Computer Graphics 05101301

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CHAPTER-3

2D and 3D Transformation

Transformation

- Transformation means changing some graphics into something else by applying rules.
- It is used to reposition the graphics on the screen and change their size or orientation.
- The basic types of transformations are …

1) Translation,

2) Rotation,

3) Scaling,

4) Shearing,

5) Reflection

When a transformation takes place on a 2D plane, it is called 2D transformation.

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Transformation

To perform any type of transformation we need to follow a sequential process –

1. Translate the coordinates,
2. Rotate the translated coordinates, and then
3. Scale the rotated coordinates to complete the composite transformation.

Homogenous Coordinate System

A 3×3 transformation matrix can be used instead of 2×2 transformation matrix. To convert a 2×2 matrix to 3×3 matrix, we have to add an extra dummy coordinate W.

Any point can be represented by using 3 numbers instead of 2 numbers, which is called **Homogenous Coordinate** system.

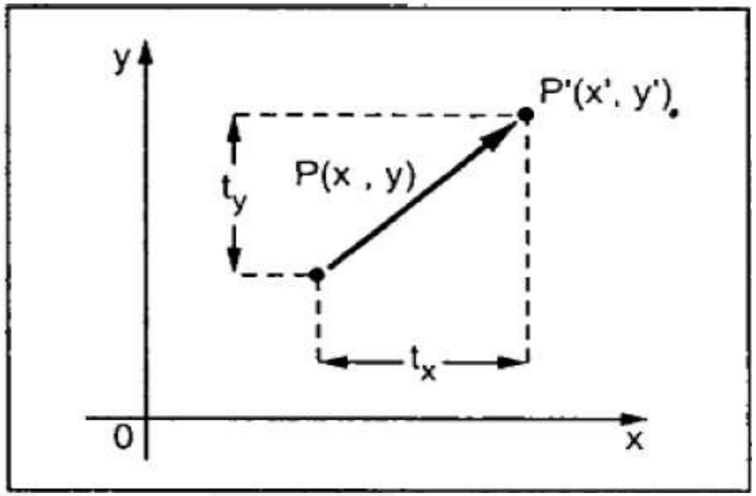
All the transformation equations can be represented in matrix multiplication. Any Cartesian point P(X, Y) can be converted to homogenous coordinates by P' (X_h, Y_h, h).

Translation

Translation refers to the shifting of a point to some other place, **whose distance with regard to the present point is known.**

A translation moves an object to a different position on the screen.

You can translate a point in 2D by adding translation coordinate (t_x, t_y) to the original coordinate (X, Y) to get the new coordinate (X', Y').



Translation

From the above figure, we can write

$$X' = X + t_x$$
$$Y' = Y + t_y$$

Where (t_x, t_y) is called the **translation vector or shift vector**.

The above equations can also be represented using the column vectors.

P = [X][Y]
P'= [X'][Y']
T = [tx][ty]

Which can be written as

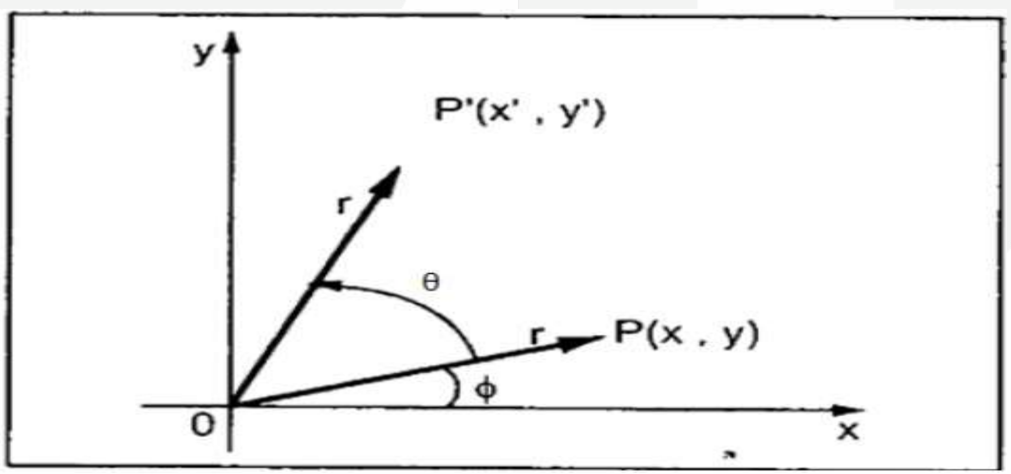
$$P' = P + T$$

Rotation

Rotation means to rotate a point about an axis.

In rotation, an object is rotated at a particular angle θ (theta) from its origin.

From the following figure, we can see that the point PX, Y is located at angle φ from the horizontal X coordinate with distance r from the origin



Rotation

Let us suppose you want to rotate it at the angle θ. After rotating it to a new location, you will get a new point P’ (X ’ , Y ’).

Using standard trigonometric the original coordinate of point P(X, Y) can be represented as –

$X=r\cos\varphi$......(1)

$Y=r\sin\varphi$......(2)

Same way we can represent the point P’ (X’, Y’) as –

$x'=r\cos(\varphi+\theta)=r\cos\varphi\cos\theta-r\sin\varphi\sin\theta$.......(3)

$y'=r\sin(\varphi+\theta)=r\cos\varphi\sin\theta+r\sin\varphi\cos\theta$.......(4)

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Rotation

Substituting equation (1) & (2) in (3) & (4) respectively, we will get

$x' = x\cos\theta - y\sin\theta$

$y' = x\sin\theta + y\cos\theta$

Representing the above equation in matrix form,

$[X'Y'] = [XY]\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ OR

$[X'Y'] = [XY]\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ OR $P' = P \cdot R$

Where R is the rotation matrix

$R = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

$R = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

Rotation

The rotation angle can be positive and negative.

