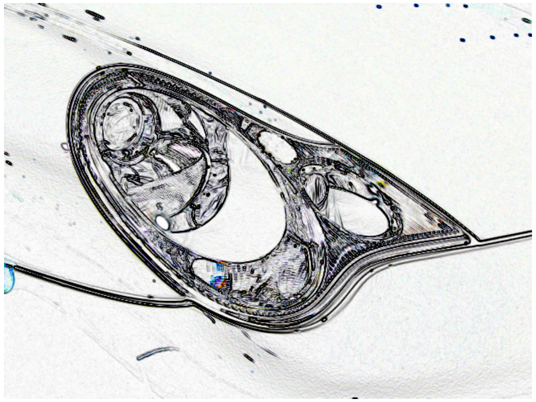
CSE578: Computer Vision

Spring 2016:

Multilabel MRF



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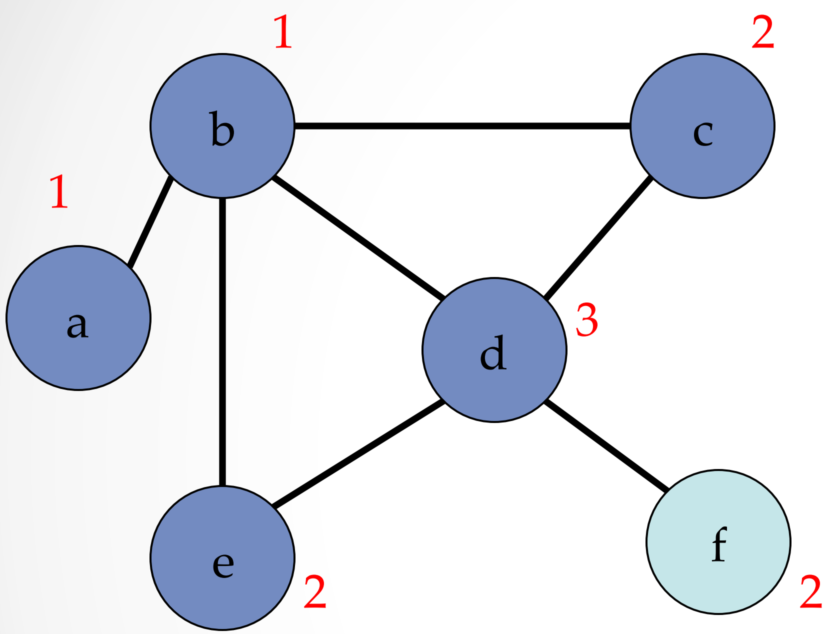
The General Problem



Graph G = (V, E)

Discrete label set L = {1,2,…,h}

Assign a label to each vertex
$f: V \rightarrow L$

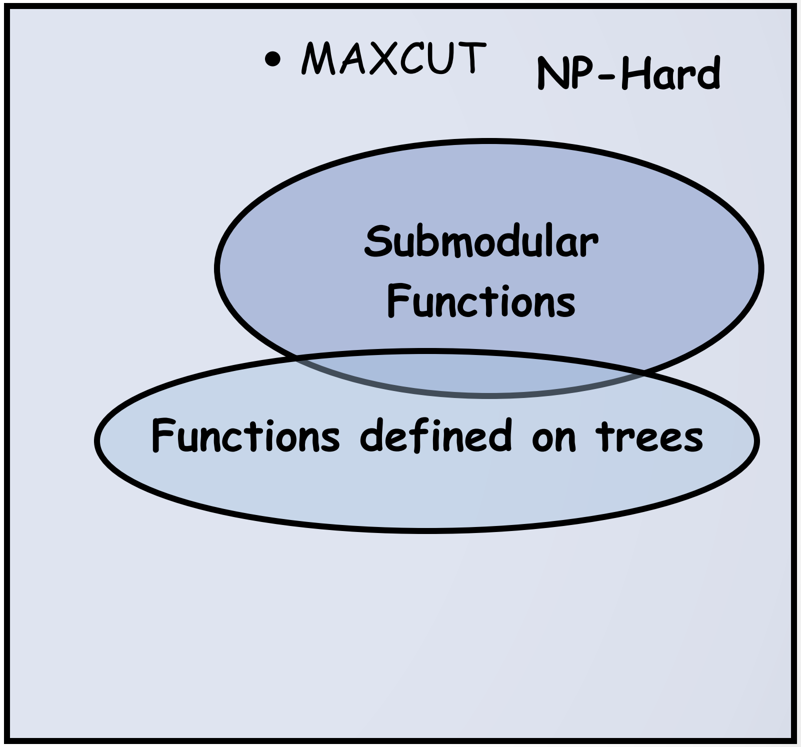
Cost of a labelling Q(f)

Unary Cost Pairwise Cost

Find f* = arg min Q(f)

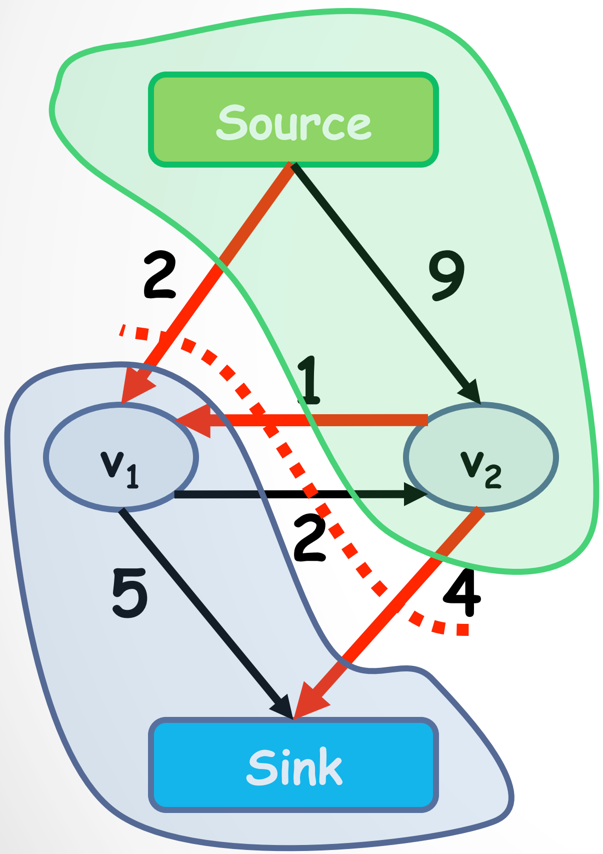
Minimizing Energy Functions

- General Energy Functions
  - NP-hard to minimize
  - Only approximate minimization possible
- Easy energy functions
  - Solvable in polynomial time
  - Submodular ~ $O(n^6)$



Space of Function Minimization Problems

The st-Mincut Problem



2 + 1 + 4 = 7

What is an st-cut?

An st-cut (**S**,**T**) divides the nodes between source and sink.

What is the cost of an st-cut?

Sum of cost of all edges going from S to T

What is the st-mincut?

st-cut with the minimum cost

Robust Interactions

- NP-hard problem (3 or more labels)
  - two labels can be solved via *s-t* cuts (Greig at. al., 1989)
- α-*expansion* approximation algorithm

  (Boykov, Veksler, Zabih 1998, 2001)
  - guaranteed approximation quality (Veksler, thesis 2001)
    - within a factor of 2 from the global minima (Potts model)
  - applies to a wide class of energies with robust interactions
    - Potts model (BVZ 1989)
    - "metric" interactions (BVZ 2001)
    - can be extended to arbitrary interactions with weaker guarantees
      - truncation (Kolmogorov et al. 2005)
      - QPBO (Boros and Hummer, 2002)
- Other "move" algorithms (e.g. α–β swap, jump-moves)

Metric and Semimetric Interactions

$$E(f) = \sum_{\{p,q\} \in \mathcal{N}} V_{p,q}(f_p, f_q) + \sum_{p \in \mathcal{P}} D_p(f_p)$$

$$\begin{aligned} V(\alpha, \beta) = 0 \quad &\Leftrightarrow \quad \alpha = \beta, \\ V(\alpha, \beta) \quad &= \quad V(\beta, \alpha) \geq 0, \\ V(\alpha, \beta) \quad &\leq \quad V(\alpha, \gamma) + V(\gamma, \beta), \end{aligned}$$

- where α,β, and γ are labels.
- Metric: If all three conditions are satisfied.
- Semi-metric: If only the first two are satisfied.
- Examples?

