	PARUL UNIVERSITY **FACULTY OF ENGINEERING AND TECHNOLOGY** **DEPARTMENT OF APPLIED SCIENCE AND HUMANITIES** **SUBJECT : BASIC MATHEMATICS (05191101)** **Unit -1 – Set theory**

- **INTRODUCTION**

Set theory is a branch of mathematical logic that studies sets, which informally are collections of objects. Although any type of object can be collected into a set, set theory is applied most often to objects that are relevant to mathematics.It is natural for us to classify items into groups, or sets, and consider how those sets overlap with each other. We can use these sets understand relationships between groups, and to analyze survey data Sets are used to define the concepts of relations and function.The study of geometry , sequences , probability ,etc requires the knowledge of sets.

Studying sets helps us categorize information. Also it allows us to make sense of a large amount of information by breaking it down into smaller groups.Sets are of much wider applications ,especially they help in preparing the program for feeding into the machine.

- **DEFINITION OF SETS**

Definition : A collection of well-defined objects is called a set.

The objects in a set are called the elements or members of the set S.

We use the bracket notation { } to refer to a set. A set is determined by its distinct elements, or members.

Notation : Usually we denote sets with upper-case letters and elements with lower-case letters. For example : A={a,b,c,d}.

The following notation is used to show set membership :

means that is a member of the set .

means that is not a member of the set .

For ex: A={a,b,c,d} then, b A but f A.because f is not an element of A.

Well defined Set : Well-defined means, it must be absolutely clear that which object belongs to the set and which does not.

Some common examples of well defined sets:

- The collection of vowels in English alphabets. This set contains five elements, namely { a, e, i, o, u }.
- N = {1,2,3,…} is the set of counting numbers, or naturals.
- Z = {…,–3,–2,–1,0,1,2,3,…} is the set of integers.

Which of the following are well-defined sets?

- Even integer from 0 to 10
- Collection of books
- Days of the week
- Good Reasurants of the city
- All the letters of PASS

1.3 REPRESENTATION OF A SET

Elements or members are listed in a set.

(i) Tabular or Roster method : Under this method we list all the elements of the sets within braces.

For e.g. (a) A= { a,e,i,o,u}

(b) N= { 1,3,5}

(ii) Selector Or Set Builder or Rule method : Under this method the elements are not listed but are indicated by description of their characteristics.

Here we choose the letter x to represent an arbitrary elements of the set and write ;
A={x | x is a prime number}

The vertical line "|" after x to be read as 'such that' .sometime we use ":" to denote 'such that'.

Exercise : **Write the following sets in the set builder form.**

- A={5,10,15,20,25,…..}

  A = {x : x is the multiple of 5}

- B={0,1,2,3,4,5,……}

  B = {x : x belongs to whole number}

- C={3,4,5,6,7}

  C = {x : x is an integer and 2<x<8 }

Write the following sets in the roster form.

- A = {x : x ∈ N, x ≤ 7}

  A={1,2,3,4,5,6,7}

- B = {x : x ∈ Z, -2 < x < 4}

  B={-1,0,1,2,3}

1.4 TYPES OF SETS

Sets may be of various types:

(i) Finite set : When elements of set can be counted then set is called Finite set.

For e.g. A={1,2,3,4,5,6,7}

B= { x | x is an even positive integer < 100}

(ii)Infinite set : When elements of set can not be counted then set is called Infinite set.

For e.g. A= {1,2,3,………… ∞}

B= { x| x is an odd integer}

(iii) Singleton set :A set containing only one element is called singleton or unit set.

For e.g. A={a}

(iv)Empty set or Null set or Void set : The set that contains no element is called the empty set or null set. The empty set is denoted by ø or by { }.

For e.g. A={x| x is a perfect square of an integer; 26<x<35}.

(v) Equal sets : Two sets A, B are equal iff they have the same elements.

or Two sets are equal if each elements of A is belongs to B and each elements of B is belongs to A .

For e.g.(a) A={1,2,3} ,B={3,1,2}

(b) A={3,5,5,9} , B={9,5,3}

(vi) Equivalent sets : Total no. of elements of one set is equal to total no. of elements of another set is called equivalent sets .

For e.g. A={a,b,c,d} and B= {1,2,3,4}.

