1D

$x_{start} = -1$

$x_{end} = +1$

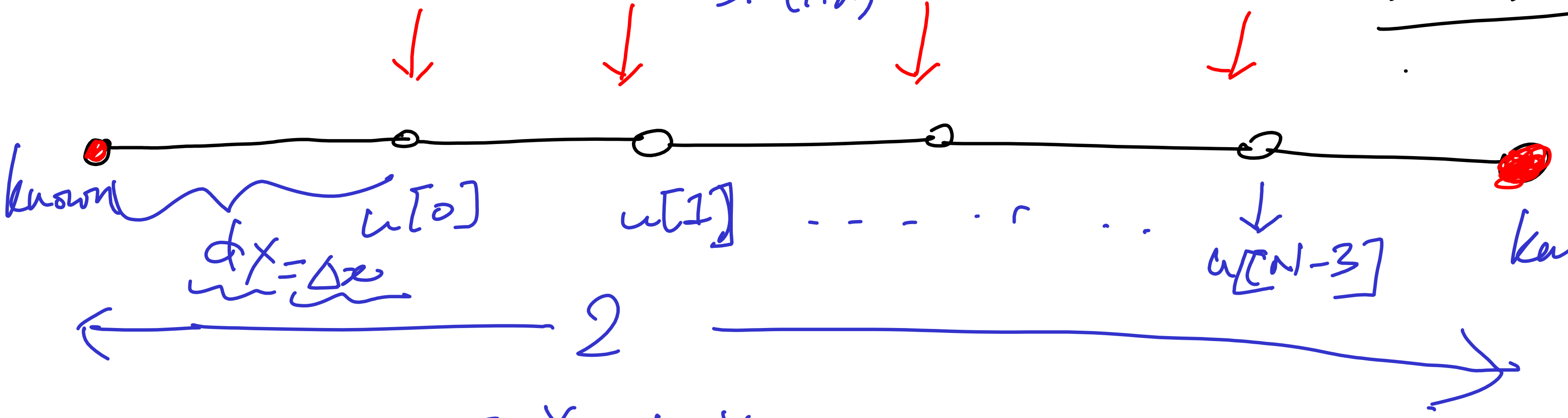
$$\frac{\partial^2 u}{\partial x^2} = f(x)$$

$\downarrow$

$\sin(\pi x)$ unknown!

$x_{end} - x_{start} = 2$

N = 30



$= X_{end} - X_{start}$

.c

.cpp

N → total # of nodes

N-1 = intervals b/w N nodes

N-2 → # of unknowns.

$$\boxed{dx = \frac{(x_{end} - x_{start})}{(N-1)}}$$

Δx

+ unknowns will be stored in an array
+ u[0], u[1], ... , u[N-3]

Size of the array will be N-2

approximation of gradients

1D

$$\left.\frac{\partial^2 u}{\partial x^2}\right|_{i} = \frac{u_{i+1} - 2u_i + u_{i-1}}{(\Delta x^2)}$$

known $= \sin(\pi(-1)) = 0$

unknown unknown

if $i = 0 \Rightarrow \left.\frac{\partial^2 u}{\partial x^2}\right|_{i=0} = \frac{u_1 - 2u_0 + u_{-1}}{(\Delta x)^2}$

$i = 1 \Rightarrow \left.\frac{\partial^2 u}{\partial x^2}\right|_{i=1} = \frac{u_2 - 2u_1 + u_0}{(\Delta x^2)}$

$\vdots$

known $= \sin(\pi(+1))$ unknown unknown

$i = N-3 \quad \left.\frac{\partial^2 u}{\partial x^2}\right|_{i=N-3} = \frac{u_{N-2} - 2u_{N-3} + u_{N-4}}{(\Delta x^2)}$

governing eqn

$$\frac{\partial^2 u}{\partial x^2} = f(x)$$

$$\underbrace{\frac{u_1 - 2u_0}{\Delta x^2}}_{\text{unknown}} = \underbrace{\sin(\pi x_0) - \frac{u_{-1}}{\Delta x^2}}_{\text{known}}$$

for each unknown points on the grid.