For positive rotation angle, we can use the above rotation matrix. However, for negative angle rotation, the matrix will change as shown below –

R=[cos(−θ)−sin(−θ)   sin(−θ)cos(−θ)]

R=[cos(−θ)sin(−θ)−sin(−θ)cos(−θ)]

=[cosθsinθ−sinθcosθ]

(∵cos(−θ)=cosθandsin(−θ)=−sinθ)

Rotation

Example

Given a triangle with corner coordinates (0, 0), (1, 0) and (1, 1). Rotate the triangle by 90 degree anticlockwise direction and find out the new coordinates.

We rotate a polygon by rotating each vertex of it with the same rotation angle.

Given-

- Old corner coordinates of the triangle = A (0, 0), B(1, 0), C(1, 1)
- Rotation angle = $\theta = 90^\circ$

Rotation

For Coordinates A(0, 0)

Let the new coordinates of corner A after rotation = (x',y').

Applying the rotation equations, we have-

$X' = x*\cos\theta - Y*\sin\theta$

$= 0 * \cos 90^\circ - 0 * \sin 90^\circ$

$= 0$

$Y' = x*\sin\theta + y*\cos\theta$

$= 0*\sin 90^\circ + 0 * \cos 90^\circ$

$= 0$

Thus, New coordinates of corner A after rotation = (0, 0).

Rotation

For Coordinates B(1, 0)

Let the new coordinates of corner B after rotation = (x',y').

$X' = x*\cos\theta - y*\sin\theta = 1 \times \cos 90^\circ - 0 \times \sin 90^\circ = 0$

$Y' = x*\sin\theta + y*\cos\theta = 1 \times \sin 90^\circ + 0 \times \cos 90^\circ = 1 + 0 = 1$

Thus, New coordinates of corner B after rotation = (0, 1).

For Coordinates C(1, 1)

Let the new coordinates of corner C after rotation = (Xnew, Ynew).

$X' = x*\cos\theta - y*\sin\theta = 1 \times \cos 90^\circ - 1 \times \sin 90^\circ = 0 - 1 = -1$

$Y' = x*\sin\theta + y*\cos\theta = 1 \times \sin 90^\circ + 1 \times \cos 90^\circ = 1 + 0 = 1$

Thus, New coordinates of corner C after rotation = (-1, 1).

Thus, New coordinates of Triangle after Rotation = A(0,0), B(0,1), C(-1,1)

Scaling

- To change the size of an object, scaling transformation is used.
- **Scaling** is the concept of increasing (or decreasing) the size of a picture in any direction. (When it is done in both directions, the increase or decrease in both directions need not be same)
- Scaling can be achieved by **multiplying the original coordinates of the object** with the **scaling factor** to get the desired result.

$\mathbf{X' = X \cdot S_X}$ **and** $\mathbf{Y' = Y \cdot S_Y}$ Where, original coordinates are (X, Y)

Scaling factors are (S_X, S_Y), and the

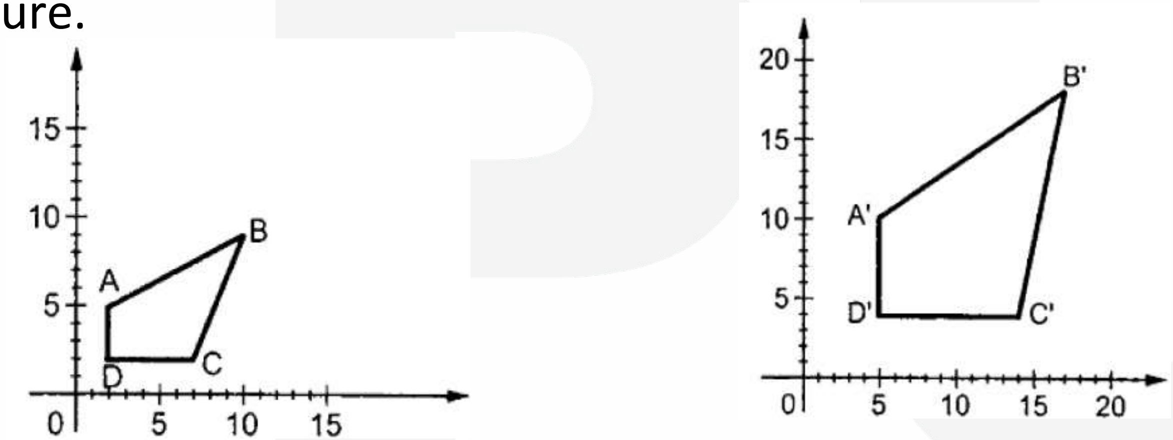
Produced coordinates are (X', Y').

Scaling

The above equations is represented in matrix form

OR **P’ = P . S**

Where S is the scaling matrix. The scaling process is shown in the following figure.



If S value is less than 1, then size of object reduces

If S greater than 1, then size of object increase

Scaling: Example

Given a square object with coordinate points A(0, 3), B(3, 3), C(3, 0), D(0, 0). Apply the scaling parameter 2 towards X axis and 3 towards Y axis and obtain the new coordinates of the object.

Old corner coordinates of the square = A (0, 3), B(3, 3), C(3, 0), D(0, 0)

Scaling factor along X axis = Sx=2

Scaling factor along Y axis = Sy=3

For Coordinates A(0, 3):x=0 and y=3

Let the new coordinates of corner A after scaling = (X_{new}, Y_{new}).

Applying the scaling equations, we have-

$X' = X*S_x = 0 \times 2 = 0$

$Y' = Y*S_y = 3 \times 3 = 9$

Thus, New coordinates of corner A after scaling = A'(0, 9).

Scaling: Example

For Coordinates B(3, 3) : B'(6,9)

Let the new coordinates of corner B after scaling = (X_{new}, Y_{new}).