(vii) Subsets: A set A is a subset of the set B, denoted by A B if every element of A is also an element of B.

We say "A is a subset of B" if i.e., all the members of A are also members of B.

For e.g. If A= {2,3,4,5} and B= {2,3,4,5,6,7} ,then we can say that A B .But B A.(B is not a subset of A).

(Viii) Proper subsets: Set A is a proper subset of B, denoted by A B. if every element of A is an element of B, But there is an atleast one element of B that is not an element of A.

For e.g. A={1,2,3} is a proper subset of B={1,2,3,4} ,i.e. A B because the element 4 is not in set A.

(ix) Power set : If A be a given set then the family of sets each of whose number is subset of the given set A is called the power set of set A and is denoted as P(S).or A set containing subsets is called power set.

P(S) = { A | A S }.

For e.g. : 1) A={1}

Then, P(S)={ ø,{1}}

2) If A={1,2}

Then, P(S)= { ø,{1},{2},{1,2}}

3) If A= {1,2,3}

Then ,P(S)={ø ,{1},{2},{3},{1,2},{1,3},{2,3},{1,2,3}}

(X)Universal set : A particular well-defined set is called Universal set and is denoted by U. For e.g. (i) set of integers.

SOLVED EXAMPLES

Example 1 : A={1,2,3} and B={1,2,3}.Then which of the following is False? Justify your answer.

(a) $A = B$
(b) $A \subseteq B$
(c) $A \subset B$

Solution : (c) is false.

because, $A \subset B$ if every element of A is an element of B, But there is atleast one element in B that is not contained in A.

Example 2 : If A={1,2,3,5,9,12},B={1,2,2,3,5,9,12,12} and C={1,2,2,3,3,5,9} then express the relation between the sets.

Solution : $C \subset A$ as every element of C is an element of A , But there is an atleast one element of A i.e. 12 is not an element of C.

$A \subseteq B$ as all the members of A are also members of B .

$B \subseteq A$ as all the members of B are also members of A .

A=B as they have same members.

Example 3 : Fill in the blank with $\subseteq$ or $\nsubseteq$ to make true statement .

a) {a, b, c} ___ { a, c, d}
b) {1, 2, 3, 4} ___ {1, 2, 3, 4}
Solution: a) {a, b, c} ⊈ { a, c, d} because b ∉ {a,c,d}
b) {1, 2, 3, 4} ⊆ {1, 2, 3, 4} because all members of set A is also a members of set B.

Example 4 : Which of the following are the examples of an empty set?

- {x | x ∈N and x < 0}
- {x|x Q, 3<x<4}
- A={0}

Example 5 : Which Are the following pairs of sets equal?

- A={3}   and B={x|x is a natural number and 2<x<5}
- A={r,s,t}   and   B={s,r,t}
- A={2,4,6,8,……}   and B={x|x is even number}
- A={ 2,3,4,5}   and B={x|x is natural number and 1<x<6}
- A={1, 4, 9,16}   B = {x | x = n^2, n ∈ N, n ≤ 5}

1.4.1 NUMBER OF SUBSETS :

- The number of subsets of a set with *n* elements is 2^n.
- The number of proper subsets of a set with *n* elements is $2^n – 1$.

For Example: Find the number of subsets and the number of proper subsets of the set {m, a, t, h, y}.

Solution: Since there are 5 elements, the number of subsets is $2^5 = 32$.and the number of proper subsets is 32 – 1 = 31.

1.4.2 SIZE OF A SET

Definition: The size of a set S, denoted by **|S|** and is defined as the number of elements contained in S.

For e.g. if S = {2, 3, 5, 7, 11, 13, 17, 19}, then |S|=8.

if S = {CSC1130, CSC2110, ERG2020, MAT2510}, then |S|=4.

if S = {{1,2}, {1,3}, {1,4}, {2,3}, {2,4}, {3,4}}, then |S|=6.

In this course we mostly focus on finite sets.

1.5 VENN DIAGRAM

Venn diagram : The Venn diagrams are named after English logician John Venn(1834-1923).Venn diagram is pictorial representation of sets. In these diagrams, the universal set U is represented by a rectangle and other sets like A,B,C are interest within the universal set are represented as circular regions.Moreover, The circles or closed curves intersect each other if there are any comon elements among them , if there is no common elements then they are shown separately as dijoints.These are useful to illustrate the set relations.

