First-Order Logic

AIMA Chapter 8, 9

Pros of propositional logic

- ☺ Propositional logic allows partial/disjunctive/negated information
  - (unlike most data structures and databases)
- ☺ Propositional logic is compositional:
  - meaning of $B_{1,1} \wedge P_{1,2}$ is derived from meaning of $B_{1,1}$ and of $P_{1,2}$
- ☺ Meaning in propositional logic is context-independent
  - (unlike natural language, where meaning depends on context)

Cons of propositional logic

- ☹ Hard to identify "individuals" (e.g., Mary, 3)
- ☹ Can't directly talk about properties of individuals or relations between individuals (e.g., "Bill is tall")
- ☹ Generalizations, patterns, regularities can't easily be represented (e.g., "all triangles have 3 sides")

First-order logic

- Whereas propositional logic assumes the world contains facts…
- First-order logic (like natural language) assumes the world contains
  – Objects: people, houses, numbers, colors, baseball games, wars, …
  – Relations: red, round, prime, bigger than, part of, comes between, …
  – Functions: father of, best friend of, one more than, …
- Also called first-order predicate logic

Syntax of FOL: Basic elements

- Logical symbols
  - Connectives $\neg, \wedge, \vee, \Rightarrow, \Leftrightarrow$
  - Quantifiers $\forall, \exists$
  - Variables x, y, a, b, ...
  - Equality =
- Non-logical symbols (ontology)
  - Constants KingArthur, 2, ShanghaiTech, ...
  - Predicates Brother, >, ...
  - Functions Sqrt, LeftLegOf, ...

Atomic sentences

Atomic sentence = *predicate* (*term*$_1$,...,*term*$_n$)
or *term*$_1$ = *term*$_2$

Term = *constant* or *variable*
or *function* (*term*$_1$,...,*term*$_n$)

Example:

Brother(KingJohn,RichardTheLionheart)

>(Length(LeftLegOf(Richard)), Length(LeftLegOf(KingJohn)))

Complex sentences

Complex sentences are made from atomic sentences using connectives

$$\neg S,\ \ S_1 \land S_2,\ \ S_1 \lor S_2,\ \ S_1 \Rightarrow S_2,\ \ S_1 \Leftrightarrow S_2,$$

Example:

$Sibling(KingJohn,Richard) \Rightarrow Sibling(Richard,KingJohn)$

$>(1,2) \lor \leq (1,2)$

$>(1,2) \land \neg >(1,2)$

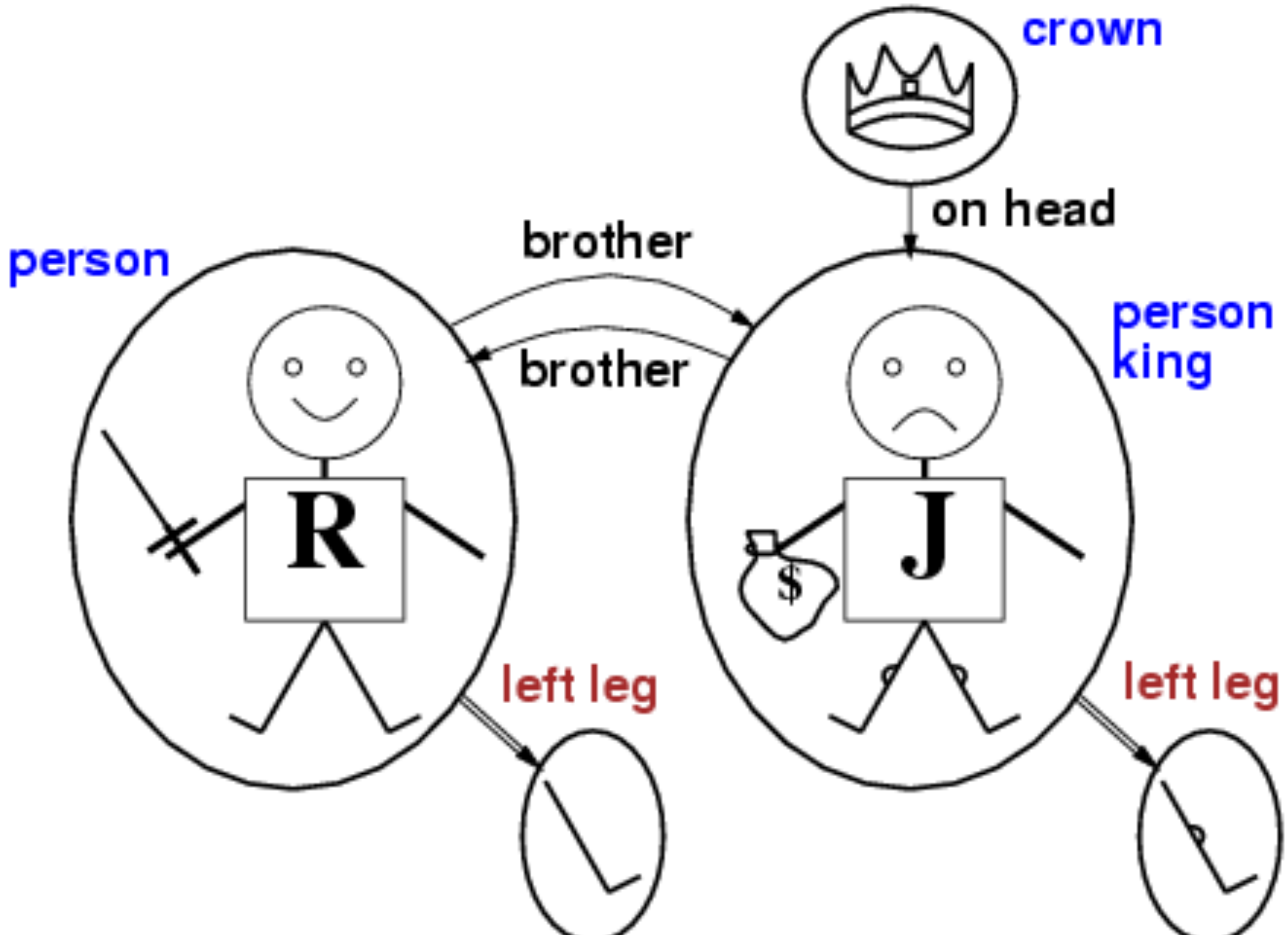
Semantics of FOL

- Sentences are true with respect to a model, which contains
  - Objects and relations among them
  - Interpretation specifying referents for

constant symbols	→	objects
predicate symbols	→	relations
function symbols	→	functional relations

- An atomic sentence *predicate*($term_1,\ldots,term_n$) is true iff the objects referred to by $term_1,\ldots,term_n$ are in the relation referred to by *predicate*

Models for FOL: Example



Models for FOL: Example

- Consider the interpretation:

  Richard → Person R

  John → Person J

  Brother → the brotherhood relation

  Under this interpretation, *Brother*(*Richard*, *John*) is true in the model.

Models for FOL

- How many models do we have? Infinite!

  Models vary in:
  - the number of objects (1 to ∞)
  - the relations among the objects
  - the mapping from constants to objects
  - the mapping from predicates to relations
  - … …

Semantics of FOL

- Complex sentences
  - Exactly the same as in propositional logic

Rules for evaluating truth with respect to a model m:

- $\neg$S is true iff S is false
- S1 $\wedge$ S2 is true iff S1 is true and S2 is true
- S1 $\vee$ S2 is true iff S1 is true or S2 is true
- S1 $\Rightarrow$ S2 is true iff S1 is false or S2 is true
- S1 $\Leftrightarrow$ S2 is true iff S1$\Rightarrow$S2 is true and S2$\Rightarrow$S1 is true

Quantifiers

- Allows us to express properties of collections of objects instead of enumerating objects by name
- Universal: “for all” $\forall$
- Existential: “there exists” $\exists$

Universal quantification

$\forall$<variables> <sentence>

Example: $\forall x \;\; At(x,STU) \Rightarrow Smart(x)$

(Everyone at ShanghaiTech is smart)

$\forall x\, P$ is true in a model m iff P is true with x being each possible object in the model

- Roughly speaking, equivalent to the conjunction of instantiations of P

$$
\begin{aligned}
& At(John,STU) \Rightarrow Smart(John) \\
\wedge \;\; & At(Richard,STU) \Rightarrow Smart(Richard) \\
\wedge \;\; & At(STU,STU) \Rightarrow Smart(STU) \\
\wedge \;\; & \ldots
\end{aligned}
$$

A common mistake to avoid

- Typically, $\Rightarrow$ is the main connective with $\forall$
- Common mistake: using $\wedge$ as the main connective with $\forall$:

  $\forall x \;\; At(x, STU) \wedge Smart(x)$

  means "Everyone is at STU and everyone is smart"

Existential quantification

$\exists$<variables> <sentence>

Example: $\exists x \;\; At(x, STU) \wedge Smart(x)$

(Someone at ShanghaiTech is smart)

$\exists x \; P$ is true in a model m iff P is true with x being some possible object in the model

- Roughly speaking, equivalent to the disjunction of instantiations of P

$$\begin{aligned} &(At(John, STU) \wedge Smart(John)) \\ \vee \;\; &(At(Richard, STU) \wedge Smart(Richard)) \\ \vee \;\; &(At(STU, STU) \wedge Smart(STU)) \\ \vee \;\; &\ldots \end{aligned}$$

Another common mistake to avoid

- Typically, $\wedge$ is the main connective with $\exists$
- Common mistake: using $\Rightarrow$ as the main connective with $\exists$:

  $\exists x\ At(x, STU) \Rightarrow Smart(x)$

  is true if there is anyone who is not at STU!

Properties of quantifiers

- $\forall x\ \forall y$ is the same as $\forall y\ \forall x$
- $\exists x\ \exists y$ is the same as $\exists y\ \exists x$
- $\exists x\ \forall y$ is not the same as $\forall y\ \exists x$
  - $\exists x\ \forall y$ Loves(x,y)

    "There is a person who loves everyone in the world"
  - $\forall y\ \exists x$ Loves(x,y)

    "Everyone in the world is loved by at least one person"
- Quantifier duality: each can be expressed using the other

  $\forall x$ Likes(x,IceCream) $\equiv$ $\neg\exists x\ \neg$Likes(x,IceCream)

  $\exists x$ Likes(x,Broccoli) $\equiv$ $\neg\forall x\ \neg$Likes(x,Broccoli)

Sentences with variables

- A variable is free in a formula if it is not quantified
  - e.g., $\forall x \; P(x,y)$
- A variable is bound in a formula if it is quantified
  - e.g., $\forall x \, \exists y \; P(x,y)$
- In a FOL sentence, every variable must be bound.

