Points and Lines in $\mathcal{P}^2$

- Points represented by: $\mathbf{x} = [x \;\; y \;\; 1]^{\mathrm{T}}$.
- Consider the line equation: $ax + by + c = 0$.
- $[a\ b\ c][x\ y\ 1]^{\mathrm{T}} = \mathbf{l}.\mathbf{x} = \mathbf{l}^{\mathrm{T}}\,\mathbf{x} = 0$, where $\mathbf{l} = [a\ b\ c]^{\mathrm{T}}$.
- Lines are represented by 3-vectors, just like points.

  Overall scale is unimportant.
- What does $\mathbf{l}^{\mathrm{T}}\,\mathbf{x} = 0$ describe?

Points/Line at Infinity

- $\mathbf{x} = [x_1 \quad x_2 \quad x_3]^{\mathrm{T}}$ represents $[{}^{x_1}/_{x_3} \quad {}^{x_2}/_{x_3}]$
- What happens when $x_3 \rightarrow 0$?
- Becomes **point at infinity**, or **vanishing point** or ideal point in the direction (x_1, x_2).
- Points at infinity can be handled like any other point in projective geometry
- $[x \quad y \quad 0]^{\mathrm{T}}$ are all points at infinity on the plane.
- What do they form together?
- What is the representation of $\mathbf{l}_\infty$?

$$\mathbf{l}_\infty = [0 \quad 0 \quad 1]^{\mathrm{T}}$$

Line joining 2 points

- Let p and q be points. We have: $\mathbf{l}^T\mathbf{p} = \mathbf{l}^T\mathbf{q} = 0$.
- Equation of $\mathbf{l}$: $y = y_1 + \frac{(y_2 - y_1)}{(x_2 - x_1)}(x - x_1)$

or: $(y_2 - y_1)x - (x_2 - x_1)y + (x_2 y_1 - x_1 y_2) = 0$

or: $\mathbf{l} = [(y_2 - y_1) \quad -(x_2 - x_1) \quad (x_2 y_1 - x_1 y_2)]^T$

- Considering them as vectors in 3-space, we want to find a vector l orthogonal to both $\mathbf{p}$ and $\mathbf{q}$.
- The cross-product $\mathbf{x} \times \mathbf{y}$ is a solution. Thus, $\mathbf{l} = \mathbf{p} \times \mathbf{q}$.
- $\mathbf{p}\times\mathbf{q} = [(y_2 - y_1) \quad -(x_2 - x_1) \quad (x_2 y_1 - x_1 y_2)]^T$

Point of Intersection of 2 lines

- Lines $\mathbf{l}$, $\mathbf{m}$ intersect at a point $\mathbf{x}$ with $\mathbf{l}^{\mathsf{T}}\mathbf{x} = \mathbf{m}^{\mathsf{T}}\mathbf{x} = 0$.
- $\mathbf{x} = \mathbf{l} \times \mathbf{m}$.
- $\mathbf{l}$: $a_1 x + b_1 y + c_1 = 0$; and $\mathbf{m}$: $a_2 x + b_2 y + c_2 = 0$.
- $x = (b_2 c_1 - b_1 c_2)/(a_2 b_1 - a_1 b_2)$.
- $y = (a_1 c_2 - a_2 c_1)/(a_2 b_1 - a_1 b_2)$.
- $\mathbf{x} = [(b_2 c_1 - b_1 c_2) \ (a_1 c_2 - a_2 c_1) \ (a_2 b_1 - a_1 b_2)]^{\mathsf{T}} = \mathbf{l} \times \mathbf{m}$.
- Duality at work: points and lines are interchangeable.

Conics: 2nd order Entities

- General quadratic entity:

$$ax^2 + bxy + cy^2 + dx + ey + f = 0.$$

- Rewrite using homogeneous coordinates as:

$$ax^2 + bxy + cy^2 + dxw + eyw + fw^2 = 0.$$

- Rewrite as:

$$[x \quad y \quad w]\begin{bmatrix} a & b/2 & d/2 \\ b/2 & c & e/2 \\ d/2 & e/2 & f \end{bmatrix}\begin{bmatrix} x \\ y \\ w \end{bmatrix} = 0$$

- A symmetric C represents a conic: $\mathbf{x}^{\mathrm{T}}\mathbf{C}\mathbf{x} = 0$. Covers circle, ellipse, parabola, hyperbola, etc.
- Degenerate conics include a line ($a = b = c = 0$) and two lines when $\mathbf{C} = \mathbf{l}\mathbf{m}^{\mathrm{T}} + \mathbf{m}\mathbf{l}^{\mathrm{T}}$.

