Homework 5

Due time: 10 p.m. Dec. 3rd, 2024

Turn in your hard-copy hand-writing homework at the entrance of Room 3-324 SIST #3 Building.

Rules:

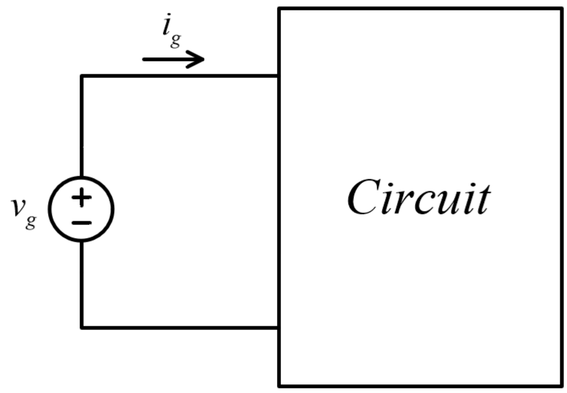
- Work on your own. Discussion is permissible, but extremely similar submissions will be judged as plagiarism.
- Please show all intermediate steps: a correct solution without an explanation will get zero credit.
- Please submit on time. No late submission will be accepted.
- Please prepare your submission in English only. No Chinese submission will be accepted.
- All final answers must be rounded to **two decimal places**.

1. The expression for the steady-state voltage and current in the terminals of the circuit seen in the figure are

$$v_g = 300\cos(5000\pi t + 78°)\,V$$
$$i_g = 6\sin(5000\pi t + 123°)\,A$$

(a) Transform the expressions of v_g and i_g into **phasor** form.

(b) What is the impedance seen by the source?



(a)

$$V_g = 300\angle 78°\,V$$
$$I_g = 6\angle 33°\,A$$

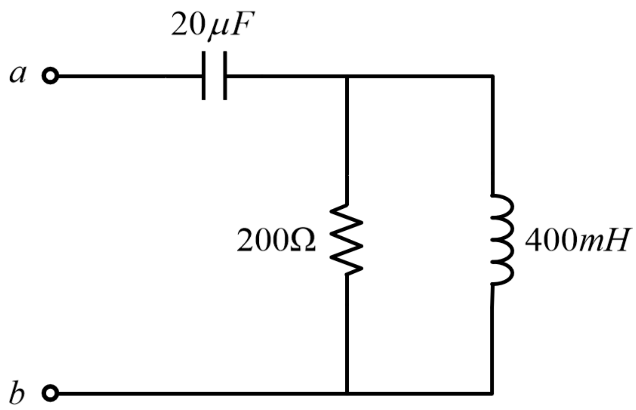
(b)

$$Z = \frac{V_g}{I_g} = \frac{300\angle 78°}{6\angle 33°} = 50\angle 45°\,\Omega$$

2. For the circuit shown below:

(a) Find the frequency (in radians per second) at which the impedance $\boldsymbol{Z_{ab}}$ is purely resistive.

(b) Find the value of $\boldsymbol{Z_{ab}}$ at the frequency of (a).



(a)

$$\frac{1}{j\omega C} + R||j\omega L = \frac{1}{j\omega C} + \frac{j\omega RL}{j\omega L + R}$$
$$= \frac{j\omega L + R - \omega^2 RLC}{j\omega C(j\omega L + R)}$$
$$= \frac{(R - \omega^2 RLC + j\omega L)(-\omega^2 LC - j\omega RC)}{(-\omega^2 LC + j\omega RC)(-\omega^2 LC - j\omega RC)}$$

Set the imaginary part to 0:

$$-\omega^3 L^2 C - \omega R^2 C + \omega^3 R^2 C^2 L = 0$$
$$\omega^2 = \frac{R^2}{R^2 LC - L^2} = 250000$$
$$\omega = 500\, rad/s$$