FOL example: kinship

- Brothers are siblings

  $\forall x,y\ Brother(x,y) \Rightarrow Sibling(x,y).$
- "Sibling" is symmetric

  $\forall x,y\ Sibling(x,y) \Leftrightarrow Sibling(y,x).$
- One's mother is one's female parent

  $\forall x,y\ Mother(x,y) \Leftrightarrow (Female(x) \wedge Parent(x,y)).$
- A first cousin is a child of a parent's sibling

  $\forall x,y\ FirstCousin(x,y) \Leftrightarrow \exists p,ps\ Parent(p,x) \wedge Sibling(ps,p) \wedge Parent(ps,y)$

FOL example: kinship

- Siblings are people with the same parents

  $\forall x,y\ Sibling(x,y) \Leftrightarrow \exists m,f\ \ Parent(m,x) \wedge Parent(f,x) \wedge Parent(m,y) \wedge Parent(f,y)$

Is this correct?

Equality

$term_1 = term_2$ is true under a given interpretation if and only if $term_1$ and $term_2$ refer to the same object

Example: Siblings are people with the same parents:

$\forall x,y\ Sibling(x,y) \Leftrightarrow [\neg(x = y) \wedge\ \exists m,f\ \ \neg(m = f) \wedge Parent(m,x) \wedge Parent(f,x) \wedge Parent(m,y) \wedge Parent(f,y)]$

FOL example

- True or false?

  $\forall x\ CollegeStudent(x) \Rightarrow Student(x)$

  $\forall x\ Student(x) \Rightarrow CollegeStudent(x)$

- Sentences are true/false with respect to a model
  - No truth-value without a model!
  - Symbols do not carry meanings by themselves

    David Hilbert: "One must be able to say at all times
    — instead of points, straight lines, and planes
    — tables, chairs, and beer mugs."

Inference in first-order logic

Universal instantiation (UI)

(Term without variables)

- For any sentence α, variable v and ground term g:

$$\frac{\forall v\ \alpha}{\text{Subst}(\{v/g\}, \alpha)}$$

Substitute v with g in α

- Every instantiation of a universally quantified sentence is entailed by it
- UI can be applied multiple times to add new sentences
- E.g., $\forall x\ King(x) \wedge Greedy(x) \Rightarrow Evil(x)$ yields:
  - $King(John) \wedge Greedy(John) \Rightarrow Evil(John)$
  - $King(Richard) \wedge Greedy(Richard) \Rightarrow Evil(Richard)$
  - $King(Father(John)) \wedge Greedy(Father(John)) \Rightarrow Evil(Father(John))$

Existential instantiation (EI)

- For any sentence α, variable v, and constant symbol k that does not appear elsewhere in the knowledge base:

$$\frac{\exists v\ \alpha}{\text{Subst}(\{v/k\},\ \alpha)}$$

- EI can be applied once to replace an existential sentence
- E.g., $\exists x\ Crown(x) \land OnHead(x, John)$ yields:

$$Crown(C_1) \land OnHead(C_1, John)$$

provided C_1 is a new constant symbol, called a Skolem constant

Skolemization

Reduction to propositional inference

- Suppose the KB contains just the following:
  - $\forall x$ King(x) $\wedge$ Greedy(x) $\Rightarrow$ Evil(x)
  - King(John)
  - Greedy(John)
  - Brother(Richard,John)

Reduction to propositional inference

- Instantiating the universal sentence in all possible ways, we have:
  - $King(John) \wedge Greedy(John) \Rightarrow Evil(John)$
  - $King(Richard) \wedge Greedy(Richard) \Rightarrow Evil(Richard)$
  - King(John)
  - Greedy(John)
  - Brother(Richard,John)
- The new KB is propositionalized: proposition symbols are
  - King(John), Greedy(John), Evil(John), King(Richard), etc.

Reduction to propositional inference

- Every FOL KB can be propositionalized so as to preserve entailment
  - i.e., a ground sentence is entailed by new KB *iff* entailed by original KB
- A naïve idea for FOL inference:
  - propositionalize KB and query, apply resolution, return result
- Problem: with function symbols, there are infinitely many ground terms,
  - e.g., *Father*(*Father*(*Father*(*John*)))

Reduction to propositional inference

- Theorem (Herbrand, 1930)
  – If a sentence α is entailed by an FOL KB, it is entailed by a finite subset of the propositionalized KB
- Idea:

For n = 0 to ∞ do

1. create a propositional KB by instantiating with depth-n terms
2. if α is entailed by this KB, return true

Function nesting levels

Does this work?

Reduction to propositional inference

- Problem
  - works if α is entailed
  - infinite loops if α is not entailed

- Theorem (Turing, 1936; Church, 1936): entailment for FOL is semi-decidable
  - algorithms exist that say yes to every entailed sentence
  - but no algorithm exists that says no to every non-entailed sentence.

Problems with propositionalization

- Propositionalization generates many irrelevant sentences
- E.g., from:
  - $\forall x\ King(x) \wedge Greedy(x) \Rightarrow Evil(x)$
  - $King(John)$
  - $\forall y\ Greedy(y)$

The query *Evil*(*John*) seems obviously true. But propositionalization produces lots of facts such as *Greedy*(*Richard*) that are irrelevant.

Unification

- Given:
  - $\forall x\ King(x) \wedge Greedy(x) \Rightarrow Evil(x)$
  - $King(John)$
  - $\forall y\ Greedy(y)$

Only variables can be substituted

- If we can find the substitution $\theta = \{x/John, y/John\}$, then we get
  - $King(John) \wedge Greedy(John) \Rightarrow Evil(John)$
  - $King(John)$
  - $Greedy(John)$

  and we can answer the query *Evil(John)* immediately
- Unification finds substitutions that make different expressions identical
  - E.g., *King(x)* vs. *King(John)*; *Greedy(x)* vs. *Greedy(y)*

Unification

- Unify(α,β) = θ if $\alpha\theta = \beta\theta$

p	q	θ
Knows(John,x)	Knows(John,Jane)	
Knows(John,x)	Knows(y,OJ)	
Knows(John,x)	Knows(y,Mother(y))	
Knows(John,x)	Knows(x,OJ)	

Unification

- Unify(α,β) = θ if αθ = βθ

p	q	θ
Knows(John,x)	Knows(John,Jane)	{x/Jane}
Knows(John,x)	Knows(y,OJ)	
Knows(John,x)	Knows(y,Mother(y))	
Knows(John,x)	Knows(x,OJ)	

Unification

- Unify(α,β) = θ if αθ = βθ

p	q	θ
Knows(John,x)	Knows(John,Jane)	{x/Jane}
Knows(John,x)	Knows(y,OJ)	{x/OJ,y/John}
Knows(John,x)	Knows(y,Mother(y))	
Knows(John,x)	Knows(x,OJ)	

Unification

- Unify(α,β) = θ if αθ = βθ

p	q	θ
Knows(John,x)	Knows(John,Jane)	{x/Jane}
Knows(John,x)	Knows(y,OJ)	{x/OJ,y/John}
Knows(John,x)	Knows(y,Mother(y))	{y/John,x/Mother(John)}
Knows(John,x)	Knows(x,OJ)	

Unification

- Unify(α,β) = θ if αθ = βθ

p	q	θ
Knows(John,x)	Knows(John,Jane)	{x/Jane}
Knows(John,x)	Knows(y,OJ)	{x/OJ,y/John}
Knows(John,x)	Knows(y,Mother(y))	{y/John,x/Mother(John)}
Knows(John,x)	Knows(x,OJ)	{fail}

If the two x are bound to different quantifiers, then they can be standardized apart: eliminate overlap of variables, e.g., Knows(z,OJ)

Unification

- To unify Knows(John,x) and Knows(y,z), θ = {y/John, x/z } or θ = {y/John, x/John, z/John}
  - The first unifier is more general than the second.

- There is a single most general unifier (MGU) that is unique up to renaming of variables.
  - MGU = { y/John, x/z }

FOL Inference

- Horn logic (the FOL case)
  - Forward chaining
  - Backward chaining
- General FOL
  - Resolution

Horn clauses in FOL

- $p_1 \wedge p_2 \wedge \ldots \wedge p_n \Rightarrow q$
  - $p_1, p_2, \ldots, p_n$, q are atomic sentences
  - All variables assumed to be universally quantified
- E.g., $human(x) \Rightarrow mortal(x)$

Generalized Modus Ponens (GMP)

$$\frac{p_1', p_2', \ldots, p_n', (p_1 \wedge p_2 \wedge \ldots \wedge p_n \Rightarrow q)}{q\theta}$$

where $p_i'\theta = p_i\theta$ for all i

Example: King(John), Greedy(y), (King(x) $\wedge$ Greedy(x) $\Rightarrow$ Evil(x))

p_1' is *King*(*John*) p_1 is *King*(*x*)

p_2' is *Greedy*(*y*) p_2 is *Greedy*(*x*)

Therefore, θ is {x/John, y/John}

q is *Evil*(*x*), so $q\theta$ is *Evil*(*John*)

Example knowledge base

- The US law says that it is a crime for an American to sell weapons to hostile nations. The country Nono, an enemy of America, has some missiles, and all of its missiles were sold to it by Colonel West, who is American.

- Prove that Col. West is a criminal

Example knowledge base contd.

