16.04.2019

Statistical Methods in AI (CSE/ECE 471)

Lecture-22: Panel Discussion

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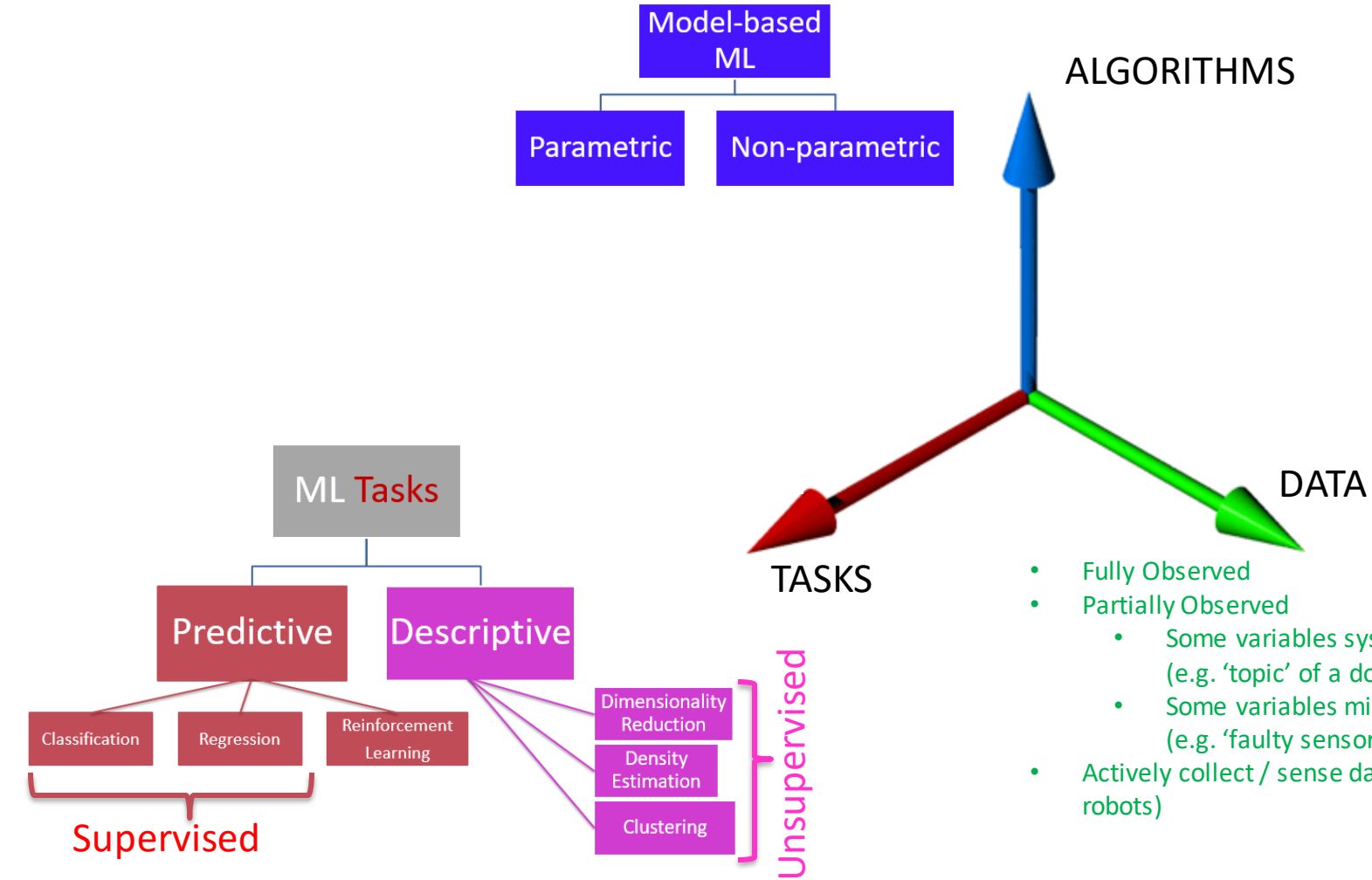
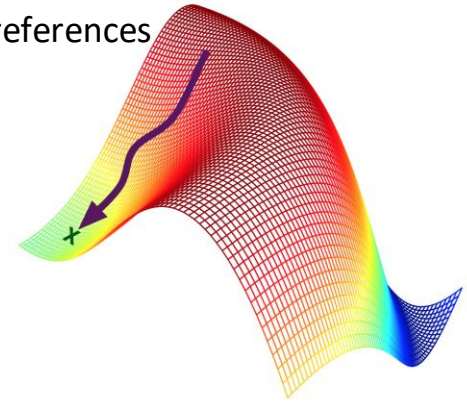


- Fully Observed
- Partially Observed
  - Some variables systematically not observed (e.g. 'topic' of a document)
  - Some variables missing some of the time (e.g. 'faulty sensor' readings)
- Actively collect / sense data (e.g. exploration robots)

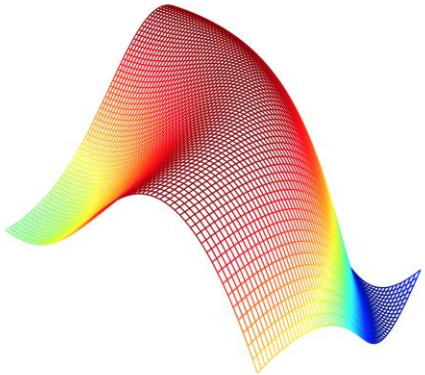
Strategy for fulfilling preferences



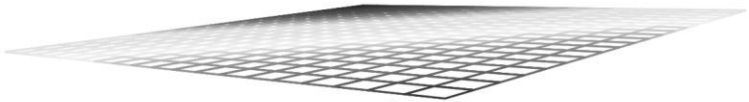
Optimization

Evaluation

Representation



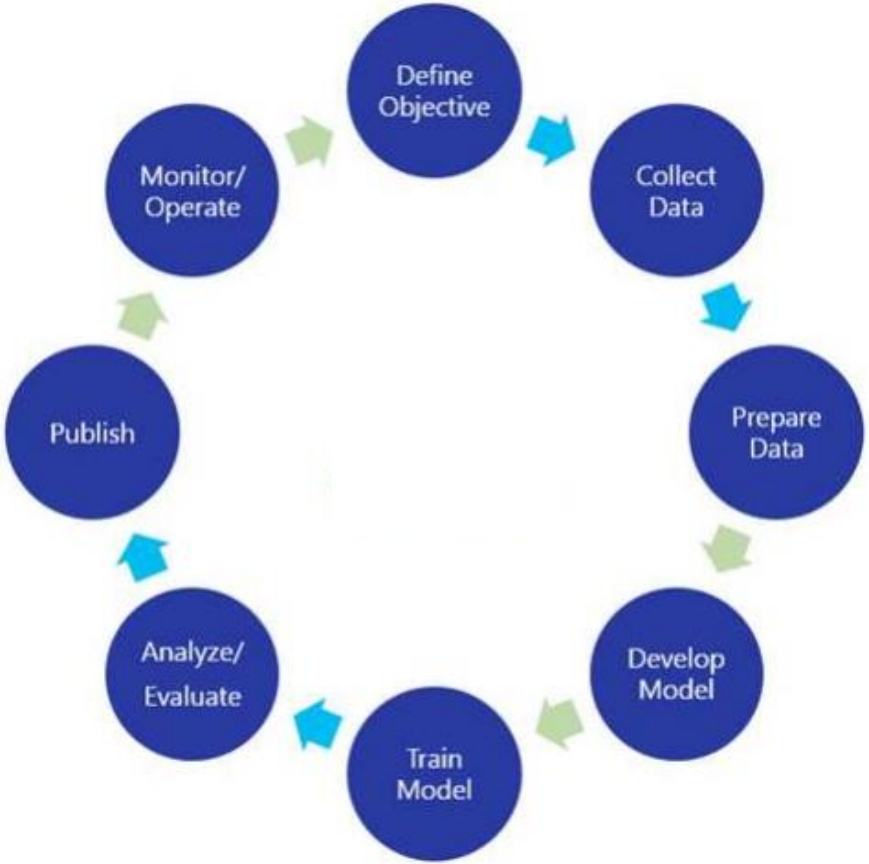
Preferences over the landscape



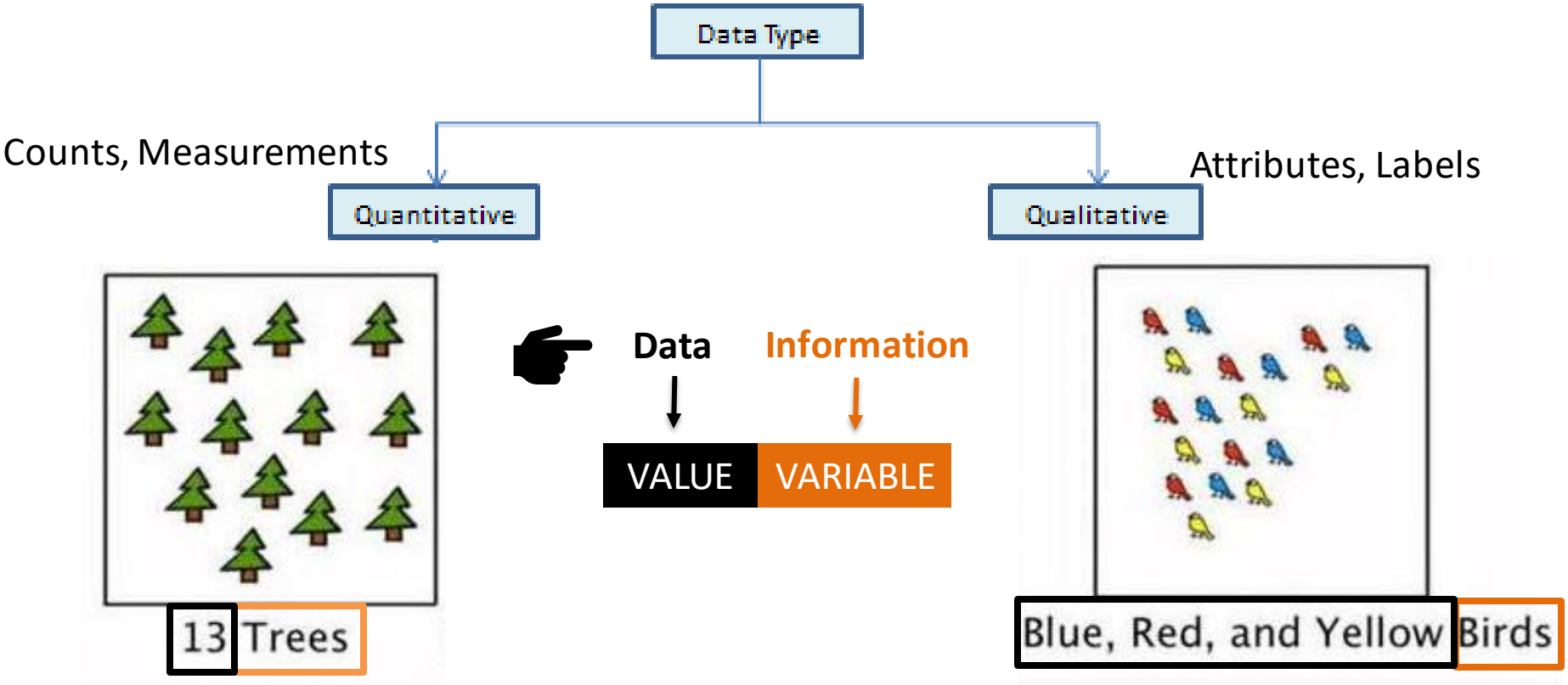
The landscape of allowed models

https://medium.com/@devnag/machine-learning-representation-evaluation-optimization-fc7b26b38fdb

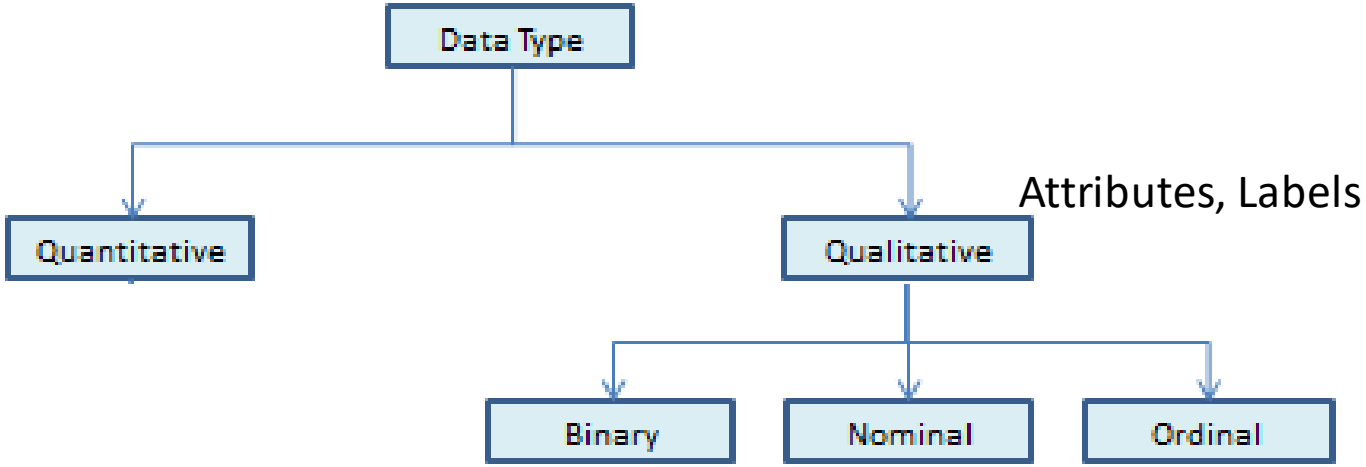
Workflow of a Machine Learning Problem



Taxonomy of data variables

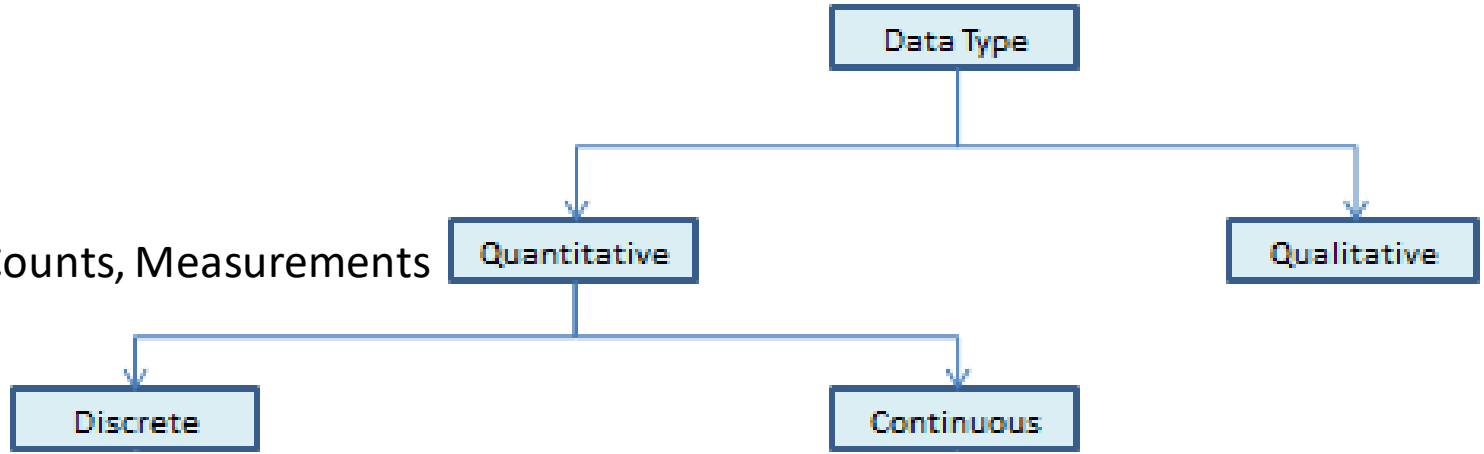


Taxonomy of data



Attributes, Labels

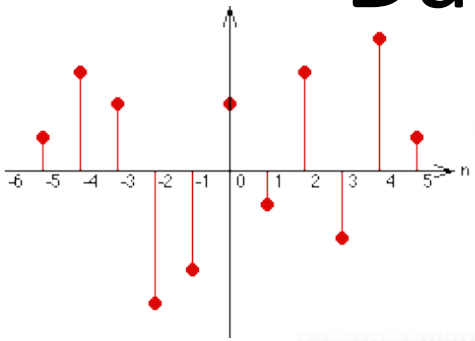
Taxonomy of data



Counts, Measurements

- # of CPU cores
- # of courses taken in a semester
- # of times word ‘sale’ appears in a doc

Data Representations



Scalars

$$x$$

Vectors

$$x = \begin{bmatrix} x_1 \\ \vdots \\ x_N \end{bmatrix}$$

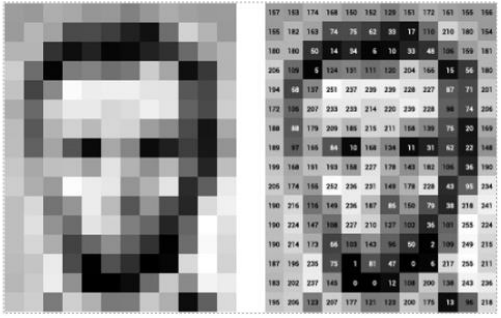
Matrix

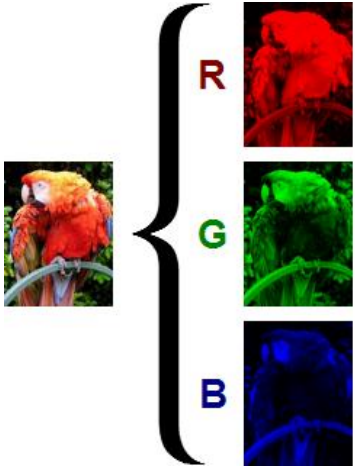
$$\mathbf{X} = \begin{bmatrix} \mathbf{x} & \cdots & \mathbf{x}_N \end{bmatrix} = \begin{bmatrix} x_{1,1} & & x_{N,1} \\ \vdots & \cdots & \vdots \\ x_{1,M} & & x_{N,M} \end{bmatrix}$$

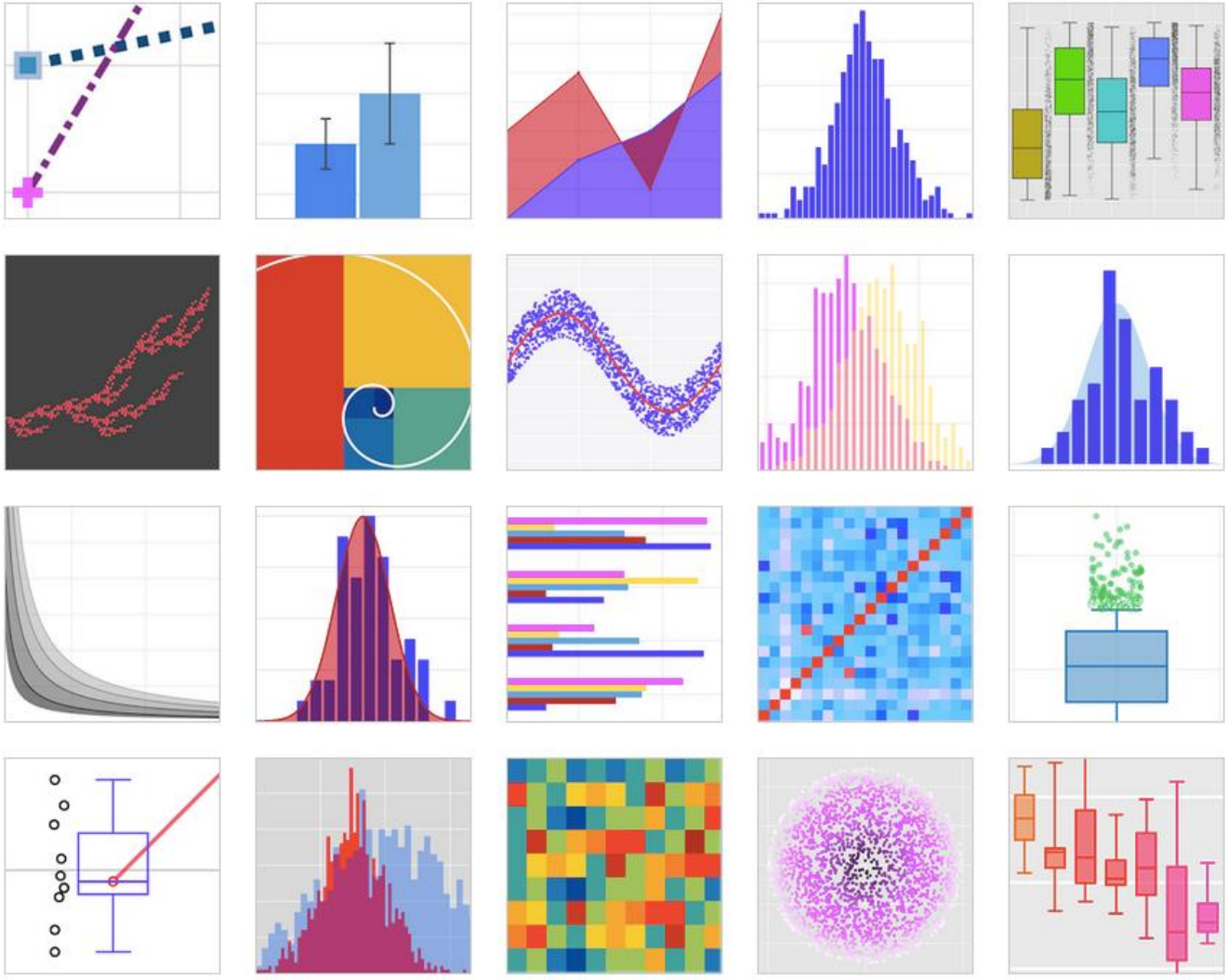
Tensor

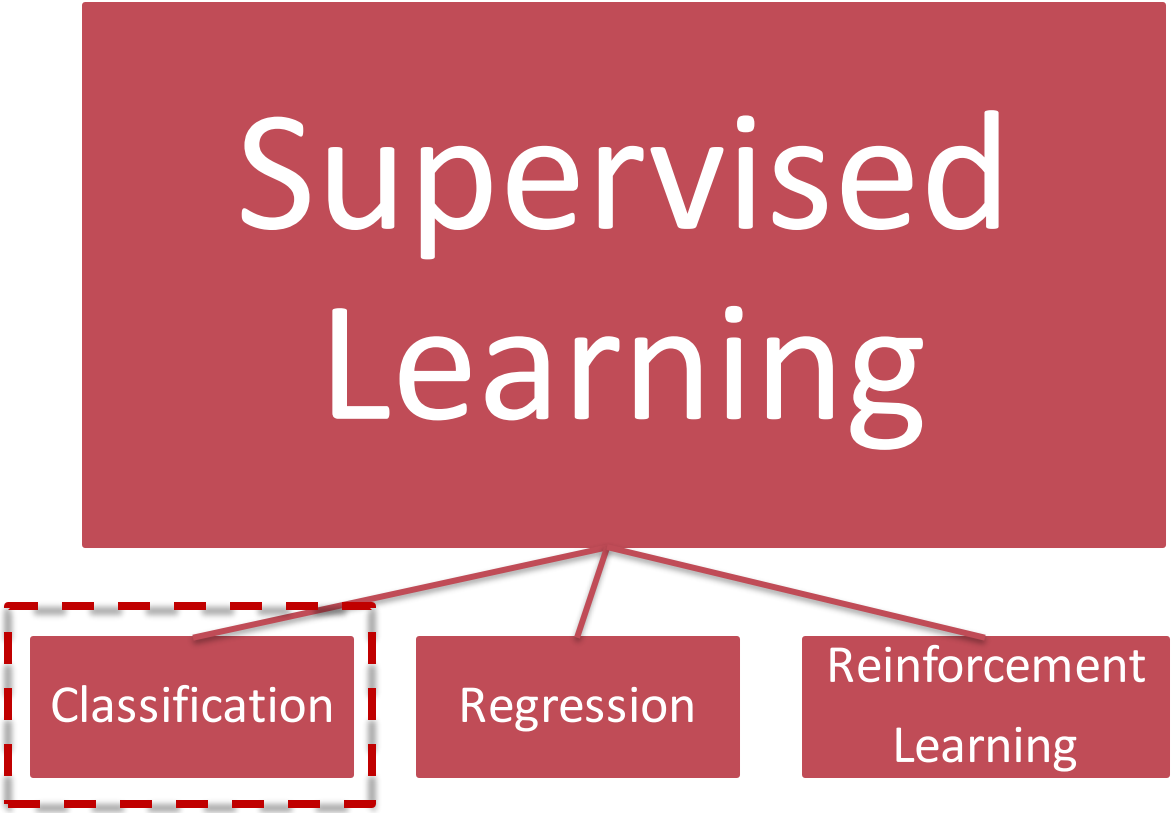
$$\mathbf{X} = \{\mathbf{X}_1, \ldots \mathbf{X}_K\} = \begin{bmatrix} x_{1,1,1} & & x_{N,1,1} \\ \vdots & \cdots & \vdots \\ x_{1,M,1} & & x_{N,M,1} \end{bmatrix} \begin{bmatrix} \ddots & & \\ & & \\ & & \end{bmatrix} \begin{bmatrix} x_{1,1,K} & & x_{N,1,K} \\ \vdots & \cdots & \vdots \\ x_{1,M,K} & & x_{N,M,K} \end{bmatrix}$$

2-d image








Supervised Learning
Classification
Regression
Reinforcement Learning

Classification

Binary

Multi-class

Multi-label

Structure

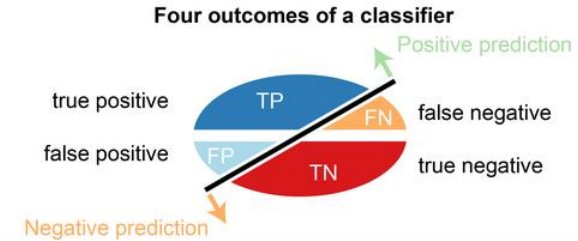
{0,1}

1-of-K

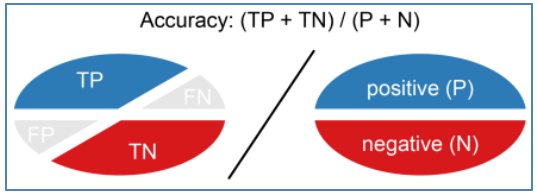
n-of-K

E.g. graph/sequence

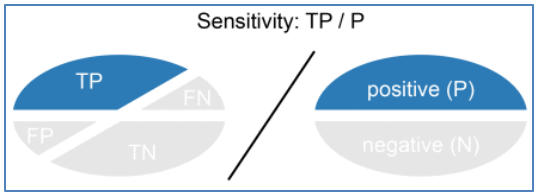
Summary of Measures



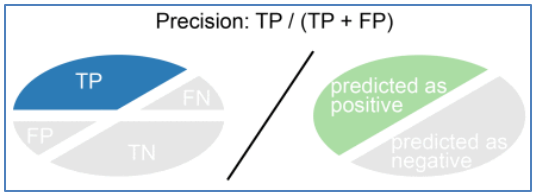
n=165	Predicted: NO	Predicted: YES	
Actual: NO	TN = 50	FP = 10	60
Actual: YES	FN = 5	TP = 100	105
	55	110	



% of correct predictions



% of + class correctly predicted
[aka Recall / TPR]



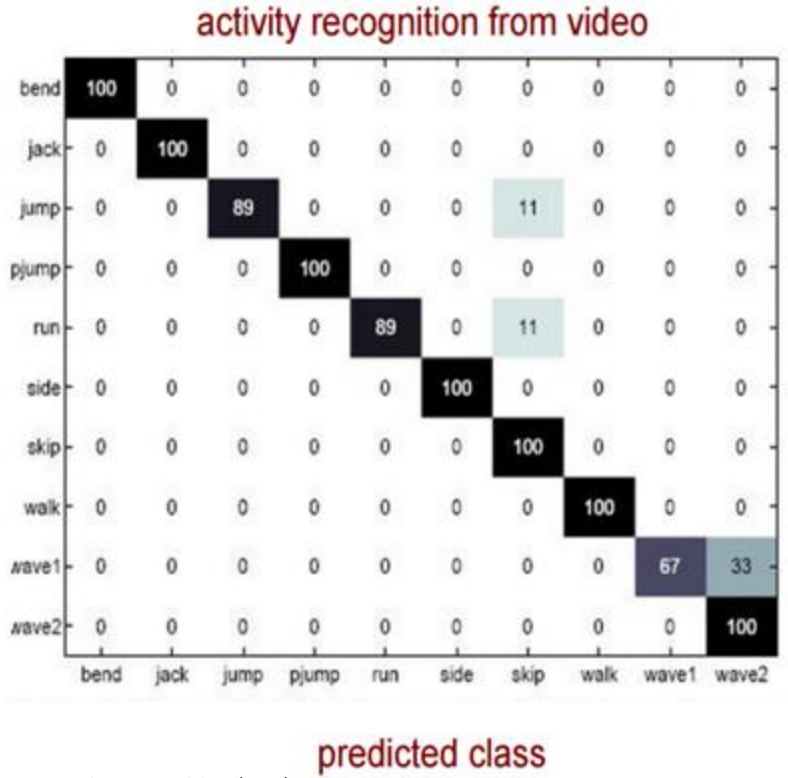
correct prediction of + class

% of – class incorrectly predicted

Multi-class problems - Confusion matrix

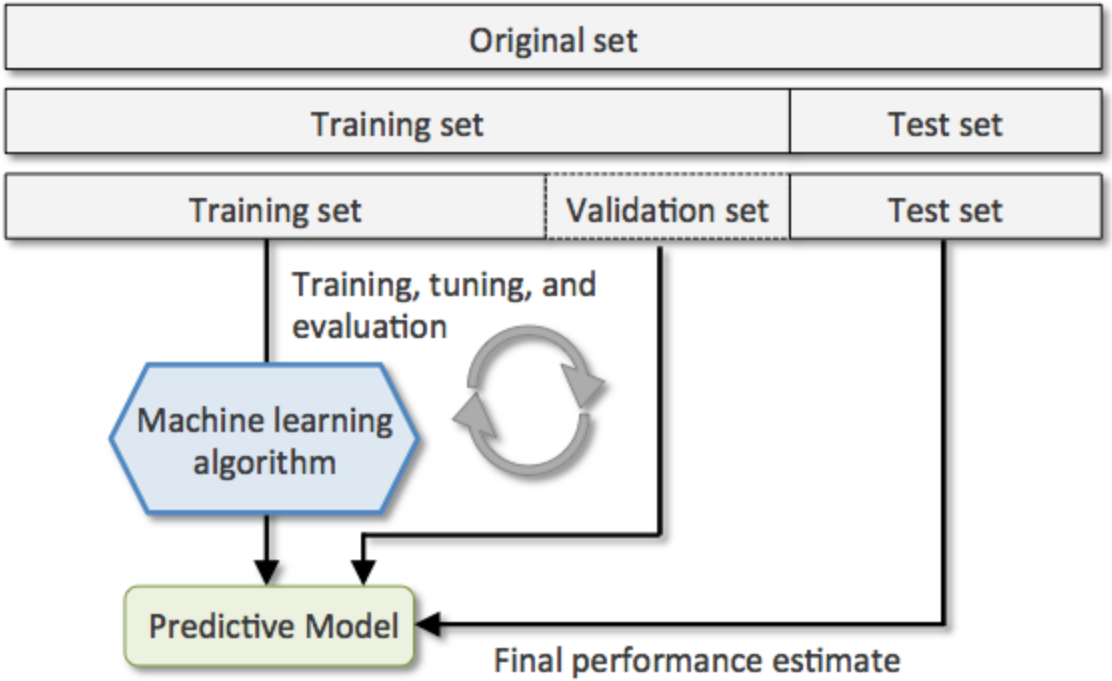
n=165	Predicted: NO	Predicted: YES	
Actual: NO	TN = 50	FP = 10	60
Actual: YES	FN = 5	TP = 100	105
	55	110	

Avg. accuracy may not be very meaningful with imbalanced class label distribution



Courtesy: vision.jhu.edu

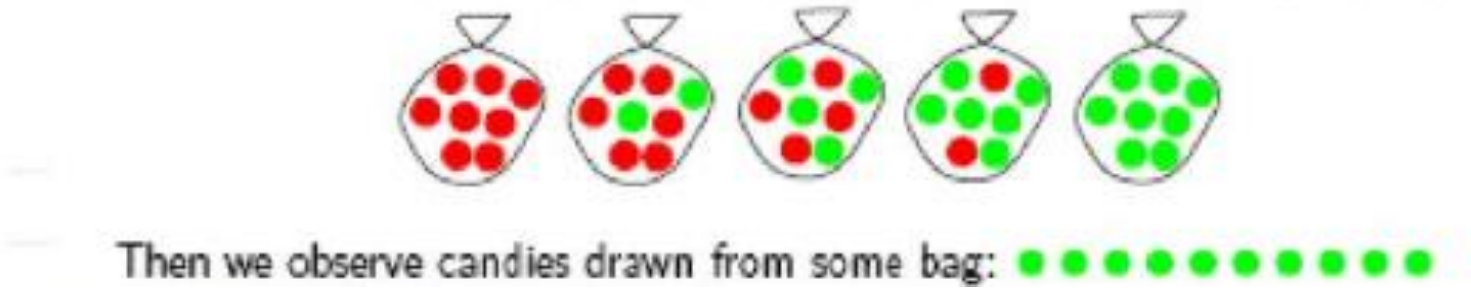
The Train-Validation-Test paradigm



- Decision Trees
- K-NN

Data – a probability-based perspective

- The basis for Statistical Learning Theory

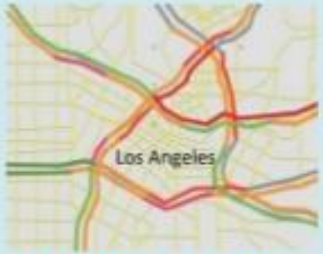


  - Domain described by random variables (r.v.)
    - X = {apple, grape}
    - $b_i \in [1,5]$
  - **Data = Instantiation of some or all r.v.'s in the domain**

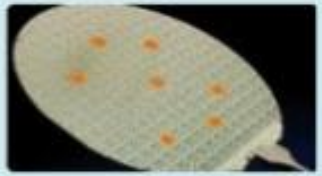
Uncertainty arises from many sources

Process Uncertainty

Processes contain "randomness"



Uncertain travel times



Semiconductor yield

Data Uncertainty

Data input is uncertain

GPS Uncertainty

Testimony {Paris Airport}

Ambiguity

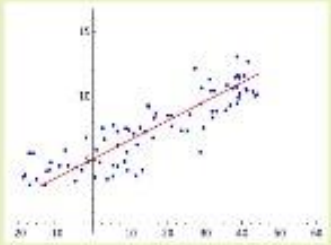
Contaminated?

