

Homework 3

*Professor: Ziyu Shao**Due: 2025/04/06 11:59am*

1. **Binary Sequences with No Adjacent 1s:** compute the expected number of 1s in a good sequence if all good sequences are equal likely ($m=150$)
 - (a) Find the analytical solution
 - (b) Implement a DTMC based MCMC algorithm for approximate computing
 - (c) Implement a CTMC based MCMC algorithm for approximate computing
2. **Power-Law Distribution:** Implement a Metropolis-Hastings algorithm to simulate from the power-law distributions shown in the lecture.
3. **Knapsack Problem:** $m=50$, $w=100$, each g_j is sampled from the discrete uniform distribution over the supporting set $\{3, 4, 5, 6, 7\}$, each w_j is chosen random from the discrete uniform distribution over the supporting set $\{2, 4, 6, 8\}$.
 - (a) Find the maximization of the total worth of the treasure by using DP method
 - (b) Find the maximization of the total worth of the treasure by using Metropolis-Hastings(MH) algorithm
 - (c) Discuss the choice of β & function of $v(x)$ in MCMC, verify your argument with numerical experiments
 - (d) Discuss the pros and cons of both MCMC(MH) and DP methods
4. **Standard Normal Distribution:** generate samples from the standard normal distribution
 - (a) Implement a Metropolis-Hastings algorithm.
 - (b) Implement a Hamiltonian Monte Carlo algorithm
 - (c) Implement with the Box-Muller Method.
 - (d) Compare the above three algorithms with corresponding pros and cons.
5. **Beta Normal Distribution:** generate samples from the beta distribution $\text{Beta}(5, 5)$
 - (a) Implement a Metropolis-Hastings algorithm.
 - (b) Implement a Hamiltonian Monte Carlo algorithm
 - (c) Implement with the Acceptance-Rejection Method.

- (d) Compare the above three algorithms with corresponding pros and cons.
- 6. **Normal-Normal Conjugacy:** Implement a Metropolis-Hastings algorithm to find the posterior mean and variance of θ after observing the value of $Y = y$. The parameter setting: $y = 3, \mu = 0, \sigma^2 = 1, \tau^2 = 4$, and $d = 0.1, 0.5, 1, 5, 10, 100$.
- 7. **Bivariate Standard Normal Distribution:** Implement a Gibbs sampler to generate samples from a bivariate standard normal distribution with correlation $\rho = 0.6, -0.6$.
- 8. **Chicken-Egg with Unknown Parameters:** The parameter setting: $\lambda = 10, a = b = 1, x = 7$. (a) Implement a Gibbs sampler to find the posterior mean and the variance of p after observing x hatched eggs. (b) Implement a Metropolis-Hastings algorithm to find the posterior mean and variance of p after observing x hatched eggs. (c) Compare such two methods of MCMC.
- 9. **Three-dimensional Joint Distribution:** Implement a Gibbs sampler to generate samples from the three-dimensional joint distribution shown in the lecture.
- 10. **Example I in the Lecture, Section 4**
 - (a) Implement a Metropolis-Hastings algorithm.
 - (b) Implement a Hamiltonian Monte Carlo algorithm
 - (c) Implement with the Acceptance-Rejection Method.
 - (d) Compare the above three algorithms with corresponding pros and cons.
- 11. **Example II in the Lecture, Section 4**
 - (a) Implement a Metropolis-Hastings algorithm.
 - (b) Implement a Hamiltonian Monte Carlo algorithm
 - (c) Implement with the Acceptance-Rejection Method.
 - (d) Compare the above three algorithms with corresponding pros and cons.
- 12. How to solve a general combinatorial optimization problem with MCMC method?
- 13. Illustrate the relationship between sampling and optimization.
- 14. **Markov Chain Monte Carlo for Wireless Networks.** Given a wireless network with 24 links and 0 – 1 interference model, i.e., any two links are either interfere with each other or not. To describe the interference relationship between wireless links, we introduce the conflict graph model. In such model, the vertex of the conflict graph represents the wireless link. An edge between two vertices means corresponding two links interfere with each other. The following Figure shows the corresponding conflict graph for 24-link wireless network. You are required to find the maximum independent set of the conflict graph, i.e., the largest set of wireless links that can simultaneously

transmit without interferences. Design the algorithm by MCMC method and evaluate your algorithm.

- Show your MCMC Design with a discrete Markov chain. Use both theory and simulation results to justify your algorithms.
- Show your MCMC Design with a continuous Markov chain. Use both theory and simulation results to justify your algorithms.
- Discuss the pros and cons for such chains.

