PARUL UNIVERSITY
FACULTY OF ENGINEERING AND TECHNOLOGYDEPARTMENT OF APPLIED SCIENCE AND HUMANITIES

SUBJECT : BASIC MATHEMATICS

Unit -6- Coordiante Geometry

Introduction

- Coordinate geometry is the system of geometry where the position of points on the plane is described using an ordered pair of numbers.
- Recall that a plane is a flat surface that goes on forever in both directions. If we were to place a point on the plane, coordinate geometry gives us a way to describe exactly where it is by using two numbers.
- Coordinate geometry is applied to many professions and used by many in real life. For example it can be used by computer programmers. Computer programmers use coordinate geometry because most of the programs that they write generate PDF files. And in a PDF file, the printed page is one big coordinate plane. Coordinate geometry is thus used to position elements on the page. PDF files that are produced contains text, images and line drawings, all of which are placed into position by using (x,y) coordinates, distances, slopes, and simple trigonometry.
- Coordinate geometry are also used in manipulating images. The selected image is like a big coordinate plane with each colour information as each individual points. Thus when the colours of the pictures are being manipulated, the points are changed.
- Coordinate geometry are also applied in scanners. Scanners make use of coordinate geometry to reproduce the exact image of the selected picture in the computer. It manipulates the points of each information in the original documents and reproduces them in soft copy.

Coordinates:

Coordinates are set of values which shows the exact position of a point in coordinate plane.

The Coordinate Plane:

The coordinate plane is a two-dimensional surface on which points are plotted and located by their x and y coordinates In coordinate geometry, points are placed on the "coordinate plane" as shown below.

Coordinate axes

The adjoining figure 6.1 shows two number lines XoX' and YoY'intersecting each other at their zeros.

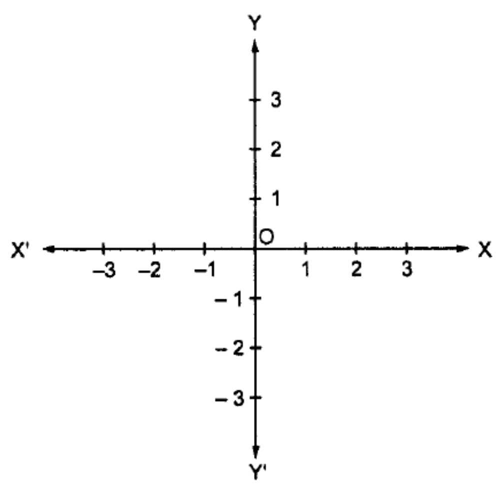


figure 6.1

XoX' and YoY' are called coordinate axes out of which XoX' is called the x-axis, YoY' is called y-axis and their point of intersection is called the origin. Number lines XoX' and YoY' are sometimes also called rectangular axes as they are perpendicular to each other.

Convention of Signs: The distance measured along OX and OY are taken as positive and those along OX' and OY' are taken as negative as shown in figure 6.1.

- **Coordinate of a point in a plane:**

Let p be a point in a plane. Let the distance of p from the y-axis = a units, and the distance of p from the x-axis = b units. Then we say that the coordinates of p are (a, b) where a is called the x-coordinate or abscissa of p and b is called the y-coordinate or ordinate of p.

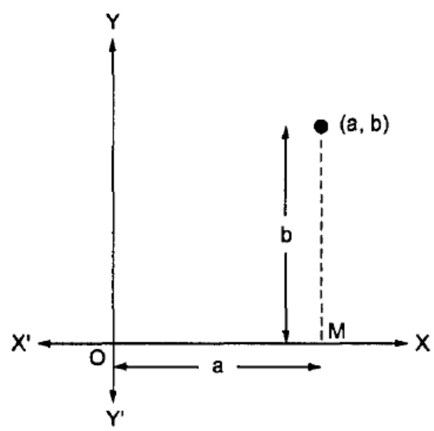


Figure 6.2

- **Quadrants:**

Let $X'OX$ and YOY' be the coordinate axes.

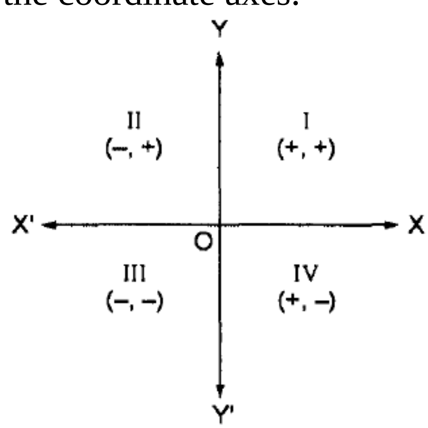


Figure 6.3

These axes divide the plane of the paper into four regions, called Quadrants. The RegionXOY, YOX',$X'OY'$ and $Y'OX$ are respectively known as first, second, third and fourth quadrant.

Using the convention of signs, we have the signs of the coordinates in various quadrant given below.

Region	**Quadrant**	**Nature of x and y**	**Signs of coordinates**
XOY	1st	$x > 0, y > 0$	(+,+)
YOX'	2nd	$x < 0, y > 0$	(-,+)
$X'OY'$	3rd	$x < 0, y < 0$	(-,-)
$Y'OX$	4th	$x > 0, y < 0$	(+,-)

- **Remarks:**

1. (x, y) and (y, x) do not represent the same point unless $x = y$
   e.g., (5,4) and (4,5) represent two different points.
2. Since at origin the value of $x - coordinates$ is 0 and the value of $y - coordinate$ is also 0. Therefore, the coordinates of origin = (0,0).
3. Since for every point on $x - axis$, its distance from $x - axis$ is 0 i.e., $y - coordinate$ is 0. Therefore, the coordinate of a point on $x - axis$ are taken as $(x, 0)$.
4. In the same way, for every point on $y - axis$ its distance from $y - axis$ is zero i.e., $x - coordinate$ is 0. Therefore, the coordinate of a point on $y - axis$ are taken as $(0, y)$

SOLVED EXAMPLES

Example 1: In which quadrant do the given point lie.

