Denoise:
Multiresolution- Pyramids & Subbands

Dr. Tushar Sandhan

Introduction

- Image analysis at different resolutions

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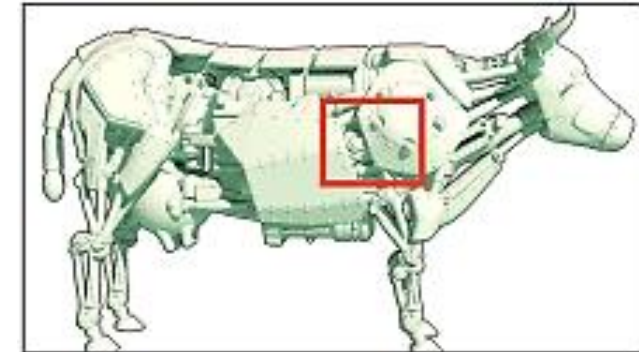
Image analysis

- Image statistics
  - change locally
  - info. is distributed at various scales
  - noise is present throughout the image
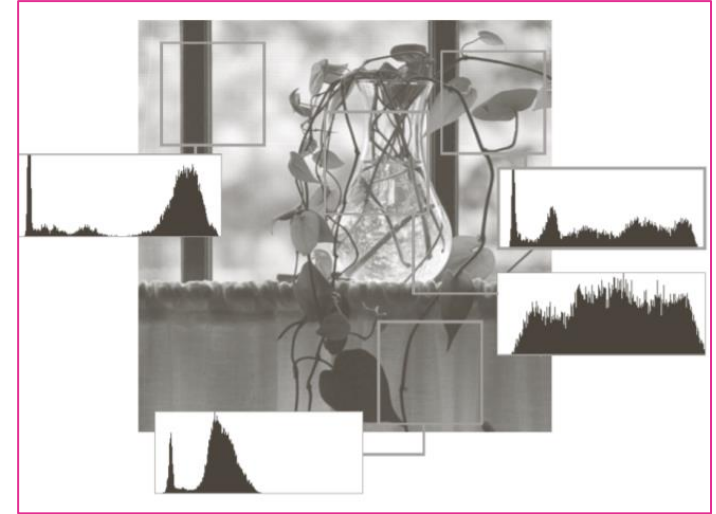

image: C. Nikou

Image analysis

- Image statistics
  - change locally
  - info. is distributed at various scales
  - noise is present throughout the image

- Larger objects can be analyzed @ coarse/low resolution
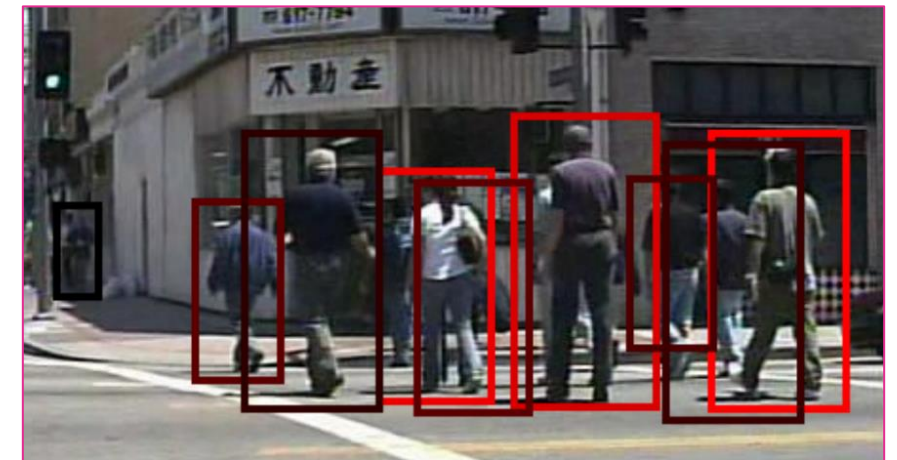
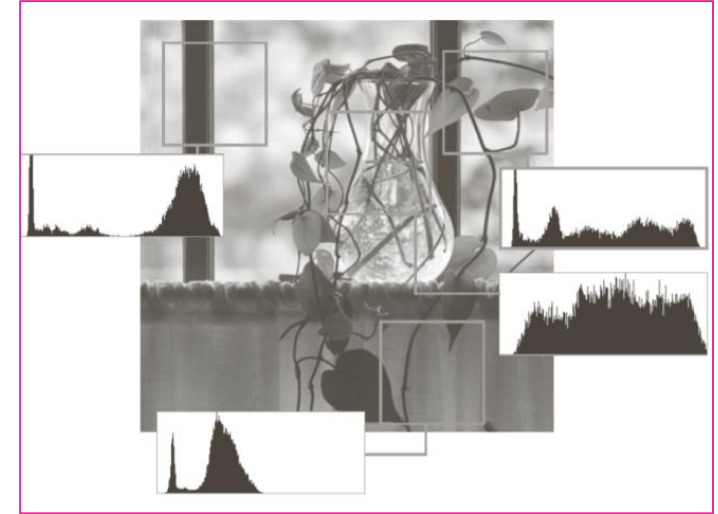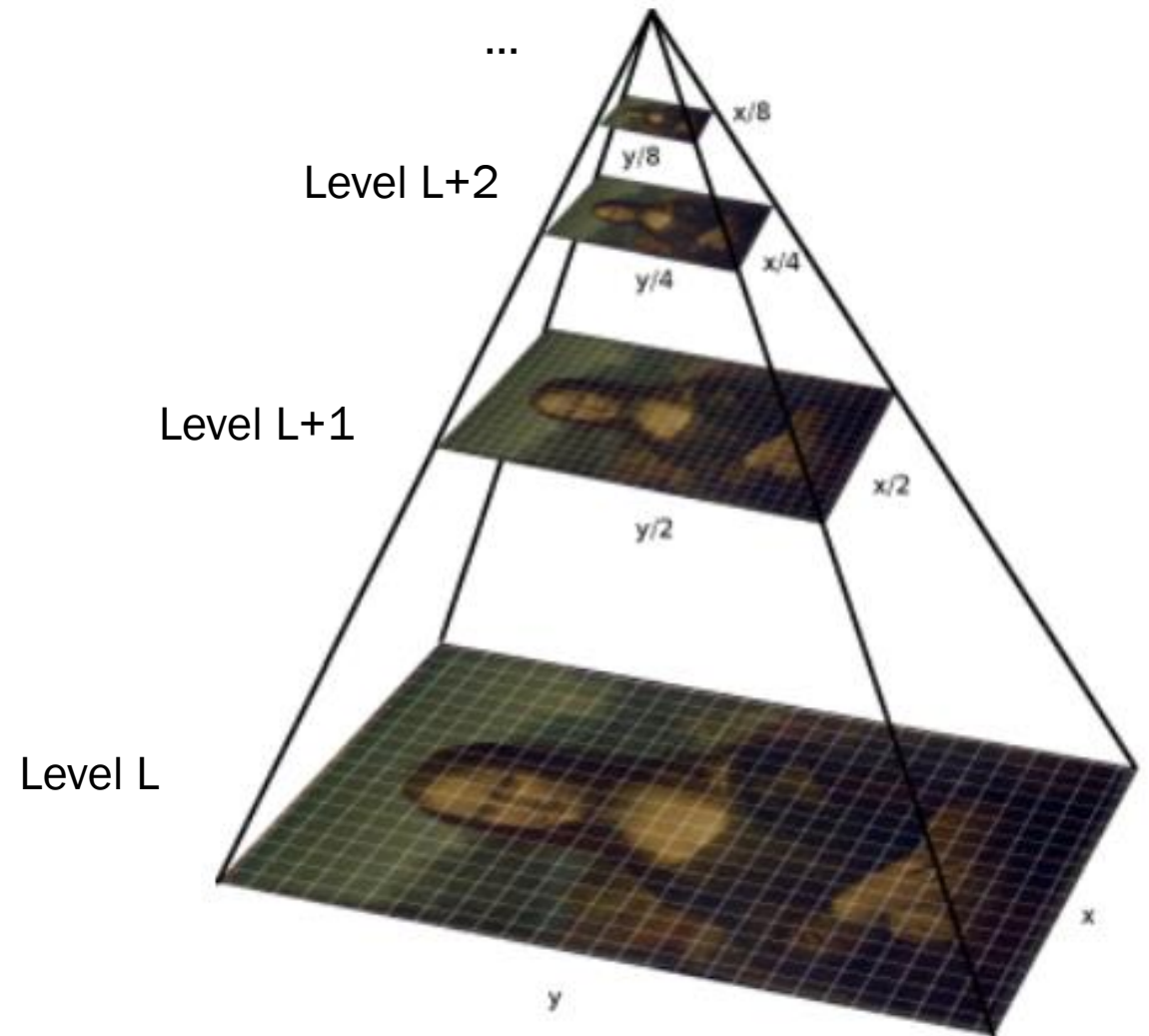- Smaller ones can be analyzed @ fine/high resolution

