CSE578: Computer Vision

Spring’17 01-Geometry: 2-View Geometry

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Case 1: Planar World

- Projection equation of points on a plane:

$$\mathbf{x} = \mathbf{K}\begin{bmatrix} \mathbf{R} & \mathbf{t} \end{bmatrix}\mathbf{X} = \mathbf{K}\begin{bmatrix} \mathbf{r}_1 & \mathbf{r}_2 & \mathbf{r}_3 & \mathbf{t} \end{bmatrix}\begin{bmatrix} X \\ Y \\ 0 \\ 1 \end{bmatrix} = \mathbf{K}\begin{bmatrix} \mathbf{r}_1 & \mathbf{r}_2 & \mathbf{t} \end{bmatrix}\begin{bmatrix} X \\ Y \\ 1 \end{bmatrix} = \mathbf{H}\mathbf{X},$$

where H is a 3 × 3 non-singular matrix

- Now if we consider two different views of the same world point, we get:

$$\mathbf{x}_1 = \mathbf{H}_1\mathbf{X}; \quad \mathbf{x}_2 = \mathbf{H}_2\mathbf{X} = \mathbf{H}_2\mathbf{H}_1^{-1}\mathbf{x}_1 = \mathbf{H}_{21}\mathbf{x}_1.$$

$$\boxed{\mathbf{x}_1 = \mathbf{H}_{21}\mathbf{x}_2; \quad \mathbf{x}_2 = \mathbf{H}_{12}\mathbf{x}_1}$$

Case 2: Same Camera Center

- Projection equation for two cameras with same C:

$$\mathbf{x}_1 = \mathbf{K}_1\mathbf{R}_1\begin{bmatrix} \mathbf{I} & \mathbf{-C} \end{bmatrix}\mathbf{X}$$

$$\mathbf{x}_2 = \mathbf{K}_2\mathbf{R}_2\begin{bmatrix} \mathbf{I} & \mathbf{-C} \end{bmatrix}\mathbf{X}$$

$$= \mathbf{K}_2\mathbf{R}_2\left(\mathbf{K}_1\mathbf{R}_1\right)^{-1}\mathbf{K}_1\mathbf{R}_1\begin{bmatrix} \mathbf{I} & \mathbf{-C} \end{bmatrix}\mathbf{X}$$

