

$$a) \hat{i} - 2\hat{j} + 2\hat{k} / 3$$

$$c) \hat{i} - 2\hat{j} + 10\hat{k}$$

$$e) 15\hat{i} - 30\hat{j} + 30\hat{k}$$

$$d) 5\hat{i} - 10\hat{j} + 10\hat{k}$$

Sarthak Gupta

BCA

041

A

$$\vec{a} = \hat{i} - 2\hat{j} + 2\hat{k}$$

$$|\vec{a}| = \sqrt{(1)^2 + (-2)^2 + (2)^2}$$

$$|\vec{a}| = \sqrt{1+4+4} \quad \Rightarrow \quad |\vec{a}| = \sqrt{9}$$

$$\text{Direction of vector} = \frac{1}{|\vec{a}|} \vec{a}$$

$$= \frac{1}{3} (\hat{i} - 2\hat{j} + 2\hat{k}) = \frac{1}{3} (\hat{i} - 2\hat{j} + 2\hat{k})$$

$$= (5\hat{i} - 10\hat{j} + 10\hat{k})$$

$\vec{a} \times \vec{b}$ is unit vector if $|\vec{a} \times \vec{b}| = 1$

$$|\vec{a} \times \vec{b}| = 1$$

$$|\vec{a}| |\vec{b}| \sin \theta = 1$$

$$2|\vec{a}| |\vec{b}| \sin \theta = 1$$

$$= \sqrt{2} \times \sqrt{2} \times \sin \theta = 1$$

$$\sin \theta = \frac{1}{\sqrt{4}}$$

$$\sin \theta = \frac{1}{\sqrt{4}}$$

$$\sin \theta = \frac{1}{2}$$

$$\theta = \frac{\pi}{6}$$

$$\textcircled{2} \quad \vec{d}_1 = 2\hat{i} + \hat{j} - 2\hat{k}$$

$$\vec{d}_2 = 3\hat{i} + \hat{j} - \hat{k}$$

$$|\vec{d}_1 \times \vec{d}_2| = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -2 \\ 3 & 1 & -1 \end{vmatrix}$$

$$= 1\hat{i} + 4\hat{j} + \hat{k}$$

Area of Parallelogram :- $\frac{1}{2} |\vec{d}_1 \times \vec{d}_2|$

$$= \frac{1}{2} |1\hat{i} + 4\hat{j} + \hat{k}|$$

$$= \frac{1}{2} \sqrt{(1)^2 + (4)^2 + (1)^2}$$

$$= \frac{1}{2} \sqrt{18}$$

$$= \frac{3}{\sqrt{2}} \text{ Ans.}$$

$$4) |\vec{a}| = 8$$

$$|\vec{b}| = 3$$

$$|\vec{a} \cdot \vec{b}| = 12\sqrt{3}$$

$$|\vec{a} \cdot \vec{b}| = |\vec{a}| |\vec{b}| \cos \theta$$

$$12\sqrt{3} = 8 \times 3 \times \cos \theta$$

$$\cos \theta = \frac{5}{2}$$

$$\cos \theta = \cos 30^\circ$$

$$\theta = 30^\circ$$

$$|\vec{a} \times \vec{b}| = |\vec{a}| |\vec{b}| \sin \theta$$

$$= 8 \times 3 \times \sin \theta$$

$$= 8 \times 3 \times \frac{1}{2}$$

$$|\vec{a} \times \vec{b}| = 12 \quad (a)$$

③ Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$
 $= 1\hat{i} + 1\hat{j} + 1\hat{k}$

A vector equally inclined on Ox, Oy, Oz .

$$|\vec{a}| = \sqrt{(1)^2 + (1)^2 + (1)^2}$$

$$|\vec{a}| = \sqrt{1+1+1}$$

$$|\vec{a}| = \sqrt{3}$$

Directions cosines of \vec{a} are

$$\left(\frac{a}{|\vec{a}|}, \frac{b}{|\vec{a}|}, \frac{c}{|\vec{a}|} \right) = \left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \right)$$

Since, the direction cosines are equal

$\vec{a} = \hat{i} + \hat{j} + \hat{k}$ is equally inclined to Ox, Oy, Oz .