{John Smith, Dallas}
{John Smith, Kansas}

Rumors Conflicting Data

Model Uncertainty

All modeling is approximate



Fitting a curve to data

Forecasting a hurricane (www.noaa.gov)

Independent vs. Dependent Events

Using the bag of marbles on the left, what is the probability of pulling a black marble two times in a row? $P(black, black)$

When you put 1st marble back in
(*Independent Events*)

$$\frac{2}{10} * \frac{2}{10}$$

$$\frac{1}{5} * \frac{1}{5} = \frac{1}{25}$$

When you KEEP 1st marble
(*Dependent Events*)

$$\frac{2}{10} * \frac{1}{9}$$

$$\frac{1}{5} * \frac{1}{9}$$

$$P(A \text{ and } B) = P(A) \times P(B)$$

$$P(A \text{ and } B) = P(A) \times P(B|A)$$

Probability of B given A

Marginal Probabilities

$$\begin{cases} x = 1 & \text{(Rains)} \\ x = 0 & \text{(Doesn't rain)} \end{cases} \qquad \begin{matrix} Pr(x = 1) = 0.6 \\ Pr(x = 0) = 0.4 \end{matrix}$$

$$\begin{cases} y = 1 & \text{(Have umbrella)} \\ y = 0 & \text{(Don't have umbrella)} \end{cases} \qquad \begin{matrix} Pr(y = 1) = 0.3 \\ Pr(y = 0) = 0.7 \end{matrix}$$

Joint Probability

$\begin{cases} x = 1 & \text{(Rains)} \\ x = 0 & \text{(Doesn't rain)} \end{cases}$ | $Pr(x=1) = 0.6$, $Pr(x=0) = 0.4$

$\begin{cases} y = 1 & \text{(Have umbrella)} \\ y = 0 & \text{(Don't have umbrella)} \end{cases}$ | $Pr(y=1) = 0.3$, $Pr(y=0) = 0.7$

Case 1: Rains but you have an umbrella

$$Pr(x=1, y=1) = Pr(x=1) \times Pr(y=1)$$
$$= 0.6 \times 0.3$$
$$= 0.18$$

Case 2: Rains but you DON'T have an umbrella

$$Pr(x=1, y=0) = Pr(x=1) \times Pr(y=0)$$
$$= 0.6 \times 0.7$$
$$= 0.42$$

$$Pr(x=0) = \sum_{y=0}^{1} Pr(x=0, y)$$
$$= Pr(x=0, y=0) + Pr(x=0, y=1)$$
$$= 0.28 + 0.12 = 0.4$$

Conditional Probability

Given

$x = 1$ (Rains)

↓

What's the Probability of

$y = 1$ (Bring umbrella)

$\begin{cases} x = 1 & \text{(Rains)} \\ x = 0 & \text{(Doesn't rain)} \end{cases}$

$\begin{cases} y = 1 & \text{(Have umbrella)} \\ y = 0 & \text{(Don't have umbrella)} \end{cases}$

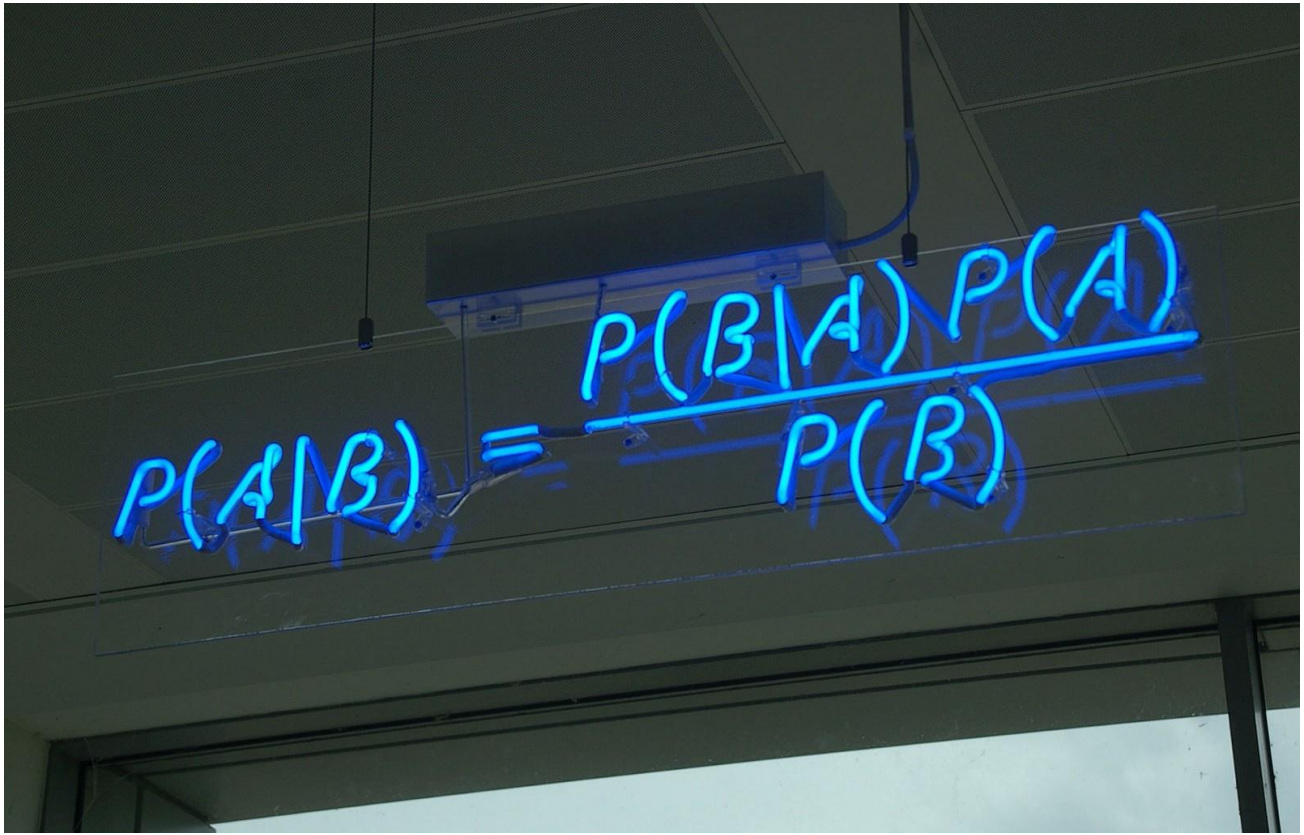
$Pr(x = 1) = 0.6$

$Pr(x = 0) = 0.4$

$Pr(y = 1) = 0.3$

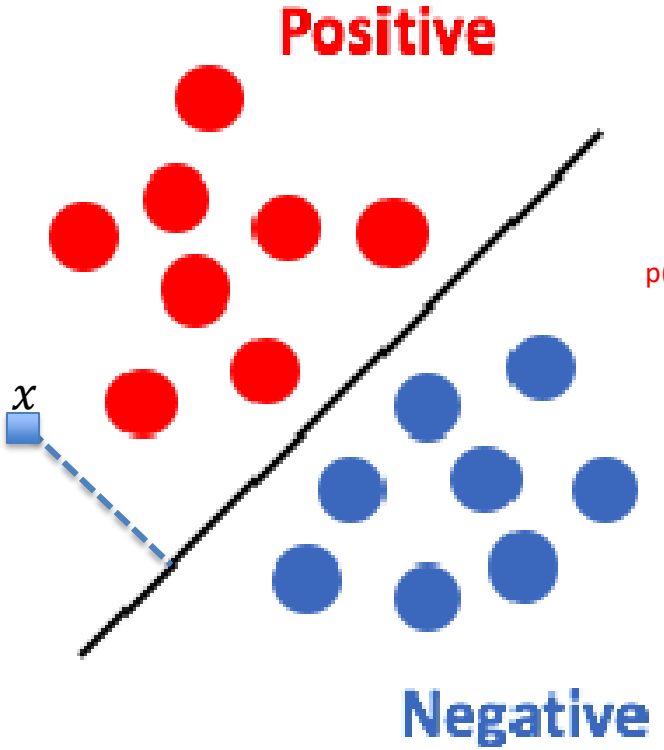
$Pr(y = 0) = 0.7$

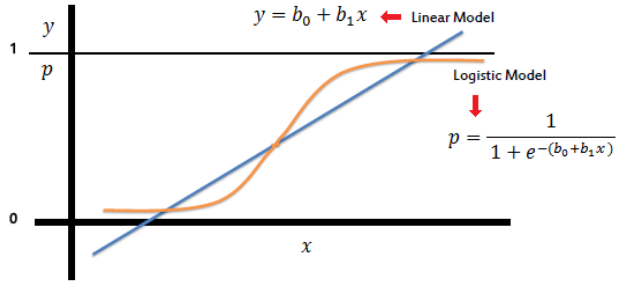
Bayes' Rule



- Decision Trees
- K-NN
- Naïve Bayes

Logistic Regression - Intuition





p(X) = p(Y=1|x) = probability that x belongs to positive class

$$\Longrightarrow p(X) = \frac{e^{(\beta_o + \beta_1 x)}}{e^{(\beta_o + \beta_1 x)} + 1}$$

$$\Longrightarrow p(e^{(\beta_o + \beta_1 x)} + 1) = e^{(\beta_o + \beta_1 x)}$$

$$\Longrightarrow p.e^{(\beta_o + \beta_1 x)} + p = e^{(\beta_o + \beta_1 x)}$$

$$\Longrightarrow p = e^{(\beta_o + \beta_1 x)} - p.e^{(\beta_o + \beta_1 x)}$$

$$\Longrightarrow p = e^{(\beta_o + \beta_1 x)}(1 - p)$$

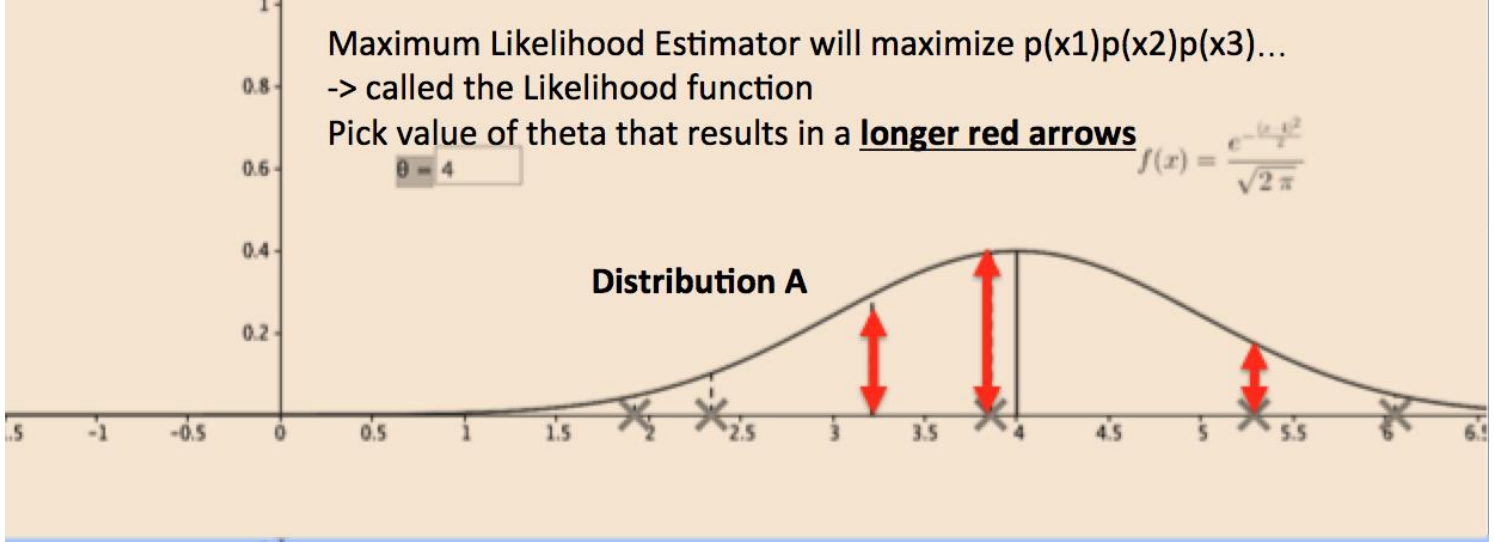
$$\Longrightarrow \frac{p}{1 - p} = e^{(\beta_o + \beta_1 x)}$$

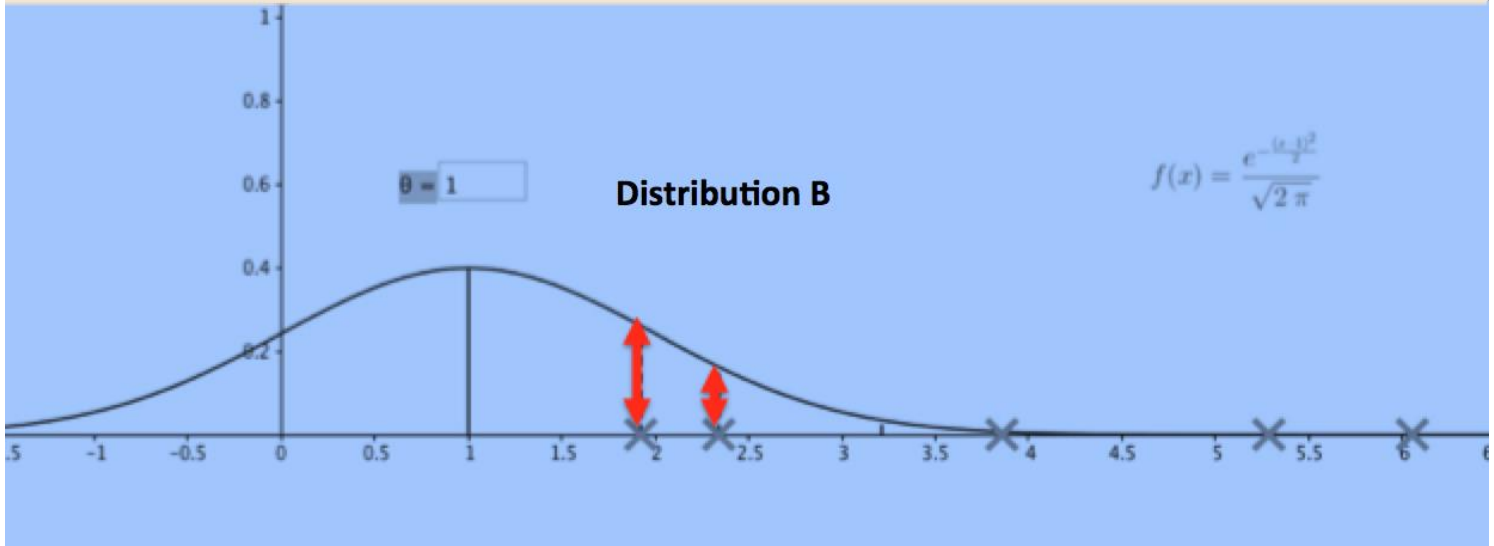
$$\Longrightarrow \ln(\tfrac{p}{1-p}) = \beta_0 + \beta_1 x$$

Distance of x from boundary

- Decision Trees
- K-NN
- Naïve Bayes
- Logistic Regression

Maximum Likelihood Estimator will maximize p(x1)p(x2)p(x3)...
-> called the Likelihood function
Pick value of theta that results in a **longer red arrows**





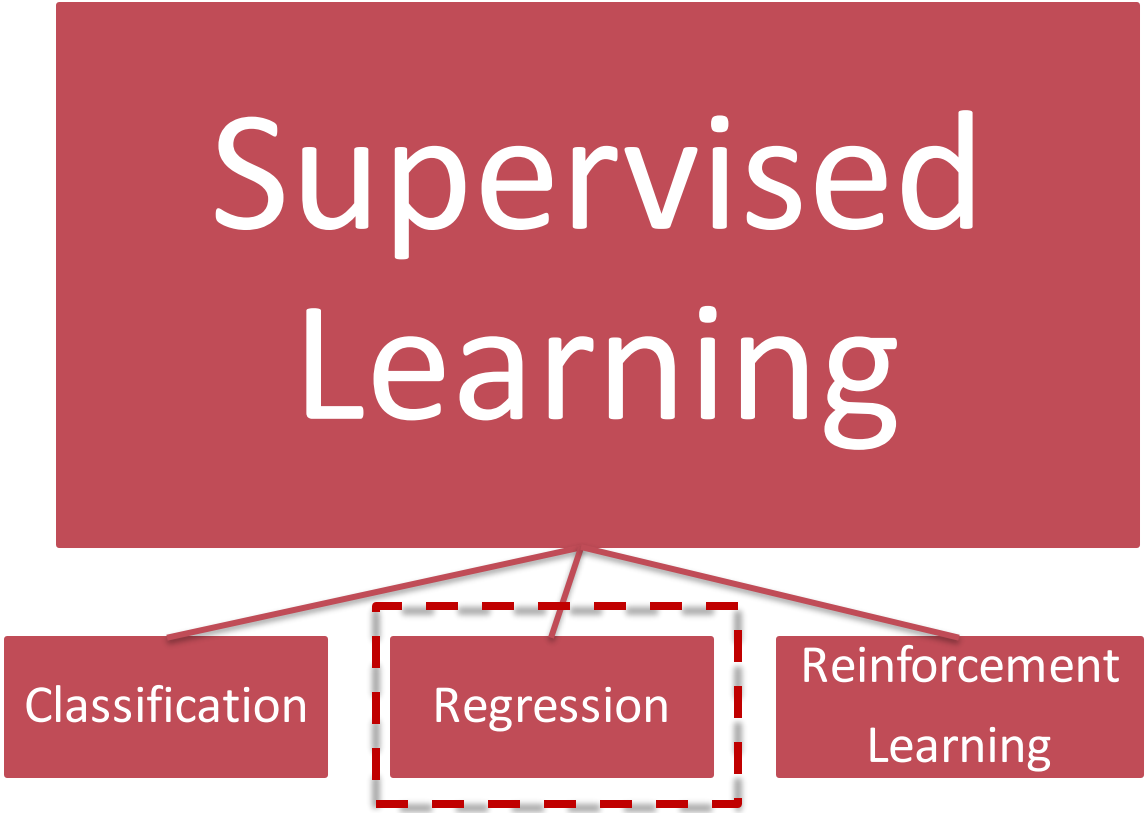
Taxonomy of classifiers : a modelling perspective

	Probabilistic	Non-Probabilistic
Discriminative	• Logistic Regression • Probabilistic neural nets • ……..	• K-nn • Linear classifier • SVM • Neural networks • ……
Generative	• Naïve Bayes • Model-based (e.g., GMM) • ……	

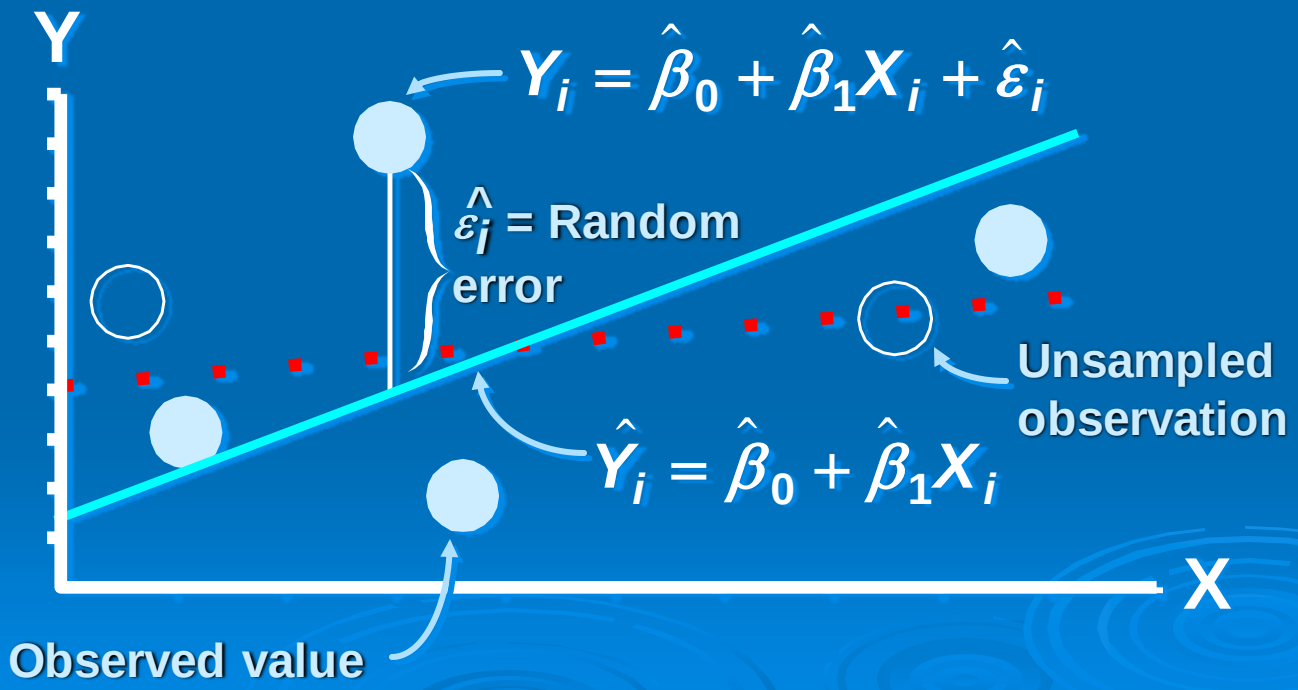
Model $p(class = C|x)$

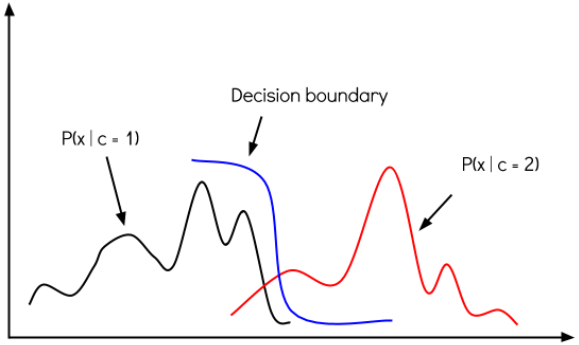
Model the classification rule directly

Model $p(x\,|class = C)$


Supervised Learning
Classification
Regression
Reinforcement Learning

Sample Linear Regression Model

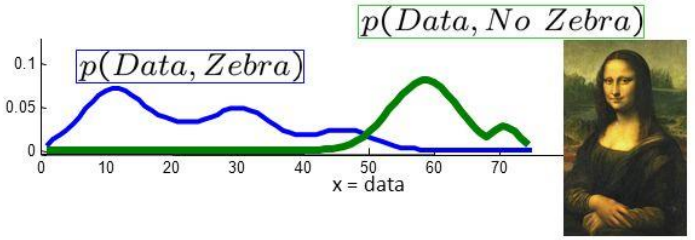




Discriminative vs. generative

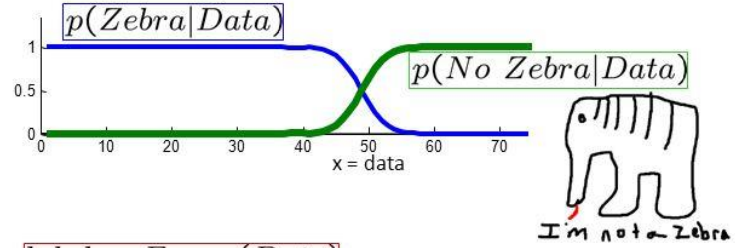
- Generative model

(*The artist*)

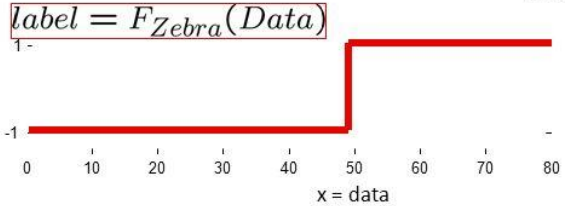


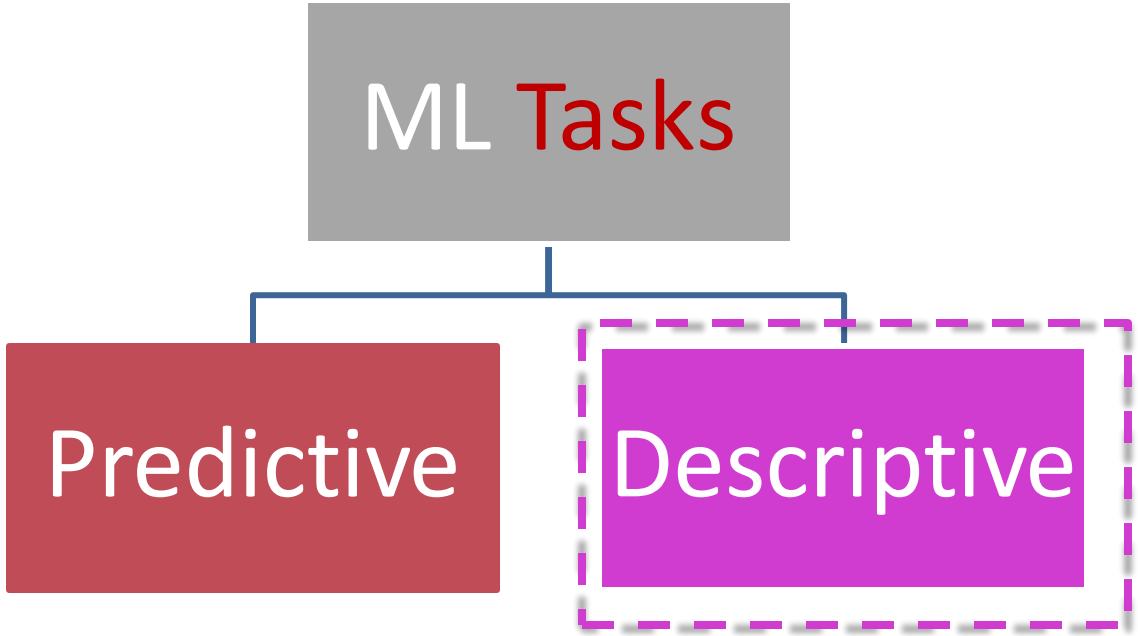
- Discriminative model

(The lousy painter)



- Classification function




ML Tasks
Predictive
Descriptive

ML::Tasks → Descriptive

- Study/Exploit the ‘structure’ of data
  - Density Estimation
  - Clustering
  - Dimensionality Reduction
- Also studied as ‘Unsupervised Learning’
  - ‘Input’ data without paired ‘Output’

$\{x^{(1)}, \ldots, x^{(m)}\}$ $\quad x^{(i)} \in \mathbb{R}^n$

The k-means clustering algorithm is as follows:

1. Initialize **cluster centroids** $\mu_1, \mu_2, \ldots, \mu_k \in \mathbb{R}^n$ randomly.
2. Repeat until convergence: {

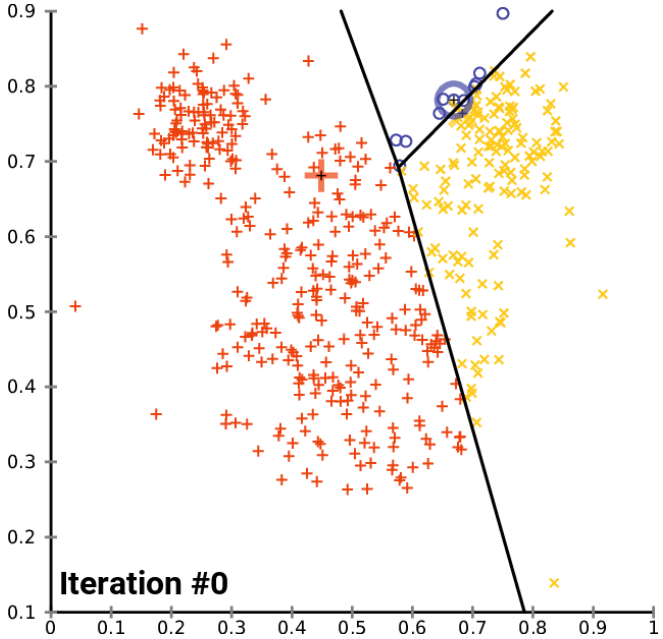
   For every i, set

   $$c^{(i)} := \arg\min_j ||x^{(i)} - \mu_j||^2.$$

   For each j, set

   $$\mu_j := \frac{\sum_{i=1}^m 1\{c^{(i)} = j\} x^{(i)}}{\sum_{i=1}^m 1\{c^{(i)} = j\}}.$$

   }



EXPECTATION MAXIMIZATION

$\{x^{(1)}, \ldots, x^{(m)}\}$ $\quad x^{(i)} \in \mathbb{R}^n$

The k-means clustering algorithm is as follows:

1. Initialize **cluster centroids** $\mu_1, \mu_2, \ldots, \mu_k \in \mathbb{R}^n$ randomly.

2. Repeat until convergence: {

   For every i, set

$$c^{(i)} := \arg\min_j ||x^{(i)} - \mu_j||^2.$$

   For each j, set

$$\mu_j := \frac{\sum_{i=1}^m 1\{c^{(i)} = j\} x^{(i)}}{\sum_{i=1}^m 1\{c^{(i)} = j\}}.$$

   }

E

Assignment step: Assign each data point to the closest cluster

M

Refitting step: Move each cluster center to the center of the data assigned to it

GMM - Algorithm

- Initialize the means μ_k, covariances Σ_k and mixing coefficients π_k
- Iterate until convergence:
  - E-step: Evaluate the responsibilities given current parameters

$$\gamma_k^{(n)} = p(z^{(n)}|\mathbf{x}) = \frac{\pi_k \mathcal{N}(\mathbf{x}^{(n)}|\mu_k, \Sigma_k)}{\sum_{j=1}^{K} \pi_j \mathcal{N}(\mathbf{x}^{(n)}|\mu_j, \Sigma_j)}$$

  - M-step: Re-estimate the parameters given current responsibilities

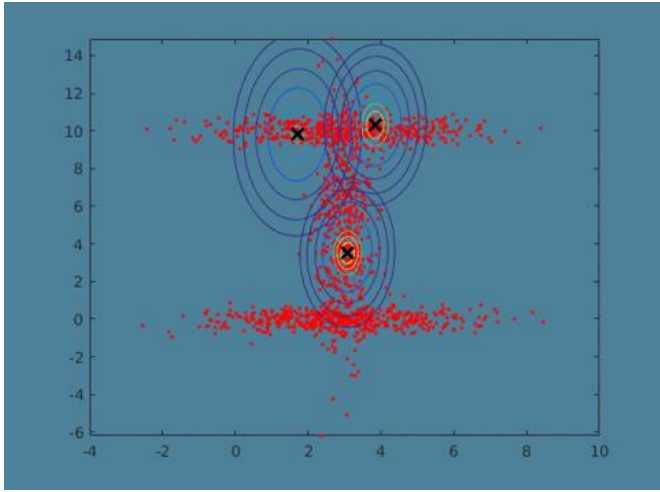
$$\mu_k = \frac{1}{N_k} \sum_{n=1}^{N} \gamma_k^{(n)} \mathbf{x}^{(n)}$$

$$\Sigma_k = \frac{1}{N_k} \sum_{n=1}^{N} \gamma_k^{(n)} (\mathbf{x}^{(n)} - \mu_k)(\mathbf{x}^{(n)} - \mu_k)^T$$

$$\pi_k = \frac{N_k}{N} \quad \text{with} \quad N_k = \sum_{n=1}^{N} \gamma_k^{(n)}$$

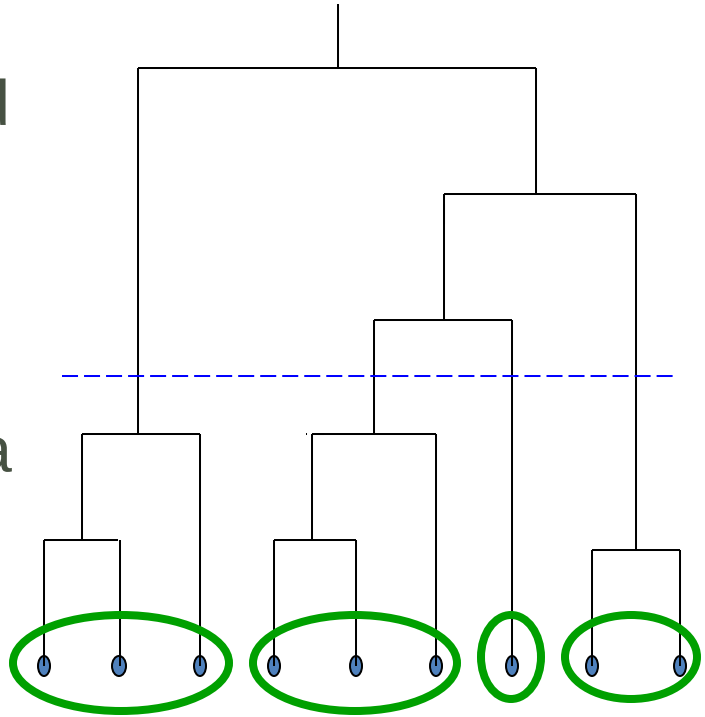
  - Evaluate log likelihood and check for convergence

$$\ln p(\mathbf{X}|\pi, \mu, \Sigma) = \sum_{n=1}^{N} \ln \left(\sum_{k=1}^{K} \pi_k \mathcal{N}(\mathbf{x}^{(n)}|\mu_k, \Sigma_k) \right)$$



Dendogram: Hierarchical Clustering

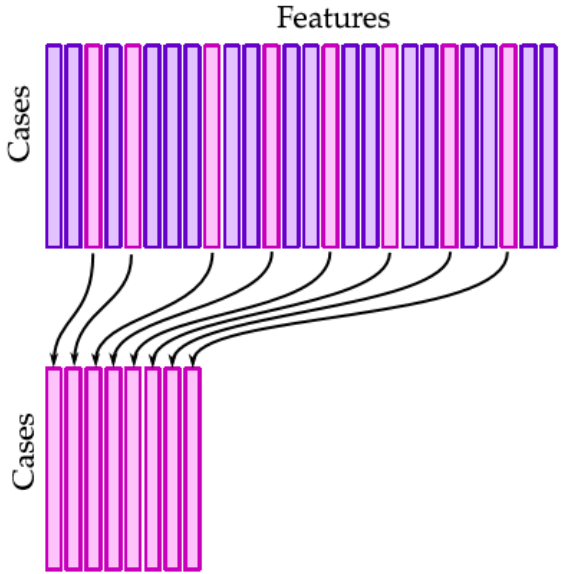
- Clustering obtained by cutting the dendrogram at a desired level: each connected component forms a cluster.



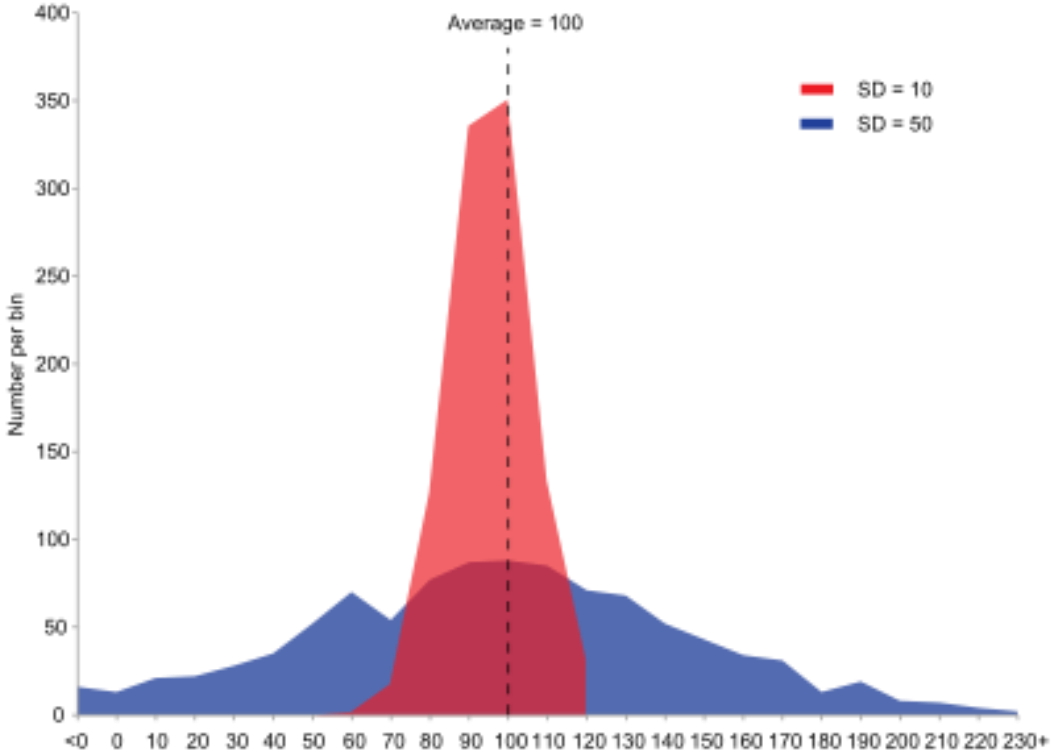
Feature Selection

- when we have a regular data set ($|T| > D$), but too flexible model, and/or
- when we have a high-dimensional data set with not enough data ($|T| < D$).

Data sets with thousands or millions of variables (features) are quite usual these days: we want to choose only those, which are needed to *construct simpler, faster, and more accurate models*.