image: C. Nikou

MRA: Image Pyramid

- Image pyramid
  - a collection of images
  - gradually decreasing resolution
  - arranged in the shape of pyramid
  - researcher use Level numbers interchangeably

    e.g. Level-0 might be bottom or top!

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Level L+2

Level L+1

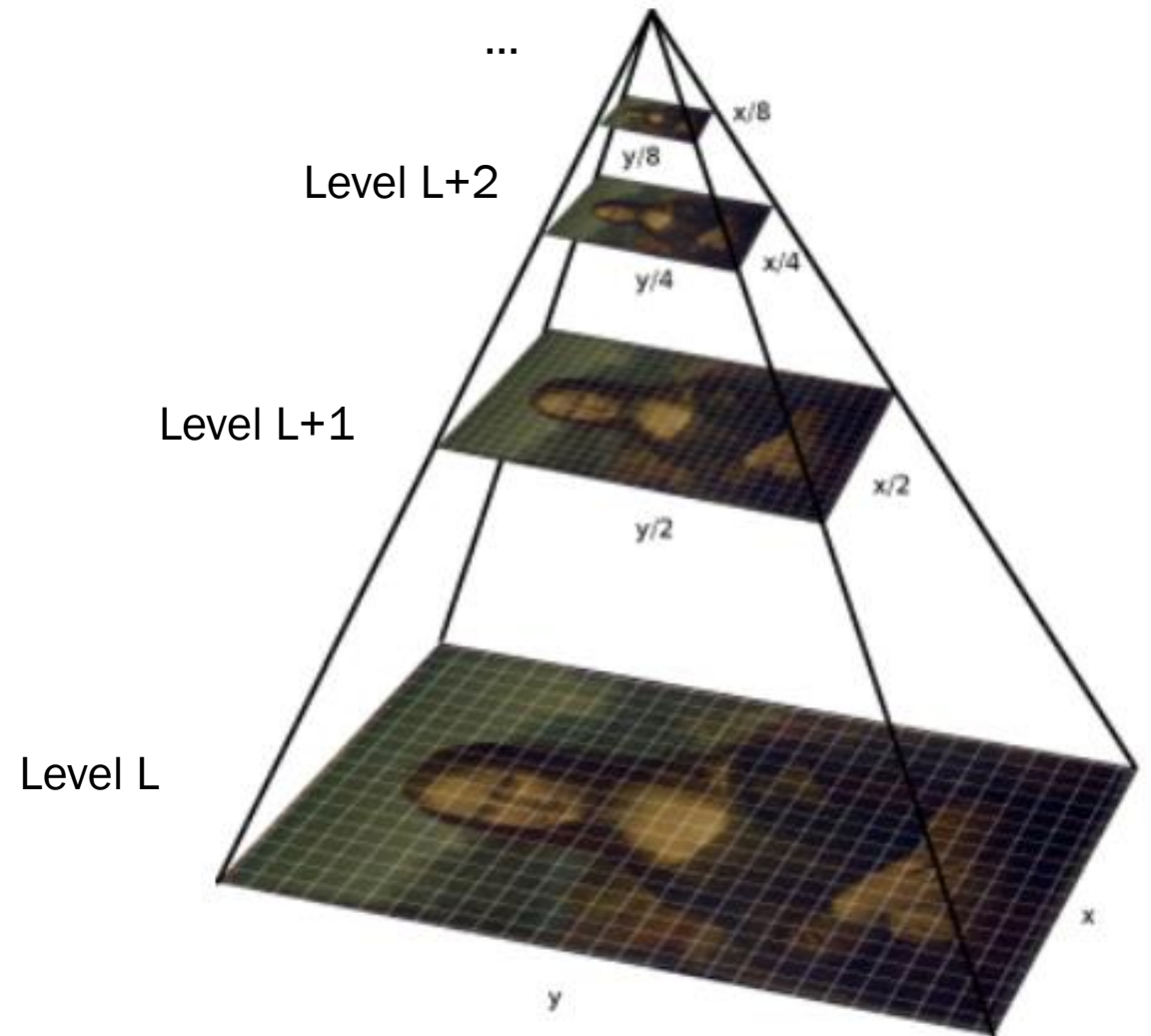Level L

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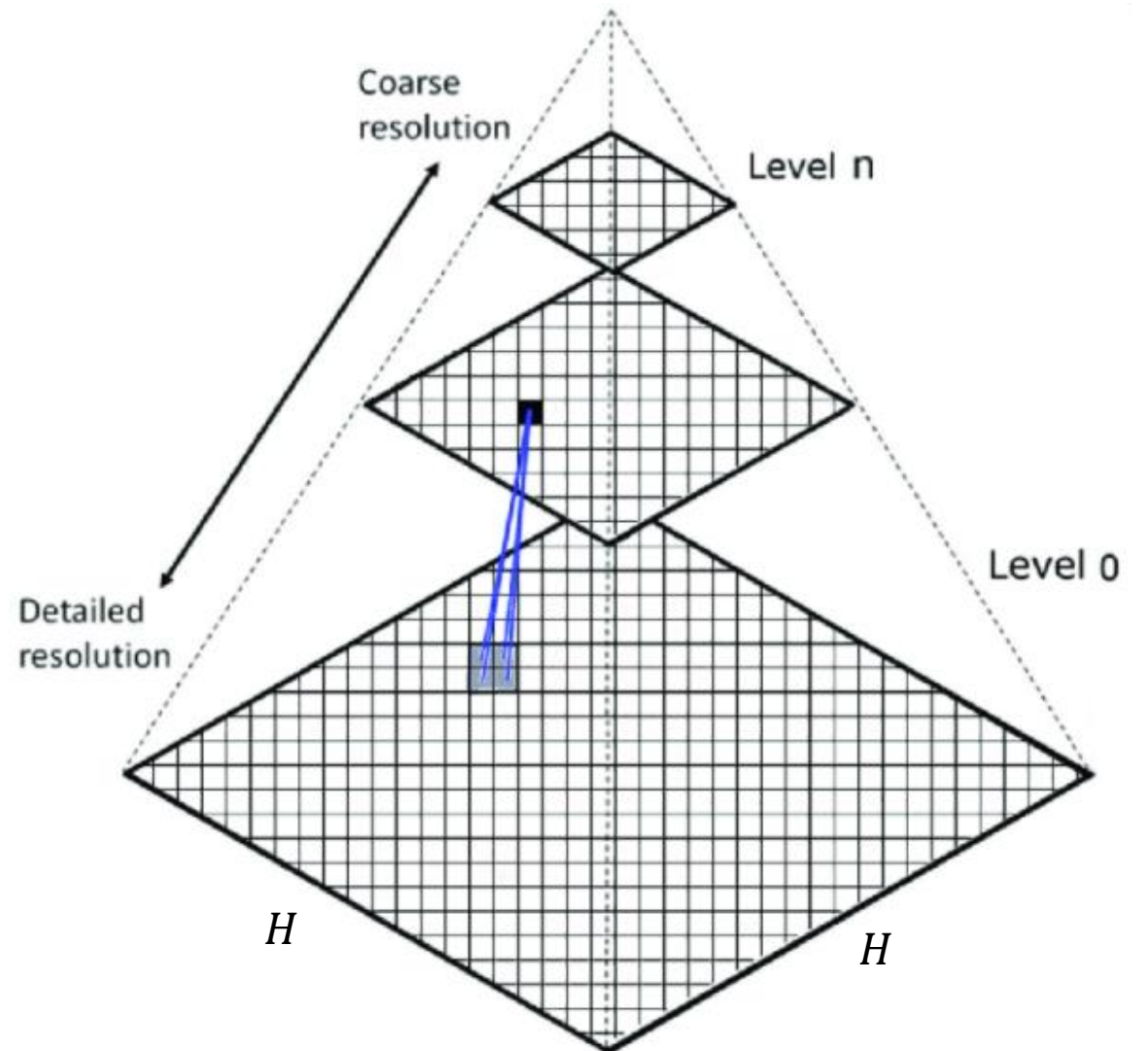 - researcher use Level numbers interchangeably