... it is a crime for an American to sell weapons to hostile nations:

$American(x) \wedge Weapon(y) \wedge Sells(x,y,z) \wedge Hostile(z) \Rightarrow Criminal(x)$

Nono … has some missiles, i.e., $\exists x\ Owns(Nono,x) \wedge Missile(x)$:

$Owns(Nono,M_1)$ and $Missile(M_1)$

… all of its missiles were sold to it by Colonel West

$Missile(x) \wedge Owns(Nono,x) \Rightarrow Sells(West,x,Nono)$

Missiles are weapons:

$Missile(x) \Rightarrow Weapon(x)$

An enemy of America counts as "hostile“:

$Enemy(x,America) \Rightarrow Hostile(x)$

West, who is American …

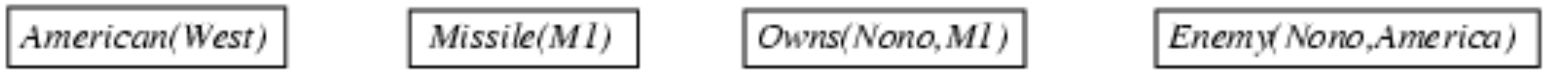
$American(West)$

The country Nono, an enemy of America …

$Enemy(Nono,America)$

Forward chaining proof

- *American(x) ∧ Weapon(y) ∧ Sells(x,y,z) ∧ Hostile(z) ⇒ Criminal(x)*
- *Missile(x) ∧ Owns(Nono,x) ⇒ Sells(West,x,Nono)*
- *Missile(x) ⇒ Weapon(x)*
- *Enemy(x,America) ⇒ Hostile(x)*
- *Owns(Nono,M_1), Missile(M_1), American(West), Enemy(Nono,America)*



Forward chaining proof

- *American(x)* $\wedge$ *Weapon(y)* $\wedge$ *Sells(x,y,z)* $\wedge$ *Hostile(z)* $\Rightarrow$ *Criminal(x)*
- *Missile(x)* $\wedge$ *Owns(Nono,x)* $\Rightarrow$ *Sells(West,x,Nono)*
- *Missile(x)* $\Rightarrow$ *Weapon(x)*
- *Enemy(x,America)* $\Rightarrow$ *Hostile(x)*
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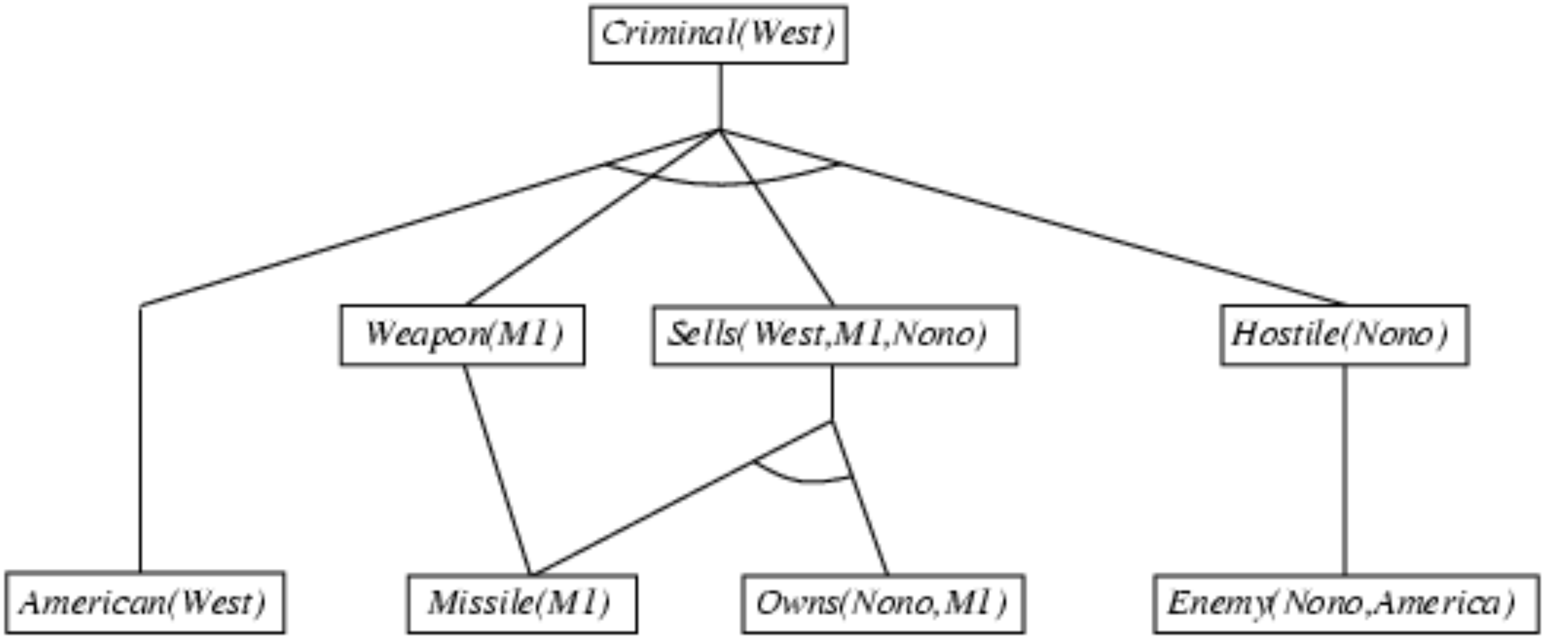

Properties of forward chaining

- Sound and complete for first-order Horn clauses
- FC terminates for first-order Horn clauses with no functions (Datalog) in finite number of iterations
- In general, FC may not terminate if α is not entailed
  - This is unavoidable: entailment with Horn clauses is also semi-decidable

Backward chaining example

- *American(x)* $\wedge$ *Weapon(y)* $\wedge$ *Sells(x,y,z)* $\wedge$ *Hostile(z)* $\Rightarrow$ *Criminal(x)*
- *Missile(x)* $\wedge$ *Owns(Nono,x)* $\Rightarrow$ *Sells(West,x,Nono)*
- *Missile(x)* $\Rightarrow$ *Weapon(x)*
- *Enemy(x,America)* $\Rightarrow$ *Hostile(x)*
- *Owns(Nono,*M_1*), Missile(*M_1*), American(West), Enemy(Nono,America)*

$$\boxed{Criminal(West)}$$

Backward chaining example

- *American(x) $\wedge$ Weapon(y) $\wedge$ Sells(x,y,z) $\wedge$ Hostile(z) $\Rightarrow$ Criminal(x)*
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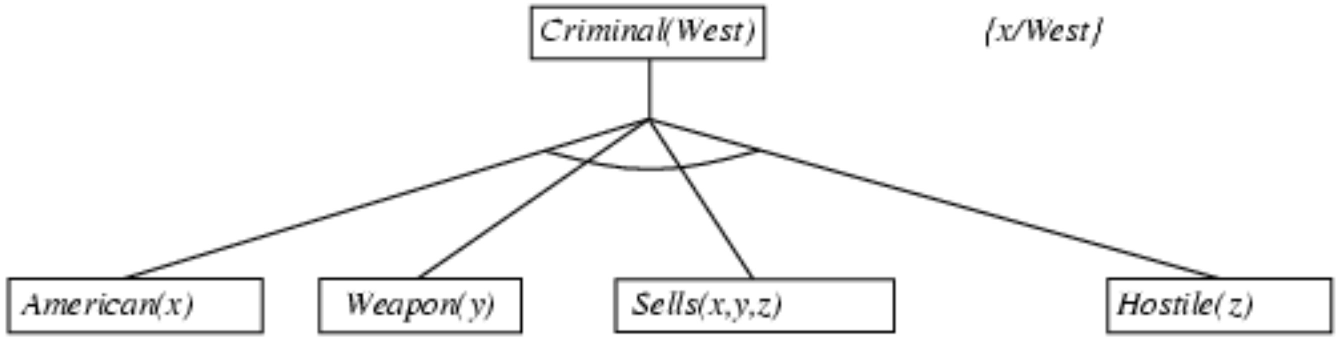

Backward chaining example

- *American(x) ∧ Weapon(y) ∧ Sells(x,y,z) ∧ Hostile(z) ⇒ Criminal(x)*
- *Missile(x) ∧ Owns(Nono,x) ⇒ Sells(West,x,Nono)*
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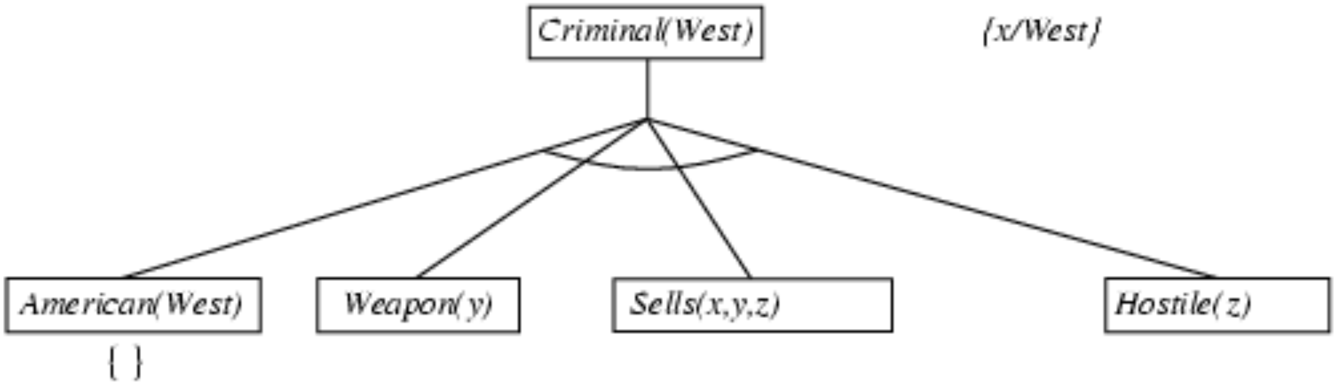

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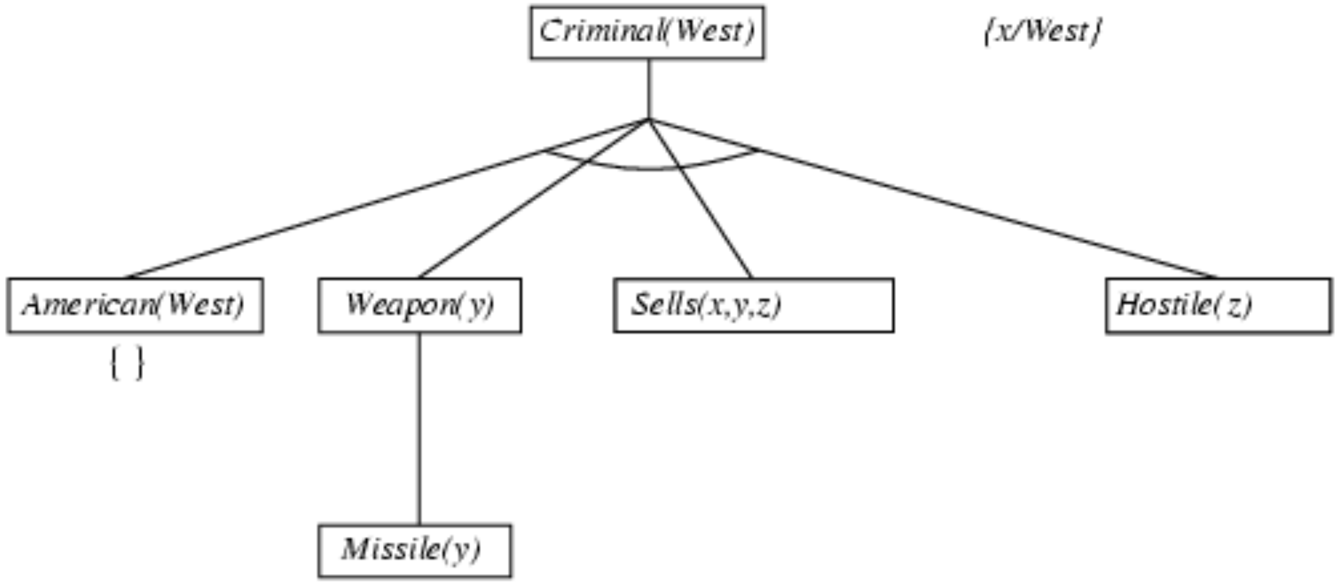

