EE101 Homework 1

Submit via Blackboard **Due：October.16th**

Question 1

In Fig.1 calculate the X-ray intensity, as a function of the incident intensity I_0, that reaches the detector for each of the three X-ray beams. The dark-shaded area represents bone, and the light-shaded area represents tissue. The linear attenuation coefficients of the bone and tissue at the average received energy of X-ray are 10 cm^{-1} and 2 cm^{-1} for bone and tissue, respectively.

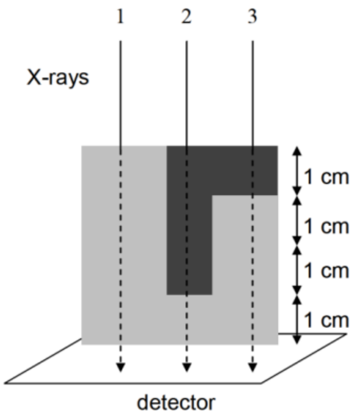


Fig.2

Solution:

Beam 1:

$$I = I_0 e^{-(2\times4)} = I_0 e^{-8} = 3.35 \times 10^{-4} I_0$$

Beam 2:

$$I = I_0 e^{-(10\times3+2\times1)} = I_0 e^{-32} = 1.27 \times 10^{-14} I_0$$

Beam 3:

$$I = I_0 e^{-(10\times1+2\times3)} = I_0 e^{-16} = 1.13 \times 10^{-7} I_0$$

Question 2

In a study for testing a new form of cancer among 500 suspected patients, multiple medical methods had been considered for auxiliary diagnosis. For each method, the number of positive and negative people corresponding to the different threshold values was given in the Table.1

Table1 Diagnosis Results

Biopsy							
Positive				200			
Negative				300			
Blood(Immune cell concentration 10^9/L)							
Threshold	3	7	8	9	10	11	15
Positive(>Thr)	500	246	198	167	131	98	0
TP	200	126	91	69	46	27	0
CT(length(cm))							
Threshold	0.5	2	2.5	3	3.5	4	8
Positive(>Thr)	500	283	233	198	169	125	0
TP	200	188	168	156	135	110	0
MRI(length(cm))							
Threshold	0.5	2	2.5	3	3.5	4	8
Positive(>Thr)	500	270	230	205	170	135	0
TP	200	190	170	160	148	120	0
Ultrasound(length(cm))							
Threshold	0.5	2	2.5	3	3.5	4	8
Positive(>Thr)	500	269	221	200	162	127	0
TP	200	173	164	154	133	112	0

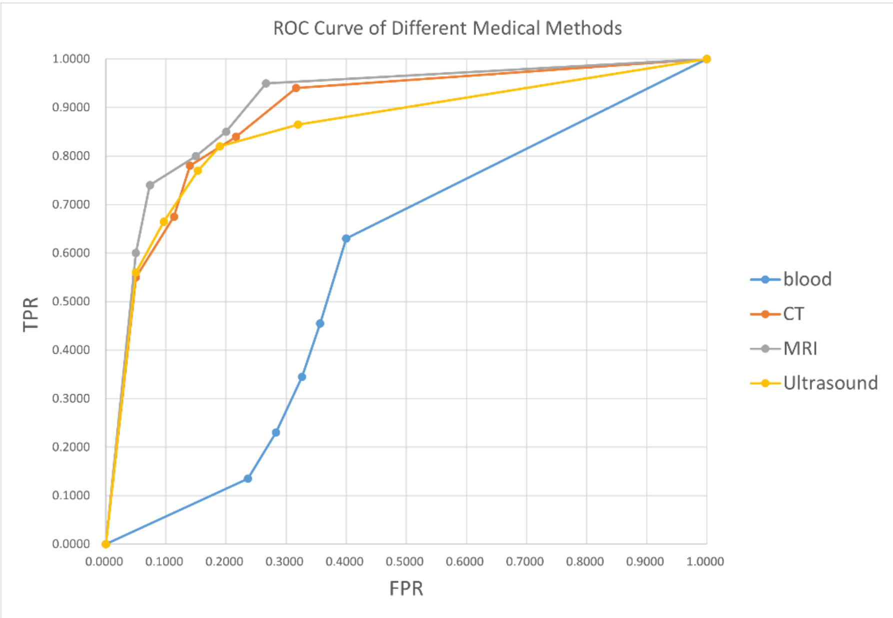
(1) Plot the ROC curve.
(2) Consider which method is best for auxiliary diagnosis and explain. List all the methods from good to bad.
(3) In clinical practice, we want to diagnose all patients as well as possible. Determine a standard Threshold for the best method from the given threshold.