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- Top level?
  - it is not necessary to go till summit of the pyramid
  - pyramid can be truncated at any level L+k
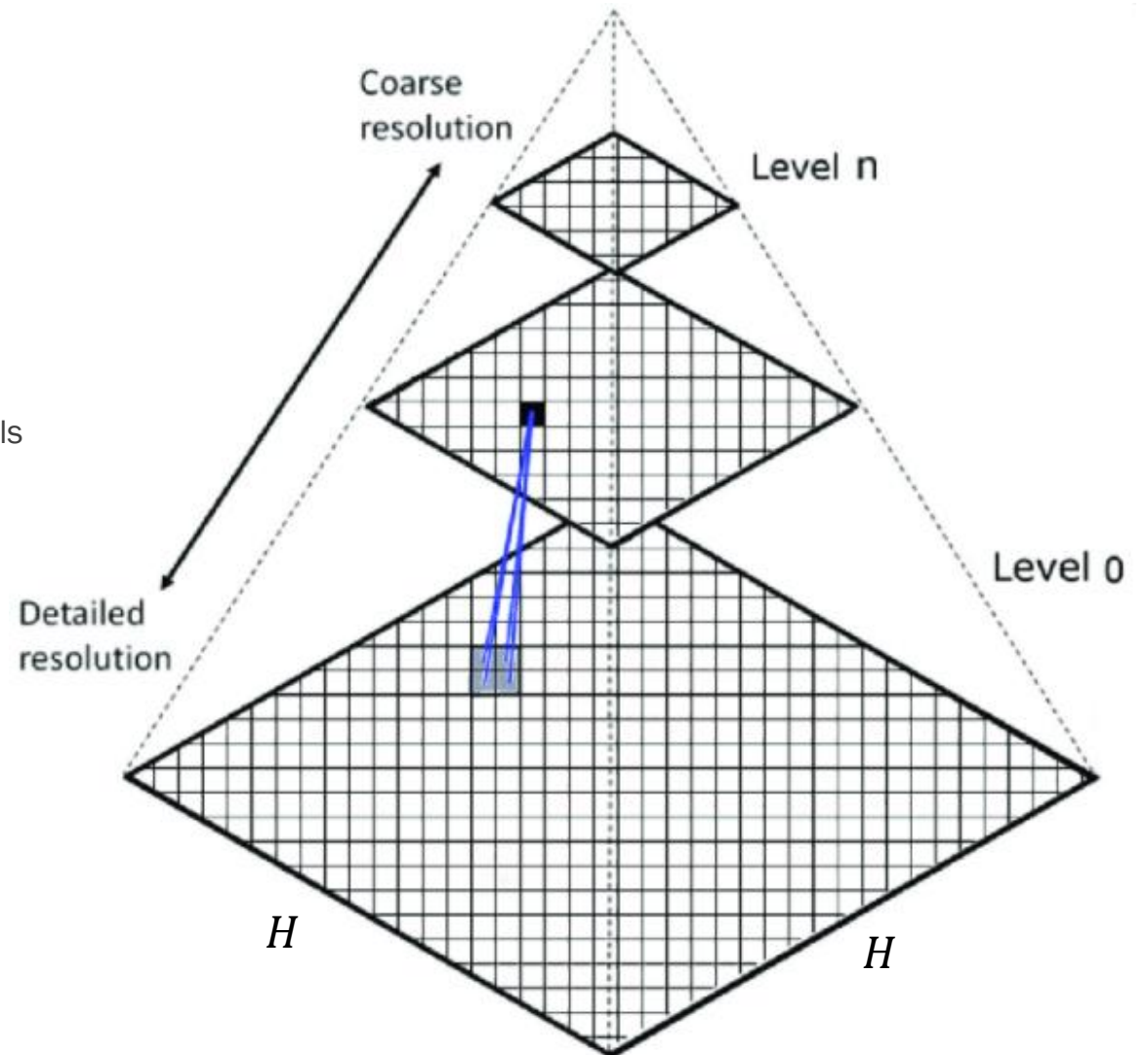 - requirement of speed & accuracy determines k

...

Level L+2

Level L+1

Level L

Pyramid

Pyramid

- Pixel correspondence among levels
  - any image patch at lower level directly corresponds to a smaller patch at upper level
  - levels can be of color image or grey scale
  - pyramid consistency
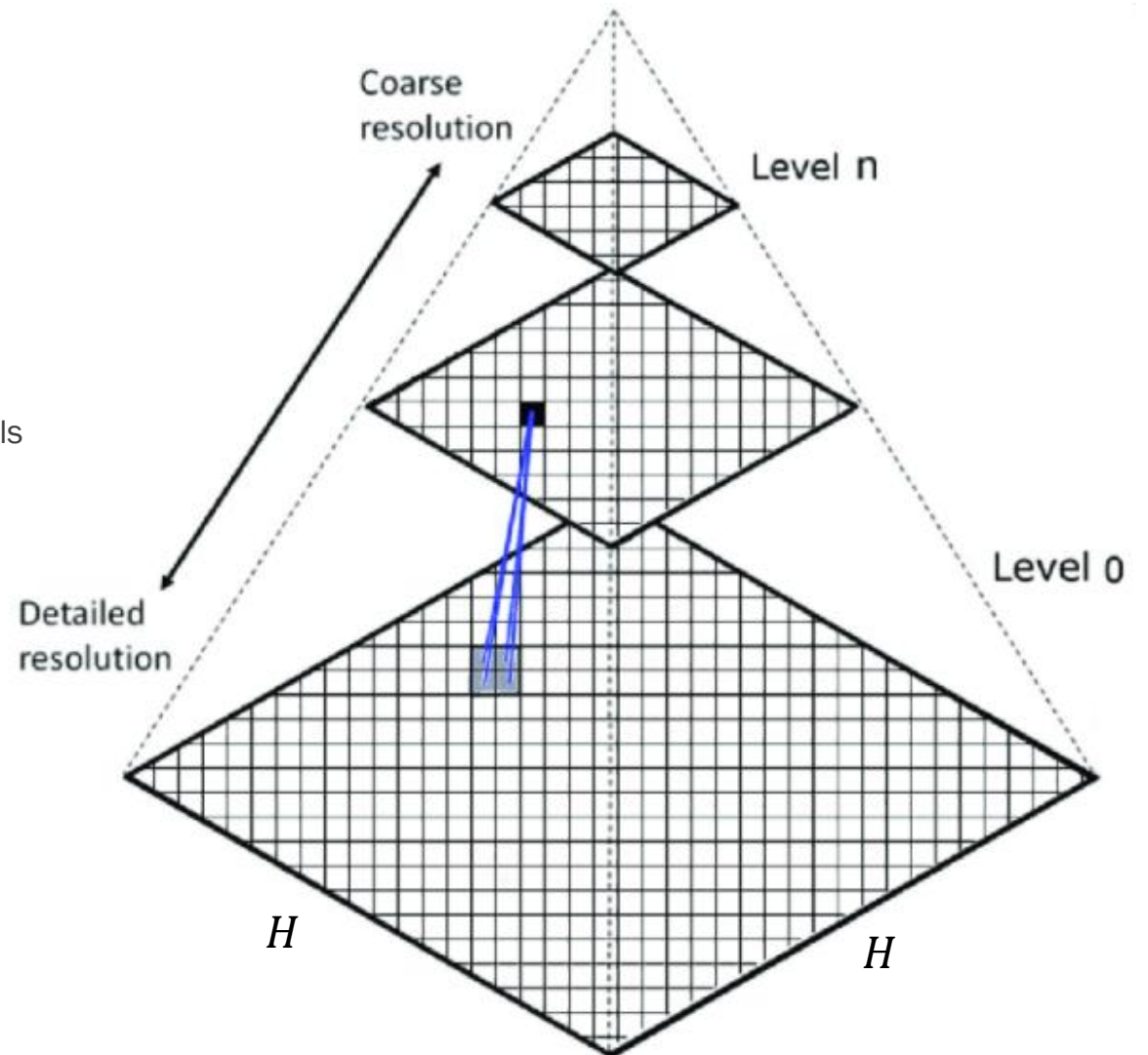   - uniformly apply image operations (e.g. crop, rotation) on all levels

