PARUL UNIVERSITY
FACULTY OF ENGINEERING AND TECHNOLOGY

DEPARTMENT OF APPLIED SCIENCE AND HUMANITIES

SUBJECT: BASIC MATHEMATICS

Chapter -3- Arithmetic and Geometric Progression

3.1 INTRODUCTION

We know number of patterns such as

(i) 1,2,3,4,...........
(ii) 2,4,6,8,...........
(iii) 1,3,5,7,9,.......
(iv) 1,4,7,10,13,........
(v) $\frac{2}{3},\frac{3}{4},\frac{4}{5},\frac{5}{6},\dots$
(vi) 1 x 3, 3 x 6, 5 x 12, 7 x 24, …

In some of these patterns, we can guess and decide the next number of pattern while in some others we are not able to. In this chapter we shall recall some of these patterns and try to arrive at next number in each pattern.

3.2 SEQUENCE

A sequence is a set of numbers arranged in a defined order according to a certain rule .It can be finite or infinite. Each number in the sequence is called term of the sequence.The terms of sequence are usually denoted by a_1 ,a_2 , a_3 ,….,a_n .Where a_1,a_2 and a_n are the first,second and n^{th} term respectively.
Sometime we use the notation T_n for the n^{th} term of the sequence.

For example:
(1) 1,2,3,4,5,……..,100 is a finite sequence.
(2) 1,2,4,8,16,…….is a infine sequence.

3.3 SERIES

A series is the sum of the terms of a sequence. A series can be finite or infinite.Suppose we have the sequence a_1 ,a_2 , a_3 ,….,a_n .Then,the series we obtain from this sequence is , $a_1 + a_2 + a_3 + \ldots + a_n$.

For example :
(1) 1+2+3+….+100 is a finite series.
(2) 1+4+9+16+……is a infinite series.

SOLVED EXAMPLES

Example 1: Write the first three terms and corresponding series of the following sequence defined by $a_n = 2n+5$.

Solution : Here, $a_n = 2n+5$. Substituting n=1,2,3.

We get;
a_1 =First term= 2(1)+5=7
a_2 = Second term = 2(2)+5=9
a_3 = third term = 2(3)+5=11

Therefore, the required terms are 7,9 and 11.and the corresponding series is 7+9+11+ …….

Example 2 : Find first five terms of sequence whose n^{th} term is n^2 .
Solution: The n^{th} term is n^2 .
Then , $a_n = n^2$
$\therefore a_1 = (1)^2 = 1$
$\therefore a_2 = (2)^2 = 4$
$\therefore a_3 = (3)^2 = 9$
$\therefore a_4 = (4)^2 = 16$
$\therefore a_5 = (5)^2 = 25$

The First five terms of the given sequence are 1,4,9,16,25.

3.4 PROGRESSION

Definition: Progression is a sequence of numbers in which each term is related to its predecessor by a uniform law.
Or An arrangement of numbers in a definite order is called “Progression”.
E.g. 1) 2,5,8,11,14,…..

2) 1,10,100,1000,…..
3) 1/2,1/4,1/8,1/16……

Types of Progression: There are 2 types of Progressions.

1. Arithmetic Progression(A.P.)
2. Geometric Progression(G.P.)

3.5 ARITHMETIC PROGRESSION (A.P.)

Arithmetic progression (A.P.) is a sequence in which each term except the first is obtained by adding a fixed number (positive or negative)to the preceding term.
Or Arithmetic Progression is the sequence of number in which each term is obtained from the preceding term by adding or substracting a fixed number called common difference 'd '.
Thus ,any sequence a_1 ,a_2 , a_3 ,….,$a_n$is called an arithmetic progression if
$a_{n+1} = a_n + d$, $n \epsilon N$. Where d is called the common difference of the A.P.,Usually we denote the first term of an A.P. by a and the last term or n^{th} term is denoted by $a_n$or l .The general term or n^{th} term of an A.P. is given by $a_n = a+(n-1)d$.
In general,if an A.P. has first term a and common difference d, thenthe A.P. can be written as a,a+d,a+2d,a+3d,………….

For e.g. The sequence 1,2,3,…… is an infinite A.P. with first term =1 and common difference =1 .and the corresponding Arithmetic series is 1+2+3+……

Ex 2 . 9,6,3,0,-3,..... is A.P. and common difference d= 6-9= -3

PROPERTIES OF A.P.

We can verify the following simple properties of an A.P. :

1. If a constant is added to each term of an A.P., the resulting sequence is also an A.P.
2. If a constant is subtracted from each term of an A.P., the resulting sequence is also an A.P.
3. If each term of an A.P. is multiplied by a constant, then the resulting sequence is also an A.P.
4. If each term of an A.P. is divided by a non-zero constant then the resulting sequence is also an A.P.

