

Edge: Operators

Dr. Tushar Sandhan

Introduction

- Who am I?



Introduction

- Who am I?



many

Introduction

- Who am I?



many

Introduction

- Who am I?



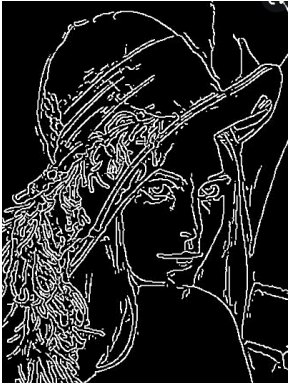
many



few, dim

Introduction

- Who am I?



many



few, dim



Introduction

- Who am I?



many



few, dim



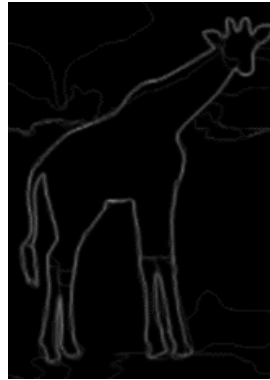
non-uniform

Introduction

■ Who am I?



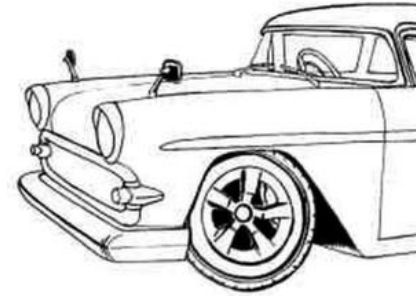
many



few, dim

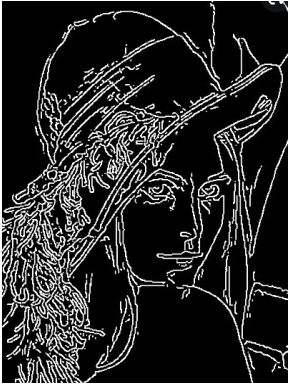


non-uniform



Introduction

■ Who am I?



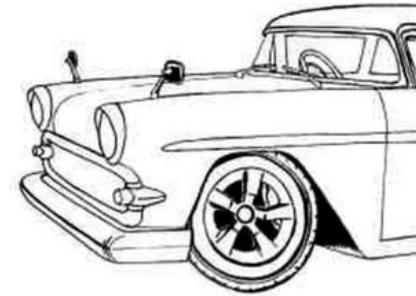
many



few, dim



non-uniform



patchy

Introduction

■ Who am I?



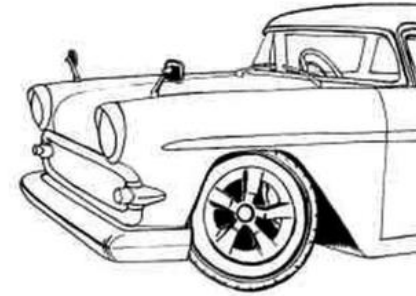
many



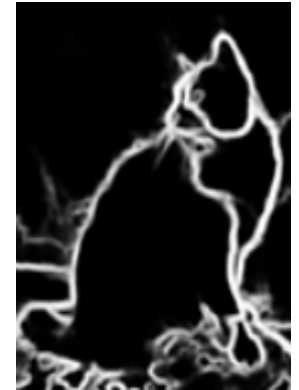
few, dim



non-uniform

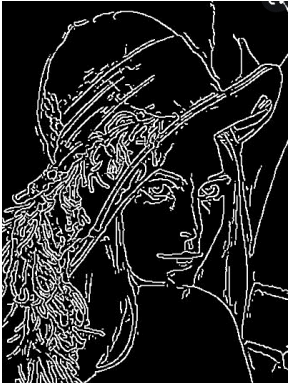


patchy

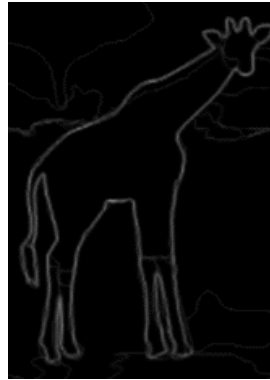


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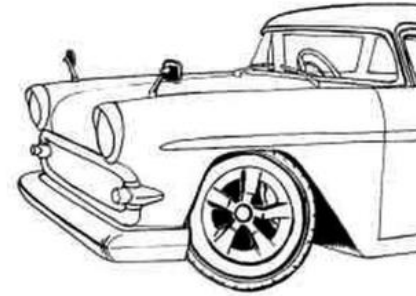
many



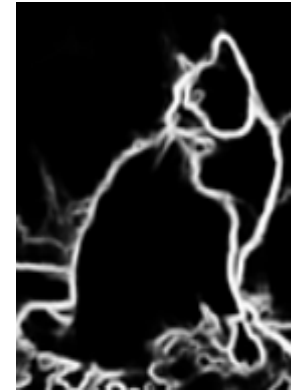
few, dim



non-uniform



patchy



mewww~

Introduction

- Who am I?



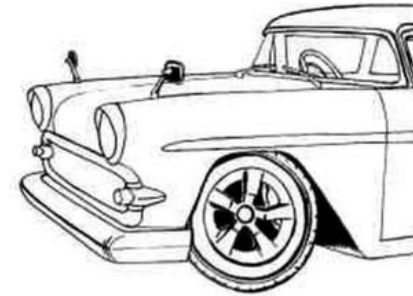
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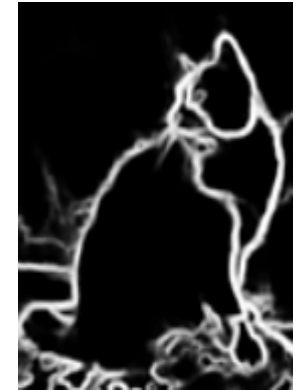
few, dim



non-uniform



patchy



mewww~

- It implies: edges convey a lot of info.
- Lossy but extremely high compression

What makes edges



What makes edges



- Discontinuity in color

What makes edges



- Discontinuity in color
- Change in surface normal

What makes edges



- Discontinuity in color
- Change in surface normal
- Change in illumination

What makes edges



- Discontinuity in color
- Change in surface normal
- Change in illumination
- Depth discontinuity

What makes edges



- Discontinuity in color
- Change in surface normal
- Change in illumination
- Depth discontinuity
- Reflectance change

What makes edges



- Discontinuity in color
- Change in surface normal
- Change in illumination
- Depth discontinuity
- Reflectance change
- Inoculation is an edge itself!

What makes edges



- Discontinuity in color
- Change in surface normal
- Change in illumination
- Depth discontinuity
- Reflectance change
- Inoculation is an edge itself!

Edge

- Taylor's edge
 - expand $f(x + \Delta x)$

Edge

- Taylor's edge

- expand $f(x + \Delta x)$

$$\begin{aligned} f(x + \Delta x) &= f(x) + \Delta x \frac{\partial f(x)}{\partial x} + \frac{(\Delta x)^2}{2!} \frac{\partial^2 f(x)}{\partial x^2} + \frac{(\Delta x)^3}{3!} \frac{\partial^3 f(x)}{\partial x^3} + \dots \\ &= \sum_{n=0}^{\infty} \frac{(\Delta x)^n}{n!} \frac{\partial^n f(x)}{\partial x^n} \end{aligned}$$

Edge

- Taylor's edge

- expand $f(x + \Delta x)$

$$\begin{aligned} f(x + \Delta x) &= f(x) + \Delta x \frac{\partial f(x)}{\partial x} + \frac{(\Delta x)^2}{2!} \frac{\partial^2 f(x)}{\partial x^2} + \frac{(\Delta x)^3}{3!} \frac{\partial^3 f(x)}{\partial x^3} + \dots \\ &= \sum_{n=0}^{\infty} \frac{(\Delta x)^n}{n!} \frac{\partial^n f(x)}{\partial x^n} \end{aligned}$$

$$\begin{aligned} f(x + 1) &= f(x) + \frac{\partial f(x)}{\partial x} + \frac{1}{2!} \frac{\partial^2 f(x)}{\partial x^2} + \frac{1}{3!} \frac{\partial^3 f(x)}{\partial x^3} + \dots \\ &= \sum_{n=0}^{\infty} \frac{1}{n!} \frac{\partial^n f(x)}{\partial x^n} \end{aligned}$$

Edge

- Taylor's edge

- expand $f(x + \Delta x)$

$$\begin{aligned} f(x + \Delta x) &= f(x) + \Delta x \frac{\partial f(x)}{\partial x} + \frac{(\Delta x)^2}{2!} \frac{\partial^2 f(x)}{\partial x^2} + \frac{(\Delta x)^3}{3!} \frac{\partial^3 f(x)}{\partial x^3} + \dots \\ &= \sum_{n=0}^{\infty} \frac{(\Delta x)^n}{n!} \frac{\partial^n f(x)}{\partial x^n} \end{aligned}$$

$$\begin{aligned} f(x + 1) &= f(x) + \frac{\partial f(x)}{\partial x} + \frac{1}{2!} \frac{\partial^2 f(x)}{\partial x^2} + \frac{1}{3!} \frac{\partial^3 f(x)}{\partial x^3} + \dots \\ &= \sum_{n=0}^{\infty} \frac{1}{n!} \frac{\partial^n f(x)}{\partial x^n} \end{aligned}$$

$$\begin{aligned} f(x - 1) &= f(x) - \frac{\partial f(x)}{\partial x} + \frac{1}{2!} \frac{\partial^2 f(x)}{\partial x^2} - \frac{1}{3!} \frac{\partial^3 f(x)}{\partial x^3} + \dots \\ &= \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \frac{\partial^n f(x)}{\partial x^n} \end{aligned}$$

Edge

- Forward

Edge

○ Forward $\frac{\partial f(x)}{\partial x} = f'(x) = f(x+1) - f(x)$

Edge

○ Forward $\frac{\partial f(x)}{\partial x} = f'(x) = f(x+1) - f(x)$

○ Backward $\frac{\partial f(x)}{\partial x} = f'(x) = f(x) - f(x-1)$

Edge

- Forward $\frac{\partial f(x)}{\partial x} = f'(x) = f(x+1) - f(x)$
- Backward $\frac{\partial f(x)}{\partial x} = f'(x) = f(x) - f(x-1)$
- Central $\frac{\partial f(x)}{\partial x} = f'(x) = \frac{f(x+1) - f(x-1)}{2}$

Edge

- Forward $\frac{\partial f(x)}{\partial x} = f'(x) = f(x+1) - f(x)$
- Backward $\frac{\partial f(x)}{\partial x} = f'(x) = f(x) - f(x-1)$
- Central $\frac{\partial f(x)}{\partial x} = f'(x) = \frac{f(x+1) - f(x-1)}{2}$
- 2nd order central $\frac{\partial^2 f(x)}{\partial x^2} = f''(x) = f(x+1) - 2f(x) + f(x-1)$

Edge

- Forward $\frac{\partial f(x)}{\partial x} = f'(x) = f(x+1) - f(x)$
 - Backward $\frac{\partial f(x)}{\partial x} = f'(x) = f(x) - f(x-1)$
 - Central $\frac{\partial f(x)}{\partial x} = f'(x) = \frac{f(x+1) - f(x-1)}{2}$
 - 2nd order central $\frac{\partial^2 f(x)}{\partial x^2} = f''(x) = f(x+1) - 2f(x) + f(x-1)$
- Image $f(x, y)$

Edge

○ Forward $\frac{\partial f(x)}{\partial x} = f'(x) = f(x+1) - f(x)$

○ Image $f(x, y)$

○ Backward $\frac{\partial f(x)}{\partial x} = f'(x) = f(x) - f(x-1)$

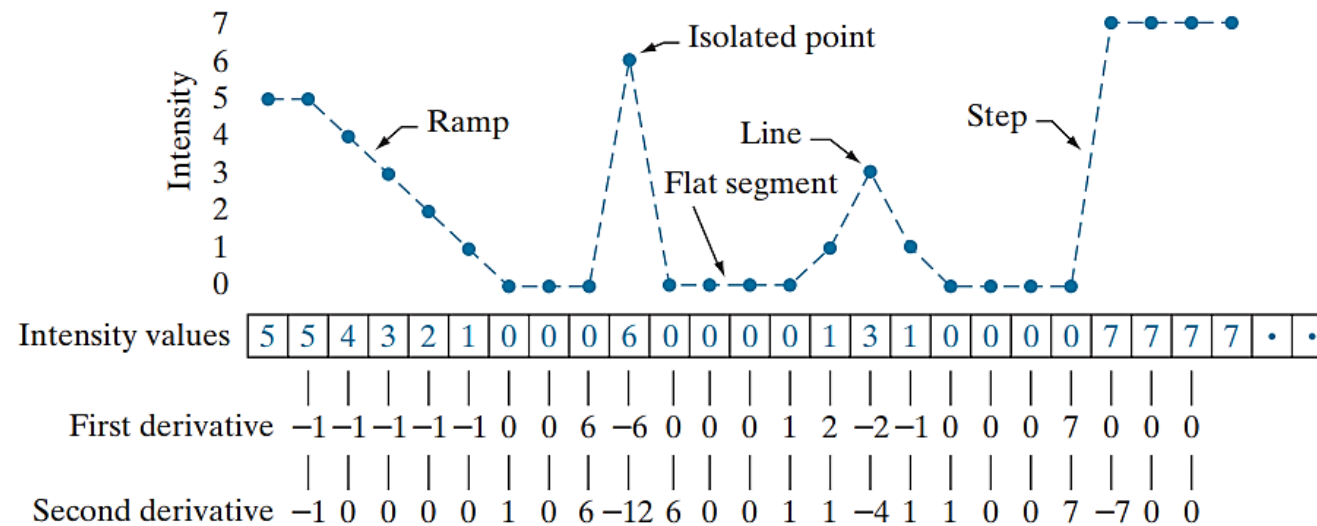
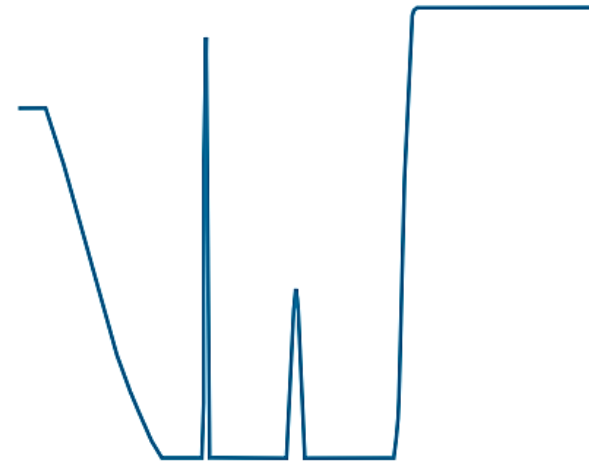
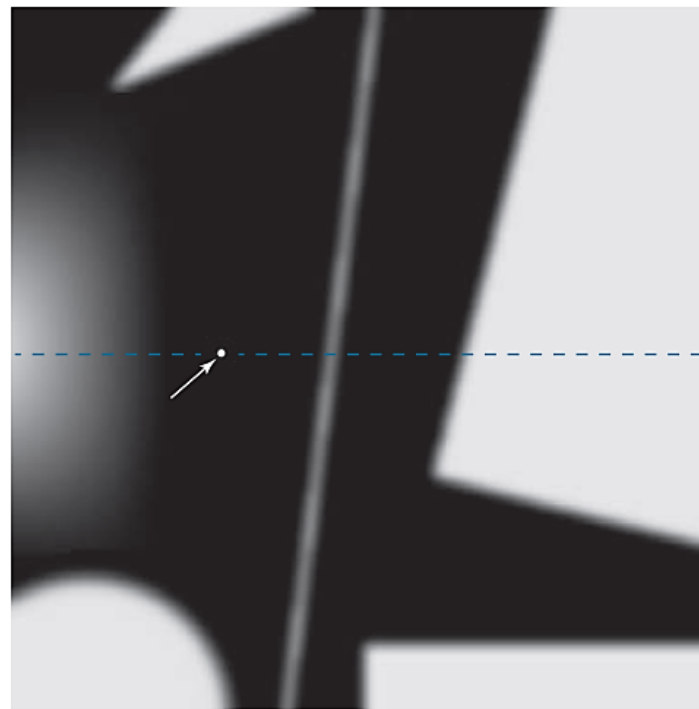
$$\frac{\partial^2 f(x, y)}{\partial x^2} = f(x+1, y) - 2f(x, y) + f(x-1, y)$$