For e.g. In below diagram ,the rectangle represents the universal set *U*, while the portion bounded by the circle represents set *A*.

Example

- If U= { set of Positive integers } ={1,2 3,4,…..}
  A={ set of even integers } ={ 2,4,6,8,….}
  B= { set of odd integers} = {1, 3, 5,7,…..}.Then draw a Venn diagram of sets A and B.

Solution : There is no common elements in set A and B then they are shown separately as disjoints.

1.6 OPERATIONS ON SETS

1.6.1 Union : The union of sets A and B is the set of those elements that are either in A or in B, or in both. Union of set A and B is denoted by A ∪ B.

i.e. .

SOLVED EXAMPLES

Example 1: Suppose S = {1,2,3}, T = {1,3,5}, and U = {2,3,4,5}. Then find S ∪T and T ∪ U.

Solution: S ∪T = {1,2,3} ∪ {1,3,5}

= {1,2,3,5}

T ∪U = {1,3,5} ∪ {2,3,4,5}

$= \{1,2,3,4,5\}$

Example 2: $A = \{1, 3, 6, 8, 10\}$ and $B = \{2, 4, 6, 7, 10\}$ then find $A \cup B$.
Solution: $A \cup B = \{1,3,6,8,10\} \cup \{2,4,6,7,10\}$
$= \{1, 2, 3, 4, 6, 7, 8, 10\}$

Exercise : **Find union of the following pairs of sets:**

- $A=\{2,3,5,8,9\}$ and $B=\{2,4,6,8,10\}$
- A={ letters in the word 'STATE'} and B={ letters in the word 'GATE'}
- $C=\{x|x$ is multiple of 3 and it is less than 10 $\}$ and $D =\{3,4,5,9\}$

<u>PROPERTIES OF UNION</u>

- $A \ (A \cup B)$ and $B \ (A \cup B)$
- $A \cup ø = A$
- $A \cup A = A$
- Commutative laws $A \cup B = B \cup A$
- Associative laws: $A \cup (B \cup C) = (A \cup B) \cup C$

Example 1 : If $A= \{1,2,3,4\}$, $B= \{2,4,5,6\}$, $C= \{3,4,6,8\}$.

Then, show that $A \cup (B \cup C) = (A \cup B) \cup C$.
Solution : $A \cup B = \{1,2,3,4,5,6\}$
Then, $(A \cup B) \cup C= \{1,2,3,4,5,6\} \cup \{3,4,6,8\}$
$= \{1,2,3,4,5,6,8\}$ _____ (1)

and $B \cup C = \{2,3,4,5,6,8\}$
Then, $A \cup (B \cup C) = \{1,2,3,4\} \cup \{2,3,4,5,6,8\}$
$=\{1,2,3,4,5,6,8\}$ _____ (2)

From (1) and (2). We get; $A \cup (B \cup C) = (A \cup B) \cup C$.

Example 2 : If $A= \{a,b,c,d\}$ and $B= \{d, e ,f\}$ then show that (i) $A \cup B = B \cup A$ and (ii) $A \cup ø = A$
Solution: (i) Given, $A= \{a,b,c,d\}$ and $B= \{d, e ,f\}$
then, $A \cup B = \{a,b,c,d\} \cup \{d, e ,f\}$
$= \{a,b,c,d,e,f\}$

and $B \cup A = \{ d, e ,f\} \cup \{a,b,c,d\}$
$=\{ a,b,c,d,e,f \}$.
so, $A \cup B = B \cup A$

(ii)$A \cup ø = \{a,b,c,d\} \cup ø$

$= \{a,b,c,d\} = A$

1.6.2 Intersection: The intersection of the sets A and B is the set of all elements that are in both A and B. Intersection of A and B is denoted by $A \cap B$.
i.e. .