SOLVED EXAMPLE

Example 1: Find the 20^{th} term of 2,5,8,11,…..

Solution : Here, a_1= a=2 and a_2 = 5.

And it is a A.P. as;

Common difference d=5-2

= 3

Then, 20^{th} term is;

a_{20} = a+(20-1)d

= 2 + 19(3)

= 59

∴20^{th} term of the given A.P. is 59.

Example 2 : Find the next four terms of -9,-2,5,…….. and also find the general formula of the sequence .

Solution : Here, a_1 = a= -9

a_2 = -2 and a_3 =5.

Common difference d= -2-(-9)= -2+9=7

Next term is a_4 = a+(4-1)d

= -9 +(3)7

∴a_4 = 12

As $a_{n+1} = a_n + d$

Then, $a_5 = a_4 + d$= 12 + 7=19

Simillarly, a_6 = 19+7 = 26 and a_7 =26+7=33

So,the next four term is 12,19,26,33.

The general formula of this sequence is a_n = a+(n-1)d

a_n = -9 + (n-1) 7

a_n = -9+7n-7

a_n = 7n-16

Exercise: Identify which of the following are arithmetic progressions. For those which are arithmetic progressions, find the common differences:

(i) $3, 6, 12, 24, \ldots$
(ii) $12, 2, -8, -18, \ldots$
(iii) $0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, \ldots$
(iv) $\frac{1}{7}, \frac{1}{9}, \frac{1}{11}, \frac{1}{13}, \ldots$
(v) $1, 1.7, 2.4, 3.1, \ldots$
(vi) $1^2, 5^2, 7^2, \ldots.$

Example 3:(a) Find the next Four terms of 0,7,14,…..

Solution: This sequence is an Arithmetic Sequence.

Here, $a = 0$ and $d = 7$

Also, $a_2 = 7$ and $a_3 = 14$,

$\therefore a_4 = a_3 + d = 14+7=21$

$a_5 = a_4 + d = 21+7=28$

$a_6 = a_5 + d = 28+7=35$

$a_7 = a_6 + d = 35+7=42$

(b) Find the next four terms of x,2x,3x, ……

Solution : This sequence is an Arithmetic Sequence.

Here, $a = x$ and $d = x$

Also, $a_2 = 2x$ and $a_3 = 3x$,

$\therefore a_4 = a_3 + d = 3x+x=4x$

$a_5 = a_4 + d = 4x+x=5x$

$a_6 = a_5 + d = 5x+x=6x$

$a_7 = a_6 + d = 6x+x=7x$

Example 4 : Find the n^{th} term and hence 11^{th} term of the A.P. 2,5,8,......

Solution: Here, the first term a=2 and common difference d=3.

$$\text{Now, } n^{th} \text{ term } a_n = a+(n-1)d$$
$$= 2+(n-1)\,3$$
$$= 2+3n-3$$
$$\therefore a_n = 3n-1$$

$$\text{And, } 11^{th} \text{ term } a_{11} = 3(11)-1$$
$$= 33-1$$
$$\therefore a_{11} = 32$$

Example 5 : Find the n^{th} term and hence T_{30} - T_{20} for the A.P. -9,-14,-19,.....

Solution : Here a= -9 and d= -14 – (-9) = -5

$$\text{Now, } n^{th} \text{ term } T_n = a+(n-1)d$$
$$T_n = -9+(n-1)(-5)$$
$$= -9 -5n +5$$
$$\therefore T_n = -5n-4$$

$$\text{Now, } T_{30} - T_{20} = [-5(30)-4]-[-5(20)-4]$$
$$= -154+104$$
$$= -50$$

Example 6 : How many terms are in the progression 12,9,6,.........,-30?

Solution: Here First term a= 12 and common difference d= -3 and n^{th} term a_n = -30.

$$\text{Using, } a_n = a+(n-1)d$$
$$-30 = 12+(n-1)-3$$
$$-30 = 12-3n +3$$
$$-30= 15-3n$$
$$-30-15=-3n$$
$$45 = 3n$$
$$\therefore n= 15$$

There are 15 terms in given progression.

Example 7: The 4^{th} term of an A.P.is 19 and its 12^{th} term is 51.Find 21^{st} term.

Solution: As,T_n= a+ (n-1)d

T_4 = a+ (4-1)d

$\therefore 19 = a+3d$ _____ (1)

And T_{12} = a+ (12-1)d
51=a+11d ______ (2)

Now,solving (1) and (2)
∴a=19-3d
∴51=19-3d+11d
∴ 51-19=8d
∴ 32=8d
∴We get; d=4
Now,Substitutiong value of d in equation (1)
∴19= a+3d
∴19 = a+3(4)
∴ a=7
And
T_{21} = a+ (21-1)d
= 7+20(4)
= 87
∴21st term is 87.