○ Central $\frac{\partial f(x)}{\partial x} = f'(x) = \frac{f(x+1) - f(x-1)}{2}$

$$\frac{\partial^2 f(x, y)}{\partial y^2} = f(x, y+1) - 2f(x, y) + f(x, y-1)$$

○ 2nd order central $\frac{\partial^2 f(x)}{\partial x^2} = f''(x) = f(x+1) - 2f(x) + f(x-1)$

Edge



Point

- Isolated point
 - 2nd derivatives
 - Laplacian

$$\nabla^2 f(x, y) = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

Point

- Isolated point
 - 2nd derivatives
 - Laplacian

$$\nabla^2 f(x, y) = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

$$\frac{\partial^2 f}{\partial x^2} = f(x+1, y) + f(x-1, y) - 2f(x, y)$$

$$\frac{\partial^2 f}{\partial y^2} = f(x, y+1) + f(x, y-1) - 2f(x, y)$$

Point

- Isolated point
 - 2nd derivatives
 - Laplacian

$$\nabla^2 f(x, y) = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

$$\frac{\partial^2 f}{\partial x^2} = f(x+1, y) + f(x-1, y) - 2f(x, y)$$

$$\frac{\partial^2 f}{\partial y^2} = f(x, y+1) + f(x, y-1) - 2f(x, y)$$

$$\nabla^2 f(x, y) = f(x+1, y) + f(x-1, y) + f(x, y+1) + f(x, y-1) - 4f(x, y)$$

Point

- Isolated point
 - 2nd derivatives
 - Laplacian

$$\nabla^2 f(x, y) = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

$$\frac{\partial^2 f}{\partial x^2} = f(x+1, y) + f(x-1, y) - 2f(x, y)$$

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$$\nabla^2 f(x, y) = f(x+1, y) + f(x-1, y) + f(x, y+1) + f(x, y-1) - 4f(x, y)$$