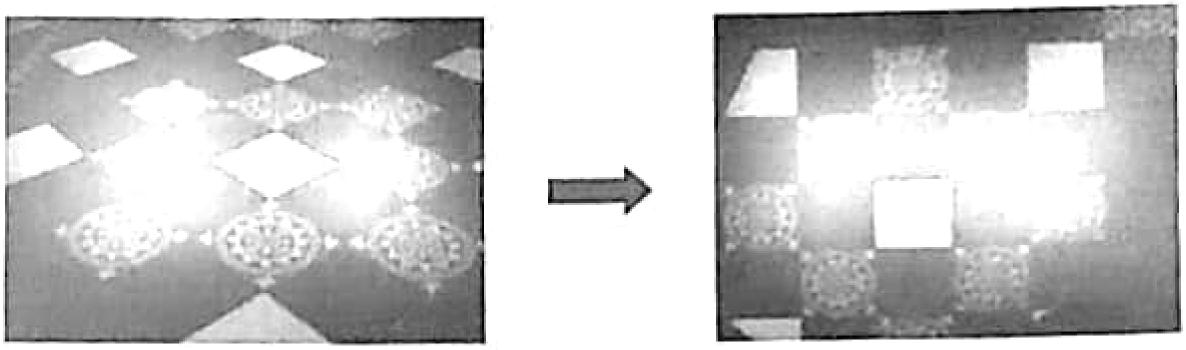
Properties of Conics

- $\mathbf{l} = \mathbf{C}\mathbf{x}$ gives the tangent line to the conic at $\mathbf{x}$.
  - A point $\mathbf{x}$ on the conic is on line $\mathbf{l} = \mathbf{C}\mathbf{x}$ as $\mathbf{x}^T(\mathbf{C}\mathbf{x}) = \mathbf{0}$.
  - If $\mathbf{l}$ intersects the conic in another point $\mathbf{y}$:
    - $\mathbf{y}^T\mathbf{C}\mathbf{y} = \mathbf{0}$ as $\mathbf{y}$ is on the conic; and
    - $(\mathbf{C}\mathbf{x})^T\mathbf{y} = \mathbf{x}^T\mathbf{C}\mathbf{y} = \mathbf{0}$ as $\mathbf{y}$ is on the line.
    - Thus, $\mathbf{C}\mathbf{y}$ is a line joining $\mathbf{x}$ and $\mathbf{y}$.
    - That is $\mathbf{C}\mathbf{y} = \mathbf{C}\mathbf{x}$ or $\mathbf{x} = \mathbf{y}$.
- Dual Conic: Conic defined by its tangent lines.
  - $\mathbf{l}^T\mathbf{C}^*\mathbf{l} = \mathbf{0}$ or $\mathbf{l}^T\mathbf{C}^{-1}\mathbf{l} = \mathbf{0}$ gives the set of lines tangential to $\mathbf{C}$.