Examples:

(1) For the sequence 7, 13, 19, 25, 31,……., find the value of the 37th term.
(2) Find the 25th term of the following arithmetic progressions:
(a) 3, 6, 9, 12, 15, …
(b) $1, \frac{5}{6}, \frac{2}{3}, \frac{1}{2}, \frac{1}{3}, \ldots$
(3) Find the 10th term of the A.P. 6, 2, -2, -6, …
(4) The fifth term of an arithmetic progression is 10 while the 15th term is 40. Write down the first 5 terms of the A.P.

3.6 <u>THE SUM OF ARITHMETIC PROGRESSION</u>

The Sum of the first n terms of an A.P. is denoted by $S_n = \frac{n}{2}[2a + (n-1)d]$. Where a is the first term and d is the common difference.

OR The Sum of the n terms of A.P. is $S_n = \frac{n}{2}[a + l]$.Where l = a_n =a+(n-1)d.

<u>SOLVED EXAMPLES</u>

Example 1 : Find sum of -19,-13,-7,........(upto 63 term).

Solution: Here, a_1= a= -19 and a_2= -13.

$\therefore$d= -13 – (-19)=6

and

a_n= a+(n-1)d

$\therefore a_{63}$= -19 +(63-1)6

= 353

Therefore, $S_n = \frac{n}{2}[a+l]$

$\therefore S_{63} = \frac{63}{2}[-19+353]$

= 10,521

Example 2: An arithmetic progression whose first term is 3 and whose n^{th} term is 171 and has the sumof its first n term equal to 3741.Find the value of n and the common difference.

Solution: Here, a=3

a_n= a+(n-1)d

$\therefore$171=3+(n-1)d

(n-1)d=168 ____ (1)

And

$S_n = \frac{n}{2}[a+l]$

$\therefore$ 3741= $= \frac{n}{2}[3+171]$ _____ (2)

Solving (2) : n= 43 _____(3)

Sub (3) into (1) we get;d=4.

Example 3 : Find the sum of 100,93,86,79,.......(upto 20 terms).

Solution: Here,a=100,d= -7 and n= 20.

$S_n = \frac{n}{2}[2a+(n-1)d]$

$\therefore S_{20} = \frac{20}{2}[2(100)+(20-1)(-7)]$

= 10[200+(-133)]

$\therefore S_{20}$ = 670

Example 4 : An auditorium has 20 raws of seats.There are 20 Seats in the first raw,21 seats in the second raw,22 seats in the third row , and so on. How many seats are there in all 20 raws?

Solution : Here, $a_1=a=20$, $a_2=21$ and $d=1$

$a_n=a+(n-1)d$

$\therefore a_{20}= 20+(20-1)(1)=39$

And

$$S_n = \frac{n}{2}[a+l]$$

$$\therefore S_{20} = \frac{20}{2}[20+39] =590$$

Example 5 : A small business sells \$10,000 worth of sports memorabilia during its first year.The owner of the business has set a goal of increasing annual sales by \$7500 each year for 19 years.Assuming that the goal is met,find the total sales during the first 20 years this business is in operation.

Solution: Here, $a_1=a=10,000$ and $d=7500$

$a_n=a+(n-1)d$

$\therefore a_{20}=10,000+19(7500)$

$=1,52,500$

And $S_n = \frac{n}{2}[a+l]$

$$\therefore S_{20} = \frac{20}{2}[10,000+1,52,500]$$

$=10(162500)$

$= 1,625,000$

So, the total sales for the first 20 years is \$ 1,625,000.

Example 6: Given an arithmetic sequence with $a_{15} = 38$ and $d= - 3$ then find a_1.

Solution: Here,n=15.

$a_n=a_1 +(n-1)d$

$\therefore 38 =a_1+(15-1)(-3)$

$\therefore a_1 = 80$

REMARKS

(i) If the sum of three numbers in A.P. is given, we have to assume the numbers
$a-d$, a, $a+d$.

(ii) If the sum of four numbers in A.P. is given , we have to assume the numbers
$a-3d$, $a-d$, $a+d$, $a+3d$.

(iii) If the sum of five number in A.P. is given ,we have to assume the numbers
$a-2d$, $a-d$, a , $a+d$, $a+2d$.

Example 7 :find three numbers in A.P. Whose sum is 9 and their product is -165.

