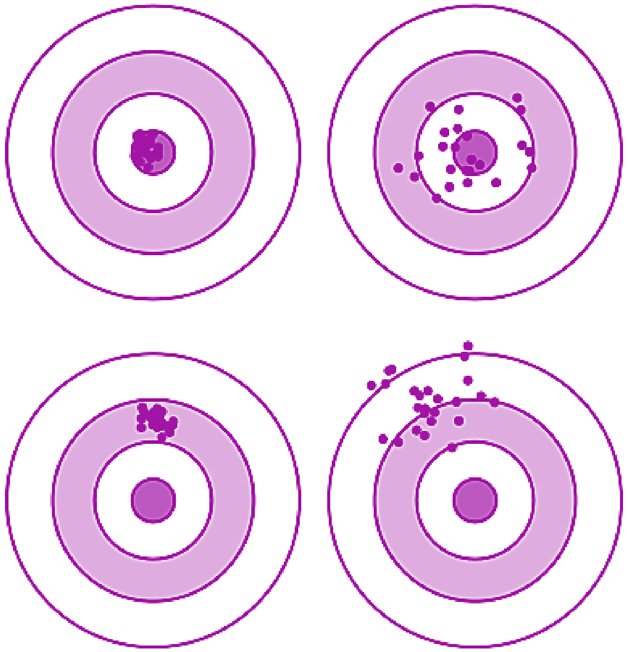
Segmentation:
Otsu's method

Dr. Tushar Sandhan

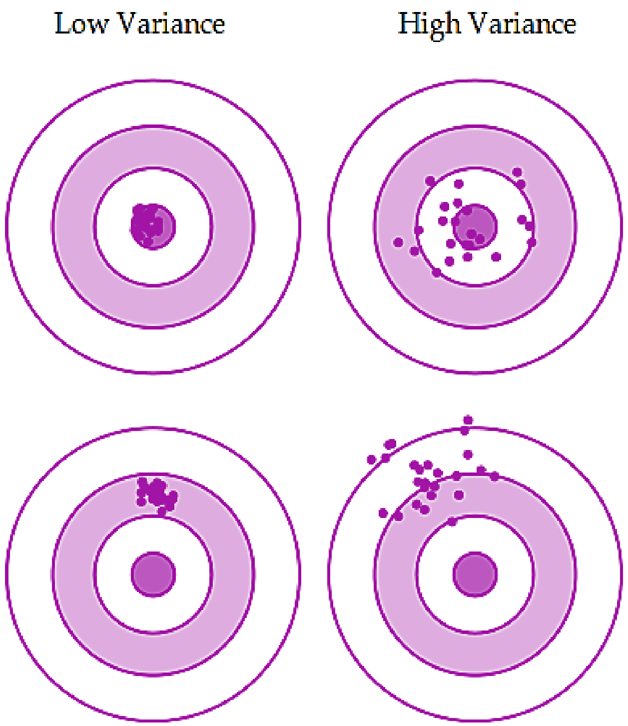
Introduction

- Variance
  - intraclass
  - interclass



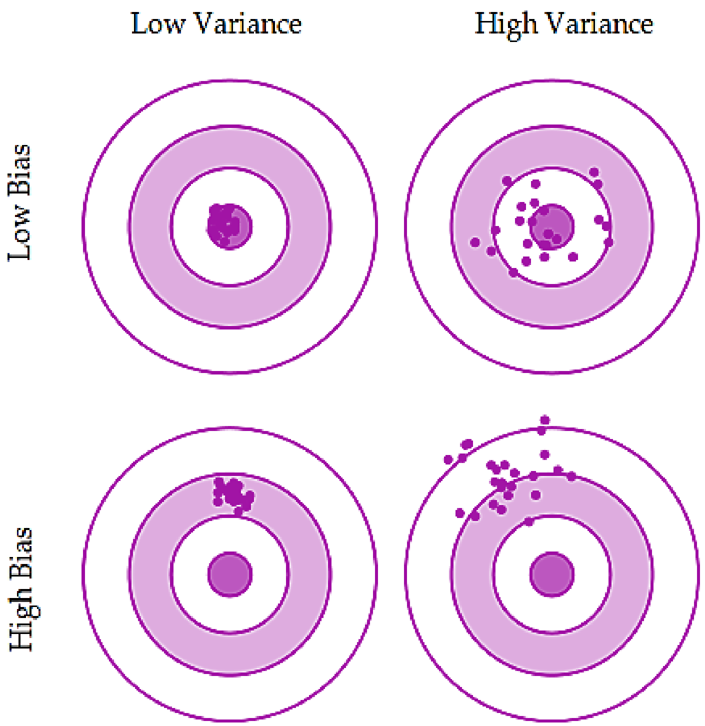
Introduction

- Variance
  - intraclass
  - interclass



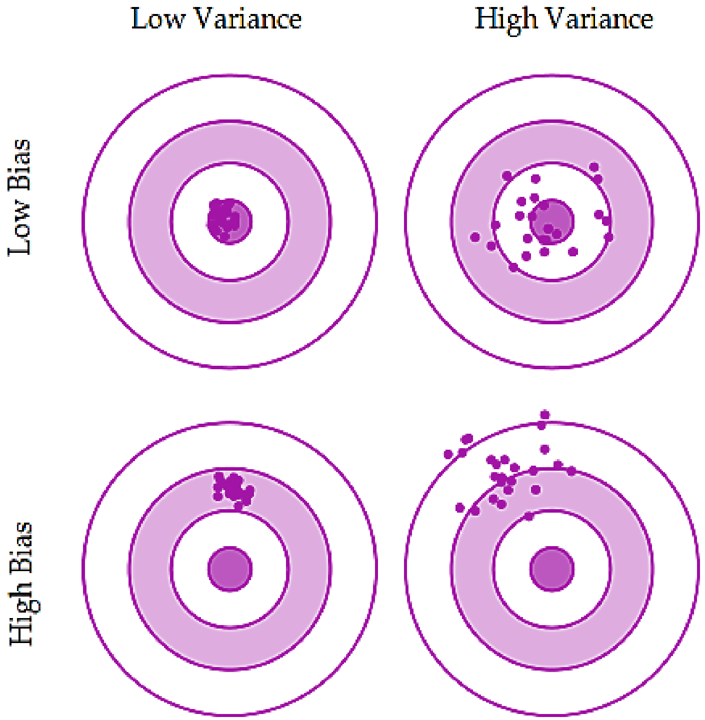
Introduction

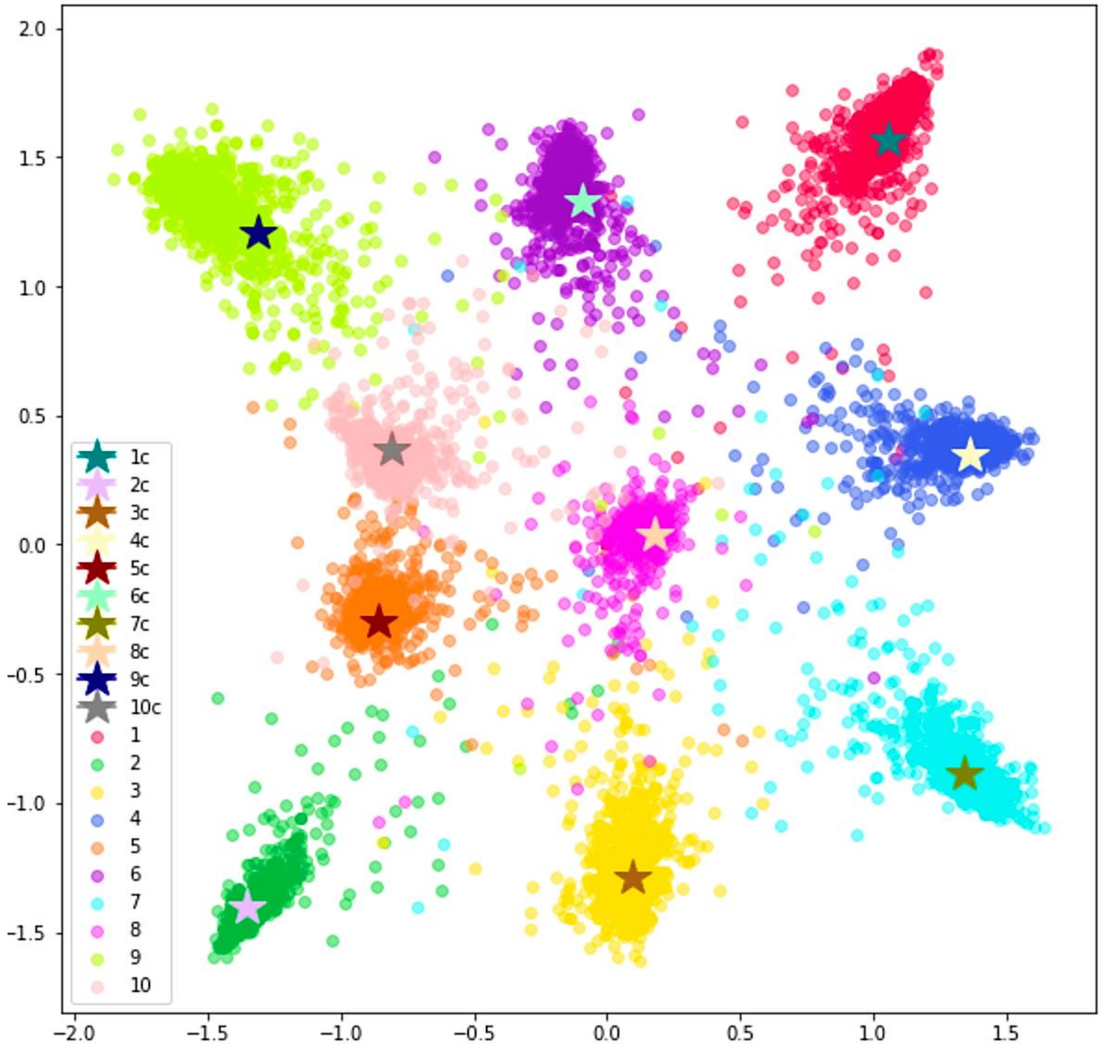
- Variance
  - intraclass
  - interclass



Introduction

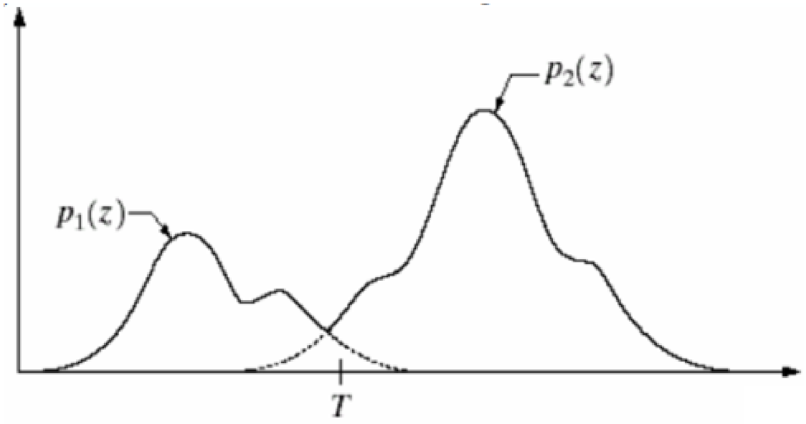
- Variance
  - intraclass
  - interclass





