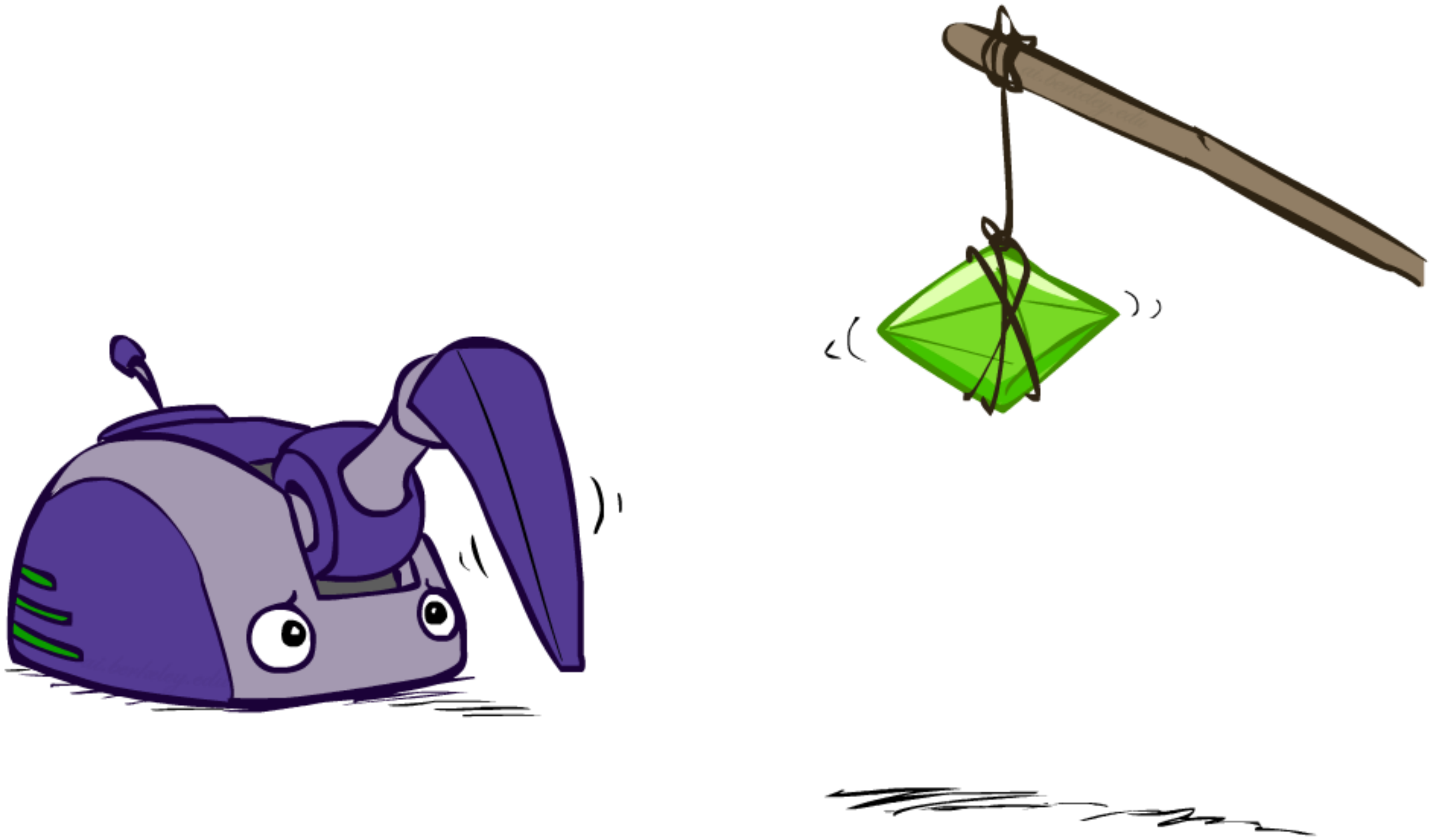
Reinforcement Learning

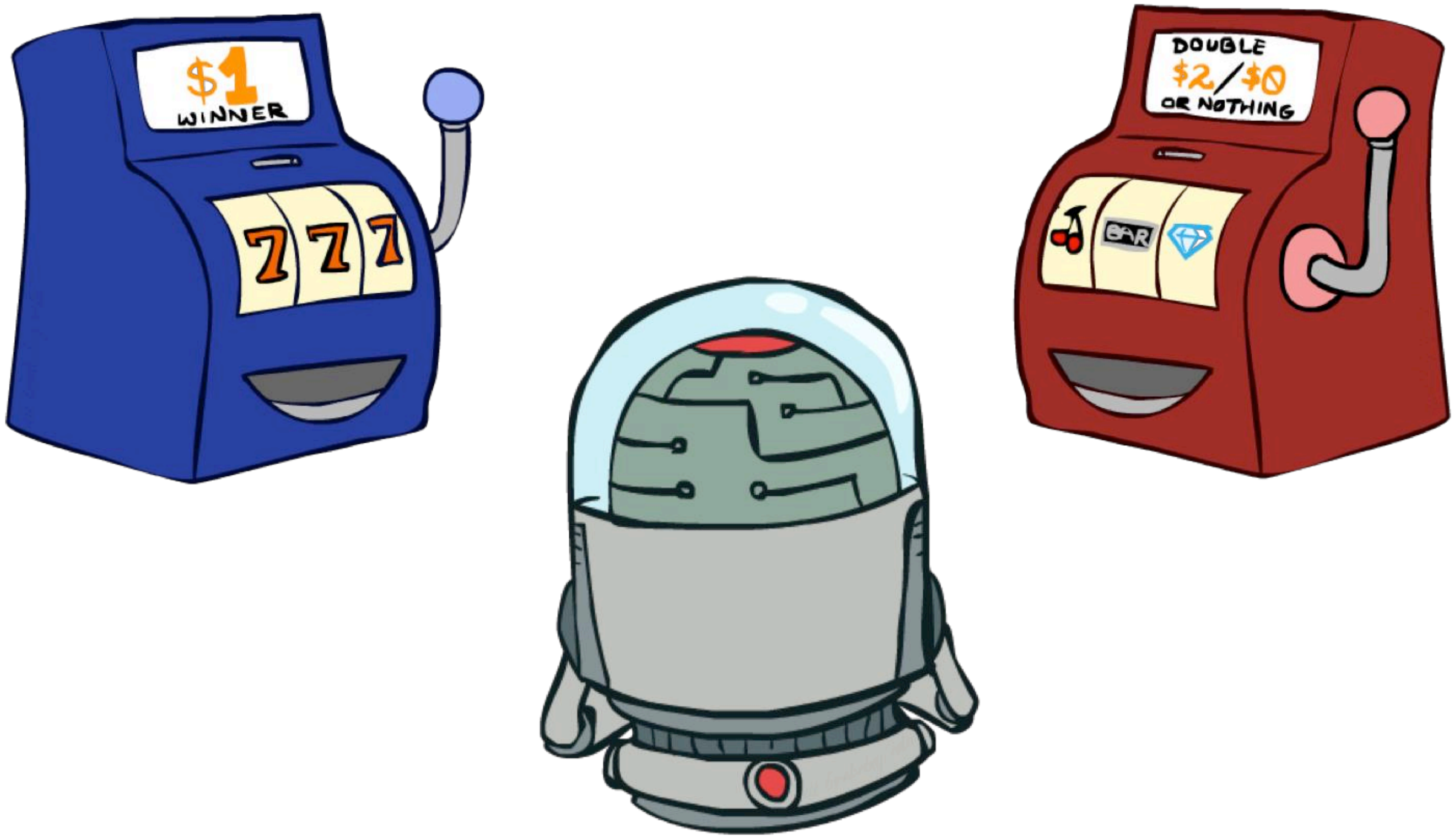


Bandits

- Exactly one state
- Set of actions: A
- Stochastic reward function: $P(r|a)$

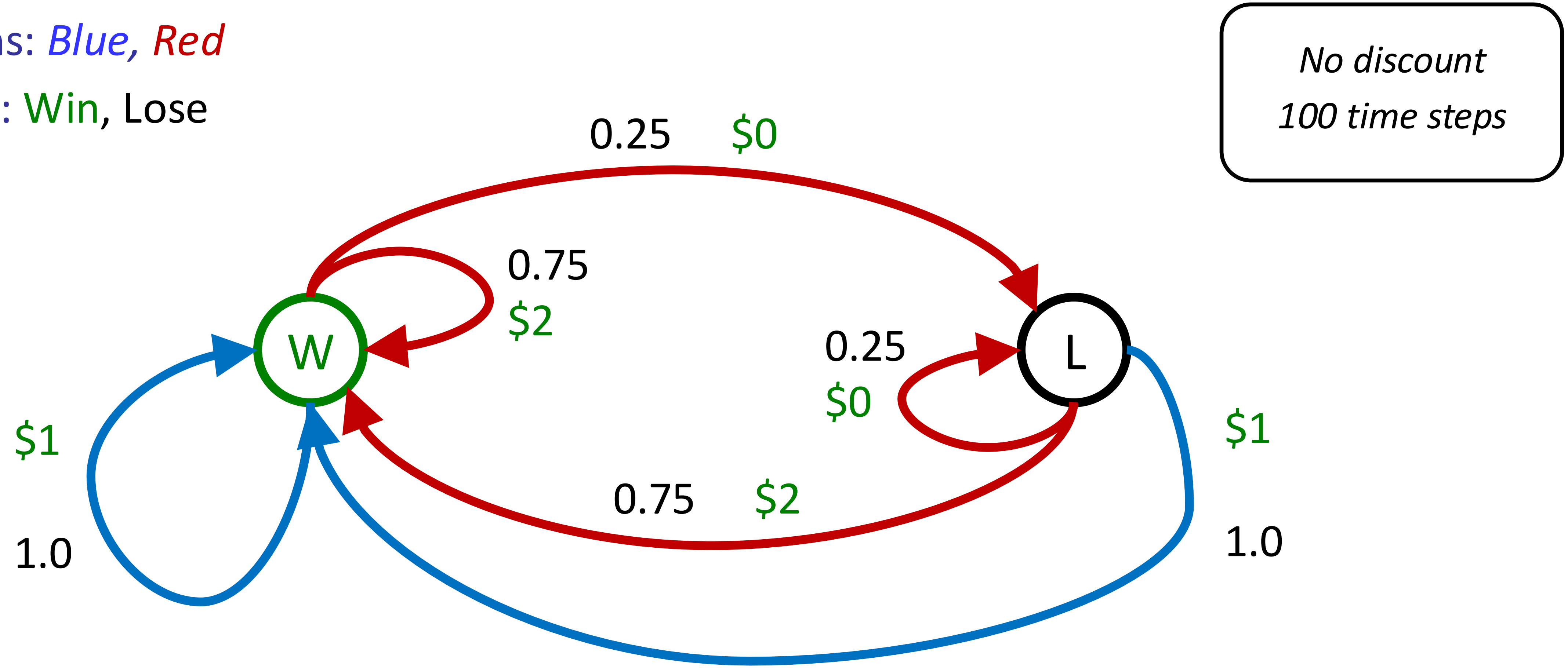
Contextual Bandits:

- Set of states $s \in S$
- Transitions always return to start state distribution $P(s'|s, a) = P_0(s')$



Double-Bandit MDP

- Actions: *Blue*, *Red*
- States: Win, Lose

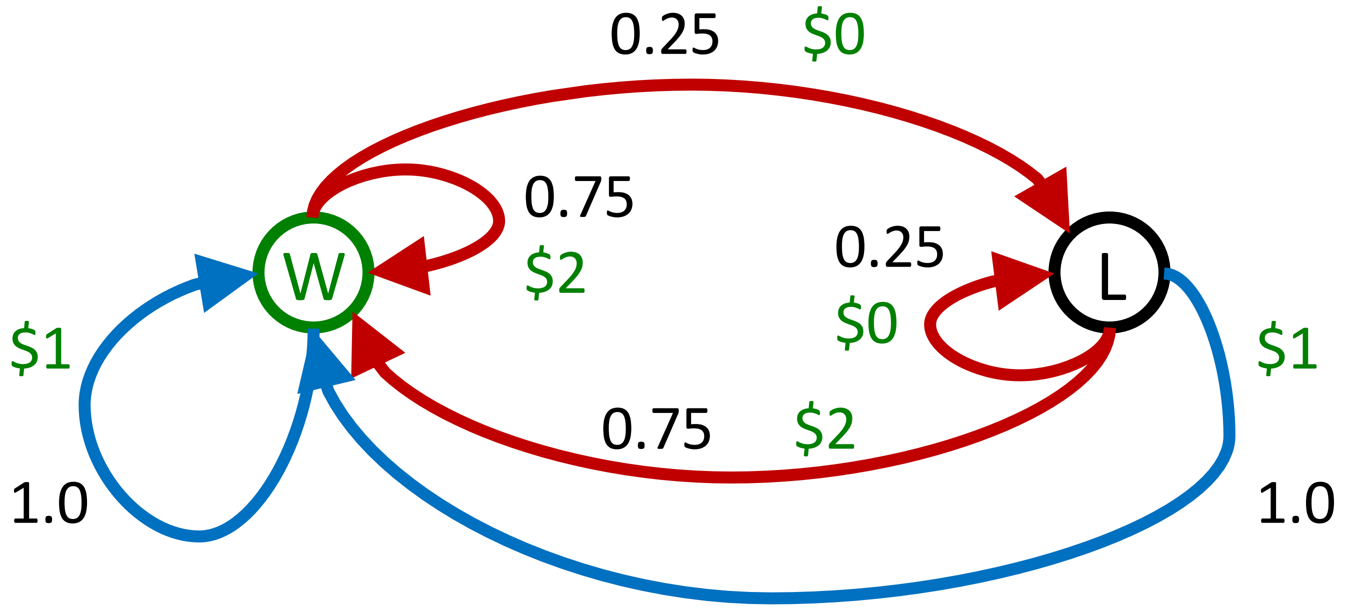


Offline Planning

- Solving MDPs is offline planning
  - You determine all quantities through computation
  - You need to know the details of the MDP
  - You do not actually play the game!