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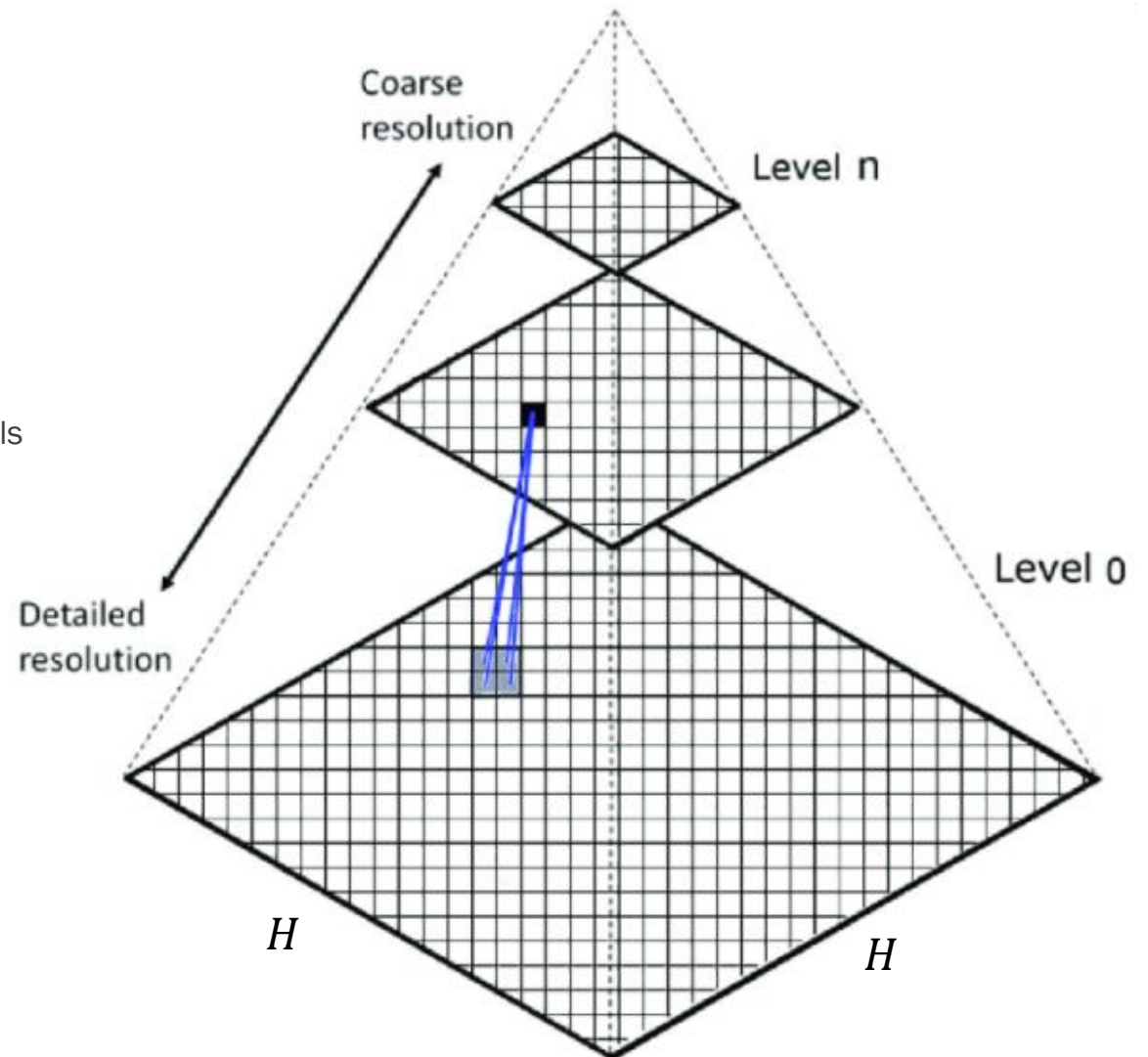
- What is the space complexity?

Pyramid

- Pixel correspondence among levels
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- What is the space complexity?

$$= N + \frac{N}{4} + \frac{N}{4^2} + \cdots + \frac{N}{4^k} \qquad N = H \times W$$

$$\leq \frac{4N}{3}$$

Coarse resolution

Level n

Level 0

Detailed resolution

H

H

Pyramid

- Tiled Multi-Resolution (or Tiled Pyramidal) TIFF
  - it is simply a tiled multi-page TIFF image
  - each resolution is stored as a separate layer within the TIFF
  - it's standard TIFF extension supported by most image procc applications including photoshop
  - 'libtiff' library is capable of reading & writing such pyramids

    ref: http://www.libtiff.org

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- Image pyramid processing
  - while using pyramid MRA, construct your own pyramids inside that image processing algorithm