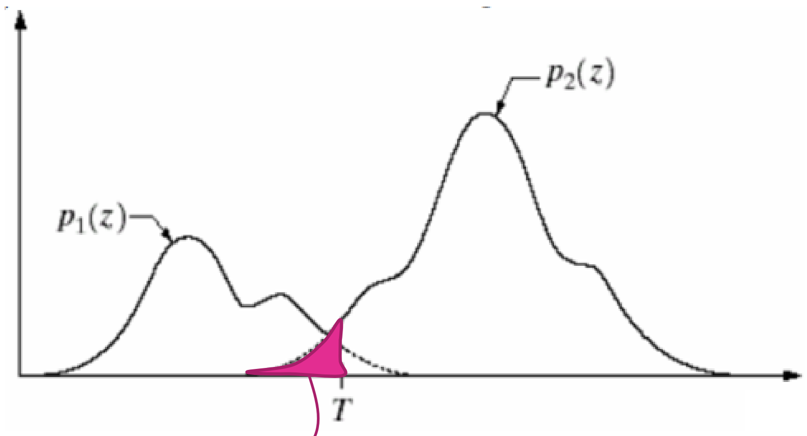
Optimal thresholding

- Global: optimal
  - probability distribution of bimodal regions (foreground & background) should to be known
  - image pdf: $p(z)$
  - P_1, P_2 : probability of occurrence of each class of pixels
  - $E_1(T)$: prob. of misclassifying class-2 as class-1
  - $E_2(T)$: prob. of misclassifying class-1 as class-2



Optimal thresholding

- Global: optimal
  - probability distribution of bimodal regions (foreground & background) should to be known
  - image pdf: $p(z)$
  - P_1, P_2 : probability of occurrence of each class of pixels
  - $E_1(T)$: prob. of misclassifying class-2 as class-1
  - $E_2(T)$: prob. of misclassifying class-1 as class-2

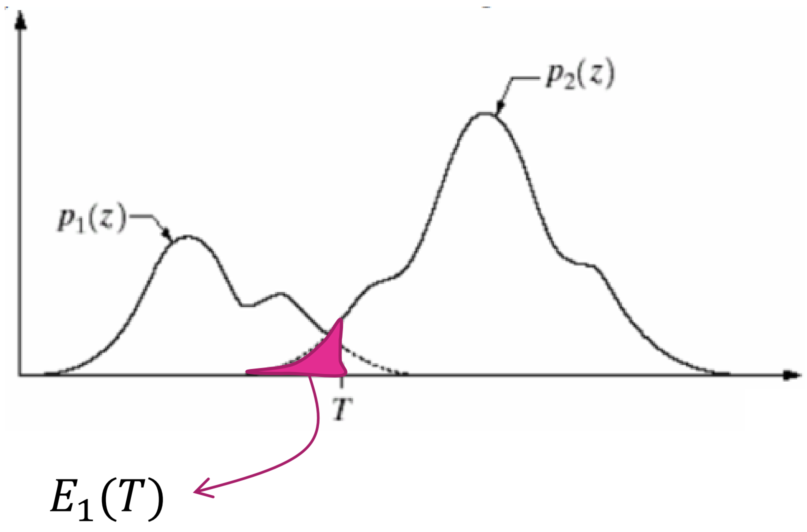


$E_1(T)$

Optimal thresholding

- Global: optimal
  - probability distribution of bimodal regions (foreground & background) should to be known
  - image pdf: $p(z)$
  - P_1, P_2 : probability of occurrence of each class of pixels
  - $E_1(T)$: prob. of misclassifying class-2 as class-1
  - $E_2(T)$: prob. of misclassifying class-1 as class-2

$$p(z) = P_1 p_1(z) + P_2 p_2(z)$$

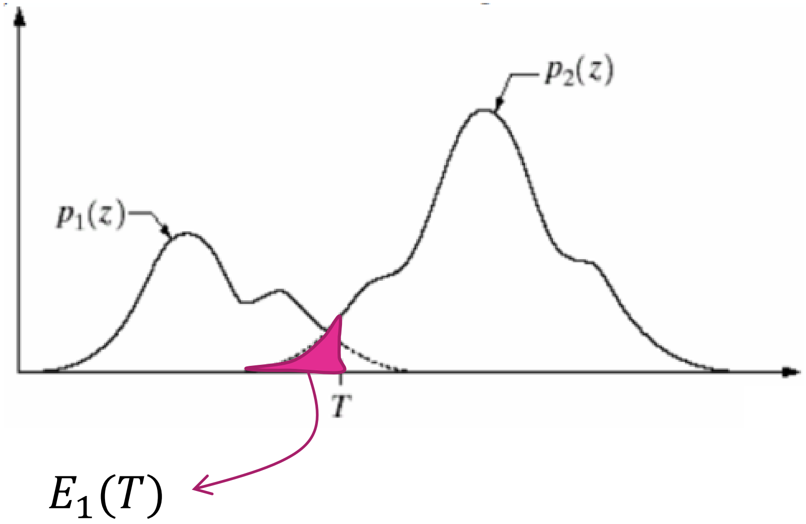


Optimal thresholding

- Global: optimal
  - probability distribution of bimodal regions (foreground & background) should to be known
  - image pdf: $p(z)$
  - P_1, P_2 : probability of occurrence of each class of pixels
  - $E_1(T)$: prob. of misclassifying class-2 as class-1
  - $E_2(T)$: prob. of misclassifying class-1 as class-2

$$p(z) = P_1 p_1(z) + P_2 p_2(z)$$

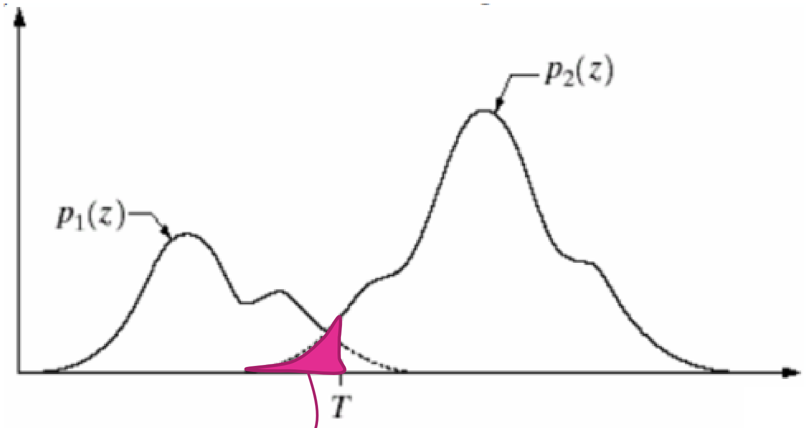
$$P_1 + P_2 = 1$$



Optimal thresholding

- Global: optimal

$$E_1(T) = \int_{-\infty}^{T} p_2(z)dz \qquad E_2(T) = \int_{T}^{\infty} p_1(z)dz$$



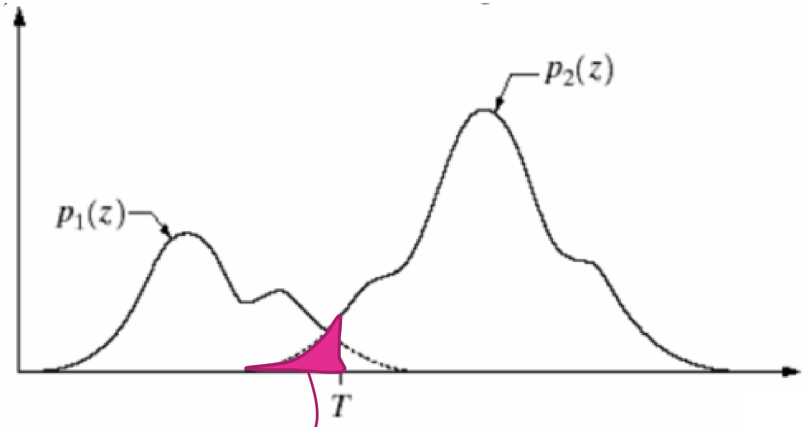
$E_1(T)$

Optimal thresholding

- Global: optimal

$$E_1(T) = \int_{-\infty}^{T} p_2(z)dz \qquad E_2(T) = \int_{T}^{\infty} p_1(z)dz$$

$$E(T) = P_2 E_1(T) + P_1 E_2(T)$$



$E_1(T)$

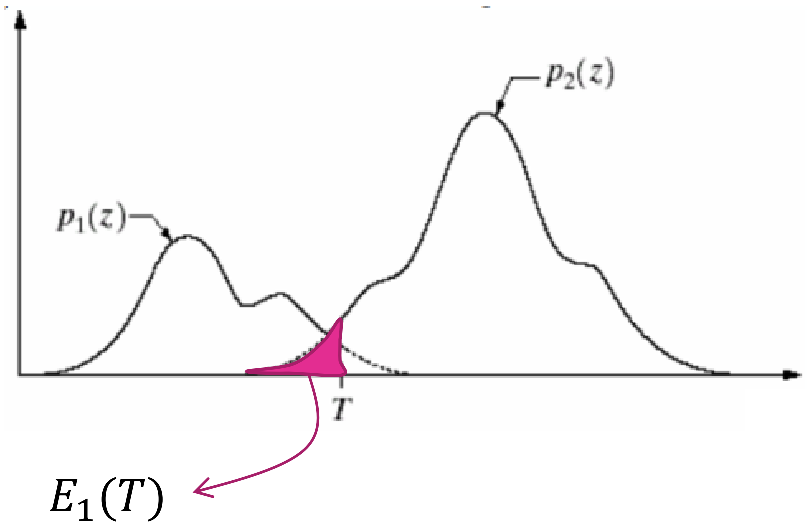
Optimal thresholding

- Global: optimal

$$E_1(T) = \int_{-\infty}^{T} p_2(z)dz \qquad E_2(T) = \int_{T}^{\infty} p_1(z)dz$$