Fill in the blanks

Q. $|\vec{a} \times \vec{c}|^2 + (\vec{a} \cdot \vec{c})^2$ is $|\vec{a}|^2 |\vec{c}|^2$

$$|\vec{a} \times \vec{c}|^2 + (\vec{a} \cdot \vec{c})^2$$

$$|\vec{a} \times \vec{c}|^2 = (\vec{a} \times \vec{c}) \cdot (\vec{a} \times \vec{c})$$

$$= (|\vec{a}| |\vec{c}| \sin \theta) \cdot (|\vec{a}| |\vec{c}| \sin \theta)$$

$$= |\vec{a}|^2 |\vec{c}|^2 \sin^2 \theta$$

$$(\vec{a} \cdot \vec{c})^2 = (|\vec{a}| |\vec{c}| \cos \theta)^2$$

$$= (|\vec{a}| |\vec{c}| \cos^2 \theta)$$

$$= |\vec{a} \times \vec{c}|^2 + (\vec{a} \cdot \vec{c})^2$$

$$= |\vec{a}|^2 |\vec{c}|^2 \sin^2 \theta + |\vec{a}| |\vec{c}| \cos^2 \theta$$

$$= |\vec{a}|^2 |\vec{c}|^2 (\sin^2 \theta + \cos^2 \theta)$$

$$= |\vec{a}|^2 + |\vec{c}|^2$$

2) if \vec{a} is any non-zero vector then $\vec{a} \cdot \vec{i}$ is

$$\text{let } \vec{a} = a_1 \hat{i} + a_2 \hat{j} + a_3 \hat{k}$$

Now, taking dot product of \vec{a} with \hat{i}

$$\vec{a} \cdot \hat{i} = (a_1 \hat{i} + a_2 \hat{j} + a_3 \hat{k}) \cdot \hat{i}$$

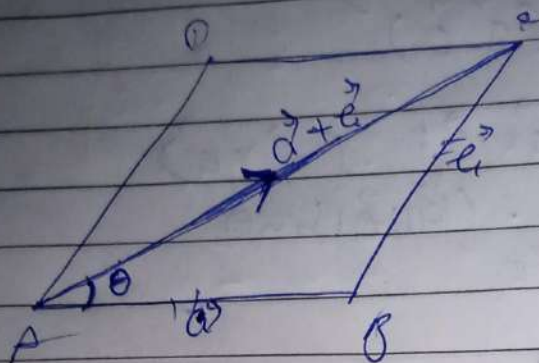
$$= a_1 \hat{i} \cdot \hat{i} + a_2 \hat{j} \cdot \hat{i} + a_3 \hat{k} \cdot \hat{i}$$

$$= a_1 \cdot 1 + a_2 \cdot 0 + a_3 \cdot 0$$

$$\vec{a} \cdot \hat{i} = a_1$$

$\vec{a} \cdot \hat{i} = a_1$

① The angle $\vec{a} + \vec{c}$ bisects the angle b/w the non-parallel sides vector \vec{a} and \vec{c} if $|\vec{a}| = |\vec{c}|$



Vector $\vec{a} + \vec{c}$ is along the diagonal of the Parallelogram.

Now, diagonal in parallelogram, the angle only it is shows

thus \vec{a} and \vec{c} forms rhombus.

$$|\vec{a}| = |\vec{c}|$$

② $|\vec{a} \times \vec{c}|^2 + |\vec{a} \cdot \vec{c}|^2 = 144$ and $|\vec{a}| = 4$ then $|\vec{c}|$ is 3

$$|\vec{a} \times \vec{c}|^2 + |\vec{a} \cdot \vec{c}|^2 = 144$$

if angle b/w \vec{a} and \vec{c} is 0

$$|\vec{a}|^2 |\vec{c}|^2 \sin^2 \theta + |\vec{a}|^2 |\vec{c}|^2 \cos^2 \theta = 144$$

$$|\vec{a}|^2 |\vec{e}|^2 = 144 \quad (\sin^2 \theta + \cos^2 \theta = 1)$$

$$|\vec{a}| |\vec{e}| = 12$$

$$4|\vec{e}| = 12 \quad (|\vec{a}| = 4)$$

$$|\vec{e}| = 3$$

Short Questions:-

$$\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$$

$$\vec{a} = 2\hat{j} + \hat{k}$$

Given unit vector needed to be found. Here we add vector \vec{a} and \vec{e} .

