N-Particle Spring Problem

The Unfinished Business with Euler-Cromer and N-particle Spring

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Content

- Equations, equations and equations …
- Properties of model
- Varying Timesteps *dt*
- Let’s try to break Euler-Cromer
- Wrap up

The Equations Involved

$$H = T + V$$

$$T = \sum_{i}^{N} \frac{p_i}{2m}$$

$$V = \sum_{i}^{N-1} \frac{k}{2}(x_{i+1} - x_i)^2 + \frac{k}{2}x_1^2 + \frac{k}{2}x_N^2$$

Hamiltonian Equations

$$\ddot{x}_1 = \frac{k}{m}(x_2 - 2x_1)$$

$$\ddot{x}_i = \frac{k}{m}(x_{i+1} + x_{i-1} - 2x_i)$$

$$\ddot{x}_N = \frac{k}{m}(x_{N-1} - 2x_N)$$

Equations of Motions

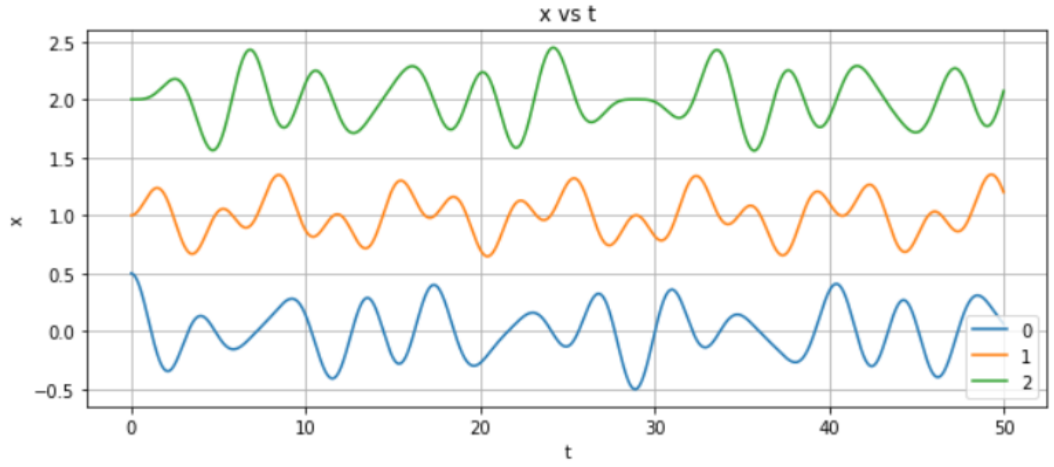
Model Properties

- Has N particles that works with N = 2, 3, 4, …
- The particles are attached to a spring in series and the particles at the ends are attached to a hard wall
- The purpose of this model is to show:
  - Conservation of total energy within system
  - The capabilities of Euler-Cromer algorithm with different timesteps

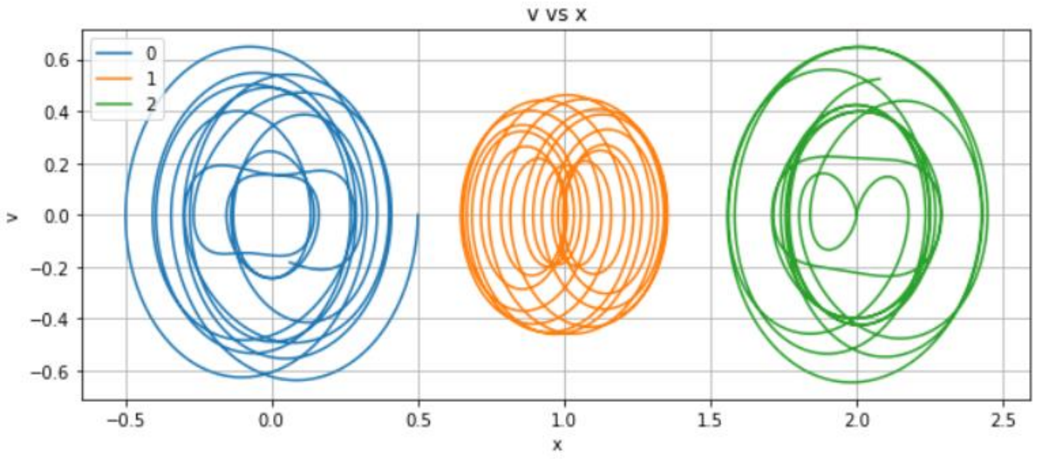
Basic Example

This is a model of 3-particle system with the following properties:

- Time is from 0 to 50
- All particles are at resting position (zero displacement) and no initial velocity **except** particle-1
- Chosen timestep, *dt* is 0.01



