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CHAPTER - 1

VECTOR

Vector Algebra

- A **vector** has direction and magnitude both but Scalar has only magnitude.
- Magnitude of a vector a is denoted by |a| or a. It is non- negative scalar.

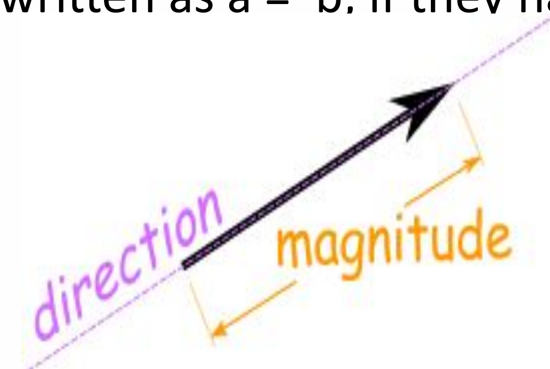
- **Equality of Vectors:**

Two vectors a and b are said to be equal written as a = b, if they have

(i) same length

(ii) the same or parallel support and

(iii) The same sense



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Types of Vectors

- **Zero or Null Vector:-** A vector whose initial aand terminal points are coincident is called zero or null vector. It is denoted by o.
- **Unit Vector:-** A vector whose magnitude is unity is called a unit vector which is denoted by nˆ
- **Free Vectors:-** If the initial point of a vector is not specified then it is said to be a free vector.
- **Negative of a Vector :-**A vector having the same magnitude as that of a given vector a and the direction opposite to that of a is called the negative of a and it is denoted by —a.
- **Like and Unlike Vectors :-**Vectors are said to be likewhen they have the same direction and unlike when they have opposite direction.

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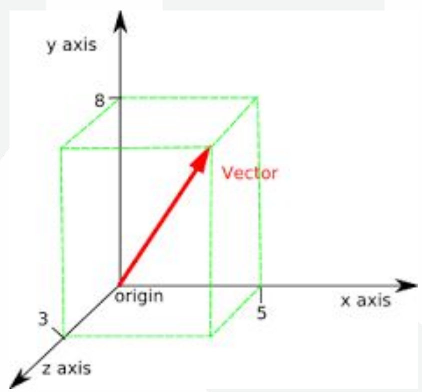
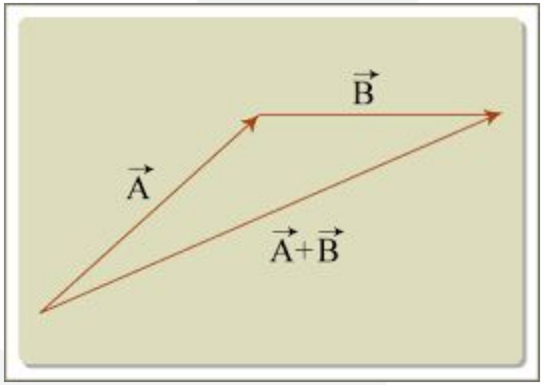
- Reciprocal of a Vector:- A vector having the same direction as that of a given vector but magnitude equal to the reciprocal of the given vector is known as the reciprocal of a. i.e., if |a| = a, then |a-1| = 1 / a.





- **Addition of Vectors**
  - Let a and b be any two vectors. From the terminal point of a, vector b is drawn. Then, the vector from the initial point O of a to the terminal point B of b is called the sum of vectors a and b and is denoted by a + b. This is called the traingle law of addition of vectors.

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Properties of Vector: Addition and Subtraction

- a + b = b + a (commutativity)
- a + (b + c)= (a + b)+ c (associativity)
- a+ O = a (additive identity)
- a + (— a) = o (additive inverse)
- (k1 + k2) a = k1 a + k2a (multiplication by scalars)
- k(a + b) = k a + k b (multiplication by scalars)
- $|a+b| \le |a| + |b|$ and $|a - b| \ge |a| - |b|$
- **Difference** (Subtraction) **of Vectors**
  - If a and b be any two vectors, then their difference a – b is defined as a + (- b).