Solution : Let three numbers in A.P. Be , $a-d,a,a+d$.

$$\text{Sum} = a-d+a+a+d=9$$

$$3a=9 \Rightarrow a=3$$

Also , product $(a-d)a(a+d)= -165$

$$a(a^2-d^2)= -165$$

$$3(9-d^2) = -165$$

$$9-d^2 = -55$$

$$d^2 = 55+9$$

$$d^2 =64 \Rightarrow d = \pm 8$$

So ,the numbers are $3-8,3,3+8$ or $3+8,3,3-8$
i.e. $-5,3,11$ or $11,3,-5$

Example 8: A man saved Rs.16,500 in ten years. In each year after the first he saved Rs.100 more than he did in the preceding year. How much did he saved in the first year?

Solution: Here a= savings in the first year = ?
n= number if years=10,d=100,S_n = 16,500
Now, using Formula;

$$S_n = \frac{n}{2}[2a+(n-1)d]$$

$$16,500 = \frac{10}{2}[2a+(10-1)100]$$

$\Rightarrow$ $16,500 = 5(2a+900)$

$\Rightarrow$ $10a=16,500-4500$

$\Rightarrow$ $10a=12,000$

$\Rightarrow$ $a=1200$

Examples

(1) Find the sum of the first 20 terms of the sequence -2, 1, 4, 7, 10,………
(2) Find the sum of the arithmetic progression 3, 8, 13, … to 10 terms.
(3) The sum of the first 8 terms of an arithmetic progression is 56 and the sum of the first 20 terms is 260. Find the first term and the common difference of the A.P.

3.7 ARITHMETIC MEAN (A.M.)

The arithmetic mean for any n positive number $a_1, a_2, a_3, \ldots, a_n$ is given by

A.M.= $\dfrac{a_1 + a_2 + a_3 + \ldots.. + a_n}{n}$.

Arithmetic mean of numbers a and b is; $\dfrac{a+b}{2}$.

For example, the A.M.of two numbers 4 and 16 is 10.(as $\dfrac{4+16}{2}$ =10). Thus we constructed an A.P. 4,10,16 by inserting a number 10 between 4 and 16.

Example 1 : Find two arithmetic means between - 4 and 5.
Solution : -4,___, _____,5.
$a_n = a_1 + (n-1)\,d$
$\therefore$ 5= -4 + (4-1) d
$\therefore$ d = 3
The two arithmetic means are -1and 2 ,since -4,-1,2,5,forms an arithmetic sequence.

INSERTION OF ARITHMETIC MEANS

Insertion of Arithmetic means : Let A_1 , A_2 , ... , A_n be the A.Ms. Between a and b .
Then a, A_1 , A_2 , ... , A_n ,b are in A.P.
b is the $(n+2)^{th}$ term of this A.P. And let d be the common difference.

$a_n = a + (n-1)d$, $a_n = b$

b=a+(n+2-1)d $\Rightarrow$ d= $\dfrac{(b-a)}{n+1}$

Hence, A_1 =a+d=a + $\frac{(b-a)}{n+1}$

$\Longrightarrow$ A_2 = a+2d = a+2 $\frac{(b-a)}{n+1}$

$\Longrightarrow A_n$ =a+nd= a+n $\frac{(b-a)}{n+1}$

Example : Find the 14 arithmetic means which can be inserted between 5 and 8 and show that their sum is 14 times the arithmetic mean between 5 and 8.

Solution: Let A_1 , A_2 , ... , A_{14} be the 14 A.Ms. Between 5 and 8. then,5, A_1 , A_2 , ... , A_{14} ,8 form an A.P. Whose first term is 5 and whose 16^{th} term is 8.Let,d be the common difference of the A.P. Then,

$a_n = a + (n-1)d$

8=5+(n-1)d$\Longrightarrow$ d= $\frac{1}{5}$

A_1 =a+d=5+ $\frac{1}{5}$ = $\frac{26}{5}$

A_2 =a+2d = $\frac{27}{5}$,.......

Hence, the fourteen A.Ms are $\frac{26}{5}$, $\frac{27}{5}$, $\frac{28}{5}$, ...

Sum of these mean is = $S_n = \frac{n}{2}[2a + (n-1)d]$ = $\frac{14}{2}[2 \times \frac{26}{5} + (14-1)\frac{1}{5}] = 91$

Also,A.M. Between 5 and 8 is $= \frac{5+8}{2} = \frac{13}{2}$

And

14 times the A.M. is $14 \times \frac{13}{2} = 91$

Hence, the sum of the 14 A.Ms, = 14 times the A.M. Between 5and 8.

3.8 GEOMETRIC PROGRESSION

Geometric Progression (G.P.) is a sequence in which each term except the first is obtained by multiplying the previous term by a non-zero constant called the common ratio.

Let us, consider a G.P. with first non-zero term a and common ratio r.

i.e. $a, ar, ar^2, ar^3, \ldots\ldots, ar^{n-1}, \ldots$

The general term or n^{th} term of G.P. is given by $l = a_n = ar^{n-1}$

Thus a G.P. can be written as $a, ar, ar^2, ar^3, \ldots\ldots, ar^{n-1}$. G.P. is finite or infinite.