Applying the scaling equations, we have-

$X_{new} = X_{old} \times S_x = 3 \times 2 = 6$

$Y_{new} = Y_{old} \times S_y = 3 \times 3 = 9$

Thus, New coordinates of corner B after scaling = (6, 9).

For Coordinates C(3, 0): C'(6,0)

- $X_{new} = X_{old} \times S_x = 3 \times 2 = 6$
- $Y_{new} = Y_{old} \times S_y = 0 \times 3 = 0$

Thus, New coordinates of corner C after scaling = (6, 0).

Scaling: Example

For Coordinates D(0, 0)

:D'(0,0)

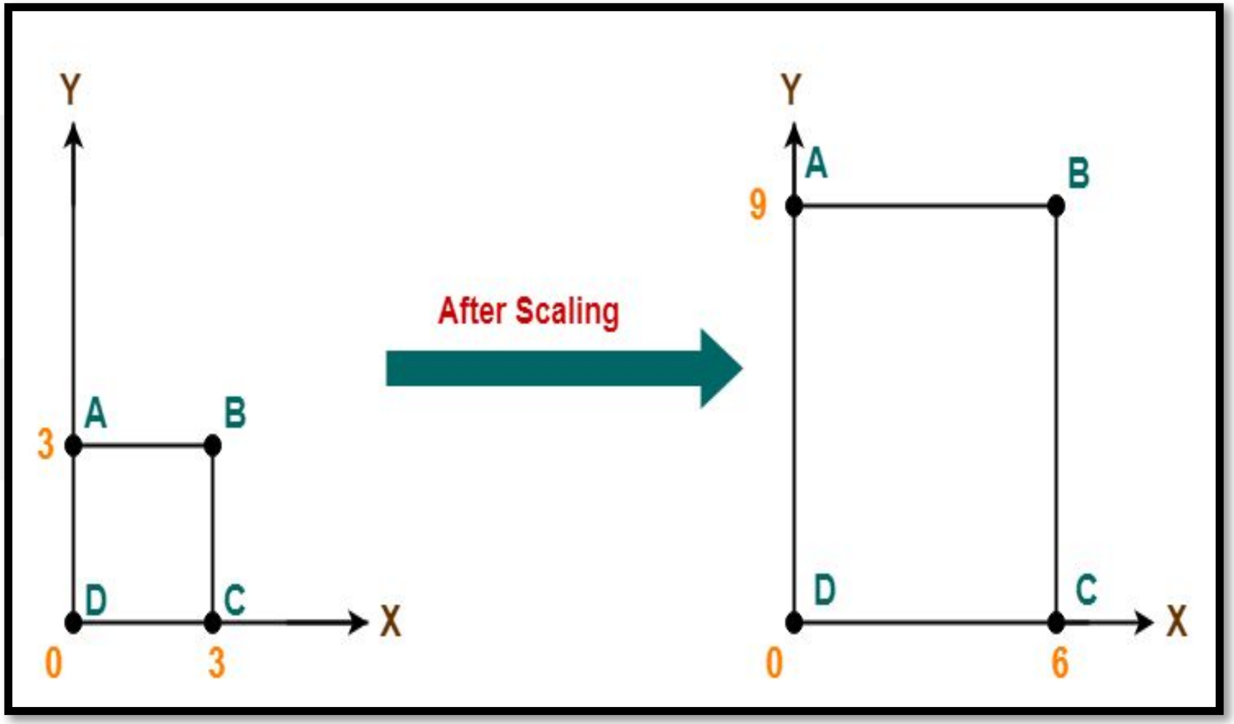
Let the new coordinates of corner D after scaling = (X_{new}, Y_{new}).

Applying the scaling equations, we have-

$X_{new} = X_{old} \times S_x = 0 \times 2 = 0$

$Y_{new} = Y_{old} \times S_y = 0 \times 3 = 0$

Thus, New coordinates of corner D after scaling = (0, 0).

