$u(x,y) = \sin \pi x \cos \pi y$, $[-1,1] \times [-1,1]$

$N = 4$

$$-\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) = f(x,y) = 2\pi^2 \sin \pi x \cos \pi y$$

$$\Delta x = \frac{x_{end} - x_{start}}{N-1} = \frac{2}{3} = \Delta y$$

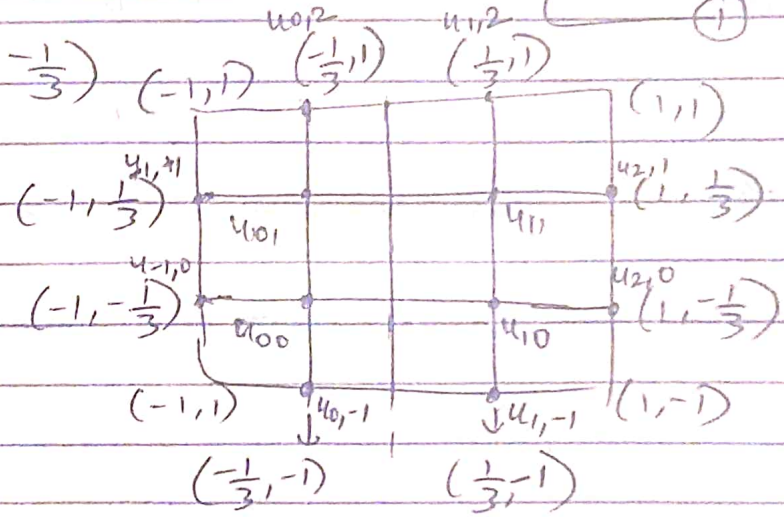
$x_{ij} = x_{start} + (i+1)\Delta x$

$y_{ij} = y_{start} + (j+1)\Delta y$

$$u_{i,j-1} + u_{i-1,j} + u_{i+1,j} + u_{i,j+1} - 4u_{i,j} = -h^2 f(x_i, y_i) \quad \text{— (1)}$$

(I)

$(i=0, j=0) \equiv \left(-\frac{1}{3}, -\frac{1}{3}\right)$



Eqn (1) ⇓

$$u_{0,-1} + u_{-1,0} + u_{1,0} + u_{0,1} - 4u_{0,0} = -\left(\frac{2}{3}\right)^2 \left(2\pi^2 \sin\left(-\frac{\pi}{3}\right)\cos\left(-\frac{\pi}{3}\right)\right) \quad \text{— (2)}$$

$u_{-1,0} = \sin(-\pi)\cos\left(-\frac{\pi}{3}\right) = 0$

$u_{1,+1} = \sin(-\pi)\cos\left(\frac{\pi}{3}\right) = 0$

$u_{0,2} = \sin\left(-\frac{\pi}{3}\right)\cos(\pi) = \frac{\sqrt{3}}{2}$

$u_{1,2} = \sin\left(\frac{\pi}{3}\right)\cos(\pi) = -\frac{\sqrt{3}}{2}$

$u_{0,-1} = \sin\left(-\frac{\pi}{3}\right)\cos(-\pi) = \frac{\sqrt{3}}{2}$

$u_{1,-1} = \sin\left(\frac{\pi}{3}\right)\cos(-\pi) = -\frac{\sqrt{3}}{2}$

$u_{2,0} = \sin(\pi)\cos\left(-\frac{\pi}{3}\right) = 0$

$u_{2,1} = \sin(\pi)\cos\left(\frac{1}{3}\right) = 0$

Eqn (2)

$$u_{0,-1} + u_{-1,0} + u_{1,0} + u_{0,1} - 4u_{0,0} = -\frac{4}{9} \times 2\pi^2 \times \left(-\frac{\sqrt{3}}{2} \times \frac{1}{2}\right)$$

$$\frac{\sqrt{3}}{2} + 0 + u_{1,0} + u_{0,1} - 4u_{0,0} = \frac{2\sqrt{3}\pi^2}{9}$$

$$u_{0,0} = \frac{1}{4}\left(u_{1,0} + u_{0,1} + \frac{\sqrt{3}}{2} - \frac{2\sqrt{3}\pi^2}{9}\right) \quad \text{— (3)}$$

II

$$(i=1, j=0) \equiv \left(\frac{1}{3}, -\frac{1}{3}\right)$$

$$u_{1,-1} + u_{0,0} + u_{2,0} + u_{1,1} - 4u_{1,0} = \left(\frac{2}{3}\right)^2 (2\pi^2) \sin\left(\frac{\pi}{3}\right) \cos\left(-\frac{\pi}{3}\right)$$

$$-\frac{\sqrt{3}}{2} + u_{0,0} + 0 + u_{1,1} - 4u_{1,0} = -\frac{4}{9} \times 2\pi^2 \times \frac{\sqrt{3}}{2} \times \frac{1}{2}$$

$$u_{1,0} = \frac{1}{4}\left(u_{0,0} + u_{1,1} - \frac{\sqrt{3}}{2} + \frac{2\sqrt{3}\pi^2}{9}\right) \quad \text{— (4)}$$

III

$$(i=0, j=1) \equiv \left(-\frac{1}{3}, \frac{1}{3}\right)$$

$$u_{0,0} + u_{-1,1} + u_{1,1} + u_{0,2} - 4u_{0,1} = \left(\frac{2}{3}\right)^2 (2\pi^2) \sin\left(-\frac{\pi}{3}\right) \cos\left(\frac{\pi}{3}\right)$$

$$u_{0,0} + 0 + u_{1,1} + \frac{\sqrt{3}}{2} - 4u_{0,1} = -\frac{4}{9} \times 2\pi^2 \times -\frac{\sqrt{3}}{2} \times \frac{1}{2}$$

$$u_{0,1} = \frac{1}{4}\left(u_{0,0} + u_{1,1} + \frac{\sqrt{3}}{2} - \frac{2\sqrt{3}\pi^2}{9}\right) \quad \text{— (5)}$$

$[-1, 1]$

(IV) $(i=1, j=1) \equiv \left(\frac{1}{3}, \frac{1}{3}\right)$

$$u_{1,0} + u_{0,1} + u_{2,1} + u_{1,2} - 4u_{1,1} = -\left(\frac{2}{3}\right)^2 (2\pi^2) \sin\frac{\pi}{3} \cos\frac{\pi}{3}$$

$$u_{1,0} + u_{0,1} + 0 + \left(\frac{-\sqrt{3}}{2}\right) - 4u_{1,1} = -\frac{4}{9} \times 2\pi^2 \times \frac{\sqrt{3}}{2} \times \frac{1}{2}$$

$$u_{1,1} = \frac{1}{4}\left(u_{1,0} + u_{0,1} - \frac{\sqrt{3}}{2} + \frac{2\sqrt{3}\pi^2}{9}\right) \quad \text{——— (6)}$$

⇒ Initial Guess $u^{\circ}_{0,0} = u^{\circ}_{0,1} = u^{\circ}_{1,0} = u^{\circ}_{1,1} = 0$

put in eqns 3, 4, 5, 6 and get

$u'_{0,0} = -0.732$

$u'_{1,0} = 0.732$

$u'_{0,1} = -0.732$

$u_{1,1} = 0.732$

⇒ Second Guess

↓

Getting same values Again.