Enhancement & Denoising:
Frequency domain filtering

Dr. Tushar Sandhan

Introduction

Input

Introduction

Input

Output

Fourier Translation & Scaling

- 2D Fourier Transform

Fourier Translation & Scaling

- 2D Fourier Transform

$$F(u,v) = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y) e^{-j2\pi(ux/M + vy/N)}$$

Fourier Translation & Scaling

- 2D Fourier Transform

$$\square \quad \text{DFT:} \quad F(u,v) = \sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y)\, e^{-j2\pi(ux/M + vy/N)}$$

Fourier Translation & Scaling

- 2D Fourier Transform

❑ DFT:

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$$u = x = 0, 1, 2, \ldots, M-1$$

Fourier Translation & Scaling

- 2D Fourier Transform

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$$u = x = 0, 1, 2, \ldots, M-1$$
$$v = y = 0, 1, 2, \ldots, N-1.$$

Fourier Translation & Scaling

■ 2D Fourier Transform

❑ DFT:

$$F(u,v) = \sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y)\, e^{-j2\pi(ux/M + vy/N)}$$

$$f(x,y) = \frac{1}{MN}\sum_{u=0}^{M-1}\sum_{v=0}^{N-1} F(u,v)\, e^{j2\pi(ux/M + vy/N)}$$

$$u = x = 0, 1, 2, \ldots, M-1$$
$$v = y = 0, 1, 2, \ldots, N-1.$$

Fourier Translation & Scaling

- 2D Fourier Transform

□ DFT:

$$F(u,v) = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y) e^{-j2\pi(ux/M + vy/N)}$$

□ IDFT:

$$f(x,y) = \frac{1}{MN} \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} F(u,v) e^{j2\pi(ux/M + vy/N)}$$

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Fourier Translation & Scaling

- 2D Fourier Transform

❑ DFT: $$F(u,v) = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y) e^{-j2\pi(ux/M + vy/N)}$$

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$$u = x = 0, 1, 2, \ldots, M-1$$
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Fourier Translation & Scaling

- Translation

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$$FT[f(x - x_0, y - y_0)] = F(u, v) \cdot \exp[-j2\pi(ux_0 + vy_0)/N]$$

Fourier Translation & Scaling

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Fourier Translation & Scaling

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$$FT[f(x - x_0, y - y_0)] = F(u, v) \cdot \exp[-j2\pi(ux_0 + vy_0)/N]$$

- Scaling

$$FT[f(ax, by)] = \frac{1}{ab} F\left(\frac{u}{a}, \frac{v}{b}\right)$$

Convolution Theorem

- Spatial filtering to frequency filtering

$$f \star g$$

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$$f \star g \quad =$$

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$$f \star g \quad = \quad \sum_m f(m)g(n-m)$$

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$$FT[f \star g] = \sum_m f(m)g(n-m)e^{-\frac{j2\pi nu}{N}}$$

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$$FT[f \star g] = \sum_n \sum_m f(m)g(n-m)\, e^{-\frac{j2\pi nu}{N}}$$

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$$FT[f \star g] = \sum_n \sum_m f(m)g(n-m)e^{-\frac{j2\pi nu}{N}}$$

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$$= \sum_m f(m)FT[g]e^{-\frac{j2\pi mu}{N}}$$

Convolution Theorem

- Spatial filtering to frequency filtering

$$FT[f \star g] = \sum_{n} \sum_{m} f(m)g(n-m) e^{-\frac{j2\pi n u}{N}}$$

$$= \sum_{m} f(m) FT[g] e^{-\frac{j2\pi m u}{N}}$$

Convolution Theorem

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$$FT[f \star g] = \sum_n \sum_m f(m)g(n-m)\,e^{-\frac{j2\pi nu}{N}}$$

$$= \sum_m f(m) FT[g] e^{-\frac{j2\pi mu}{N}}$$

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Convolution Theorem

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$$FT[f \star g] = \sum_n \sum_m f(m)g(n-m)e^{-\frac{j2\pi nu}{N}}$$

$$= \sum_m f(m)FT[g]e^{-\frac{j2\pi mu}{N}}$$

$$= FT[g] \sum_n f(m)e^{-\frac{j2\pi mu}{N}}$$

$$= FT[f] \cdot FT[g]$$

Convolution Theorem

- Spatial filtering to frequency filtering

$$FT[f \star g] = \sum_{n} \sum_{m} f(m) g(n-m) e^{-\frac{j2\pi nu}{N}}$$

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Convolution Theorem

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$$FT[f \star g] = \sum_n \sum_m f(m)g(n-m)\,e^{-\frac{j2\pi nu}{N}}$$

$$= \sum_m f(m)FT[g]e^{-\frac{j2\pi mu}{N}}$$

$$= FT[g]\sum_n f(m)e^{-\frac{j2\pi mu}{N}}$$

$$FT[f \star g] = FT[f] \cdot FT[g]$$

2D Fourier Transform

- Repositioning the quadrants

2D Fourier Transform
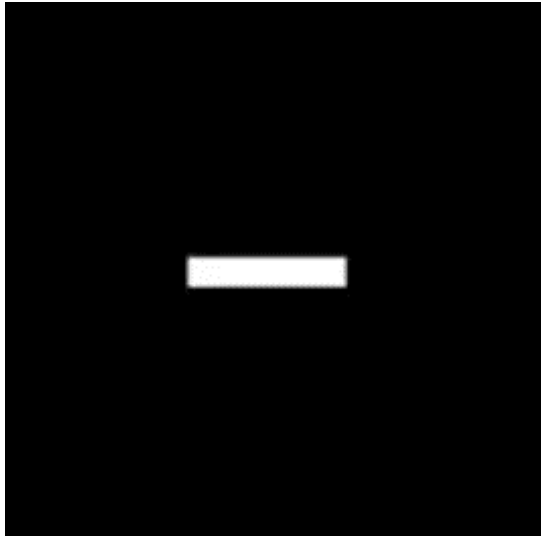
- Repositioning the quadrants

$f(x, y)$

2D Fourier Transform

- Repositioning the quadrants

$f(x, y)$

2D Fourier Transform

- Repositioning the quadrants

$$f(x, y)$$

$$|F(u, v)|$$
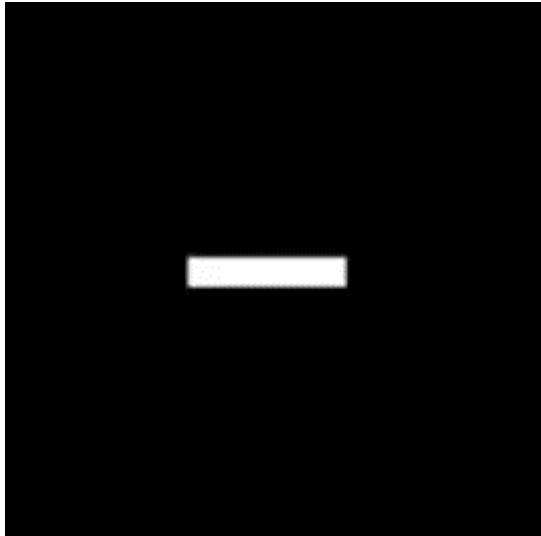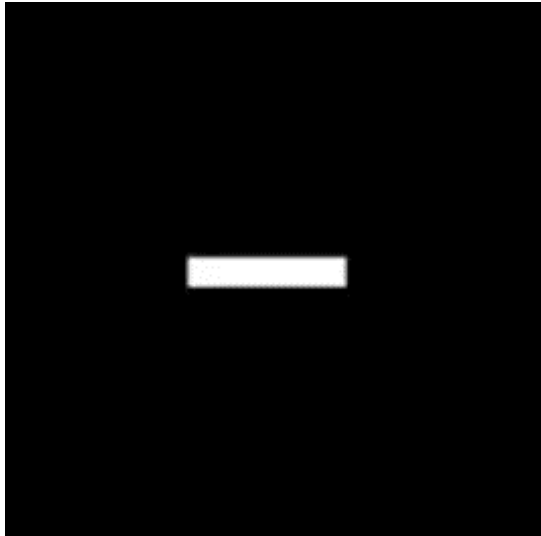
2D Fourier Transform

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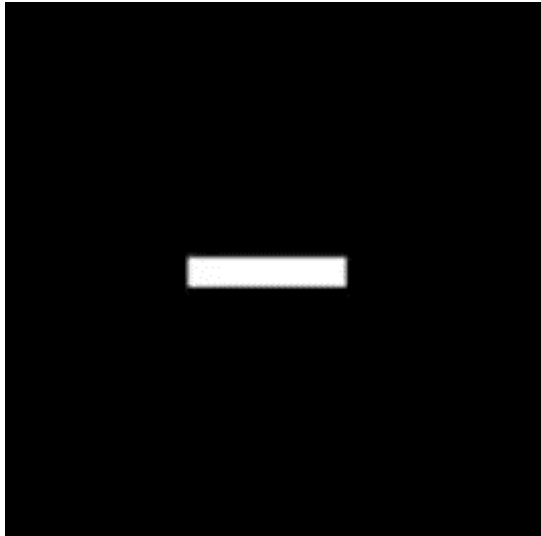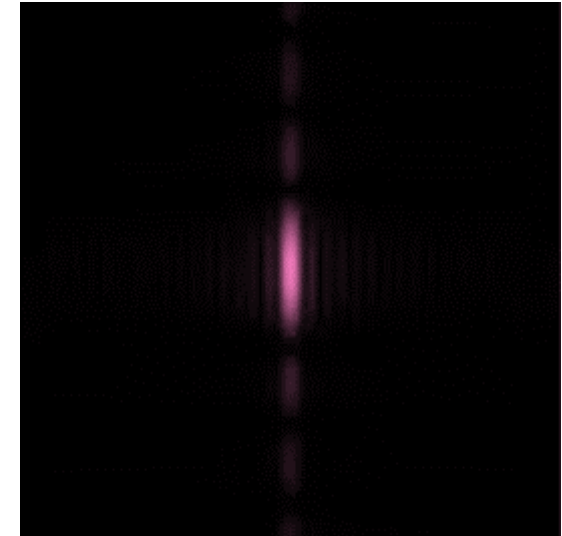
$$f(x, y) \qquad\qquad |F(u, v)|$$

2D Fourier Transform

- Repositioning the quadrants

$$f(x, y)$$
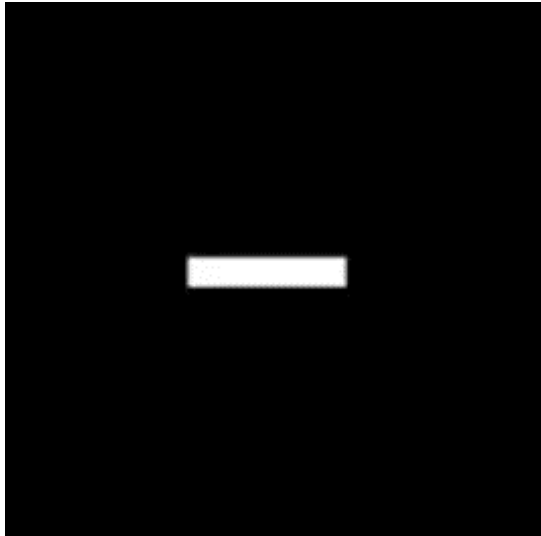
$$|F(u, v)|$$

$$Shift(|F(u, v)|)$$

2D Fourier Transform

- FT as image & intensity transformations

2D Fourier Transform

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2D Fourier Transform

- FT as image & intensity transformations

FT

2D Fourier Transform

- FT as image & intensity transformations

FT

2D Fourier Transform

- FT as image & intensity transformations

FT

Thresholding

2D Fourier Transform

- FT as image & intensity transformations

FT

Thresholding

2D Fourier Transform

- FT as image & intensity transformations

FT

Thresholding

Scaling

2D Fourier Transform

- FT as image & intensity transformations

FT

Thresholding

Scaling

2D Fourier Transform

- FT as image & intensity transformations

FT

Thresholding

Scaling

Log

2D Fourier Transform

- FT as image & intensity transformations

FT

Thresholding

Scaling

Log

2D Fourier Transform

- FT as image & intensity transformations

FT

Thresholding

Scaling

Log

Log+Scaling

2D Fourier Transform

- FT as image & intensity transformations

2D Fourier Transform

- FT as image & intensity transformations

FT
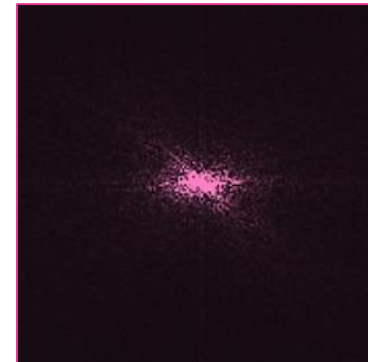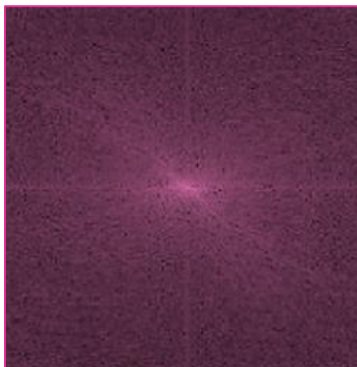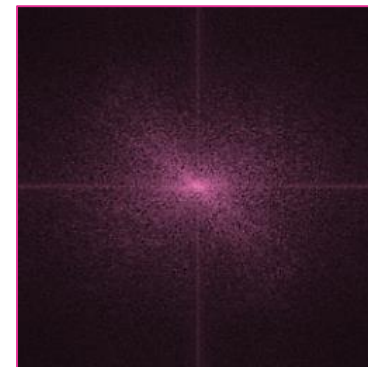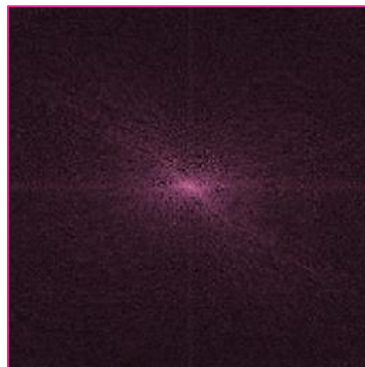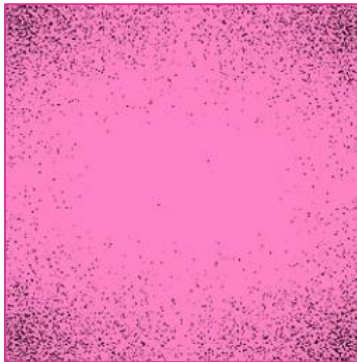
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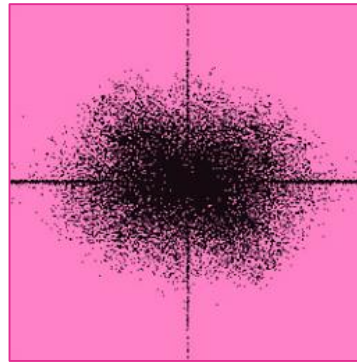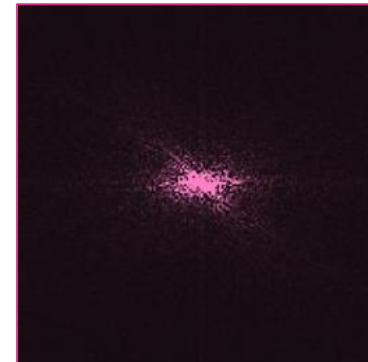
Scaling

Log
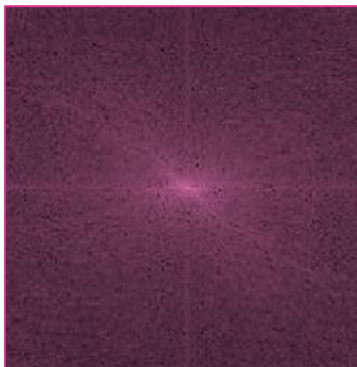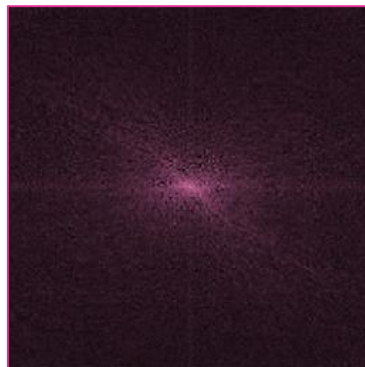
Log+Scaling

Log+Scaling+Histeq
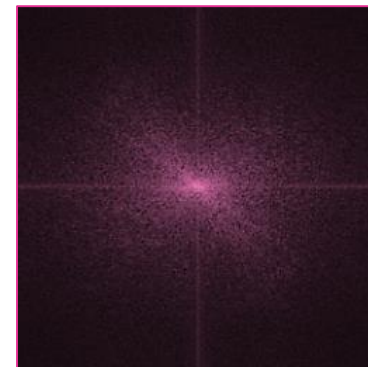
2D Fourier Transform
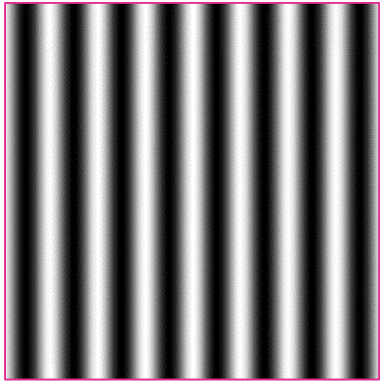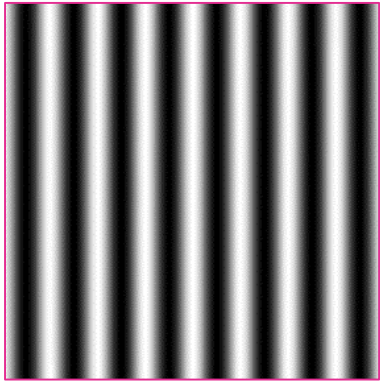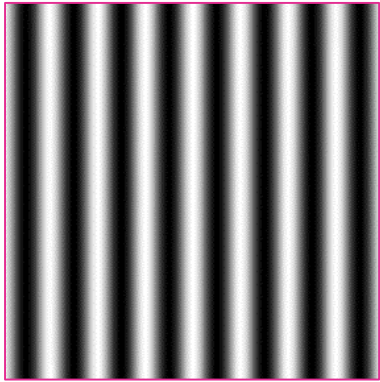
$f_1(x, y)$

