Shock and Vibration Characteristics of Tennis Rackets

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Abstract

Tennis Rackets, among other sporting equipment, have been rigorously tested and analyzed by industry for years, in hopes of achieving more efficient and higher performing equipment. Rackets in particular can be categorized by many different attributes, including but not limited to its; size, shape, curvature, material, aerodynamic characteristics, string tension, string pattern, weight, center of gravity, head design, neck design, handle design, bending moment, modulus, racket stiffness, [1] and what we call its Peak Shock[2]. Peak Shock is a measurement of the resonant frequency that develops when a racket is struck impulsively and consequent vibrations propagate throughout the body. In general, Peak Shock is inherent to each individual racket design. Through our testing we hope to find that Peak Shock can be used as an accurate value to differentiate one racket from another.

Objective

The concept behind this type of data acquisition is not a new idea, however there does not seem to be an extensive amount of research in the area, and there is by no means any standardized way to run these tests. One book, The Physics and Technology of Tennis[3], which has an overview on many of the physical aspects behind the game of tennis, touches on the subjects slightly and has a couple of pages explaining their testing procedure.

Instead of solely capturing the accelerometer data we also recorded the magnitude of the impulsive force used to create the vibration in the racket. Doing so allowed us to take our data in the time domain and create a transfer function in the frequency domain that was independent of the magnitude of the impulsive force, allowing us to attain more consistent and accurate results. Also, it is important to note that public data for this type of testing is readily available and was used to validate our results.

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[1]Tennis Warhouse - http://twu.tennis-warehouse.com/learningcenter /vibrationexplanation.php

[2]Peak Shock - Coined by Dr. Karl Hedrick and his Undergraduate Research Team (Dylan Vassberg and Aman Khan) to describe the characteristic frequency and magnitude that describes each racket's main mode of vibration. It can be used to quantify the vibration numerically and simply as it is a data point that makes for easy comparison to other rackets. Peak Shock is comprised of two values, the Peak Shock Frequency (Hz) at which the vibration occurs and the Peak Shock Magnitude (g/N) which quantifies the amount of vibration.

[3]Howard Brody, Rodney Cross, and Crawford Lindsey. *The Physics and Technology of Tennis*. Solana Beach, CA: Racquet Tech Pub., 2002.

Procedure

In order to obtain the Peak Shock of a racket, the overall testing procedure is to impart an impulsive strike on the racket followed by measurements of the consequent vibrations that propagate throughout the racket's body. In practice this can be done many different ways and is a bit more complicated than the general principle stated above. For instance, it is impossible to measure the vibrations in the racket without actually altering the vibrations themselves, the key is to minimize this alteration by creating a testing setup that does not physically carry vibrations from surrounding objects or damp the vibrations on the racket incidentally. To alleviate some of these issues we created a testing rig that suspended the racket in air, leaving as much freedom to vibrate as possibly could be. We hung the rackets symmetrically[4] using two strings tied around the racket frame placed vertically a string above the Upper Strike Zone. [5] To minimize the vibrations from the racket being transferred to the cage/surroundings we used cotton string instead of fishing line or another polymer based string because it tends to not carry vibrations as easily. It is also important to note that the way we hung our racket symmetrically was to allow for vibrations in the directions that they would most likely occur during game play, while maintaining symmetry to not allow for off axis movement that could affect our data. In comparison with Rod Cross' method[3] of testing the racket hung by the racket head and handle (horizontally hung), our approach does not allow for off axis motion that would be captured in the vibration data.

As far as actually capturing the vibration data, we used a piezoelectric accelerometer[6] attached to the grip placed 4.5 inches from base of the handle. Please note the intentional use of the piezoelectric accelerometer, which does not use springs, and allows for all 3 dimensions of data to be accurately captured. Another reason to use a piezoelectric accelerometer is that spring based accelerometers have orientations that must be accounted for when collecting data. Also, the piezoelectric accelerometer is smaller in size than other accelerometers and therefore does not affect the actual vibration of the racket as much a more bulky/heavier accelerometer would.[7] The placement of the accelerometer 4.5 inches from the base of the handle is intentional, not only for consistency of data captures, but also strategically placed in order to capture the vibrations that a hand would encounter while gripping the racket.

To capture the magnitude of the impulsive strike dealt to the face of the racket we used a force hammer,[8] which, opposed to other forms of striking the

[4]See Figure 1 for Testing Setup and Hang Symmetry
[5]See Figure 2 for Upper Strike Zone
[6]ICP ®Accelerometer (100 mV/g)
[7]See Figure 3 for Relative Size of the Accelerometer
[8]ICP ©Impact Hammer (2.5 mV/N)

racket that could have been more consistent, is actually more useful for our application. The reason being that we need the magnitude data in the time domain along with the accelerometer data, which is also captured in the time domain, in order to create a transfer function in the frequency domain that yields data that is independent of the impulsive strike magnitude altogether.