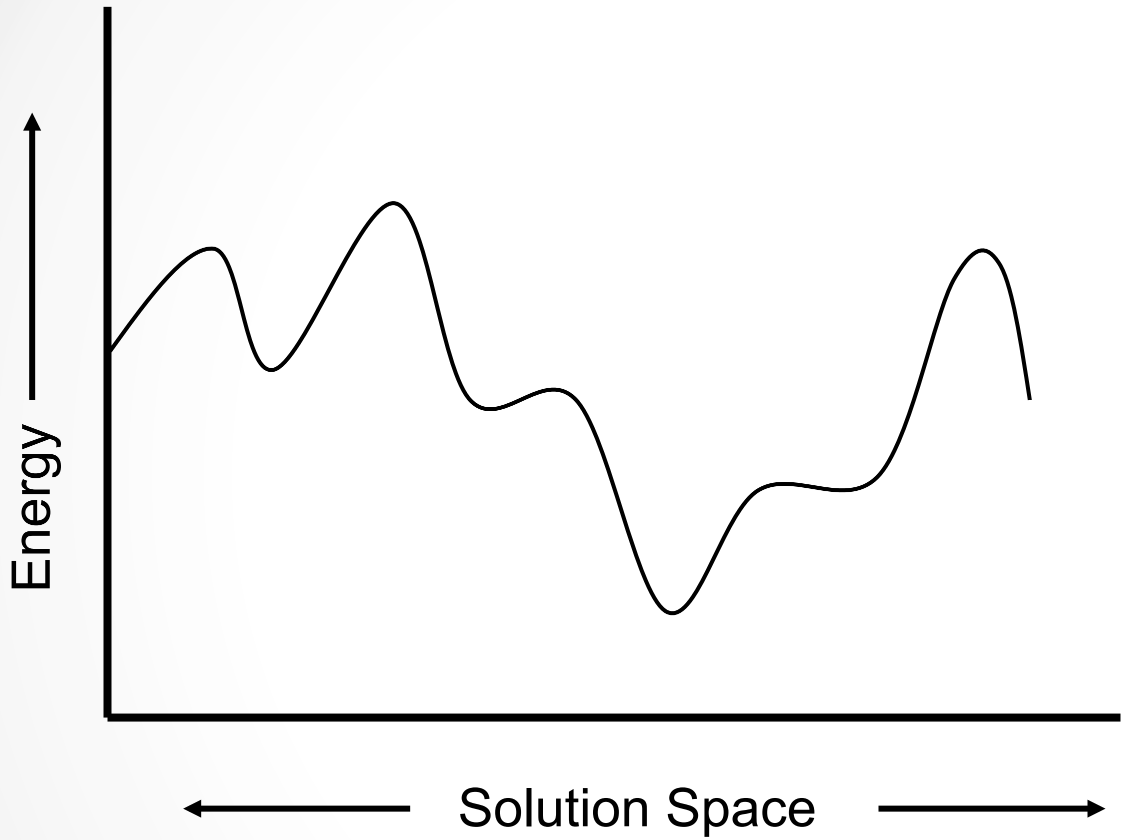
Multi-label Graphcut: Move Making

$$E(\mathbf{x}) = \sum_{i} \theta_i(x_i) + \sum_{i,j} \theta_{ij}(x_i, x_j)$$

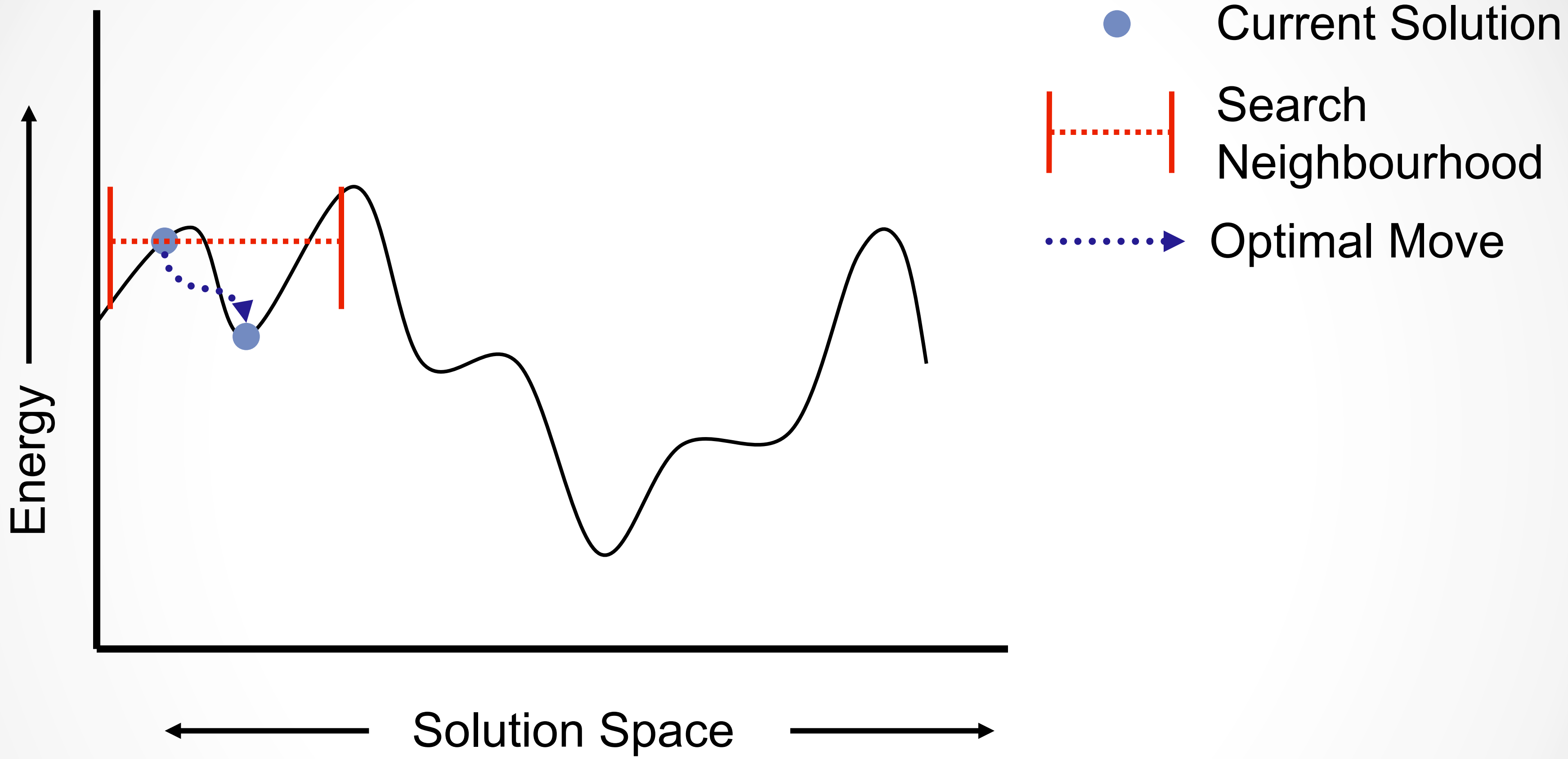
$x \in$ Labels $L = \{l_1, l_2, \dots, l_k\}$

- Commonly used for solving **non-submodular** multi-label problems
- Extremely efficient and produce good solutions
- Not Exact: Produce local optima

