Homework - 4

Ques 1. Bayes theorem.

The probability of two event A & B happening is P(A) & P(B), $P(A \cap B)$, is the probability of A intersection B
$P(B|A) \rightarrow$ Probability of B given that A has occured

$$P(A \cap B) = P(A)\,P(B|A) \quad \text{——} (1)$$

On the other hand, the probability of A & B is also equal to the probability of B times the probability of A given B.

$$P(A \cap B) = P(B)\,P(A|B) \quad \text{——} (2)$$

Comparing (1) & (2) equation

$$P(B)\,P(A|B) = P(A)\,P(B|A)$$

2)

$$P(A|B) = P(A)\,\frac{P(B|A)}{P(B)}$$

This equation is the Baye's theorem.

Bayes' theorem is useful for machine learning problems because it can update probability based on new data or evidence. It provides a framework for inferring uncertain events in the presence of incomplete or changing information. In machine learning, it is commonly used in Bayesian statistics and probabilistic modelling and can be applied to problems such as classification, regression, & decision making under uncertainty.
One of the main advantages of Baye's theorem in machine learning is the ability to incorporate prior knowledge and beliefs. This is useful when data is limited or noisy. By incorporating prior probability. Bayes' theorem allows for more robust & adaptive modelling especially in situations where data are spare or uncertain. it also provides a way to update probabilities as new data becomes available, so machine learning models can continously adjust and improve their prediction over time.
It is also used for Naive Bayes ~~theor~~ Model

Ques 2

$P(\text{cancer}) = 0.008$

$P(-\text{cancer}) = 0.992$

$P(\oplus / \text{cancer}) = 0.98$

$P(\oplus \mid -\text{cancer}) = 0.03$

As, the two tests are independent, we get

$P(\oplus\oplus \mid \text{cancer}) = P(\oplus \mid \text{cancer}) \times P(\oplus / \text{cancer})$

$= 0.98 \times 0.98$

$= 0.9604$

$P(\oplus\oplus \mid -\text{cancer}) = P(\oplus \mid -\text{cancer}) \times P(\oplus / \text{cancer})$

$= 0.03 \times 0.03$

$= 0.0009$

By law of total probability that both the first & and second test returns a positive result

$\Rightarrow$

$P(\oplus\oplus) = P(\oplus\oplus / \text{cancer})\, P(\text{cancer}) + P(\oplus\oplus / -\text{cancer})\, P(-\text{cancer})$

$\Rightarrow 0.9604 \times 0.008 + 0.0009 \times 0.992$

$\Rightarrow 0.008576$

Given the + results in both the tests,
Probability of cancer =

$$P(\text{cancer} \mid ++) = \frac{P(++ \mid \text{cancer})\,P(\text{cancer})}{P(++)}$$

(By Bayes theorem)

$$\Rightarrow \frac{0.9604 \times 0.008}{0.008576}$$

$$\Rightarrow 0.8958955$$

Given the positive results in both the test, probability of no cancer $\Rightarrow$

$$P(-\text{cancer} \mid ++) = \frac{P(++ \mid -\text{cancer})\,P(-\text{cancer})}{P(++)}$$

$$\Rightarrow \frac{0.0009 \times 0.992}{0.008576}$$

$$\Rightarrow 0.1041045$$

If the second test returns a positive results as well, the posterior probability of cancer is 0.8958955 & the posterior probability of no cancer is 0.1041045.

Ques 3

P(Play tennis = Yes) = 8/12
P(Play Tennis = No) = 4/12

P(Outlook = sun | Play tennis = Yes) = 2/8

P(outlook = sun | Play tennis = No) = 3/4

P(temprature = Cool | Play tennis = Yes) = 3/8
P(temperature = cool | Play tennis = No) = 1/4

P(Hummidity = High | Play tennis = Yes) = 3/8
P(Humidity = high | Play tennis = No) = 3/4

P(wind = strong | Play tennis = Yes) = 3/8
P(wind = strong | Play tennis = No) = 2/4

P(Play tennis = Yes | Outlook = Sun, temprature = Cool, Humidity = High, wind = strong)
⇒ P(Play tennis = Yes) * P(Outlook = Yes) * P(Humidity = High | Play tennis = Yes) * P(wind = strong | Play = Yes)

$\Rightarrow \frac{8}{12} * \frac{2}{8} * \frac{3}{8} * \frac{3}{8} * \frac{3}{8} \Rightarrow 0.0088$

P(Play tennis = No | outlook = Sun, temprature = Cool, Humidity = High, wind = strong) ⇒

$\Rightarrow \frac{4}{12} * \frac{3}{4} * \frac{1}{4} * \frac{3}{4} * \frac{2}{4} \Rightarrow 0.023$

$0.023 > 0.0088$ theorefore the Ans is No.