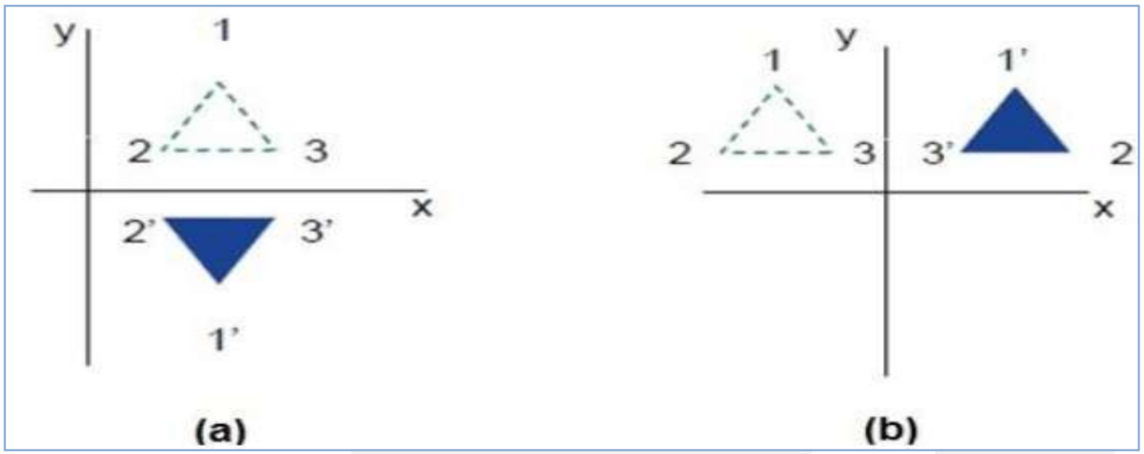


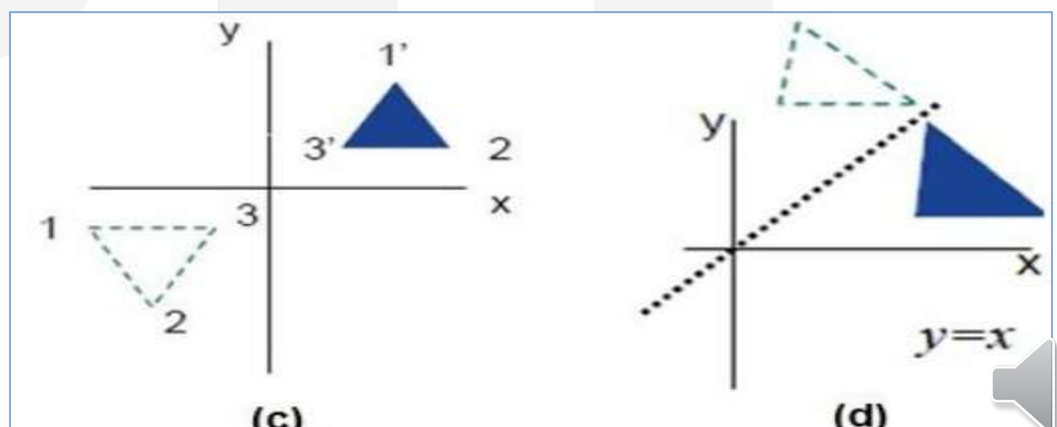
Reflection

- Reflection is the mirror image of original object. In other words, we can say that it is a rotation operation with 180°. In reflection transformation, the size of the object does not change.

- The following figures show reflections with respect to X and Y axes, and about the origin respectively.

Reflection





Reflection

Reflection On X-Axis:

This reflection is achieved by using the following reflection equations- (X’ and Y’ are the new coordinates)

X’ = X
Y’ = -Y

In Matrix form, the above reflection equations may be represented as-

$$\begin{bmatrix} X_{new} \\ Y_{new} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \end{bmatrix}$$

Reflection Matrix
(Reflection Along X Axis)

Reflection

Reflection On Y-Axis:

This reflection is achieved by using the following reflection equations- (X’ and Y’ are the new coordinates)

X’ = -X
Y’ = Y
In Matrix form, the above reflection equations may be represented as-

$$\begin{bmatrix} X_{new} \\ Y_{new} \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \end{bmatrix}$$

Reflection Matrix
(Reflection Along Y Axis)

Reflection: Example

Given a triangle with coordinate points A(3, 4), B(6, 4), C(5, 6). Apply the reflection on the X axis and obtain the new coordinates of the object.

Given-

Old corner coordinates of the triangle = A (3, 4), B(6, 4), C(5, 6)

Reflection has to be taken on the X axis

Reflection: Example

For Coordinates A(3, 4)

Let the new coordinates of corner A after reflection = (X_{new}, Y_{new}).

Applying the reflection equations, we have-

$X_{new} = X_{old} = 3$

$Y_{new} = -Y_{old} = -4$

Thus, New coordinates of corner A after reflection = (3, -4).

Reflection: Example

For Coordinates B(6, 4)

Let the new coordinates of corner B after reflection = (X_{new}, Y_{new}).

Applying the reflection equations, we have-

$X_{new} = X_{old} = 6$

$Y_{new} = -Y_{old} = -4$

Thus, New coordinates of corner B after reflection = (6, -4).

Reflection: Example

For Coordinates C(5, 6)

Let the new coordinates of corner C after reflection = (X_{new}, Y_{new}).

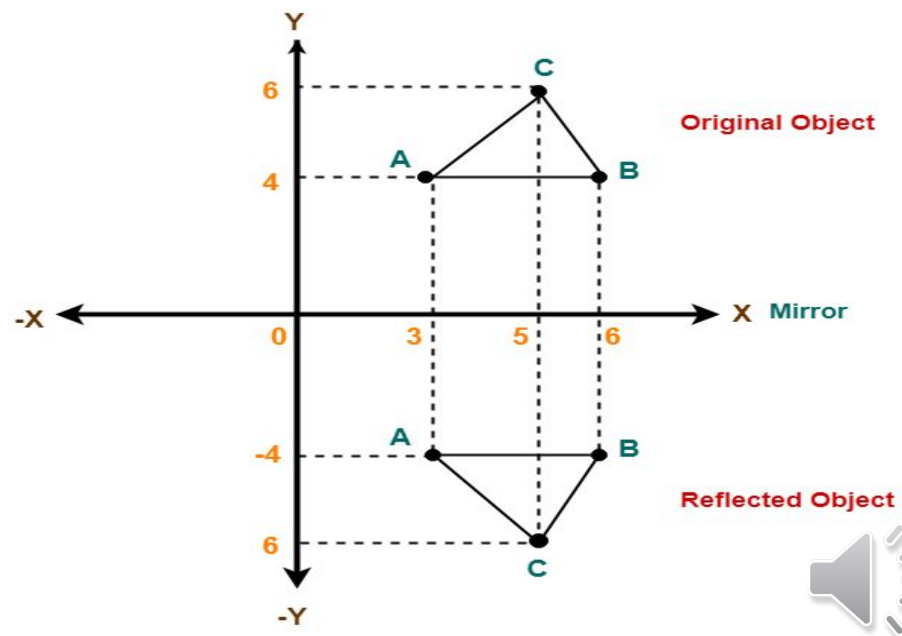
Applying the reflection equations, we have-

$X_{new} = X_{old} = 5$

$Y_{new} = -Y_{old} = -6$

Thus, New coordinates of corner C after reflection = (5, -6).

Thus, New coordinates of the triangle after reflection = A (3, -4), B(6, -4), C(5, -6).



Shear

A transformation that slants the shape of an object is called the shear transformation.