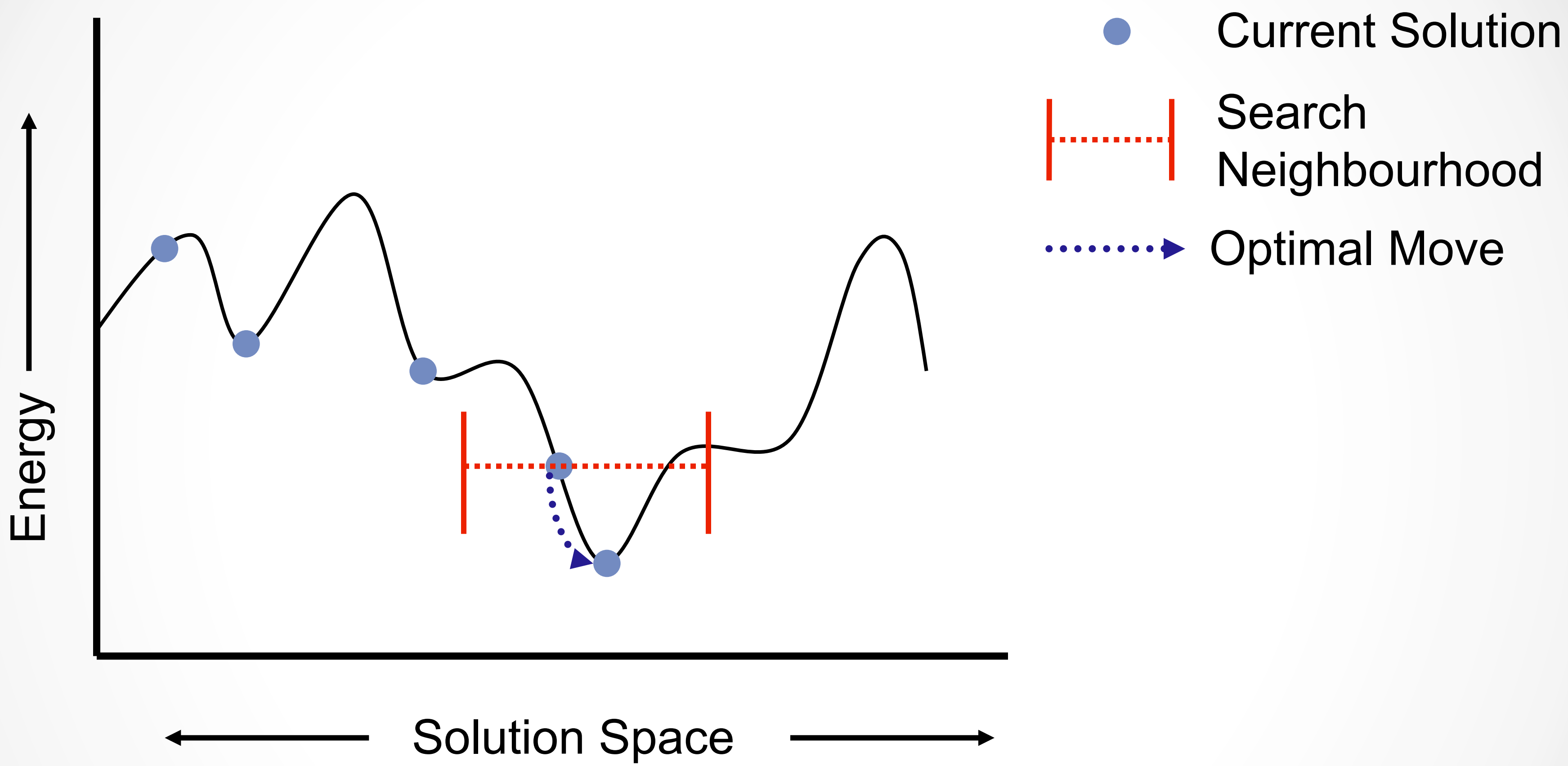
Move Making Algorithms



Move Making Algorithms



Move Making Algorithms



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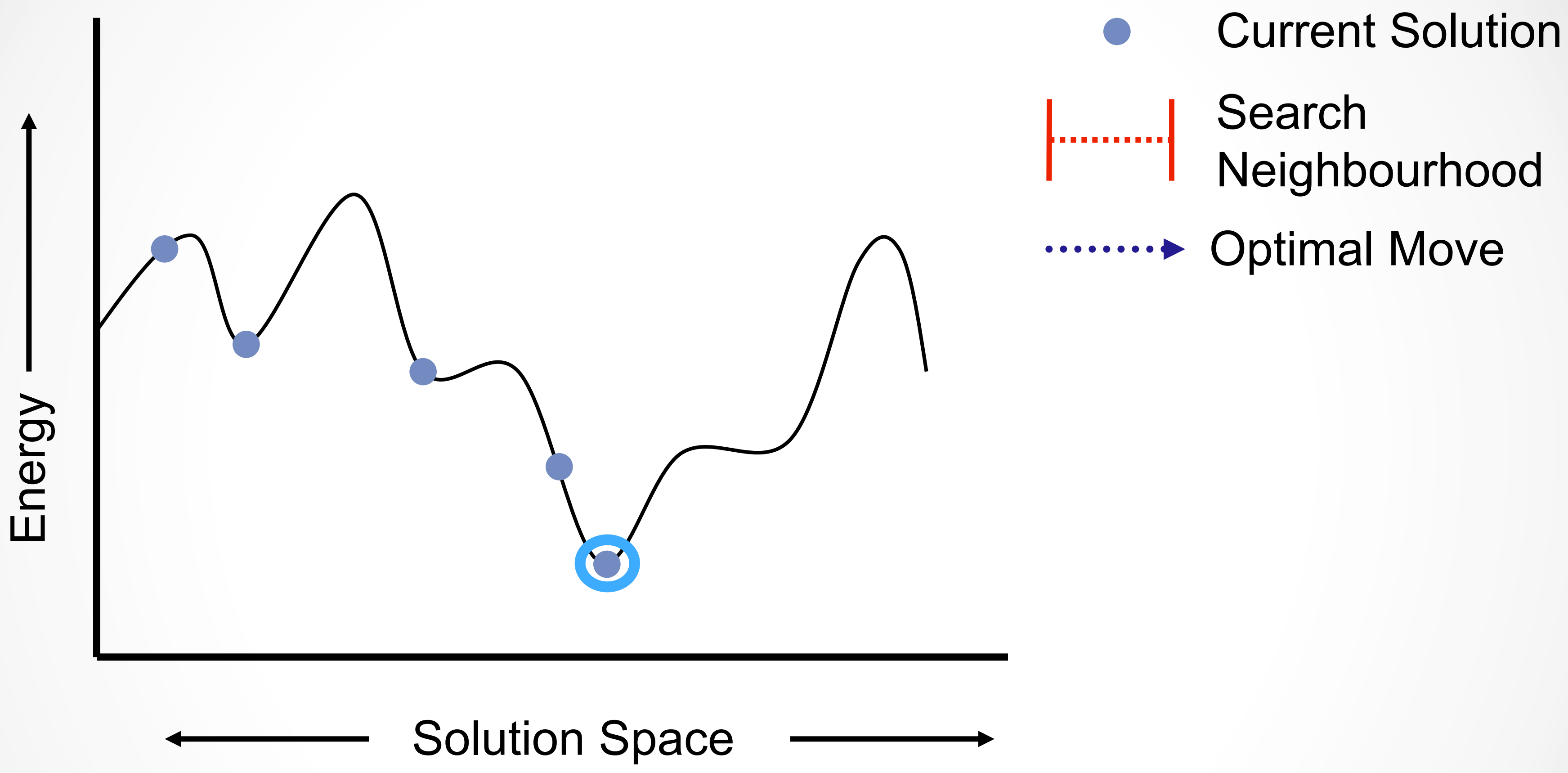
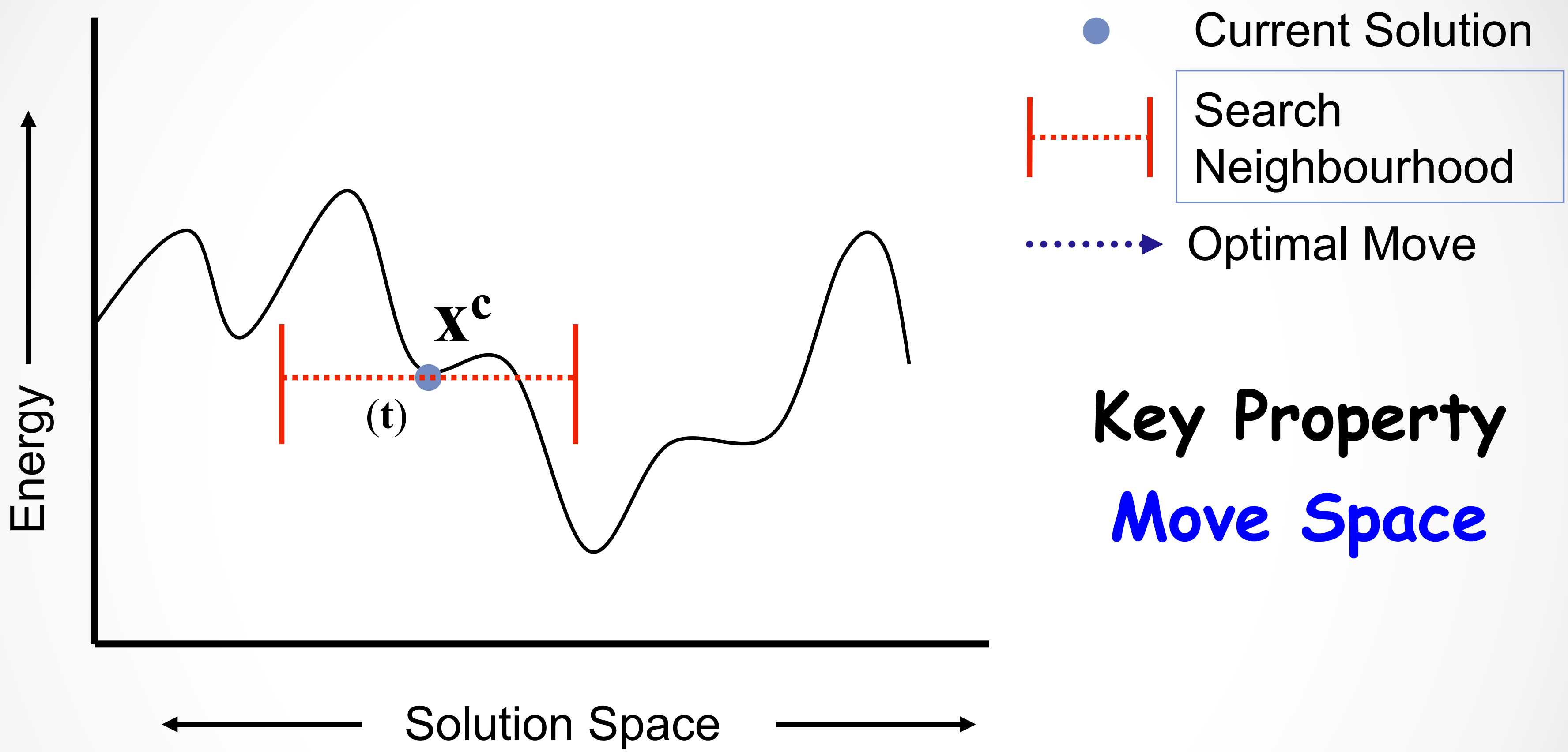


Computing the Optimal Move



Key Property

Move Space

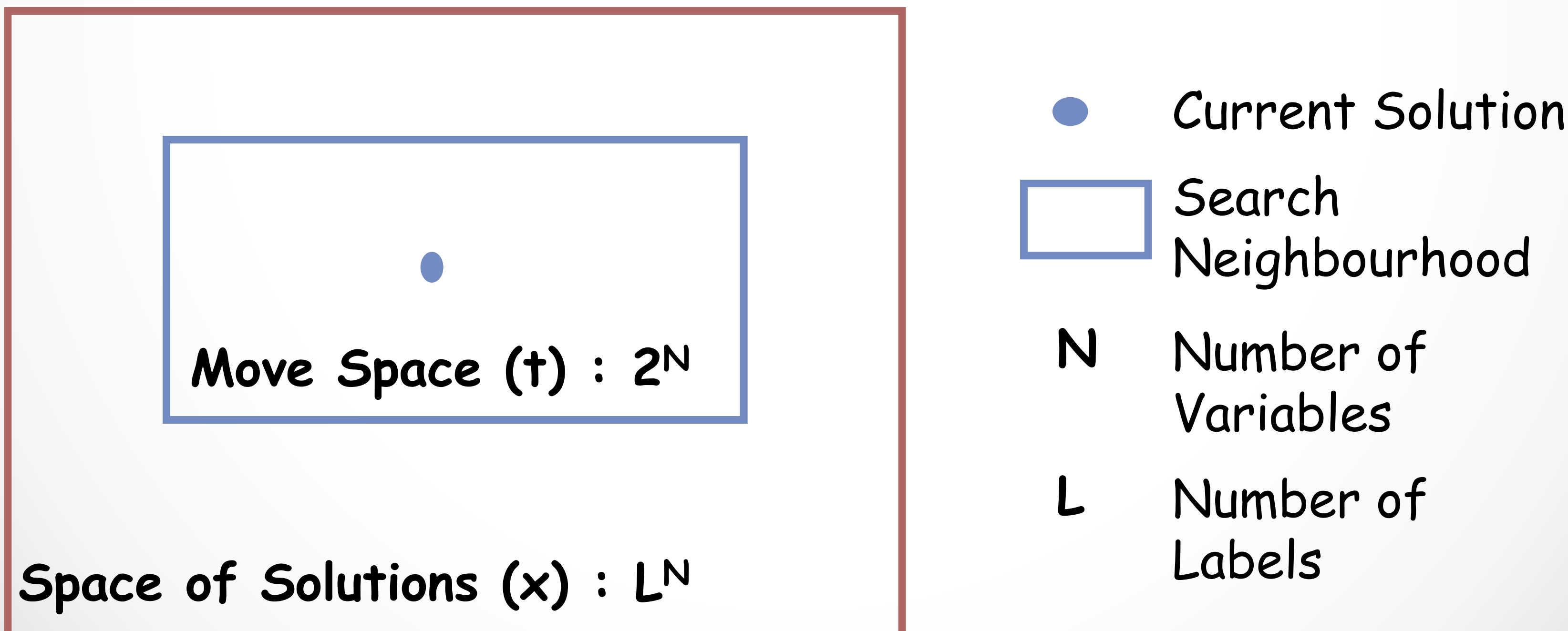
Bigger move space →
- **Better solutions**
- **Finding the optimal move hard**

