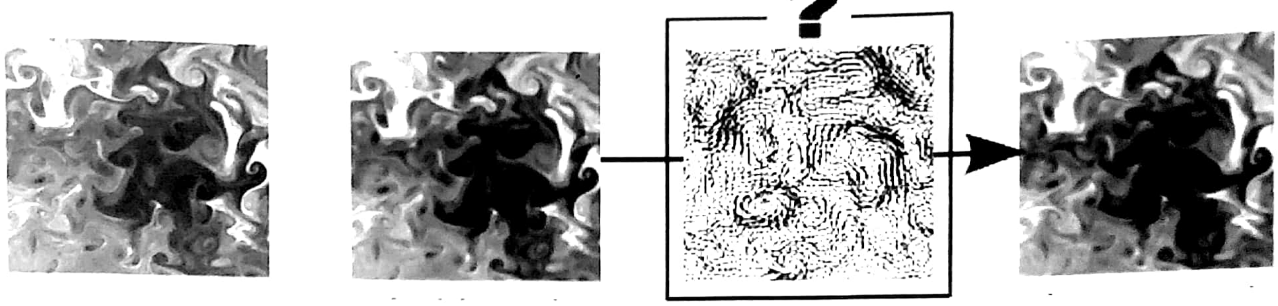
CSE578: Computer Vision

Spring 2019:

Optical Flow and Tracking



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Center for Visual Information Technology
IIIT Hyderabad, IN[illegible]

[Slides Generously Borrowed f[illegible] Sources]

Feature Tracking vs. Optical Flow

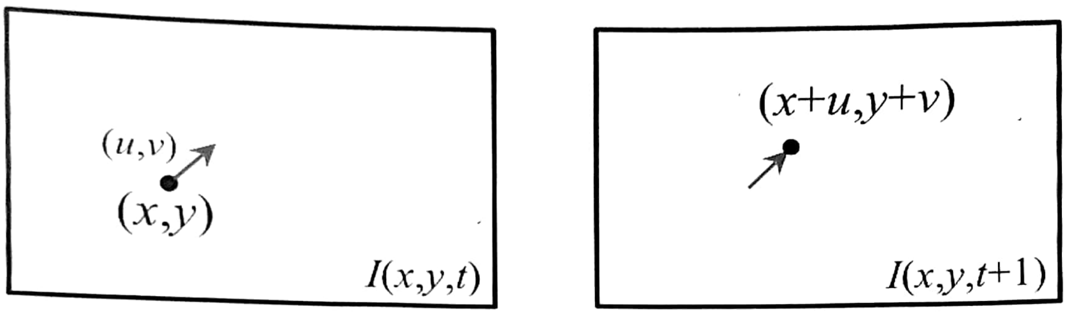
Feature Tracking: Extract visual features (corners, textured areas) and "track" them over multiple frames (**sparse corr.**)

Optical Flow: Recover image motion at each pixel from spatio-temporal image brightness variations (**dense corr.**)

- Relationship to stereo matching, SFM

[redacted]

Optical Flow



- Brightness Constancy Assumption

$$I(x, y, t) = I(x + u, y + v, t + 1)$$

- [redacted]

A Linear Motion Model

Take Taylor expansion of $I(x+u, y+v, t+1)$ at (x,y,t) to linearize the right side:

$$I(x,y,t) = I(x+u, y+v, t+1)$$

Image derivative along x   Difference over frames

$$I(x+u, y+v, t+1) \approx I(x,y,t) + \boxed{I_x} \cdot u + I_y \cdot v + \boxed{I_t}$$

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$$I(x+u, y+v, t+1) - I(x, y, t) \approx I_x \cdot u + I_y \cdot v + I_t$$

Hence, $I_x \cdot u + I_y \cdot v + I_t \approx 0 \quad \rightarrow \nabla I \cdot [u \;\; v]^T + I_t = 0$

Ambiguity of Motion

Can we use this equation to recover image motion (u,v) at each pixel?

$$\nabla I \cdot [u \;\; v]^T + I_t = 0$$

- How many equations and unknowns per pixel?
  - One equation (this is a scalar equation!), two unknowns (u,v)

The component of the motion perpendicular to the gradient (i.e., parallel to the edge) cannot be measured

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so does $(u+u', v+v')$ if

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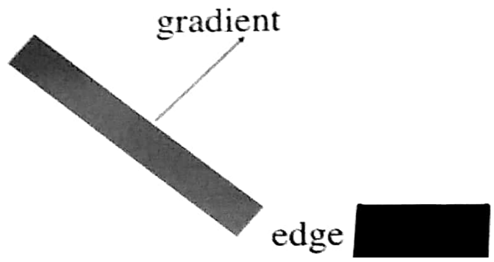

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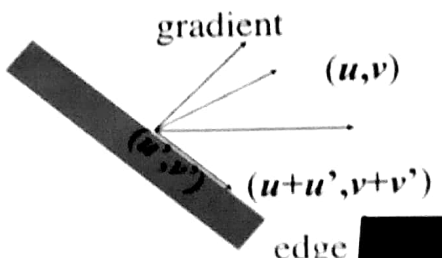
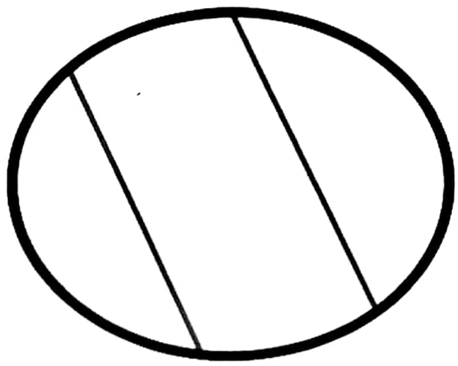