Variance, Covariance and Correlation



Population

Variance:

$$s^2 = \frac{\sum(\overline{X} - X_i)^2}{N}$$

Standard Deviation:

$$s = \sqrt{\frac{\sum(\overline{X} - X_i)^2}{N}}$$

Sample Variance

$$s^2 = \frac{\sum(x - \bar{x})^2}{n - 1}$$

Sample Standard Deviation

$$s = \sqrt{\frac{\sum(x - \bar{x})^2}{n - 1}}$$

Covariance

Vectors 1 and 3

Cell (3, 1) or (1, 3)

$$A = \begin{bmatrix} 1 & 3 & 5 \\ 5 & 4 & 1 \\ 3 & 8 & 6 \end{bmatrix}$$

Covariance

$$\begin{bmatrix} 2.67 & 0.67 & -2.67 \\ 0.67 & 4.67 & 2.33 \\ -2.67 & 2.33 & 4.67 \end{bmatrix}$$

Covariance Matrix

Variance:

$$s^2 = \frac{\sum (\overline{X} - X_i)^2}{N}$$

$$cov(\boldsymbol{X}, \boldsymbol{Y}) = \frac{1}{n} \sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})$$

$$\begin{bmatrix} 0.39701 & 0.51117 \\ 0.55582 & 0.93003 \\ 0.59403 & 0.96645 \\ 0.51544 & 0.29759 \\ 0.85313 & 0.18118 \\ 0.88564 & 0.69114 \end{bmatrix}$$

$$\begin{pmatrix} & M1 & M2 & M3 & \dots & Mn \\ S1 & q_{1,1} & q_{1,2} & q_{1,3} & \dots & q_{1,n} \\ S2 & q_{2,1} & q_{2,2} & q_{2,3} & \dots & q_{2,n} \\ S3 & q_{3,1} & q_{3,2} & q_{3,3} & \dots & q_{3,n} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ Sm & q_{m,1} & q_{m,2} & q_{m,3} & & q_{m,n} \end{pmatrix}$$

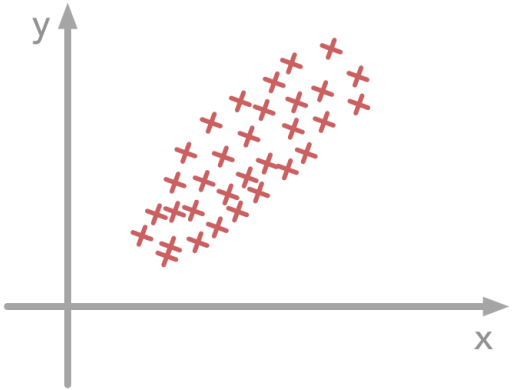
$$Cov(M_a, M_b) = \frac{1}{m} \sum_{i=1}^{m} (q_{i,a} - \overline{q_a})(q_{i,b} - \overline{q_b})$$

$$C = \begin{pmatrix} \operatorname{cov}(M_1, M_1) & \operatorname{cov}(M_1, M_2) & \cdots & \operatorname{cov}(M_1, M_n) \\ \operatorname{cov}(M_2, M_1) & \operatorname{cov}(M_2, M_2) & \cdots & \operatorname{cov}(M_2, M_n) \\ \vdots & \vdots & \ddots & \vdots \\ \operatorname{cov}(M_n, M_1) & \operatorname{cov}(M_n, M_2) & \cdots & \operatorname{cov}(M_n, M_n) \end{pmatrix}_{n \times n}$$

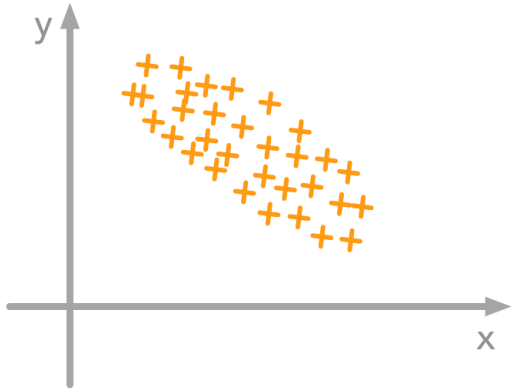
n-dimensional Covariance Matrix

Covariance

Positive covariance



Negative covariance



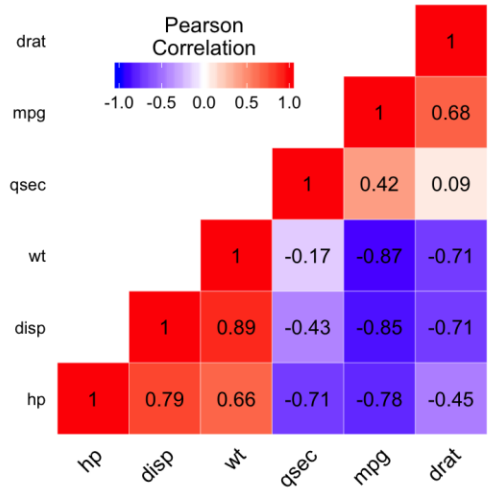
Variance:

$$s^2 = \frac{\sum (\overline{X} - X_i)^2}{N}$$

$$cov(\boldsymbol{X}, \boldsymbol{Y}) = \frac{1}{n} \sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})$$

Correlation

$$cov(\boldsymbol{X},\boldsymbol{Y}) = \frac{1}{n}\sum_{i=1}^{n}(x_i-\bar{x})(y_i-\bar{y})$$



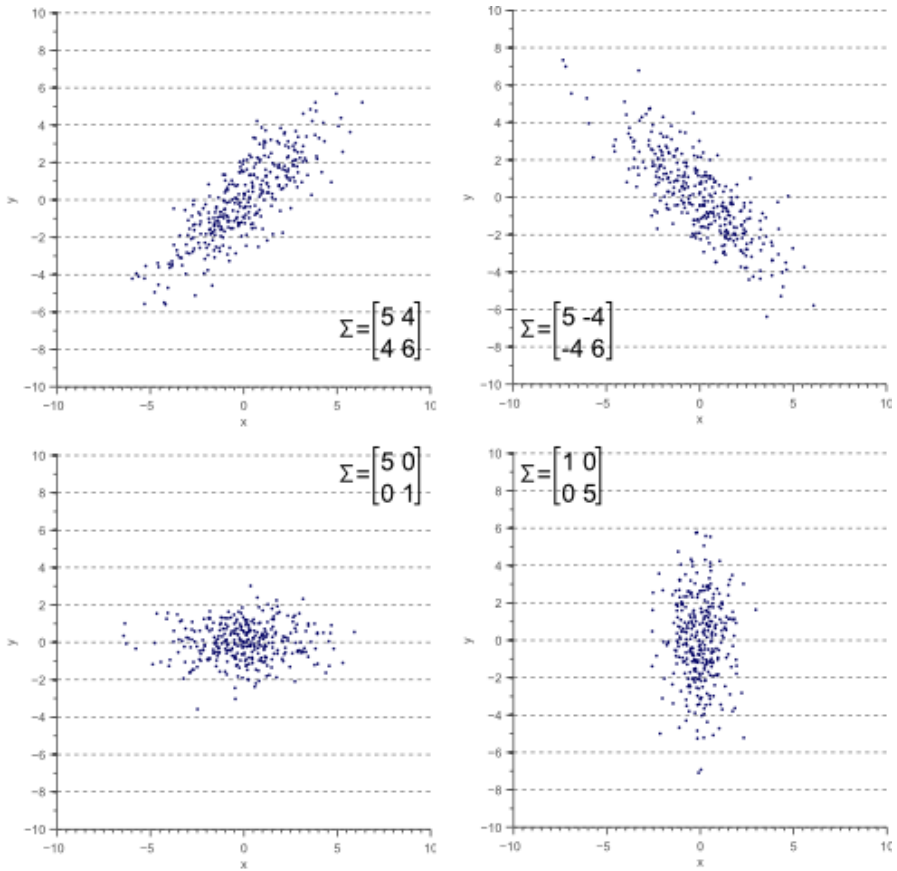
$$r_{xy} = \frac{\sum_{i=1}^{n}(x_i-\bar{x})(y_i-\bar{y})}{\sqrt{\sum_{i=1}^{n}(x_i-\bar{x})^2\sum_{i=1}^{n}(y_i-\bar{y})^2}}$$

$$\rho_{AB} = \frac{\mathrm{cov}_{AB}}{\sigma_A\sigma_B}$$

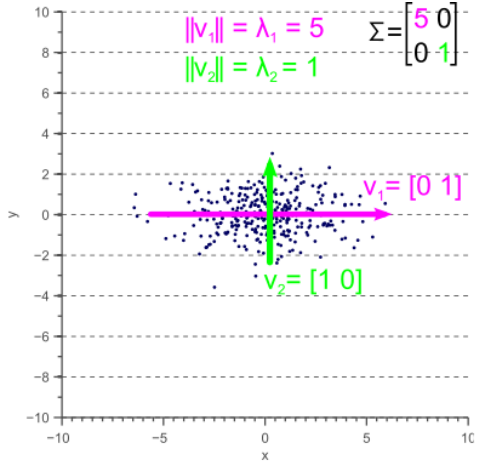
$$r = \frac{1}{n-1}\sum_{i=1}^{n}\left(\frac{x_i-\bar{x}}{s_x}\right)\left(\frac{y_i-\bar{y}}{s_y}\right)$$

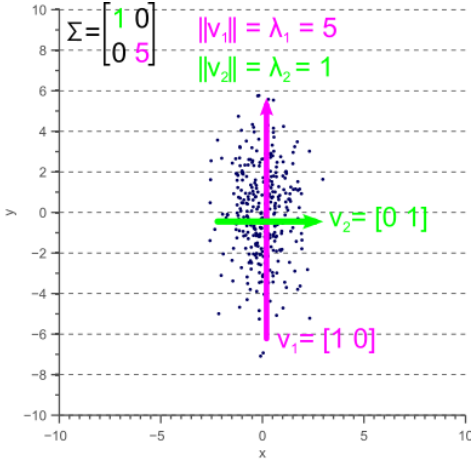
$$\boldsymbol{X}' = \frac{\boldsymbol{X}-\bar{x}}{\sigma_{\boldsymbol{X}}}$$

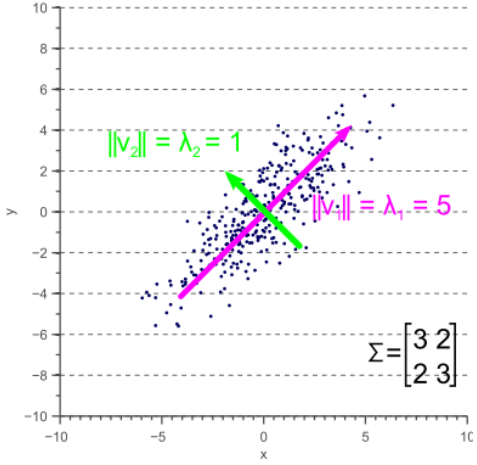
Covariance Matrix encodes spread and orientation of data

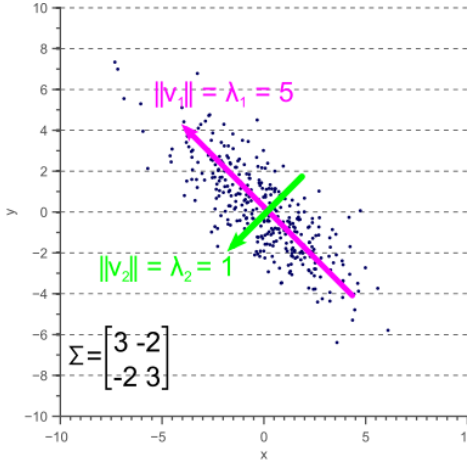


$$\Sigma\vec{v} = \lambda\vec{v}$$

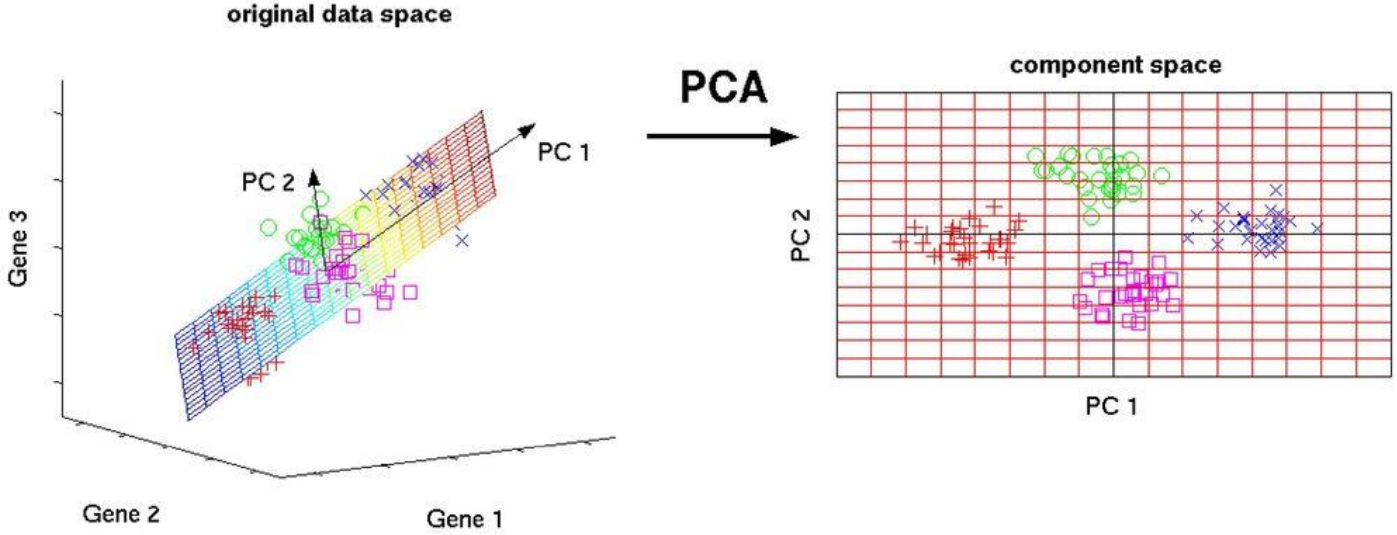




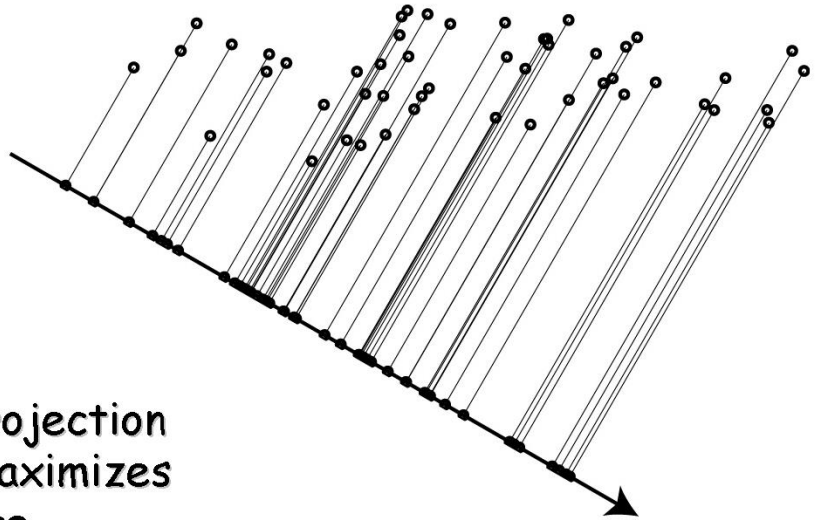




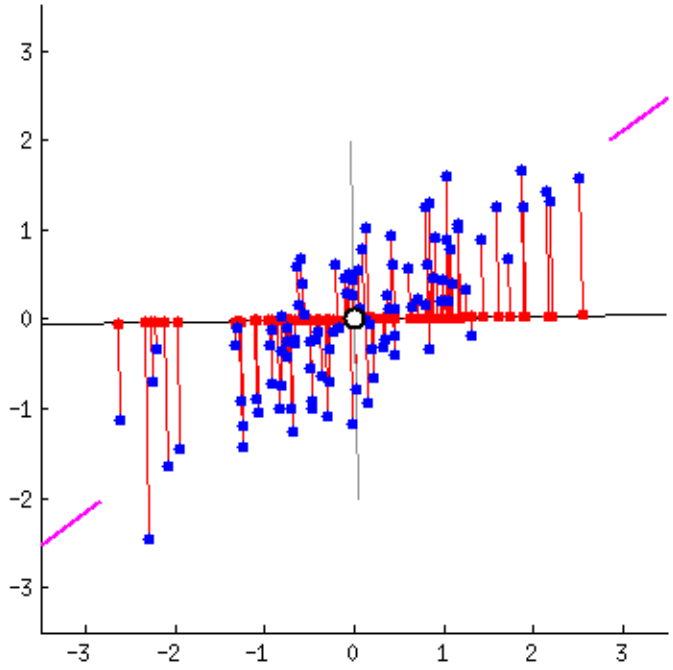
PCA finds new variables which are linear combinations of the original variables such that in the new space, the data has fewer dimensions.



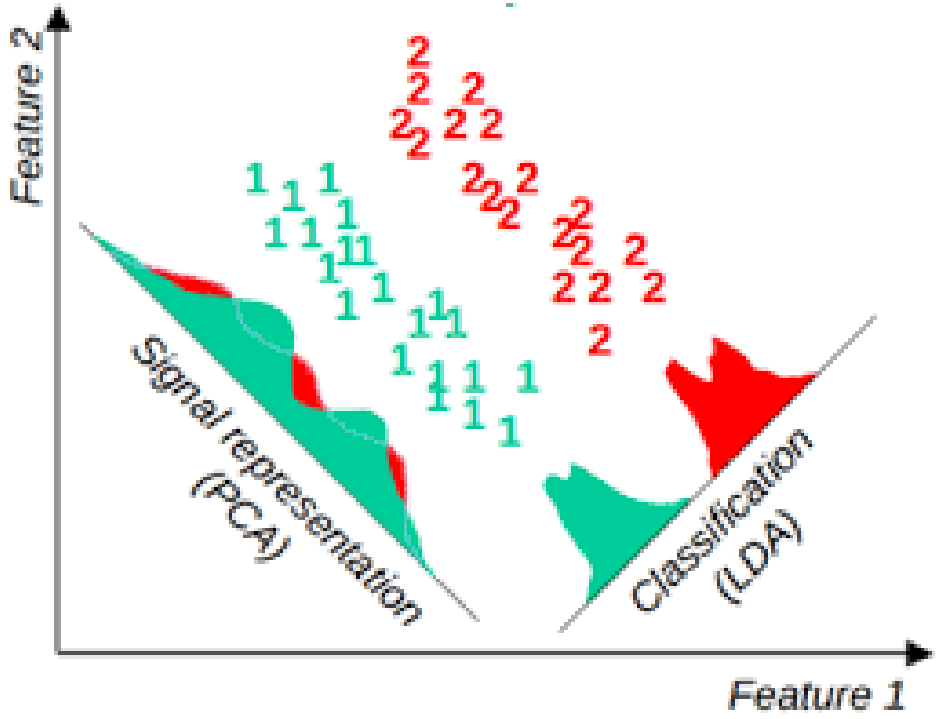
One-dimensional projection



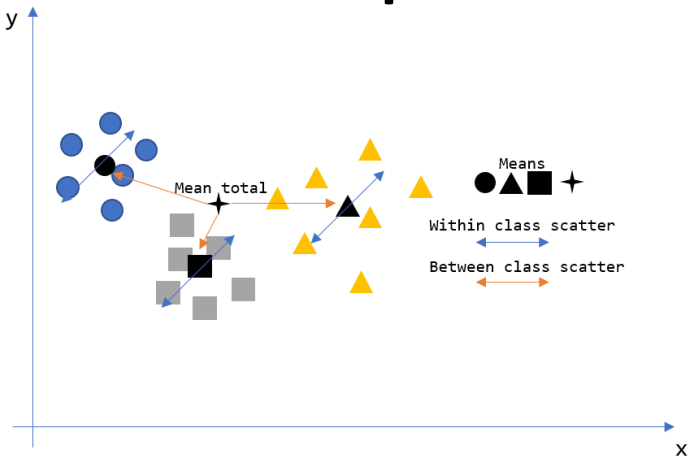
find projection
that maximizes
variance


3
2
1
0
-1
-2
-3
-3
-2
-1
0
1
2
3

Linear Discriminant Analysis (LDA)



Multiple Discriminant Analysis (MDA)



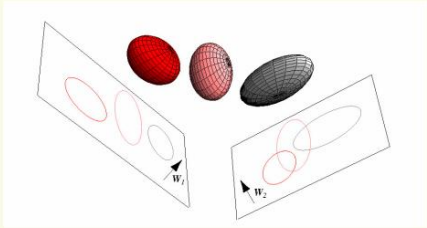
mean vector for each class: $\mathbf{m}_c = \frac{\sum_{i=1}^{N_c} \mathbf{x}_i}{N_c} \in R^{d\times 1}$

total mean vector: $\mathbf{m} = \frac{\sum_{i=1}^{N} \mathbf{x}_i}{N} \in R^{d\times 1}$

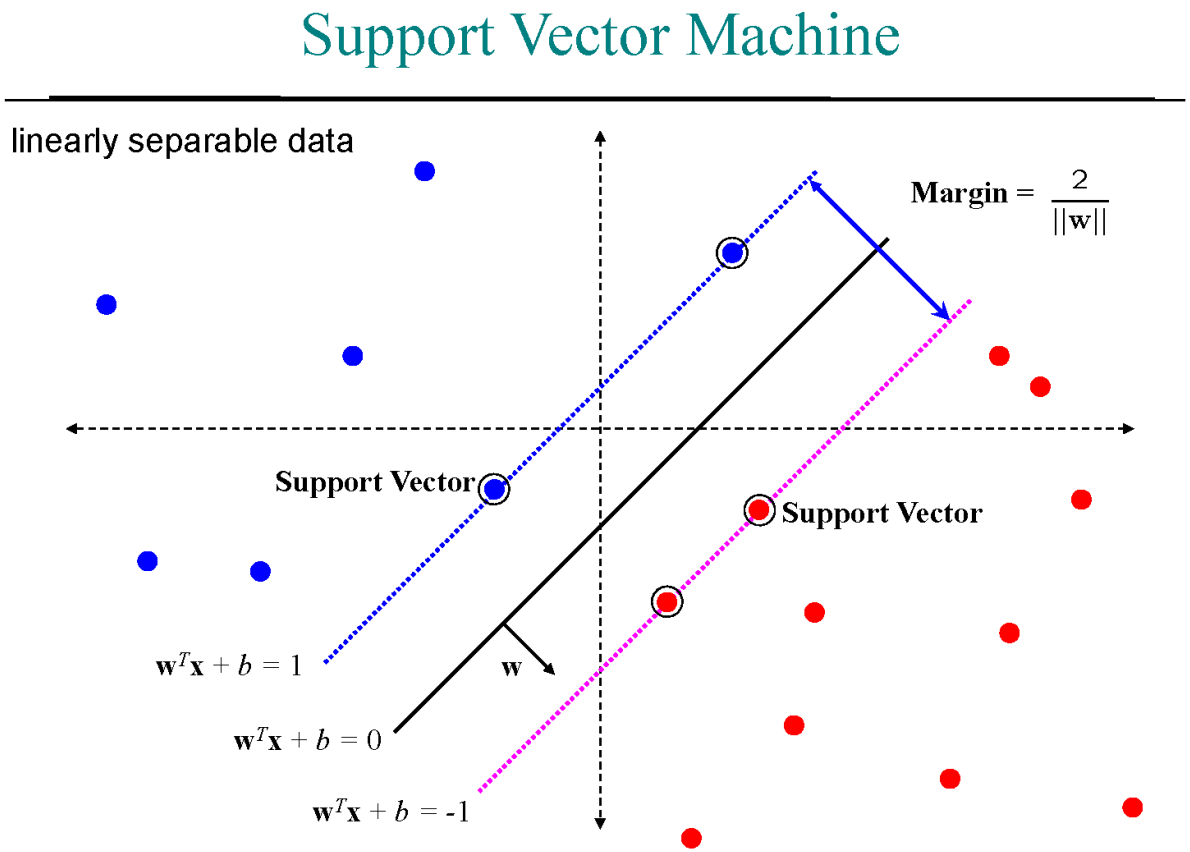
within-class scatter: $S_w = \sum_{c=1}^{C} \sum_{j=1}^{N_c} (\mathbf{x}_j - \mathbf{m}_c)(\mathbf{x}_j - \mathbf{m}_c)^T \in R^{d\times d}$

between-class scatter: $S_b = \sum_{c=1}^{C} N_c(\mathbf{m}_c - \mathbf{m})(\mathbf{m}_c - \mathbf{m})^T \in R^{d\times d}$

- Can generalize FLD to multiple classes
- In case of ***c*** classes, can reduce dimensionality to 1, 2, 3,…, ***c***-1 dimensions
- Project sample $\boldsymbol{x_i}$ to a linear subspace $\boldsymbol{y_i = V^t x_i}$
  - ***V*** is called projection matrix



How to find support vectors ?



SVM – Optimization

- Learning the SVM can be formulated as an optimization:

$$\max_{\mathbf{w}} \frac{2}{||\mathbf{w}||} \text{ subject to } \mathbf{w}^\top \mathbf{x}_i + b \begin{array}{ll} \geq 1 & \text{if } y_i = +1 \\ \leq -1 & \text{if } y_i = -1 \end{array} \text{ for } i = 1 \ldots N$$

- Or equivalently

$$\min_{\mathbf{w}} ||\mathbf{w}||^2 \text{ subject to } y_i \left(\mathbf{w}^\top \mathbf{x}_i + b \right) \geq 1 \text{ for } i = 1 \ldots N$$

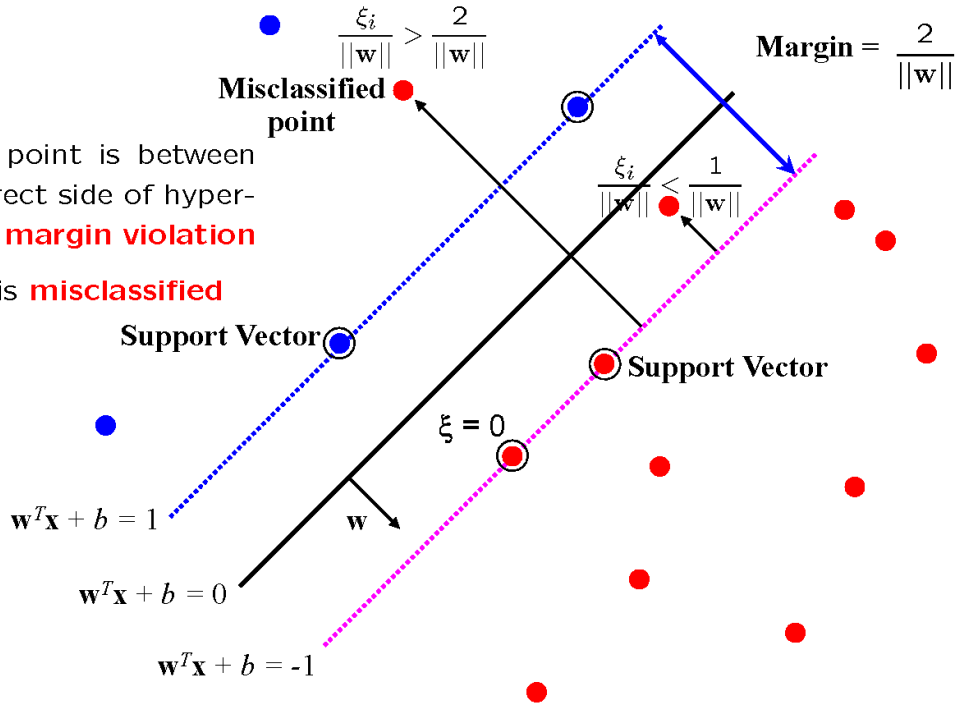
- This is a quadratic optimization problem subject to linear constraints and there is a unique minimum

Quadratic programming in python: https://scaron.info/blog/quadratic-programming-in-python.html

Introduce "slack" variables

$\xi_i \geq 0$

- for $0 < \xi \leq 1$ point is between margin and correct side of hyper-plane. This is a **margin violation**
- for $\xi > 1$ point is **misclassified**



"Soft" margin solution

The optimization problem becomes

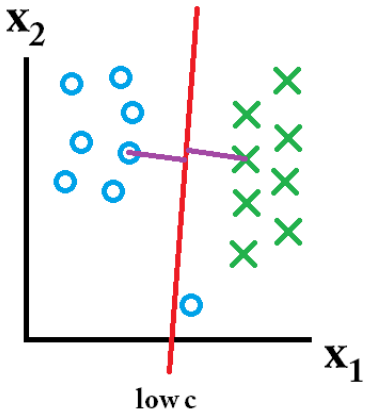
$$\min_{\mathbf{w}\in\mathbb{R}^d,\xi_i\in\mathbb{R}+} ||\mathbf{w}||^2 + C\sum_i^N \xi_i$$

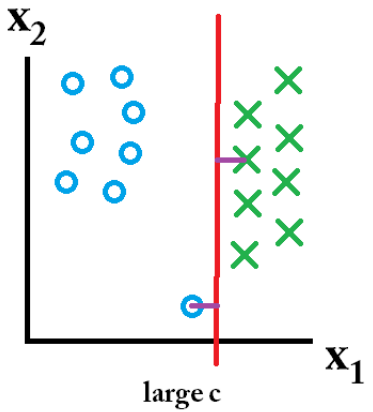
subject to

$$y_i\left(\mathbf{w}^\top \mathbf{x}_i + b\right) \geq 1 - \xi_i \text{ for } i = 1\ldots N$$

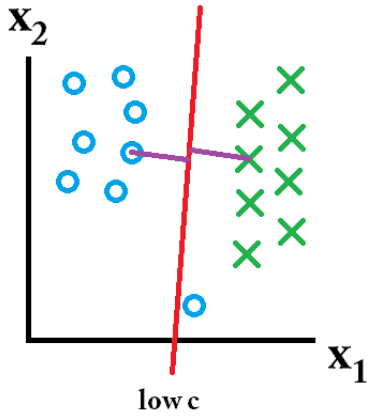
- Every constraint can be satisfied if ξ_i is sufficiently large
- C is a regularization parameter:
  - small C allows constraints to be easily ignored → large margin
  - large C makes constraints hard to ignore → narrow margin
  - $C = \infty$ enforces all constraints: hard margin
- This is still a quadratic optimization problem and there is a unique minimum. Note, there is only one parameter, C.

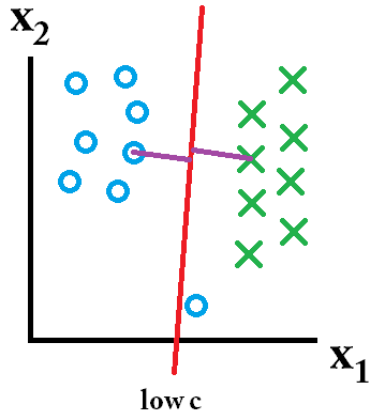
C = 0 ???

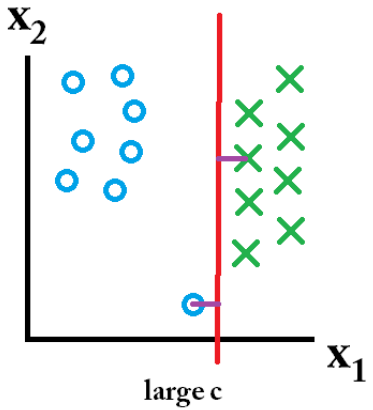


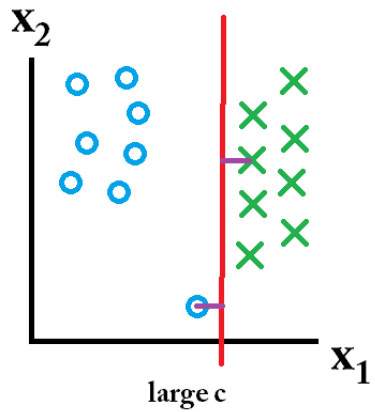


NOTE: Best margin = That which generalizes to ‘unseen’ data







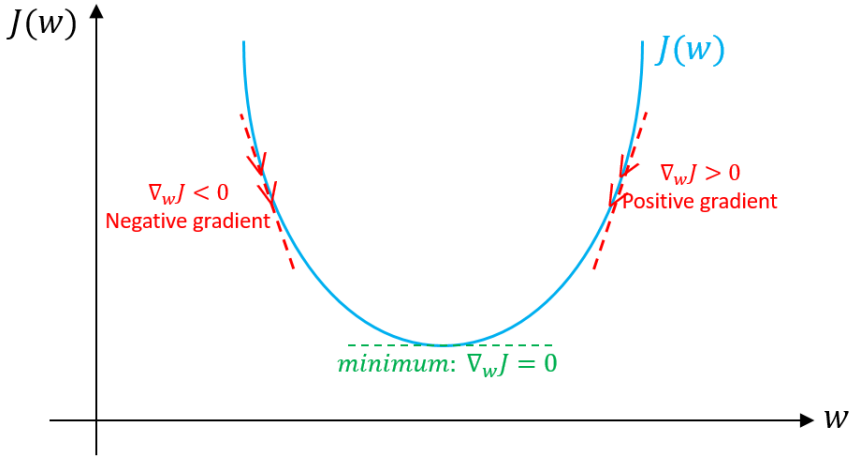


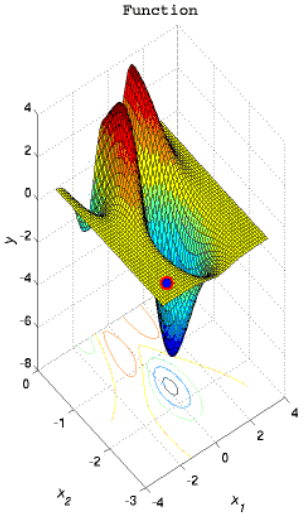
Gradient (or steepest) descent algorithm for SVM

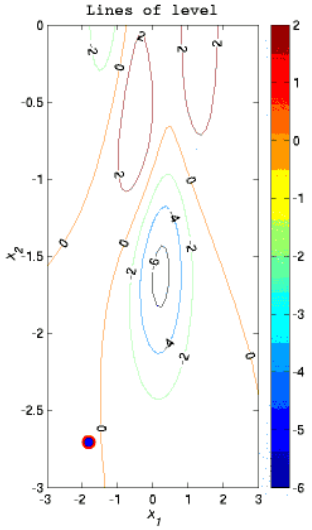
To minimize a cost function $\mathcal{C}(\mathbf{w})$ use the iterative update

$$\mathbf{w}_{t+1} \leftarrow \mathbf{w}_t - \eta_t \nabla_{\mathbf{w}} \mathcal{C}(\mathbf{w}_t)$$

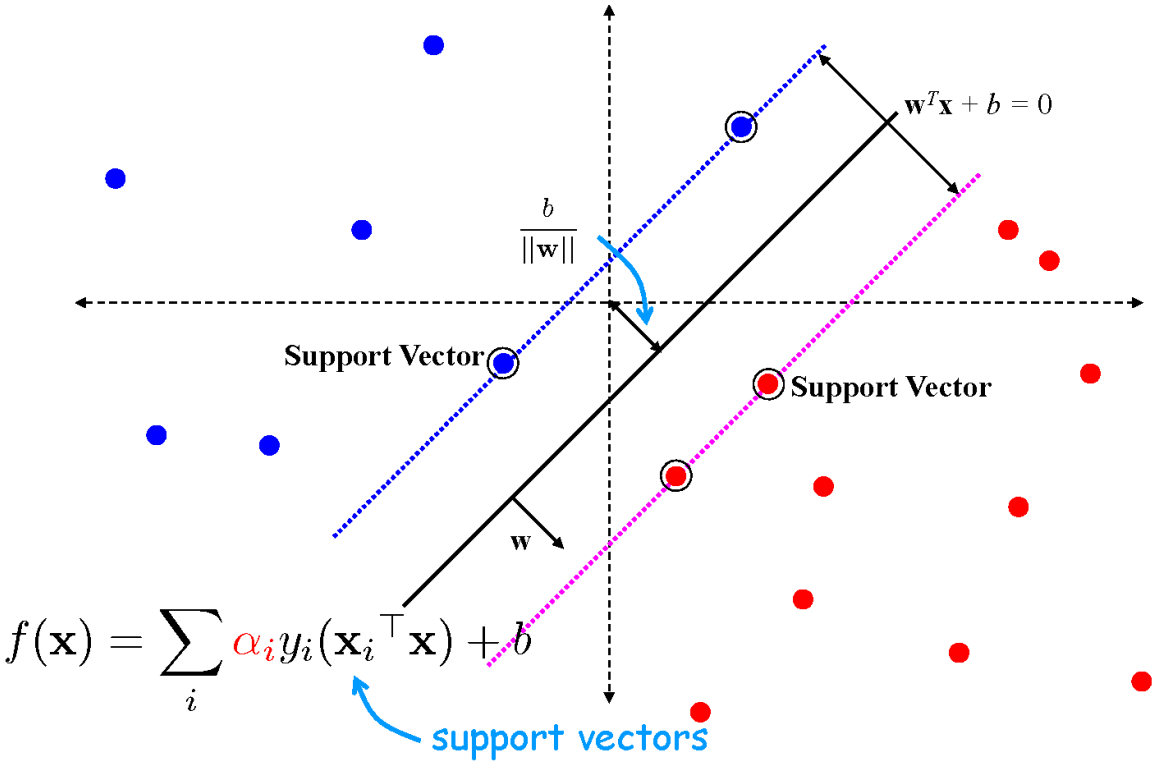
where η is the learning rate.



