EE150 - Homework 6

Problem 1

(20 points)

(a) (10 points) Given a band-limited input $x(t)$, and the frequency response of two ideal low-pass filters $H_1(j\omega)$ (zero-phase) and $H_2(j\omega)$ with cut-off frequency ω_c, plot the output signal $y_1(t)$ and $y_2(t)$ filtered by $H_1(j\omega)$ and $H_2(j\omega)$, respectively. Note that the maximum frequency of $x(t)$ is lower than ω_c.

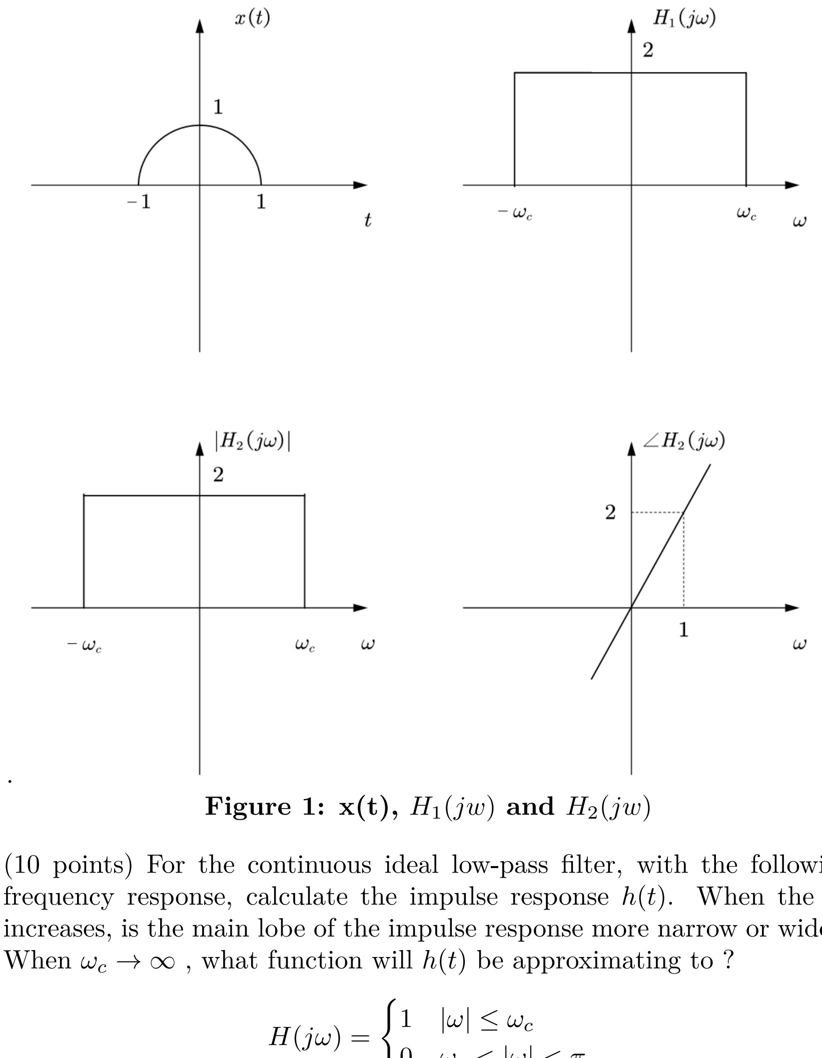

Figure 1: x(t), $H_1(jw)$ and $H_2(jw)$

(b) (10 points) For the continuous ideal low-pass filter, with the following frequency response, calculate the impulse response $h(t)$. When the ω_c increases, is the main lobe of the impulse response more narrow or wider? When $\omega_c \to \infty$, what function will $h(t)$ be approximating to ?

$$H(j\omega) = \begin{cases} 1 & |\omega| \leq \omega_c \\ 0 & \omega_c < |\omega| < \pi \end{cases}$$

Problem 2

(20 points) Figure 2 shows the frequency response $H(j\omega)$ of a continuous-time filter referred to as a lowpass differentiator. For the input signal $x(t) = \cos(\pi t + \phi) + \sin(2\pi t + \phi) + \sin(4\pi t + \phi)$, determine the filtered output signal $y(t)$.

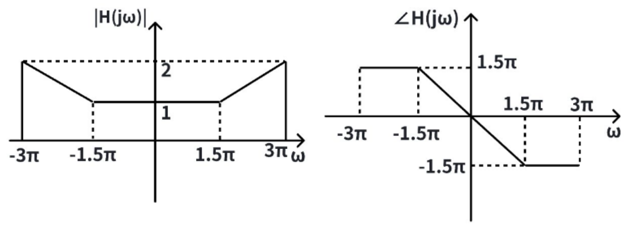

Figure 2: The magnitude and phase spectrum of H(jw)

Problem 3

(20 points) Given the following properties of a causal LTI system of first-order:

A.If the input signal is $x(t) = a[e^{-(\omega_0-1)t}u(t) - e^{-(\omega_0+1)t}u(t)]$, the output will be $y(t) = be^{-(\omega_0-1)t}u(t) - e^{-(\omega_0)t)}u(t) + be^{-(\omega_0+1)t}u(t)$, where $a, b, \omega_0 \neq 0$, and they are real numbers.

