Lecture 12 – Ultrasound Physics

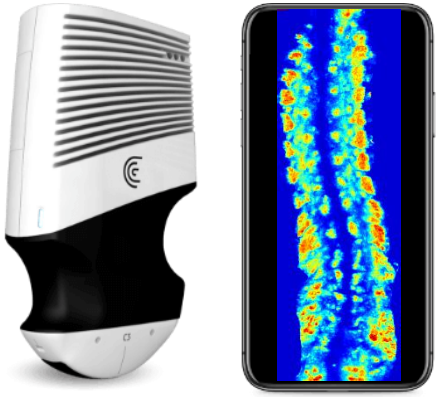
This lecture will cover: *(CH4.1-4.4)*

- Introduction to ultrasound imaging
- Ultrasound waves propagation
  - Wave types and parameters
  - Reflection and transmission
  - Scattering
  - Absorption and attenuation
- Thin medium layer

(Supplementary reading: The Essential Physics of Medical Imaging CH14.1-14.2)

Introduction to ultrasound imaging

- Least expensive, most portable, no ionizing radiation and magnetic field, continuous images at real time;
- Morphological and structural information, blood flow;
- Frequency for clinical use: 1-15MHz
- Detecting reflected energy from the boundaries between tissues with different acoustic and physical properties; Doppler effect;



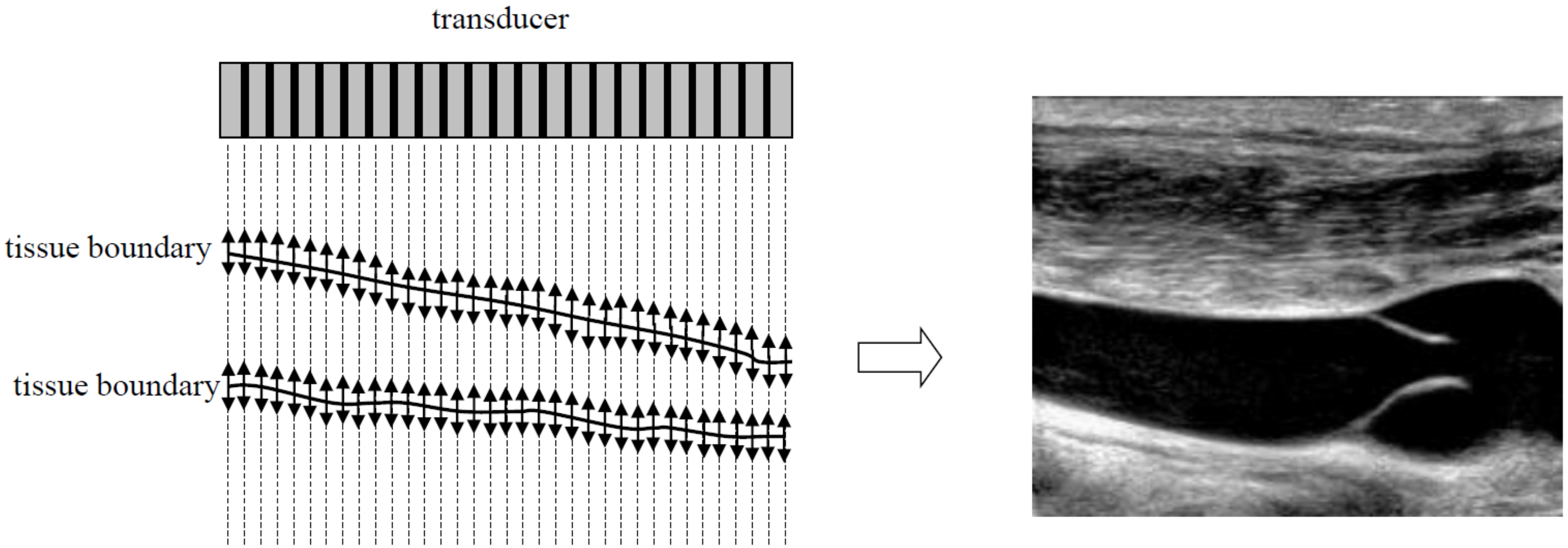


Fig. (left) Basic principle of ultrasound imaging. A transducer sends a series of pressure waves through the tissue. At boundaries between tissues, a small fraction of the energy is backscattered towards the transducer where it is detected. Using the speed of sound through tissue, the depth of the tissue boundary can be determined. Electronic steering of the beam across the sample builds up successive lines which form the image. (right) The intensity of each pixel in the image is proportional to the strength of the detected signal reflected from that point.