Types of shear transformations :

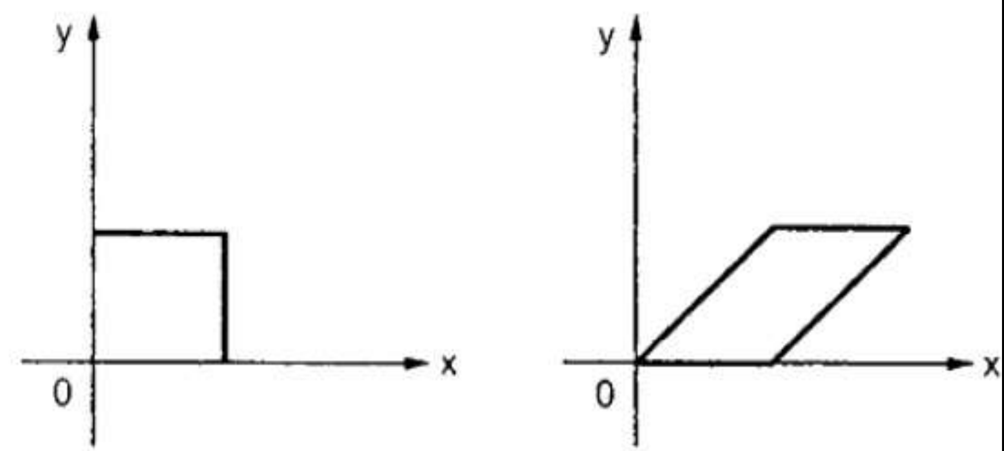
X-Shear and **Y-Shear**.

One shifts X coordinates values and other shifts Y coordinate values. However; in both the cases only one coordinate changes its coordinates and other preserves its values. Shearing is also termed as **Skewing**.

X-Shear

The X-Shear preserves the Y coordinate and changes are made to X coordinates, which causes the vertical lines to tilt right or left as shown in below figure.

The transformation matrix for X-Shear can be represented as –



(a) Original object (b) Object after x shear

Transformation Matrix:

$$XS=\begin{bmatrix} 1 & 0 & 0 \\ Sx & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

x1= x + Sx.y

y1=y

X-Shear

Shearing in X Axis-

$X_{new} = X_{old} + Sh_x \times Y_{old}$

$Y_{new} = Y_{old}$

The transformation matrix for X-Shear can be represented as –

$$\begin{bmatrix} X_{new} \\ Y_{new} \end{bmatrix} = \begin{bmatrix} 1 & Sh_x \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \end{bmatrix}$$

Shearing Matrix

(In X axis)

Y-Shear

The Y-Shear preserves the X coordinates and changes the Y coordinates which causes the horizontal lines to transform into lines which slopes up or down as shown in the following figure.

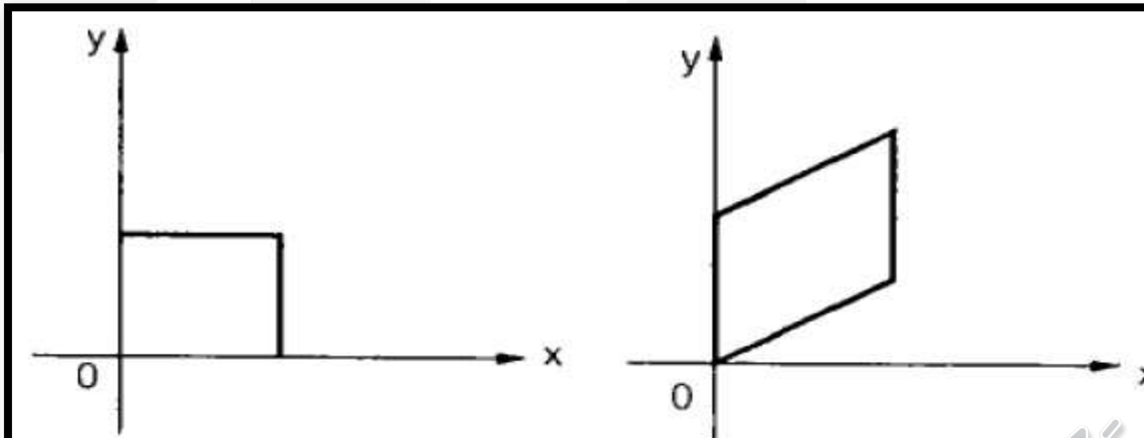
The Y-Shear can be represented in matrix from as:

Transformation Matrix:

$$XY = \begin{bmatrix} 1 & Sy & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$x1 = x$

$y1 = y + Sx$



(a) Original object (b) Object after y shear

Y-Shear

Shearing in Y axis is achieved by using the following shearing equations-

- $X_{new} = X_{old}$
- $Y_{new} = Y_{old} + Sh_y \times X_{old}$

The Y-Shear can be represented in matrix from as –

$$\begin{bmatrix} X_{new} \\ Y_{new} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ Sh_y & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \end{bmatrix}$$

Shearing Matrix

(In Y axis)

Shear - Example

Given a triangle with points (1, 1), (0, 0) and (1, 0). Apply shear parameter 2 on X axis and 2 on Y axis and find out the new coordinates of the object.

Given-

Old corner coordinates of the triangle = A (1, 1), B(0, 0), C(1, 0)

Shearing parameter towards X direction (Sh_x) = 2

Shearing parameter towards Y direction (Sh_y) = 2

Shear - Example

Shearing in X Axis- For Coordinates A(1, 1)

Let the new coordinates of corner A after shearing = (X_{new}, Y_{new}).

Applying the shearing equations, we have-

$X_{new} = X_{old} + Sh_x \times Y_{old} = 1 + 2 \times 1 = 3$

$Y_{new} = Y_{old} = 1$

Thus, New coordinates of corner A after shearing = (3, 1).

Shear - Example

For Coordinates B(0, 0)

Let the new coordinates of corner B after shearing = (X_{new}, Y_{new}).

Applying the shearing equations, we have-

$X_{new} = X_{old} + Sh_x \times Y_{old} = 0 + 2 \times 0 = 0$

$Y_{new} = Y_{old} = 0$

Thus, New coordinates of corner B after shearing = (0, 0).

