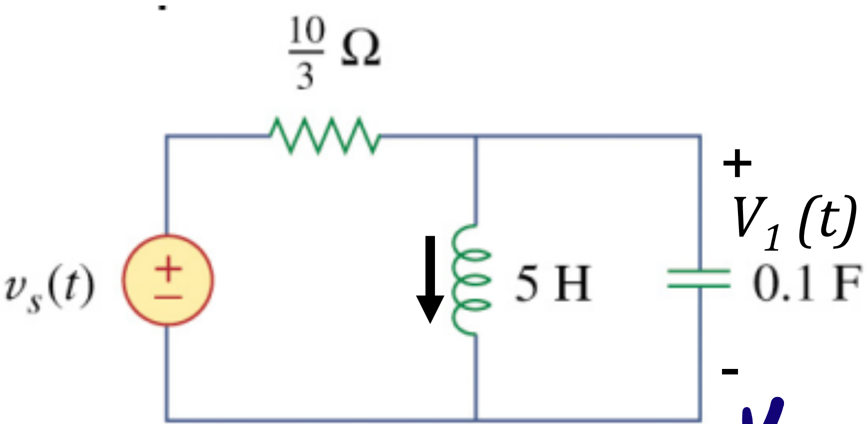
Example 2

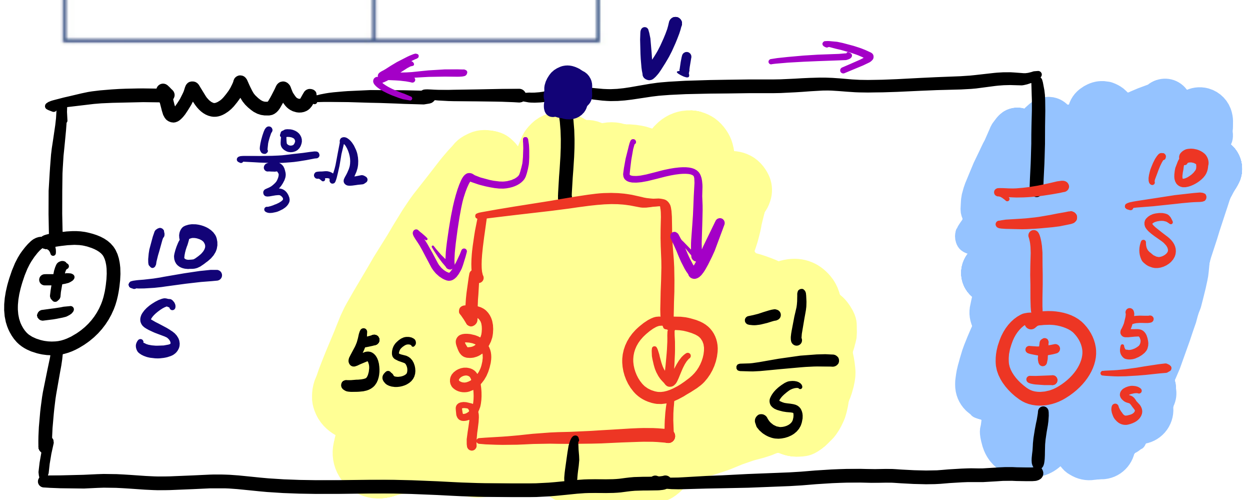
Find (1) the voltage across the capacitor ✓

(2) current through the inductor

assuming that $v_s(t) = 10u(t)$ V, and assume that at $t = 0$, *-1 A* flows through the inductor and +5 V is across the capacitor.

$-1A \cdot u(t) \rightarrow \frac{-1}{s}$





$$\frac{V_1-\frac{10}{S}}{\frac{10}{3}}+\frac{V_1-0}{5S}+\frac{-1}{S}+\frac{V_1-\frac{5}{S}}{\frac{10}{S}}=0$$

$$\Longrightarrow \boxed{V_1^{(S)}=\frac{40+5S}{(S+1)(S+2)}=\frac{k_1}{S+1}+\frac{k_2}{S+2}}$$

For k_1:

$$\frac{(40+5S)\cancel{(S+1)}}{\cancel{(S+1)}(S+2)}=\frac{k_1\cancel{(S+1)}}{\cancel{S+1}}+\frac{k_2(S+1)}{S+2}$$