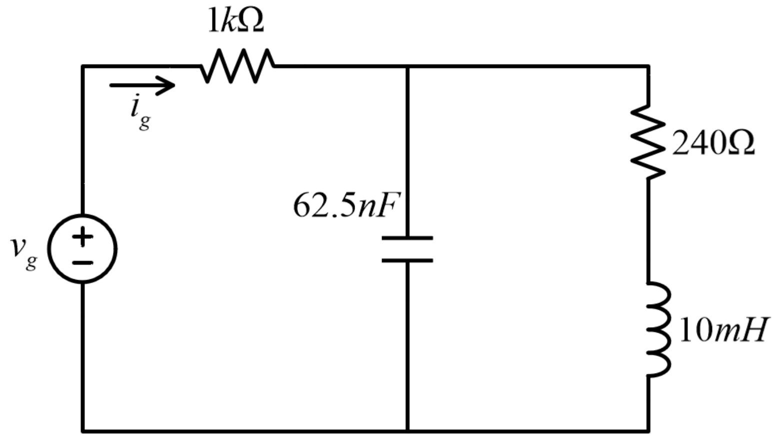
(b)

$$Z_{ab}(500) = -j100 + \frac{200\ \cdot (j200)}{200 + j200} = 100\,\Omega$$

3. The frequency of the sinusoidal voltage source in the circuit is adjusted until i_g is in phase with v_g.

(a) What is the value of w in radians per second.

(b) If $V_g = 15coswt\,V$ (where w is the frequency found in (a)), what is the steady-state expression for i_g in **time domain**?



(a)

$$Z_{eq} = \frac{1}{j\omega C}||(R + j\omega L)$$
$$= \frac{(R + j\omega L)(1 - \omega^2 LC - j\omega RC)}{(1 + \omega^2 LC)^2 + \omega^2 R^2 C^2}$$

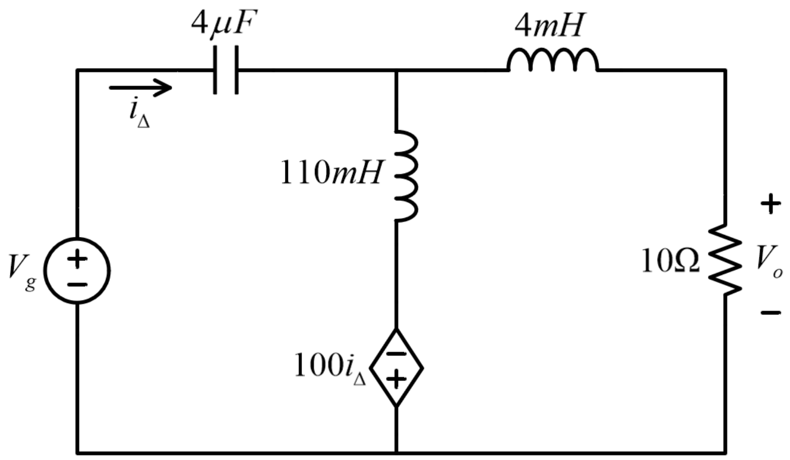
The voltage and current are in phase when the impedance to the left of the 1kω resistor is purely real. Set the imaginary part of Zeq to zero:

$$-\omega R^2 C + \omega L - \omega^3 L^2 C = 0$$
$$\omega = 32000\,rad/s$$

(b)

$$Z_t = 1000 + (-j500)||(240 + j320) = 1666.67\ \Omega$$
$$I_g = \frac{V_g}{Z_t} = 9\angle 0°\ mA$$
$$i_g = 9\cos(32000t)\,mA$$

4. Use the nodal or mesh method to find $\boldsymbol{V_o}$ in **phasor domain** in the circuit if $V_g = 75cos5000t\,V$.



$$j\omega L_1 = j5000(4\times 10^{-3}) = j20\ \Omega$$
$$j\omega L_2 = j5000(110\times 10^{-3}) = j550\ \Omega$$
$$\frac{1}{j\omega C} = \frac{-j}{(5000)(4\times 10^{-6})} = -j50\ \Omega$$

Mesh method:

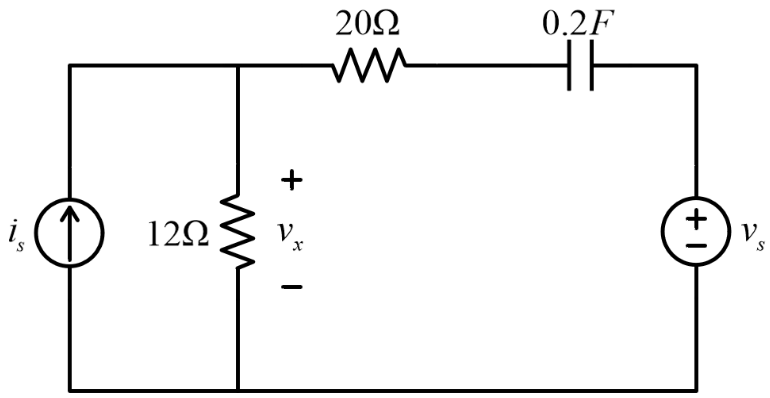
The current in the left mesh is i_Δ, assume the current in the right mesh is i_a.