CSE578: Computer Vision

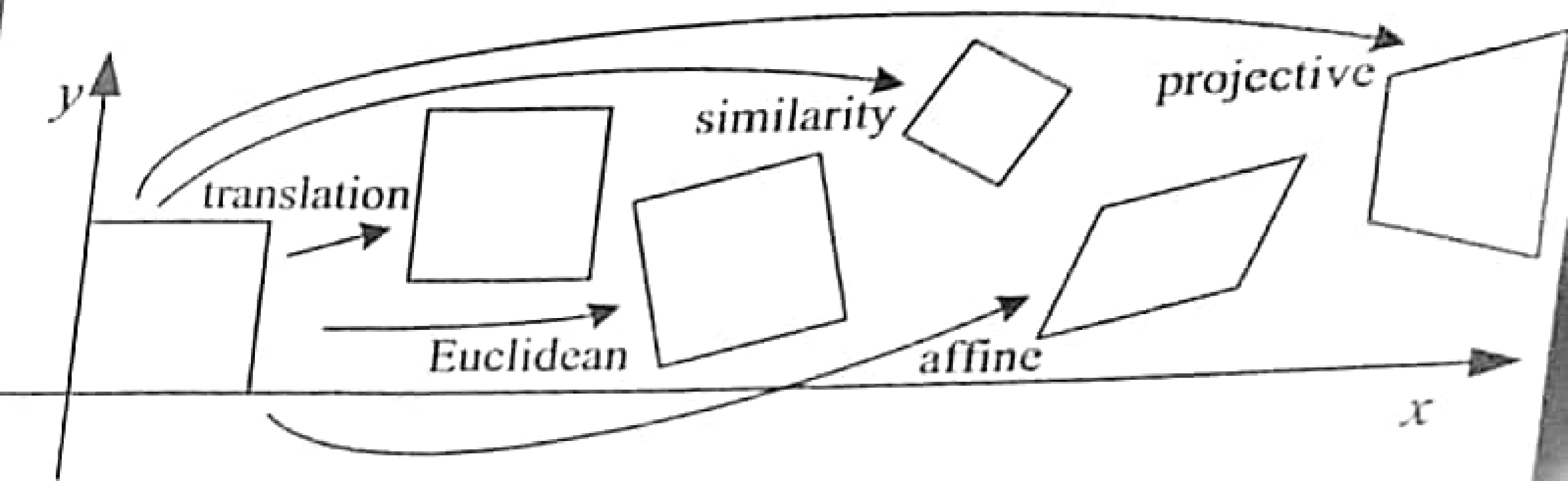
Spring'19

Image Rectification:
Recovering Structure from Single Image



Anoop M. Namboodiri
Center for Visual Information Technology
IIIT Hyderabad, INDIA

Hierarchy of Transformations



Hierarchy of Transformations

- Translation (2) $\longrightarrow \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \end{bmatrix}$
- Euclidean (3) or Rigid Body $\longrightarrow \begin{bmatrix} c_\theta & -s_\theta & t_x \\ s_\theta & c_\theta & t_y \end{bmatrix}$
- Similarity (4) $\longrightarrow \begin{bmatrix} 1+a & -b & t_x \\ b & 1+a & t_y \end{bmatrix}$

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- Affine (6) $\longrightarrow \begin{bmatrix} 1+a & b & t_x \\ c & 1+d & t_y \end{bmatrix}$
- Projective (8) $\longrightarrow \begin{bmatrix} 1+h_{00} & h_{01} & h_{02} \\ h_{10} & 1+h_{11} & h_{12} \\ h_{20} & h_{21} & 1 \end{bmatrix}$

Invariants of Transformations

Property	Euclidean	Similarity	Affine	Projective
Length	Yes	No	No	No
Angle	Yes	Yes	No	No
Length Ratio	Yes	Yes	No	No
Area Ratio	Yes	Yes	Yes	No
Parallelism	Yes	Yes	Yes	No
Centroid	Yes	Yes	Yes	No
Ratio of len. ratio	Yes	Yes	Yes	Yes
Collinearity	Yes	Yes	Yes	Yes

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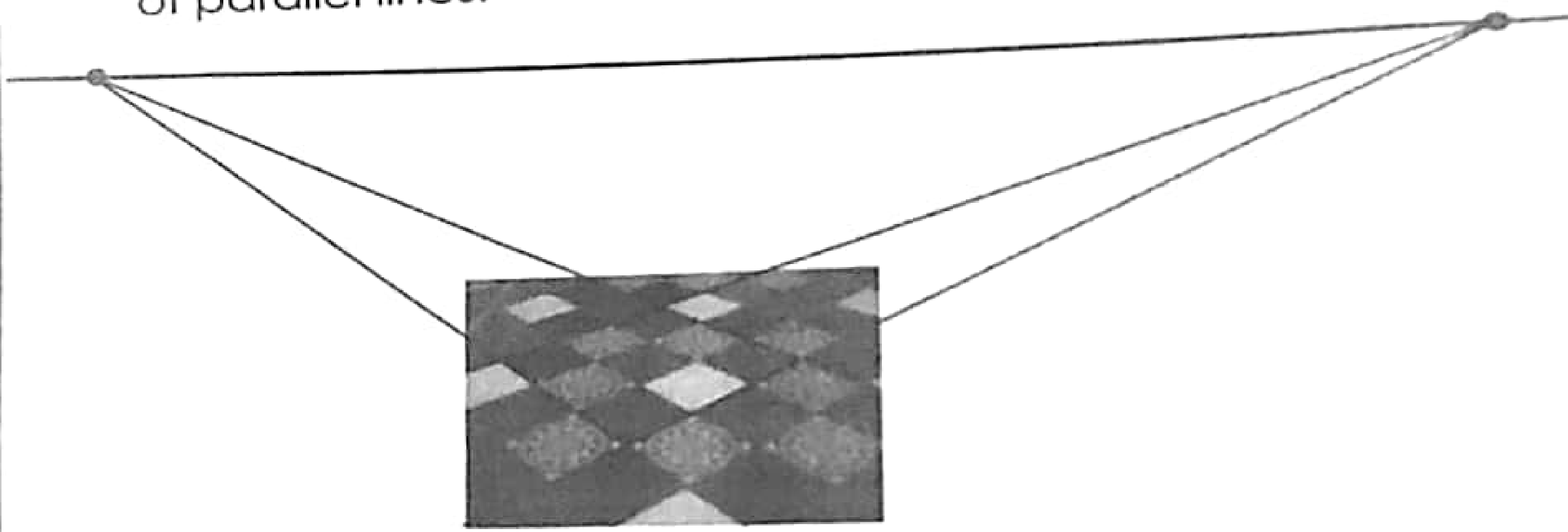
Circular Points