$$\vec{c} = \vec{a} + \vec{e}$$

$$\vec{c} = (2\hat{i} - \hat{j} + \hat{k}) + (2\hat{j} + \hat{k})$$

$$\vec{c} = 2\hat{i} - \hat{j} + \hat{k} + 2\hat{j} + \hat{k}$$

$$\vec{c} = 2\hat{i} + \hat{j} + 2\hat{k}$$

$$\vec{a} \times \vec{e} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 1 \\ 0 & 2 & 1 \end{vmatrix}$$

- 2) Find the area of a parallelogram whose adjacent sides are given by the vectors $\vec{a} = 3\hat{i} + \hat{j} + 4\hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + \hat{k}$

$$\vec{a} = 3\hat{i} + \hat{j} + 4\hat{k}$$

$$\vec{b} = \hat{i} - \hat{j} + \hat{k}$$

$$\text{Area of parallelogram} = |\vec{a} \times \vec{b}|$$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & 4 \\ 1 & -1 & 1 \end{vmatrix}$$

$$\vec{a} \times \vec{b} = 5\hat{i} + \hat{j} - 4\hat{k}$$

hence,

$$|\vec{a} \times \vec{b}| = 5\hat{i} + \hat{j} - 4\hat{k}$$

$$= \sqrt{(5)^2 + (1)^2 + (-4)^2}$$

$$= \sqrt{25 + 1 + 16}$$

$$|\vec{a} \times \vec{b}| = \sqrt{42}$$

$\sqrt{42}$ sq. units.

4, if $|\vec{a}|=8$ and $|\vec{b}|=3$ and $|\vec{a} \cdot \vec{b}|=12\sqrt{3}$ then the value of $|\vec{a} \times \vec{b}|$ is

a, 12

b, $12\sqrt{3}$

c, 0

d, $4\sqrt{3}$

u) $|\vec{a}|=8$

$|\vec{b}|=3$

$|\vec{a} \cdot \vec{b}|=12\sqrt{3}$

$$|\vec{a} \cdot \vec{b}| = |\vec{a}| |\vec{b}| \cos \theta$$

$$12\sqrt{3} = 8 \times 3 \times \cos \theta$$

$$\frac{12\sqrt{3}}{24} = \cos \theta$$

$$\cos \theta = \frac{\sqrt{3}}{2}$$

$$\cos 30^\circ$$

$$\cos \theta = \cos 30^\circ$$

$$\theta = 30^\circ$$

$$|\vec{a} \times \vec{b}| = |\vec{a}| |\vec{b}| \sin \theta$$

$$= 8 \times 3 \times \sin 30^\circ$$

$$= 8 \times 3 \times \frac{1}{2}$$

$$|\vec{a} \times \vec{b}| = 12$$

Unit:- 3

MCQs :-

- 1) Which of the following statistics measures the most frequently occurring value in a set of data?
a, median ☒ b, mode c, mean d, none of above
- 2) The mode of a data set is the value that occurs most often.
- 3) The mean temperature for the past ten days was 22°C . If the sum of the temperatures for the first nine days was 200° , what was the temperature on day 10?
a, 22 b, 32 c, 10 ☒ d, 20

4)

mean temperature for last 10 days:- 22°C

sum of temperature for (1-9) days:- 200°C

mean temp. $\therefore 22^{\circ}\text{C}$

$$\text{Mean} = \frac{\text{sum of total No.}}{\text{total No.}}$$

Thus,

$$\text{sum of temp. for all ten days} = 22 \times 10 \\ = 220^{\circ}\text{C}$$

sum of (1-9) days $\therefore 208^{\circ}\text{C}$

$$\text{10th day temp.} = 220 - 208 \\ = 20^{\circ}\text{C}$$

- 3) The method used to compute the average or central value of the collected data is considered as
- a) measures of positive variation
 - ✓ b) measures of central tendency
 - c) measures of negative skewness
 - ✗ d) measures of negative variation

measures of positive variation are considered as a method used to compute average or central value of collected data

5, let $(100, -20), (103, -10), (107, -5), (109, 1), (111, 30)$ be the ordered pairs of the variables (u, v) . What is the value of the rank correlation coefficient between u and v ?