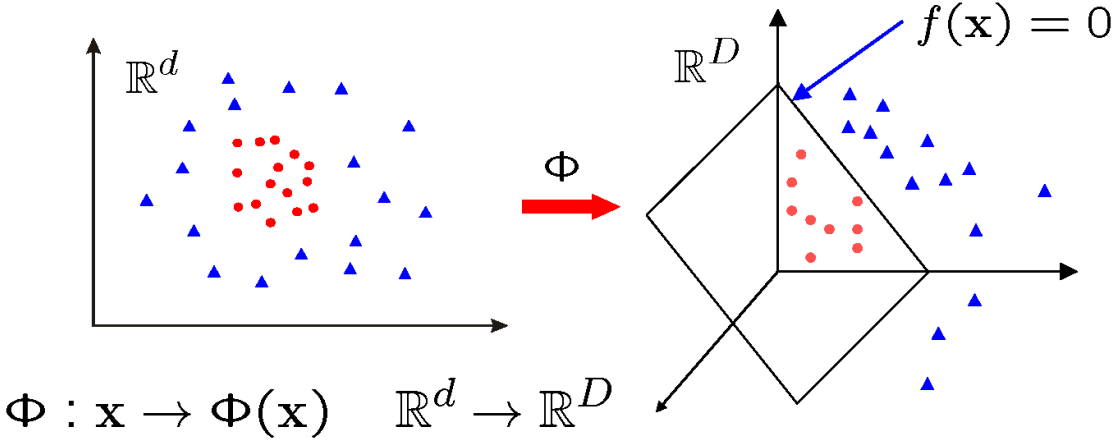




Support Vector Machine



SVM classifiers in a transformed feature space



$\Phi : \mathbf{x} \rightarrow \Phi(\mathbf{x}) \quad \mathbb{R}^d \rightarrow \mathbb{R}^D$

Learn classifier linear in $\mathbf{w}$ for $\mathbb{R}^D$:

$$f(\mathbf{x}) = \mathbf{w}^\top \Phi(\mathbf{x}) + b$$

$\Phi(\mathbf{x})$ is a feature map

Dual Classifier in transformed feature space

Classifier:

$$f(\mathbf{x}) = \sum_{i}^{N} \alpha_i y_i \, \mathbf{x}_i{}^{\top}\mathbf{x} + b$$

$$\rightarrow f(\mathbf{x}) = \sum_{i}^{N} \alpha_i y_i \, \mathbf{\Phi}(\mathbf{x}_i)^{\top}\mathbf{\Phi}(\mathbf{x}) + b$$

Learning:

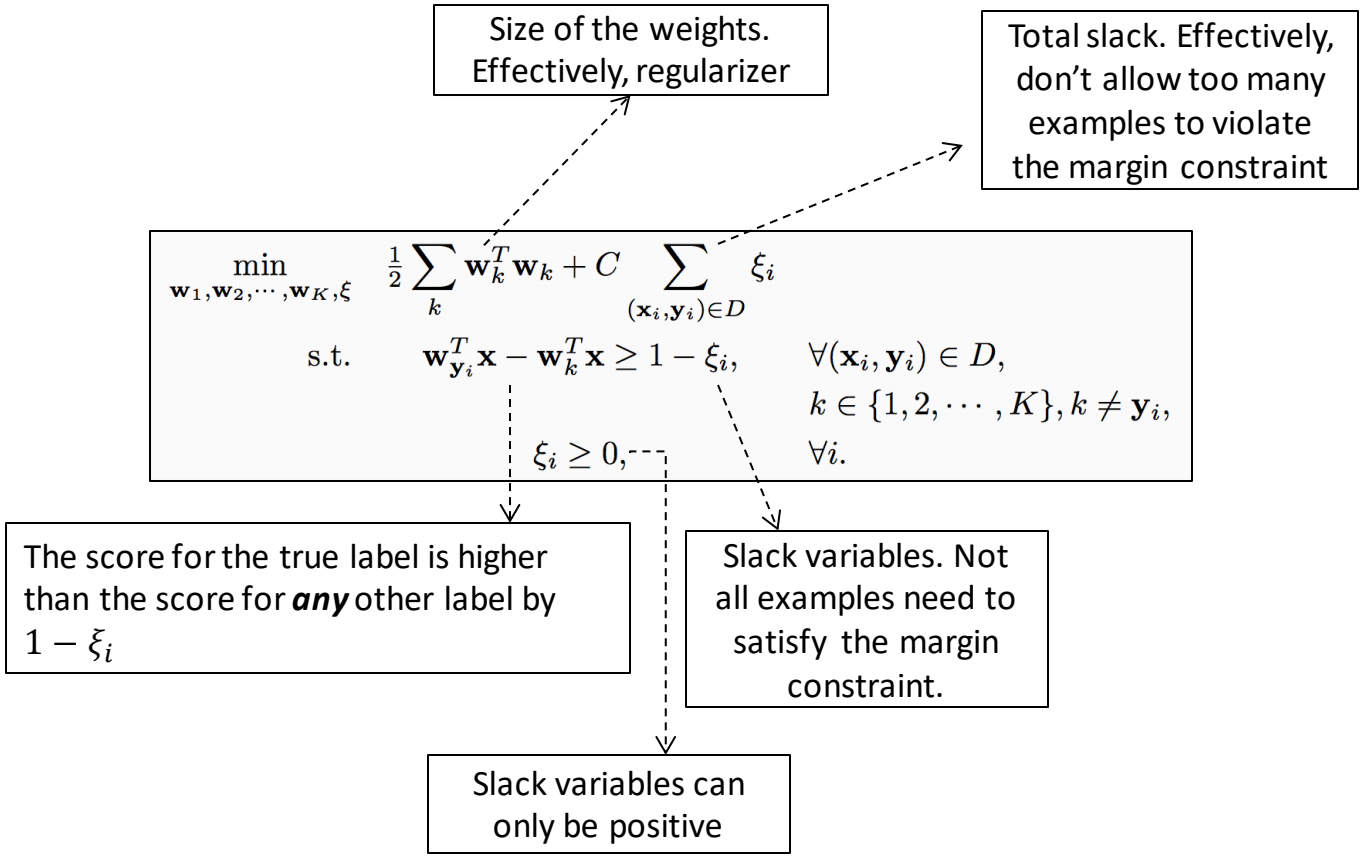
$$\max_{\alpha_i \geq 0} \sum_i \alpha_i - \frac{1}{2} \sum_{jk} \alpha_j \alpha_k y_j y_k \mathbf{x}_j{}^{\top}\mathbf{x}_k$$

$$\rightarrow \max_{\alpha_i \geq 0} \sum_i \alpha_i - \frac{1}{2} \sum_{jk} \alpha_j \alpha_k y_j y_k \mathbf{\Phi}(\mathbf{x}_j)^{\top}\mathbf{\Phi}(\mathbf{x}_k)$$

subject to

$$0 \leq \alpha_i \leq C \text{ for } \forall i, \text{ and } \sum_i \alpha_i y_i = 0$$

Multiclass SVM: General case



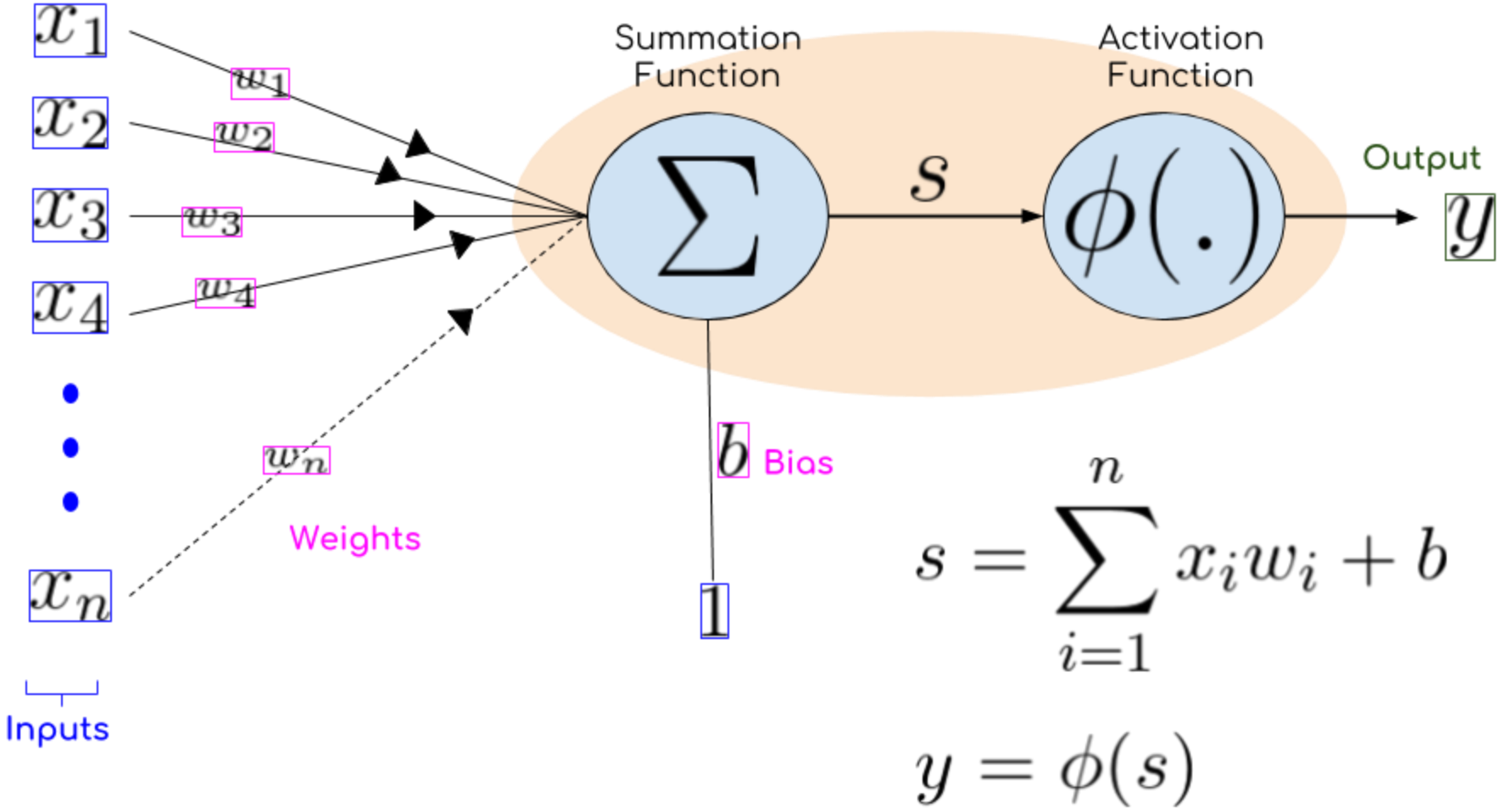
Multiclass SVM: Summary

- Training:
  - Optimize the SVM objective

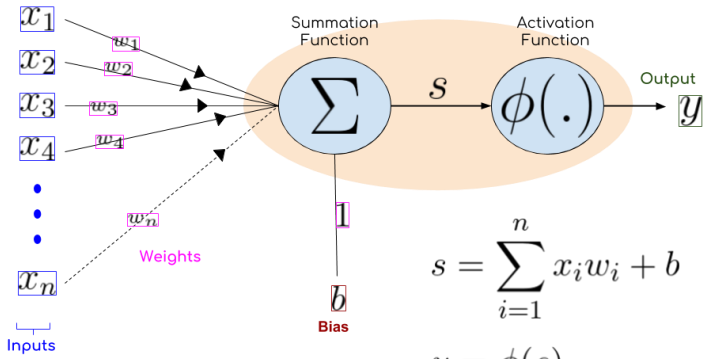
- Prediction:
  - Winner takes all

  $\text{argmax}_i \mathbf{w}_i^T \mathbf{x}$

Neural Networks

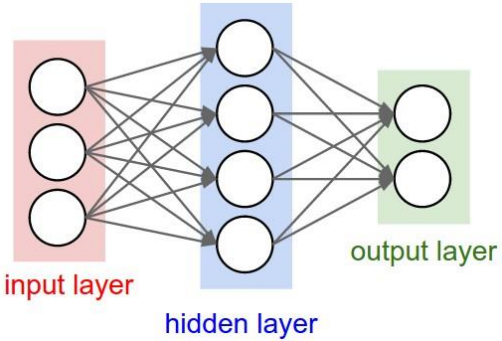

x_1
x_2
x_3
x_4
x_n
w_1
w_2
w_3
w_4
w_n
Weights
Inputs
Summation Function
$\sum$
s
Activation Function
$\phi(.)$
Output
y
b Bias
1
$s = \sum_{i=1}^{n} x_i w_i + b$
$y = \phi(s)$

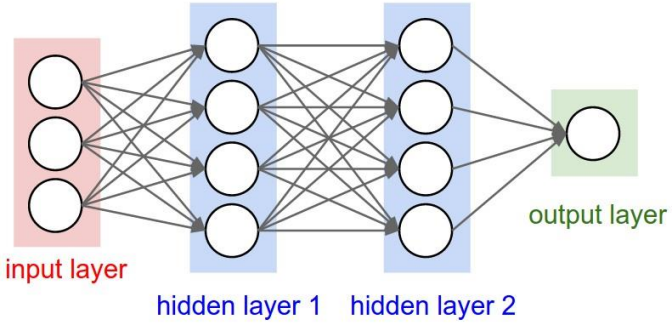
ACTIVATION FUNCTIONS

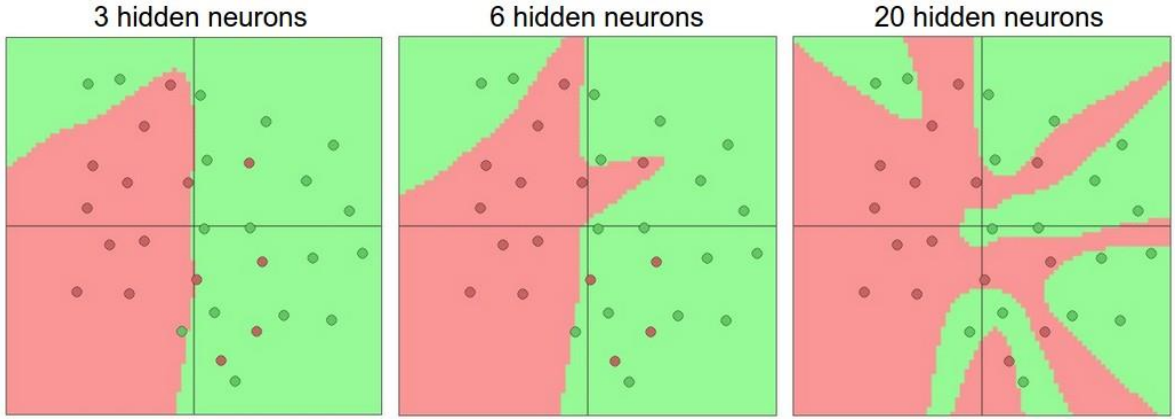


Activation Function	Equation	Example	1D Graph
Linear	$\phi(z) = z$	Adaline, linear regression	
Unit Step (Heaviside Function)	$\phi(z) = \begin{cases} 0 & z < 0 \\ 0.5 & z = 0 \\ 1 & z > 0 \end{cases}$	Perceptron variant	
Sign (signum)	$\phi(z) = \begin{cases} -1 & z < 0 \\ 0 & z = 0 \\ 1 & z > 0 \end{cases}$	Perceptron variant	
Piece-wise Linear	$\phi(z) = \begin{cases} 0 & z \leq -½ \\ z + ½ & -½ \leq z \leq ½ \\ 1 & z \geq ½ \end{cases}$	Support vector machine	
Logistic (sigmoid)	$\phi(z) = \frac{1}{1 + e^{-z}}$	Logistic regression,	
Hyperbolic Tangent (tanh)	$\phi(z) = \frac{e^{z} - e^{-z}}{e^{z} + e^{-z}}$		
ReLU	$\phi(z) = \begin{cases} 0 & z < 0 \\ z & z > 0 \end{cases}$		

Why Use Only One Neuron ?

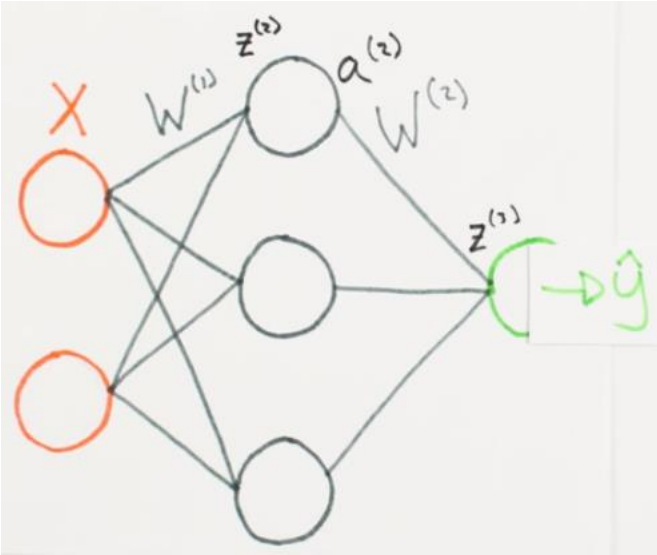






Multi-Neuron Networks :: FORWARD PROPAGATION

```
In [7]: NN = Neural_Network()

In [8]: yHat = NN.forward(X)

In [9]: yHat

Out[9]: array([[ 0.59470263],
               [ 0.58177822],
               [ 0.50641742]])

n [10]: y

ut[10]: array([[ 0.75],
               [ 0.82],
               [ 0.93]])
```



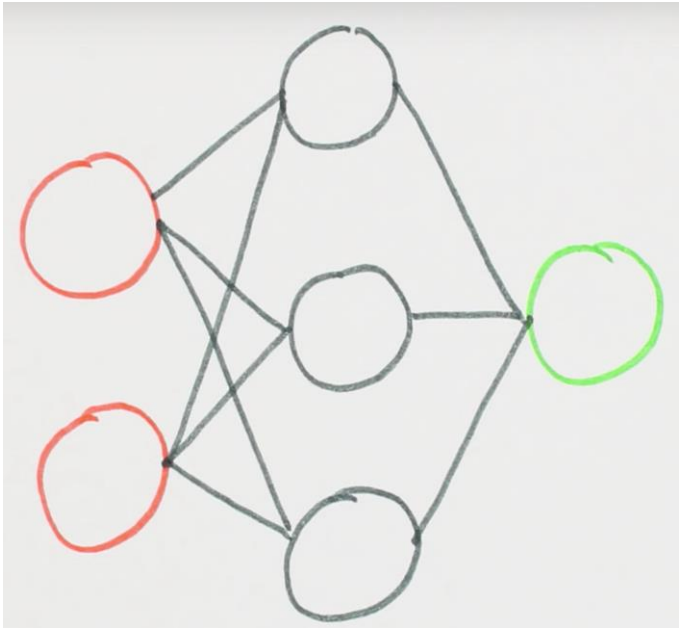
$$z^{(2)} = XW^{(1)} \quad (1)$$

$$a^{(2)} = f(z^{(2)}) \quad (2)$$

$$z^{(3)} = a^{(2)}W^{(2)} \quad (3)$$

$$\hat{y} = f(z^{(3)}) \quad (4)$$

Multi-Neuron Networks :: Gradient Descent

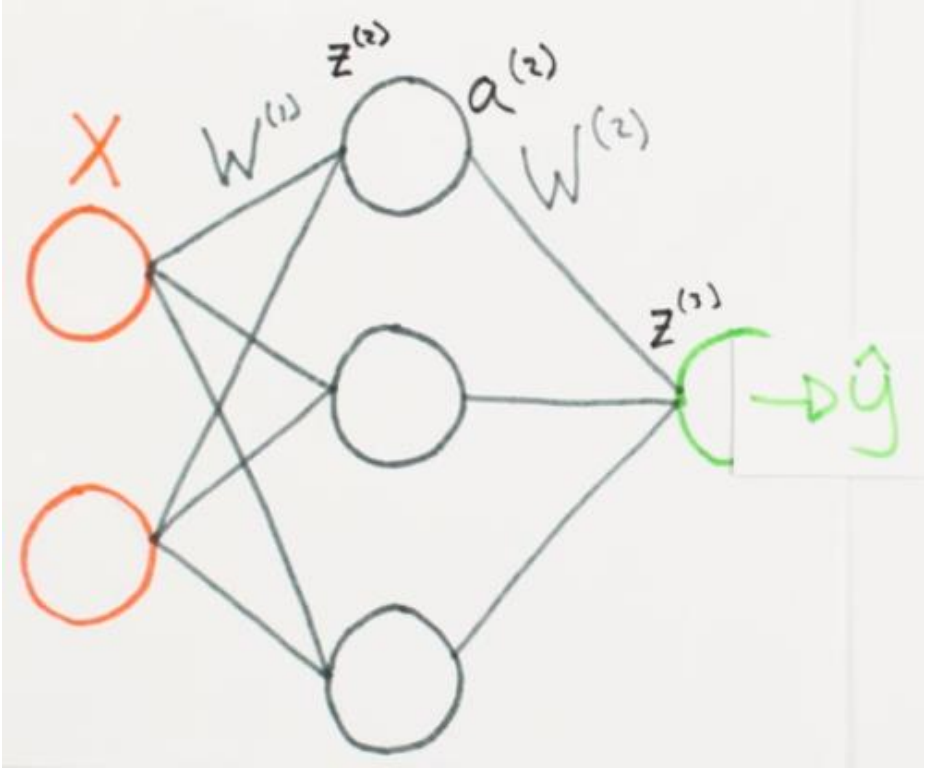


Training a Network

=

Minimizing a Cost Function

Multi-Neuron Networks :: Gradient Descent



$$z^{(2)} = XW^{(1)} \quad (1)$$

$$a^{(2)} = f(z^{(2)}) \quad (2)$$

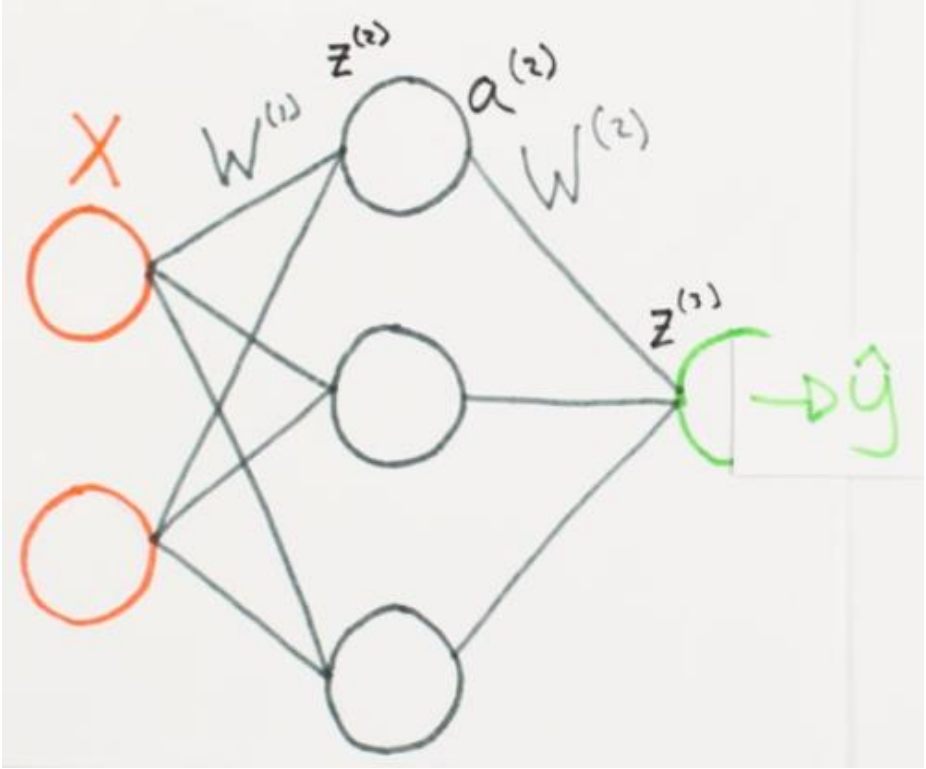
$$z^{(3)} = a^{(2)}W^{(2)} \quad (3)$$

$$\hat{y} = f(z^{(3)}) \quad (4)$$

$$J = \sum \frac{1}{2}(y - \hat{y})^2 \quad (5)$$

$$J = \sum \frac{1}{2}(y - f(f(XW^{(1)})W^{(2)}))^2$$

Multi-Neuron Networks :: Gradient Descent

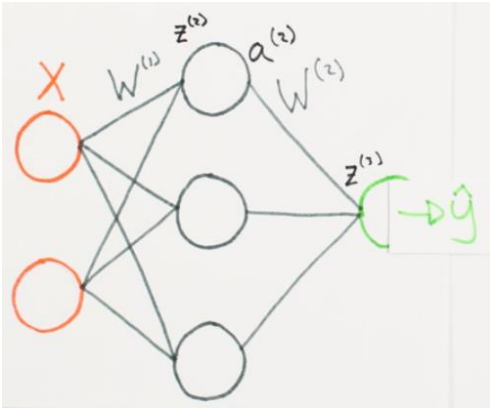


$$J = \sum \frac{1}{2}(y - f(f(XW^{(1)})W^{(2)}))^2$$

HOW DOES THIS CHANGE IF I CHANGE THESE?

$$\frac{\partial J}{\partial W}$$

Multi-Neuron Networks :: Backpropagation



$$J = \sum \frac{1}{2}(y - f(f(XW^{(1)})W^{(2)}))^2$$

HOW DOES THIS CHANGE IF I CHANGE THESE?

$$\frac{\partial J}{\partial W}$$

$$W^{(1)} = \begin{bmatrix} W_{11}^{(1)} & W_{12}^{(1)} & W_{13}^{(1)} \\ W_{21}^{(1)} & W_{22}^{(1)} & W_{23}^{(1)} \end{bmatrix}$$

$$\frac{\partial J}{\partial W^{(1)}} = \begin{bmatrix} \frac{\partial J}{\partial W_{11}^{(1)}} & \frac{\partial J}{\partial W_{12}^{(1)}} & \frac{\partial J}{\partial W_{13}^{(1)}} \\ \frac{\partial J}{\partial W_{21}^{(1)}} & \frac{\partial J}{\partial W_{22}^{(1)}} & \frac{\partial J}{\partial W_{23}^{(1)}} \end{bmatrix}$$

$$W^{(2)} = \begin{bmatrix} W_{11}^{(2)} \\ W_{21}^{(2)} \\ W_{31}^{(2)} \end{bmatrix}$$

$$\frac{\partial J}{\partial W^{(2)}} = \begin{bmatrix} \partial J/\partial W_{11}^{(2)} \\ \partial J/\partial W_{21}^{(2)} \\ \partial J/\partial W_{31}^{(2)} \end{bmatrix}$$

Multi-Neuron Networks :: Backpropagatio

$$J = \sum \frac{1}{2}(y - \hat{y})^2$$

$$J = \sum \frac{1}{2}(y - f(f(XW^{(1)})W^{(2)}))^2$$

HOW DOES THIS CHANGE IF I CHANGE THESE?

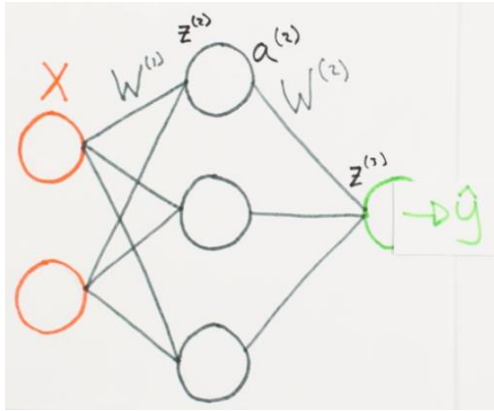
$$\frac{\partial J}{\partial W}$$

$$z^{(2)} = XW^{(1)} \quad (1)$$

$$a^{(2)} = f(z^{(2)}) \quad (2)$$

$$z^{(3)} = a^{(2)}W^{(2)} \quad (3)$$

$$\hat{y} = f(z^{(3)}) \quad (4)$$



$$\frac{\partial J}{\partial W^{(2)}} = (a^{(2)})^T \delta^{(3)}$$

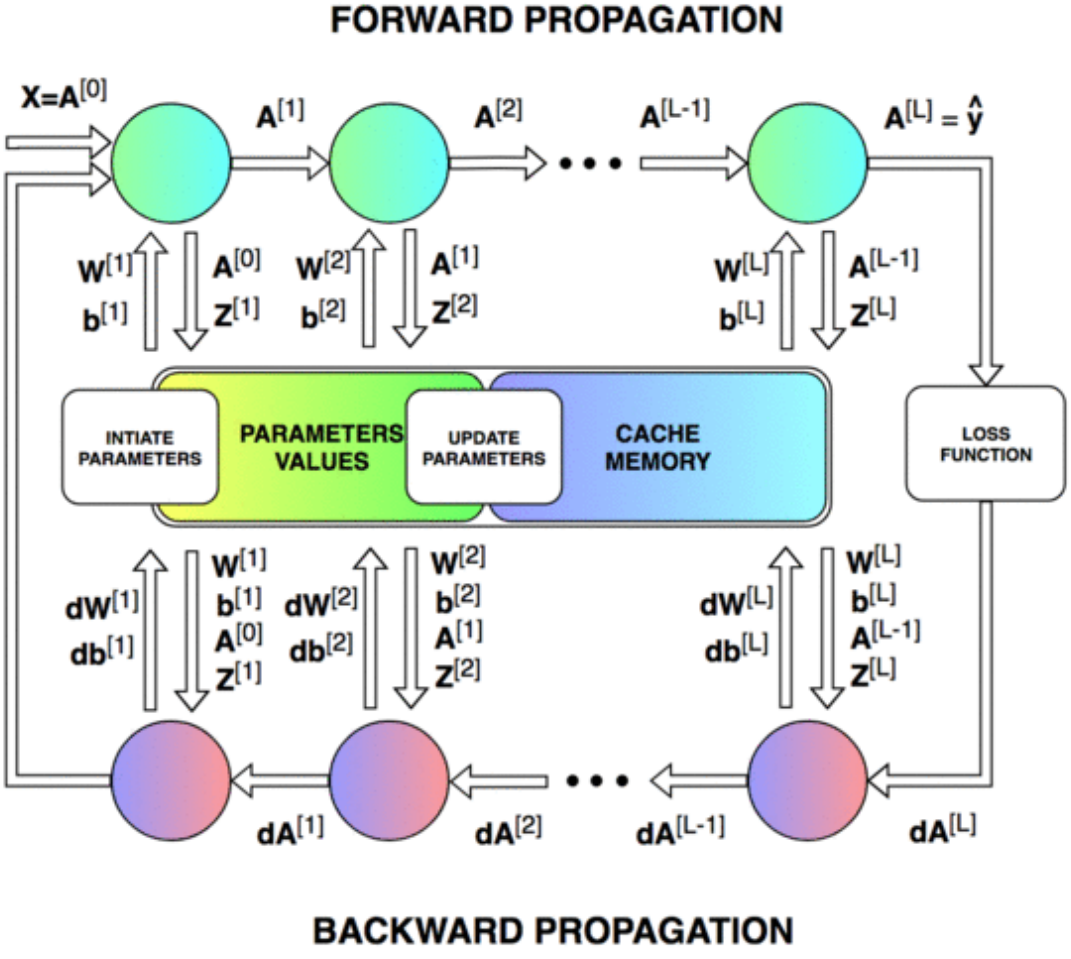
$$\delta^{(3)} = -(y - \hat{y}) f'(z^{(3)})$$

$$\frac{\partial J}{\partial W^{(1)}} = X^T \delta^{(2)}$$

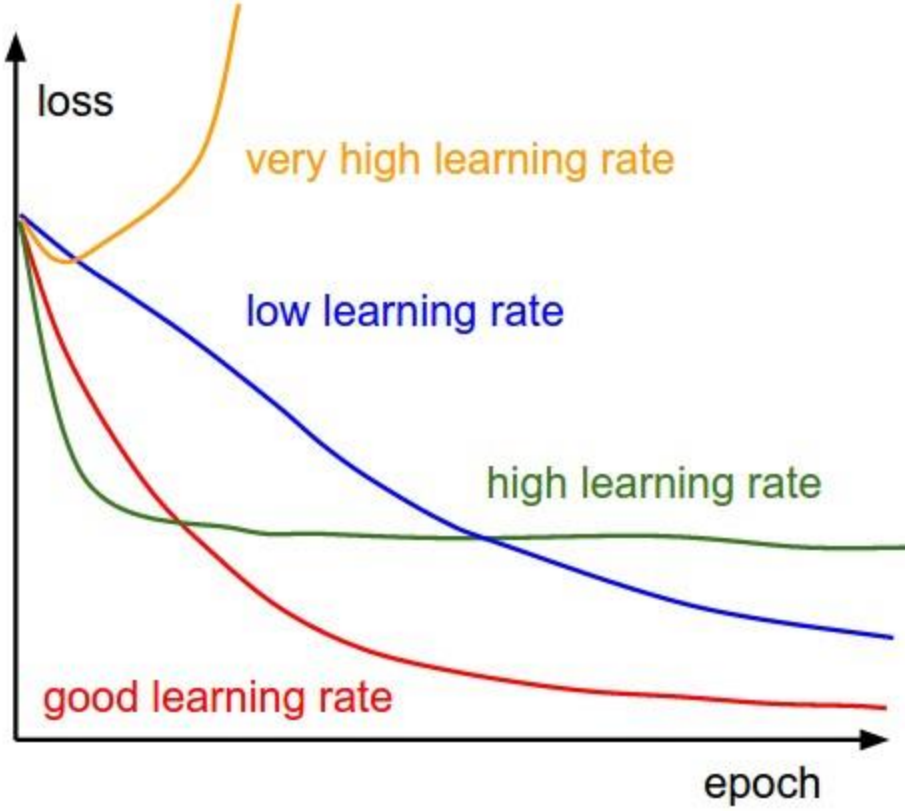
$$\delta^{(2)} = \delta^{(3)}(W^{(2)})^T f'(z^{(2)})$$

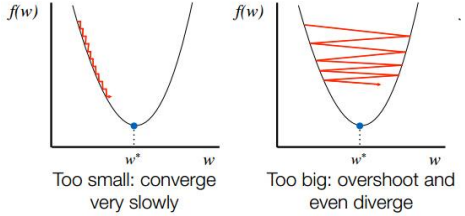
$$W^{(1)} = \begin{bmatrix} W_{11}^{(1)} & W_{12}^{(1)} & W_{13}^{(1)} \\ W_{21}^{(1)} & W_{22}^{(1)} & W_{23}^{(1)} \end{bmatrix} \qquad \frac{\partial J}{\partial W^{(1)}} = \begin{bmatrix} \frac{\partial J}{\partial W_{11}^{(1)}} & \frac{\partial J}{\partial W_{12}^{(1)}} & \frac{\partial J}{\partial W_{13}^{(1)}} \\ \frac{\partial J}{\partial W_{21}^{(1)}} & \frac{\partial J}{\partial W_{22}^{(1)}} & \frac{\partial J}{\partial W_{23}^{(1)}} \end{bmatrix}$$