(i) $(4, -2)$ (ii) $(-3,7)$ (iii) $(-1, -2)$ (iv) $(3,6)$

Solution:(i) Fourth quadrant

(ii) Second quadrant

(iii)Third quadrant

(iv) First quadrant

Example 2: On which axis do the given point lie?

(i) $(7,\ 0)$ (ii) $(0,\ -3)$ (iii) $(0,\ 6)$ (iv) $(-5\ , 0)$

Solution:(i) In $(7,\ 0)$, we have $y - coordinate$ is 0

$\therefore$ $(7,\ 0)$ lies on the $x - axis$.

(ii) In$(0, -3)$ we have $x - coordinate$ is 0.

$\therefore$ $(0, -3)$ lies on the $y - axis$.

(iii) In $(0,6)$ we have $x - coordinate$ is 0.

$\therefore$ $(0,6)$ lies on the $y - axis$.

(iv) In $(-5,0)$ we have $y - coordinate$ is 0.

$\therefore$ $(-5,0)$lies on the x$-axis$.

Distance between two points:

The distance between the points (x_1, y_1) and (x_2,y_2) (or the length of the line segment joining them) is

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

- **Notes:** 1) Distances are always positive, or zero if the points coincide. The distance from A to B is the same as the distance from B to A.

  2) The distance of a point P$(x,\ y)$ from the origin O$(0,0)$ is given by

$$OP = \sqrt{(x-0)^2 + (y-0)^2} = \sqrt{x^2 + y^2}$$

SOLVED EXAMPLES

Example 1: Find the distance between $A(1,\ 3)$ and$B(0,\ -4)$.

Solution: Using distance formula $d = AB = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

$$= \sqrt{(1-0)^2 + (3+4)^2}$$

$= \sqrt{1+49} = \sqrt{50}$
$= \sqrt{25 \times 2} = 5\sqrt{2}$

Example 2: Find the distance between $A(7,\ -5)$ and $B(3,\ -2)$.

Solution: Using distance formula $d = AB = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

$$= \sqrt{(7-3)^2 + (-5+2)^2}$$
$$= \sqrt{16+9} = \sqrt{25} = 5$$

Example 3: $d[(3,2), (-1,1)] =$ ________

Solution: Using distance formula $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

$$= \sqrt{(3+1)^2 + (2-1)^2}$$
$$= \sqrt{16+1}$$
$$= \sqrt{17}$$

Example 4: If $A(-5,4)$ and $B(7,-1)$ then $AB =$ ______

Solution: Using distance formula $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

$$= \sqrt{(-5-7)^2 + (4+1)^2}$$
$$= \sqrt{144 + 25}$$
$$= \sqrt{169} = 13$$

Example 5: If the distance between $(5,7)$ and $(-3, m)$ is 10. find the values of m.

Solution: Using distance formula $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

$$10 = \sqrt{(5+3)^2 + (7-m)^2}$$
$$(10)^2 = 64 + (49 - 14m + m^2)$$
$$100 - 64 - 49 = m^2 - 14m$$
$$m^2 - 14m + 13 = 0$$
$$\Rightarrow\ m^2 - 13m - m + 13 = 0$$
$$\Rightarrow m(m-13) - 1(m-13) = 0$$
$$\Rightarrow\ (m-13)(m-1) = 0$$
$$\Rightarrow m = 13\ or\ 1$$

<u>Mid-point of any two points:</u>

Suppose point A has coordinates (x_1, y_1) and the point B has coordinates (x_2, y_2). then the midpoint M of AB is

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$A(x_1, y_1)$ $\qquad$ $M(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2})$ $\qquad$ $B(x_2, y_2)$

SOLVED EXAMPLES

Example 1: Find the mid-point of $A(-2,-1)$ and $B(4,3)$

Solution: By using Mid-point formula, $M = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$$= \left(\frac{-2+4}{2}, \frac{-1+3}{2}\right)$$

$$= \left(\frac{2}{2}, \frac{2}{2}\right) = (1,1)$$

Example 2: Find the mid-point of $A(4,2)$ and $B(6,-2)$

Solution: By using Mid-point formula, $M = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$$= \left(\frac{4+6}{2}, \frac{2-2}{2}\right)$$

$$= \left(\frac{10}{2}, \frac{0}{2}\right) = (5,0)$$

Example 3: If the mid-point of line segment AB is $(1,1)$ and $B(4,3)$, then find the coordinate of $A(x_1, y_1)$.

Solution: By using Mid-point formula, $M = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$$(1,1) = \left(\frac{x_1+4}{2}, \frac{y_1+3}{2}\right)$$

$\Rightarrow \frac{x_1+4}{2} = 1$ and $\frac{y_1+3}{2} = 1$

$\Rightarrow x_1 + 4 = 2$ and $y_1 + 3 = 2$

$\Rightarrow x_1 = 2 - 4$ and $y_1 = 2 - 3$

$\Rightarrow x_1 = -2$ and $y_1 = -1$

Example : If the mid-point of line segment AB is $(2,3)$ and $A(5,7)$, then find the coordinate of $B(x_2, y_2)$.

<u>Collinear points:</u>

Three points $A(x_1, y_1), B(x_2, y_2)$ and $C(x_3, y_3)$ are said to be collinear if they lie on the same line.

