# Segmentation:

## Regions & K-means

Dr. Tushar Sandhan

# Introduction

- Goal of segmentation


Image credit: J. Jordan

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# Region growing

- Partition the image $I$ into $m$ regions
  - every pix belong to some region
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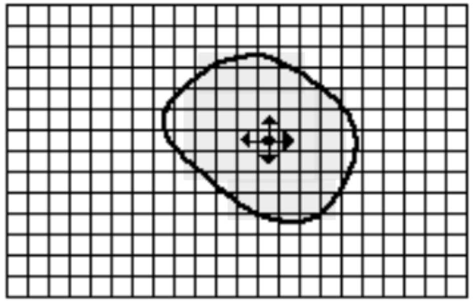

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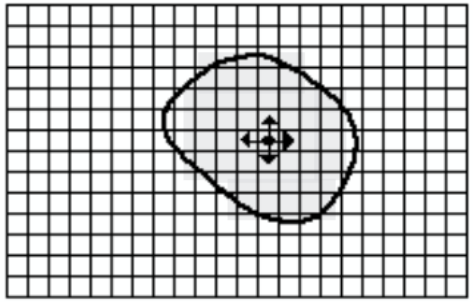

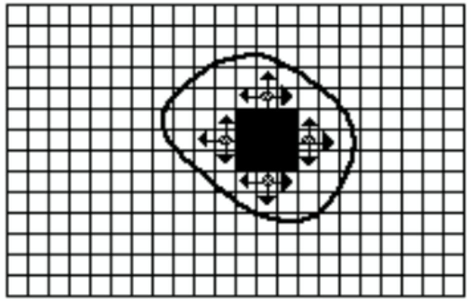

after few iterations

# Region growing

- Partition and grow

---

- Start with a seed point $s_j$ for region $R_j$
- For every $s_j$
  - Initialize mean intensity of each region: $\mu_j = s_j$
  - Initialize region: $R_j = \{s_j\}$
- For each point $p$ in $R_j$
  - Get its 4-connect neighborhood: $\mathcal{N}_i(p),\ i = 1, 2, 3, 4$
  - If $|\mathcal{N}_i(p) - \mu_j| < \tau$ , $\mathcal{N}_i(p) \notin \mathcal{R}_k \quad j \neq k$
    - $\mathcal{R}_j \leftarrow \mathcal{R}_j \bigcup \mathcal{N}_i(p)$
    - update $\mu_j$
  - Stop growing when no neighborhood pixel matches
- Move to the next seed point, until the whole image is partitioned.

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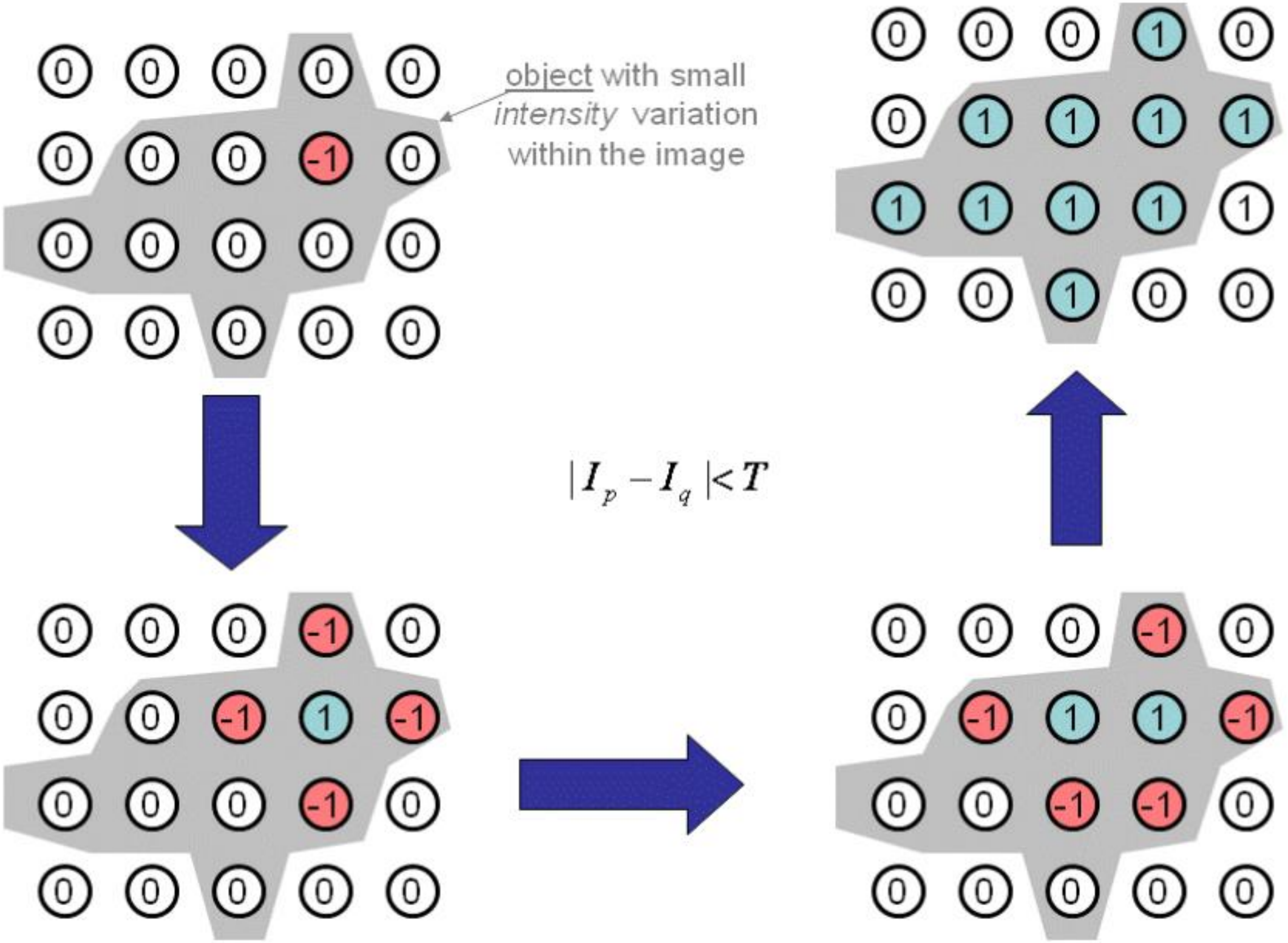
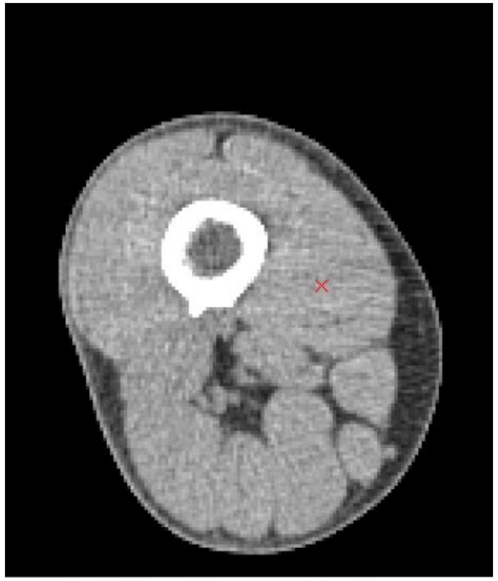
# Region growing



