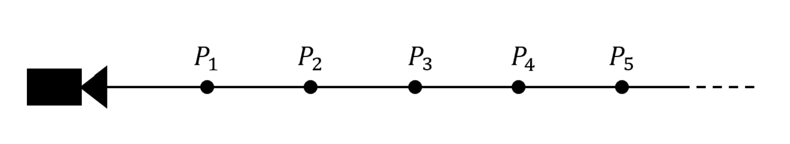
1. Ray marching is a common technique to achieve volume rendering. Now consider a single ray, and the front-to-back compositing scheme is used. The sample points during the ray marching process are P_1, P_2, P_3, P_4 and P_5 (assuming that only these points are used for volume rendering), as shown in the figure below.

   After sampling the volume data and applying the transfer function, the output color $\mathbf{C}_i$ and alpha α_i are listed below:

$$\mathbf{C}_1 = (0.0, 0.2, 0.1),\ \alpha_1 = 0.2$$
$$\mathbf{C}_2 = (0.1, 0.3, 0.2),\ \alpha_2 = 0.5$$
$$\mathbf{C}_3 = (0.3, 0.0, 0.2),\ \alpha_3 = 0.5$$
$$\mathbf{C}_4 = (0.5, 0.2, 0.1),\ \alpha_4 = 0.8$$
$$\mathbf{C}_5 = (0.1, 0.2, 0.5),\ \alpha_5 = 1.0$$



   Assuming that we apply early ray termination when the accumulated beam transmittance (transparency) is lower than 0.1, please give the resulting color of this ray and show the entire derivation process.

   Answer:

   Initial: $\hat{\mathbf{C}} = (0, 0, 0)$, $\hat{T} = 1.0$

   After P_1: $\hat{\mathbf{C}} = (0, 0.2, 0.1)$, $\hat{T} = 0.8$

   After P_2: $\hat{\mathbf{C}} = (0.08, 0.44, 0.26)$, $\hat{T} = 0.4$

   After P_3: $\hat{\mathbf{C}} = (0.2, 0.44, 0.34)$, $\hat{T} = 0.2$

   After P_4: $\hat{\mathbf{C}} = (0.3, 0.48, 0.36)$, $\hat{T} = 0.04$

   Ray terminates here. So the answer is $(0.3, 0.48, 0.36)$.

2. A transfer function is needed when rendering the volume data, which maps the volume data at any point into the corresponding optical properties. The transfer function determines what the volume data looks like.

   Assume we have a scalar value v as the volume data at some point, and the optical properties should contain the opacity α. Please design the transfer function $\alpha = f(v)$ such that the iso-surface of the volume with the iso-value v_0 could be rendered with volume ray casting, in which the iso-surface has an opacity α_0. Please describe it, and explain why.

   Answer: Any reasonable answer is acceptable. A reference one (normal distribution): $f(v) = \frac{\alpha_0}{\sqrt{2\pi}} e^{-\frac{(v-v_0)^2}{2}}$.