0	1	0
1	-4	1
0	1	0

Point

- Isolated point
 - 2nd derivatives
 - Laplacian

$$\nabla^2 f(x, y) = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

$$\frac{\partial^2 f}{\partial x^2} = f(x+1, y) + f(x-1, y) - 2f(x, y)$$

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0	1	0
1	-4	1
0	1	0



Point

- Isolated point
 - 2nd derivatives
 - Laplacian

$$\nabla^2 f(x, y) = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

$$\frac{\partial^2 f}{\partial x^2} = f(x+1, y) + f(x-1, y) - 2f(x, y)$$

$$\frac{\partial^2 f}{\partial y^2} = f(x, y+1) + f(x, y-1) - 2f(x, y)$$

$$\nabla^2 f(x, y) = f(x+1, y) + f(x-1, y) + f(x, y+1) + f(x, y-1) - 4f(x, y)$$

0	1	0
1	-4	1
0	1	0

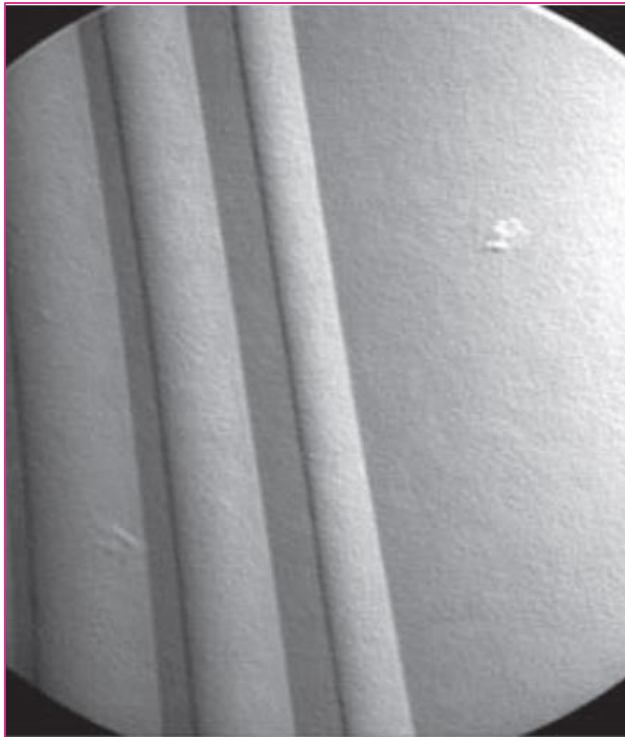


1	1	1
1	-8	1
1	1	1

Point

- Turbine blade under X-ray
 - Laplace operator

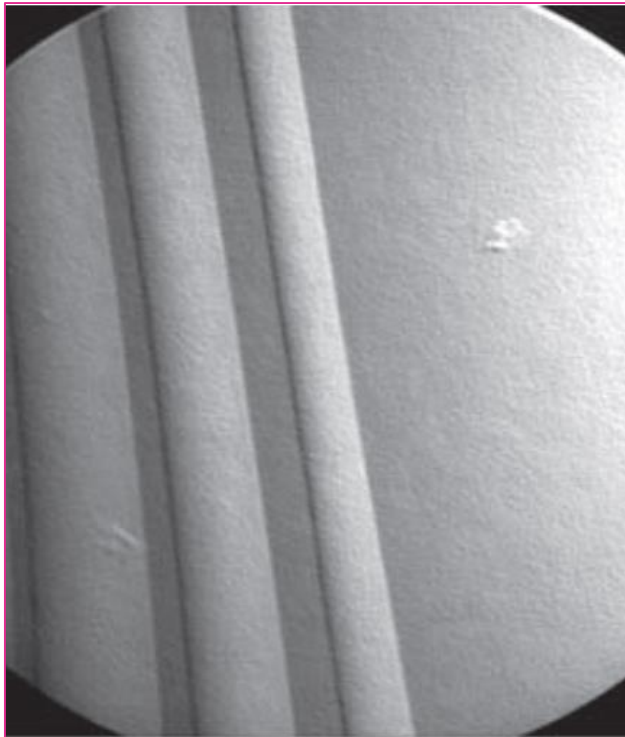
Input



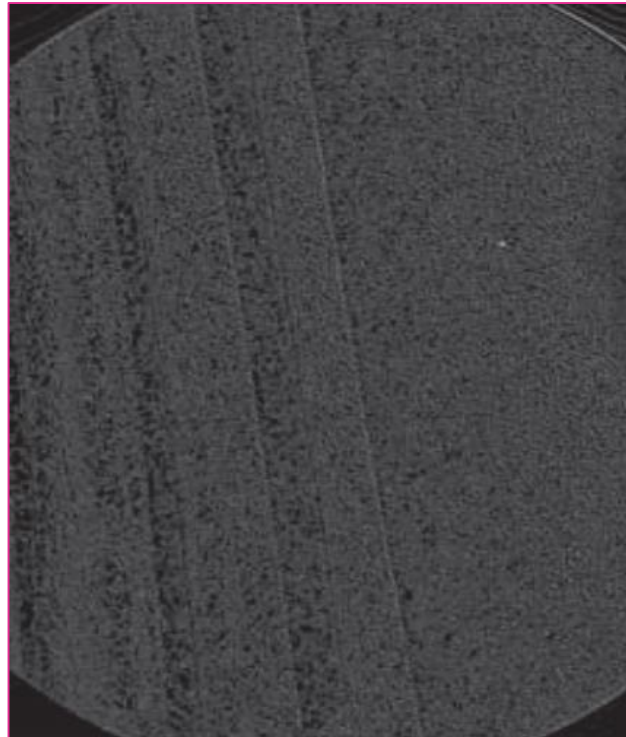
Point

- Turbine blade under X-ray
 - Laplace operator

Input



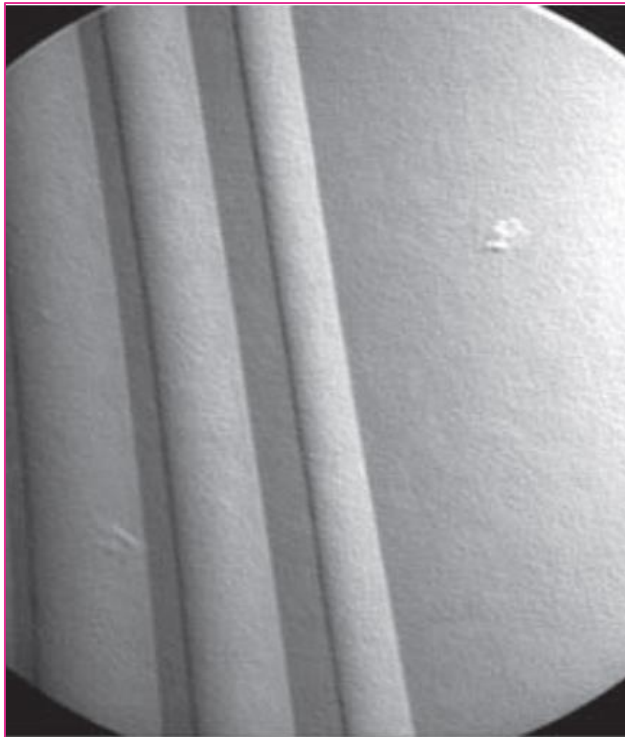
Laplacian



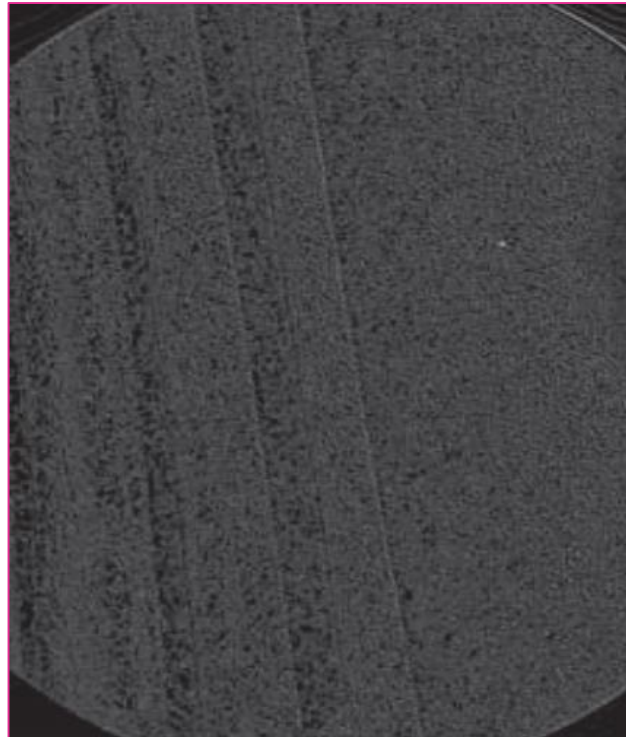
Point

- Turbine blade under X-ray
 - Laplace operator

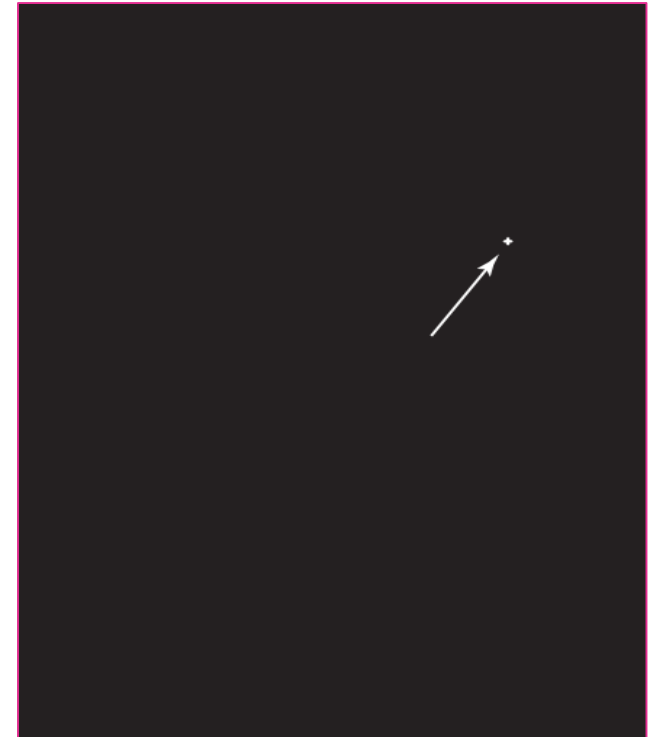
Input



Laplacian



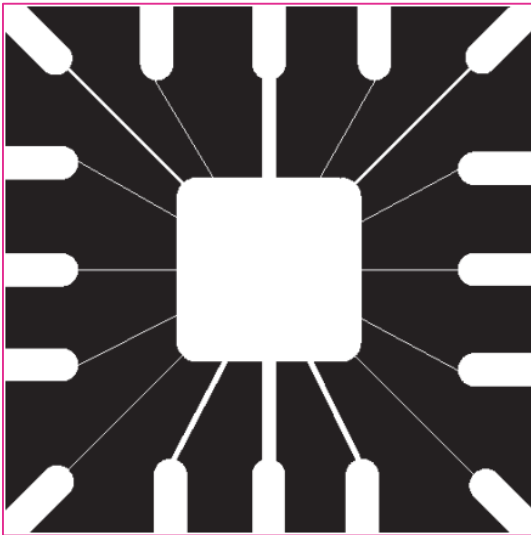
Thresholding



Line

- PCB wire bonds
 - Laplace operator

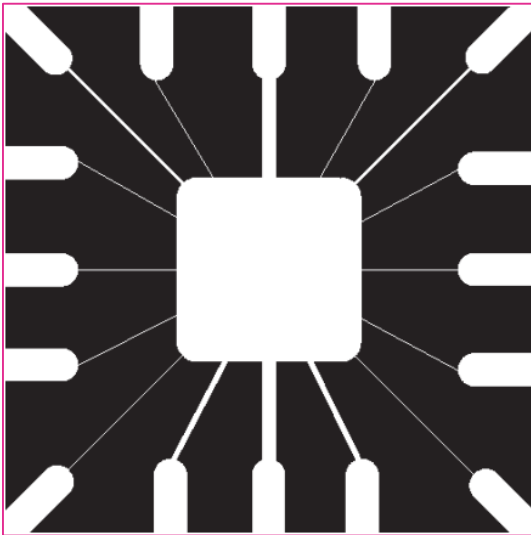
Input



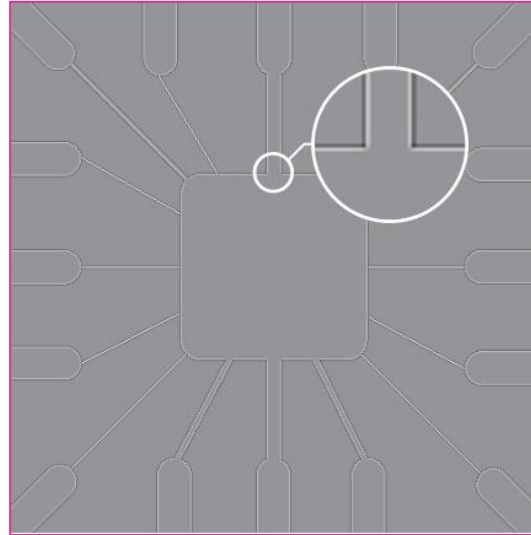
Line

- PCB wire bonds
 - Laplace operator

Input



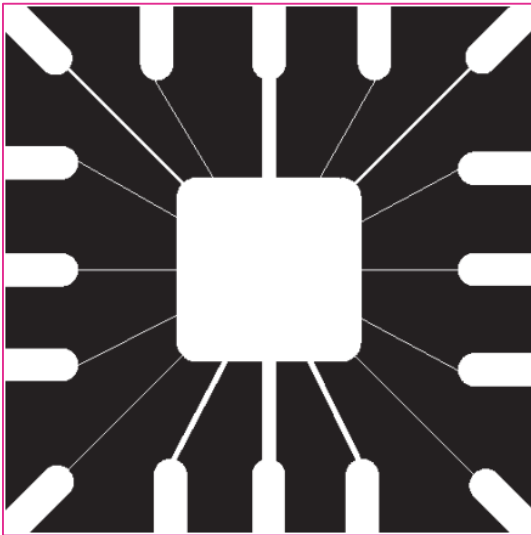
Laplacian



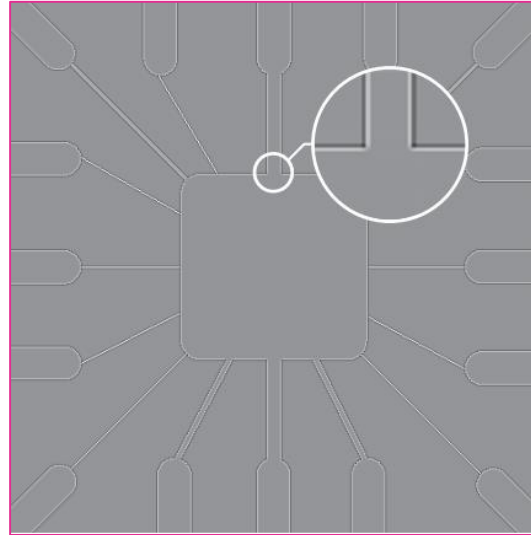
Line

- PCB wire bonds
 - Laplace operator

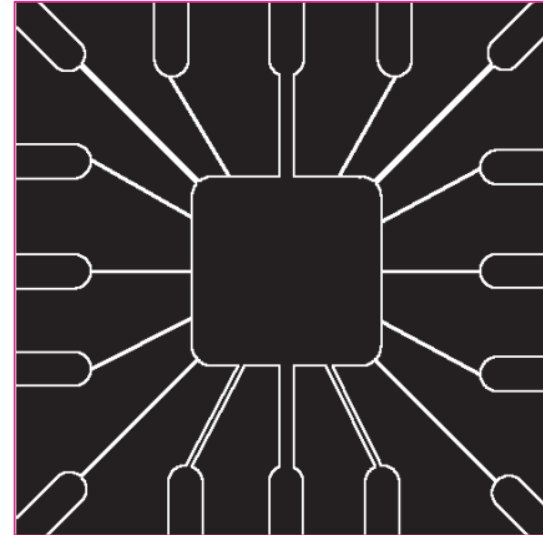
Input



Laplacian



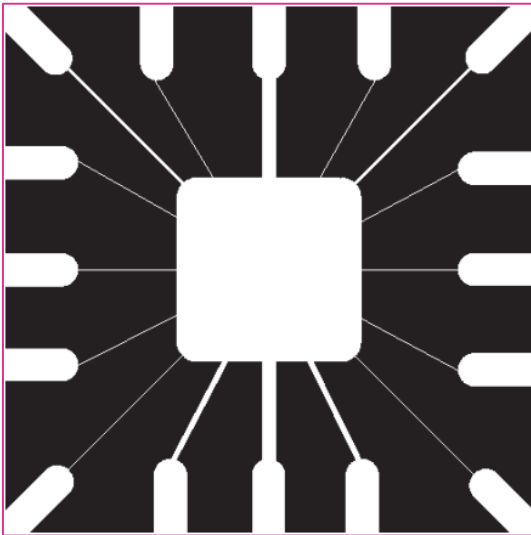
| Laplacian |



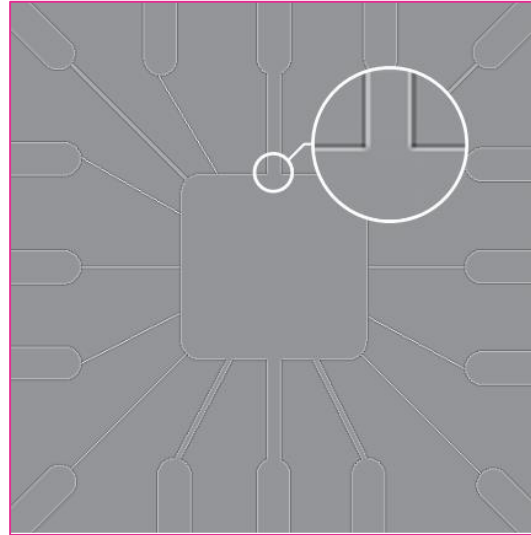
Line

- PCB wire bonds
- Laplace operator

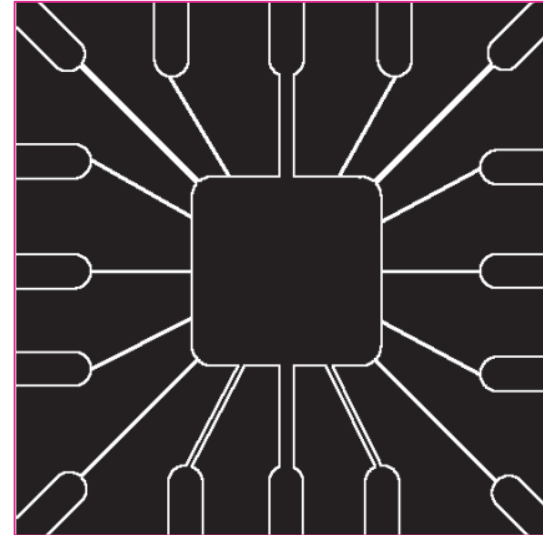
Input



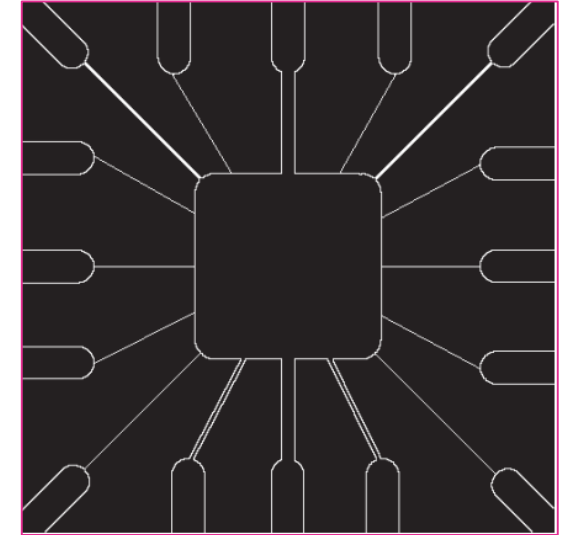
Laplacian



|Laplacian|



max(0, Laplacian)



