Question Bank

Solve the following questions

1. If A = $\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$, then calculate the value of A^{50}
2. The rank of a matrix $\begin{pmatrix} k & -2 & 0 \\ 0 & k & -2 \\ -2 & 0 & k \end{pmatrix}$ is 2, then find the value of k
3. Find the rank of the matrix $\begin{pmatrix} 1 & 2 & -2 & 3 \\ 2 & 5 & -4 & 6 \\ -1 & -3 & 2 & -2 \\ 2 & 4 & -1 & 6 \end{pmatrix}$
4. For what values of k, the following equations $x + y + z = 1,\ 2x + y + 4z = k, 4x + y + 10z = k^2$ have solutions and solve them completely in each case.
5. If A = $\begin{pmatrix} 1 & 2 & 1 \\ 1 & -1 & 1 \\ 2 & 3 & -1 \end{pmatrix}$ then verify that A satisfies its own characteristic equation. Hence find A^{-1}.
6. If $y = Sin\ (m\ \sin^{-1} x\ )$, prove that

   $(1\text{-}x^2)y_{n+2} - (2n + 1)xy_{n+1}\ \text{-}(n^2 - m^2\ )y_n = 0$

7. $\frac{x}{x+1} < \log(1 + \mathrm{x}) < \mathrm{x}$, if $\mathrm{x}\ > 0$

   or

   $\mathrm{x}\ <\ \sin^{-1}\mathrm{x}\ < \frac{\mathrm{x}}{\sqrt{1-\mathrm{x}^2}}$ if $0\ < \mathrm{x}\ < 1$

8. Evaluate $\int_0^{\pi/2} \cos^9 x\ dx$

9. Expanding the determinant by Laplace's method, Show that

$$\begin{vmatrix} 0 & a & b & c \\ -a & 0 & d & e \\ -b & -d & 0 & f \\ -c & -e & -f & 0 \end{vmatrix} = (af - be + cd)^2$$

10. Find the values of a and b such that $\lim_{x \to 0} \frac{x(1+\cos x) - b\sin x}{x^3} = 1$

11. Show that $\int_{-\infty}^{\infty} xe^{-x^2}\, dx = 0$

12. Check whether the matrix is diagonalisable or not

$$B = \begin{pmatrix} 1 & 1 & 1 \\ -1 & -1 & -1 \\ 0 & 0 & 1 \end{pmatrix}$$

13. Let S={ (x, y, z) ϵ $\mathbb{R}^3$: $3x - y + z = 2$ }. Show that S is not a subspace of $\mathbb{R}^3$.

14. Find whether the following transformation is linear or not
    T: $\mathbb{R}^3 \to \mathbb{R}^3$ defined by T(x, y, z) = $(x + y,\ y + z, x + y)$

15. Find the matrix of the linear transformation T, where T: V → W, such that T(α) = α+β+γ, T(β) = α+β, T(γ) = α, where {α, β, γ} is the basis of V.

16. Use the Gram- Schmidt process to obtain an orthonormal basis from the basis set {(1, 0, 1), (1, 1, 1), (1, 3, 4)} of the Euclidean space $\mathbb{R}^3$ with a standard inner product.

17. Prove that the set of vectors {(1, 2, 2), (2, -2, 1), (2, 1, -2)} is the orthogonal basis of $\mathbb{R}^3$.

18. Show that the vectors in $\mathbb{R}^3$ $\{(1, -1, 1), (1, 1, 0), (1, 2, -3), (1, 0, 1)\}$ are linearly dependent.