Moves using Graph Cuts

Expansion and Swap move algorithms

[Boykov, Veksler and Zabih, PAMI 2001]

- **Makes a series of changes to the solution (moves)**
- **Each move results in a solution with smaller energy**



Moves using Graph Cuts

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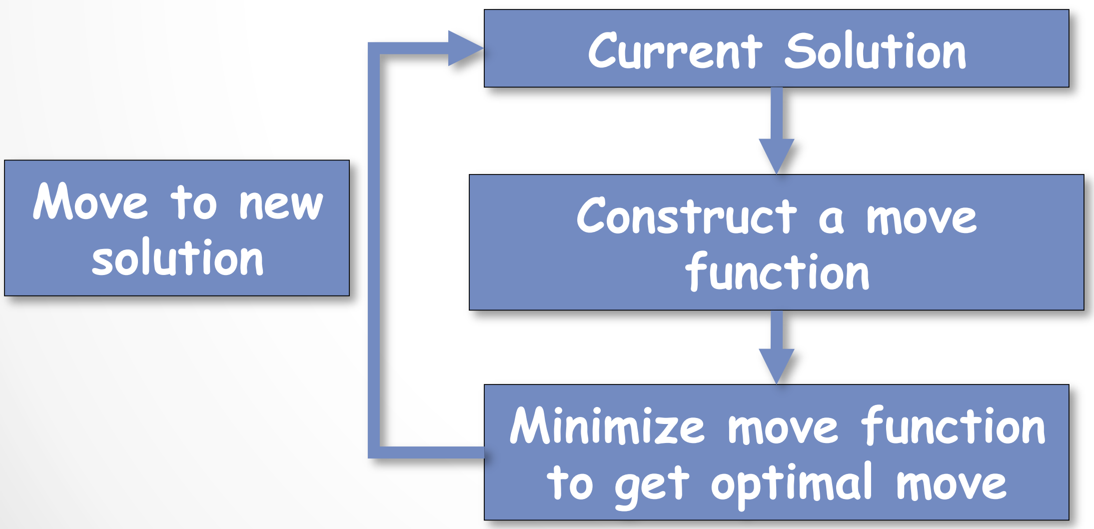


How to minimize move functions?

α-expansion algorithm

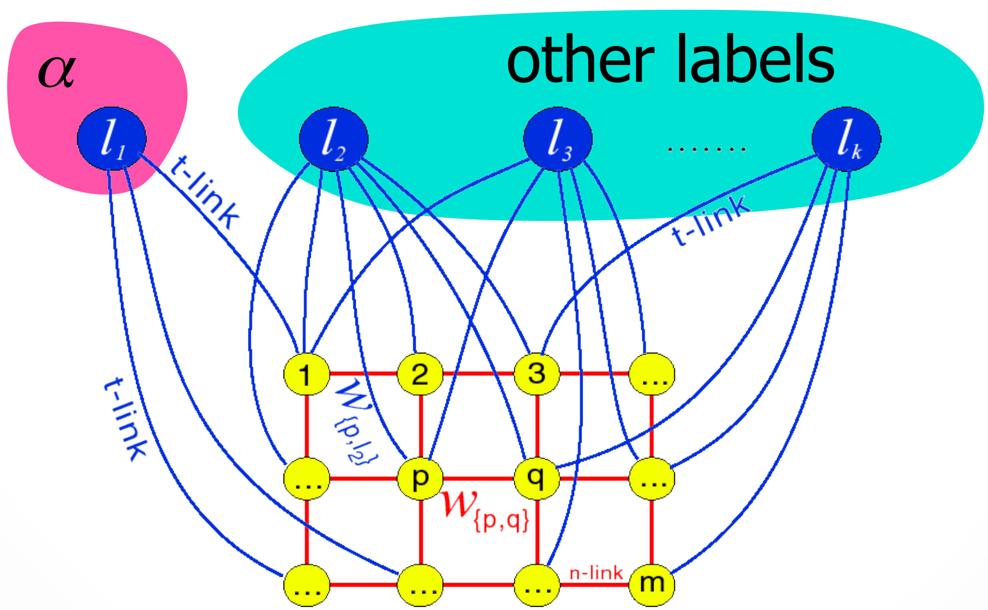
1. Start with any initial solution
2. For each label "α" in any (e.g. random) order

    1. Compute optimal α-expansion move (s-t graph cuts)
    2. Decline the move if there is no energy decrease

3. Stop when no expansion move would decrease energy

α-expansion move

Basic idea: break multi-way cut computation into a **sequence of binary *s-t* cuts**



α-expansion move

In each α-expansion a given label "α" grabs space from other labels

initial solution

-expansion

-expansion

-expansion

-expansion

-expansion

-expansion

-expansion

For each move we choose expansion that gives the largest decrease in the energy: **binary optimization problem**

Metric and Semimetric Interactions

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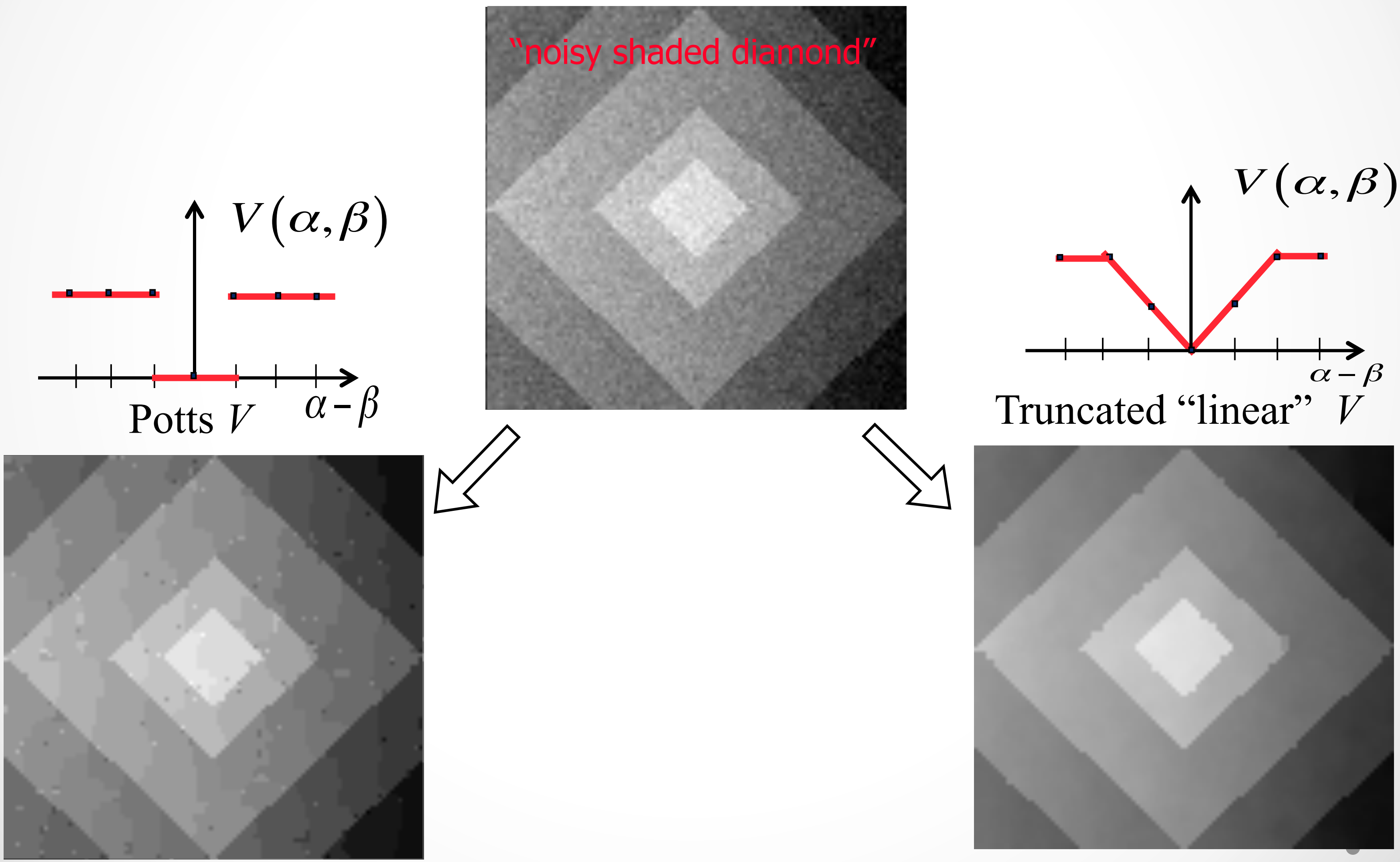
- Examples?