$a, r = 1$ $b, r = 0.5$ $c, r = 0$ $d, r = -1$

Fill in the blanks:-

- 1) Range of correlation coefficient is -1 to 1.
- 2) A positive r values indicates that as one variable increases.
- 3) The arithmetic mean is 12 and the number of observations are 20 then the sum of all the values are

$$\begin{aligned} \text{Sum of all the values are} &= 12 \times 20 \\ &= \underline{240} \end{aligned}$$

4. given the following set of data, what is the variance? $[2, 8, 8, 3, 7, 9, 1, 4]$
- a, 40 b, 12.74 c, 5 d, 7.5

$$\text{mean is } M = \frac{2+8+8+3+7+9+1+4}{8}$$

$$= \frac{40}{8}$$

$$= 5$$

$$\text{Variance is } \frac{(2-5)^2 + (8-5)^2 + (8-5)^2 + (3-5)^2 + (7-5)^2 + (9-5)^2 + (1-5)^2 + (4-5)^2}{8}$$

$$= \frac{9+1+9+4+4+16+16+1}{8}$$

$$= \frac{60}{8}$$

$$= 7.5$$

5) The difference of marks scored in social science and science is

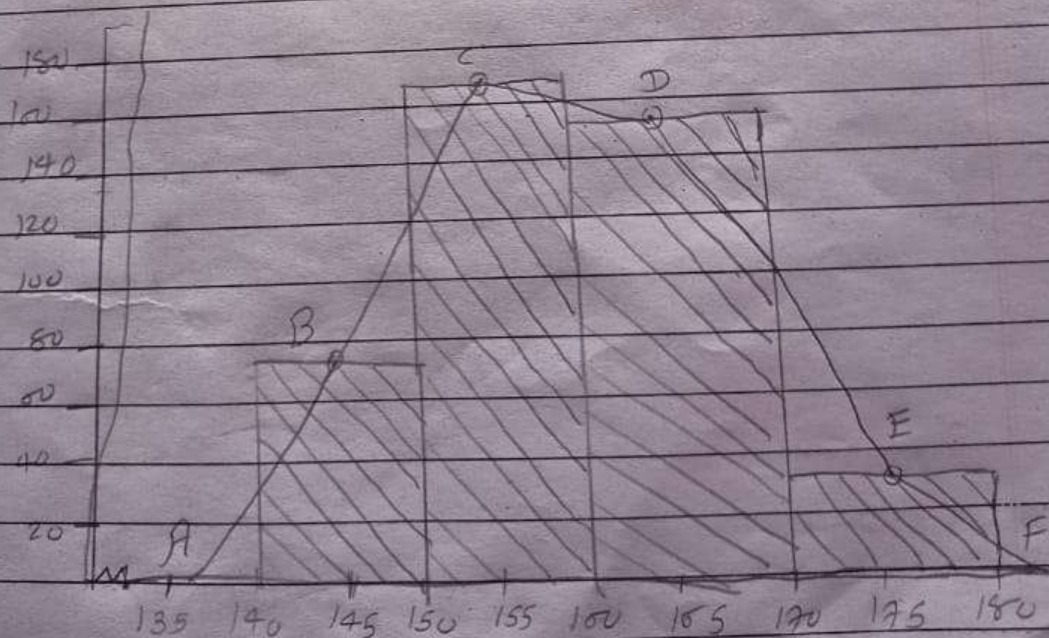
1.37.5 2.40 2.40 3.20 4.15

Social science and science: $\frac{(80+85) \times 900}{300}$

= 37.5

4) In a batch of 400 students, the height of student is given in the following table. Represent it through a frequency polygon

1) Which of the following statistics measures the most frequently occurring value in a set of data?
 a) median b) mode c) mean d) none of above