- **<u>Step Knowledge:</u>**

In order to show that three given points $A, B\ and\ C$ are collinear, we find distance AB,BC and CA

If the sum of any two of these distances is equal to the third distance then the given points are collinear.

I.e.,

A B C

If $AB + BC = CA$ then points are said to be collinear.

SOLVED EXAMPLES

Example 1: Prove that the points $A(1,4), B(3, -2)$ and $C(-3,16)$ are collinear.

Solution: By using distance formula,

$$AB = \sqrt{(1-3)^2 + (4+2)^2}$$

$$= \sqrt{4+36} = \sqrt{40} = 2\sqrt{10}$$

$$BC = \sqrt{(3+3)^2 + (-2-16)^2}$$

$$= \sqrt{36+324} = \sqrt{360} = 6\sqrt{10}$$

and $CA = \sqrt{(-3-1)^2 + (16-4)^2}$

$$= \sqrt{16+144} = \sqrt{160} = 4\sqrt{10}$$

$\therefore AB + CA = 2\sqrt{10} + 4\sqrt{10} = 6\sqrt{10} = BC$, Hence points are collinear.

Example 2: Prove that the points $A(-1,4), B(2,3)$ and $C(8,1)$ are collinear

Solution: By using distance formula,

$$AB = \sqrt{(-1-2)^2 + (4-3)^2}$$

$$= \sqrt{9+1} = \sqrt{10}$$

$$BC = \sqrt{(2-8)^2 + (3-1)^2}$$

$$= \sqrt{36+4} = \sqrt{40} = \sqrt{4X10} = 2\sqrt{10}$$

and $CA = \sqrt{(8+1)^2 + (1-4)^2}$

$$= \sqrt{81+9} = \sqrt{90} = 3\sqrt{10}$$

$\therefore AB + BC = \sqrt{10} + 2\sqrt{10} = 3\sqrt{10} = CA$, Hence points are collinear.

Section Formula

The coordinates of the point $P(x, y)$ which divides the line segment joining $A(x_1, y_1)$ and $B(x_2, y_2)$ internally in the ratio $m{:}\,n$ are given by

$$x = \frac{mx_2+nx_1}{m+n},\ \ y = \frac{my_2+ny_1}{m+n}$$

Or

$$P(x, y) = \left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$$

The coordinates of the point $P(x, y)$ which divides the line segment joining$A(x_1, y_1)$ and $B(x_2, y_2)$ externally in the ratio $m:n$ are given by

$$x = \frac{mx_2 - nx_1}{m-n},\ y = \frac{my_2 - ny_1}{m-n}$$

Or $\quad P(x, y) = \left(\frac{mx_2 - nx_1}{m-n}, \frac{my_2 - ny_1}{m-n}\right)$

SOLVED EXAMPLES

Example 1: Find the coordinates of the point which divide the line segment joining the points $(6,3)$ and $(-4,5)$ in the ration $3:2$ (i) internally and (ii) externally.

Solution: Here, $(6,3)$, $B(-4,5)$ and $m:n = 3:2$

Let P be the required point.

(i) for internal division, we have

$$x = \frac{mx_2 + nx_1}{m+n} = \frac{3\times(-4)+2\times6}{3+2} = 0,\ y = \frac{my_2 + ny_1}{m+n} = \frac{3\times5+2\times3}{3+2} = \frac{21}{5}$$

(ii) for external division, we have

$x = \frac{mx_2 - nx_1}{m-n} = \frac{3\times(-4)-2\times6}{3-2} = -24,\ y = \frac{my_2 - ny_1}{m-n} = \frac{3\times5-2\times3}{3-2} = 9$

Example 2: Find the coordinates of the point which divides the line segment joining the points (4, – 3) and (8, 5) in the ratio 3:1 internally and Externally.

Solution: Let$P(x, y)$ be the required point.

Using the section formula, ($\frac{m_1x_2+m_2x_1}{m_2+m_1}$, $\frac{m_1y_2+m_2y_1}{m_2+m_1}$)

We get;

x= $\frac{3(8)+1(4)}{3+1}$ =7, y= $\frac{3(5)+1(-3)}{3+1}$ =3

Therefore, (7,3) is the required point.

Example : Find the coordinates of the point which divide the line segment joining the points $(7{,}4)$ and (-2,3) in the ration $3{:}\,4$ internally and Exernally.

Example 3: In what ratio the point $P(-2{,}3)$ divide the line segment joining the points $A(-3{,}5)$ and $B(4, -9)$ Internally.

Solution: Let the required ratio be $k{:}\,1$

Comparing $x - coordinate$

$$x = \frac{mx_2 + nx_1}{m+n} \Rightarrow -2 = \frac{k\times4 + 1\times(-3)}{k+1} \Rightarrow -2 = \frac{4k-3}{k+1}$$

$$\Rightarrow -2k - 2 = 4k - 3 \Rightarrow 6k = 1 \Rightarrow k = \frac{1}{6}$$

Comparing $y - coordinate$

$$y = \frac{my_2 + ny_1}{m+n} \Rightarrow 3 = \frac{k \times (-9) + 1 \times 5}{k+1} \Rightarrow 3k + 3 = -9k + 5 \Rightarrow 12k = 2 \Rightarrow k = \frac{1}{6}$$

Hence, the required ratio is $1{:}\,6$.

LOCUS and Equation to a LOCUS

LOCUS: The curve described by a point which moves under given condition or conditions is called its LOCUS.