Set $S=-1$ $\Longrightarrow$

$$k_1=\frac{40-5}{-1+2}=35$$

$k_2=-30.$

$$\Longrightarrow V_1=\frac{35}{S+1}-\frac{30}{S+2}$$

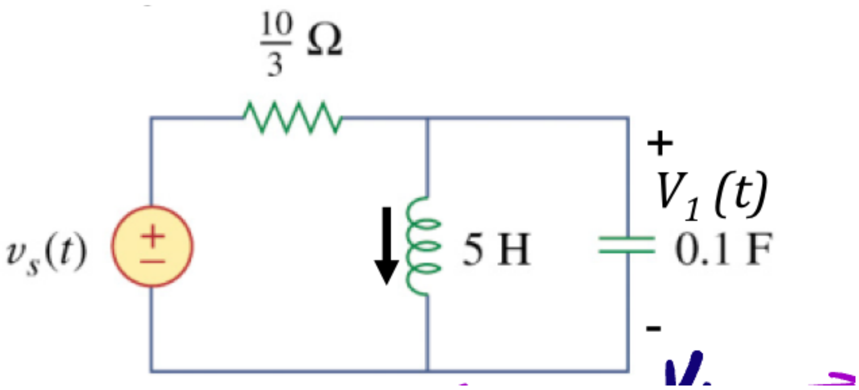
$\hookrightarrow \mathcal{L}^{-1}$ $f(t)$: $v_1(t)=(35\cdot e^{-t}-30\cdot e^{-2t})u(t)$ V

S.-domain

$$I_L(s)=\frac{V_1^{(S)}-0}{5S}+\frac{-1}{S}=\frac{40+5S}{(S+1)(S+2)\cdot 5S}+\frac{-1}{S}$$

$$=\frac{-S^2-2S+6}{S(S+1)(S+2)}=\frac{k_3}{S}^{=3}+\frac{k_4}{S+1}^{=-7}+\frac{k_5}{S+2}^{=3}$$

$\mathcal{L}^{-1}$ $i_L(t) = (3 - 7e^{-t} + 3e^{-2t})\, u(t)$ A



$$i_L(t) = i_0 + \frac{1}{L}\int_0^t V_1(t)\,dt$$

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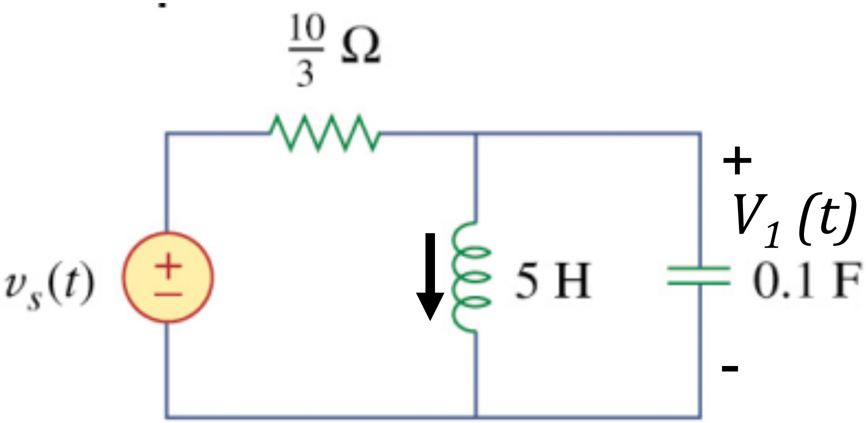
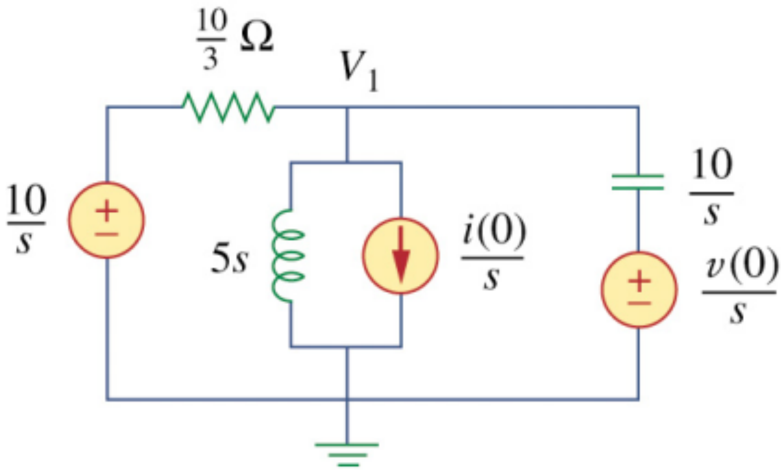
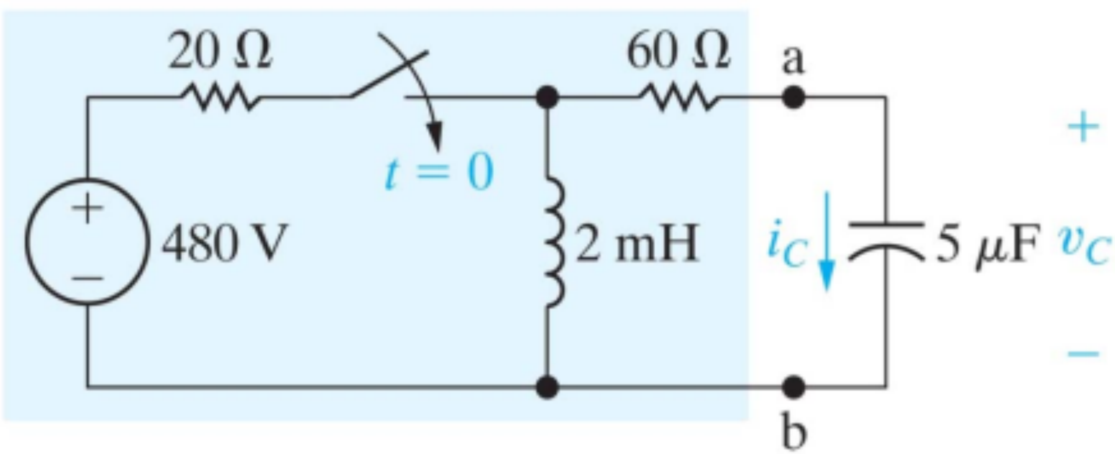


