

Project One

MAT350: Applied Linear Algebra

Aman Jemal

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Problem 1

Develop a system of linear equations for the network by writing an equation for each router (A, B, C, D, and E). Make sure to write your final answer as $Ax=b$ where A is the 5×5 coefficient matrix, x is the 5×1 vector of unknowns, and b is a 5×1 vector of constants.

Solution:

Put your math/explanation here...

We are working with the presumption that total incoming and transmitted data at each point is equal.

At point A 100 is in coming and x_1 and $2x_2$ are transmitted out therefore the equation is $x_1 + 2x_2 = 100$

At point B x_1 and x_2 are incoming while x_3 and x_5 are transmitted out therefore the equation is $x_5 + x_3 - x_1 - x_2 = 0$ which we can rewrite as $-x_1 - x_2 + x_3 + x_5 = 0$

At point C x_2 and 50 are the incoming while x_3 and x_5 are transmitted out therefore the equation is $x_5 + x_3 - x_2 = 50$ which we can rewrite as $-x_2 + x_3 + x_5 = 50$

At point D x_4 and x_5 are the incoming while x_2 and 120 are transmitted out therefore the equation is $x_2 - x_4 - x_5 = -120$

At point E x_5 is the incoming while x_4 is transmitted out therefore the equation is $x_5 - x_4 = 0$

Given that we have five equations and five variables matrix A will be a 5 by 5 matrix while our constant matrix b will be a 5 by 1

$$A = [1, 2, 0, 0, 0; -1, -1, 1, 0, 1; 0, -1, 1, 0, 1; 0, 1, 0, -1, -1; 0, 0, 0, 1, -1]$$

$$b = [100; 0; 50; -120; 0]$$

$$x = [x_1; x_2; x_3; x_4; x_5]$$

$$Ax = b$$

$$[1, 2, 0, 0, 0; -1, -1, 1, 0, 1; 0, -1, 1, 0, 1; 0, 1, 0, -1, -1; 0, 0, 0, 1, -1][x_1; x_2; x_3; x_4; x_5] = [100; 0; 50; -120; 0]$$

$$\begin{array}{ccccc|cc}
 1 & 2 & 0 & 0 & 0 & x_1 & 100 \\
 -1 & -1 & 1 & 0 & 1 & x_2 & 0 \\
 0 & -1 & 1 & 0 & 1 & \times x_3 & = 50 \\
 0 & 1 & 0 & -1 & -1 & x_4 & -120 \\
 0 & 0 & 0 & 1 & -1 & x_5 & 0
 \end{array}$$

```
A=[1,2,0,0,0;-1,-1,1,0,1;0,-1,1,0,1;0,1,0,-1,-1;0,0,0,1,-1]
```

```
A = 5x5
     1     2     0     0     0
    -1    -1     1     0     1
     0    -1     1     0     1
     0     1     0    -1    -1
     0     0     0     1    -1
```

```
b=[100;0;50;-120;0]
```

```
b = 5x1
    100
     0
     50
    -120
     0
```

```
x=A\b
```

```
x = 5x1
    50.0000
    25.0000
     2.5000
    72.5000
    72.5000
```

Problem 2

Use MATLAB to construct the augmented matrix $[A \ b]$ and then perform row reduction using the `rref()` function. Write out your **reduced matrix and identify the free and basic variables of the system.**

Solution:

```
%code
AugmA=[A b]
```

```
AugmA = 5x6
     1     2     0     0     0    100
    -1    -1     1     0     1     0
     0    -1     1     0     1     50
     0     1     0    -1    -1    -120
     0     0     0     1    -1     0
```

```
0    0    0    1   -1    0
```

```
rref(AugmA)
```

```
ans = 5x6
 1.0000    0    0    0    0  50.0000
    0    1.0000    0    0    0  25.0000
    0    0    1.0000    0    0   2.5000
    0    0    0    1.0000    0  72.5000
    0    0    0    0    1.0000  72.5000
```

```
%although we know that it has no free variables if it did this is how we
%would find it
```

```
[numeqsA,numvarA]=size(A)
```

```
numeqsA = 5
numvarA = 5
```

```
NumfreevarA= numvarA-numeqsA
```

```
NumfreevarA = 0
```

Problem 3

Use MATLAB to **compute the LU decomposition of A**, i.e., find $A = LU$. For this decomposition, find the transformed set of equations $Ly = b$, where $y = Ux$. Solve the system of equations $Ly = b$ for the unknown vector y .