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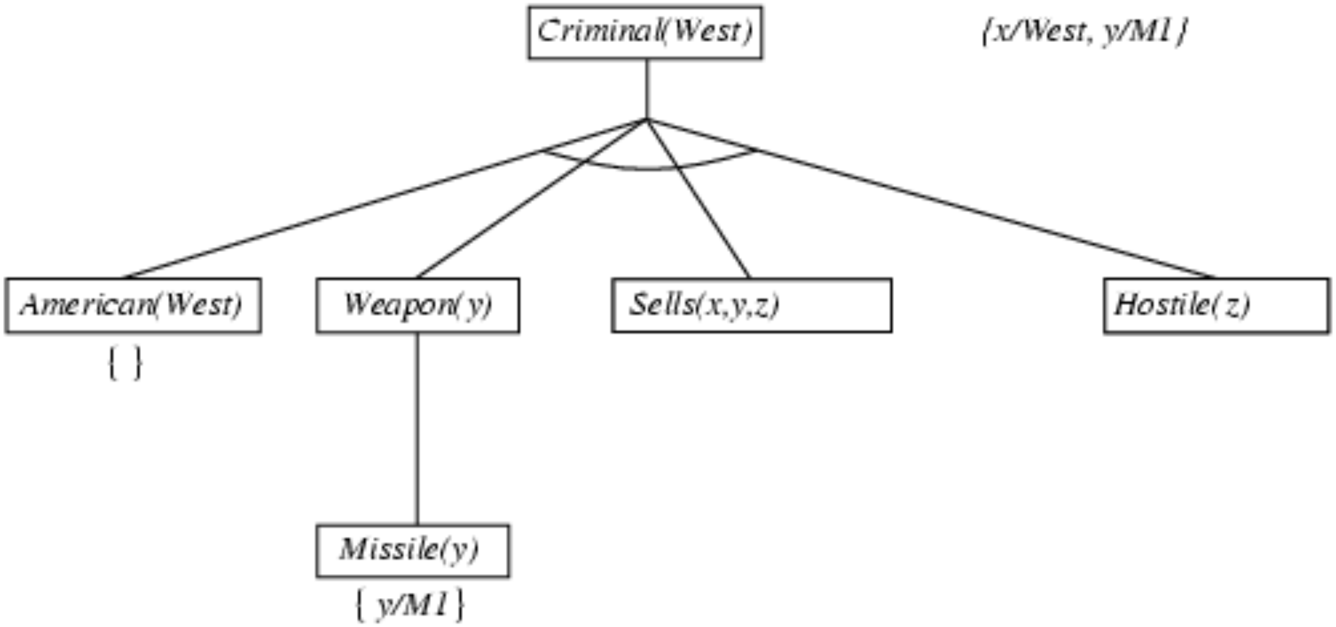
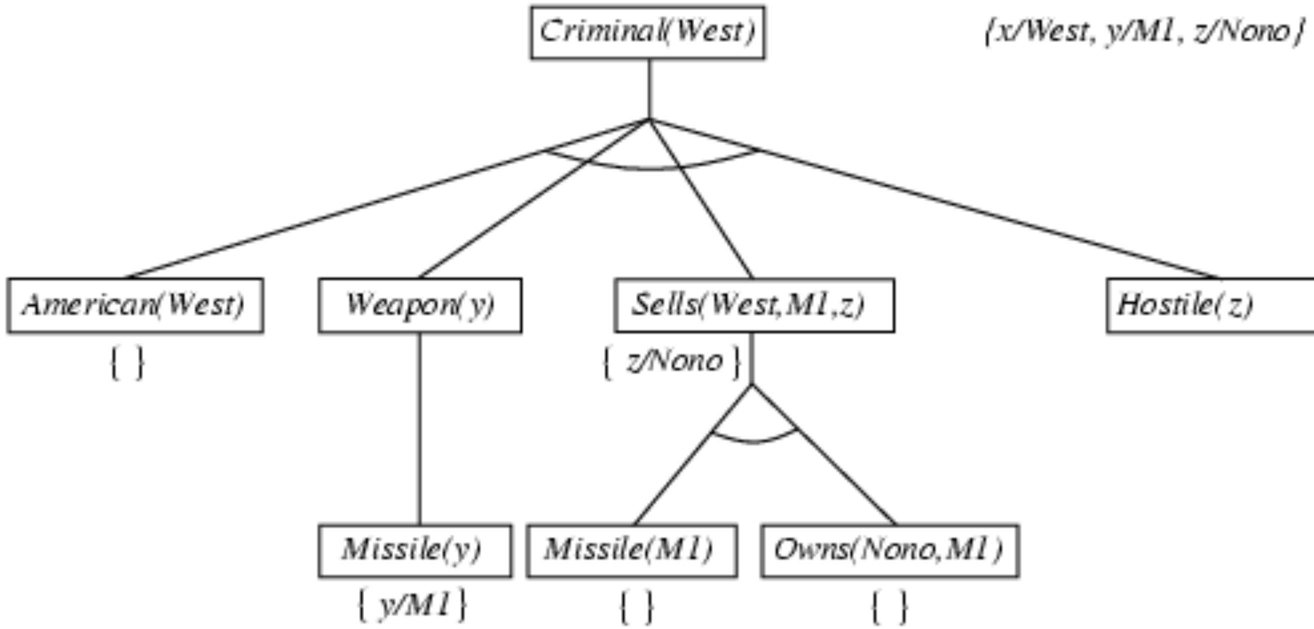


Backward chaining example

- $American(x) \wedge Weapon(y) \wedge Sells(x,y,z) \wedge Hostile(z) \Rightarrow Criminal(x)$
- $Missile(x) \wedge Owns(Nono,x) \Rightarrow Sells(West,x,Nono)$
- $Missile(x) \Rightarrow Weapon(x)$
- $Enemy(x,America) \Rightarrow Hostile(x)$
- $Owns(Nono,M_1)$, $Missile(M_1)$, $American(West)$, $Enemy(Nono,America)$



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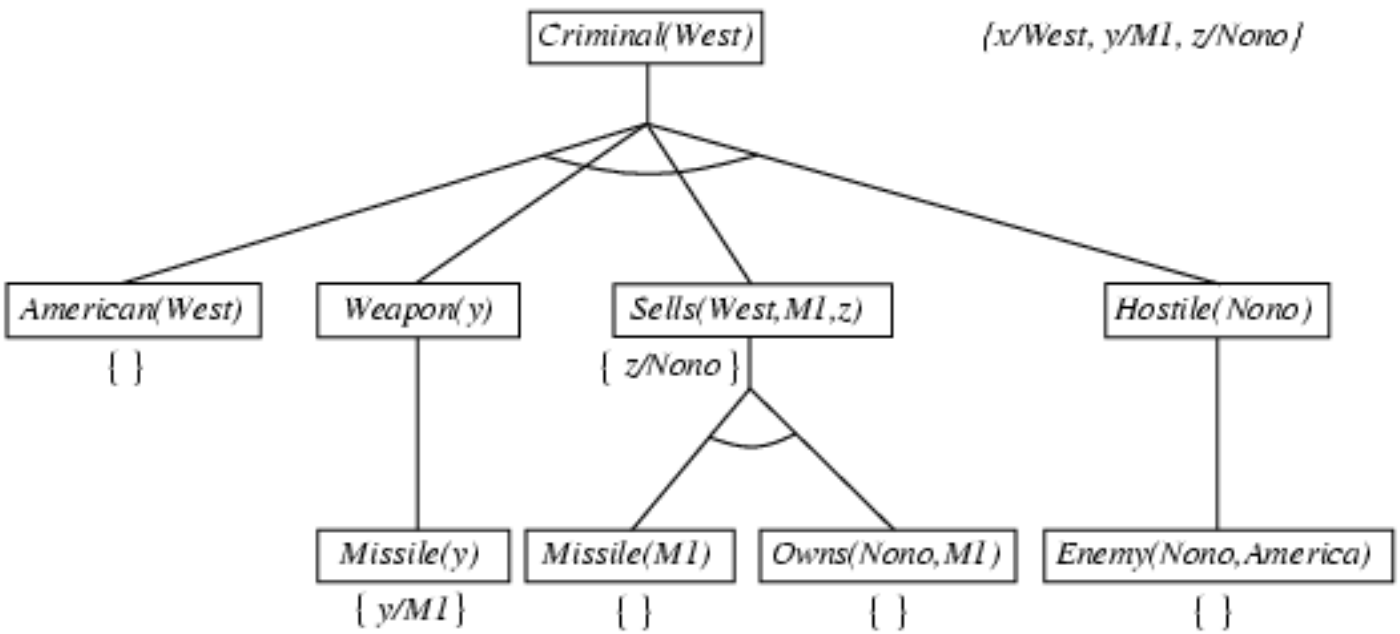

Backward chaining

- Depth-first recursive proof search: space is linear in size of proof
- Avoid infinite loops by checking current goal against every goal on stack
- Avoid repeated subgoals by caching previous results

- Widely used for logic programming

Logic programming

- Ordinary programming
  - Identify problem
  - Assemble information
  - Figure out solution
  - Encode solution
  - Encode problem instance as data
  - Apply program to data
- Logic programming
  - Identify problem
  - Assemble information
  - <coffee break> ☺
  - Encode info in KB
  - Encode problem instances as facts
  - Ask queries (run SAT solver)

Logic programming: Prolog

- Was widely used in Europe, Japan (basis of 5th Generation project)
- Basis: backward chaining with Horn clauses
  - Program = set of Horn clauses
  - Inference: depth-first, left-to-right backward chaining
- Additions:
  - Built-in predicates for arithmetic etc., e.g., X is Y*Z+3
  - Built-in predicates that have side effects (e.g., input and output predicates, assert/retract predicates)
- Closed-world assumption ("negation as failure")

Resolution

- Full first-order version:

$$\frac{\ell_1 \vee \cdots \vee \ell_k, \qquad m_1 \vee \cdots \vee m_n}{(\ell_1 \vee \cdots \vee \ell_{i-1} \vee \ell_{i+1} \vee \cdots \vee \ell_k \vee m_1 \vee \cdots \vee m_{j-1} \vee m_{j+1} \vee \cdots \vee m_n)\theta}$$

  where `Unify`(ℓ_i, $\neg m_j$) = θ.