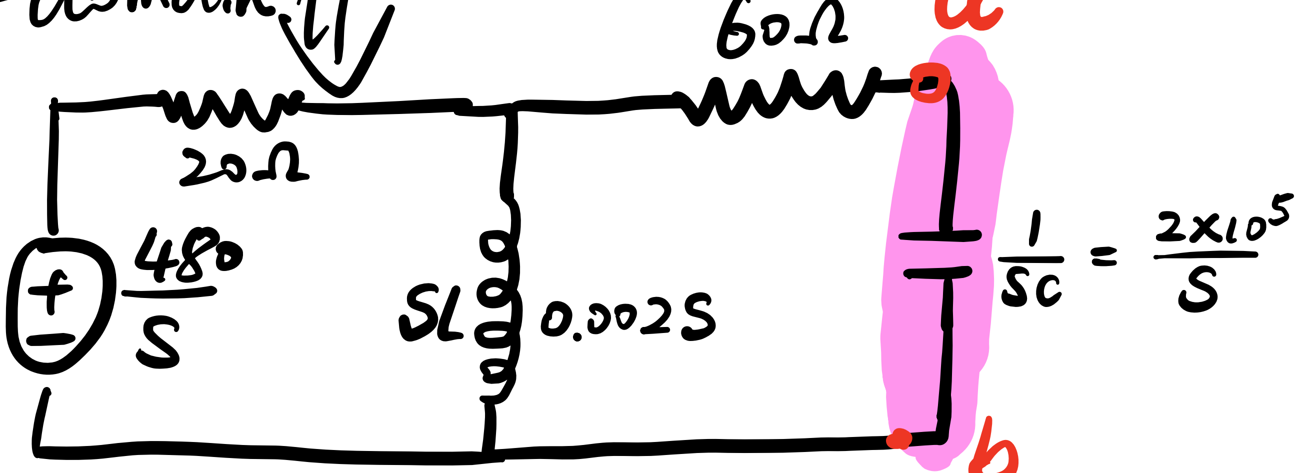


Example 3

- Use Thevenin's equivalent circuit w.r.t. terminals *a*-*b* to find current $i_C(t)$ for t>0.



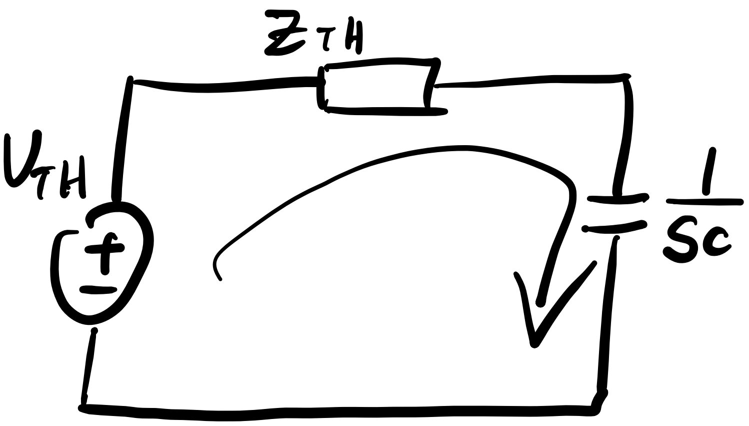
S-domain



Thevenin Equivalent Circuit:

$$V_{TH} = \frac{0.002S}{20+0.002S} \cdot \frac{480}{S} = \frac{480}{S+10^4}$$

$$Z_{TH} = 60 + \frac{0.002S \cdot 20}{0.002S+20} = \frac{80(S+7500)}{S+10^4}$$



$$I_c(s) = \frac{V_{TH}}{Z_{TH}+\frac{1}{Sc}}$$

$$= \frac{6S}{(S+5000)^2} = \frac{A}{(S+5000)^2} + \frac{B}{S+5000}$$

For A: $\frac{6S\cancel{(S+5000)^2}}{\cancel{(S+5000)^2}} = \frac{A\cancel{(S+5000)^2}}{\cancel{(S+5000)^2}} + \frac{B\cancel{(S+5000)}^2}{\cancel{(S+5000)}}$