Multiplication of a Vector by a Scalar

- Let a be a given vector and λ be a scalar. Then, the product of the vector a by the scalar λ is λ a and is called the multiplication of vector by the scalar.

- **Important Properties**
- $|\lambda a| = |\lambda|\ |a|$
- λ O(zero) = O(zero)
- $m(-a) = -ma = -(m a)$
- $(-m)(-a) = m a$
- $m(n a) = mn a = n(m a)$
- $(m + n)a = m a + n a$
- $m(a+b) = m a + m b$

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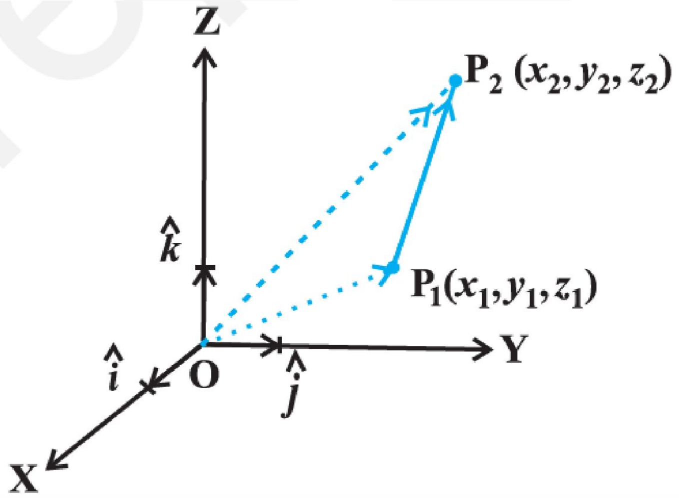
- **Vector Equation of Joining by Two Points :** Let P1 (x1, y1, z1) and P2 (x2, y2, z2) are any two points, be the vector joining P1 and P2 is the vector P1 P2. i.e., P1 P2 = (x2i + y2j + z2k) – (x1i + y1j + z1k) = (x2 – x1) i + (y2 – y1) j + (z2 – z1) k Its magnitude is P1 P2 = √(x2 – x1)2 + (y2 – y1)2 + (z2 – z1)2



Position vector

Consider a point P in space having co ordinates (X,Y,Z) with respect to origin O(0,0,0), then the Vector $\overrightarrow{OP}$ having O and P its initial and terminal points, respectively is called the position vector of point P with respect to O , Using distance formula magnitude of $\overrightarrow{OP}$ is given by

$$|\overrightarrow{OP}| = \sqrt{x^2 + y^2 + z^2}$$

Position vector of A, B , C with respect to origin is given by $\vec{a}$, $\vec{b}$, $\vec{c}$