Solution:

(1) At first, we have to do the calculations.

Method	threshold	Positive	TP	FP	FN	TN	TPR(Y)	FPR (X)	distance^2	distance	min_distance	area	AUC	sensitivity
	3	500	200	300	0	0	1.0000	1.0000	1.0000	1.0000		0.489		
	7	246	126	120	74	180	0.6300	0.4000	0.2969	0.5449		0.023508333		
	8	198	91	107	109	193	0.4550	0.3567	0.4242	0.6513		0.012		
Blood	9	167	69	98	131	202	0.3450	0.3267	0.5357	0.7319	0.5449	0.012458333	0.561458333	
	10	131	46	85	154	215	0.2300	0.2833	0.6732	0.8205		0.008516667		
	11	98	27	71	173	229	0.1350	0.2367	0.8042	0.8968		0.015975		
	15	0	0	0	200	300	0.0000	0.0000	1.0000	1.0000		0		
	0.5	500	200	300	0	0	1.0000	1.0000	1.0000	1.0000		0.662833333		
	2	283	188	95	12	205	0.9400	0.3167	0.1039	0.3223		0.089		
	2.5	233	168	65	32	235	0.8400	0.2167	0.0725	0.2693		0.0621		
CT	3	198	156	42	44	258	0.7800	0.1400	0.0680	0.2608	0.2608	0.0194	0.885875	
	3.5	169	135	34	65	266	0.6750	0.1133	0.1185	0.3442		0.038791667		
	4	125	110	15	90	285	0.5500	0.0500	0.2050	0.4528		0.01375		
	8	0	0	0	200	300	0.0000	0.0000	1.0000	1.0000		0		
	0.5	500	200	300	0	0	1.0000	1.0000	1.0000	1.0000		0.715		1
	2	270	190	80	10	220	0.9500	0.2667	0.0736	0.2713		0.06		0.95
	2.5	230	170	60	30	240	0.8500	0.2000	0.0625	0.2500		0.04125		0.85
MRI	3	205	160	45	40	255	0.8000	0.1500	0.0625	0.2500	0.2500	0.059033333	0.905916667	0.8
	3.5	170	148	22	52	278	0.7400	0.0733	0.0730	0.2701		0.015633333		0.74
	4	135	120	15	80	285	0.6000	0.0500	0.1625	0.4031		0.015		0.6
	8	0	0	0	200	300	0.0000	0.0000	1.0000	1.0000		0		0
	0.5	500	200	300	0	0	1.0000	1.0000	1.0000	1.0000		0.6341		
	2	269	173	96	27	204	0.8650	0.3200	0.1206	0.3473		0.109525		
	2.5	221	164	57	36	243	0.8200	0.1900	0.0685	0.2617		0.02915		
US	3	200	154	46	46	254	0.7700	0.1533	0.0764	0.2764	0.2617	0.040658333	0.856016667	
	3.5	162	133	29	67	271	0.6650	0.0967	0.1216	0.3487		0.028583333		
	4	127	112	15	88	285	0.5600	0.0500	0.1961	0.4428		0.014		
	8	0	0	0	200	300	0.0000	0.0000	1.0000	1.0000		0		

Based on the calculations, plot the ROC curve:



(2) Quantitatively speaking, AUC (area under curve) proves to be an appropriate measure. Comparing AUC of different methods, we will jump to the conclusion that MRI is the best for auxiliary diagnosis.

Method	Blood	CT	**MRI**	Ultrasound
AUC	0.5615	0.8859	**0.9059**	0.8560

The list is MRI>CT>Ultrasound>Blood.

(3) A standard threshold must optimize both sensitivity and specificity. Since sensitivity = TPR and specificity = 1 – FPR, the nearer the distance between the threshold point to the upper-left corner point, the better the threshold point is. However, there are two points have the same distance. In order to diagnose all patients as well as possible, we preferentially select parameters with higher sensitivity. The standard threshold for the best method is 2.5.