$$A\Big|_{S=-5000} = 6S\Big|_{S=-5000} = -30000$$

For B: $\left[\dfrac{6s\cancel{(s+5000)^2}}{\cancel{(s+5000)^2}} = \dfrac{A\cancel{(s+5000)^2}}{\cancel{(s+5000)^2}} + \dfrac{B(s+5000)^{\cancel{2}}}{\cancel{(s+5000)}}\right]'$

$$(6s)' = [A + B(s+5000)]'$$

$$B = (6s)' = 6$$

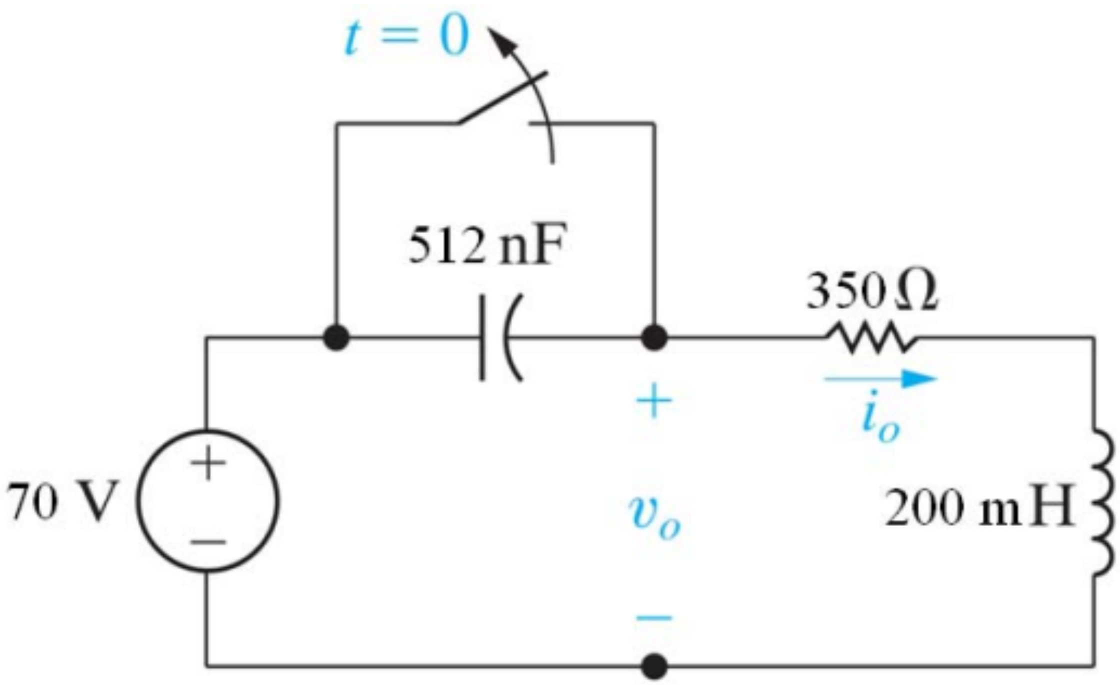
$$I_c(s) = \frac{-30000}{(s+5000)^2} + \frac{6}{s+5000}$$