2D Fourier Transform

$f_1(x, y)$

2D Fourier Transform

$f_1(x, y)$　　　　$\log(|F_1(u, v)|)$

2D Fourier Transform
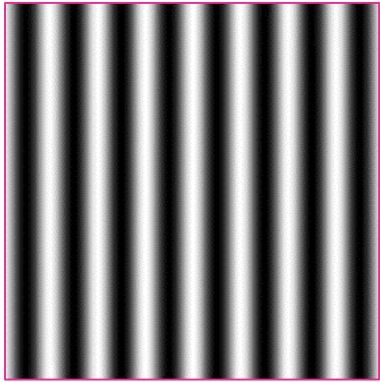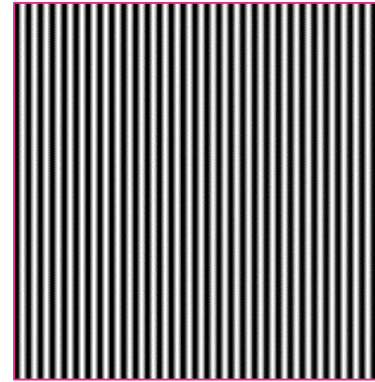
$$f_1(x, y)$$

$$\log(|F_1(u, v)|)$$

2D Fourier Transform
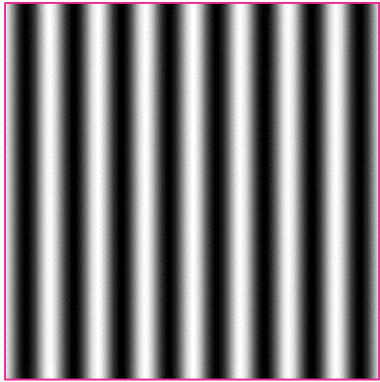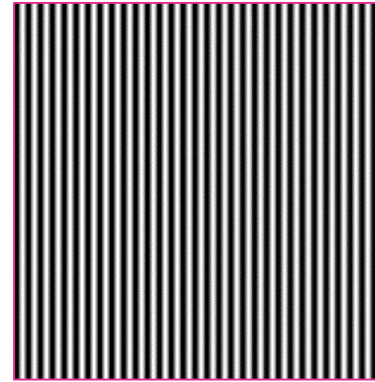
$f_1(x, y)$ $\log(|F_1(u, v)|)$ $f_2(x, y)$

2D Fourier Transform
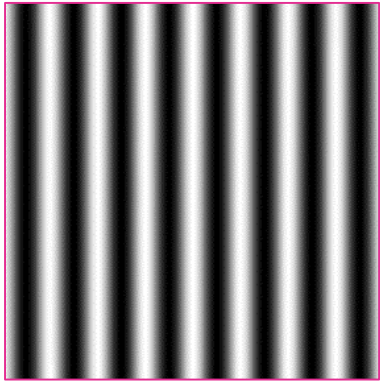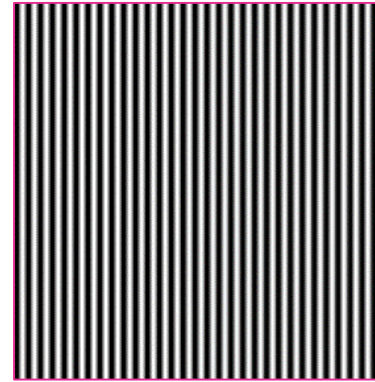
$$f_1(x, y) \qquad\qquad \log(|F_1(u, v)|) \qquad\qquad f_2(x, y)$$

2D Fourier Transform
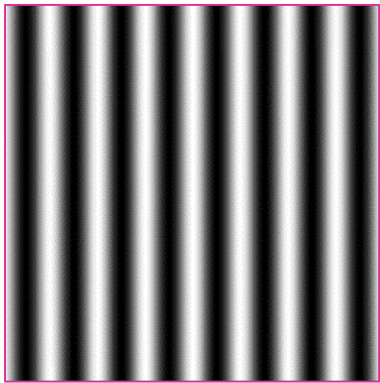
$$f_1(x, y)$$

$$\log(|F_1(u, v)|)$$

$$f_2(x, y)$$

$$\log(|F_2(u, v)|)$$

2D Fourier Transform
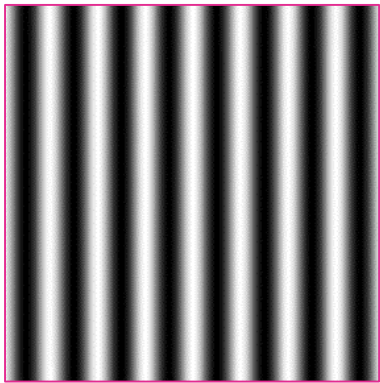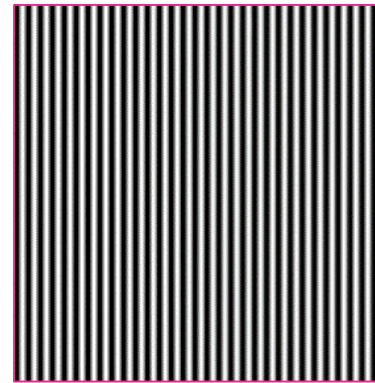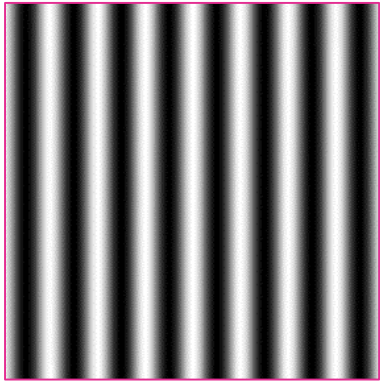
$f_1(x, y)$ $\log(|F_1(u, v)|)$ $f_2(x, y)$ $\log(|F_2(u, v)|)$

$f_1(x, y) + f_2(x, y)$

2D Fourier Transform
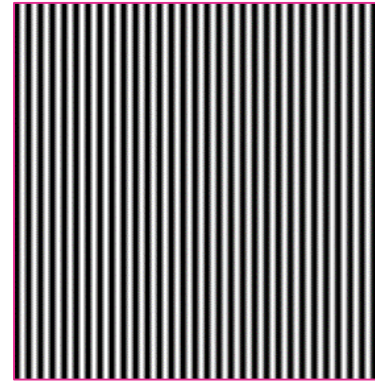
$f_1(x, y)$ $\qquad$ $\log(|F_1(u, v)|)$ $\qquad$ $f_2(x, y)$ $\qquad$ $\log(|F_2(u, v)|)$
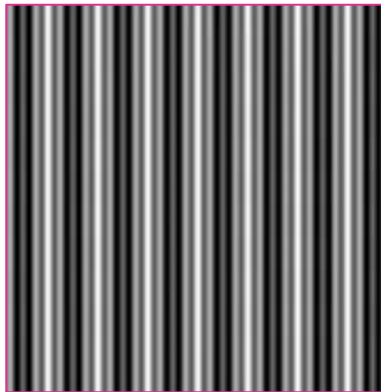
$f_1(x, y) + f_2(x, y)$

2D Fourier Transform
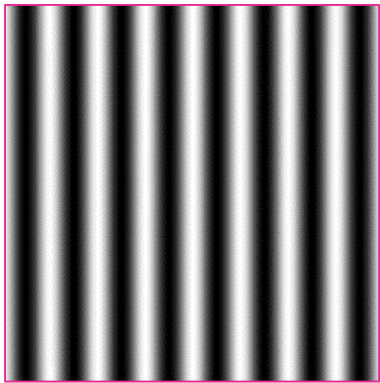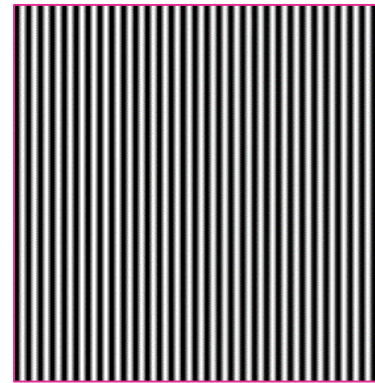
$f_1(x, y)$ $\log(|F_1(u, v)|)$ $f_2(x, y)$ $\log(|F_2(u, v)|)$
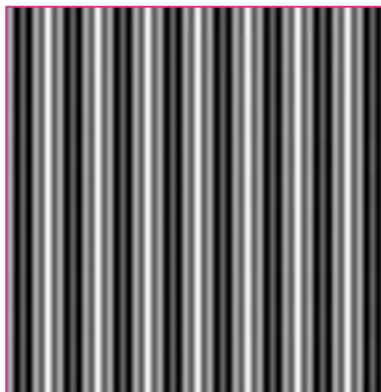
$f_1(x, y) + f_2(x, y)$

2D Fourier Transform
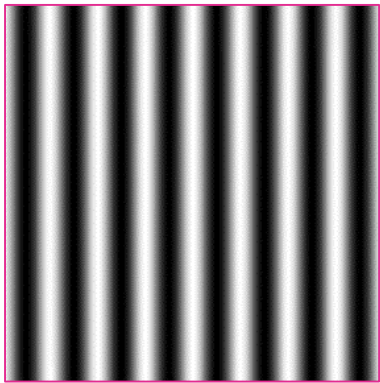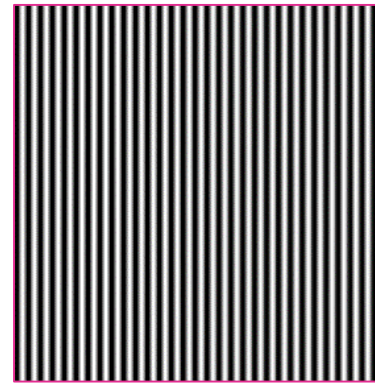
$$f_1(x, y) \qquad \log(|F_1(u, v)|) \qquad\qquad f_2(x, y) \qquad \log(|F_2(u, v)|)$$
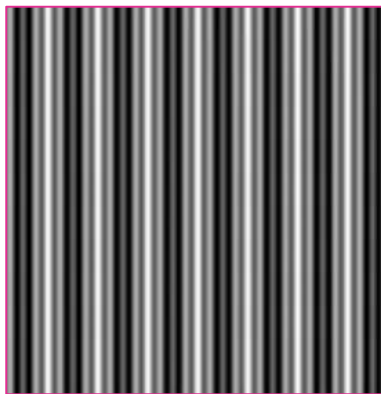
$$f_1(x, y) + f_2(x, y)$$

2D Fourier Transform
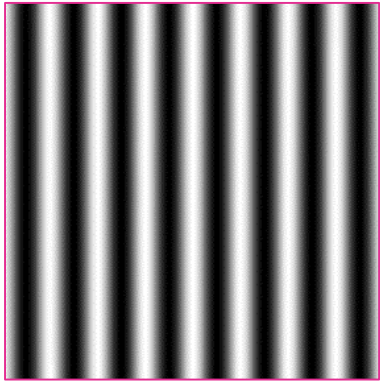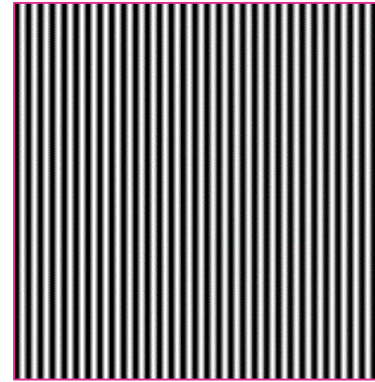
$f_1(x, y)$ $\log(|F_1(u, v)|)$ $f_2(x, y)$ $\log(|F_2(u, v)|)$

$f_1(x, y) + f_2(x, y)$

$\log(|F_1(u, v) + F_2(u, v)|)$

2D Fourier Transform
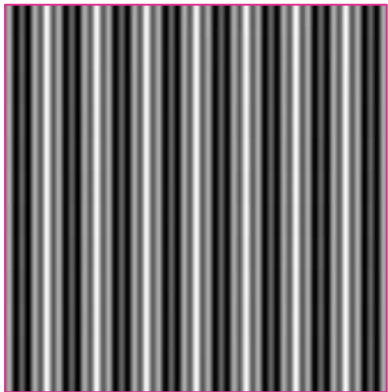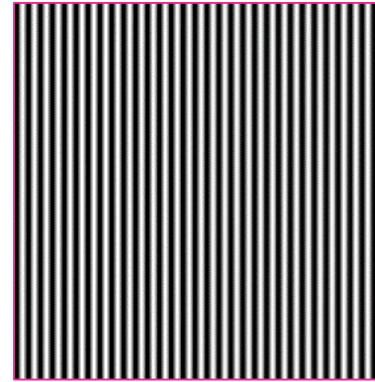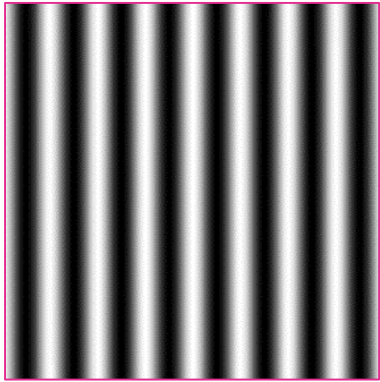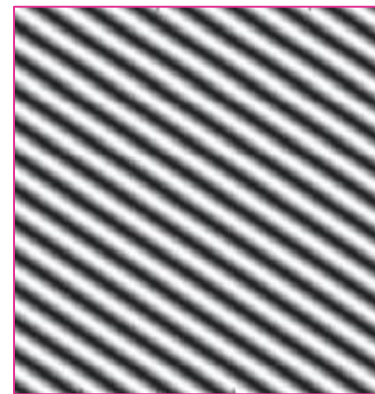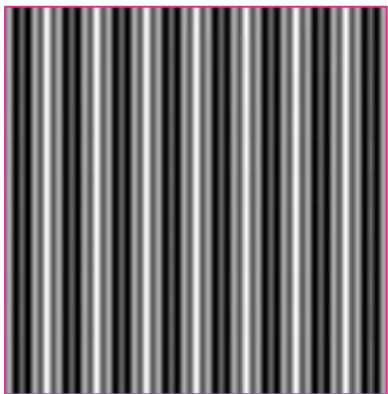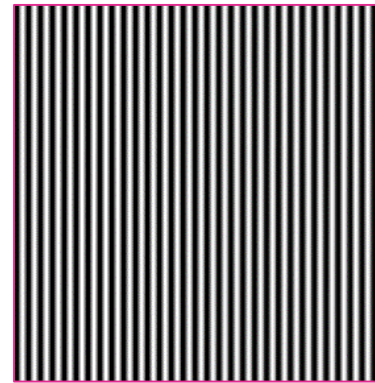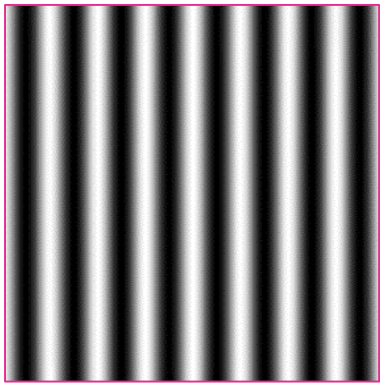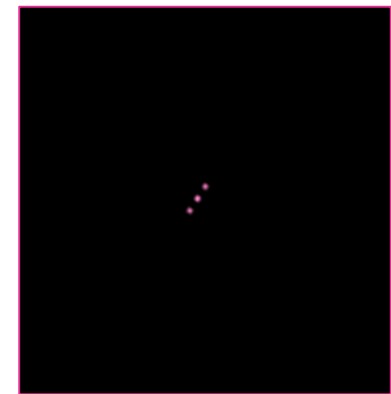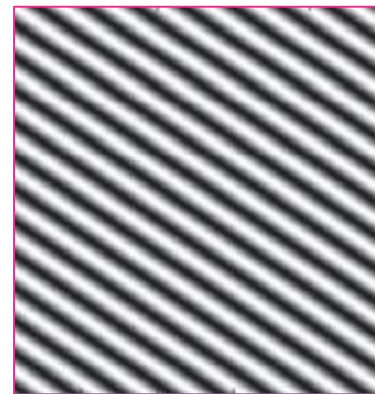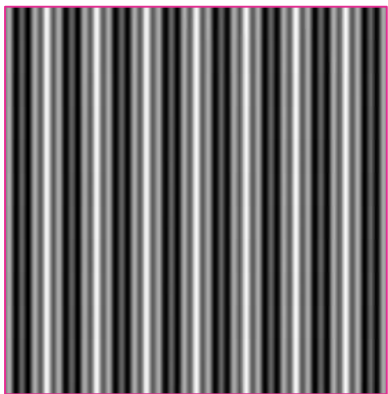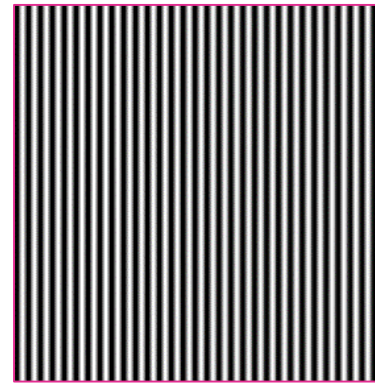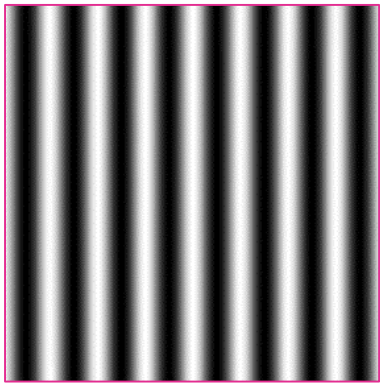
2D Fourier Transform

