

## **Homework 7**

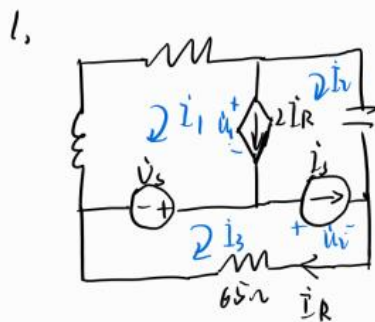
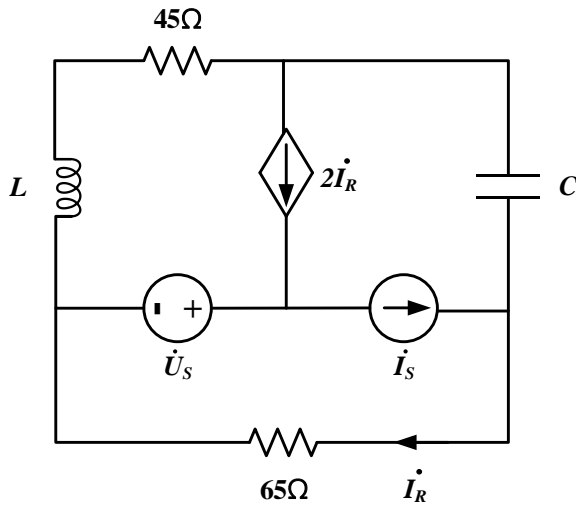
Due date: Jan. 2<sup>nd</sup>, 2024

Turn in your hard-copy hand-writing homework in class

Rules:

- Work on your own. Discussion is permissible, but extremely similar submissions will be judged as plagiarism.
- Please show all intermediate steps: a correct solution without an explanation will get zero credit.
- Please submit on time. No late submission will be accepted.
- Please prepare your submission in English only. No Chinese submission will be accepted.

1. For the circuit below,  $U_S=100\angle 0^\circ$  V (rms),  $I_S=2\angle 0^\circ$  A,  $\omega L=30\ \Omega$ ,  $1/\omega C=90\ \Omega$ , find the complex power delivered by **voltage and current source**.



$$\dot{I}_3 = \dot{I}_R$$

$$Z_L = j\omega L = 30j\ \Omega \quad Z_C = \frac{1}{j\omega C} = -90j\ \Omega \quad 2'$$

$$\begin{cases} \dot{U}_S + 45 + 30j\dot{I}_1 + \dot{u}_1 = 0 \\ -\dot{u}_1 - \dot{u}_2 - 90j\dot{I}_2 = 0 \\ -\dot{u}_2 + \dot{u}_3 + 65\dot{I}_R = 0 \\ \dot{I}_1 - \dot{I}_2 = 2\dot{I}_R \\ \dot{I}_3 = \dot{I}_R - \dot{I}_2 = 2\angle 0^\circ \end{cases} \quad 5'$$

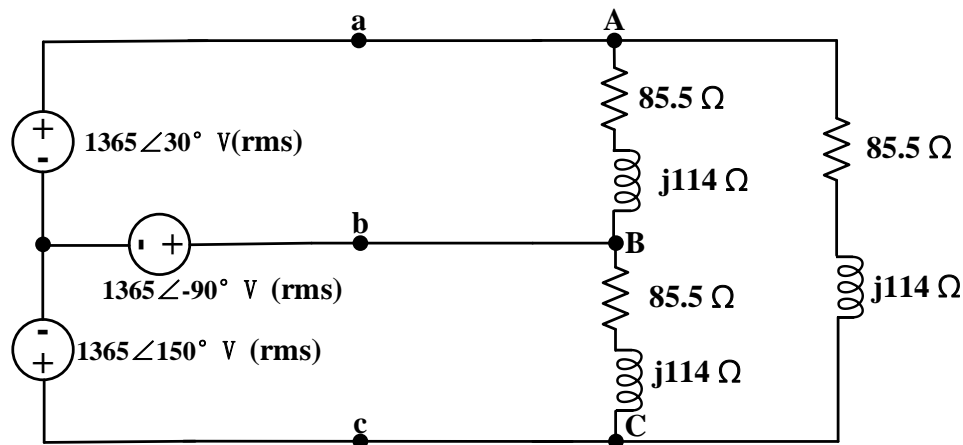
$$\Rightarrow \begin{cases} \dot{I}_1 = -0.65 - 1.8j \\ \dot{I}_2 = -1.55 - 0.6j \\ \dot{I}_R = 0.45 - 0.6j \end{cases}$$

$$\begin{cases} \dot{u}_1 = -124.75 + 100.5j \\ \dot{u}_2 = 70.75 + 39j \end{cases} \quad 4'$$