The Aperture Problem



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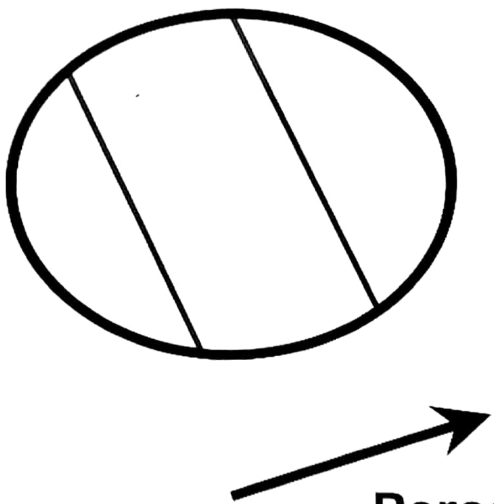
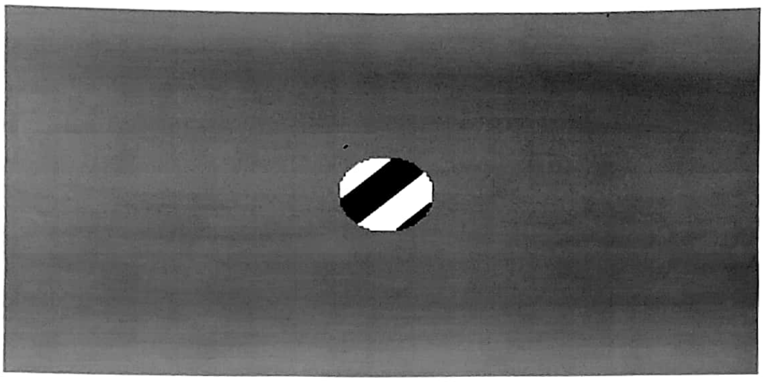


Perceived motion

[redacted]

Motion Ambiguity



- http://en.wikipedia.org/wiki/Barberpole_illusion

Motion Ambiguity

- Motion perpendicular to gradient direction is not discernible.

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Solving the Ambiguity...

B. Lucas and T. Kanade. An iterative image registration technique with an application to stereo vision. In *Proceedings of the International Joint Conference on Artificial Intelligence*, pp. 674–679, 1981.

- How to get more equations for a pixel?
- **Spatial coherence constraint**
- Assume the pixel's neighbors have the same (u,v)
  - If we use a 5x5 window, that gives us 25 equations per pixel

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$$\begin{bmatrix} I_x(\mathbf{p}_1) & I_y(\mathbf{p}_1) \\ I_x(\mathbf{p}_2) & I_y(\mathbf{p}_2) \\ \vdots & \vdots \\ I_x(\mathbf{p}_{25}) & I_y(\mathbf{p}_{25}) \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = - \begin{bmatrix} I_t(\mathbf{p}_1) \\ I_t(\mathbf{p}_2) \\ \vdots \\ I_t(\mathbf{p}_{25}) \end{bmatrix}$$

[redacted]

Matching Patches Across Images

- Overconstrained linear system

$$\begin{bmatrix} I_x(\mathbf{p}_1) & I_y(\mathbf{p}_1) \\ I_x(\mathbf{p}_2) & I_y(\mathbf{p}_2) \\ \vdots & \vdots \\ I_x(\mathbf{p}_{25}) & I_y(\mathbf{p}_{25}) \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = - \begin{bmatrix} I_t(\mathbf{p}_1) \\ I_t(\mathbf{p}_2) \\ \vdots \\ I_t(\mathbf{p}_{25}) \end{bmatrix} \qquad \underset{25\times 2}{A}\ \underset{2\times 1}{d} = \underset{25\times 1}{b}$$

Least squares solution for d given by $\quad (A^T A)\ d = A^T b$

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$$\underbrace{\begin{bmatrix} \sum I_x I_x & \sum I_x I_y \\ \sum I_x I_y & \sum I_y I_y \end{bmatrix}}_{A^T A} \begin{bmatrix} u \\ v \end{bmatrix} = - \underbrace{\begin{bmatrix} \sum I_x I_t \\ \sum I_y I_t \end{bmatrix}}_{A^T b}$$

The summations are over all pixels in the K x K window

[redacted]

Conditions for Solvability

Optimal (u, v) satisfies Lucas-Kanade equation

$$\underbrace{\begin{bmatrix} \sum I_x I_x & \sum I_x I_y \\ \sum I_x I_y & \sum I_y I_y \end{bmatrix}}_{A^T A} \begin{bmatrix} u \\ v \end{bmatrix} = - \underbrace{\begin{bmatrix} \sum I_x I_t \\ \sum I_y I_t \end{bmatrix}}_{A^T b}$$

When is this solvable? I.e., what are good points to track?

- **A^TA** should be invertible
- **A^TA** should not be too small due to noise
  - eigenvalues λ_1 and λ_2 of **A^TA** should not be too small
- **A^TA** should be well-conditioned
  - λ_1 / λ_2 should not be too large (λ_1 = larger eigenvalue)

Does this remind you of anything?