For Coordinates C(1, 0)

Thus, New coordinates of corner C after shearing = (1, 0).

New coordinates of the triangle after shearing in X axis = A (3, 1), B(0, 0), C(1, 0).

Shear - Example

<u>Shearing in Y Axis-</u>

<u>For Coordinates A(1, 1)</u>

Let the new coordinates of corner A after shearing = (X_{new}, Y_{new}).

Applying the shearing equations, we have-

$X_{new} = X_{old} = 1$

$Y_{new} = Y_{old} + Sh_y \times X_{old} = 1 + 2 \times 1 = 3$

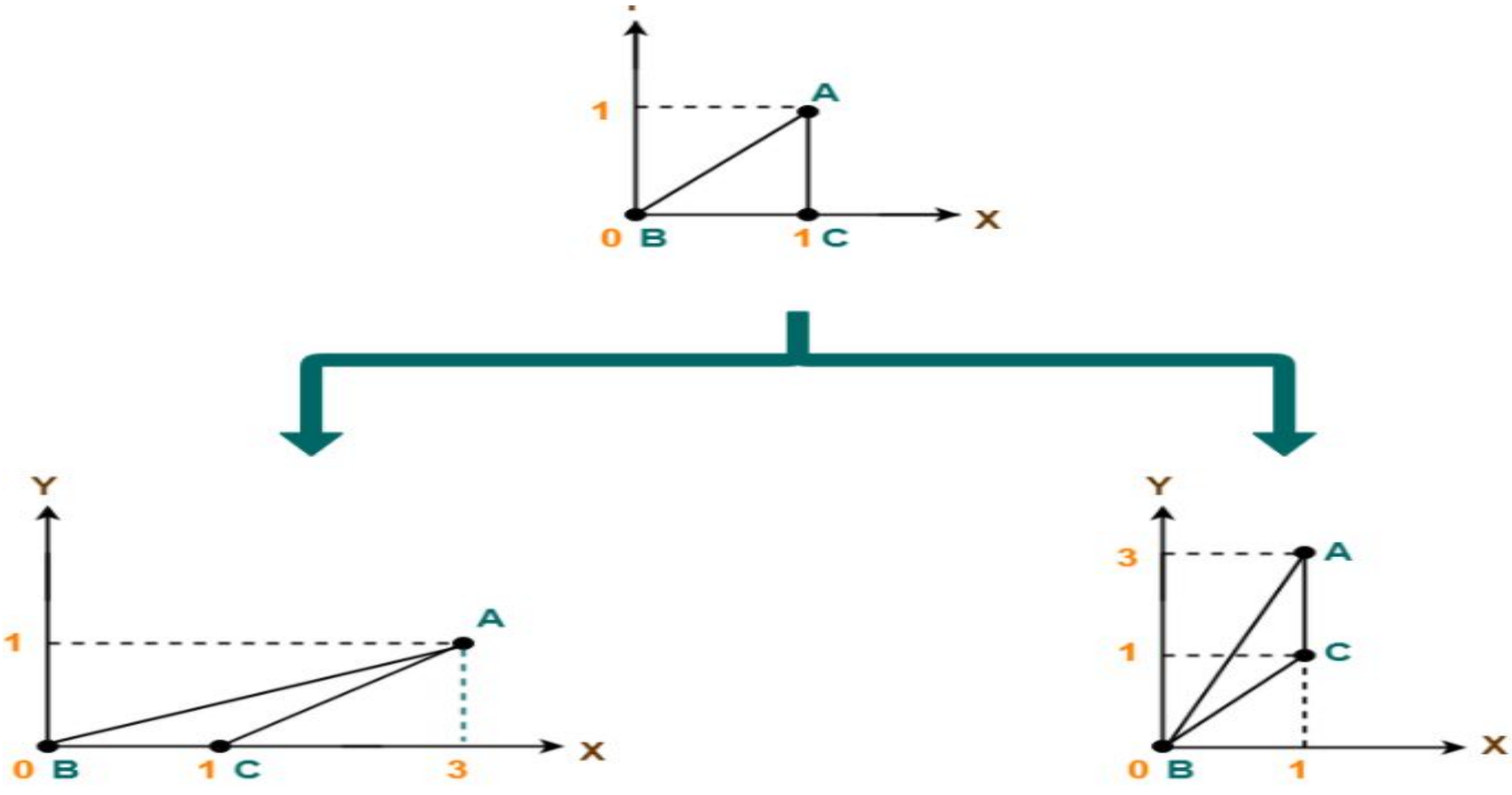
Thus, New coordinates of corner A after shearing = (1, 3).

<u>For Coordinates B(0, 0),</u> New coordinates of corner B after shearing = (0, 0).

<u>For Coordinates C(1, 0),</u> New coordinates of corner B after shearing = (1, 2).

New coordinates of the triangle after shearing in Y axis = A (1, 3), B(0, 0), C(1, 2).

Shear - Example



3D Transformations

The geometric transformations play a vital role in generating images of three Dimensional objects with the help of these transformations. The location of objects relative to others can be easily expressed. Sometimes viewpoint changes rapidly, or sometimes objects move in relation to each other. For this number of transformation can be carried out repeatedly.