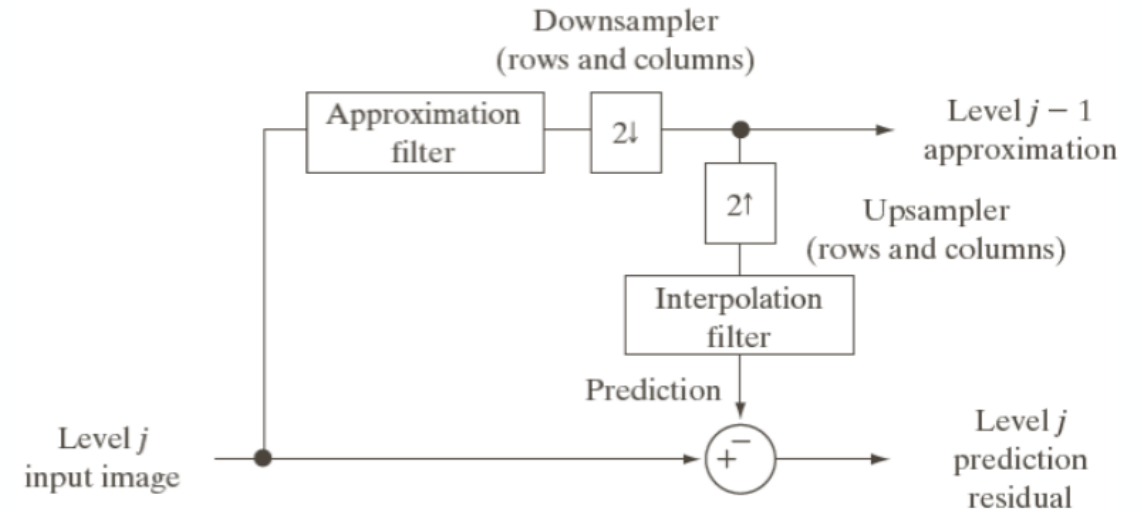# Pyramid construction

- Build the pyramid using below cornerstones

  1. Approximation filter
     - Gaussian filter
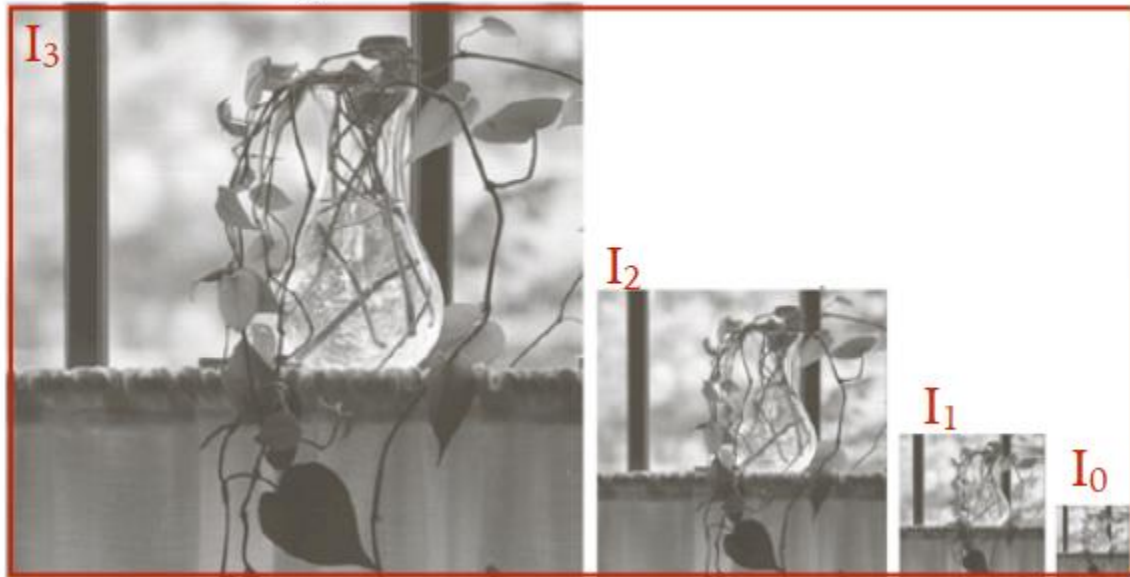   - Averaging/mean filter
     - Low pass filter

  2. Detailed filter
     - Bilinear interpolation
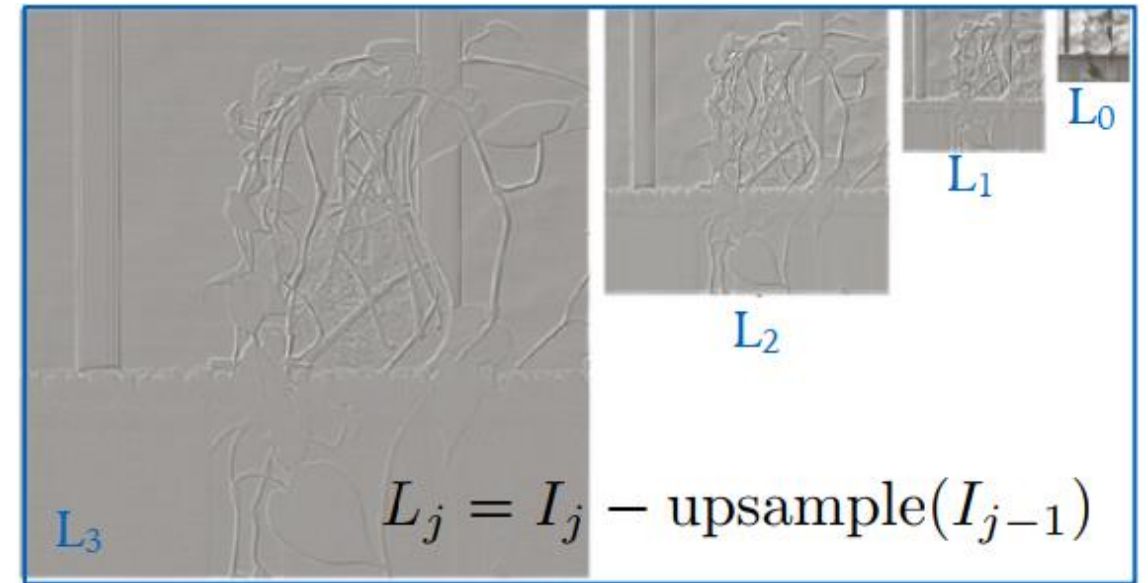   - Bicubic interpolation
     - Upsampler

Gaussian Pyramid

$\text{original} = I_j \qquad I_{j-1} = \text{downsample}(I_j * G_\sigma)$

Approximation pyramid

$L_j = I_j - \text{upsample}(I_{j-1})$

Residual pyramid

Pyramid representation

- We mostly use residual format
  - note level-0 is same in approximation & residual
  - efficient representation

$$I_0 = L_0$$

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Pyramid representation

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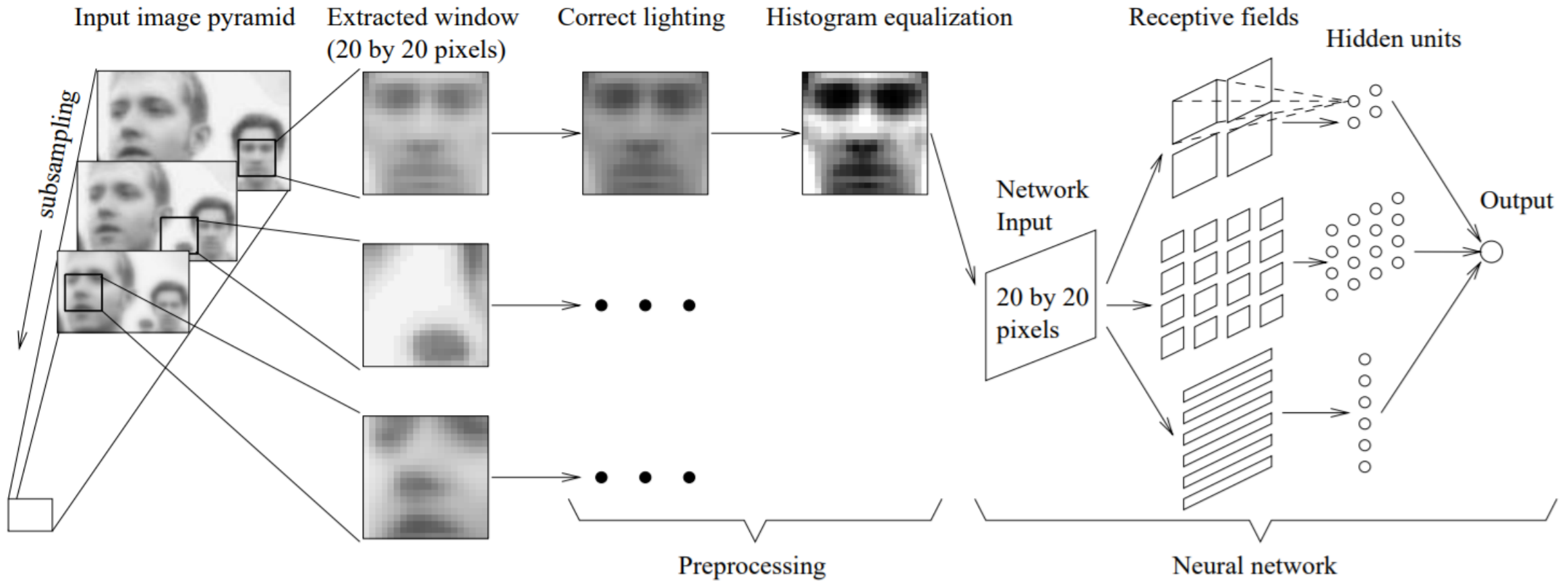 - efficient representation

$$I_0 = L_0$$

$$L_j = I_j - \text{upsample}(I_{j-1})$$

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$$I_j = L_j + upsample(L_{j-1} + upsample(I_{j-2}))$$