$$75\angle 0^\circ = j500I_\Delta - 100I_\Delta - j550I_\Delta$$
$$0 = (10 + j20)I_\text{a} + 100I_\Delta + j550(I_\text{a} - I_\Delta)$$

Solving,

$$I_\text{a} = j2.5\ A$$
$$V_o = 10I_\text{a} = j25 = 25\angle 90^\circ\ V$$

5. Use the superposition principle to obtain the steady-state expression for v_x in **time domain** in the circuit. Assume $v_s = 50sin2t\,V$ and $i_s = 12\cos(6t + 10^{\circ})A$.



Step 1. Only i_s.

$$\omega = 6\,rad/s$$
$$I_s = 12\angle 10°\,A$$
$$Z_c = \frac{1}{j\omega C} = -j0.83\,\Omega$$
$$Z = 12||(20 - j0.83) = 7.50\angle -0.89°\,\Omega$$
$$V_{x1} = I_s \cdot Z = 90.05\angle 9.11°\,V$$

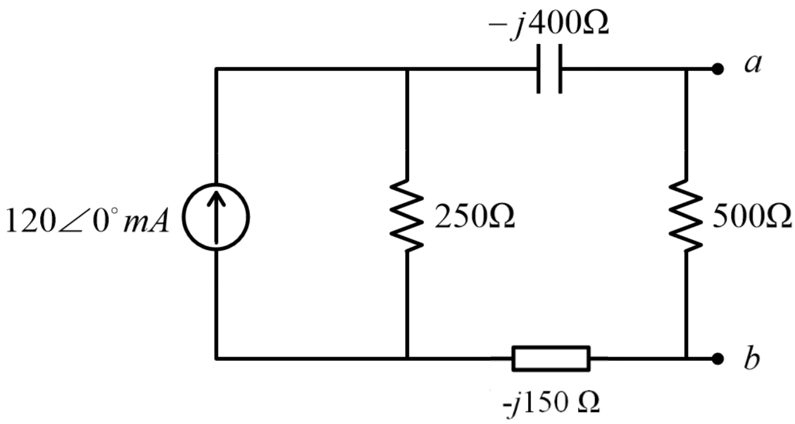
Step 2. Only vs.

$$\omega = 2\,rad/s$$
$$V_s = 50\angle -90°\,V$$
$$Z_c = \frac{1}{j\omega C} = -j2.5\,\Omega$$
$$V_{x2} = \frac{12}{12 + 20 - j2.5} \cdot V_s = 18.69\angle -85.53°\,V$$

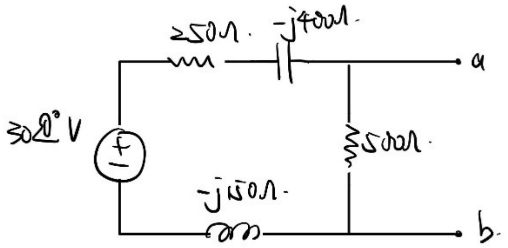
Step 3. Superposition.

$$v_x(t) = 90.05\cos(6t + 9.11°) + 18.69\cos(2t - 85.53°)\,V$$

6. Use source transformations to find the Thevenin equivalent circuit with respect to the terminals a, b for the circuits shown below.

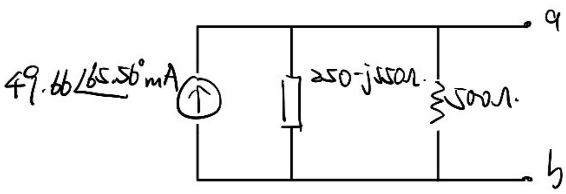


$$250 \cdot 0.12\angle 0° = 30\angle 0°\ V$$



$$250 - j400 - j150 = 250 - j550\ \Omega$$

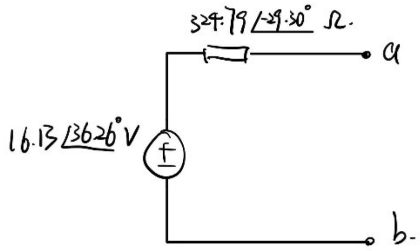
$$\frac{30\angle 0°}{250 - j550} = 49.6\angle 65.56\ mA$$