- Affinity maps l_∞ to itself. Conversely, any transformation that does that is an affine one
- General projectivity can map l_∞ to a finite line and vice versa
- A circle intersects l_∞ at **circular points**. Canonical (Euclidean) circle is: $x^2 + y^2 + dxw + eyw + fw^2 = 0$.
- Points on l_∞ have $w = 0$. Thus, $x^2 + y^2 = 0$.
- Circular points are given canonically by:

$$I = \begin{bmatrix} 1 \\ i \\ 0 \end{bmatrix} \quad \text{and} \quad J = \begin{bmatrix} 1 \\ -i \\ 0 \end{bmatrix}$$

Finding Line at Infinity

- Line at infinity can be found in the image from 2 sets of parallel lines.



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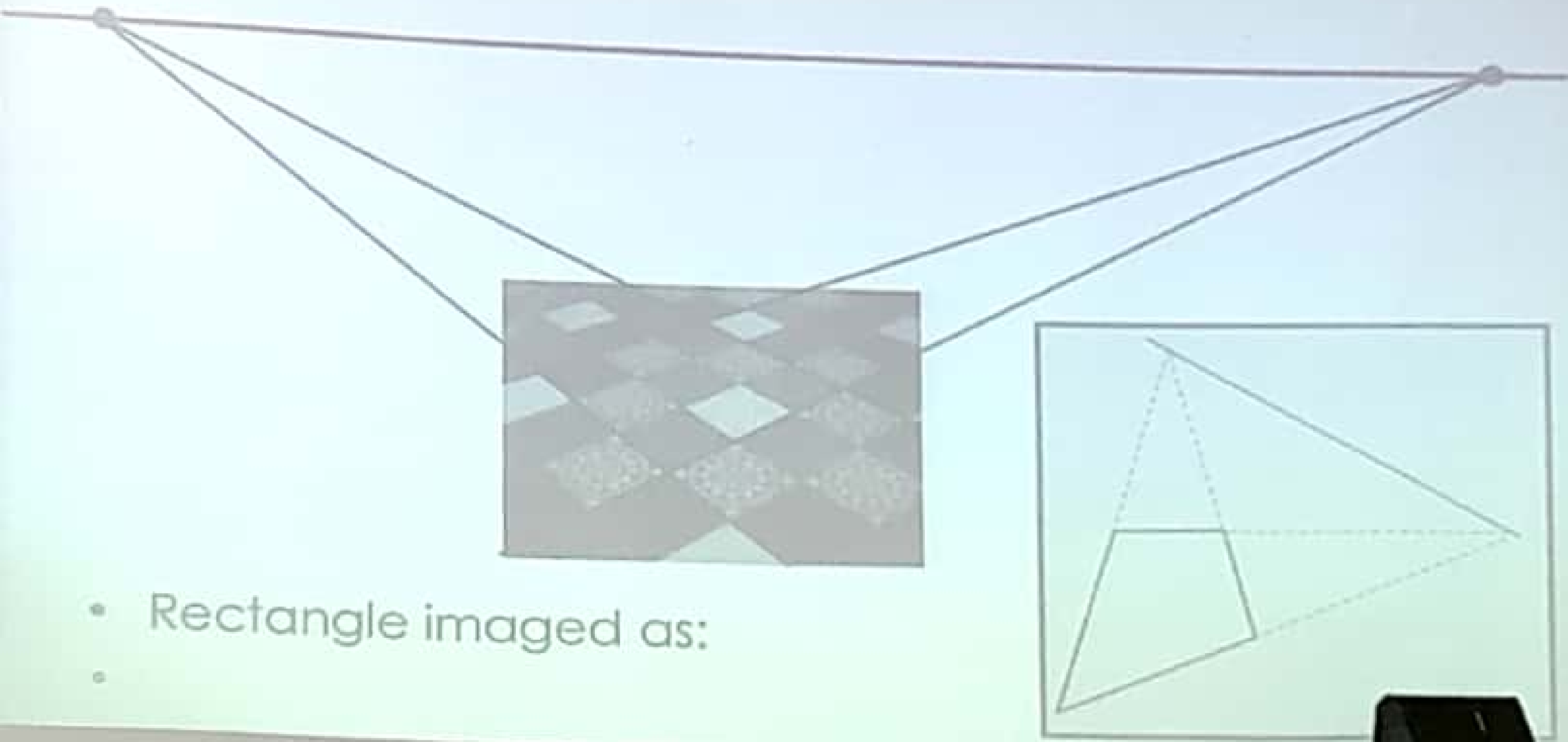