Question 3:

(1) Given $LSF(y) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp(-\frac{(y-y_0)^2}{2\sigma^2})$, deduce the FWHM of LSF.

(2) Prove: Taking N averages when obtaining an image, the signal-to-noise ratio (SNR) is $\sqrt{N}$ times the SNR of a single image.

Solution:

(1)

$$LSF_{max} = \frac{1}{\sqrt{2\pi\sigma^2}}$$

$$\frac{1}{2} LSF_{max} = \frac{1}{\sqrt{2\pi\sigma^2}} \exp(-\frac{(y-y_0)^2}{2\sigma^2})$$

$$\frac{1}{2} = \exp(-\frac{(y-y_0)^2}{2\sigma^2})$$

$$2\sigma^2 ln2 = (y-y_0)^2$$

$$y_1 = \sqrt{2\sigma^2 ln2} + y_0 \quad y_2 = -\sqrt{2\sigma^2 ln2} + y_0$$

$$FWHM = y_1 - y_2 = 2\sqrt{2\sigma^2 ln2}$$

(2) The noise variance of image after average σ_A^2 is:

$$\sigma_A^2 = \left[\frac{1}{N}\sum_N (x_i - \bar{x})\right]^2$$

$$= \frac{1}{N^2}\left[\sum_N x_i - N\bar{x}\right]^2$$

$$= \frac{1}{N^2}[(x_1 - \bar{x}) + (x_2 - \bar{x}) + \cdots + (x_N - \bar{x})]^2$$

Where x_i is the result of one measurement, and according to the covariance formula, we have:

$$\sigma_A^2 = \frac{1}{N^2}\left[\sum_N (x_i - \bar{x})^2\right] + \frac{1}{N^2} var(x_i - \bar{x})$$

where $var(x_i - \bar{x})$ is the cross-correlation function and equal to 0,

$$\sigma_A^2 = \frac{1}{N^2} N \sigma^2 = \frac{\sigma^2}{N}$$

$$\sigma_A = \frac{\sigma}{\sqrt{N}}$$

Therefore, after N times of sampling, the standard deviation is reduced to $\frac{1}{\sqrt{N}}$ times the original, so the SNR is $\sqrt{N}$ times the original.

Question 4

(1) Given Equation (1), derive equation (3).

$$\Delta\lambda = \frac{\mathrm{h}}{\mathrm{m}_0 c}(1 - cos\theta)\ (1)$$

$$\Delta\mathrm{E} = \mathrm{E}_{\mathrm{X,inc}} - E_{X,scat} = \frac{hc}{\lambda_{inc}} - \frac{hc}{\lambda_{scat}}\ (2)$$

$$\mathrm{E}_{\mathrm{X,scat}} = \frac{E_{X,inc}}{1 + \left(\frac{E_{X,inc}}{mc^2}\right)(1 - cos\theta)}\ (3)$$

(2) Given incident X-ray energy $\mathrm{E}_{\mathrm{X,inc}} = 67keV$, $(\mathrm{m_e c^2} = 511keV)$, calculate the scatter energy $\mathrm{E}_{\mathrm{X,scat}}$ when $\theta = 0°, 15°, 45°, 60°, 90°$. Then plot $\mathrm{E}_{\mathrm{X,scat}}$ vs scatter angle.

(3) **Bonus question**: try to derive equation (1) from conservation of energy and conservation of momentum. (**Hint:** the incident and outgoing photons have similar masses.)

Solution:

(1) The incident and scattered X-ray energy can be represented as:

$$\mathrm{E}_{\mathrm{X,inc}} = hf_{inc} = \frac{hc}{\lambda_{inc}}\ (4)$$

$$\mathrm{E}_{\mathrm{X,scat}} = hf_{scat} = \frac{hc}{\lambda_{scat}}\ (5)$$

From (1), we know that:

$$\lambda_{\mathrm{scat}} = \lambda_{inc} + \Delta\lambda = \lambda_{inc} + \frac{h}{m_0 c}(1 - \cos\theta)\ (6)$$