α-expansions: Examples of *metric* interactions



Ink-Bleed Removal

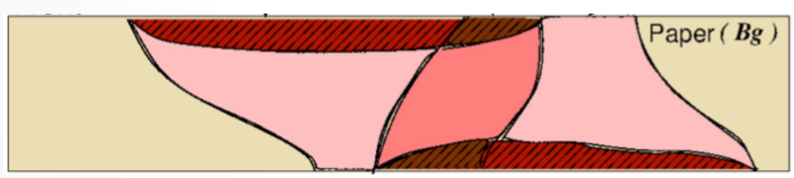
Yi Huang Michael S. Brown Dong Xu

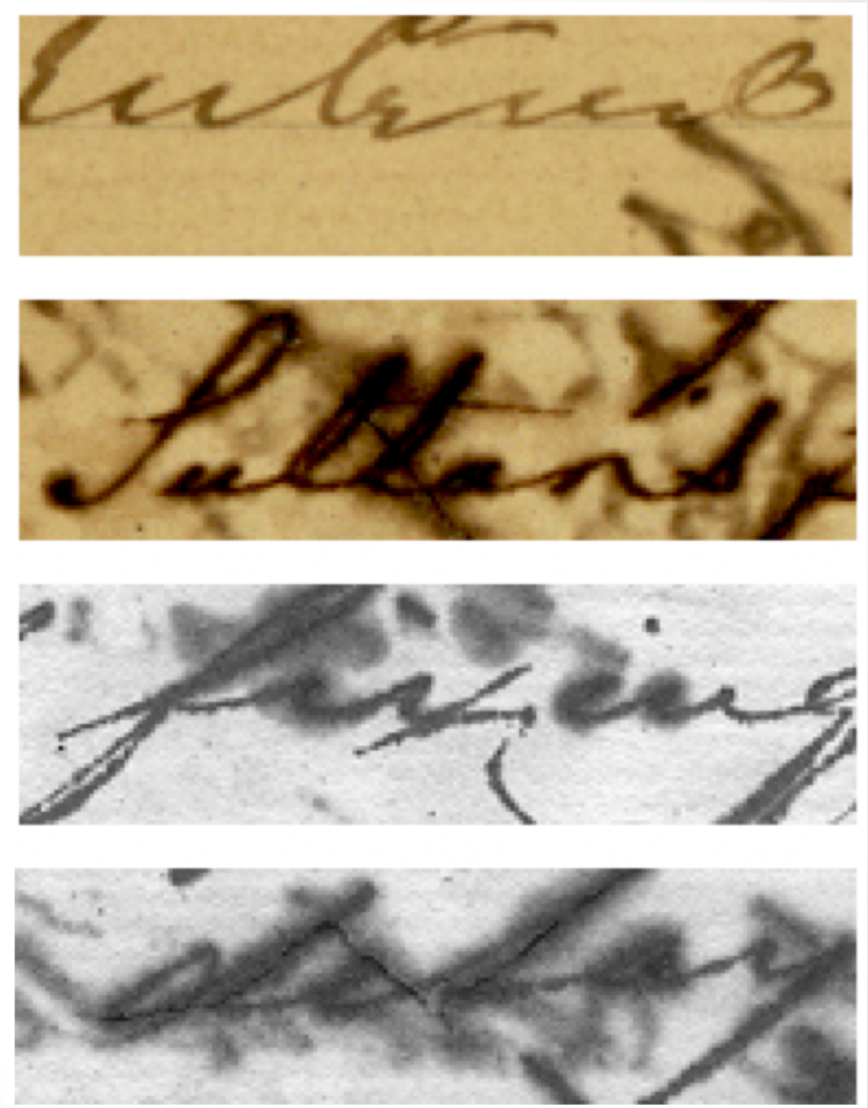
“A Framework for Reducing Ink-Bleed in Old Documents”,

Proc. CVPR, June 2008, Anchorage, AK, USA.

What is Ink Bleed?

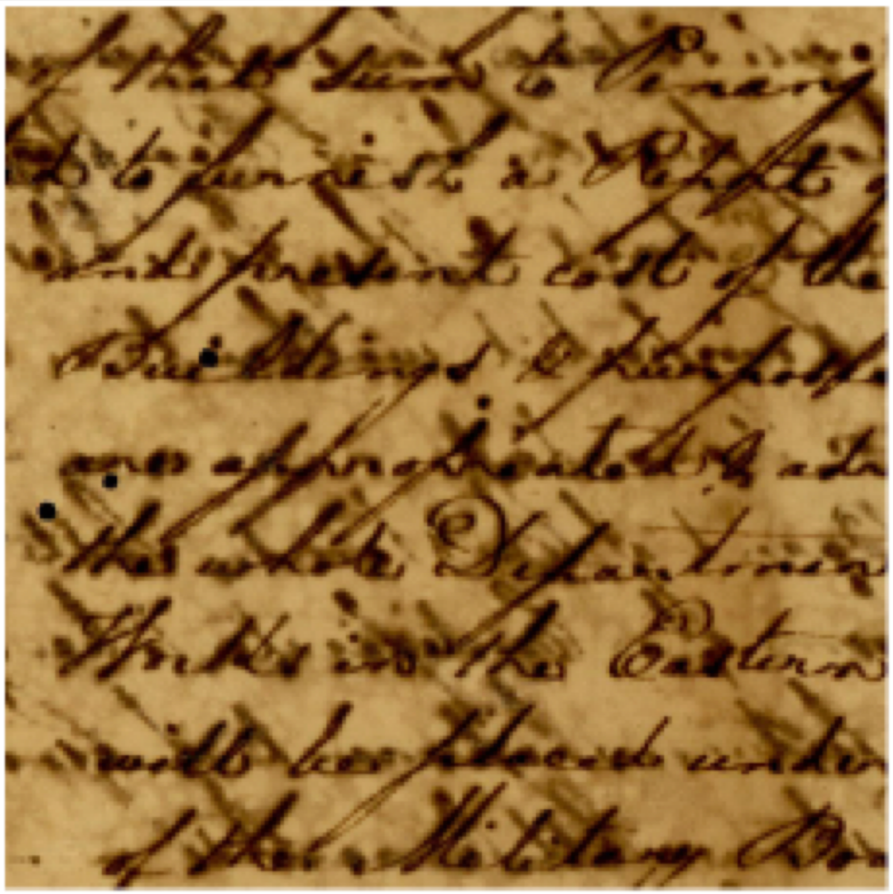
- Ink bloats through paper and appears on the reverse side