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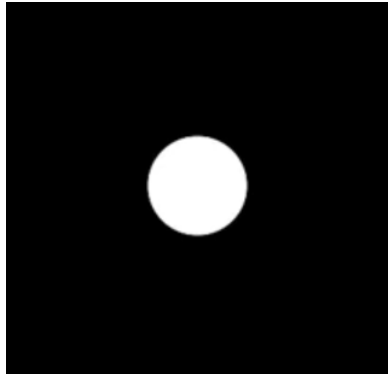
2D Fourier Transform
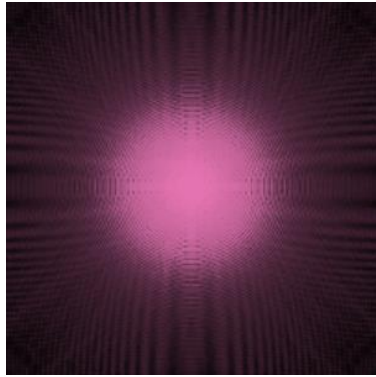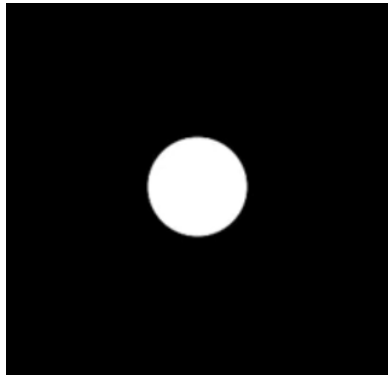
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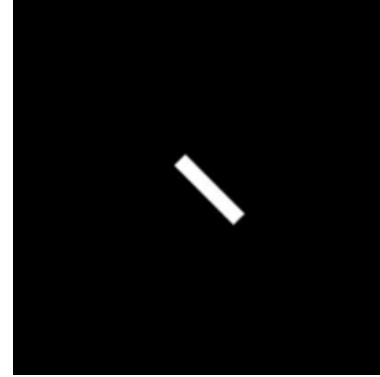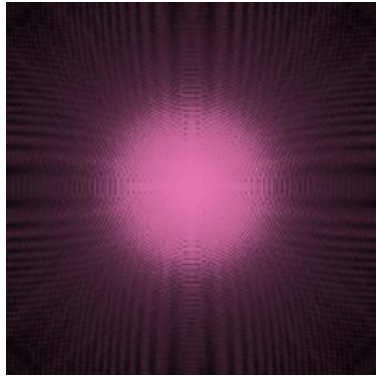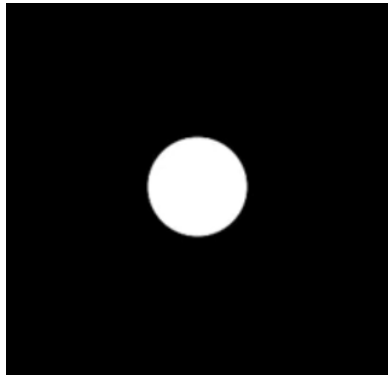
2D Fourier Transform
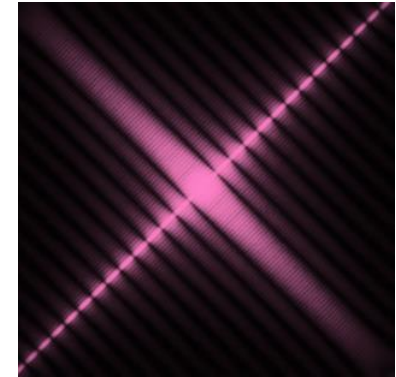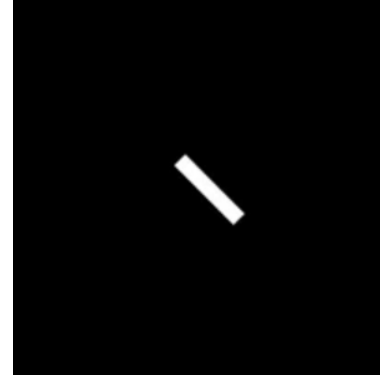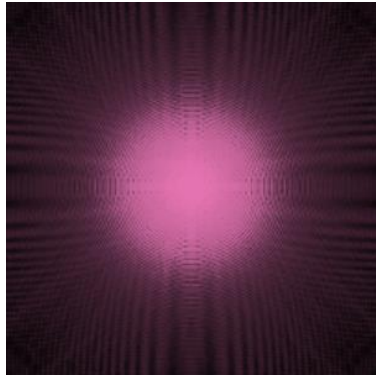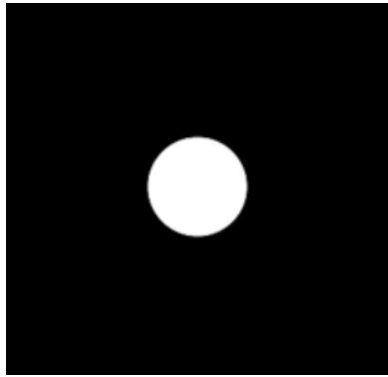
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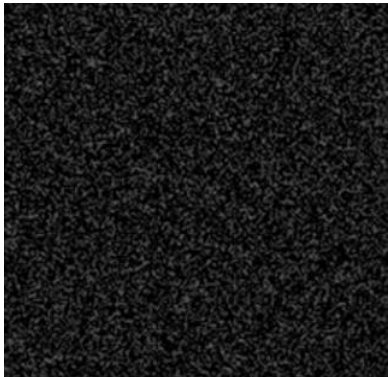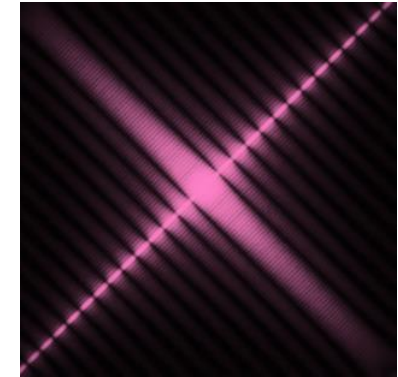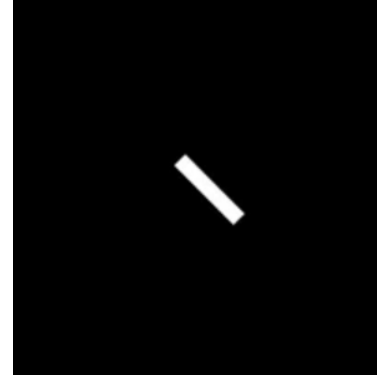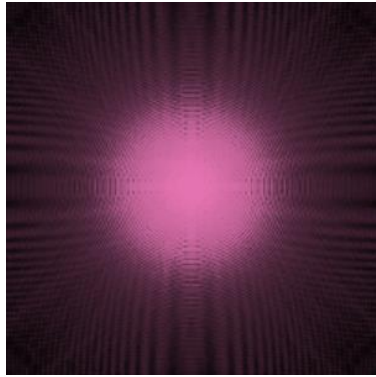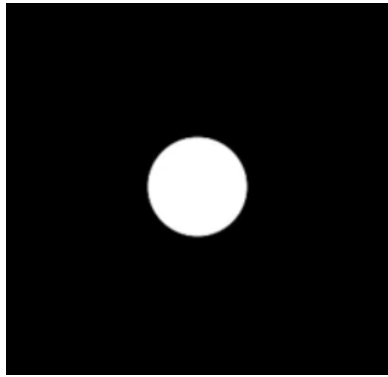
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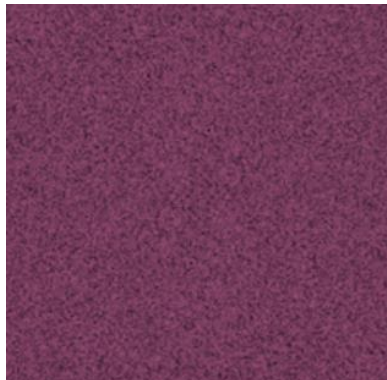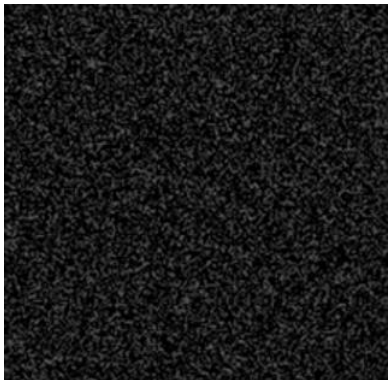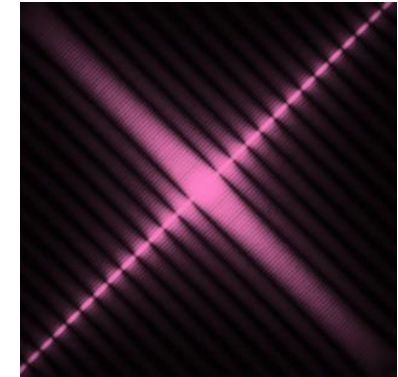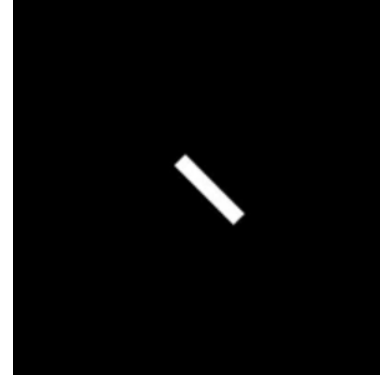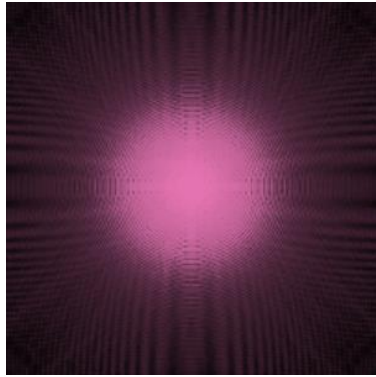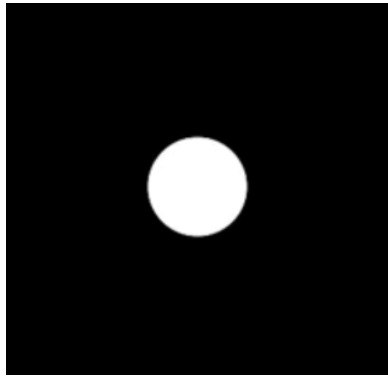
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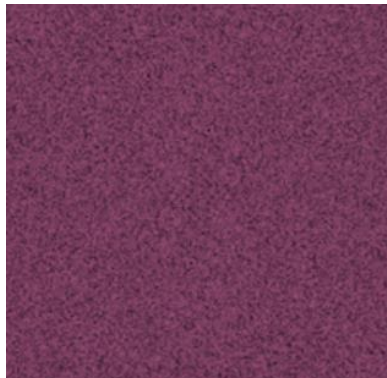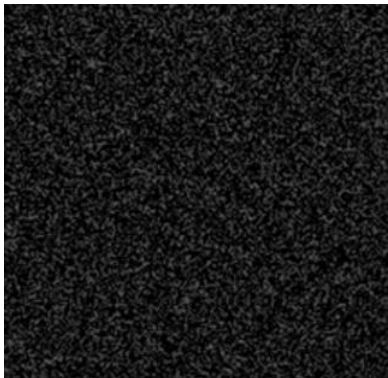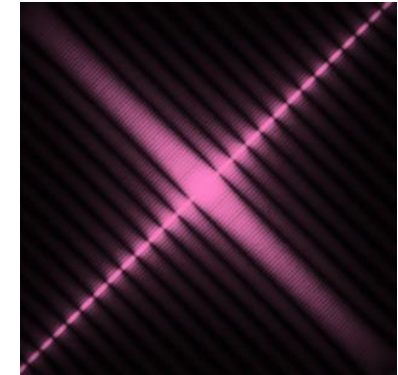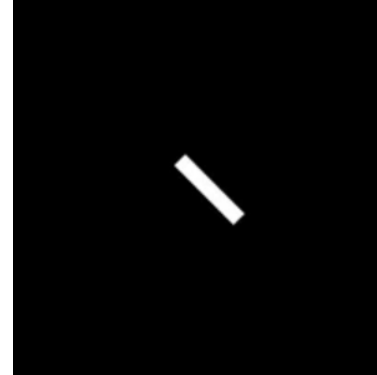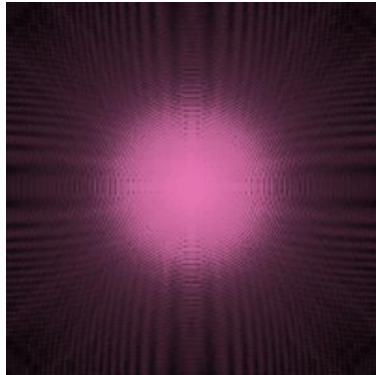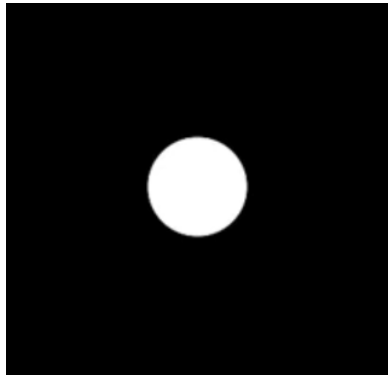
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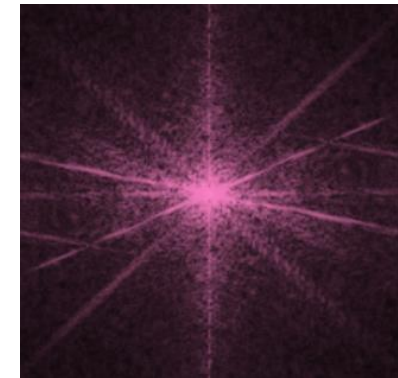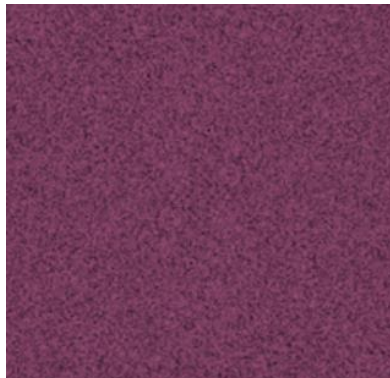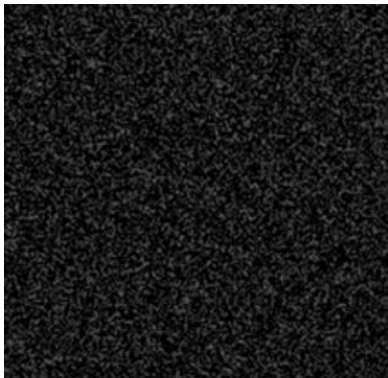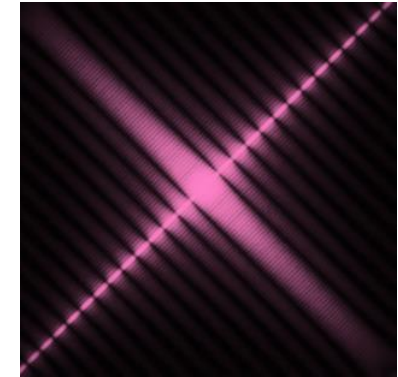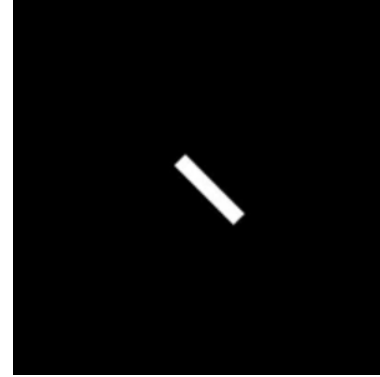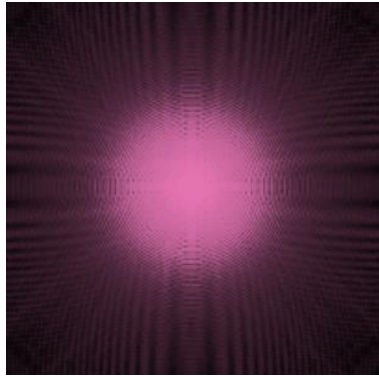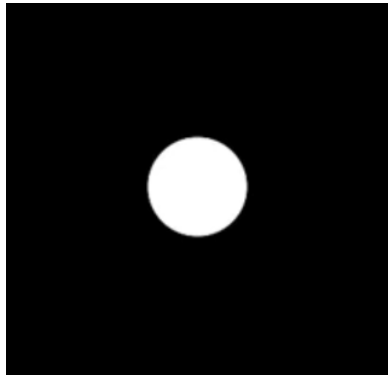
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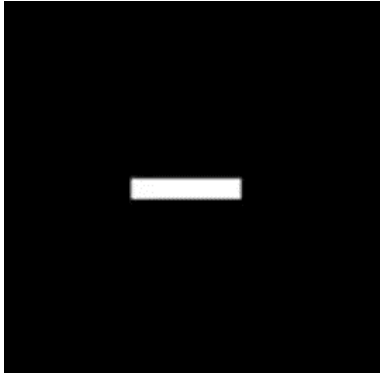
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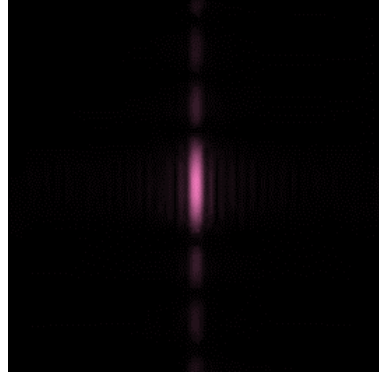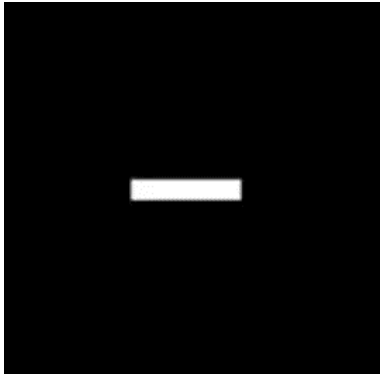
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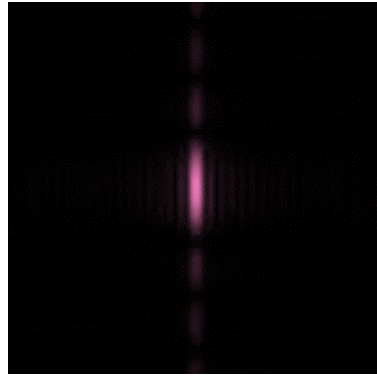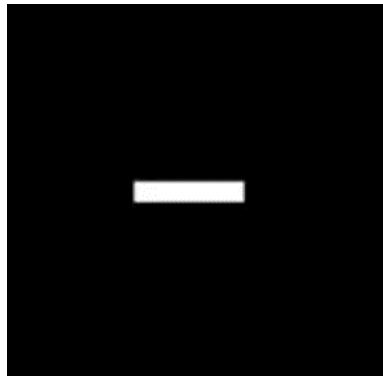
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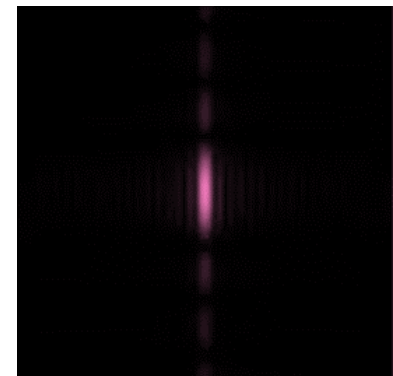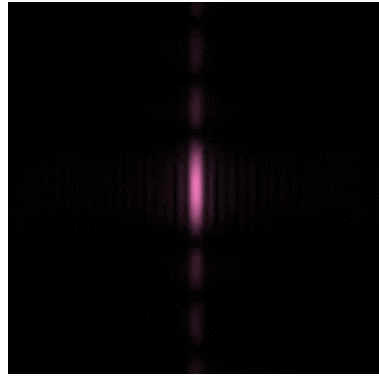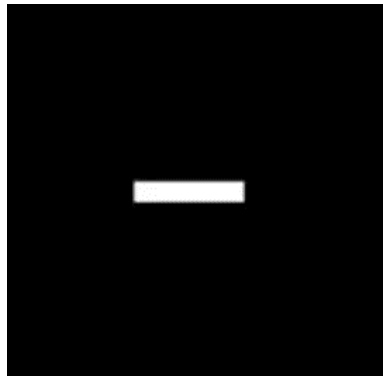
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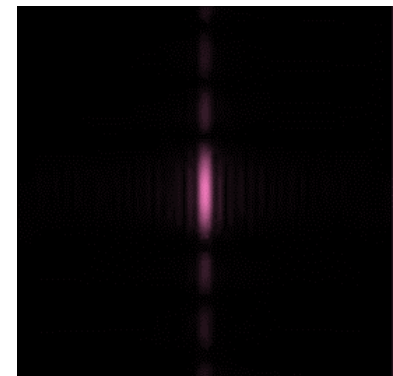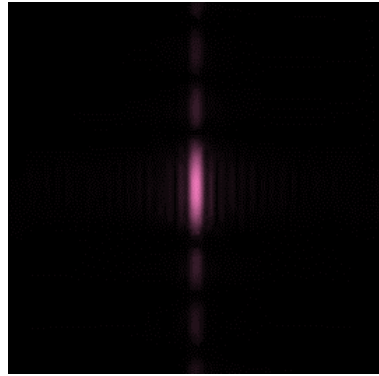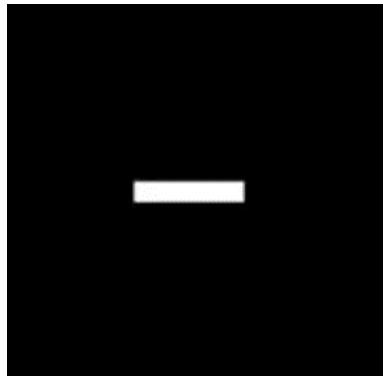
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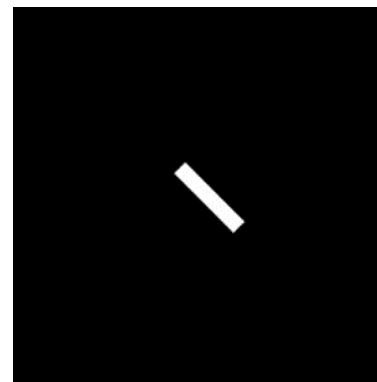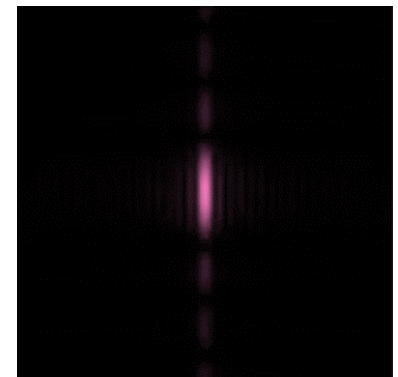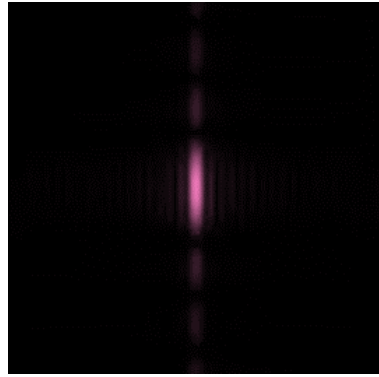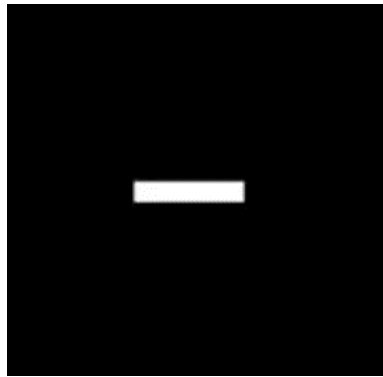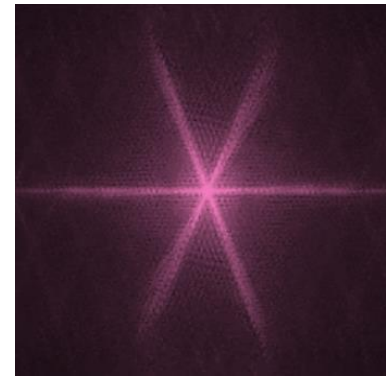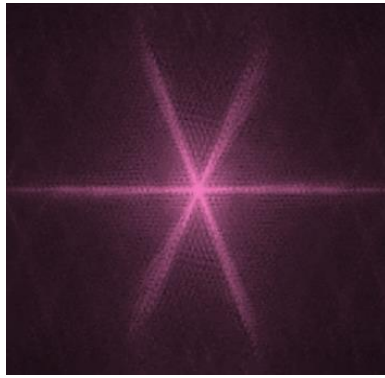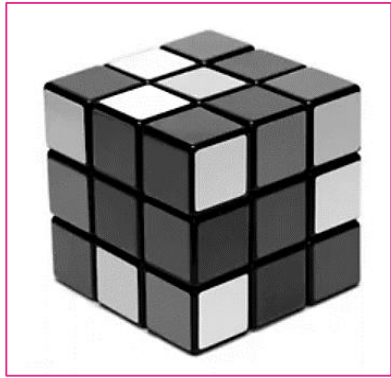
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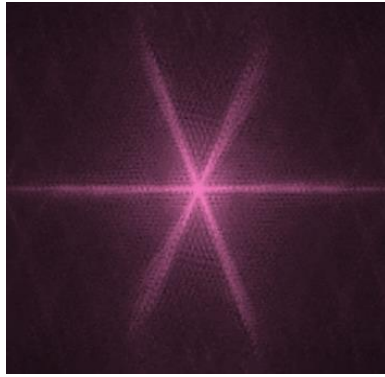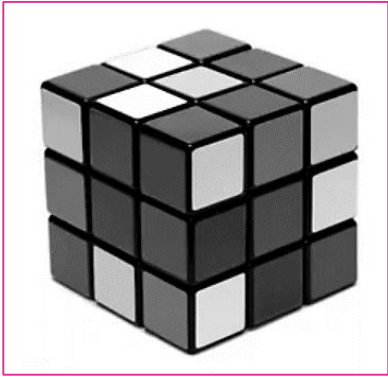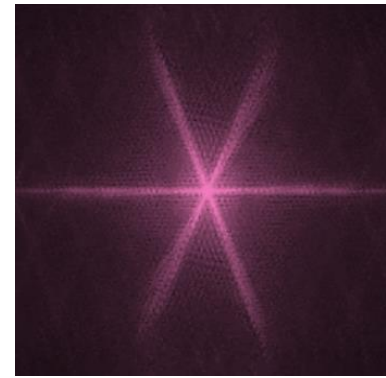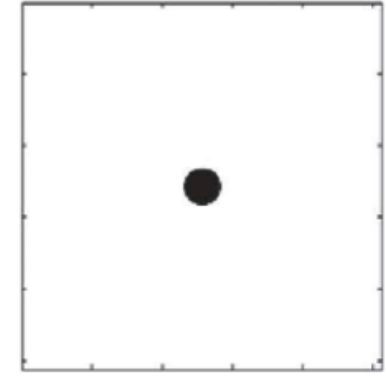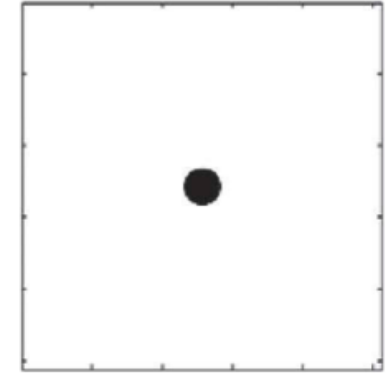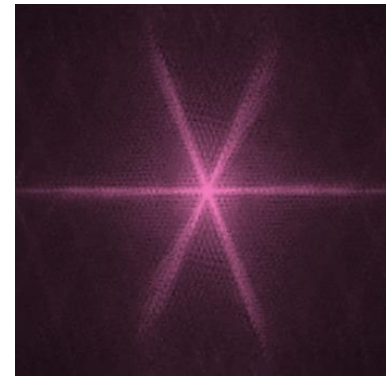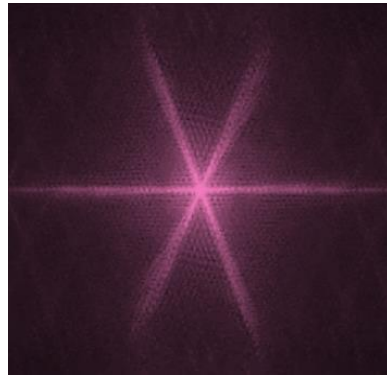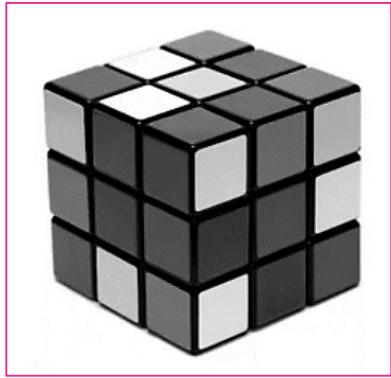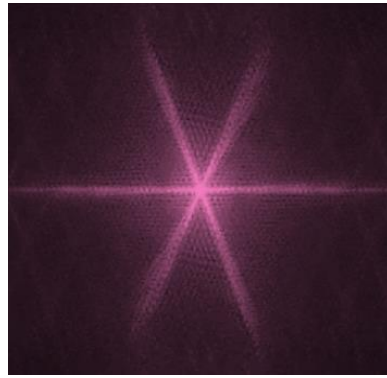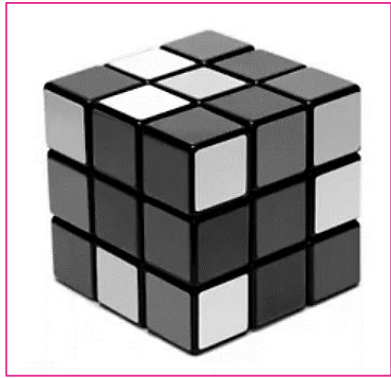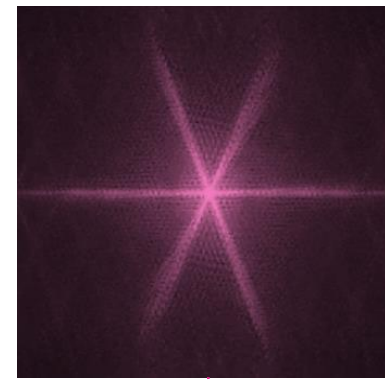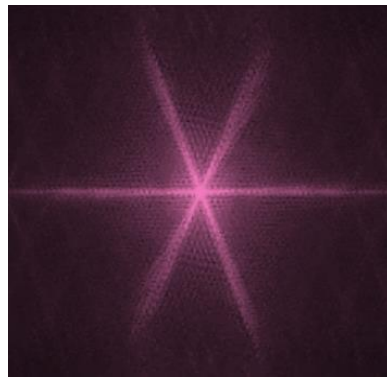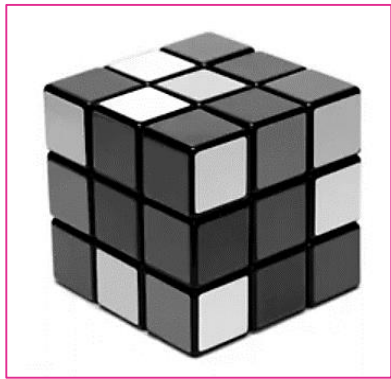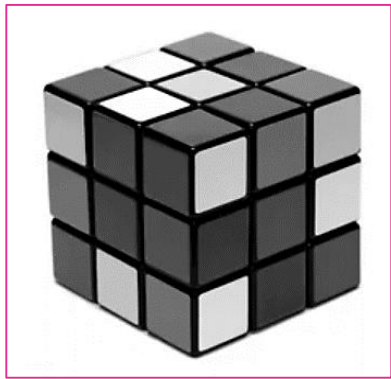
Frequency Filtering