Line

- With orientation

Horizontal

-1	-1	-1
2	2	2
-1	-1	-1

Line

- With orientation

Horizontal

-1	-1	-1
2	2	2
-1	-1	-1

+45 degrees

2	-1	-1
-1	2	-1
-1	-1	2

Line

- With orientation

Horizontal

-1	-1	-1
2	2	2
-1	-1	-1

+45 degrees

2	-1	-1
-1	2	-1
-1	-1	2

Vertical

-1	2	-1
-1	2	-1
-1	2	-1

Line

- With orientation

Horizontal

-1	-1	-1
2	2	2
-1	-1	-1

+45 degrees

2	-1	-1
-1	2	-1
-1	-1	2

Vertical

-1	2	-1
-1	2	-1
-1	2	-1

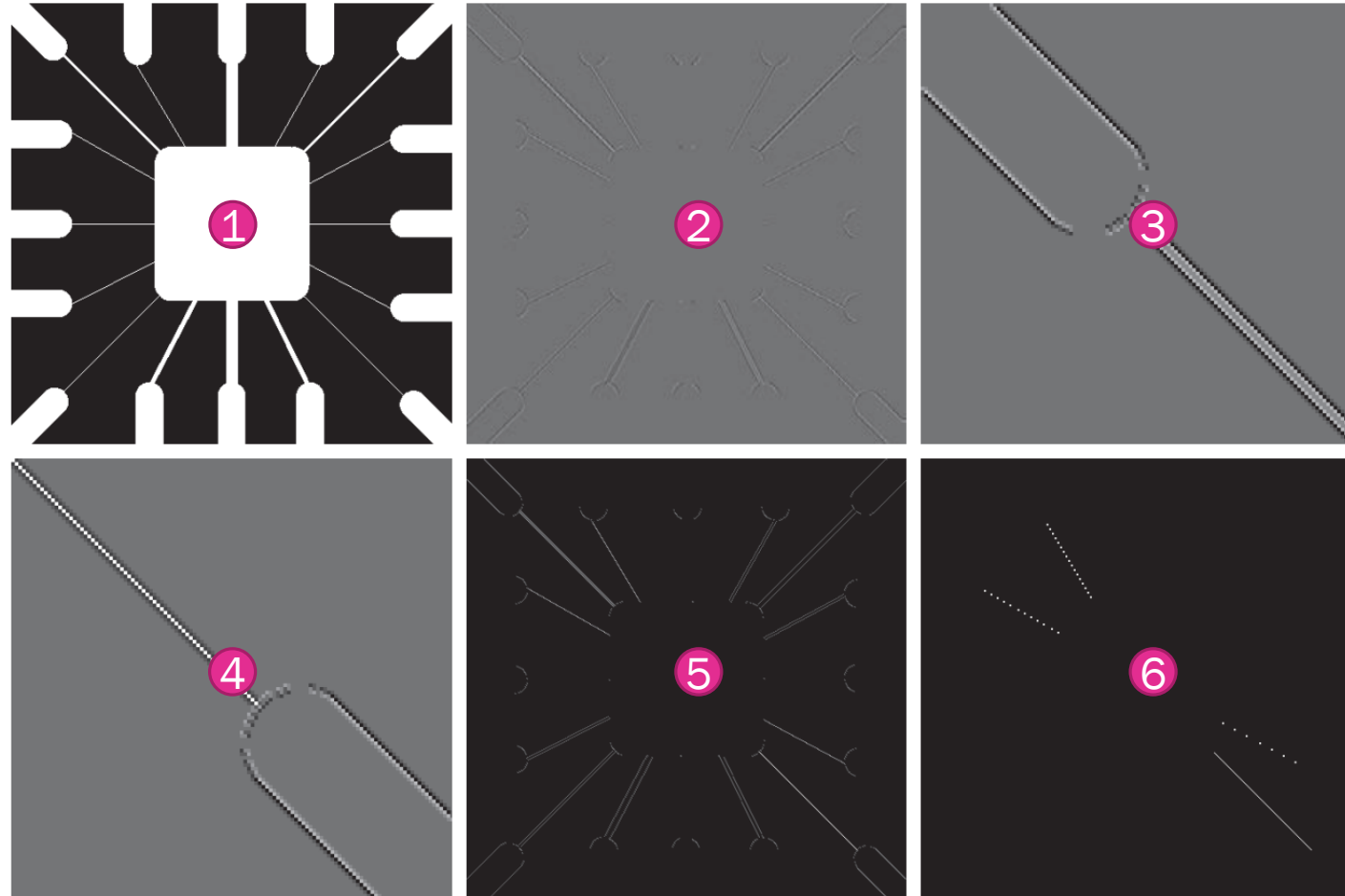
-45 degrees

-1	-1	2
-1	2	-1
2	-1	-1

Line

- With orientation

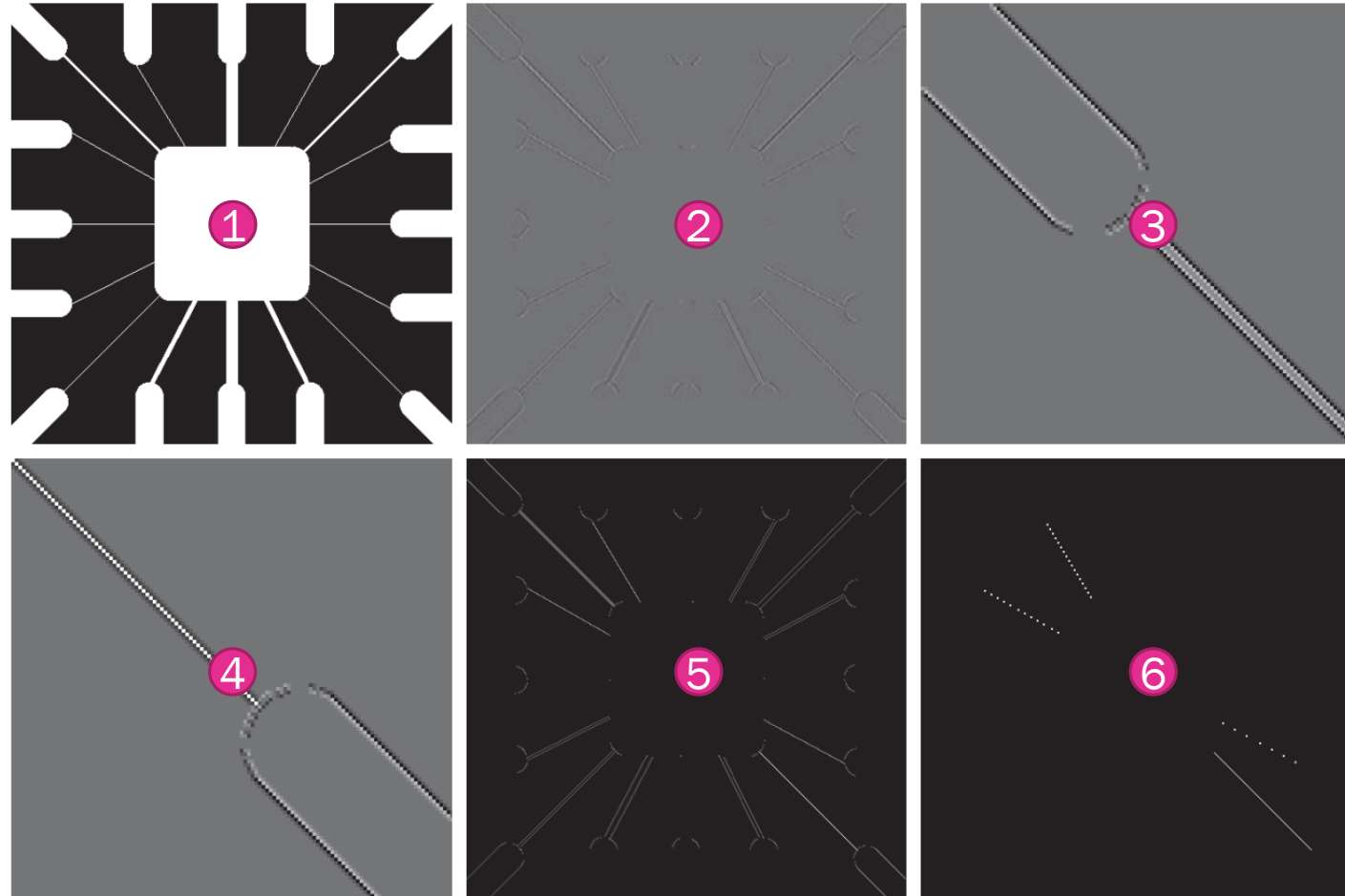
- +45d line
det kernel



Line

- With orientation

- +45d line
det kernel



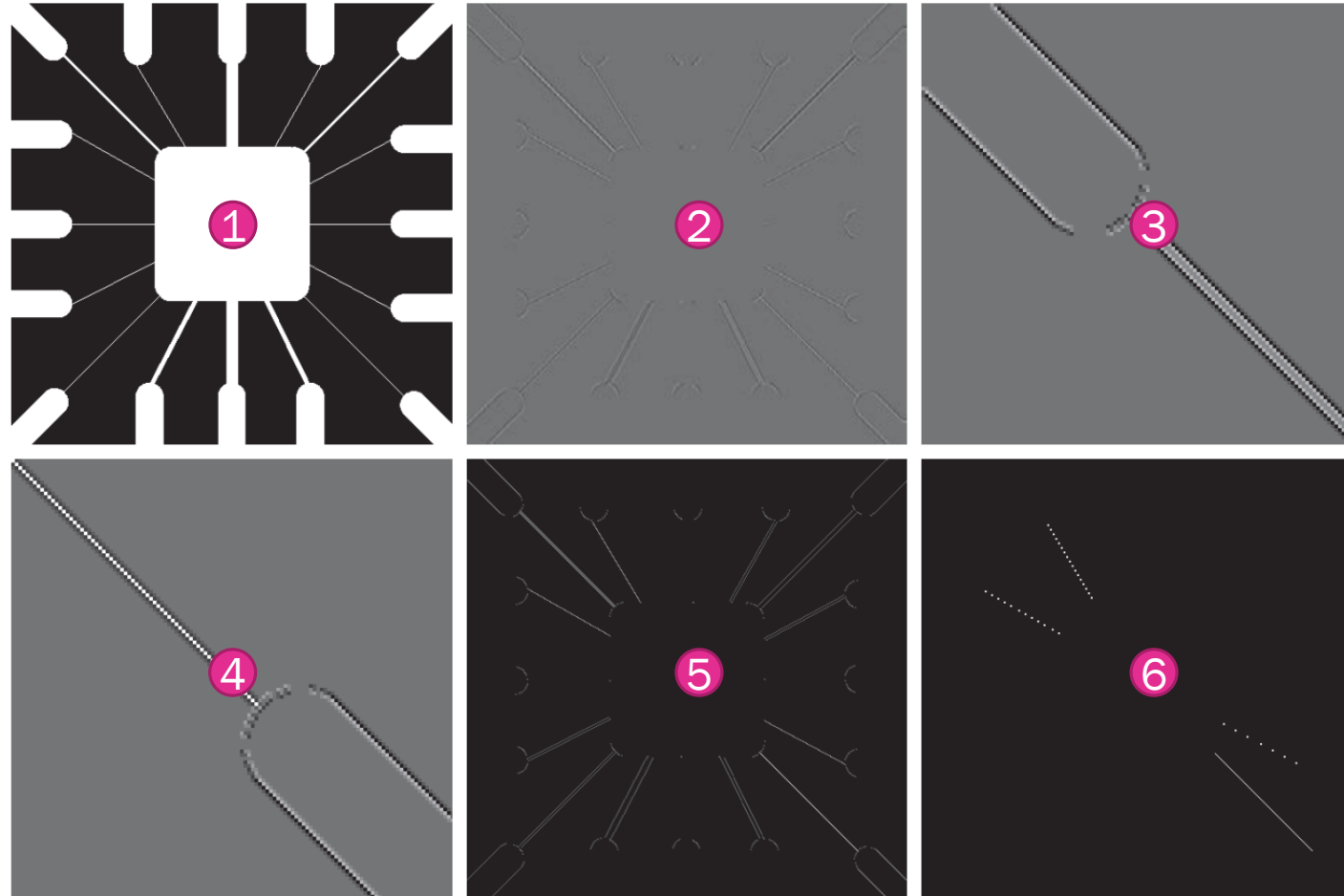
+45 degrees

2	-1	-1
-1	2	-1
-1	-1	2

Line

■ With orientation

- +45d line
det kernel



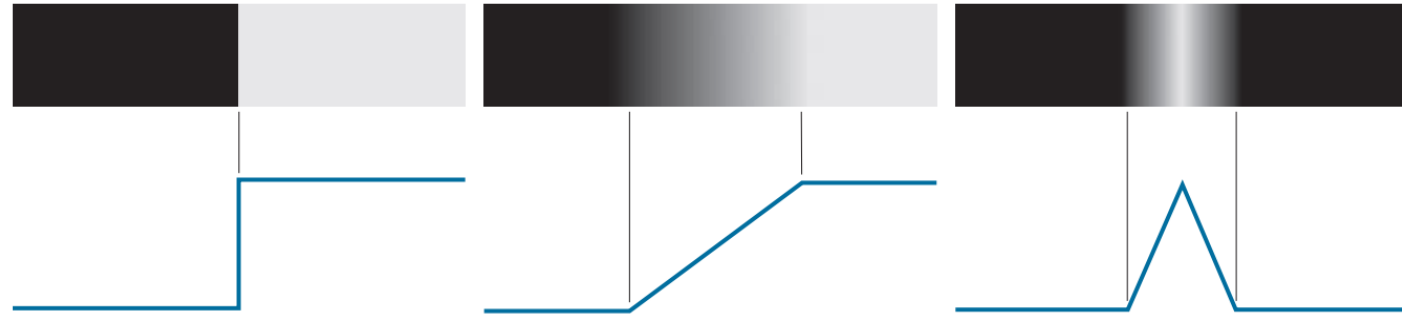
+45 degrees

2	-1	-1
-1	2	-1
-1	-1	2

1. Input
2. 45d det
3. Top zoom
4. Bottom zoom
5. Max(0, Lap)
6. Thresholding

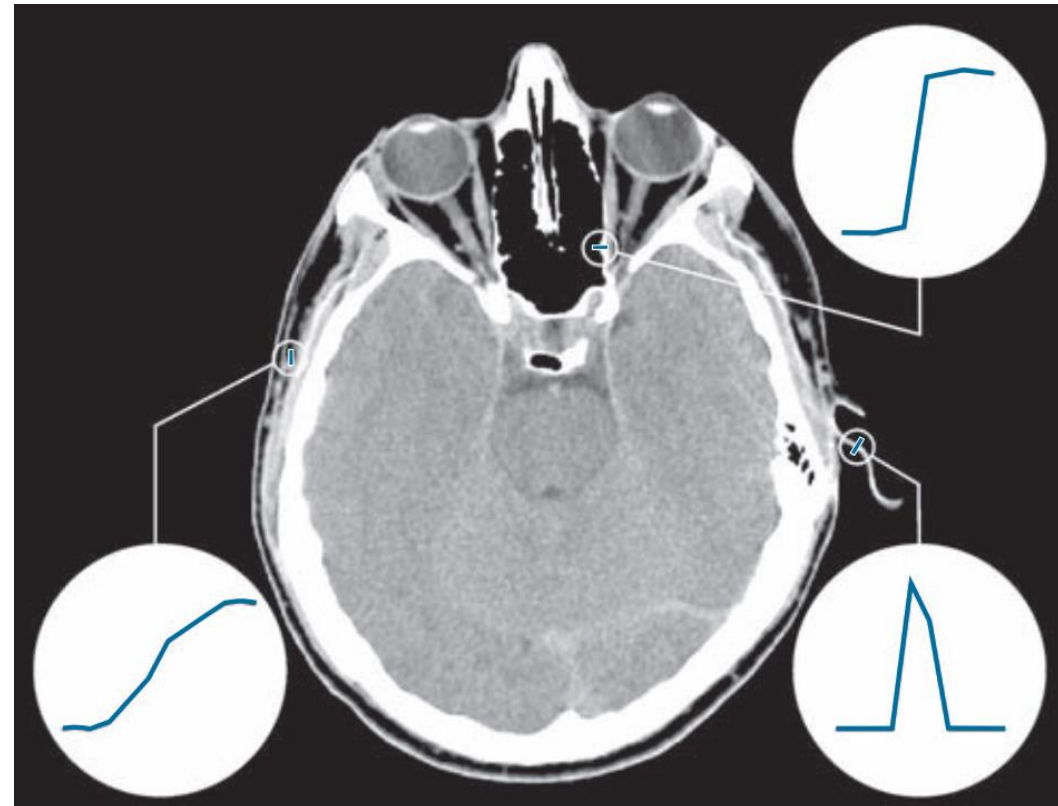
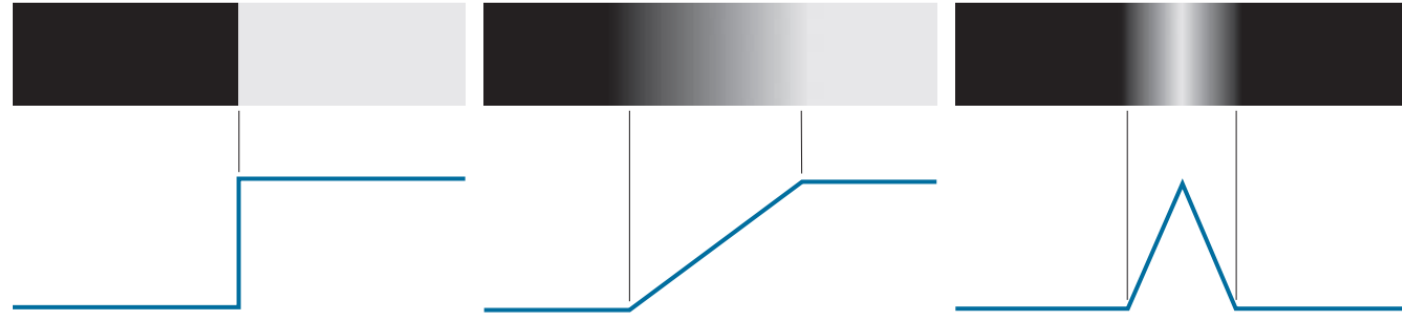
Edge