Query first iteration.

$net_c = w_{c0} + (a * w_{ca}) + b * w_{cb})$

$\Rightarrow 0.1 + 0.1 + 0$

$\Rightarrow 0.2$

$O_c = \frac{1}{1+e_c^{-net}} = \frac{1}{1+e^{-0.2}} = 0.55$

$net_d = w_{d0} + (O_c * w_{dc})$

$\Rightarrow 0.1 + (0.55 * 0.1)$

$\Rightarrow 0.155$

Similarly.

$O_d = \frac{1}{1+e_d^{-net}} = \frac{1}{1+e^{-0.155}} = 0.539$

Using Back propagation,

$\delta_d = O_d * (1 - O_d) * (t_a - O_a)$

$\Rightarrow 0.539 * (1 - 0.539) * (1 - 0.539)$

$\Rightarrow 0.115$

$\Delta w_{dc} = \eta * \delta_d * O_c + \alpha * 0$

$\Rightarrow 0.3 * 0.115 * 0.55$
$= 0.019.$

$\Delta w_{do} = 0.034$

$\therefore w_{dc} = w_{dc} + \Delta w_{dc}$
$\Rightarrow 0.1 + 0.019$
$\Rightarrow 0.119.$

$\therefore w_{do} = w_{do} + \Delta w_{do}$
$\Rightarrow 0.1 + 0.034$
$\Rightarrow 0.134.$

$\delta_c = O_c * (1 - O_c) * (w_a * \delta d)$
$\Rightarrow 0.55 * (1 - 0.55) * (0.1 * (0.115)$
$\Rightarrow 0.005$

$\Delta w_{ca} = \eta * \delta_c * x_a + \alpha * 0$
$\Rightarrow 0.3 * 0.003 * 1$
$\Rightarrow 0.001$

$\Delta w_{co} = 0.001 \qquad \Delta w_{cb} = 0$

$w_{co} = w_{co} + \Delta w_{co} = 0.1 + 0.001 = 0.101$
$w_{ca} = w_{ca} + \Delta w_{ca} = 0.1 + 0.001 = 0.101$
$w_{cb} = w_{cb} + \Delta w_{cb} = 0.1 + 0 = 0.1$

Second iteration

$net_c = w_{c0} + a * w_{ca} + b * w_{cb}$

$\Rightarrow 0.101 + 0 + 0.1$

$\Rightarrow 0.201$

$$O_c = \frac{1}{1+e^{-0.201}} \Rightarrow 0.55$$

$net_d = w_{d0} + (O_c * w_{dc})$

$\Rightarrow 0.134 + (0.55 * 0.119)$

$\Rightarrow 0.1994$

Similarly,

$$O_d = \frac{1}{1+e^{-0.1994}} = 0.5496$$

Using Backpropagation:

$\delta d = O_d * (1 - O_d) * (1 - O_d)$

$\Rightarrow 0.5496 * (1 - 0.5496) * (1 - 0.5496)$

$\Rightarrow -0.136$

$\Delta w_{dd} = \eta * \delta d * O_c + \alpha * \Delta dc$

$\Rightarrow -0.0053$

$\Delta d0 = -0.01$

$\therefore w'dc = w_{dc} + \Delta w_{dc}$

$\Rightarrow 0.119 + (-0.0053)$

$= 0.113.$

$w_{do} = w_{do} + \Delta w_{do} \Rightarrow 0.134 + (-0.01)$
$\Rightarrow 0.124$

$\delta_c = O_c * (1 - O_c) * (w_{dc} * \delta d)$
$\Rightarrow 0.55(1-0.55)(0.113 * (-0.1367))$
$\Rightarrow -0.0038$

$\Delta w_{ca} = \eta * \delta_c * X_a + \alpha * \Delta w_{ca}$
$\Rightarrow 0.3 * (-0.0038) * 0 + 0.9 * 0.101$
$= 0.0009$

$\Delta w_{co} = 0$

$\Delta w_{cb} = -0.001$

$\therefore w_{co} = w_{co} + \Delta w_{co} = 0.101 + 0 = 0.101$

$w_{ca} = w_{ca} = 0.101 + 0.0009 = 0.102$

$w_{cb} = w_{cb} + \Delta w_{cb} = 0.100 + (-0.001) = 0.09$

$w_{co} = 0.101 \quad w_{ca} = 0.102 \quad w_{cb} = 0.099$

$w_{do} = 0.124 \quad w_{dc} = 0.113$