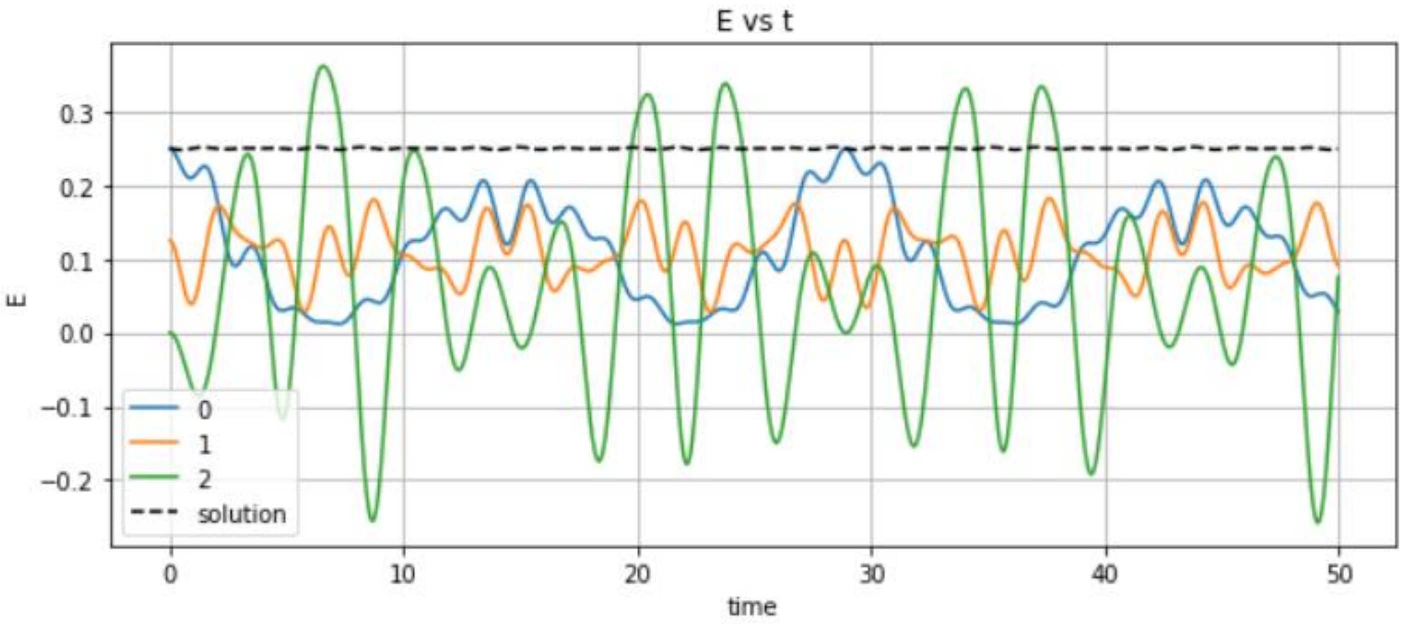
Displacement against Time



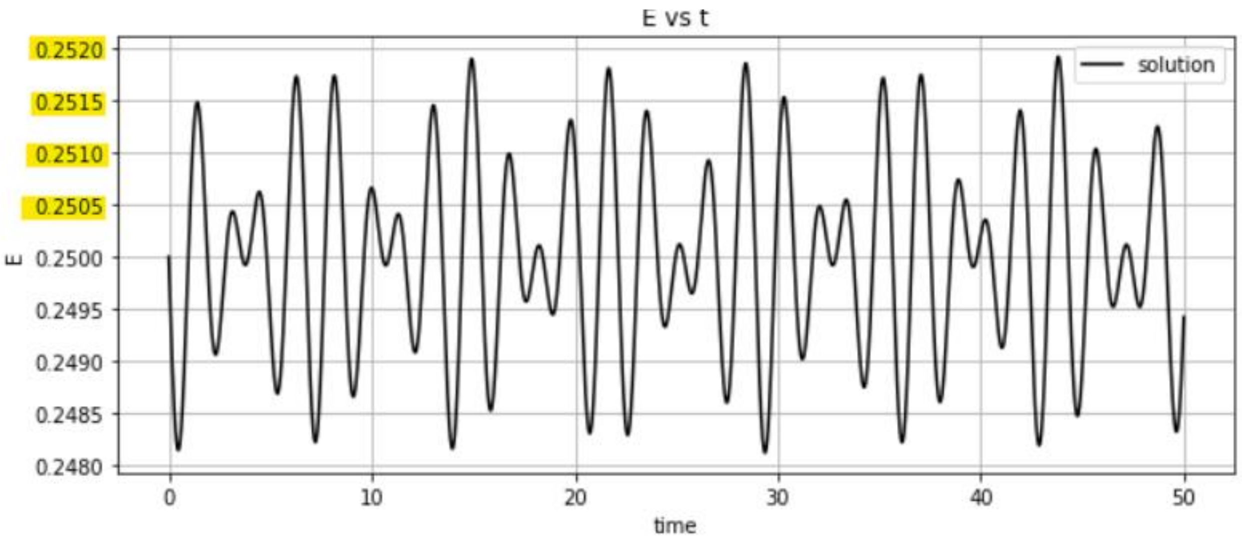
Phase Diagram

Energy

The interesting part is where the energy diagram is at. It shows how well Euler-Cromer has the energy conservation property in its algorithm. As we can see, the total energy is somewhat a tiny bit wiggly if not perfectly *flat*.



Zooming in to its scale, we can see how it “wiggles”. It seems significant but the actual value is not that bad.



Varying Timesteps

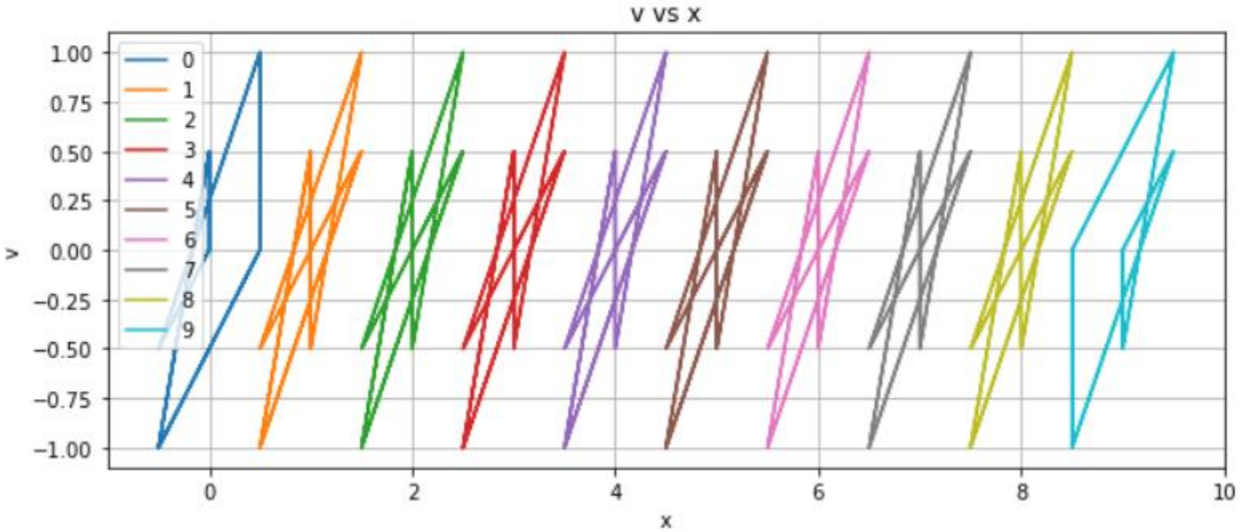
Now, we will try to see how varying timesteps affects the energy curve. For this sake of model, we will stick with these properties:

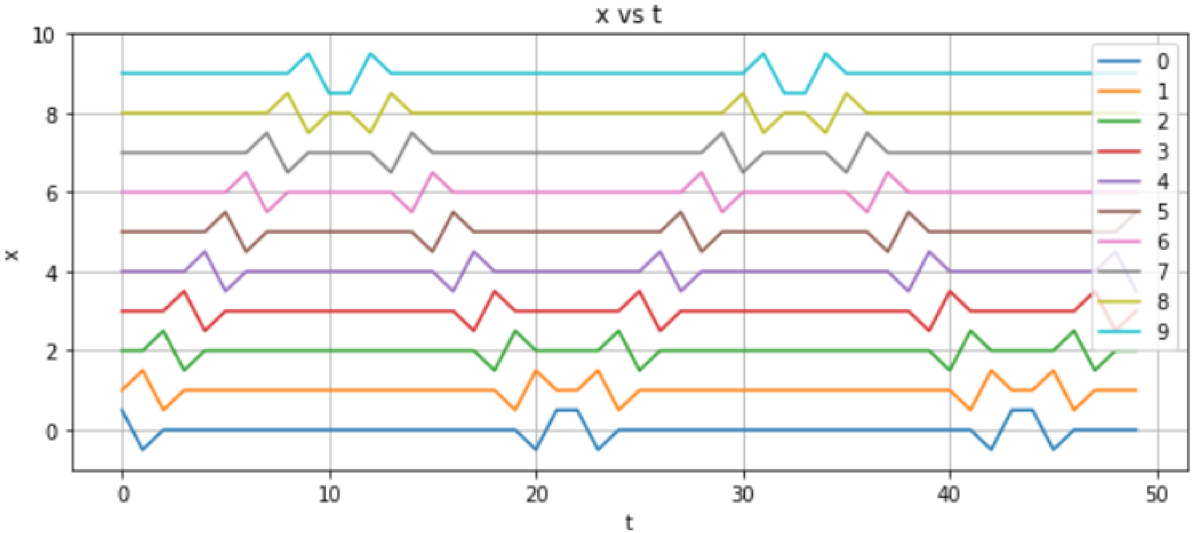
- N = 10
- Time from 0 to 50
- Initial condition is set on particle-1 with +1 displacement and 0 velocity

