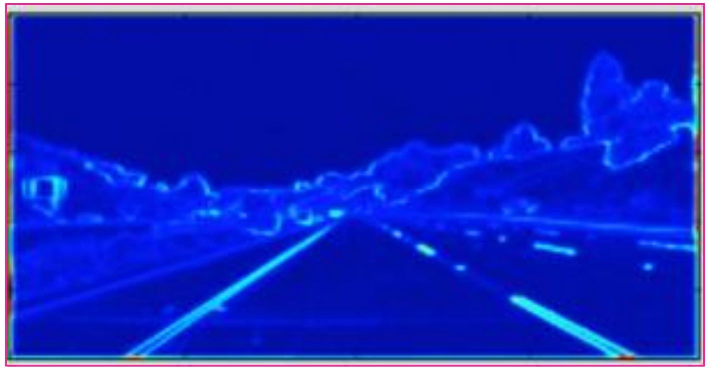
Edge:
Hough transform

Dr. Tushar Sandhan

Introduction



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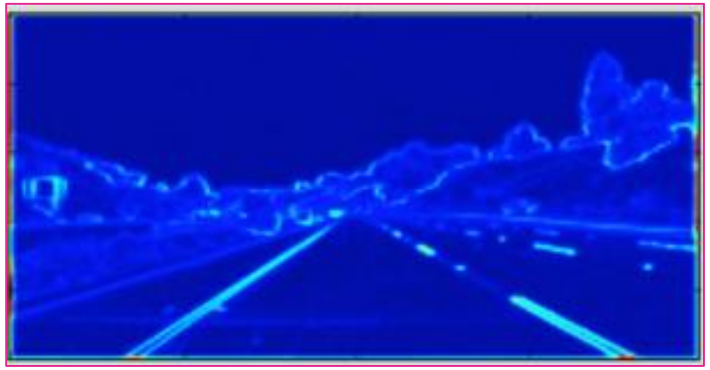

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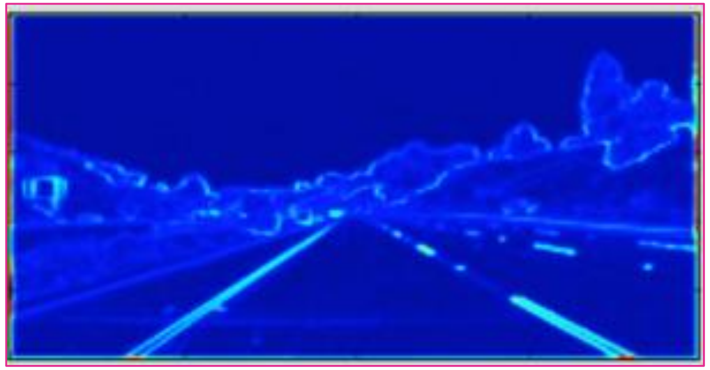

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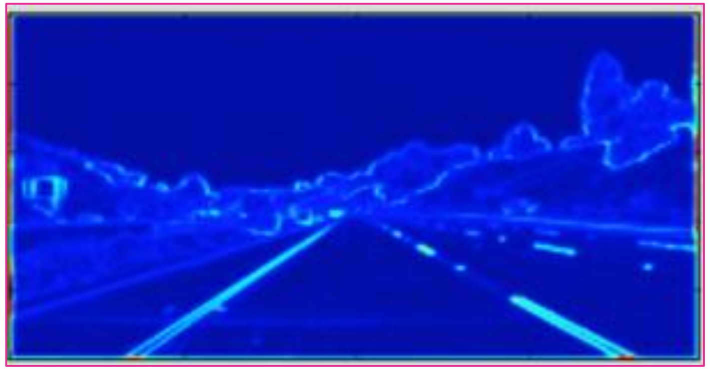

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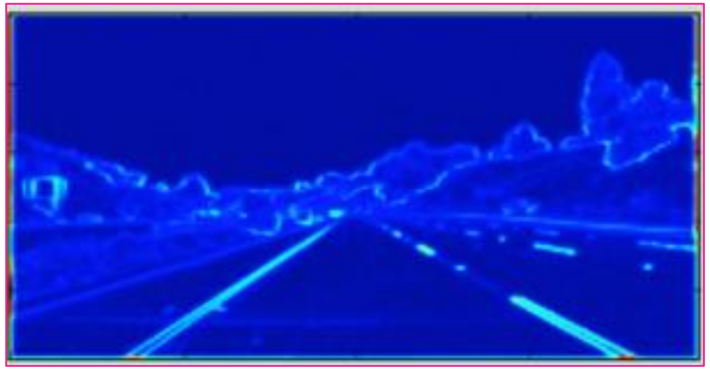

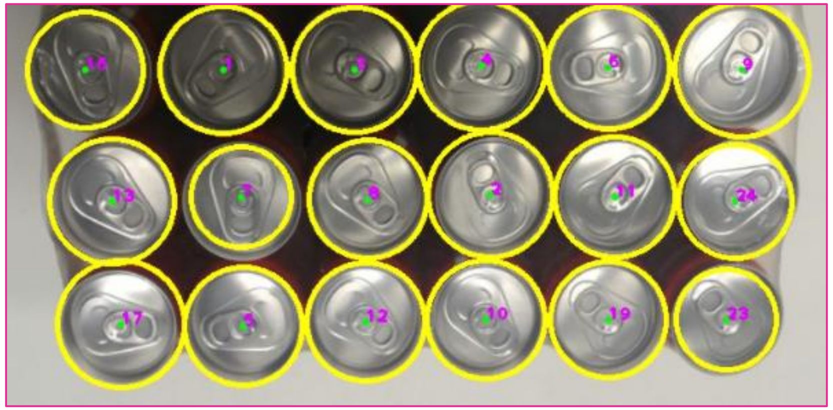

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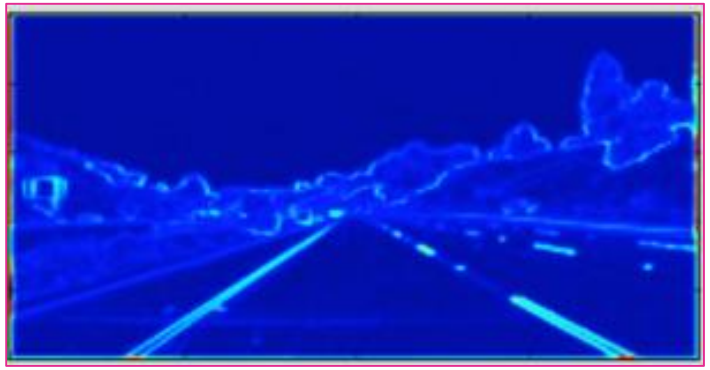

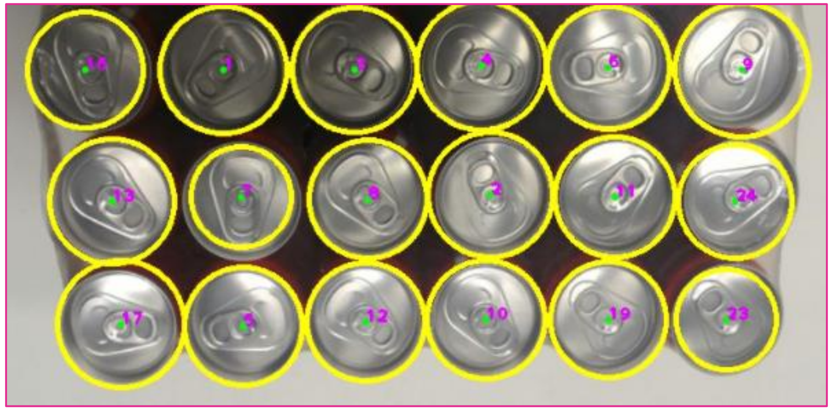

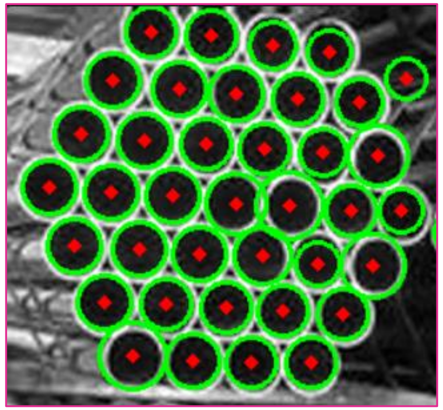

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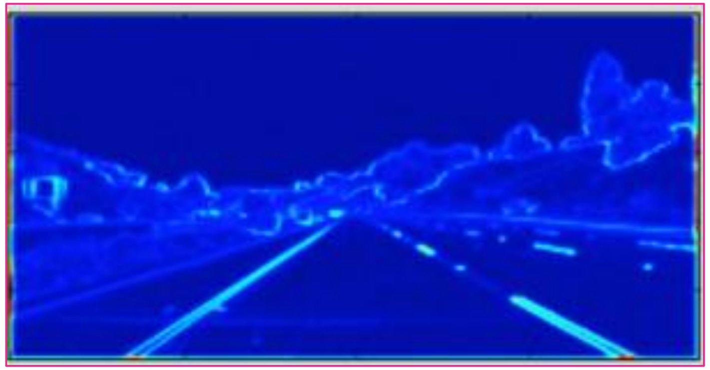

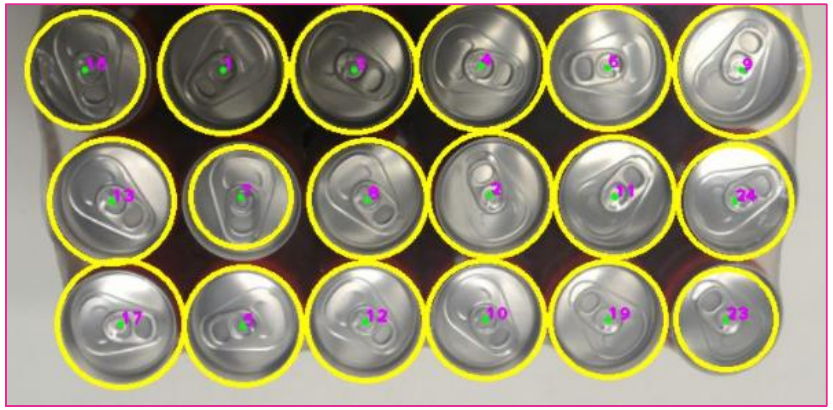

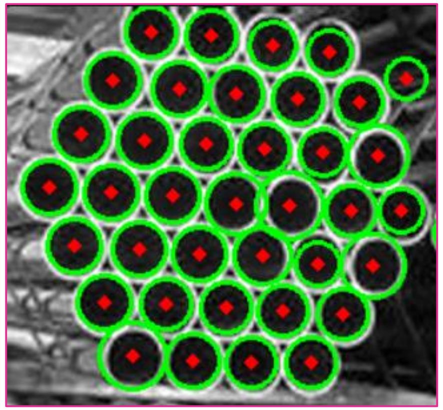

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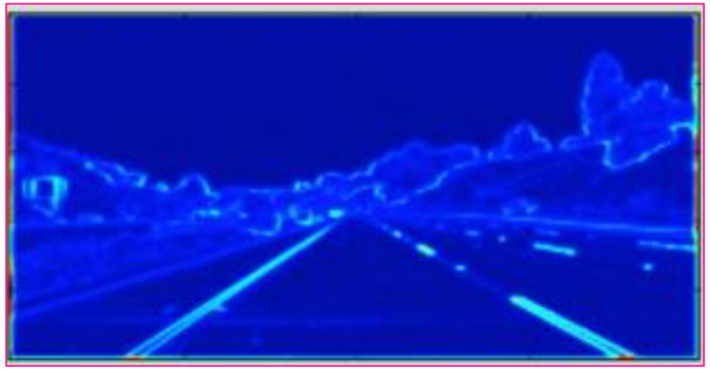