Frequency Filtering

Frequency Filtering

Frequency Filtering
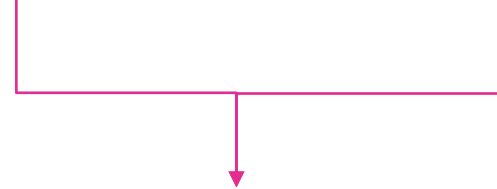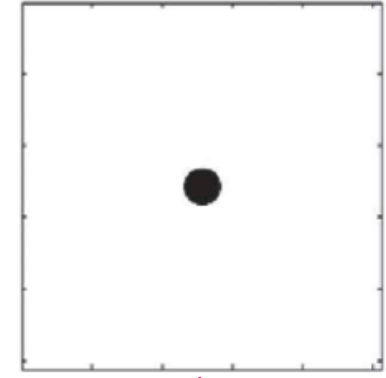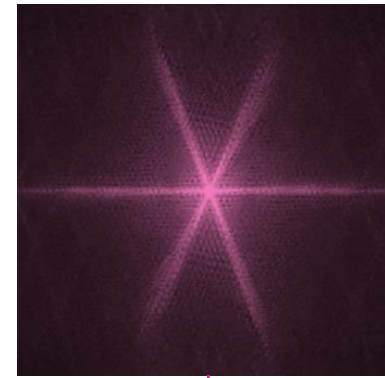
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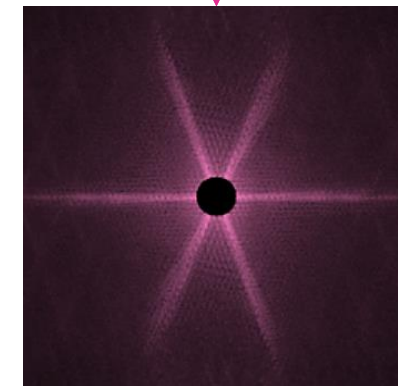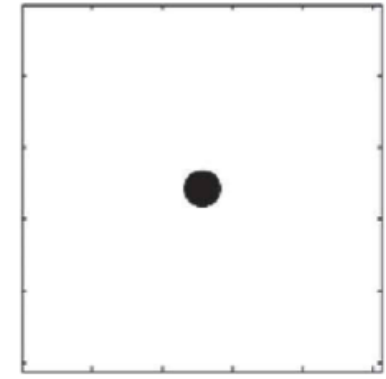
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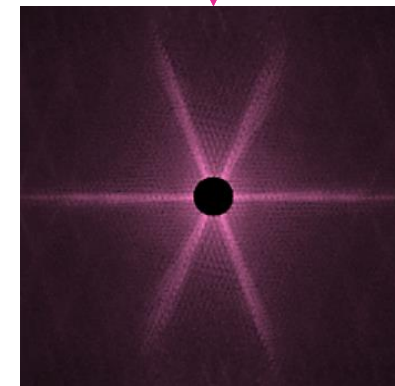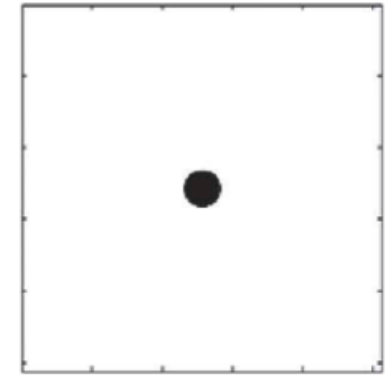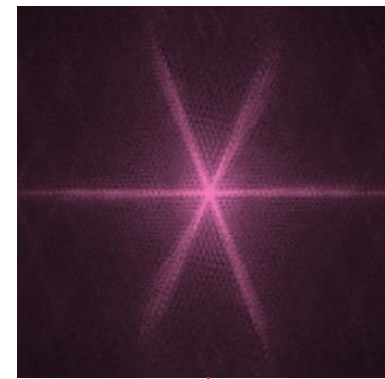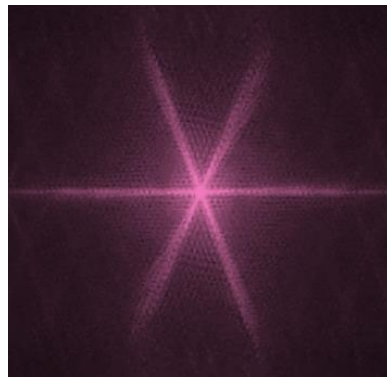
Inverse