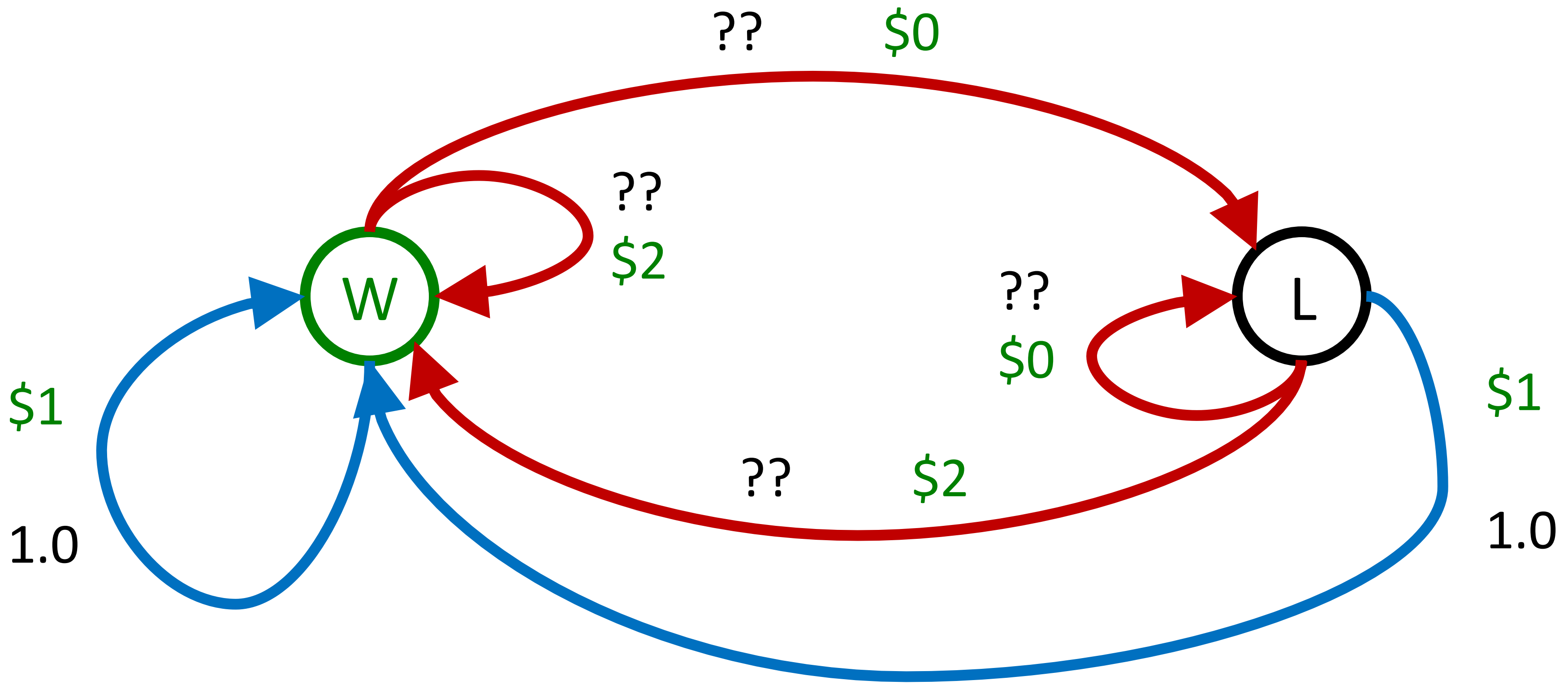
No discount
100 time steps

	Value
Play Red	150
Play Blue	100

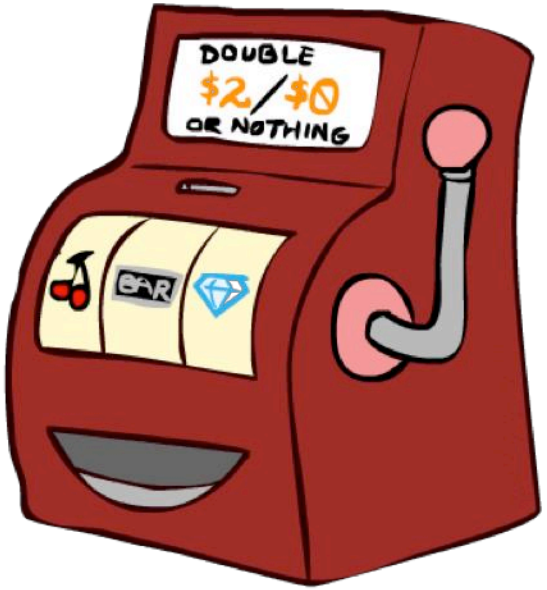


Online Planning

- Rules changed! Red's win chance is different.



Let’s Play!



$0 $0 $0 $2 $0

$2 $0 $0 $0 $0

What Just Happened?

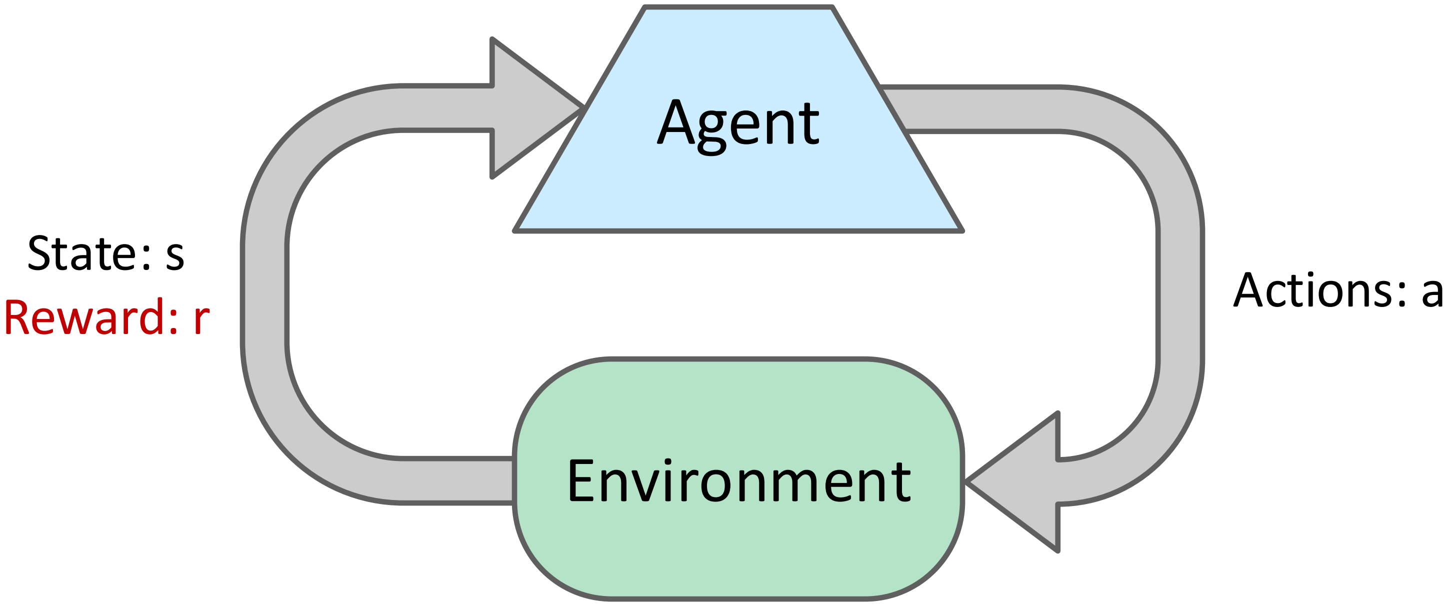
- That wasn't planning, it was learning!
  - Specifically, reinforcement learning
  - There was an MDP, but you couldn't solve it with just computation
  - You needed to actually act to figure it out

Reinforcement Learning

- Still assume a Markov decision process (MDP):
  1. A set of states $s \in S$
  2. A set of actions (per state) A
  3. A model $T(s,a,s')$
  4. A reward function $R(s,a,s')$
- Still looking for a policy $\pi(s)$

- New twist: don't know T or R
  - I.e. we don't know which states are good or what the actions do
  - Must actually try actions and states out to learn

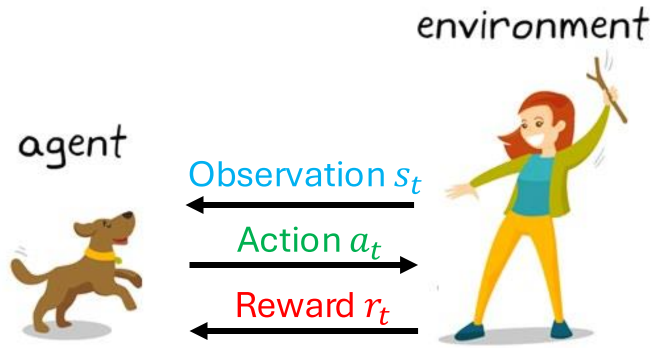
Reinforcement Learning



- Basic idea:
  - Receive feedback in the form of rewards
  - Agent's utility is defined by the reward function
  - Must (learn to) act so as to maximize expected rewards
  - All learning is based on observed samples of outcomes!

Basis of reinforcement learning*

- Markov decision process (MDP) assumption
- At timestep t, agent following a policy $\pi_\omega(s_t)$,
  - Obtains an observation s_t of the surrounding environment,
  - produces action a_t,
  - then, environment will transmit to s_{t+1},
  - and agent will receive reward r_t,



  - The goal of the learning agent is to *maximize* the expected cumulative rewards as

$$R = \mathrm{E}_\pi\left[\sum_{t=0}^{T-1} r_t(s_t, a_t)\right]$$

Interaction data stored in the transition dataset.

$$\{s_0, a_0, r_0, s_1, a_1, r_1, \dots, \underbrace{s_t, a_t, r_t, s_{t+1}}_{\text{Transition}}, \dots s_{T-1}, a_{T-1}, r_{T-1}, \}$$

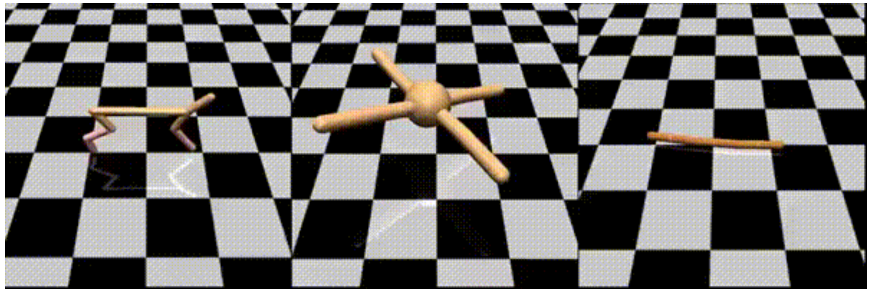
* Episodic environment with limited timesteps

Reinforcement Learning

- What if the MDP is initially unknown? Lots of things change!
  - ***Exploration***: you have to ***try unknown actions*** to get information
  - ***Exploitation***: eventually, you have to use what you know
  - ***Regret***: early on, you inevitably "make mistakes" and lose reward
  - ***Sampling***: you may need to repeat many times to get good estimates
  - ***Generalization***: what you learn in one state may apply to others too

Practical examples of sequential decision making

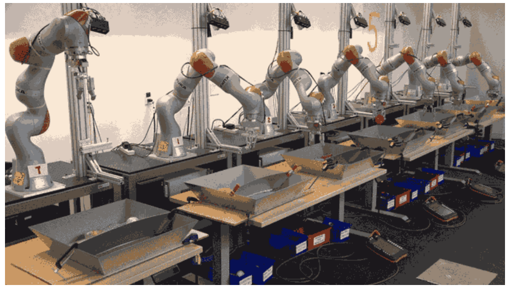
Logistics system



MuJoCo Robot Control

Video Games

Intelligent financial investment

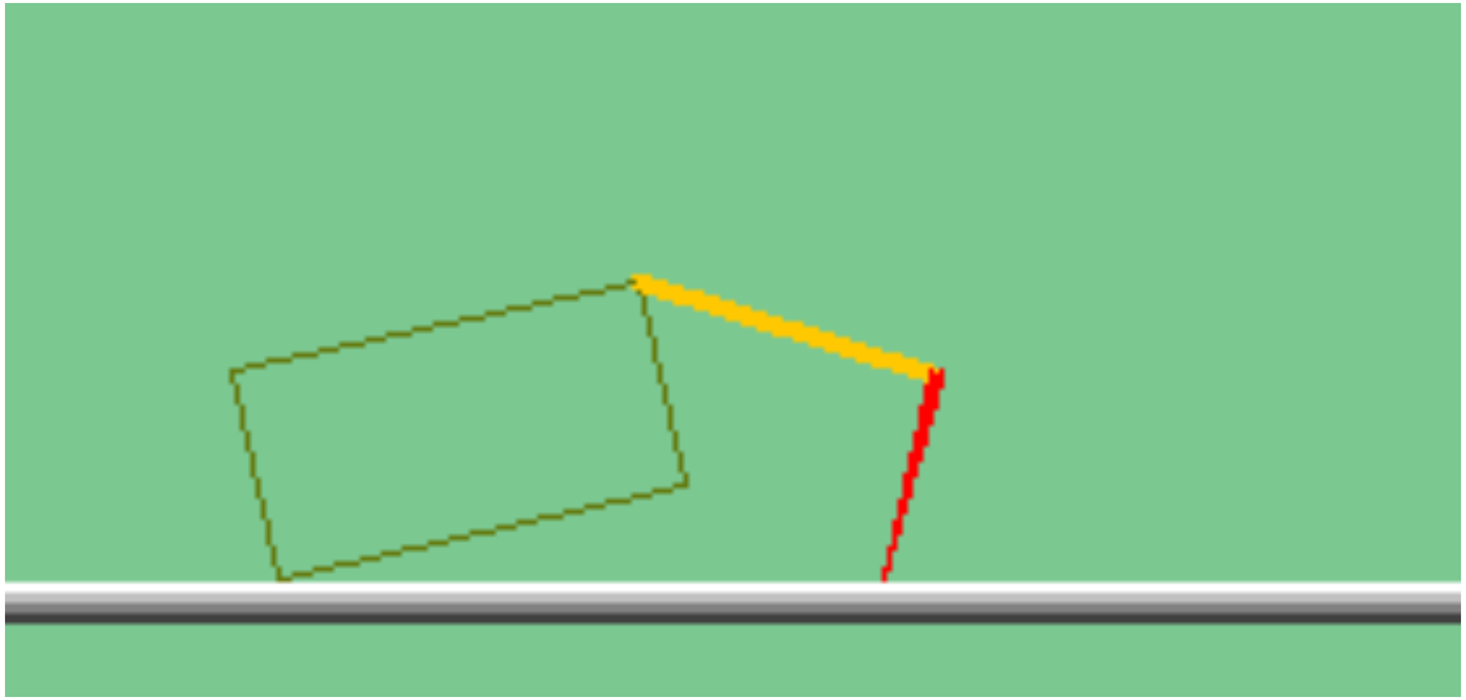


Industrial production

Recommendation and ads

Cheetah

The Crawler!



[Demo: Crawler Bot (L10D1)] [You, in Project 6]

Example: Learning to Walk

Initial

[Kohl and Stone, ICRA 2004]

[Video: AIBO WALK – initial]

Example: Learning to Walk

Finished

[Kohl and Stone, ICRA 2004]

[Video: AIBO WALK – finished]

Example: Breakout (DeepMind)

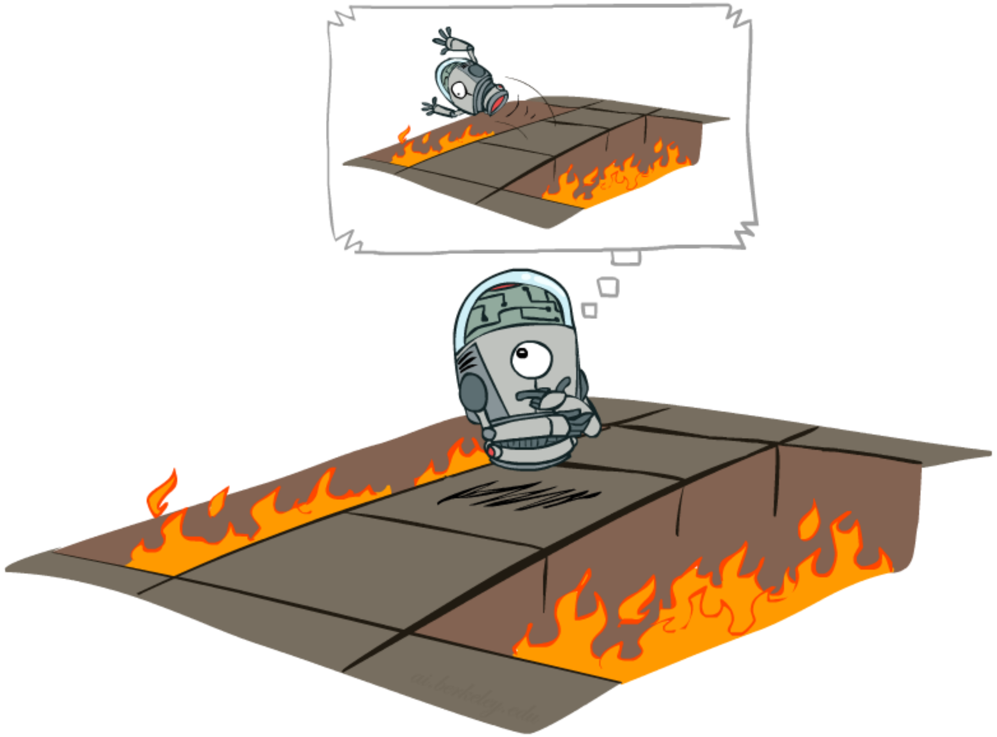
[© TwoMinuteLectures]