Then taking (6) into (5), we can write:

$$\mathrm{E}_{\mathrm{X,scat}} = \frac{hc}{\lambda_{inc} + \frac{h}{m_0 c}(1 - \cos\theta)} = \frac{\frac{hc}{\lambda_{inc}}}{1 + \frac{\frac{hc}{\lambda_{inc}}}{m_0 c^2}(1 - \cos\theta)}$$

$$= \frac{E_{X,inc}}{1 + \left(\frac{E_{X,inc}}{mc^2}\right)(1 - cos\theta)}\ (7)$$

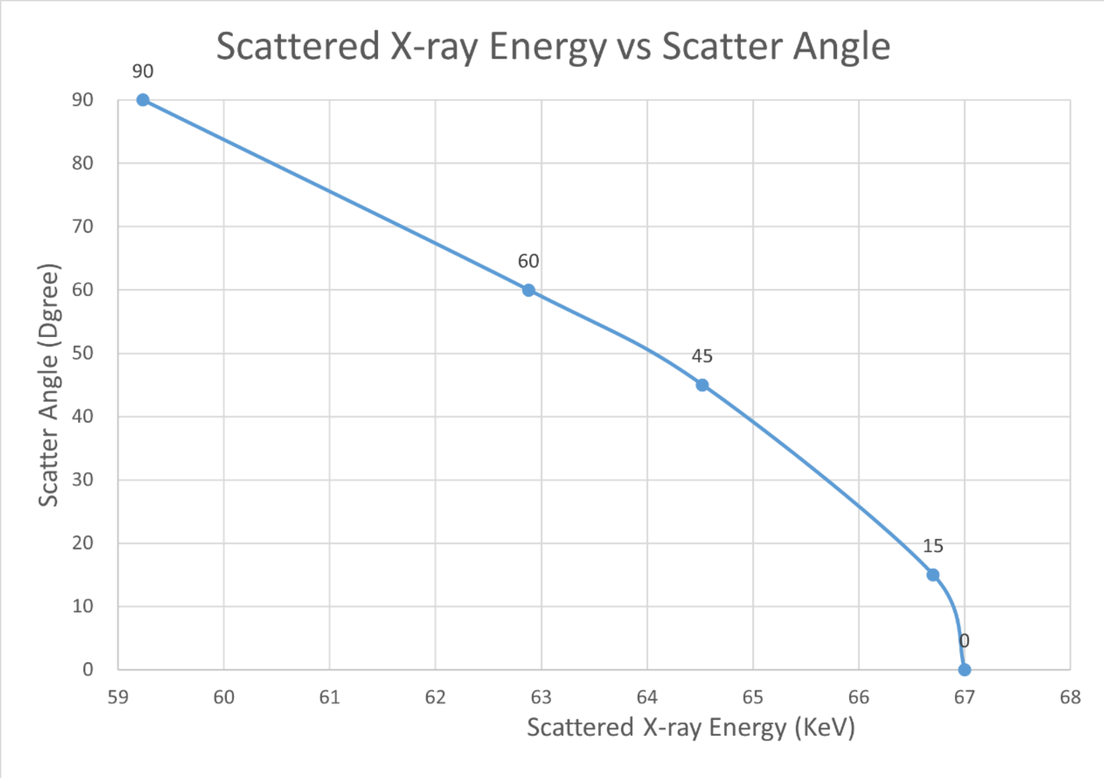
(2) Take $\mathrm{E}_{\mathrm{X,inc}} = 67keV$ and $\mathrm{m_e}c^2 = 511keV$ into (8), the following relationship is obtained:

$$\mathrm{E}_{\mathrm{X,scat}} = \frac{67keV}{1 + \left(\frac{67}{511}\right)(1 - cos\theta)}$$