$$E(T) = P_2 E_1(T) + P_1 E_2(T)$$

$$T^* = \underset{T}{\operatorname{argmin}} \; E(T)$$



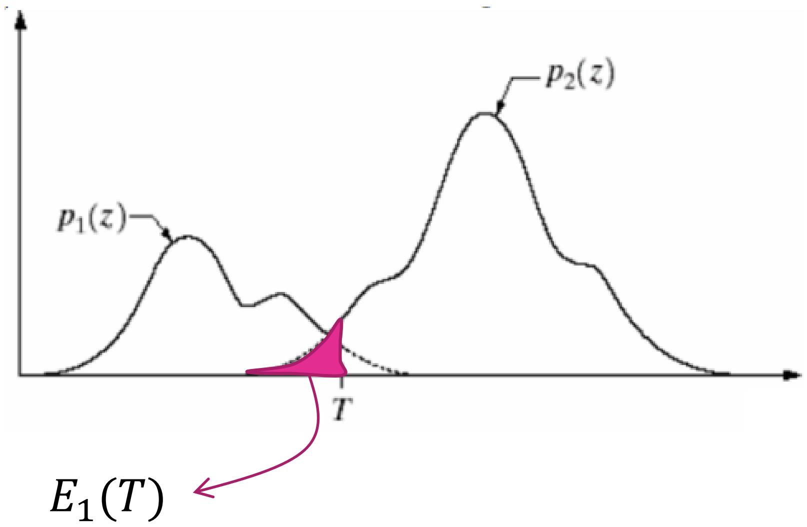
Optimal thresholding

- Global: optimal

$$E_1(T) = \int_{-\infty}^{T} p_2(z)dz \qquad E_2(T) = \int_{T}^{\infty} p_1(z)dz$$

$$E(T) = P_2E_1(T) + P_1E_2(T)$$

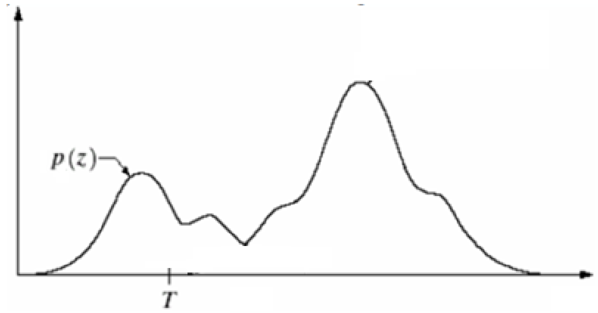
$$T^* = \underset{T}{\operatorname{argmin}}\ E(T)$$



  - differentiate w.r.t. T and set it to 0

Otsu's thresholding

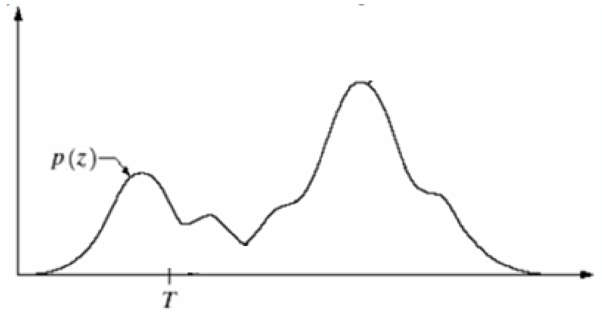
- Global: adaptive Otsu's threshold
  - exhaustively searches $\forall\, T$ that minimizes intra-class variance
  - min. intra-class var. is equivalent to max. inter-class var.



$$\sigma_r^2(T) = P_1(T)\sigma_1^2(T) + P_2(T)\sigma_2^2(T)$$

Otsu’s thresholding

- Global: adaptive Otsu’s threshold
  - exhaustively searches ∀ T that minimizes intra-class variance
  - min. intra-class var. is equivalent to max. inter-class var.

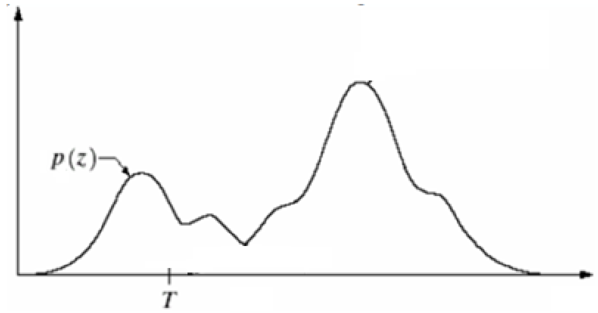


$$\sigma_r^2(T) = P_1(T)\sigma_1^2(T) + P_2(T)\sigma_2^2(T)$$

$$P_1(T) = \sum_{t=0}^{T-1} p\,(t)$$

Otsu's thresholding

- Global: adaptive Otsu's threshold
  - exhaustively searches $\forall\, T$ that minimizes intra-class variance
  - min. intra-class var. is equivalent to max. inter-class var.

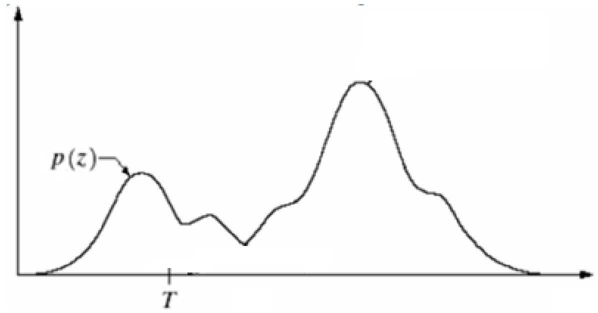


$$\sigma_r^2(T) = P_1(T)\sigma_1^2(T) + P_2(T)\sigma_2^2(T)$$

$$P_1(T) = \sum_{t=0}^{T-1} p\,(t) \qquad P_2(T) = \sum_{t=T}^{L-1} p(t)$$

Otsu's thresholding

- Global: adaptive Otsu's threshold
  - exhaustively searches $\forall\, T$ that minimizes intra-class variance
  - min. intra-class var. is equivalent to max. inter-class var.



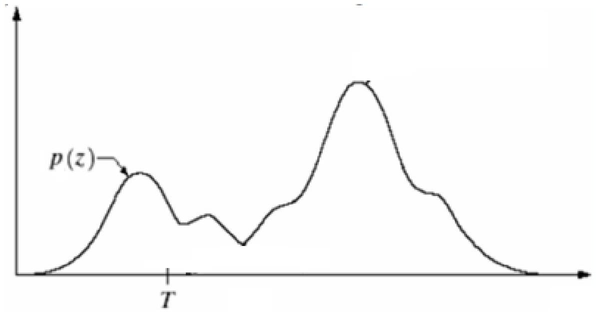
$$\sigma_r^2(T) = P_1(T)\sigma_1^2(T) + P_2(T)\sigma_2^2(T)$$

$$P_1(T) = \sum_{t=0}^{T-1} p\,(t) \qquad\qquad P_2(T) = \sum_{t=T}^{L-1} p(t)$$

$$\mu_1(T) = \sum_{t=0}^{T-1} \frac{t \cdot p(t)}{P_1(T)}$$

Otsu's thresholding

- Global: adaptive Otsu's threshold
  - exhaustively searches $\forall\, T$ that minimizes intra-class variance
  - min. intra-class var. is equivalent to max. inter-class var.



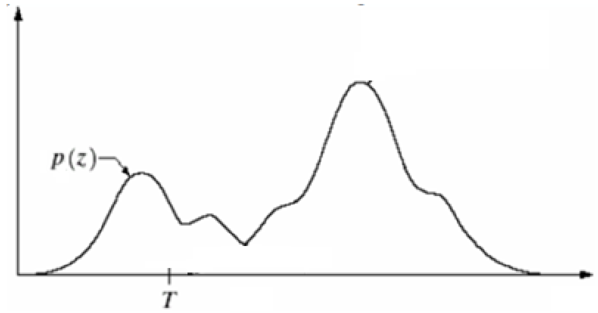
$$\sigma_r^2(T) = P_1(T)\sigma_1^2(T) + P_2(T)\sigma_2^2(T)$$

$$P_1(T) = \sum_{t=0}^{T-1} p\,(t) \qquad P_2(T) = \sum_{t=T}^{L-1} p(t)$$

$$\mu_1(T) = \sum_{t=0}^{T-1} \frac{t \cdot p(t)}{P_1(T)} \qquad \mu_2(T) = \sum_{t=T}^{L-1} \frac{t \cdot p(t)}{P_2(T)}$$

Otsu’s thresholding

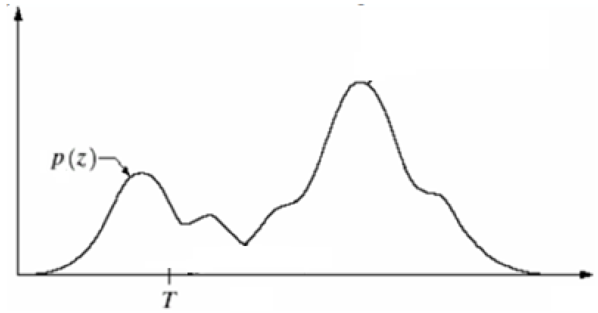
- Global: adaptive Otsu’s threshold
  - exhaustively searches $\forall\, T$ that minimizes intra-class variance
  - min. intra-class var. is equivalent to max. inter-class var.



$$\sigma_r^2(T) = P_1(T)\sigma_1^2(T) + P_2(T)\sigma_2^2(T)$$

Otsu's thresholding

- Global: adaptive Otsu's threshold
  - exhaustively searches $\forall\, T$ that minimizes intra-class variance
  - min. intra-class var. is equivalent to max. inter-class var.

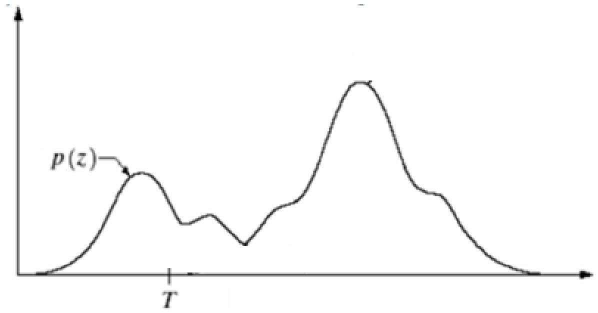


$$\sigma_r^2(T) = P_1(T)\sigma_1^2(T) + P_2(T)\sigma_2^2(T)$$

$$P_1(T) = \sum_{t=0}^{T-1} p\,(t) \qquad P_2(T) = \sum_{t=T}^{L-1} p(t) \qquad \mu_1(T) = \sum_{t=0}^{T-1} \frac{t \cdot p(t)}{P_1(T)} \qquad \mu_2(T) = \sum_{t=T}^{L-1} \frac{t \cdot p(t)}{P_2(T)}$$

Otsu's thresholding

- Global: adaptive Otsu's threshold
  - exhaustively searches $\forall\, T$ that minimizes intra-class variance
  - min. intra-class var. is equivalent to max. inter-class var.



