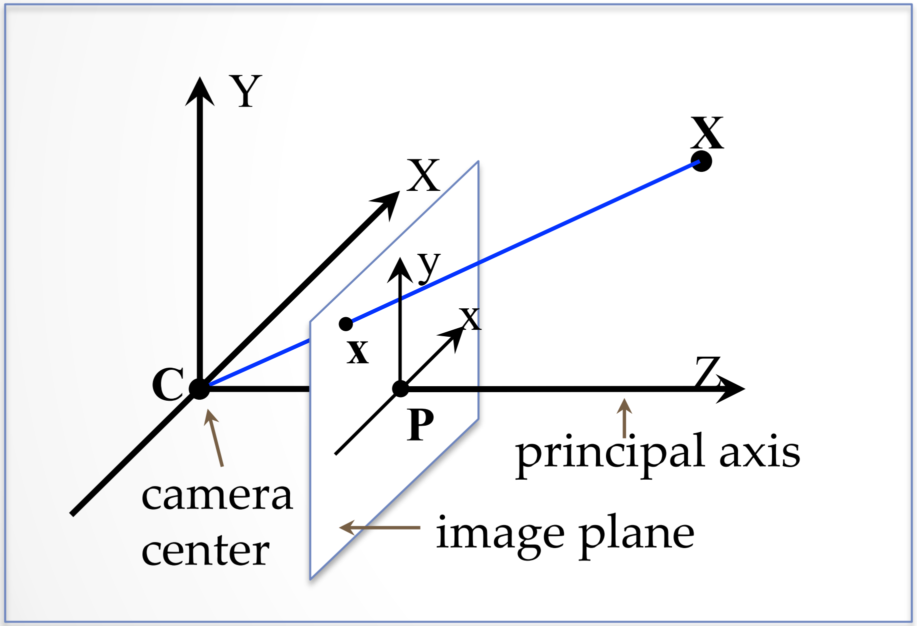
CSE578: Computer Vision

Spring 2017

01-Geometry: Camera Matrix and Properties



$$\mathbf{P} = \left[\left[\begin{array}{ccc:c} \alpha_x & s & x_0 & t_1 \\ 0 & \alpha_y & y_0 & t_2 \\ 0 & 0 & 1 & 0 \end{array}\right]\right]$$

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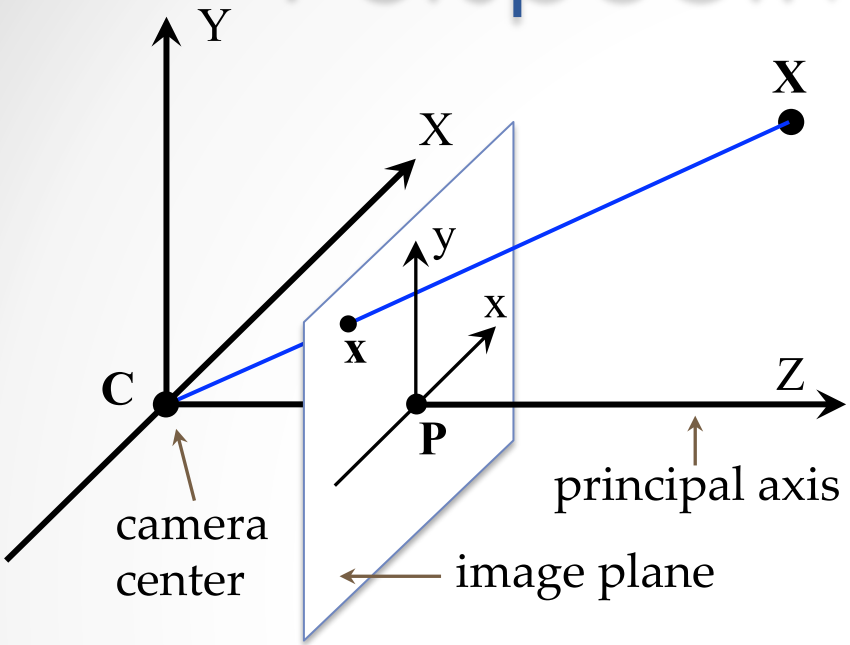
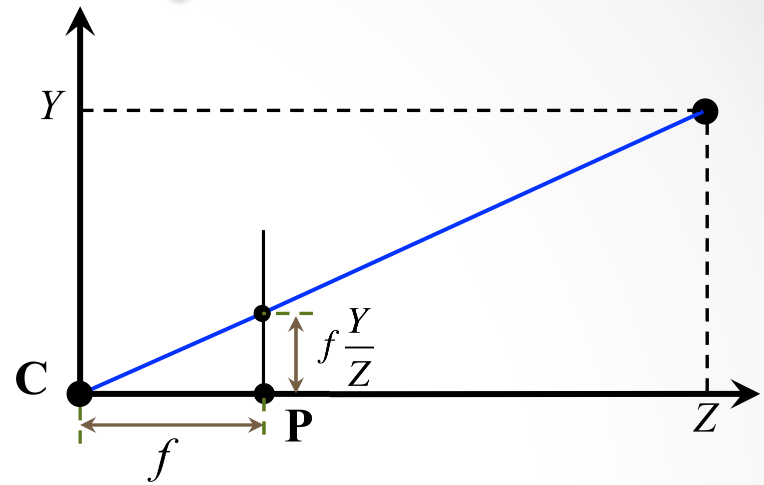
Perspective Projection