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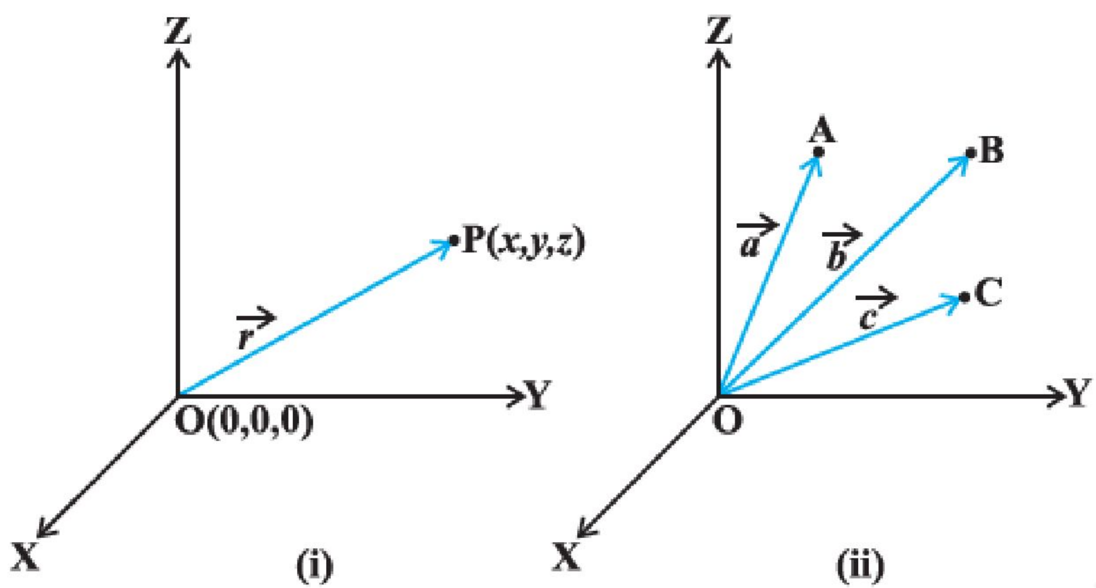
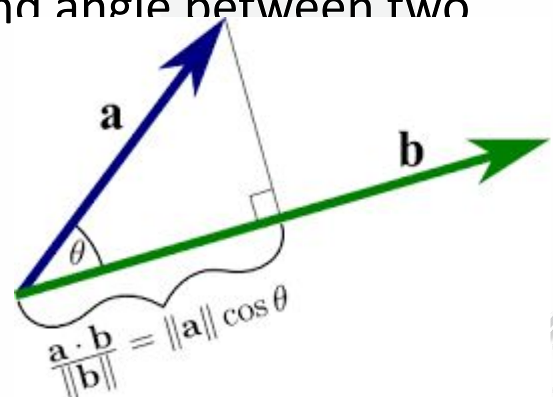
Graphical representation of position vector



Scalar Product of Two Vectors

- If a and b are two non-zero vectors, then the scalar or dot product of a and b is denoted by a * b and is defined as a * b = |a| |b| cos θ, where θ is the angle between the two vectors and 0 < θ < π
- The angle between two vectors a and b is defined as the smaller angle θ between them, when they are drawn with the same initial point. Usually, we take 0 < θ < π. Angle between two like vectors is O and angle between two unlike vectors is π .



- If either a or b is the null vector, then scalar product of the vector is zero.
- If a and b are two unit vectors, then a * b = cos θ.
- The scalar product is commutative i.e., a * b= b * a

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- If i , j and k are mutually perpendicular unit vectors i , j and k, then i * i = j * j = k * k =1 and i * j = j * k = k * i = 0
- (vi) The scalar product of vectors is distributive over vector addition.
- (a) a * (b + c) = a * b + a * c (left distributive)
- (b) (b + c) * a = b * a + c * a (right distributive)
- Note Length of a vector as a scalar product
- If a be any vector, then the scalar product a * a = |a| |a| cos θ
- Condition of perpendicularity a * b = 0 <=> a ⊥b, a and b being non-zero vectors.

Important Points to be Remembered

- $(a + b) * (a – b) = |a|^2 – |b|^2$
- $|a + b|^2 = |a|^2 + |b|^2 + 2 (a * b)$
- $|a – b|^2 = |a|^2 + |b|^2 – 2 (a * b)$
- $|a + b|^2 + |a – b|^2 = (|a|^2 + |b|^2)$ and $|a + b|^2 –|a\text{-}b|= 4 (a * b)$ or $a * b = 1 / 4 [|a + b|^2 – |a – b|^2]$
- If $|a + b| = |a| + |b|$, then a is parallel to b.
- If $|a + b| = |a| – |b|$, then a is parallel to b.
- $(a * b)^2 \leq |a|^2 |b|^2$
- If a = a1i + a2j + a3k, then $|a|^2 = a * a$