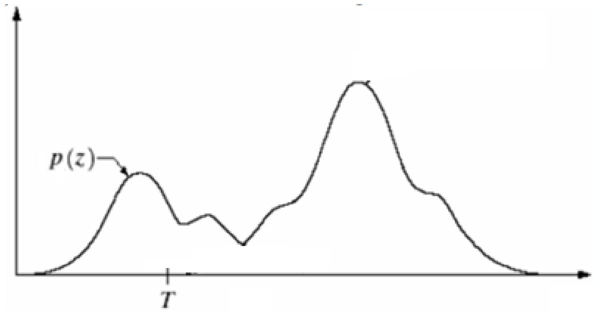
$$\sigma_r^2(T) = P_1(T)\sigma_1^2(T) + P_2(T)\sigma_2^2(T)$$

$$P_1(T) = \sum_{t=0}^{T-1} p\,(t) \qquad P_2(T) = \sum_{t=T}^{L-1} p(t) \qquad \mu_1(T) = \sum_{t=0}^{T-1} \frac{t \cdot p(t)}{P_1(T)} \qquad \mu_2(T) = \sum_{t=T}^{L-1} \frac{t \cdot p(t)}{P_2(T)}$$

$$\sigma_1(T) = \sum_{t=0}^{T-1} \frac{\left(t - \mu_1(T)\right)^2 \cdot p(t)}{P_1(T)} \qquad \sigma_2(T) = \sum_{t=T}^{L-1} \frac{\left(t - \mu_2(T)\right)^2 \cdot p(t)}{P_2(T)}$$

Otsu’s thresholding

- Global: adaptive Otsu’s threshold
  - exhaustively searches $\forall\, T$ that minimizes intra-class variance
  - min. intra-class var. is equivalent to max. inter-class var.



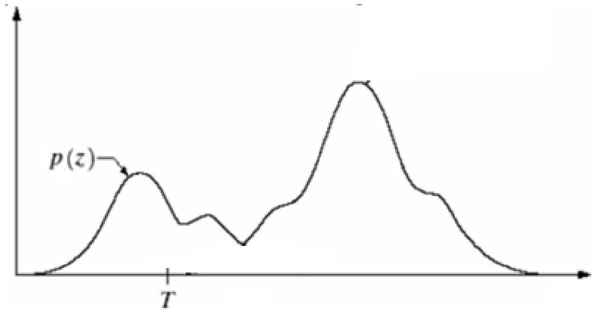
$$\sigma_r^2(T) = P_1(T)\sigma_1^2(T) + P_2(T)\sigma_2^2(T)$$

$$P_1(T) = \sum_{t=0}^{T-1} p\,(t) \qquad P_2(T) = \sum_{t=T}^{L-1} p(t) \qquad \mu_1(T) = \sum_{t=0}^{T-1} \frac{t \cdot p(t)}{P_1(T)} \qquad \mu_2(T) = \sum_{t=T}^{L-1} \frac{t \cdot p(t)}{P_2(T)}$$

$$\sigma_1(T) = \sum_{t=0}^{T-1} \frac{\big(t - \mu_1(T)\big)^2 \cdot p(t)}{P_1(T)} \qquad \sigma_2(T) = \sum_{t=T}^{L-1} \frac{\big(t - \mu_2(T)\big)^2 \cdot p(t)}{P_2(T)}$$

Otsu's thresholding

- Global: adaptive Otsu's threshold
  - exhaustively searches $\forall\, T$ that minimizes intra-class variance
  - min. intra-class var. is equivalent to max. inter-class var.



$$\sigma_r^2(T) = P_1(T)\sigma_1^2(T) + P_2(T)\sigma_2^2(T)$$

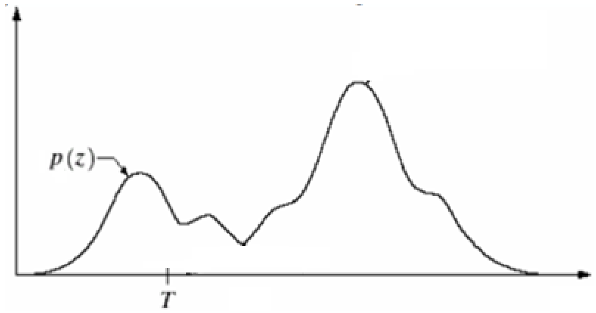
$$P_1(T) = \sum_{t=0}^{T-1} p\,(t) \qquad P_2(T) = \sum_{t=T}^{L-1} p(t) \qquad \mu_1(T) = \sum_{t=0}^{T-1} \frac{t \cdot p(t)}{P_1(T)} \qquad \mu_2(T) = \sum_{t=T}^{L-1} \frac{t \cdot p(t)}{P_2(T)}$$

$$\sigma^2 = \sigma_e^2(T) + \sigma_r^2(T)$$

$$\sigma_1(T) = \sum_{t=0}^{T-1} \frac{\left(t - \mu_1(T)\right)^2 \cdot p(t)}{P_1(T)} \qquad \sigma_2(T) = \sum_{t=T}^{L-1} \frac{\left(t - \mu_2(T)\right)^2 \cdot p(t)}{P_2(T)}$$

Otsu's thresholding

- Global: adaptive Otsu's threshold
  - exhaustively searches $\forall\, T$ that minimizes intra-class variance
  - min. intra-class var. is equivalent to max. inter-class var.



$$\sigma_r^2(T) = P_1(T)\sigma_1^2(T) + P_2(T)\sigma_2^2(T)$$

$$P_1(T) = \sum_{t=0}^{T-1} p\,(t) \qquad P_2(T) = \sum_{t=T}^{L-1} p(t) \qquad \mu_1(T) = \sum_{t=0}^{T-1} \frac{t \cdot p(t)}{P_1(T)} \qquad \mu_2(T) = \sum_{t=T}^{L-1} \frac{t \cdot p(t)}{P_2(T)}$$

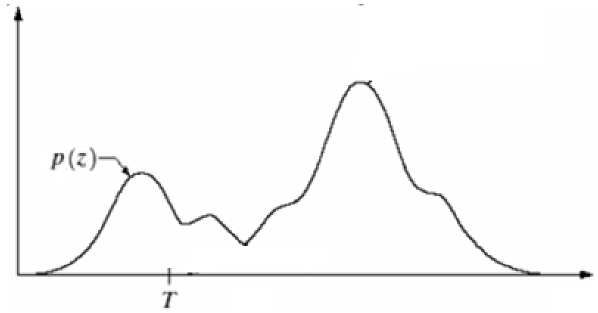
$$\sigma^2 = \sigma_e^2(T) + \sigma_r^2(T)$$

$$\sigma_e^2(T) = \sigma^2 - \sigma_r^2(T)$$

$$\sigma_1(T) = \sum_{t=0}^{T-1} \frac{\left(t - \mu_1(T)\right)^2 \cdot p(t)}{P_1(T)} \qquad \sigma_2(T) = \sum_{t=T}^{L-1} \frac{\left(t - \mu_2(T)\right)^2 \cdot p(t)}{P_2(T)}$$

Otsu's thresholding

- Global: adaptive Otsu's threshold
  - exhaustively searches $\forall\, T$ that minimizes intra-class variance
  - min. intra-class var. is equivalent to max. inter-class var.



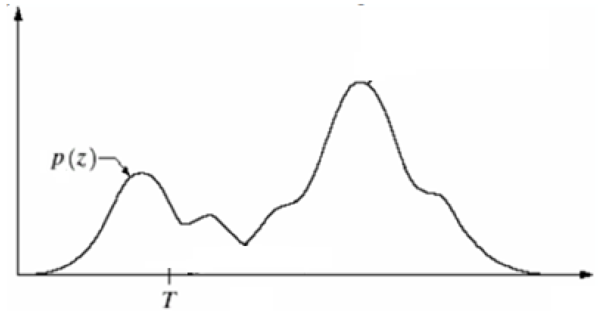
$$\sigma_r^2(T) = P_1(T)\sigma_1^2(T) + P_2(T)\sigma_2^2(T)$$

$$P_1(T) = \sum_{t=0}^{T-1} p\,(t) \qquad P_2(T) = \sum_{t=T}^{L-1} p(t) \qquad \mu_1(T) = \sum_{t=0}^{T-1} \frac{t \cdot p(t)}{P_1(T)} \qquad \mu_2(T) = \sum_{t=T}^{L-1} \frac{t \cdot p(t)}{P_2(T)}$$

$$\sigma_1(T) = \sum_{t=0}^{T-1} \frac{\left(t - \mu_1(T)\right)^2 \cdot p(t)}{P_1(T)} \qquad \sigma_2(T) = \sum_{t=T}^{L-1} \frac{\left(t - \mu_2(T)\right)^2 \cdot p(t)}{P_2(T)}$$

Otsu's thresholding

- Global: adaptive Otsu's threshold
  - exhaustively searches $\forall\, T$ that minimizes intra-class variance
  - min. intra-class var. is equivalent to max. inter-class var.



$$\sigma_r^2(T) = P_1(T)\sigma_1^2(T) + P_2(T)\sigma_2^2(T)$$

$$P_1(T) = \sum_{t=0}^{T-1} p\,(t) \qquad P_2(T) = \sum_{t=T}^{L-1} p(t) \qquad \mu_1(T) = \sum_{t=0}^{T-1} \frac{t \cdot p(t)}{P_1(T)} \qquad \mu_2(T) = \sum_{t=T}^{L-1} \frac{t \cdot p(t)}{P_2(T)}$$

$$\sigma_e^2(T) = \sigma^2 - \sigma_r^2(T)$$

$$\boxed{\sigma_e^2(T) = P_1(T)P_2(T)\big(\mu_1(T) - \mu_2(T)\big)^2}$$

$$\sigma_1(T) = \sum_{t=0}^{T-1} \frac{\big(t - \mu_1(T)\big)^2 \cdot p(t)}{P_1(T)} \qquad \sigma_2(T) = \sum_{t=T}^{L-1} \frac{\big(t - \mu_2(T)\big)^2 \cdot p(t)}{P_2(T)}$$