Fourier

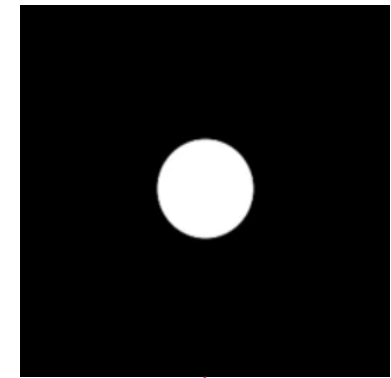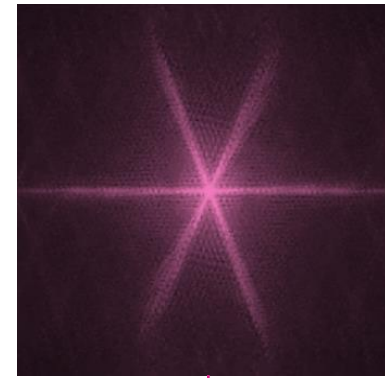# Frequency Filtering
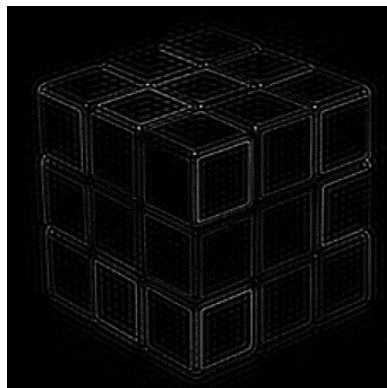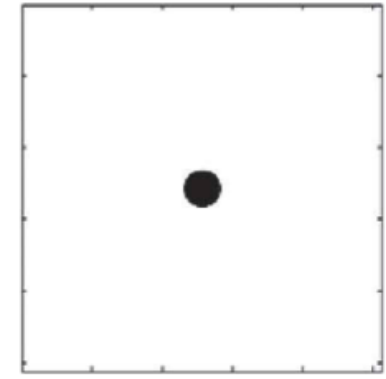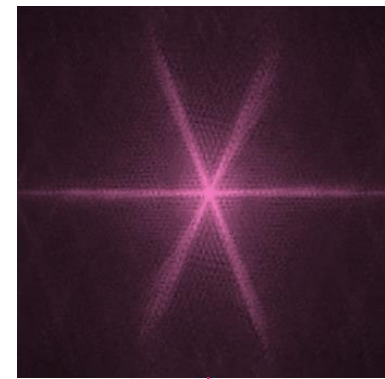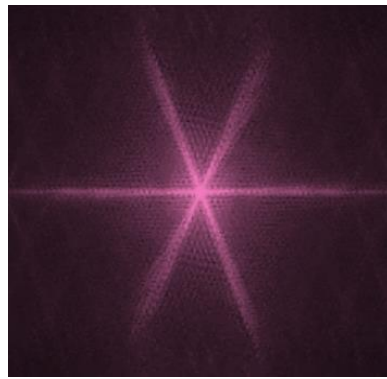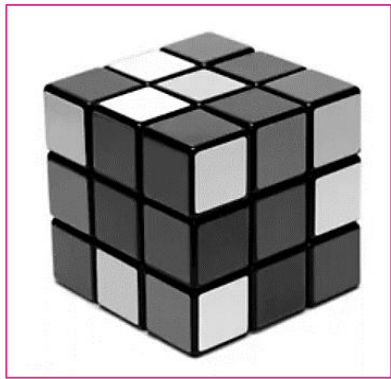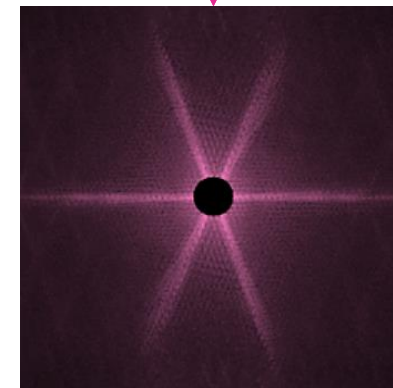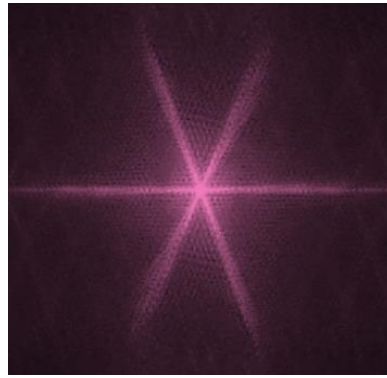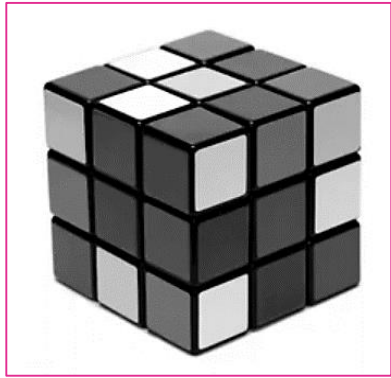

Inverse

Fourier

Frequency Filtering
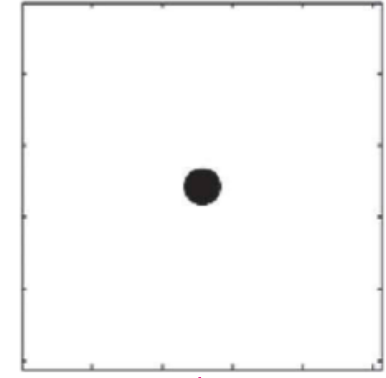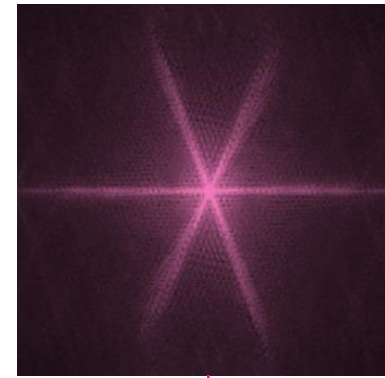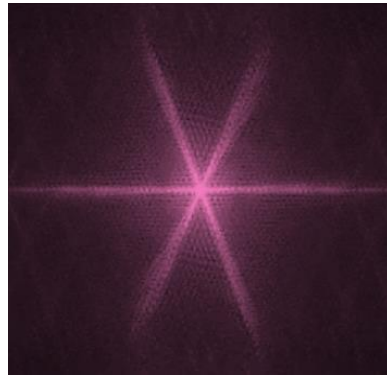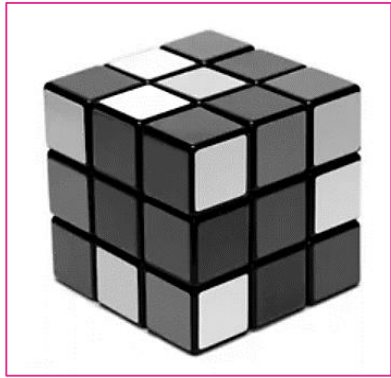
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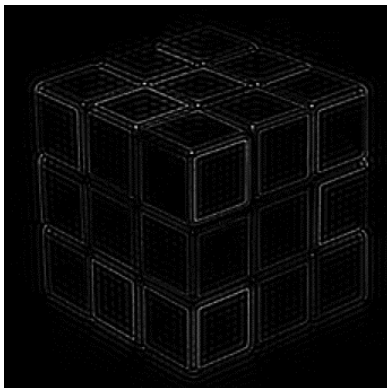
Inverse

Fourier
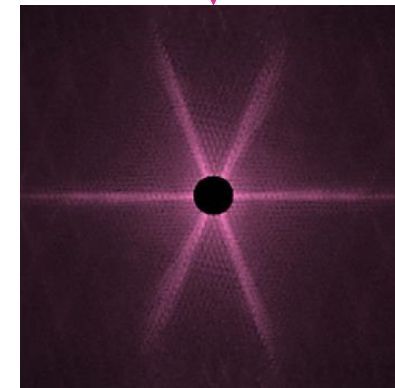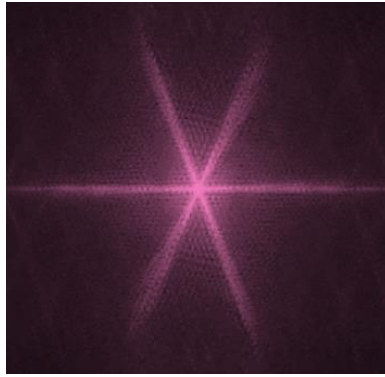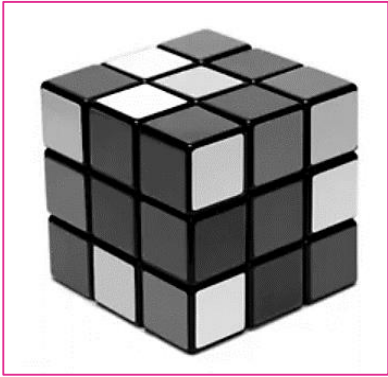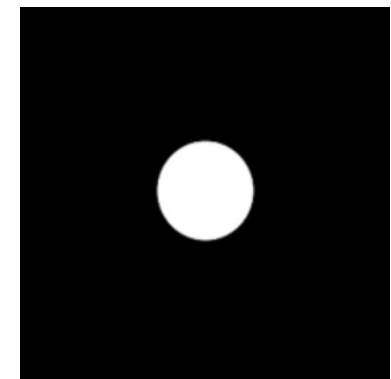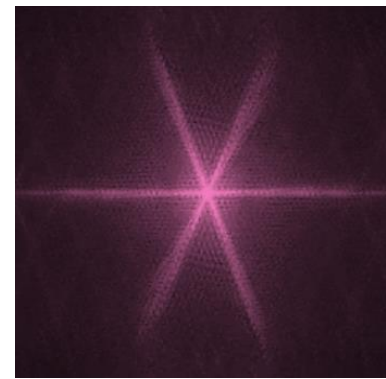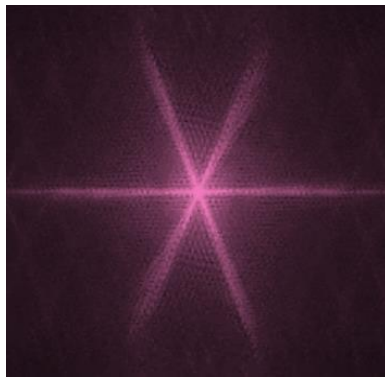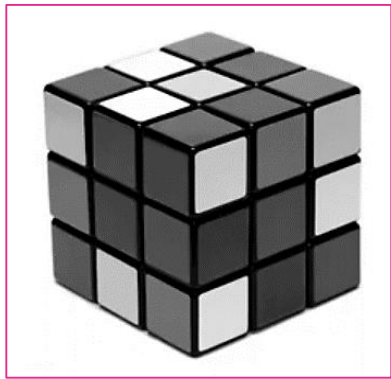
Frequency Filtering

Frequency Filtering

Frequency Filtering

Frequency Filtering
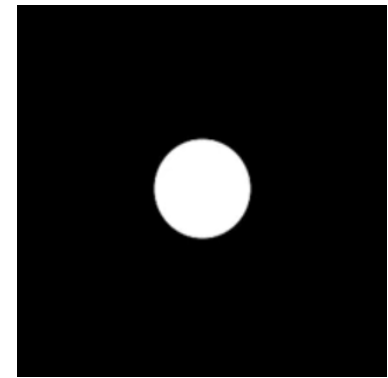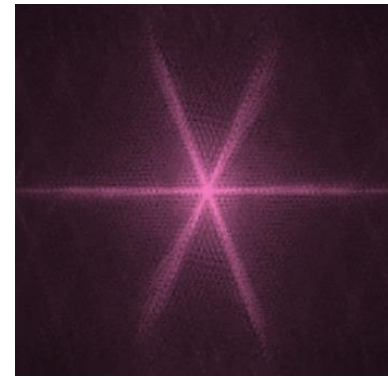
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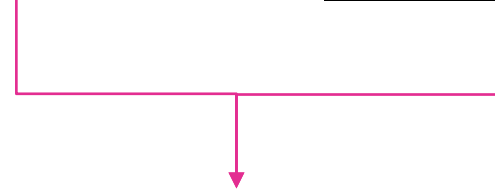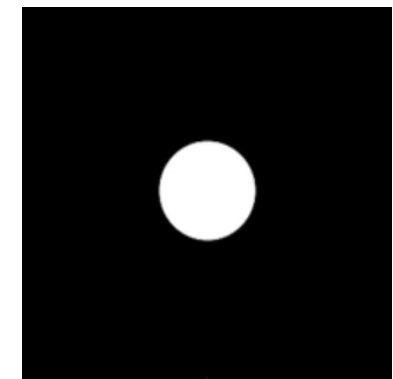
Frequency Filtering
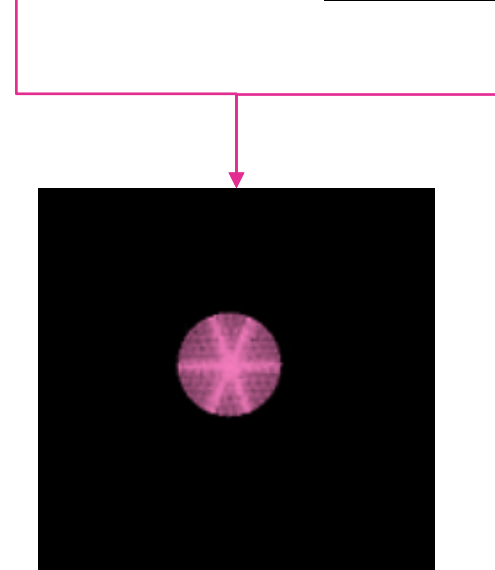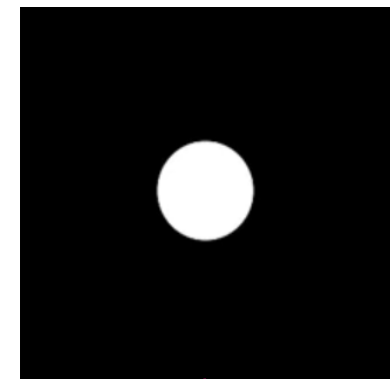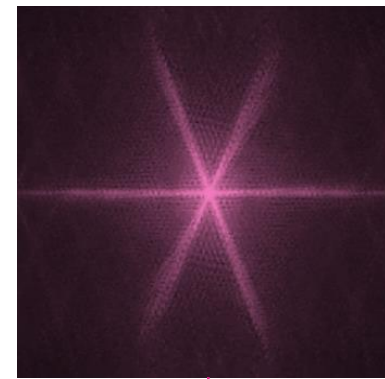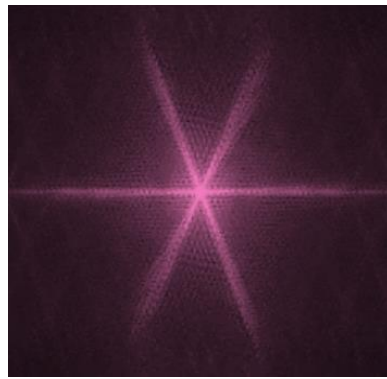
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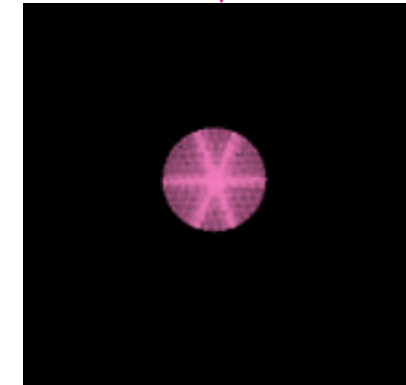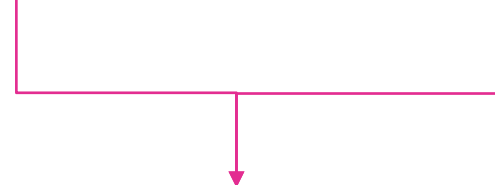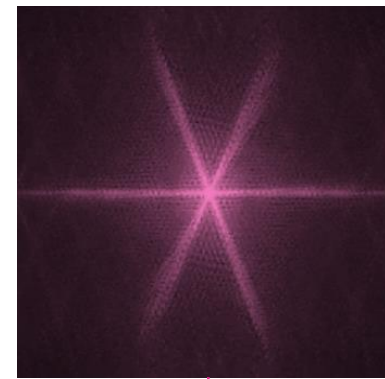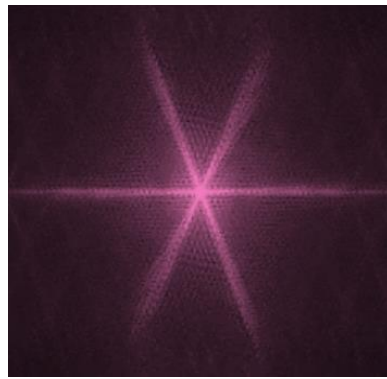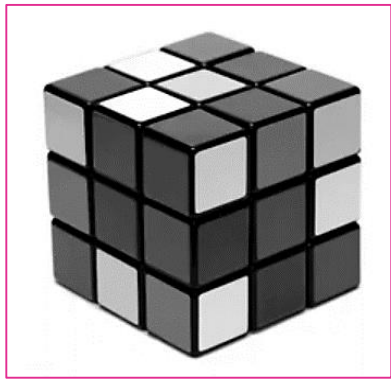