- Rectangle imaged as:

-

Affine Structure from Images

- Affine structure gives parallelism, ratio of areas, centroid, etc., and can be the basis of many decisions.
- Find l_∞ in image using parallel lines.
- Apply a transformation H that maps the line to $[0\ 0\ 1]^T$
- $H_A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ l_1 & l_2 & l_3 \end{bmatrix}$ sends points on $\begin{bmatrix} l_1 \\ l_2 \\ l_3 \end{bmatrix}$ to $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ as

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ l_1 & l_2 & l_3 \end{bmatrix}^{-T} \begin{bmatrix} l_1 \\ l_2 \\ l_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -l_1/l_3 \\ 0 & 1 & -l_2/l_3 \\ 0 & 0 & 1/l_3 \end{bmatrix} \begin{bmatrix} l_1 \\ l_2 \\ l_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Circular Points to Similarity

- Circular points are fixed under similarity

$$\begin{bmatrix} s\mathbf{R} & t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ -i \\ 0 \end{bmatrix} = s(cos\theta + isin\theta) \begin{bmatrix} 1 \\ -i \\ 0 \end{bmatrix}$$

- Conversely, any transformation that fixes circular points is a similarity.
- Thus, a transformation **H** that sends the circular points to their canonical form ***I*** and ***J*** leaves only a similarity transformation.

Dual Conic to Circular Points

- $C^*_\infty = IJ^T + JI^T$ is a dual conic defined by the circular points. It is also fixed under similarity.
- In canonical or Euclidean frame, $C^*_\infty = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$
- We can see that

$$\begin{bmatrix} s\mathbf{R}^T & 0 \\ t^T & 1 \end{bmatrix} C^*_\infty \begin{bmatrix} s\mathbf{R} & t \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} s^2 & 0 & 0 \\ 0 & s^2 & 0 \\ 0 & 0 & 0 \end{bmatrix} \equiv C^*_\infty$$

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- $C_\infty^* = \mathrm{IJ}^\mathrm{T} + \mathrm{JI}^\mathrm{T}$ is a dual conic defined by the circular points. It is also fixed under similarity.