- Cartesian image coordinates: $x = f\dfrac{X}{Z},\quad y = f\dfrac{Y}{Z}$
- In matrix form (homogeneous):

$$\mathbf{x} = \begin{bmatrix} x \\ y \\ w \end{bmatrix} = \begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} = \mathbf{PX}$$

Basic Camera Equation

A pinhole camera projects a 3D point $\mathbf{X_c}$ in camera coords to an image point $\mathbf{x}$ via the 3x4 camera matrix $\mathbf{P}$ as:

$$\mathbf{x} = \mathbf{P}\mathbf{X_c} = \begin{bmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{bmatrix} [\mathbf{I} \mid \mathbf{0}]\mathbf{X_c} = \mathbf{K}[\mathbf{I} \mid \mathbf{0}]\mathbf{X_c},$$

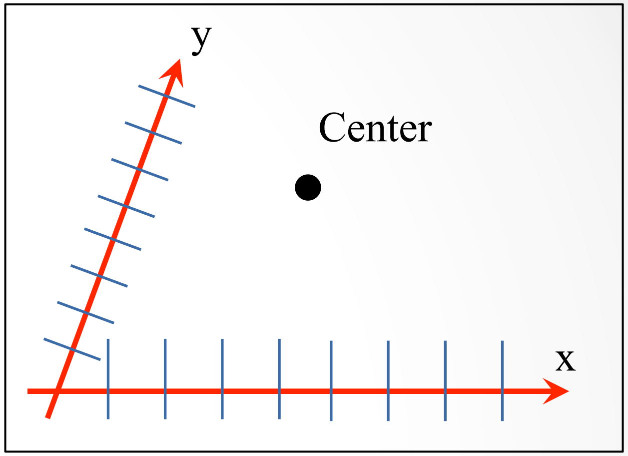
where $\mathbf{K}$ is the internal camera calibration matrix.

Note that:

- The camera is at origin
- Z is the Camera or Optical axis
- Principal Point: Center of the image
- Focal length in pixel units
- Orthogonal image axes with uniform scale

A General Camera

Image center at (x_0, y_0), Non-orthogonal axes with skew s, and different scales for axes with focal lengths, α_x and α_y.