3, If the marks scored by the student are 137.5, then the subject is

1. English 2. Hindi 3. Mathematics 4. Science

13 subject :-
subject marks :- x

$$\text{subject} = \frac{x}{360} \times 900$$

$$137.5 = \frac{x}{360} \times 900$$

$$x = \frac{137.5 \times 360}{900}$$

$$x = 55$$

subject marks :- 55

then the subject is English

4, total marks scored in social science and English is

1. 400 2. 350 3. 500 4. 300

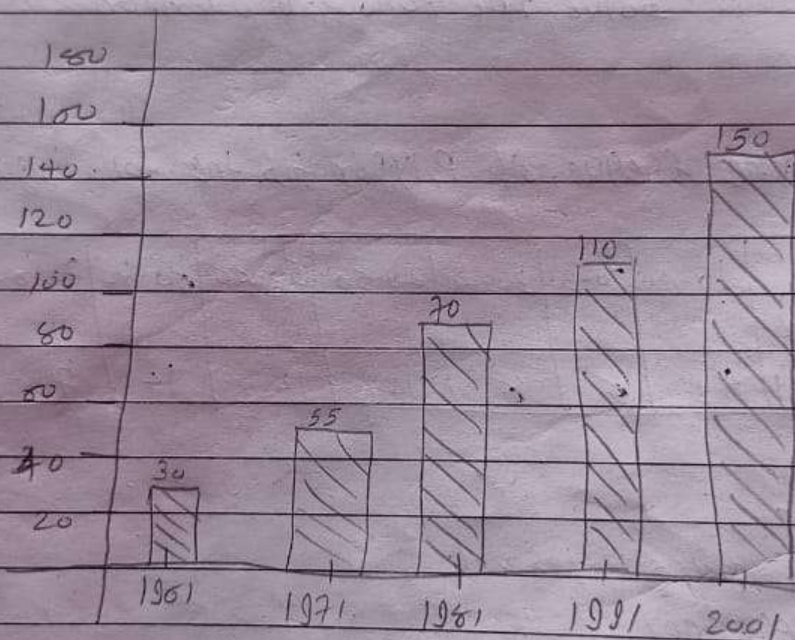
$$\text{Social science and English} = \frac{(55 + 55) \times 900}{360}$$

$$= \frac{120 \times 900}{360}$$

$$= 300$$

2) The population of Delhi state in different census years is as given below.

Census Year	1951	1971	1981	1991	2001
Population (Lakh)	30	55	70	110	150



3) refers to the pie chart given below and answer the questions that follow.

3) If the total marks were 3000, then marks in mathematics would be

1. 800 2. 750 3. 850 4. 900

mathematics would be:- $\frac{90}{360} \times 3000$

$$= 750$$

3) The marks scored in English and mathematics is less than than the marks scored in science and Hindi by

1. 5% 2. 4.33% 3. 3.33% 4. 6%

English and mathematics:- $\frac{(55+90)}{360} \times 900$

$$= 362.5 \text{ marks}$$

Hindi and science:- $\frac{(70+80)}{360} \times 900$

$$= 375$$

% percent decrease:- $\frac{12.5}{375} \times 100 \Rightarrow 3.33$

4) A frequency polygon is a line curve
↳ straight line

5)

5) diagrams are used for comparisons
↳ bar

question:- solve the following question

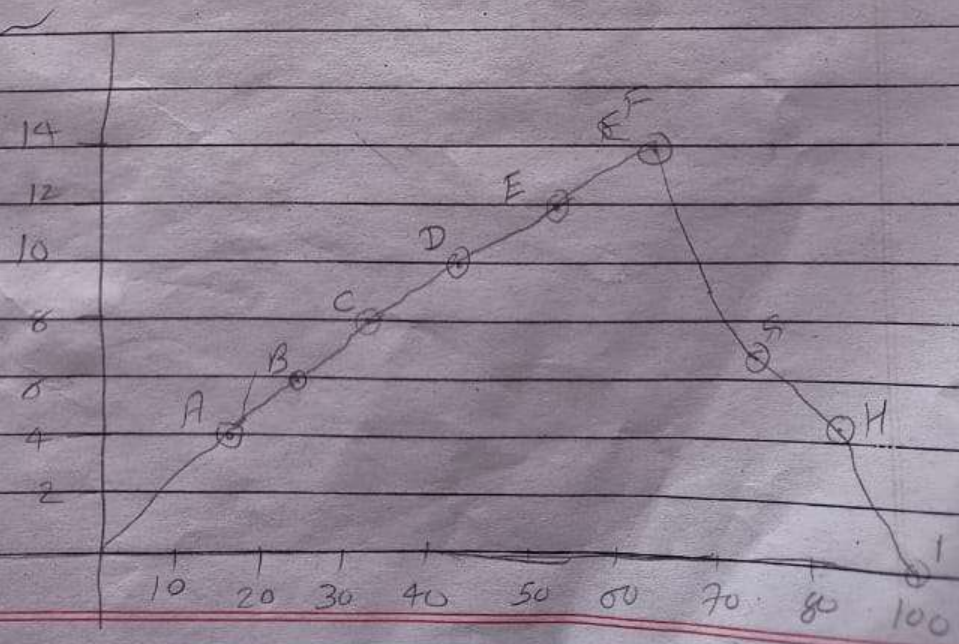
1) draw the frequency Polygon for the following data

Class	10-20	20-30	30-40	40-50	50-60	60-70	70-80	80-90
Interval								
Frequency	4	6	8	10	12	14	7	5

~~x-axis:-~~

~~or~~ x:- class interval

y:- frequency



3) what is statistical inference?

a) The process of drawing conclusions about a sample based on population data.

b) The process of drawing conclusions about a statistic based on a parameter.