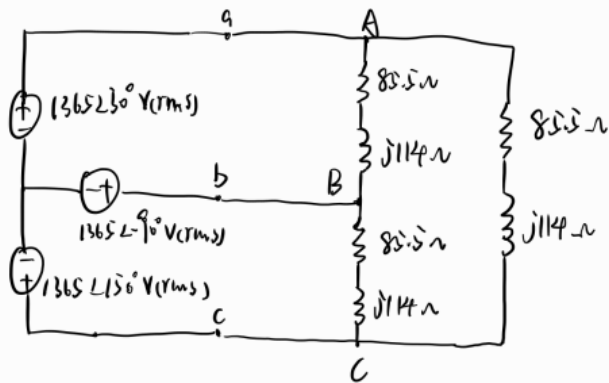
delivered power:  $P_U = 100 (1.1 - j1.1) = (110 - 120j) \text{ VA} \quad 2'$

$$P_C = (-70.75 - 39j) \times 2 \times (1 + 1.5 - 1.8j) \text{ VA} \quad 2'$$

2. For the circuit below, find  $I_{AB}$ ,  $I_{BC}$ ,  $I_{CA}$ ,  $V_{CA}$  and in the circuit.



2.



$$\dot{I}_{AB} (85.5 + j114) = 1365 \angle 30^\circ - 1365 \angle 90^\circ$$

$$\dot{I}_{BC} (85.5 + j114) = 1365 \angle -90^\circ - 1365 \angle 150^\circ$$

$$\dot{I}_{CA} (85.5 + j114) = 1365 \angle 150^\circ - 1365 \angle 30^\circ = \dot{V}_{CA}$$

$$\Rightarrow \begin{cases} \dot{I}_{AB} = 16.59 \angle 6.87^\circ \text{ A} = 16.47 + j1.98 \text{ A} \\ \dot{I}_{BC} = 16.59 \angle -113.13^\circ \text{ A} = -6.50 - j15.46 \text{ A} \\ \dot{I}_{CA} = 16.59 \angle 126.8^\circ \text{ A} = -9.95 + j14.47 \text{ A} \end{cases}$$

$$\dot{V}_{CA} = 2364.75 \angle 180^\circ \text{ V}$$

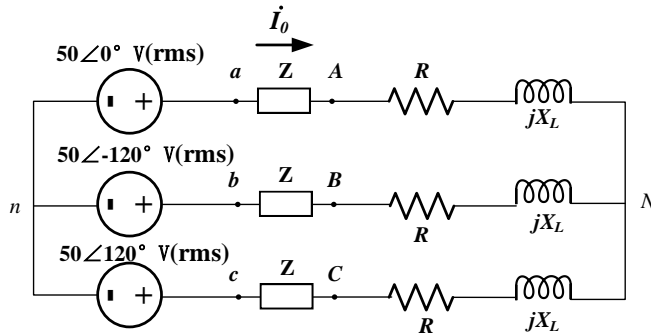
6'

6'

3'

3. Consider the following three-phase circuit.  $R=20\ \Omega$ ,  $jX_L=j5\ \Omega$ ,  $Z=(1+j0.5)\ \Omega$ .

- (1) Calculate the **line current**  $I_0$ .
- (2) Calculate **the voltage**  $U_{AB}$ .
- (3) Calculate the total complex power absorbed by **all the loads including Z, R and  $jX_L$** .



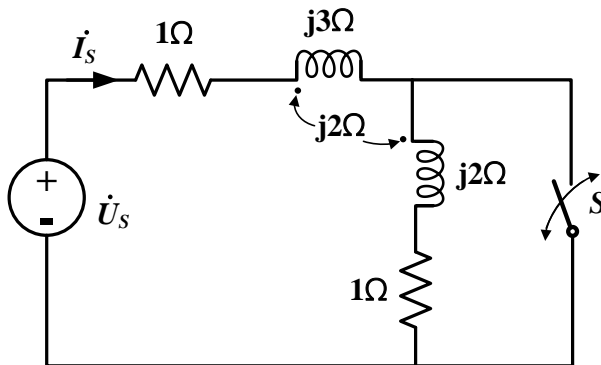
$$\begin{aligned} \text{3.4, Using KCL} \quad & \begin{cases} 50\angle 0^\circ - \dot{I}_0(1+j0.5) - (\dot{I}_0 - \dot{I}_1)(1+j0.5) - 50\angle -120^\circ = 0 \\ 50\angle -120^\circ - (\dot{I}_1 - \dot{I}_0)(1+j0.5) - \dot{I}_1(1+j0.5) - 50\angle 120^\circ = 0 \end{cases} \\ \text{3.4} \Rightarrow & \begin{cases} \dot{I}_0 = 2.30\angle -14.68^\circ = 2.22 - 0.58j \\ \dot{I}_1 = 2.32\angle -74.68^\circ \end{cases} \end{aligned}$$