To conduct an actual test the following procedure takes place:

1. The Lower, Middle, and Upper Strike Zones are determined by measuring the racket head height (longest direction of the oval shape), finding the center which corresponds to the Middle Strike Zone and then finding the half way point from the center to the base of the head and the top of the head, corresponding to the Lower and Upper Strike Zones respectively.

2. The racket is hung as shown in Figure 1 by cotton strings placed one string higher than the string on which the Upper Strike Zone resides.

3. The accelerometer is attached 4.5 inches from the base of the handle using adhesive wax to attach it to the grip, a rubber band is attached around the grip and accelerometer wire as shown in Figure 3 to reduce data picked up from the moving accelerometer wire.

4. All inputs and outputs (Force Hammer and Accelerometer) are connected to a power source and the computer, and allowed to reach equilibrium before testing starts. (Piezoelectric accelerometers need about a minute to reach equilibrium after being connected to a power source, collecting data during this time would yield invalid data)

5. The MATLAB™ code is run that captures the data during a specified time window, in which the racket (in equilibrium) is struck by the force hammer and allowed to vibrate for the duration of this time window undisturbed.

6. The MATLAB™ code then analyzes the Magnitude vs. Time, and the Acceleration vs. Time data using a Discrete Fourier Transform and creates a transfer function of the data in the frequency domain.

7. From the plot and saved data of the transfer function the characteristic frequency and magnitude, or Peak Shock, is easy to determine visually by comparing data from different tests on the same racket.

Theory

After running a successful test and collecting the data from the accelerometer and the force hammer in the time domain we utilized the built in MATLAB™ function *"fft"* to calculate the Discrete Fourier Transform of the time domain data for both inputs. After these transformations were complete we had force hammer and acceleration data now described in the frequency domain. From this we were able to create a transfer function that displayed the relative g's/N (Acceleration/Force) that could describe the vibration that the racket endured during the test, which was also independent of the magnitude at which the racket was struck.

$$TF = \frac{ACC_{FFT}}{FH_{FFT}}$$

Where TF is the transfer function, ACC_{FFT} is the Discrete Fourier Transform of the time domain accelerometer data, and FH_{FFT} is the Discrete Fourier Transform of the time domain force hammer data.

Results and Discussion

After running numerous tests on each Strike Zone (Lower, Middle, Upper) and obtaining transfer functions for each test we were able to visually locate the Peak Shock by comparison. From this we tabulated the Peak Shock for each test taken, found averaged Peak Shock values for each individual racket, as well as created averaged transfer functions to validate the averaged Peak Shock values we obtained.[9] Our aggregate data can be seen in Figure 5 in the Appendix.

We found that the Peak Shock Frequency only varied at most +/- 0.2 Hz between each of the Strike Zones, meaning that the Peak Shock Frequency is inherent to the racket and not a function of where the impact occurs. Judging between rackets of the same model but with different string tensions we can tell that the Peak Shock Frequency is a function of string tension. Although it is important to note that the largest tension differential between two rackets of the same model that we compared was 8 lbs. In general most Peak Shock Frequencies only changed about 1 Hz with the varying string tension, except for the 2013 Ojoee model, which changed by 8 Hz with a tension differential of 5 lbs. As far as we are concerned this case is an outlier, which could have been caused by a number of things including a difference in grip material, an unseen issue with the equipment, or even just a manufacturing difference between the two rackets (since we were using two different rackets of the same model with different string tensions rather than using the same racket and restringing it at different tensions).

[9] See Figure 4 for Averaged Peak Shock Values (Ojoee vs. Wilson vs. Babolat)

Looking at the Peak Shock Magnitude, which describes the amount of vibration and force due to vibration, we can see that there is a great deal of change due to differences in racket model, string tension, and Strike Zone. The first two of these variables makes sense intuitively, different rackets and different string tensions should produce different Peak Shock Magnitudes due to differences in geometry and internal forces respectively. What is quite interesting about the Peak Shock Magnitude data, is that when you inspect the changes due to Strike Zone while keeping the geometry of the racket in mind you find an interesting correlation. In general, the racket's lowest Peak Shock Magnitude occurs when the impulse is delivered to the Middle Strike Zone. Once the strikes move away either to the Lower or Upper Strike Zones the Peak Shock Magnitudes tend to increase, meaning that a player would feel more force from the racket when hitting a tennis ball if struck in these zones. When looking at all three data points for a single racket at each zone, there is a trend that the Lower and Upper Strike Zone Peak Shock Magnitude values are relatively similar, this makes sense physically because they are equidistant from the Middle Strike Zone. What is interesting, in particular with the Ojoee rackets that utilize the bridge technology, is that the Peak Shock Magnitudes calculated at the Upper Strike Zone are almost and average of the Peak Shock Magnitude values calculated at the Lower and Middle Strike Zones, instead of being extremely close to the Lower Strike Zone value. In comparison to other rackets this means that a higher hit on the face of the racket will generally result in less shock translated to the player's hand than another racket without this technology hit in the same spot.