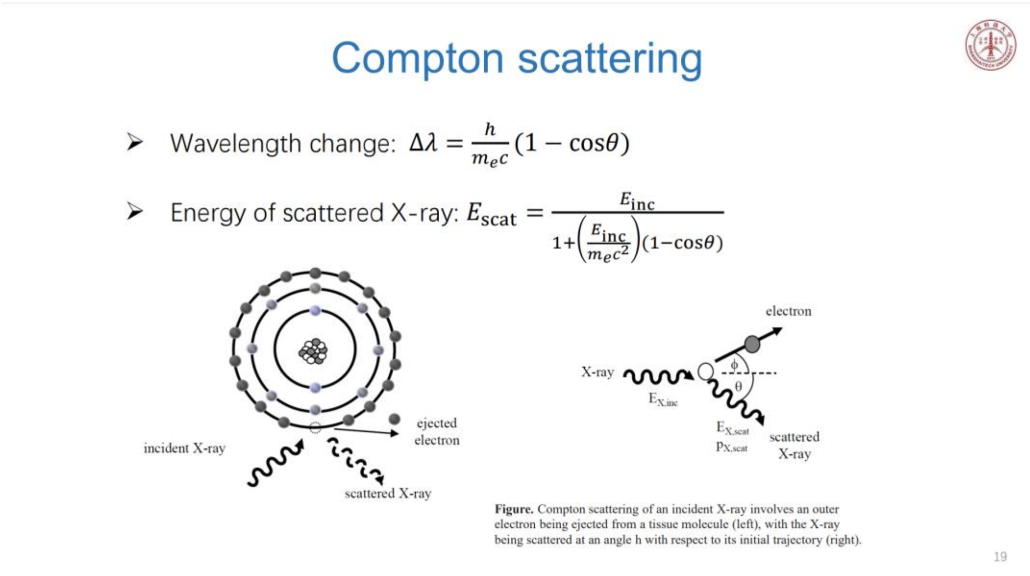
Take $\theta = 0, 15°, 45°, 60°, 90°$, we can get:

$$E_{X,scat} = \begin{cases} 67.0000keV, & \theta = 0° \\ 66.7020keV, & \theta = 15° \\ 64.5222keV, & \theta = 45° \\ 62.8779keV, & \theta = 60° \\ 59.2336keV, & \theta = 90° \end{cases}$$

The $E_{X,scat}$ vs scatter angle plot is as follows:



(3)



Based on conservation of energy and conservation of kinetic energy, we have:

$$\begin{cases} E_{inc} = h\upsilon = E_{sca} + \frac{1}{2}m_e v_e^2 & (1) \\ m_e v_e \cdot \sin\varphi = \mathrm{m_{sca}}c \cdot sin\theta & (2) \\ m_{inc}c = m_e v_e \cdot cos\varphi + m_{sca}c \cdot cos\theta & (3) \\ E_{inc} = \frac{hc}{\lambda_{inc}} & (4) \\ E_{sca} = \frac{hc}{\lambda_{sca}} & (5) \end{cases}$$

Where m_{inc}, m_{sca}, m_e, and v_e are respectively the incident photon mass, scattered photon mass, electron mass, and electron velocity. Derive from (2) and (3), we have:

$$m_e^2 v_e^2 = m_{sca}^2 c^2 + m_{inc}^2 c^2 - 2m_{inc}m_{sca}c^2 \cdot cos\theta \ (6)$$

Meanwhile, we have:

$$\Delta\lambda = \lambda_{sca} - \lambda_{inc} = hc(\frac{1}{E_{sca}} - \frac{1}{E_{inc}})$$

$$= \frac{h}{m_e c}(\frac{m_e c^2}{E_{sca}} - \frac{m_e c^2}{E_{inc}})$$

$$= \frac{h}{m_e c}\left(m_e c^2 \frac{E_{inc} - E_{sca}}{E_{inc}E_{sca}}\right) \quad (7)$$

Then take (1) into (7), we have:

$$\Delta\lambda = \frac{h}{m_e c}\left(m_e c^2 \frac{\frac{1}{2}m_e v_e^2}{E_{inc}E_{sca}}\right) = \frac{h}{m_e c}\left(\frac{c^2 m_e^2 v_e^2}{2E_{inc}E_{sca}}\right) \ (8)$$

The energies of E_{sca} and E_{inc} are much greater than the kinetic energy of the electron, so the two energies can be regarded as approximately equal under certain conditions. In this case, we can assume the incident and outgoing photons have similar masses *m*, from (6) and (8), we have:

$$m_e^2 v_e^2 = 2m^2 c^2 (1 - cos\theta) \ (9)$$

$$\Delta\lambda = \frac{h}{m_e c}\left(\frac{c^2 m_e^2 v_e^2}{2mc^2 \cdot mc^2}\right) \ (10)$$

Take (9) into (10), we have:

$$\Delta\lambda = \frac{h}{m_e c}\left(\frac{c^2 2m^2 c^2 (1 - cos\theta)}{2m^2 c^4}\right)$$

$$\Delta\lambda = \frac{h}{m_e c}(1 - cos\theta) \ (11)$$

Question 5

Fig.3 is the X-ray image before the angiography, and Fig.4 is the X-ray image after the angiography. Calculate the digital silhouette angiography image, and enhance the image with a power law transformation.