Solution:

```
%code
%since I have used x as variable before I will use x1 this time
[L, U]=lu(A)
```

```
L = 5x5
 1.0000    0    0    0    0
-1.0000    1.0000    0    0    0
    0   -1.0000    1.0000    0    0
    0    1.0000   -0.5000    1.0000    0
    0    0    0   -1.0000    1.0000

U = 5x5
 1    2    0    0    0
 0    1    1    0    1
 0    0    2    0    2
 0    0    0   -1   -1
 0    0    0    0   -2
```

```
y=L\b
```

```
y = 5x1
 100
 100
 150
-145
-145
```

```
x1=U\y
```

```
x1 = 5x1
50.0000
25.0000
2.5000
72.5000
72.5000
```

Problem 4

Use MATLAB to **compute the inverse** of U using the inv() function.

Solution:

```
%code
Uinv=inv(U)
```

```
Uinv = 5x5
1.0000   -2.0000    1.0000         0         0
0         1.0000   -0.5000         0         0
0          0        0.5000         0        0.5000
0          0          0       -1.0000        0.5000
0          0          0         0       -0.5000
```

Problem 5

Compute the solution to the original system of equations by transforming **y** into **x**, i.e., compute $\mathbf{x} = \text{inv}(\mathbf{U})\mathbf{y}$.

Solution:

```
%code
%since we have used x and x1 we will use x2 this time
x2=Uinv*y
```

```
x2 = 5x1
50.0000
25.0000
2.5000
72.5000
72.5000
```

Problem 6

Check your answer for x_1 using Cramer's Rule. Use MATLAB to compute the required determinants using the det() function.

Solution:

```
%let x_1 be x11 to avoid confusing it code
A1=A
```

A1 = 5×5

1	2	0	0	0
-1	-1	1	0	1
0	-1	1	0	1
0	1	0	-1	-1
0	0	0	1	-1

A2=A

A2 = 5×5

1	2	0	0	0
-1	-1	1	0	1
0	-1	1	0	1
0	1	0	-1	-1
0	0	0	1	-1

A3=A

A3 = 5×5

1	2	0	0	0
-1	-1	1	0	1
0	-1	1	0	1
0	1	0	-1	-1
0	0	0	1	-1

A4=A

A4 = 5×5

1	2	0	0	0
-1	-1	1	0	1
0	-1	1	0	1
0	1	0	-1	-1
0	0	0	1	-1

A5=A

A5 = 5×5

1	2	0	0	0
-1	-1	1	0	1
0	-1	1	0	1
0	1	0	-1	-1
0	0	0	1	-1

A1(:,1)=b

A1 = 5×5

100	2	0	0	0
0	-1	1	0	1
50	-1	1	0	1
-120	1	0	-1	-1
0	0	0	1	-1

A2(:,2)=b

A2 = 5×5

1	100	0	0	0
-1	0	1	0	1
0	50	1	0	1
0	-120	0	-1	-1
0	0	0	1	-1

A3(:,3)=b

```
A3 = 5x5
     1     2    100     0     0
    -1    -1     0     0     1
     0    -1     50     0     1
     0     1   -120    -1    -1
     0     0     0     1    -1
```

A4(:,4)=b

```
A4 = 5x5
     1     2     0    100     0
    -1    -1     1     0     1
     0    -1     1     50     1
     0     1     0   -120    -1
     0     0     0     0    -1
```

A5(:,5)=b

```
A5 = 5x5
     1     2     0     0    100
    -1    -1     1     0     0
     0    -1     1     0     50
     0     1     0    -1   -120
     0     0     0     1     0
```

x_1=det(A1)/det(A)

x_1 = 50.0000

x_2=det(A2)/det(A)

x_2 = 25.0000

x_3=det(A3)/det(A)

x_3 = 2.5000

x_4=det(A4)/det(A)

x_4 = 72.5000

x_5=det(A5)/det(A)

x_5 = 72.5000

Problem 7

The Project One Table Template, provided in the Project One Supporting Materials section in Brightspace, shows the recommended throughput capacity of each link in the network. Put your solution for the system of

equations in the third column so it can be easily compared to the maximum capacity in the second column. In the fourth column of the table, provide recommendations for how the network should be modified based on your network throughput analysis findings. The modification options can be No Change, Remove Link, or Upgrade Link. In the final column, explain how you arrived at your recommendation.

Solution:

Fill out the table in the original project document and export your table as an image. Then, use the **Insert** tab in the MATLAB editor to insert your table as an image.

MAT 500 Project One Table Template

Complete this template by replacing the bracketed text with the relevant information.



Network Link	Recommended Capacity (Mbps)	Solution	Recommendation	Explanation
x ₁	60	[50]	Keep it as it is	The speed is within the capacity
x ₂	50	[25]	Keep it as it is	The speed is within the capacity
x ₃	100	[2.5]	Swapping its capacity with x ₅ would be more efficient.	It is performing way below the capacity performing way above its capacity thus their capacities would improve the overall efficiency.
x ₄	100	[72.5]	Keep it as it is	The speed is within the capacity
x ₅	50	[72.5]	Needs to be upgraded	The capacity needs to be increased to network needs.