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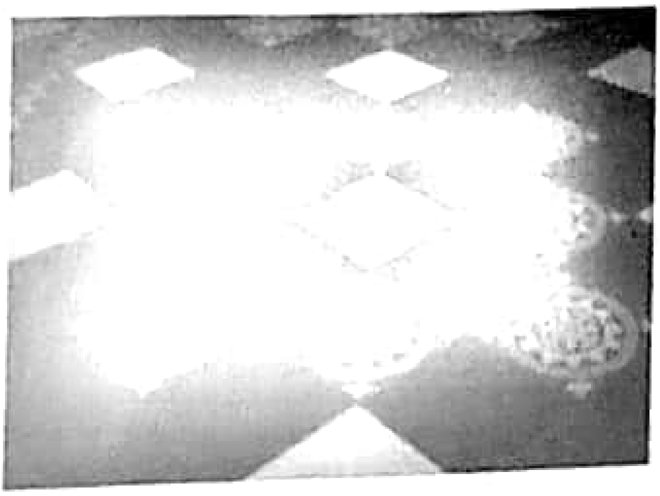
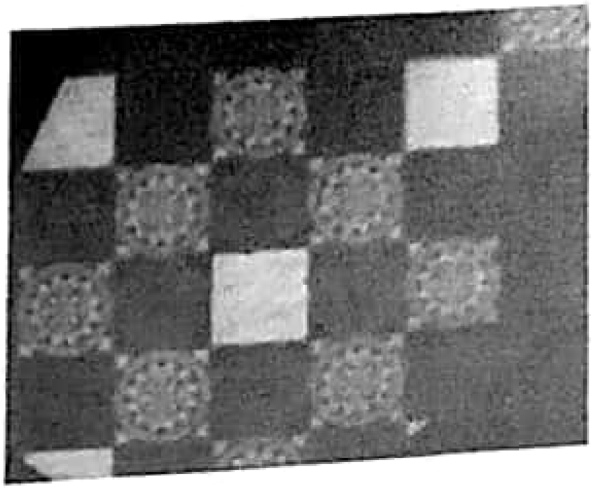
Metric Structure from Images

- Identify circular points in the image. This can be done by finding a world circle in the image as a conic, finding the l_∞ in the image and finding their intersection
- Map one circular point to I and the other to J. The transformation H that does it metric rectifies the image.
- l_∞ gives affine structure, the circle gives metric structure.
- Can be done using 2 non-parallel orthogonal line pairs instead of a circle or 5 orthogonal line pairs from projective!