[redacted]

Low-Texture Region

$$\sum \nabla I (\nabla I)^T$$

– gradients have small magnitude

– small λ_1, small λ_2

[redacted]

Edge

$$\sum \nabla I (\nabla I)^T$$

- gradients very large or very small
- large λ_1, small λ_2

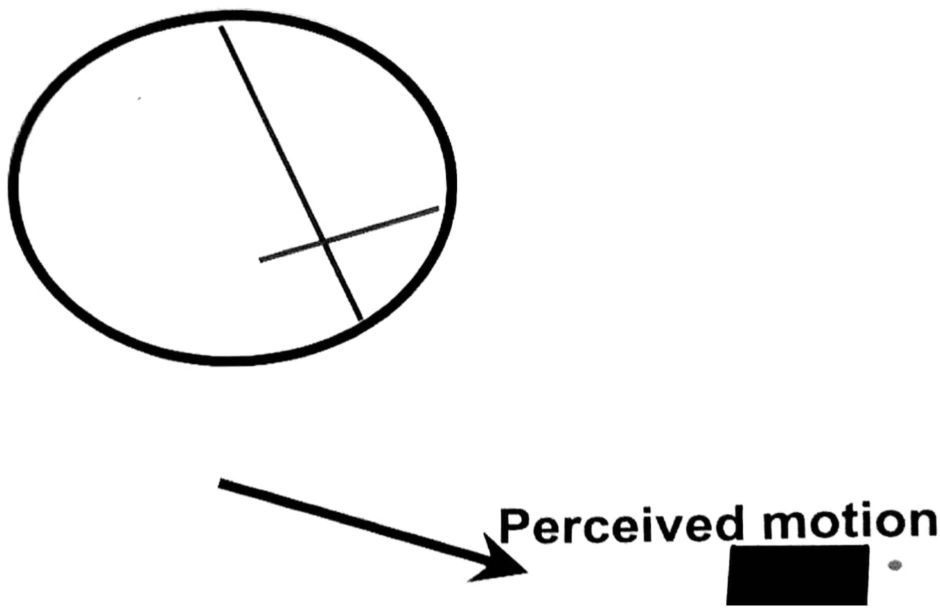
High-Texture Region

$$\sum \nabla I (\nabla I)^T$$

– gradients are different, large magnitudes

– large λ_1, large λ_2

The aperture problem resolved



Dealing with Larger Movements: Iterative Refinement

Original (x,y) position

1. Initialize (x',y') = (x,y)
2. Compute (u,v) by

$$I_t = I(x', y', t+1) - I(x, y, t)$$

$$\begin{bmatrix} \sum I_x I_x & \sum I_x I_y \\ \sum I_x I_y & \sum I_y I_y \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = - \begin{bmatrix} \sum I_x I_t \\ \sum I_y I_t \end{bmatrix}$$

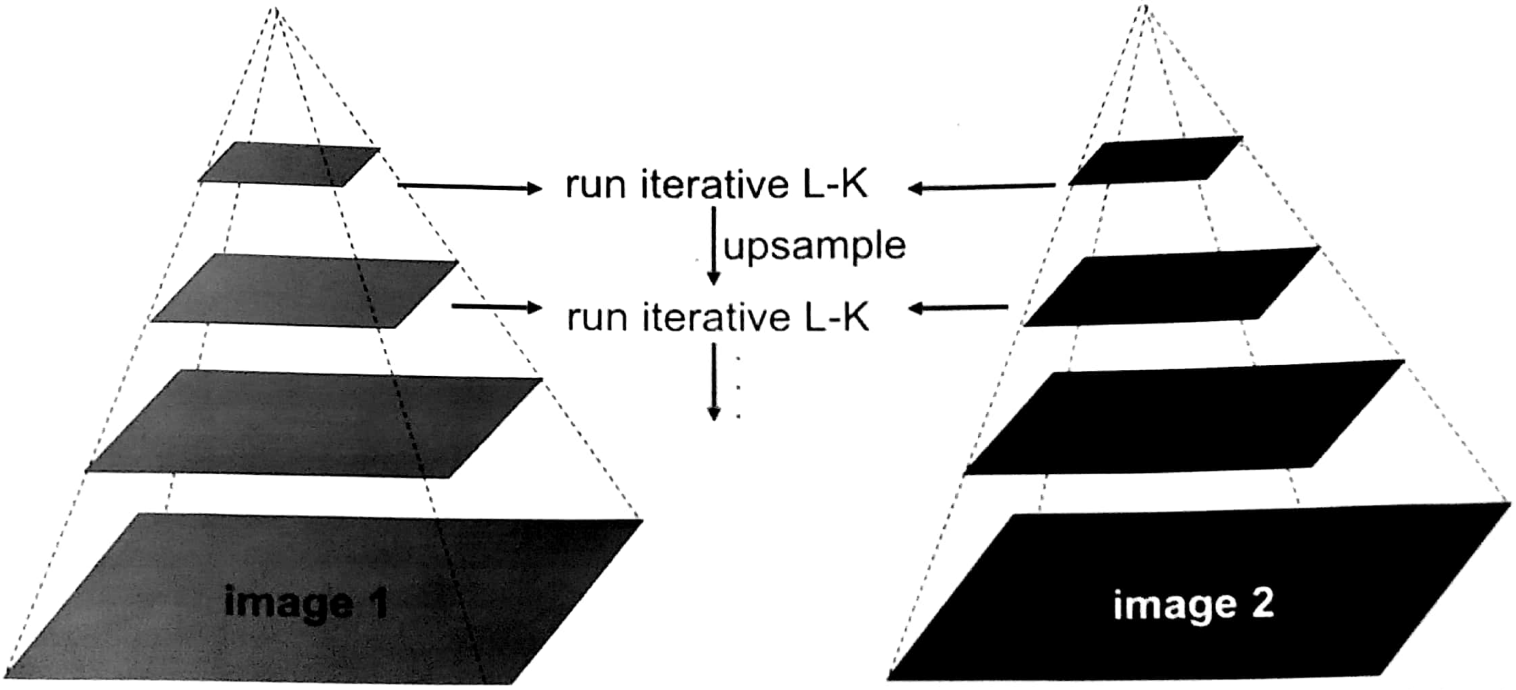
2nd moment matrix for feature patch in first image