- Types
 - step
 - ramp
 - roof



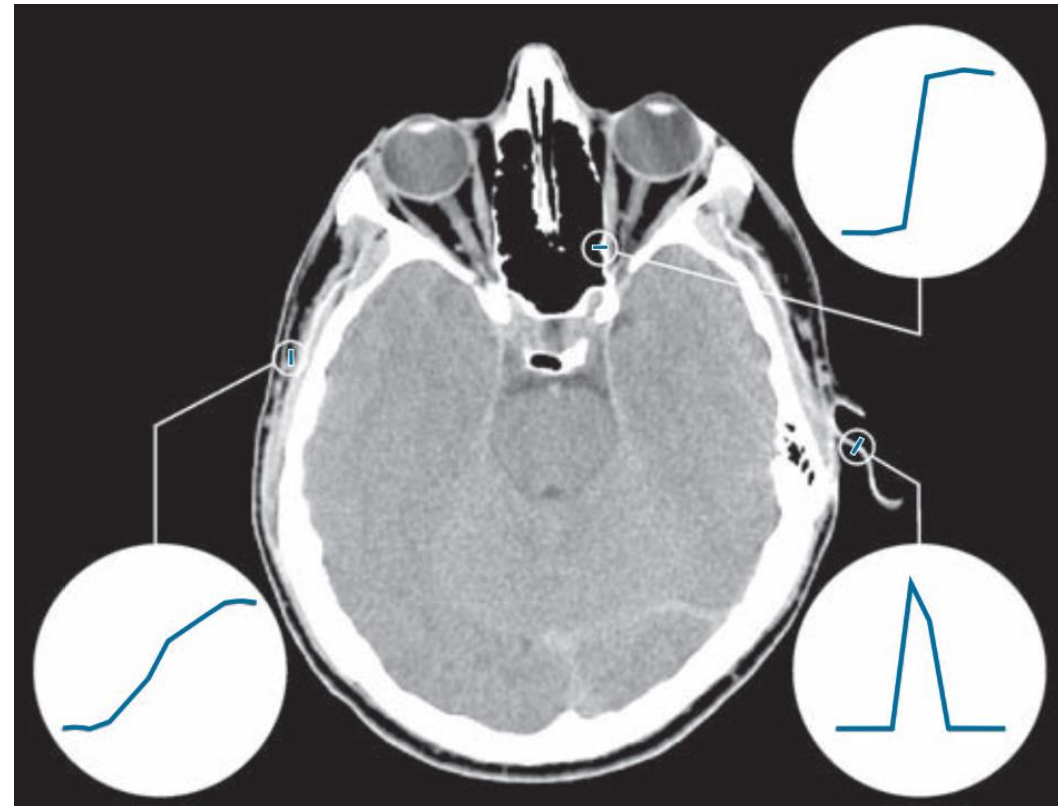
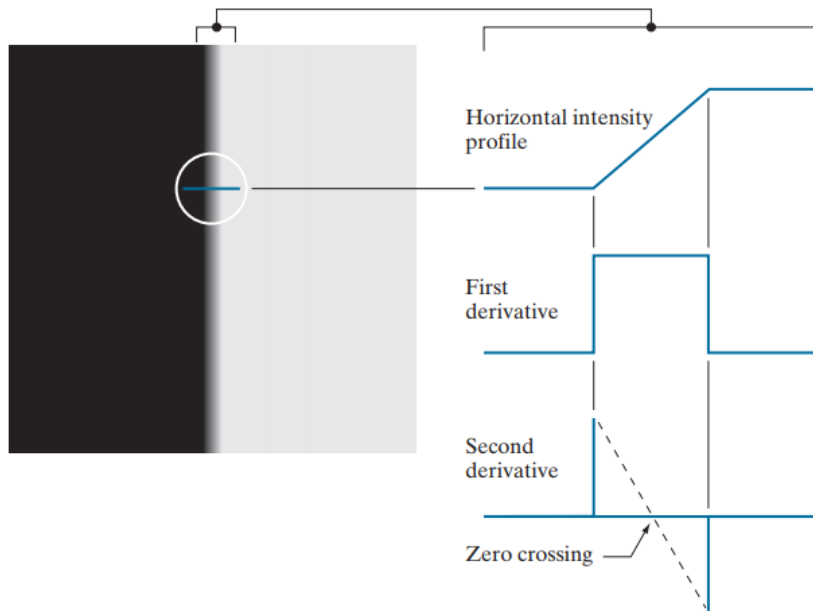
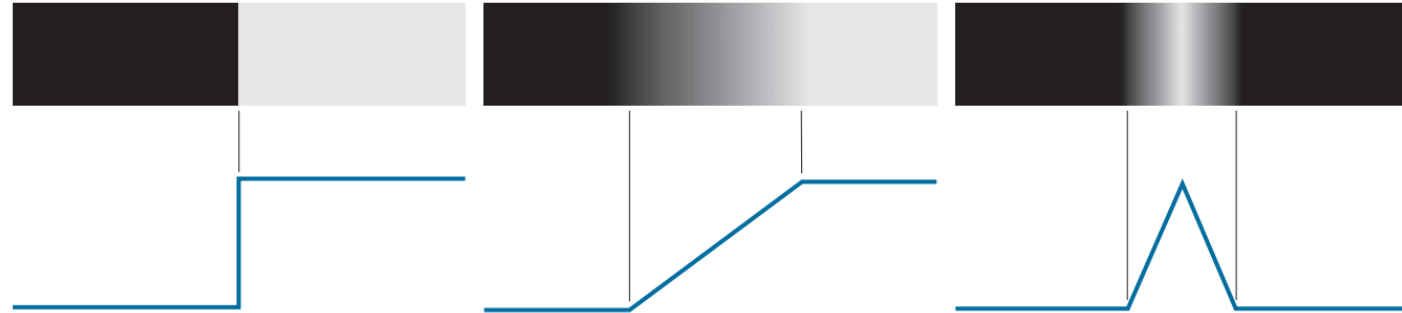
Edge

- Types
 - step
 - ramp
 - roof



Edge

- Types
 - step
 - ramp
 - roof



Edge

- Gradients

$$\nabla f(x, y) \equiv \text{grad}[f(x, y)] \equiv \begin{bmatrix} g_x(x, y) \\ g_y(x, y) \end{bmatrix} = \begin{bmatrix} \frac{\partial f(x, y)}{\partial x} \\ \frac{\partial f(x, y)}{\partial y} \end{bmatrix}$$

Edge

- Gradients

$$\nabla f(x, y) \equiv \text{grad}[f(x, y)] \equiv \begin{bmatrix} g_x(x, y) \\ g_y(x, y) \end{bmatrix} = \begin{bmatrix} \frac{\partial f(x, y)}{\partial x} \\ \frac{\partial f(x, y)}{\partial y} \end{bmatrix}$$

$$M(x, y) = \|\nabla f(x, y)\| = \sqrt{g_x^2(x, y) + g_y^2(x, y)}$$

$$\alpha(x, y) = \tan^{-1} \left[\frac{g_y(x, y)}{g_x(x, y)} \right]$$

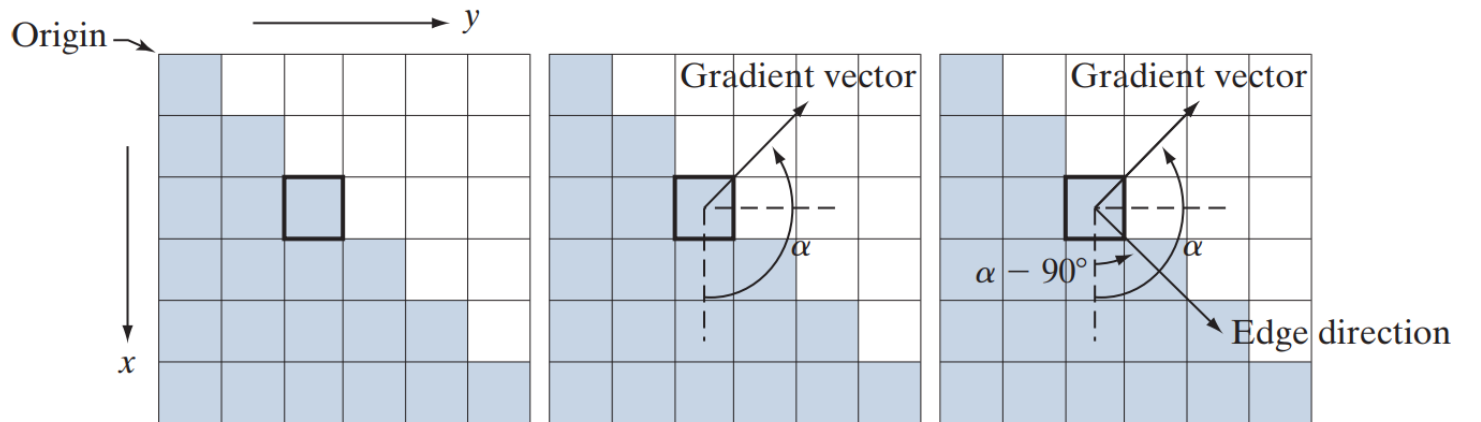
Edge

- Gradients

$$\nabla f(x, y) \equiv \text{grad}[f(x, y)] \equiv \begin{bmatrix} g_x(x, y) \\ g_y(x, y) \end{bmatrix} = \begin{bmatrix} \frac{\partial f(x, y)}{\partial x} \\ \frac{\partial f(x, y)}{\partial y} \end{bmatrix}$$

$$M(x, y) = \|\nabla f(x, y)\| = \sqrt{g_x^2(x, y) + g_y^2(x, y)}$$

$$\alpha(x, y) = \tan^{-1} \left[\frac{g_y(x, y)}{g_x(x, y)} \right]$$



Edge

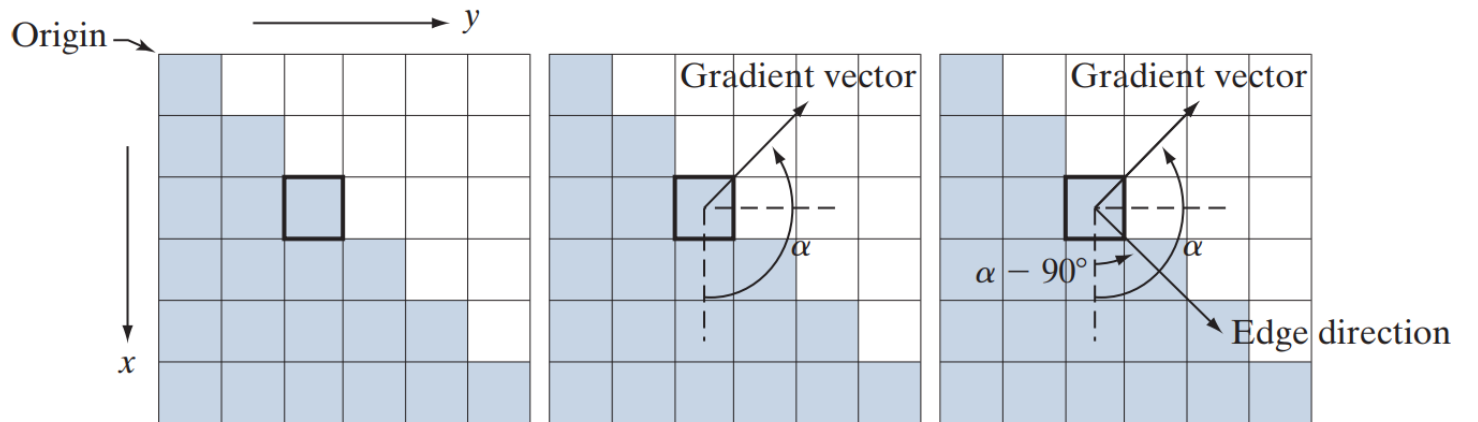
- Gradients

$$\nabla f(x, y) \equiv \text{grad}[f(x, y)] \equiv \begin{bmatrix} g_x(x, y) \\ g_y(x, y) \end{bmatrix} = \begin{bmatrix} \frac{\partial f(x, y)}{\partial x} \\ \frac{\partial f(x, y)}{\partial y} \end{bmatrix}$$

$$M(x, y) = \|\nabla f(x, y)\| = \sqrt{g_x^2(x, y) + g_y^2(x, y)}$$

$$\alpha(x, y) = \tan^{-1} \left[\frac{g_y(x, y)}{g_x(x, y)} \right]$$

$$M(x, y) \approx |g_x| + |g_y|$$



Edge

- Sobel operator
 - derivatives via kernel
 - separable
 - diagonal direction points are not greatly discriminatory

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$$\mathbf{M}_y = \begin{bmatrix} +1 & +2 & +1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix}$$

$$M = \sqrt{(M_x^2 + M_y^2)}$$

$$\theta = \tan^{-1}(M_y, M_x)$$

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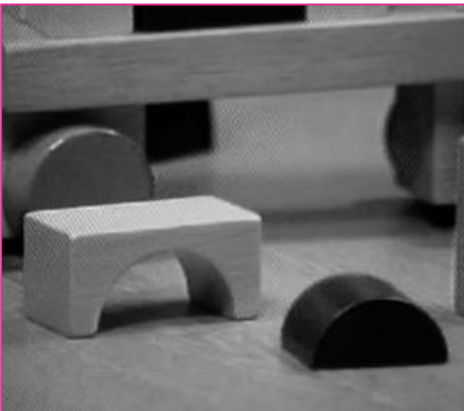
$$M_x = \begin{bmatrix} +1 & 0 & -1 \\ +2 & 0 & -2 \\ +1 & 0 & -1 \end{bmatrix}$$

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Input



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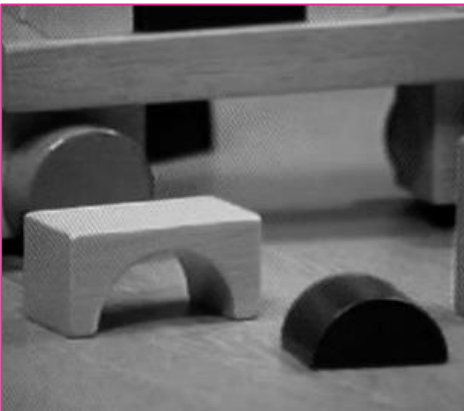
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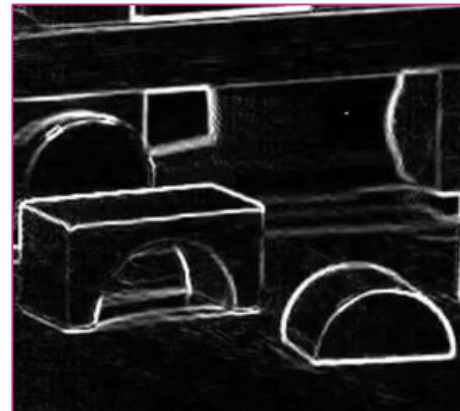
$$M = \sqrt{(M_x^2 + M_y^2)}$$