For 2D transformation □ use 2 X 2 matrix (2 row n 2columns)

For 3D transformation □ use 3 X 3 matrix for linear algebraic equations/ use 4 X 4 matrix

3D Transformations - Translation

- It is the movement of an object from one position to another position.
- There are three vectors in 3D instead of two. These vectors are in x, y, and z directions. Translation in the x-direction is represented using T_x.
- The translation is y-direction is represented using T_y.
- The translation in the z- direction is represented using T_z.
- If P is a point having co-ordinates in three directions (x, y, z) is translated, then after translation its coordinates will be (x^1 y^1 z^1) after translation. T_x T_y T_z are translation vectors in x, y, and z directions respectively.

$$x^1=x+T_x$$
$$y^1=y+T_y$$
$$z^1=z+T_z$$

3D Transformations - Translation

Matrix for translation

$$\left\{\begin{matrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ T_x & T_y & T_z & 1 \end{matrix}\right\} \quad \text{or} \quad \left\{\begin{matrix} 1 & 0 & 0 & T_x \\ 0 & 1 & 0 & T_y \\ 0 & 0 & 1 & T_z \\ 0 & 0 & 0 & 1 \end{matrix}\right\}$$

Matrix representation of point translation

Point shown in fig is (x, y, z). It become (x^1, y^1, z^1) after translation. T_x T_y T_z are translation vector.

$$\begin{bmatrix} x^1 \\ y^1 \\ z^1 \\ 1 \end{bmatrix} = \begin{pmatrix} 1 & 0 & 0 & T_x \\ 0 & 1 & 0 & T_y \\ 0 & 0 & 1 & T_z \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

3D Transformations – Translation Example

Given a 3D object with coordinate points A(0, 3, 1), B(3, 3, 2), C(3, 0, 0), D(0, 0, 0). Apply the translation with the distance 1 towards X axis, 1 towards Y axis and 2 towards Z axis and obtain the new coordinates of the object.

Given-

Old coordinates of the object = A (0, 3, 1), B(3, 3, 2), C(3, 0, 0), D(0, 0, 0)

Translation vector = $(T_x, T_y, T_z) = (1, 1, 2)$

<u>For Coordinates A(0, 3, 1)</u>

Let the new coordinates of A = $(X_{new}, Y_{new}, Z_{new})$.

3D Transformations– Translation Example

Applying the translation equations, we have-

$X_{new} = X_{old} + T_x = 0 + 1 = 1$

$Y_{new} = Y_{old} + T_y = 3 + 1 = 4$

$Z_{new} = Z_{old} + T_z = 1 + 2 = 3$

Thus, New coordinates of A = (1, 4, 3).
Same way calculation will be done for coordinates of B,C and D

New coordinates of the object = A (1, 4, 3), B(4, 4, 4), C(4, 1, 2), D(1, 1, 2).

3D Transformations - Scaling

Scaling is used to change the size of an object. The size can be increased or decreased. The scaling three factors are required S_x S_y and S_z.

S_x=Scaling factor in x- direction

S_y=Scaling factor in y-direction

S_z=Scaling factor in z-directio

Matrix for Scaling

$$\left\{\begin{matrix} S_x & 0 & 0 & 0 \\ 0 & S_y & 0 & 0 \\ 0 & 0 & S_z & 0 \\ 0 & 0 & 0 & 1 \end{matrix}\right\}$$

This scaling is achieved by using the following scaling equations-

$X_{new} = X_{old} \times S_x$

$Y_{new} = Y_{old} \times S_y$

$Z_{new} = Z_{old} \times S_z$

3D Transformations - Scaling

$$\begin{bmatrix} X_{new} \\ Y_{new} \\ Z_{new} \\ 1 \end{bmatrix} = \begin{bmatrix} S_x & 0 & 0 & 0 \\ 0 & S_y & 0 & 0 \\ 0 & 0 & S_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \\ Z_{old} \\ 1 \end{bmatrix}$$

3D Scaling Matrix

Following are steps performed when scaling of objects with fixed point (a, b, c). It can be represented as below:

1. Scale the object relative to the origin
2. Translate object back to its original position.
3. Translate fixed point to the origin

3D Transformations - Rotation

Always Rotation is applied to corresponding axis, that axis new coordinate value will

Be same as the old coordinate value

Rotation is applied across X axis : X'=X , Y? and Z?

Rotation is applied across Y axis :Y'=Y , X? and Z?

Rotation is applied across Z axis :Z'=Z, X? and Y?

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3D Transformations

For X-Axis Rotation-

This rotation is achieved by using the following rotation equations-

X’ = X

Y’ = Ycosθ – Zsinθ

Z’ = Ysinθ + Zcosθ

In Matrix form, the above rotation equations may be represented as-

3D-Rotation Matrix applied across X axis

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} X_{new} \\ Y_{new} \\ Z_{new} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \\ Z_{old} \\ 1 \end{bmatrix}$$

3D Rotation Matrix

(For X-Axis Rotation)

O’ = R * O

3D Transformations

<u>For Y-Axis Rotation-</u>

This rotation is achieved by using the following rotation equations-

$X' = Z\sin\theta + X\cos\theta$

$Y' = Y$

$Z' = Y\cos\theta - X\sin\theta$

In Matrix form, the above rotation equations is

$$\begin{bmatrix} X_{new} \\ Y_{new} \\ Z_{new} \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \\ Z_{old} \\ 1 \end{bmatrix}$$

3D Rotation Matrix

(For Y-Axis Rotation)

3D - Rotation Transformation matrix applied across Y axis

$$\begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3D Transformations

For Z-Axis Rotation-

This rotation is achieved by using the following rotation equations-

X’ = Xcosθ – YSinθ

Y’ = Xsinθ + Ycosθ

Z’ = Z

In Matrix form, the above rotation equations may be represented as-

$$\begin{bmatrix} X_{new} \\ Y_{new} \\ Z_{new} \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \\ Z_{old} \\ 1 \end{bmatrix}$$

3D Rotation Matrix

(For Z-Axis Rotation)

3D Rotation Transformation Z axis

$$\begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3D Transformations - Shearing

- **Shearing in X direction**
- **Shearing in Y direction**
- **Shearing in Z direction**

Consider a point object O has to be sheared in a 3D plane.