Then series $a+ar+ar^2+ar^3+\ldots+ar^{n-1}+\ldots$ are called infinite geometric Series.

e.g. A sequence 1,2,4,8,16,…..is an infinite G.P. with first term =1and common ratio =2 and the corresponding geometric series is 1+2+4+8+16,…..

PROPERTIES OF GEOMETRIC PROGRESSION

1. If the terms of a G.P. are multiplied or divided by the same non-zero constant ($k \neq 0$),they still remain in G.P.
2. If $a_1, a_2, a_3, \ldots$ are in G.P. , then $a_1k, a_2k, a_3k, \ldots$ and $\frac{a_1}{k}, \frac{a_2}{k}, \frac{a_3}{k}, \ldots$ are also in G.P. with same common ratio,in particularly $a_1, a_2, a_3, \ldots$ are in G.P. , then $\frac{1}{a_1}, \frac{1}{a_2}, \frac{1}{a_3}, \ldots$ are also in G.P.
3. If $a_1, a_2, a_3, \ldots$ and If $b_1, b_2, b_3, \ldots$ are two G.Ps. then $a_1b_1, a_2b_2, a_3b_3, \ldots$ and $\frac{a_1}{b_1}, \frac{a_2}{b_2}, \frac{a_3}{b_3}, \ldots$ are also in G.P.

SOLVED EXAMPLES

Example 1: Find the 10^{th} and n^{th} terms of the G.P. 5,25,125,........

Solution : Here a=5 and r=5.

Thus, n^{th} terms of the G.P is

$$a_n = ar^{n-1}$$
$$= 5(5)^{n-1}$$
$$= 5^n$$

10^{th} term is $a_{10} = 5(5)^{10-1}$

$$= 5(5)^9$$

$=5^{10}$

Example 2: In a G.P. , the 3rd term is 24 and the 6th term is 192.Find the 10th term.

Solution :The n^{th} terms of the G.P.is $a_n= ar^{n-1}$

Then $a_3= ar^2=24$ _____(1)

$a_6= ar^5= 192$ _____(2)

Now, dividing (2) by (1) we get; r=2.Substituting r=2 in (1).

We get;

$4a=24 \Longrightarrow a=6.$

Hence, $a_{10}= ar^9= 6(2)^9=3072$

Example 3 : Find the next three terms of 2,3, $\frac{9}{2}$,.....

Solution : a=2 and r= $\frac{3}{2}$ = 1.5 . Then, the next three terms are;

$a_4= ar^3= 2(\frac{3}{2})^3 = \frac{27}{4}$

$a_5= ar^4= 2(\frac{3}{2})^4 = \frac{81}{8}$

$a_6= ar^5= 2(\frac{3}{2})^5 = \frac{243}{16}$

So,the G.P. is 2,3, $\frac{9}{2}$, $\frac{27}{4}$, $\frac{81}{8}$, $\frac{243}{16}$.

Example 4: Find a G.P. Whose 3rd term and 6th term are 1 and -1/8 respectively.

Solution : Given , 3rd term = 6 so, $ar^2 =1$ ______(1)

and , 5th term = -1/8 so, $ar^5 = -1/8$ ______(2)

Now, dividing (1) by (2) we get;

$$\frac{ar^2}{ar^5} = \frac{1}{(\frac{-1}{8})} \Rightarrow \frac{1}{r^3} = \frac{8}{-1} \Rightarrow r^3 = \left(\frac{-1}{8}\right)$$

$$\Rightarrow r^3 = \left(\frac{-1}{2}\right)^3$$

$$\Rightarrow r = \left(\frac{-1}{2}\right)$$

Now, $ar^2=1$ so, $a(-1/2)^2 = 1 \Rightarrow a\,(1/4)=1 \Rightarrow a=4$.

so, G.P is 4,-2,1,-1/2,.......

Examples:

(1) For the sequence 2, 6, 18, 54, 162,……., find the value of the 12th term.

(2) Find the 8^{th} term of the GP 27, -18, 12, -18, …

(3) Which term of the GP: 36, -12, 4, … is $\frac{4}{81}$?

(4) Find the first term of a GP whose 10^{th} term is $\frac{3}{1024}$ and whose common ratio is $\frac{1}{2}$.

3.9. <u>THE SUM OF GEOMETRIC PROGRESSION</u>

Let the first term of a G.P. be a and the common ratio be r and S_n be the sum to first n terms of G.P. then;

$S_n = \frac{a(r^n-1)}{r-1}$, when r>1

$= \frac{a(1-r^n)}{1-r}$, when r<1.