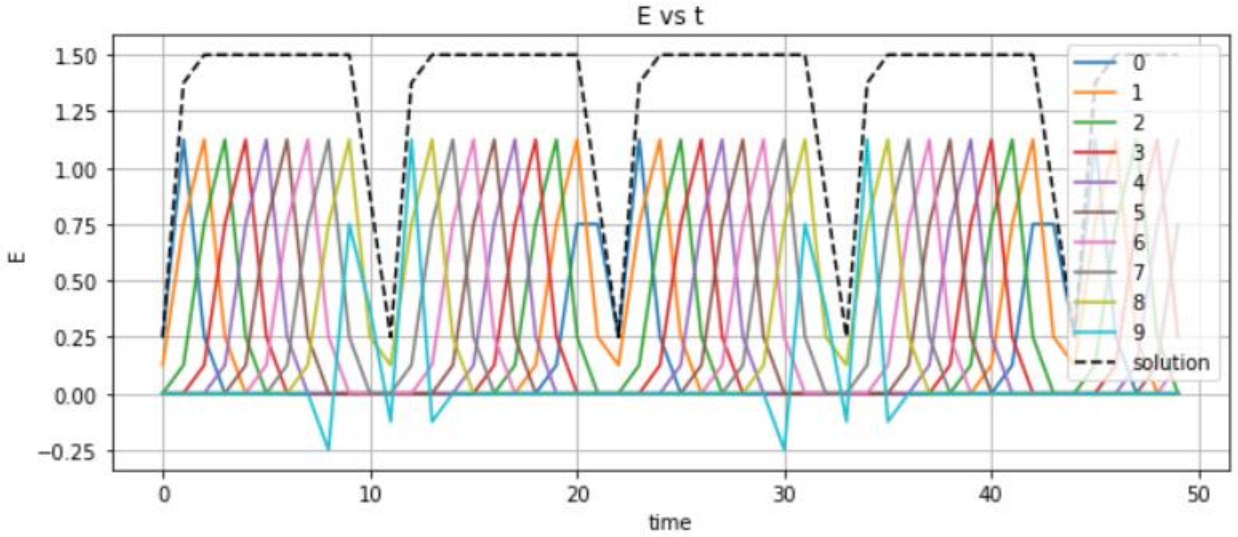
We will vary from 1, 0.1, and 0.01

dt = 1

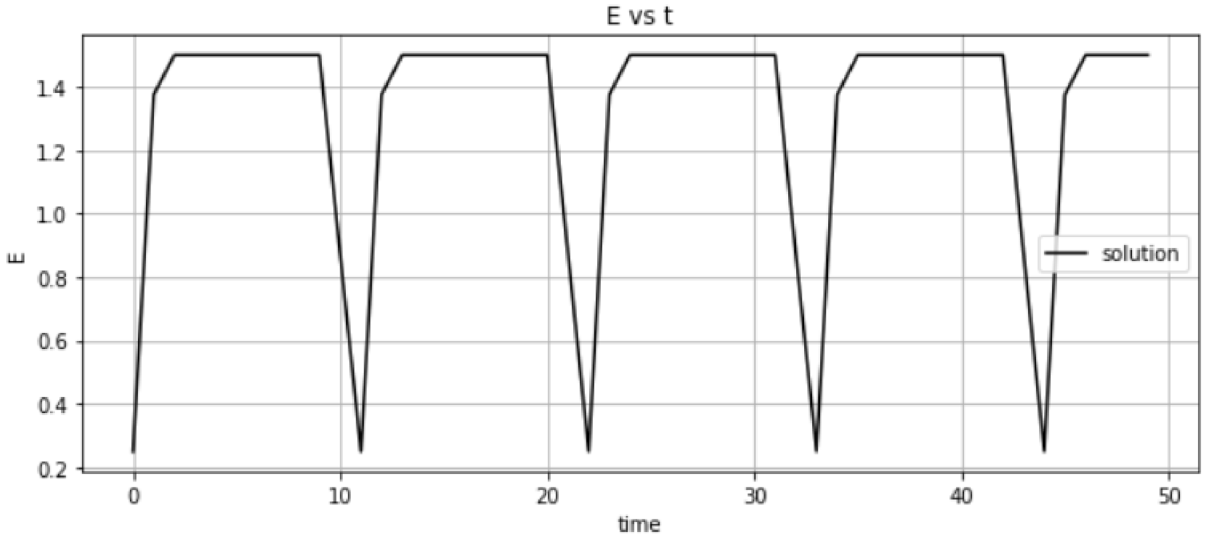
With dt=1, there are 50 iterations from 0 to 50