# Region growing: CT scan

- Seed-1



stricter similarity

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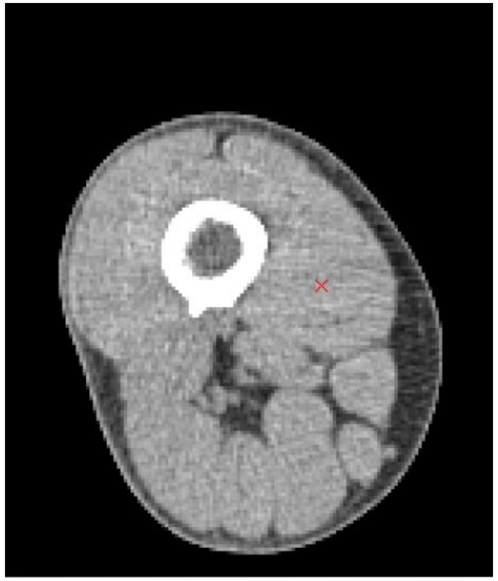

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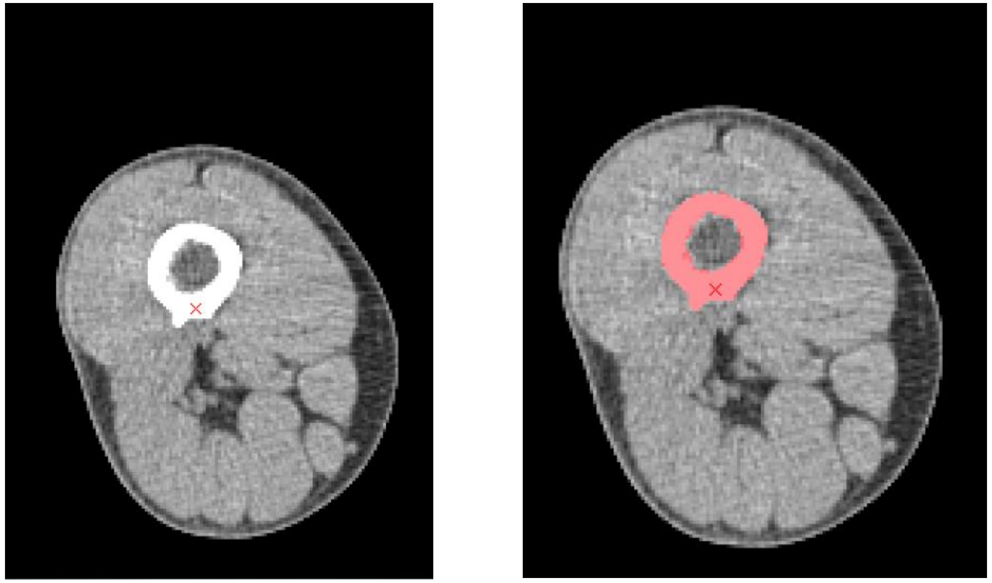
- Seed-2



# Comparative example

| | | | | | | |
|---|---|---|---|---|---|---|
| 3 | 5 | 7 | 3 | 4 | 2 | 1 |
| 2 | 4 | 9 | 10 | 22 | 9 | 3 |
| 3 | 5 | 12 | 11 | 15 | 10 | 3 |
| 5 | 6 | 11 | 9 | 17 | 19 | 1 |
| 2 | 3 | 11 | 12 | 18 | 16 | 2 |
| 3 | 6 | 8 | 10 | 18 | 9 | 5 |
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| | | | | | | |
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threshold $T \geq 10$

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threshold $T \geq 12$

| | | | | | | |
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region growing with variance of 2 in respect to value 11 with reference to threshold $T \geq 11$

# Region splitting & merging

- Split
  - sub-quadrants
    - e.g. 4 parts: quadregions
    - quadtree (having leaves as quadregions or quadimages)
  - continuous splitting
    - adjacent quadimages will be having identical properties

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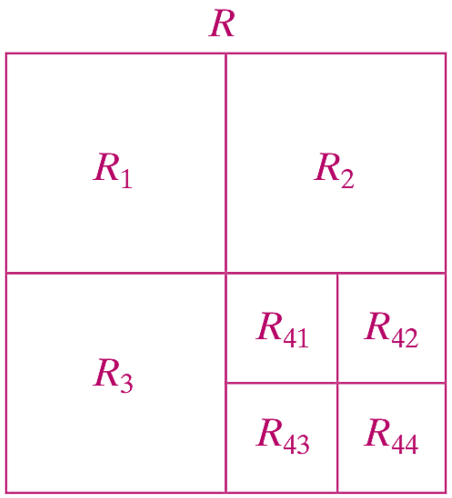