<u>SOLVED EXAMPLES</u>

Example 1 : Find the sum of first 5 terms of the geometric series

$1+\frac{2}{3}+\frac{4}{9}+......$

Solution : Here ,a=1 and r= $\frac{2}{3}$.

Therefore $S_n = \frac{a(r^n - 1)}{r - 1}$

$\therefore S_5 = \frac{1[\left(\frac{2}{3}\right)^5 - 1]}{(\frac{2}{3} - 1)}$

$\therefore S_5 = \frac{211}{81}$

Example 2 :A person has 2 parents,4 grandparents,8 great grandparents, and so on. Find the number of his ancestors during the ten generations preceding his own.

Solution : Here a=2 , r=2 and n=10.

Using sum formula ,

$S_n = \frac{a(r^n - 1)}{r - 1}$

$S_{10} = 2(2^{10} - 1)$

= 2046

Hence, the number of ancestors preceding the person is 2046.

Example 3 : How many terms of the G.P. 3, $\frac{3}{2}$, $\frac{3}{4}$,…….. are needed to give the sum $\frac{3069}{512}$?

Solution : Let n be the number of terms needed. Given that a=3,r= $\frac{1}{2}$ and $S_n = \frac{3069}{512}$.

Since,

$S_n = \frac{a(r^n - 1)}{r - 1}$

$$\frac{3069}{512} = \frac{3[\left(\frac{1}{2}\right)^n - 1]}{(\frac{1}{2}-1)}$$

$$\frac{3069}{512} = [1 - \frac{1}{(2)^n}]$$

$2^n = 1024 = 2^{10}$.

Which gives n=10.

Example 4 : If $a_1 = \frac{1}{2}$ and $r = \frac{2}{3}$.Then find a_9.

Solution :$a_9 = a_1\, r^{9-1}$

$= \frac{1}{2} (\frac{2}{3})^{9-1}$

$= \frac{1 \times 2^8}{2 \times 3^8}$

$= \frac{128}{6561}$

Example 5 : Find which term of the series 0.004+0.02+0.1+......is 12.5?

Solution: Here, $a_1 = 0.004$ and $a_2 = 0.02$.

And . $\frac{a_2}{a_1} = \frac{0.02}{0.004} = 5$ Therefore, the series is a G.P. with r=5.

Now, suppose 12.5 is n^{th} term.

But, $a_n = ar^{n-1} = 0.004 \times 5^{n-1}$

$0.004 \times 5^{n-1} = 12.5$

$\Rightarrow\ 5^{n-1} = \frac{12.5}{0.004} = \frac{12500}{4} = 3125 = 5^5$

$\Rightarrow\ 5^{n-1} = 5^5$

$\Rightarrow\ n-1=5$

$\Rightarrow\ n=6$

Hence, 12.5 is the 6^{th} term.

REMARKS

(i) If 3 numbers are given in the G.P., then suppose the numbers $\frac{a}{r}$,a,ar.

(ii) If 4 numbers are given in the G.P., then suppose the numbers $\frac{a}{r^3}$, $\frac{a}{r}$,ar, ar^3 .

(iii) If 5 numbers are given in the G.P., then suppose the numbers $\frac{a}{r^2}$, $\frac{a}{r}$,a,ar, ar^2 .

Example :3 numbers in G.P.,their sum and product are 28 and 512 respectively . find the numbers.

Solution : Suppose, the three numbers are $\frac{a}{r}$,a,ar.

Now, it is given $\frac{a}{r} \times$ a $\times$ ar = 512

$\therefore a^3 = 512$

$\therefore$ a= 8

Their sum = 28

$\therefore \frac{a}{r}$ + a + ar = 28

$\therefore \frac{8}{r}$ + a + ar = 28

$\therefore \frac{8+8r+8r^2}{r} = 28$

$\therefore 8+8r+8r^2 = 28r$

$\therefore 8r^2 - 20r + 8 = 0$

$\therefore 8r^2 - 16r - 4r + 8 = 0$

$\therefore 8r(r-2) - 4(r-2) = 0$

$\therefore (8r-4)(r-2) = 0$

$\therefore r-2=0$ or $8r-4=0$

$\therefore$ r=2 and $r = \frac{1}{2}$

If a=8 and r=2 then G.P. is 4,8,16

And if a=8 and $r = \frac{1}{2}$ then G.P. is 16,8,4

3.10<u>GEOMETRIC MEAN (G.M.)</u>

The Geometric mean of two positive numbers a and b is the number $\sqrt{ab}$.

For example , the geometric mean of 2 and 8 is 4. We observe that the three numbers 2,4,8 are consecutive terms of a G.P.

This leads to a generalisation of the concept of geometric mens of two numbers.

Example : Find the AM and GM of 8 and 32.