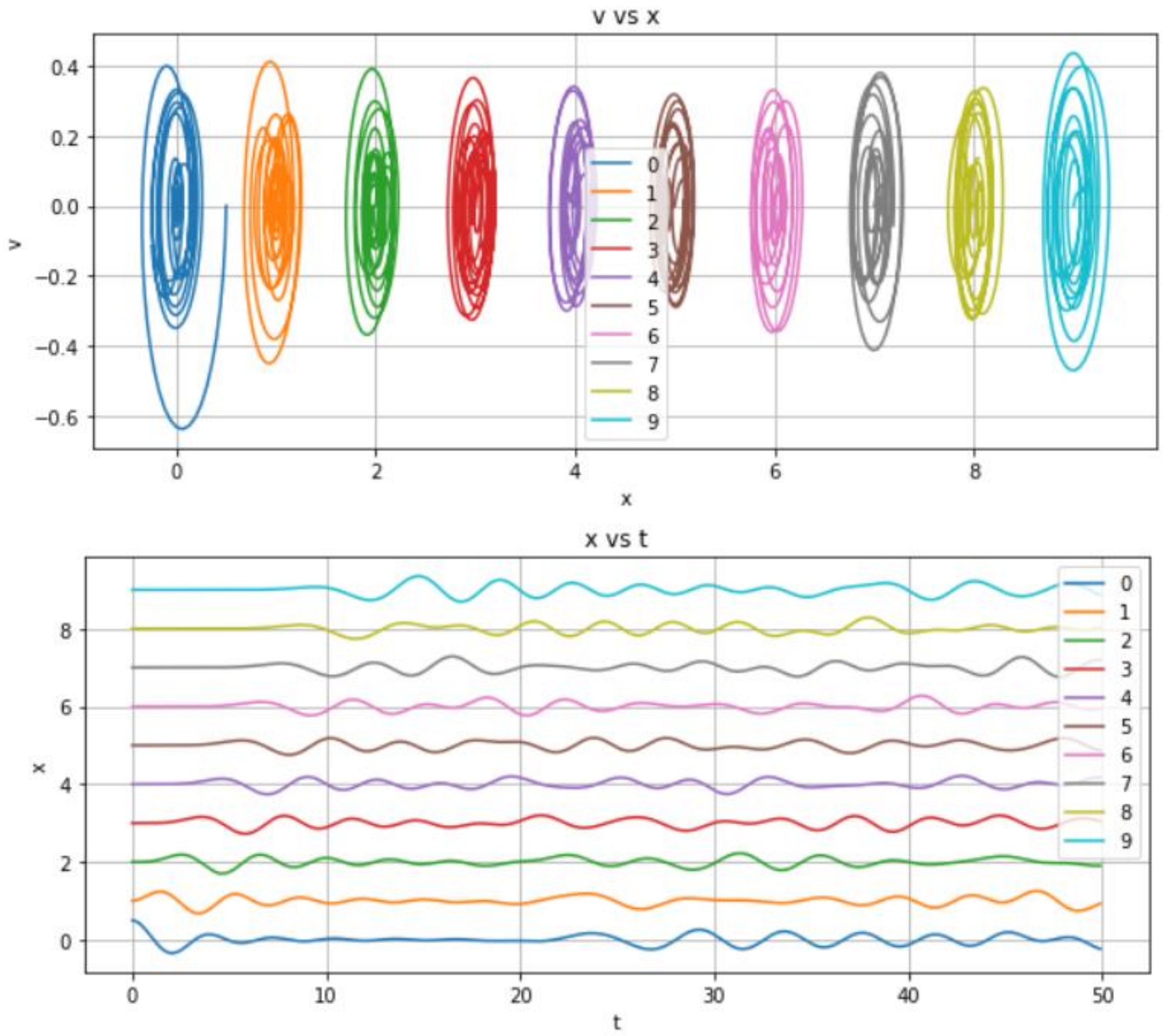


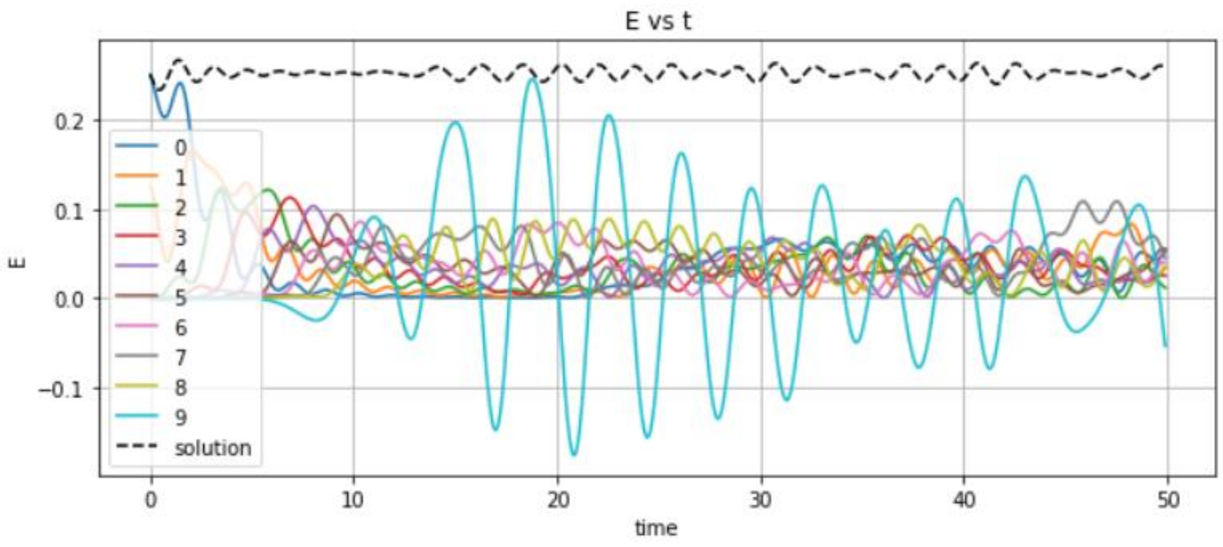
Here the difference between scale in the energy diagram is 2e-1