☒ c) The process of drawing conclusions about a Population based on sample data.

d) ~~none~~ none of the above.

Inferential

Statistical inference: the process of using data analysis to infer properties of an underlying distribution of probability.

4) If you have data on the Yearly average temperature at Cape Town International Airport from 1900 to 2000, and if you are particularly interested in change over time what is the most effective graphical display?

a) Histogram

b) bar graph

c) Pie diagram

☒ d) Line graph

5) A histogram is a graphical representation of which of the following.

- a) an ogive
- ✓ b) A frequency distribution
- c) A cumulative relative frequency distribution
- d) All of the above.

A histogram is a graphical representation of a frequency distribution with class.

Fill in the blanks

1) Table is an orderly arrangement of data in rows and column.

2)

is a visual form of presenting
a) diagram

3) vertical rectangles representing class frequencies in a frequency distribution is called Histogram

4)

MCQ

1) Which of the following is a ~~quant~~ qualitative variable?

a) Annual salary

c) Age

b) weight

✓ d) eye colour

Qualitative: A variable

qualitative data are data which can assume
value that manifest the concept of attributes.

2) A population value or characteristic that is of
interest to us and that we would like to
estimate is _____

↳ a) hypothesis

b) statistic

c) population

✓ d) parameter

3) ~~The process of drawing conclusions about a~~
sample based on population data.

4. Find mode of the following data

x										
No of										

x	0-	10-	20-	30-	40-	50-	60-	70-	80-	90-100
	10	20	30	40	50	60	70	80	90	
y	3	5	7	10	12	15	12	5	2	8

↳

CLASS(x)	y
0-10	3
10-20	5
20-30	7
30-40	10
40-50	12
50-60	15
60-70	12
70-80	5
80-90	2
90-100	8

$$L = 50$$

$$n = 60 - 50$$

$$= 10$$

$$f = 15$$

$$f_1 = 12$$

$$f_2 = 12$$

Here

$$n=5, \sum x_i = 30, \sum y_i = 40$$

$$\sum x_i^2 = 220, \sum y_i^2 = 340$$

$$\sum x_i y_i = 214$$

Now

$$\bar{x} = \frac{\sum x}{n} = \frac{30}{5} = 6$$

$$\bar{y} = \frac{\sum y}{n} = \frac{40}{5} = 8$$

$$s = \frac{1}{5} \sum x_i y_i - \bar{x} \bar{y}$$

$$\sqrt{\left[\frac{\sum x_i^2}{n} - \bar{x}^2 \right]} \cdot \sqrt{\left[\frac{\sum y_i^2}{n} - \bar{y}^2 \right]}$$

$$= \frac{1}{5} \times 214 - 6 \times 8$$

$$\sqrt{\frac{220 - 36}{5}} \times \sqrt{\frac{340 - 64}{5}}$$

$$= 42.8 - 48$$

$$\sqrt{44 - 36} \times \sqrt{88 - 64}$$

$$= \frac{-52}{4\sqrt{2}}$$

$$r = -0.92$$

3) find mean, median mode for below observation.
15, 17, 12, 13, 14, 16, 1, 6, 18, 14

$$\text{mean} = \frac{15 + 17 + 12 + 13 + 14 + 16 + 1 + 6 + 18 + 14}{10}$$

$$= \frac{128}{10}$$

$$= 12.8$$

median

$$n = 10$$

$$= \frac{[(n/2) + \text{term}] + \{[(n/2) + 1] + \text{term}\}}{2}$$

$$= \frac{[(10/2) + \text{term}] + \{[(10/2) + 1] + \text{term}\}}{2}$$

$$= \frac{5^{\text{th}} \text{ term} + 6^{\text{th}} \text{ term}}{2}$$

$$= \frac{14 + 16}{2}$$

$$= 15 \Rightarrow 14$$

$$\boxed{\text{median} = 14}$$

mode:- 15, 17, 12, 13, 14, 16, 1, 6, 18, 14

Here, 14 occurs the maximum number of times, hence it's the mode.

$$\boxed{\text{mode} = 14}$$

2. From the following table calculate the coefficient of correlation by Karl Pearson's method. Arithmetic means of x and y series are 8 and 8 respectively.