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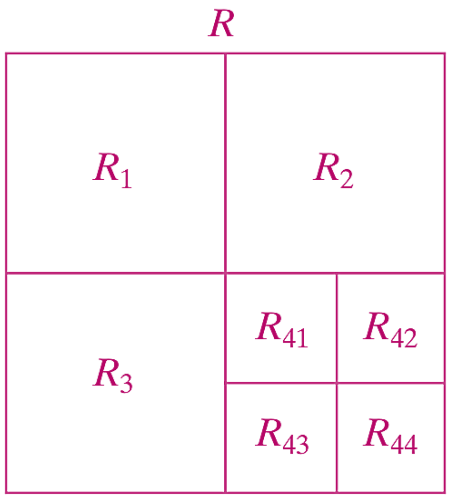


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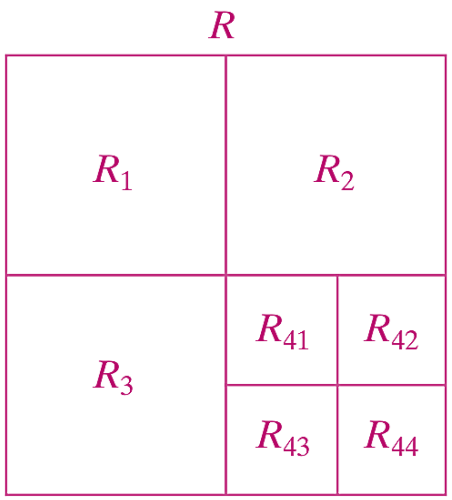


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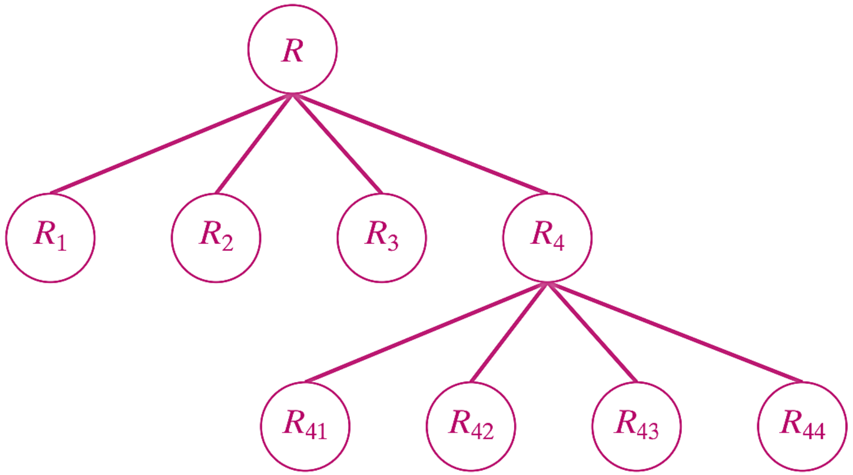

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- Segment the ring of supernova
  - quadimages size 32x32, 16x16 & 8x8
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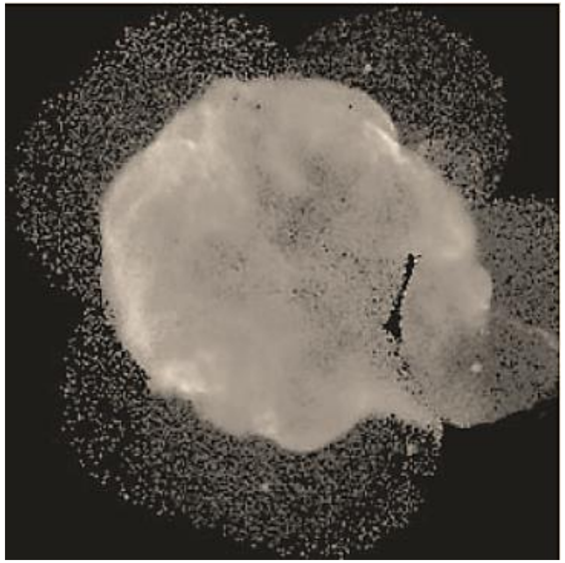
X-ray band image



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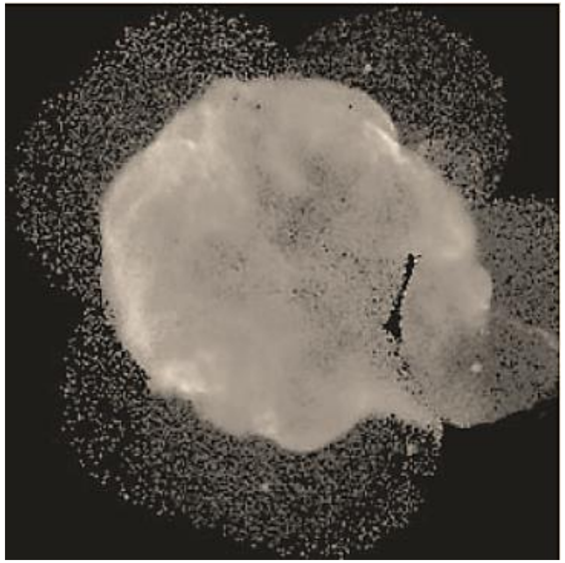

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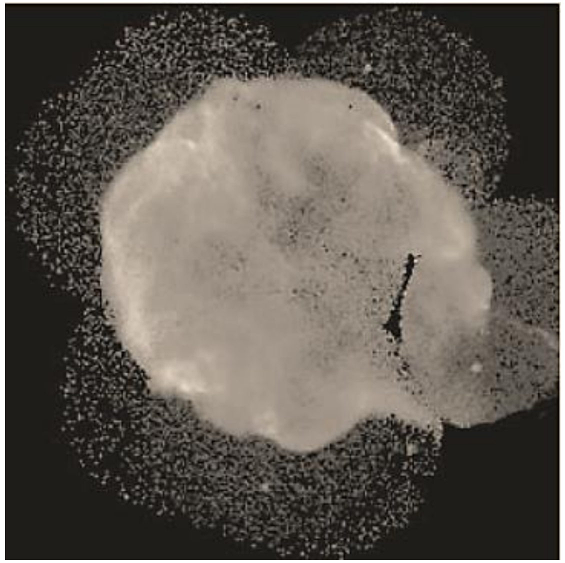