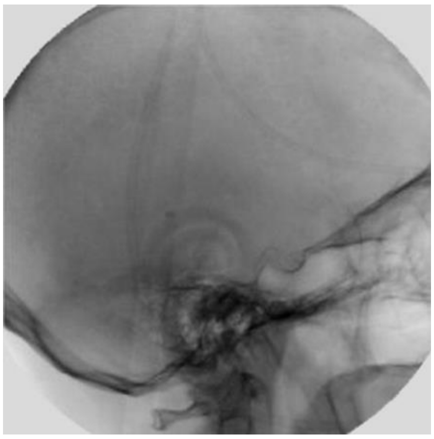


Fig.3

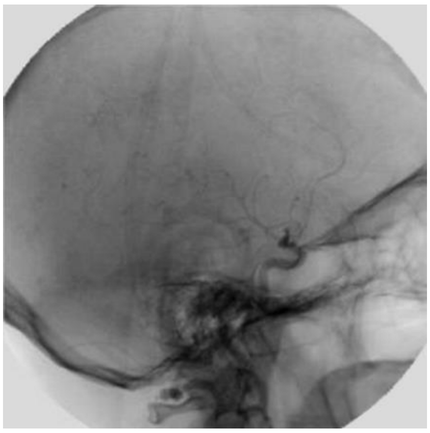
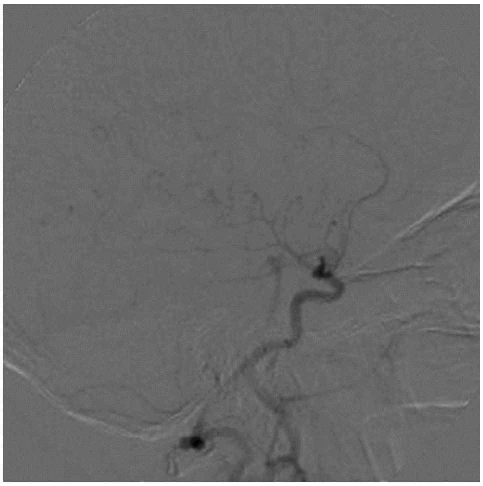
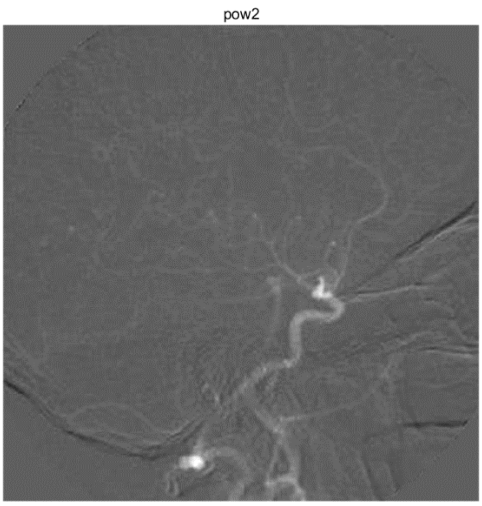


Fig.4

Solution:

Subtraction and Image enhancement:





Question 6

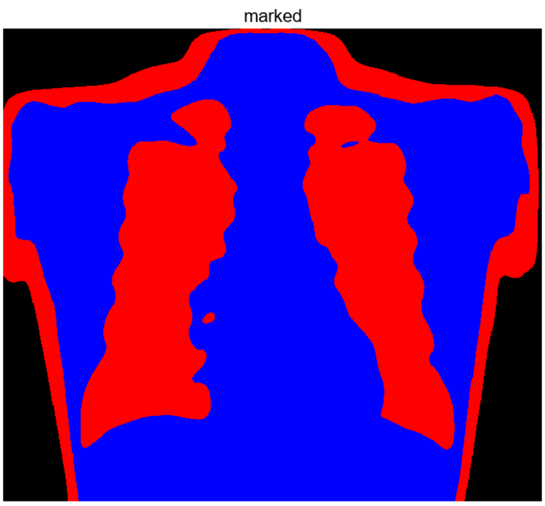
An X-ray image of the chest was provided. Use the threshold method to segment the lungs. (Histogram can be used to determine the threshold.)