For example:

Suppose C is a point in the plane of the paper and P is a variable point in the plane of the paper such that its distance from C is always equal to $r(say)$. Obviously, all the positions of the moving point P lie on the circumference of a circle whose radius is r. The circumference of this circle is therefore the Locus of the point $O$when it moves under the condition that its distance from point C is always equal to constant r.

- **Equation of the locus of a point:**

The equation of the locus of a point is the relation which is satisfied by the coordinates of every point on the locus of the point.

SOLVED EXAMPLES

Example 1: Find the equation of a point $P(x, y)$ is moving under the condition

$PA^2 + PB^2 = 100$, where $A(3,4)$ and $B(-3,-4)$.

Solution: Given $PA^2 + PB^2 = 100$

$$(x-3)^2 + (y-4)^2 + (x+3)^2 + (y+4)^2 = 100$$

$$x^2 - 6x + 36 + y^2 - 8y - 16 + x^2 + 6x + 9 + y^2 + 8y + 16 = 100$$

$$2x^2 + 2y^2 + 50 = 100$$

$x^2 + y^2 = 50$, which is a required equation.

Example 2: Find the equation to the locus of a point which equidistance from

the points $A(1,3)$ and $B(-2,1)$

Solution: Let $P(x, y)$ be any point on the locus. Then

$$PA = PB \Rightarrow PA^2 = PB^2 \Rightarrow (x-1)^2 + (y-3)^2 = (x+2)^2 + (y-1)^2$$

$$\Rightarrow x^2 - 2x + 1 + y^2 - 6y + 9 = x^2 + 4x + 4 + y^2 - 2y + 1$$

$$\Rightarrow 6x + 4y = 5$$

Hence, locus(x, y) is $6x + 4y - 5 = 0$.

Example : Find the equation to Locus at a point ,so that the join of (-5,1) and (3,2) subtends a right angle at the moving point.

Introduction to the straight lines

A straight line is the locus of all those points which are collinear with two given points. Since, we know that one and only one line can be drawn from any two given points. So straight line Is a curve such that every point on the line segment joining any two points on it lies on it.

- **Notes:**

1) Every first degree in x, y represents a straight line.

2) The $x - axis$ and all lines parallel to it are called horizontal lines.

3) The $y - axis$ and all lines parallel to it are called vertical lines.

- **Slope or Gradient of a line :**

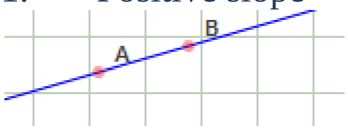
The gradient is a measure of the steepness of line.
The gradient of a line is defined to be the gradient of any interval within the line.
This definition depends on the fact that two intervals on a line have the same gradient.
Suppose $A(x_1, y_1)$ and $B(x_2, y_2)$ are two points on the line. Then, by definition, the gradient of the interval AB is;

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{Change\ in\ y}{change\ in\ x}$$

SLOPE DIRECTION:

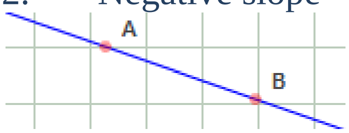
The slope of a line can positive, negative, zero or undefined.

1. Positive slope



Here, y *increases* as *x* increases, so the line slopes upwards to the right. The slope will be a positive number. The line on the right has a slope of about +0.3, it goes *up* about 0.3 for every step of 1 along the x-axis. If $0 < \theta < 90°$, then $\tan\theta > 0$ and the line has positive slope.

2. Negative slope

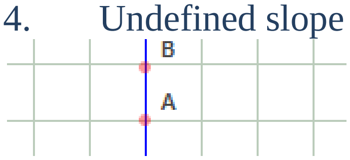


Here, y *decreases* as *x* increases, so the line slopes downwards to the right. The slope will be a negative number. The line on the right has a slope of about -0.3, it goes *down* about 0.3 for every step of 1 along the *x*-axis. If $90° < \theta < 180°$, then the line has negative slope.

3. Zero slope

Here, y *does not change* as *x* increases, so the line in exactly horizontal. The slope of any horizontal line is always zero. The line on the right goes neither up nor down as *x* increases, so its slope is zero.

4. Undefined slope



When the line is exactly vertical, it does not have a defined slope. The two *x* coordinates are the same, so the difference is zero.

When you divide anything by zero the result has no meaning. The line above is exactly vertical, so it has no defined slope. We say "the slope of the line AB is undefined".

SOLVED EXAMPLES

Example 1: A line passes through the points (1, 2) and (5, 10). Find its gradient.

Solution: Here, $(x_1, y_1) = (1,2)$ and $(x_2, y_2) = (5,10)$

Therefore, Gradient = $m = \frac{y_2 - y_1}{x_2 - x_1}$

$$= \frac{10-2}{5-1} = \frac{8}{4} = 2$$

Example 2: If slope of the line passing through $(x_1, 5)$ and $(3,4)$ is 5 then find the value of x_1.

Solution: Here, $(x_1, y_1) = (x_1, 5)$ and $(x_2, y_2) = (3,4)$

Therefore, slope = $m = \frac{y_2 - y_1}{x_2 - x_1} \Rightarrow 5 = \frac{4-5}{3-x_1} \Rightarrow 15 - 5x_1 = -1$

$$\Rightarrow 16 = 5x_1 \Rightarrow x_1 = \frac{16}{5}$$

- **General Equation of a line**

  General form of an Equation of a line is given by $ax + by + c = 0$
  e.g., $2x + 3y + 5 = 0$ represents a general equation of a line.