Example: AlphaGo (2016)

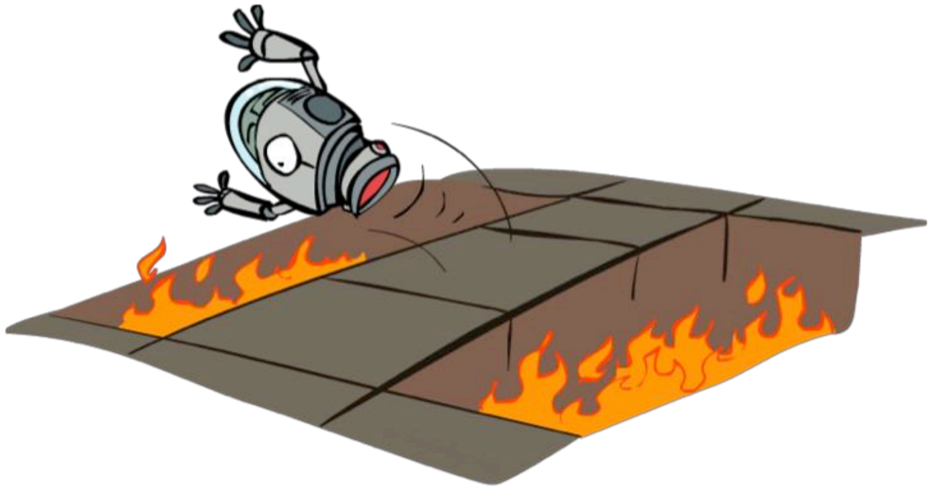
Approaches to reinforcement learning

1. Model-based: Learn the model, solve it, execute the solution
2. Learn values from experiences, use to make decisions
   a. Direct evaluation
   b. Temporal difference learning
   c. Q-learning
3. Optimize the policy directly

Offline (MDPs) vs. Online (RL)

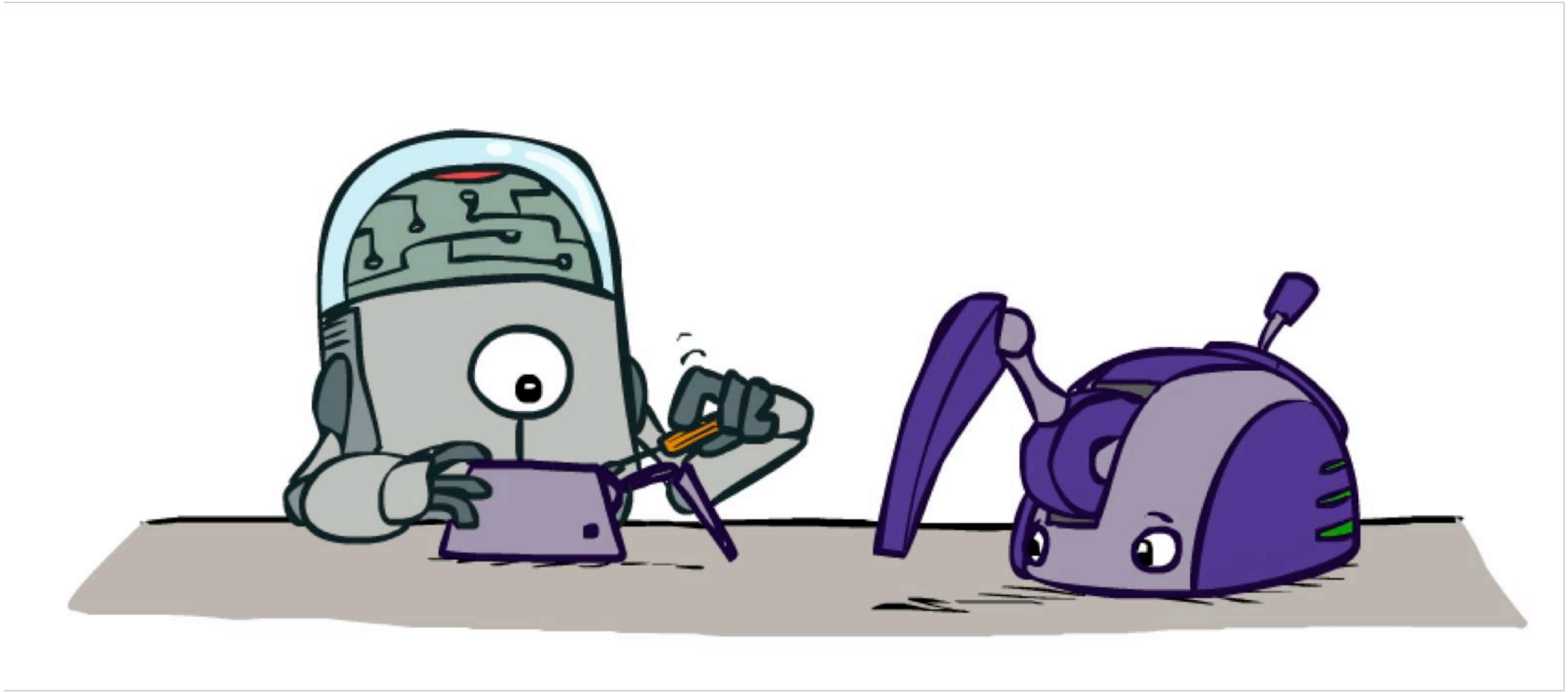


Offline Solution



Online Learning

Model-Based RL

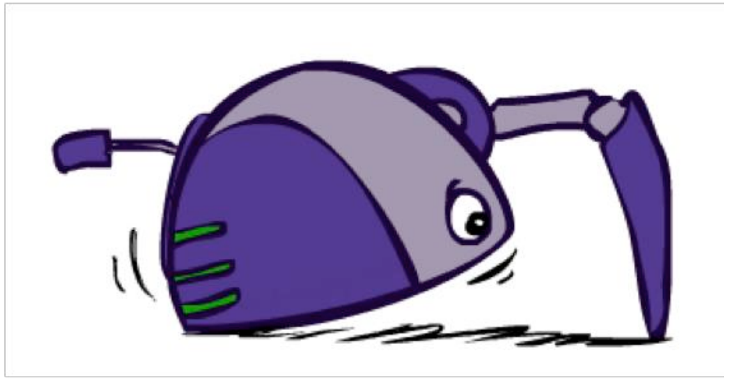


Model-Based Learning

- Model-Based Idea:
  - Learn an approximate model based on experiences
  - Solve for values as if the learned model were correct

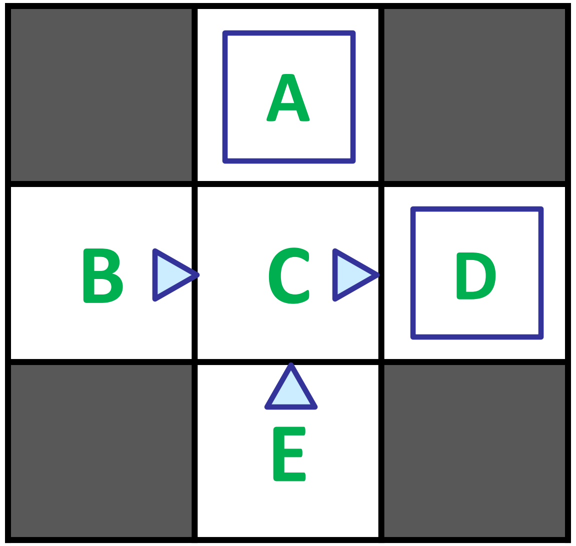
- Step 1: Learn empirical MDP model
  - Count outcomes *s'* for each *s*, *a*
  - Directly estimate each entry in *T(s,a,s')* from counts
  - Discover each *R(s,a,s')* when we experience the transition

- Step 2: Solve the learned MDP
  - Use, e.g., value or policy iteration, as before



Example: Model-Based Learning

Input Policy π



Assume: $\gamma = 1$

Observed Episodes (Training)

Episode 1

B, east, C, -1
C, east, D, -1
D, exit, x, +10

Episode 2

B, east, C, -1
C, east, D, -1
D, exit, x, +10

Episode 3

E, north, C, -1
C, east, D, -1
D, exit, x, +10

Episode 4

E, north, C, -1
C, east, A, -1
A, exit, x, -10

Learned Model

T(s,a,s')

T(B, east, C) = 1.00
T(C, east, D) = 0.75
T(C, east, A) = 0.25
…

R(s,a,s')

R(B, east, C) = -1
R(C, east, D) = -1
R(D, exit, x) = +10
…

Pros and cons

- Pro:
  - Makes efficient use of experiences (low ***sample complexity***)
- Con:
  - May not scale to large state spaces
    - Solving MDP is intractable for very large $|S|$
  - RL feedback loop tends to magnify small model errors
  - Much harder when the environment is partially observable

Basic idea of model-free methods

- To approximate expectations with respect to a distribution, you can either
  - Estimate the distribution from samples, compute an expectation
  - Or, bypass the distribution and estimate the expectation from samples directly

Model-Based vs. Model-Free

Goal: Compute expected age of ShanghaiTech students

Known P(A)

$$E[A] = \sum_a P(a) \cdot a \qquad = 0.35 \times 20 + \ldots$$

Without P(A), instead collect samples [a_1, a_2, … a_N]

Unknown P(A): "Model Based"

$$\hat{P}(a) = \frac{\text{num}(a)}{N}$$

$$E[A] \approx \sum_a \hat{P}(a) \cdot a$$

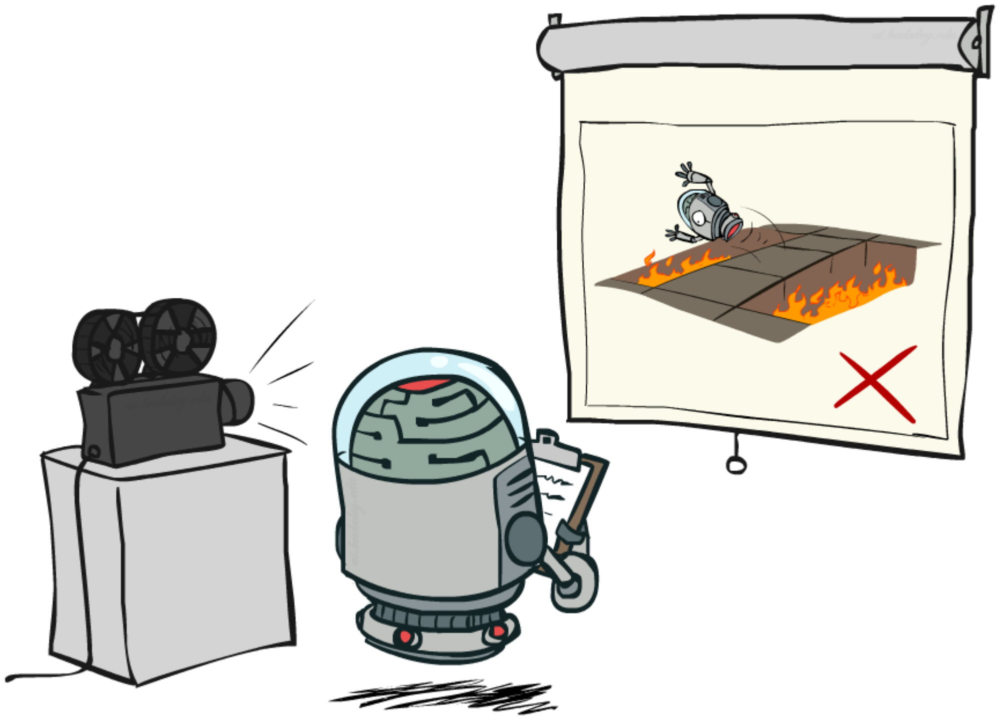
Why does this work? Because eventually you learn the right model.

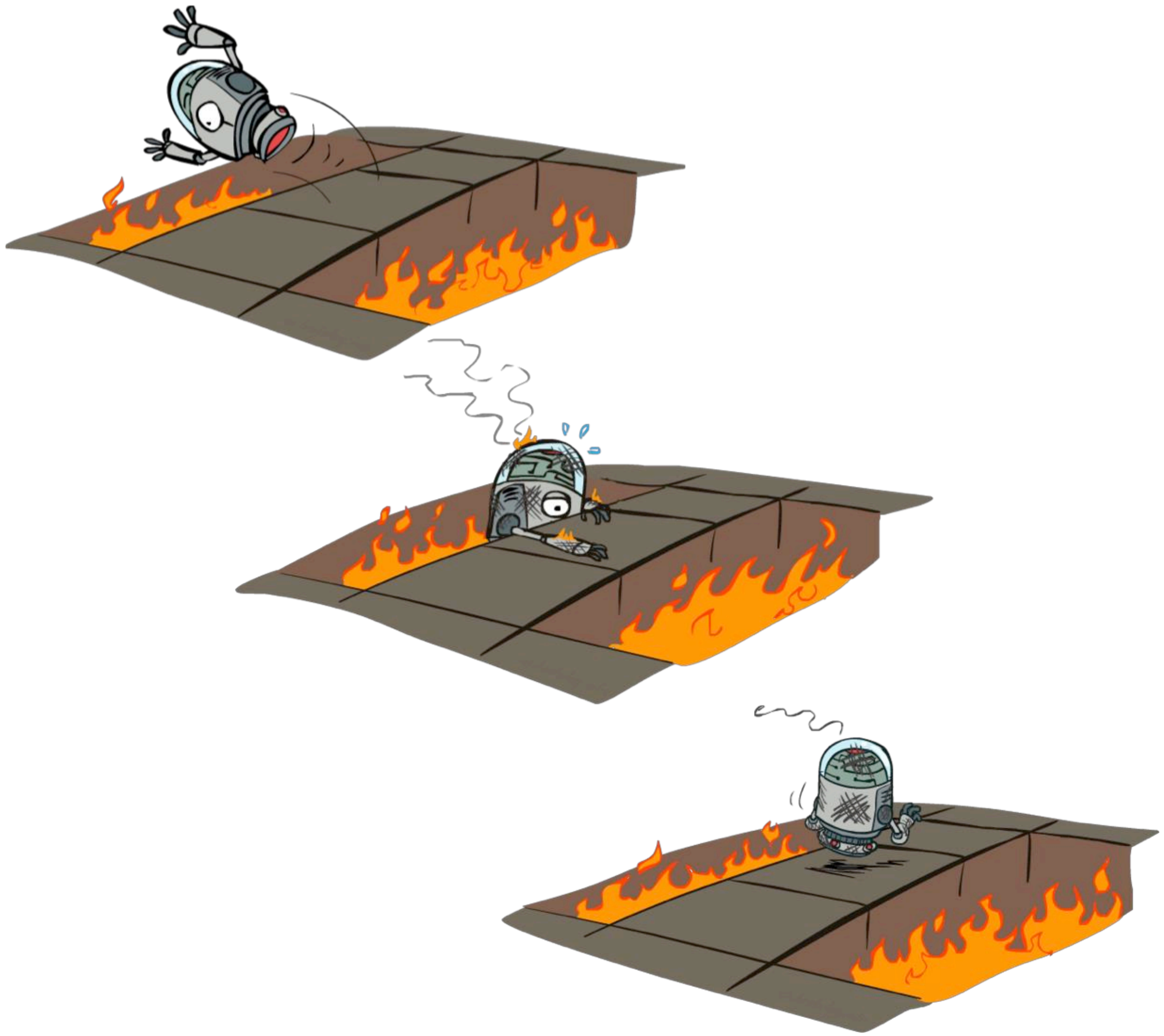
Unknown P(A): "Model Free"

$$E[A] \approx \frac{1}{N} \sum_i a_i$$

Why does this work? Because samples appear with the right frequencies.