➢ Entire approximation pyramid can be obtained via residual pyramid

Pyramid processing

Input image pyramid | Extracted window (20 by 20 pixels) | Correct lighting | Histogram equalization | Receptive fields | Hidden units

subsampling

Network Input

20 by 20 pixels

Output

Preprocessing

Neural network

IEEE CVPR, HA.Rowley 1996

Sub-bands

- Sub-bands are the bandpass filters
  - a series of bandpass filters also known as filter bank
  - Equi-rate: equal bandwidth sub-bands
  - Multi-rate: different bandwidth sub-bands
  - similar to image pyramid, we use below corner stones :

1. Analysis
   - decompose image into set of images of different freq bands

2. Synthesis
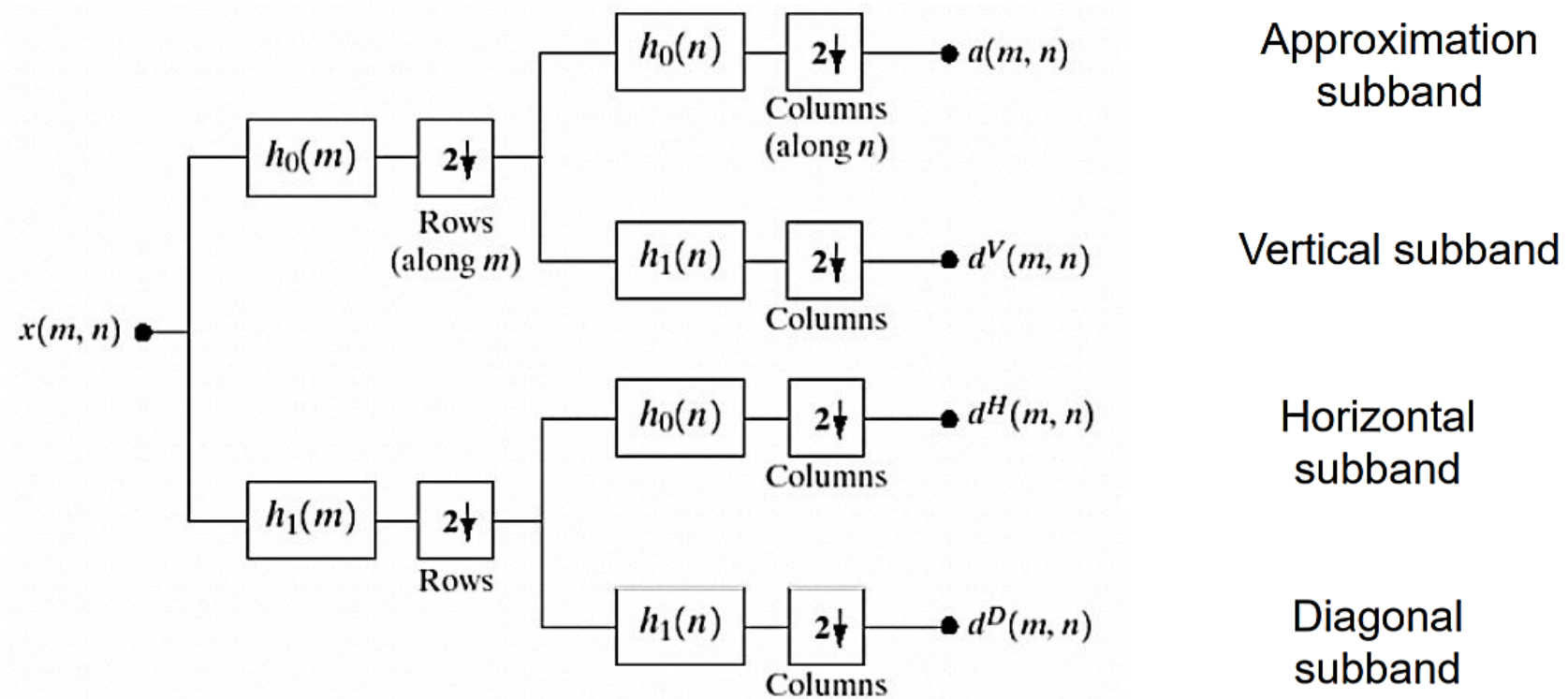   - original image should be reconstructed from bands of analysis stage