SOLVED EXAMPLES

Example 1 : Suppose $S = \{1,2,3,5\}$, $T = \{1,3,4,5\}$, and $U =\{2,3,4,5\}$. Then find $S \cap T$, $T \cap U$ and $S \cap U$.
Solution : $S \cap T=\{1,2,3,5\} \cap \{1,3,4,5\}$
$= \{1,3,5\}$
$T \cap U=\{1,3,4,5\} \cap \{2,3,4,5\}$
$= \{3,4,5\}$
$S \cap U=\{1,2,3,5\} \cap \{\{2,3,4,5\}$
$= \{2,3,5\}$

Example 2 : $A = \{1, 3, 6, 8, 10\}$ and $B = \{2, 4, 6, 7, 10\}$ then find $A \cap B$.
Solution :$A \cap B=\{1,3,6,8,10\} \cap \{2,4,6,7,10\}$
$= \{6,10\}$

Exercise : **Find intersection of the following pairs of sets:**
- A={ letters in the word 'STATE'} and B={ letters in the word 'GATE'}
- $C = \{x|x$ is an odd number $\}$ and $D =\{x|x$ is an even number$\}$
- $D= \{ 2,4,7,8,9\}$ and $E=\{3,4,7,10\}$

PROPERTIES OF INTERSECTION

- $(A \cap B) \subseteq A$ and $(A \cap B) \subseteq B$
- $A \cap ø = ø$
- $A \cap A = A$
- Commutative laws $A \cap B = B \cap A$

- Associative laws: $A \cap (B \cap C) = (A \cap B) \cap C$

Example 1 : Let A= { 1,2,3,4} , B= {2,4,5,6} and C= {3,4,6,8,} .
then show that $A \cap (B \cap C) = (A \cap B) \cap C$.

Solution : Here, A= { 1,2,3,4} , B= {2,4,5,6} and C= {3,4,6,8,} .
then, $B \cap C$ = {2,4,5,6} ∩ {3,4,6,8,}
= { 4,6 }
and $A \cap (B \cap C)$ = { 1,2,3,4} ∩ { 4,6 }
= {4} _____(1)

Given, , A= { 1,2,3,4} , B= {2,4,5,6} and C= {3,4,6,8,}
then, $(A \cap B)$ = { 1,2,3,4} ∩ {2,4,5,6}
={2,4}
and $(A \cap B) \cap C$ = {2,4} ∩ {3,4,6,8,}
= {4} ______(2)

From (1) and (2) we get ;$A \cap (B \cap C) = (A \cap B) \cap C$.

Example 2 : Let A= { 3,4,7} , B= { 4,6,8,9} then show that (i) $A \cap B = B \cap A$ and (ii) $A \cap ø = ø$

Solution: (i) Here, A= { 3,4,7} and B= { 4,6,8,9}
then, $A \cap B$ = { 3,4,7} ∩ { 4,6,8,9}
= {4}
and, $B \cap A$ = { 4,6,8,9} ∩ {3,4,7}
= {4}
so, $A \cap B = B \cap A$

(ii) $A \cap ø$= {3,4,7} ∩ ø = ø

<u>UNION AND INTERSECTION USING VENN DIAGRAM</u>

In venn diagram the circles intersects each other if there areany common elements among them ,if there are no commonelements then they are shown seperately as disjoints.

Example 1: Let, U ={0,1,2,3,…..9} ,A={4,7,9} and B={ 1,2,3,4,5}. Then label $A \cup B$ and A∩Bin Venn diagram.

Solution :$A \cup B$ ={4,7,9} ∪ { 1,2,3,4,5}
= { 1,2,3,4,5,7,9}

and $A \cap B$ = {4,7,9} ∩ { 1,2,3,4,5}
= {4}

Example 2: Let A={1,2,3} and B={3,4,5}.Draw and Label a Venn diagram of sets A &BSolution : A ∪ B = {1,2,3,4,5}
and A ∩ B = {3}

Using ,Venn diagram

1.6.3 Complement : The complement of a set is the set of all those elements which do not belong to that set. It is denoted by A'or A or A^c .

i.e. A = A^c=

SOLVED EXAMPLES

Example 1 : Let U={1,2,3,4,5,6} ,A={1,2,3} and B={3,4,5,6} .Then find B'.
Solution: Given U={1,2,3,4,5,6} and B={3,4,5,6} .
So, B'={1,2}.