Otsu's thresholding

- Global: adaptive Otsu's threshold

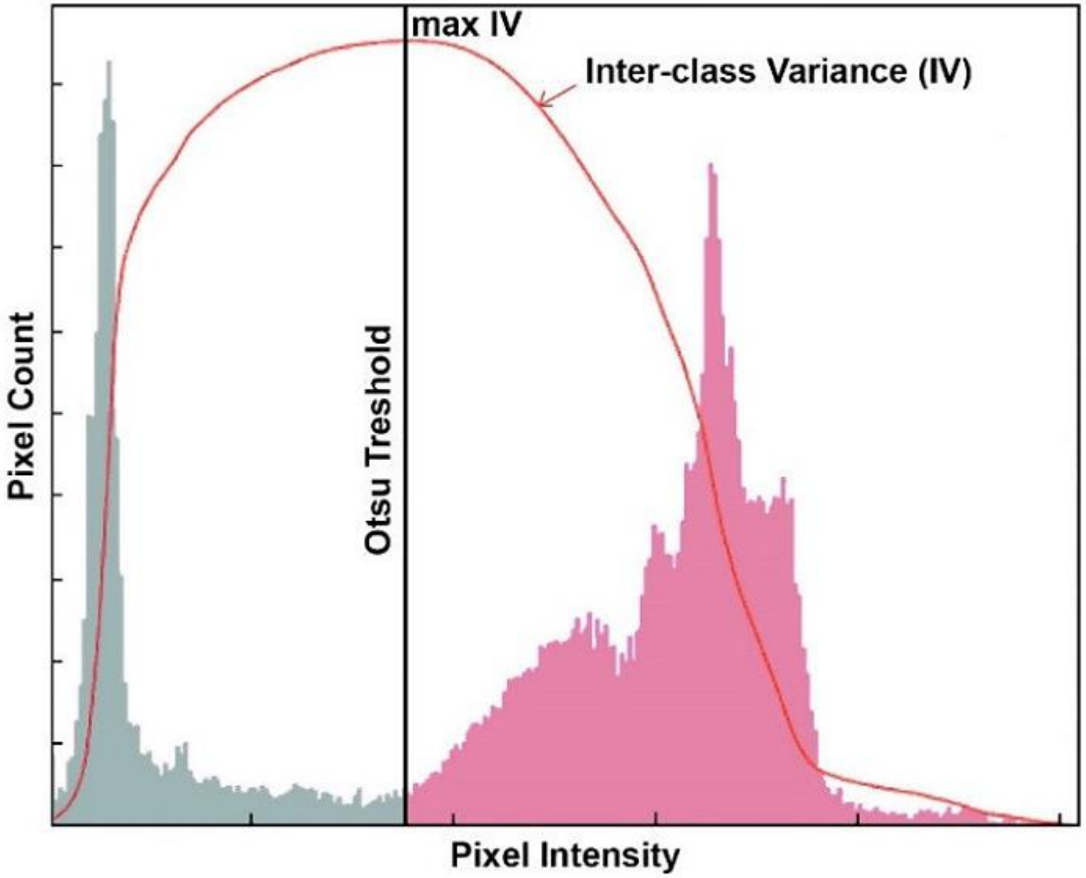
$$\boxed{\sigma_e^2(T) = P_1(T)P_2(T)\big(\mu_1(T) - \mu_2(T)\big)^2}$$

1. Compute histogram and probabilities of each intensity level t
2. Set up initial $P_i(0)$ and $\mu_i(0)$
3. Step through all possible thresholds $T = 1, \ldots, L$
    1. Update $P_i(T)$ and $\mu_i(T)$
    2. Compute $\sigma_e^2(T)$
4. Desired threshold T^* corresponds to the maximum $\sigma_e^2(T)$

Otsu's thresholding

- Variance variation
  - inter-class var maximization

$$\sigma_e^2(T) = P_1(T)P_2(T)\big(\mu_1(T) - \mu_2(T)\big)^2$$

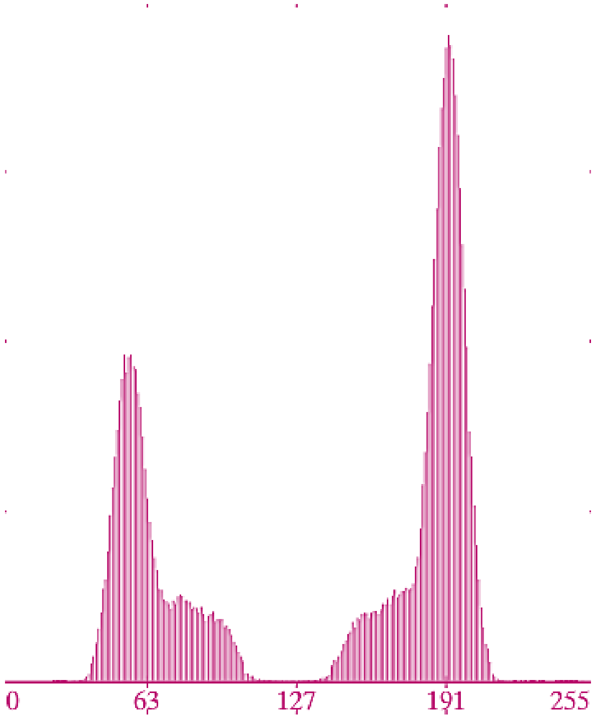


Otsu’s thresholding

- Global: iterative adapting threshold: TH = 125
- Global: Otsu’s thresholding: TH = 125

Otsu's thresholding

- Global: iterative adapting threshold: TH = 125
- Global: Otsu's thresholding: TH = 125



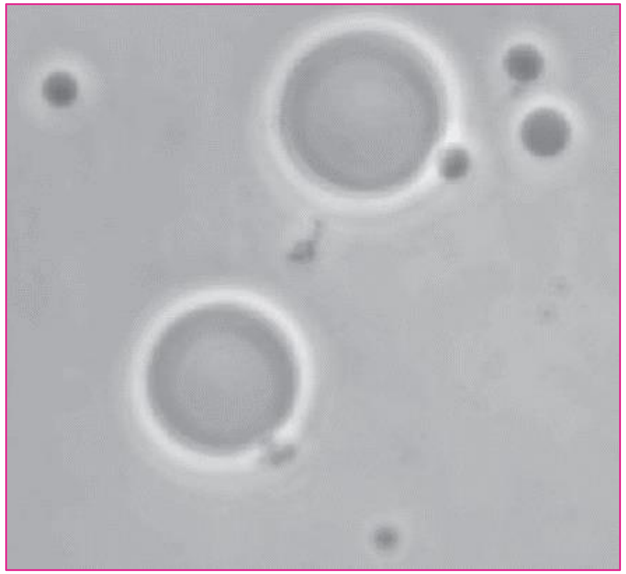
Otsu’s thresholding

- Global: iterative adapting threshold: TH = 125
- Global: Otsu’s thresholding: TH = 125

Otsu’s thresholding

- Example
  - microscopic image (polymer cells)

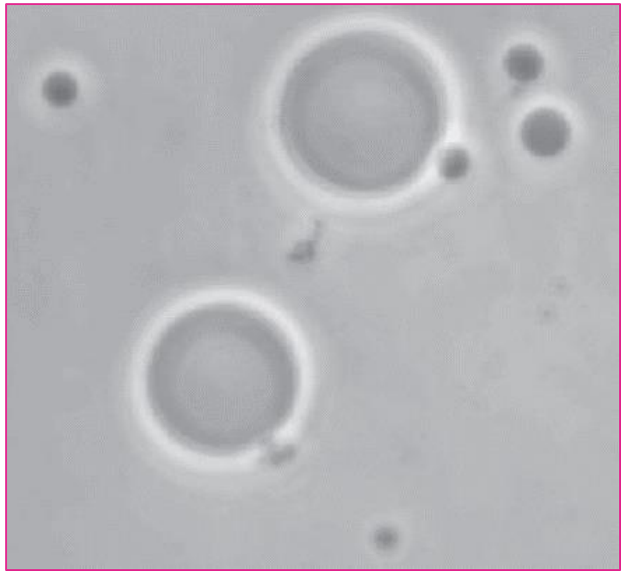
Input

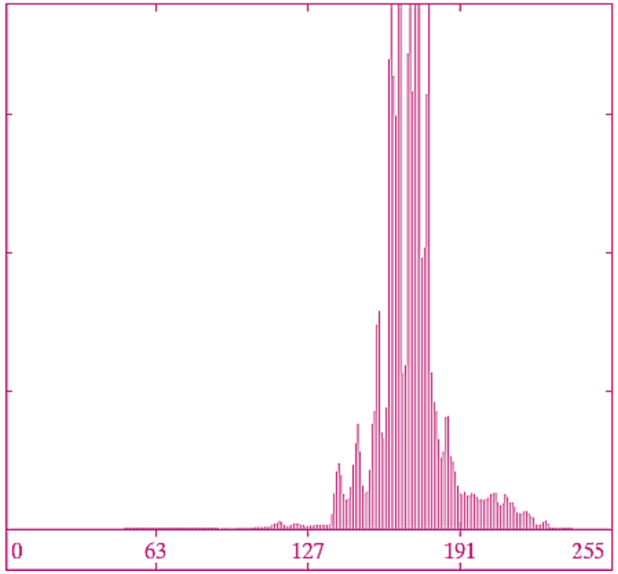


Otsu's thresholding

- Example
  - microscopic image (polymer cells)

Input

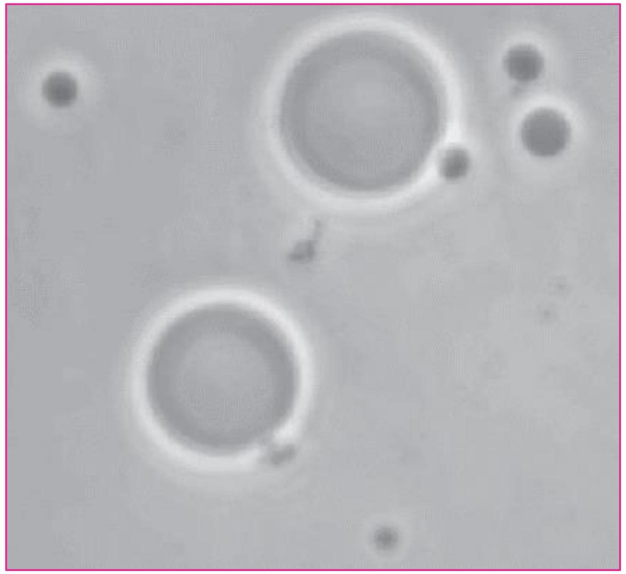




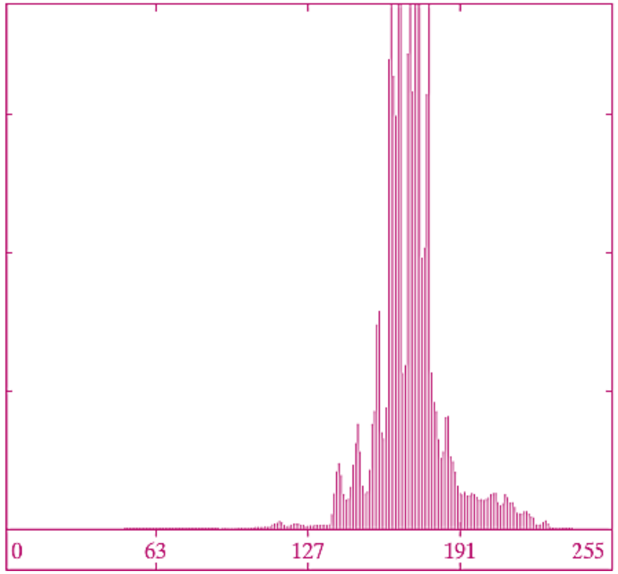
Otsu’s thresholding

- Example
  - microscopic image (polymer cells)

