EE150-Signals and System, FALL 2024

Homework Set #4

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Problem 1. (20 points) Determine the Fourier transform of the following signals:

(a) $x(t) = \begin{cases} \cos(\pi t), & |t| \leq 1 \\ 0, & |t| > 1 \end{cases}$

(b) $x(t) = \cos\left(6t + \frac{\pi}{4}\right)$

(c) As shown in the Figure 1, $x(t)$ is a continuous periodic signal with fundamental period T = 6:

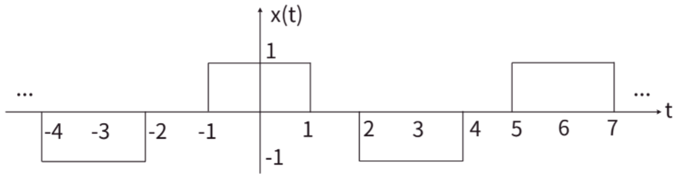

Figure 1

Problem 2. (15 points) Determine the inverse Fourier transform of $X(j\omega)$:

(a) $X(j\omega) = u(\omega - 2) - u(\omega - 4)$

(b) $X(j\omega) = 2\cos(3\omega)$

(c) $X(j\omega)$ as shown in the Figure 2:

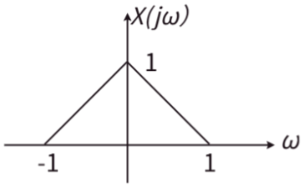

Figure 2

Problem 3. (15 points)

(a) Determine the Fourier transform of the following signal:

$$x(t) = te^{-2|t|}$$

(b) Use the result from part (a), along with the duality property, to determine the Fourier transform of the following signal:

$$f(t) = \frac{8t}{(4 + t^2)^2}$$

Problem 4. (15 points) Let $X(j\omega)$ denotes the Fourier transform of the signal $\mathrm{x(t)}$ depicted in the Figure 3:

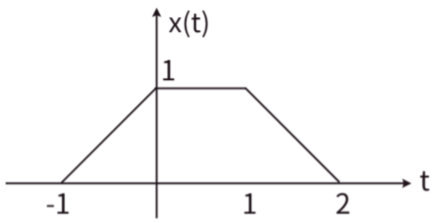

Figure 3

(a) Determine $\int_{-\infty}^{\infty} X(j\omega)d\omega$

(b) Sketch the inverse Fourier transform of $Re\{X(j\omega)\}$ and $Im\{X(j\omega)\}$

Problem 5. (15 points) Consider a signal x(t) with Fourier transform $X(j\omega)$. Suppose we are given the following facts:

(a) $x(t)$ is real and nonnegative

(b) $Ae^{-t}u(t) \overset{\mathcal{F}}{\leftrightarrow} (1 + j\omega)X(j\omega)$

(c) $\int_{-\infty}^{\infty} \mid X(j\omega) \mid^2 d\omega = 2\pi$

Determine a closed-form expression for x(t)

Problem 6. (20 points) A causal and stable LTI system has the frequency response:

$$H(jw) = \frac{j\omega - 1}{(j\omega)^2 + 5j\omega + 6}$$

(a) Determine a differential equation relating the input x(t) and output y(t) of the LTI system

(b) What is the output of the LTI system when the input is $x(t) = e^{-t}u(t)$

(c) What is the output of the LTI system when the input is $x(t) = \sqrt{3}\sin\left(\mathrm{t} + \frac{\pi}{4}\right)$