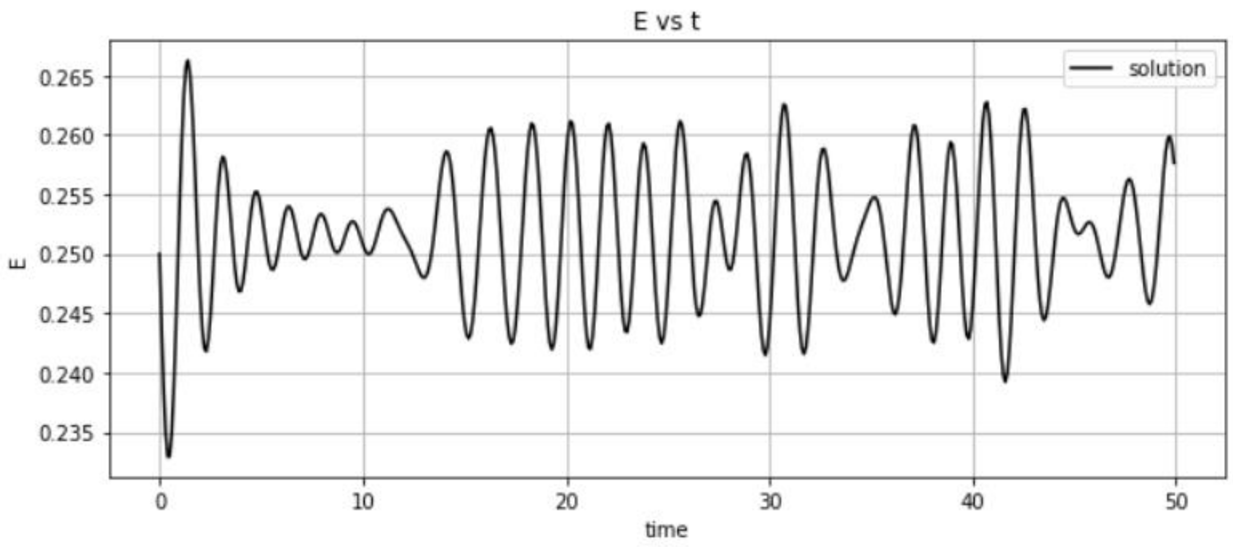
dt = 0.1

With dt=0.1, there are 500 iterations from 0 to 50



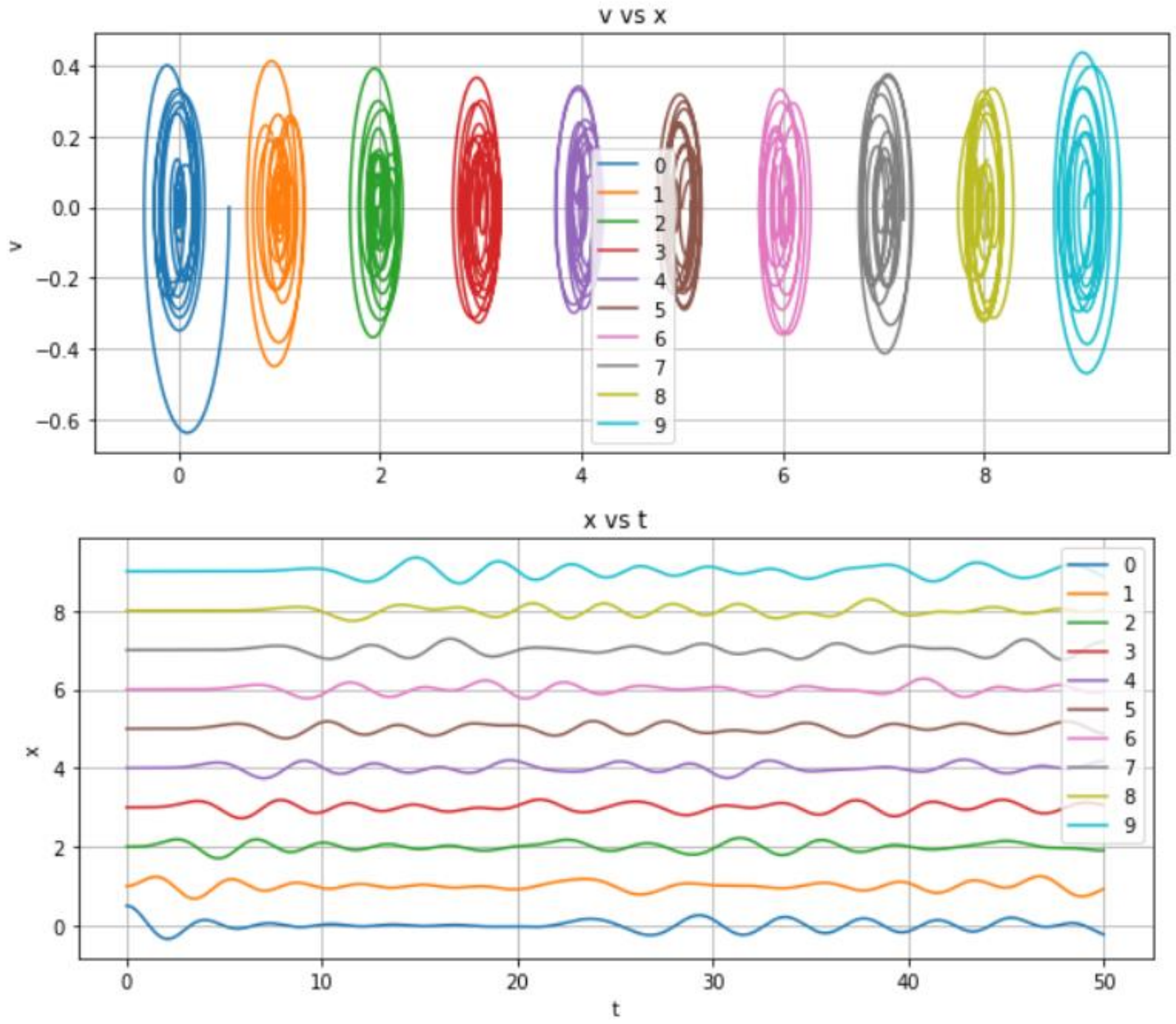


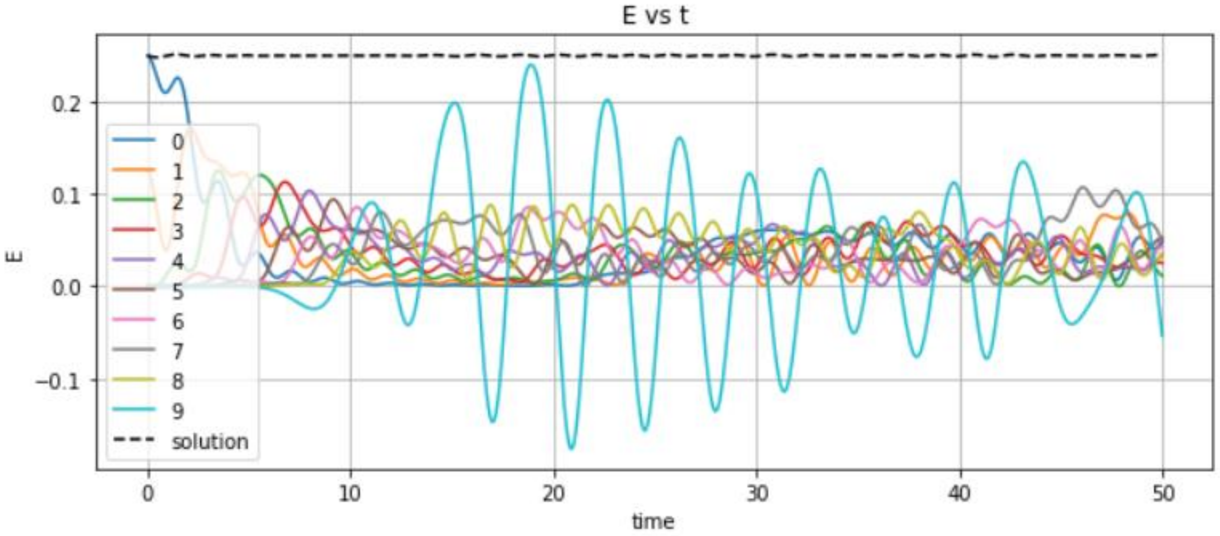
Here the difference between scale in the energy diagram is 5e-2