Example 2 : Let the universal set be the letters {a,e,i,o,u} and A={i,u}.Then find A'.
Solution: Given U={a,e,i,o,u} and A={i,u} .
So, A'={a,e,o}

PROPERTIES OF COMPLEMENT

- A∪ A' = U
- A ∩ A' = ø
- U'= ø
- ø' = U

- (A')'= A

Example : Let U= {1,2,3,4,5,6} ,A= {4,6} and B = { 1,2,3,4} .Then find A ∩ A' and (A')'.

Solution : A={4,6} and U= {1,2,3,4,5,6}

A'={ 1,2,3,5}

A ∩ A'= ø

and A' = {1,2,3,5}

(A')'={4,6}

= A

- **De morgan's law**

Complement of Union is equals to the Intersection of the complements .
i.e. (A∪ B)' = A' ∩ B'
and
Complement of Intersection is equals to the Union of the complements .
i.e. (A ∩B)' = A' ∪ B'.

Example : Let U = {1,2,3,4,5,6} ,A={4,6} and B={1,2,3,4},Then Verify De Morgan's law.
Solution: Using , De Morgan's law;(AB)'=A' B'

A B = {4,6} {1,2,3,4}

= {1,2,3,4,6}

(A B)' = {5}

Also,A'={1,2,3,5} and B'={5,6}
A' B'={1,2,3,5}{5,6}

= {5}

(A B)'= A' B'

And A B={4}

(A B)'={1,2,3,5,6}

Also,A'={1,2,3,5} and B'={5,6}

A' B'={1,2,3,5,6}

(A B)' = A' B'

1.6.4 Set Difference : The set difference of two sets Aand B is the set of all elements which belongs to A and not to B is denoted by .

i.e. and

Note that, $A-B \neq B-A$

SOLVED EXAMPLES

Example 1 : Let U={1,2,3,4,5,6} ,A={1,2,3} ,B={3,4,5,6}.Then =____
Solution: = the set of elements that are in B, and not in A.
i.e. ={4,5,6}

Example 2: Prove that for A={a,b,c,d,e,f,g,h} and B={a,e,i,o,u}
Solution: Here,A={a,b,c,d,e,f,g,h}
And B={a,e,i,o,u}
{b,c,d,f,g,h}
And {i,o,u}

- **De Morgan's law on Difference of sets**
  Let A,B,C any sets then, and
  .

  Example :A={1,2,3,4,5,6,7,8,9},B={3,5,7} and C={2,4,6}.Then prove that .
  Solution: = {3,5,7} {2,4,6}
  = { 2,3,4,5,6,7}
  = {1,2,3,4,5,6,7,8,9} {2,3,4,5,6,7}
  = {1,8,9}
  and
  ={1,2,4,6,8,9} {1,3,5,7,8,9}
  = {1,8,9}

- **Algebra of a Sets**

  Identity laws: $A \cup \varnothing = A$

  AU=A

  Domination laws: $A \cup U = U$

  A ø = ø

  Idempotent laws : $A \cup A = A$

  AA=A

  Commutative laws $A \cup B = B \cup A$

  AB = BA

Associative laws: $A\cup(B\cup C) = (A\cup B)\cup C$

$A(BC) = (AB)C$

Distributive laws: $A\cup(B\ C)=(A \cup B)\ \ (A \cup C)$

$A\ (B \cup C)=(A\ \ B) \cup (A\ \ C)$

Absorption laws: $A\cup(AB) = A$

$A\ (A\cup B) = A$

1.6.5 Symmetric Difference: A difference set iscalled symmetric difference of two sets if it contains all those elements which are in set A and not in set B or those which are in set B and not in set A.For example , the symmetric difference of two sets A and B will be denoted by .

Example :A={1,2,3,4,5} and B={6,7,8}.then,find .
Solution : Given,A={1,2,3,4,5,6} and B={6,7,8}
={1,2,3,4,5}
And = {7,8}

Now,={1,2,3,4,5} {7,8}
= {1,2,3,4,5,7,8}

={1,2,3,4,5,7,8}

<u>NUMBER OF ELEMENTS IN FINITE SET</u>

Let n(A) be a number of elements in set A.and n(B) be a number of elements in set B.Then, $n(AB)=n(A)+n(B)\ \ n(AB)$.
Similarly,$n(ABC)=n(A)+n(B)+ n(C)\ \ n(AB)\ n(BC)\ n(AC)+n(ABC)$.