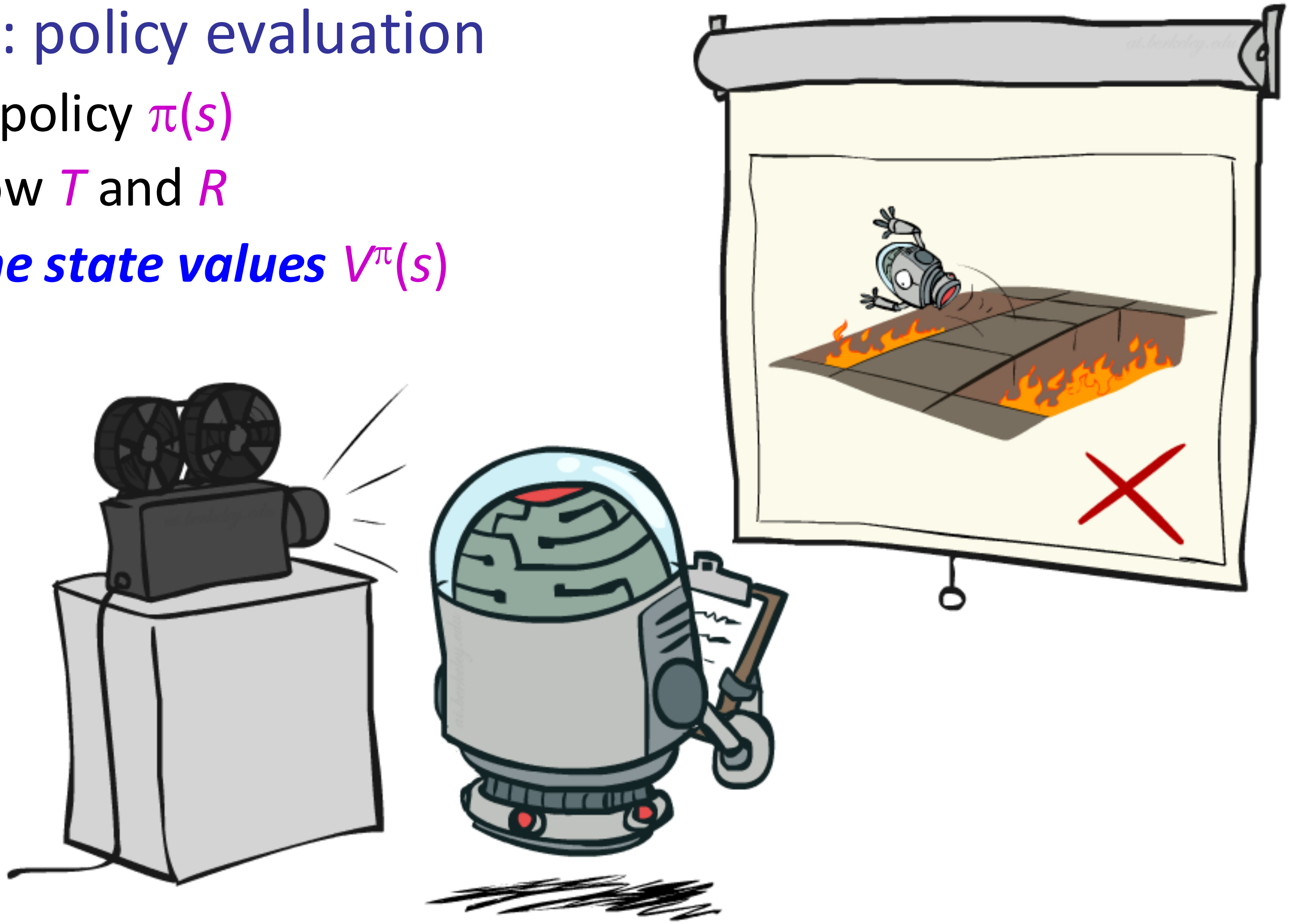
Passive vs Active Reinforcement Learning





Passive Reinforcement Learning

- Simplified task: policy evaluation
  - Input: a fixed policy $\pi(s)$
  - You don't know T and R
  - ***Goal: learn the state values*** $V^{\pi}(s)$

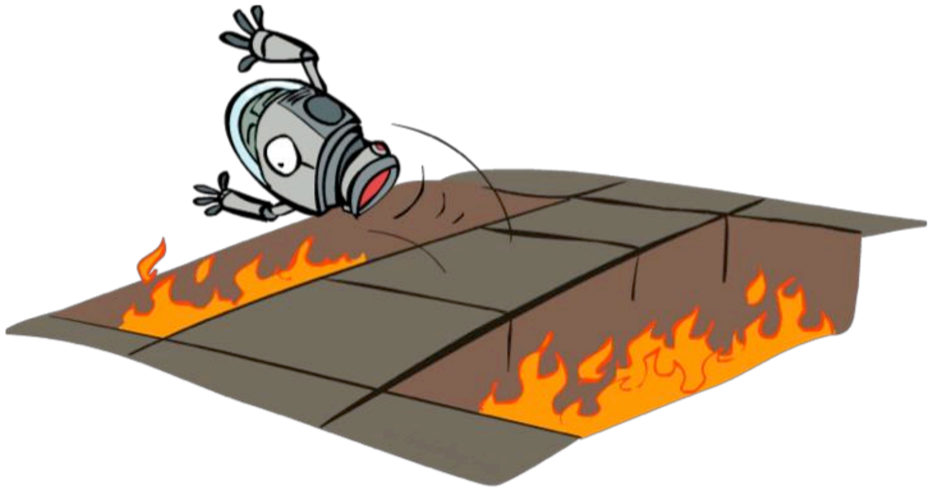


Direct evaluation

- Goal: Estimate $V^{\pi}(s)$, i.e., expected total discounted reward from s onwards

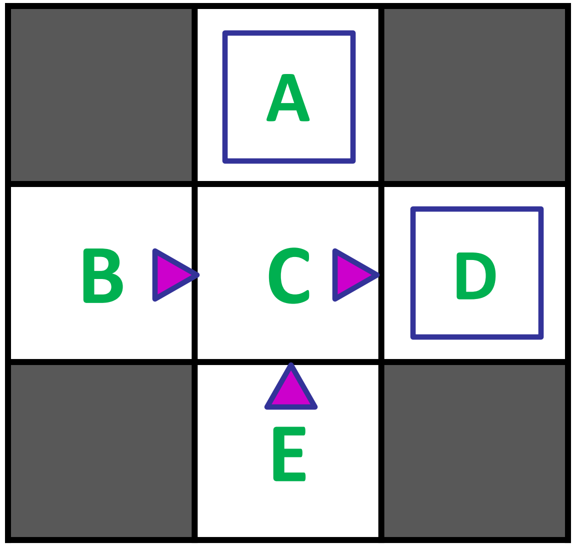
- Idea:
  - Use ***returns***, the *actual* sums of discounted rewards from s
  - Average over multiple trials and visits to s

- This is called ***direct evaluation*** (or direct utility estimation)



Example: Direct Estimation

Input Policy π



Assume: $\gamma = 1$

Observed Episodes (Training)

Episode 1

B, east, C, -1
C, east, D, -1
D, exit, x, +10

Episode 2

B, east, C, -1
C, east, D, -1
D, exit, x, +10

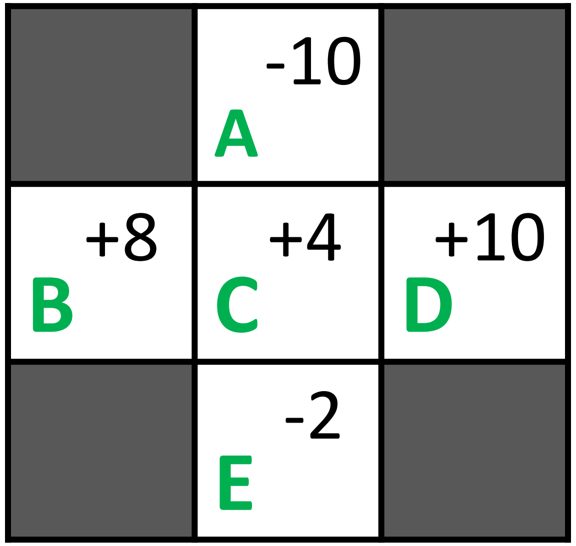
Episode 3

E, north, C, -1
C, east, D, -1
D, exit, x, +10

Episode 4

E, north, C, -1
C, east, A, -1
A, exit, x, -10

Output Values

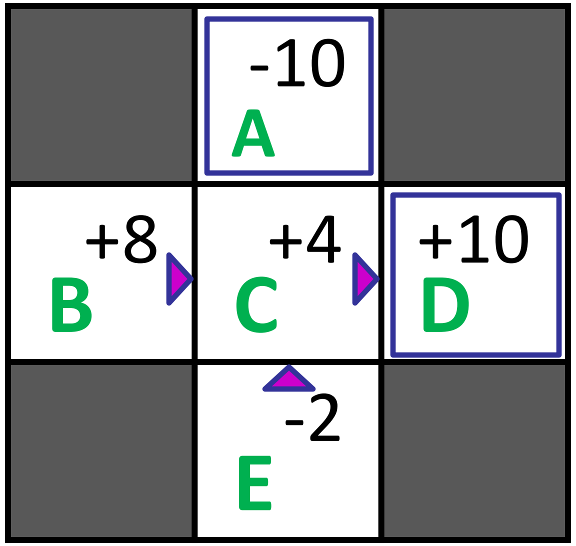


Problems with Direct Estimation

- What's good about direct estimation?
  - It's easy to understand
  - It doesn't require any knowledge of *T* and *R*
  - It converges to the right answer in the limit

- What's bad about it?
  - Each state must be learned separately (fixable)
  - It ***ignores information about state connections***
  - So, it takes a long time to learn

Output Values



If B and E both go to C under this policy, how can their values be different?

E.g., B=at home, study hard
E=at library, study hard
C=know material, go to exam

Approaches to reinforcement learning

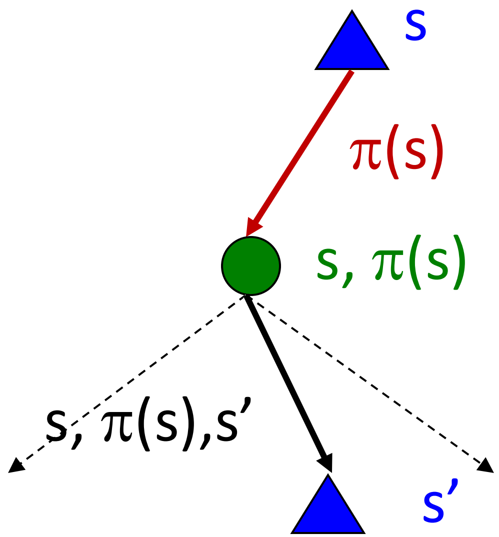
1. Model-based: Learn the model, solve it, execute the solution
2. Learn values from experiences, use to make decisions
   a. Direct evaluation
   b. Temporal difference learning
   c. Q-learning
3. Optimize the policy directly

Why Not Use Policy Evaluation?

- Simplified Bellman updates calculate V for a fixed policy:

$$V_0^\pi(s) = 0$$

$$V_{k+1}^\pi(s) \leftarrow \sum_{s'} T(s,\pi(s),s')[R(s,\pi(s),s') + \gamma V_k^\pi(s')]$$

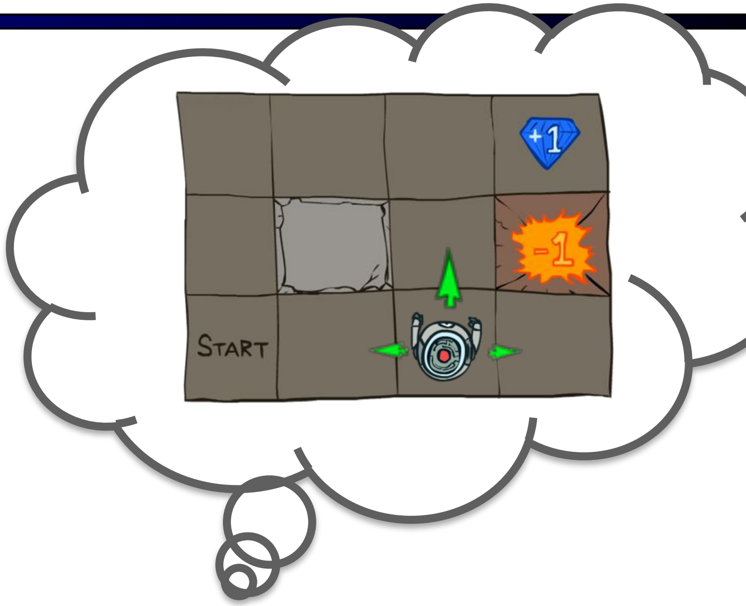


  - This approach fully exploits the connections between the states
  - Unfortunately, we need T and R to do it!

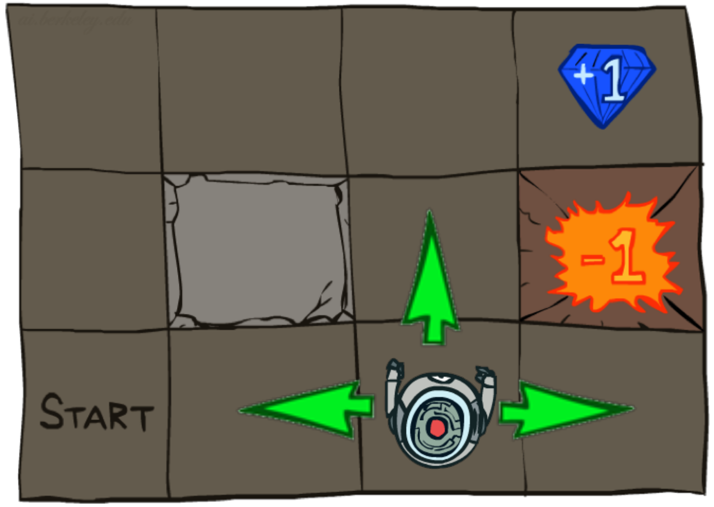
- Key question: how can we do this update to V without knowing T and R?
  - In other words, how do we take a weighted average without knowing the weights?

Sample-Based Policy Evaluation?

- Given a fixed policy, the value of a state is an expectation over next-state values:
  - $V^{\pi}(s) = \sum_{s'} T(s,\pi(s),s') \, [R(s,\pi(s),s') + \gamma V^{\pi}(s')]$



- Idea 1: Use actual samples to estimate the expectation:
  - $\text{sample}_1 = R(s,\pi(s),s_1') + \gamma V^{\pi}(s_1')$
  - $\text{sample}_2 = R(s,\pi(s),s_2') + \gamma V^{\pi}(s_2')$
  - …
  - $\text{sample}_N = R(s,\pi(s),s_N') + \gamma V^{\pi}(s_N')$
  - $V^{\pi}(s) \leftarrow 1/N \sum_i \text{sample}_i$



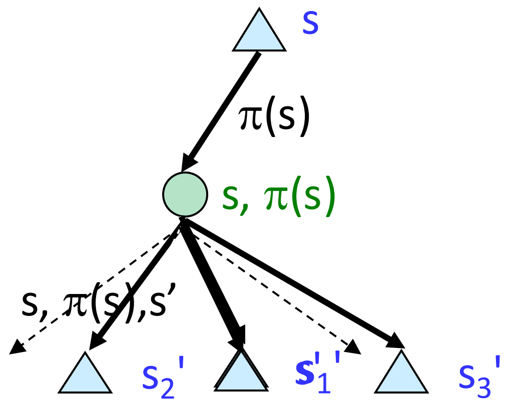
Sample-Based Policy Evaluation?