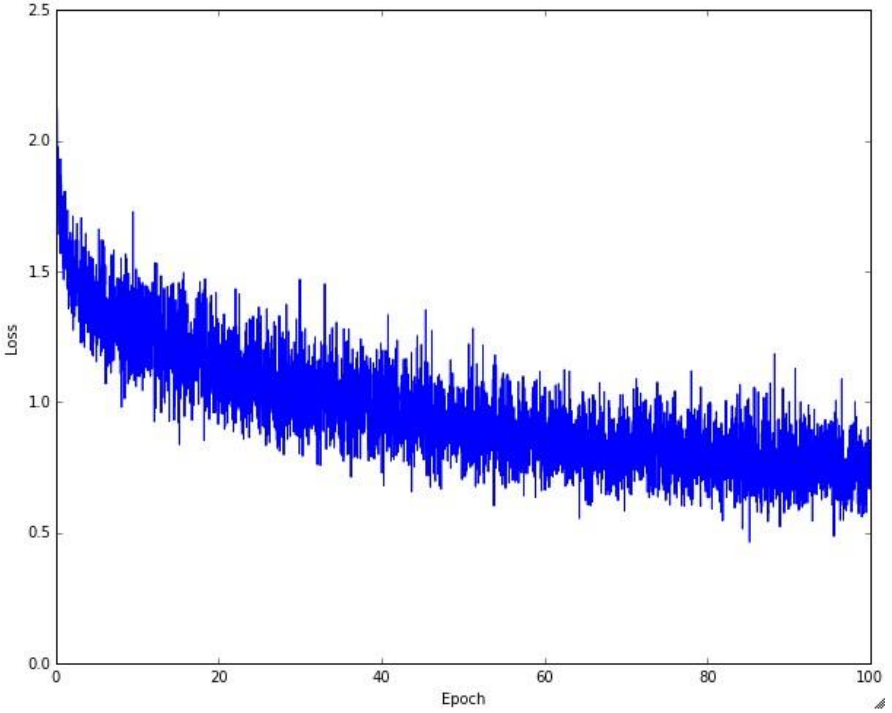
$$W^{(2)} = \begin{bmatrix} W_{11}^{(2)} \\ W_{21}^{(2)} \\ W_{31}^{(2)} \end{bmatrix} \qquad \frac{\partial J}{\partial W^{(2)}} = \begin{bmatrix} \partial J/\partial W_{11}^{(2)} \\ \partial J/\partial W_{21}^{(2)} \\ \partial J/\partial W_{31}^{(2)} \end{bmatrix}$$


FORWARD PROPAGATION
X=A[0]
A[1]
A[2]
A[L-1]
A[L] = ŷ
W[1]
b[1]
A[0]
Z[1]
W[2]
b[2]
A[1]
Z[2]
W[L]
b[L]
A[L-1]
Z[L]
INTIATE PARAMETERS
PARAMETERS VALUES
UPDATE PARAMETERS
CACHE MEMORY
LOSS FUNCTION
dW[1]
db[1]
W[1]
b[1]
A[0]
Z[1]
dW[2]
db[2]
W[2]
b[2]
A[1]
Z[2]
dW[L]
db[L]
W[L]
b[L]
A[L-1]
Z[L]
dA[1]
dA[2]
dA[L-1]
dA[L]
BACKWARD PROPAGATION

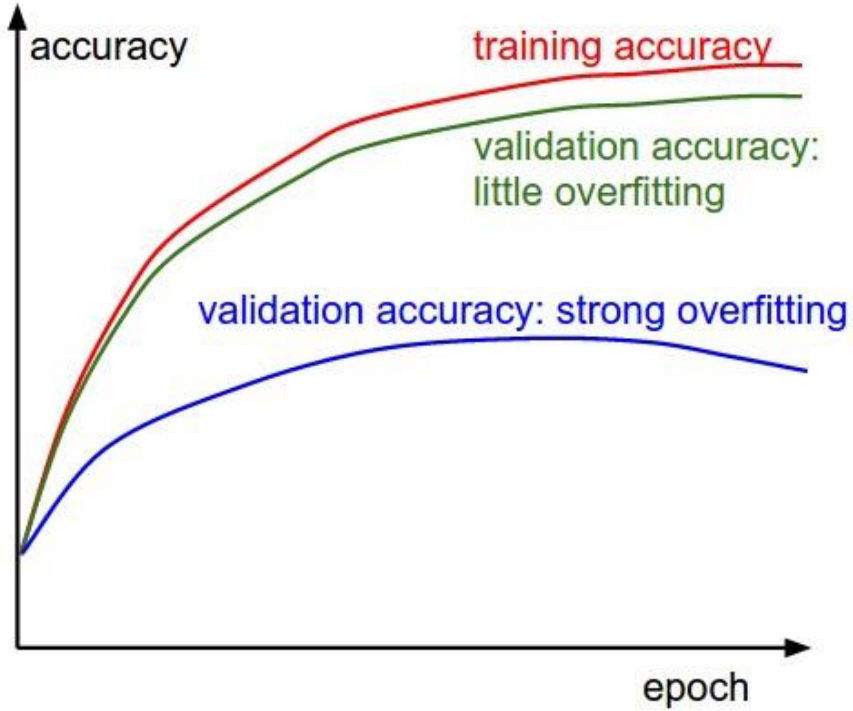
Training – setting learning rate

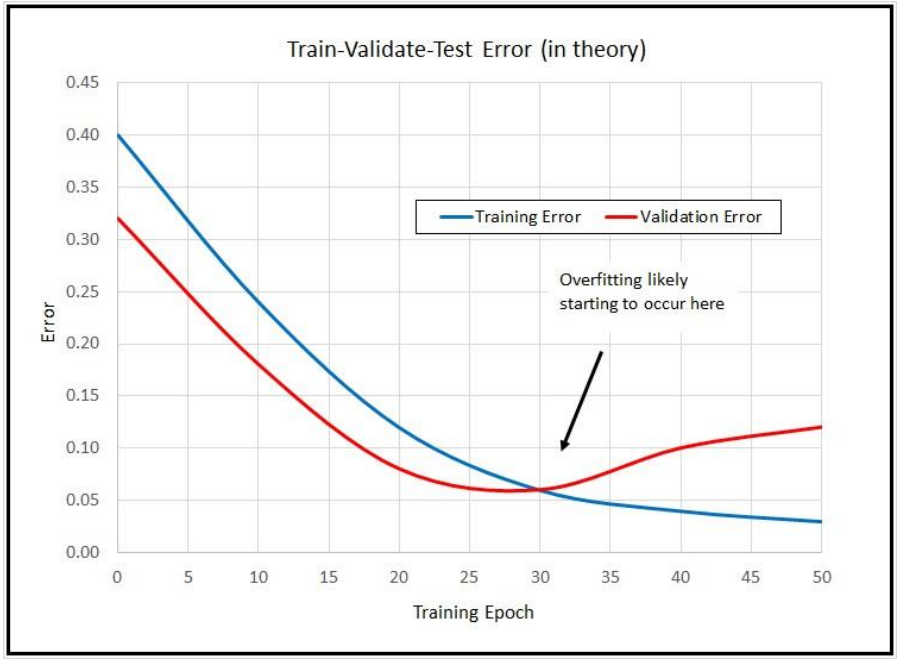






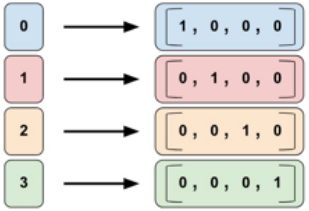
When to stop training

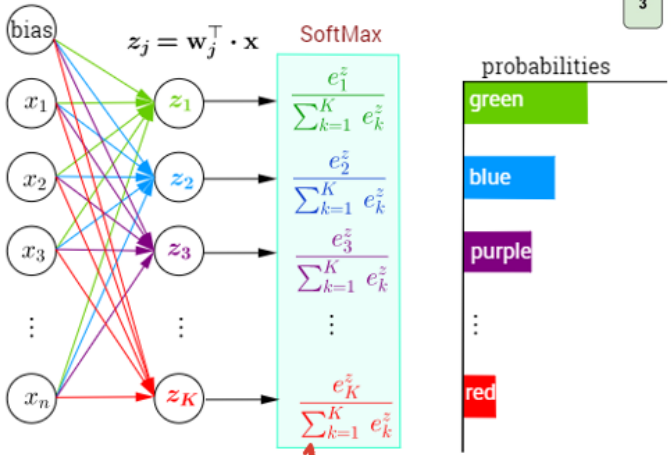


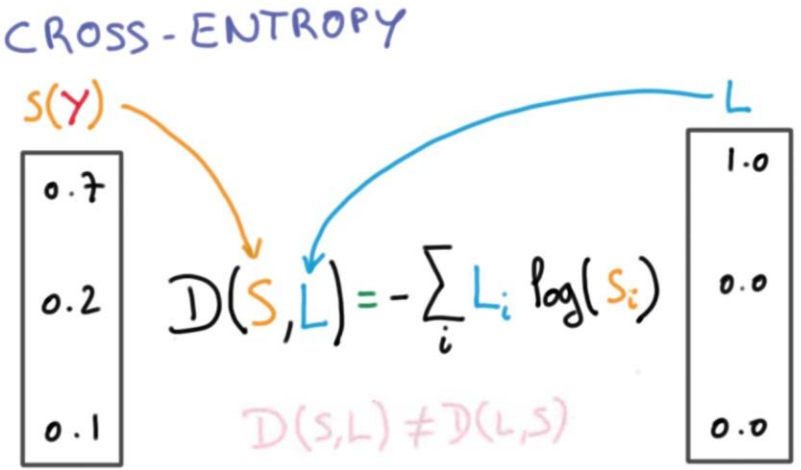


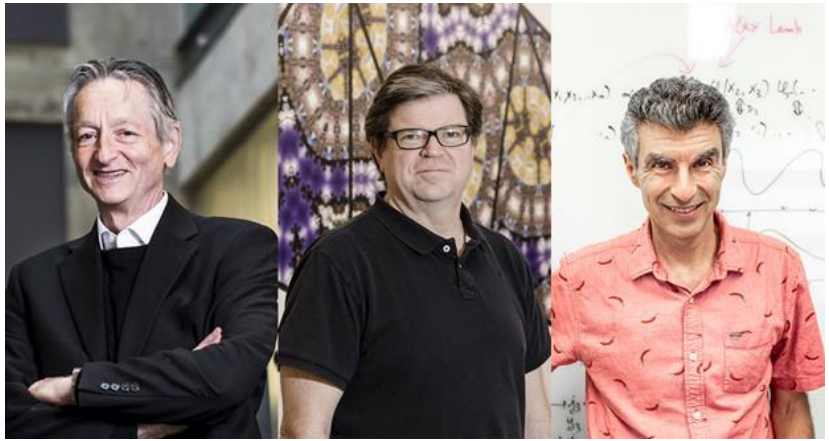
Classification Loss

$$
\begin{aligned}
D_{\mathrm{KL}}(p|q) &= \sum_i p_i \log \frac{p_i}{q_i} \\
&= \sum_i (-p_i \log q_i + p_i \log p_i) \\
&= -\sum_i p_i \log q_i + \sum_i p_i \log p_i \\
&= -\sum_i p_i \log q_i - \sum_i p_i \log \frac{1}{p_i} \\
&= -\sum_i p_i \log q_i - H(p) \\
&= \sum_i p_i \log \frac{1}{q_i} - H(p)
\end{aligned}
$$









G. Hinton (U.Toronto, Google DeepMind)

Yann LeCun (Facebook AI)

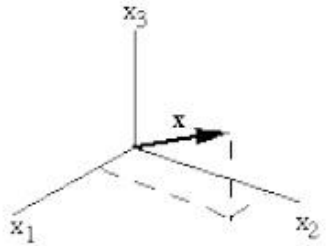
Y. Bengio (MILA, Montreal)

Feature Vectors

- Usually a single object can be represented using several features, e.g.
  - **x1** = shape (e.g. nr of sides)
  - **x2** = size (e.g. some numeric value)
  - **x3** = color (e.g. rgb values)
  - ...
  - **xd** = some other (numeric) feature.

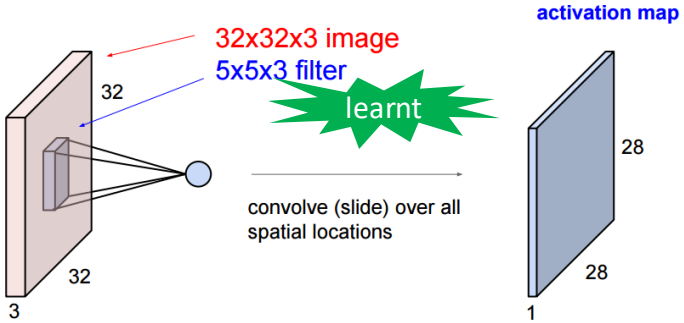
- **X** becomes a feature vector
  - x is a point in a d-dimensional **feature space**.

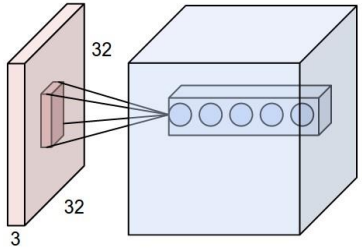
$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ \cdot \\ \cdot \\ \cdot \\ x_d \end{bmatrix}$$



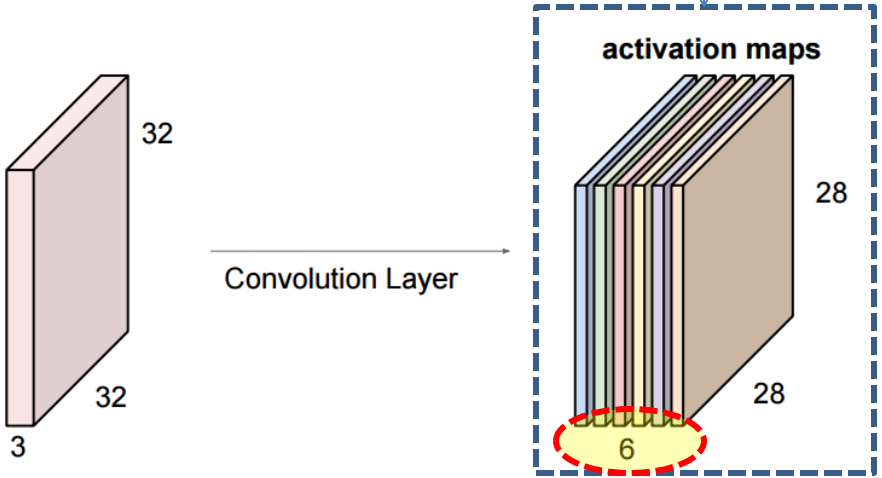
Source: https://www.slideshare.net/lgustavomartins/introduction-to-pattern-recognition

Convolution Layer

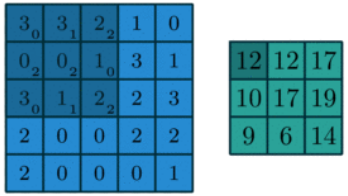


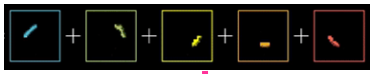


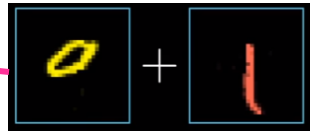
Input to next layer

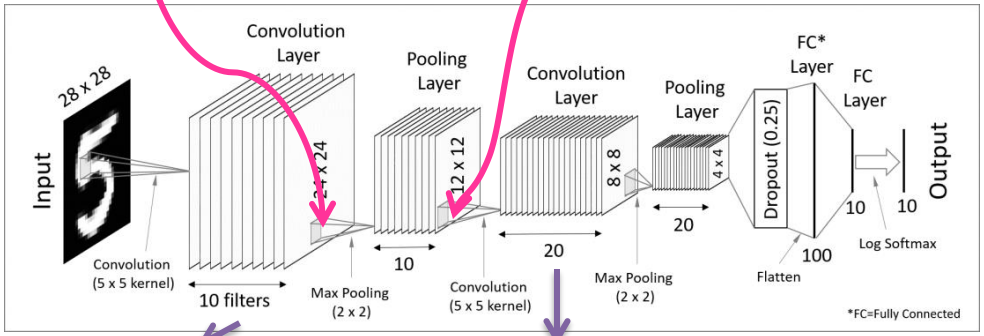


Design choices : Filter size, # of filters





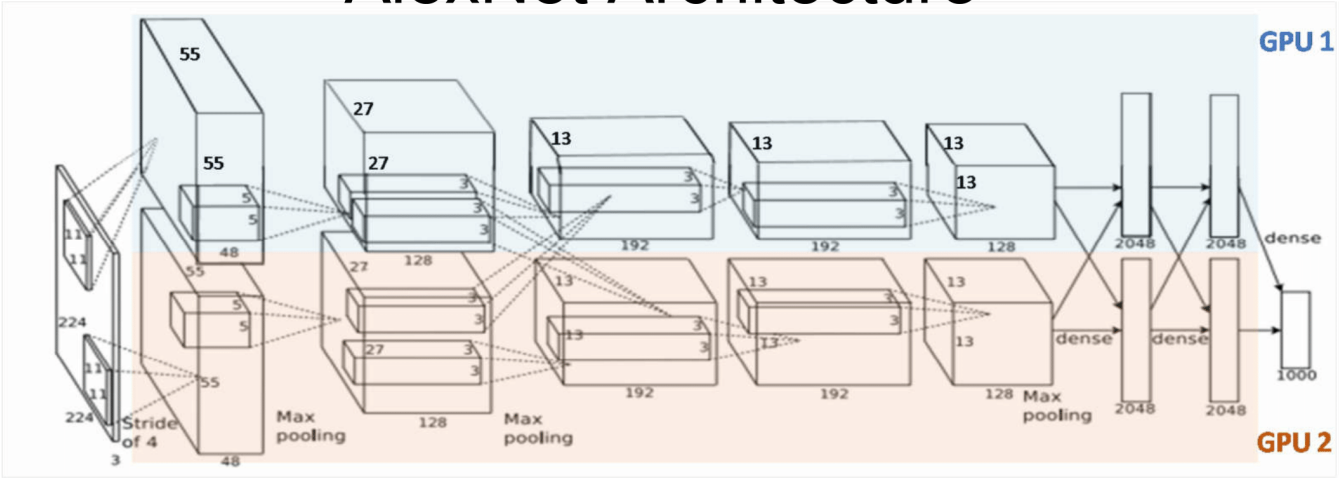


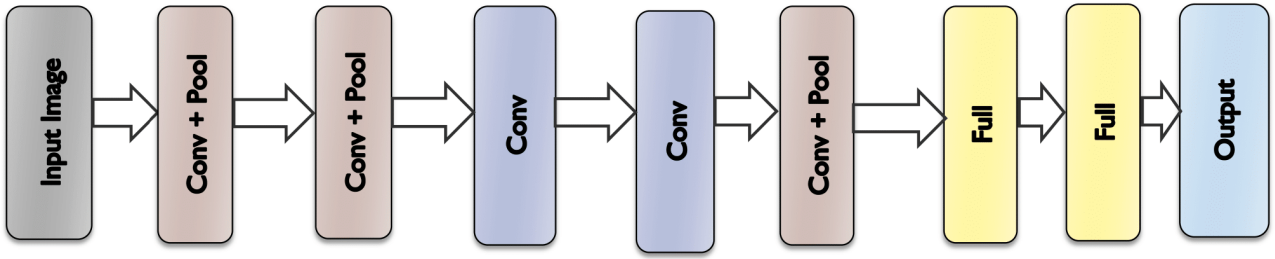


~ Alphabets

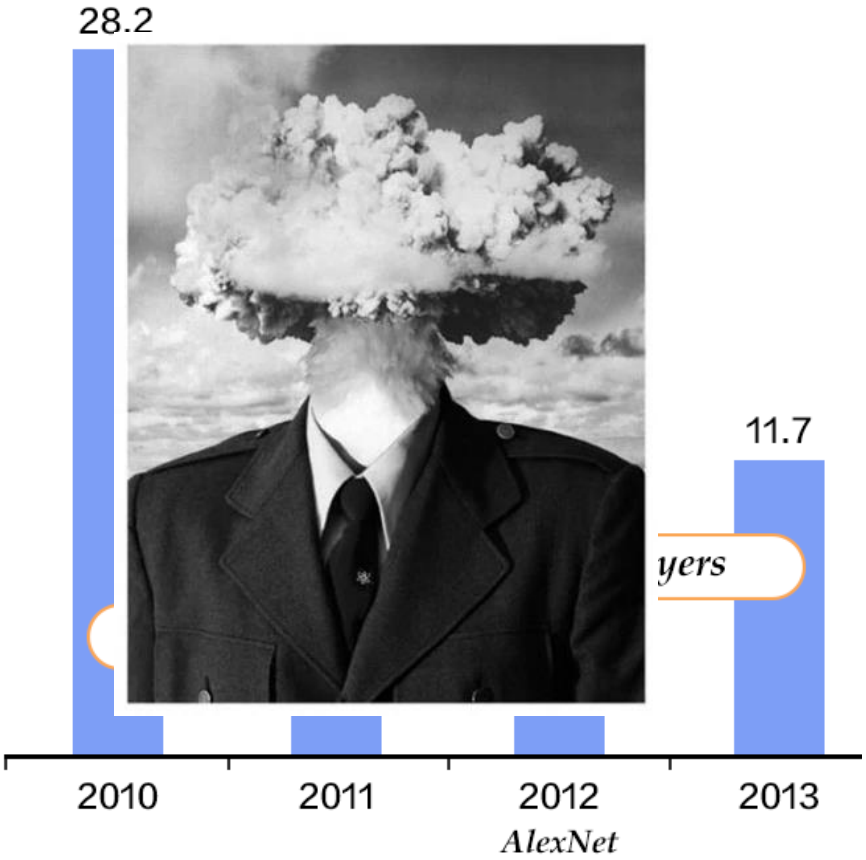
~ Words

AlexNet Architecture



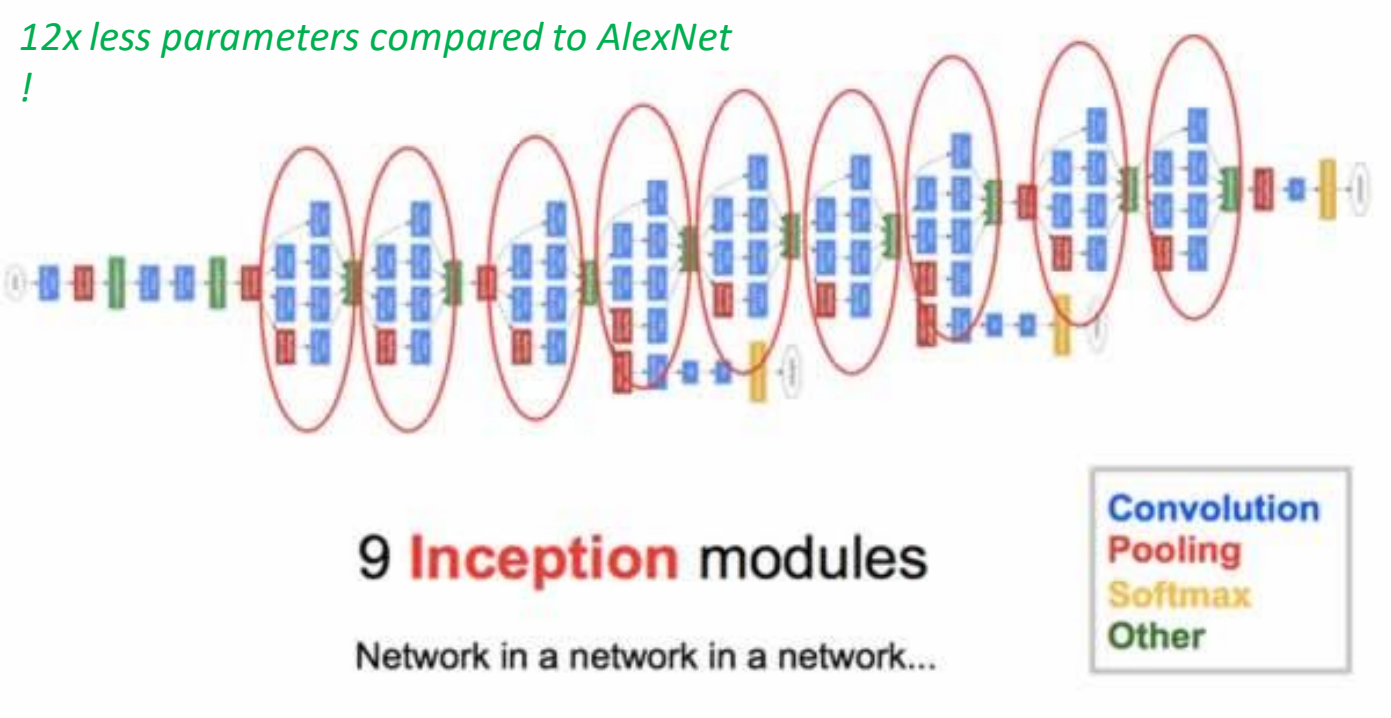


Top-5 misclassification error rate



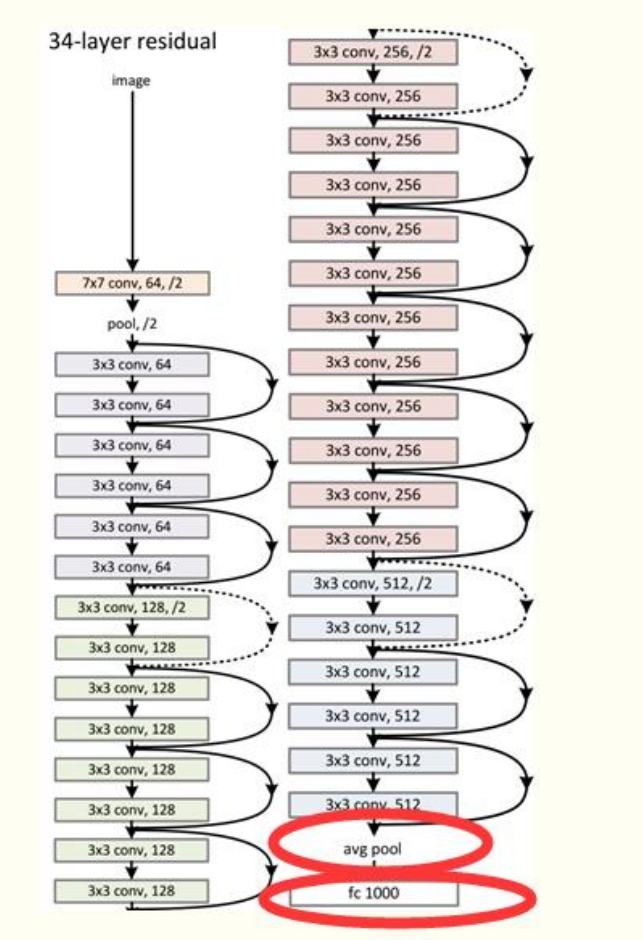
GoogLeNet

12x less parameters compared to AlexNet !



https://www.youtube.com/watch?v=LUaRa65J_Ms

Residual Networks



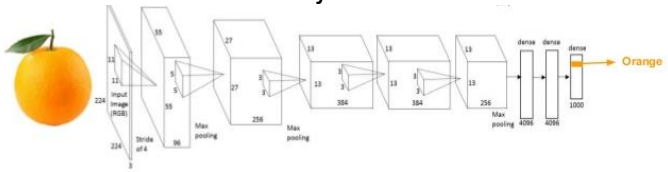
Gradients can be supplied to shallower layers directly if needed (via skips)

Transfer Learning

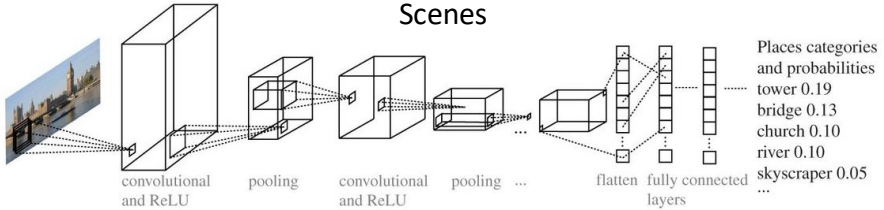
Landscape of CNNs : Architectures

Image → Label

Objects



Scenes



Faces

DeepFace Architecture

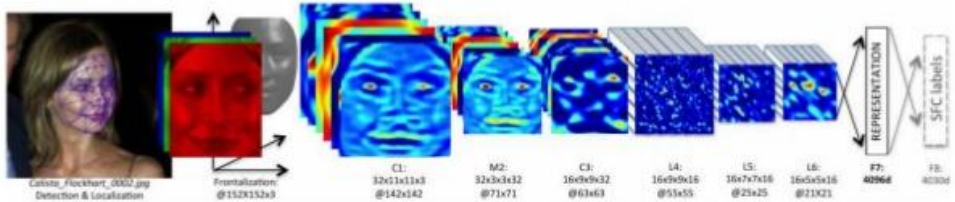
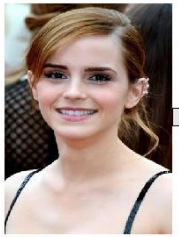
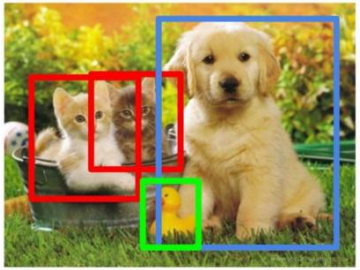


Image → Number



Face Detection → Face Alignment → Deep Feature Representation (from face-identification task) → 3-layer Neural Network → Age

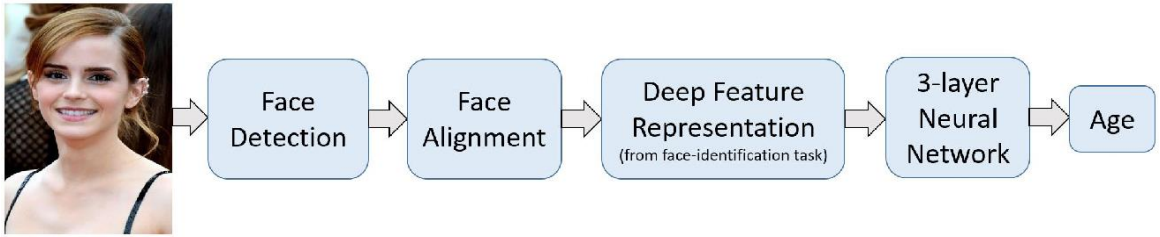
Object Detection



CAT, DOG, DUCK

Image → Number

Age Estimation



Object detection

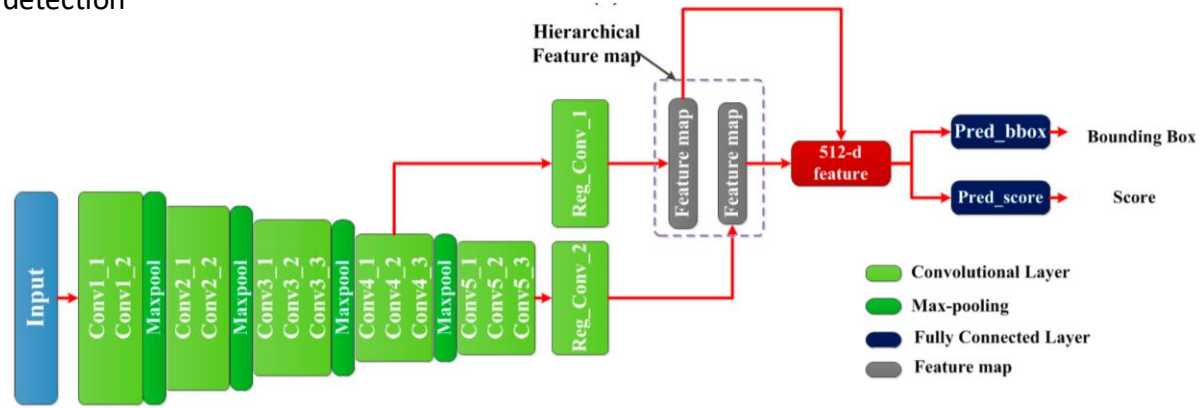
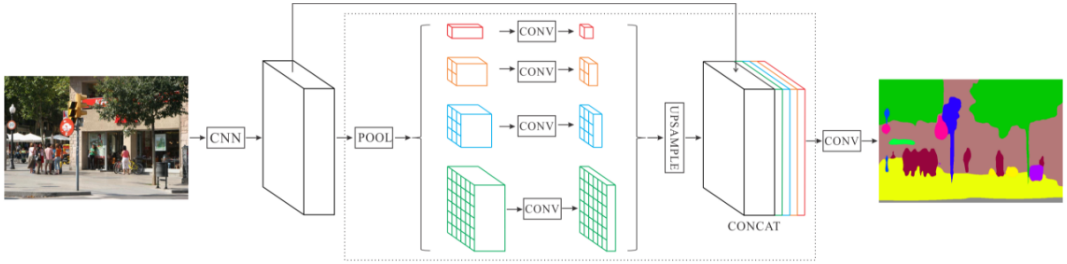


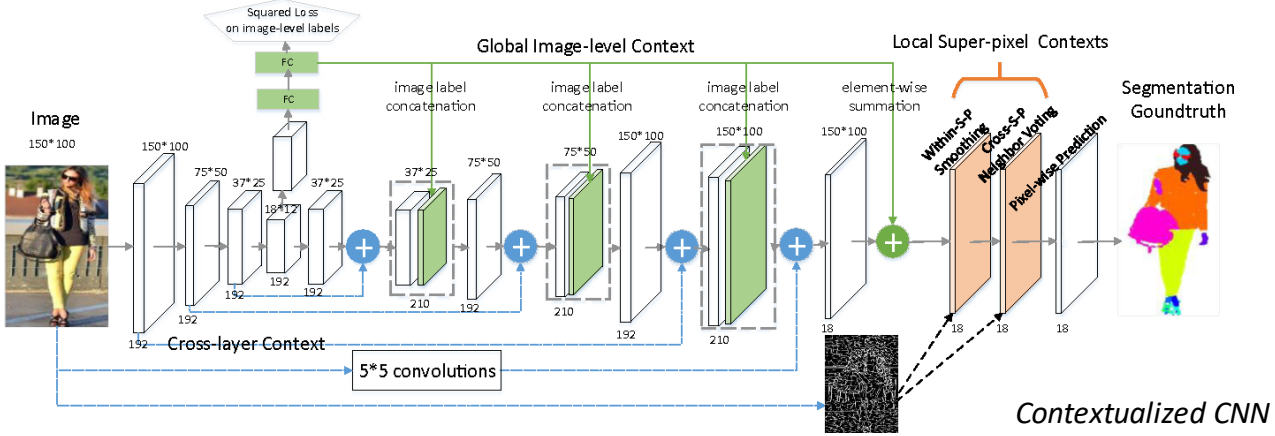
Image → Label Image

Scene Parsing



PSPNet

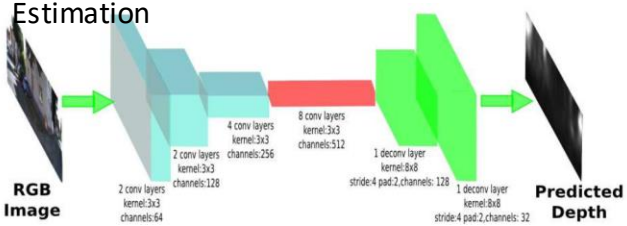
Object Parsing



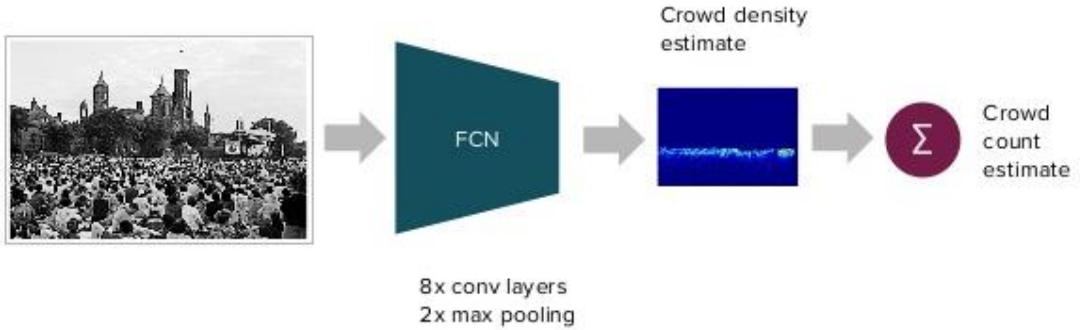
Contextualized CNN

Image → Image

Depth Estimation



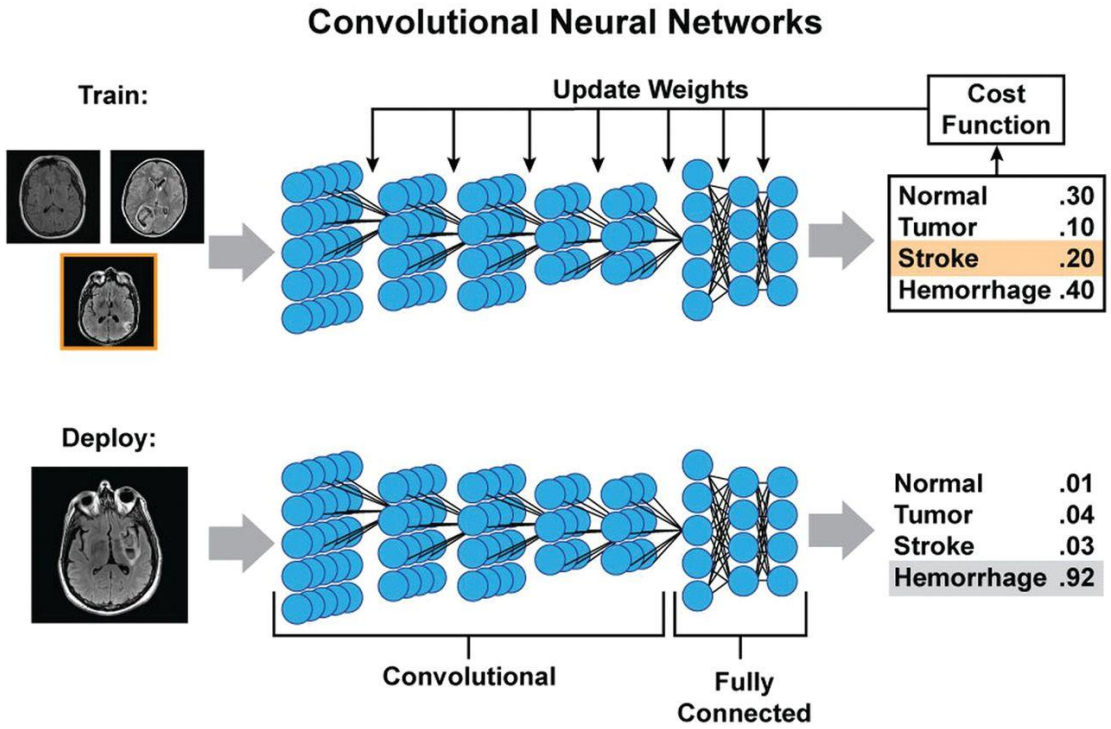
Crowd Counting



Marsden et al. **Fully convolutional crowd counting on highly congested scenes.** VISAPP 2017
https://arxiv.org/abs/1612.00220