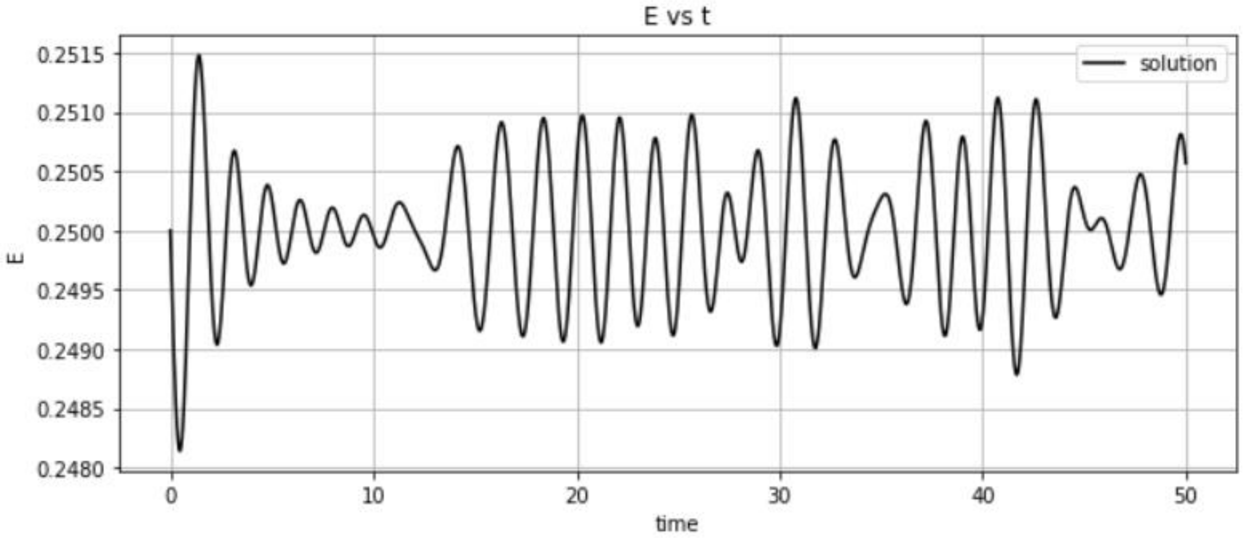
dt = 0.01

With dt=0.01, there are 5000 iterations from 0 to 50



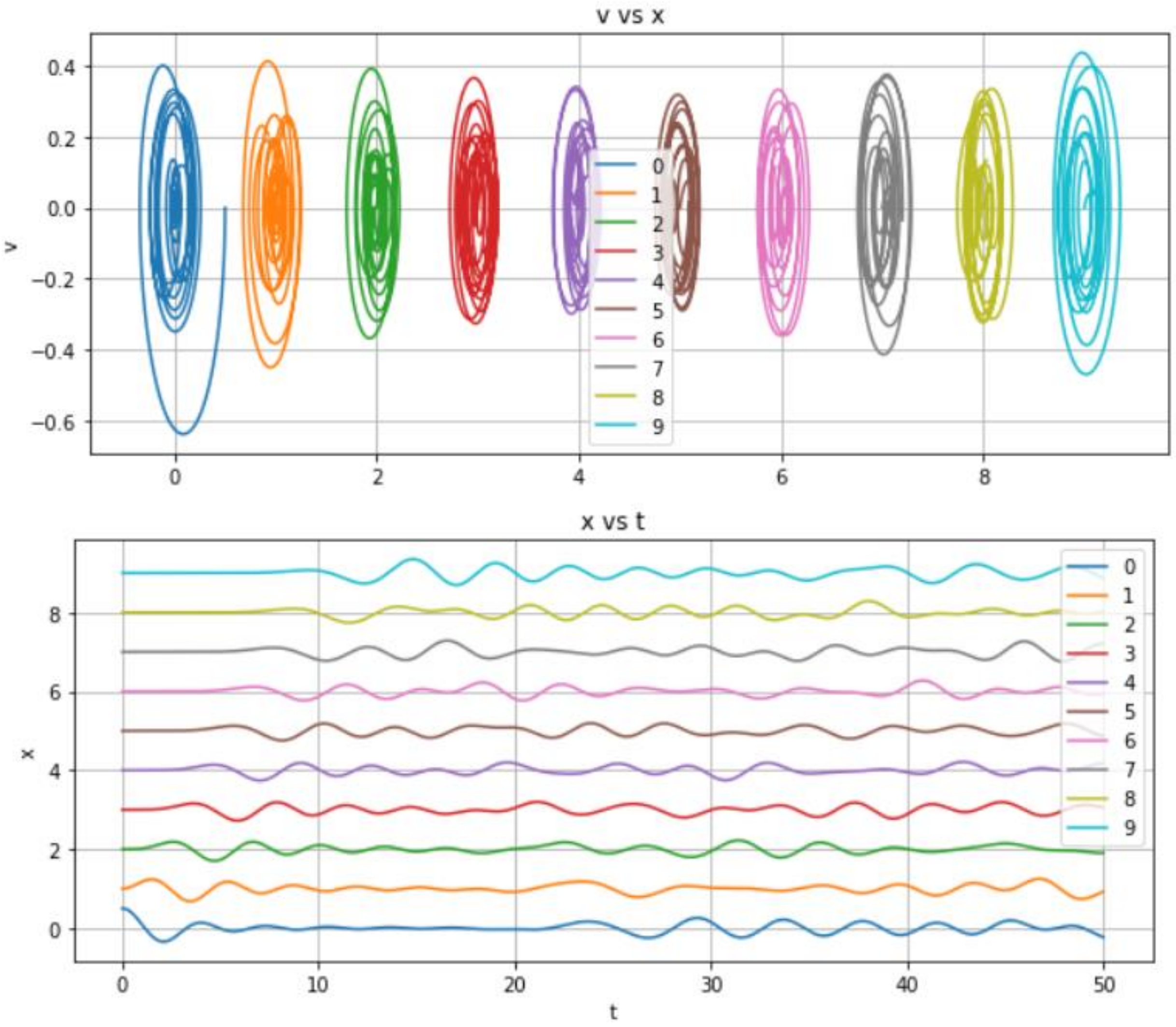


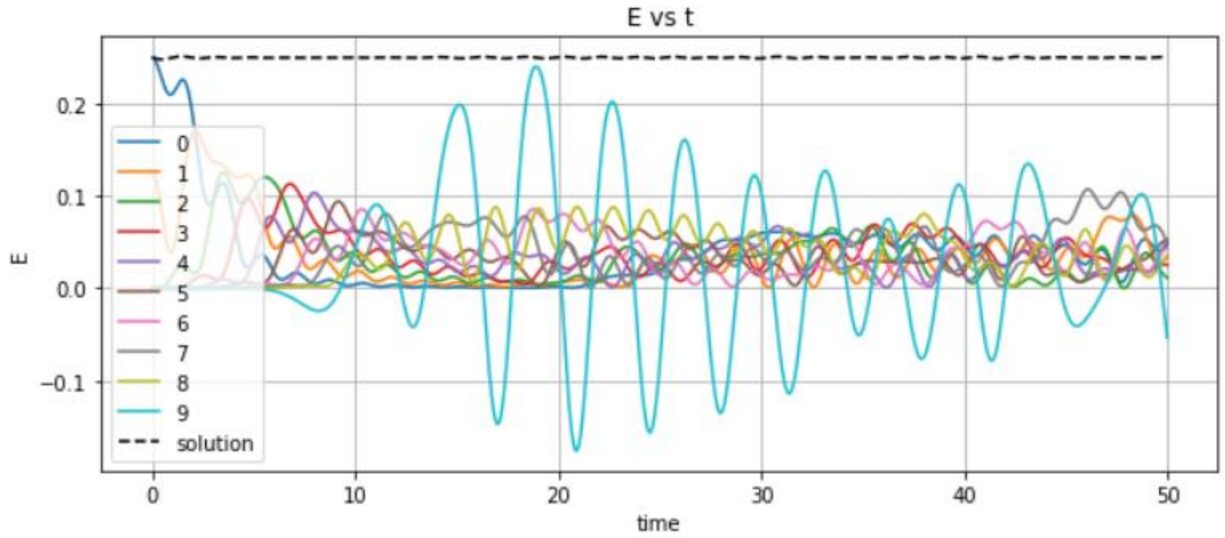
Here the difference between scale in the energy diagram is 5e-4