Inverse

Fourier

Frequency Filtering
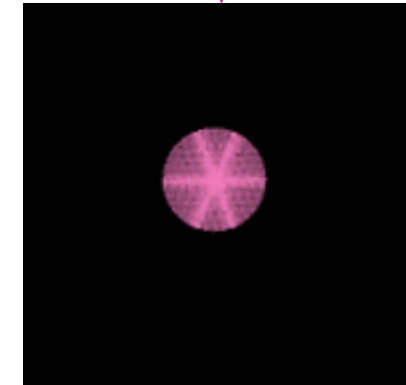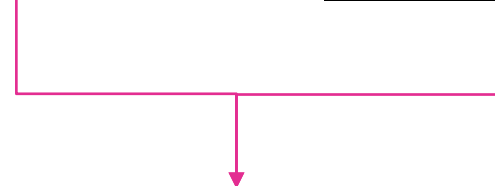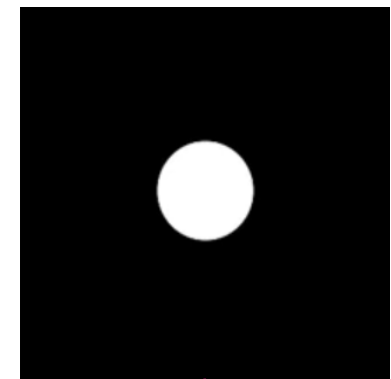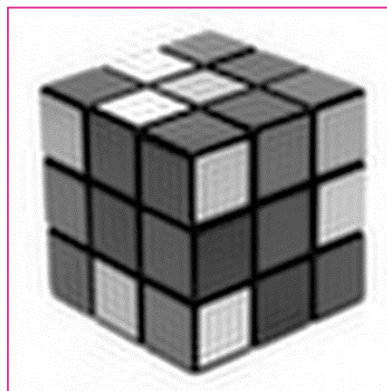
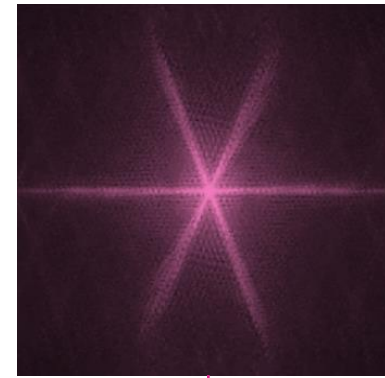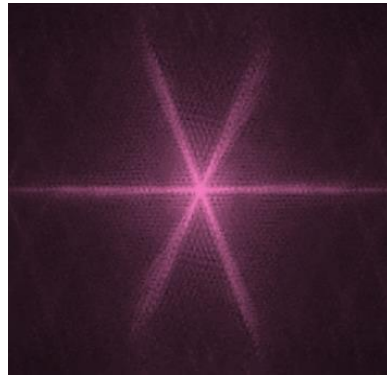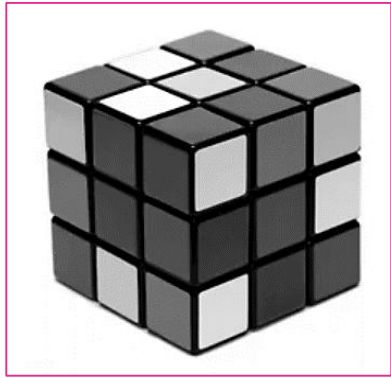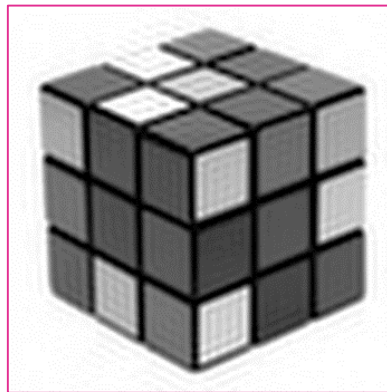
Inverse

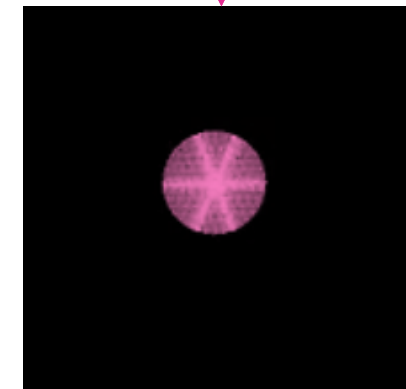Fourier

Frequency Filtering
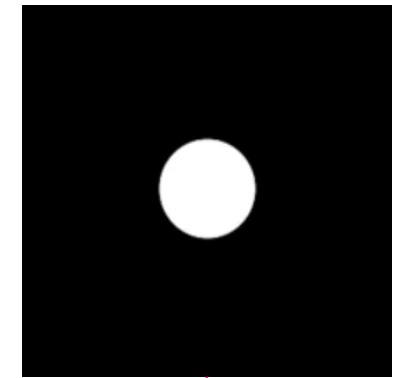
- LPF

Inverse

Fourier

Frequency Filtering

Input

Frequency Filtering

Input

Frequency Filtering

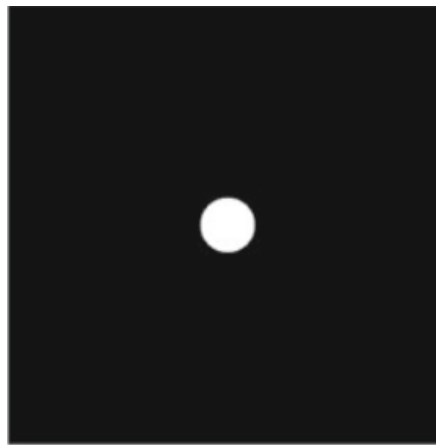Input   LPF

Frequency Filtering

Input

LPF

Frequency Filtering

Input

LPF

Frequency Filtering

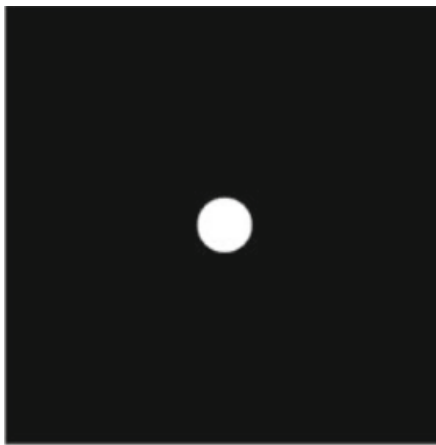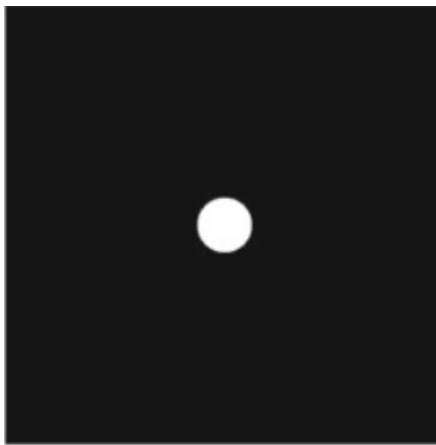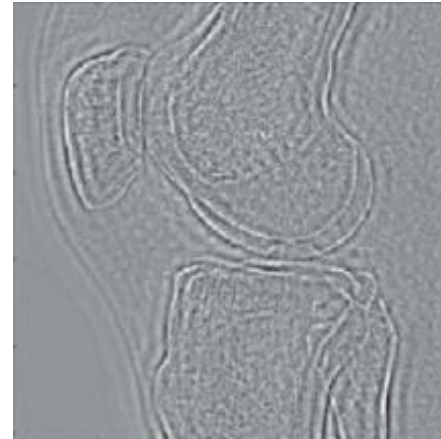Input   LPF   HPF

Frequency Filtering

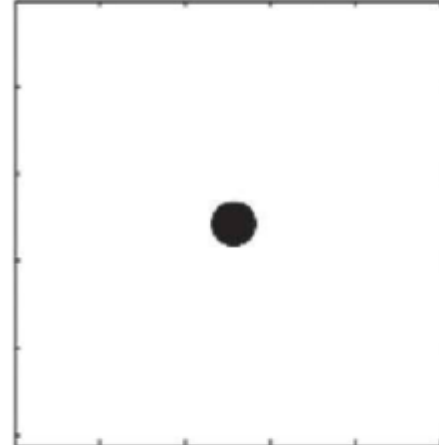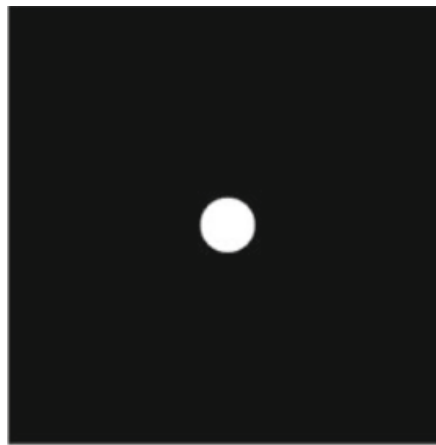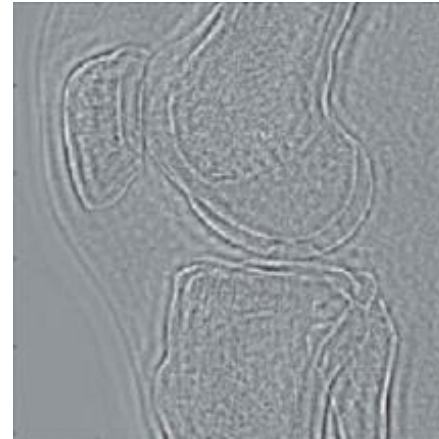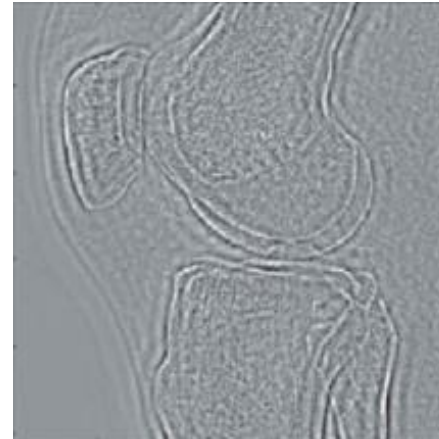Input   LPF   HPF

Frequency Filtering

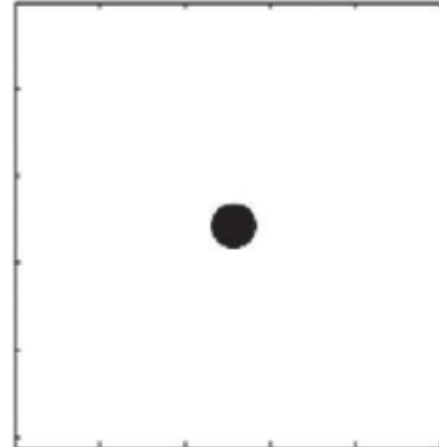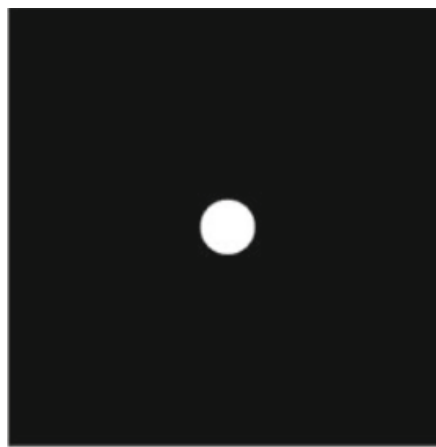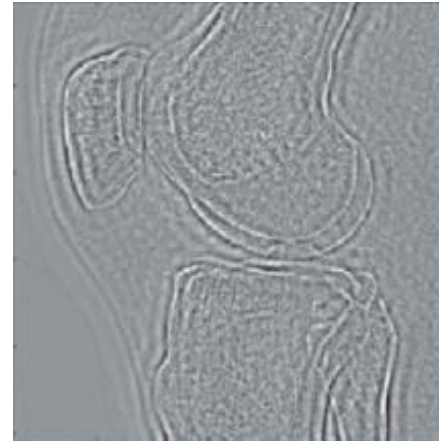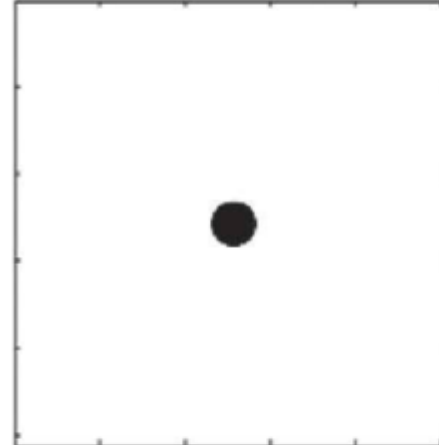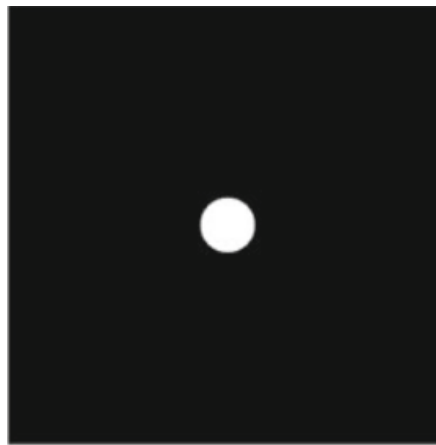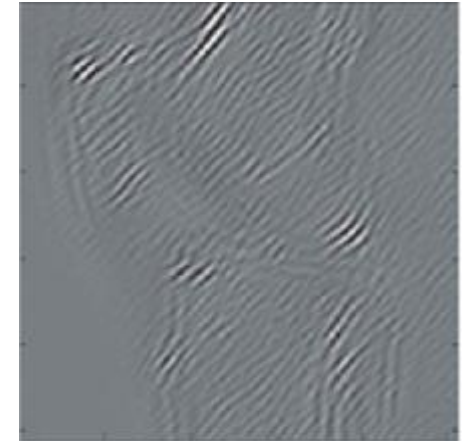Input   LPF   HPF