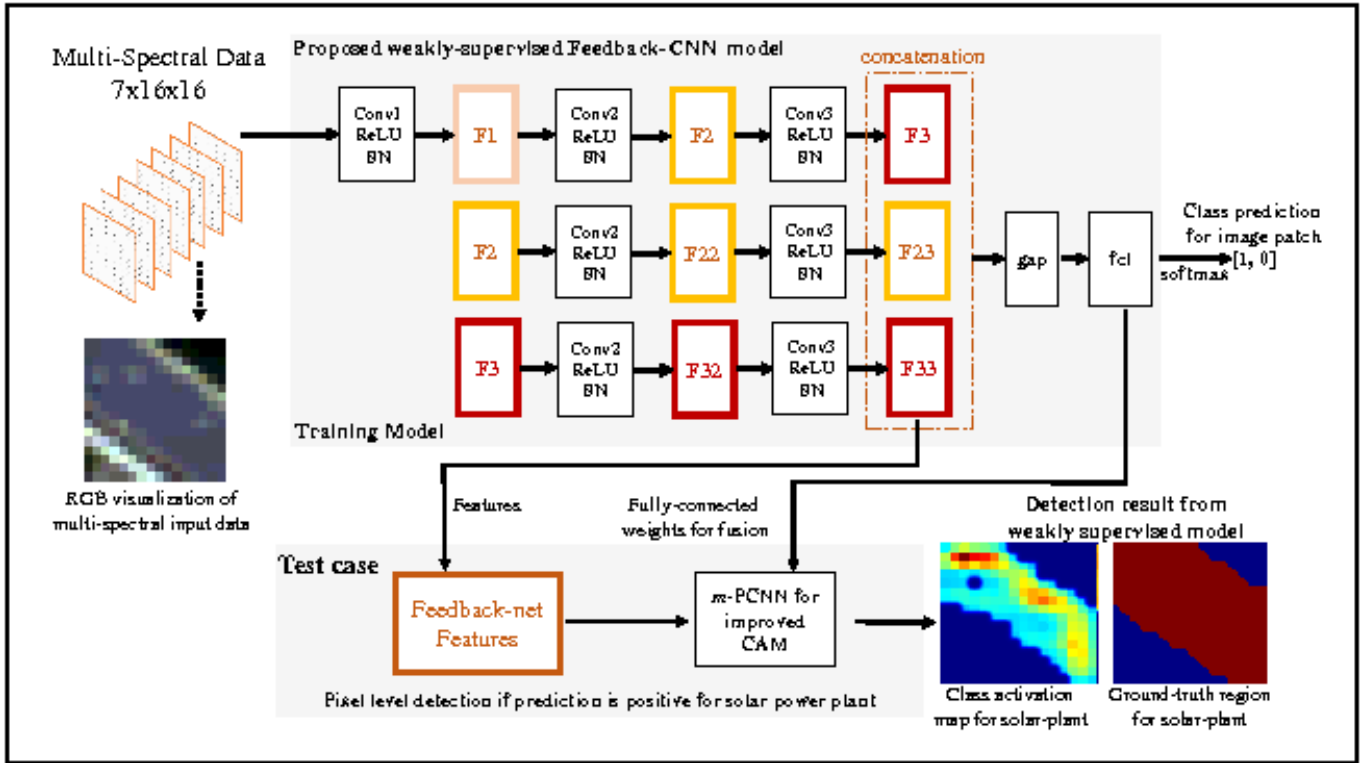
Multi-channel input

NeuroRadiology (fMRI)



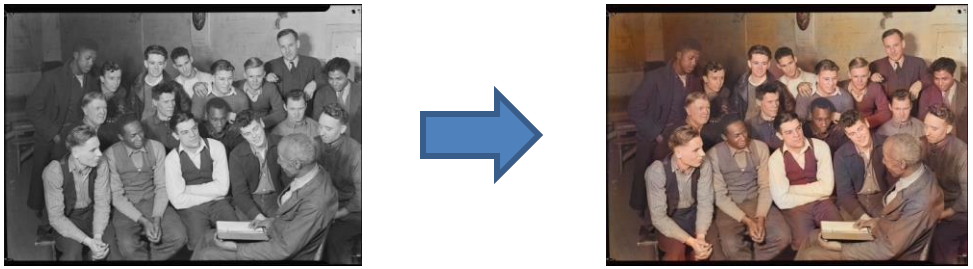
Multi-channel input

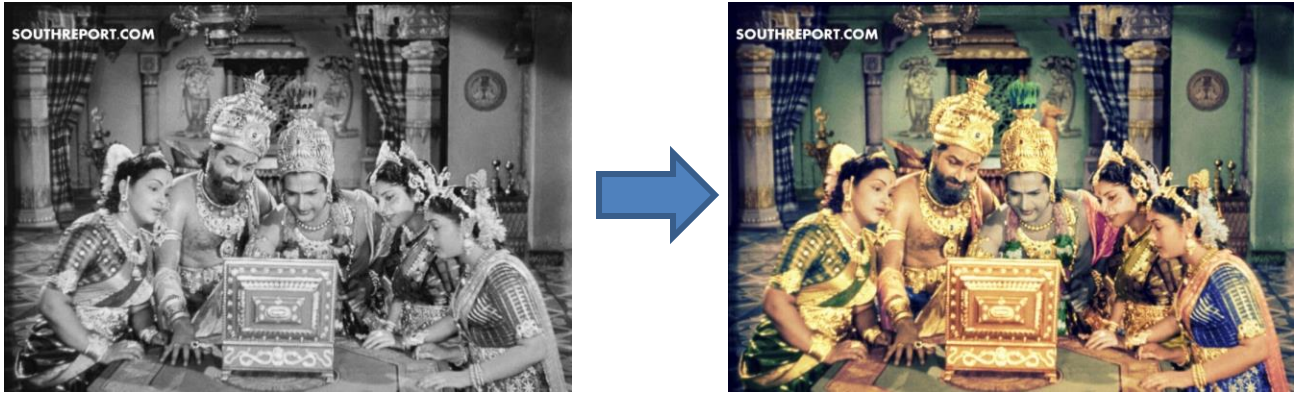
Solar-power plant detection from multi-spectral data



Multi-branch input

Image Colorization

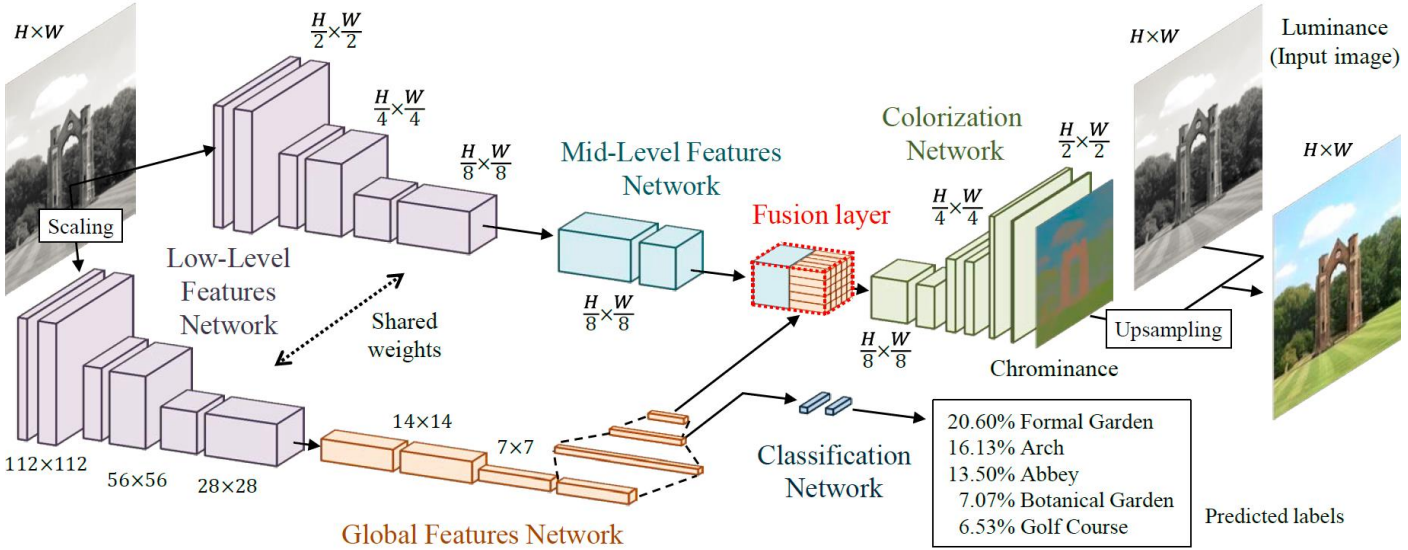




https://algorithmia.com/algorithms/deeplearning/ColorfulImageColorization

Multi-branch input

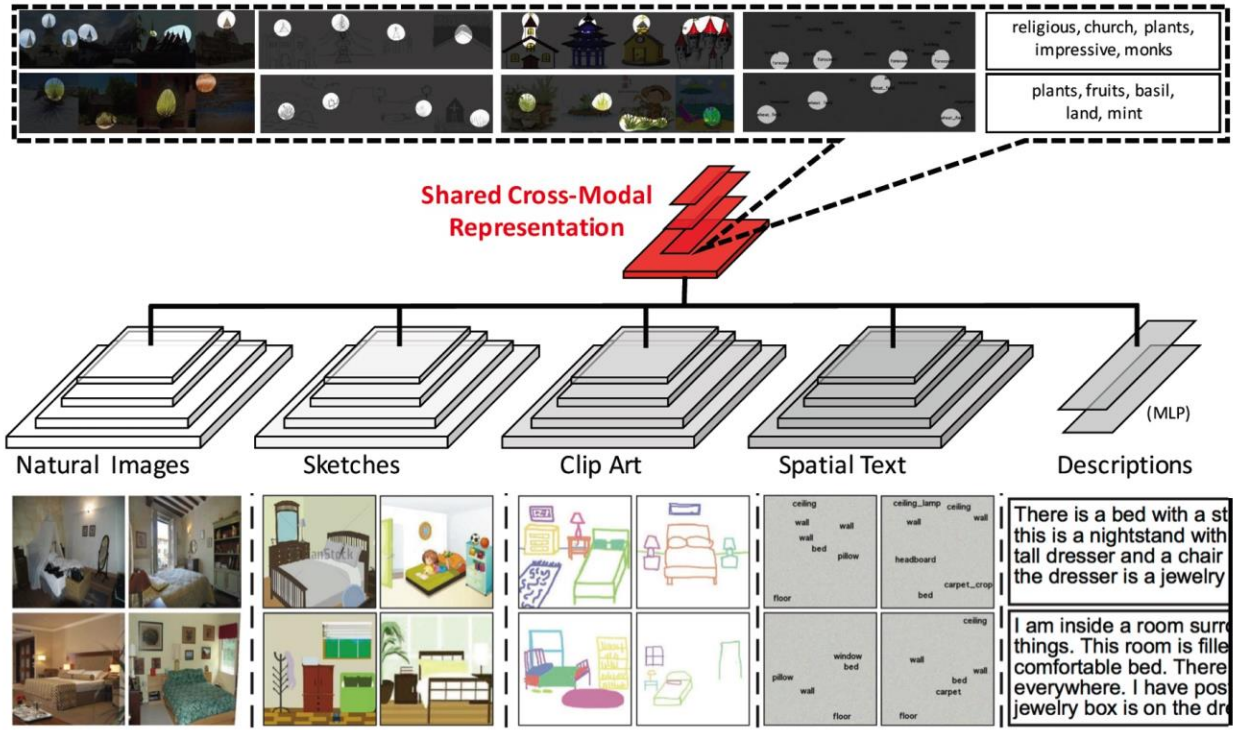
Image Colorization



http://hi.cs.waseda.ac.jp/~iizuka/projects/colorization/en/

Multi-branch input

Cross-modal Scene Classification



http://projects.csail.mit.edu/cmplaces/

Multi-branch input

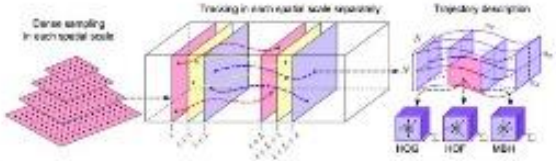
Activity Recognition

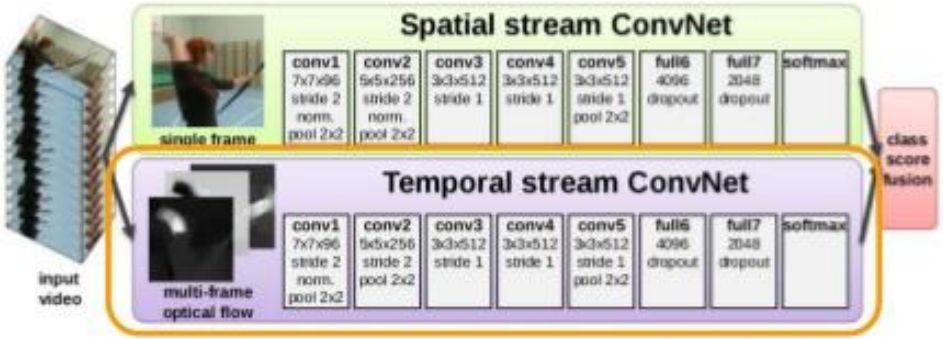
Recognition: Two stream

Two CNNs in paralel:

- One for RGB images
- One for Optical flow (hand-crafted features)

Fusion after the softmax layer

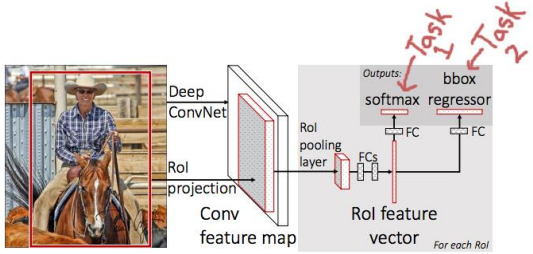




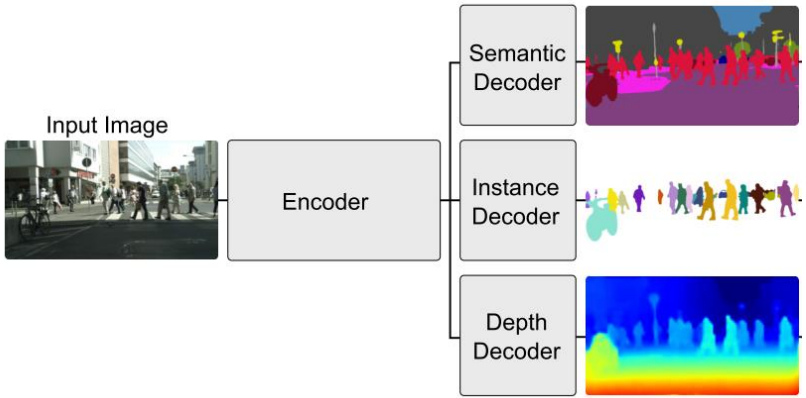
Simonyan, Karen, and Andrew Zisserman. "Two-stream convolutional networks for action recognition in videos." NIPS 2014.

Multi-branch output

Object Detection, Classification



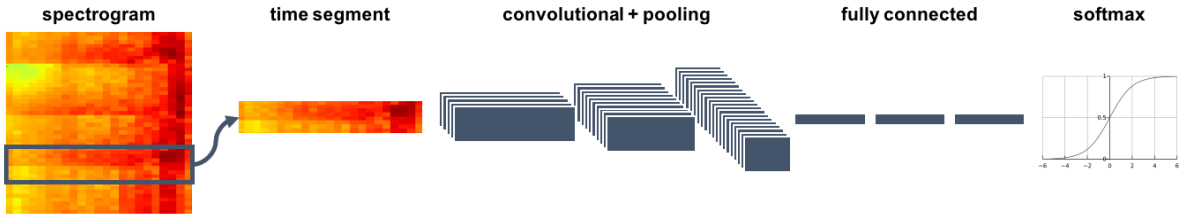
Scene Parsing



http://ruder.io/multi-task/

Image CNNs for non-image data

Audio Beat Detection



http://tommymullaney.com/projects/rhythm-games-neural-networks

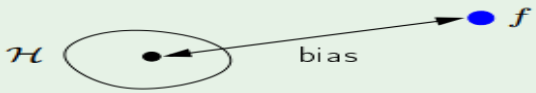
$$\mathbb{E}_{\mathcal{D}}\left[\left(g^{(\mathcal{D})}(\mathbf{x}) - f(\mathbf{x})\right)^2\right] = \underbrace{\mathbb{E}_{\mathcal{D}}\left[\left(g^{(\mathcal{D})}(\mathbf{x}) - \bar{g}(\mathbf{x})\right)^2\right]}_{\text{var}(\mathbf{x})} + \underbrace{\left(\bar{g}(\mathbf{x}) - f(\mathbf{x})\right)^2}_{\text{bias}(\mathbf{x})}$$

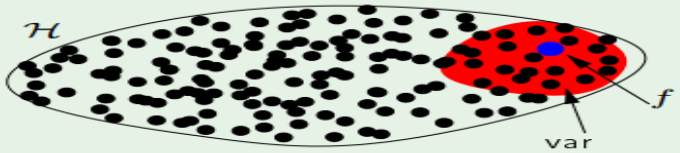
$$\mathbb{E}_{\mathbf{x}}[\text{bias}(\mathbf{x}) + \text{var}(\mathbf{x})]$$

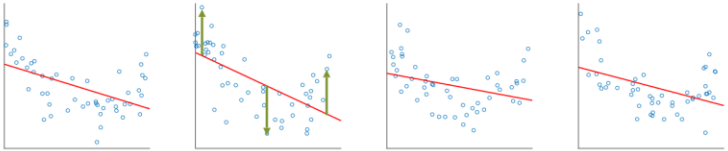
The tradeoff

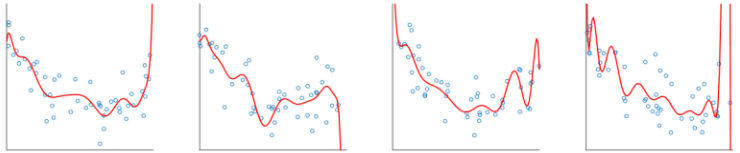
$$\text{bias} = \mathbb{E}_{\mathbf{x}}\left[\left(\bar{g}(\mathbf{x}) - f(\mathbf{x})\right)^2\right]$$

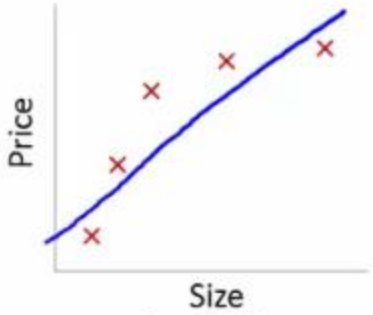
$$\text{var} = \mathbb{E}_{\mathbf{x}}\left[\mathbb{E}_{\mathcal{D}}\left[\left(g^{(\mathcal{D})}(\mathbf{x}) - \bar{g}(\mathbf{x})\right)^2\right]\right]$$





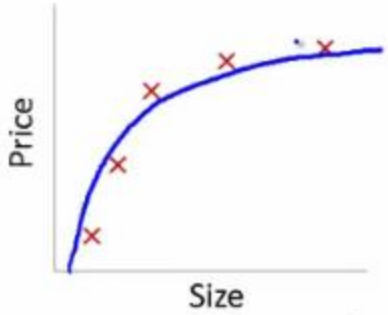






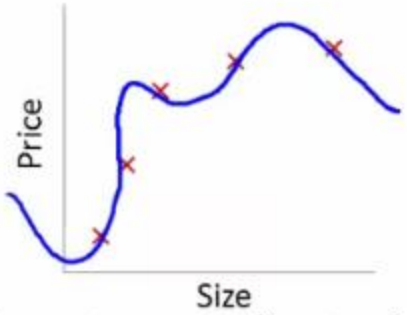
$\theta_0 + \theta_1 x$

High bias
(underfit)



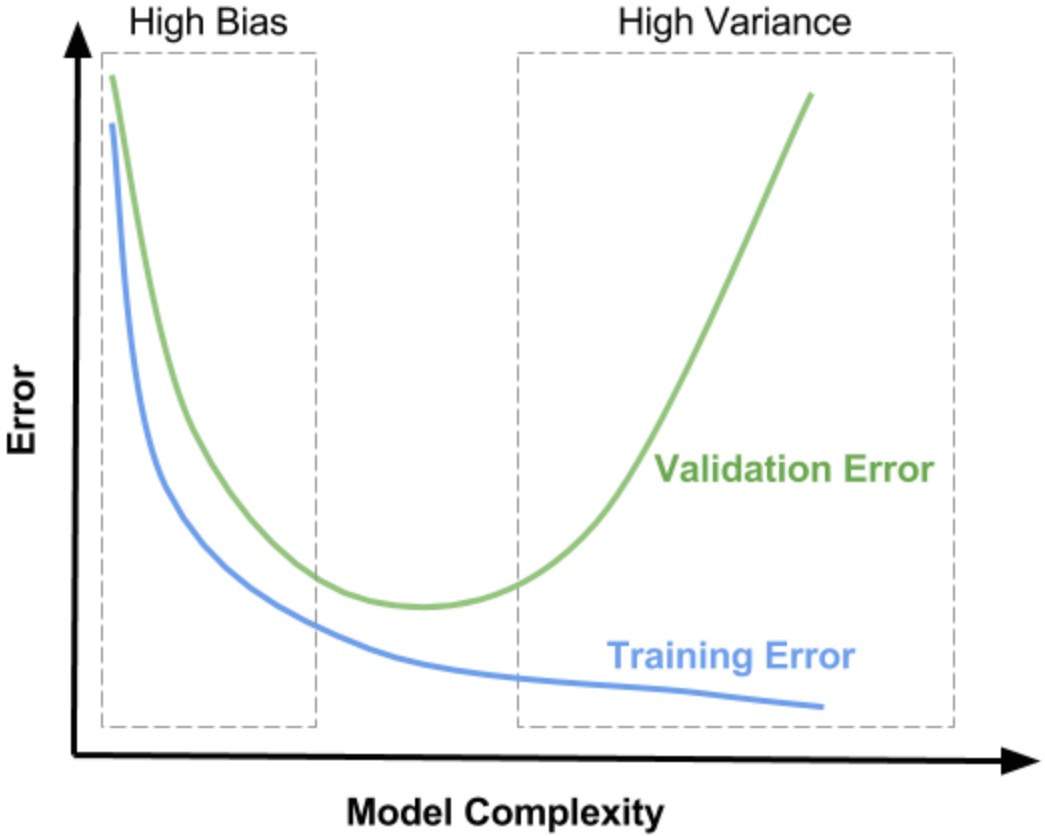
$\theta_0 + \theta_1 x + \theta_2 x^2$

"Just right"



$\theta_0 + \theta_1 x + \theta_2 x^2 + \theta_3 x^3 + \theta_4 x^4$

High variance
(overfit)


High Bias
High Variance
Error
Validation Error
Training Error
Model Complexity

Regularization

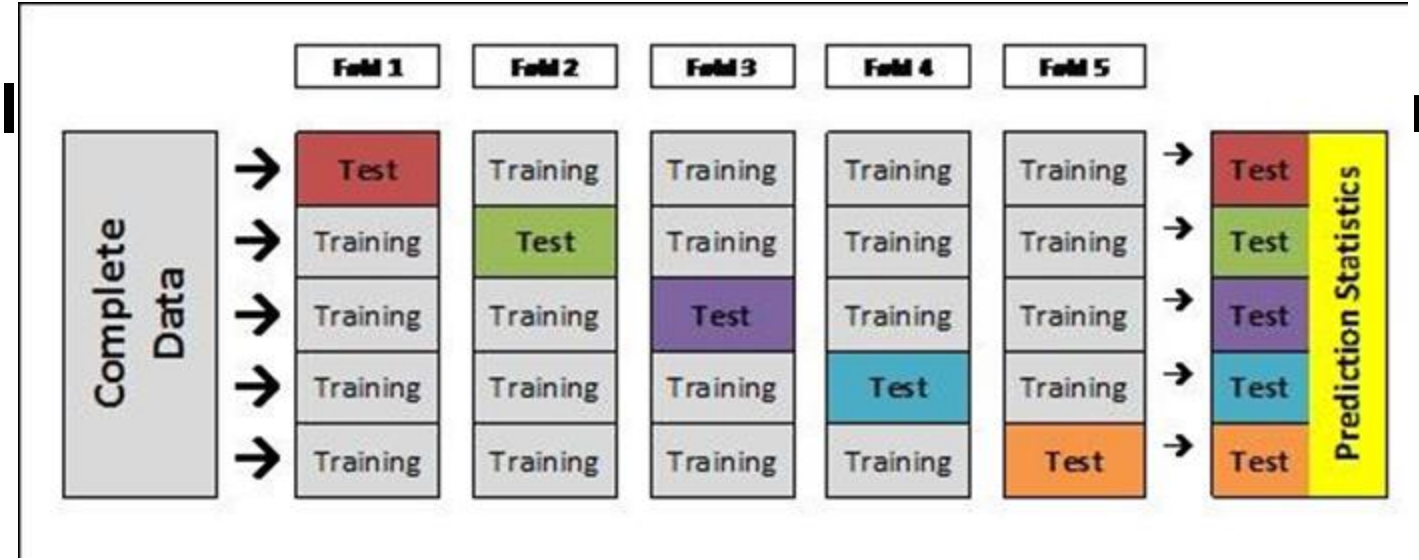
- Remember the intuition: complicated hypotheses lead to overfitting
- Idea: change the error function to *penalize hypothesis complexity*:

$$J(\mathbf{w}) = J_D(\mathbf{w}) + \lambda J_{pen}(\mathbf{w})$$

  This is called *regularization* in machine learning and *shrinkage* in statistics
- λ is called *regularization coefficient* and controls how much we value fitting the data well, vs. a simple hypothesis

Fold → bias

K-fold Cross Validation



Assess model's bias-variance characteristics using k-fold Cross Validation?

- With *k*-fold CV , you get *k* different estimates of your model's error- $e_1, e_2, e_3, \ldots, e_k$
- Mean(errors) ~ 0 ➔ low bias
- Variance(errors) ~ 0 ➔ low variance

Input:
Dataset L
Ensemble size T

```
1.for t=1 to T:
2.    sample = BootstrapSample(L)
3.    h_t = TrainClassifier(sample)
```

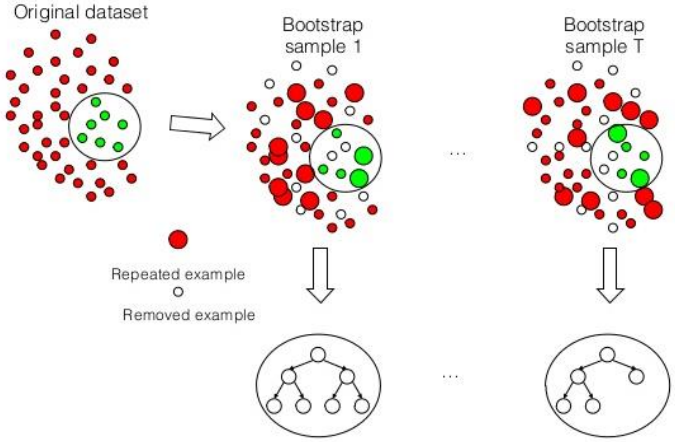
Bootstrap

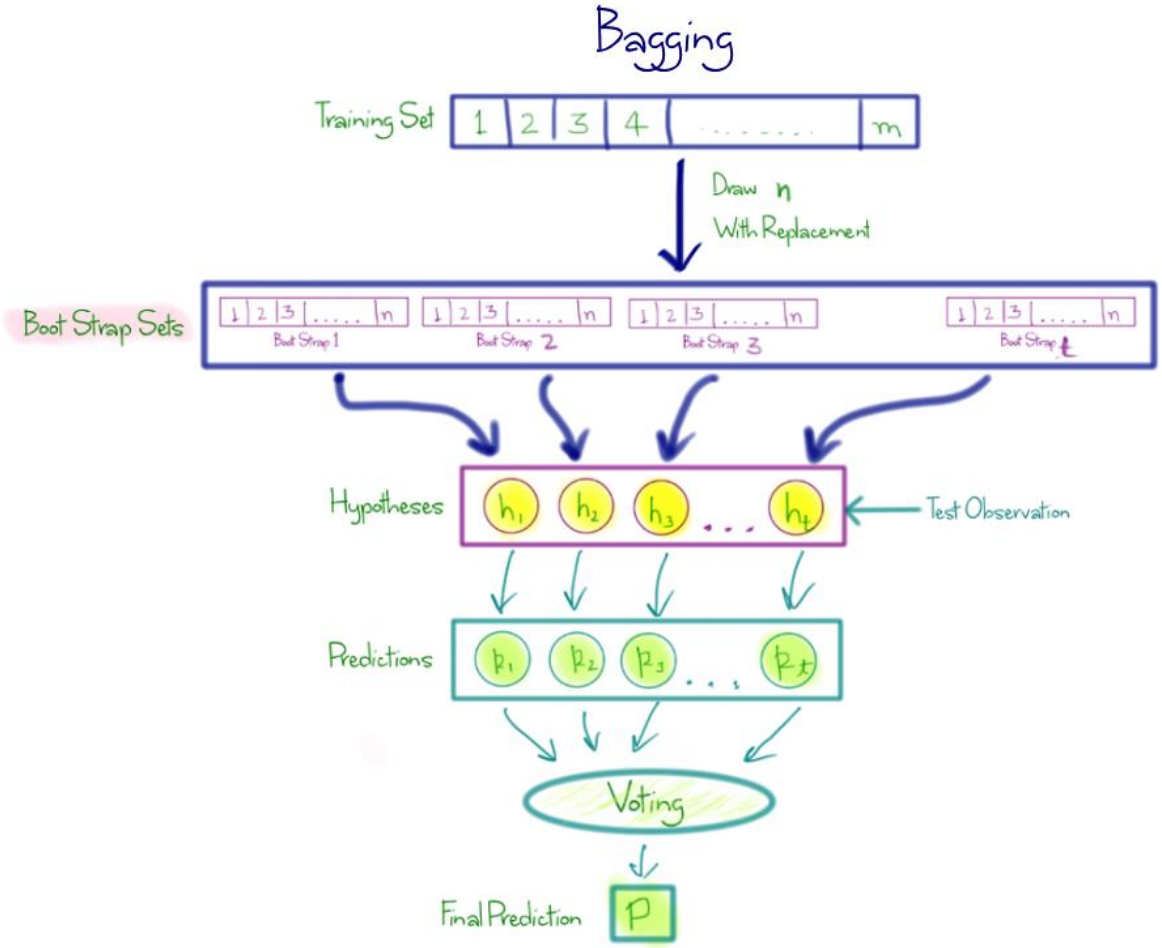
+

Output:

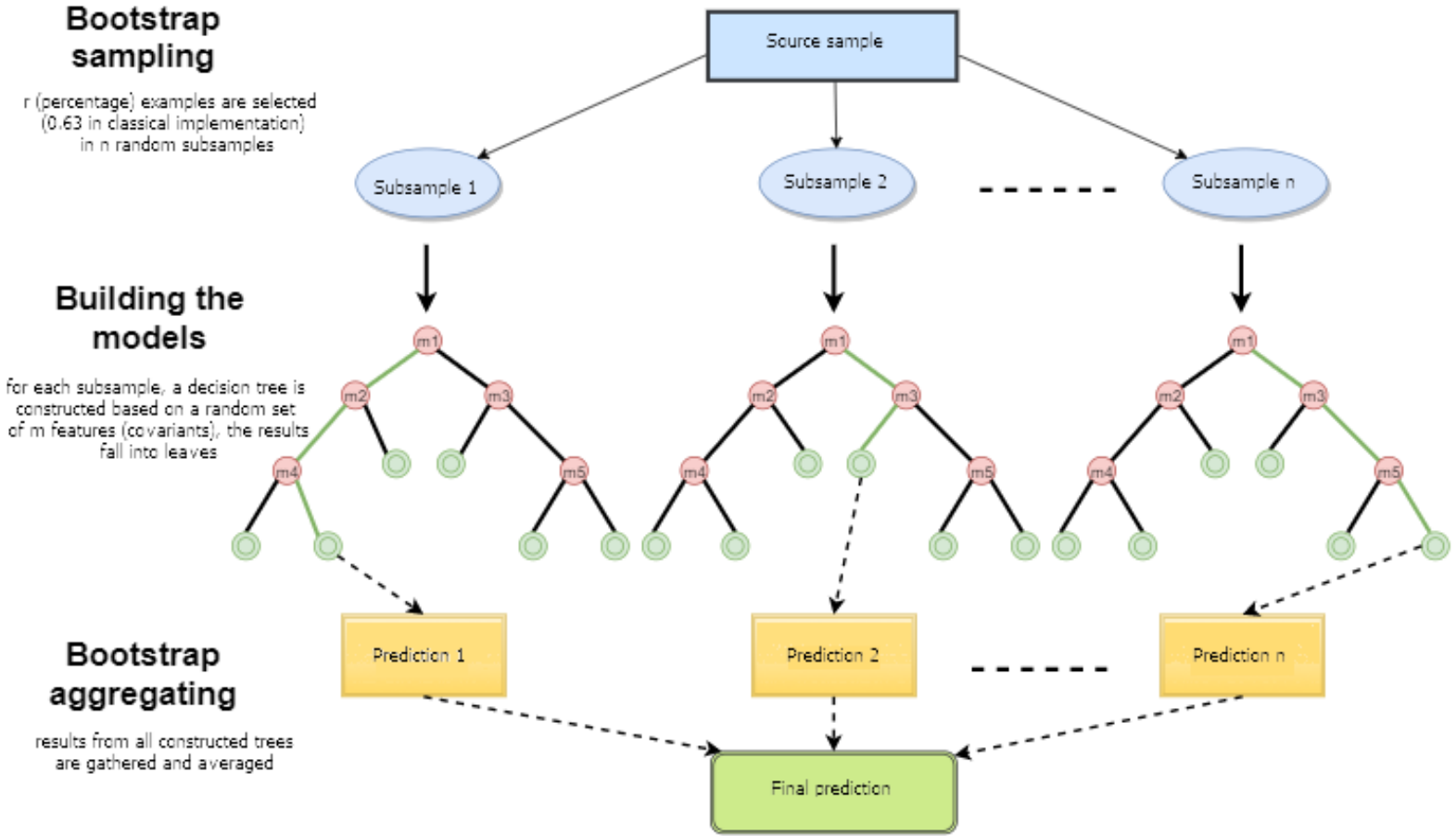
$$H(\mathbf{x}) = \underset{j}{\operatorname{argmax}} \left(\sum_{t=1}^{T} I(h_t(\mathbf{x}) = j) \right)$$