Angle between Two Vectors

- If θ is angle between two non-zero vectors, a, b, then We have

a * b = |a| |b| cos θ

- cos θ = a * b / |a| |b|
- If a = a1i + a2j + a3k and b = b1i + b2j + b3k Then, the angle θ between a and b is given by

cos θ = a * b / |a| |b|

Projection and Component of a Vector

- Projection of a on b = a * b / |a|
- Projection of b on a = a * b / |a|
- Vector component of a vector a on b Similarly the vector component of b on a = ((a * b) / $|a|^2$) * a

Work done by Force

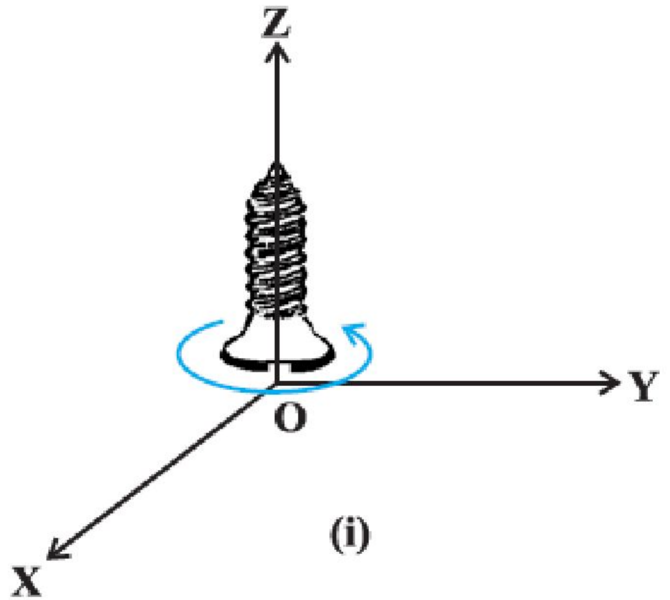
- The work done by a force is a scalar quantity equal to the product of the magnitude of the force and the resolved part of the displacement. ∴ $F * S$ = dot products of force and displacement.
- Suppose F1, F1,..., Fn are n forces acted ona particle,then during the displacement S of the particle, the separate forces do quantities of work $F1 * S$, $F2 * S$, $Fn * S$. Here, system of forces were replaced by its resultant R.

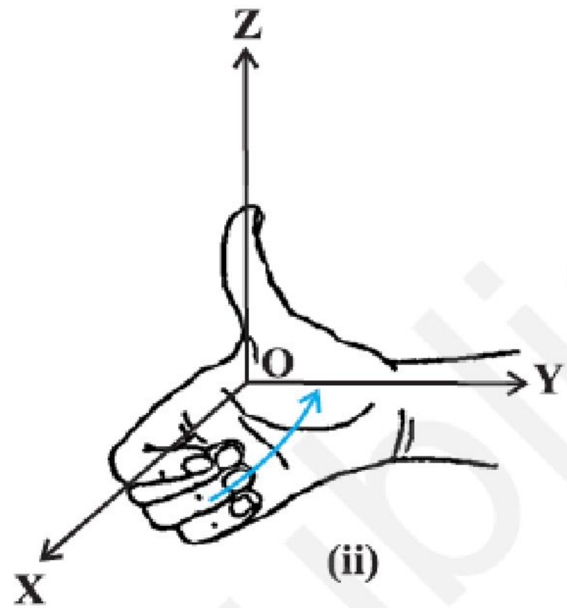
Vector or Cross Product of TwoVectors

- The vector product of the vectors a and b is denoted by $a \times b$ and it is defined as $a \times b = (|a|\ |b| \sin \theta)\, n = ab \sin \theta\, n$ where, $a = |a|$, $b = |b|$, θ is the angle between the vectors a and b and n is a unit vector which is perpendicular to both a and b, such that a, b and n form a right-handed triad of vectors.