$i=0$

$$\left.\frac{\partial^2 u}{\partial x^2}\right|_{i=0} = f(x)\Big|_{i=0} \Rightarrow \frac{u_1 - 2u_0 + u_{-1}}{(\Delta x)^2} = \sin(\pi x_0)$$

$$x_0 = \underbrace{(x_{start} + \Delta x)}$$

$i=1$

$$\left.\frac{\partial^2 u}{\partial x^2}\right|_{i=1} = f(x)\Big|_{i=1} \Rightarrow \frac{u_2 - 2u_1 + u_0}{(\Delta x^2)} = \sin(\pi x_1)$$

$$x_1 = x_{start} + 2\Delta x$$

$\vdots$

$i = N-3$

$$\frac{u_{N-2} - 2u_{N-3} + u_{N-4}}{(\Delta x^2)} = \sin(\pi x_{N-3})$$

$$x_{N-3} = x_{start} + (N-2)\Delta x$$

$$\frac{-2u_{N-3}}{\Delta x^2} + \frac{u_{N-4}}{\Delta x^2} = \sin(\pi x_{N-3}) - \frac{u_{N-2}}{\Delta x^2}$$

S_0

$$-2u_0 + u_1 = \Delta x^2 \sin(\pi x_0) - u_{-1} \quad (1)$$

$$u_0 - 2u_1 + u_2 = \Delta x^2 \sin(\pi x_1) \quad (2)$$

S_1

$$u_1 - 2u_2 + u_3 = \Delta x^2 \sin(\pi x_2) \quad (3)$$

S_2

$\vdots$

S_{N-3}

$$u_{N-4} - 2u_{N-3} = \Delta x^2 \sin(\pi x_{N-3}) - u_{N-2}$$

$$
\begin{bmatrix}
-2 & 1 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 \\
1 & -2 & 1 & 0 & 0 & 0 & 0 & \cdots & 0 \\
0 & 1 & -2 & 1 & 0 & 0 & 0 & \cdots & \\
0 & 0 & 1 & -2 & 1 & 0 & 0 & \cdots & 0 \\
 & & & & & & & & \\
\cdot & - & - & \cdot & - & - & \cdot & 1 & -2
\end{bmatrix}
\begin{bmatrix}
u_0 \\ u_1 \\ u_2 \\ \vdots \\ \vdots \\ u_{N-3}
\end{bmatrix}
=
\begin{bmatrix}
s_0 \\ s_1 \\ s_2 \\ \vdots \\ \vdots \\ s_{N-3}
\end{bmatrix}
$$

matrix for 1D

direct methods - suitable for small matrices

iterative methods - suitable for large matrices

Jacobi method : is a type of iterative method.

all unknown u_s are zero in the beggining (initial guess)

~~u_0 grid points~~

$$u_0^{(k+1)} = \Delta x^2 \sin(\pi x_0) - \underbrace{u_{-1}}_{\text{know}} - u_1^{(k)}$$

$$u_0^{(1)} = (\Delta x^2)\sin(\pi x_0) - u_{-1} - u_1^{(k)} \quad ; \quad u_1^{(1)} \; ; \quad u_2^{(1)} \; ; \; \ldots \; u_{N-3}^{(1)}$$