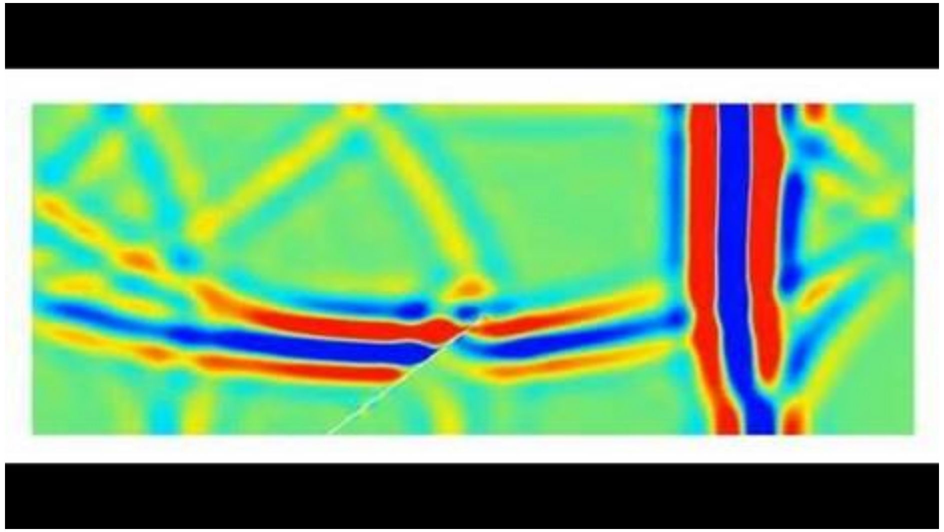
Wave propagation

- **Wave equation：**

$$\rho\frac{\partial^2 u}{\partial t^2} = (\lambda+\mu)\frac{\partial\theta}{\partial x} + \mu\nabla^2 u$$

$$\rho\frac{\partial^2 v}{\partial t^2} = (\lambda+\mu)\frac{\partial\theta}{\partial y} + \mu\nabla^2 v$$

$$\rho\frac{\partial^2 w}{\partial t^2} = (\lambda+\mu)\frac{\partial\theta}{\partial z} + \mu\nabla^2 w$$



where

λ and μ : lamé constant and rigidity

t : time

ρ : density.

$\mathbf{u}(u,v,w)$: displacement vector

θ: the divergence $\theta = \nabla\cdot\mathbf{u} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}$

∇^2: Laplacian operators

$$\nabla = \frac{\partial}{\partial x}\mathbf{i} + \frac{\partial}{\partial y}\mathbf{j} + \frac{\partial}{\partial z}\mathbf{k}$$

$$\nabla\cdot\nabla = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

Wave propagation

- **Ultrasound parameters**
  - Velocity, Speed of sound (SOS)
  - Acoustic impedance
- **Wave propagation**
  - Wave reflection and transmission;
  - Scattering by small structures
  - Absorption and attenuation

Ultrasound wave types

- **Longitudinal/compressional/bulk wave (纵波)**
  - Particles oscillate in the direction of wave propagation;
  - Propagate in solid, liquid and air;
- **Shear wave (横波、剪切波):**
  - Particles oscillate perpendicular to the direction of wave propagation;
  - Propagate in solid only;