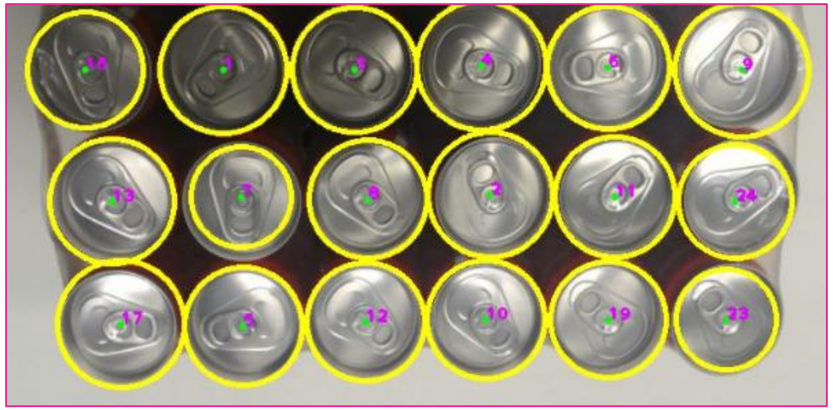

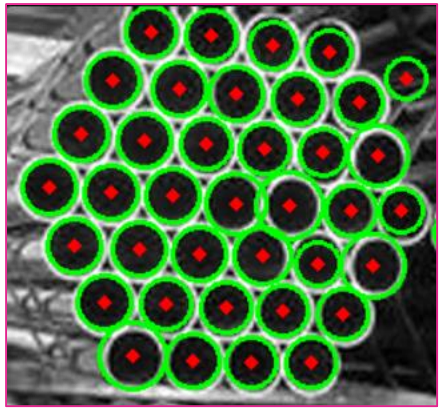

Hough transform

- Image derivatives
  - input image $f(x, y)$
  - (optional) smoothed $f_s(x, y)$
  - get the gradients $g_x(x, y)$, $g_y(x, y)$
  - get thresholded edge map M_T

$$G(x, y) = e^{-\frac{x^2 + y^2}{2\sigma^2}}$$

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  - considers shape of the object as prior info.
  - shape is defined as a function and parametrized

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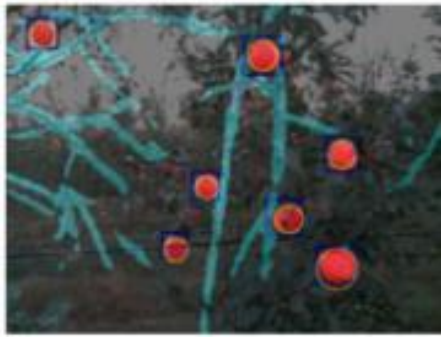

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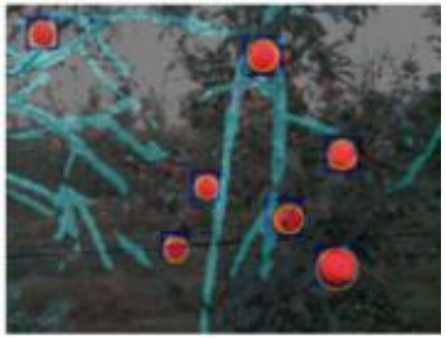
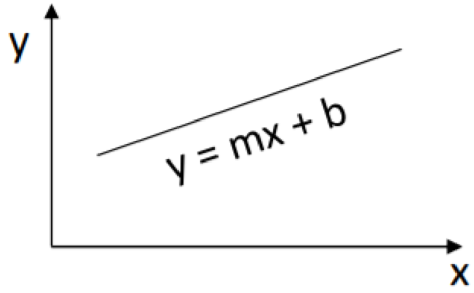


- Lines



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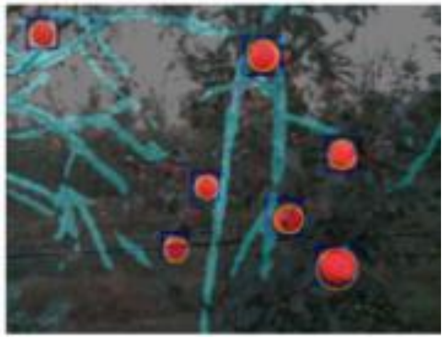

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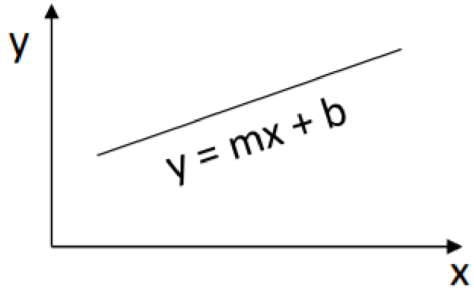

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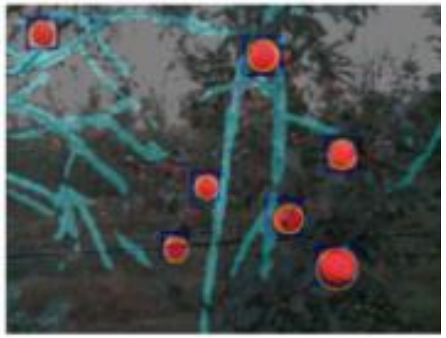

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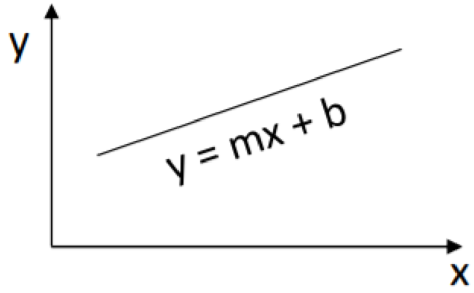

- Hough Transform
  - a space of parameters

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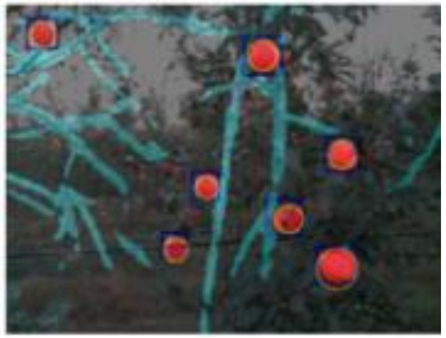

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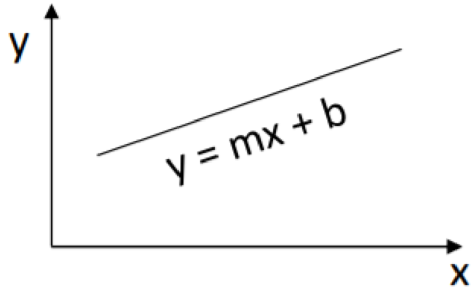

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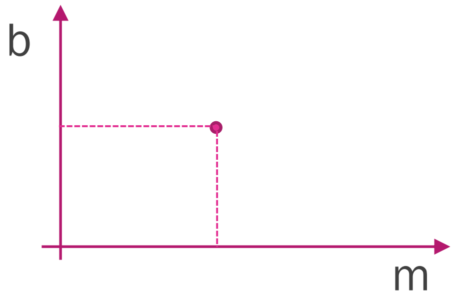

Hough transform

- Hough transform duality
  - Lines in the image space becomes a point in the Hough space
  - A point in the image space becomes ____ in the Hough space

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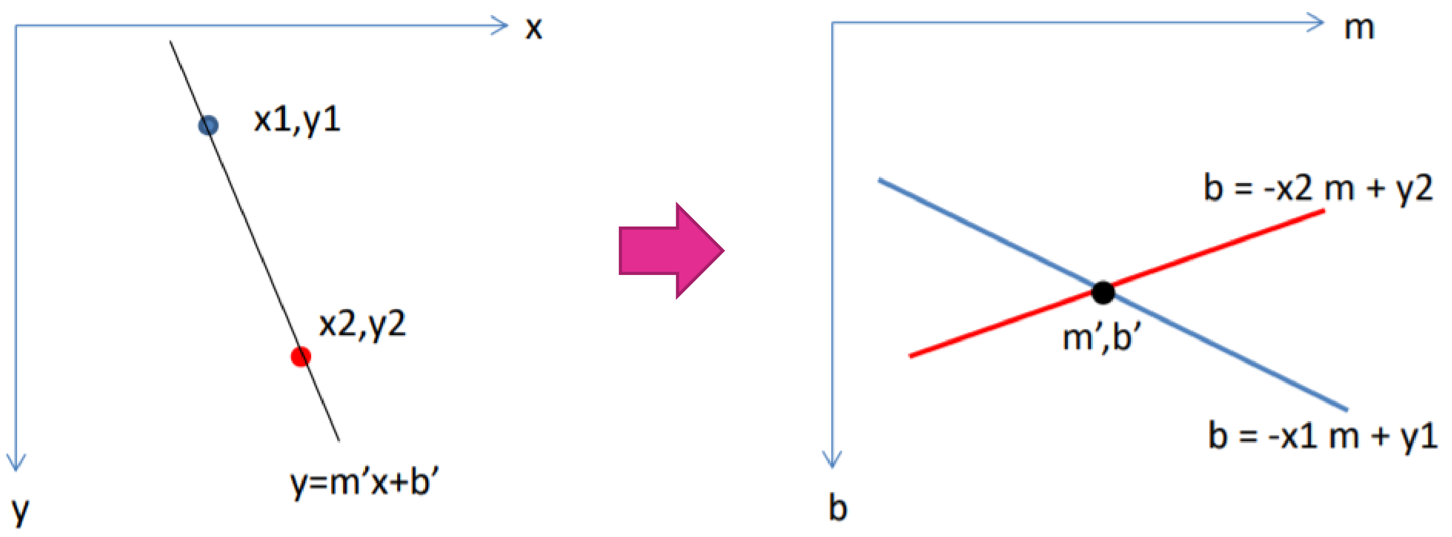