In theory, to acquire accurate data, the impulsive strikes to the racket face must be consistent in both duration and magnitude. In reality this is almost impossible to do, and is the main reason why we used a force hammer that captures the magnitude of the impulsive strike in the time domain. Having this information is of course what allows us to create the transfer function, by dividing the ACC_{FFT} by the FH_{FFT}, which yields results independent of the actual striking force. As far as impulse duration is concerned, a strike from the force hammer (including the duration of deformation and restitution) on average lasted 1 - 1.25 milliseconds. This is consistent enough where the duration of impact can be ignored as an important factor in our results. If we were to use a tennis ball for the impacts, the impulsive duration can vary anywhere from 3.5 - 6 milliseconds.[10] This range of duration is considerable and would have to be factored in when accounting for variability between tests. In comparison to using a tennis ball or any other type of striking device the force hammer was chosen for this type of testing due to its reliability, consistency, and data capturing ability.

One issue important to note that we ran into during testing was the instruments picking up ambient voltages, which resulted in static and unclear

[10] *The Physics and Technology of Tennis.* Page 92.

transfer functions. To solve this issue we created a low pass filter that eliminated all frequencies higher than 1000 Hz, which worked well because all of our pertinent data lied between 0 - 500 Hz. After the low pass filter was installed on the accelerometer our results became much more refined and accurate up to the 1000 Hz threshold. This yielded accurate Peak Shock data, which lies between 100 - 200 Hz.

After our results were tabulated we calculated averaged transfer functions as well as averaged the data that we obtained for each test, and compared the two. Our findings showed that our method of averaging the Peak Shock values numerically was equally as accurate as creating an averaged transfer function and then obtaining the Peak Shock from that information. Furthermore, to validate the numbers we were getting we compared our data with publicly available data on tennis racket "Vibration Frequency"[11] and found that our frequencies were extremely close to theirs,[12] generally within +/- 5 Hz. Although, it is important to note that Tennis Warehouse does not provide any information about how their Vibration Frequencies were calculated.

[11]"Vibration Frequency" is the term Tennis Warehouse uses to describe their rackets. It is equivalent to the "Peak Shock Frequency" - http://twu.tennis-warehouse.com/cgi-bin/vibfrequency.cgi

[12]See Table 1 for Comparison of Peak Shock Frequency to Tennis Warehouse's Vibration Frequency

Summary

- Created test setup that allows for large range of motion, that does not critically interfere with the vibrational aspects of the tennis rackets.
- Collected force hammer magnitude (N) and accelerometer (g) data in the time domain over a certain time span in which the test was performed.
- Ran Discrete Fourier Transform on both sets of time domain data.
- Used both Discrete Fourier Transforms to create a transfer function in the form of:

$$TF = \frac{ACC_{FFT}}{FH_{FFT}}$$

- Visually interpreted transfer functions in order to determine which spike on graph was the Peak Shock and recorded the magnitude and frequency of each Peak Shock.
- Calculated Peak Shock for each racket multiple times for the Lower, Middle, and Upper Strike Zones.
- Averaged values of Peak Shocks and compared to averaged transfer functions to validate averaging method.
- Determined that Ojoee rackets have a larger range of area on the racket face between the Middle and Upper Strike Zones that was more forgiving in regards to the Peak Shock Magnitude.

Appendix

Figure 1: Front View of the Testing Setup

Racket Name	Tennis Warehouse (Hz)	Peak Shock Frequency (Hz)*
Babolat Pure Drive (2012)	170	171.8(50 N), 171(58 N)
Head YOUTEK Graphene Instinct S	170	167.6(50 N), 166.1(58 N)
Wilson BLX Juice 100	173	174.2(50 N), 175.3(54 N)

Table 1: Comparison of our Data with Tennis Warehouse's Data

*Peak Shock Frequency calculated with impulses taken at the center of the racket head (Middle Strike Zone). It is unclear what location and string tension Tennis Warehouse used to acquire their data.