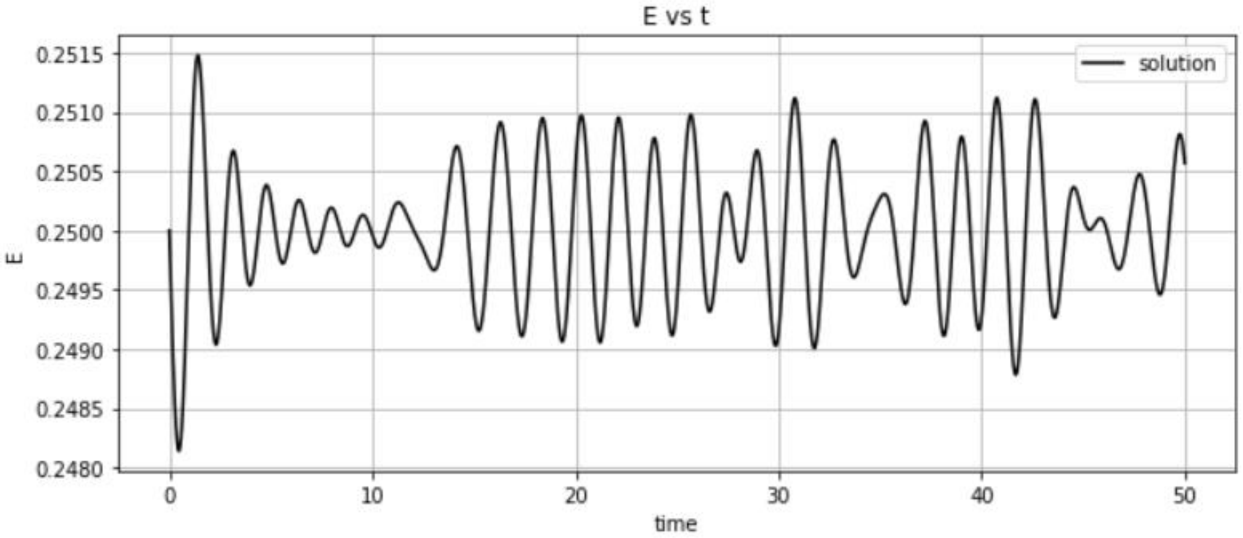
dt = 0.01

With dt=0.01, there are 5000 iterations from 0 to 50





Here the difference between scale in the energy diagram is 5e-4

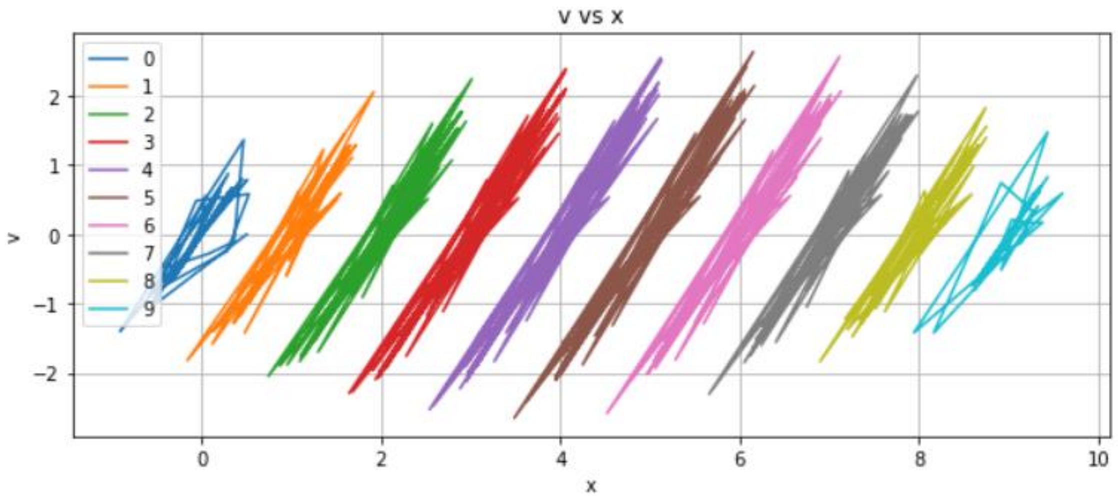


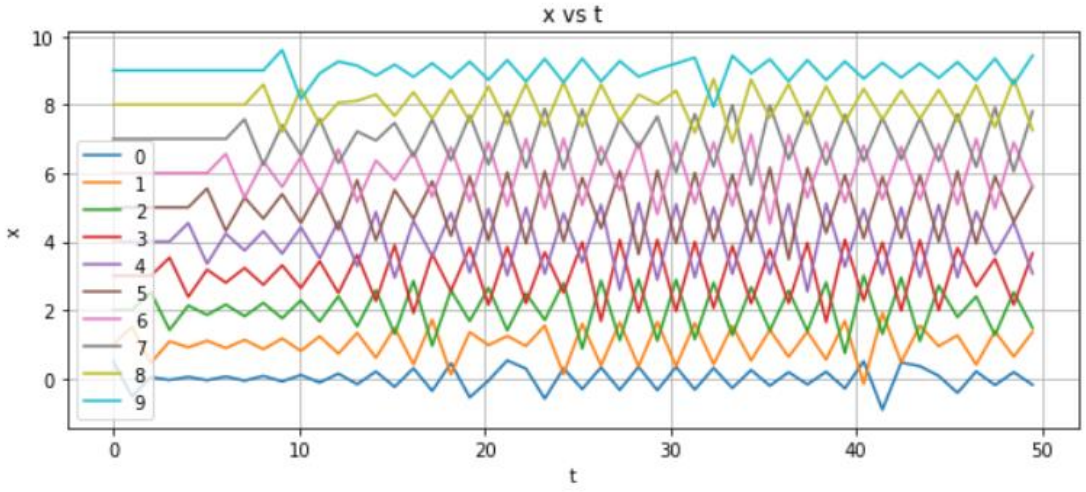
Breaking the Euler-Cromer

I have put up two timesteps that I try it out to see at what point the Euler-Cromer will not have a good enough total energy curve that its average might not look accurate. I did one with dt = 1.01 and another with dt = 1.015

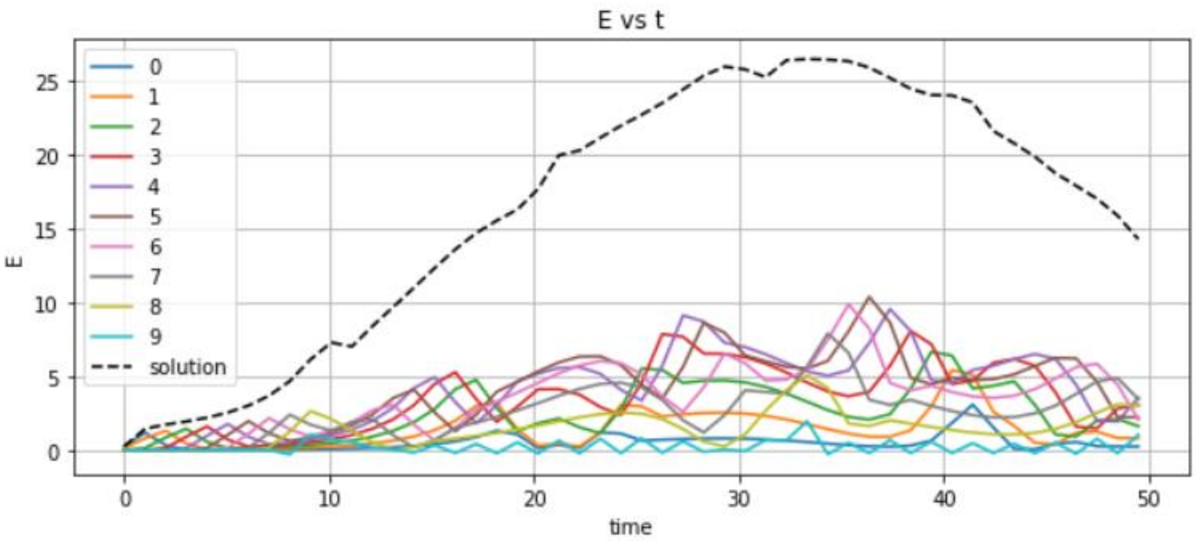
dt = 1.01

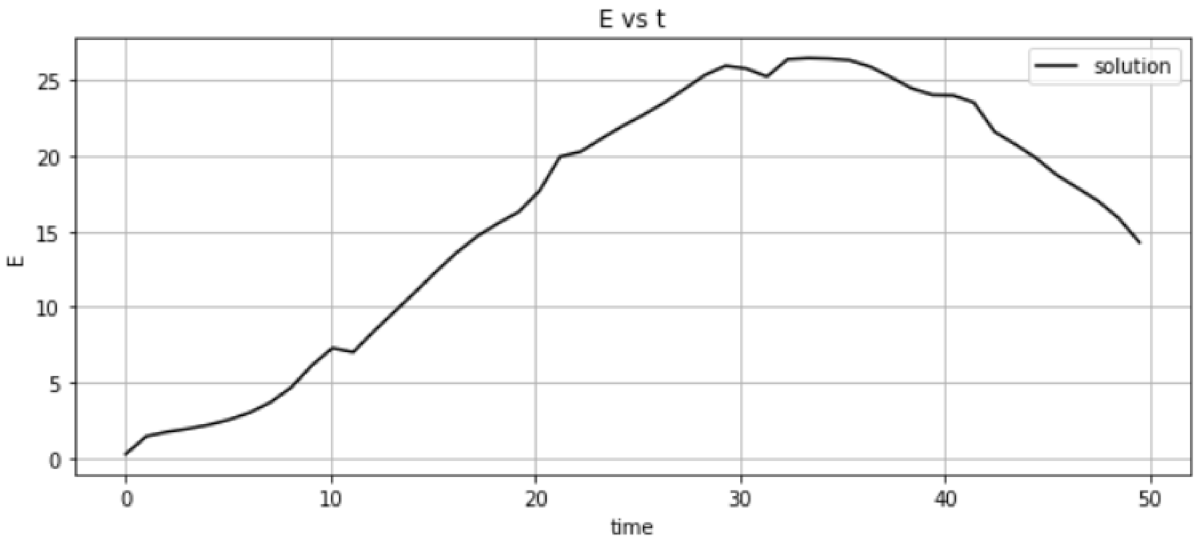
With dt=1.01, there are 49.5 iterations from 0 to 50