Sub-bands

- Double bands
  - 2 band decomposition
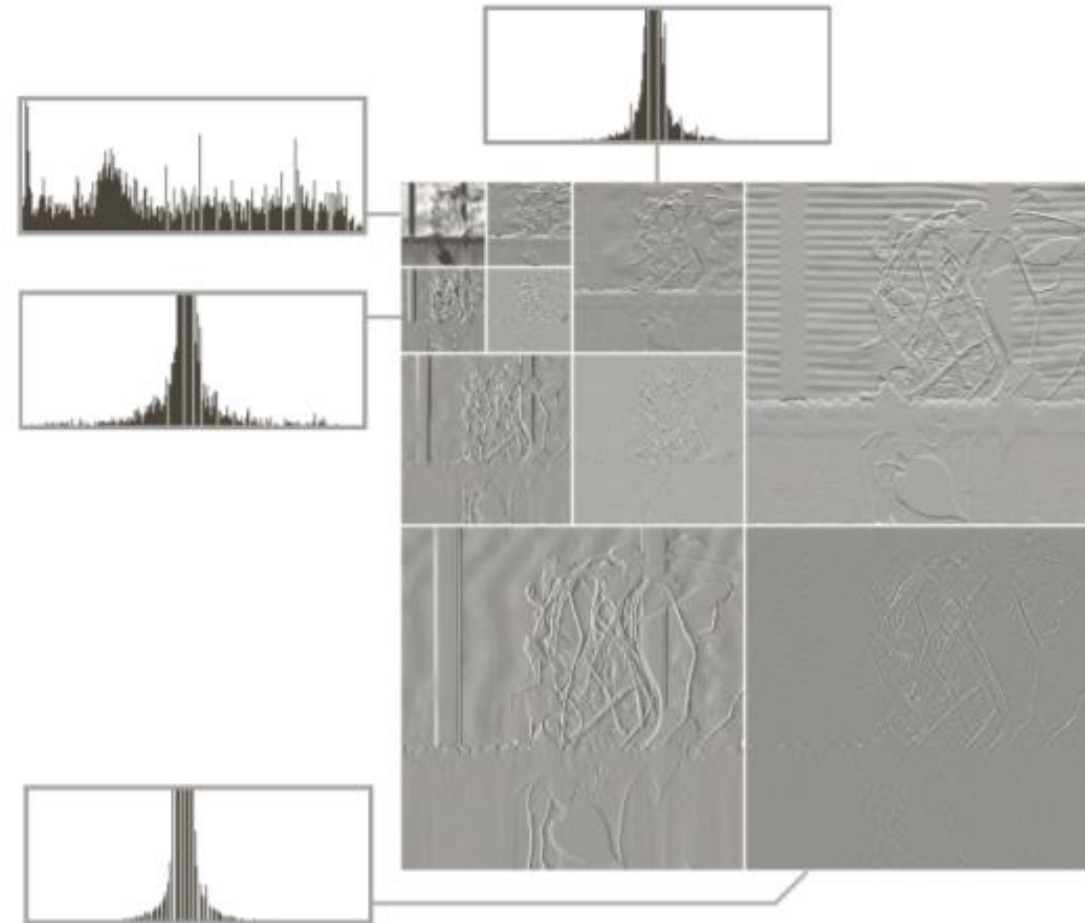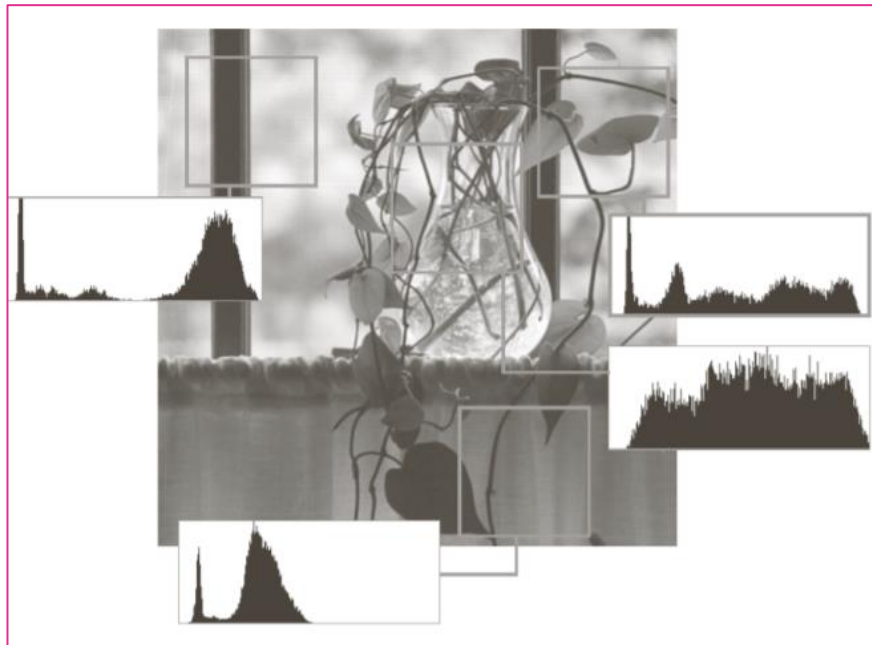 - filtering + 2↓
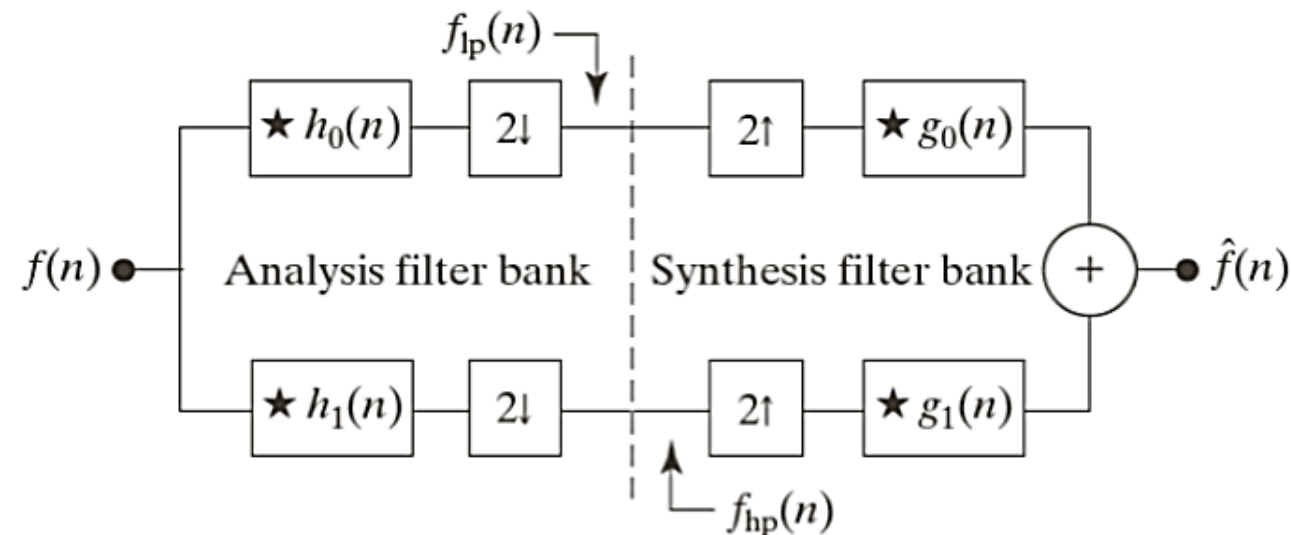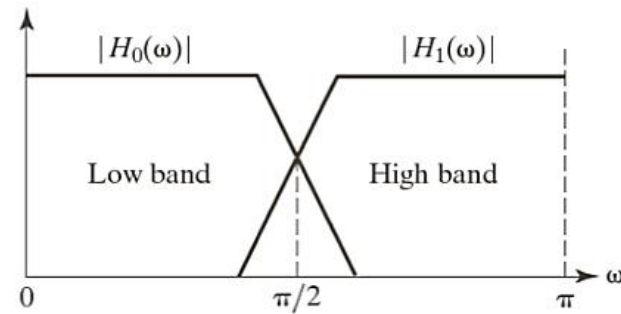
$g_0(n) = (-1)^n h_1(n)$

$g_1(n) = (-1)^{n+1} h_0(n)$

Sub-bands

- sub-band coding
  - 2 band decomposition expanded recursively

Sub-band decomposition

Sub-band denoise

- Different bit-rates or different coding technique can be used for each sub-band
  - useful in compression as well

- Denoise
  - noise types
    - uniform
    - speckle
    - band-limited

  - sub-band denoise
    - different bands show different response to noise types
    - it allows errors to be distributed across sub-bands
    - filter each sub-band image separately with specific filtering parameters dedicated to those bands

Sub-band denoising

original   noisy   denoised

Multiresolution BF

Multiresolution BF

R

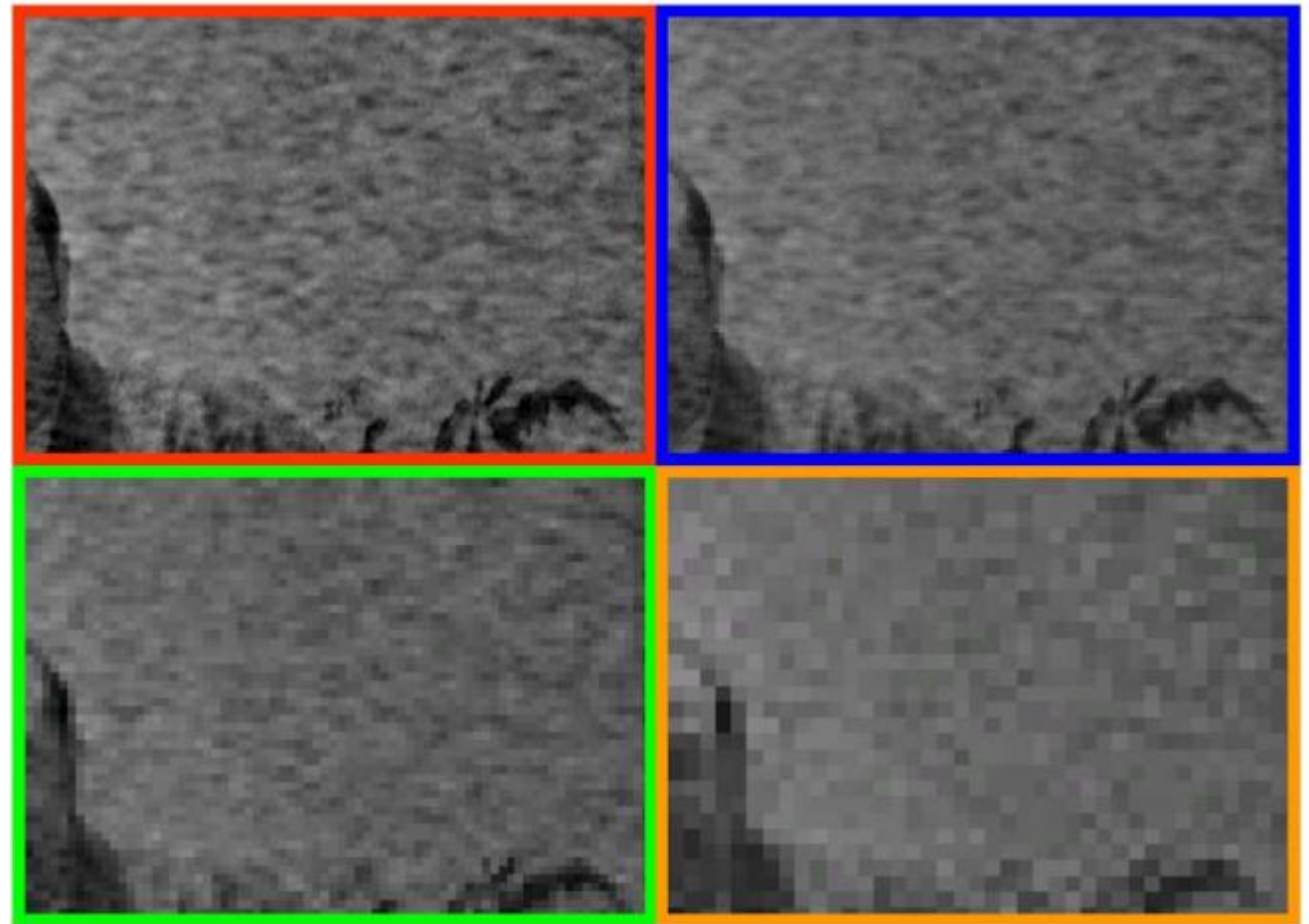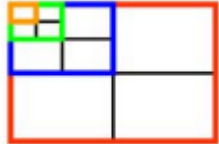# Multiresolution BF