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Input



M



Edge

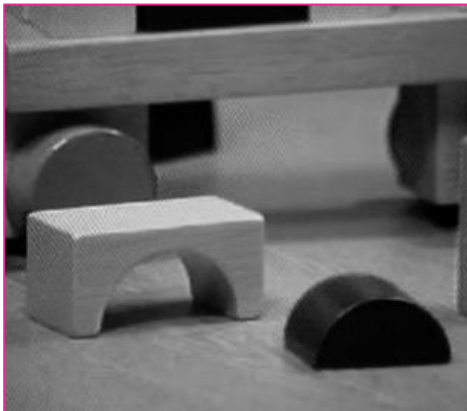
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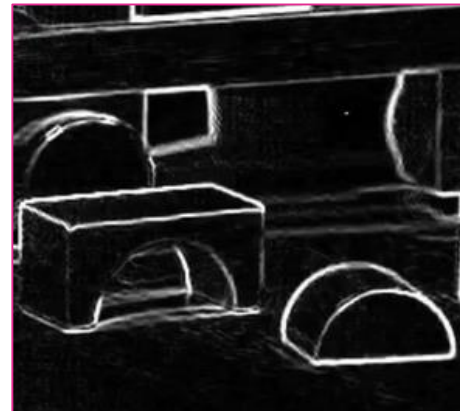
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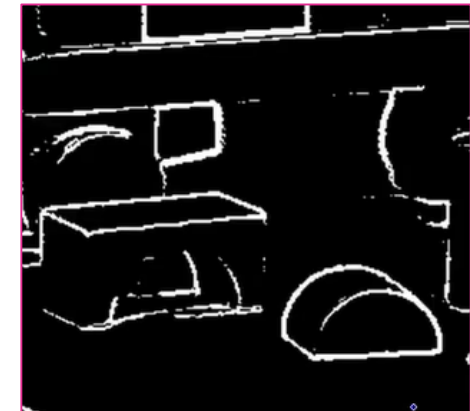
Input



M



Threshold on M



Edge

- Roberts operator
 - discriminatory diagonals
 - fast

$$M_x = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \quad M_y = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

input



no TH



with TH



Edge

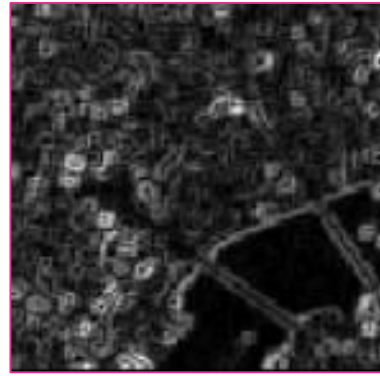
- Prewitt operator
 - high sensitivity than Sobel

$$M_x = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix} \quad M_y = \begin{bmatrix} -1 & -1 & -1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

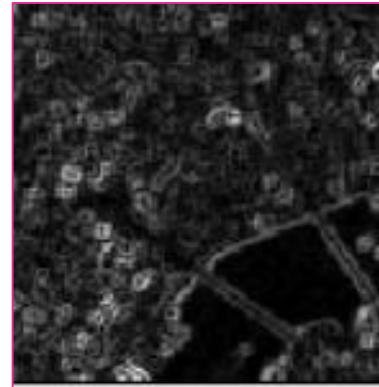
Input



Prewitt



Sobel



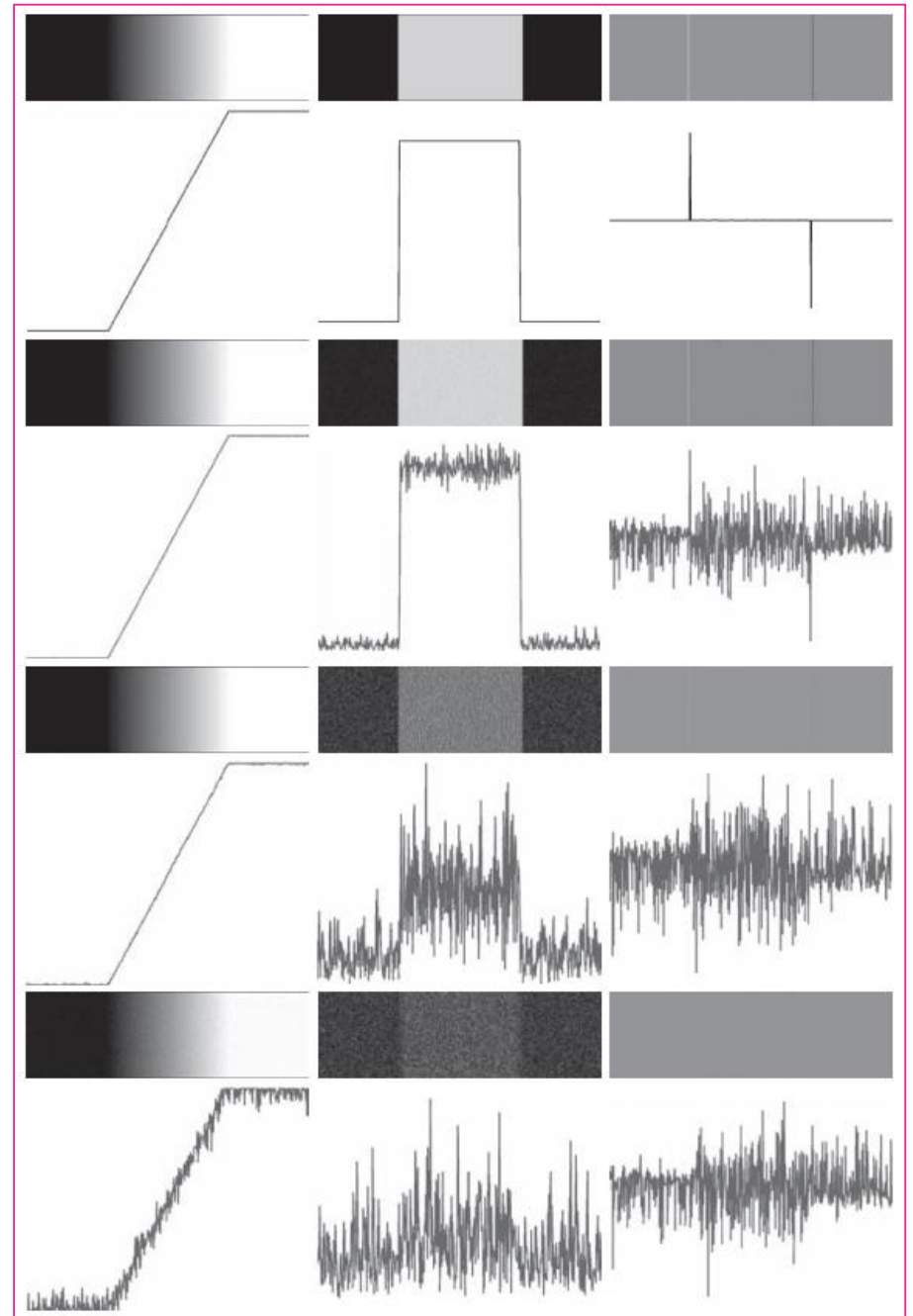
Roberts



Edge

- Edge Sensitivity

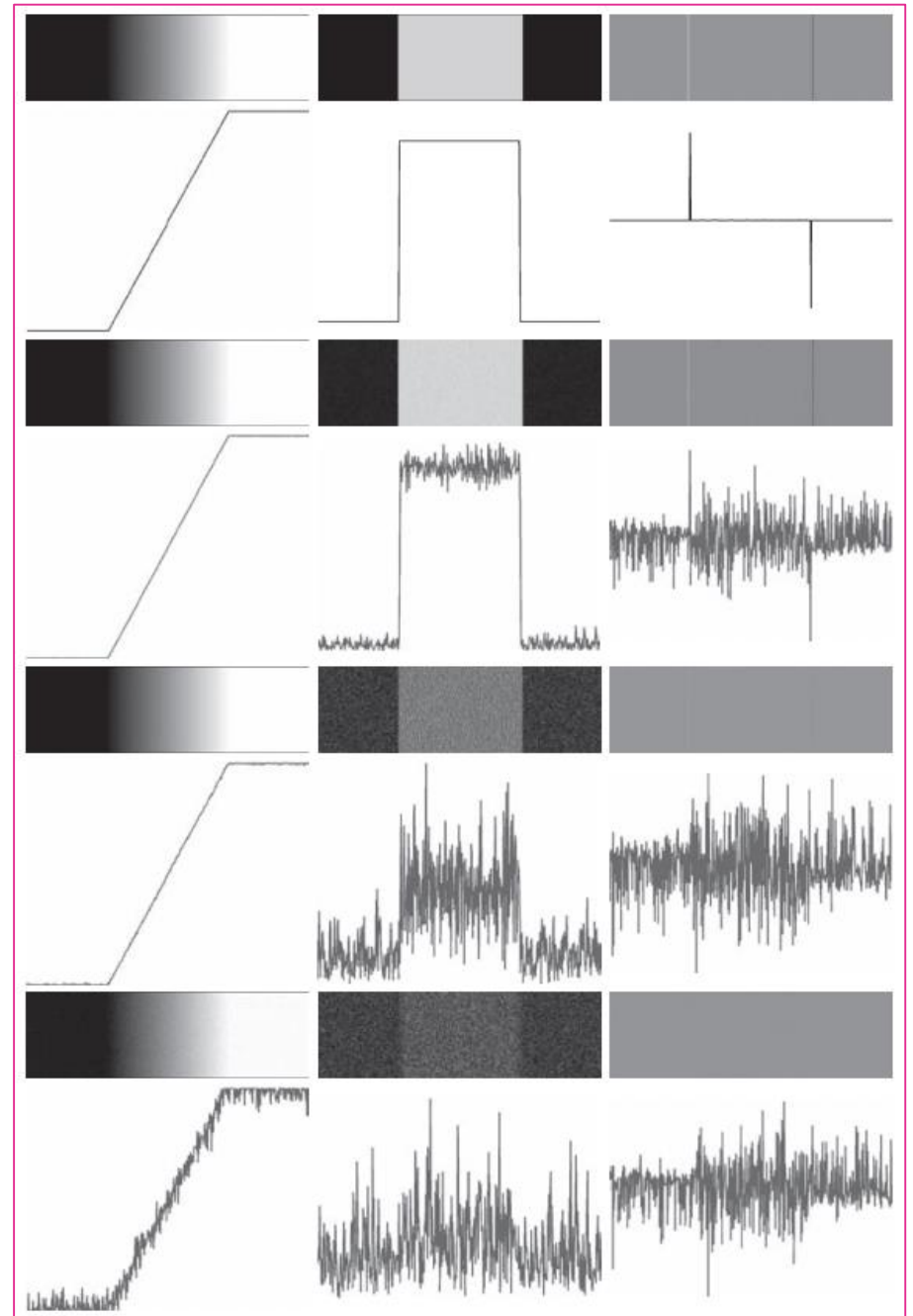
- Edge point is the peak in M in θ direction



Edge

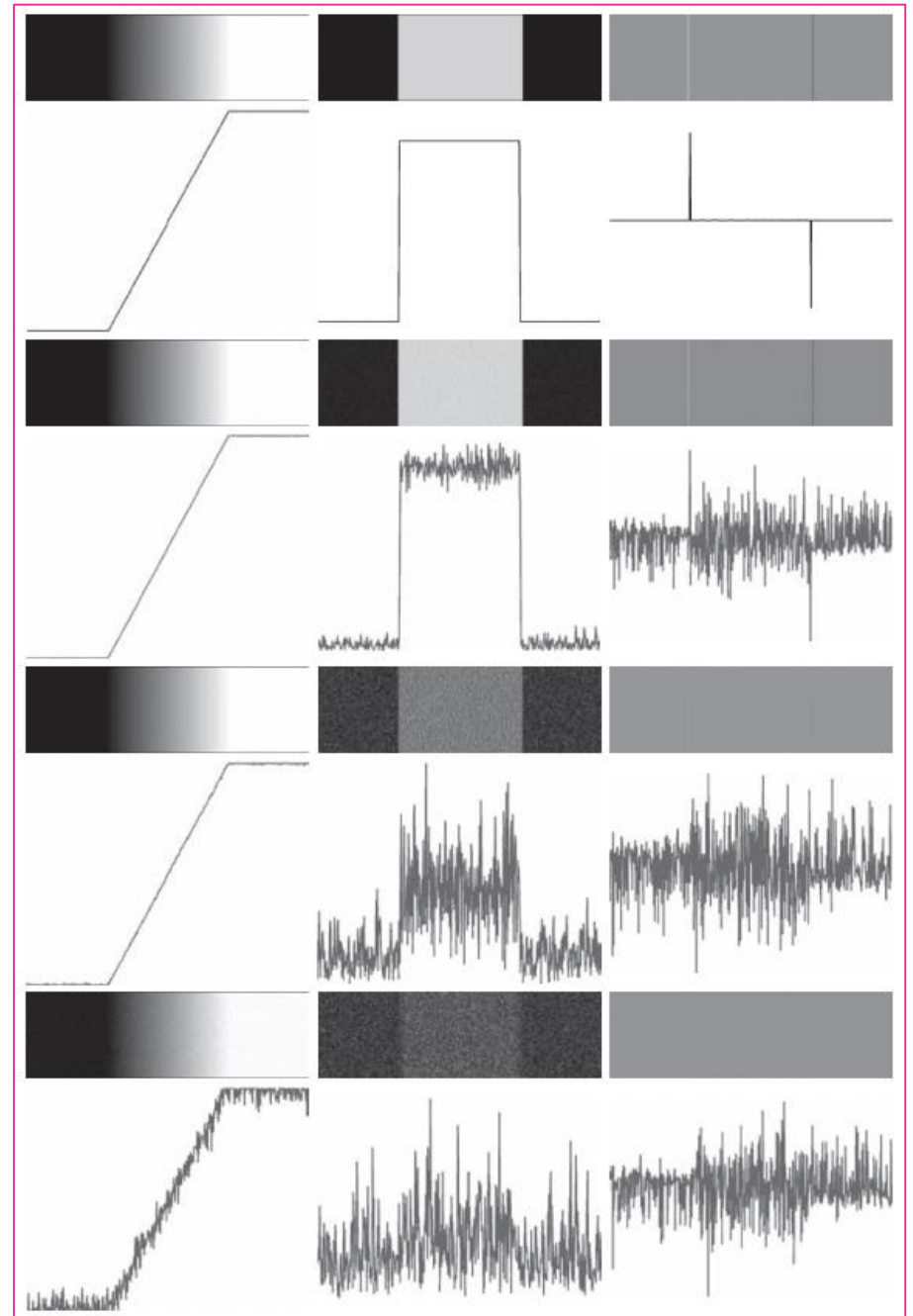
- Edge Sensitivity

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- Edges are highly sensitive to the noise