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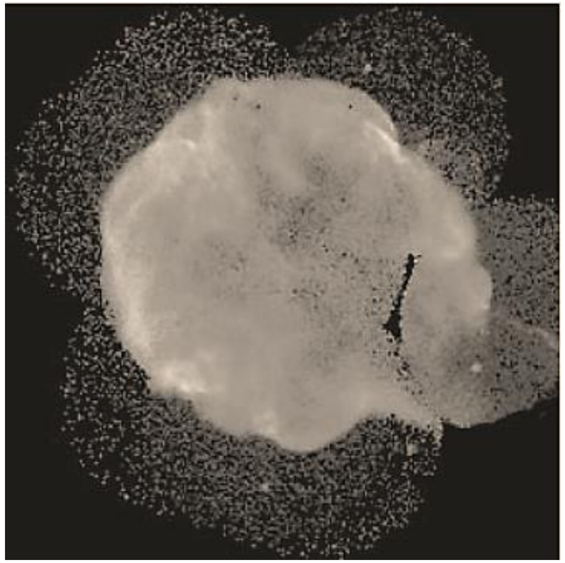

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  - intra-class variance is low (high similarity)
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- Unsupervised learning paradigm
  - finding class labels directly from data
  - training data labels are not available
- What are similarity measures:
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- K-means clustering
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  - detects united patterns e.g. groups of text topics, regions of images
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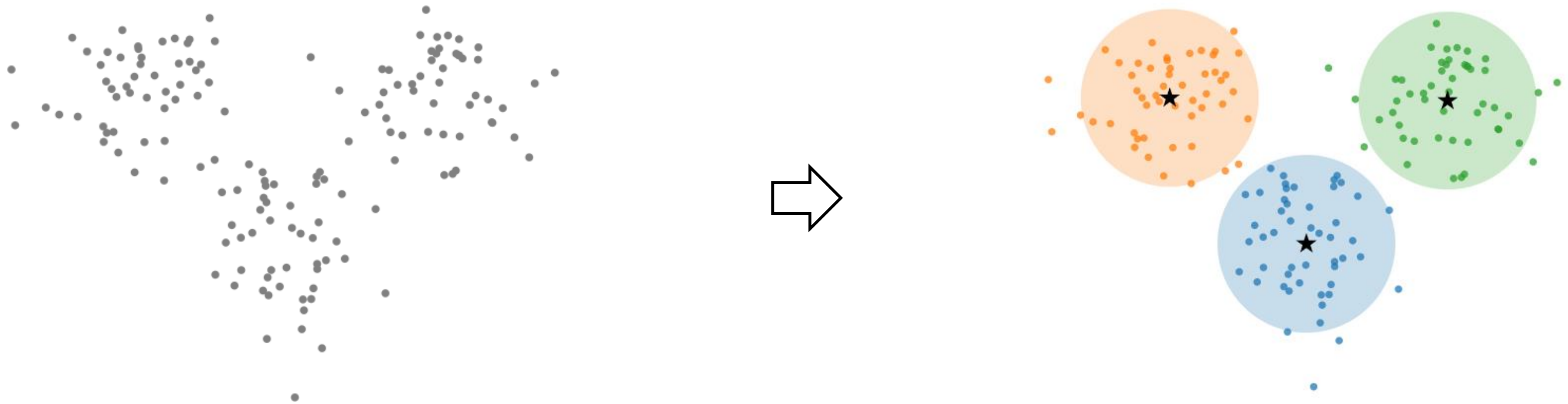

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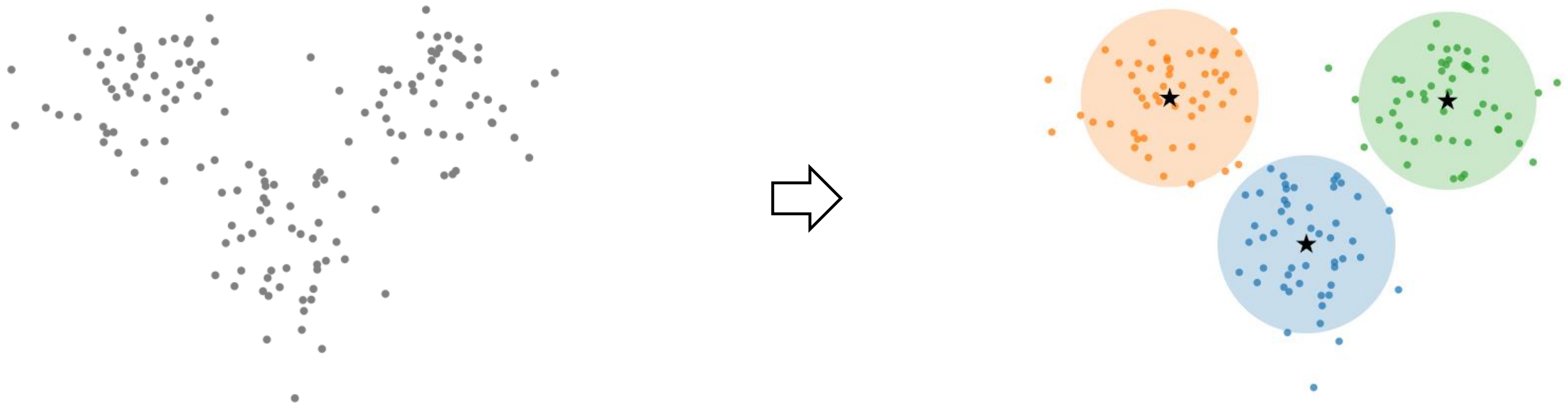

# K-Means

- **Input:** $x^{(1)}, x^{(2)}, ..., x^{(n)}$
- **Output:** Set of clusters $C_1, C_2, ...C_k$
- **Initialization:** Randomly pick ***k*** centroids $z^{(1)}, z^{(2)}, \ldots, z^{(k)}$
- **Itereate** until convergence or up to iterations $T$
  - **Assignment:** Assign each point to its closest centroid

    $$\text{for each } j = 1, .., k$$
    $$C_j = \{i | \text{s.t. } x^{(i)} \text{ is closest to } z^{(j)}\}$$