Frequency Filtering

Input   LPF   HPF   BPF

Frequency Filtering

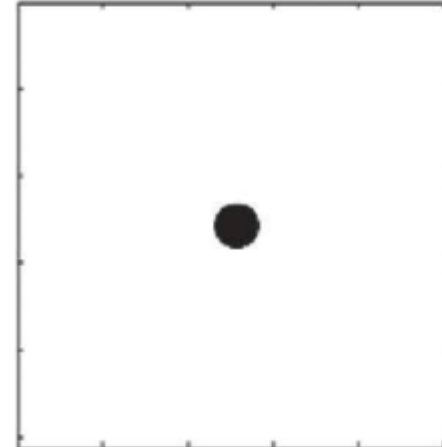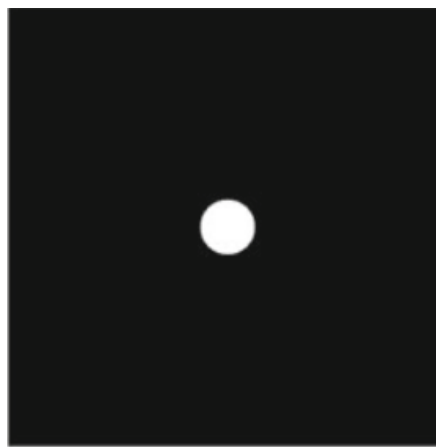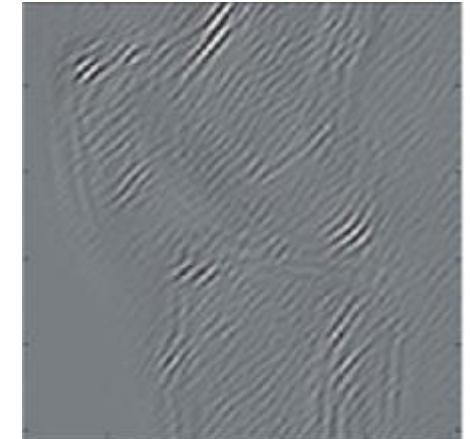| Input	LPF	HPF	BPF

Frequency Filtering
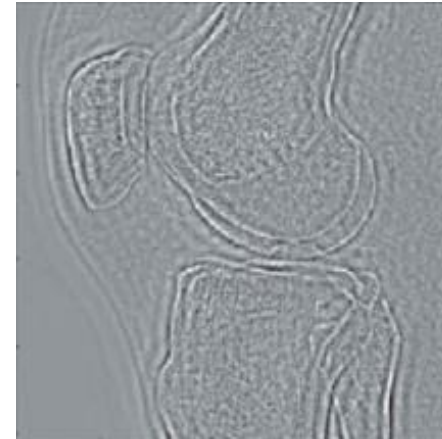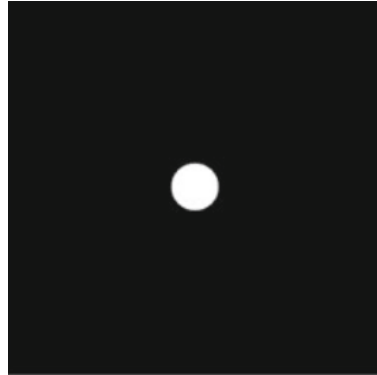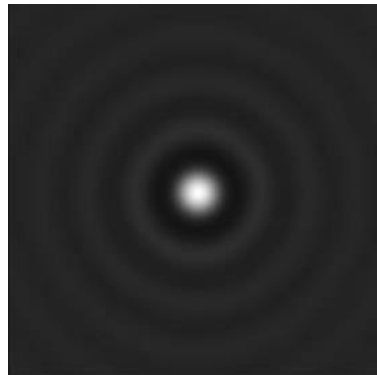
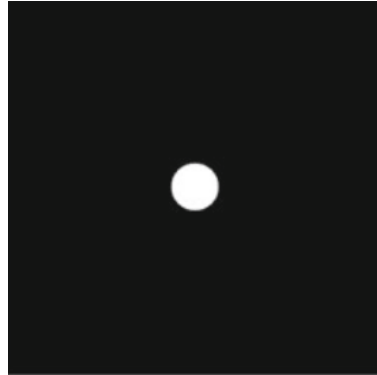Input   LPF   HPF   BPF

Frequency-Spatial Filtering

Input $f(x, y)$

W_1

Frequency-Spatial Filtering
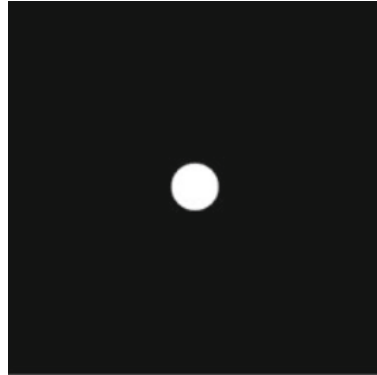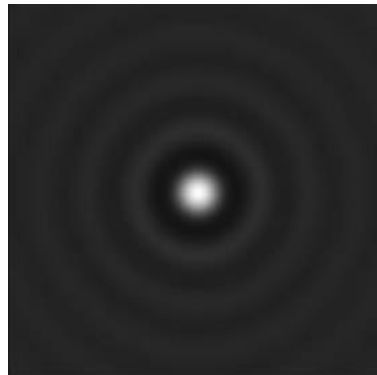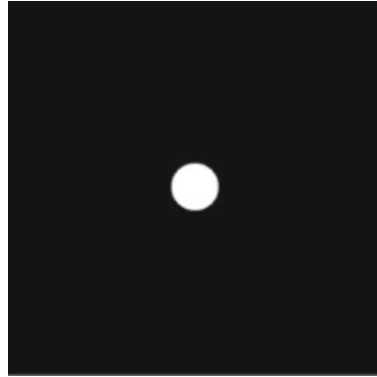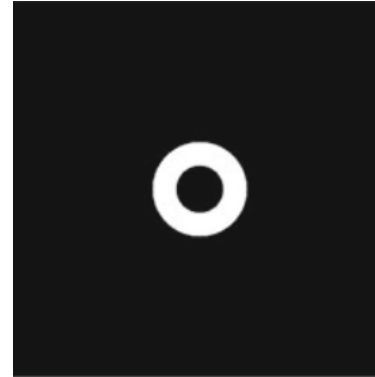
Input $f(x, y)$ W_1

Frequency-Spatial Filtering

Input $f(x, y)$

W_1

w_1

Frequency-Spatial Filtering

Input $f(x, y)$

W_1

W_2

w_1

Frequency-Spatial Filtering

Input $f(x, y)$

W_1

W_2

w_1

Frequency-Spatial Filtering

Input $f(x,y)$

W_1

W_2

w_1

w_2

Frequency-Spatial Filtering

Input $f(x, y)$

W_1

W_2

w_1

w_2

Frequency-Spatial Filtering

Input $f(x, y)$

W_1
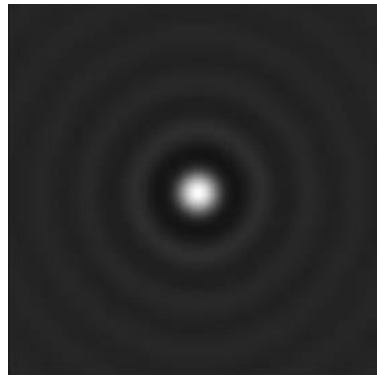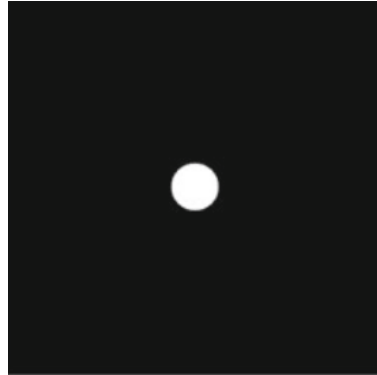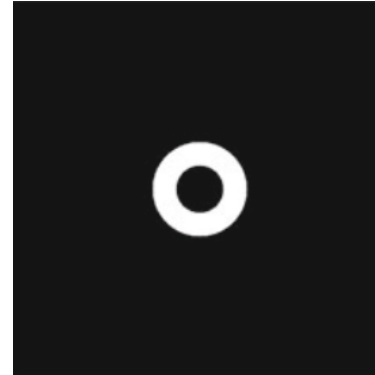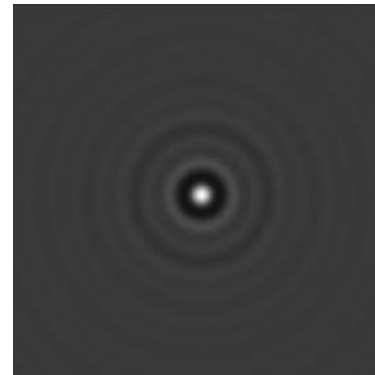
W_2

$w_1 \star f$

w_1

w_2

Frequency-Spatial Filtering

Input $f(x, y)$
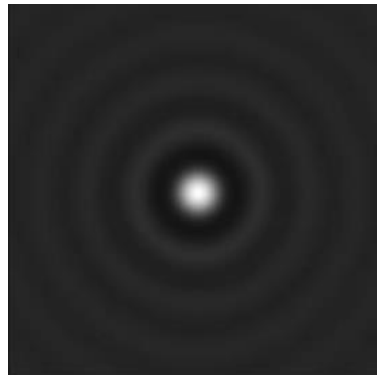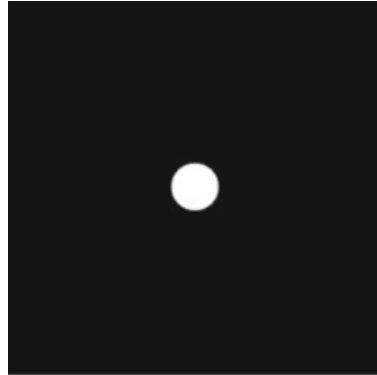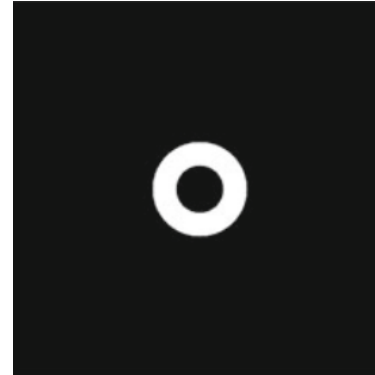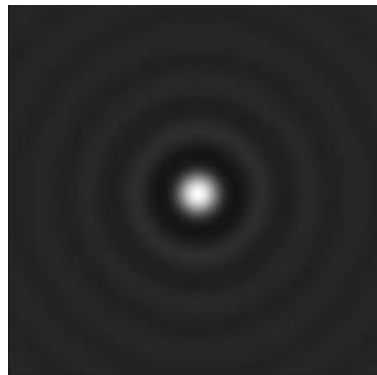
W_1

W_2

$w_1 \star f$

w_1

w_2
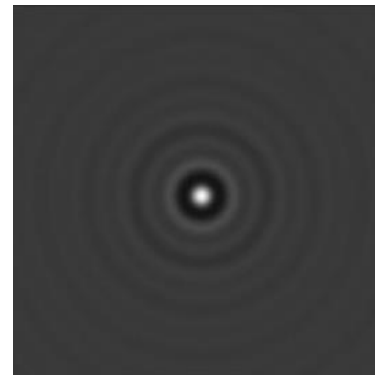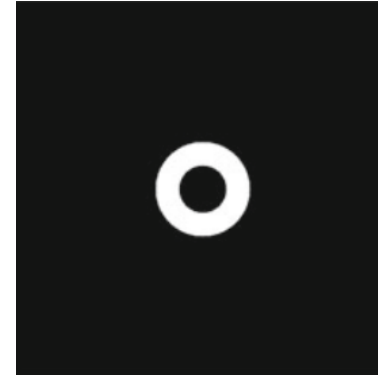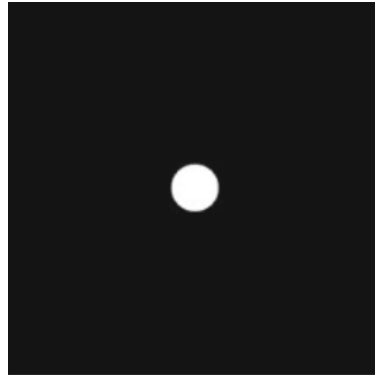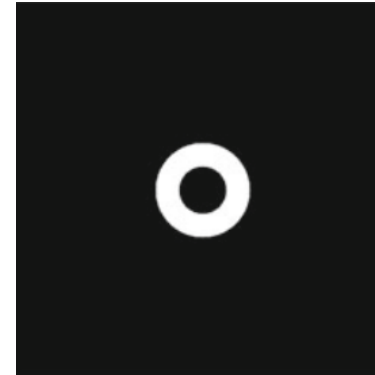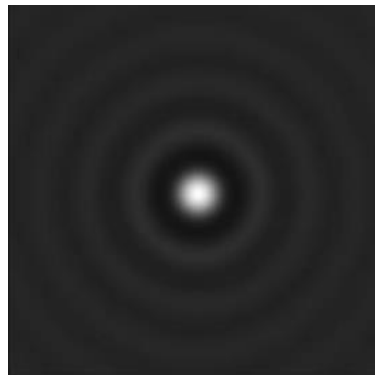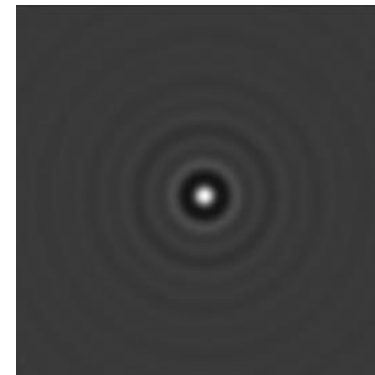
Frequency-Spatial Filtering

Input $f(x,y)$

W_1

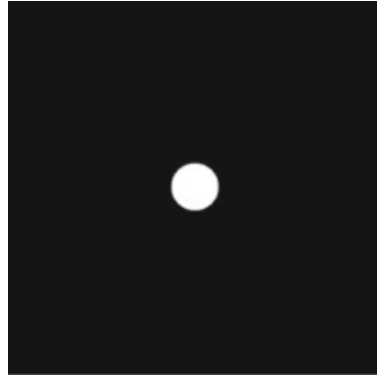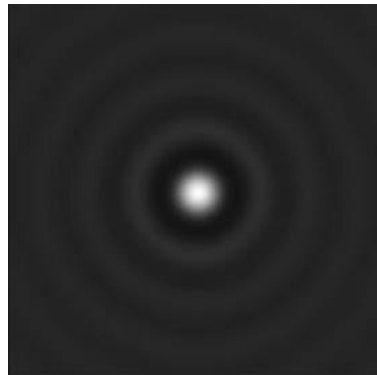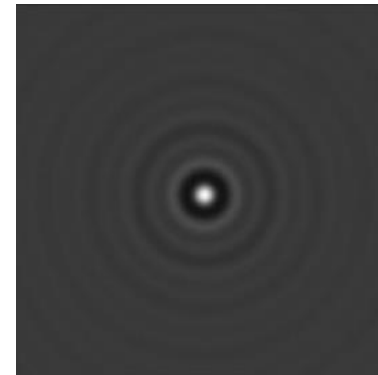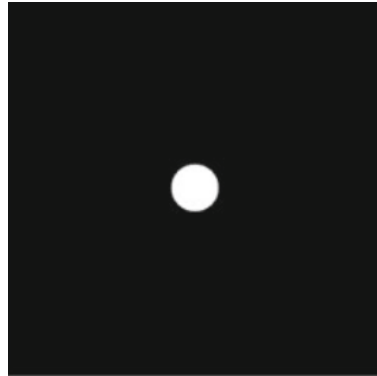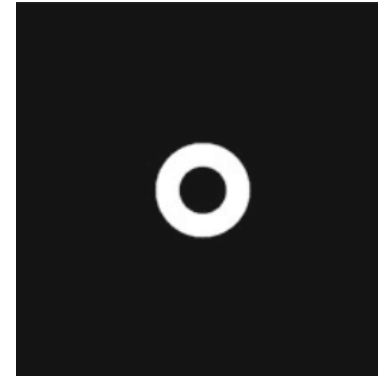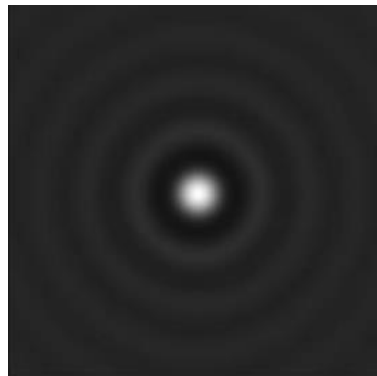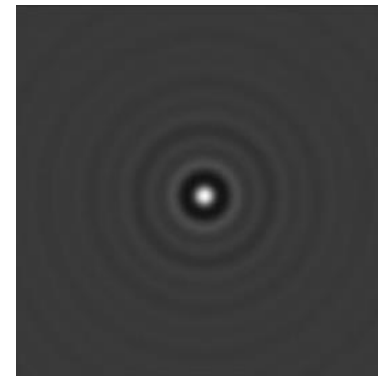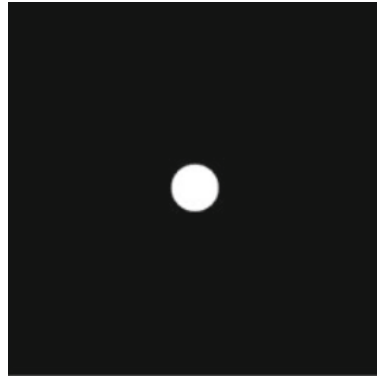W_2
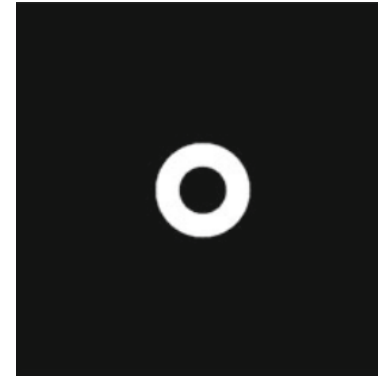
$w_1 \star f$

w_1
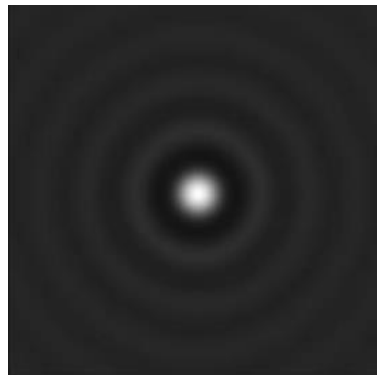
w_2
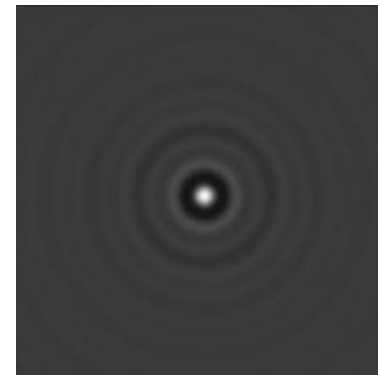
$w_2 \star f$
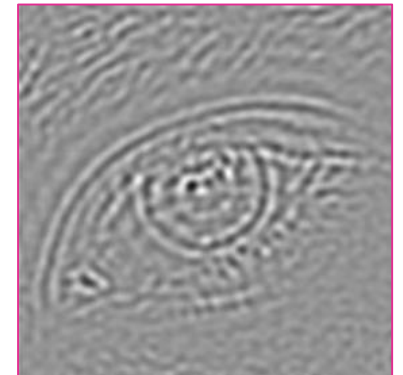
Frequency-Spatial Filtering
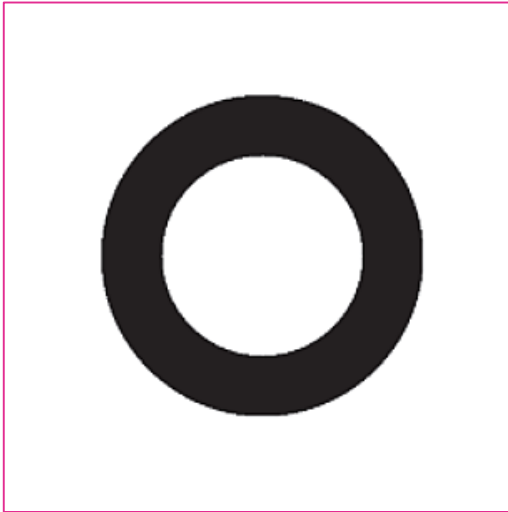
- Prior smoothing to reduce ripple effects