Aggregation



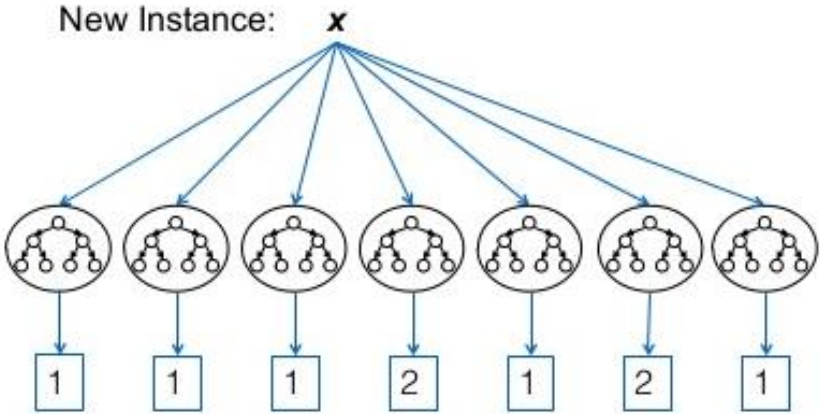


Random Forests

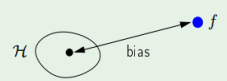


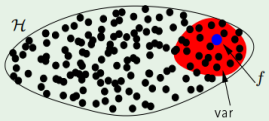
- **Ensemble methods that minimize variance**
  - **Classifier Bagging**
  - **Classifier + Feature Bagging (e.g. Random Forests)**

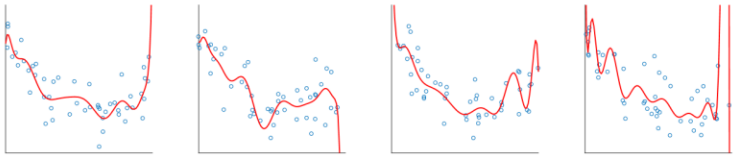
$$\mathbb{E}_{\mathcal{D}}\left[\left(g^{(\mathcal{D})}(\mathbf{x}) - f(\mathbf{x})\right)^2\right] = \underbrace{\mathbb{E}_{\mathcal{D}}\left[\left(g^{(\mathcal{D})}(\mathbf{x}) - \bar{g}(\mathbf{x})\right)^2\right]}_{\text{var}(\mathbf{x})} + \underbrace{\left(\bar{g}(\mathbf{x}) - f(\mathbf{x})\right)^2}_{\text{bias}(\mathbf{x})}$$



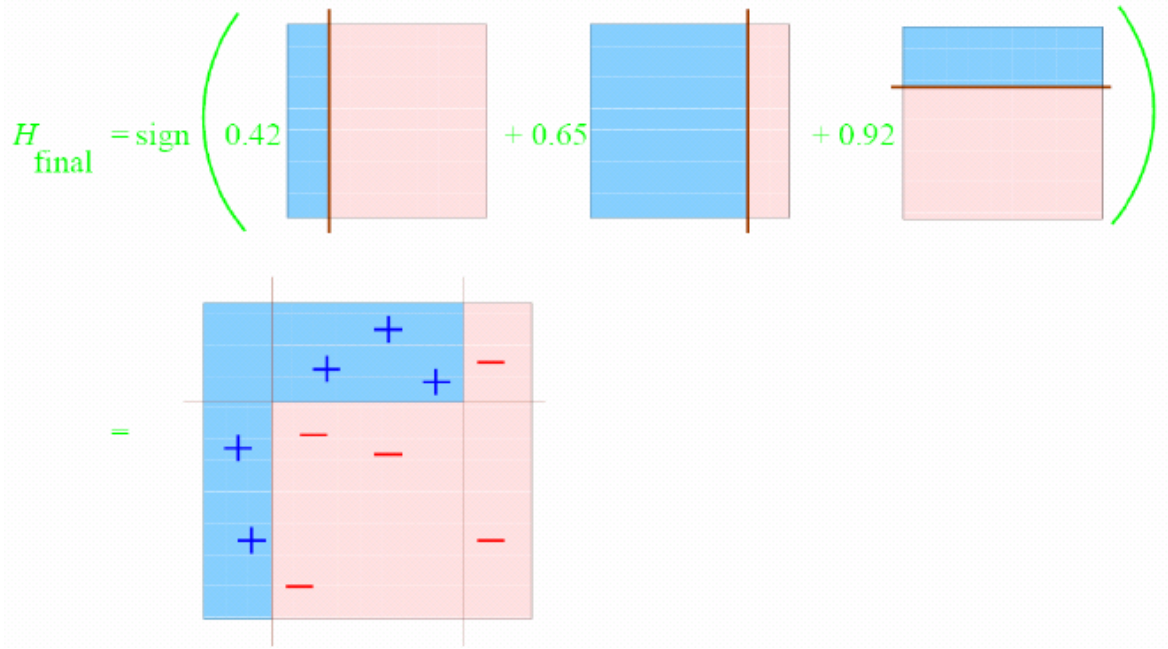
$$\text{bias} = \mathbb{E}_{\mathbf{x}}\left[\left(\bar{g}(\mathbf{x}) - f(\mathbf{x})\right)^2\right] \qquad \text{var} = \mathbb{E}_{\mathbf{x}}\left[\mathbb{E}_{\mathcal{D}}\left[\left(g^{(\mathcal{D})}(\mathbf{x}) - \bar{g}(\mathbf{x})\right)^2\right]\right]$$







A toy example(cont'd)



Final Classifier: integrate the three "weak" classifiers and obtain a final strong classifier.

Given: $(x_1, y_1), \ldots, (x_m, y_m)$ where $x_i \in \mathcal{X}$, $y_i \in \{-1, +1\}$.

Initialize $D_1(i) = 1/m$ for $i = 1, \ldots, m$. ← Initial Distribution of Data

For $t = 1, \ldots, T$:

- Train weak learner using distribution D_t.
- Get weak hypothesis $h_t : \mathcal{X} \to \{-1, +1\}$. ← Train model
- Aim: select h_t with low weighted error:

$$\varepsilon_t = \Pr_{i \sim D_t}\left[h_t(x_i) \neq y_i\right].$$ ← Error of model

- Choose $\alpha_t = \frac{1}{2} \ln\left(\frac{1 - \varepsilon_t}{\varepsilon_t}\right)$. ← Coefficient of model
- Update, for $i = 1, \ldots, m$:

$$D_{t+1}(i) = \frac{D_t(i) \exp(-\alpha_t y_i h_t(x_i))}{Z_t}$$ ← Update Distribution

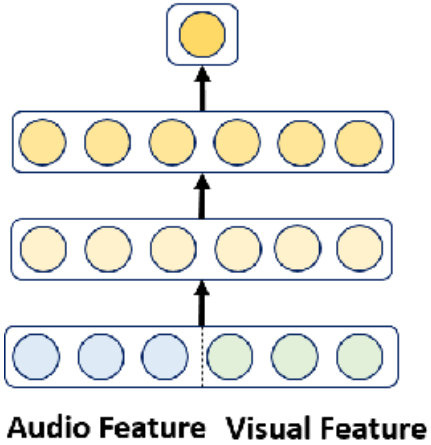
where Z_t is a normalization factor (chosen so that D_{t+1} will be a distribution).

Output the final hypothesis:

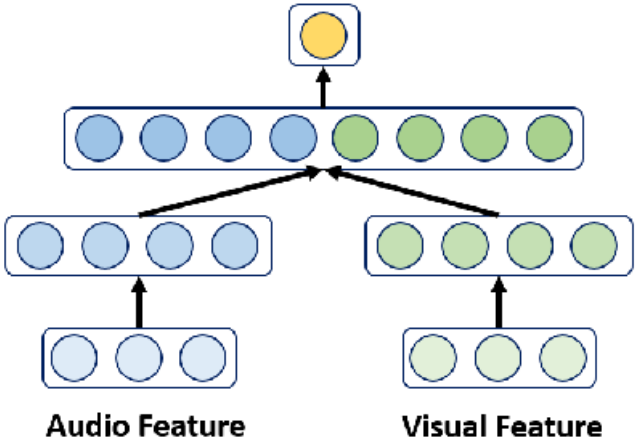
$$H(x) = \text{sign}\left(\sum_{t=1}^{T} \alpha_t h_t(x)\right).$$ ← Final average

Theorem: training error drops exponentially fast

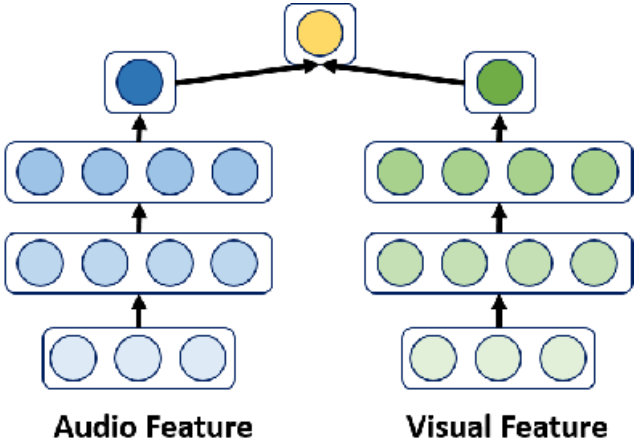
https://www.cs.princeton.edu/~schapire/papers/explaining-adaboost.pdf



(a) Early Fusion



(b) Model-level Fusion



(c) Late Fusion

Eyes detector

Mouth detector

Ears detector

Skin detector

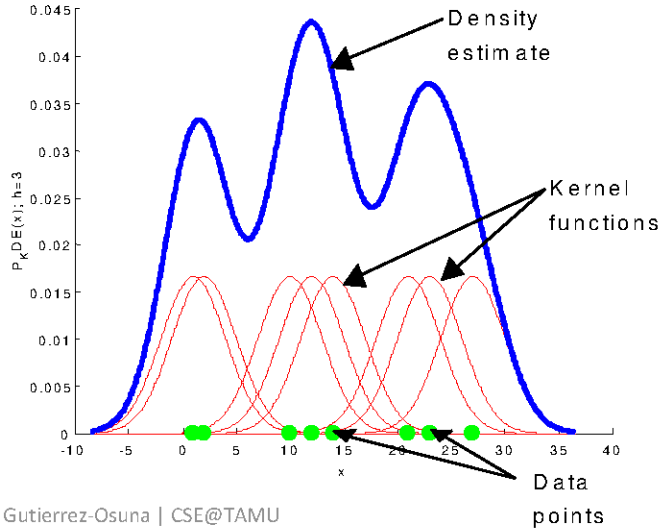
Meta Classifier

High accuracy!

Low individual accuracy
Computationally efficient

Interpretation

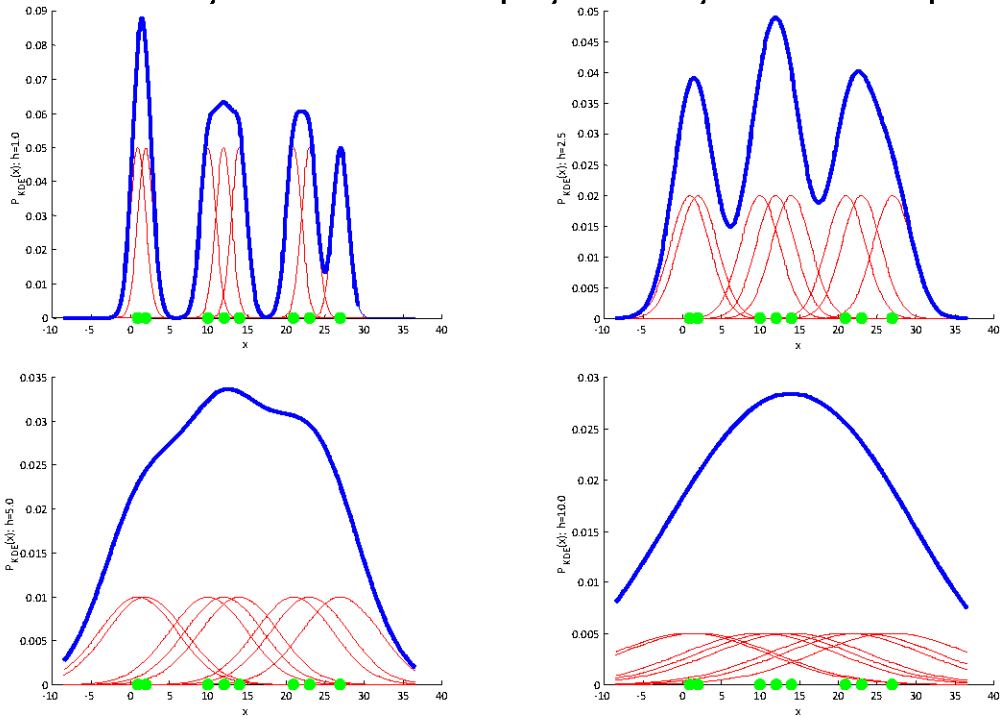
- Just as the Parzen window estimate can be seen as a sum of boxes centered at the data, the smooth kernel estimate is a sum of "bumps"
- The kernel function determines the shape of the bumps
- The parameter h, also called the smoothing parameter or bandwidth, determines their width



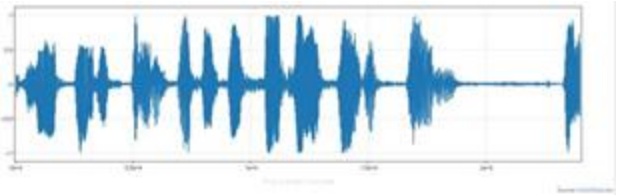
Bandwidth selection

The problem of choosing h is crucial in density estimation

- A large h will over-smooth the DE and mask the structure of the data
- A small h will yield a DE that is spiky and very hard to interpret



Sequential data

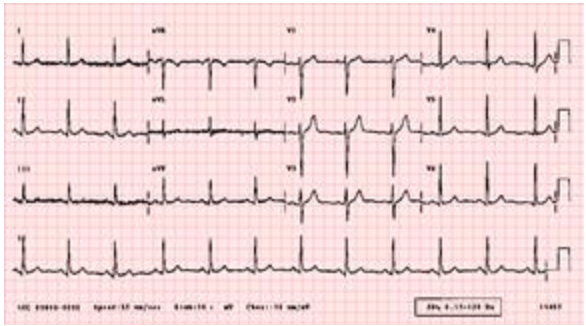


Audio

Video

O Nachiketa, after pondering well the pleasures that are or seem to be delightful, you have renounced them all. You have not taken the road abounding in wealth, where many men sink. *(Kathopanishad, II:3)*

Text



Biological

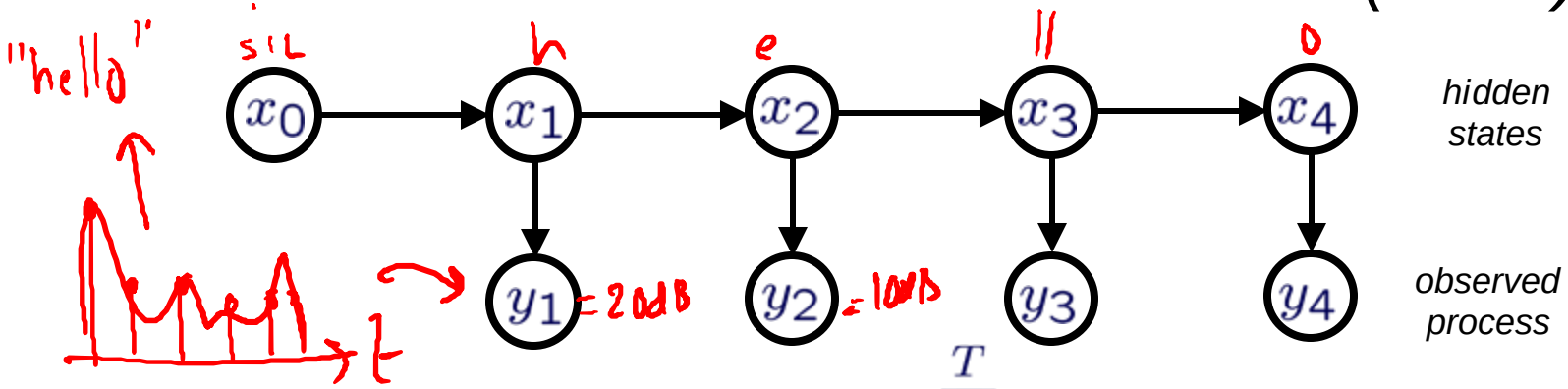
many more ……

Hidden Markov Models

- Few realistic time series directly satisfy the assumptions of Markov processes:

"Conditioned on the present,
the past & future are independent"

- Motivates *hidden Markov models (HMM):*



$$p(x_0, x_1, \ldots, x_T) = p(x_0) \prod_{t=1}^{T} p(x_t \mid x_{t-1}) p(y_t \mid x_t)$$

Discrete HMMs: Observations

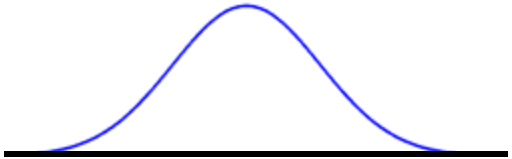
Discrete Observations

$$y_t \in \{1, 2, \ldots, M\}$$

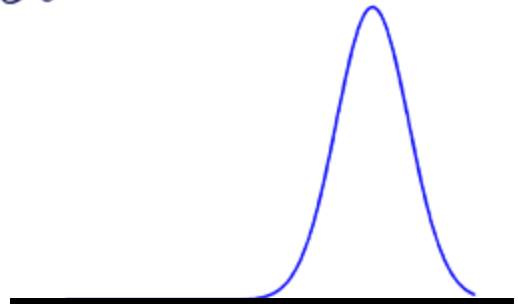
$$p(y_t \mid x_t = 1) = \begin{bmatrix} 0.3 \\ 0.1 \\ 0.5 \\ 0.1 \end{bmatrix} \qquad p(y_t \mid x_t = 2) = \begin{bmatrix} 0.2 \\ 0.2 \\ 0.1 \\ 0.5 \end{bmatrix}$$

Continuous Observations

$$y_t \in \mathbb{R}^k$$

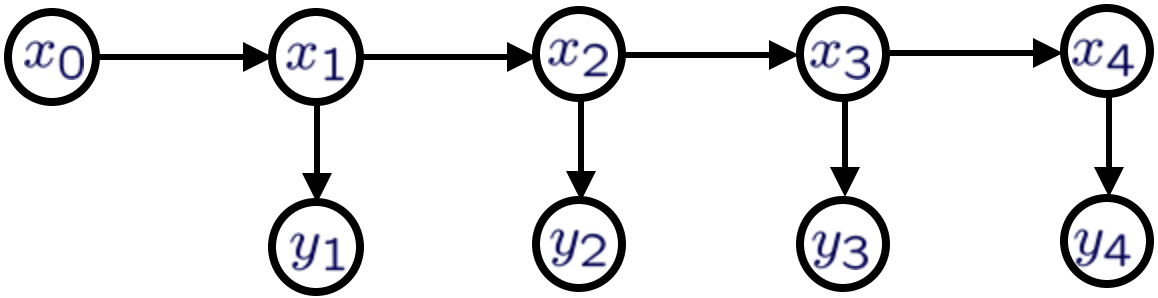


$p(y_t \mid x_t = 1)$



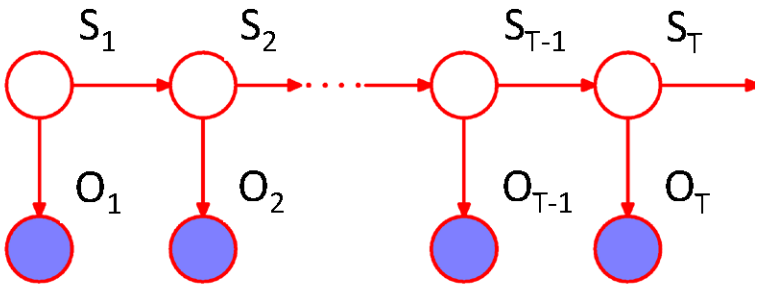
$p(y_t \mid x_t = 2)$

Specifying an HMM



- Observation model: $P(y_i|x_i)$
- Transition model: $P(x_i|x_{i-1})$
- Initial state distribution: $P(x_0)$

Hidden Markov Models



$$p(S_1, \ldots, S_T, O_1, \ldots, O_T) = \prod_{t=1}^{T} p(O_t|S_t) \prod_{t=1}^{T} p(S_t|S_{t-1})$$

$$p(A|B) = \frac{p(A,B)}{p(B)} = \frac{p(A,B)}{\sum_A p(A,B)}$$

1st order Markov assumption on hidden states $\{S_t\}$ t = 1, …, T
(can be extended to higher order).

Note: O_t depends on all previous observations $\{O_{t-1}, \ldots O_1\}$

Hidden Markov Models

- Parameters – stationary/homogeneous markov model (independent of time t)

Initial probabilities

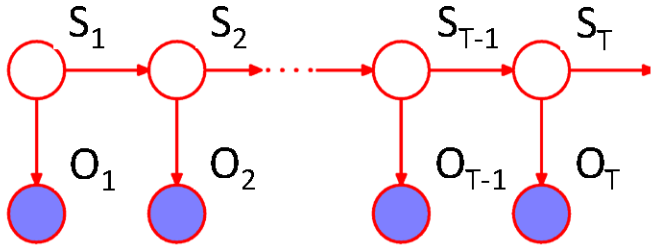
$p(S_1 = i) = \pi_i$

Transition probabilities

$p(S_t = j \mid S_{t-1} = i) = p_{ij}$

Emission probabilities

$p(O_t = y \mid S_t = i) = q_i^y$



$$p(\{S_t\}_{t=1}^T, \{O_t\}_{t=1}^T) = p(S_1) \prod_{t=2}^{T} p(S_t|S_{t-1}) \prod_{t=1}^{T} p(O_t|S_t)$$

Three main problems in HMMs

O_1 → 12455264621461461361366616646616366163661636161565156151151461235623 44 ← O_T

- Evaluation – Given HMM parameters & observation seqn $\{O_t\}_{t=1}^T$

  find $p(\{O_t\}_{t=1}^T|\theta)$ prob of observed sequence

  - How likely is this sequence, given our model of how the casino works?
- Decoding – Given HMM parameters & observation seqn $\{O_t\}_{t=1}^T$

  find $\arg\max_{s_1,\ldots,s_T} p(\{S_t\}_{t=1}^T|\{O_t\}_{t=1}^T,\theta)$ most probable

  sequence of hidden states

  - What portion of the sequence was generated with the fair die, and what portion with the loaded die?
- Learning – Given HMM with unknown parameters and $\{O_t\}_{t=1}^T$ observation sequence

  find $\arg\max_\theta p(\{O_t\}_{t=1}^T|\theta)$ parameters that maximize

  likelihood of observed data

  - How "loaded" is the loaded die? How "fair" is the fair die? How often does the casino player change from fair to loaded, and back?

HMM Algorithms

- Evaluation – What is the probability of the observed sequence? Forward Algorithm

- Decoding – What is the probability that the third roll was loaded given the observed sequence? Forward-Backward Algorithm

  – What is the most likely die sequence given the observed sequence? Viterbi Algorithm

- Learning – Under what parameterization is the observed sequence most probable? Baum-Welch Algorithm (EM)

Recurrent Neural Network

We can process a sequence of vectors **x** by applying a recurrence formula at every time step:

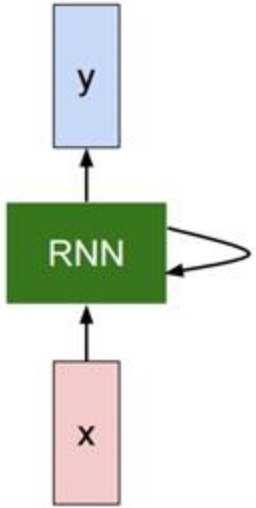
$$h_t = f_W(h_{t-1}, x_t)$$

new state

some function with parameters W

old state

input vector at some time step

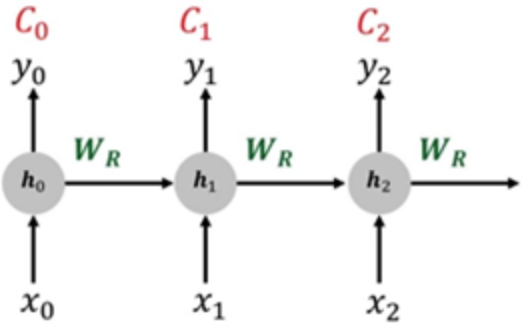


Task-specific abstraction for portion of sequence seen up to this point ($x_1, x_2 \ldots x_{t-1}$)

http://cs231n.stanford.edu/slides/winter1516_lecture10.pdf

Training RNNs - Backpropagation through time

$$\boldsymbol{h}^{(t)} = g_h(W_I \boldsymbol{x}^{(t)} + W_R \boldsymbol{h}^{(t-1)} + \boldsymbol{b}_h)$$
$$\boldsymbol{y}^{(t)} = g_y(W_y \boldsymbol{h}^{(t)} + \boldsymbol{b}_y)$$



$$\frac{\partial C}{\partial W_R} = \sum_{t=1}^{T} \frac{\partial C_t}{\partial W_R}$$

$$\frac{\partial C_t}{\partial W_R} = \sum_{k=1}^{t} \frac{\partial C_t}{\partial y_t} \frac{\partial y_t}{\partial h_t} \frac{\partial h_t}{\partial h_k} \frac{\partial h_k}{\partial W_R}$$

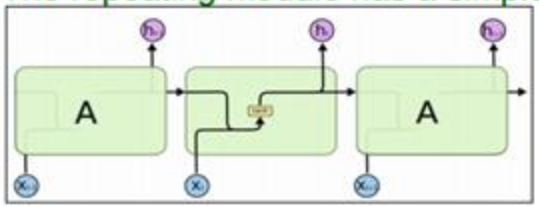
$$\frac{\partial h_t}{\partial h_k} = \prod_{i=k+1}^{t} \frac{\partial h_i}{\partial h_{i-1}} = \prod_{i=k+1}^{t} W_R^T \mathrm{diag}(g_h'(W_I x^{(i)} + W_R h^{(i-1)} + b_h))$$

- (Potentially) multiple, intermediate costs C_0, C_1, C_2
- Shared weights $\mathbf{W_R}$

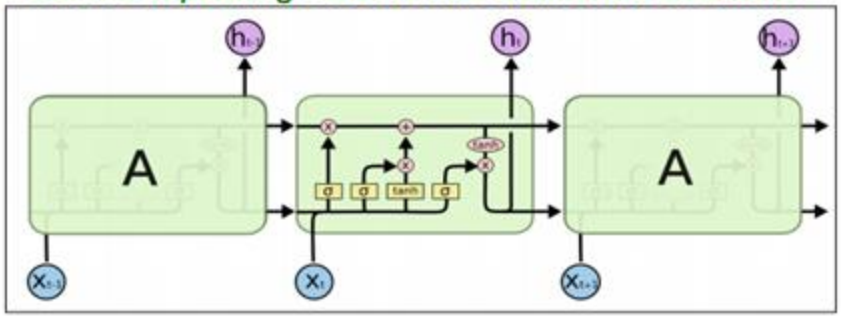
https://www.youtube.com/watch?v=Ukgii7Yd_cU

Long Short Term Memory (LSTM)

- Explicitly designed to avoid the long-term dependency problem
- RNNs have the form of a repeating chain structure
  - The repeating module has a simple structure such as tanh



- LSTMs also have a chain structure
  - but the repeating module has a different structure

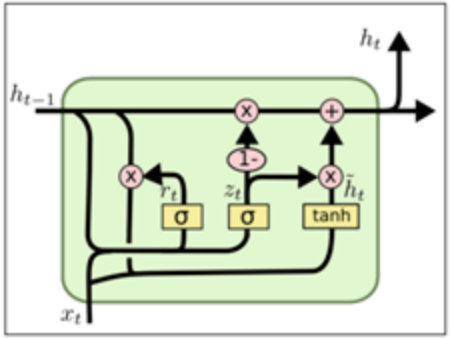


http://www.cedar.buffalo.edu/~srihari/CSE676/LSTM.pdf

GRU: a LSTM variant

Gated Recurrent Unit (GRU)

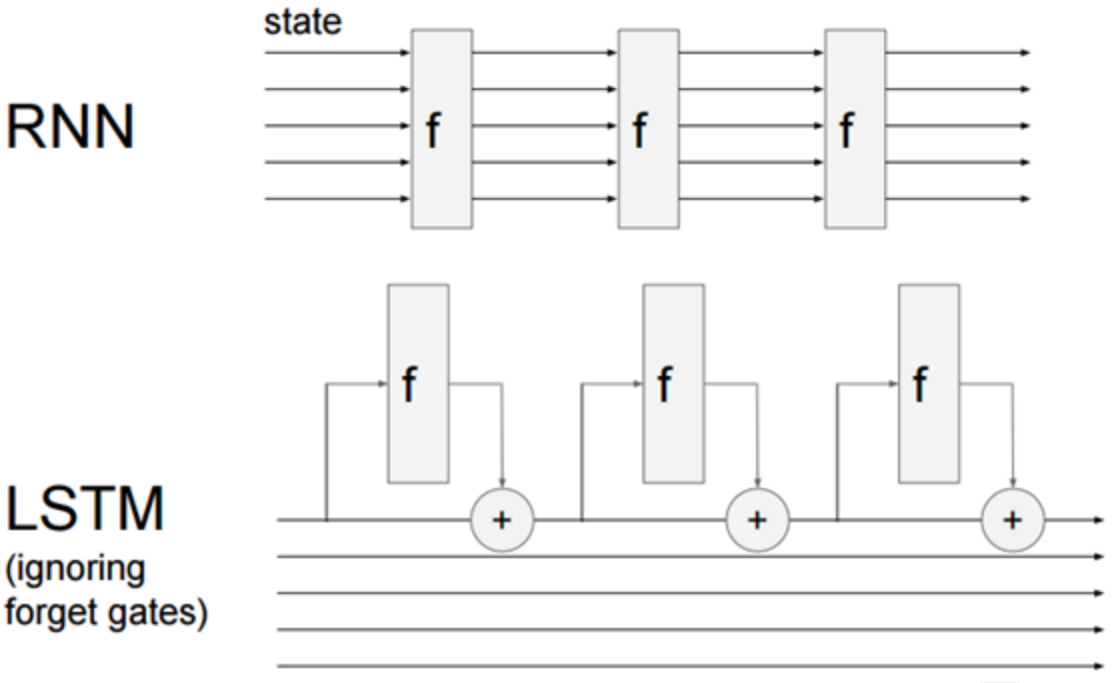
- A dramatic variant of LSTM
  - It combines the forget and input gates into a single update gate
  - It also merges the cell state and hidden state, and makes some other changes
  - The resulting model is simpler than LSTM models
  - Has become increasingly popular



$$z_t = \sigma\left(W_z \cdot [h_{t-1}, x_t]\right)$$
$$r_t = \sigma\left(W_r \cdot [h_{t-1}, x_t]\right)$$
$$\tilde{h}_t = \tanh\left(W \cdot [r_t * h_{t-1}, x_t]\right)$$
$$h_t = (1 - z_t) * h_{t-1} + z_t * \tilde{h}_t$$

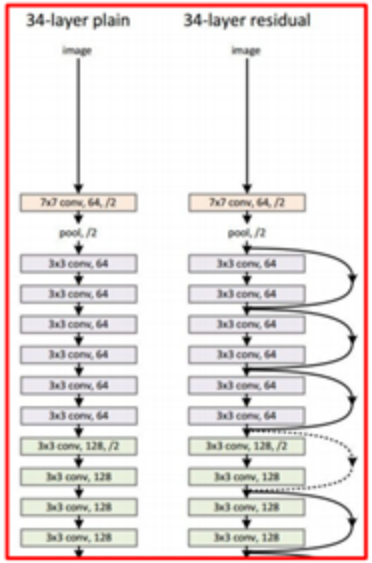
http://www.cedar.buffalo.edu/~srihari/CSE676/LSTM.pdf

Long Short Term Memory (LSTM)



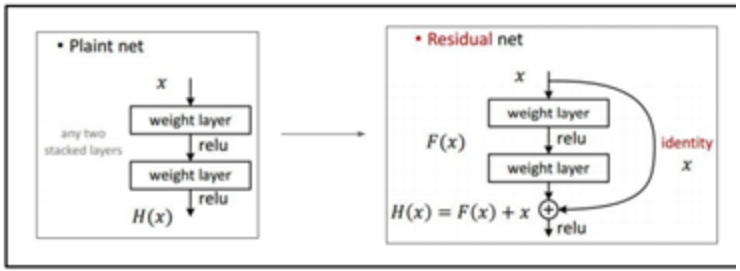
http://cs231n.stanford.edu/slides/winter1516_lecture10.pdf

Long Short Term Memory (LSTM)



Recall:
"PlainNets" vs. ResNets

ResNet is to PlainNet what LSTM is to RNN, kind of.



http://cs231n.stanford.edu/slides/winter1516_lecture10.pdf

Getting targets when modeling sequences

- When applying machine learning to sequences, we often want to turn an input sequence into an output sequence that lives in a different domain.
  - *E. g.* turn a sequence of sound pressures into a sequence of word identities.

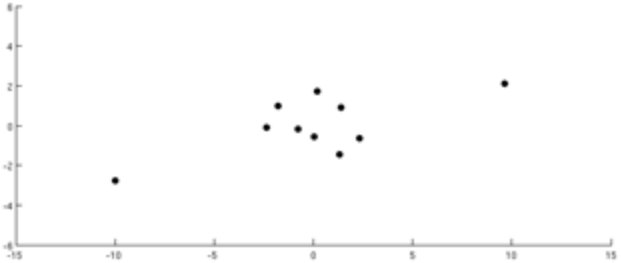
- When there is no separate target sequence, we can get a teaching signal by trying to predict the next term in the input sequence.
  - The target output sequence is the input sequence with an advance of 1 step.
  - This seems much more natural than trying to predict one pixel in an image from the other pixels, or one patch of an image from the rest of the image.
  - For temporal sequences there is a natural order for the predictions.

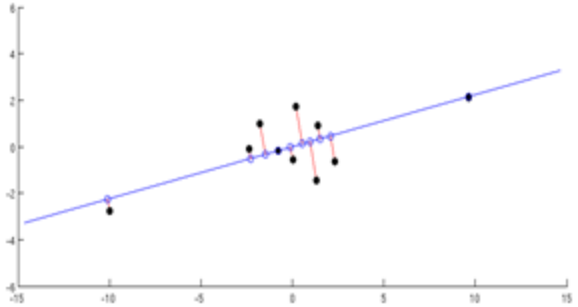
- Predicting the next term in a sequence blurs the distinction between supervised and unsupervised learning.
  - It uses methods designed for supervised learning, but it doesn't require a separate teaching signal.

https://www.cs.toronto.edu/~hinton/csc2535/notes/lec10new.pdf

Is PCA minimizing the right objective function ?