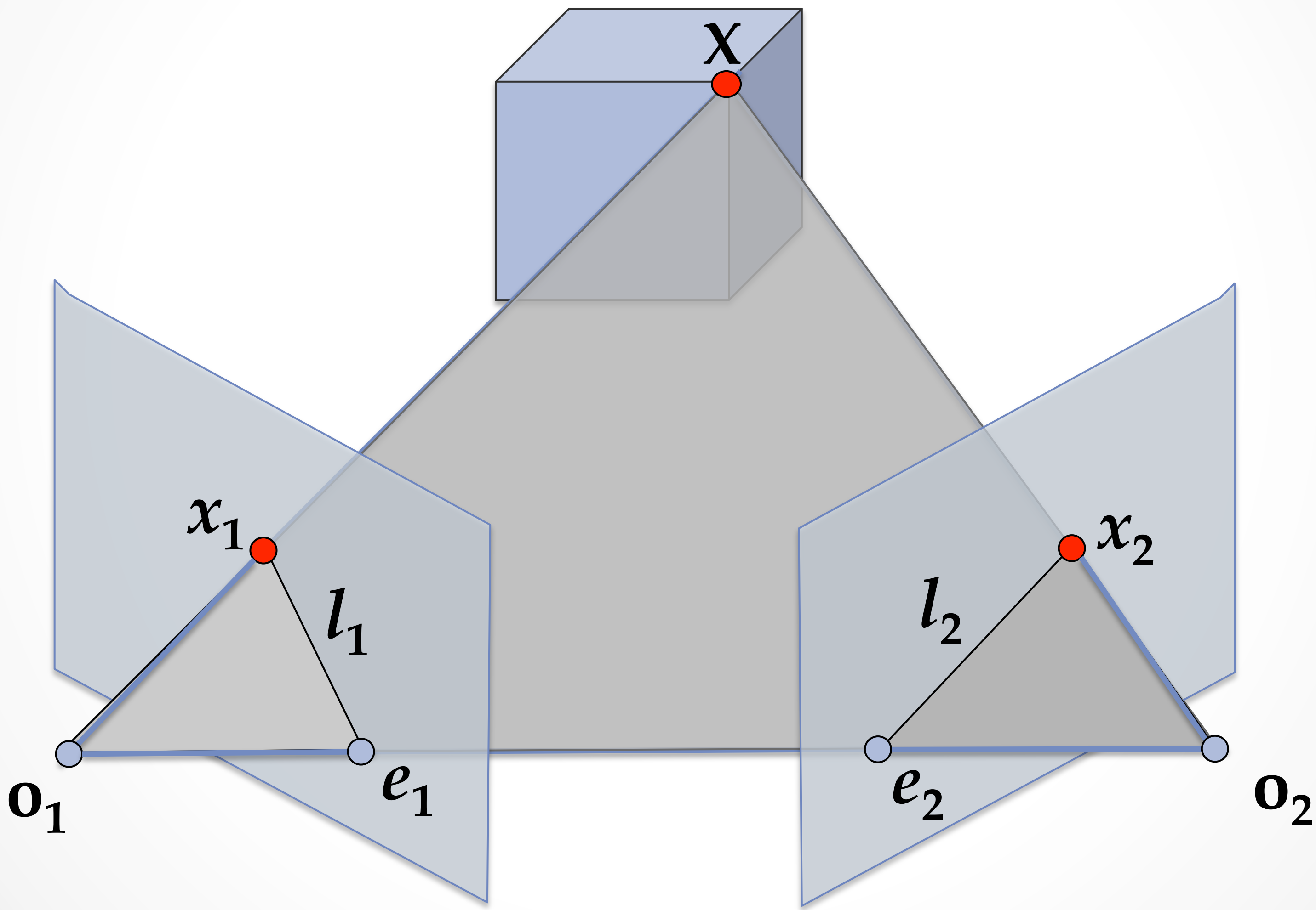
$$= \mathbf{K}_2\mathbf{R}_2\left(\mathbf{K}_1\mathbf{R}_1\right)^{-1}\mathbf{x}_1$$