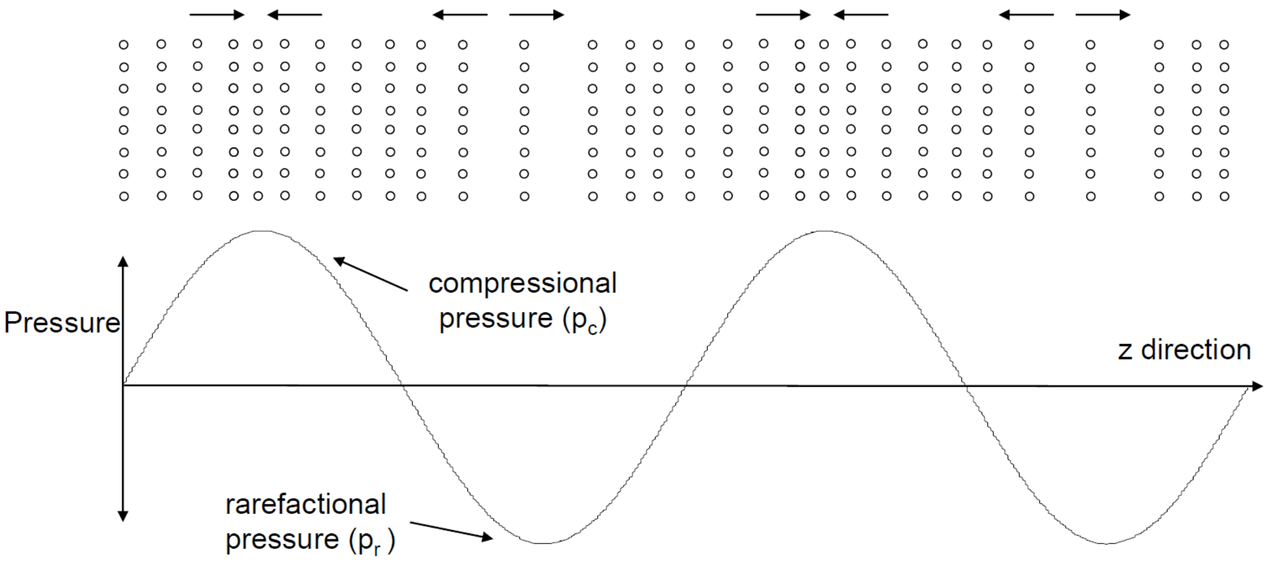


Fig. The effect of the passage of an ultrasound wave on the displacement of the molecules within tissue. The maximum positive pressure of the wave corresponds to the maximum compressional force, pushing the molecules together. The corresponding maximum negative pressure represents a rarefactional force.

Ultrasound wave parameters

- **Frequency (f)**
  - A large frequency bandwidth is covered for sharp pulse;
  - Typical center frequencies 1-15MHz;
- **Velocity of ultrasound wave**

$$c = \frac{1}{\sqrt{\kappa\rho}} = \sqrt{\frac{Y}{\rho}} \text{ or } \sqrt{\frac{G}{\rho}}$$

where ρ: the tissue density, κ: the compressibility (压缩系数), $Y(G)$: elastic modulus

- **Wavelength :** $\lambda = \frac{c}{f}$

Ultrasound wave parameters

- **The particle velocity:**

$$u_z = \frac{dw}{dt}$$

where w: particle displacement

- **The pressure of ultrasound wave at a particular point in the z-direction**

$$p = \rho c u_z \quad \Longrightarrow \quad Z = \frac{p}{u_z}$$

- **Acoustic impedance (声阻抗) :**

$$Z = \rho c = \sqrt{\frac{\rho}{\kappa}}$$

Acoustic properties

Table Acoustic properties of biological tissues

	$Z \times 10^5$ ($g\ cm^{-2}\ s^{-1}$)	Speed of sound ($m\ s^{-1}$)	Density (gm^{-3})	Compressibility $x10^{11}$ ($cm\ g^{-1}\ s^2$)
Air	0.00043	330	1.3	70 000
Blood	1.59	1570	1060	4.0
Bone	7.8	4000	1908	0.3
Fat	1.38	1450	925	5.0
Brain	1.58	1540	1025	4.2
Muscle	1.7	1590	1075	3.7
Liver	1.65	1570	1050	3.9
Kidney	1.62	1560	1040	4.0

Reflection and transmission

- For the incident, reflected and transmitted angles

$$\theta_i = \theta_r \qquad \frac{\sin\theta_i}{\sin\theta_t} = \frac{c_1}{c_2}$$

- The reflection and transmission coefficients for pressure

$$R_{\mathrm{p}} = \frac{Z_2\cos\theta_i - Z_1\cos\theta_t}{Z_2\cos\theta_i + Z_1\cos\theta_t} \qquad T_{\mathrm{p}} = \frac{2Z_2\cos\theta_i}{Z_2\cos\theta_i + Z_1\cos\theta_t}$$

$$T_{\mathrm{p}} = R_{\mathrm{p}} + 1$$

- The reflection and transmission coefficients for intensity

$$R_{\mathrm{I}} = \frac{(Z_2\cos\theta_i - Z_1\cos\theta_t)^2}{(Z_2\cos\theta_i + Z_1\cos\theta_t)^2} \qquad T_{\mathrm{I}} = \frac{4Z_1Z_2\cos\theta_i\cos\theta_t}{(Z_2\cos\theta_i + Z_1\cos\theta_t)^2}$$

