AMPL Excercise: Dog Bed Production

ORIE 4140 / 6140 Fall 2019

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1 A production planning problem

A firm produces various types of dog beds. Our goal is to optimize the production planning of the firm during one week. We consider the week as starting on Monday ($t = 1$) and make decisions every day, up to and including Sunday ($t = 7$). We think of the time horizon as $\{1, 2, \ldots, T\}$ with $T = 7$; time zero $t = 0$ represents the Sunday preceding the week of interest.

The firm keeps track of various resources, each in one of three categories. *Raw materials* W are physical resources not produced in-house, *products* P are resources produced in-house, and *capacities* C are non-inventoriable resources such as worker-time available on a given day. The relevant resources for our production problem listed below.

Key	Variable	Category
PadFab	Padded fabric (square inches)	Raw
FlecFab	Fleece fabric (square inches)	Raw
MetRod	Metal rod (inches)	Raw
PVCRod	PVC rod (inches)	Raw
MetLeg	Metal leg	Raw
PVCLeg	PVC leg	Raw
Package	Packaging material	Raw
MetalCut	Metal cutting (machinist) time (minutes)	Cap
PVCCut	PVC cutting time (minutes)	Cap
FabCut	Fabric cutting time (minutes)	Cap
FabSew	Fabric sewing time (minutes)	Cap
KitAssm	Kit assembly time (minutes)	Cap
36x30M	36x30 inch metal frame	Prod
30x24M	30x24 inch metal frame	Prod
30x24PVC	30x24 inch PVC frame	Prod
24x18PVC	24x18 inch PVC frame	Prod
36x30Flec	36x30 inch fleece bedding	Prod
36x30Pad	36x30 inch padded bedding	Prod
30x24Flec	30x24 inch fleece bedding	Prod
30x24Pad	30x24 inch padded bedding	Prod
24x18Flec	24x18 inch fleece bedding	Prod
24x18Pad	24x18 inch padded bedding	Prod
36x30MPad	Complete large metal frame with padded bedding	Prod
36x30MFlec	Complete large metal frame with fleece bedding	Prod
30x24MPad	Complete medium metal frame with padded bedding	Prod
30x24MFlec	Complete medium metal frame with fleece bedding	Prod
30x24PVCPad	Complete medium PVC frame with padded bedding	Prod
30x24PVCFlec	Complete medium PVC frame with fleece bedding	Prod
24x18PVCPad	Complete small PVC frame with padded bedding	Prod
24x18PVCFlec	Complete small PVC frame with fleece bedding	Prod

There are many ways to partition the resources, but here we consider two additional categories. First, let the set of *resources*, denoted R, be the union of products, raw material, and capacities $R = P \cup W \cup C$; this is the set of everything in the list above. Let the set of *materials* M be the set of products and raw material, $M = P \cup M$; these are all resources but the capacities.

To define the problem, a number of parameters and decision variables are needed. The *parameters* (also called *problem data*, or *data*) are given; in our case these are found in the data tables. The *decision variables* are the ones we want to choose optimally; these represent various decisions our firm can take. Parameters and variables are summarized in the table below.

Variable	Mathematical notation	Type
Supply of raw material/capacity $i \in W \cup C$ at time t	b_{it}	Parameter
Demand for material $i \in M$ at time t (or later)	d_{it}	Parameter
Revenue from selling material $i \in M$ to demand in time t	c_{it}	Parameter
Scrapping cost for resource $i \in R$ at time t	e_{it}	Parameter
Inventory cost of storing material $i \in M$ from t to $t+1$	f_{it}	Parameter
Units of material $i \in M$ inherited from last week ($t = 0$)	u_{i0}	Parameter
Resources $j \in R$ used to make product $i \in P$	$a_i = (a_{ij})_{j \in P}$	Parameter
Units of material $i \in M$ sold to demand in t at $t' \geq t$	$y_{it}^{t'}$	Variable
Units of product $i \in P$ made in t	x_{it}	Variable
Units of resource $i \in R$ scrapped in t	v_{it}	Variable
Units of $i \in M$ inventoried in t	u_{it}	Variable

Notice the following points. First, if a customer demands one unit of material $i \in M$ at time t, we can sell material i to said customer earliest at time t (this corresponds to $y_{it}^t = 1$. We can also wait and sell it later, at some time $t' > t$ (indicated by letting $y_{it}^{t'} = 1$). So far we have only specified the revenue c_{it} of selling item i to a demand in time t at time t, and not any later. We will introduce a constraint to reflect our revenue from selling late. In addition, note that the last listed parameters, the *bill of resources*, is of dimension $|P| \times |R|$ and is supplied in its own table.

We use these in the following following model:

$$\text{Profit} = \sum_{t=0}^{T} \left(\sum_{i \in M} \left(\sum_{\tau=t}^{T} c_{it}^{\tau} y_{it}^{\tau} \right) - \sum_{j \in R} e_{jt} v_{jt} - \sum_{j \in M} f_{jt} u_{jt} \right) \tag{1.1}$$

$$\text{s.t.} \quad \sum_{i \in M} a_{ij} x_{it} + v_{jt} + u_{jt} + \sum_{\tau=0}^{t} y_{j\tau}^{t} = b_{jt} + x_{jt} + u_{j(t-1)} \qquad \forall j \in M, t = 1, \ldots, T \tag{1.2}$$

$$\sum_{i \in M} a_{ij} x_{it} + v_{jt} = b_{jt} \qquad \forall j \in C, t = 1, \ldots, T \tag{1.3}$$

$$\sum_{i \in M} a_{ij} x_{it} + v_{jt} + u_{jt} + \sum_{\tau=0}^{t} y_{j\tau}^{t} = b_{jt} + u_{j(t-1)} \qquad \forall j \in W, t = 1, \ldots, T \tag{1.4}$$

$$\sum_{\tau=t}^{T} y_{it}^{\tau} \leq d_{it} \qquad \forall i \in M, t = 1, \ldots, T \tag{1.5}$$

$$c_{it}^{\tau} = c_{it} \cdot 0.95^{(\tau - t)} \qquad \forall i \in M, t = 1, \ldots, T, \ \tau \geq t \tag{1.6}$$

$$y_{it}^{\tau} \geq 0, \quad u_{it} \geq 0, \ \forall i \in M \qquad t = 1, \ldots, T,$$

$$v_{it} \geq 0 \ \forall i \in R, \quad x_{it} \geq, \ \forall i \in P \qquad t = 1, \ldots, T \tag{1.7}$$

The model model is largely explained in the lecture slides, with one notable addition. The constraint (1.6) gives the revenue from selling item i late, in period τ instead of t (where $T \geq \tau \geq t$). If in period τ item i is sold to satisfy demand that first appeared in period t, we reduce the original on-time revenue by a factor of $(0.95)^{(\tau - t)}$. In other words we incur a 5% penalty in revenue for every day the customer waits on her product. Implement and solve this model in AMPL with the data given. What do you find?