Input



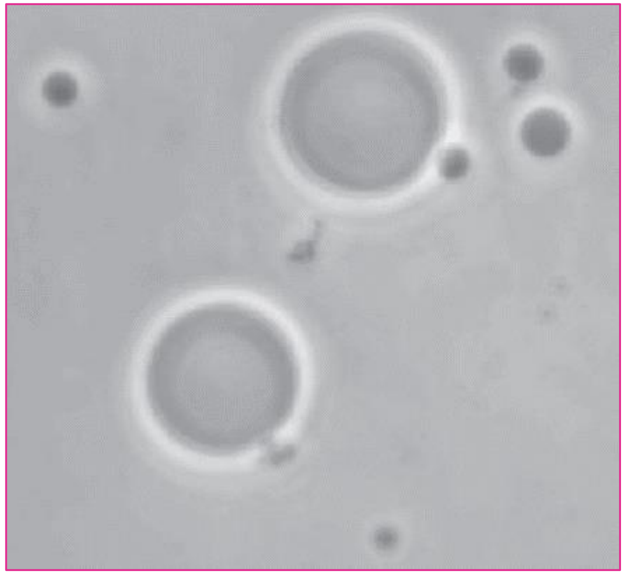
Hist



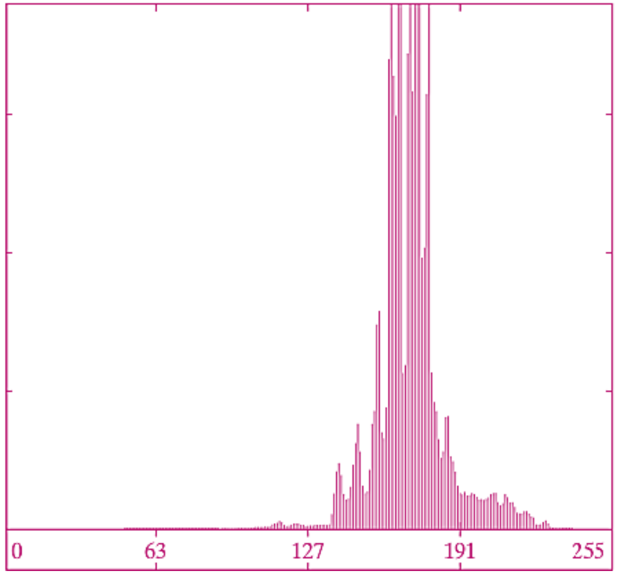
Otsu's thresholding

- Example
  - microscopic image (polymer cells)

Input



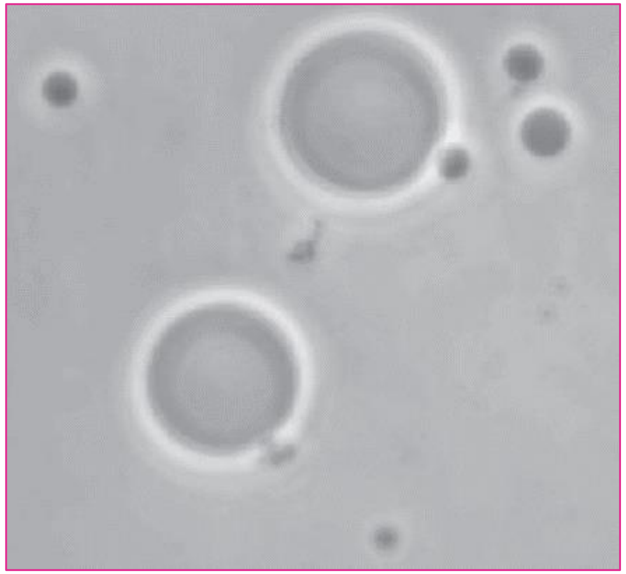
Hist



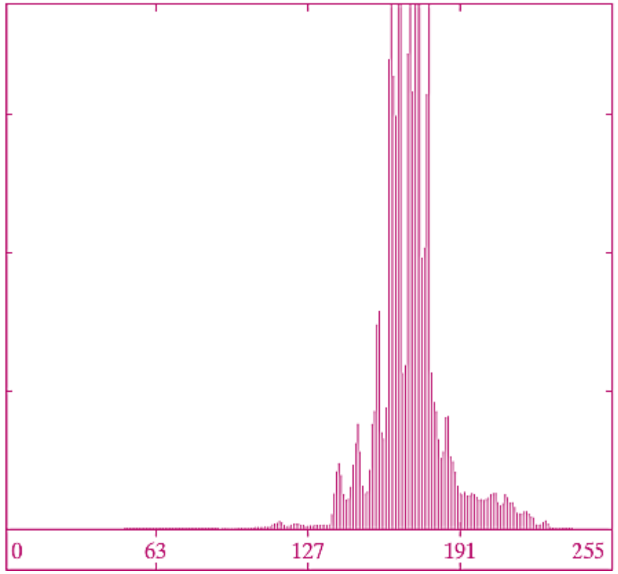
Otsu's thresholding

- Example
  - microscopic image (polymer cells)

Input



Hist

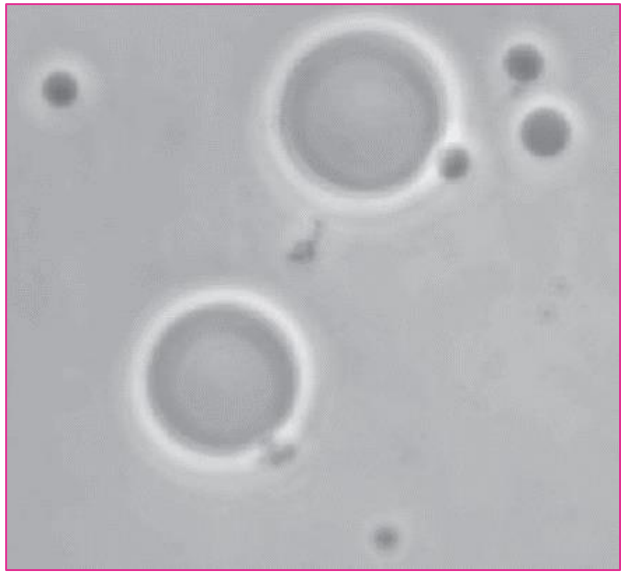


Global: iterative adaptive TH

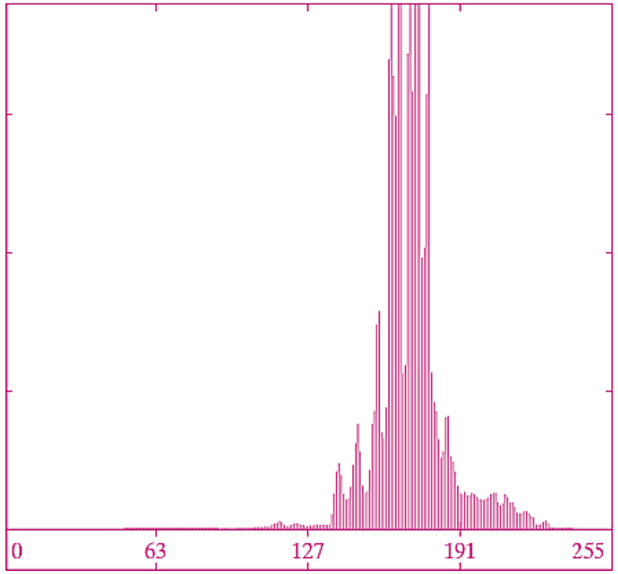
Otsu’s thresholding

- Example
  - microscopic image (polymer cells)

Input



Hist

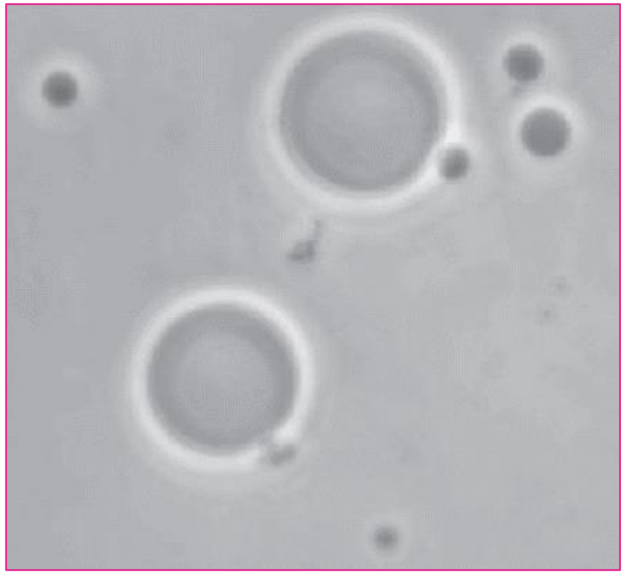


Global: iterative adaptive TH

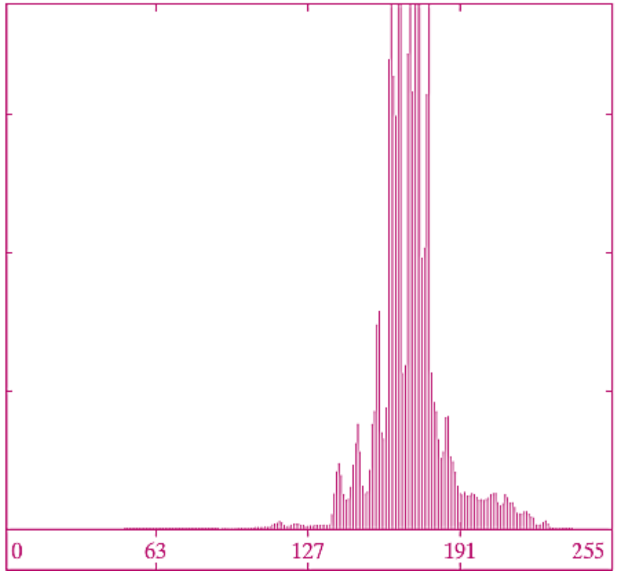
Otsu's thresholding

- Example
  - microscopic image (polymer cells)

