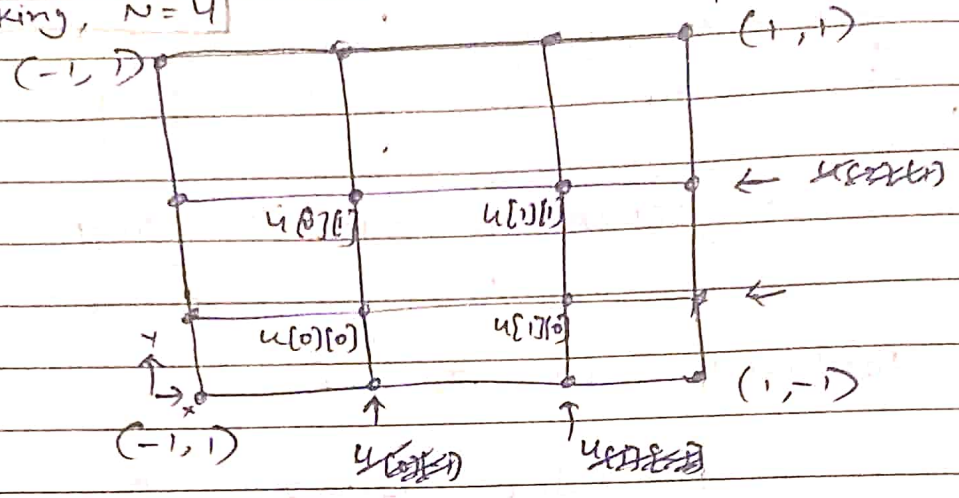
$$-\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) = f(x,y)$$

$f(x,y) = \sin(\pi x)\cos(\pi y)$

Taking, $N = 4$



$$\Delta x = \frac{x_{end} - x_{start}}{N-1} = \frac{2}{3} \qquad \Delta y = \frac{y_{end} - y_{start}}{N-1} = \frac{2}{3}$$

and *
$$\begin{cases} x_{ij} = x_{start} + (i+1)\Delta x \\ y_{ij} = y_{start} + (j+1)\Delta y \end{cases}$$

Now, the approximation

$$u_{i,j-1} + u_{i-1,j} - 4u_{i,j} + u_{i+1,j} + u_{i,j+1} = -h^2 f(x,y) \quad \text{— (1)}$$

$i=0, \; j=0$

$$x_{0,0} = -1 + \frac{2}{3} = -\frac{1}{3} \qquad y_{0,0} = -1 + \frac{2}{3} = -\frac{1}{3}$$

$$u_{0,-1} + u_{-1,0} - 4u_{0,0} + u_{1,0} + u_{0,1} = -\left(\frac{2}{3}\right)^2 \sin\left(-\frac{\pi}{3}\right)\cos\left(-\frac{\pi}{3}\right)$$

$$4u_{0,0} = u_{1,0} + u_{0,1} + \frac{7\sqrt{3}}{18}$$

$u[0][0]\left(-\frac{1}{3}, -\frac{1}{3}\right)$

$$u_{0,0} = \frac{1}{4}\left(u_{1,0} + u_{0,1} + \frac{7\sqrt{3}}{18}\right)$$

-2

$i=1,\ j=0$

$$u_{1,-1} + u_{0,0} - 4u_{1,0} + u_{2,0} + u_{1,1} = -\frac{4}{9}\sin\left(\frac{2\pi}{3}\right)\cos\left(-\frac{\pi}{3}\right)$$

$$-\frac{\sqrt{3}}{2} + u_{0,0} + u_{1,1} + \frac{\sqrt{3}}{9} = 4u_{1,0}$$

$u[1]\,u[0]\left(\frac{1}{3}, -\frac{1}{3}\right)$

$$\boxed{u_{1,0} = \frac{1}{4}\left(u_{0,0} + u_{1,1} - \frac{7\sqrt{3}}{18}\right)} \quad (3)$$

$i=0,\ j=1$

$$u_{0,0} + u_{-1,1} - 4u_{0,1} + u_{1,1} + u_{0,2} = \frac{-4}{9}\sin\left(-\frac{\pi}{3}\right)\cos\left(\frac{\pi}{3}\right)$$

$$u_{0,0} + u_{1,1} + \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{9} = 4u_{0,1}$$

$u[0]\,u[1]\left(-\frac{1}{3}, \frac{1}{3}\right)$

$$\boxed{u_{0,1} = \frac{1}{4}\left(u_{0,0} + u_{1,1} + \frac{7\sqrt{3}}{18}\right)} \quad (4)$$

$i=1,\ j=1$

$$u_{1,0} + u_{0,1} - 4u_{1,1} + u_{2,1} + u_{1,2} = -\left(\frac{2}{3}\right)^2 \sin\left(\frac{2\pi}{3}\right)\cos\left(\frac{\pi}{3}\right)$$

$u[1][1]\left(\frac{1}{3}, \frac{1}{3}\right)$

$$\boxed{u_{1,1} = \frac{1}{4}\left(u_{1,0} + u_{0,1} - \frac{7\sqrt{3}}{18}\right)} \quad (5)$$

Initial Guess: $u^0_{0,0}, u^0_{1,0}, u^0_{0,1}, u^0_{1,1} = 0$

$$u'_{0,0} = \frac{1}{4}\left(0 + 0 + \frac{7\sqrt{3}}{18}\right) = 0.168 \qquad u^2_{0,0} = 0.168$$

$$u'_{1,0} = -0.168 \qquad u^2_{1,0} = -0.168$$

$$u'_{0,1} = 0.168 \qquad u^2_{0,1} = 0.168$$

$$u'_{1,1} = -0.168 \qquad u^2_{1,1} = -0.168$$