- We want to compute these averages:

$$V_{k+1}^{\pi}(s) \leftarrow \sum_{s'} T(s,\pi(s),s')[R(s,\pi(s),s') + \gamma V_k^{\pi}(s')]$$

- Idea: Take samples of outcomes s' (by doing the action!) and average

$$sample_1 = R(s,\pi(s),s_1') + \gamma V_k^{\pi}(s_1')$$
$$sample_2 = R(s,\pi(s),s_2') + \gamma V_k^{\pi}(s_2')$$
$$\dots$$
$$sample_n = R(s,\pi(s),s_n') + \gamma V_k^{\pi}(s_n')$$



$$V_{k+1}^{\pi}(s) \leftarrow \frac{1}{n}\sum_i sample_i$$

Any problems?

But we can't rewind time to get sample after sample from state s!

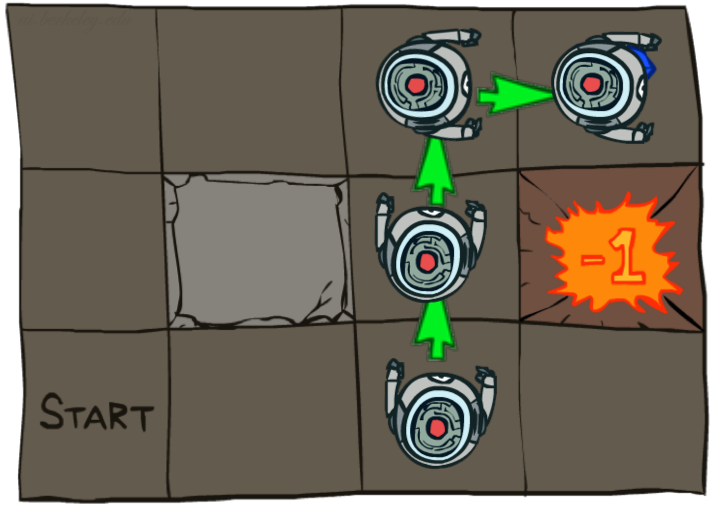
Sample-Based Policy Evaluation?

- Idea 2: Update value of s after each transition s,a,s',r :

- Update V^{π} ([3,1]) based on *R*([3,1],up,[3,2]) and γV^{π}([3,2])
- Update V^{π} ([3,2]) based on *R*([3,2],up,[3,3]) and γV^{π}([3,3])
- Update V^{π} ([3,3]) based on *R*([3,3],right,[4,3]) and γV^{π}([4,3])

Any problems?

One sample estimation may not be accurate.
Early estimation function may not be accurate.



Sample-Based Policy Evaluation?

- Idea 3: Update values by maintaining a ***running average***

Running averages

- How do you compute the average of 1, 4, 7?
- Method 1: add them up and divide by N
  - 1+4+7 = 12
  - average = 12/N = 12/3 = 4
- Method 2: keep a running average μ_n and a running count n
  - $n=0 \quad \mu_0=0$
  - $n=1 \quad \mu_1 = (0 \cdot \mu_0 + x_1)/1 = (0 \cdot 0 + 1)/1 = 1$
  - $n=2 \quad \mu_2 = (1 \cdot \mu_1 + x_2)/2 = (1 \cdot 1 + 4)/2 = 2.5$
  - $n=3 \quad \mu_3 = (2 \cdot \mu_2 + x_3)/3 = (2 \cdot 2.5 + 7)/3 = 4$
  - General formula: $\mu_n = ((n-1) \cdot \mu_{n-1} + x_n)/n$
  - $= [(n-1)/n]\, \mu_{n-1} + [1/n]\, x_n$ (weighted average of old mean, new sample)

Running averages contd.

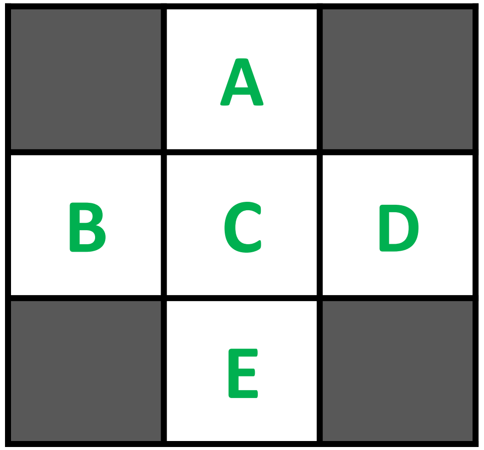
- What if we use a weighted average with a fixed weight?
  - $\mu_n = (1-\alpha)\,\mu_{n-1} + \alpha\, x_n$
  - n=1 $\mu_1 = x_1$
  - n=2 $\mu_2 = (1-\alpha) \cdot \mu_1 + \alpha x_2 = (1-\alpha) \cdot x_1 + \alpha x_2$
  - n=3 $\mu_3 = (1-\alpha) \cdot \mu_2 + \alpha x_3 = (1-\alpha)^2 \cdot x_1 + \alpha(1-\alpha)x_2 + \alpha x_3$
  - n=4 $\mu_4 = (1-\alpha) \cdot \mu_3 + \alpha x_4 = (1-\alpha)^3 \cdot x_1 + \alpha(1-\alpha)^2 x_2 + \alpha(1-\alpha)x_3 + \alpha x_4$
- I.e., ***exponential forgetting*** of old values
- μ_n is a convex combination of sample values (weights sum to 1)
- $E[\mu_n]$ is a convex combination of $E[X_i]$ values, hence unbiased

TD as approximate Bellman update

- Idea 3: Update values by maintaining a ***running average***
  - sample = $R(s,\pi(s),s') + \gamma V^{\pi}(s')$
  - $V^{\pi}(s) \leftarrow (1-\alpha) \cdot V^{\pi}(s) + \alpha \cdot \text{sample}$
  - $V^{\pi}(s) \leftarrow V^{\pi}(s) + \alpha \cdot [\text{sample} - V^{\pi}(s)]$
  - This is the ***temporal difference learning rule***
  - $[\text{sample} - V^{\pi}(s)]$ is the "TD error"
  - α is the ***learning rate***
- Observe a sample, move $V^{\pi}(s)$ a little bit to make it more consistent with its neighbor $V^{\pi}(s')$

Example: Temporal Difference Learning

States

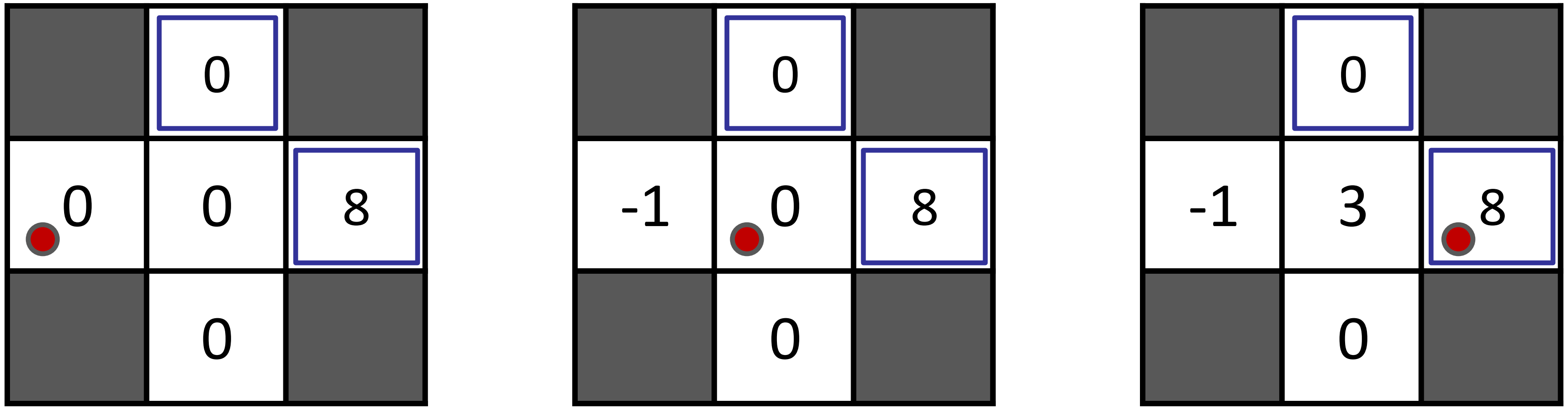


Assume: $\gamma = 1$, $\alpha = 1/2$

Observed Transitions

B, east, C, -2

C, east, D, -2



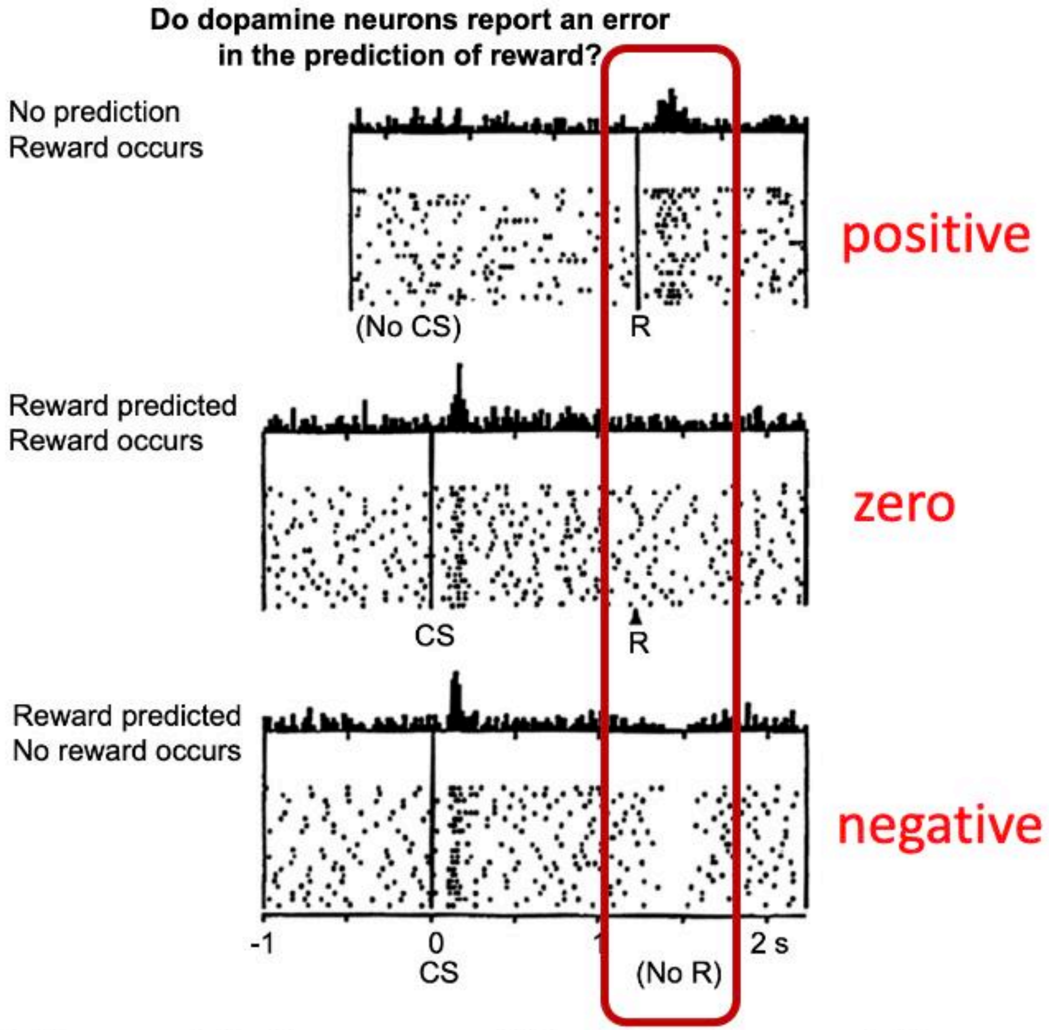
$$V^{\pi}(s) \leftarrow (1-\alpha)\, V^{\pi}(s) + \alpha \cdot [R(s,\pi(s),s') + \gamma V^{\pi}(s')]$$

TD Learning Happens in the Brain!

- Neurons transmit *Dopamine* to encode reward or value prediction error:

  $$\delta_i = \left(r_i + \gamma V^\pi(s_i')\right) - V^\pi(s_i).$$

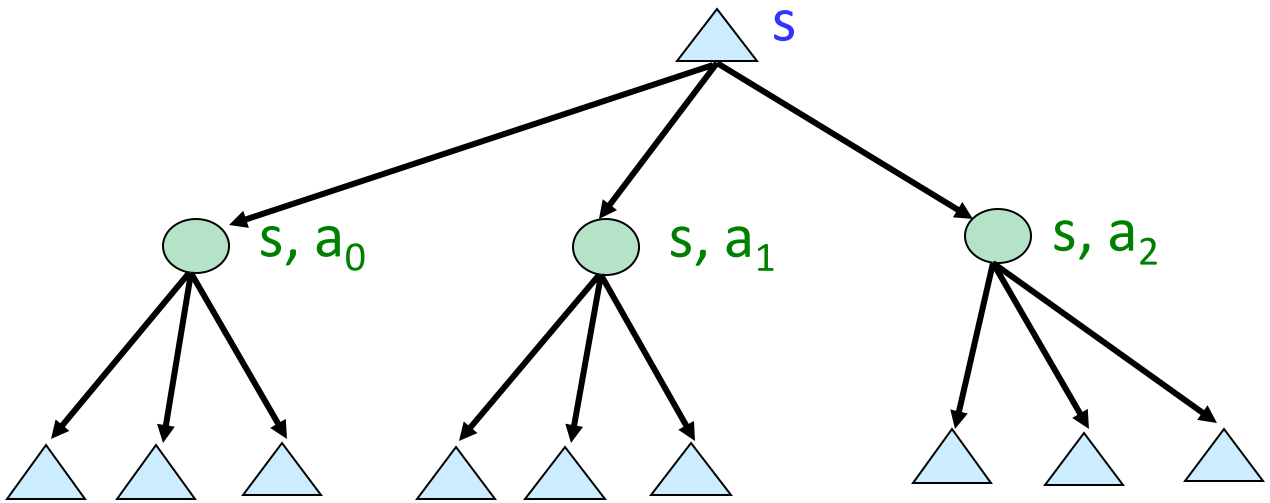
- Example of Neuroscience & AI informing each other



[A Neural Substrate of Prediction and Reward. Schultz, Dayan, Montague. 1997]

Problems with TD Value Learning

- Model-free policy evaluation! 🎉
- Bellman updates with running sample mean! 🎉



- Need the transition model to improve the policy! 😱

Detour: Q-Value Iteration

- Value iteration: find successive (depth-limited) values
  - Start with $V_0(s) = 0$, which we know is right
  - Given V_k, calculate the depth k+1 values for all states:

$$V_{k+1}(s) \leftarrow \max_a \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma V_k(s') \right]$$

- But Q-values are more useful, so compute them instead
  - Start with $Q_0(s,a) = 0$, which we know is right
  - Given Q_k, calculate the depth (k+1) q-values for all q-states:

$$Q_{k+1}(s,a) \leftarrow \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma \max_{a'} Q_k(s',a') \right]$$

Q-learning as approximate Q-iteration

- Recall the definition of Q values:
  - $Q^*(s,a)$ = expected return from doing a in s and then behaving *optimally* thereafter; and $\pi^*(s) = \max_a Q^*(s,a)$
- Bellman equation for Q values:
  - $Q^*(s,a) = \sum_{s'} T(s,a,s')[R(s,a,s') + \gamma \max_{a'} Q^*(s',a')]$
- *Approximate* Bellman update for Q values:
  - $Q(s,a) \leftarrow (1-\alpha) \cdot Q(s,a) + \alpha \cdot [R(s,a,s') + \gamma \max_{a'} Q(s',a')]$
- We obtain a policy from learned $Q(s,a)$, with no model!
  - (No free lunch: $Q(s,a)$ table is $|A|$ times bigger than $V(s)$ table)