- **Notes:**
  1) The x $-$ *intercept* of an equation of a line $ax + by + c = 0$ is given by $-\frac{c}{a}$
  i.e., $x - intercept = -\frac{c}{a}$

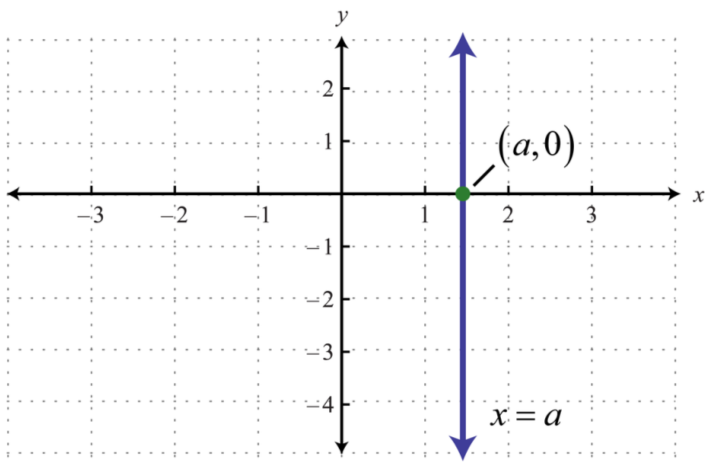
2) The $y - intercept$ of an equation of a line $ax + by + c = 0$ is given by $-\frac{c}{b}$

i.e., $y - intercept = -\frac{c}{b}$

3) The slope of an equation of a line $ax + by + c = 0$ is given by $-\frac{a}{b}$

i.e., slope= $m = -\frac{a}{b}$

Vertical line:



- Definition: A line on the coordinate plane where all points on the line have the same x- coordinate.
- A vertical line is one the goes straight up and down, parallel to the y-axis of the coordinate plane.
- A vertical line has no slope. Another way, for a vertical line the slope is undefined (∞)
- The equation of a vertical line is ***x = a***

  Where:

  x is the coordinate of any point on the line

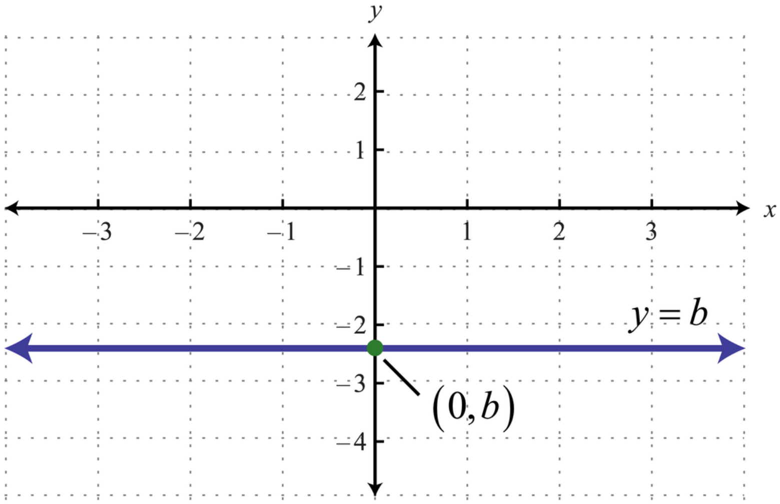
  a is where the line crosses the x-axis (x intercept).

Notice that the equation is independent of y. Any point on the vertical line satisfies the
equation.

<u>Example:</u>

Find the equation of a vertical line passing through (-7, 3).

Horizontal line



- Definition: A straight line on the coordinate plane where all points on the line have the same y- coordinate.
- A horizontal line is one the goes left-to-right, parallel to the x-axis of the coordinate plane.
- A horizontal line has a slope of zero. As you move to the right along the line, it does not rise or fall at all. As you drag the points above, you can see that when the line is horizontal, the points both have the same y-coordinate, and the slope is zero.
- The equation of a horizontal line is ***y = b***
  Where:
  y is the coordinate of any point on the line
  b is where the line crosses the y-axis (y intercept).

Notice that the equation is independent of x. Any point on the horizontal line satisfies the equation.

Example:

Find the equation of a horizontal line passing through (5, -2).

SOLVED EXAMPLES

Example 1: Find $x - intercept,\ y - intercept$ and slope of a line

$$3x - 5y + 8 = 0$$

Solution: Given $3x - 5y + 8 = 0$ by comparing with $ax + by + c = 0$ we have

$a = 3, b = -5$ and $c = 8$

$$\therefore\ x - intercept = -\frac{c}{a} = -\frac{8}{3}$$

$$y - intercept = -\frac{c}{b} = -\frac{8}{(-5)} = \frac{8}{5}$$

and slope$= -\frac{b}{a} = -\frac{(-5)}{3} = \frac{5}{3}$

Example 2: Find$x - intercept,\ y - intercept$ and slope of a line $2x + 3y - 4 = 0$

Solution: Given $2x + 3y - 4 = 0$ by comparing with $ax + by + c = 0$ we have

$a = 2, b = 3$ and $c = -4$

$$\therefore\ x - intercept = -\frac{c}{a} = -\frac{(-4)}{2} = 2$$

$$y - intercept = -\frac{c}{b} = -\frac{(-4)}{3} = \frac{4}{3}$$

and slope$= -\frac{b}{a} = -\frac{3}{2} = -\frac{3}{2}$

Example 3: Find the equation of a line whose $y - intercept$ is c and slope is

$m.$

Solution:

