



$$\text{Theorem: } \delta(f(x)) = \sum_i \frac{1}{|f'(x_i)|} \delta(x - x_i)$$

where x_1, \dots, x_n let $f(x_i) = 0$

$$\text{pf: } \int \delta(f(x)) g(x) dx$$

$$= \sum_i \int_{U_i} \delta(f(x)) g(x) dx$$

use change of variable, let $a_i = f(x)$

$$\Rightarrow = \sum_i \int_{f(U_i)} \delta(a_i) g(f^{-1}(a_i)) df^{-1}(a_i)$$

$$= \sum_i \int_{f(U_i)} \delta(a_i) \frac{g(f^{-1}(a_i))}{f'(f^{-1}(a_i))} da_i$$

$$\Rightarrow \int \delta(f(x)) g(x) dx = \sum \frac{1}{|f'(x_i)|} g(x_i)$$

$$\Rightarrow \delta(f(x)) = \sum_i \frac{1}{|f'(x_i)|} \delta(x - x_i)$$

So, back to your question.

$$\delta(x^2 - d^2) = \delta(f(d))$$

$$= \frac{1}{2|d|} \left[\delta(x-d) + \delta(x+d) \right]$$