R

G

Multiresolution BF

R

G

B

Multiresolution BF

- Noise decomposition

M. Zhang, IEEE TIP'08

Multiresolution BF

- Denoising

$$\tilde{I}(\mathbf{x}) = \frac{1}{C} \sum_{\mathbf{y} \in \mathcal{N}(\mathbf{x})} e^{\frac{-\|\mathbf{y}-\mathbf{x}\|^2}{2\sigma_d^2}} e^{\frac{-|I(\mathbf{y})-I(\mathbf{x})|^2}{2\sigma_r^2}} I(\mathbf{y})$$
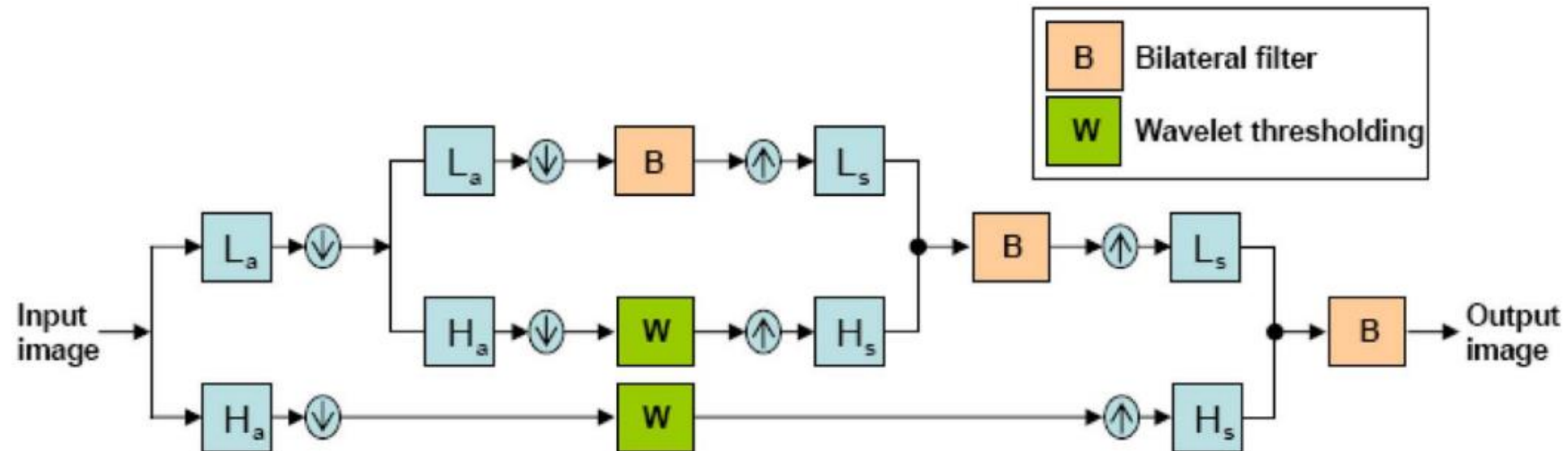
$$C = \sum_{\mathbf{y} \in \mathcal{N}(\mathbf{x})} e^{\frac{-\|\mathbf{y}-\mathbf{x}\|^2}{2\sigma_d^2}} e^{\frac{-|I(\mathbf{y})-I(\mathbf{x})|^2}{2\sigma_r^2}}$$

Multiresolution BF

- Denoising

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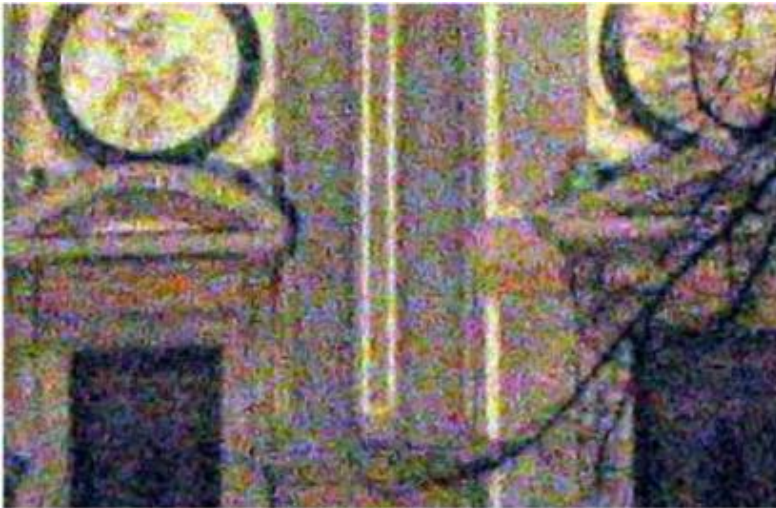
$$C = \sum_{\mathbf{y} \in \mathcal{N}(\mathbf{x})} e^{\frac{-\|\mathbf{y}-\mathbf{x}\|^2}{2\sigma_d^2}} e^{\frac{-|I(\mathbf{y})-I(\mathbf{x})|^2}{2\sigma_r^2}}$$

Multiresolution BF

- Results
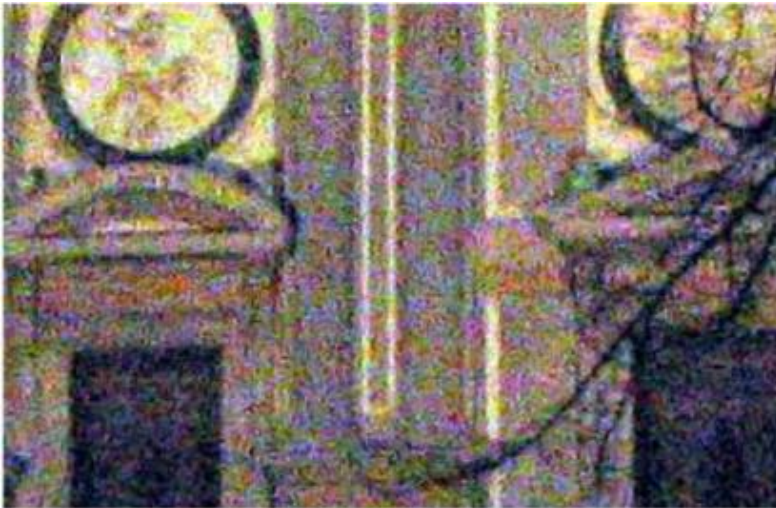
input

Multiresolution BF

- Results

input

BF (psnr 29.6)

Multiresolution BF

- Results

input   BF (psnr 29.6)   MRBF (psnr 31.4)

Conclusion

- Denoising

- Multiresolutions
    - Pyramids
    - Sub-bands

BF

MRBF

Conclusion

- Denoising

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❑ Multiresolutions
- Pyramids
- Sub-bands

❑ Denoising
- Even minute imperceptible improvement in the denoising is very important for scientific images

BF

MRBF

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  - Pyramids
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  ▪ Pyramids
  ▪ Sub-bands

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  ▪ Even minute imperceptible improvement in the denoising is very important for scientific images

BF

MRBF