$$T_{\mathrm{I}} = 1 - R_{\mathrm{I}}$$

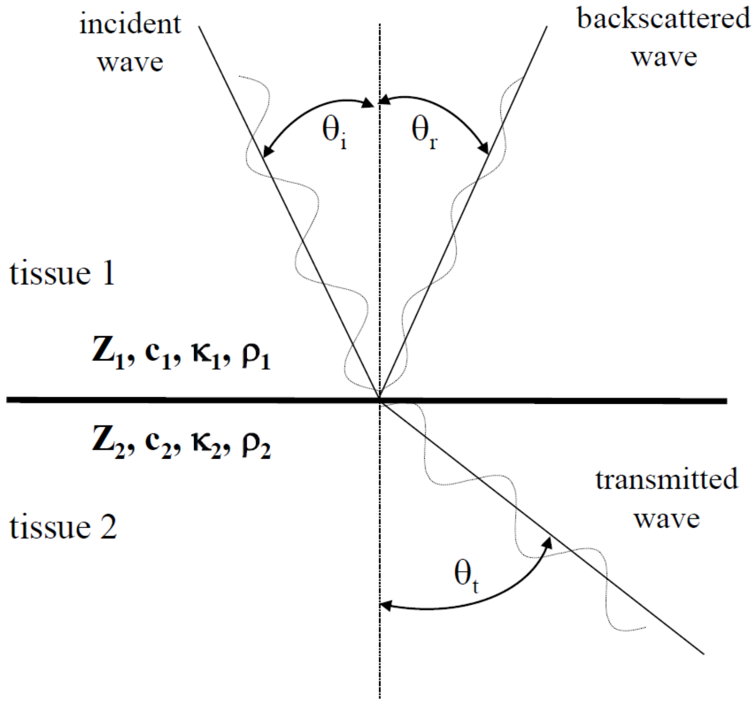


Fig. Showing the behavior of an ultrasound beam when it strikes the boundary between two tissues with different acoustic properties. A certain fraction of the wave is backscattered/reflected back towards the transducer, with the remaining fraction being transmitted through the boundary deeper into tissue.

Normal incidence

- For the normal incidence case,

$$\theta_i = \theta_r = \theta_t = 0$$

- The reflection and transmission coefficients for pressure

$$R_{\mathrm{p}} = \frac{Z_2 - Z_1}{Z_2 + Z_1} \qquad T_{\mathrm{p}} = \frac{2Z_2}{Z_2 + Z_1} \qquad T_{\mathrm{p}} = R_{\mathrm{p}} + 1$$

- The reflection and transmission coefficients for intensity

$$R_{\mathrm{I}} = \frac{(Z_2 - Z_1)^2}{(Z_2 + Z_1)^2} \qquad T_{\mathrm{I}} = \frac{4Z_1 Z_2}{(Z_2 + Z_1)^2} \qquad R_{\mathrm{I}} + T_{\mathrm{I}} = 1$$

- The reflection and transmission coefficients for intensity

① $Z_1 \gg Z_2$: muscle/air interface

② $Z_1 \sim Z_2$: liver/kidney interface

③ $Z_1 \ll Z_2$: muscle/bone interface

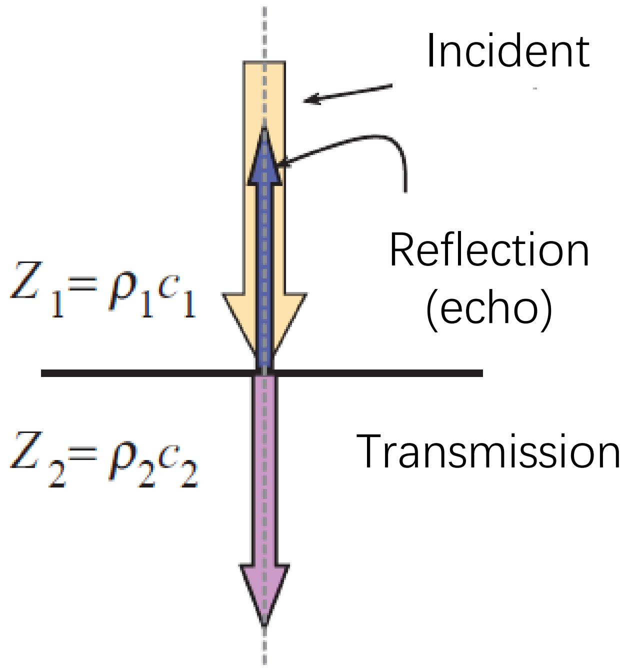


Fig. With perpendicular incidence (90 degrees), a fraction of the beam is transmitted and a fraction of the beam is reflected to the source at a tissue boundary.