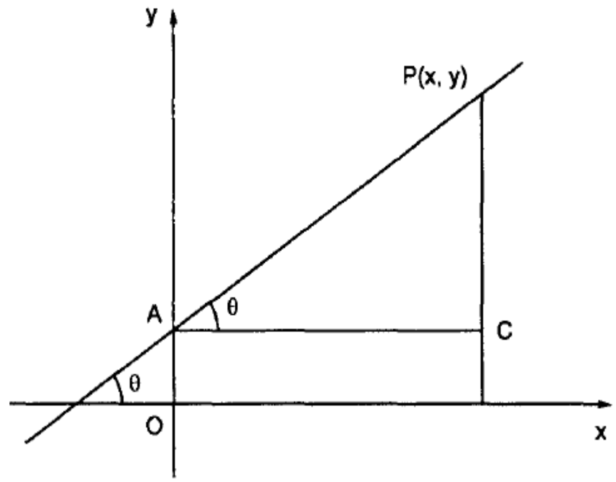


Figure 6.4

Let the given line meet $y - axis$ in A and Let

$P(x, y)$ be any point on it. As the $y - intercept$ of the line is$c$

$\therefore$ coordinates of A are $(0, c)$

Now the slope between the points $A(0, c)$ and $P(x, y)$ is:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{y - c}{x - 0} = \frac{y - c}{x}$$

$mx = y - c \Rightarrow y = mx + c$, which is a standard form of an equation of a line.

- **Remarks:**

1) If c becomes zero, the equation $y = mx + c$ reduces to $y = mx$ which is the equation of a line through the origin
2) If $m = 0, c \neq 0$, then equation $y = mx + c$ reduces to $y = c$ which is an equation of a line parallel to $x - axis$ at a distance c from it.

3) If $m = 0, c = 0$, then equation becomes $y = 0$ which represents the $x-axis$.

Slope-point form of an equation

To find the equation of a line passing through the given point (x_1, y_1) and having slope m:

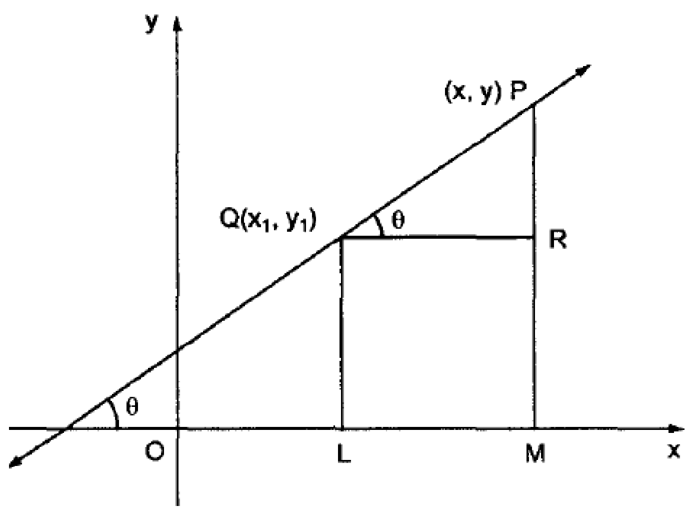


Figure 6.5

Let the point(x_1, y_1) be represented by Q.

Let $P(x, y)$ be any point on the line.

Then the slope between $P \ and \ Q$ is:

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{y - y_1}{x - x_1}$

$$\Rightarrow m(x - x_1) = y - y_1$$

$\therefore y - y_1 = m(x - x_1)$, is known as a slope-point form.

$\boxed{\textbf{SOLVED EXAMPLES}}$

Example 1: Find the equation of the line through (3,4) with slope 5.

Solution: Let $P(x_1, y_1) = P(3,4)$ be the given point and slope$= m = 5$.

Then by Slope-point form of an equation is:

$$y - y_1 = m(x - x_1) \Rightarrow y - 4 = 5(x - 3)$$
$$\Rightarrow y - 4 = 5x - 15$$
$$\Rightarrow 5x - y - 15 + 4 = 0$$

$\Rightarrow 5x - y - 11 = 0$, which is a required equation.

Example 2: Find the equation of a line passing through point $P(1,2)$ and having

slope 1.

Solution: Let$P(x_1, y_1) = P(1,2)$ be the given point and slope$= m = 1$.

Then by Slope-point form of an equation is:

$$y - y_1 = m(x - x_1) \Rightarrow y - 2 = 1(x - 1)$$

$$\Rightarrow y - 2 = x - 1$$

$$\Rightarrow x - y - 1 + 2 = 0$$

$\Rightarrow x - y + 1 = 0$, which is a required equation.

Two-point form of an equation

To find the equation of the straight line passing through two points:

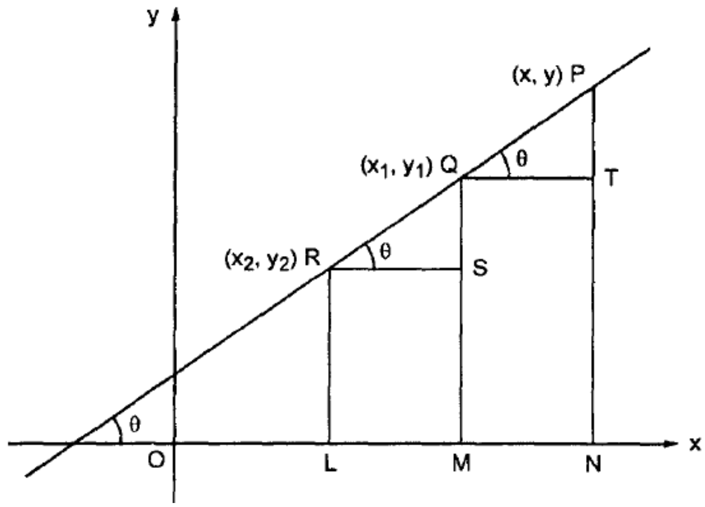


Figure 6.6

Let the two given points be $Q(x_1, y_1)$ and $R(x_2, y_2)$.