displacement

1. Shift window by (u, v): $x' = x' + u;\ y' = y' + v;$
2. Recalculate I_t
3. Repeat steps 2-4 until small change
   - Use interpolation for subpixel values

[redacted]

Dealing with Larger Movements: Coarse-to-Fine Registration



Gaussian pyramid of image 1 (t)

Gaussian pyramid of i[redacted] (t+1)

Shi-Tomasi Feature Tracker

- Find good features using eigenvalues of 2nd-moment matrix (e.g., Harris detector or threshold on the smallest eigenvalue)
  - Key idea: "good" features to track are the ones whose motion can be estimated reliably

J. Shi and C. Tomasi. Good Features to Track. CVPR 1994. [redacted]

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- Track from frame to frame with Lucas-Kanade
  - This amounts to assuming a translation model for frame-to-frame feature movement

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- Track from frame to frame with Lucas-Kanade
  - This amounts to assuming a translation model for frame-to-frame feature movement

- Check consistency of tracks by *affine* registration to the first observed instance of the feature
  - Affine model is more accurate for larger displacements
  - Comparing to the first frame helps to minimize drift

J. Shi and C. Tomasi. Good Features to Track. CVPR 1994. [redacted]

Tracking Example

Figure 1: Three frame details from Woody Allen's *Manhattan*. The details are from the 1st, 11th, and 21st frames of a subsequence from the movie.

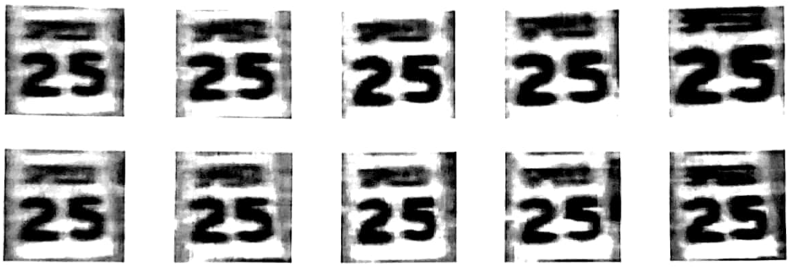


Figure 2: The traffic sign windows from frames 1,6,11,16,21 as tracked (top), and warped by the computed deformation matrices (bottom).

J. Shi and C. Tomasi. Good Features to Track. CVPR 1994. [redacted]

Summary of KLT Tracking

- Find a good point to track (harris corner)
- Use intensity second moment matrix and difference across frames to find displacement
- Iterate and use coarse-to-fine search to deal with larger movements
- When creating long tracks, check appearance of registered patch against appearance of initial patch to find points that have drifted

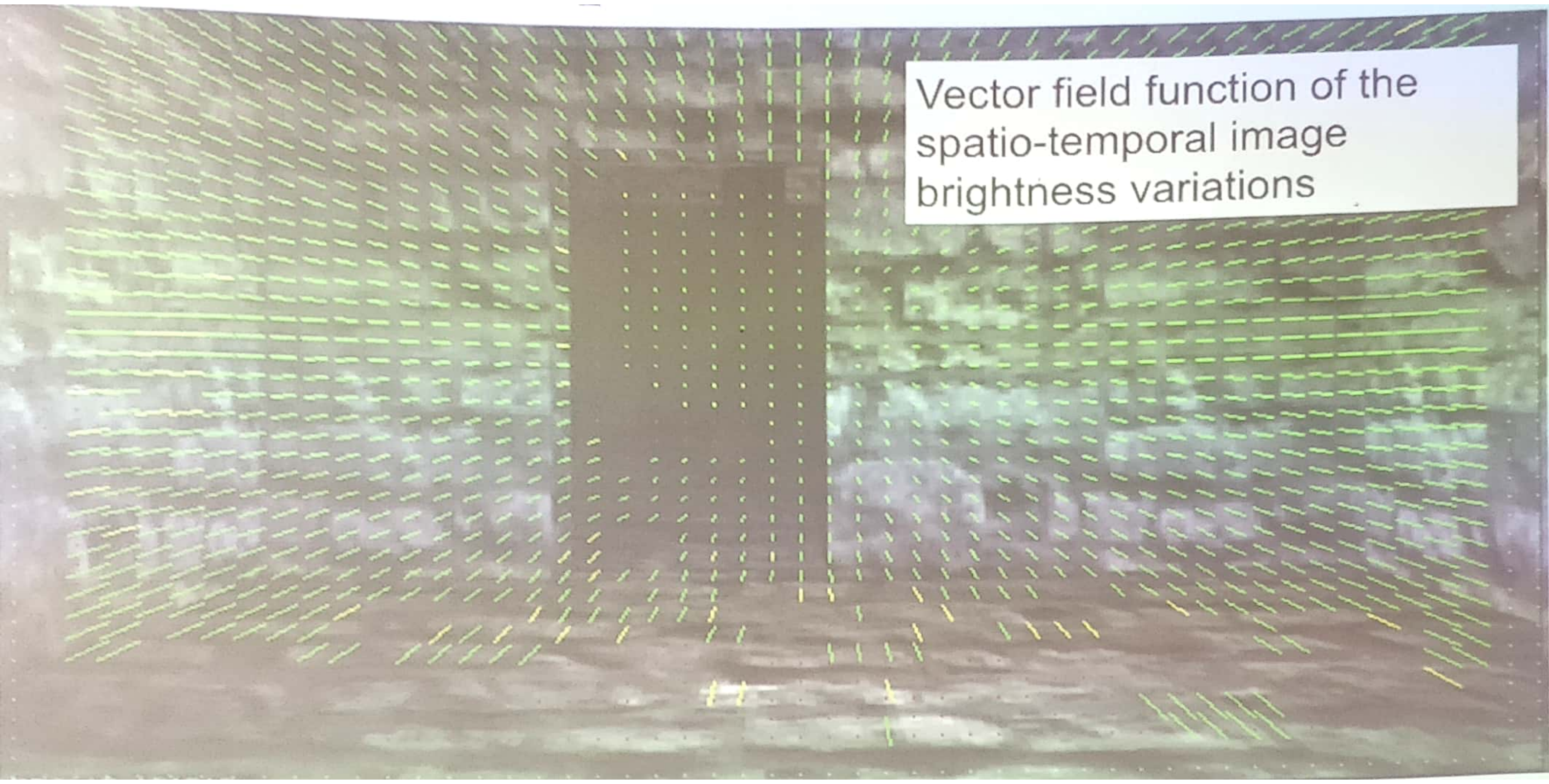
[redacted]