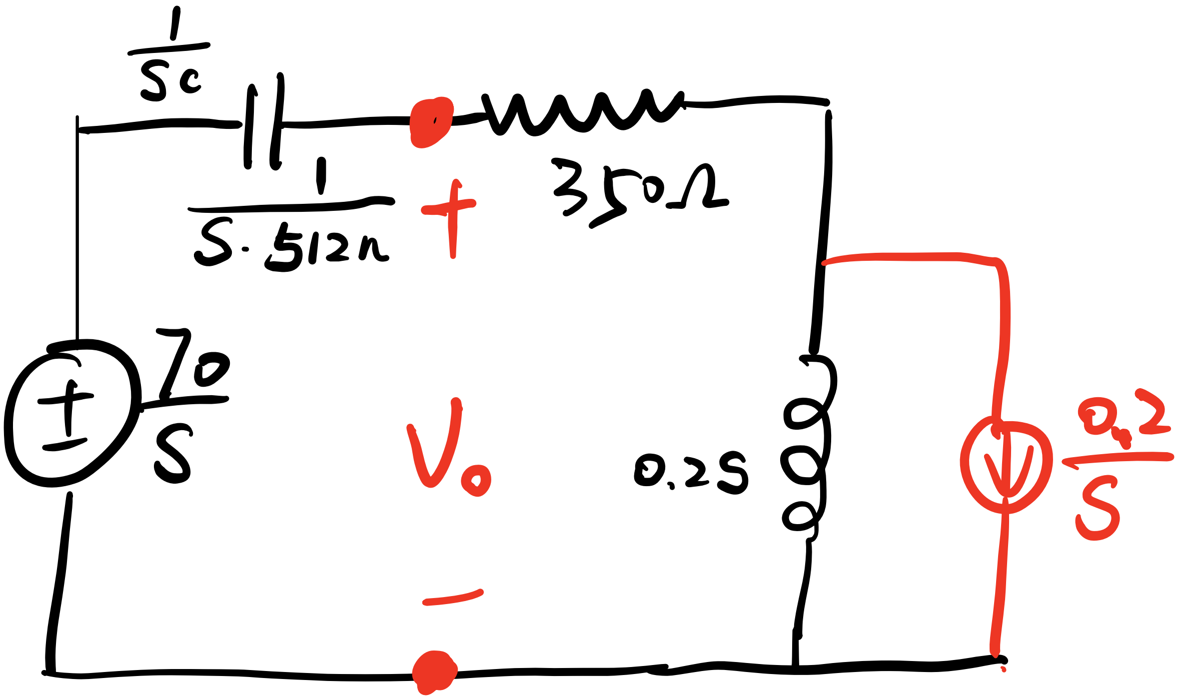
$$\Big\downarrow \mathcal{L}^{-1}$$

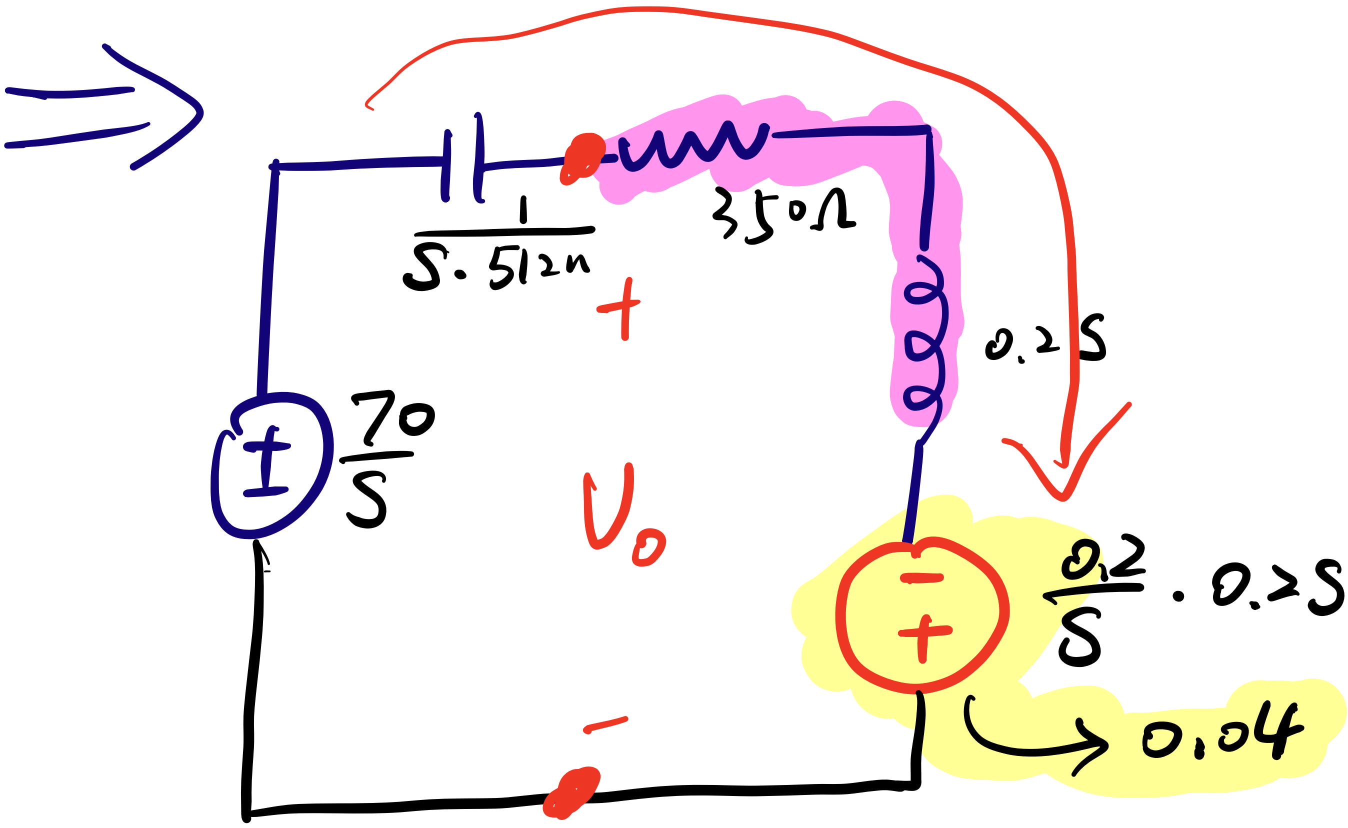
$$i_c(t) = \left[-30000\, t \cdot e^{-5000t} + 6 \cdot e^{-5000t}\right] u(t) \ \text{A}.$$