Fig.1

Requirement: Please attach the segmented image and your code below, and hand in your code in a separate file.

Solution:

```
clear;clf;
I = imread("q5.png");
figure
imhist(I);
figure
imshow(I);title("origin");
figure
Iblur1 = imgaussfilt(I,7);
imshow(Iblur1);title("gauss");
figure
thresholds = [90 150]
newImg = grayslice(Iblur1,thresholds);
cmap = [0 0 0
    1 0 0
    0 0 1
    1 1 0
    ];
imshow(newImg,colormap(cmap)); title("marked");
```



Question 7

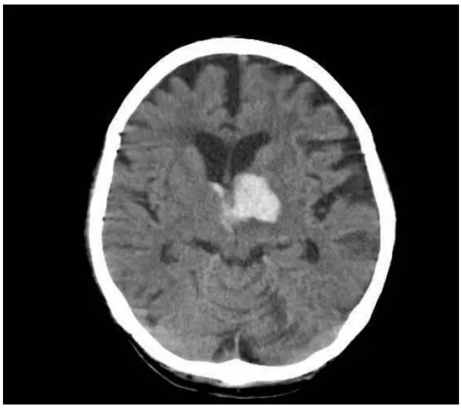


Fig.5

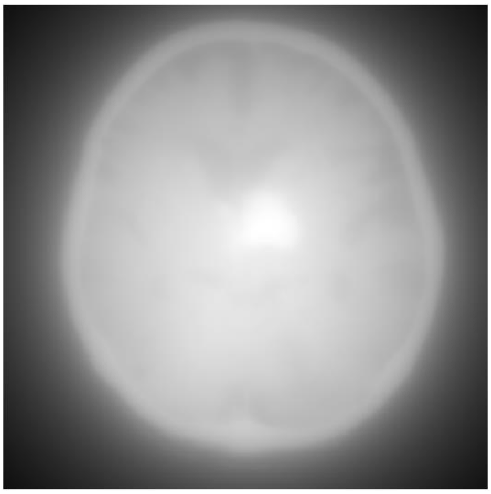
(1) Please generate the sinogram using radon transformation from the given slice.
(2) Do the back projection (BP) from the sinogram.
(3) Do the filtered back projection (FBP) from the sinogram.

Solution:

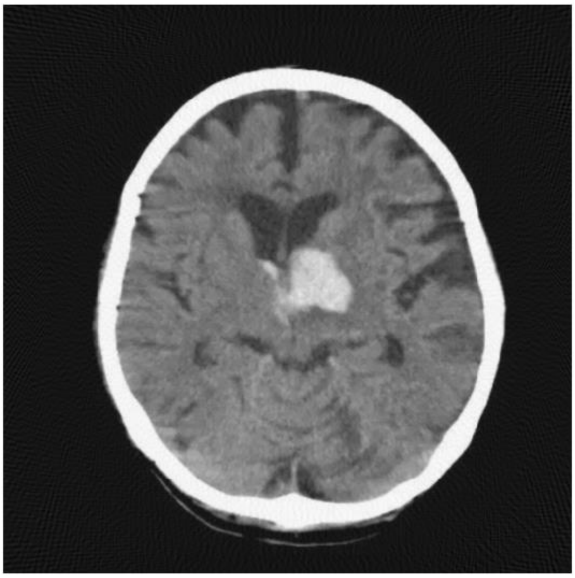
(1)

(2)

EE101 Medical Imaging
Homework 1



(3)



Code:

```
clc,clear;

clear all;

% P = phantom(256);
% imshow(P);
coronal = im2gray(imread('E:\Master\2024国庆\TA EE101\2024-Homework\Homework1\q7.jpg'));
theta = 0:1:179;
R = radon(coronal,theta);
imshow(R,[]);

recon = iradon(R,0:179,'linear','none');

imshow(recon,[]);

recon = iradon(R,0:179,'Ram-Lak');

imshow(recon,[]);
```