  – The two clauses are assumed to be standardized apart so that they share no variables.

- Example:

$$\frac{\neg Rich(x) \vee Unhappy(x) \qquad Rich(Ken)}{Unhappy(Ken)}$$

  with θ = {x/Ken}

- Inference algorithm: applying resolution steps to CNF(KB $\wedge \neg\alpha$)
- Resolution is sound and complete for FOL

Conversion to CNF

$\forall x\ [\forall y\ Animal(y) \Rightarrow Loves(x,y)] \Rightarrow [\exists y\ Loves(y,x)]$

1. Eliminate biconditionals and implications

   $\forall x\ [\neg\forall y\ \neg Animal(y) \vee Loves(x,y)] \vee [\exists y\ Loves(y,x)]$

2. Move $\neg$ inwards: $\neg\forall x\ p \equiv \exists x\ \neg p$, $\neg\exists x\ p \equiv \forall x\ \neg p$

   $\forall x\ [\exists y\ \neg(\neg \mathrm{Animal}(y) \vee \mathrm{Loves}(x,y))] \vee [\exists y\ \mathrm{Loves}(y,x)]$

   $\forall x\ [\exists y\ \neg\neg \mathrm{Animal}(y) \wedge \neg \mathrm{Loves}(x,y)] \vee [\exists y\ \mathrm{Loves}(y,x)]$

   $\forall x\ [\exists y\ \mathrm{Animal}(y) \wedge \neg \mathrm{Loves}(x,y)] \vee [\exists y\ \mathrm{Loves}(y,x)]$

3. Standardize variables: each quantifier should use a different variable

   $\forall x\ [\exists y\ \mathrm{Animal}(y) \wedge \neg \mathrm{Loves}(x,y)] \vee [\exists z\ \mathrm{Loves}(z,x)]$

Conversion to CNF contd.

$\forall x\ [\exists y\ Animal(y) \wedge \neg Loves(x,y)] \vee [\exists z\ Loves(z,x)]$

4. Skolemize: a more general form of existential instantiation. Each existential variable is replaced by a Skolem function of the enclosing universally quantified variables:

   $\forall x\ [Animal(F(x)) \wedge \neg Loves(x,F(x))] \vee Loves(G(x),x)$

5. Distribute $\vee$ over $\wedge$:

   $\forall x\ [Animal(F(x)) \vee Loves(G(x),x)] \wedge [\neg Loves(x,F(x)) \vee Loves(G(x),x)]$

6. Drop universal quantifiers

Semantic Web: Application of Logic to WWW

acm

MORE ACM AWARDS

A.M. TURING AWARD

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FEATURED AWARD WINNER:

Sir Tim Berners

For inventing the World
the first web browser, an
fundamental protocols an
allowing the Web to scale.

More on Sir Tim Berners-Lee and his
work can be found here

Inventor of World Wide Web Receives ACM A.M. Turing Award

Sir Tim Berners-Lee Designed Integrated Architecture and Technologies that Underpin the Web

ACM named Sir Tim Berners-Lee, a Professor at Massachusetts Institute of Technology and the University of Oxford, the recipient of the 2016 ACM A.M. Turing Award. Berners-Lee was cited for

The Semantic Web

Since the late 1990s Berners-Lee's primary focus has been on trying to get Web publishers and technology companies to add a set of capabilities he called the "Semantic Web." Berners-Lee defined his idea as follows: "The Semantic Web is an extension of the current Web in which information is given well-defined meaning, better enabling computers and people to work in cooperation."

commerce, and perform many other important activities.

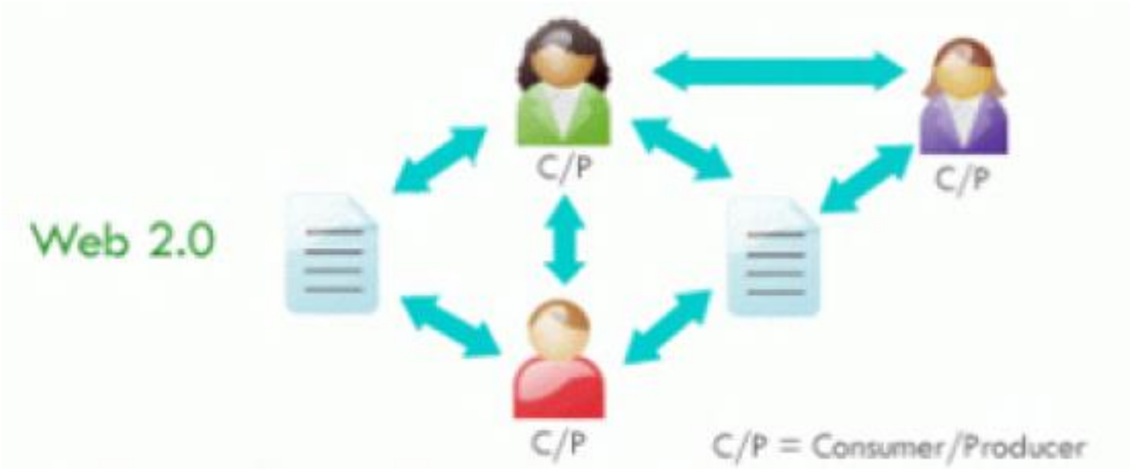
The ACM Turing Award, often referred to as the "Nobel Prize of

Web 1.0

- Information flows from providers to consumers
  - The majority of WWW users are consumers
  - Webpages are static and read-only

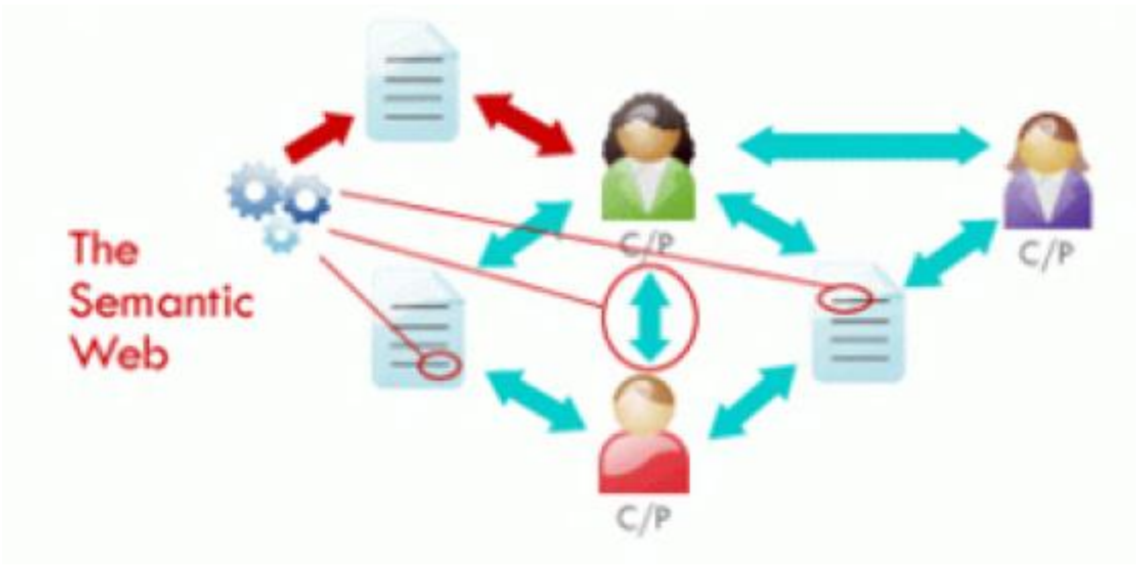
Web 2.0

- WWW users are both providers and consumers
  - Webpages are dynamic and interactive



Semantic Web (Web 3.0)

- Computers mediate the information flow, helping users publish and acquire data
  - Semantic annotations (i.e., logic sentences) on data that can be understood and processed by computers
  - (Semantic Web is Symbolic AI applied to WWW)



RDF

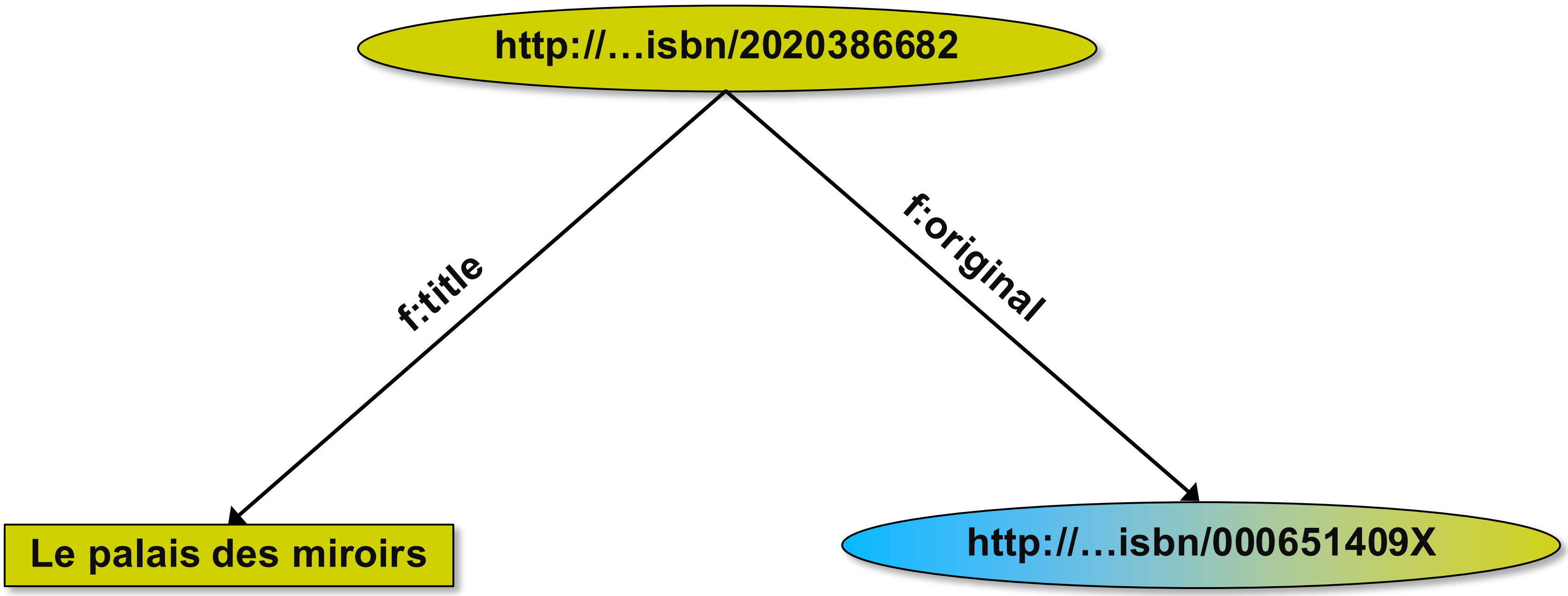
- RDF (Resource Description Framework)
  - The basic language of semantic web
- RDF represents information with triples (s,p,o)
  - s/p/o : “subject”, “property”, and “object”
  - p(s,o) in FOL

```
(<http://…isbn…6682>, <http://…/title>, Le palais des mirroirs)
```