x	5	2	10	4	8
y	9	11	?	8	7

Value of a from a

2)

$$\bar{y} = \frac{\sum y}{n}$$

$$8 = \frac{9 + 11 + a + 8 + 7}{5} \quad (\text{Value from } a) \quad (\because \bar{y} = 8)$$

$$40 = 35 + a$$

$$a = 5$$

x	5	2	10	4	8
y	9	11	5	8	7

f_i	y_i	$x_i y_i$	x_i^2	y_i^2
5	9	54	36	81
2	11	22	4	121
10	5	55	100	25
4	8	32	16	64
8	7	56	64	49

$\sum x_i = 30$	$\sum y_i = 40$	$\sum x_i y_i =$	$\sum x_i^2 =$	$\sum y_i^2 = 340$
		214	220	

$$= L + \frac{f - f_2}{2F - f_1 - f_2} \times n$$

$$= 50 + \frac{15 - 12}{2 \times 15 - 12 - 12} \times 10$$

$$= 50 - \frac{3 \times 10}{6}$$

$$= 55$$

- ⑤ The scores for student are 49, 45, 49, 53, 61, 55, 71, 79, 85, 91. What is the percentile for score 71?

↳ No of score below the value 71

$$n = 6$$

total no. of scores is $N = 10$

The formula for Percentile is given as below

$$P = \frac{n}{N} \times 100$$

$$= \frac{6}{10} \times 100$$

$$= 60$$

- 5) The central tendency media to be measured must lie in second quartile
- ↳ second quartile

- 6) _____ correlation between variable u and v
 if $(-2, 4), (-1, 8), (0, 12), (1, 16), (2, 20)$
- ↳ Positive correlation

Question:-1

- 1) Ten competitors in a beauty contest are ranked by three judges in the following order. Use rank correlation coefficient to determine which of the two judges have similar approach

2) 1 st judge	1	5	5	10	3	2	4	9	7	8
2 nd judge	3	5	8	4	7	10	2	1	6	9
3 rd judge	6	4	9	8	1	2	3	10	5	7

↳ 1 st judge	2 nd judge	3 rd judge	O_{12} ($R_1 - R_2$)	O_{12}^2	O_{23} ($R_2 - R_3$)	O_{23}^2	O_{13} ($R_1 - R_3$)	O_{13}^2
1	3	6	-2	4	-3	9	-5	25
5	5	4	-1	1	1	1	2	4
5	8	9	-3	9	-1	1	-4	16
10	4	8	1	1	-4	16	2	4
3	7	1	-3	9	6	36	2	4
2	10	2	8	64	8	64	0	0
4	2	3	-2	4	-1	1	1	1
9	1	10	-8	64	-9	81	-1	1
7	6	5	2	4	1	1	4	16
8	9	7	8	64	4	16	1	1
			$\Sigma = 0$	$\Sigma = 200$	$\Sigma = 0$	$\Sigma = 214$	$\Sigma = 0$	$\Sigma = 60$

$$R_{12} = \frac{1 - (\sigma \epsilon \sigma_{12}^2)}{(N(N^2 - 1))}$$

$$= \frac{1 - (\sigma \times 200)}{(10 \times 99)}$$

$$= 0.212$$

$$R_{23} = \frac{1 - (\sigma \epsilon \sigma_{23}^2)}{(N(N^2 - 1))}$$

$$= \frac{1 - (\sigma \times 214)}{10 \times 99}$$

$$= 0.297$$

$$R_{13} = \frac{1 - (\sigma \epsilon \sigma_{13}^2)}{(N(N^2 - 1))}$$

$$= \frac{1 - (\sigma \times 80)}{10 \times 99}$$

$$= 0.630$$

first and third judge have similar approach

⑥ calculate quartile-2, percentiles-45 from the following data 85, 96, 90, 108, 85, 80, 100, 85, 70, 95

↳ here

$$n=10$$

$$d_2 = \left(\frac{2(n+1)}{4} \right) + n \text{ value}$$

$$= \left(2 \left(\frac{11}{4} \right) \right) + n \text{ value}$$

$$= 5.5^{\text{th}} \text{ value}$$

$$= 5^{\text{th}} \text{ value} + 0.5[6^{\text{th}} - 5^{\text{th}}]$$

$$= 85 + 0.5[85 - 85]$$

$$= 85 + 0$$

$$= 85$$

$$= 85$$