Edge

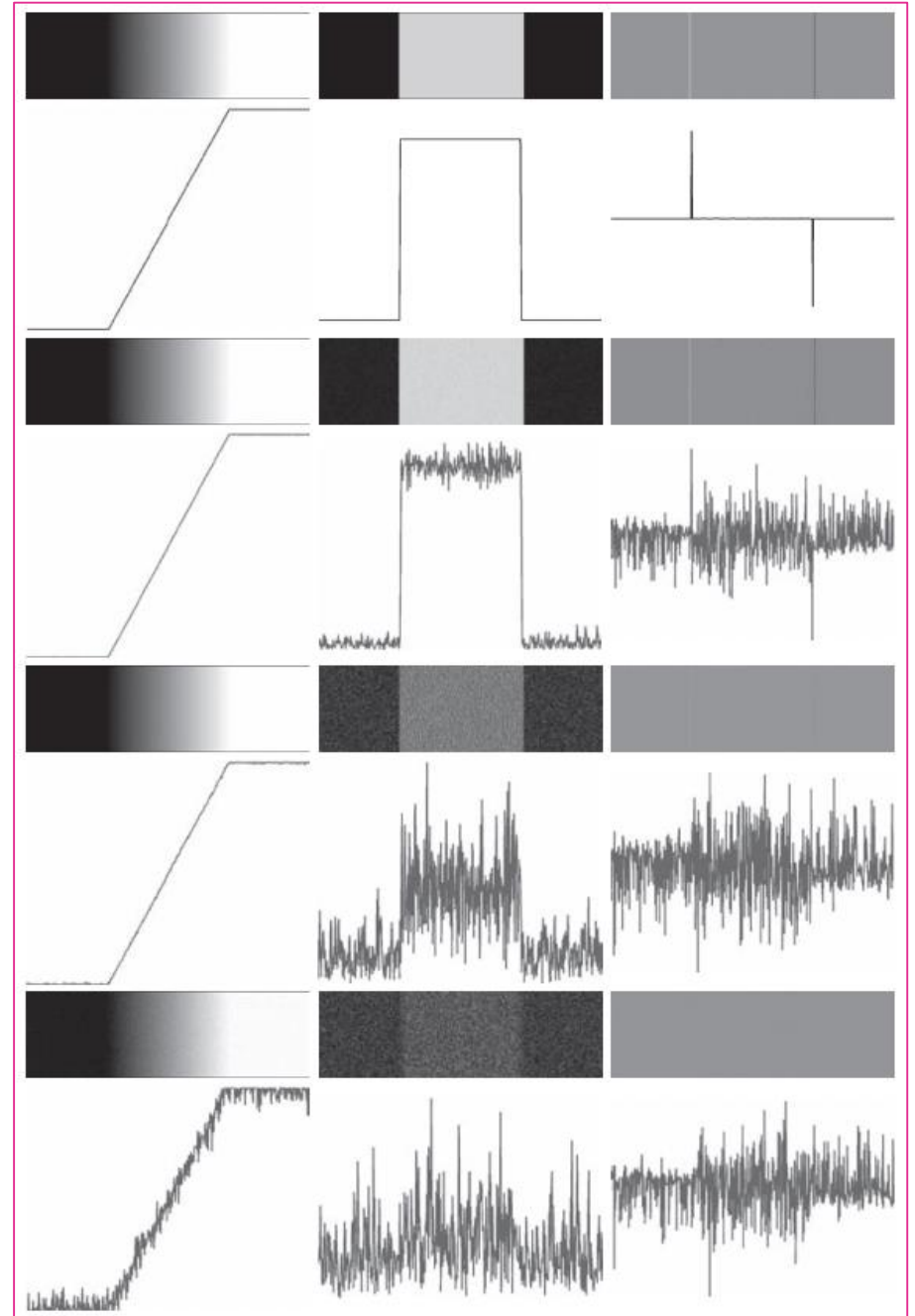
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Edge

■ Edge Sensitivity

- Edge point is the peak in M in θ direction
- Edges are highly sensitive to the noise
- Derivatives amplify noise
- How to reduce this sensitivity?



Edge

- Stability
 - refers to less sensitivity to noise
- Solution: apply smoothing filter G before finding edges

Edge

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... Δ is derivative (1^{st} or 2^{nd}) operator

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$$E = \Delta * \left\{ G_{\sigma} * I \right\}$$

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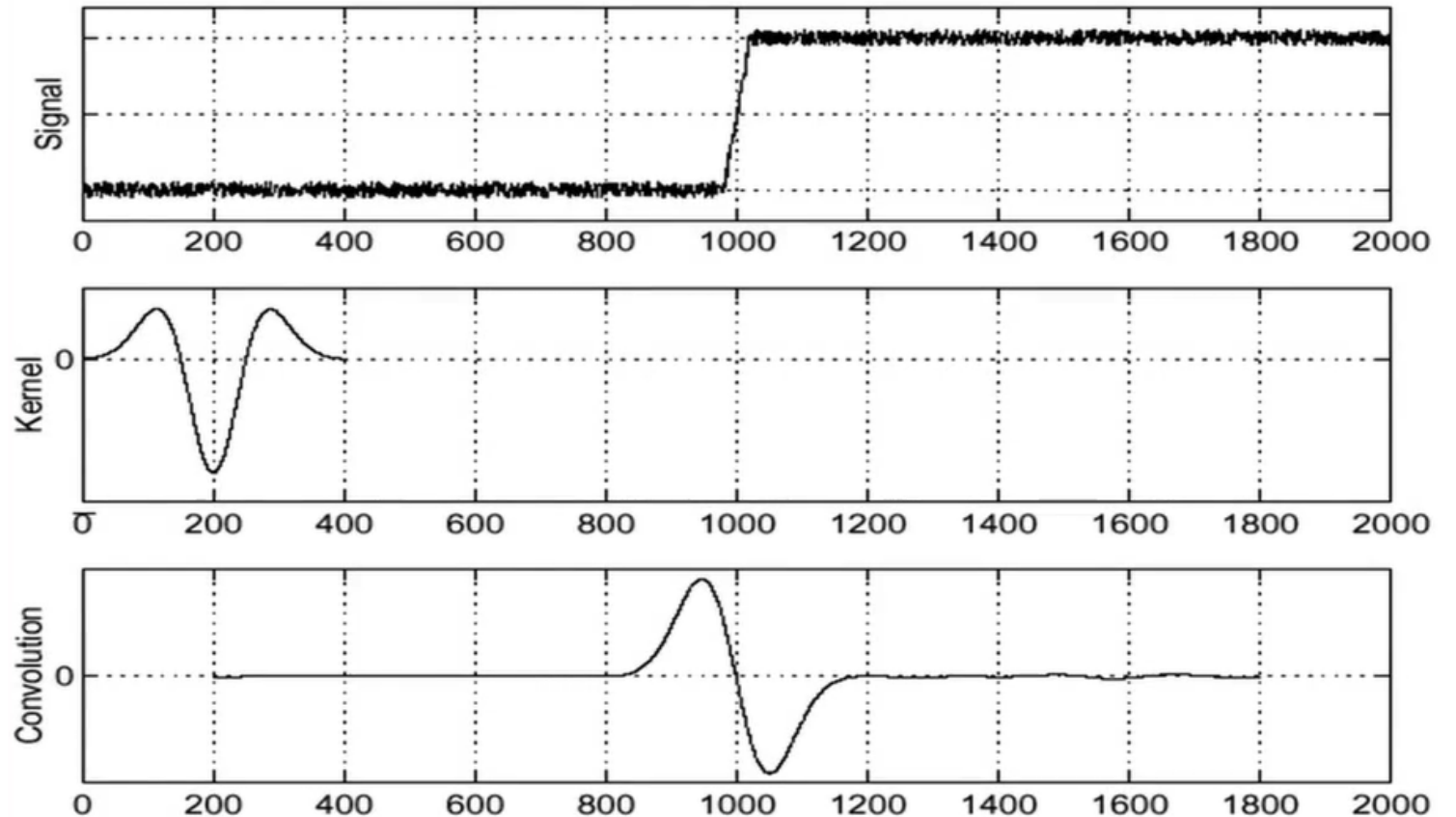
$$E = \left\{ \Delta * G_{\sigma} \right\} * I$$

... Conv. is associative

Edge

- Edge at zero crossings

$$\frac{\delta^2}{\delta x^2} \left\{ \begin{array}{c} \text{Kernel} \\ \begin{array}{|c|} \hline \text{Graph of Kernel} \\ \hline \end{array} \end{array} \right\} \rightarrow$$



Edge

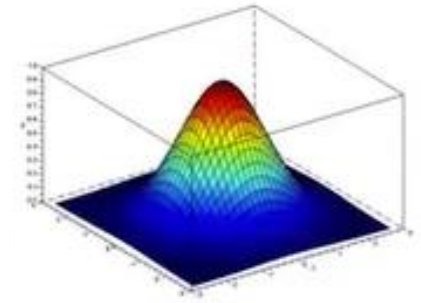
$$G(x, y) = e^{-\frac{x^2 + y^2}{2\sigma^2}}$$

- LoG
 - Laplacian of Gaussian

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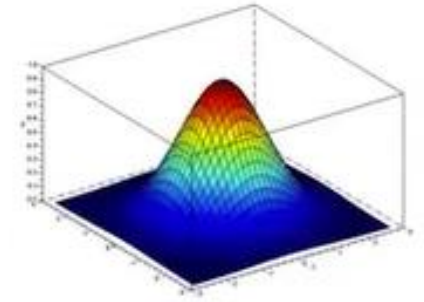


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$$\nabla^2 G(x, y) = \frac{\partial^2 G(x, y)}{\partial x^2} + \frac{\partial^2 G(x, y)}{\partial y^2}$$



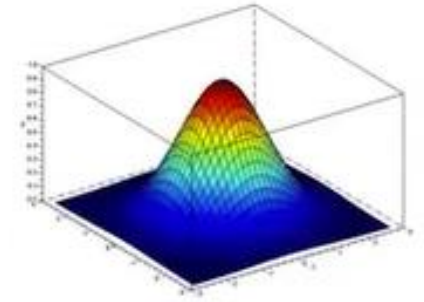
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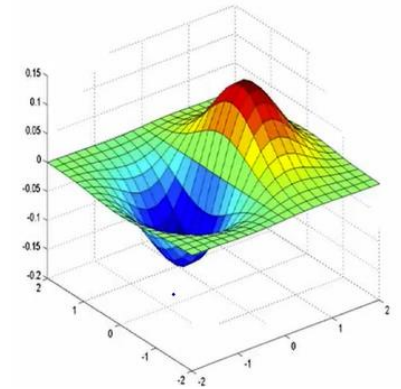
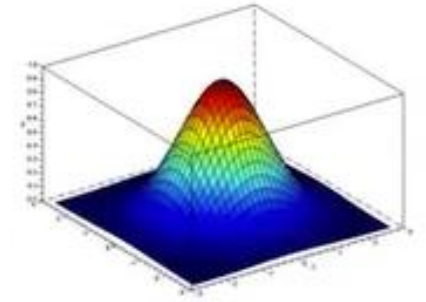
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Edge

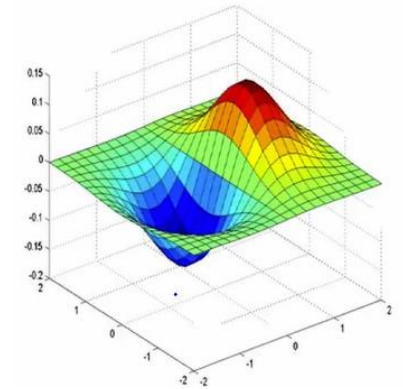
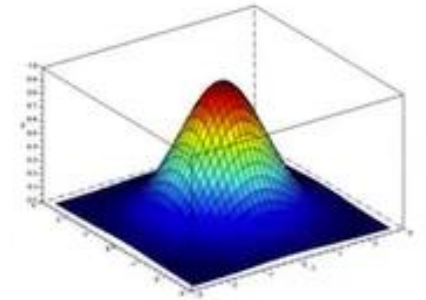
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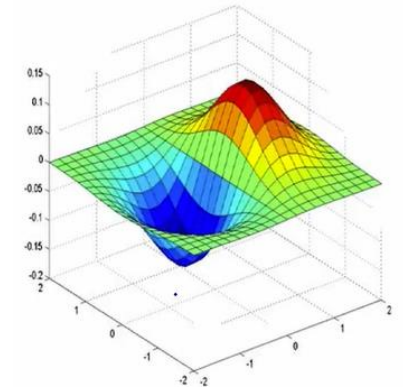
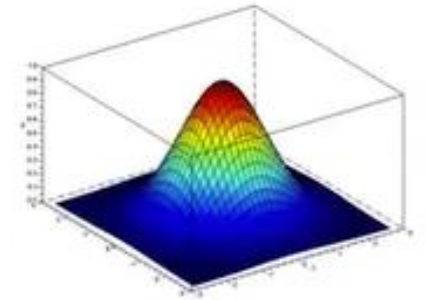
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$$= \left(\frac{x^2 + y^2 - 2\sigma^2}{\sigma^4} \right) e^{-\frac{x^2 + y^2}{2\sigma^2}}$$