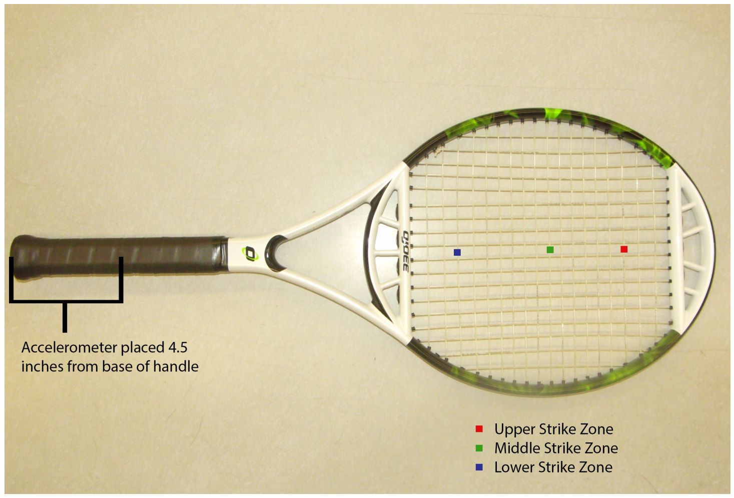

Figure 2: Closeup of Racket and Strike Zones

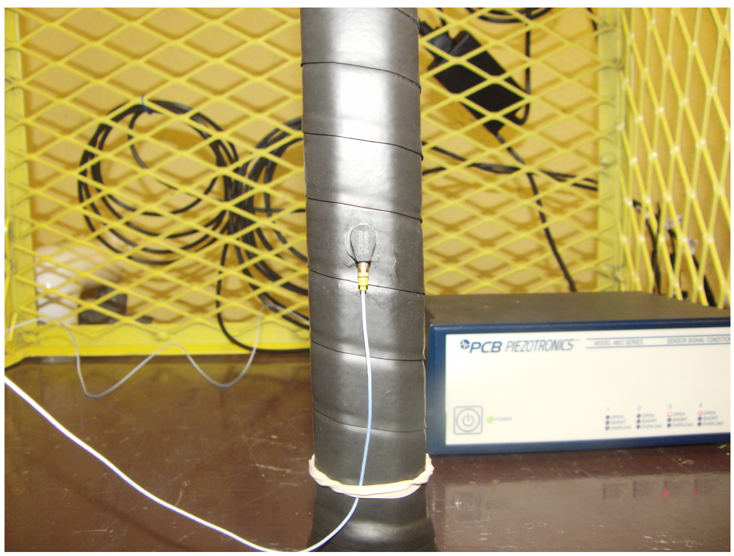

Figure 3: Closeup of Accelerometer Placement

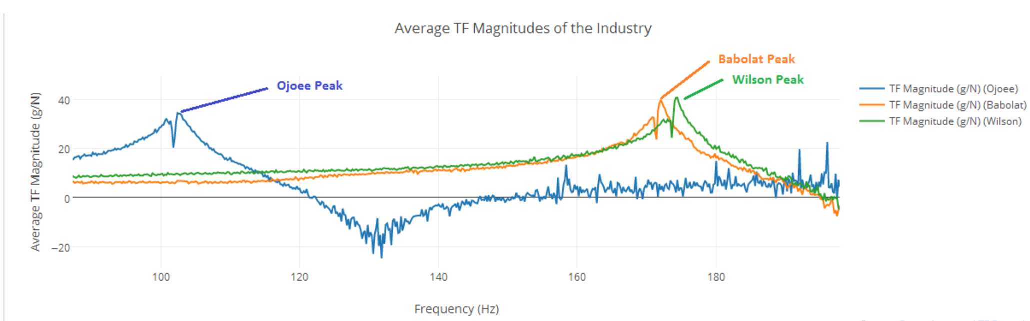

Figure 4: Averaged Transfer Functions

Aggregate String Strike Peak Shock Data						
Racket	Lower Area Average Peak Shock		Middle Area Average Peak Shock		Upper Area Average Peak Shock	
	Hz	g/N	Hz	g/N	Hz	g/N
Babolat 50	171.80	54.62	171.80	42.97	171.73	54.88
Babolat 58	170.87	55.96	171.00	43.28	170.93	55.16
Head 50	166.39	57.13	167.60	57.23	167.73	56.25
Head 55	166.13	53.13	166.13	39.56	166.33	51.64
Ojoee 50 (2013)	101.80	48.99	101.87	39.62	101.87	44.39
Ojoee 55 (2013)	109.47	50.65	109.47	41.71	109.60	46.52
Ojoee Silver (2014)	106.40	52.25	106.40	44.23	106.53	47.03
Ojoee White (2014)	107.33	55.74	107.60	48.15	107.47	48.11
Wilson 50	174.00	56.83	174.20	43.57	174.13	56.58
Wilson 58	175.20	53.50	175.33	45.90	175.27	50.92

Figure 5: Aggregate Peak Shock Data