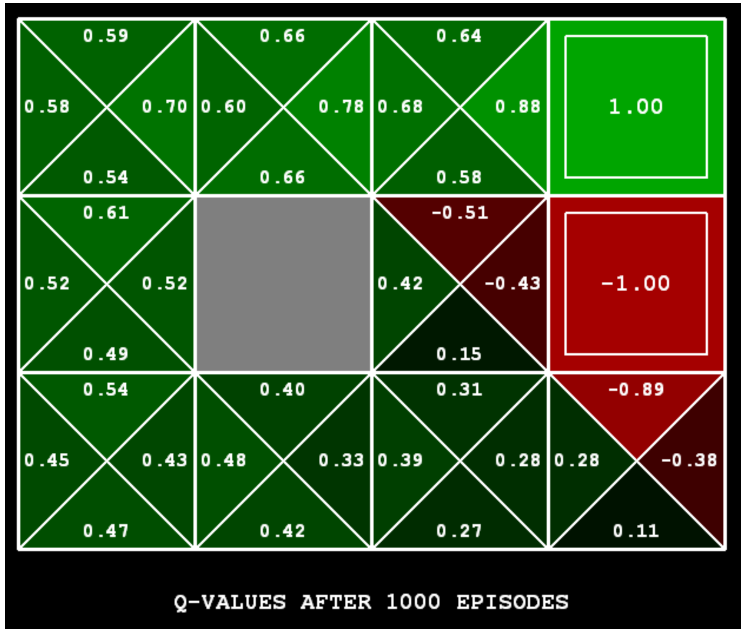
Q-Learning

- Learn Q(s,a) values as you go
  - Receive a sample (s,a,s',r)
  - Consider your old estimate: $Q(s,a)$
  - Consider your new sample estimate:

    $$\textit{sample} = R(s,a,s') + \gamma \max_{a'} Q(s',a')$$

  - Incorporate the new estimate into a running average:

    $$Q(s,a) \leftarrow (1-\alpha)\, Q(s,a) + \alpha \cdot [\textit{sample}]$$



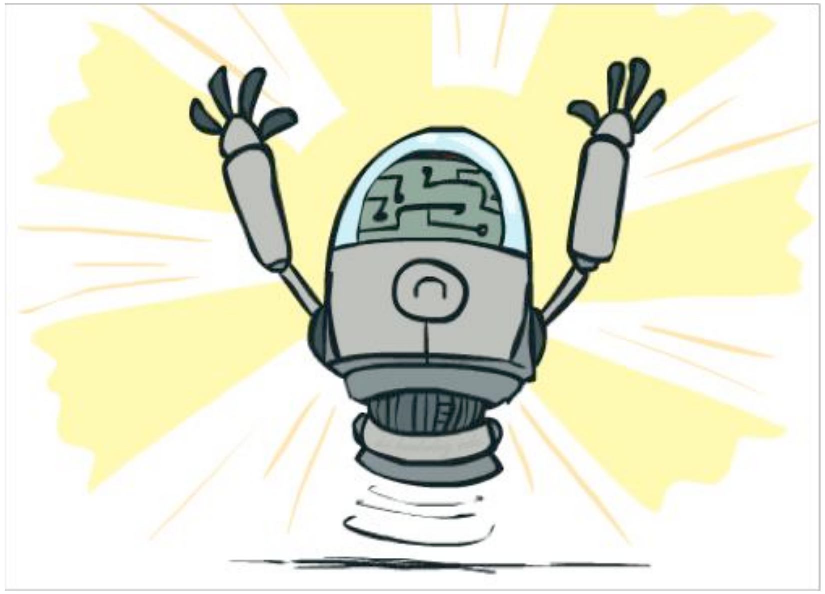
[Demo: Q-learning – gridworld (L10D2)]
[Demo: Q-learning – crawler (L10D3)]

Video of Demo Q-Learning -- Gridworld

Video of Demo Q-Learning -- Crawler

Q-Learning Properties

- Amazing result: Q-learning converges to optimal policy -- even if samples are generated from a suboptimal policy!
- This is called off-policy learning
- Caveats:
  - You have to explore enough
  - You have to eventually make the learning rate small enough
  - … but not decrease it too quickly
  - Basically, in the limit, it doesn't matter how you select actions (!)



The Story So Far: MDPs and RL

Known MDP: Offline Solution

Goal	Technique
Compute V*, Q*, π*	Value / policy iteration
Evaluate a fixed policy π	Policy evaluation

Unknown MDP: Model-Based

Goal	Technique
Compute V*, Q*, π*	VI/PI on approx. MDP
Evaluate a fixed policy π	PE on approx. MDP

Unknown MDP: Model-Free

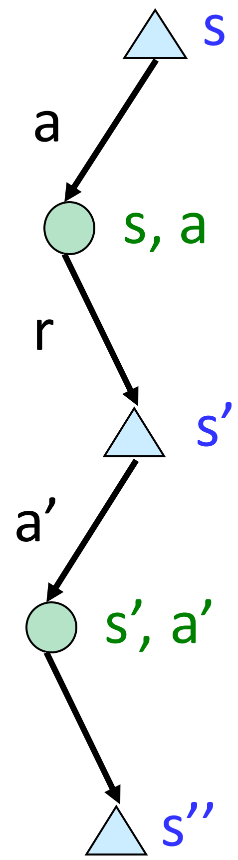
Goal	Technique
Compute V*, Q*, π*	Q-learning
Evaluate a fixed policy π	TD Value Learning

Model-Free Learning

- Model-free (temporal difference) learning
  - Experience world through episodes

$$(s, a, r, s', a', r', s'', a'', r'', s'''' \ldots)$$

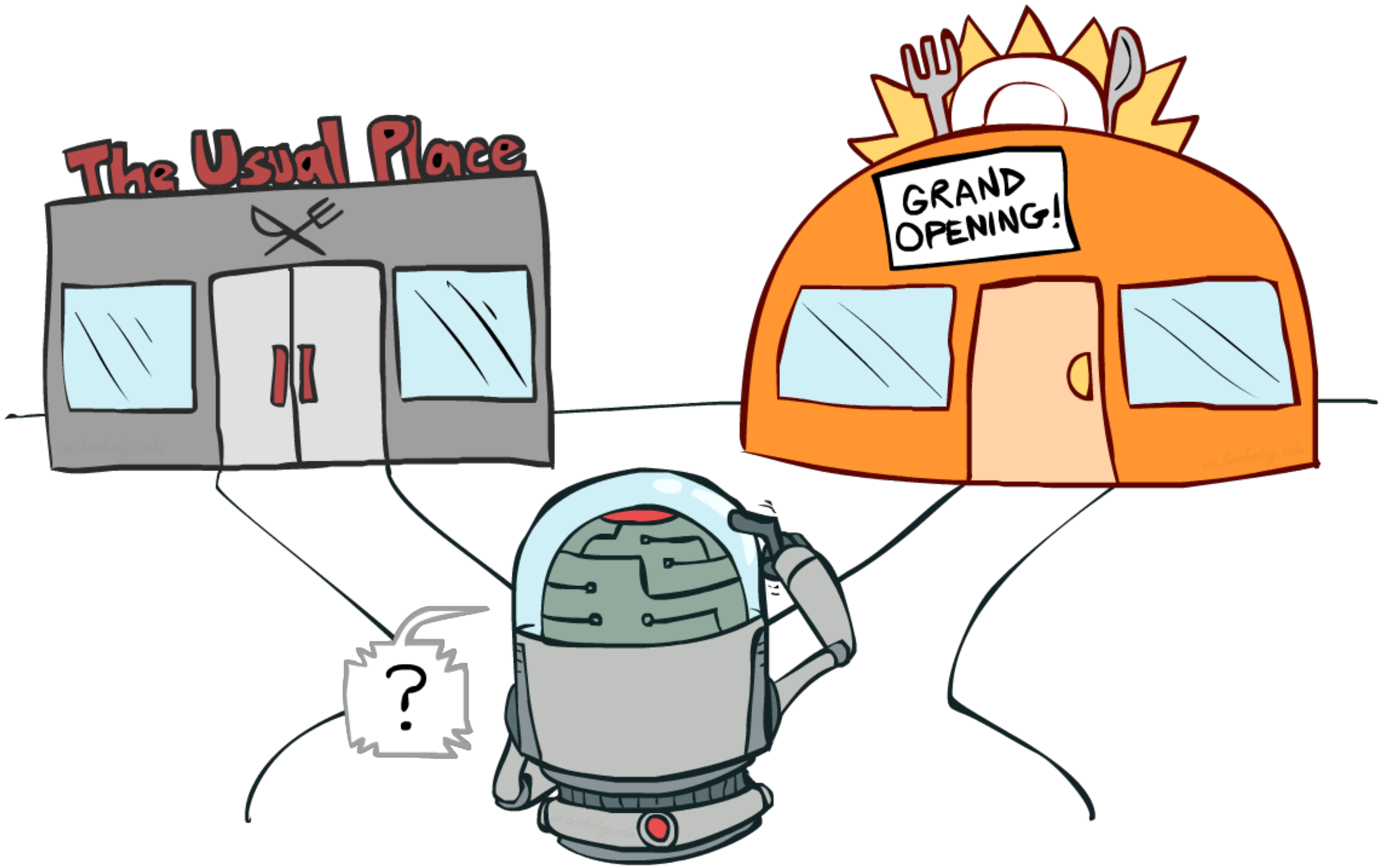
  - Update estimates each transition (s, a, r, s')
  - Over time, updates will mimic Bellman updates



Summary of previous RL knowledge

- RL solves MDPs via direct experience of transitions and rewards
- There are several approaches:
  - Learn the MDP model and solve it
  - Learn V directly from sums of rewards, or by TD local adjustments
    - Still need a model to make decisions by lookahead
  - Learn Q by local Q-learning adjustments, use it directly to pick actions
  - (and about 100 other variations)
- Big missing pieces:
  - How to explore without too much regret?
  - How to scale this up to Tetris (10^{60}), Go (10^{172}), StarCraft ($|A|=10^{26}$)?

Exploration vs. Exploitation



Exploration vs. Exploitation

- ***Exploration***: try new things
- ***Exploitation***: do what's best given what you've learned so far
- Key point: pure exploitation often gets ***stuck in a rut*** and never finds an optimal policy!

Exploration method 1: ε-greedy

- ε-greedy exploration
  - Every time step, flip a biased coin
  - With (small) probability ε, act randomly
  - With (large) probability 1-ε, act on current policy

- Properties of ε-greedy exploration
  - Every s,a pair is tried infinitely often
  - Does a lot of stupid things
    - Jumping off a cliff *lots of times* to make sure it hurts
  - Keeps doing stupid things for ever
    - Decay ε towards 0

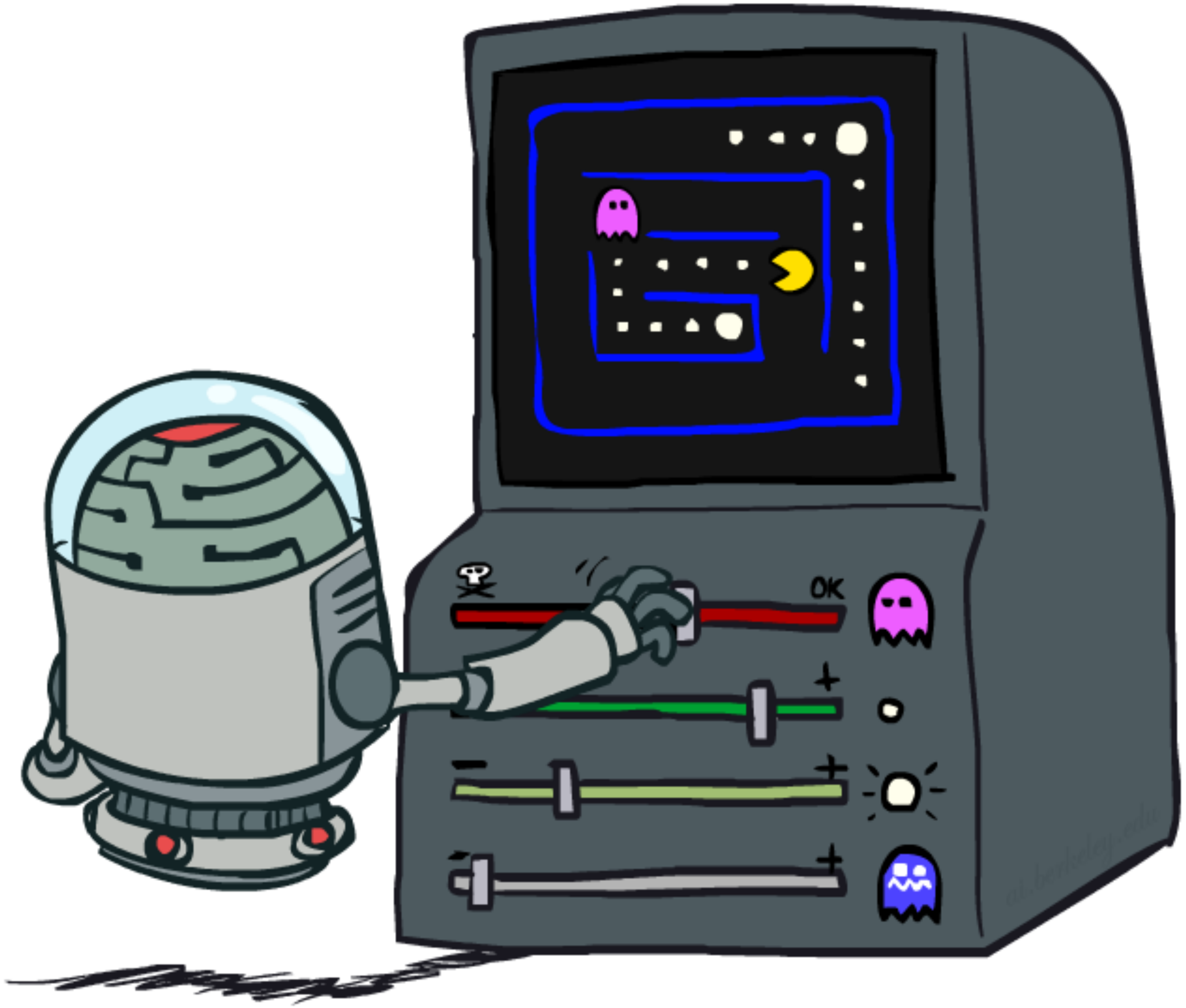
Demo Q-learning – Epsilon-Greedy – Crawler

Method 2: Optimistic Exploration Functions

- ***Exploration functions*** implement this tradeoff
  - Takes a value estimate u and a visit count n, and returns an optimistic utility, e.g., $f(u,n) = u + k/\sqrt{n}$
- Regular Q-update:
  - $Q(s,a) \leftarrow (1-\alpha) \cdot Q(s,a) + \alpha \cdot [R(s,a,s') + \gamma \max_a Q(s',a)]$
- Modified Q-update:
  - $Q(s,a) \leftarrow (1-\alpha) \cdot Q(s,a) + \alpha \cdot [R(s,a,s') + \gamma \max_a f(Q(s',a'),n(s',a'))]$
- Note: this propagates the "bonus" back to states that lead to unknown states as well!

