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Solutions

Ans1: Proof of Bayes Theorem:

Using Conditional Probability,

$P(A \cap B) = P(A|B) * P(B)$

Rearranging above equation,

$P(A|B) = \frac{P(A \cap B)}{P(B)}$ -------(1)

Similarly, again using Conditional Probability,

$P(B|A) = \frac{P(A \cap B)}{P(A)}$ -------(2)

Now combining both equations (1) and (2),

$P(A|B) * P(B) = P(B|A) * P(A)$ -------(3)

Now, dividing both sides by $P(B)$ for solving $P(A|B)$, we get:

$P(A|B) = \frac{P(B|A) * P(A)}{P(B)}$ -------(4)

Hence Proved. This above equation is knowns as Bayes Theorem which relates the conditional

Probability of B given A.

Bayes theorem is a powerful statistical tool that helps us to update our beliefs about an event or hypothesis based on new evidence or information. This makes it particularly useful for machine learning problems, where we often need to update our models as we receive new data.
In machine learning, Bayes theorem is used in various ways, including:

1. **Bayesian Inference:** This is a technique used to estimate the probability distribution of model parameters. By updating prior knowledge about model parameters based on new evidence, Bayesian inference provides a principled way to learn from data.
2. **Bayesian Networks:** These are graphical models that represent probabilistic relationships between variables. Bayesian networks can be used to model complex systems, and they are particularly useful in situations where there are many variables and limited data.
3. **Bayesian Optimization:** This is a technique used to optimize the hyperparameters of a machine learning model. By using Bayesian inference to update our beliefs about the optimal hyperparameters, we can efficiently search the hyperparameter space and find the best values for our model.

Overall, Bayes theorem provides a principled and flexible framework for reasoning under uncertainty, which makes it a useful tool for many machine learning problems.

Ans2: $P(Cancer|+) = \frac{P(+|Cancer) * P(Cancer)}{P(+|Cancer) * P(Cancer) + P(+|\neg Cancer) * P(\neg Cancer)}$

$$= \frac{0.98 * 0.008}{(0.98 * 0.008) + (0.03 * 0.992)}$$

$$= \frac{0.0078}{0.0078 + 0.0298}$$

$$= \frac{0.0078}{0.0376}$$

$$= 0.2074$$

$$= 0.21$$

And,

$$P(Cancer|+\,+) = \frac{P(+|Cancer) * P(Cancer|+)}{P(+|Cancer) * P(Cancer|+) + P(+|\neg Cancer) * P(\neg Cancer|+)}$$

We know that,

$$P(Cancer|+) = 0.21$$

$$P(\neg Cancer|+) = 1 - P(Cancer|+) = 1 - 0.21 = 0.79$$

Hence,

$$P(Cancer|+\,+) = \frac{0.98 * 0.21}{0.98 * 0.21 + 0.03 * 0.79}$$

$$= \frac{0.2058}{0.2058 + 0.0237}$$

$$= \frac{0.2058}{0.2295}$$

$$= 0.8967$$

$$P(\neg Cancer|+\,+) = 1 - P(Cancer|+\,+) = 1 - 0.8967 = 0.103$$

Note: Rounding off may cause different results.

Ans3:

$$P(PlayTennis = Yes) = \frac{8}{12}$$

$$P(PlayTennis = No) = \frac{4}{12}$$

$$P(Outlook = Sun \mid PlayTennis = Yes) = \frac{2}{8}$$

$$P(Outlook = Sun \mid PlayTennis = No) = \frac{3}{4}$$

$$P(Temperature = Cool \mid PlayTennis = Yes) = \frac{3}{8}$$

$$P(Temperature = Cool \mid PlayTennis = No) = \frac{1}{4}$$

$$P(Humidity = High \mid PlayTennis = Yes) = \frac{3}{8}$$

$$P(Humidity = High \mid PlayTennis = No) = \frac{3}{4}$$

$$P(Wind = Strong \mid PlayTennis = Yes) = \frac{3}{8}$$

$$P(Wind = Strong \mid PlayTennis = No) = \frac{2}{4}$$

$$P(PlayTennis = Yes \mid Outlook = Sun, Temperature = Cool, Humidity = High, Wind = Strong)$$
$$= P(PlayTennis = Yes) * P(Outlook = Sun \mid PlayTennis = Yes)$$
$$* P(Temperature = Cool \mid PlayTennis = Yes) * P(Humidity = High \mid PlayTennis = Yes)$$
$$* P(Wind = Strong \mid PlayTennis = Yes)$$

$$= \frac{8}{12} * \frac{2}{8} * \frac{3}{8} * \frac{3}{8} * \frac{3}{8}$$

$$= 0.6667 * 0.25 * 0.375 * 0.375 * 0.375$$

$$= 0.0088$$

$$P(PlayTennis = No \mid Outlook = Sun, Temperature = Cool, Humidity = High, Wind = Strong)$$
$$= P(PlayTennis = Yes) * P(Outlook = Sun \mid PlayTennis = Yes)$$
$$* P(Temperature = Cool \mid PlayTennis = Yes) * P(Humidity = High \mid PlayTennis = Yes)$$
$$* P(Wind = Strong \mid PlayTennis = Yes)$$

$$= \frac{4}{12} * \frac{3}{4} * \frac{1}{4} * \frac{3}{4} * \frac{2}{4}$$

$$= 0.3333 * 0.75 * 0.25 * 0.75 * 0.5$$

$$= 0.0234$$

$$= 0.023$$

As $0.023 > 0.0088$

Therefore, based on the Naive Bayes algorithm, the predicted value for PlayTennis for the same new instance $< Outlook = Sun, Temperature = Cool, Humidity = High, Wind = Strong >$ is No.