Let-

Initial coordinates of the object O = (X, Y, Z)

Shearing parameter towards X direction = Sh_x

Shearing parameter towards Y direction = Sh_y

Shearing parameter towards Z direction = Sh_z

New coordinates of the object O after shearing = (X', Y', Z')

3D Transformations - Shearing

Shearing in X Axis-

Shearing in X axis is achieved by using the following shearing equations-

$X' = X$

$Y' = Y + Sh_y * X$

$Z' = Z + Sh_z * X$

In Matrix form, the above shearing equations may be represented as-

$$\begin{bmatrix} X_{new} \\ Y_{new} \\ Z_{new} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ Sh_y & 1 & 0 & 0 \\ Sh_z & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \\ Z_{old} \\ 1 \end{bmatrix}$$

3D Shearing Matrix
(In X axis)

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3D Transformations - Shearing

Shearing in Y Axis-

Shearing in X axis is achieved by using the following shearing equations-

$X' = X + Sh_x * Y$

$Y' = Y$

$Z' = Z + Sh_z * Y$

In Matrix form, the above shearing equations may be represented as-

$$\begin{bmatrix} X_{new} \\ Y_{new} \\ Z_{new} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & Sh_x & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & Sh_z & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \\ Z_{old} \\ 1 \end{bmatrix}$$

3D Shearing Matrix

(In Y axis)

3D Transformations - Shearing

Shearing in Z Axis-

Shearing in X axis is achieved by using the following shearing equations-

$X' = X + Sh_x \times Z$

$Y' = Y + Sh_y * Z$

$Z' = Z$

In Matrix form, the above shearing equations may be represented as-

$$\begin{bmatrix} X_{new} \\ Y_{new} \\ Z_{new} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & Sh_x & 0 \\ 0 & 1 & Sh_y & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \\ Z_{old} \\ 1 \end{bmatrix}$$

3D Shearing Matrix
(In Z axis)

3D Transformations - Reflection

- Reflection is a kind of rotation where the angle of rotation is 180 degree.
- The reflected object is always formed on the other side of mirror.
- The size of reflected object is same as the size of original object.

Consider a point object O has to be reflected in a 3D plane.

Let-

- Initial coordinates of the object O = (X, Y, Z)
- New coordinates of the reflected object O after reflection = (X’, Y’,Z’)

3D Transformations - Reflection

In 3 dimensions, there are 3 possible types of reflection-

Reflection relative to XY plane : X'=X and Y'=Y and Z'=-Z

Reflection relative to YZ plane : Y'=y and Z'=Z and X'=-X

Reflection relative to XZ plane : X'=X and Y'=-Y and Z'=Z

<u>Reflection Relative to XY Plane:</u>

This reflection is achieved by using the following reflection equations-

X' = X

Y' = Y

Z'= -Z

$$\begin{bmatrix} X_{new} \\ Y_{new} \\ Z_{new} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \\ Z_{old} \\ 1 \end{bmatrix}$$

3D Reflection Matrix

(Reflection Relative to XY plane)

3D Transformations - Reflection

Reflection Relative to YZ Plane:

This reflection is achieved by using the following reflection equations-

X' = -X

Y' = Y

Z' = Z

In Matrix form, the above reflection equations may be represented as-

$$\begin{bmatrix} X_{new} \\ Y_{new} \\ Z_{new} \\ 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \\ Z_{old} \\ 1 \end{bmatrix}$$

3D Reflection Matrix

(Reflection Relative to YZ plane)

3D Transformations - Reflection

Reflection Relative to XZ Plane:

This reflection is achieved by using the following reflection equations-

$X' = X$

$Y' = -Y$

$Z' = Z$

In Matrix form, the above reflection equations may be represented as-

O' = R * O

$$\begin{bmatrix} X_{new} \\ Y_{new} \\ Z_{new} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \\ Z_{old} \\ 1 \end{bmatrix}$$

3D Reflection Matrix

(Reflection Relative to XZ plane)

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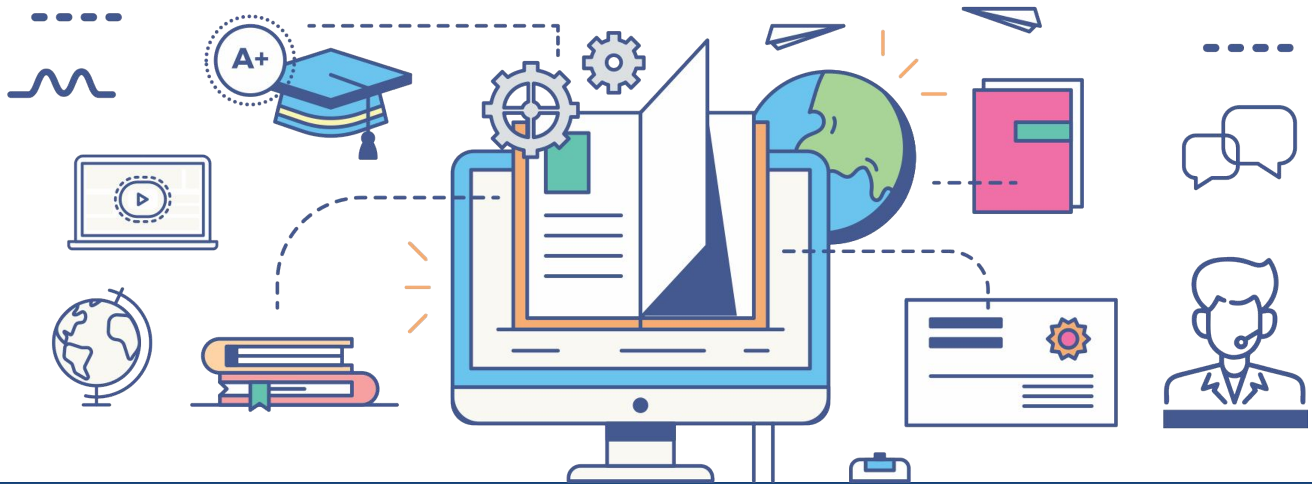

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