$$= \mathbf{H}_{12}\mathbf{x}_1$$

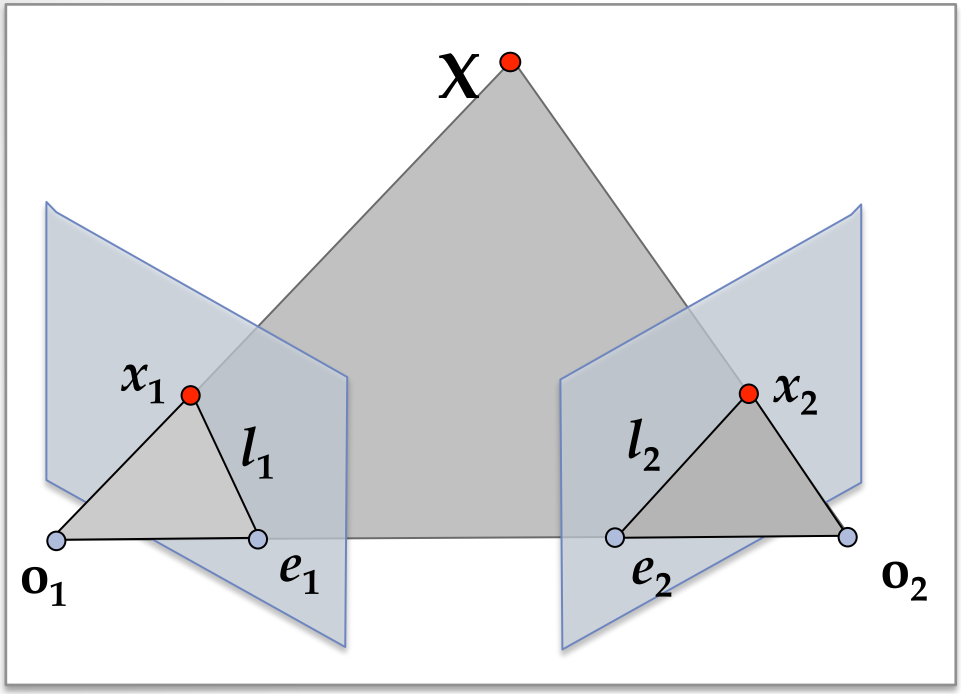
where H is a 3 × 3 non-singlular matrix

$$\boxed{\mathbf{x}_1 = \mathbf{H}_{21}\mathbf{x}_2; \quad \mathbf{x}_2 = \mathbf{H}_{12}\mathbf{x}_1}$$

Case 3: Generic World and Cameras



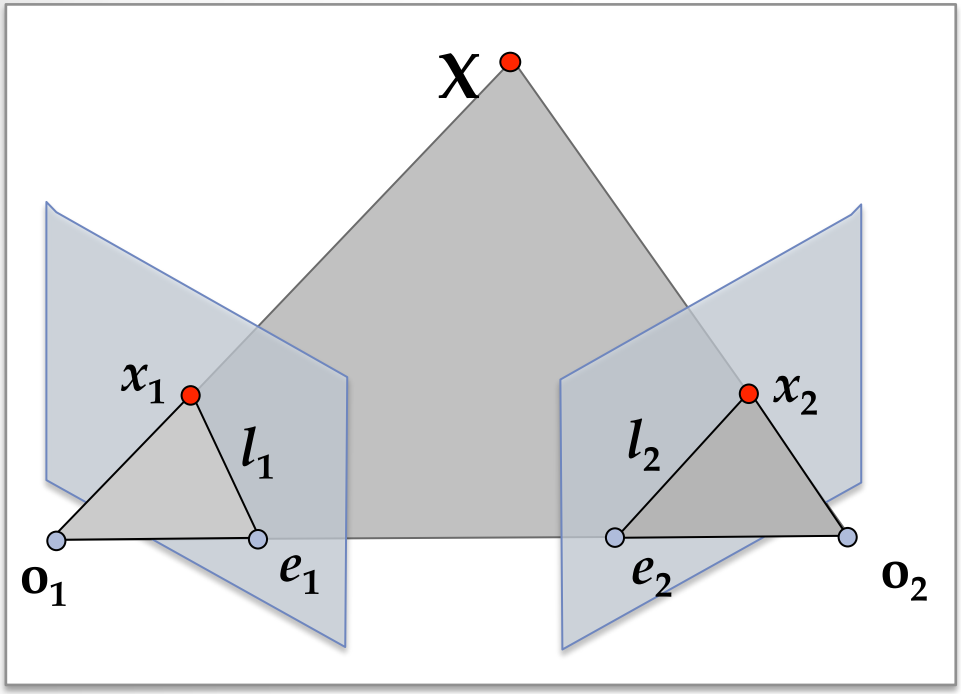
Epipolar Geometry



- All world points that map to $\boldsymbol{x}_1$ in $\boldsymbol{I}_1$ (pre-image of $\boldsymbol{x}_1$) map to a line $\boldsymbol{l}_2$ in $\boldsymbol{I}_2$, which is called an epipolar line. (and vice-versa)
- The image of $\boldsymbol{o}_1$ in $\boldsymbol{I}_2$ ($\boldsymbol{e}_2$) is an epipole (and vice-versa)

- The plane containing these is called the epipolar plane.
- These result in a set of constraints, which are referred to as the *epipolar constraints* and the resulting geometry is referred to as the *epipolar geometry*.