  - **Update:** Recompute centroids with newly assigned points

    $$z^{(j)} = \frac{1}{|C_j|} \sum_{i \in C_j} x^{(i)}$$

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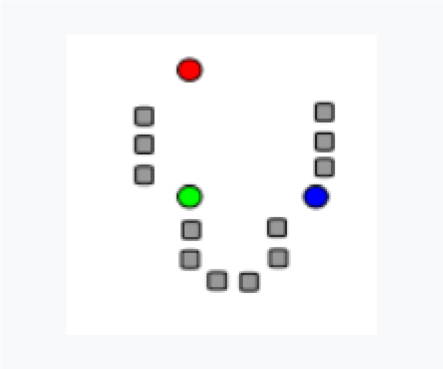

Courtesy: wiki

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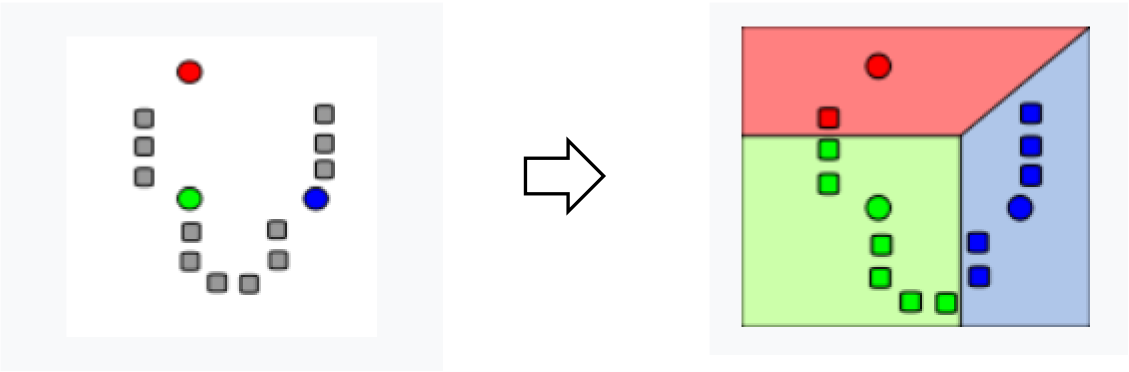

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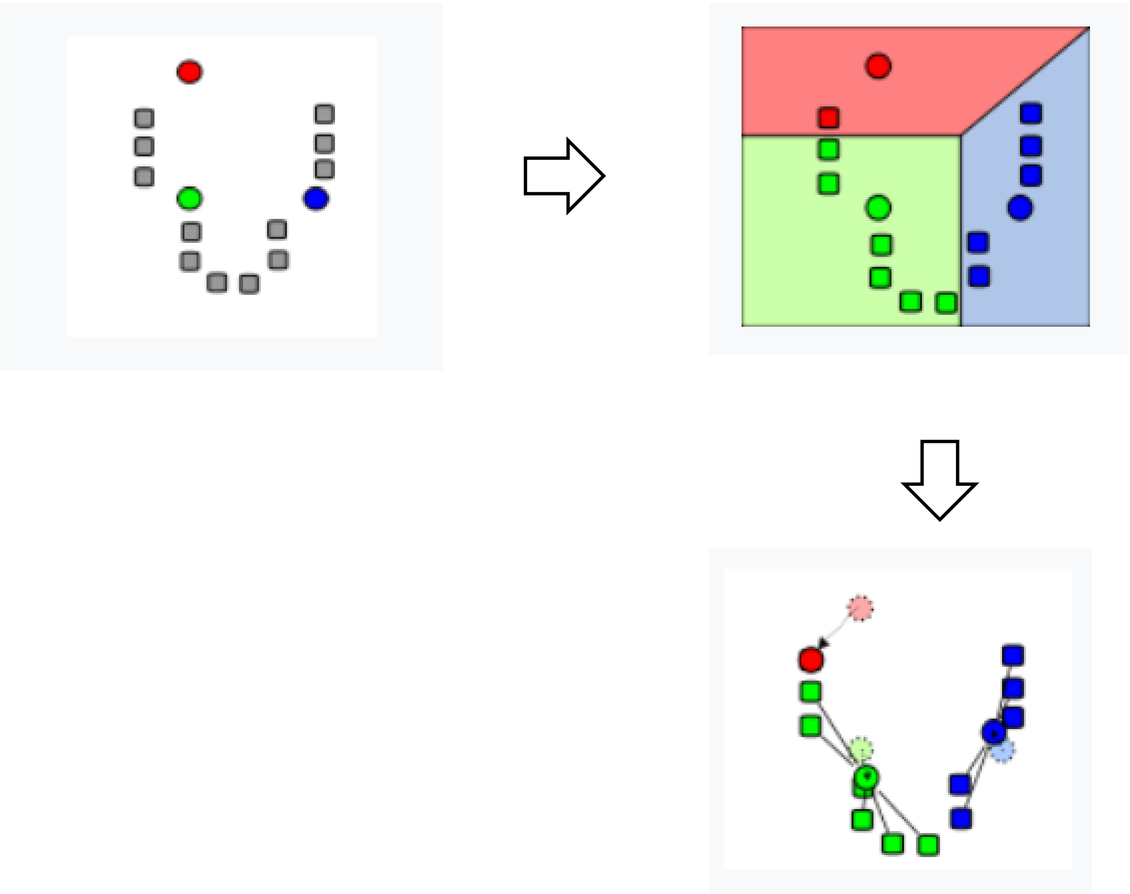