Vector (Cross) product of two vector

In a right handed co ordinate system , the thumb of right hand points in the direction of the positive z axis when the figures are curled in the direction away from the positive x axis towards the positive y axis



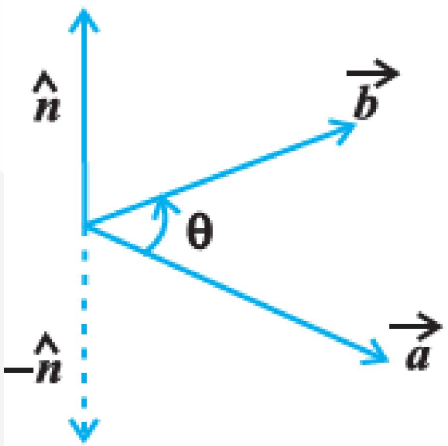


Vector (Cross) product of two vector

The vector product of two non zero vector $\vec{a}$ and $\vec{b}$ is denoted by

$$a \times b = (|a|\ |b|\ \sin\theta)\hat{n}$$

Where θ is angle between $\vec{a}$ and $\vec{b}$ and $\hat{n}$ is unit vector perpendicular to both $\vec{a}$ and $\vec{b}$



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Important Points to be Remembered

Let $a = a1i + a2j + a3k$ and $b = b1i + b2j + b3k$

- If $a = b$ or if a is parallel to b, then $\sin\theta = 0$ and so $a \times b = 0$
- The direction of $a \times b$ is regarded positive, if the rotation from a to b appears to be anticlockwise.
- a b is perpendicular to the plane, which contains both a and b. Thus, the unit vector
- perpendicular to both a and b or to the plane containing is given by

$n = a \times b / |a \times b| = a \times b / ab \sin\theta$

- Vector product of two parallel or collinear vectors is zero
- If $a \times b = 0$, then $a = 0$ or $b = 0$ or a and b are parallel collinear

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Vector Product of Two Perpendicular Vectors

If θ = 900, then sin θ = 1, i.e. , a ×b = (ab)n or |a × b| = |ab n| = ab

Vector Product of Two Unit Vectors

If a and b are unit vectors, then a = |a| = 1, b = |b| = 1 ∴ a × b = ab sin θ n = (sin theta;).n

Vector Product is not Commutative

The two vector products a * b and b * a are equal in magnitude but opposite in direction i.e., b × a =- a × b

The vector product of a vector a with itself is null vector, i. e., a * a= 0.

Distributive Law

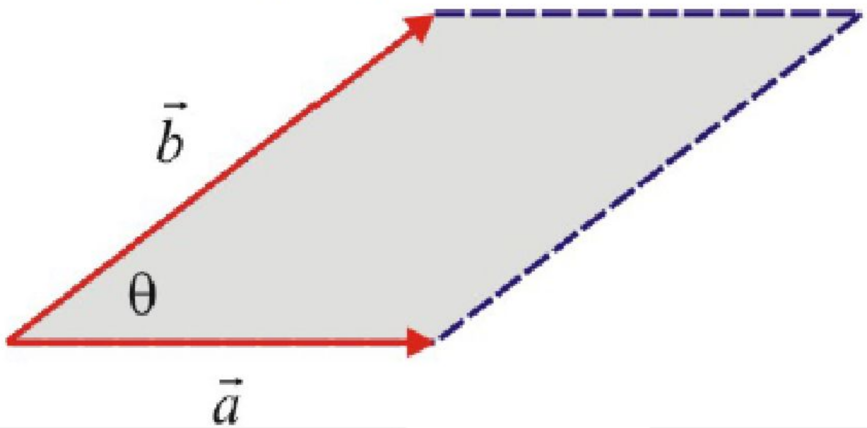
For any three vectors a, b, c a × (b + c) = (a × b) + (a × c)