URI: a unique string used
to identify a resource

- RDF triples form a directed, labeled graph

Example

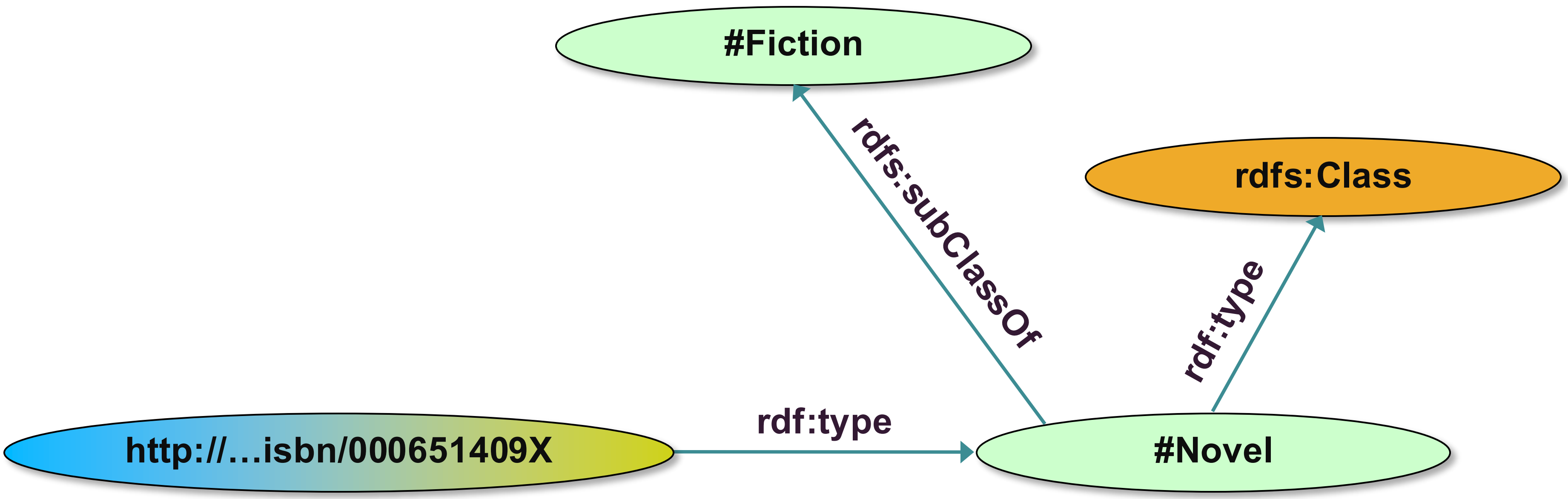


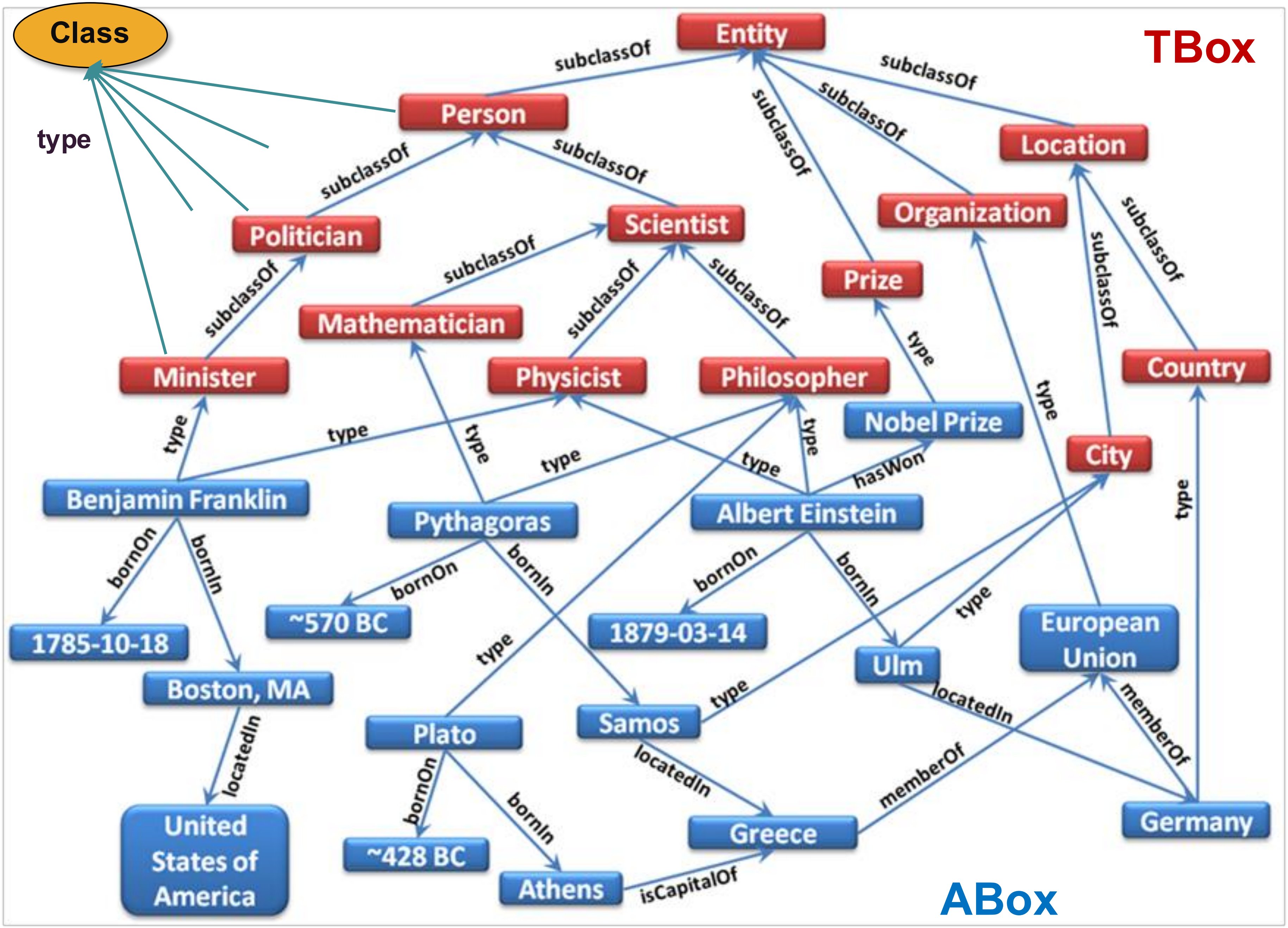

```
<rdf:Description rdf:about="http://…/isbn/2020386682">
    <f:title xml:lang="fr">Le palais des mirroirs</f:title>
    <f:original rdf:resource="http://…/isbn/000651409X"/>
</rdf:Description>
```

RDFS

- RDFS (RDF Schema) is used to specify the ontology (a set of constants and predicates)
  - "rdfs:Class": a collection of resources
    - novel, fiction, …
  - "rdf:type": an individual belongs to a specific class
    - "«http://.../000651409X» is a novel"
  - "rdfs:subClassOf": all instances of one are also the instances of the other
    - "every novel is a fiction"
  - rdfs:domain, rdfs:range, rdfs:subPropertyOf, etc.

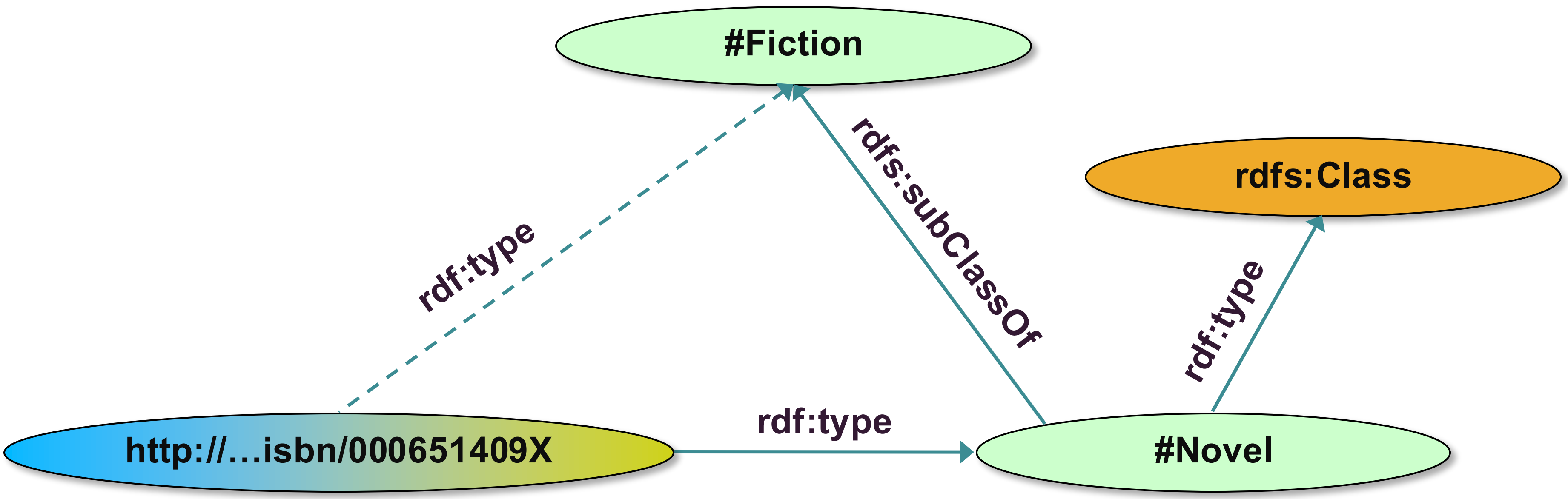
Classes, Resources, …





Class
type
TBox
Entity
Person
Location
Politician
Scientist
Organization
Prize
Mathematician
Minister
Physicist
Philosopher
Nobel Prize
Country
City
Benjamin Franklin
Pythagoras
Albert Einstein
1785-10-18
~570 BC
1879-03-14
Ulm
European Union
Boston, MA
Samos
Plato
United States of America
~428 BC
Athens
Greece
Germany
subclassOf
type
bornOn
bornIn
hasWon
locatedIn
memberOf
isCapitalOf
ABox

Inference



`(<http://.../isbn/000651409X> rdf:type #Fiction)`

New triples can be inferred from RDFS rules!