Implementation Issues

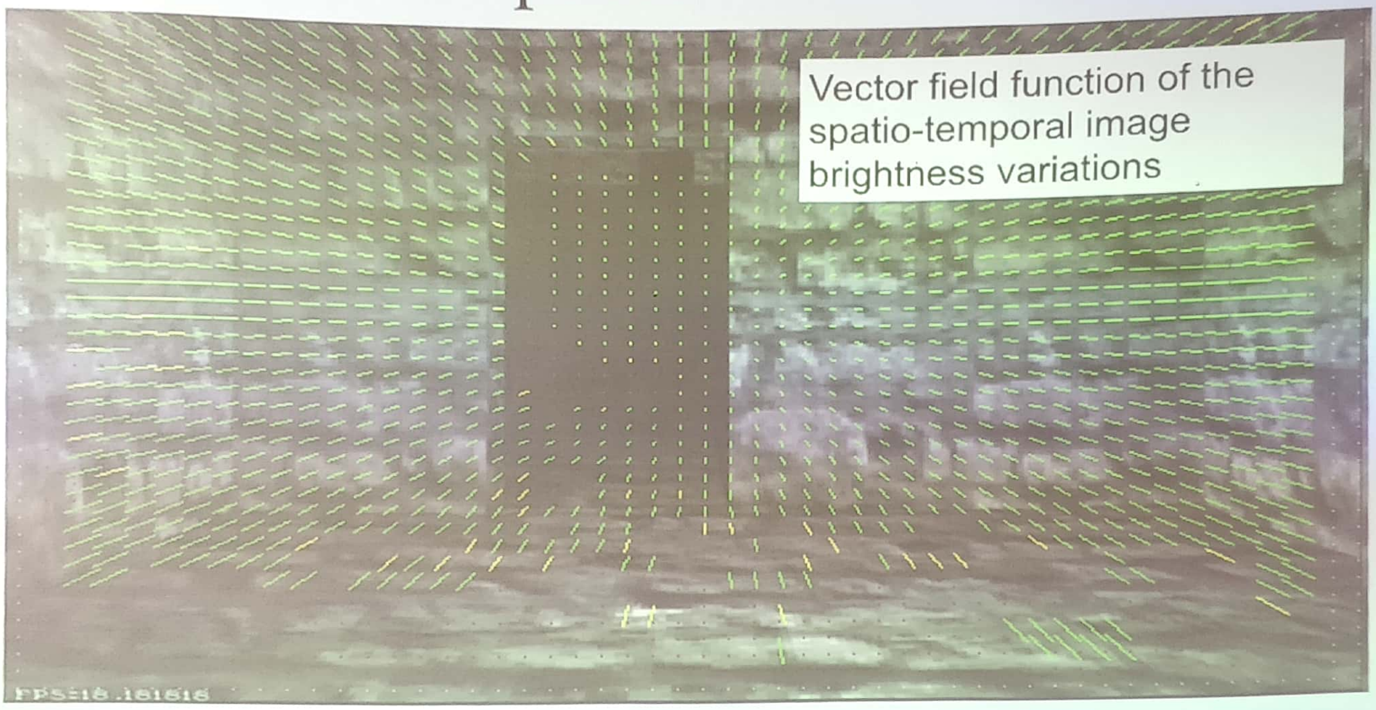
- Window size
  - Small window more sensitive to noise and may miss larger motions (without pyramid)
  - Large window more likely to cross an occlusion boundary (and it's slower)
  - 15x15 to 31x31 seems typical

- Weighting the window
  - Common to apply weights so that center matters more (e.g., with Gaussian)

[redacted]