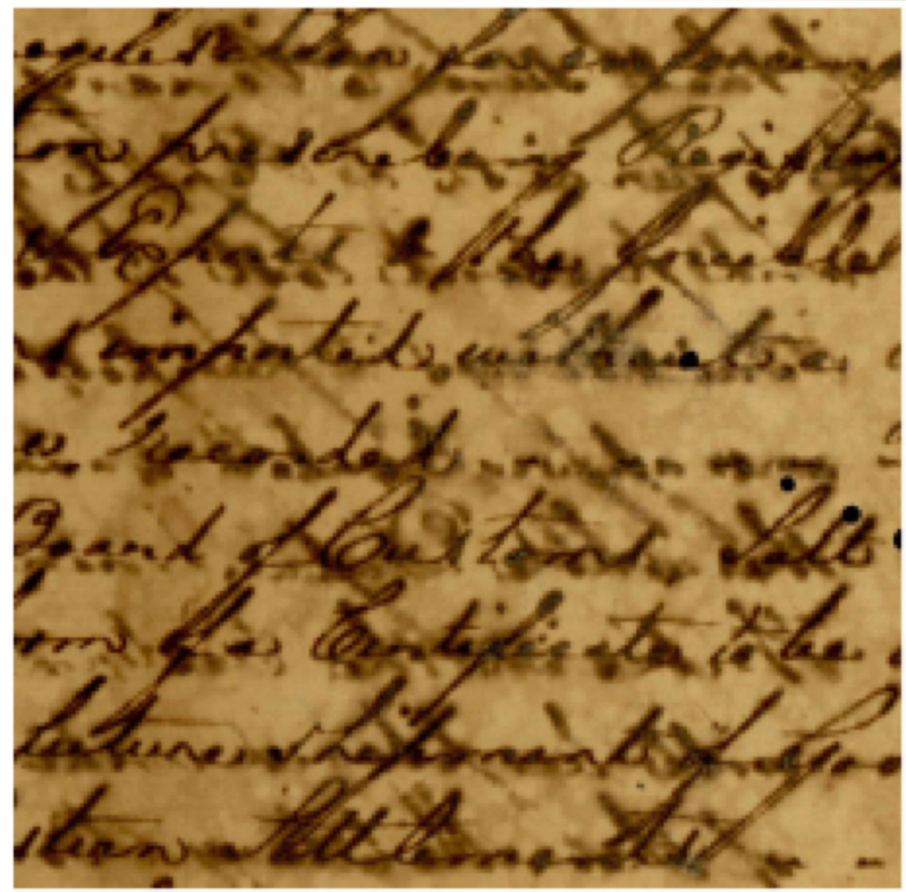




Recto and Verso Images

Recto (front) Side ⟺ Verso (back) Side

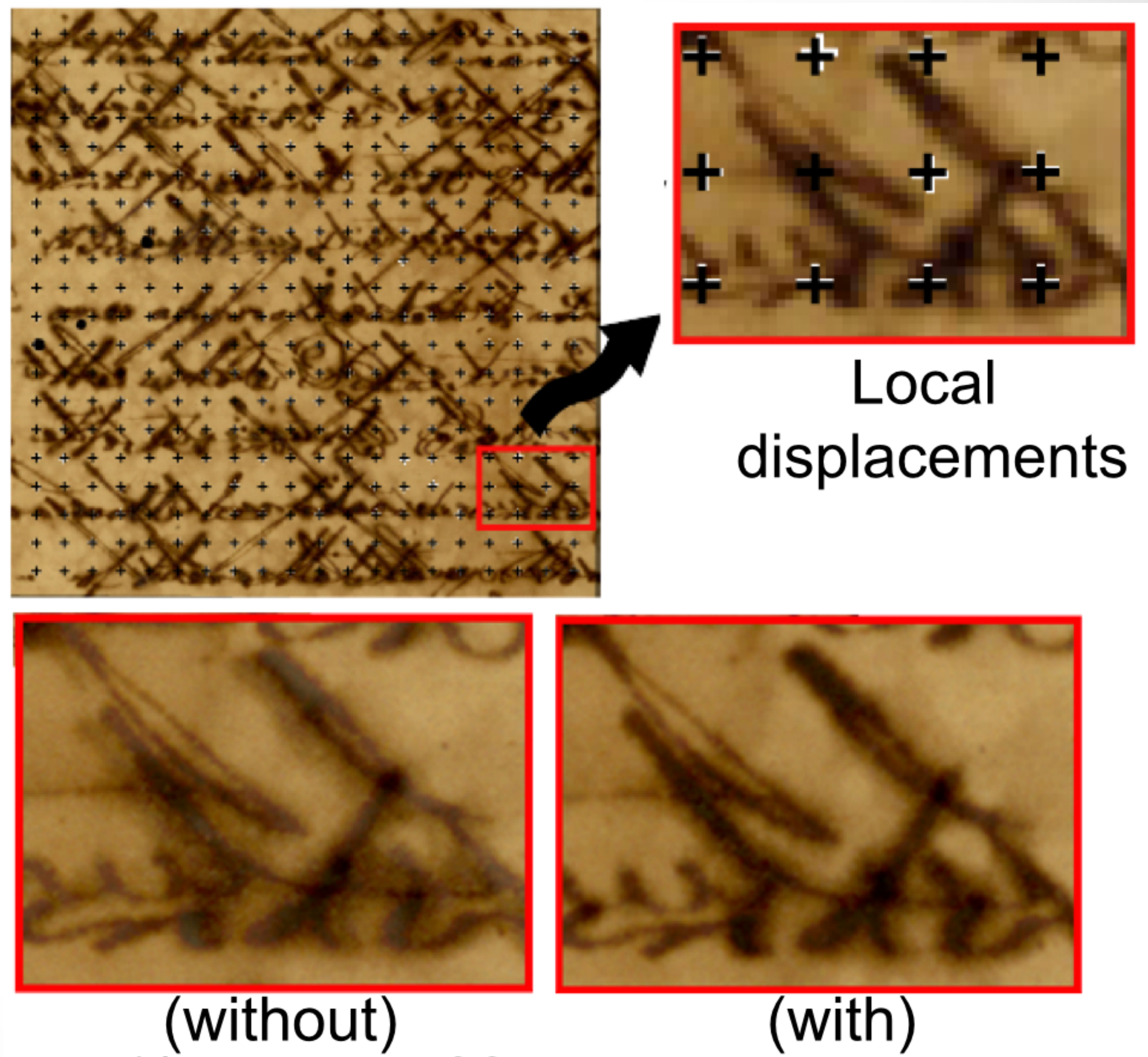




- Treat the two restoration problems together

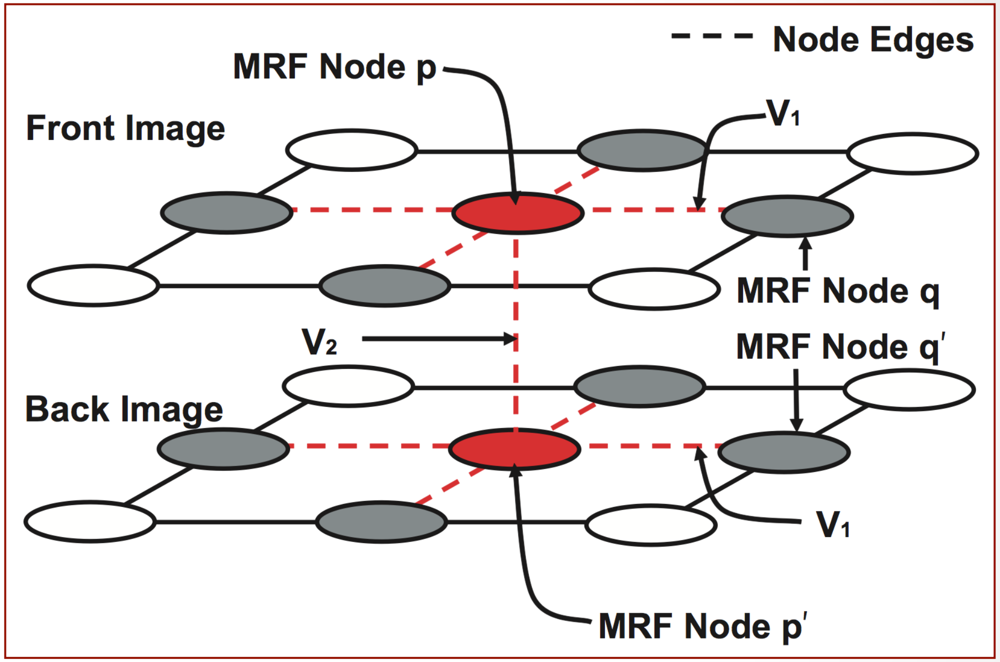
Local Alignment

- Compute correlations at a set of grid points: 60x60 patch; [-10,10] displacement
- Smoothen the local displacements (TPS)
- Warp the verso image



Dual Layer MRF

- Consider two sets of nodes, one for recto and one for verso
- The pixels/nodes of the images are aligned
- Connect corresponding pixels from either side.



Defining Potentials

- Class Similarity:

$$
\begin{aligned}
S_{\mathcal{F}} &= \sum_{m \in \pi^{\mathcal{F}}} \exp(-d_{pm}^2 / d_p^2) \\
S_{\mathcal{I}} &= \sum_{m \in \pi^{\mathcal{I}}} \exp(-d_{pm}^2 / d_p^2) \\
S_{\mathcal{B}} &= \sum_{m \in \pi^{\mathcal{B}}} \exp(-d_{pm}^2 / d_p^2).
\end{aligned}
$$

- Unary:

$$
\begin{aligned}
E_d(l_p = \mathcal{F}) &= \frac{S_{\mathcal{I}} + S_{\mathcal{B}}}{2 \times (S_{\mathcal{F}} + S_{\mathcal{I}} + S_{\mathcal{B}})} \\
E_d(l_p = \mathcal{I}) &= \frac{S_{\mathcal{F}} + S_{\mathcal{B}}}{2 \times (S_{\mathcal{F}} + S_{\mathcal{I}} + S_{\mathcal{B}})} \\
E_d(l_p = \mathcal{B}) &= \frac{S_{\mathcal{F}} + S_{\mathcal{I}}}{2 \times (S_{\mathcal{F}} + S_{\mathcal{I}} + S_{\mathcal{B}})}.
\end{aligned}
$$

Defining Potentials

- Pairwise: $$E_s = \sum_{p,q \in \mathcal{N}} V_1(l_p, l_q) + \sum_{p,p' \in \mathcal{M}} V_2(l_p, l_{p'})$$

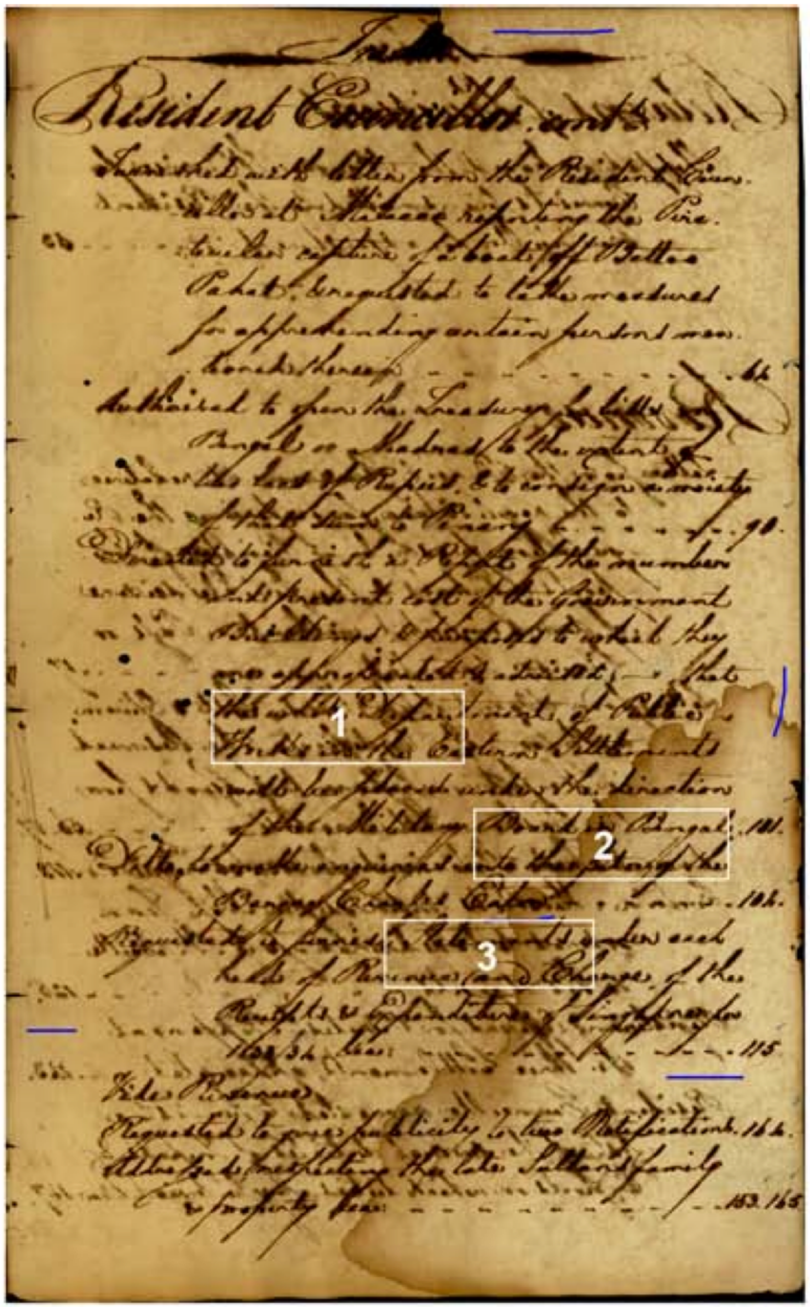
- Intra-Layer: $$V_1(l_p, l_q) = \frac{1}{1 + (\xi_{pq})^2}$$

l_p	l_q Foreground	Ink-Bleed	Background
Foreground	∞	d^ρ_{pq}	d^c_{pq}
Ink-Bleed	d^ρ_{pq}	∞	d^ρ_{pq}
Background	d^c_{pq}	d^ρ_{pq}	∞

- Inter-Layer:

l_p	$l_{p'}$ Foreground	Ink-Bleed	Background
Foreground	0	0	0
Ink-Bleed	0	∞	∞
Background	0	∞	2ω