B.The group delay imposed by the system to the input signal is $\tau(\omega) = \frac{5}{25+\omega^2}$. (Hint: $\frac{\mathbf{d}\arctan x}{\mathbf{d}x} = \frac{1}{1+x^2}$)

C.The total energy of the input signal $x(t)$ specified in property(A) is $E_x = \frac{1}{120}$.

please answer the following questions:

1.Find the value of a, b, ω_0.

2.Write out the differential equation of the system in terms of $y(t), x(t)$. And find the frequency response $H(j\omega)$, sketch its Bode plot (use the asymptotic approximation). Note: the lateral axis of the Bode plot should be $\log_{10}(\omega)$.

Problem 4

(20 points) In many practical situations a signal is recorded in the presence of an echo, which we would like to remove by appropriate processing. For example, in Figure 4, we illustrate a system in which a receiver simultaneously receives a signal $x(t)$ and an echo represented by an attenuated delayed replication of $x(t)$. Thus, the receiver output is $s(t) = x(t) + \alpha x(t - T_0)$, where $|\alpha| < 1$. This output is to be processed to recover $x(t)$ by first converting to a sequence and then downsampled by N, and then using an appropriate digital filter $h[n]$, as indicated in Figure 4(b).

Assume that $x(t)$ is band limited [i.e., $X(j\omega) = 0$ for $|\omega| > \omega_M$] and that $|\alpha| < 1$.

(a) If $T_0 < \frac{\pi}{\omega_M}$, and the sampling period is taken to be equal to T_0 (i.e., $NT = T_0$), determine the difference equation for the digital filter $h[n]$ such that $y_c(t)$ equals $x(t)$. (note: for the filter h[n], $s_2[n]$ is the input, y[n] is the output)

(b) Now suppose that $\frac{\pi}{\omega_M} < T_0 < \frac{2\pi}{\omega_M}$. Determine a choice for the sampling period T, the lowpass filter gain A, and the frequency response for the digital filter $h[n]$ such that $y_c(t)$ equals $x(t)$.

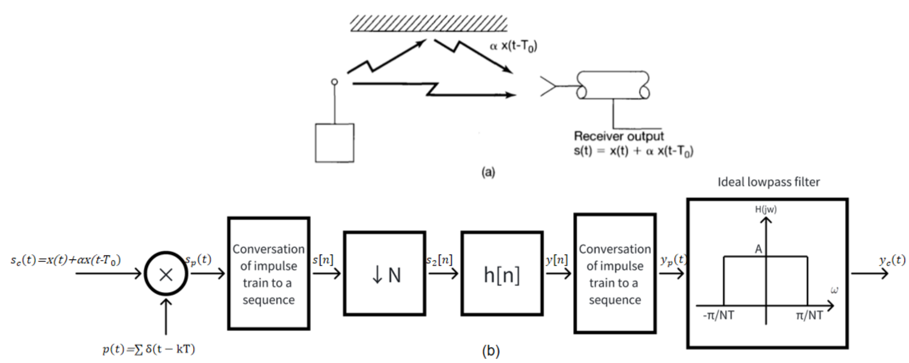

Figure 4.Problem 4

Problem 5

(20 points)

In Figure 5, we have an input signal $x_c(t) = \frac{\sin(0.5t)}{\pi t} + \sin(0.75t)$, sampling function $p(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT)$, given that $T = \frac{\pi}{2}$. After converting $x_p(t)$ to discrete time,, we obtain the sequence $x_d[n]$. We then down sampling the sequence $x_d[n]$ with $M = 2$ to obtain $x_{d2}[n]$. Please plot the spectrum $X_c(j\omega)$, $X_p(j\omega)$, $X_d(e^{j\Omega})$, $X_{d2}(e^{j\Omega})$, $Y_d(e^{j\Omega})$ and $Y_p(j\omega)$. Then write the expression of $y_c(t)$. $H(e^{j\Omega})$ in one period is given below:

$$H(e^{j\Omega}) = \begin{cases} 1, & -\frac{5\pi}{8} \leq \Omega \leq \frac{5\pi}{8} \\ 0, & \text{otherwise} \end{cases}$$

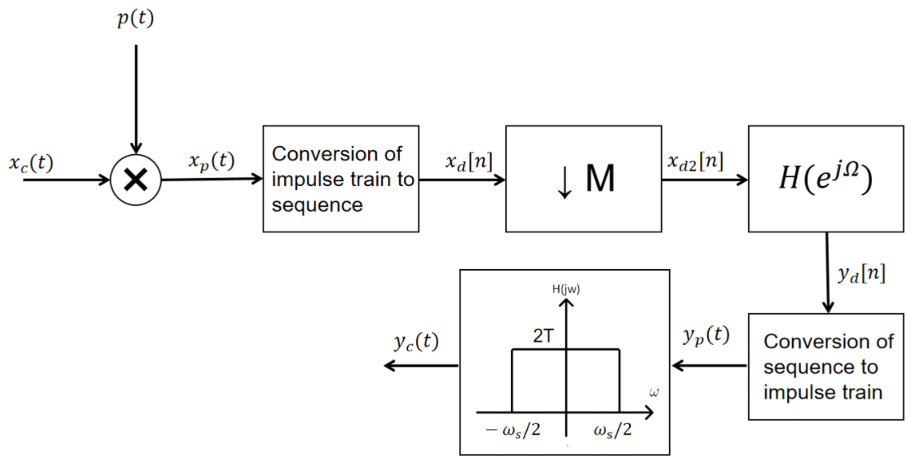

Figure 5. Problem 5