Input



Hist



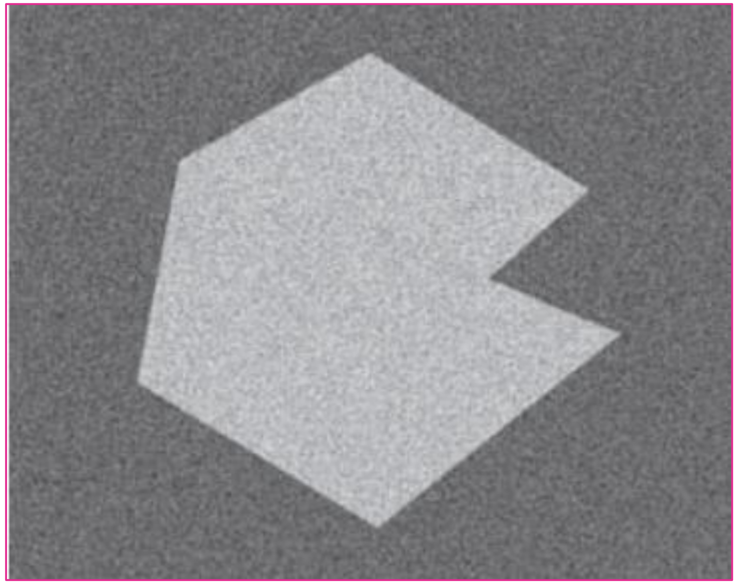
Global: iterative adaptive TH

Global: Otsu's TH

Otsu’s thresholding

- Example
  - noisy input as it is

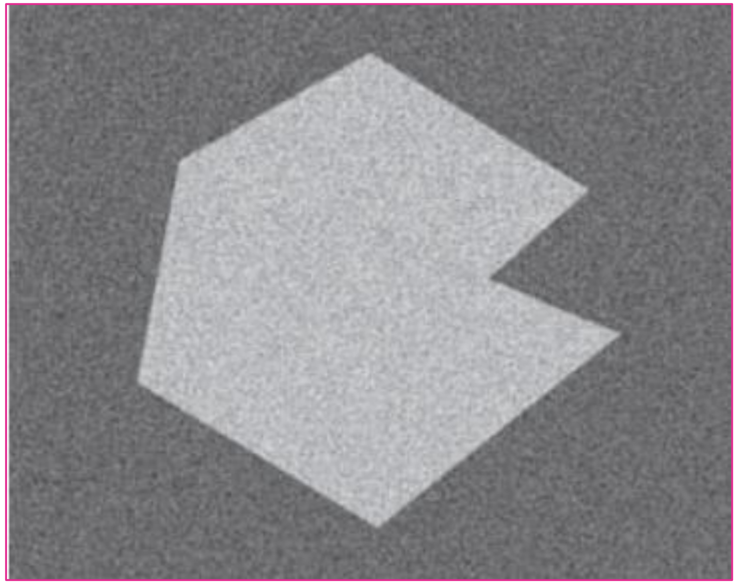
Input

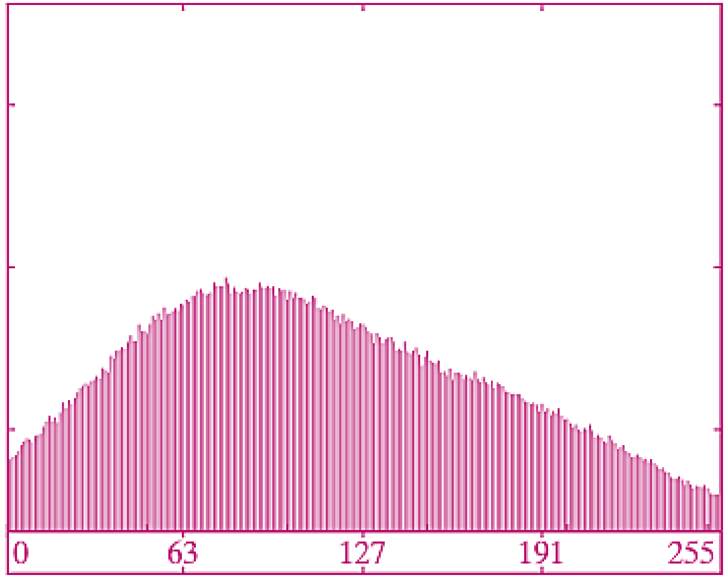


Otsu's thresholding

- Example
  - noisy input as it is

Input

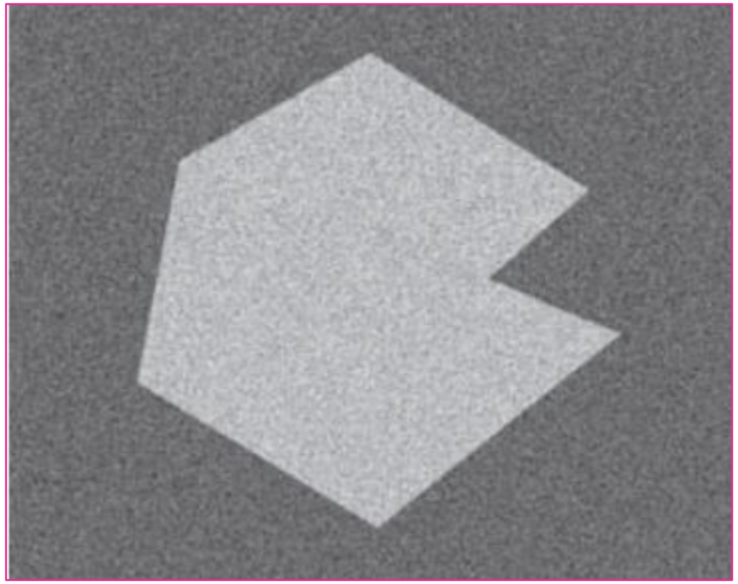




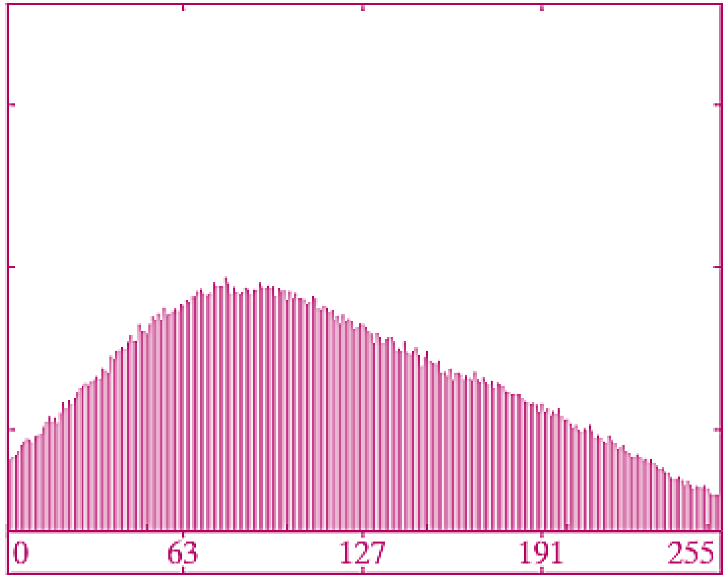
Otsu's thresholding

- Example
  - noisy input as it is

Input



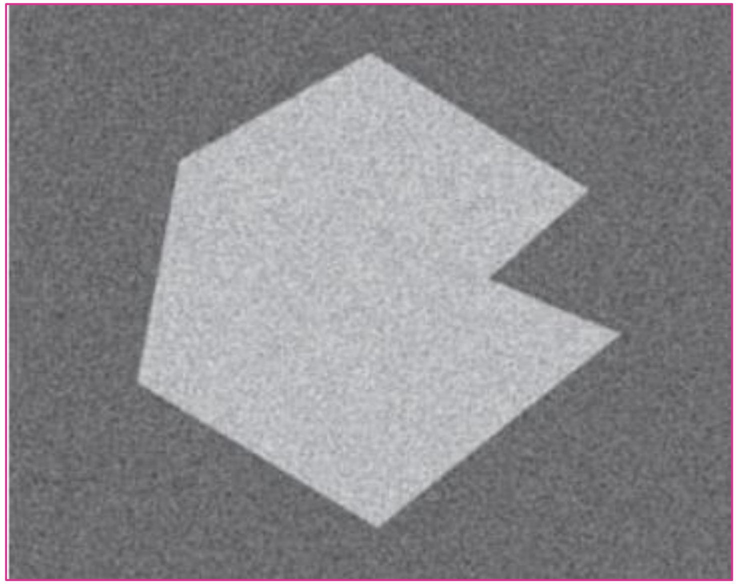
Hist



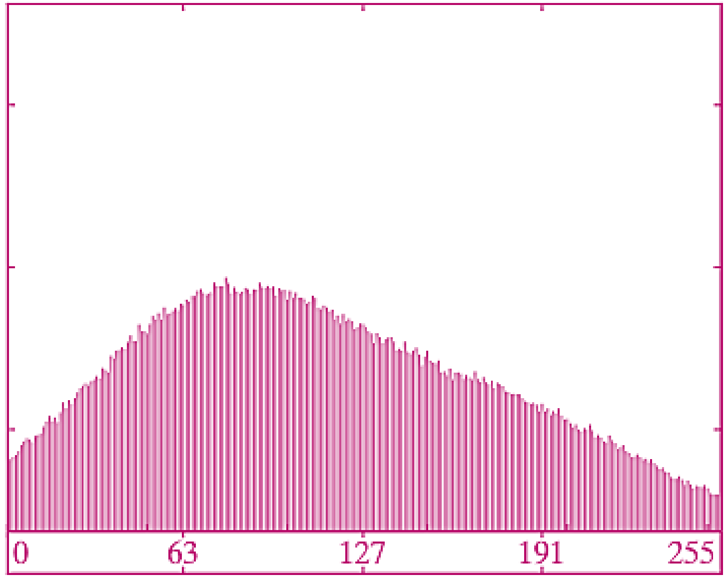
Otsu’s thresholding

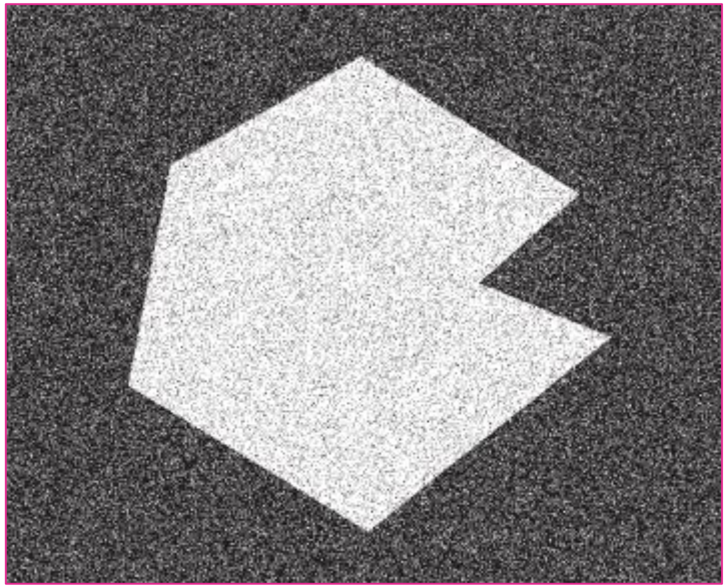
- Example
  - noisy input as it is

Input