Solution: Arithmetic Mean of two numbers is $\frac{a+b}{2}$.

Here, a=8 and b= 32

AM= $\frac{a+b}{2}$

= $\frac{8+32}{2}$

= 20

Geometric Mean of two number is $\sqrt{ab}$

GM= $\sqrt{ab}$

= $\sqrt{8 \times 32}$

= $\sqrt{256}$

= 16

<u>INSERTION OF GEOMETRIC MEANS</u>

Insertion of Geometric Means : Let G_1 , G_2 ,....., G_n be the n geometric means between a an b .Then a, G_1, G_2 ,.....,G_n ,b are in G.P. Let r denote the common ratio of this G.P. , including the given terms a and b, there are (n+2) terms in this b= (n+2)th term of the G.P.= $ar^{(n+2)-1}$ =ar^{n+1}

r^{n+1} = $\frac{b}{a}$

i.e. r = $\left(\frac{b}{a}\right)^{\frac{1}{n+1}}$ or $\sqrt[n+1]{\frac{b}{a}}$

$\Rightarrow$ G_1 = ar=a $\left(\frac{b}{a}\right)^{\frac{1}{n+1}}$

$\Rightarrow$ G_2 = ar^2 = a $\left(\frac{b}{a}\right)^{\frac{2}{n+1}}$ and G_n = a $\left(\frac{b}{a}\right)^{\frac{n}{n+1}}$ respectively.

Example : Insert 5 geometric means between 320 and 5.

Solution: We have in all 7 terms of which the first term is 320 and 7th term is 5.Therefore, using the formula

r^{n+1} = $\frac{l}{a}$ where, l = 5 and a=320

We have; r^6 = $\frac{1}{64}$,i.e., = $\sqrt[6]{\frac{1}{64}} = \left(\frac{1}{64}\right)^{\frac{1}{6}}$

$\Rightarrow$ r= $\frac{1}{2}$, which is the common ratio.

Therefore, the series is 320,160,80,40,20,10 and 5, and the geometric means are 160,80,40,20,10.

3.11 <u>SUM OF INFINITE SERIES</u>

A Series is said to be convergent (when | r | < 1) if the value of the sum to infinity exists as a finite number.

Otherwise, it is divergent (when | r | > 1).

In general, S(or S_∞) denote the sum to infinity.

and S=$\frac{a}{1-r}$, if | r |<1.

Example 1: Find the sum of the series 1+ $\frac{1}{2}$ + $\frac{1}{4}$ + $\frac{1}{8}$ +..... ∞

$\frac{\left(\frac{1}{2}\right)}{1}$ $\frac{\left(\frac{1}{2}\right)}{\left(\frac{1}{4}\right)}$ $\frac{1}{2}$

Solution: Here, r = = =

As r = $\frac{1}{2}$ then $-1 \le r \le 1$

∴Sum is possible.

S_∞ = $\frac{a}{1-r}$ = $\frac{1}{1-\frac{1}{2}}$ = 2

∴ S = 2

Example 2 : Find the sum of the series : $2\sqrt{2}$ +8+$16\sqrt{2}$ +…..∞

Solution : Here r = $\frac{8}{2\sqrt{2}}$ = $\frac{16\sqrt{2}}{2}$ = $2\sqrt{2}$

∴ r $\notin$ [-1,1]

∴ Sum is not possible.

Example 3: Find the sum of the series : 10+5+$\frac{5}{2}$ +….∞

Solution : Here, r = $\frac{5}{10}$ = $\frac{\left(\frac{5}{2}\right)}{5}$ = $\frac{1}{2}$

As r = $\frac{1}{2}$ then $-1 \le r \le 1$

∴ S_∞ = $\frac{a}{1-r}$ = $\frac{10}{1-\frac{1}{2}}$ = 20

- **Example 4:**Find the infinite G.P. whose first term is and the sum is .

Solution : Here, a = $\frac{1}{4}$ and S = $\frac{1}{3}$

Using, Formula of Sum;