Let $P(x_1, y_1)$ be any point on the line.

From Figure, ΔRQS and ΔQTP both are similar

$\therefore$ Slope between R and Q = Slope between Q and P

$$\Rightarrow \frac{y_2 - y_1}{x_2 - x_1} = \frac{y - y_1}{x - x_1}$$

$\Rightarrow y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$, which is known as Two-point form of an equation.

SOLVED EXAMPLES

Example 1: Find the equation of line passing through $A(5,7)$ and $B(-2,1)$.

Solution: Let $A(x_1, y_1) = A(5,7)$ and $B(x_2, y_2) = B(-2,1)$

We know that the two-point form of an equation:

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$\Rightarrow y - 7 = \frac{1 - 7}{-2 - 5}(x - 5)$$

$$\Rightarrow y - 7 = \frac{-6}{-7}(x - 5)$$

$$\Rightarrow -7(y - 7) = -6(x - 5)$$

$$\Rightarrow -7y + 49 = -6x + 30$$

$$\Rightarrow 6x - 7y + 49 - 30 = 0$$

$\Rightarrow 6x - 7y + 19 = 0.$

Example 2: Find the equation of line passing through $A(1,3)$ and $B(4,-2)$.

Solution: Let $A(x_1, y_1) = A(1,3)$ and $B(x_2, y_2) = B(4,-2)$

We know that the two-point form of an equation:

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$\Rightarrow y - 3 = \frac{-2 - 3}{4 - 1}(x - 1)$$

$$\Rightarrow y - 3 = \frac{-5}{3}(x - 5)$$

$$\Rightarrow 3(y - 3) = -5(x - 5)$$

$$\Rightarrow 3y - 9 = -5x + 25$$

$$\Rightarrow 5x - 3y - 9 - 25 = 0$$

$\Rightarrow 5x - 3y - 34 = 0.$

Intercept from of an equation

To find the equation of the line which cuts off intercepts a and b on $x-axis$ and $y-axis$ respectively.

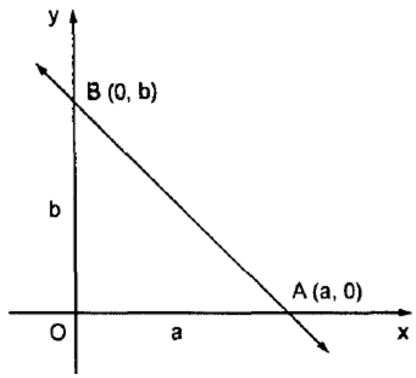


Figure 6.7

Let the line meet $x-axis$ at point A and $y-axis$ at point B. As the respective

intercepts are a and b. So $OA = a$ and $OB = b$.

$\therefore$ Coordinates of A and B are $(a, 0)$ and $(0, b)$ respectively.

Now, by using two-point form of an equation is:

$$\frac{x-a}{0-a} = \frac{y-0}{b-0} \Rightarrow \frac{x-a}{-a} = \frac{y}{b} \Rightarrow -\frac{x}{a} + 1 = \frac{y}{b}$$

$\Rightarrow \frac{x}{a} + \frac{y}{b} = 1$, which is known as intercept form of an equation.

SOLVED EXAMPLES

Example 1: Find the equation of the straight line which makes equal intercepts on the axes and passes through the point $(3, -5)$.

Solution: Let the equation of the straight line be

$\frac{x}{a} + \frac{y}{b} = 1$ (1)

The line (1) makes equal intercepts on the axes, i.e., $a = b$.

$$\therefore \frac{x}{a} + \frac{y}{a} = 1 \; or \; x + y = a$$

If this line passes through the point $(3, -5)$, then

$3 - 5 = a \Rightarrow a = -2$.

Hence, the required equation is $x + y = -2 \; or \; x + y + 2 = 0$.

Example 2: Find the equation of the straight line, the portion of which intercepted between the axes is divided by the point $(-2,6)$ in the ratio $3:2$

Solution: Let the equation of the straight line be

$\frac{x}{a} + \frac{y}{b} = 1$ (1)

The line (1) meet $x - axis$ at the point $A(a, 0)$ and $y - axis$ at the point $B(0, b)$.

Then the point $(-2,6)$ divides the line AB is the ratio $3:2$

By section formula, we have

$$(-2,6) = \left(\frac{2a + 3 \times 0}{2 + 3}, \frac{2 \times 0 + 3 \times b}{2 + 3}\right) \Rightarrow -2 = \frac{2a}{5} \; and \; 6 = \frac{3b}{5} \Rightarrow a = -5 \; and \; b = 10$$

Putting the values of $a \; and \; b$ in (1), the required equation of the line is

$\frac{x}{-5} + \frac{y}{10} = 1 \Rightarrow y - 2x = 10$.

- **<u>Summery:</u>**

In the below table the summery of all the forms of an equation are mentioned:

Equation of the straight line

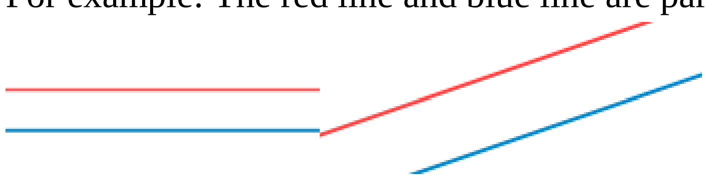
Form	Equation
General Form	$ax + by + c = 0$
Standard Form	$y = mx + c$
Slope-point Form	$y - y_1 = m(x - x_1)$
Two-point Form	$y - y_1 = \dfrac{y_2 - y_1}{x_2 - x_1}(x - x_1)$
Intercept Form	$\dfrac{x}{a} + \dfrac{y}{b} = 1$

<u>Parallel and Perpendicular lines</u>

- Two lines are said to be a parallel lines if

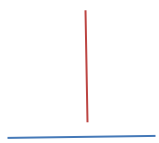
$$m_1 = m_2$$

For example: The red line and blue line are parallel in both these examples:

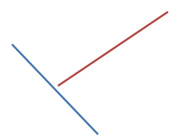


Example 1 Example 2

- Two lines are said to be perpendicular lines if
$$m_1 m_2 = -1$$
For example: The red line and blue line are perpendicular in both these examples:



Example 1



Example 2

SOLVED EXAMPLES

Example 1: Prove that the lines $2x + 3y + 6 = 0$ and $4x + 6y + 9 = 0$ are parallel to each other.