Example 4

- Find $V_o\,(t)$ *for* $t>0$







$$I(s) = \frac{\frac{70}{S} + 0.04}{\frac{1}{S \cdot 512n} + 350 + 0.2S}$$

$$V_o(s) = I(s) \cdot (350 + 0.2S) - 0.04$$

$s^2+1750s+\cdots=0$

$\alpha=-875$

$\omega=3000j$

$s_1, s_2=-875\pm 3000j$

$=\frac{K_1}{s-(-875+3000j)}+\frac{K_2}{s-(-875-3000j)}$

$$V_0(s)=\frac{70s-268,125}{s^2+1750s+9,765,625}=\frac{K_1}{(s+875-j3000)}+\frac{K_2}{(s+875+j3000)}$$

$$K_1=\frac{70s-268,125}{(s+875+j3000)}\bigg|_{s=-875+j3000}=\frac{70(-875+j3000)-268,125}{[(-875+j3000)+875+j3000]}=\underbrace{65.1}_{|K_1|}\angle\underbrace{57.48^\circ}_{\varphi_K}$$

$$K_2=\frac{70s-268,125}{(s+875-j3000)}\bigg|_{s=-875-j3000}=\frac{70(-875-j3000)-268,125}{[(-875-j3000)+875+-j3000]}=65.1\angle-57.48^\circ$$

$$V_0(s)=\frac{65.1\angle 57.48^\circ}{(s+875-j3000)}+\frac{65.1\angle-57.48^\circ}{(s+875+j3000)}$$

$$v_0(t)=2(65.1)e^{-875t}\cos(3000t+57.48^\circ)=130.2e^{-875t}\cos(3000t+57.48^\circ)u(t)\text{ V}$$