Note : n(A) represents the number of elements in set A or size of A and it is Also known as **Cardinal number** of set .
For example : if A= {2,4,7,9} then cardinal number (or n(A)) of set A is 4,because there are 4 elements in set A.

Example : 1.If A={0}

Then , Cardinality of set A is 1. because 0 is an element of the set .and also it is not an empty set.

- If B={ }

Then Cardinality of set B is 0 because the set B is empty set.

<u>SOLVED EXAMPLES</u>

Example 1 : Let A and B are two finite sets such that n(A)=20,n(B)=28 and n(A B)=36,find n(AB).
Solution : Using,the formula,n(AB)=n(A)+n(B) n(AB)
36=20+28 n(AB)
n(AB)=12

Example 2 : In a group of 60 people,27 like cold drink and 42 like hot drinks also each person likes at least oneof the twodrinks.How many likes both drinks?
Solution: Let, A=Set of people who like cold drinks
B=Set of people who like hot drinks
n(AB)=60,n(A)=27 and n(B)=42
Using,n(AB)= n(A)+n(B) n(AB)
60= 27+42 n(AB)
n(AB) = 9
9 people likes both drinks.

Example 3: In a group of 100 person, 72 people can speak English and 43 can speak French. How many can speak English only ? How many can speak French only. How many can speak both language ?
Solution : Let A= Set of person can speak English
B = Set of person can speak French
A-B= Set of people can speak English only.

B-A = Set of people can speak French only.

and $A \cap B$ = Set of people who speak both English and French.

Given, n(A) = 72 ,n(B)= 43, $n(A \cup B)$ =100

Now, $n(A \cap B) = n(A) + n(B) - n(A \cup B)$

$= 72 + 43 - 100$

$n(A \cap B) = 15$

Then, $n(B - A) = n(B) - n(A \cap B)$

= 43 – 15

= 28

= people speaking French only

And, n(A– B) = n(A) – n(A ∩ B)

= 72 – 15

= 57

= people speaking English only

Exercise : Find the cardinal number of the following sets:

- {6,7,4}
- {x|x is the letters in word 'FREE'}
- {2,2,4,5,6,7}

<u>ORDERED PAIR</u>

Ordered pair of two objects consist of two elements a & b written in parentheses (a,b) such that one of them ,say a is first member and b is second member.
For e.g. The Natural number and their squares can be represented by ordered pair in following manner :

(1,1),(2,4),(3,9),(4,16),.......

Two ordered pair (a,b) & (c,d) will be equal if and only if a=c & b=d. i.e.

If(a,b) = (c,d) then a=c and b=d.

The points in the plane can be represented by ordered pair like (x,y).

1.7 <u>CARTESIAN PRODUCT</u>

If A & B be any two sets then the Set of all ordered pairs whose first member belongs to Set A and second member belongs to set B is called Cartesian product of A and B. and it is denoted by A B.
i.e.

Ordered pairs means the ordering is important.For e.g. (1,2) ≠ (2,1)

For example: If A = {1,2} and B = {x,y,z}
Then AB = {(1,x), (1,y), (1,z), (2,x), (2,y), (2,z)}
And BA = {(x,1), (x,2), (y,1), (y,2), (z,1), (z,2)}

Example 1: If A={1,3},B={2,4,6} Then AB=____
Solution: IAI =n(A)=2 and |B|=3
Then, |A B|=2*3=6

So, the cartesian product AxB contains 6 order pairs.
AB={(1,2),(1,4),(1,6),(3,2),(3,4),(3,6)}.

PROPERTIES OF CARTESIAN PRODUCT

- If A B and B A have same number of elements but A B $\neq$ B A unless A=B
- If Set A consist m elements & set B consist n elements , then AxB consist mn elements.

Example : If A= {1,4} , B= {2,3} , C= {3,5} . Then prove that A B $\neq$ B A. Also find (AB) ∩ (A C).
Solution: A B={(1,2),(1,3),(4,2),(4,3)}
B A = {(2,1),(2,4),(3,1),(3,4)}
A B $\neq$ B A

and A C= {(1,3),(1,5),(4,3),(4,5)}
(A B) ∩ (A C) ={(1,2),(1,3),(4,2),(4,3)} ∩{(1,3),(1,5),(4,3),(4,5)}
= { (1,3),(4,3)}