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- Hough space voting

courtesy: W. Hoff

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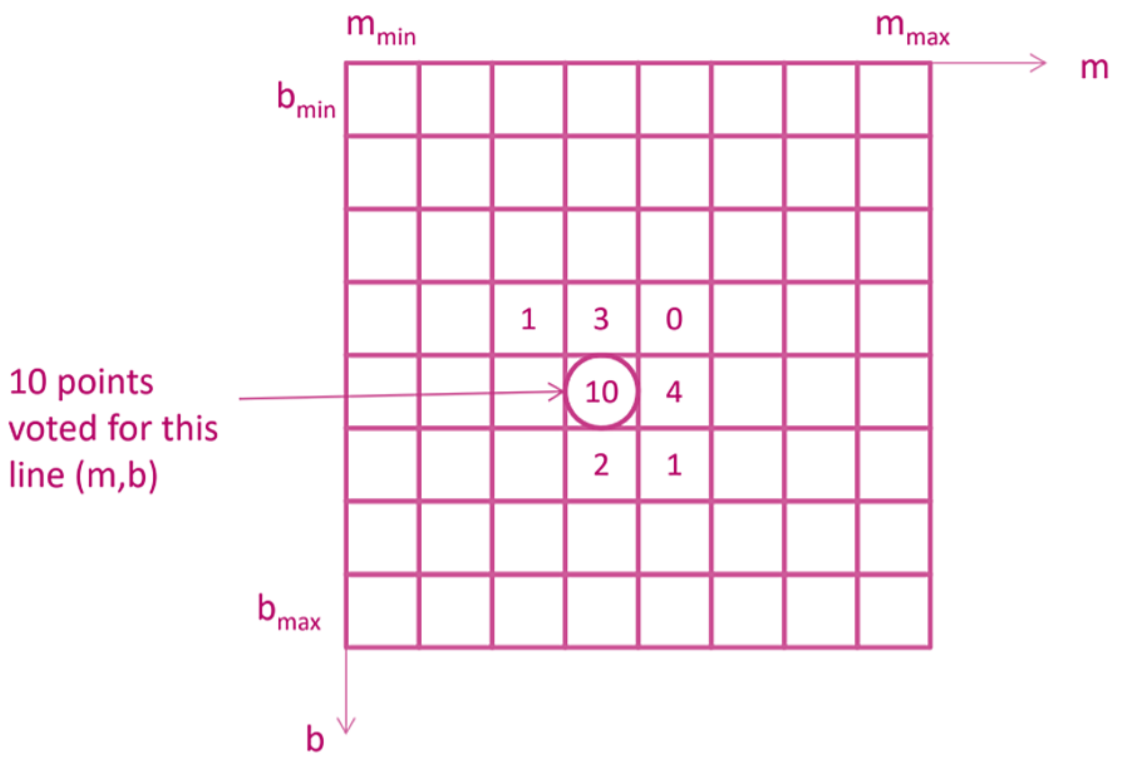

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  - initialize accumulator A(m,b) → 0
  - for each edge element,
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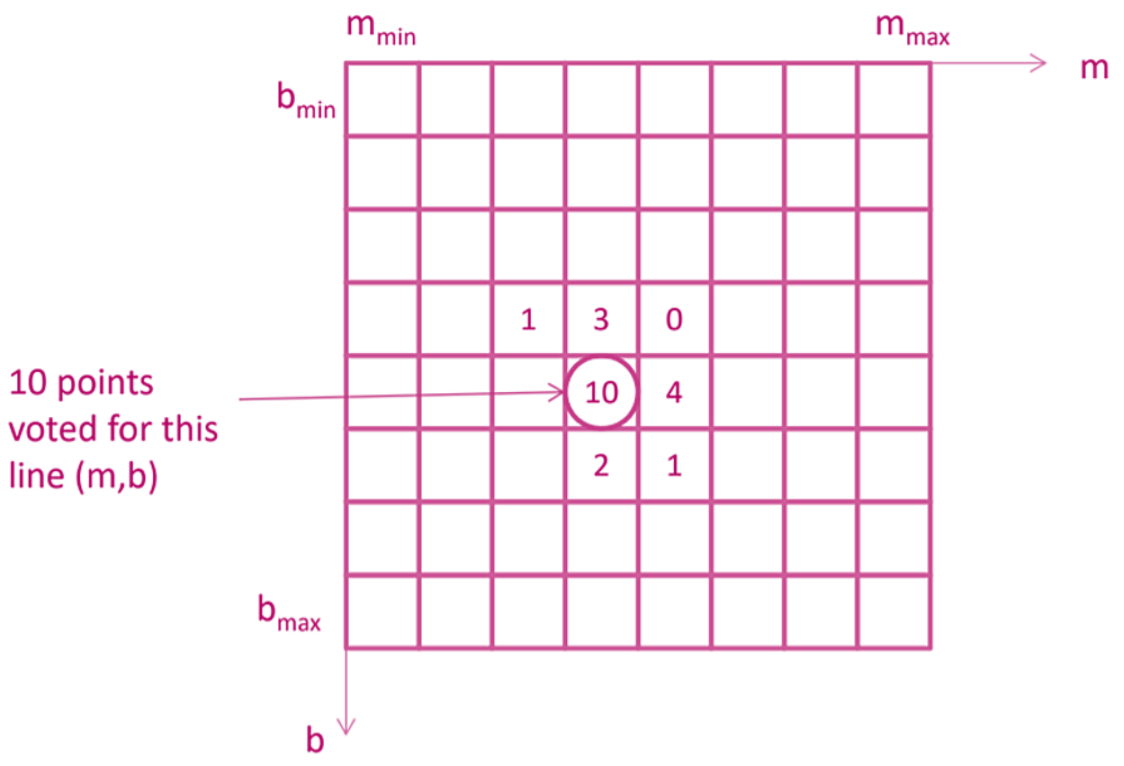

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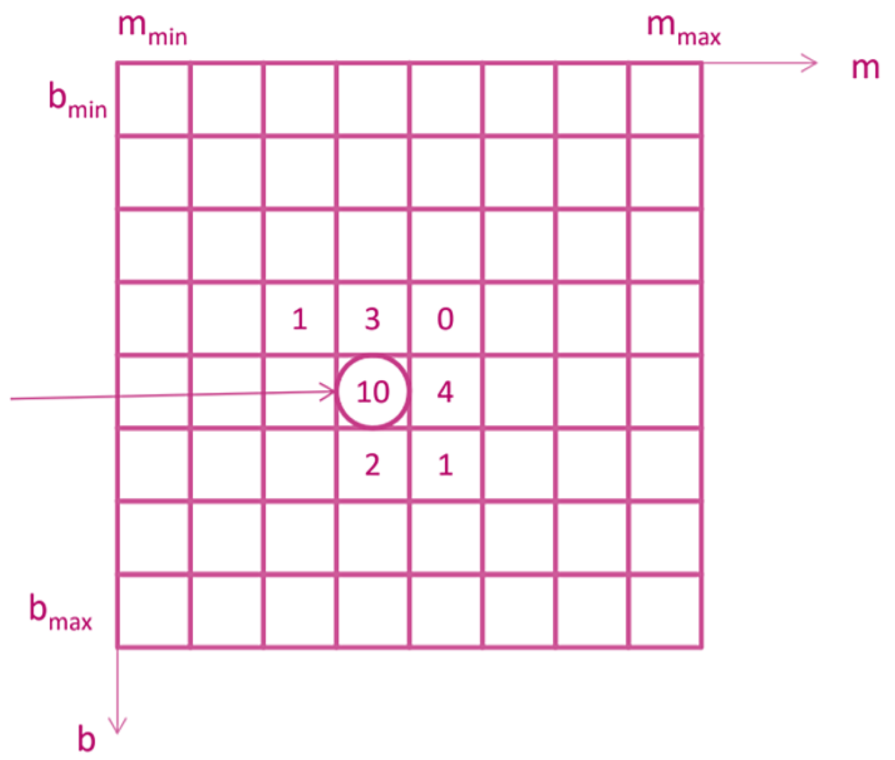
  - local maxima in A(m,b) correspond to lines

10 points voted for this line (m,b)



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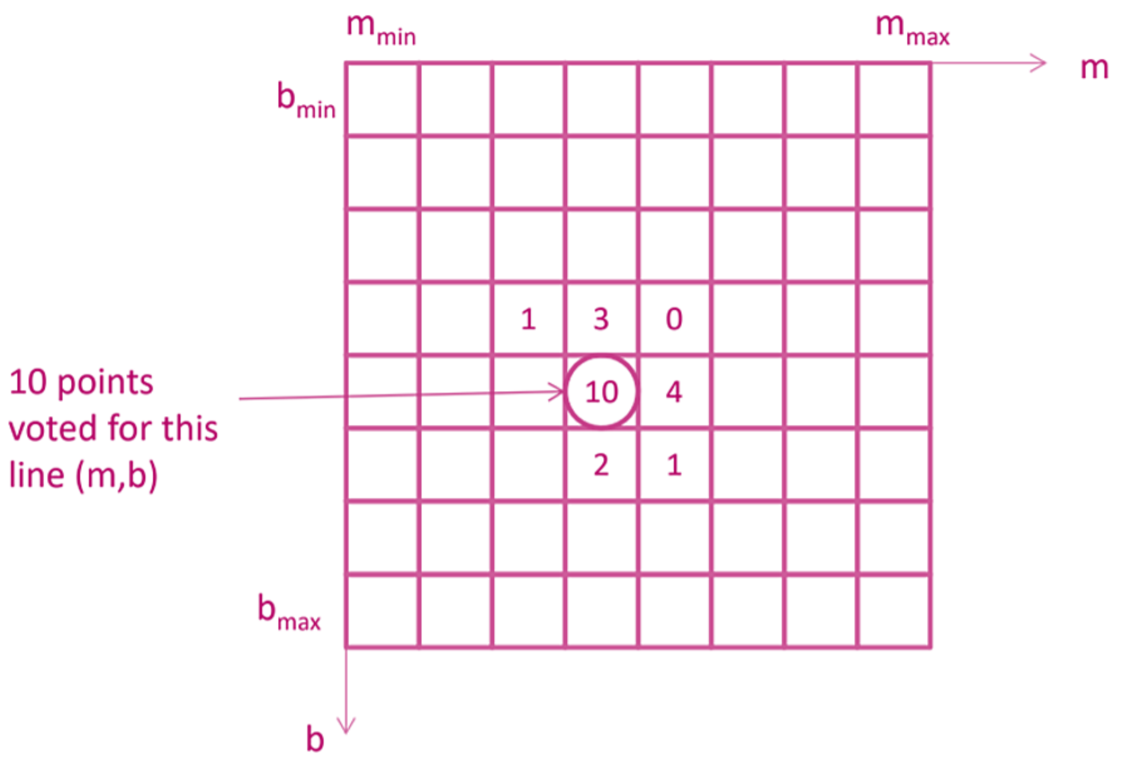