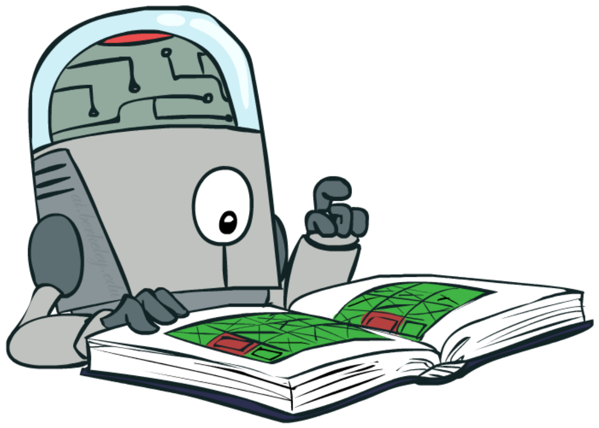
Demo Q-learning – Exploration Function – Crawler

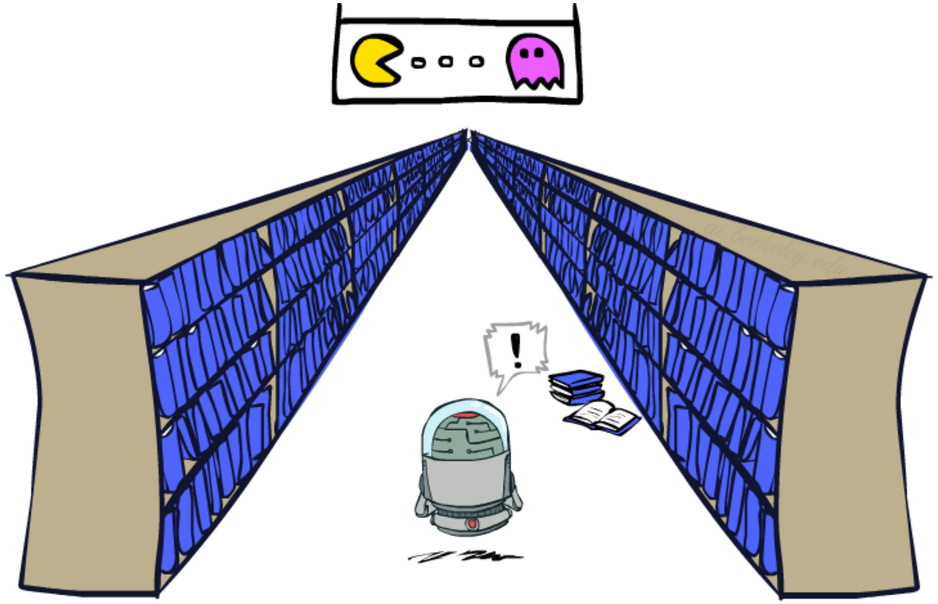
Approximate Q-Learning



Generalizing Across States

- Basic Q-Learning keeps a table of all Q-values
- In realistic situations, we cannot possibly learn about every single state!
  - Too many states to visit them all in training
  - Too many states to hold the Q-tables in memory
- Instead, we want to generalize:
  - Learn about some small number of training states from experience
  - Generalize that experience to new, similar situations
  - Can we apply some *machine learning* tools to do this?

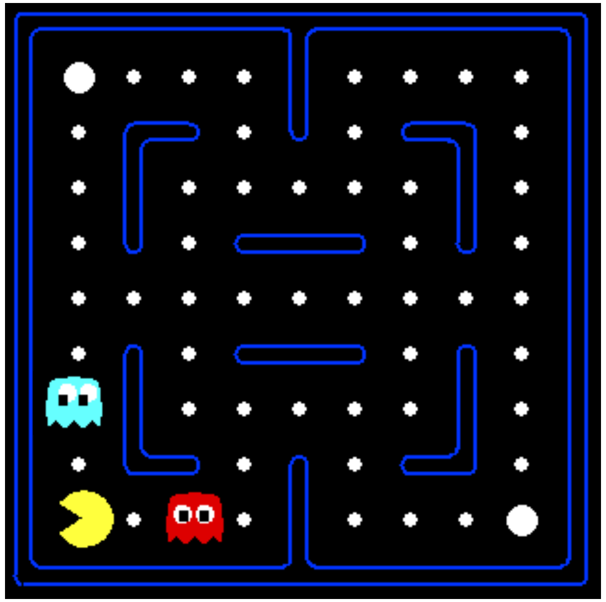




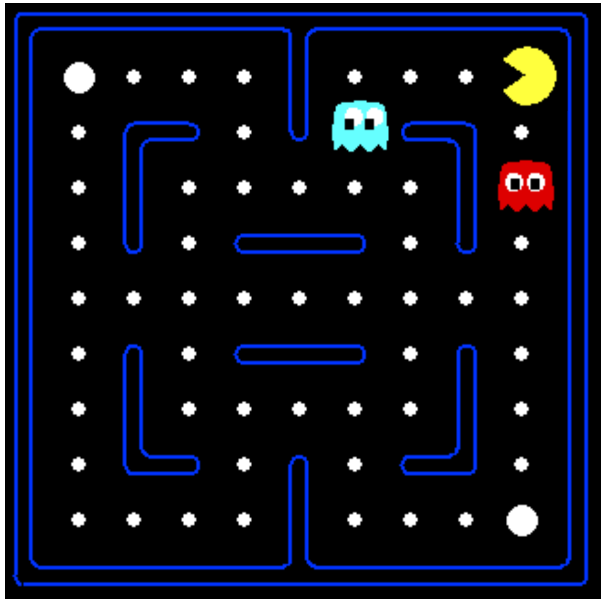
[demo – RL pacman]

Example: Pacman

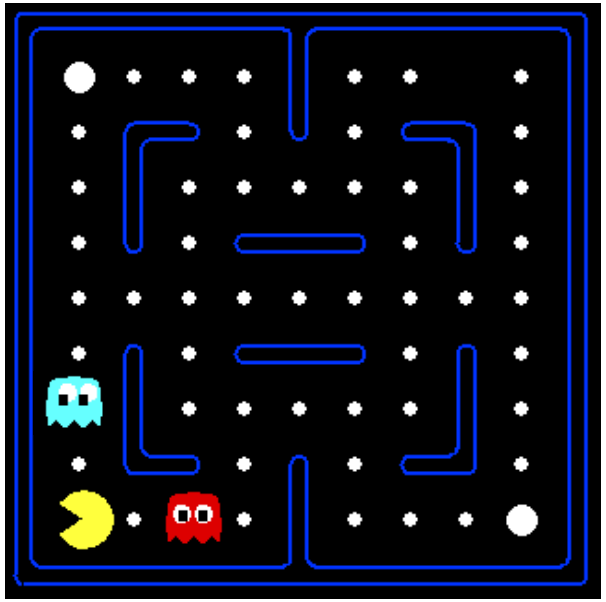
Let's say we discover through experience that this state is bad:



In naïve q-learning, we know nothing about this state:



Or even this one!



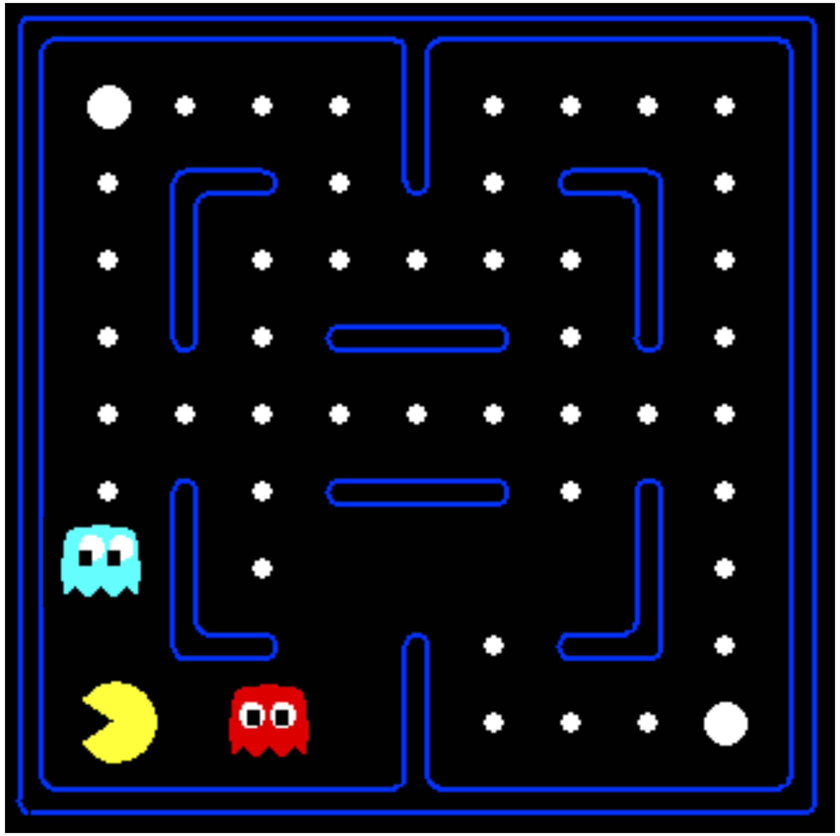
Demo Q-Learning Pacman – Tiny – Watch All

Demo Q-Learning Pacman – Tiny – Silent Train

Demo Q-Learning Pacman – Tricky – Watch All

Feature-Based Representations

- Solution: describe a state using a vector of ***features***
  - Features are functions from states to real numbers (often 0/1) that capture important properties of the state
  - Example features:
    - Distance to closest ghost f_{GST}
    - Distance to closest dot
    - Number of ghosts
    - 1 / (distance to closest dot) f_{DOT}
    - Is Pacman in a tunnel? (0/1)
    - …… etc.
  - Can also describe a q-state (s, a) with features (e.g., action moves closer to food)



Linear Value Functions

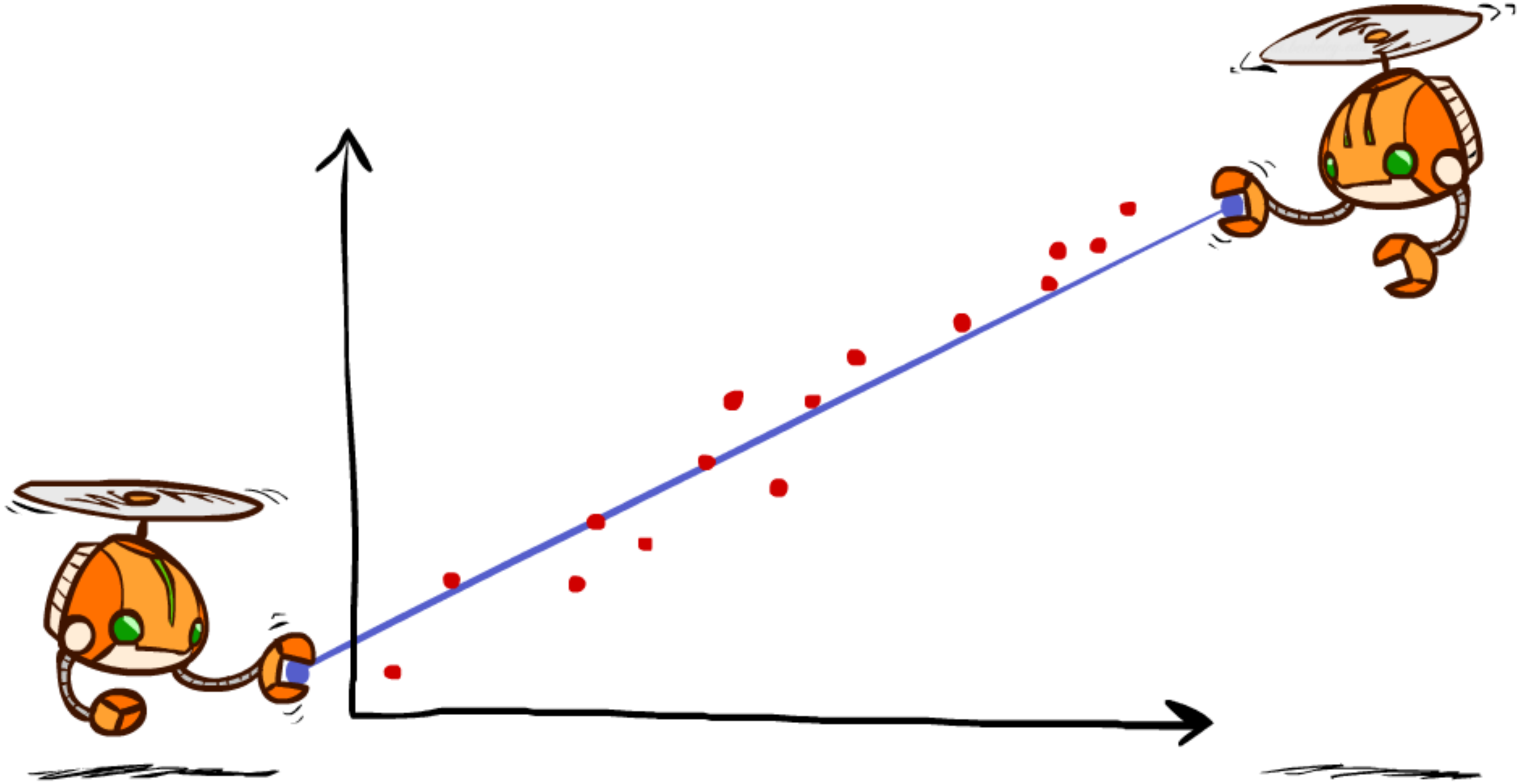
- We can express V and Q (approximately) as weighted linear functions of feature values:
  - $V_{\mathbf{w}}(s) = w_1 f_1(s) + w_2 f_2(s) + \ldots + w_n f_n(s)$
  - $Q_{\mathbf{w}}(s,a) = w_1 f_1(s,a) + w_2 f_2(s,a) + \ldots + w_n f_n(s,a)$
- Advantage: our experience is summed up in a few powerful numbers
  - Can compress a value function for chess (10^{43} states) down to about 30 weights!
- Disadvantage: states may share features but have very different expected utility!