Epipolar Constraint: Essential Matrix



$$\lambda_1 \mathbf{x}_1 = \mathbf{X}$$

$$\lambda_2 \mathbf{x}_2 = \mathbf{R}\mathbf{X} + \mathbf{T}$$

$$= \mathbf{R}\left(\lambda_1 \mathbf{x}_1\right) + \mathbf{T}$$

$$\hat{\mathbf{T}}\lambda_2 \mathbf{x}_2 = \hat{\mathbf{T}}\mathbf{R}\lambda_1 \mathbf{x}_1 + 0$$

$$\lambda_2 \mathbf{x}_2^T \hat{\mathbf{T}} \mathbf{x}_2 = \lambda_1 \mathbf{x}_2^T \hat{\mathbf{T}}\mathbf{R}\mathbf{x}_1$$

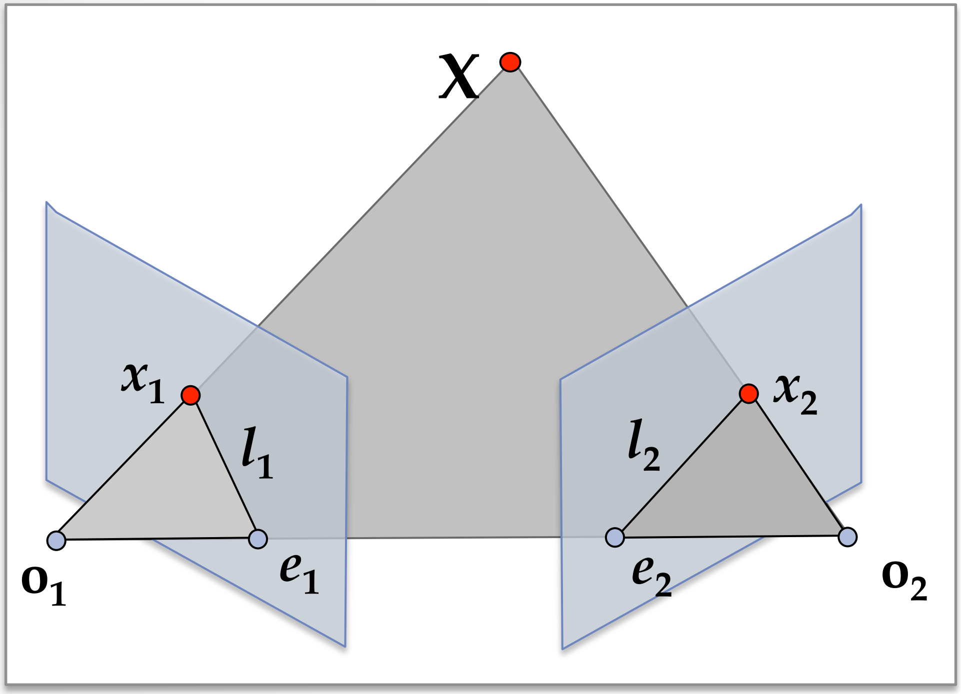
$$\mathbf{x}_2^T \hat{\mathbf{T}}\mathbf{R}\mathbf{x}_1 = 0$$

$$\mathbf{x}_2^T \mathbf{E}\mathbf{x}_1 = 0 \quad \text{or} \quad \mathbf{x}_1^T \mathbf{E}\mathbf{x}_2 = 0$$

Cross product of any two vectors: AxB can be written as the matrix product ÂB, where:

$$\hat{\mathbf{A}} = \begin{bmatrix} 0 & -A_z & A_y \\ A_z & 0 & -A_x \\ A_y & A_x & 0 \end{bmatrix}$$