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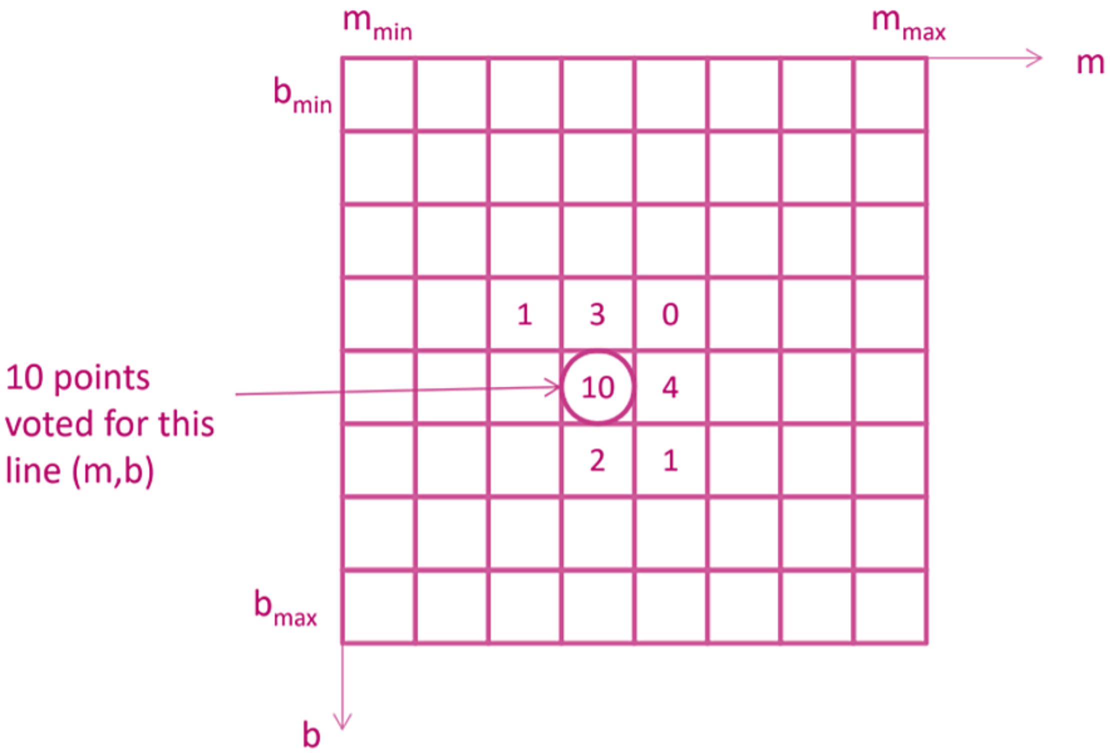

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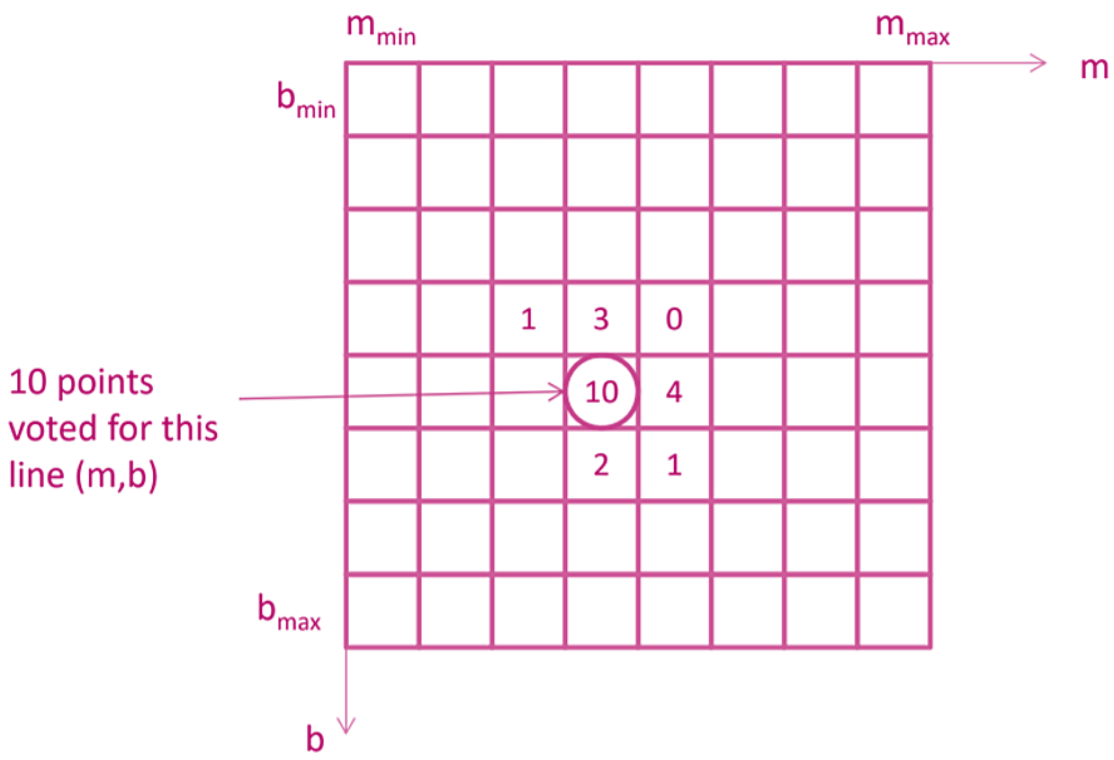

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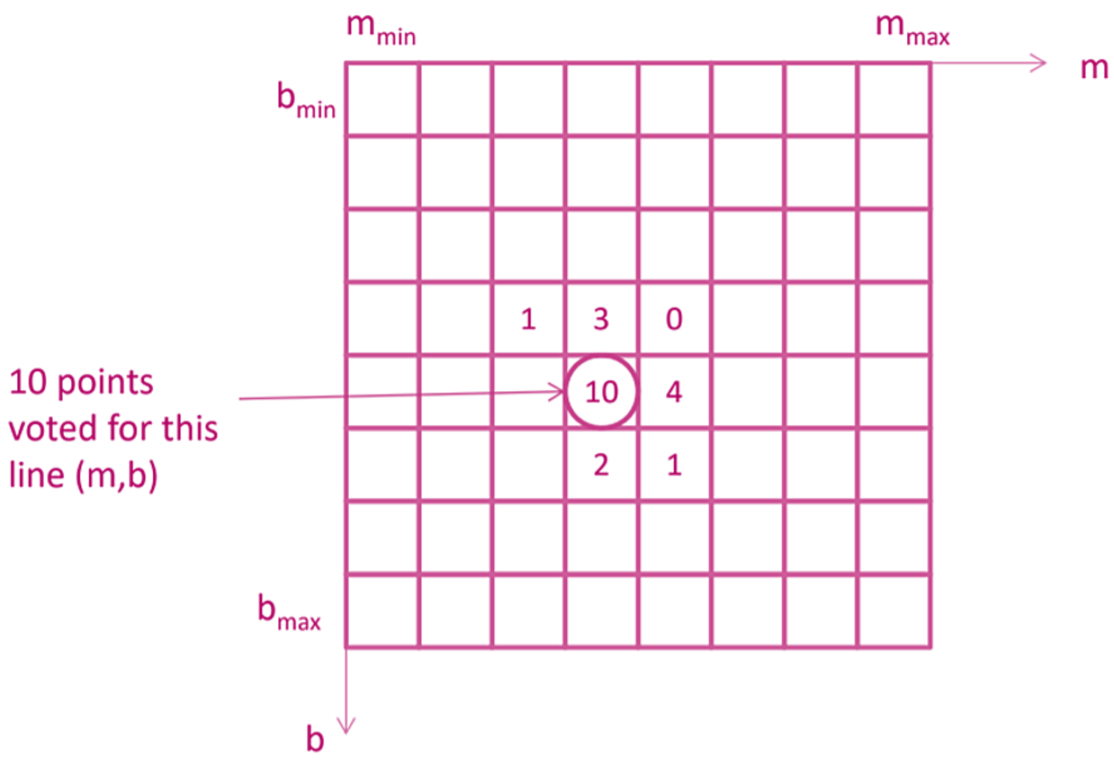
      - m → ∞



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- Horizontal lines
  - $\theta = 0^o$

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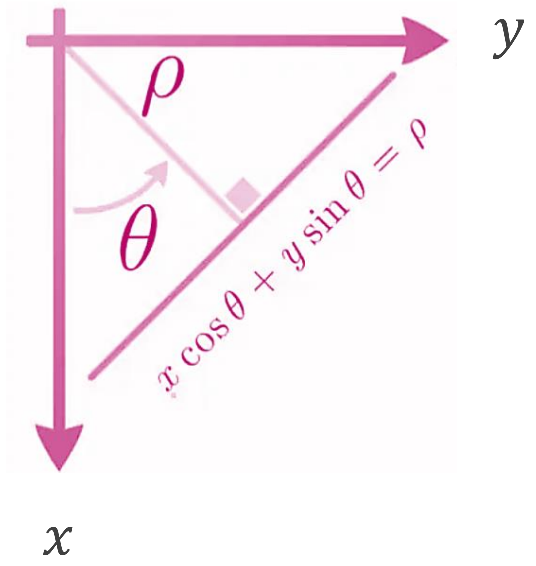

Hough transform

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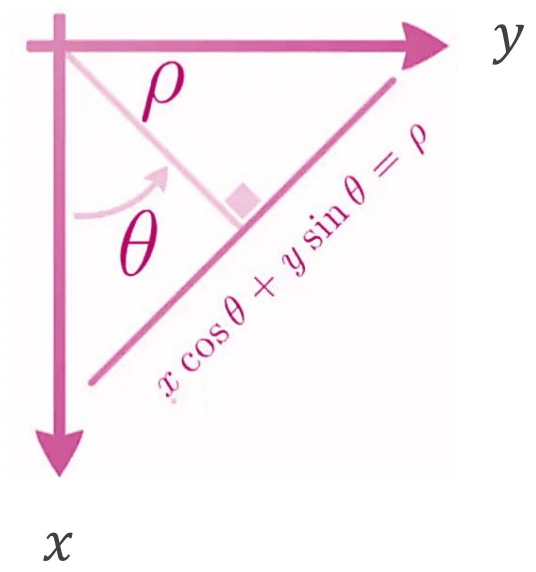
  - $\theta = 0^o$

- Vertical lines
  - $\theta = 90^o$



Hough transform

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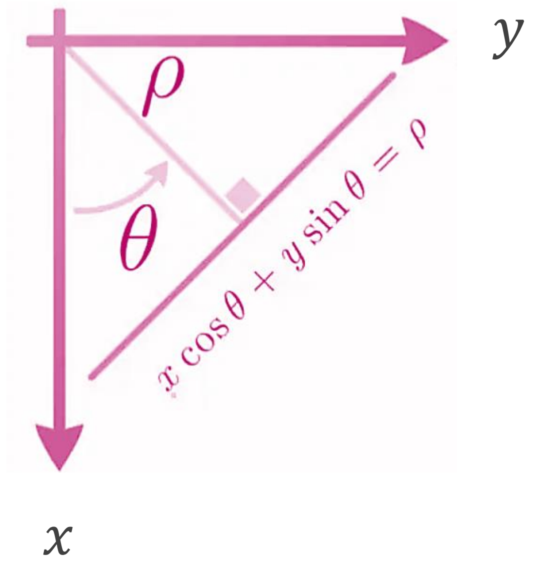
- Vertical lines
  - $\theta = 90^o$

- Ranges:
  - $\theta \in [-90^o, 90)$
  - $\rho \in [-\text{dmax}, +\text{dmax}]$
  - dmax?