Vector field function of the
spatio-temporal image
brightness variations

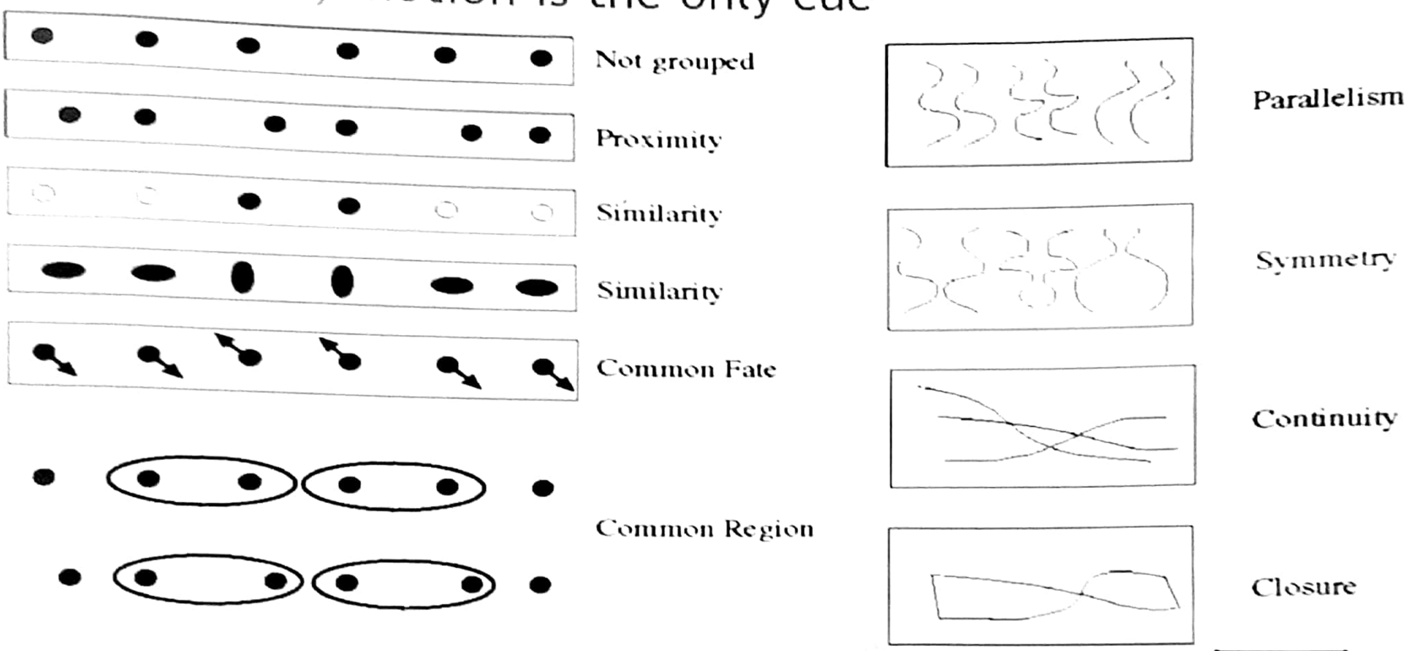
Optical flow



Picture courtesy of Selim Temizer - Learning and Intelligent Systems (LIS) Group, MIT

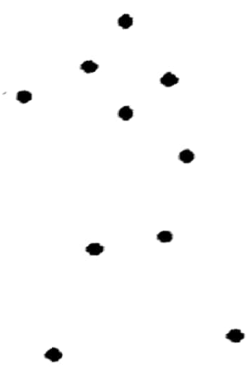
Motion and Perceptual Organization

- Sometimes, motion is the only cue



Motion and Perceptual Organization

- Even "impoverished" motion data can evoke a strong percept



G. Johansson, "Visual Perception of Biological Motion and a Model For Its Analysis", *Perception and Psychophysics 14, 201-211, 1973.* [redacted]

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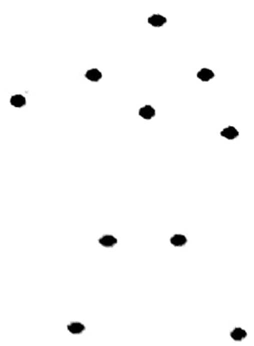