Metric Rectification

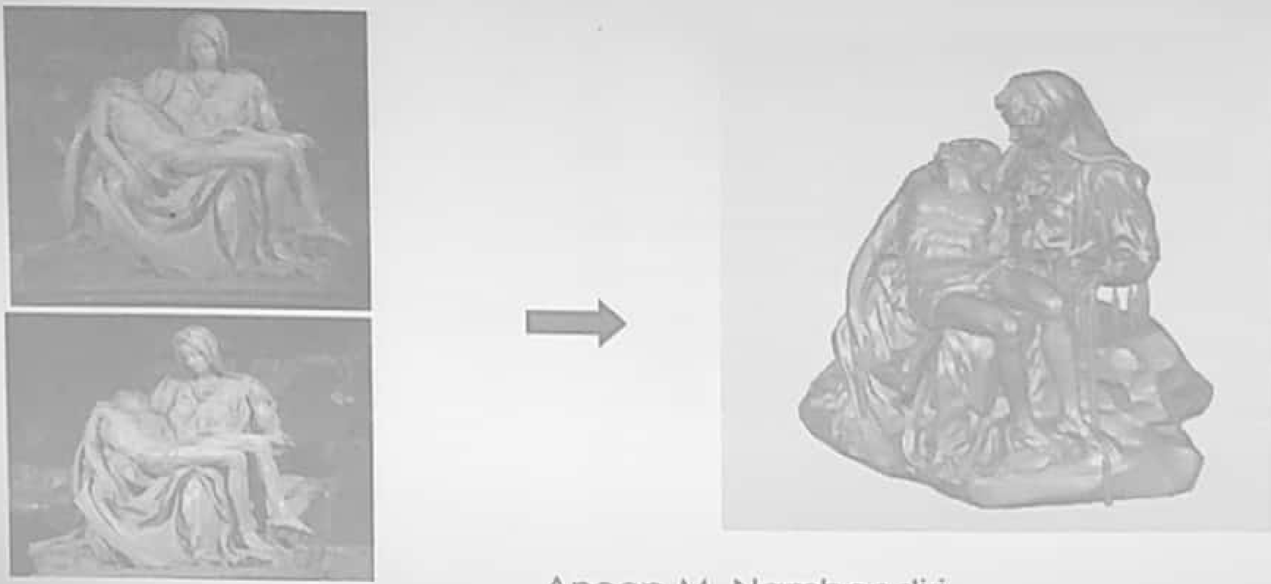




- Resulting image is a scale away from actual image

CSE578: Computer Vision

3D Reconstruction:
Recovering Structure from Single/Multiple Images

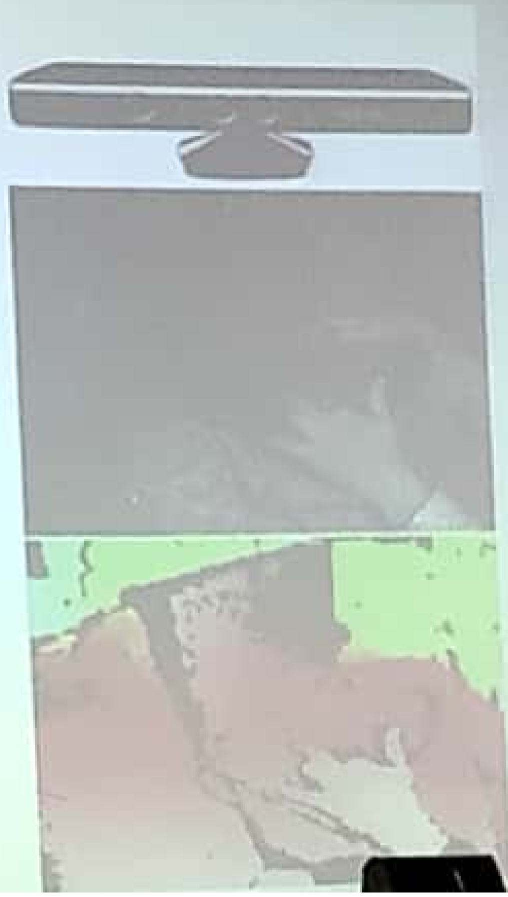


Anoop M. Namboodiri
Center for Visual Information Technology
IIIT Hyderabad, INDIA

Xbox Kinect

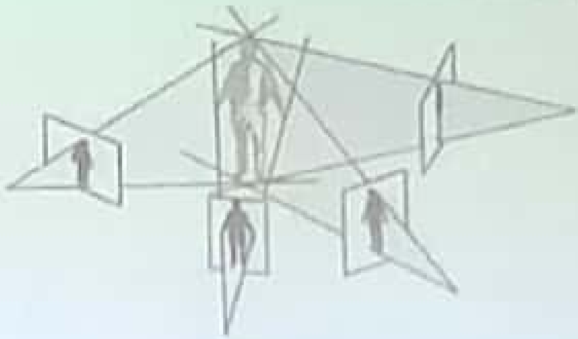
IR-based range sensor for Xbox

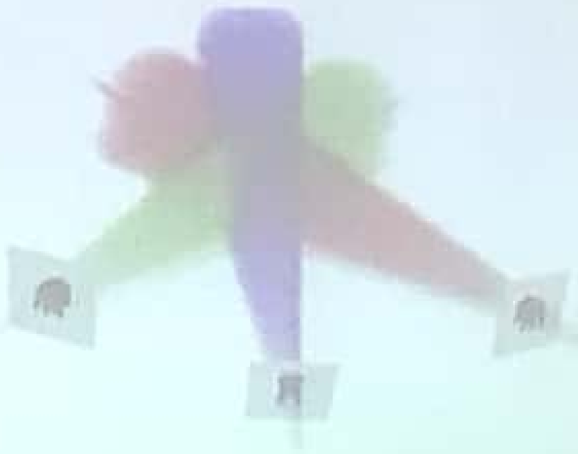
- Aligned depth and RGB images at 640 × 480
- Original goal: Interact with games in full 3D
- Computer vision happy with real-time depth and image
    - Games, HCI, etc
    - Action recognition
    - Image based modelling of dynamic scenes
- Fastest selling electronic appliance ever!!
- Other products that use PrimeSense sensor



Visual Hull

- Object silhouette represents a generalized cone with the camera centre as the apex
- Intersect these cones for multiple views in the 3D space
- Visual Hull, like convex hull
- Cannot get fine details like concavities
- Gives a very good, approximate shape, without scene modification!





Space Carving

- Reason directly in a volumetric voxel space
- If a voxel is filled, it projects to similar colours in all cameras
- If a voxel is empty, its projections will have different appearances
- Colour consistency: filled or empty?
- Assume all filled initially; carve out empty ones by going over the images, guessing visibility, etc.