-

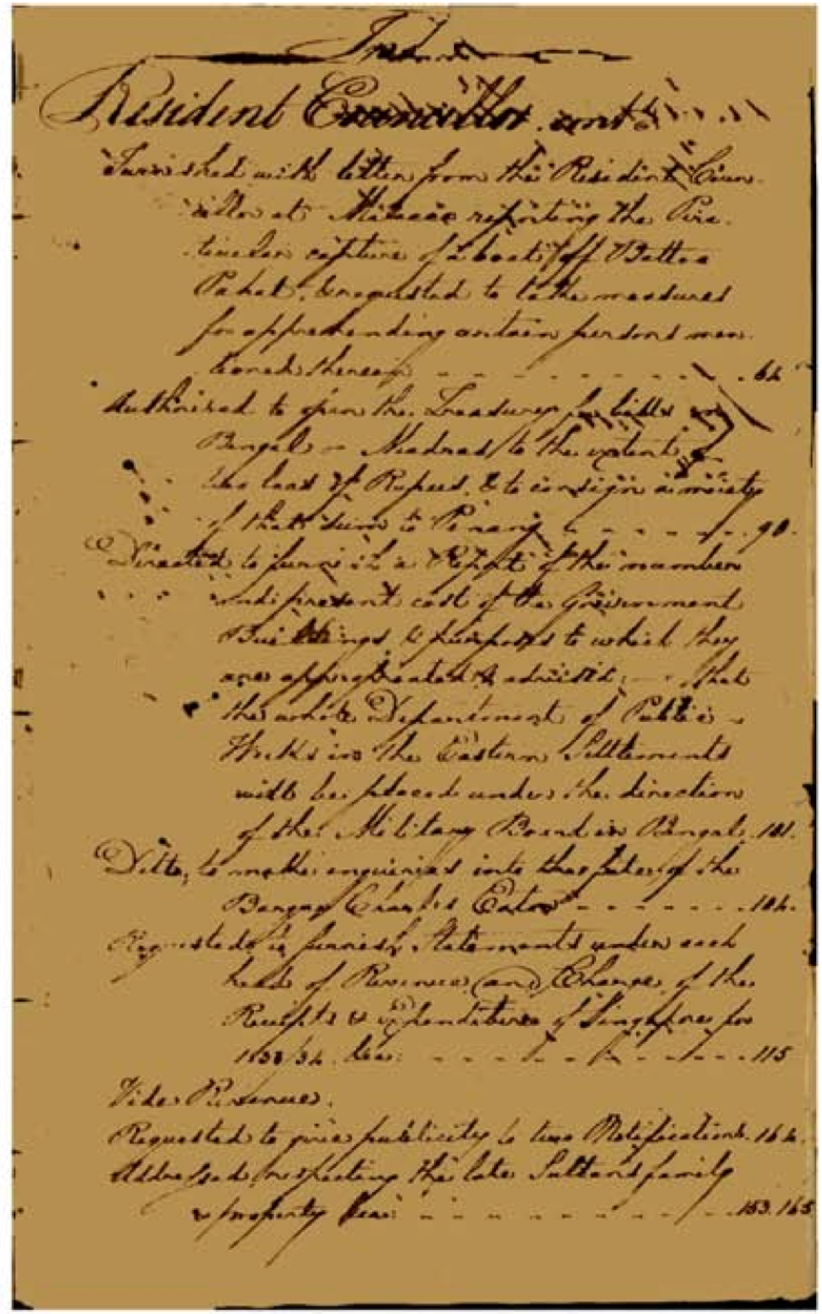


Ind.

Resident Councillor and

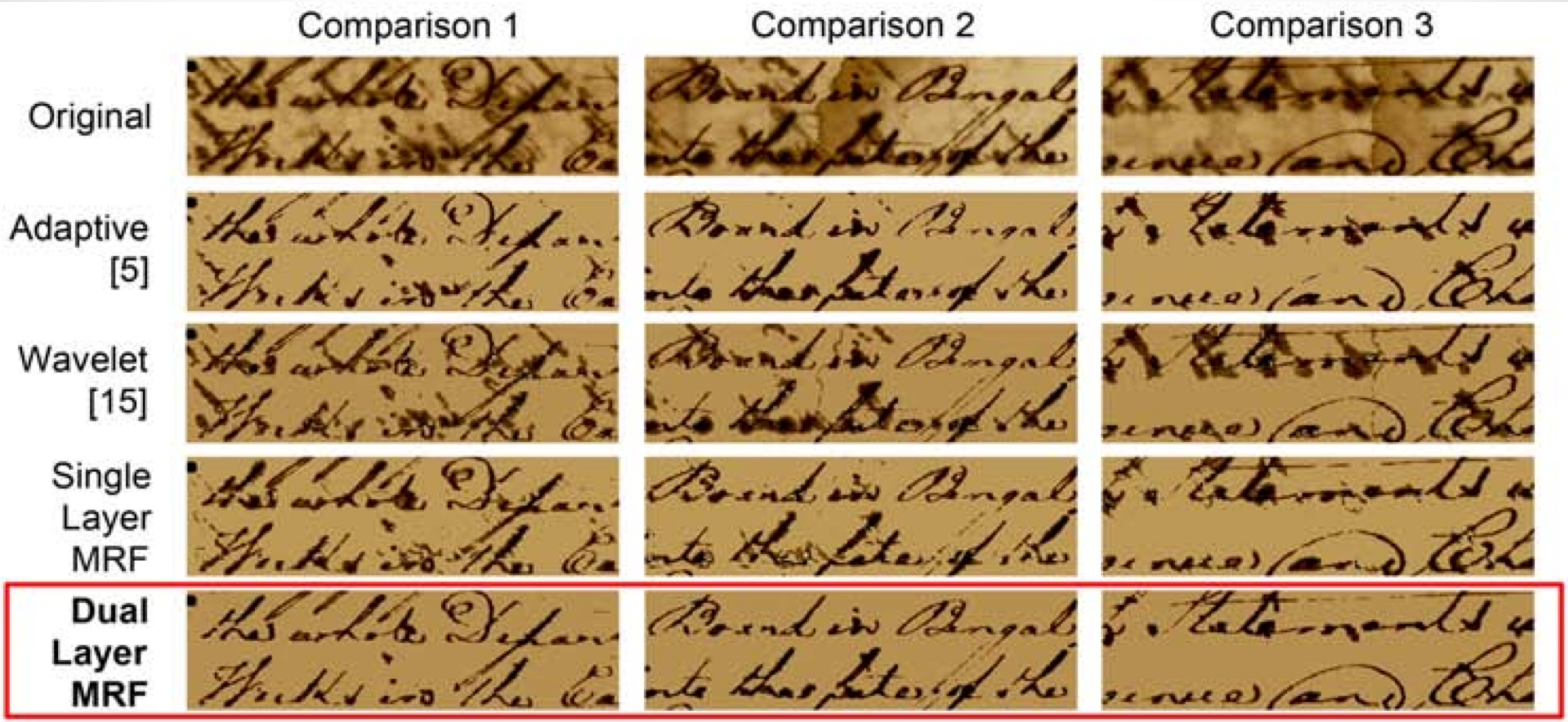
Furnished with letter from the Resident Coun-
cillor at Malacca reporting the Pira-
tical capture of a boat off Batta
Pahat, requested to take measures
for apprehending certain persons men-
tioned therein - - - - - - 66
Authorized to open the Treasury for bills on
Bengal or Madras to the extent of
the last of Report, & to consign a moiety
of that sum to Penang - - - - f. 90.
Directed to furnish a Report of the number
and present cost of the Government
Buildings & purposes to which they
are appropriated & advised that
the whole Department of Public
Works in the Eastern Settlements
will be placed under the direction
of the Military Board in Bengal. 101.
Ditto, to make enquiries into the fate of the
Bungay Chandi Cater - - - - - - 104.
Requested to furnish Statements under each
head of Revenue and Charges of the
Receipts & Expenditures of Singapore for
1833/34 Vide - - - - - - - 115
Vide Revenue.
Requested to give publicity to two Notifications. 164.
Address to respecting the late Sultan's family
& property - - - - - - - - 153. 165