$|K_1|$ $\quad\alpha\quad$ ω $\quad\varphi_K$

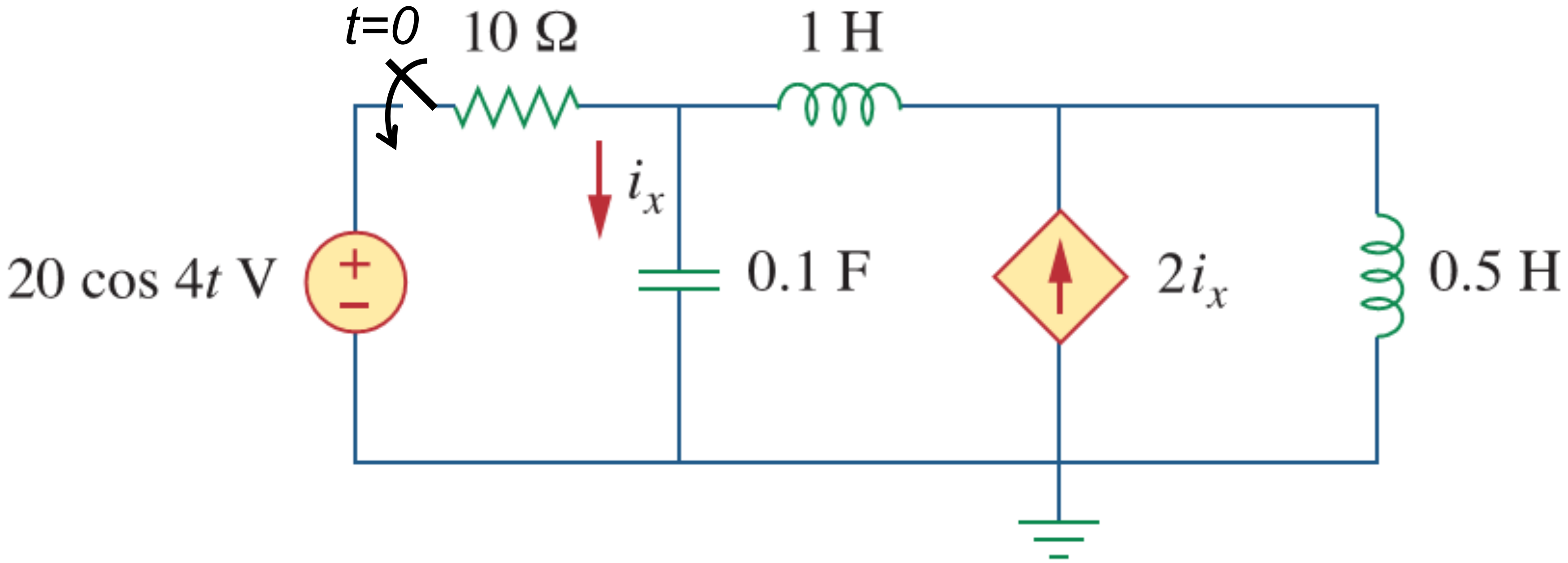
$\alpha=-875$

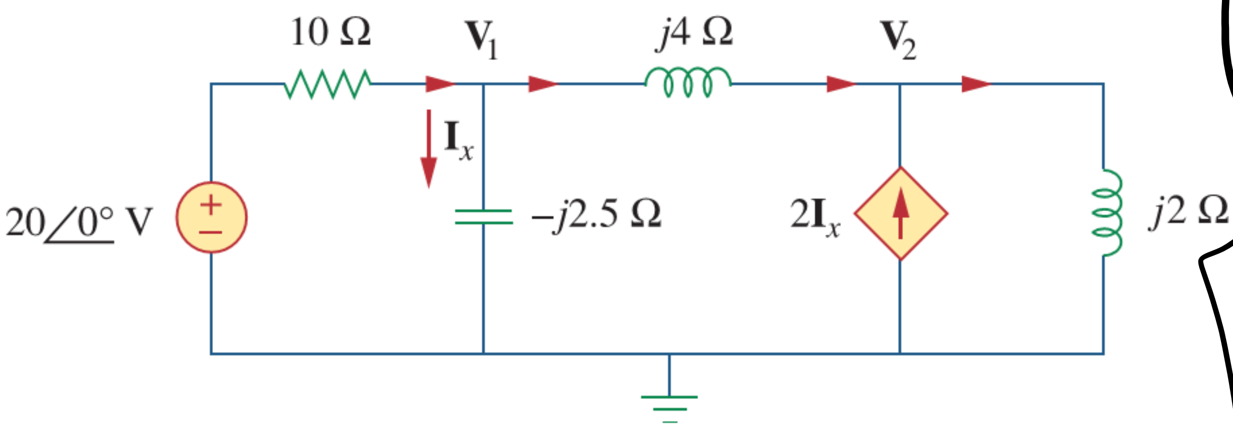
Example 5

- Example---Find i_x(s.s.) assuming no initial energy stored

Using (1)phasor method

(2)Laplace transform method

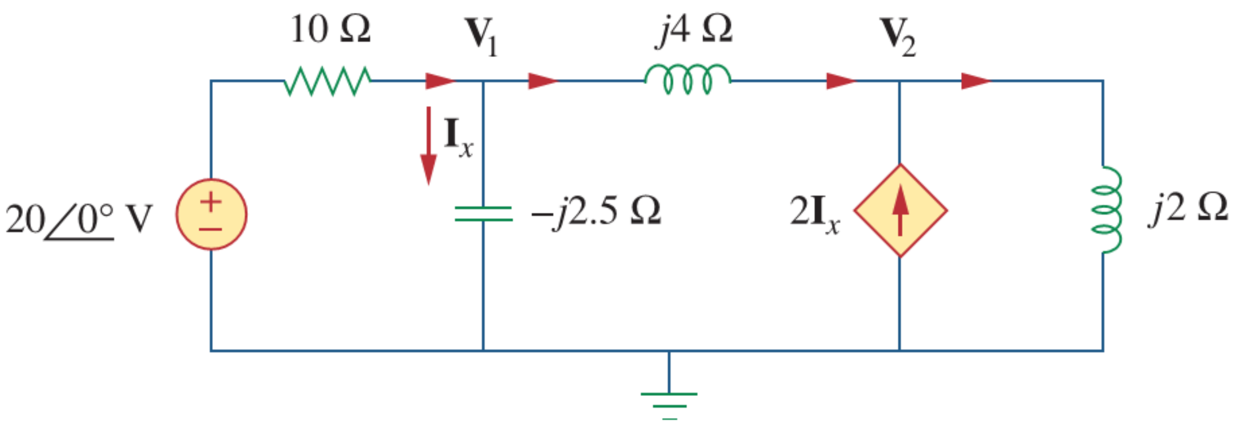




$$\frac{20 - \mathbf{V}_1}{10} = \frac{\mathbf{V}_1}{-j2.5} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{j4}$$

$$2\mathbf{I}_x + \frac{\mathbf{V}_1 - \mathbf{V}_2}{j4} = \frac{\mathbf{V}_2}{j2}$$