Updating a linear value function

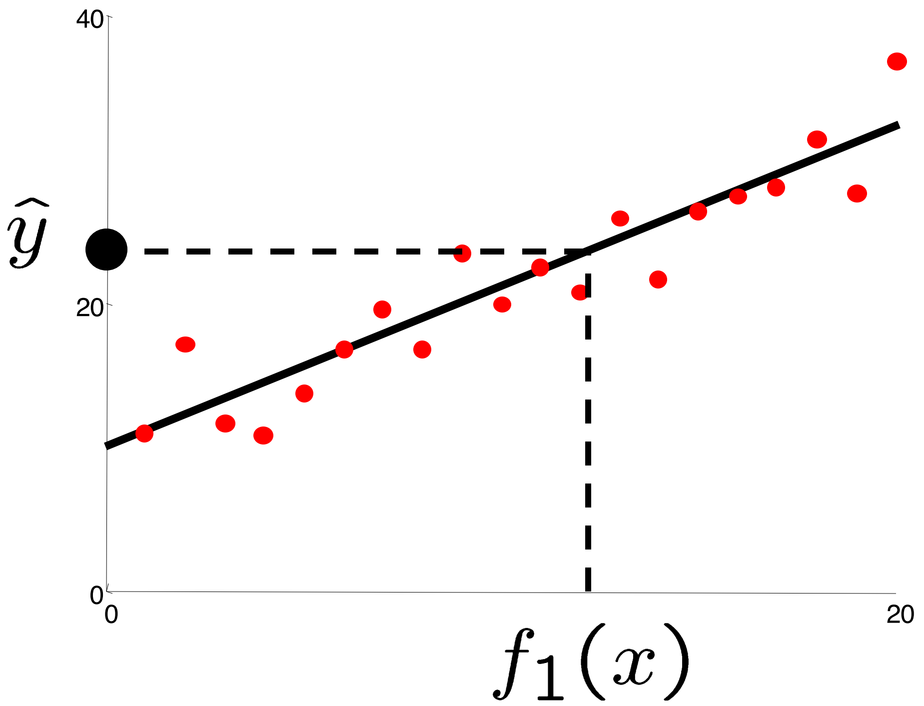
- Original Q-learning rule tries to reduce prediction error at s,a:
  - $Q(s,a) \leftarrow Q(s,a) + \alpha \cdot [R(s,a,s') + \gamma \max_{a'} Q(s',a') - Q(s,a)]$
- Instead, we update the weights to try to reduce the error at s,a:
  - $w_i \leftarrow w_i + \alpha \cdot [R(s,a,s') + \gamma \max_{a'} Q(s',a') - Q(s,a)] \, \partial Q_{\mathbf{w}}(s,a)/\partial w_i$

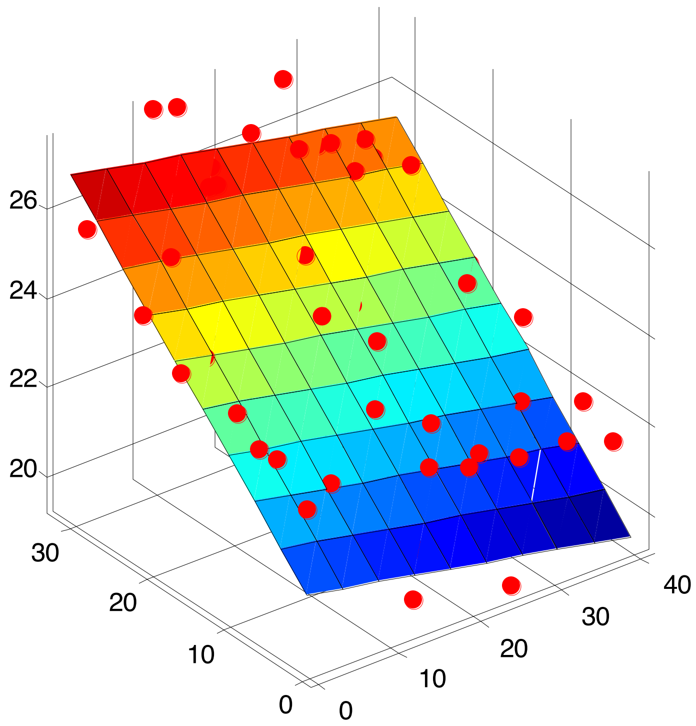
    $= w_i + \alpha \cdot [R(s,a,s') + \gamma \max_{a'} Q(s',a') - Q(s,a)] \, f_i(s,a)$
- Intuitive interpretation:
  - Adjust weights of active features
  - If something bad happens, blame the features we saw; decrease value of states with those features. If something good happens, increase value!

Q-Learning and Least Squares



Linear Approximation: Regression*





Prediction:

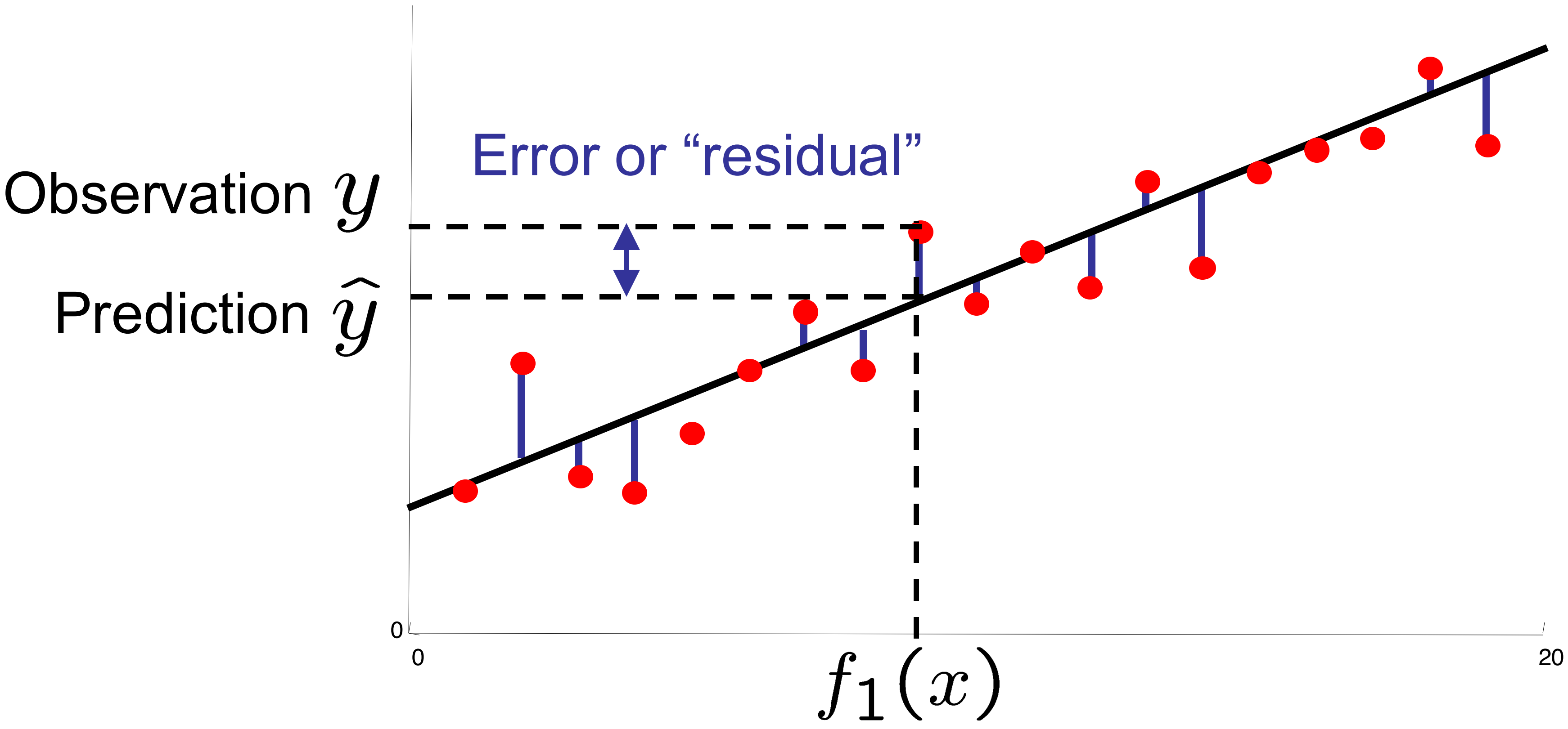
$$\hat{y} = w_0 + w_1 f_1(x)$$

Prediction:

$$\hat{y}_i = w_0 + w_1 f_1(x) + w_2 f_2(x)$$

Optimization: Least Squares*

$$\text{total error} = \sum_i (y_i - \hat{y}_i)^2 = \sum_i \left(y_i - \sum_k w_k f_k(x_i) \right)^2$$



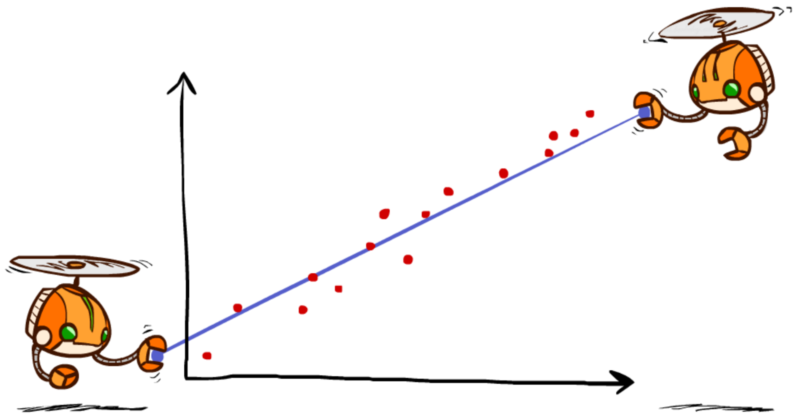
Minimizing Error*

Imagine we had only one point x, with features f(x), target value y, and weights w:

$$\text{error}(w) = \frac{1}{2}\left(y - \sum_k w_k f_k(x)\right)^2$$

$$\frac{\partial\ \text{error}(w)}{\partial w_m} = -\left(y - \sum_k w_k f_k(x)\right) f_m(x)$$

$$w_m \leftarrow w_m + \alpha \left(y - \sum_k w_k f_k(x)\right) f_m(x)$$



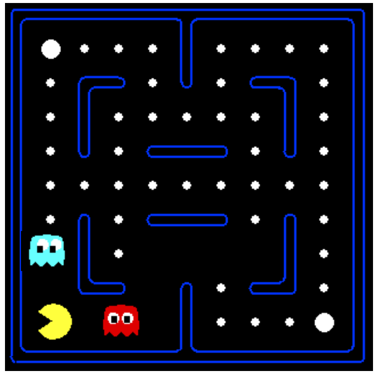
Approximate q update explained:

$$w_m \leftarrow w_m + \alpha \left[\underbrace{r + \gamma \max_a Q(s',a')}_{\text{“target”}} - \underbrace{Q(s,a)}_{\text{“prediction”}}\right] f_m(s,a)$$

Example: Q-Pacman

$Q(s,a) = 4.0\, f_{DOT}(s,a) - 1.0\, f_{GST}(s,a)$

s

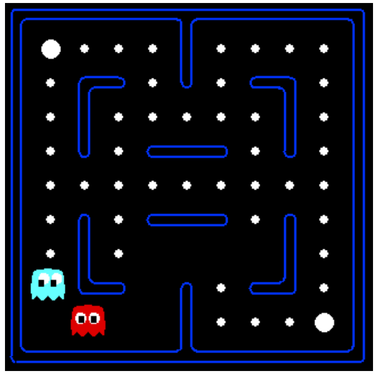


$f_{DOT}(s,\text{NORTH}) = 0.5$

$f_{GST}(s,\text{NORTH}) = 1.0$

a = NORTH
r = –500

s'



$Q(s,\text{NORTH}) = +1$

$r + \gamma \max_{a'} Q(s',a') = -500 + 0$

$Q(s',\cdot) = 0$

difference = –501

$w_{DOT} \leftarrow 4.0 + \alpha[-501]0.5$

$w_{GST} \leftarrow -1.0 + \alpha[-501]1.0$

$Q(s,a) = 3.0\, f_{DOT}(s,a) - 3.0\, f_{GST}(s,a)$

Demo Approximate Q-Learning -- Pacman

More Powerful Functions

Linear: $Q(s,a) = w_1 f_1(s,a) + w_2 f_2(s,a) + \ldots + w_n f_n(s,a)$

Polynomial: $Q(s,a) = w_{11} f_1(s,a) + w_{12} f_1(s,a)^2 + w_{13} f_1(s,a)^3 + \ldots$

Neural network: $Q(s,a) = w_1 f_1(s,a) + w_2 f_2(s,a) + \ldots + w_n f_n(s,a)$

learn these too

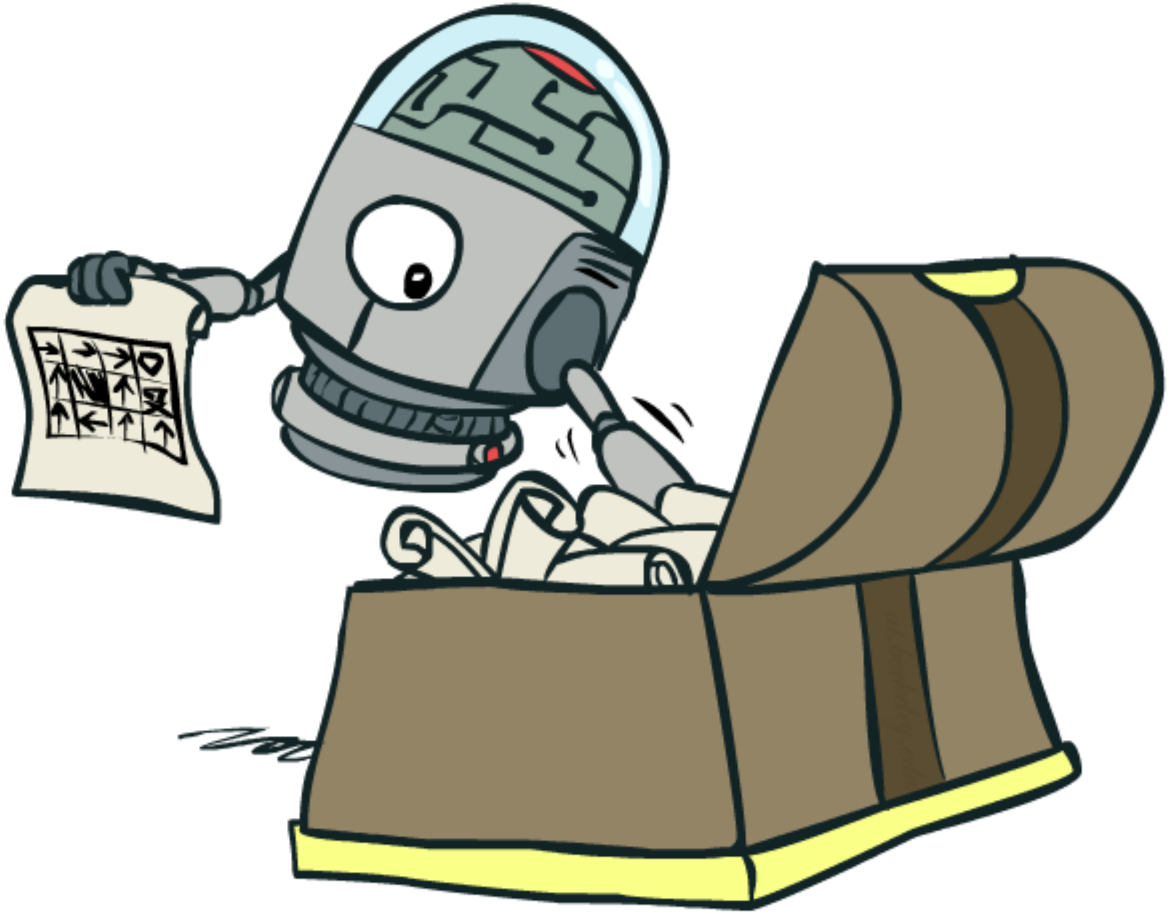
$$w_m \leftarrow w_m + \alpha \left[r + \gamma \max_a Q(s',a') - Q(s,a) \right] \frac{dQ}{dw_m}(s,a)$$

$= f_m(s,a)$ in linear case

Approaches to reinforcement learning

1. Model-based: Learn the model, solve it, execute the solution
2. Learn values from experiences, use to make decisions
   a. Direct evaluation
   b. Temporal difference learning
   c. Q-learning
3. Optimize the policy directly

Policy Search



Policy Search

- Problem: often the feature-based policies that work well (win games, maximize utilities) aren't the ones that approximate V / Q best
  - E.g. your value functions from project 2 were probably horrible estimates of future rewards, but they still produced good decisions
  - Q-learning's priority: get Q-values close (modeling)
  - Action selection priority: get ordering of Q-values right (prediction)

- Solution: learn policies that maximize rewards, not the values that predict them

- Policy search: start with an ok solution (e.g. Q-learning) then fine-tune by **hill climbing** (or gradient ascent!) on feature weights

Policy Search

- Simplest policy search:
  - Start with an initial linear value function or Q-function
  - Nudge each feature weight up and down and see if your policy is better than before

- Pros:
  - Works well for partial observability / stochastic policies

- Cons:
  - How do we tell the policy got better?
  - Need to run many sample episodes!
  - If there are a lot of features, this can be impractical

Policy Search

[Andrew Ng]

[Video: HELICOPTER]

Policy gradient

- We will cover it in "deep reinforcement learning" part.

Summary

- RL solves MDPs via direct experience of transitions and rewards
- There are several approaches:
  - Learn the MDP model and solve it
  - Learn V directly from sums of rewards, or by TD local adjustments
    - Still need a model to make decisions by lookahead
  - Learn Q by local Q-learning adjustments, use it directly to pick actions
  - Optimize the policy directly
- Scaling up with feature representations and approximation