Epipolar Constraint: Fundamental Matrix



$$\mathbf{x}_1 = \mathbf{K}_1\mathbf{X}$$

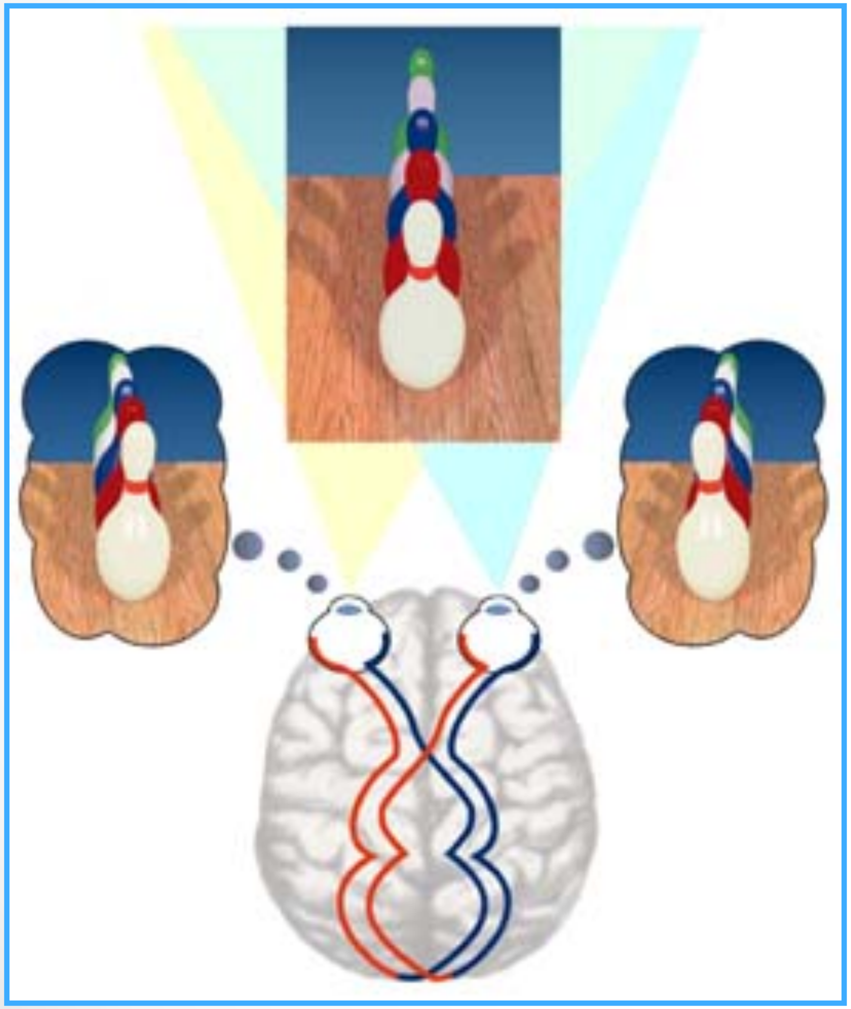
$$\mathbf{x}_2 = \mathbf{K}_2\mathbf{X}$$

$$\mathbf{x}_2^T\mathbf{K}_2^{-T}\hat{\mathbf{T}}\mathbf{R}\mathbf{K}_1^{-1}\mathbf{x}_1 = 0$$

$$\mathbf{x}_1^T F \mathbf{x}_2 = 0$$

Both Essential and Fundamental matrices are 3x3 and are independent of the world point.