Application of dot and cross product of vectors

The *area* of a *parallelogram* defined by the vectors $\vec{a}$ and $\vec{b}$ is determined by the formula:

$$A = \|\vec{a} \times \vec{b}\| = \|\vec{a}\| \, \|\vec{b}\| \sin\alpha$$

$$\alpha = \angle(\vec{a}, \vec{b})$$

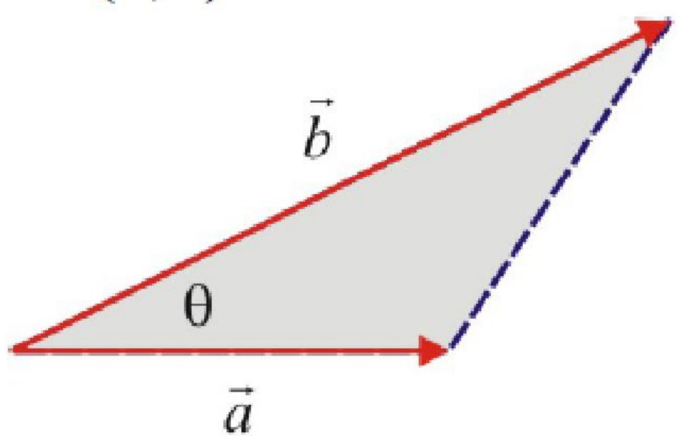


Application of dot and cross product of vectors

The *area* of a *triangle* defined by the vectors $\vec{a}$ and $\vec{b}$ is given by:

$$A = \frac{1}{2} \| \vec{a} \times \vec{b} \| = \frac{1}{2} \| \vec{a} \| \| \vec{b} \| \sin\alpha$$

$$\alpha = \angle(\vec{a}, \vec{b})$$



Vector Moment of a Force about a Point

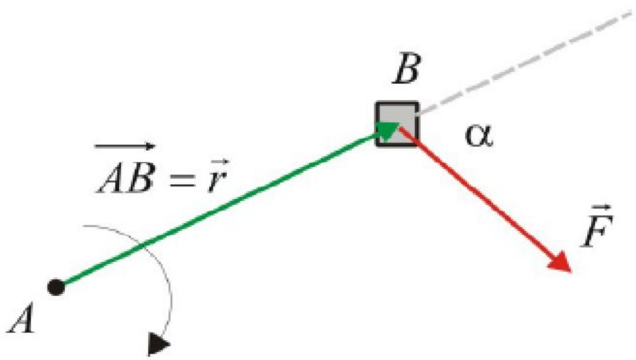
- The vector moment of torque M of a force F about the point O is the vector whose magnitude is equal to the product of |F| and the perpendicular distance of the point O from the line of action of F.
  ∴ M = r * F where, r is the position vector of A referred to O.
- If several forces are acting through the same point A, then the vector sum of the moments of the separate forces about a point O is equal to the moment of their resultant force about O.

Application of dot and cross product of vectors

The torque (rotational or turning effect) about the point A, created by a force $\vec{F}$ acting on an object located at the point B is given by:

$$\vec{\tau} = \overrightarrow{AB} \times \vec{F} = \vec{r} \times \vec{F}$$

$$\| \vec{\tau} \| = r\, F \sin \alpha$$



where:

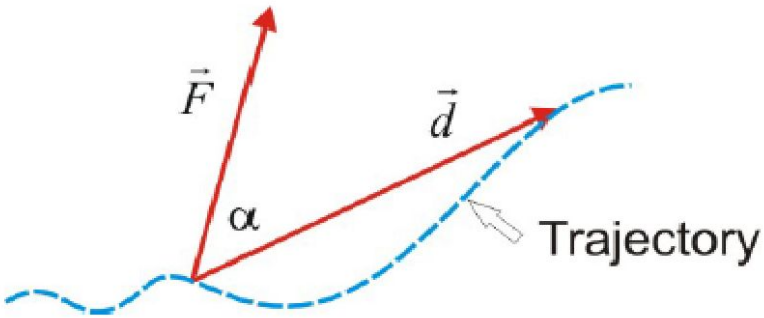
$$\alpha = \angle(\vec{F}, \vec{r})$$

$$\vec{r} = \overrightarrow{AB}\ (meter)$$

Application of dot and cross product of vectors

The work W done by a *constant force* $\vec{F}$ acting on an object during a *displacement* $\vec{d}$ is given by:

$$W = \vec{F} \cdot \vec{d} = F\, d \cos\alpha$$



where

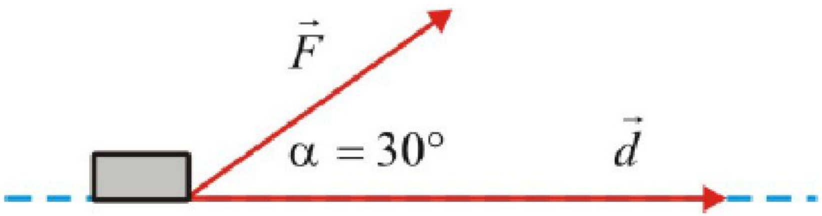
$$\alpha = \angle(\vec{F}, \vec{d})$$

$$[\vec{F}]_{SI} = N\ (Newton) \quad [\vec{d}]_{SI} = m\ (meter)$$

$$[W]_{SI} = J\ (Joule)$$

Examples

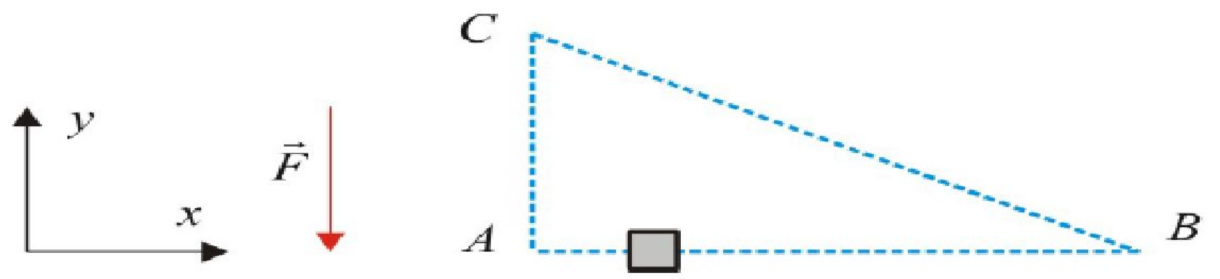
Ex 1. A box is pulled a horizontal distance of $100m$ by a force of $500N$ applied at an angle of $30°$ to the horizontal line. Calculate the amount of work done.



$$W = F\ d\ \cos\alpha = (500N)(100m)\cos 30° = 25000\sqrt{3}J \cong 43.3kJ$$

Examples

Ex 2. An object with a weight of $50N$ is moved, in vertical plane, along to the path $ABCA$ as presented in the next figure where $AB = 6m$ and $AC = 3m$.



Find the work done by the force of gravity:

a) from A to B

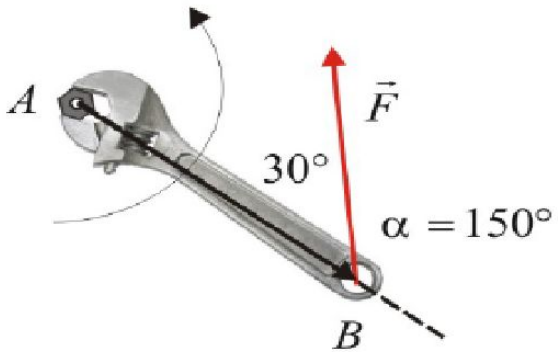
$\vec{F} = (0,-50)N$

$\vec{d} = \overrightarrow{AB} = (6,0)m$

$W = \vec{F} \cdot \vec{d} = (0)(6) + (-50)(0) = 0J$

Examples

Ex 3. A wrench $30cm$ long is used to loose a bolt by applying a force of $20N$ (see the figure below). Find the magnitude of the torque.



$$r = AB = 30cm = 0.3m$$

$$F = 20N$$

$$\alpha = \angle(\vec{F}, \overrightarrow{AB}) = 180° - 30° = 150°$$

$$\|\vec{\tau}\| = r\,F\sin\alpha = (0.3)(20)\sin 150° = 3Nm$$

$$\therefore \|\vec{\tau}\| = 3Nm$$

Examples

Ex 4. Find the area of the parallelogram defined by the vectors $\vec{a}=(1,-1,0)$ and $\vec{b}=(0,1,2)$.