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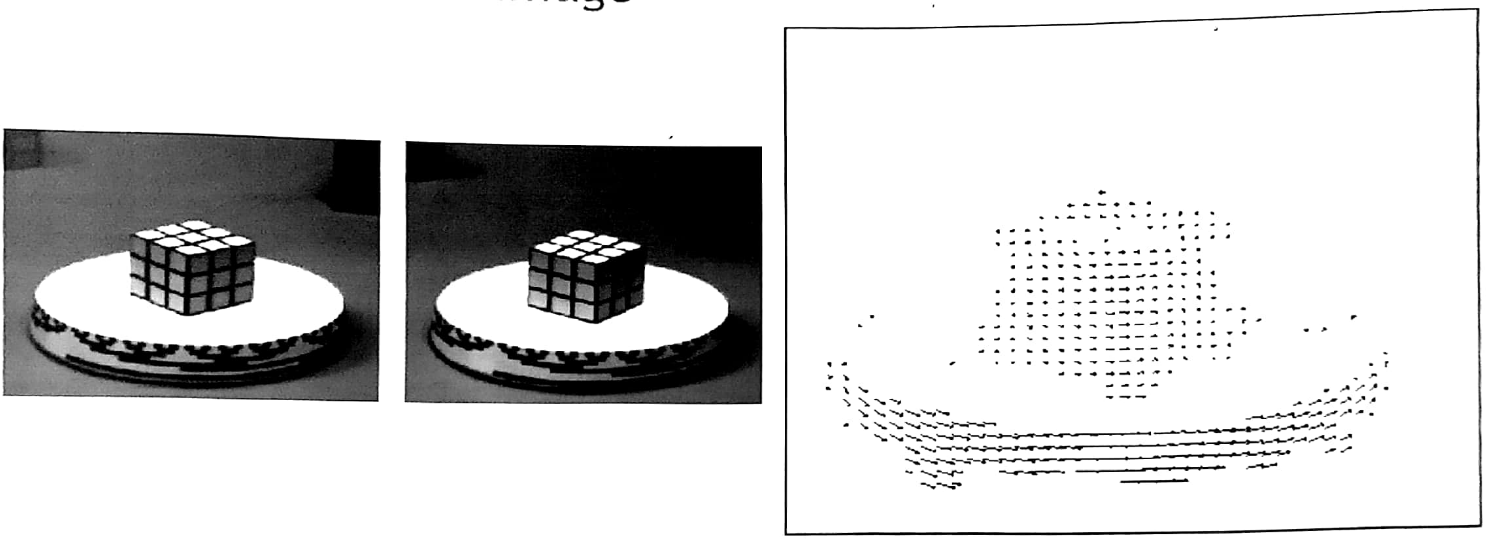
Uses of Motion Estimation

- Estimating 3D structure
- Segmenting objects based on motion cues
- Learning and tracking dynamical models
- Recognizing events and activities
- Improving video quality (motion stabilization)
- Video Compression (MPEG-4)

[redacted]

Motion Field

- The motion field is the projection of the 3D scene motion into the image



Optical Flow vs Motion Field

- Definition: optical flow is the *apparent* motion of brightness patterns in the image
- Ideally, optical flow would be the same as the motion field

[redacted]

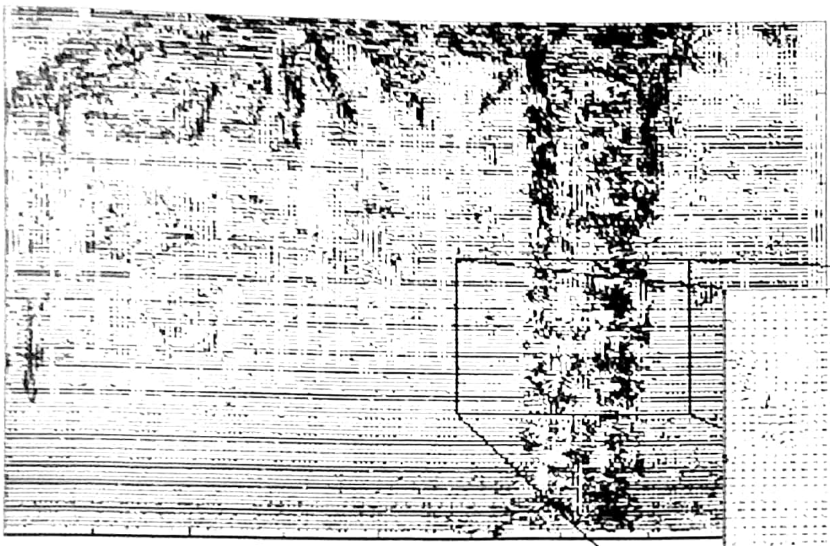
Lucas-Kanade Optical Flow

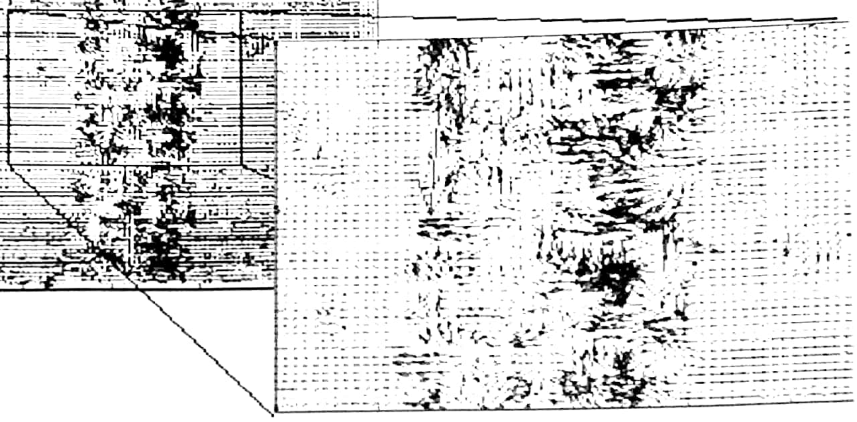
- Same as Lucas-Kanade feature tracking, but for each pixel
  - As we saw, works better for textured pixels
- Operations can be done one frame at a time, rather than pixel by pixel
  - Efficient

Iterative Refinement

- Iterative Lukas-Kanade Algorithm
  1. Estimate displacement at each pixel by solving Lucas-Kanade equations
  2. Warp I(t) towards I(t+1) using the estimated flow field
     - Basically, just interpolation
  3. Repeat until convergence