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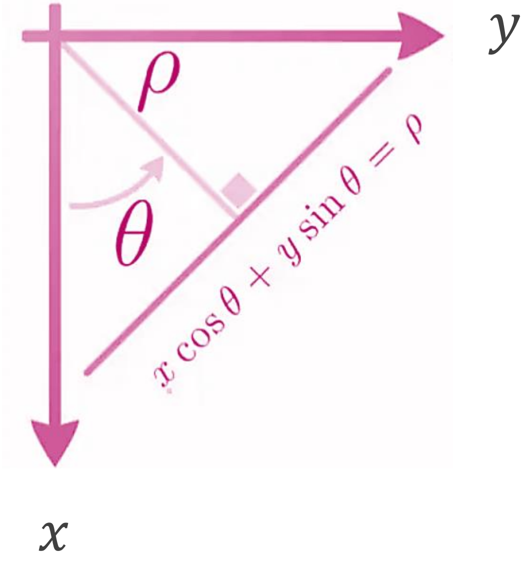
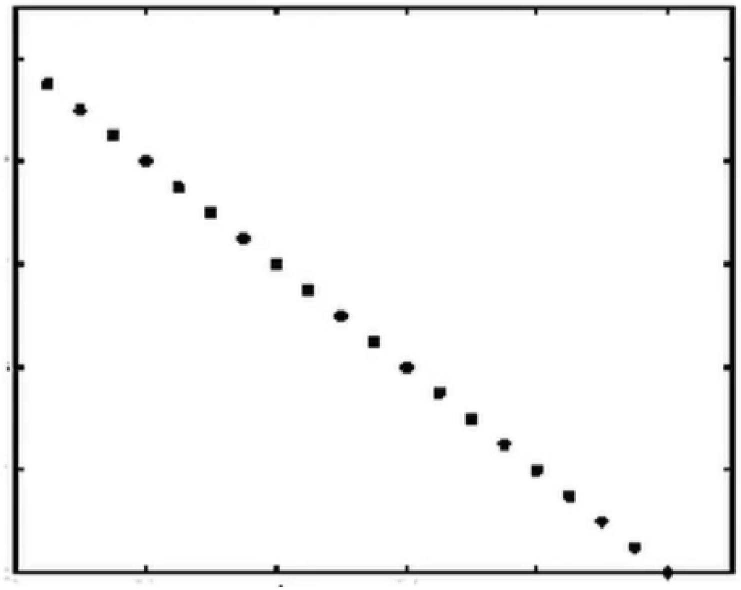


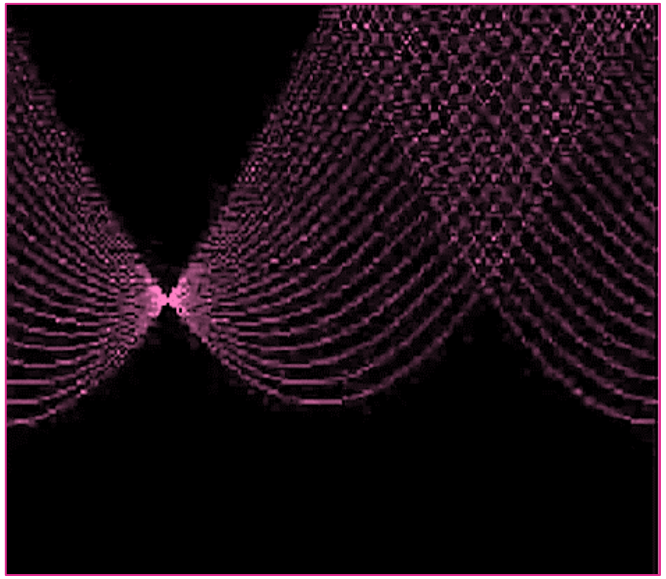
Hough transform

- HT example:

y



x

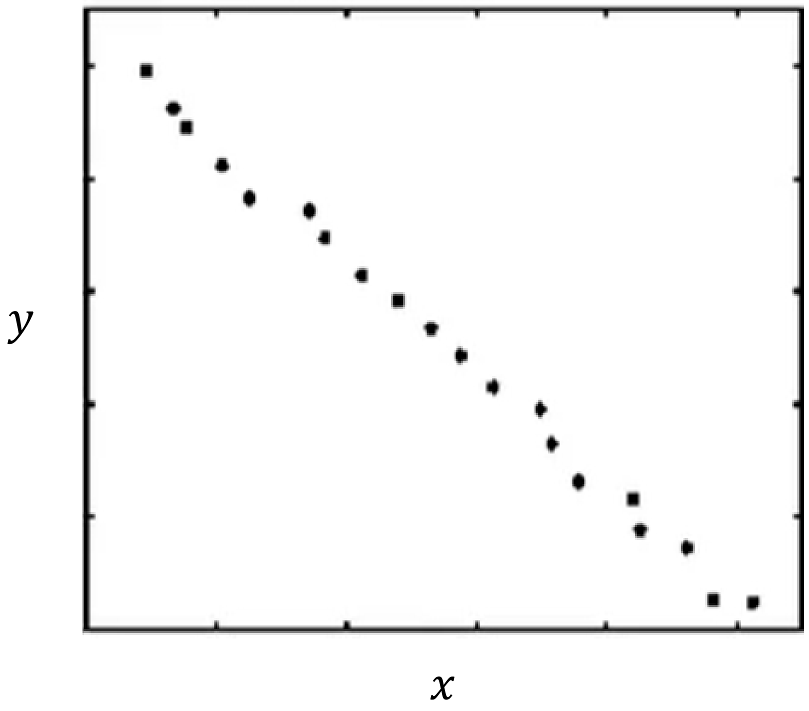


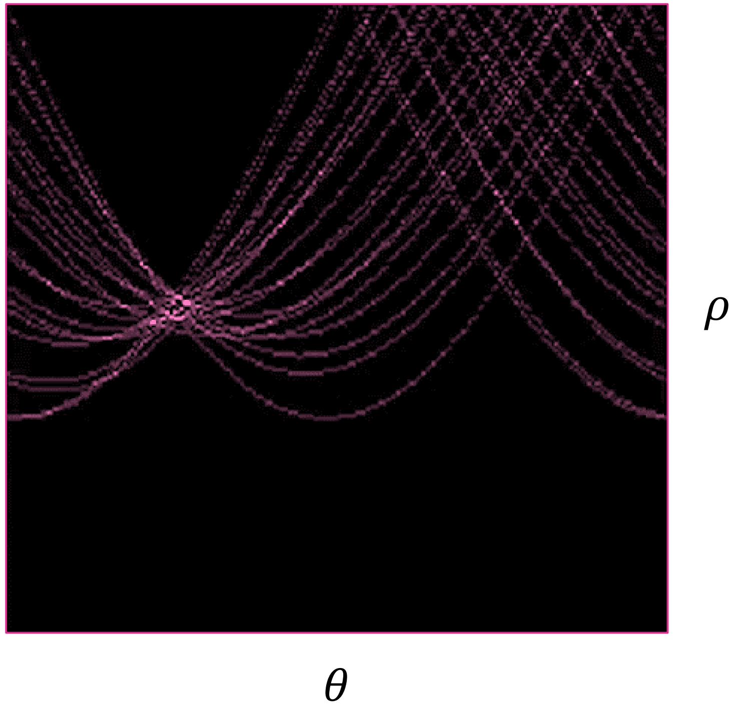
ρ

θ

Hough transform

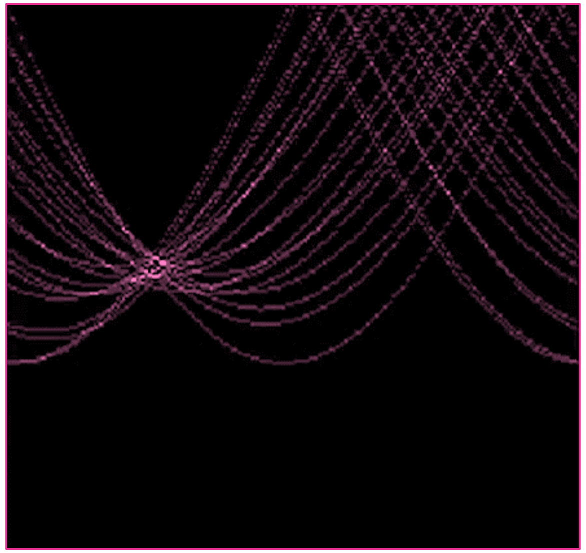
- HT example: with a greater noise





Hough transform

- HT example: with a greater noise
  - tackling the noise



ρ

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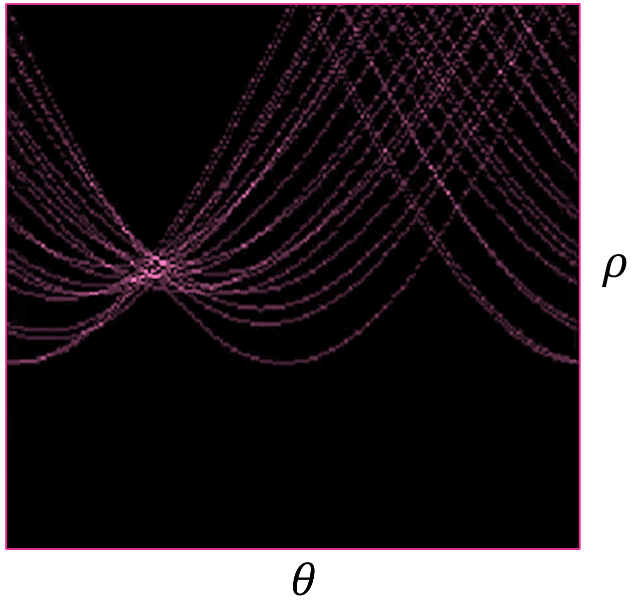
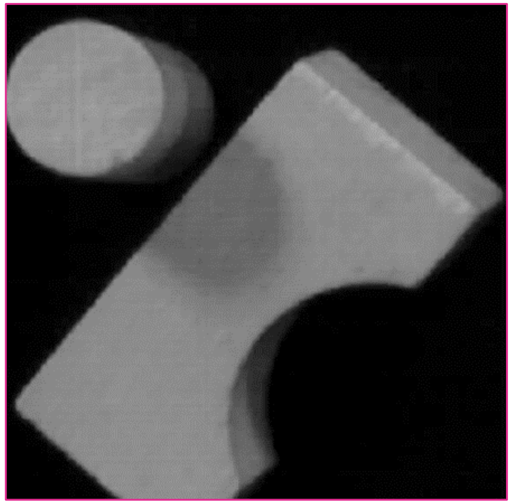


- Image processing in the Hough space
  - smoothing
  - thresholding
  - zoom in the space
  - re-quantize zoomed space
  - redo HT in zoomed space

Hough transform

input



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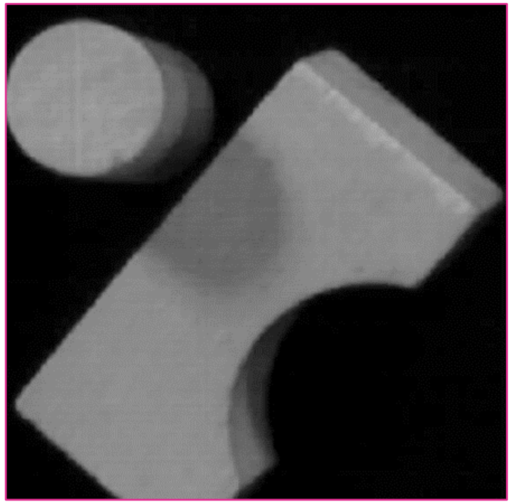