$$=$$

$$u_0^{(2)} = (\Delta x^2)\sin(\pi x_1) - u_{-1} - u_1^{(1)}$$

$\vdots$

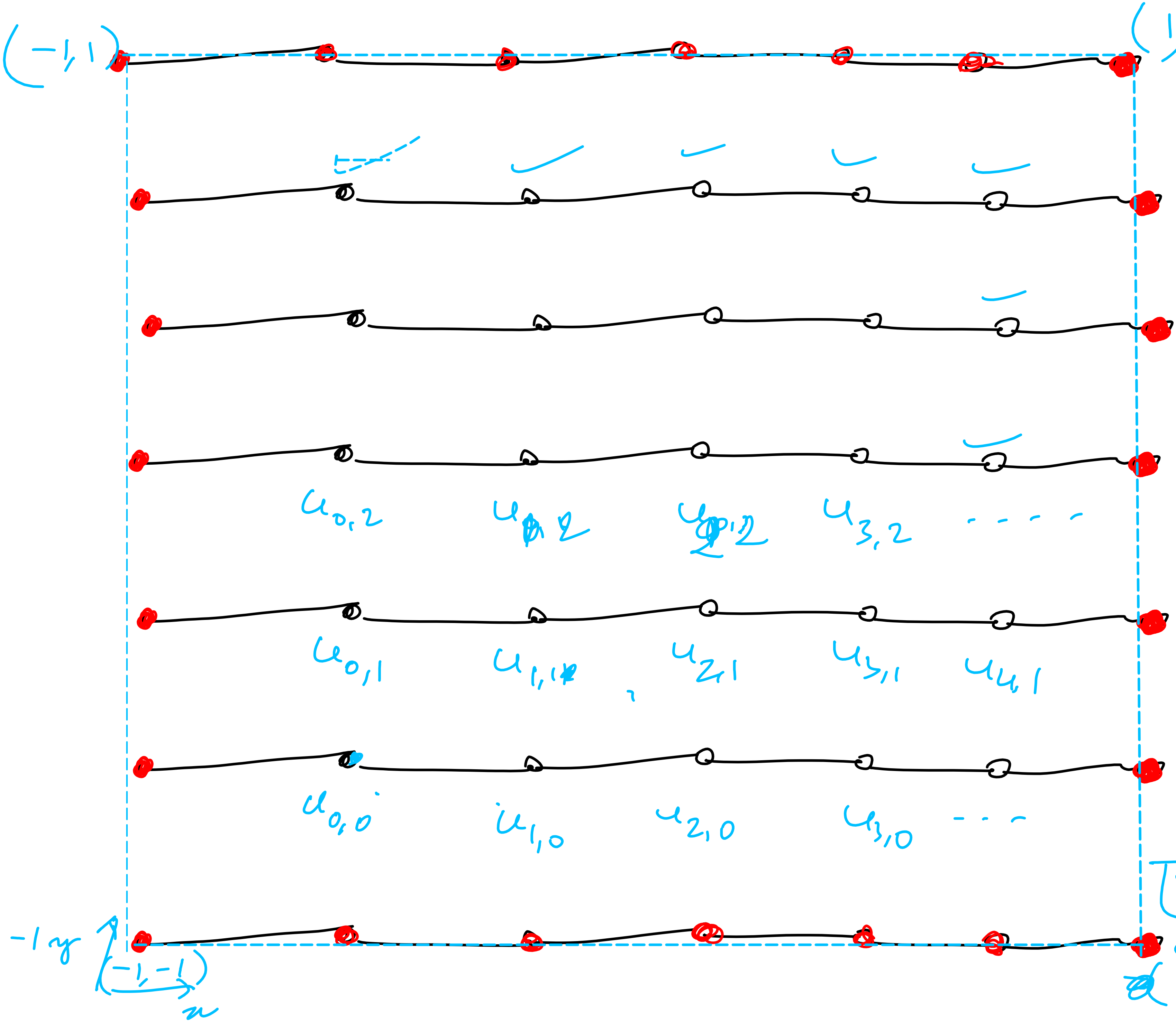
$$u_0^{(k)} = - - - - -$$

stopping crietria

$$\left[u_0^{(k)} - u_0^{(k-1)}\right] = \cancel{1e6}\; 1\times 10^{-6}$$

$1.e-6$

$\boxed{\text{double etol} = 1.e-6;}$



reds - known
whites - unknown

$$\Delta x = \frac{x_{end} - x_{start}}{N_x - 1}$$

$$\Delta y = \frac{y_{end} - y_{start}}{N_y - 1}$$

$$x_{0,0} = x_{start} + \Delta x$$

$$x_{1,0} = x_{start} + 2\Delta x$$

$$\boxed{x_{i,j} = x_{start} + (i+1)\Delta x}$$

$$y_{i,j} = y_{start} + (j+1)\Delta y$$

$(0, 0) \Rightarrow \quad \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \cancel{} \, f(x, y)$

$= \sin(\pi x)\cos(\pi y)$