Front+Markup

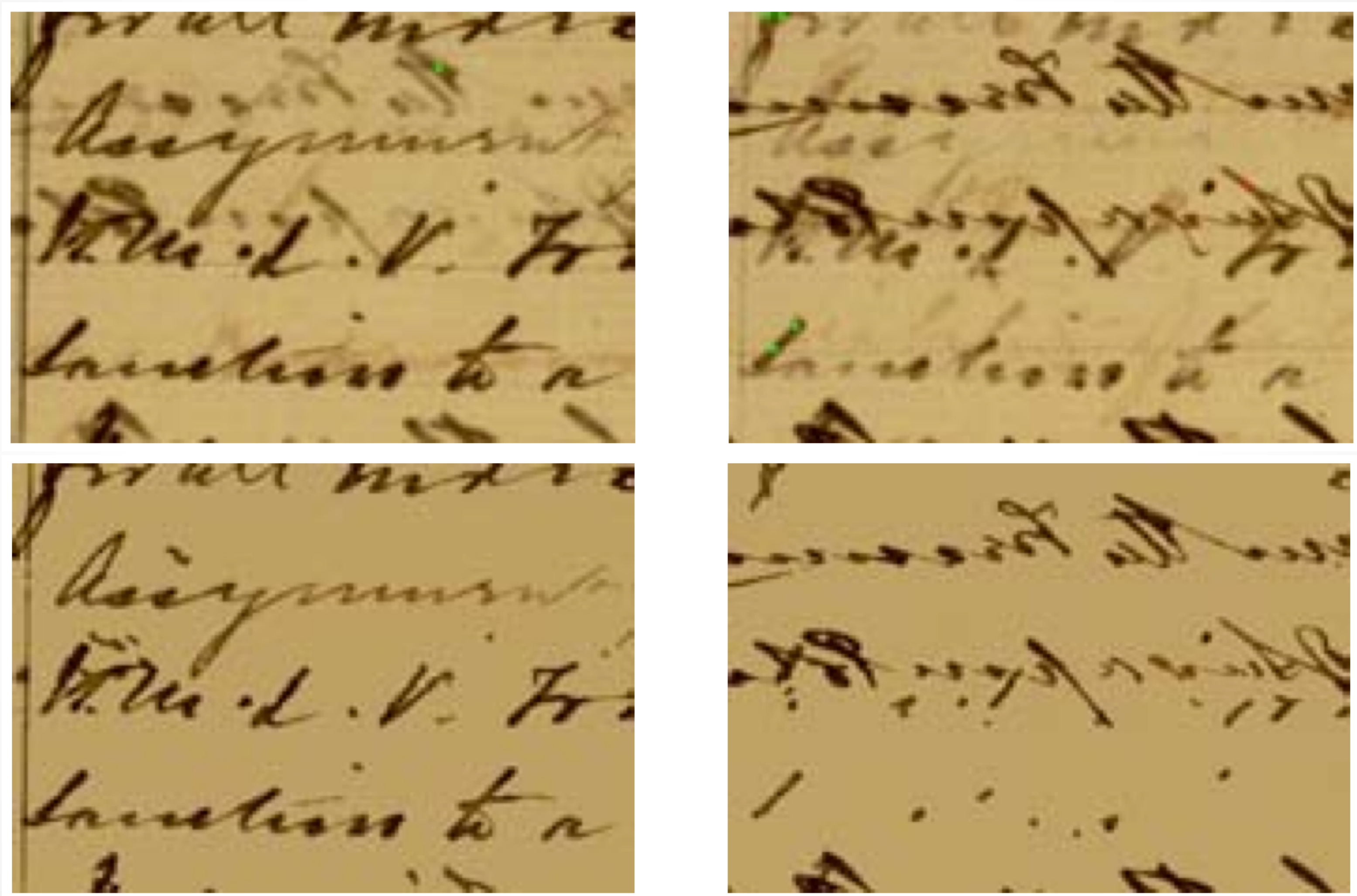


Corrected

Results: Comparison

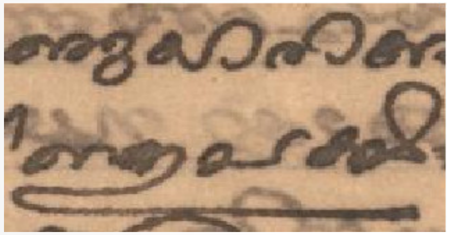


A Closer Look



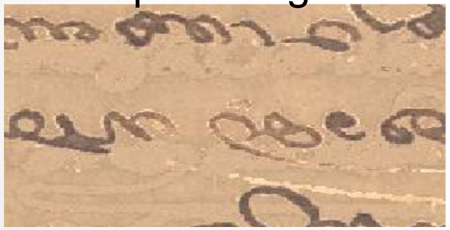
Going Beyond Segmentation

- Can we separate the layers or de-bleed the document?

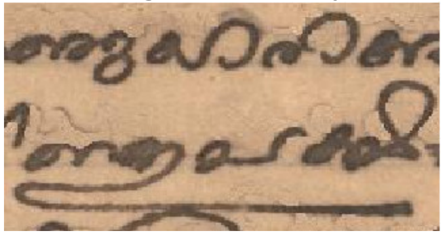


Input Image

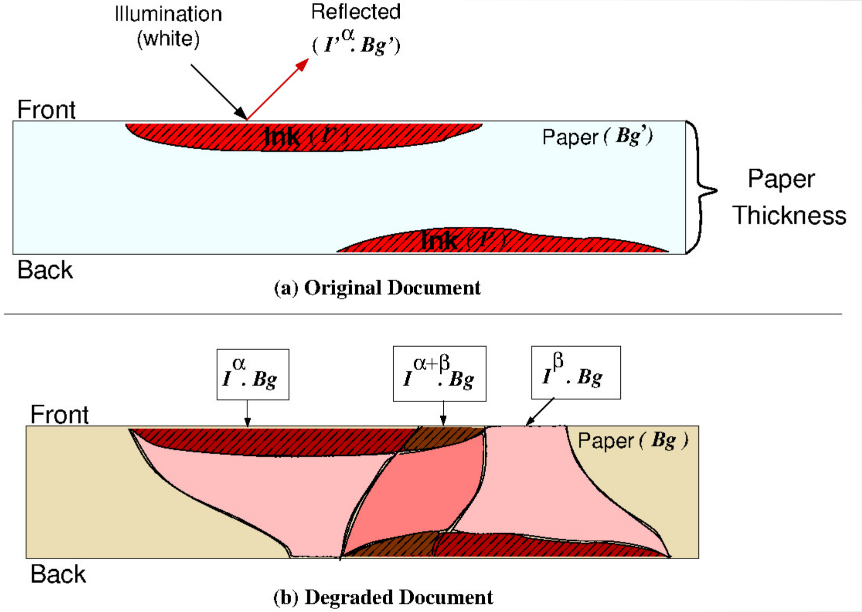
Background Layer



Bleed Layer



Output(Ink Layer)



Each pixel contain different levels of ink, bleed and background.

Shrikant Baronia and Anoop M. Namboodiri, "Ink-Bleed Reduction using Layer Separation", Proc. ICDAR, Aug. 2013, Washington DC, USA.

Multi-Label Classification Model

(a) α=1, β=.25 (b) α=.8, β=.3 (c) α=1, β=1

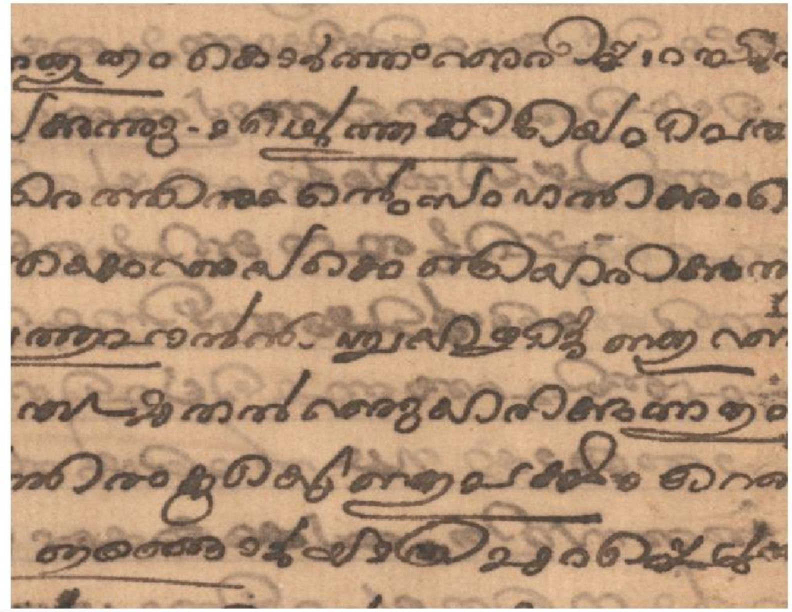
$$Ink_p = I^{(\alpha_p + \beta_p)} Bg_p$$

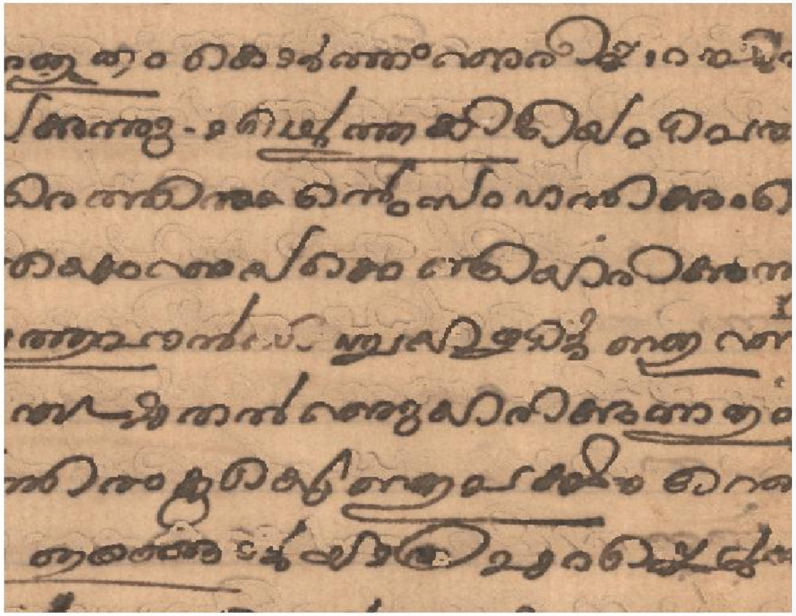
- Assume α and β are discrete $\{0,0.25,0.5,0.75,1\}$
- Reduces to 20 possible labels for a pixel.
- Define the energy terms and solve for labels.

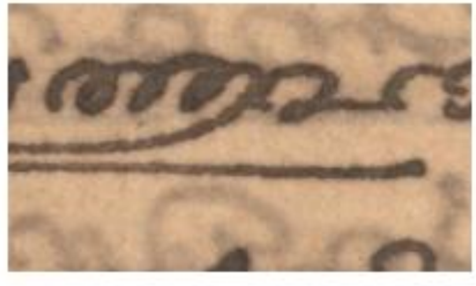
Defining Potentials

- Total Energy: $$E = E_d + \phi E_s$$
- Unary: $$E_d(l^p = l_i) = \frac{\sum_{j=1}^{20} S_j - S_i}{19 * \sum_{j=1}^{20} S_j}$$
- Pairwise: $$E_s = \sum_{p,q \in \mathcal{N}} V_1(l^p, l^q) + \sum_{p,p' \in \mathcal{M}} V_2(l^p, l^{p'})$$
- Solved using α-expansion move of graph cuts.

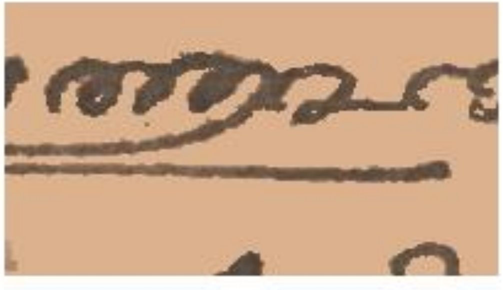
Results



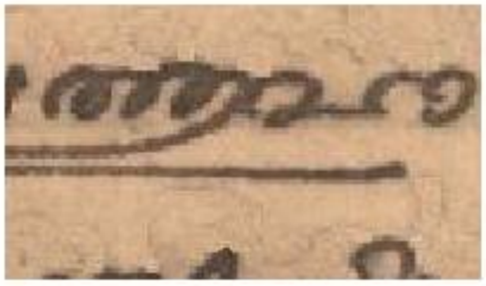




Input



Binary DMRF



Multilabel DMRF