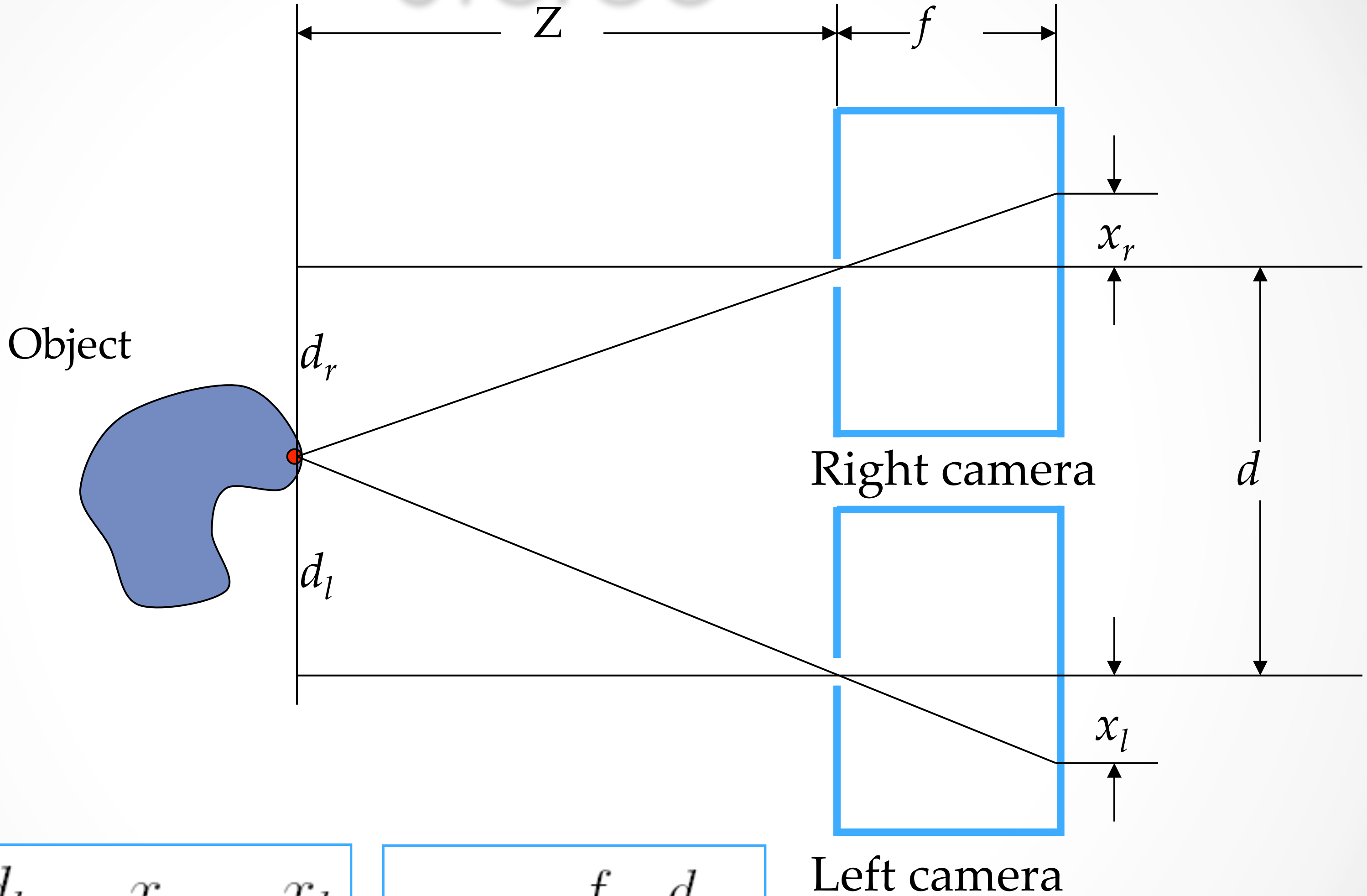
Stereo



1. We see a slightly different image of the world through the two eyes.
2. The shift in image is proportional to the the distance to the object.
3. The roadside trees are left behind, while the farther mountains follow you.

http://www.vision3d.com/stereo.html

Stereo



$$\frac{d_r}{Z} + \frac{d_l}{Z} = \frac{x_r}{f} + \frac{x_l}{f}$$

$$Z = \frac{f \,.\, d}{(x_r + x_l)}$$