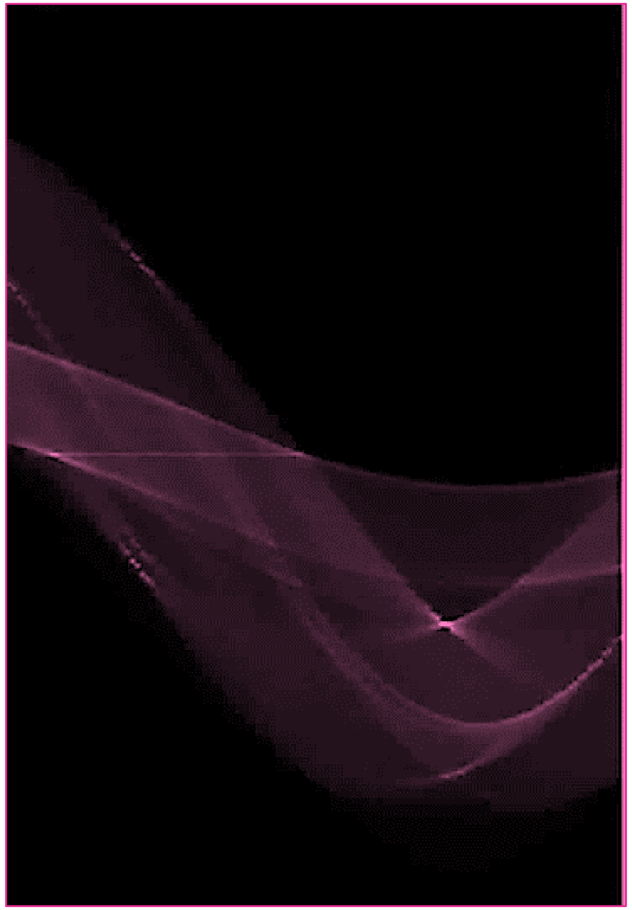

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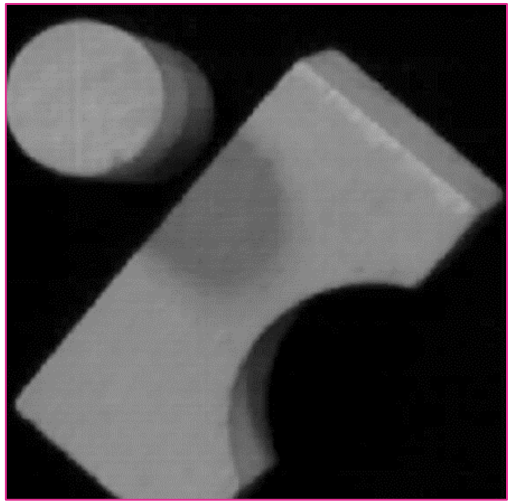

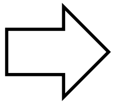

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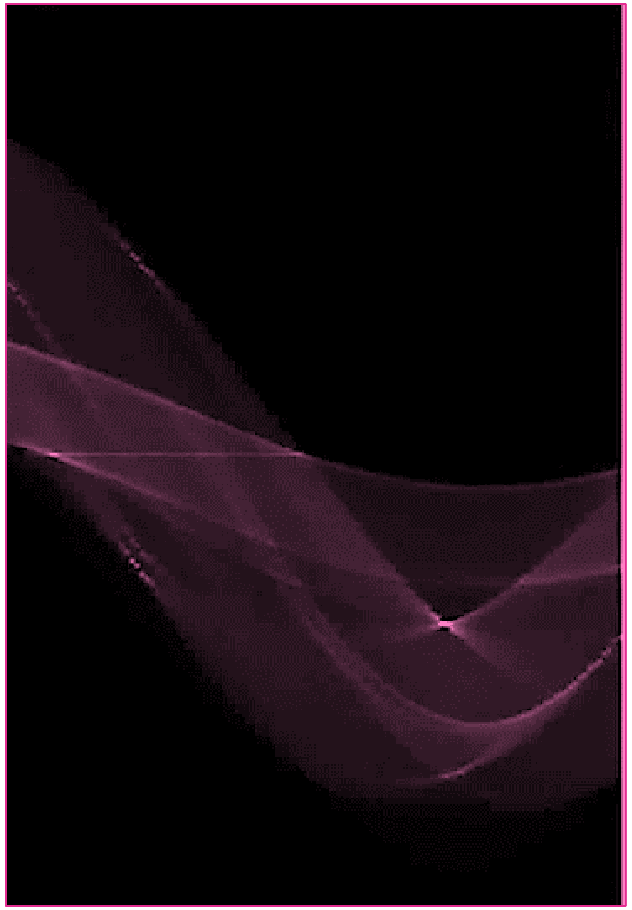
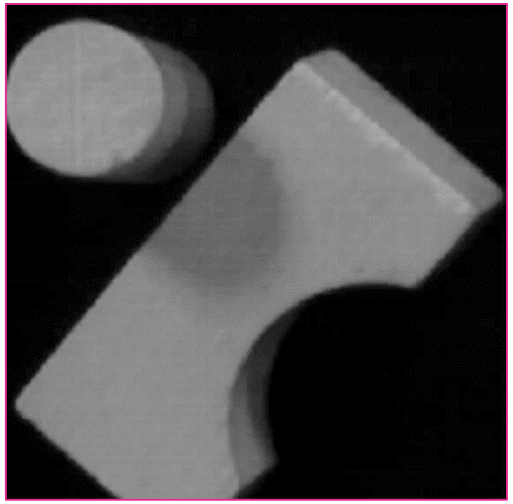
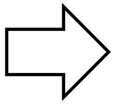


Hough transform

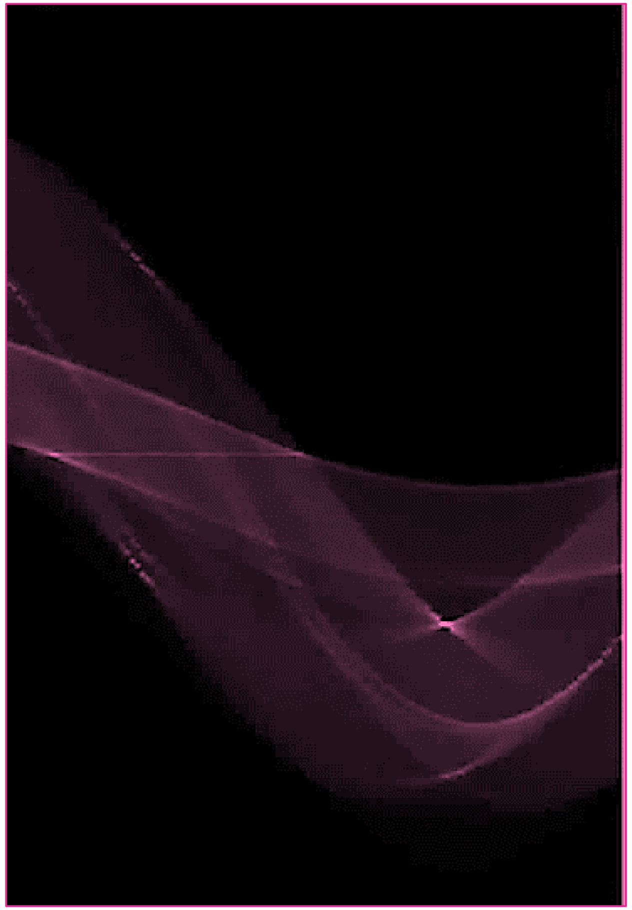
- bright spot in HT corresponds to?

input





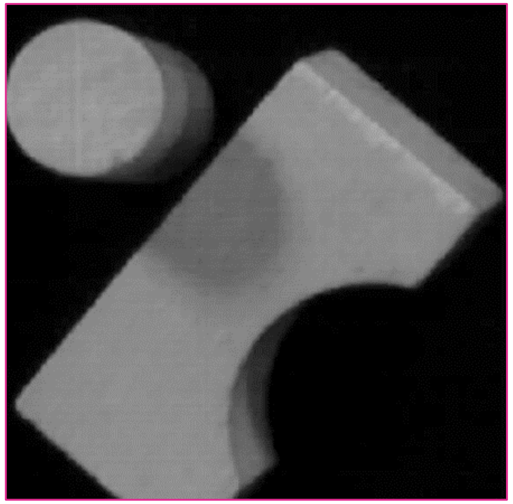
Hough transform

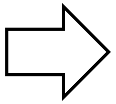


Hough transform

- bright spot in HT corresponds to?
- can circles be detected?

input





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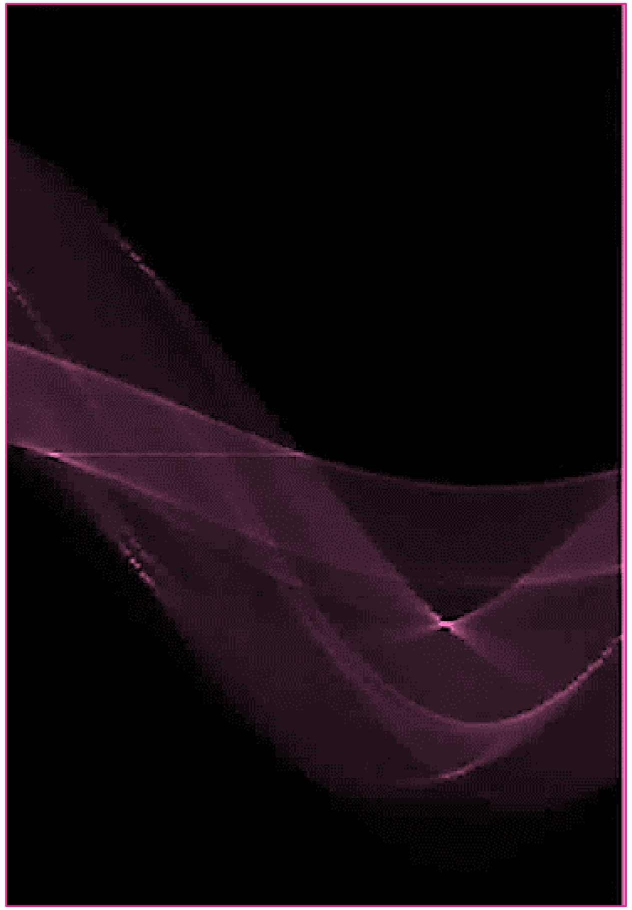
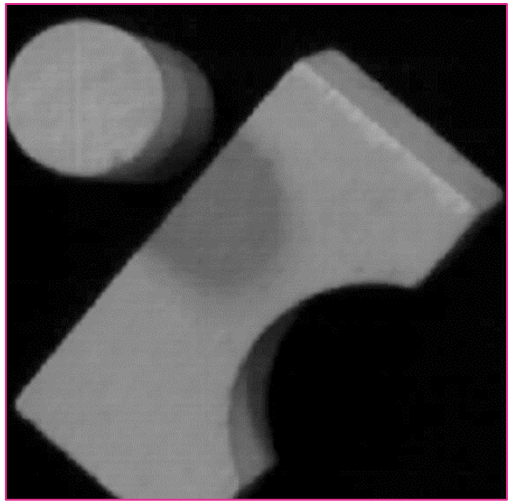
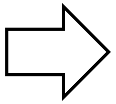