Scattering

- ➢ The size of structure is approximate or smaller than wavelength;
- ➢ The wave is scattered in all directions (point source)
- ➢ The scattered energy increases as f^4

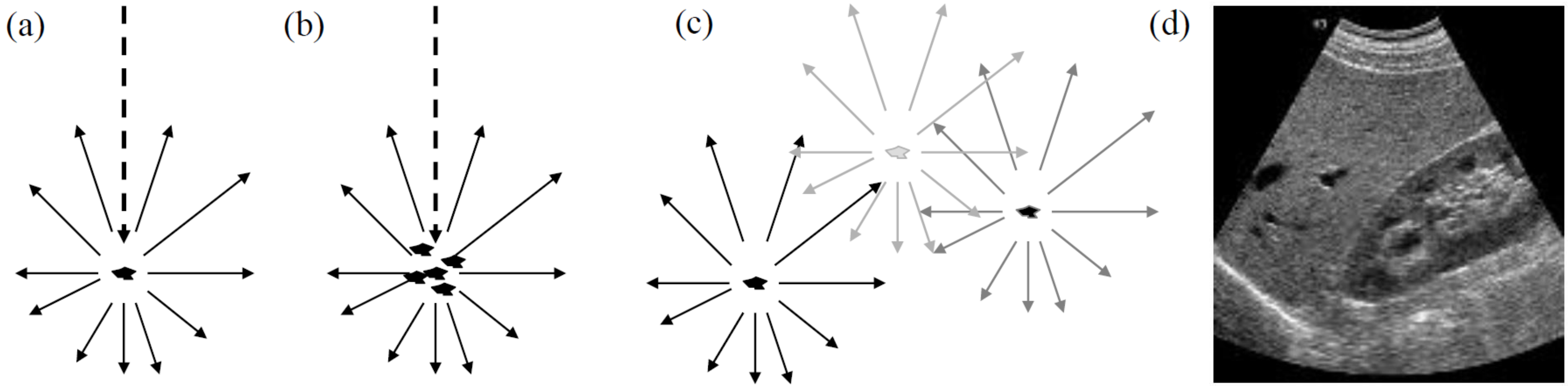


Fig. (a) Rayleigh scattering of an ultrasound beam by a structure which is physically small compared to the wavelength of the ultrasound beam. (b) Scattering from several structures which are close together produces scattered waves which add constructively. (c) Scattering structures which are relatively far from one another produce scattering patterns which add constructively at certain locations and destructively at others, thus producing areas of high and low image intensity, as illustrated in the image in (d).

Wave attenuation

- The wave energy decreases with its travel distance in the tissue;
- Causes of wave attenuation
  - Diffusion: inverse of square law

$$I = I_0 \frac{r_0^{\ 2}}{r^2}$$

  - Scattering and refraction
  - Absorption:
    - ✓ Mostly covert the energy of ultrasound beam into heat
    - ✓ viscosity, heating, relaxation（弛豫）

Relaxation absorption

$$\beta_{\mathrm{r,tissue}} \propto \sum_{n} \frac{f^2}{1+\left(\frac{f}{f_{\mathrm{r,n}}}\right)^2}$$