$$S = \frac{a}{1-r} = \frac{\frac{1}{4}}{1-r}$$

$$\therefore \frac{1}{3} = \frac{\frac{1}{4}}{1-r}$$

$\therefore$ 4- 4r =3   4r=1

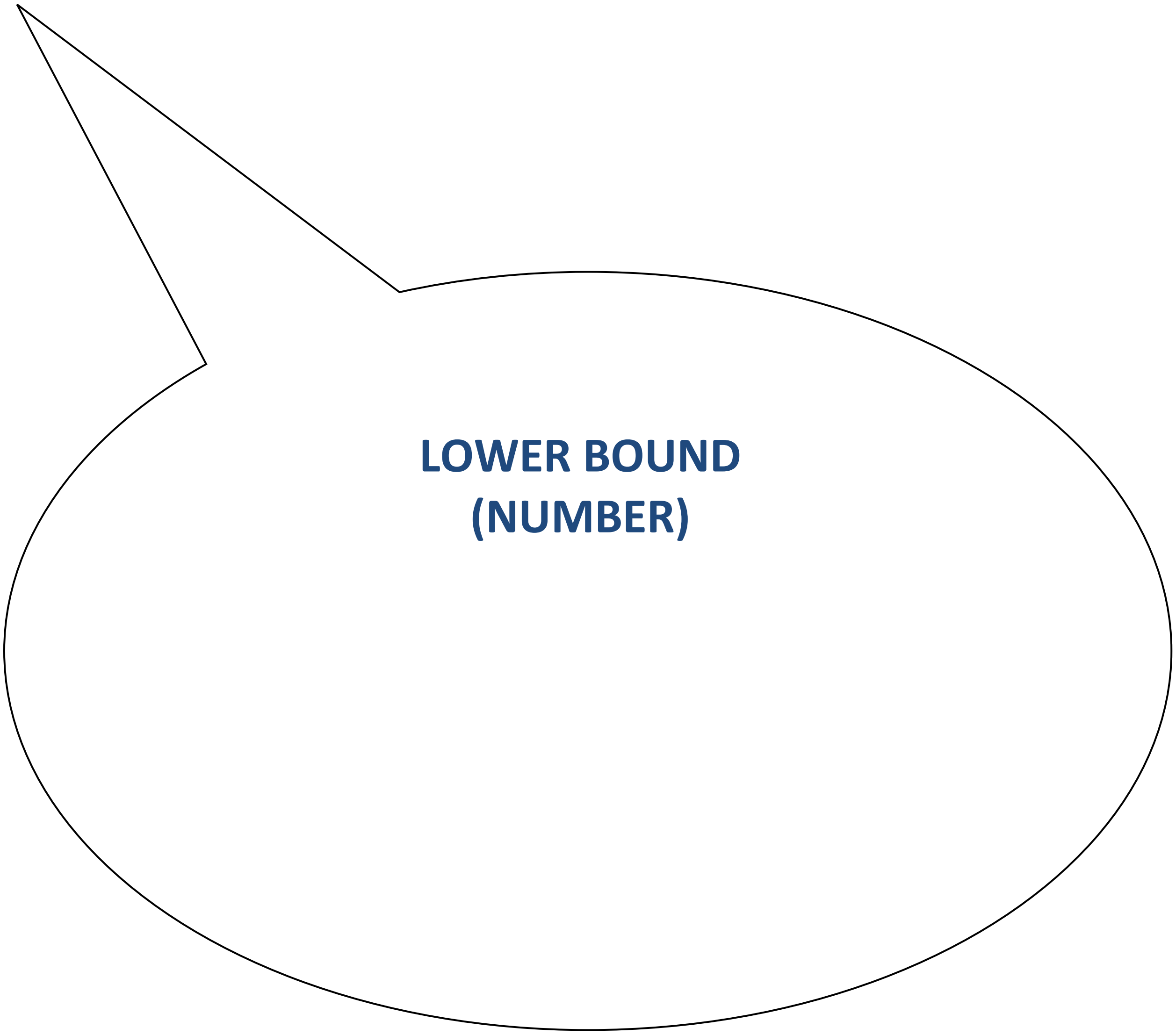
$\therefore$ r = $\frac{1}{4}$

$\therefore$ The G.P. is, $\frac{1}{16}$, $\frac{1}{64}$, ….$\frac{1}{4}$

Examples:

1. Find the sum of the first 8 terms of the G.P. 3, 2, $\frac{4}{3}$, $\frac{8}{9}$, ….
2. Find the sums to infinity of the following geometric series
   (a) 6 + 2 + $^{2}/_{3}$ +…,   (b) 1 - ½ + ¼ +…,

<u>SIGMA NOTATION</u>



Example 1 : Rewrite using sigma notation: 3 + 6 + 9 + 12

Solution : It is Arithmetic . And d= 3.

$\therefore$ $a_n = a_1 + (n-1)\, d$

$\therefore$ $a_n = 3 + (n-1)\, 3$

$a_n = 3n$

$\therefore 3+6+9+12 = \sum_{n=1}^{4} 3n$

Example 2 : Rewrite using sigma notation: 16 + 8 + 4 + 2 + 1

Solution : It is Geometric . and $r = \frac{1}{2}$

$\therefore$ $a_n = a_1\, (r)^{n-1}$

$a_n = 16 \left(\frac{1}{2}\right)^{n-1}$

$\therefore 16 + 8 + 4 + 2 + 1 = \sum_{n=1}^{5} 16\left(\frac{1}{2}\right)^{n-1}$