

Solution:

Since the bonds pay 12% annually, it means that they pay 6% at a semi-annual rate. Thus, every half year, the bond holder receives $\$10,000 \times 0.06 = \600 per half year.

The bonds will mature in 3 years i.e. after 6 payments. The full lump sum of \$10,000 will be paid after this period. Thus, the present value of the bond for a market rate of $r\%$ per payment, is calculated as follows:

- (a) The payments are like an annuity; thus, their present value becomes:

$$\begin{aligned} PV \text{ of annuity payments} &= 600 \left(\frac{1}{1+r} + \frac{1}{(1+r)^2} + \dots + \frac{1}{(1+r)^6} \right) \\ &= \frac{600}{r} \left(1 - \frac{1}{(1+r)^6} \right) \end{aligned}$$

- (b) The lumpsum amount paid at the end, which can be written as:

$$PV \text{ of the lumpsum amount} = \frac{10,000}{(1+r)^6}$$

Thus, the present value of the bond will be given by:

$$PV \text{ of bond} = \frac{600}{r} \left(1 - \frac{1}{(1+r)^6} \right) + \frac{10,000}{(1+r)^6}$$

1. For a market rate of 12% (6% semi-annually), we get the present bond value as:

$$PV \text{ of bond @ 12\%} = \$10,000.00$$

- The interest part: \$2,950.40
- The principal part: \$7,049.60

2. For a market rate of 14% (7% semi-annually), we get the present bond value as:

$$PV \text{ of bond @ 14\%} = \$9,523.35$$

- The interest part: \$2,859.93
- The principal part: \$6,663.42

3. For a market rate of 10% (5% semi-annually), we get the present bond value as:

$$PV \text{ of bond @ 10\%} = \$10,507.57$$

- The interest part: \$3,045.42
- The principal part: \$7,462.15

As expected, in (2) the bond is sold at a discount of \$476.65, while in (3), the bond is sold at a premium of \$507.57 respectively.