Where $\beta_{\mathrm{r,tissue}}$: relaxation absorption coefficient

$f_{\mathrm{r,n}}$: the relaxation frequency

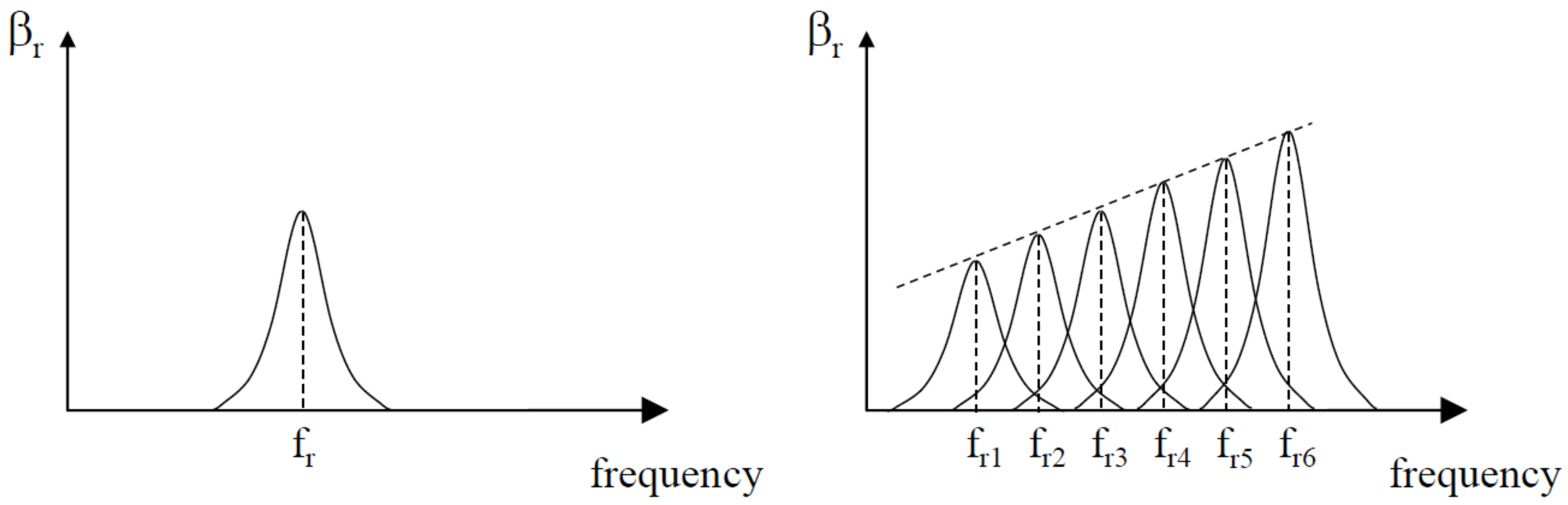


Fig. Plots of the relaxation absorption coefficient, β_r, for a completely homogeneous tissue (left) and for a realistic tissue (right) in which there are many relaxation frequencies. The overall absorption coefficient increases linearly with frequency.

Attenuation coefficient

- Characterized by an exponential decrease in both pressure and intensity as a function of its propagation distance;

$$I(z) = I_0 e^{-\mu z}$$

$$P(z) = P_0 e^{-\alpha z}$$

Where α: pressure attenuation coefficient

μ: intensity attenuation coefficient and $\mu = 2\alpha$

unit: dB/cm or cm^{-1}, and μ(dB/cm)=4.343μ(cm^{-1})

- 3dB corresponds to a reduction in intensity by a factor of 2

- The frequency dependence of μ is 1 dB/cm/MHz for soft tissue, 45dB/cm/MHz for air, and 8.7dB/cm/MHz for bone.

Thin medium layer

The transmission coefficient of intensity for a thin medium layer:

$$t_I = \frac{4Z_1Z_3}{(Z_1 + Z_3)^2 \cos^2\left(\frac{2\pi}{\lambda_2}d\right) + \left(Z_2 + \frac{Z_1Z_3}{Z_2}\right)^2 \sin^2\left(\frac{2\pi}{\lambda_2}d\right)}$$

- $Z_2 \ll Z_1Z_3$: $t_I \to 0$, no penetration, air between two layers
- $d = \frac{\lambda_2}{2}, \lambda_2, \frac{3\lambda_2}{2}, \cdots$ or $d \ll \lambda_2$: $t_I = \frac{4Z_1Z_3}{(Z_1+Z_3)^2}$, penetrate as no middle layer exists;
- $Z_2 = \sqrt{Z_1Z_3}$ and $d = \frac{(2n+1)\lambda_2}{4}$: $t_I = 1$,
    - totally penetrate without energy loss;
    - for coupling (耦合) design;

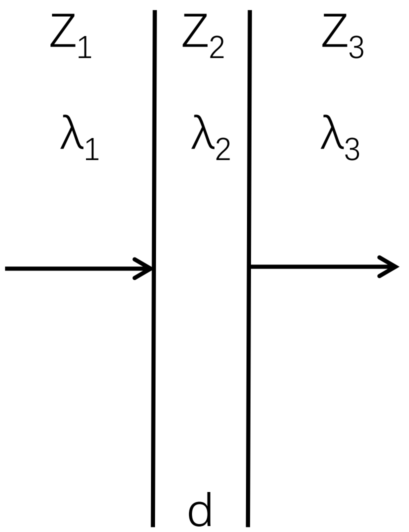


Fig. Ultrasound beam pass through a thin medium layer .