Hough transform

- bright spot in HT corresponds to?
- can circles be detected?
- why does Hough image is not the same size as input?

input





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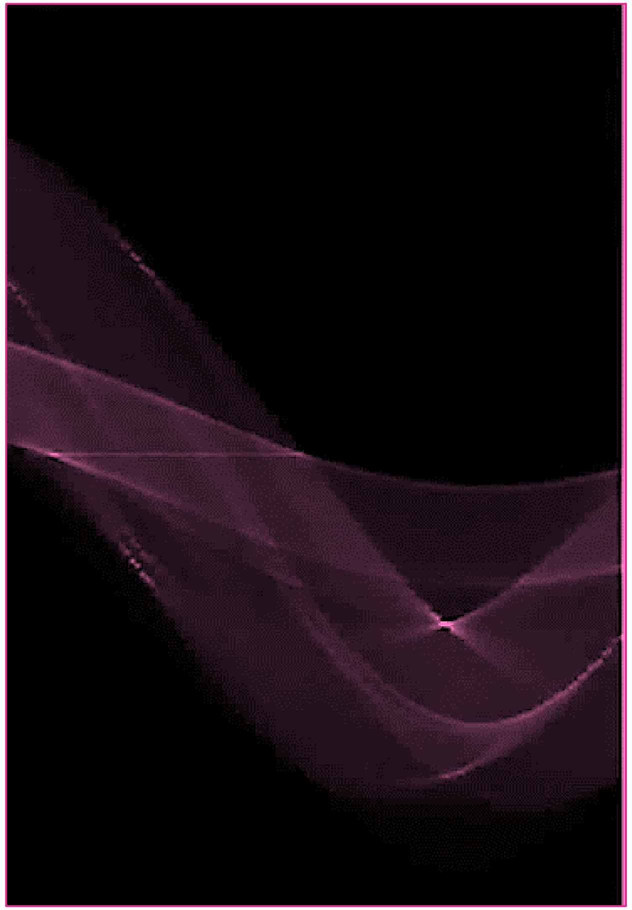


Hough transform

Input

Hough transform

Input


Hough transform

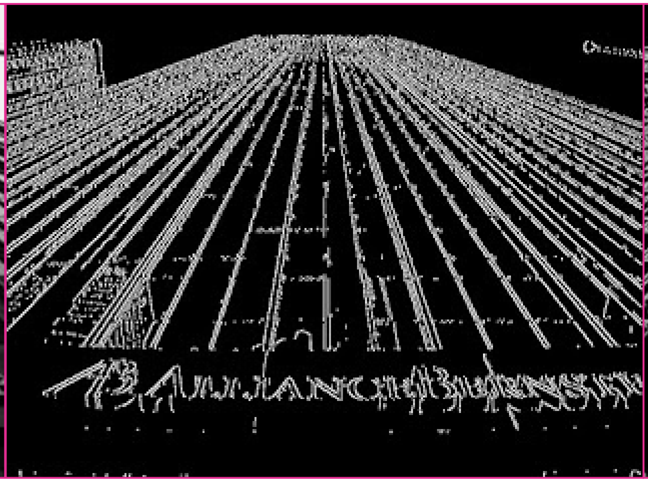
Input

Sobel

Hough transform

Input

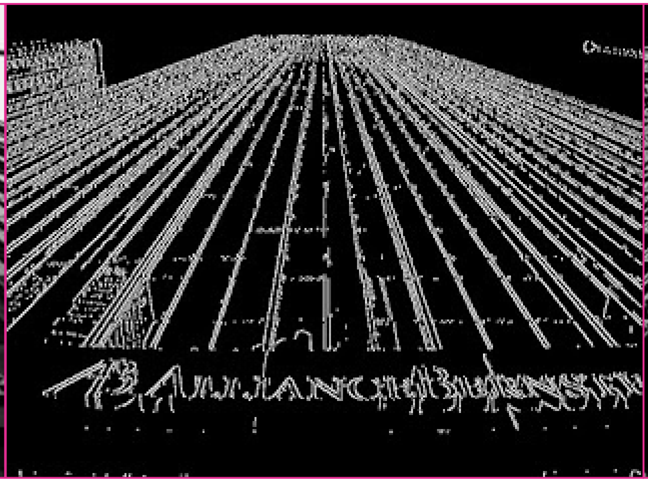
Sobel



Hough transform

Input

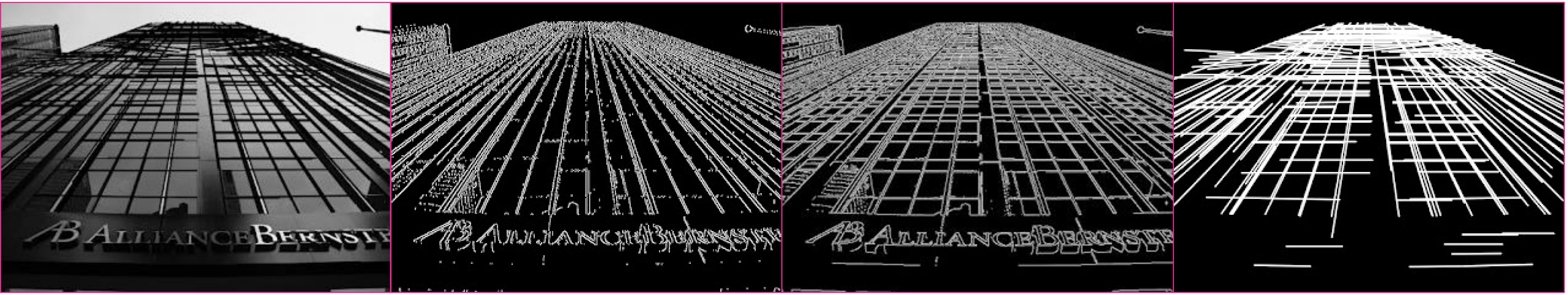
Sobel



Canny

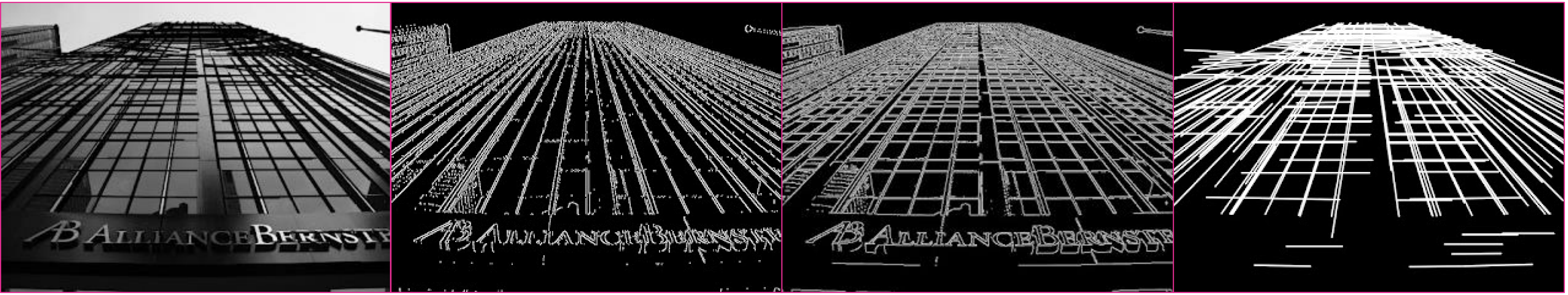
Hough transform

Input Sobel Canny



Hough transform

Input | Sobel | Canny | Hough



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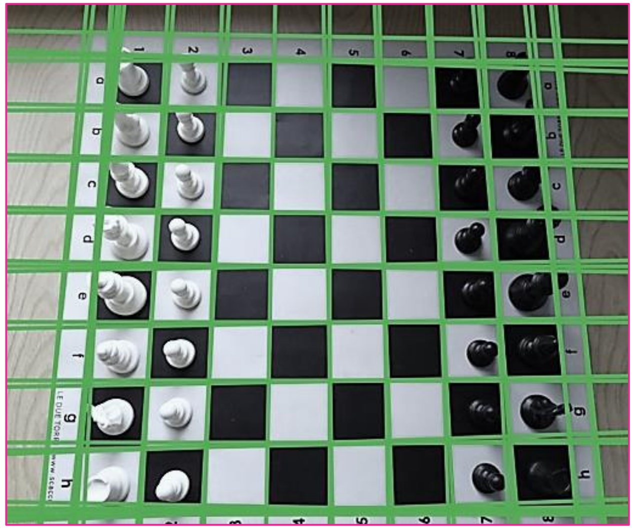
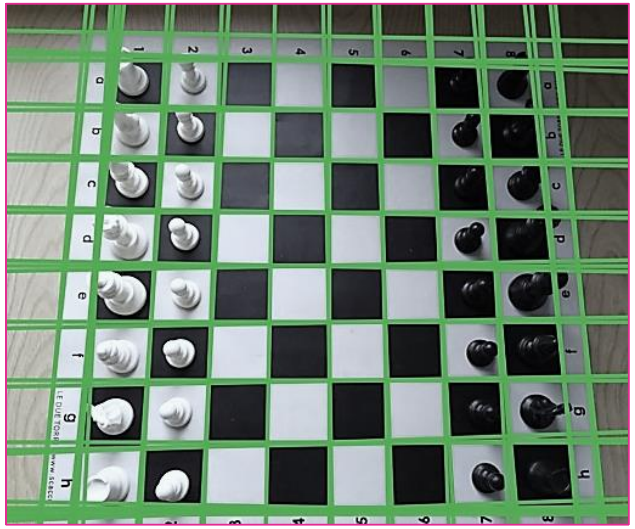
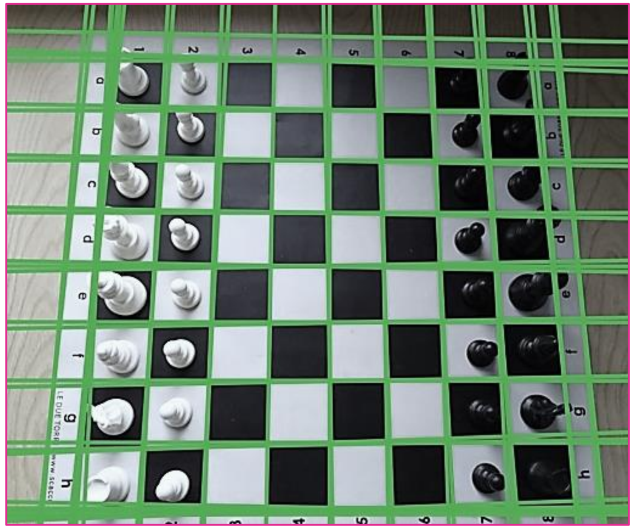