$$\text{4.1 a) } \dot{U}_{AB} = (20+j5)(3.52\angle 34.48^\circ - 1.33\angle 135.52^\circ) = 82.28\angle 27.36^\circ \text{ V} = 71.71 + 40.34j \text{ V}$$

$$\text{4.1 b) } \tilde{S}_{\text{total}} = 3 \cdot 50\angle 0^\circ \cdot (2.30\angle -14.68^\circ)^* = 354.22 + 87.55j \text{ V}\cdot\text{A}$$

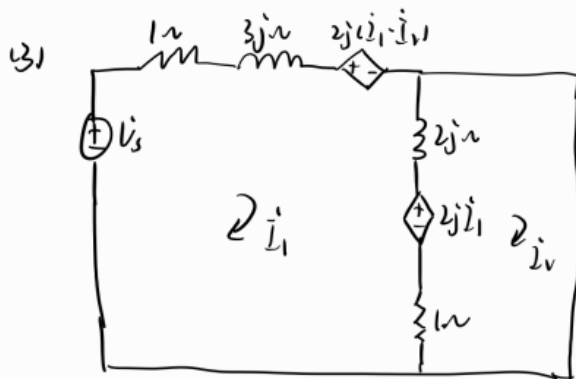
4. For the circuit below,  $U_s = 80 \angle 30^\circ$  V (phasor expressed by amplitude), find

- (1) The current  $I_s$  for switch  $S$  is off.
- (2) The complex power delivered by voltage source when  $S$  is off.
- (3) The current  $I_s$  when the switch  $S$  is on.



4. (1)  $L = L_1 + L_2 + 2M = j\Omega \sim \checkmark$   $\dot{I}_s = \frac{80 \angle 30^\circ}{2 + j1} = 8.08 \angle -47.47^\circ = 5.87 - 6.39j$  A 3'

(2)  $\dot{S} = \frac{1}{2} \dot{U}_s \cdot \dot{I}_s^* = 75.54 + j38.76$  V·A 4'

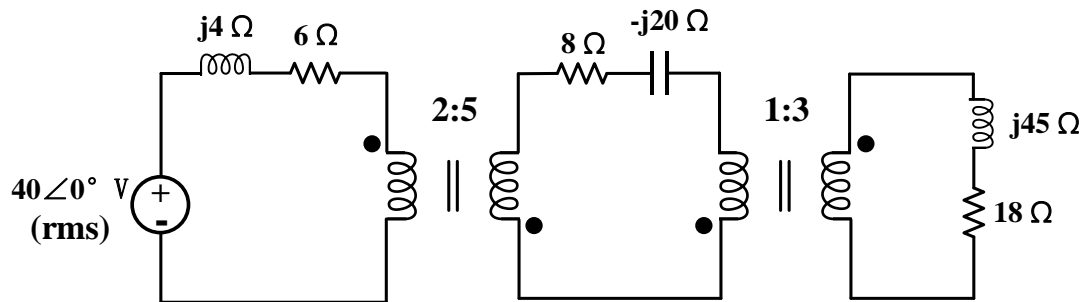


$$\begin{cases} -\dot{U}_s + (1+j3)\dot{I}_1 + 2j(\dot{I}_1 - \dot{I}_v) + (\dot{I}_1 - \dot{I}_v)(2j+1) + 2j\dot{I}_1 = 0 \\ -2j\dot{I}_1 + (\dot{I}_v - \dot{I}_1)(2j+1) = 0 \end{cases}$$
 6'

$\Rightarrow \dot{I}_s = 35.08 \angle -78.7^\circ$  A  $\approx 34.75 - 4.89j$  A. 4'

5. For the circuit below, please find:

- The average power released by the source.
- The average power delivered to the  $18\Omega$  resistor.