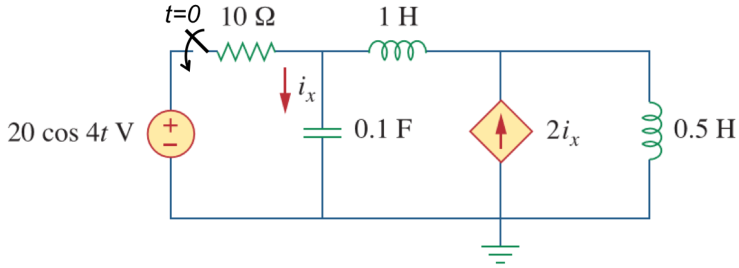
$$i_x = \frac{V_1}{-2.5j}$$



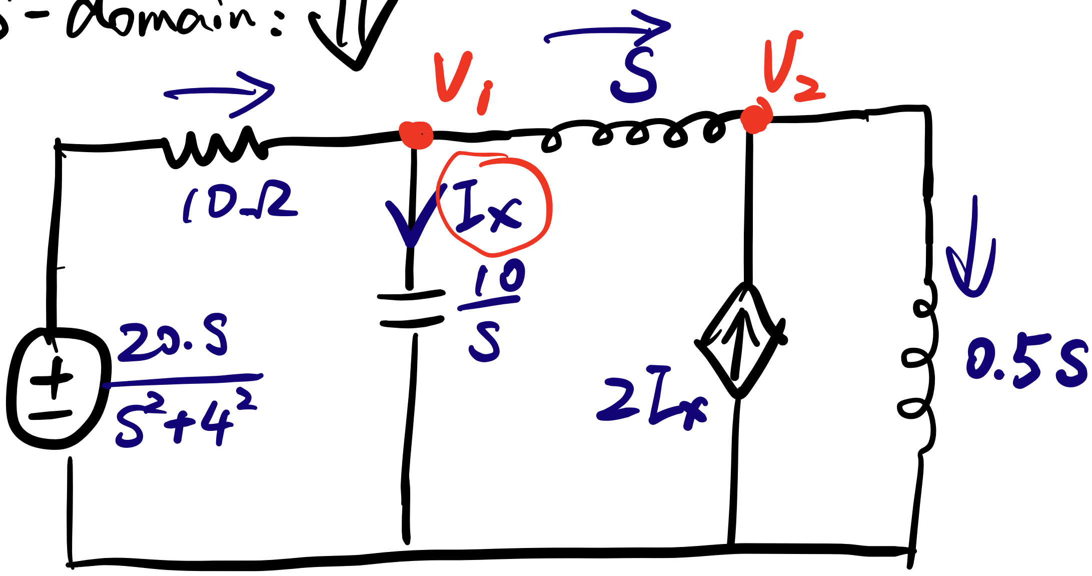
$$\frac{20-\mathbf{V}_1}{10}=\frac{\mathbf{V}_1}{-j2.5}+\frac{\mathbf{V}_1-\mathbf{V}_2}{j4}$$

$$2\mathbf{I}_x+\frac{\mathbf{V}_1-\mathbf{V}_2}{j4}=\frac{\mathbf{V}_2}{j2}$$

$$i_x = 7.59\cos(4t + 108.4°)\ \text{A}$$



S-domain:



$$\frac{\frac{20 \cdot s}{s^2+4^2} - V_1}{10} = \frac{V_1}{\frac{10}{s}} + \frac{V_1 - V_2}{s}$$

$$\frac{V_1 - V_2}{s} + 2I_x = \frac{V_2}{0.5s}$$

$$I_x = \frac{V_1}{\frac{10}{s}} = \frac{V_1 \cdot s}{10}$$

$$I_x = \frac{6s^3}{(s^2+4^2)(s^2+3s+20)}$$

$$= \frac{k_1}{(s-(4j))} + \frac{k_1^*}{(s-(-4j))}$$

$\alpha = 0$

$\omega = 4$

$$+ \frac{k_2}{s-(-1.5+4.2j)} + \frac{k_2^*}{s-(-1.5-4.2j)}$$

$\alpha = -1.5$

$\omega = 4.2j$

$$e^{\alpha t} \begin{cases} \alpha = 0 \rightarrow \text{S.S.} \\ \alpha < 0 \rightarrow \text{t.S.} \end{cases}$$

$\hookrightarrow e^{\alpha t}\cos(\omega t + \varphi)$

$$K_1 = 3.79 \angle 108.43^\circ$$

$$K_2 = 5.0 \angle -146.2^\circ$$

$$= \frac{K_1}{(s-(4j))} + \frac{K_1^*}{(s-(-4j))}$$

$\alpha_1 = 0$

$\omega_1 = 4$

$$+ \frac{K_2}{s-(-1.5+4.2j)} + \frac{K_2^*}{s-(-1.5-4.2j)}$$

$\alpha_2 = -1.5$

$\omega_2 = 4.2j$

φ^{-1}

$$v(t) = 2|K_1| \cdot e^{\alpha_1 t} \cos(\omega_1 t + \varphi_{K_1})$$

$$+ 2|K_2| e^{\alpha_2 t} \cos(\omega_2 t + \varphi_{K_2})$$

Example 6

- There is no initial energy stored in this circuit. Find $i(t)$ if
- $v(t) = e^{-0.6t} \sin 0.8t$ V.