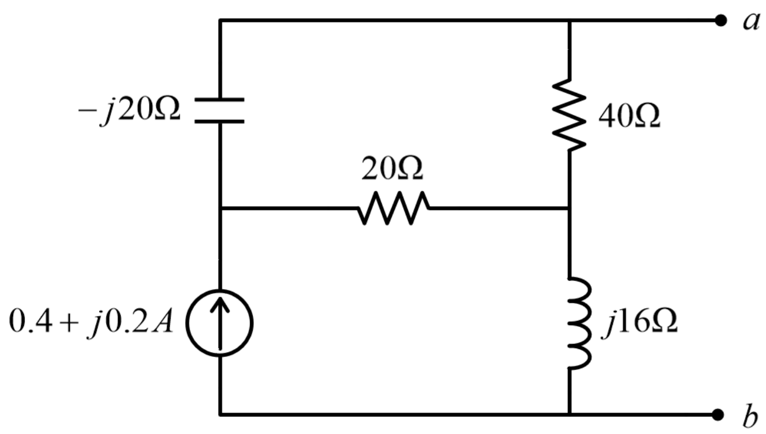
$$R_{th} = (250 - j550)||500 = 324.79\angle -29.30°\ \Omega$$

$$V_{th} = 49.6\angle 65.56 \cdot 324.79\angle -29.30° = 16.13\angle 36.26°\ V$$

The Thevenin equivalent circuit is shown below:



7. Find the Thevenin equivalent circuit with respect to the terminals a, b for the circuit.



Open circuit voltage:

Assume the current in the upper mesh is I_a:

$$-j20I_a + 40I_a + 20(I_a - 0.4 - j0.2) = 0$$

Solving:

$$I_a = \frac{20(0.4 + j0.2)}{60 - j20} = 0.1 + j0.1\ A$$

$$V_{oc} = 40I_a + j16(0.4 + j0.2) = 0.8 + j10.4 = 10.43\angle 85.60°\ V$$

Shor circuit current:

Mesh:

$$-j20I_a + 40(I_a - I_{sc}) + 20(I_a - 0.4 - j0.2) = 0$$

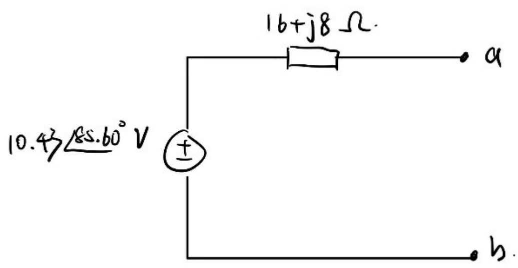
$$40(I_{sc} - I_a) + j16(I_{sc} - 0.4 - j0.2) = 0$$

Solving:

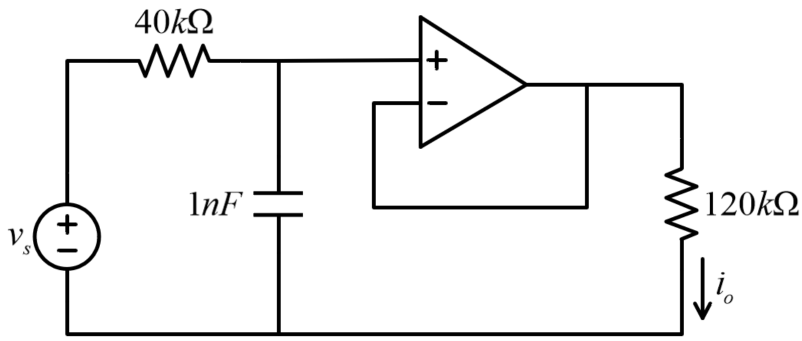
$$I_{sc} = 0.3 + j0.5\ A$$

$$Z_{th} = \frac{V_{oc}}{I_{sc}} = 16 + j8 = 17.89\angle 26.57°\ \Omega$$

The Thevenin equivalent circuit:



8. Compute $i_o(t)$ in the operational amplifier circuit if $v_s = 4\cos(10^4 t)\ V$.



$$V_s = 4\angle 0°\ V$$
$$Z_c = \frac{1}{j\omega C} = -j100\ k\Omega$$

KCL:

$$\frac{V_s - V_o}{40\ k\Omega} = \frac{V_o}{-j100\ k\Omega}$$

Solving:

$$V_o = 3.71\angle -21.80°\ V$$
$$I_o = \frac{V_o}{120\ k\Omega} = 3.09 \times 10^{-5}\angle -21.80°\ A$$
$$i_o(t) = 3.09 \times 10^{-5}\cos(10^4 t - 21.80°)\ A$$