ZZCA6510 3: Quantitative Decision Analysis

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1 Product Composition

Peanut for Life, a food manufacturer, seeks to produce a chanachur snack product comprising three mixtures: puffed rice, nuts and cereal. Each ingredient mixture has a different cost per kilogram (see Table 1); therefore, Peanut for Life is aiming to minimise the total cost of each package of chanachur by formulating a linear programming model.

Table 1: Cost of each ingredient.

Ingredient	Cost ($/kg)
Puffed rice mix	0.35
Nut mix	0.50
Cereal mix	0.20

If x_1, x_2 and x_3 represent the weight (in kilograms) of puffed rice, nuts and cereal respectively, then the total cost of one package of chanachur would be:

$$total\_cost = 0.35x_1 + 0.5x_2 + 0.2x_3 \tag{1}$$

The objective is to *minimise* this quantity. However, there is not just the cost to consider. Peanut for Life must also ensure the product is commercially viable; it can do this by composing the product in such a way that it is both nutritionally balanced and attractive to consumers. To this end, the following constraints on the final composition are given:

- The chanachur package must hold between 3 and 4 cups of product.
- One package cannot contain more than 1000 calories in food energy.
- One package cannot contain more than 25 grams of fat.
- At least 20% of the product volume must comprise puffed rice mixture.
- No more than 15% of the product weight may comprise nuts.

Using the nutritional information provided in Table 2, these constraints may be modelled mathematically as inequalities 2 to 7, presented in *canonical form*.

Table 2: Nutritional information per kg.

Ingredient	Volume (cups)	Fat (g)	Calories
Puffed rice mix	0.25	0	150
Nut mix	0.375	10	400
Cereal mix	1.0	1	50

$$-0.25x_1 - 0.375x_2 - x_3 \leq -3 \tag{2}$$

$$0.25x_1 + 0.375x_2 + 1x_3 \leq 4 \tag{3}$$

$$150x_1 + 400x_2 + 50x_3 \leq 1000 \tag{4}$$

$$10x_2 + 1x_3 \leq 25 \tag{5}$$

$$-0.2x_1 + 0.075x_2 + 0.2x_3 \leq 0 \tag{6}$$

$$-0.15x_1 + 0.85x_2 - 0.15x_3 \leq 0 \tag{7}$$

One additional constraint is required; that is, a nonnegativity constraint for the weight of each ingredient:

$$x_i \geq 0, \text{ where } i \in \{1, 2, 3\} \tag{8}$$

Equation 1, together with inequations 2 to 8, completely define the linear programming model for optimising the composition of the new chanachur product. However, when it is solved using the Simplex algorithm, the optimal solution does not include any nuts at all. This is likely due to the high fat and energy content of the nut mixture (as shown in Table 2) encouraging the solver to favour the less nutritionally dense puffed rice and cereal mixtures.

From a marketability standpoint, this is an unacceptable result, as nuts cannot be excluded from a commercially viable chanachur product. For this reason, a minimum amount of nut mixture must be enforced through an additional constraint. A cursory examination of popular chanachur recipes reveals that most small batches contain between 0.5 and 1.0 cups of nuts; approximately 75-113 grams. Therefore, for the present exercise, a nominal minimum of 100 grams has been chosen, and represented by inequation 9 in *canonical form*.

$$-x_2 \leq -0.1 \tag{9}$$

When the model is solved with this new constraint, a more suitable composition is achieved. This is given in Table 3. The corresponding minimal cost per package is \$1.36.

Table 3: Optimal composition of chanachur.

Ingredient	Optimal weight (kg)
Puffed rice mix (x_1)	2.4
Nut mix (x_2)	0.1
Cereal mix (x_3)	2.363

Appendix XXXX contains the model construction in Microsoft Excel.

2 Staff Scheduling

Chris Stokes, the rostering manager at Orient Computer Manufacturer, has been tasked with developing a staff schedule that minimises the total weekly cost of salary payments. Employees at Orient work in weekly shifts that include two days off, and their weekly salary depends on which days they are rostered on, as shown in Table 4. In addition, Mr Stokes has estimated the number of workers required on the factory floor every day of the week; this is shown in Table 5.

Table 4: Weekly salary per shift.

Shift	Days off	Salary ($)
1	Sun and Mon	900
2	Mon and Tue	850
3	Tues and Wed	920
4	Wed and Thu	860
5	Thu and Fri	780
6	Fri and Sat	910
7	Sat and Sun	850

Table 5: Workers required per day.

Day	No. workers required
Sun	18
Mon	13
Tue	15
Wed	18
Thu	21
Fri	18
Sat	21

Mr Stokes sees that a linear programming approach is ideally suited to this scenario. To this end, if x_1 represents the number of employees assigned to shift 1, x_2 represents the number of employees assigned to shift 2, and so on, then the total salary cost for one week would be:

$$total\_salary = 900x_1 + 850x_2 + 920x_3 + 860x_4 + 780x_5 + 910x_6 + 850x_7 \tag{10}$$

The objective is to *minimise* this cost while ensuring that the staff requirement for every day is met. This can be done by representing the number of workers required every day, given in Table 5, as a set of constraints. These manifest themselves as inequations 11 to 17.

$$x_2 + x_3 + x_4 + x_5 + x_6 \geq 18 \tag{11}$$

$$x_3 + x_4 + x_5 + x_6 + x_7 \geq 13 \tag{12}$$

$$x_1 + x_4 + x_5 + x_6 + x_7 \geq 15 \tag{13}$$

$$x_1 + x_2 + x_5 + x_6 + x_7 \geq 18 \tag{14}$$

$$x_1 + x_2 + x_3 + x_6 + x_7 \geq 21 \tag{15}$$

$$x_1 + x_2 + x_3 + x_4 + x_7 \geq 18 \tag{16}$$

$$x_1 + x_2 + x_3 + x_4 + x_5 \geq 21 \tag{17}$$

There are additional constraints; namely, that each five-day shift must be assigned at least one worker, and that the number of workers assigned to each shift is a nonnegative integer. While part-time assignments are possible in other scenarios, Orient states that its employees are entitled to two days off per week, implying that its entire staff body works full time i.e. five days per week.

The additional constraints are represented by inequations 18 and 19. It is also conventional to explicitly require nonnegativity (see inequation 20) but inequation 18 is sufficiently restrictive.

$$x_i \geq 1, \text{ where } i \in \{1, 2, .., 7\} \tag{18}$$

$$x_i \in \mathbb{Z}, \text{ where } i \in \{1, 2, .., 7\} \tag{19}$$

$$x_i \geq 0, \text{ where } i \in \{1, 2, .., 7\} \tag{20}$$

The objective function, given by equation 10, together with these constraints form a complete mathematical model suitable for linear programming. When this model is solved, an optimal number of employees working each five-day shift is obtained (see Table 6). The corresponding minimal salary cost is $22,410.

Table 6: Optimal shift allocation.

Shift	Optimal allocation
1	5
2	8
3	3
4	1
5	4
6	2
7	3