Edge

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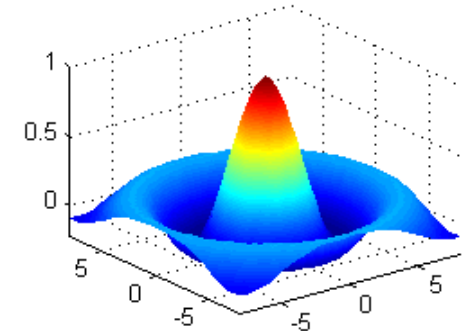
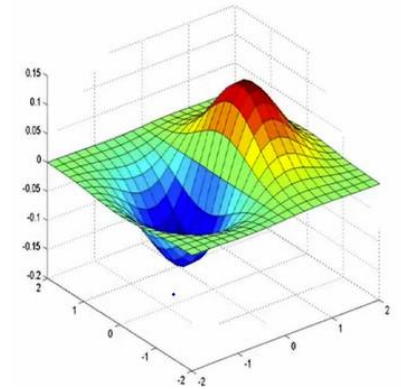
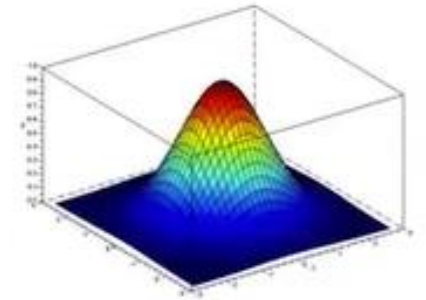
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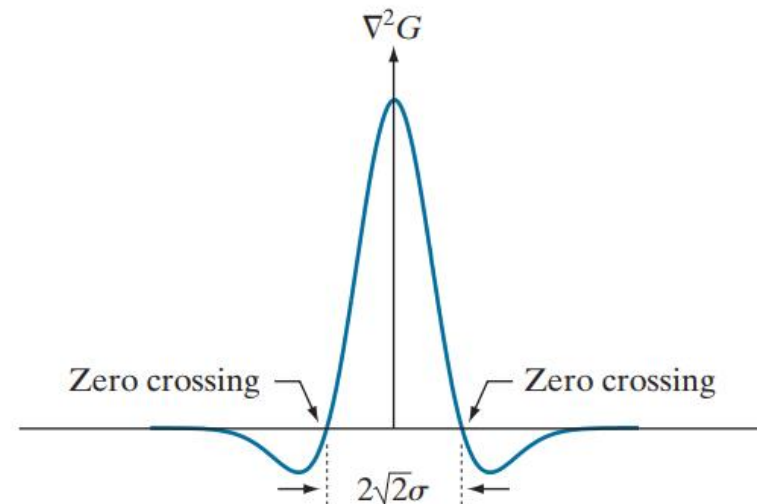
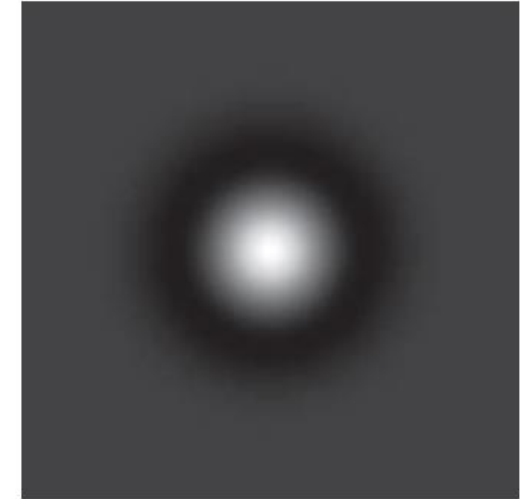
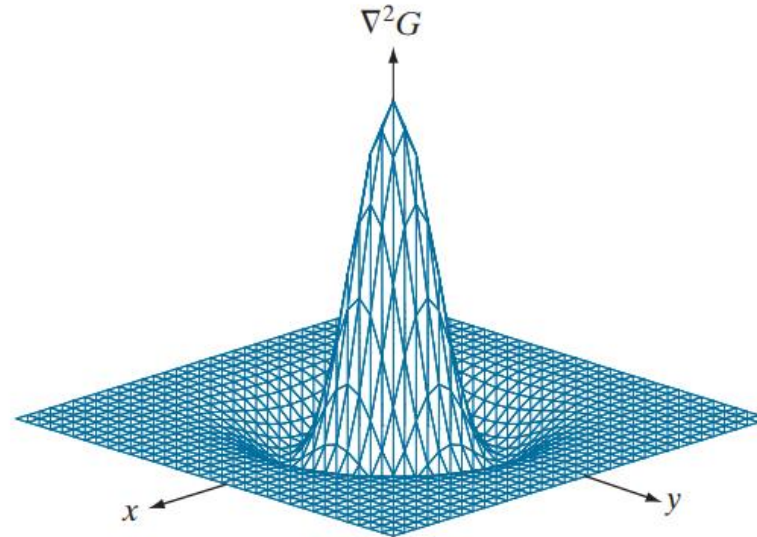
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Edge

■ LoG

- Laplacian of Gaussian
- for convenience negative of LoG are plotted



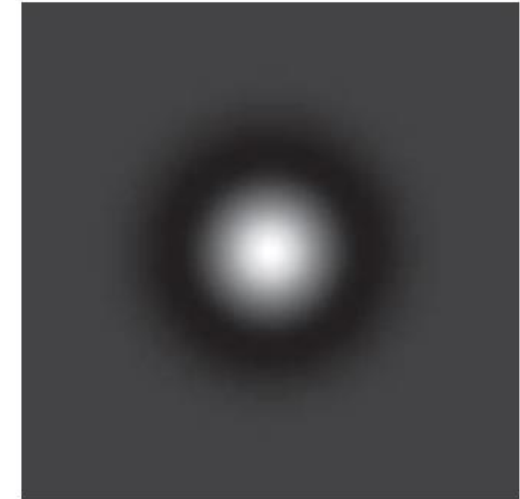
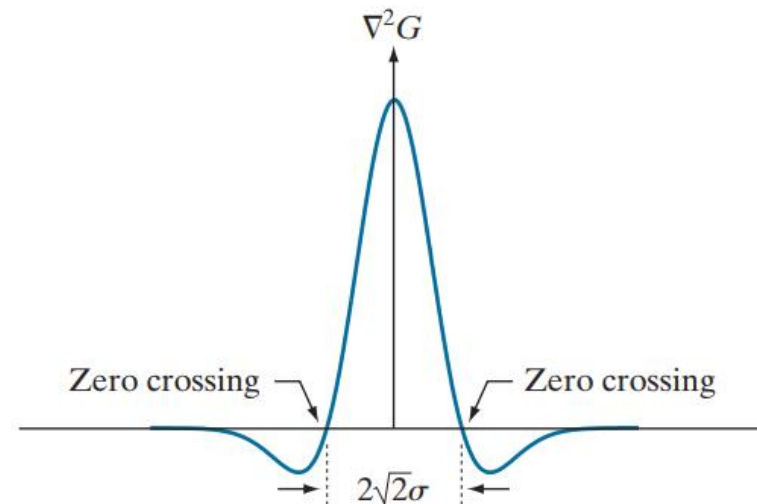
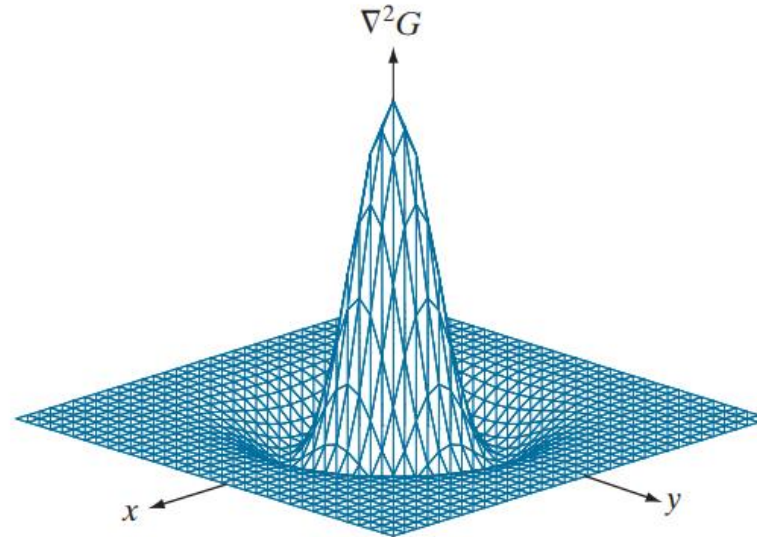
0	0	-1	0	0
0	-1	-2	-1	0
-1	-2	16	-2	-1
0	-1	-2	-1	0
0	0	-1	0	0

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$$g(x, y) = [\nabla^2 G(x, y)] \star f(x, y)$$



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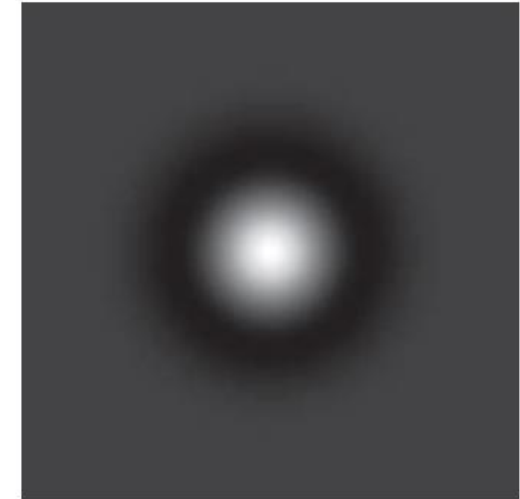
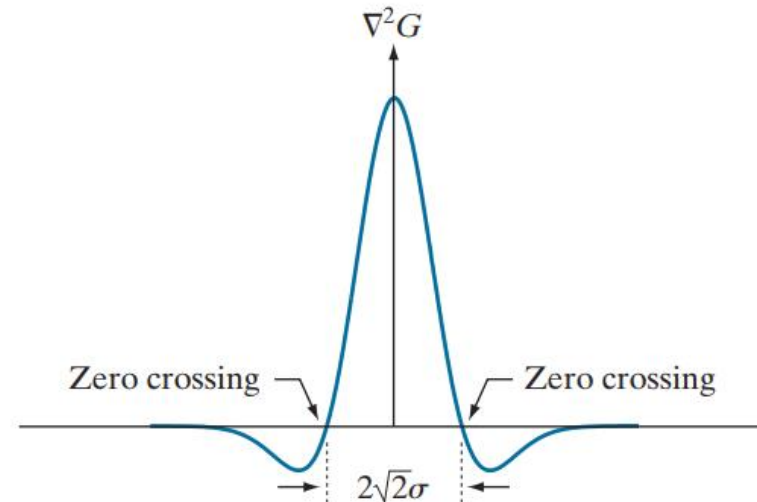
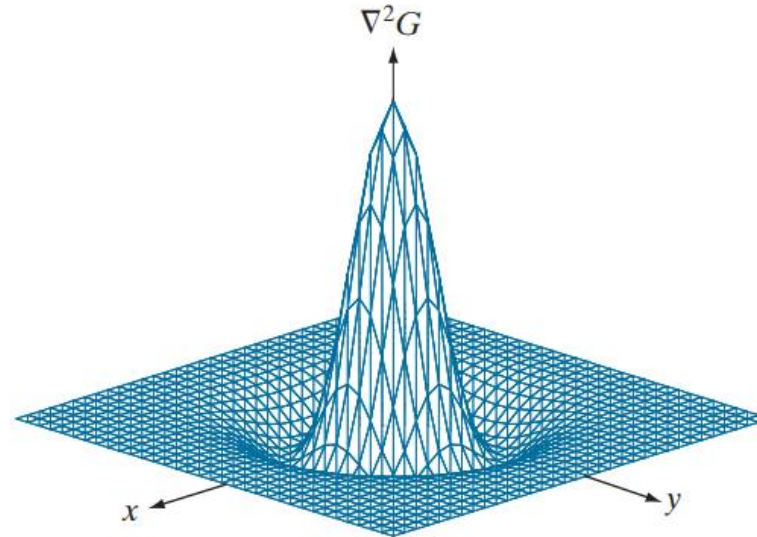
Edge

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Edge

- LoG

input



LoG



zero crossings



Edge

- LoG are approximately DoGs

Edge

- LoG are approximately DoGs
 - to speed up computations

Edge

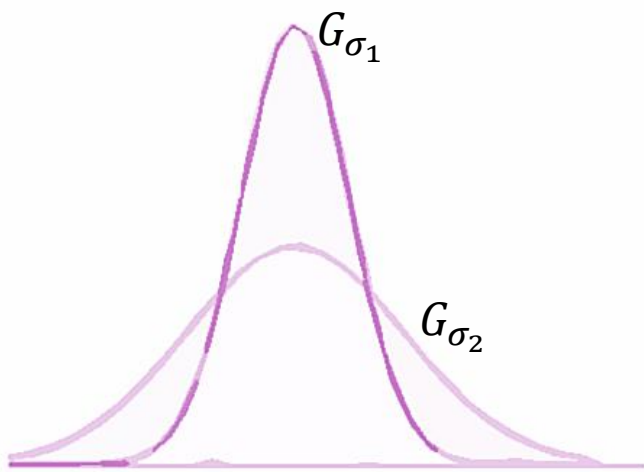
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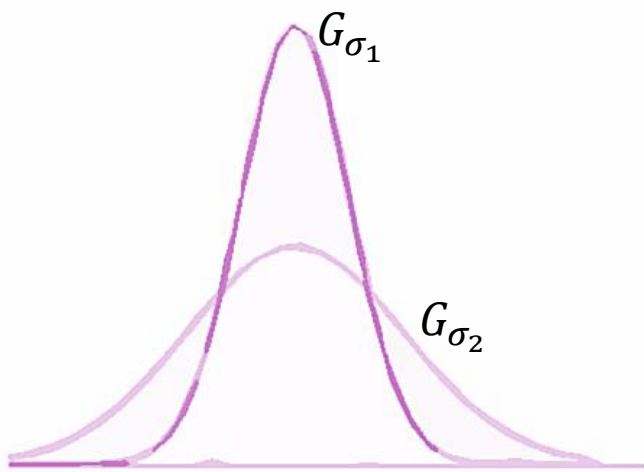


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- What is the best DOG?
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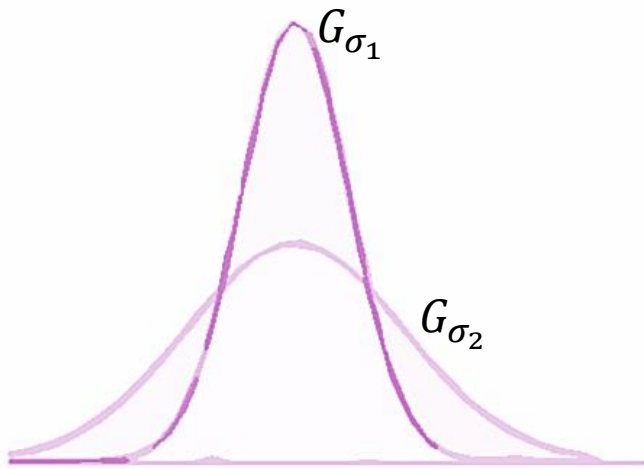


Edge

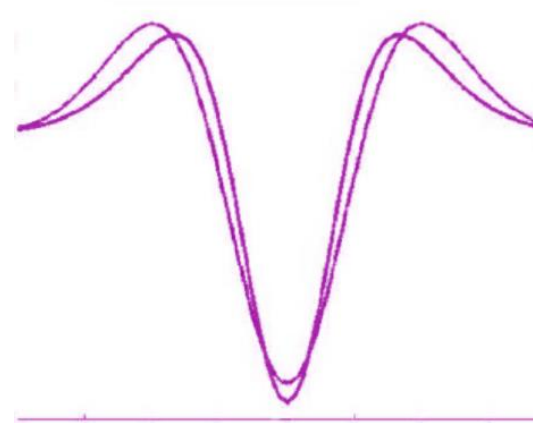
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 - the one who obeys the LoG closely



$$\sigma_1 = \frac{\sigma}{\sqrt{2}} \quad \sigma_2 = \sqrt{2} \sigma$$



Conclusion

- Operators

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□ There is no definition about what is a perfect edge

□ depending upon applications, edge definition changes

- Sobel
- Roberts
- Prewitt
- Laplacian
- LoG
- DoG

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- DoG

Who was I ?