- PCA is mainly concerned with preserving **large** pairwise distances in the map

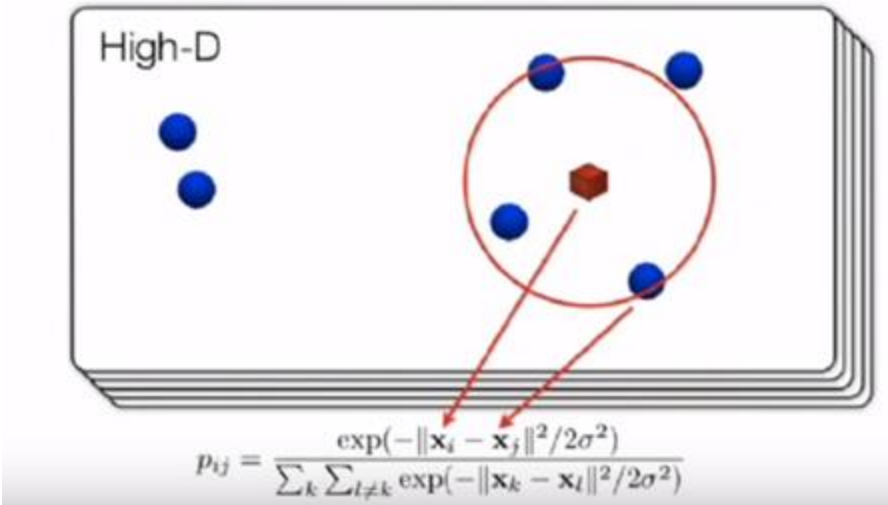




- “crowding” problem for smaller pairwise distances

t-SNE : Modeling Similarities in High-D

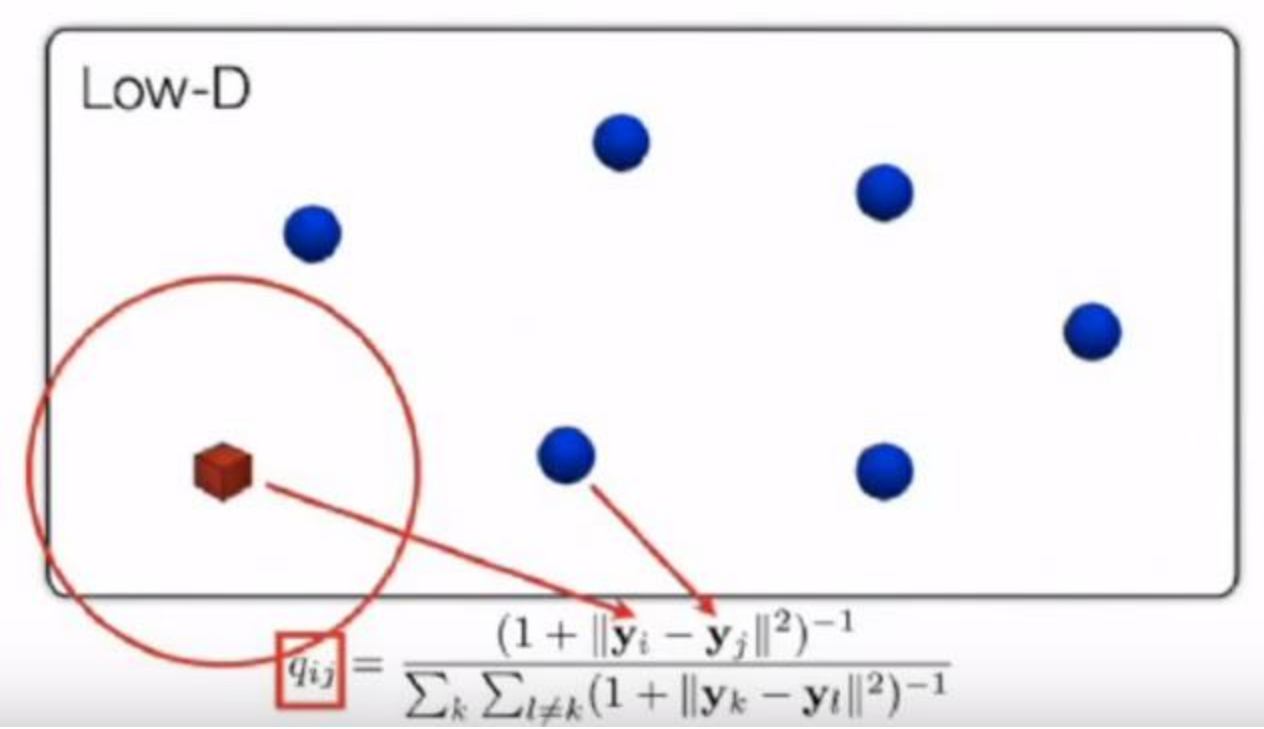
Center a gaussian under each point i



Prob of picking point pair (i,j) p_ij [their similarity]

Nearby points=>Large p_ij, Far points => Infinitesimal p_ij

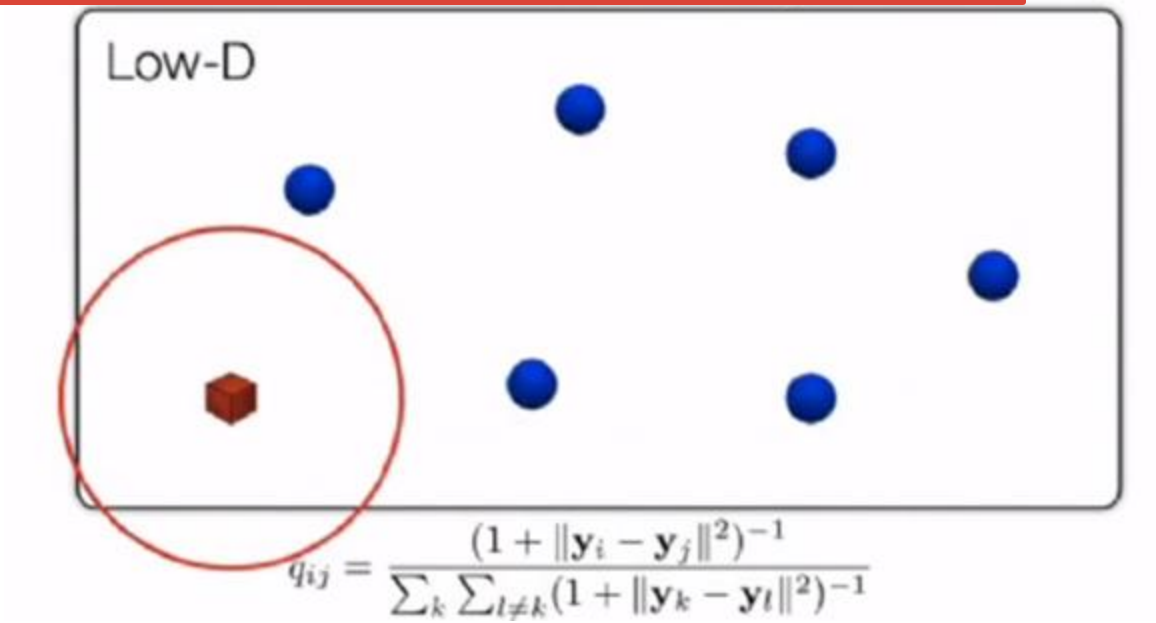
t-SNE : Modeling Similarities in Low-D



Lay out points in 2D/3D such that q_ij ~ p_ij

t-SNE : Modeling Similarities in Low-D

- Move points around to minimize: $KL(P||Q) = \sum_i \sum_{j \neq i} p_{ij} \log \frac{p_{ij}}{q_{ij}}$



Lay out points in 2D/3D such that q_ij ~ p_ij

t-SNE: Optimization

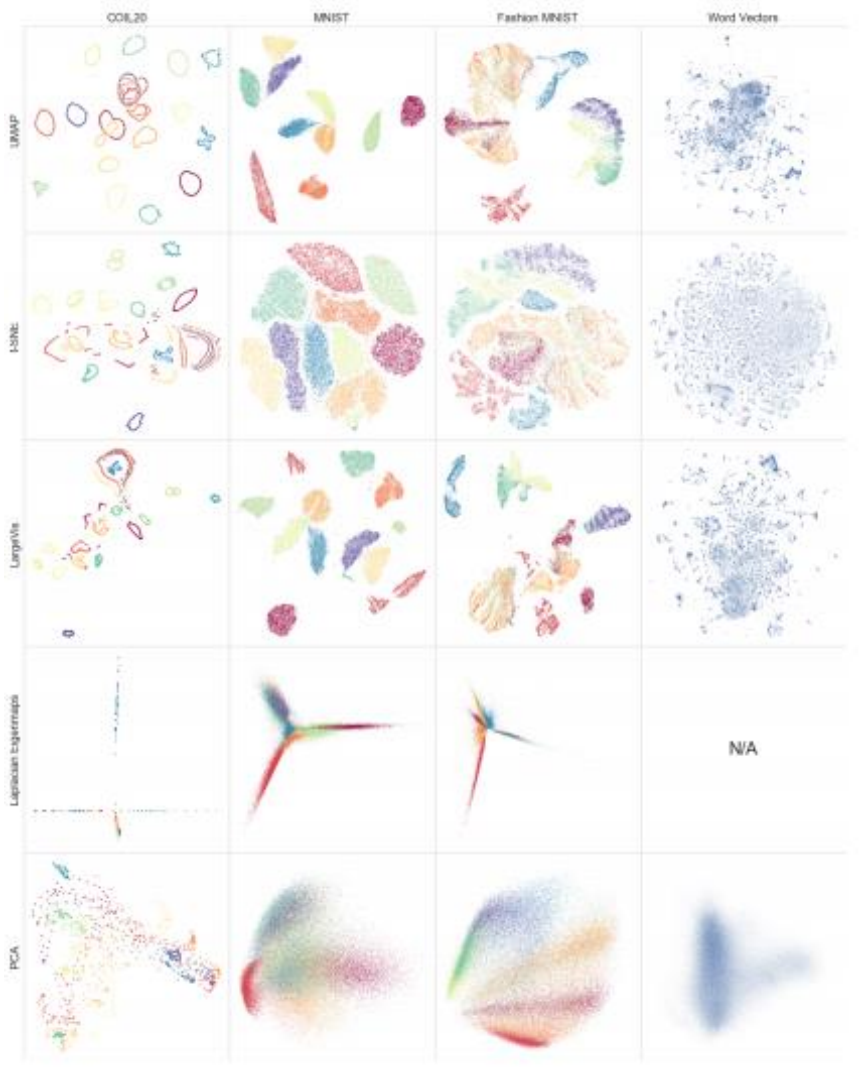
$$p_{j|i} = \frac{\exp(-\|\mathbf{x}_i - \mathbf{x}_j\|^2/2\sigma_i^2)}{\sum_{j' \neq i} \exp(-\|\mathbf{x}_i - \mathbf{x}_{j'}\|^2/2\sigma_i^2)} \qquad q_{ij} = \frac{(1 + \|\mathbf{y}_i - \mathbf{y}_j\|^2)^{-1}}{\sum_k \sum_{l \neq k} (1 + \|\mathbf{y}_k - \mathbf{y}_l\|^2)^{-1}}$$

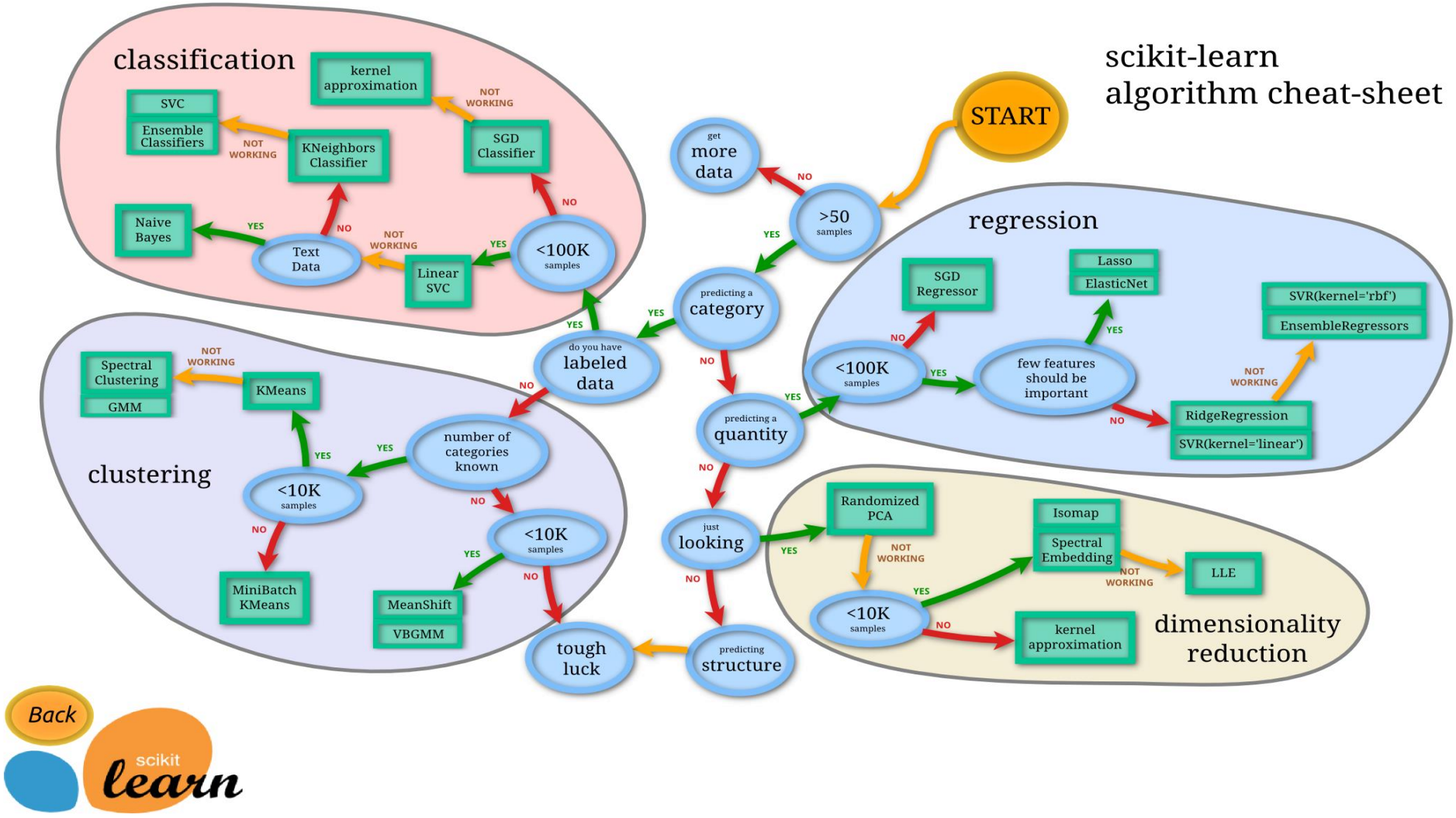
$$C = \sum_i KL(P_i||Q_i) = \sum_i \sum_j p_{j|i} \log \frac{p_{j|i}}{q_{j|i}}$$

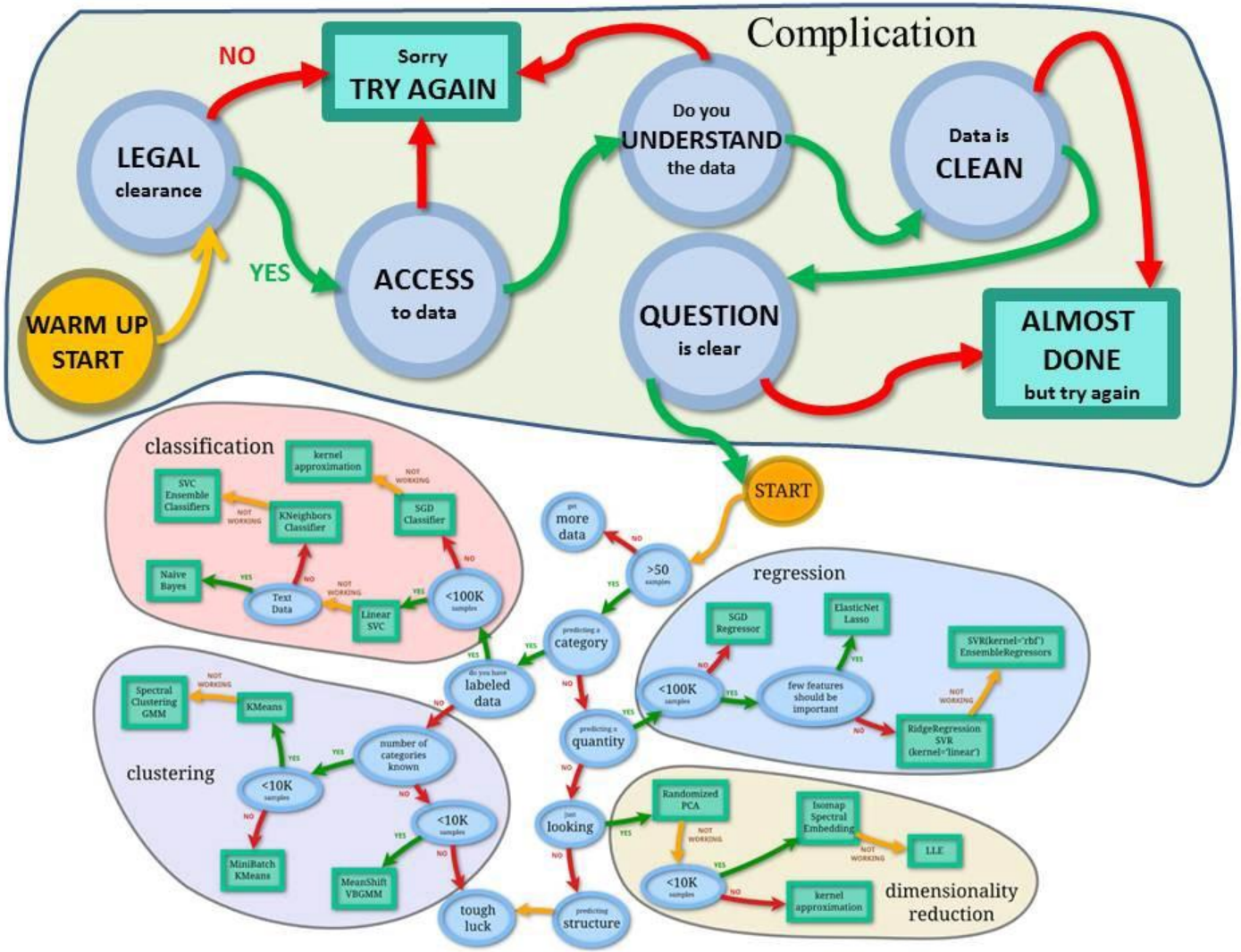
$$\frac{\delta C}{\delta y_i} = 2 \sum_j (p_{j|i} - q_{j|i} + p_{i|j} - q_{i|j})(y_i - y_j)$$

$$\mathcal{Y}^{(t)} = \mathcal{Y}^{(t-1)} + \eta \frac{\delta C}{\delta \mathcal{Y}} + \alpha(t) \left(\mathcal{Y}^{(t-1)} - \mathcal{Y}^{(t-2)} \right)$$

UMAP
(Universal Manifold approximation)



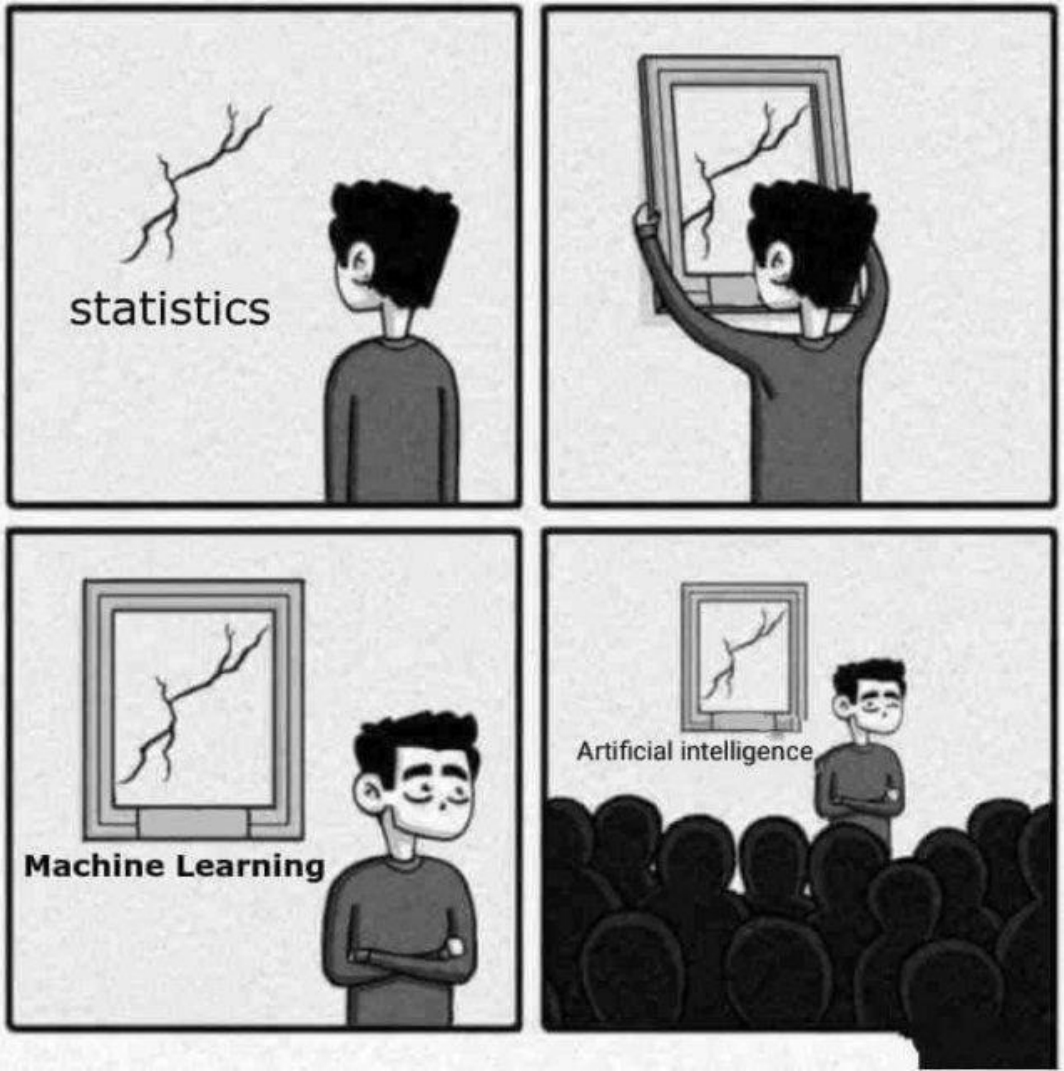

scikit-learn
algorithm cheat-sheet
START
get more data
NO
>50 samples
YES
predicting a category
YES
NO
do you have labeled data
YES
NO
predicting a quantity
YES
NO
just looking
YES
NO
predicting structure
tough luck
classification
kernel approximation
NOT WORKING
SVC
Ensemble Classifiers
NOT WORKING
KNeighbors Classifier
SGD Classifier
NO
Naive Bayes
YES
Text Data
NO
NOT WORKING
Linear SVC
YES
<100K samples
YES
regression
SGD Regressor
Lasso
ElasticNet
SVR(kernel='rbf')
EnsembleRegressors
NO
<100K samples
YES
few features should be important
YES
NOT WORKING
NO
RidgeRegression
SVR(kernel='linear')
clustering
Spectral Clustering
NOT WORKING
GMM
KMeans
YES
number of categories known
YES
NO
<10K samples
NO
MiniBatch KMeans
<10K samples
YES
NO
MeanShift
VBGMM
dimensionality reduction
Randomized PCA
YES
NOT WORKING
Isomap
Spectral Embedding
NOT WORKING
LLE
YES
<10K samples
NO
kernel approximation
Back
scikit learn


Complication
NO
Sorry
TRY AGAIN
LEGAL
clearance
YES
ACCESS
to data
Do you
UNDERSTAND
the data
Data is
CLEAN
WARM UP
START
QUESTION
is clear
ALMOST
DONE
but try again
START
classification
kernel approximation
NOT WORKING
SVC Ensemble Classifiers
NOT WORKING
KNeighbors Classifier
SGD Classifier
NO
Naive Bayes
YES
NO
Text Data
NOT WORKING
Linear SVC
YES
<100K samples
NO
more data
NO
>50 samples
YES
regression
SGD Regressor
ElasticNet Lasso
SVR(kernel='rbf') EnsembleRegressors
NO
<100K samples
YES
few features should be important
YES
NOT WORKING
RidgeRegression SVR (kernel='linear')
NO
predicting a category
YES
do you have labeled data
NO
clustering
Spectral Clustering GMM
NOT WORKING
KMeans
NO
number of categories known
YES
<10K samples
YES
NO
<10K samples
NO
YES
MiniBatch KMeans
MeanShift VBGMM
tough luck
predicting a quantity
NO
just looking
YES
NO
predicting structure
Randomized PCA
NOT WORKING
Isomap Spectral Embedding
NOT WORKING
LLE
YES
<10K samples
NO
kernel approximation
dimensionality reduction

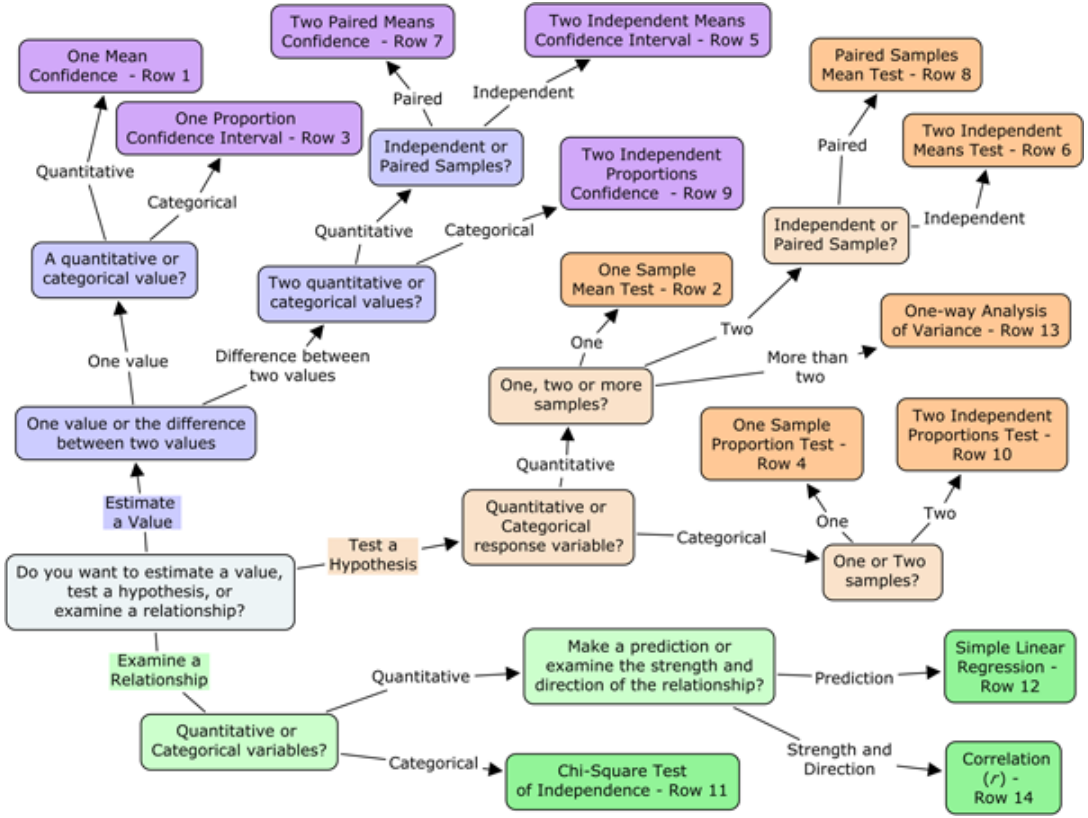
SMAI (Statistical Methods in AI)

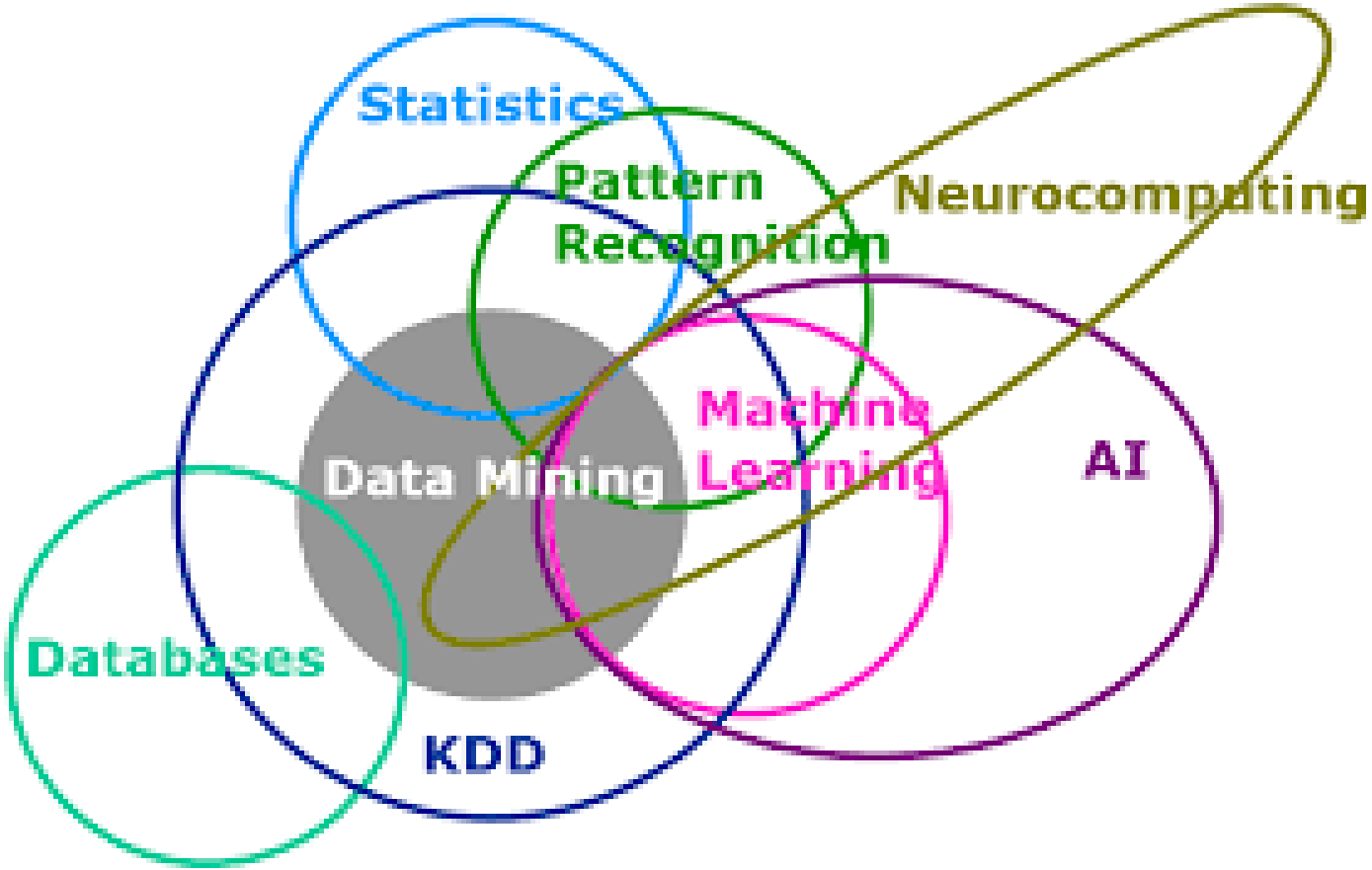
- SMAI ~ Introduction to Machine Learning

- Good news: One half is already familiar to you [Machine !]
- Other half = What this course is about !


statistics
Machine Learning
Artificial intelligence

Statistical Methods - cheatsheet





https://www.analyticsvidhya.com/blog/2015/07/difference-machine-learning-statistical-modeling/

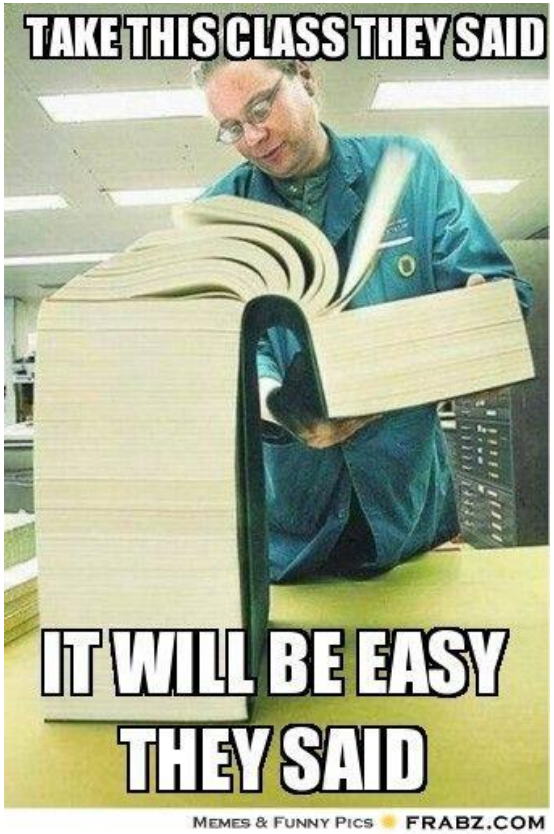
Know the **practical** limits of ML

Course Objectives

- Determine whether ML is suitable for a problem
- Formulate a problem as a ML problem (data ,representations, tasks, algorithms)
- **Understand** and apply ML method(s)
- Be aware of ML pitfalls, follow best practices
- Be ready to dive deeper (into ML theory or applied areas)

About the course – Grading Policy

- Assessment
  - 2 mid semester exams (2x15% = 30%)
  - 1 Final Exam (30%)
  - 13 Assignments (13x2% = 26%)
  - 1 Project (14%)

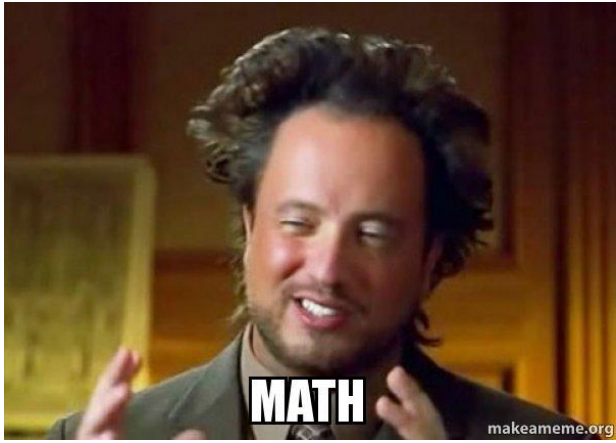


About the course – collaboration policy

- OK to discuss assignment questions and approaches
- But work must be your own (no copying – partially or fully)
- If you worked with someone, mention their name(s)
- We will be checking for copying/plagiarism
- Better to own up than be caught !

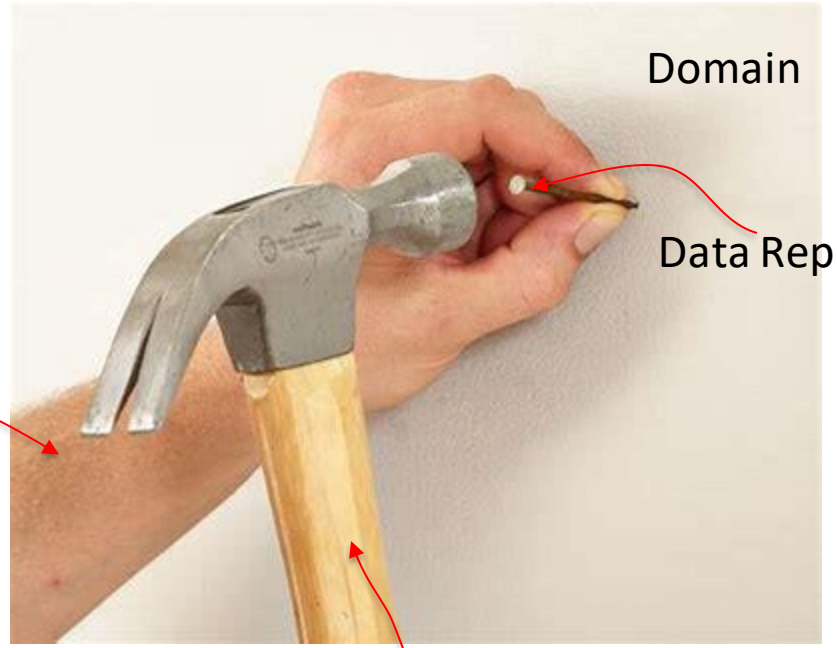
https://twitter.com/terago_networks/status/707283921323429889

Additionally …



- **Understand**, don’t just memorize
- Love the math, not the toolbox !
- Capture the broad ideas and insights (useful years down the line)
- Implement ! No substitute for experience.
- Just the beginning ….

Projects



Projects

- Problem/Task-first thinking
- Come up with a top-3 list and why the problem is interesting / novel to study
- Be creative / original