Optical Flow Results





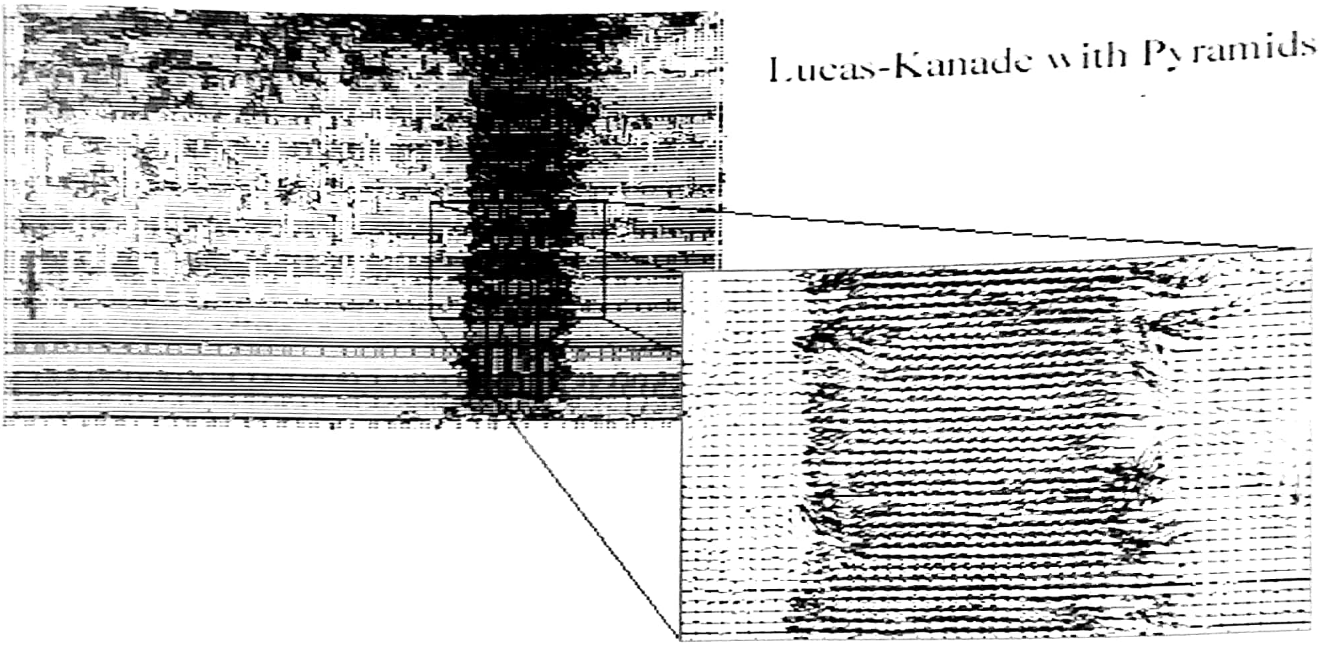
Lucas-Kanade without pyramids

Fails in areas of large motion

* From Khurram Hassan-Shafique CAP5415 [redacted] n 2003

Optical Flow Results

Lucas-Kanade with Pyramids



* From Khurram Hassan-Shafique CAP5415 [redacted] n 2003

Errors in Lucas-Kanade

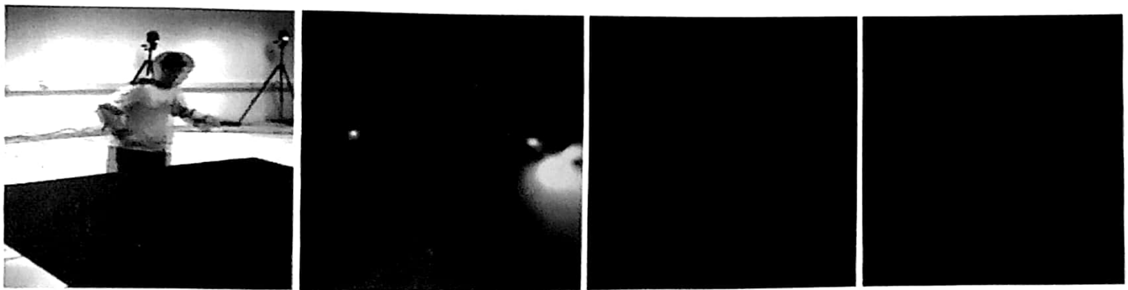
- The motion is large
  - Possible Fix: Keypoint matching
- A point does not move like its neighbors
  - Possible Fix: Region-based matching
- Brightness constancy does not hold
  - Possible Fix: Gradient constancy

[redacted]

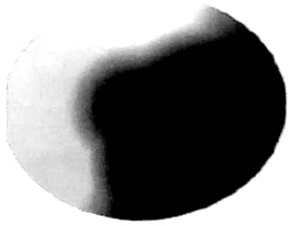
Other methods for optical flow

Start with something similar to Lucas-Kanade

+ gradient constancy
+ energy minimization with smoothing term
+ region matching
+ keypoint matching (long-range)



Region-based +Pixel-based +Keypoint-based



Color map used to visualize flow fields. Smaller v[redacted] and color indicates [redacted]

Large displacement optical flow, Brox et al., CVPR 2009

Scanned by CamScanner

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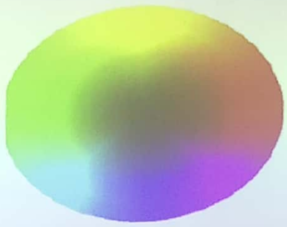

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Large displacement optical flow, Brox et al., CVPR 2009

Summary

- Major contributions from Kanade Lucas, Tomasi
  - Tracking feature points
  - Optical flow

- Key ideas
  - By assuming brightness constancy, truncated Taylor expansion leads to simple and fast patch matching across frames
  - Coarse-to-fine registration

[redacted]