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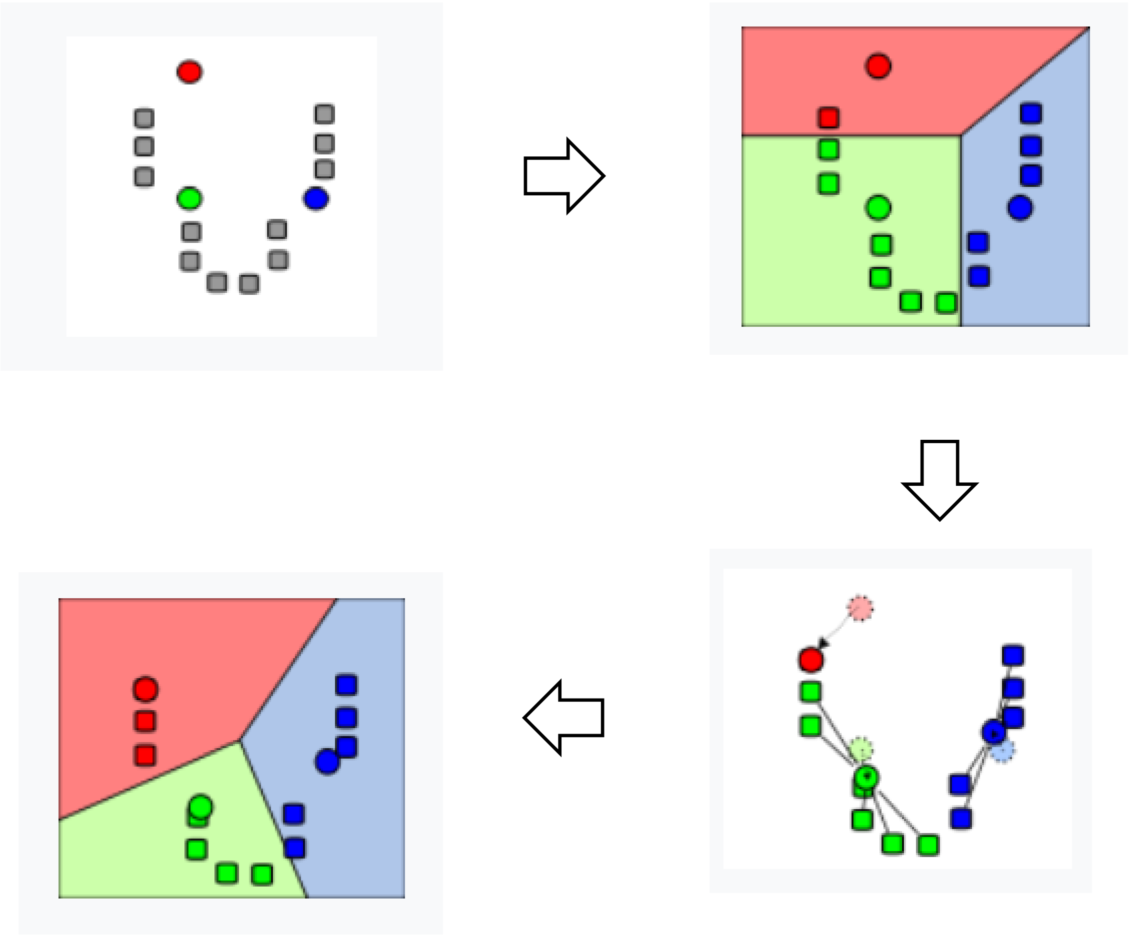

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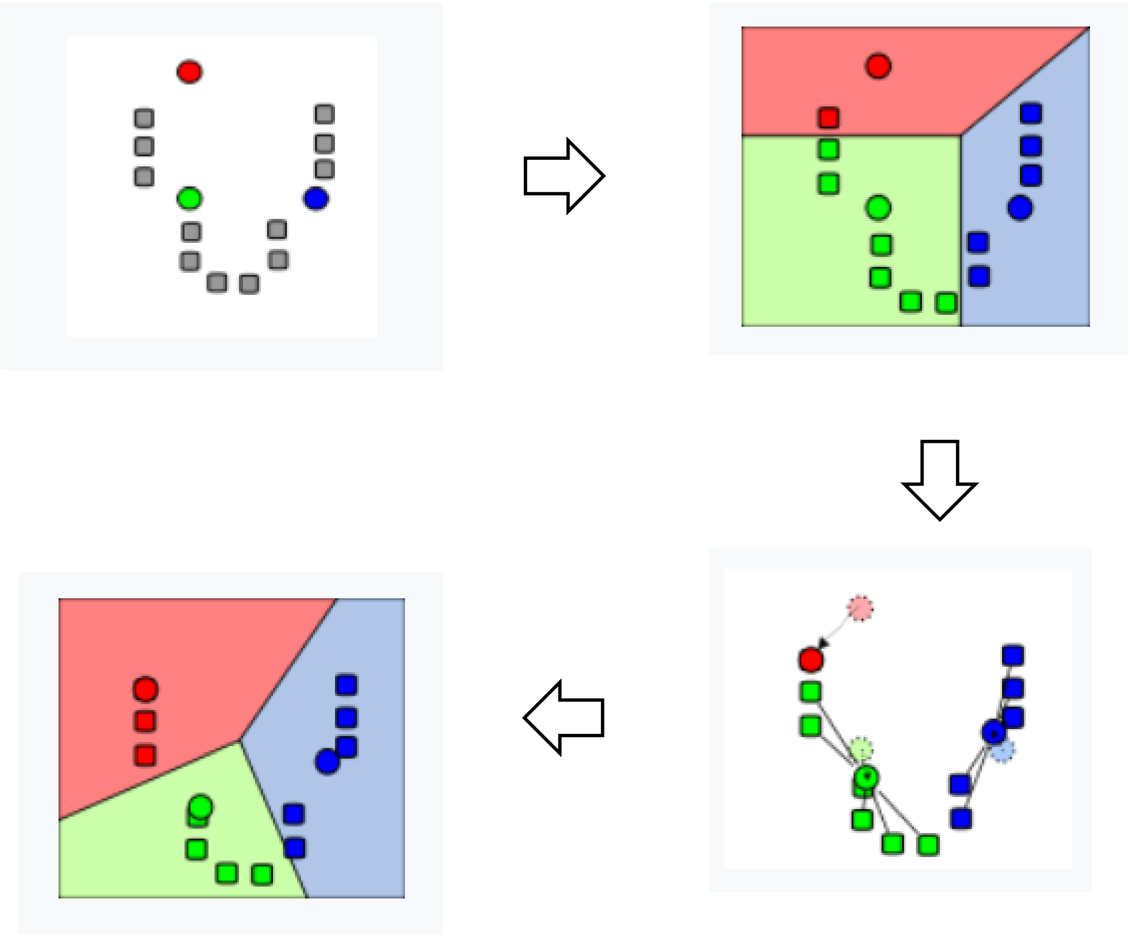