Ans4: First Iteration,

$$net_c = w_{co} + (a * w_{ca}) + (b * w_{cb})$$

$$= 0.1 + 0.1 + 0$$

$$= 0.2$$

$$O_c = \frac{1}{1 + e_c^{-net}} = \frac{1}{1 + e^{-0.2}} = 0.55$$

$$net_d = w_{do} + (O_c * w_{dc})$$

$$= 0.1 + (0.55 * 0.1)$$

$$= 0.155$$

Similarly,

$$O_d = \frac{1}{1 + e_d^{-net}} = \frac{1}{1 + e^{-0.155}} = 0.539$$

Using Backpropagation,

$$\delta_d = O_d * (1 - O_d) * (t_a - O_a)$$

$$= 0.539 * (1 - 0.539) * (1 - 0.539)$$

$$= 0.115$$

$$\Delta w_{dc} = \eta * \delta_d * O_c + \alpha * O$$

$$= 0.3 * 0.115 * 0.55$$

$$= 0.019$$

$$\Delta w_{d0} = 0.034$$

$$\therefore w_{dc} = w_{dc} + \Delta w_{dc}$$

$$= 0.1 + 0.019$$

$$= 0.119$$

$$\therefore w_{d0} = w_{d0} + \Delta w_{d0}$$

$$= 0.1 + 0.034$$

$$= 0.134$$

$$\delta_c = O_c * (1 - O_c) * (w_c * \delta_d)$$

$$= 0.55 * (1 - 0.55) * (0.1 * 0.115)$$

$$= 0.003$$

$$\Delta w_{ca} = \eta * \delta_c * x_a + \alpha * 0$$

$$= 0.3 * 0.003 * 1$$

$$= 0.001$$

$$\Delta w_{c0} = 0.001$$

$$\Delta w_{cb} = 0$$

$$w_{c0} = w_{c0} + \Delta w_{c0} = 0.1 + 0.001 = 0.101$$

$$w_{ca} = w_{ca} + \Delta w_{ca} = 0.1 + 0.001 = 0.101$$

$$w_{cb} = w_{cb} + \Delta w_{cb} = 0.1 + 0 = 0.1$$

Second Iteration

$$net_c = w_{c0} + a * w_{ca} + b * w_{cb}$$

$$= 0.101 + 0 + 0.1$$

$$= 0.201$$

$$O_c = \frac{1}{1 + e^{-0.201}} = 0.55$$

$$net_d = w_{d0} + (O_c * w_{dc})$$

$$= 0.134 + (0.55 * 0.119)$$

$$= 0.1994$$

Similarly,

$$O_d = \frac{1}{1 + e^{-0.1994}} = 0.5496$$

Using Backpropagation,

$$\delta_d = O_d * (1 - O_d) * (1 - O_d)$$

$$= 0.5496 * (1 - 0.5496) * (1 - 0.5496)$$

$$= -0.136$$

$$\Delta w_{dc} = \eta * \delta_d * O_c + \alpha * \Delta w_{dc}$$

$$= (0.3 * (-0.136) * 0.55) + (0.9 * 0.019)$$

$$= -0.0053$$

$$\Delta w_{d0} = -0.01$$

$$\therefore w_{dc} = w_{dc} + \Delta w_{dc}$$

$$= 0.119 + (-0.0053)$$

$$= 0.113$$

$$w_{d0} = w_{d0} + \Delta w_{d0}$$

$$= 0.134 + (-0.01)$$

$$= 0.124$$

$$\delta_c = O_c * (1 - O_c) * (w_{dc} * \delta_d)$$

$$= 0.55(1 - 0.55)(0.113 * -0.136)$$

$$= -0.0038$$

$$\Delta w_{ca} = \eta * \delta_c * x_a + \alpha * \Delta w_{ca}$$

$$= 0.3 * (-0.0038) * 0 + 0.9 * 0.101$$

$$= 0.0009$$

$$\Delta w_{c0} = 0$$

$$\Delta w_{cb} = -0.001$$

$$\therefore w_{c0} = w_{c0} + \Delta w_{c0} = 0.101 + 0 = 0.101$$

$$w_{ca} = w_{ca} + \Delta w_{ca} = 0.101 + 0.0009 = 0.102$$

$$w_{cb} = w_{cb} + \Delta w_{cb} = 0.100 + (-0.001) = 0.099$$

Final Results:

$$w_{c0} = 0.101$$

$$w_{ca} = 0.102$$

$$w_{cb} = 0.099$$

$$w_{d0} = 0.124$$

$$w_{dc} = 0.113$$