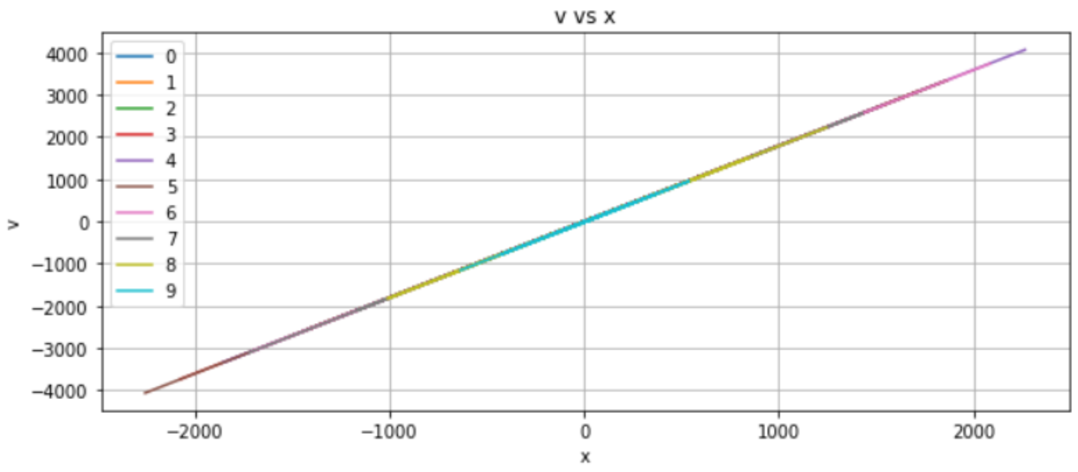
Here the difference between scale in the energy diagram is just the same as the one with all the energy of each particle has

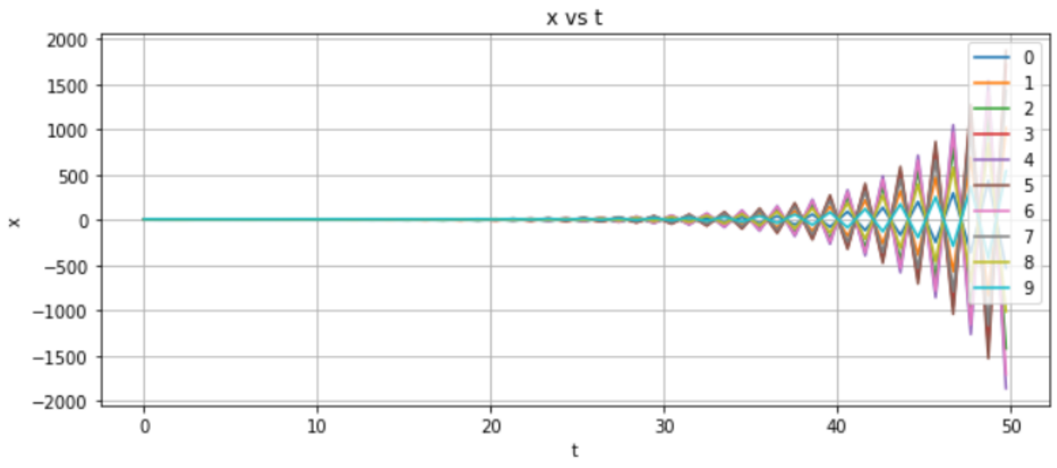




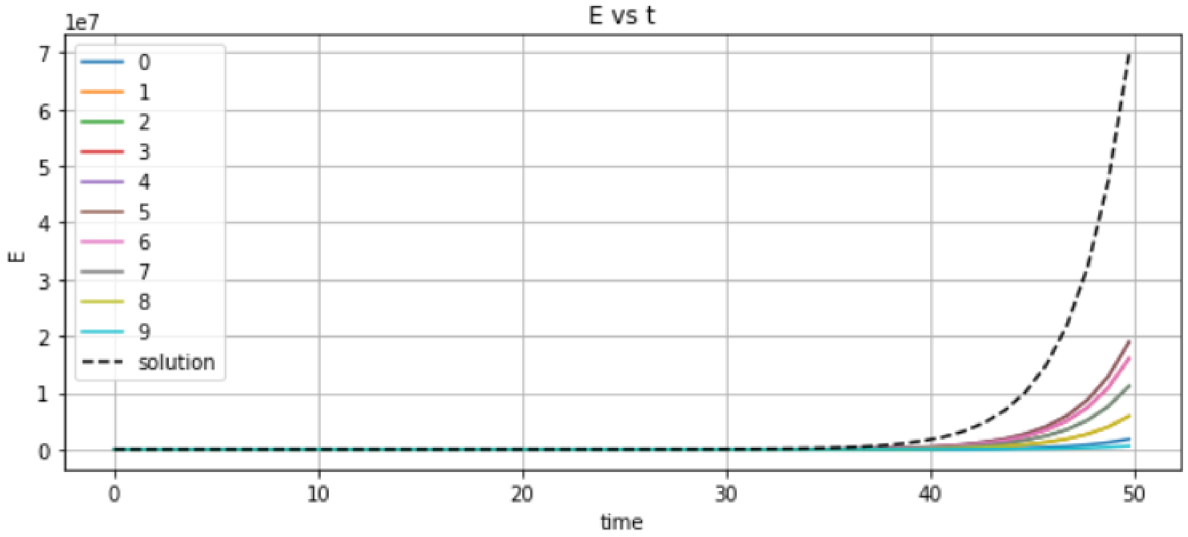
dt = 1.015

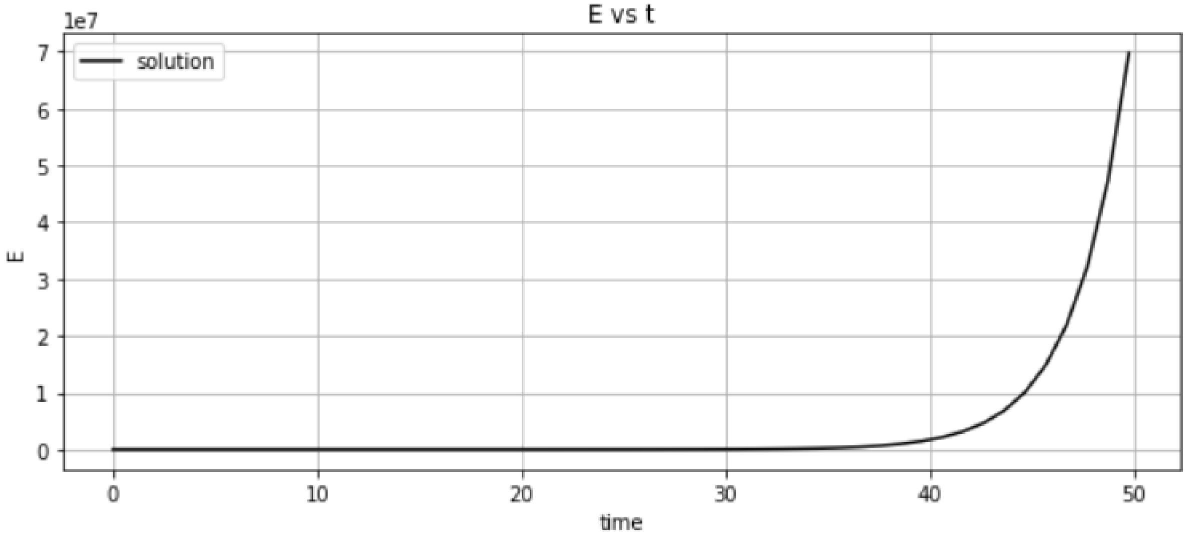
With dt=1.015, there are iterations from 0 to 50





Here the difference between scale in the energy diagram is just the same as the one with all the energy of each particle has too





Conclusion

- Euler-Cromer has been proven to conserve energy well
- With small change of timestep, we can get large precision improvements.
- The algorithm can break with large enough timestep that it will be unable to conserve energy properly and the phase diagram cannot make a good loop with the energy diagram has increase of energy in system