Solution: Given $l_1: 2x + 3y + 6 = 0$ and $l_2: 4x + 6y + 9 = 0$

Slope $m_1 = -\frac{2}{3}$ and Slope $m_2 = -\frac{4}{6} = -\frac{2}{3}$

Here, $m_1 = m_2 \Rightarrow$ lines are parallel.

Example 2: Prove that the lines $3x + 7y + 9 = 0$ and $7x - 3y + 10 = 0$ are perpendicular to each other.

Solution: Given $l_1: 3x + 7y + 9 = 0$ and $l_2: 7x - 3y + 10 = 0$

Slope $m_1 = -\frac{3}{7}$ and Slope $m_2 = -\frac{7}{(-3)} = \frac{7}{3}$

Here, $m_1 m_2 = -1 \Rightarrow$ lines are perpendicular.

Example 3: Find the equation of line passing through $(1,1)$ and parallel to $2x - 3y + 1 = 0$.

Solution: Let $P(x_1, y_1) = (1,1)$ and $l_1: 2x - 3y + 1 = 0$

Then Slope $m_1 = \frac{2}{3}$

Now, we wish to find the equation of a line which is parallel to l_1 and passing

through $(1,1)$.

For parallel lines, $m_1 = m_2 \Rightarrow m_2 = \frac{2}{3}$

$\therefore$ The equation of a line passing through $(1,1)$ having slope $\frac{2}{3}$ is given by:

$$y - y_1 = m(x - x_1) \Rightarrow y - 1 = \frac{2}{3}(x - 1)$$

$$\Rightarrow 3y - 3 = 2x - 2$$

$\Rightarrow 2x - 3y + 1 = 0$, which is a required equation.

Example 4: Find the equation of line passing through $(1,4)$ and perpendicular to

$$3x + 5y - 7 = 0$$

Solution: Let $P(x_1, y_1) = (1,4)$ and $l_1: 3x + 5y - 7 = 0$

Then Slope $m_1 = -\frac{3}{5}$

Now, we wish to find the equation of a line which is parallel to l_1 and passing through $(1,4)$.

For parallel lines, $m_1 m_2 = -1 \Rightarrow m_2 = \frac{5}{3}$

$\therefore$ The equation of a line passing through (1,4) having slope $\frac{5}{3}$ is given by:

$$y - y_1 = m(x - x_1) \Rightarrow y - 4 = \frac{5}{3}(x - 1)$$

$$\Rightarrow 3y - 12 = 5x - 5$$

$\Rightarrow 5x - 3y + 7 = 0$, which is a required equation.

Angle between two points

The angle between two lines $l_1: ax + by + c = 0$ and $l_2: px + qy + r = 0$ having slopes

$m_1 = -\frac{a}{b}$ and $m_2 = -\frac{p}{q}$ respectively can be given by

$$tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

Notes:

1) If two lines are parallel then $m_1 = m_2 \Rightarrow tan\theta = 0 \Rightarrow \theta = 0$

2) if two lines are perpendicular then $m_1 m_2 = -1 \Rightarrow tan\theta = \infty \Rightarrow \theta = \frac{\pi}{2}$

SOLVED EXAMPLES

Example 1: Find the angle between $2x + 3y - 6 = 0$ and $5x + 10y - 11 = 0$

Solution: Given lines $l_1: 2x + 3y - 6 = 0$ and $l_2: 5x + 10y - 11 = 0$

$\therefore$ Slopes are $m_1 = -\frac{2}{3}$ and $m_2 = -\frac{5}{10} = -\frac{1}{2}$

The angle between l_1 and l_2 is given by

$$tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

$$= \left|\frac{-\frac{2}{3} + \frac{1}{2}}{1 + \left(-\frac{2}{3}\right)\left(-\frac{1}{2}\right)}\right| = \left|\frac{-\frac{1}{2}}{\frac{4}{3}}\right|$$

$$= \frac{3}{8} \Rightarrow \theta = tan^{-1}\frac{3}{8}$$

Example 2: Find the angle between lines passing through A, B and B, C where $A(-2,1)$, $B(2,3)$ and $C(-2,-4)$

Solution: The slope of the line passing through A, B is $m_1 = \frac{3-1}{2+2} = \frac{2}{4} = \frac{1}{2}$

The slope of the line passing through B, C is $m_2 = \frac{-4-3}{-2-2} = \frac{-7}{-4} = \frac{7}{4}$

∴ The angle between lines A, B and B, C is given by

$$tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

$$= \left|\frac{\frac{1}{2} - \frac{7}{4}}{1 + \left(\frac{1}{2}\right)\left(\frac{7}{4}\right)}\right| = \left|\frac{-\frac{10}{8}}{\frac{15}{8}}\right|$$

$$= \frac{2}{3} \Rightarrow \theta = tan^{-1}\frac{2}{3}$$