Ideal Mask

Frequency-Spatial Filtering

- Prior smoothing to reduce ripple effects
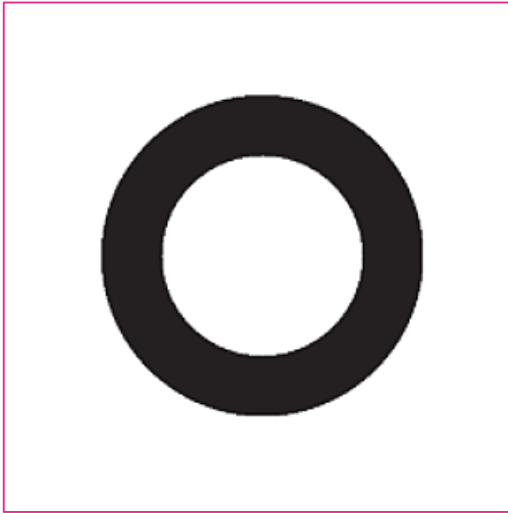
Ideal Mask
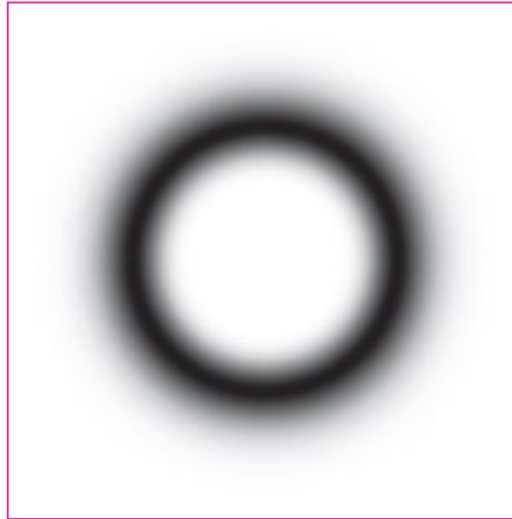
Smooth Mask-1

Frequency-Spatial Filtering

- Prior smoothing to reduce ripple effects
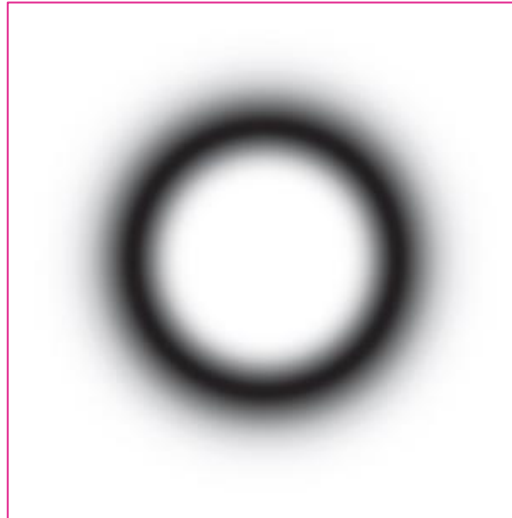
Ideal Mask   Smooth Mask-1   Smooth Mask-2

Frequency-Spatial Filtering

Forget me, but
don't forget my car!

Frequency-Spatial Filtering

Forget me, but
don't forget my car!

Frequency-Spatial Filtering

Forget me, but
don't forget my car!

Frequency-Spatial Filtering

Forget me, but
don't forget my car!

Frequency-Spatial Filtering

Forget me, but
don't forget my car!

Frequency Filtering & HVS

credit: A. Oliva

Frequency Filtering & HVS
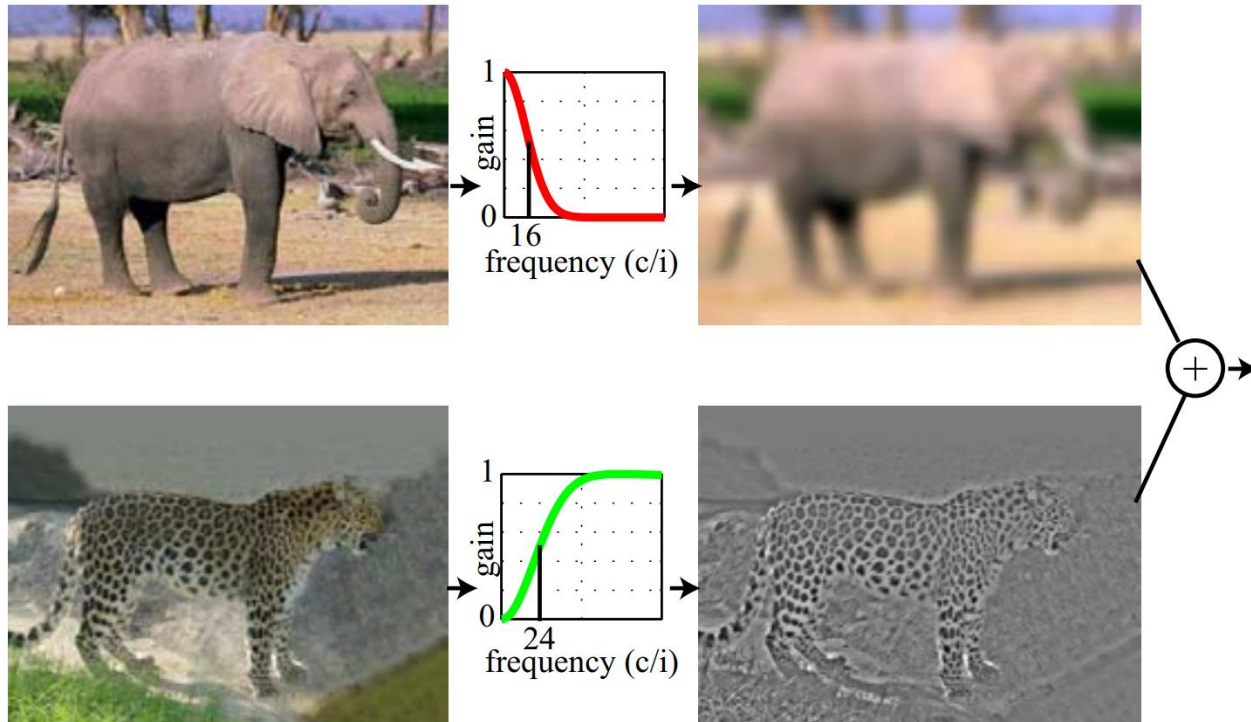
credit: A. Oliva

Frequency Filtering & HVS

credit: A. Oliva

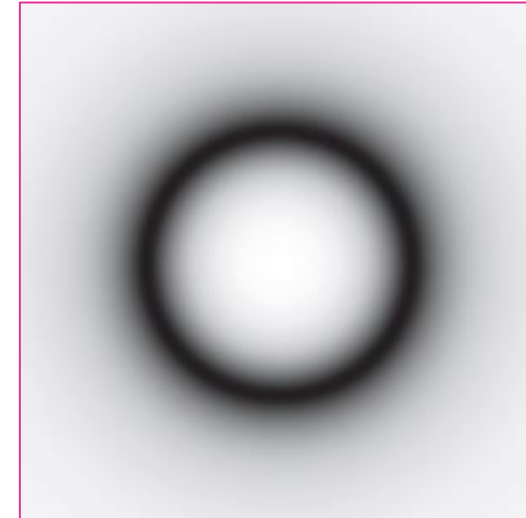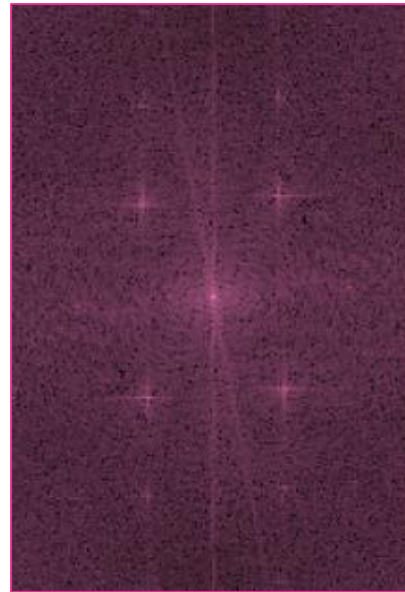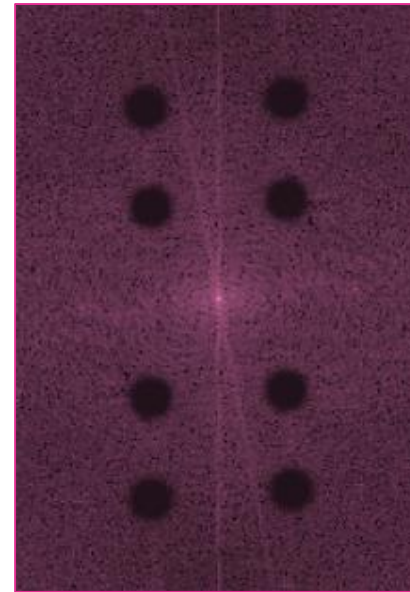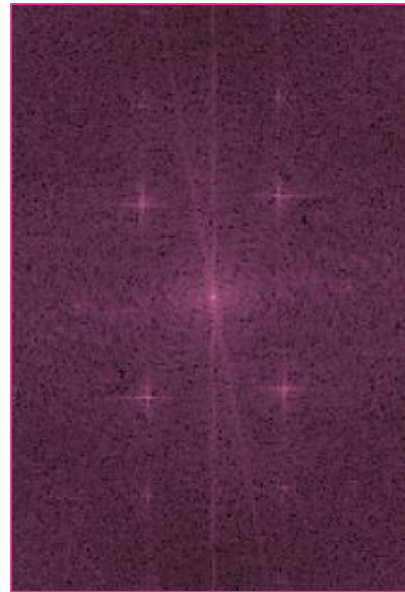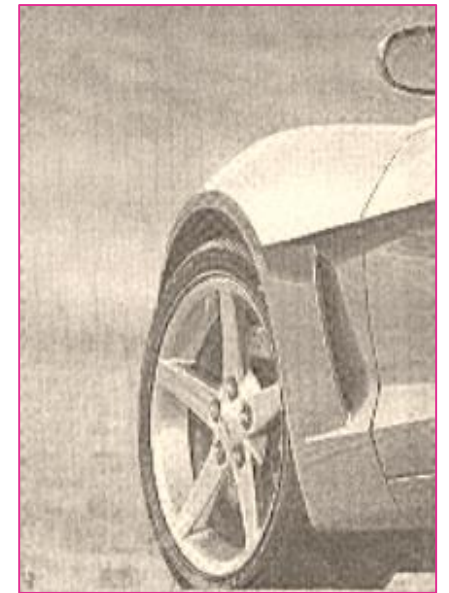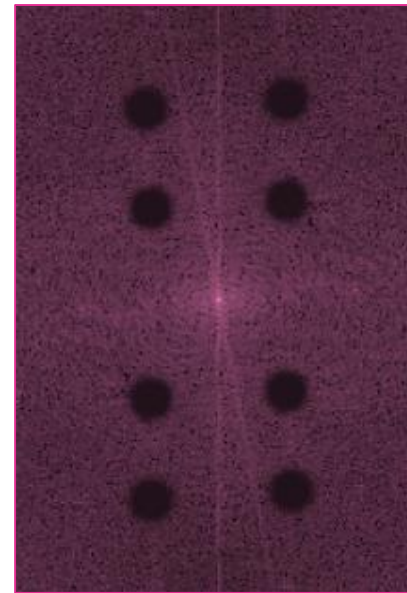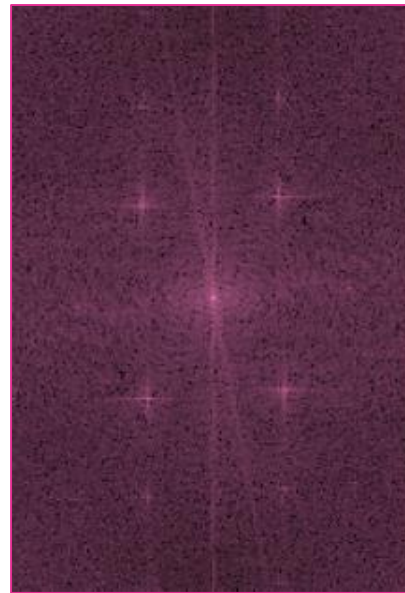# Conclusion

- 2D FT properties & images

- Frequency filtering

Conclusion

- 2D FT properties & images

- Frequency filtering

❑ 2D Fourier Transform

- Properties

- Convolution theorem

- 2D FT images

❑ Frequency filtering

- Filtering in FT domain

- Freq-spatial filtering

- Freq-mixing

Conclusion

- 2D FT properties & images

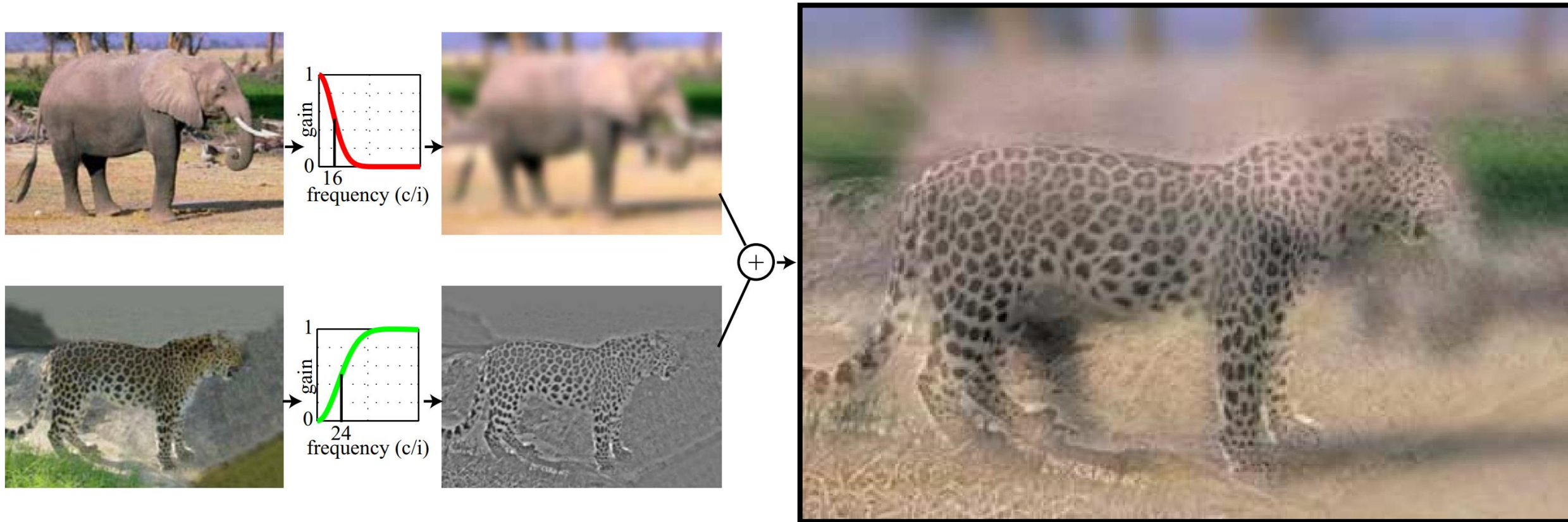- Frequency filtering

❑ 2D Fourier Transform
  - Properties
  - Convolution theorem
  - 2D FT images

❑ Frequency filtering
  - Filtering in FT domain
  - Freq-spatial filtering
  - Freq-mixing