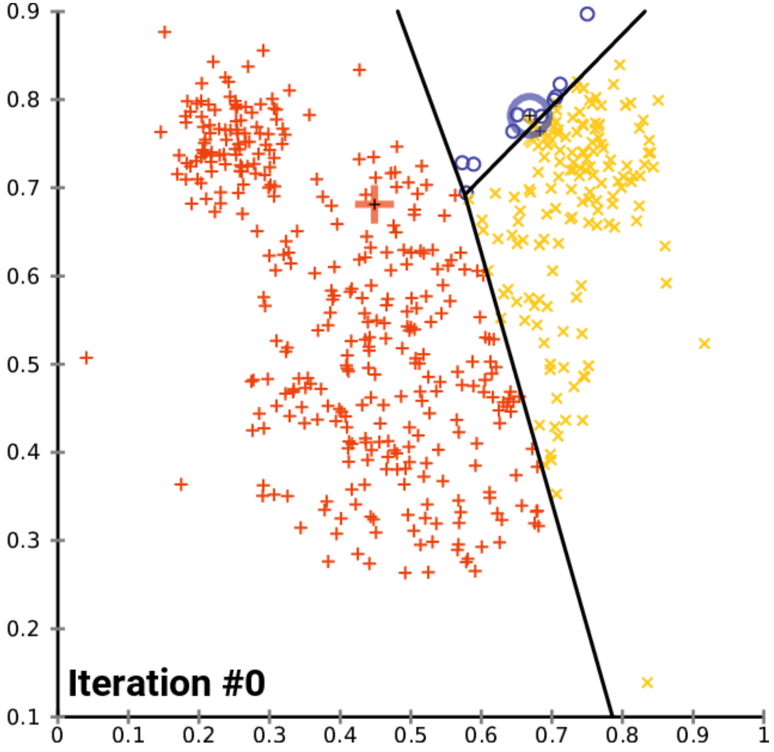


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# K-Means

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  - guaranteed to converge in a finite iterations
    - at each iteration the error reduces
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    - data assignment to the closest cluster: $O(kN)$
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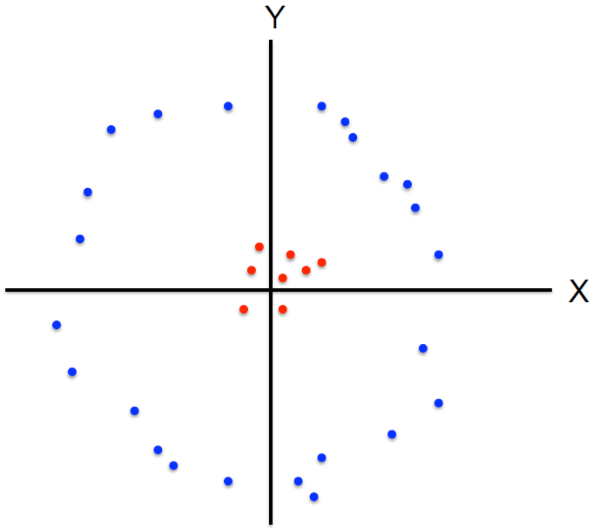

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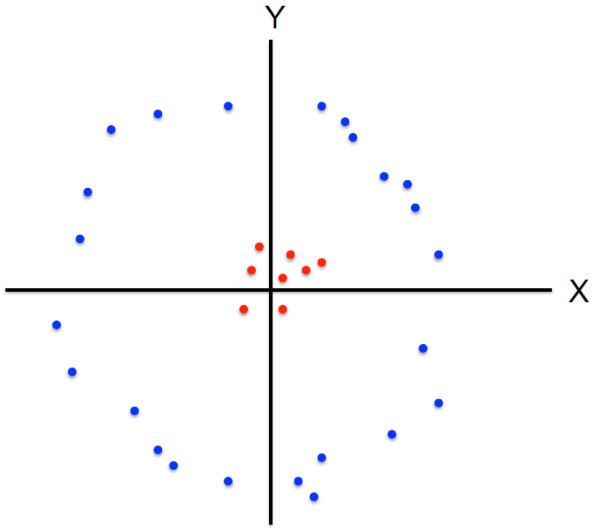


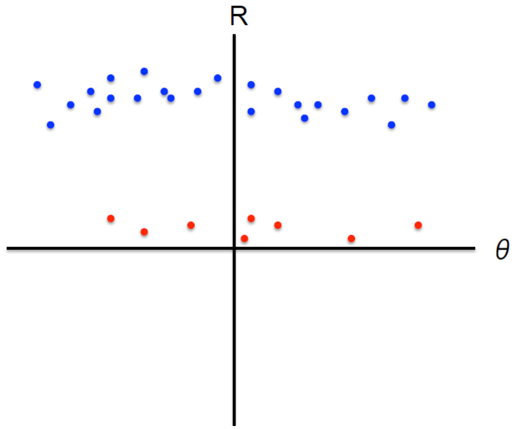

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- Convergence

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  - update: fix $C$, optimize for $z$

$$J(z) = \min_{z^{(1)},\dots,z^{(k)}} \sum_{j=1}^{k} \sum_{i \in C_j} \|x^{(i)} - z^{(j)}\|^2$$