Inference

- The RDF Semantics document has a list of (33) inference rules:
  - “if such and such triples are in the graph, add this and this”

```
If:
  uuu rdfs:subClassOf xxx .
  vvv rdf:type uuu .
Then add:
  vvv rdf:type xxx .
```

  - Can do that recursively until the graph does not change
    - *Forward chaining!*

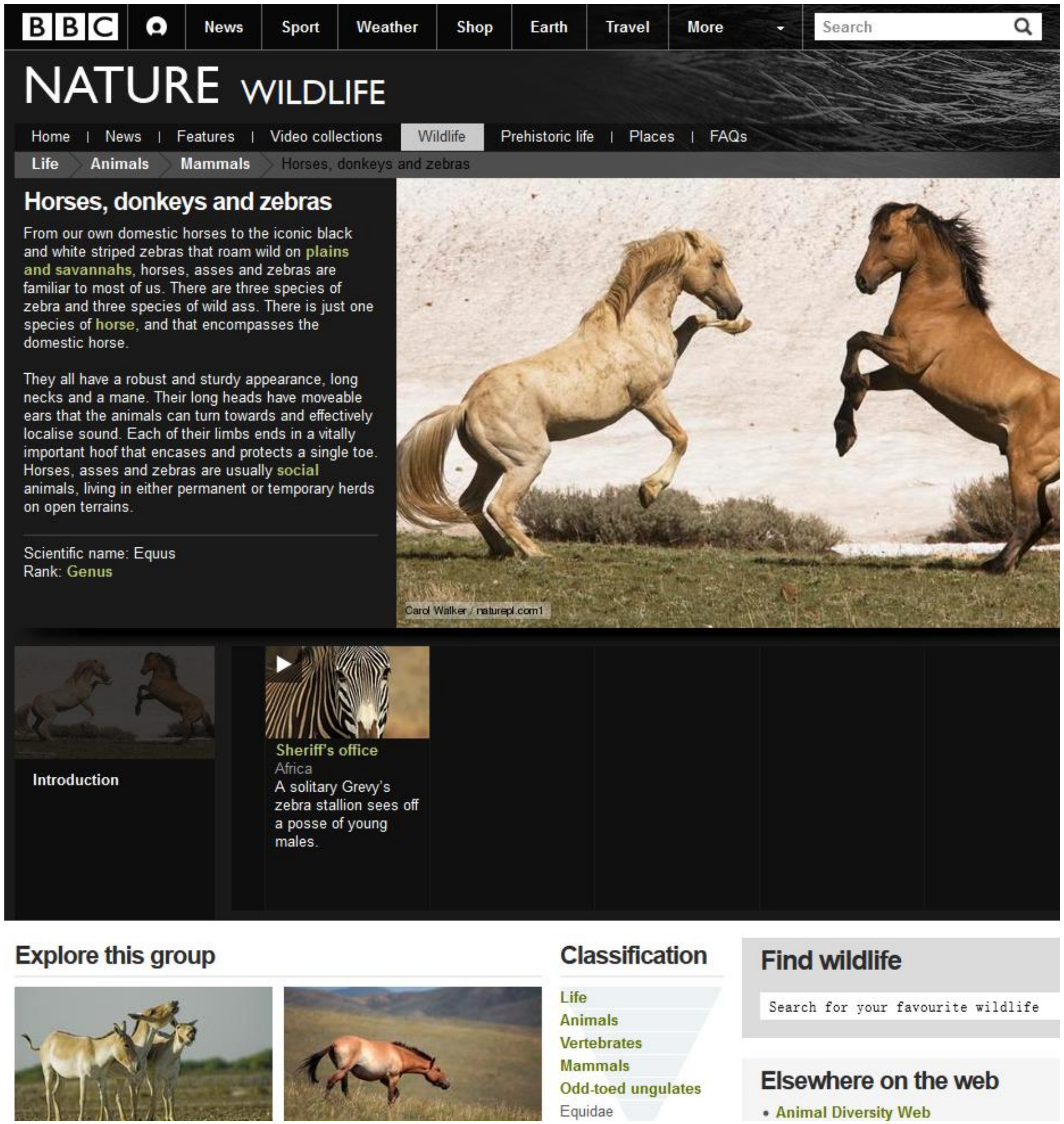
BBC Wildlife

- A BBC website on wildlife
- The whole website is automatically created from an RDF knowledge base
  - Every webpage is based on an RDF file

- Unfortunately, the site is offline since 2018 ☹
- An archive is available at https://github.com/rdmpage/bbc-wildlife

Offline since 2018 ☹

http://www.bbc.co.uk/nature/life/Equus_(genus)



Offline since 2018 ☹

http://www.bbc.co.uk/nature/life/Equus_(genus).rdf

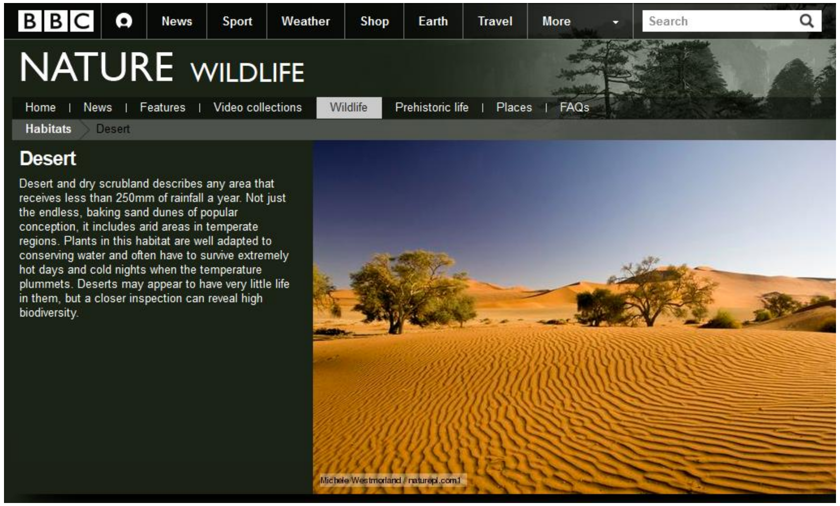

```
- <rdf:RDF>
  - <rdf:Description rdf:about="/nature/genus/Equus_(genus)">
      <foaf:primaryTopic rdf:resource="/nature/genus/Equus_(genus)#genus"/>
      <rdfs:seeAlso rdf:resource="/nature/genus"/>
    </rdf:Description>
  - <wo:Genus rdf:about="/nature/life/Equus_(genus)#genus">
      <rdfs:label>Horses, donkeys and zebras</rdfs:label>
      <wo:name rdf:resource="http://www.bbc.co.uk/nature/genus/Equus_(genus)#name"/>
      <foaf:depiction rdf:resource="http://ichef.bbci.co.uk/naturelibrary/images/ic/640x360/e/eq/equus_genus
      /equus_genus_1.jpg"/>
    - <dc:description>
        From our own domestic horses to the iconic black and white striped zebras that roam wild on
        <a href="/nature/habitats/Temperate_grasslands%2C_savannas%2C_and_shrublands">plains and savannahs</a>
        , horses, asses and zebras are familiar to most of us. There are three species of zebra and three species of
        wild ass. There is just one species of
        <a href="/nature/life/Wild_horse">horse</a>
        , and that encompasses the domestic horse.
        <br/>
        <br/>
        They all have a robust and sturdy appearance, long necks and a mane. Their long heads have moveable ears
        that the animals can turn towards and effectively localise sound. Each of their limbs ends in a vitally
        important hoof that encases and protects a single toe. Horses, asses and zebras are usually
        <a href="/nature/adaptations/Presociality">social</a>
        animals, living in either permanent or temporary herds on open terrains.
      </dc:description>
      <owl:sameAs rdf:resource="http://dbpedia.org/resource/Equus_(genus)"/>
      <wo:adaptation rdf:resource="/nature/adaptations/Hearing_(sense)#adaptation"/>
      <wo:adaptation rdf:resource="/nature/adaptations/Herbivore#adaptation"/>
      <wo:adaptation rdf:resource="/nature/adaptations/Parental_investment#adaptation"/>
      <wo:adaptation rdf:resource="/nature/adaptations/Presociality#adaptation"/>
```

Offline since 2018 ☹

http://www.bbc.co.uk/nature/life/Equus_(genus).rdf

- Connects to other resources in the dataset

```
Equus_(genus) wo:livesIn Deserts_and_xeric_shrublands
```



www.bbc.co.uk/nature/habitats/Deserts_and_xeric_shrublands

www.bbc.co.uk/nature/life/Equus_(genus)