$$\begin{array}{lccccc} & \vec{i} & \vec{j} & \vec{k} & \vec{i} & \vec{j} \\ \vec{a}\times\vec{b}= & 1 & -1 & 0 & 1 & -1 = \vec{i}(-2-0)+\vec{j}(0-2)+\vec{k}(1-0) \\ & 0 & 1 & 2 & 0 & 1 \end{array}$$

$$=(-2,-2,1)$$

$$A=\|\vec{a}\times\vec{b}\|=\sqrt{(-2)^2+(-2)^2+1^2}=\sqrt{9}=3$$

$$\therefore A=3$$

Examples

Ex 5. Find the area of the triangle ΔABC where $A(0,1,2)$, $B(-1,0,2)$, and $C(1,-2,0)$.

$$\overrightarrow{AB} = (-1,0,2) - (0,1,2) = (-1,-1,0)$$

$$\overrightarrow{AC} = (1,-2,0) - (0,1,2) = (1,-3,-2)$$

$$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{array}{ccccc} \vec{i} & \vec{j} & \vec{k} & \vec{i} & \vec{j} \\ -1 & -1 & 0 & -1 & -1 \\ 1 & -3 & -2 & 1 & -3 \end{array} = \vec{i}\,(2-0) + \vec{j}\,(0-2) + \vec{k}\,(3+1)$$

$$= (2,-2,4)$$

$$A = \frac{1}{2} \| \vec{a} \times \vec{b} \| = \frac{1}{2}\sqrt{2^2 + (-2)^2 + 4^2} = \frac{1}{2}\sqrt{24} = \sqrt{6}$$

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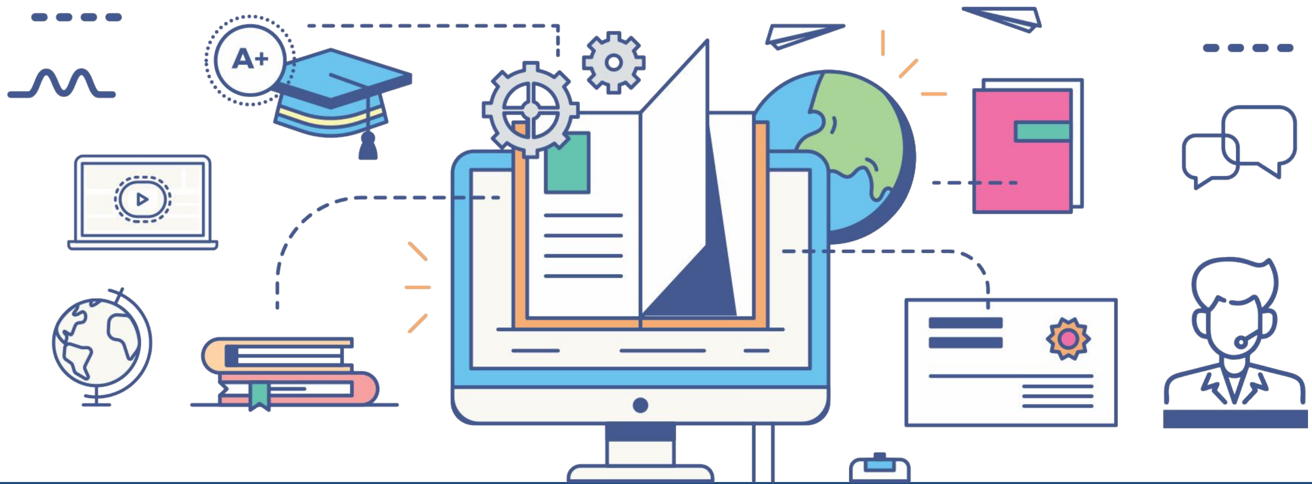

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