5. (a)  $Z_L' = 8 - 20j + \underbrace{(18 + 45j)}_{3'}$  2'

$Z_{in} = 6 + 4j + \frac{10 - 15j}{n^2} = 7.6 + 1.6j = 7.77 \angle 11.8^\circ$ ,  $n = \frac{5}{2} = 2.5$  2'

$I_1 = \frac{40}{Z_{in}} = 5.15 \angle -11.8^\circ$  2'

$\dot{S} = V_s I_1^* = 206 \angle 11.8^\circ \text{ V}\cdot\text{A} = 201.58 + 42.44j \text{ V}\cdot\text{A}$  3'

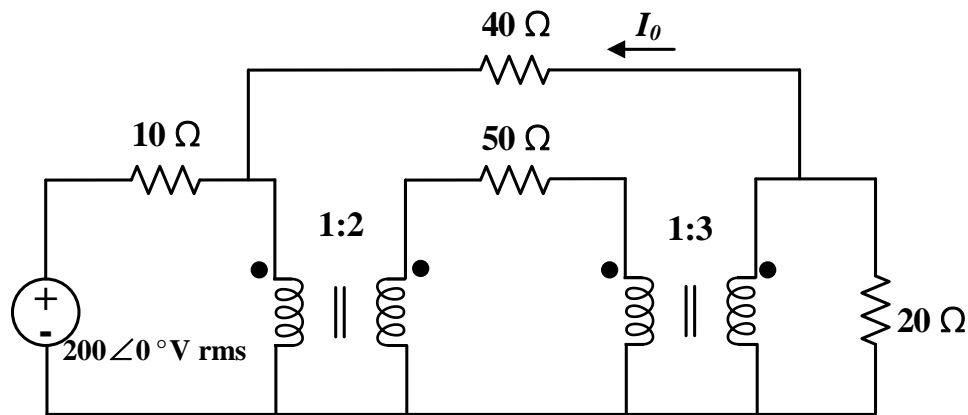
$P = \text{Re}\{\dot{S}\} = 201.58 \text{ W}$  1'

(b).  $I_2 = \frac{-I_1}{n}$ ,  $n = 2.5$

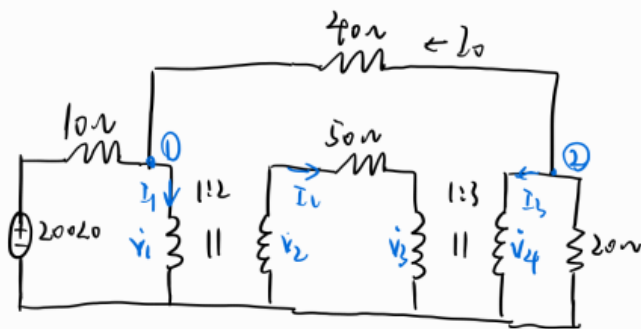
$I_3 = \frac{-I_2}{n'}$ ,  $n = 3$

$\therefore I_3 = 0.169 \angle -11.8^\circ$  5'  $P = |I_3|^2 \cdot 18 = 8.49 \text{ W}$

6. For the following circuit, please find current  $I_0$



6.



$$\frac{\dot{V}_2}{\dot{V}_1} = \frac{2}{1} \quad \frac{\dot{I}_2}{\dot{I}_1} = \frac{1}{2} \quad \frac{\dot{V}_4}{\dot{V}_3} = \frac{3}{1} \quad \frac{\dot{I}_3}{\dot{I}_4} = \frac{1}{3} \quad 2'$$

$$\frac{200 - \dot{V}_1}{10} = \frac{\dot{V}_1 - \dot{V}_4}{40} + \dot{I}_1$$

$$\Rightarrow 200 = 1.25 \dot{V}_1 - 0.25 \dot{V}_4 + 10 \dot{I}_1$$

$$8' \quad \frac{\dot{V}_1 - \dot{V}_4}{40} = \frac{\dot{V}_4}{20} + \dot{I}_3$$

$$\dot{V}_1 = 3\dot{V}_4 + 40\dot{I}_3$$

$$\dot{V}_3 = \dot{V}_2 - 50\dot{I}_2$$

$$\begin{cases} 200 = 1.25 \dot{V}_1 - 0.25 \dot{V}_4 + 10 \dot{I}_1 \\ \dot{V}_1 = 3\dot{V}_4 + 40 \cdot \frac{1}{6} \dot{I}_1 \\ \frac{\dot{V}_4}{3} = 2\dot{V}_1 - 25 \dot{I}_1 \end{cases}$$

$$6' \Rightarrow \begin{cases} \dot{V}_1 = 107.54 \text{ V} \\ \dot{V}_4 = 53.38 \text{ V} \\ \dot{I}_1 = 7.89 \text{ A} \end{cases}$$

$$\dot{I}_0 = \frac{\dot{V}_4 - \dot{V}_1}{40} = -1.15 \text{ A} \quad 4'$$