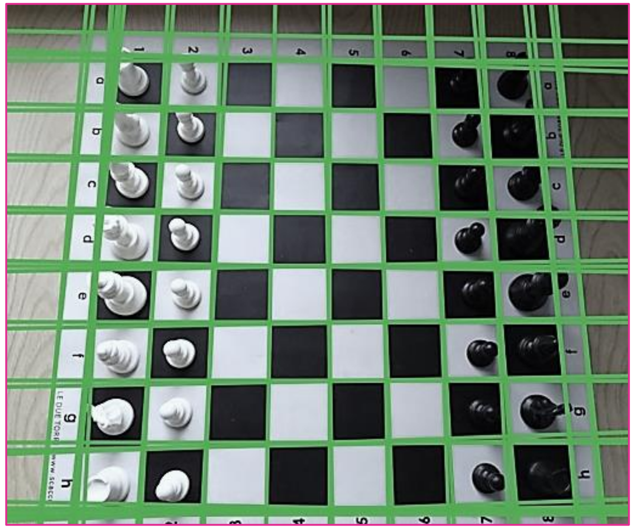
Hough transform



Hough transform

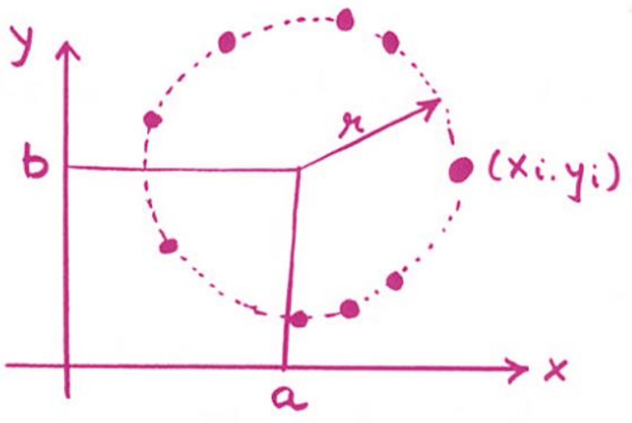


Hough transform



Hough transform

- Equation of circle



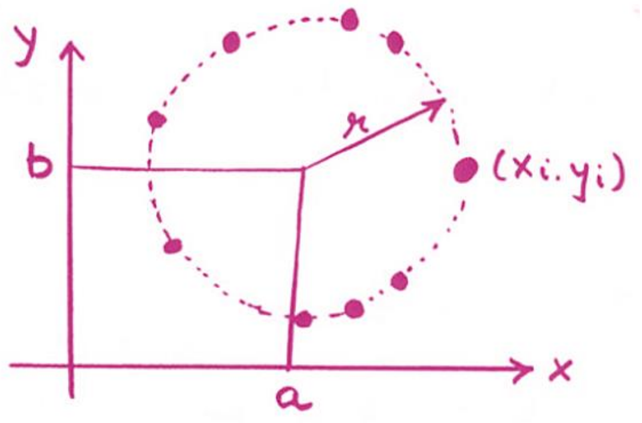
Hough transform

- Equation of circle

$$(x_i - a)^2 + (y_i - b)^2 = r^2$$

If radius is known: (2D Hough Space)

Accumulator Array $A(a,b)$



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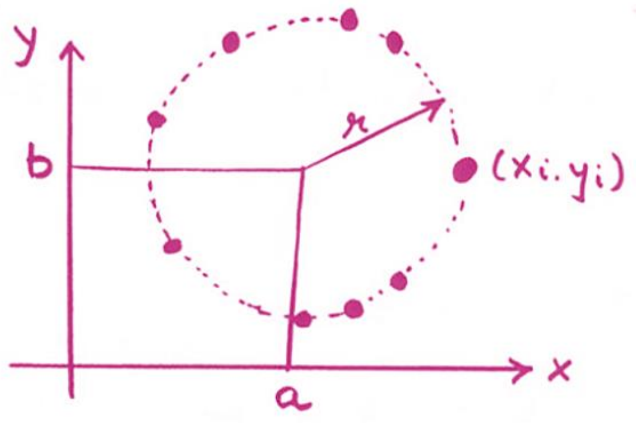

- Approx. object's size are known
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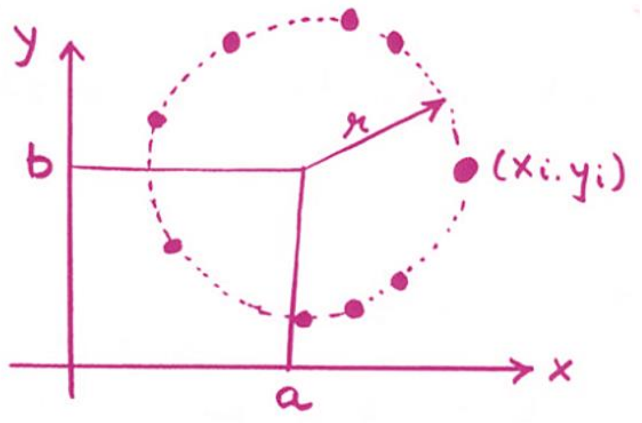

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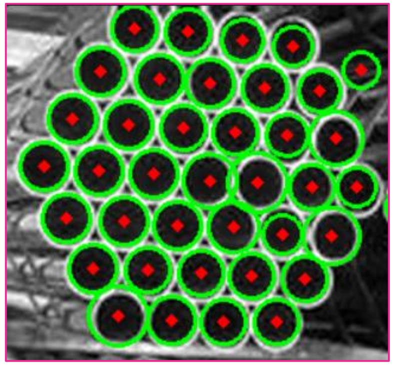


Hough transform

- If radii are not known, exhaustively search all possibilities
- Pseudocode:

$: A(\quad) = 0$

$: \forall(x, y)$

$: if\ M_T(x, y)$

$: \forall(a, b)$

$: r = sqrt\{\ (x-a)^2 + (y-b)^2\ \}$

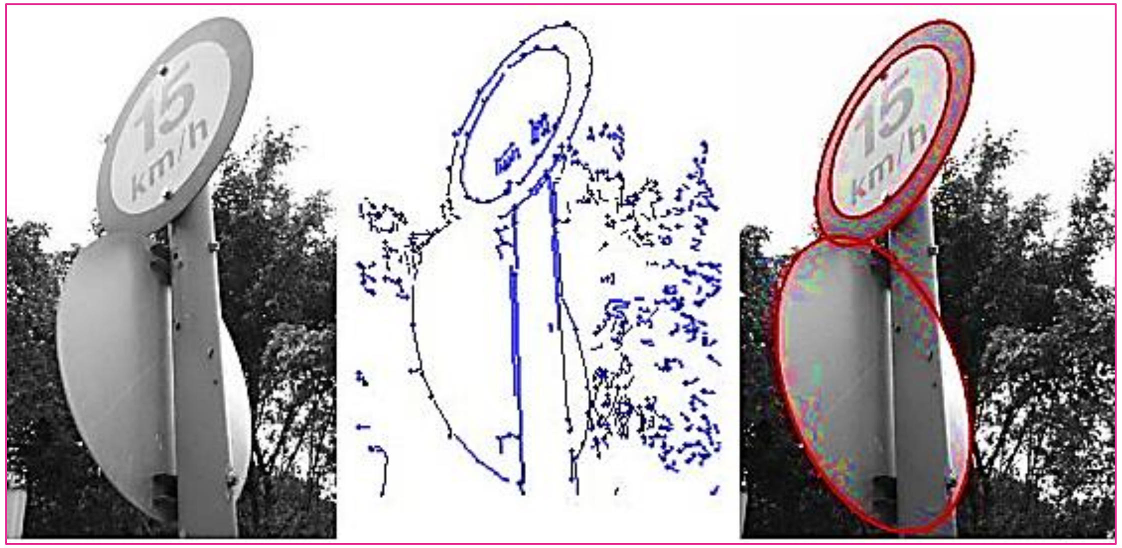
$: A(a, b, r) + +$

$: find\ maximas\ in\ A(\quad)$

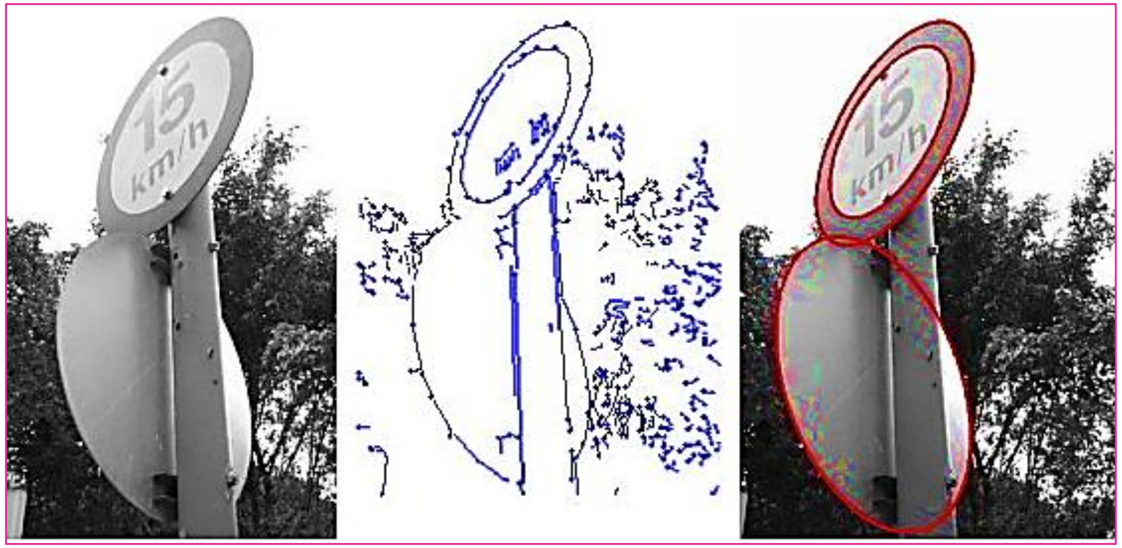
Hough transform

Hough transform

Hough transform



Hough transform



Hough Transform

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Hough Transform

- Space complexity

  - With k quantized bins, it exponentially increases with number of parameters $n \rightarrow k^n$

  - Separate quantization for each dimension can be performed

- Time complexity

  - Voting is linearly proportional to # of edge points
  - Time complexity is constant in # of edge points detected

Hough transform: other shapes

	parameters	
Line	ρ, θ	$x\cos\theta + y\sin\theta = \rho$
Circle	x_0, y_0, ρ	$(x-x_o)^2+(y-y_0)^2=r^2$
Parabola	x_0, y_0, ρ, θ	$(y-y_0)^2=4\rho(x-x_o)$
Ellipse	x_0, y_0, a, b, θ	$(x-x_o)^2/a^2+(y-y_0)^2/b^2=1$

Conclusion

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- It's tolerant to noise in some extent and highly tolerant to the missing points in an edge

- Space complexity increases with the number of parameters

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  - finer the quantization,
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  - but slower will be the procc. speed

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Hough? →