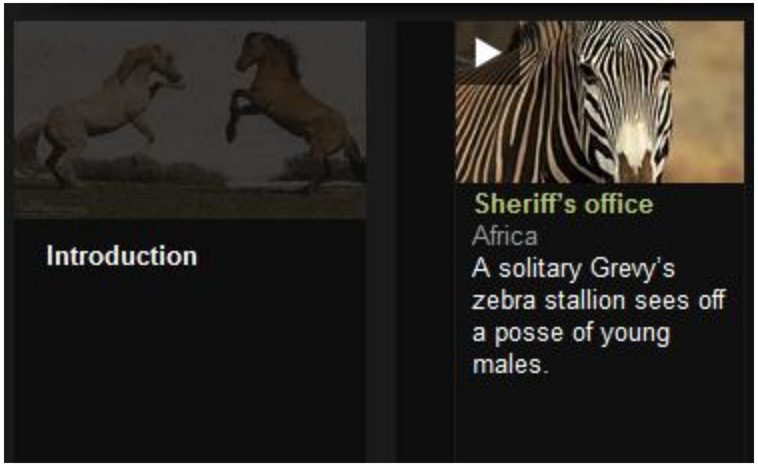
Offline since 2018 ☹

http://www.bbc.co.uk/nature/life/Equus_(genus).rdf

- Connects to resources in other BBC datasets

```
Equus_(genus) po:Clip programmes/p014dcrj
```

http://www.bbc.co.uk/programmes/p014dcrj

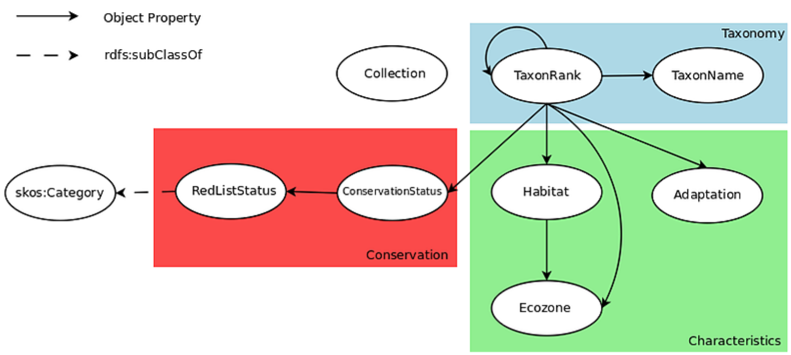


www.bbc.co.uk/nature/life/Equus_(genus)

BBC **Wildlife Ontology**

Ontology Diagram

The following diagram illustrates the relationships between the key classes in the ontology. A number of classes, e.g. sub classes of TaxonRank, Habitat and Adaptation have been omitted for clarity.



Ontology Terms

Automatically generated documentation for the ontology terms.

Classes

Adaptation	
URI	http://purl.org/ontology/wo/Adaptation
Description	An adaptation is any feature of an animal or plant which makes it better suited for a particular habitat or to do a particular task. For instance, being streamlined is an adaptation to swimming fast and being able to survive on very little water is an adaptation to life in the desert.
Subclasses	ExtremesAdaptation, BehaviouralPattern, CommunicationAdaptation, EcosystemRole, FeedingHabit, LifeCycle, LocomotionAdaptation, MorphologyAdaptation, PredationStrategy, ReproductionStrategy, SocialBehaviour, SurvivalStrategy
Properties	adaptation

Adapted to Extremes	

Adapted to Extremes	
URI	http://purl.org/ontology/wo/ExtremesAdaptation
Description	Organisms that are adapted to extremes (known as Extremophiles) are organisms that thrives in and even may require physically or geochemically extreme conditions that are detrimental to the majority of life on Earth.
Superclasses	Adaptation

Animal Intelligence	
URI	http://purl.org/ontology/wo/AnimalIntelligence
Description	Animal Intelligence or animal cognition is the title given to a modern approach to the mental capacities of non-human animals. It has developed out of comparative psychology, but has also been strongly influenced by the approach of ethology, behavioral ecology, and evolutionary psychology.

Behavioural Pattern	
URI	http://purl.org/ontology/wo/BehaviouralPattern
Description	Behavioural pattern describes an animal's dominant way of life. Arboreal animals, for example, live in trees and nocturnal animals are active at night.
Superclasses	Adaptation

Class	
URI	http://purl.org/ontology/wo/Class
Description	A class is a scientific way to group related organisms together, some examples of classes being jellyfish, reptiles and sea urchins. Classes are big groups and contain within them smaller groupings called orders, families, genera and species.
Superclasses	TaxonRank
Properties	class

Collection	
URI	http://purl.org/ontology/wo/Collection
Description	A collection of resources, including documents, multimedia files, programme clips and their associated taxa, which aims to showcase a particular aspect of natural history film-making, or illustrate aspects of the natural world. A collection provides an alternate way to organize content over and above the basic taxonomic hierarchy.
Superclasses	http://purl.org/dc/dcmitype/Collection
Properties	collection

Communication Adaptation	
URI	http://purl.org/ontology/wo/CommunicationAdaptation

http://www.bbc.co.uk/ontologies

- 14 domain ontologies
  - Core Concepts Ontology
  - Programmes Ontology
  - Business News Ontology
  - Politics Ontology
  - Food Ontology
  - Sport Ontology
  - Wildlife Ontology
  - ……

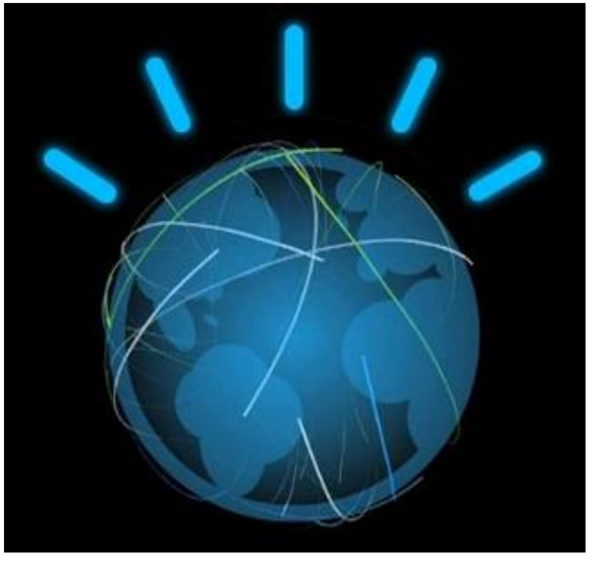
Google Knowledge Graph

- Google uses a knowledge base to enhance its
  - search engine
  - personal assistant
  - smart speaker
- KB size
  - (2012) over 570 million objects and more than 18 billion facts
  - (2016) over 70 billion facts.

IBM Watson

- A computer system capable of answering questions posed in natural language.

- The computer system was initially developed to answer questions on the popular quiz show Jeopardy! and in 2011, the Watson computer system competed on Jeopardy! against champions Brad Rutter and Ken Jennings, *winning the first-place prize of 1 million USD*.



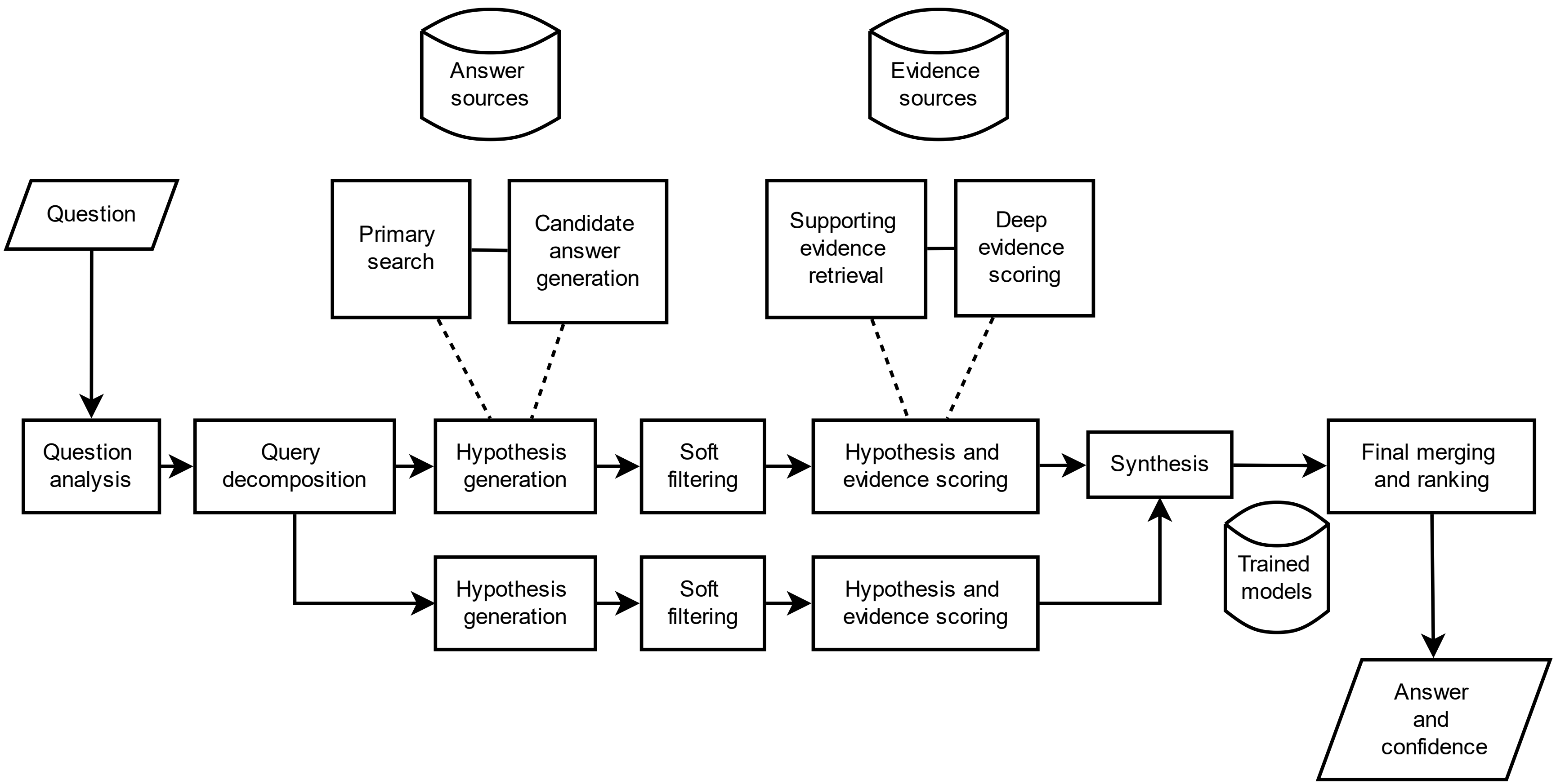
IBM Watson

- A computer system capable of answering questions posed in natural language.

https://www.ibm.com/history/watson-jeopardy

IBM Watson

- The whole framework



Summary

- First-order logic:
  - objects and relations are semantic primitives
  - syntax: constants, functions, predicates, quantifiers
- Inference
  - Unification
  - Forward/backward chaining
  - Resolution
- Semantic web
  - Application of predicate logic to WWW