$$z^{(j)} = \frac{1}{|C_j|} \sum_{i \in C_j} x^{(i)}$$

$$\frac{\delta J(z)}{\delta z^{(j)}} \to 0$$

# K-Means

input

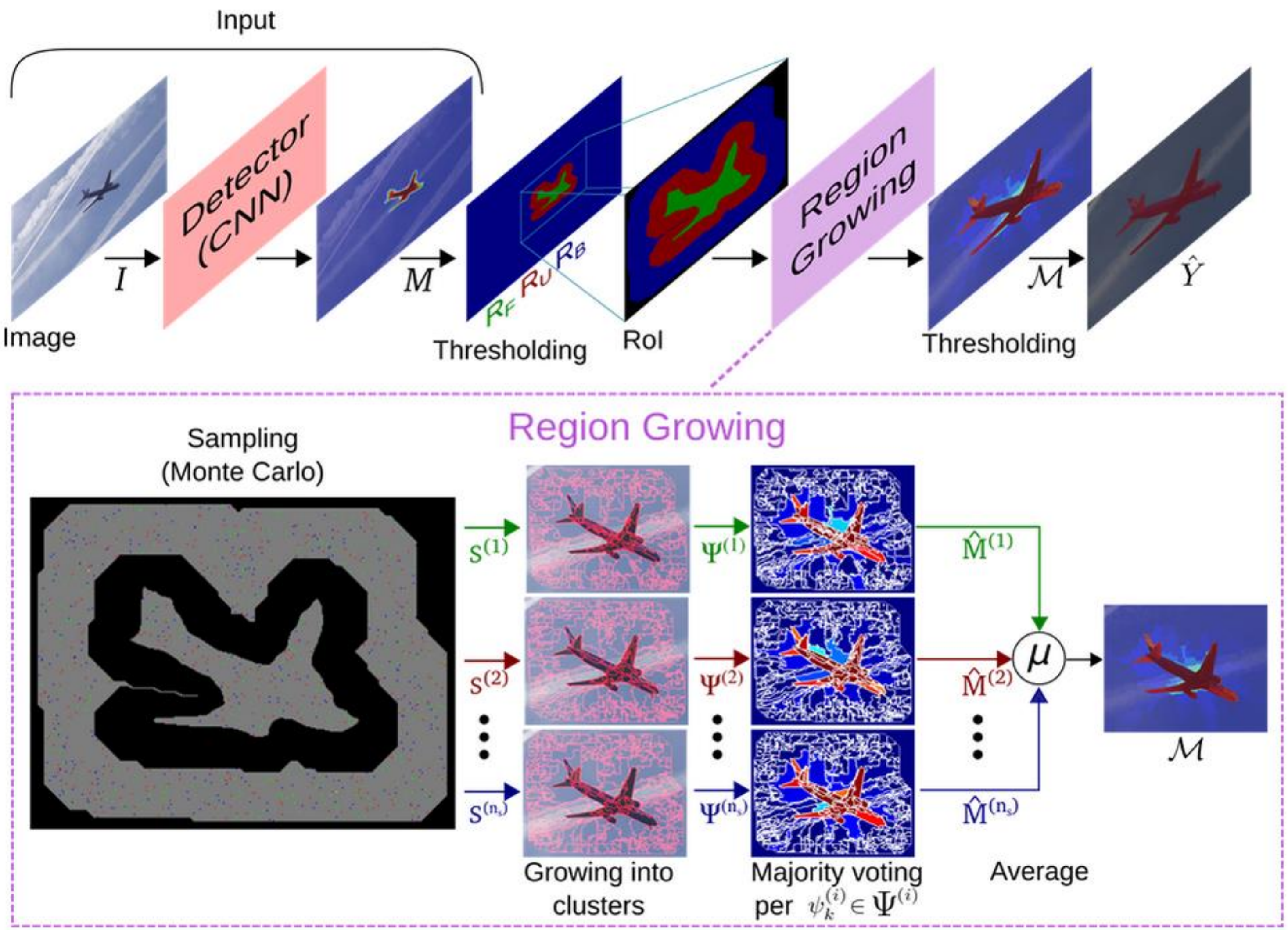
K = 2

K = 3

K = 10

Courtesy: D. Sontag

# Region growing in feature space

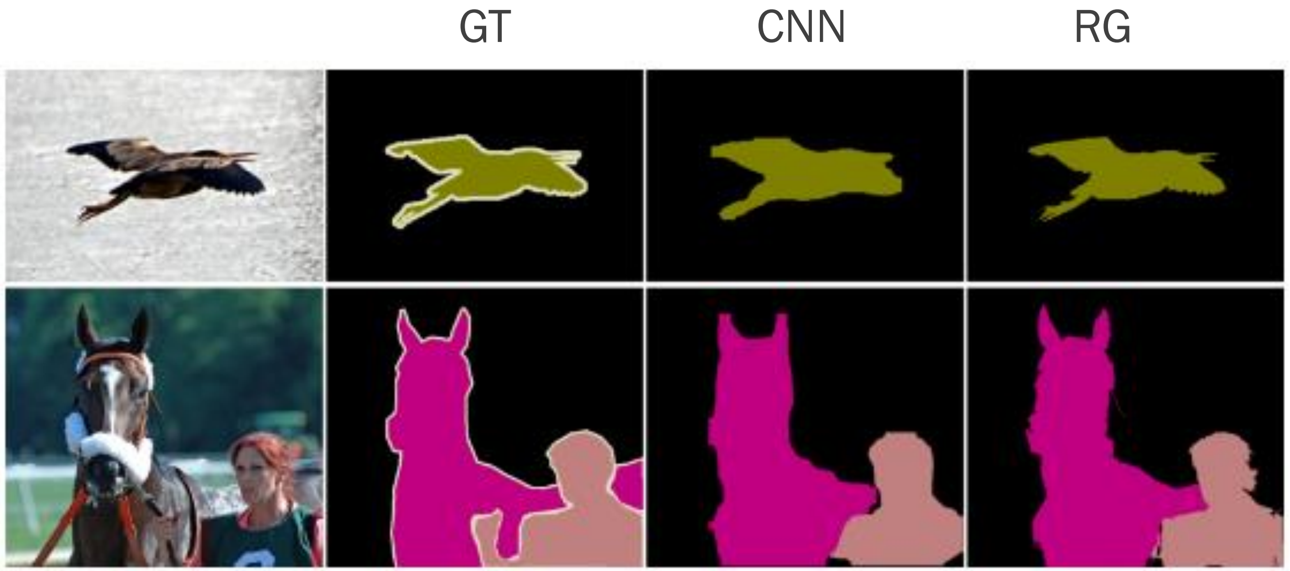



$$R_F = \{p_j | M(p_j) > \tau_F\},$$
$$R_U = \{p_j | \tau_B < M(p_j) < \tau_F\},$$
$$R_B = \{p_j | M(p_j) < \tau_B\},$$

Image credit: PA Dias

# Conclusion

- Regions
- Clustering



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- Region growing
- Region splitting & merging
- Clustering
  - K-means clustering