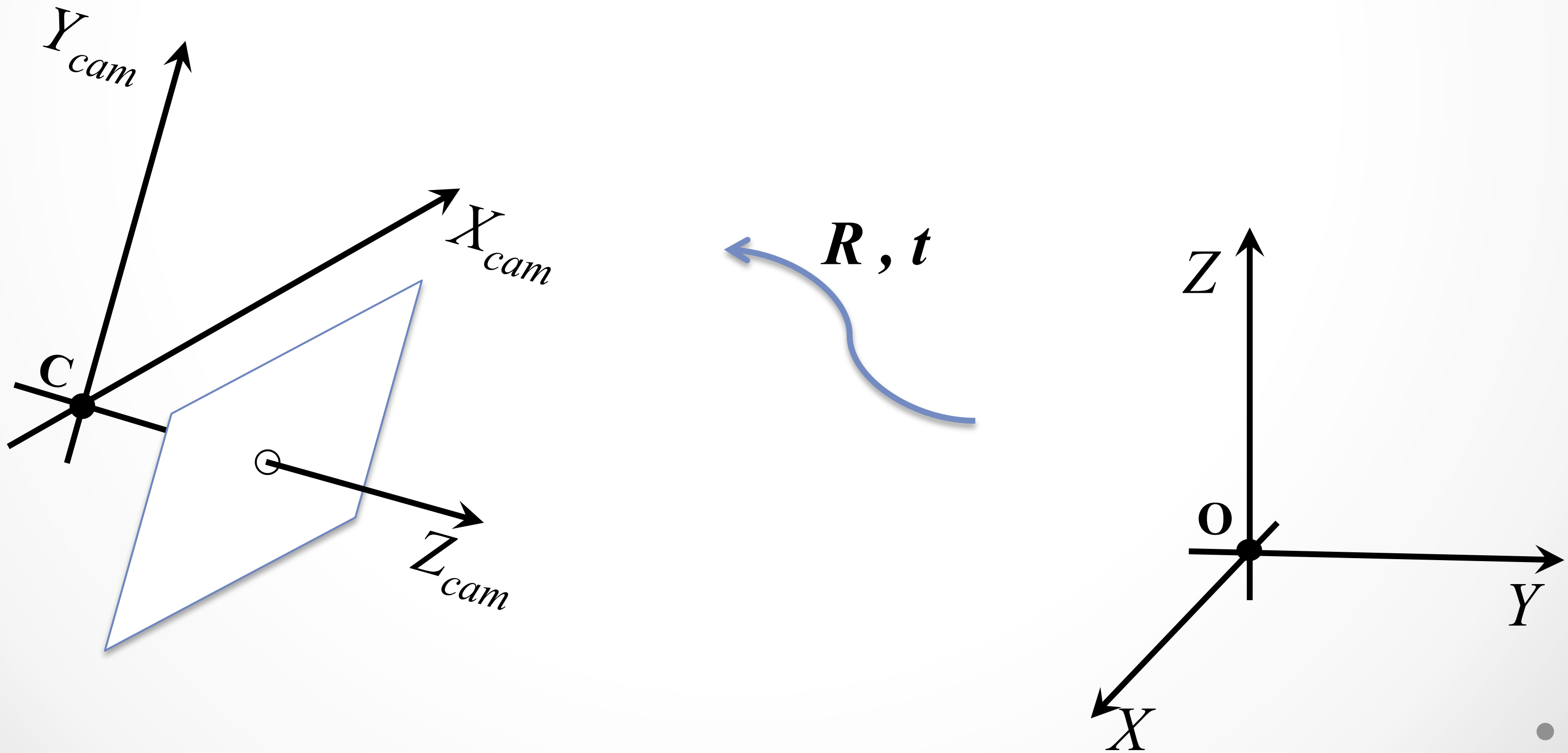


$$\mathbf{K} = \begin{bmatrix} \alpha_x & s & x_0 \\ 0 & \alpha_y & y_0 \\ 0 & 0 & 1 \end{bmatrix}$$

K an upper diagonal matrix with 5 degrees of freedom.

Moving the Camera from Origin

- General Setting: Camera is not at origin and Z is not the optical axis.
- Camera is at a point C in world coordinates. The camera axes are also rotated by a matrix R.



General Camera Equation

- Camera and world are related by:

$$\mathbf{X_c} = \begin{bmatrix} \mathbf{R} & \mathbf{-RC} \\ \mathbf{0} & \mathbf{1} \end{bmatrix} \mathbf{X_w}$$

- 2D projection $\mathbf{x}$ of a 3D point $\mathbf{X_w}$ given by:
  - $\mathbf{x = K[\,I \mid 0\,]\,X_c = K\,[\,R \mid -RC\,]\,X_w}$
- $\mathbf{x = PX_w}$; camera matrix $\mathbf{P = [KR \mid -KRC] = [M \mid p_4]}$
- Common K:

$$\begin{bmatrix} f & 0 & x_0 \\ 0 & f & y_0 \\ 0 & 0 & 1 \end{bmatrix}$$

General K:

$$\begin{bmatrix} \alpha_x & s & x_0 \\ 0 & \alpha_y & y_0 \\ 0 & 0 & 1 \end{bmatrix}$$

General Camera Equation

- General projection equation in world coordinates:

$$\mathbf{x} = \mathbf{K}\,[\mathbf{R} \mid -\mathbf{RC}]\,\mathbf{X_w} = [\mathbf{KR} \mid -\mathbf{KRC}]\,\mathbf{X_w} = [\mathbf{M} \mid \mathbf{p_4}]\,\mathbf{X_w}$$

- 3x4 matrix $\mathbf{P}$ maps/projects World-C to Image-C
  - Left 3 × 3 submatrix is non-singluar for finite cameras
  - Orthographic projection: left submatrix is singular
- Any 3 × 4 matrix P with non-singular left submatrix represents a camera! It can be decomposed as:
  - A non-singular upper diagonal matrix K
  - An orthonormal matrix R and a vector C with the usual meanings!!