Hist

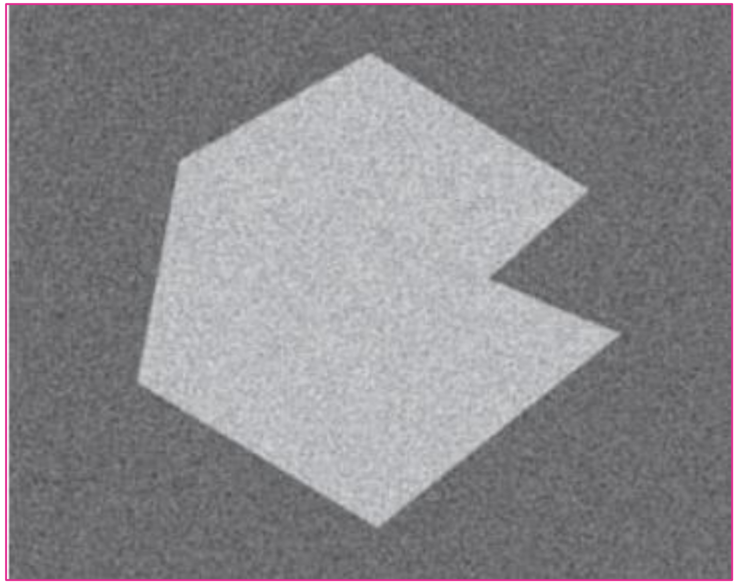




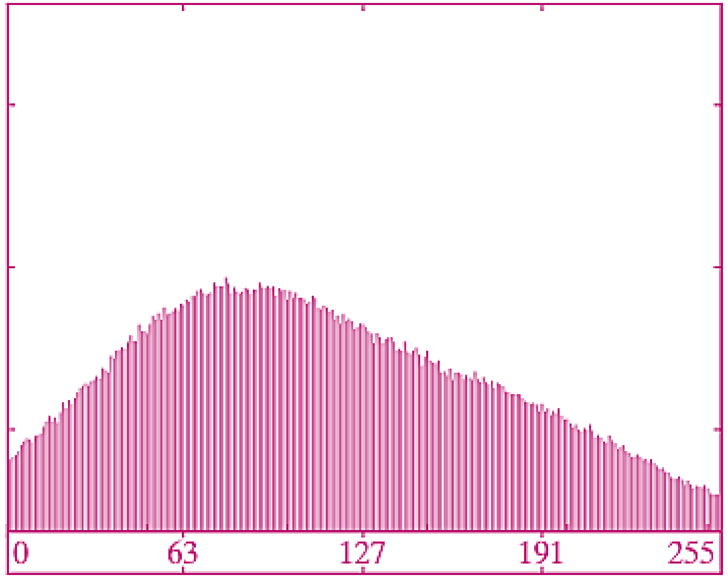
Otsu's thresholding

- Example
  - noisy input as it is

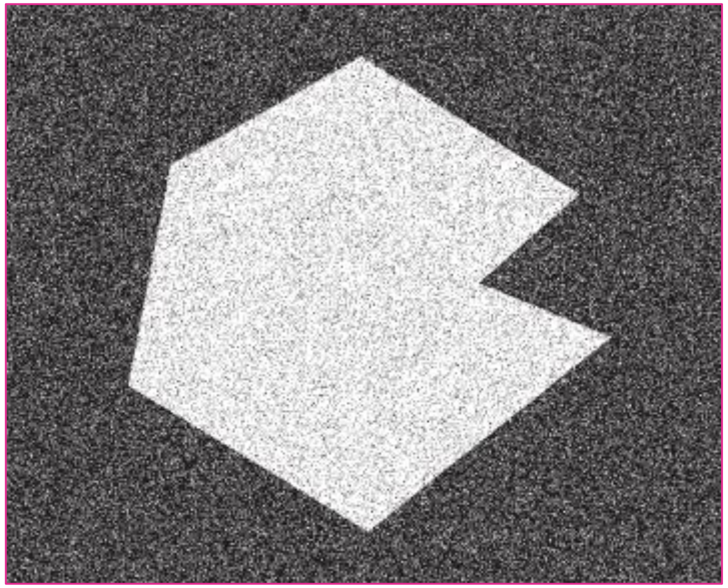
Input



Hist



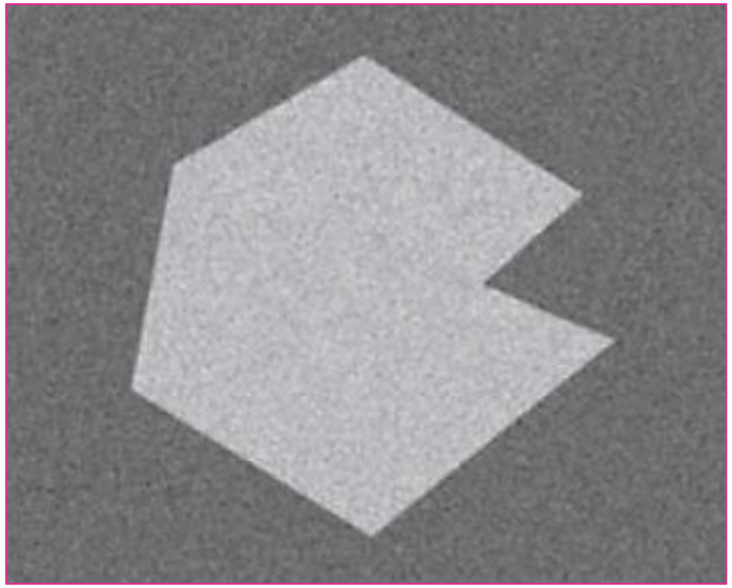
Global: Otsu's TH



Otsu's thresholding

- Example
  - noisy input after minor smoothing

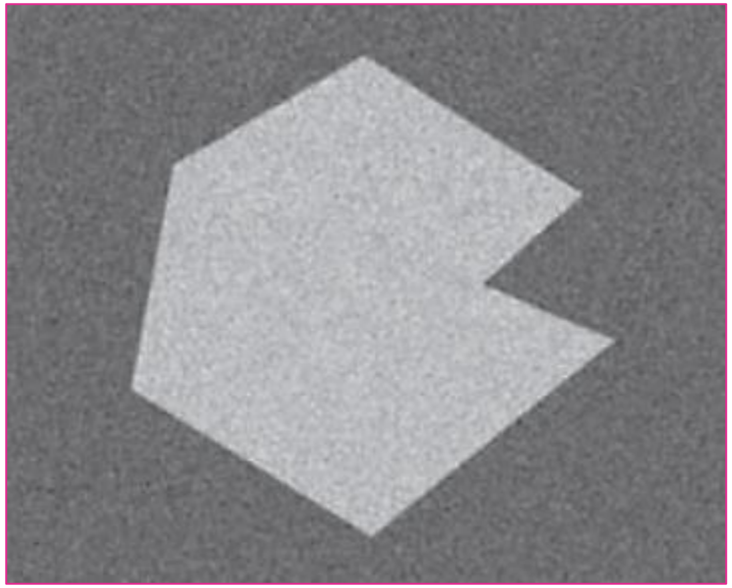
Input

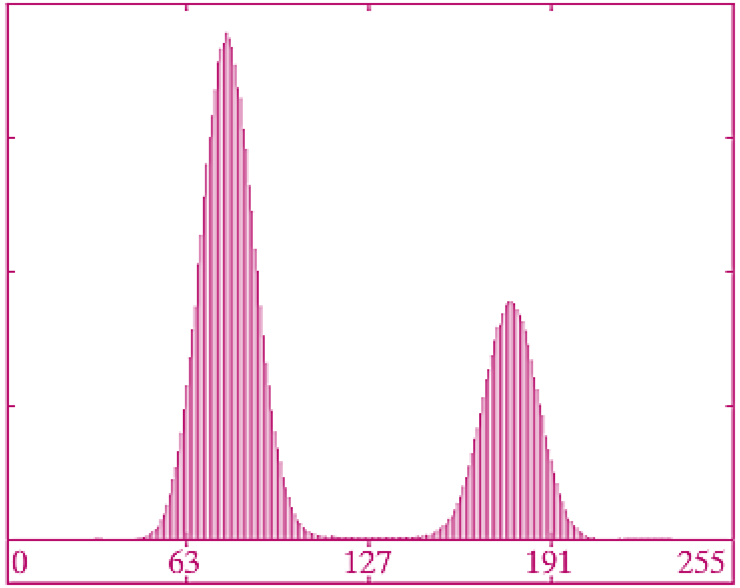


Otsu’s thresholding

- Example
  - noisy input after minor smoothing

Input

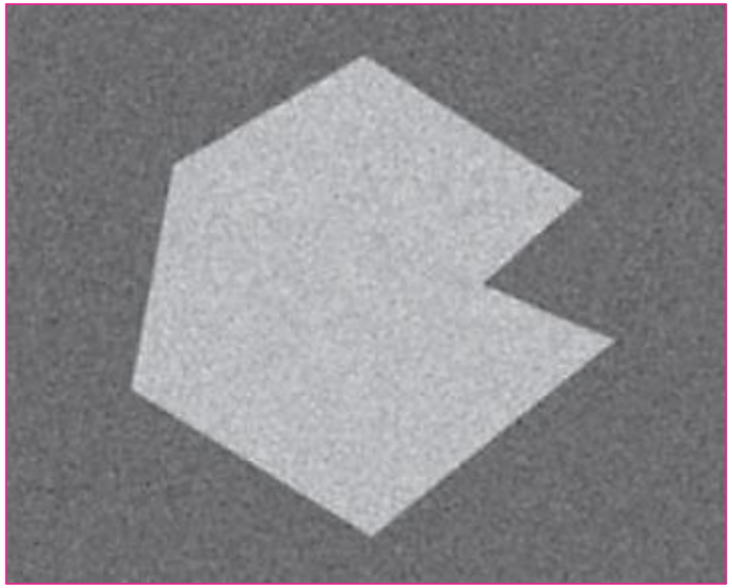




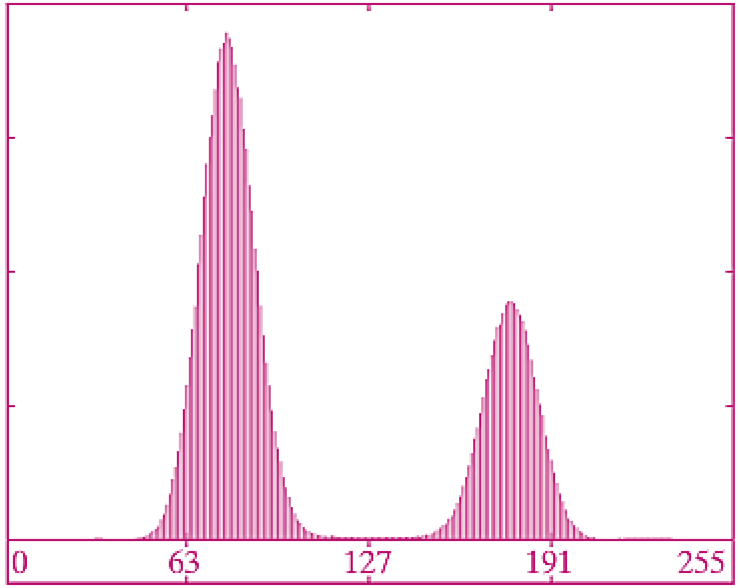
Otsu’s thresholding

- Example
  - noisy input after minor smoothing

Input



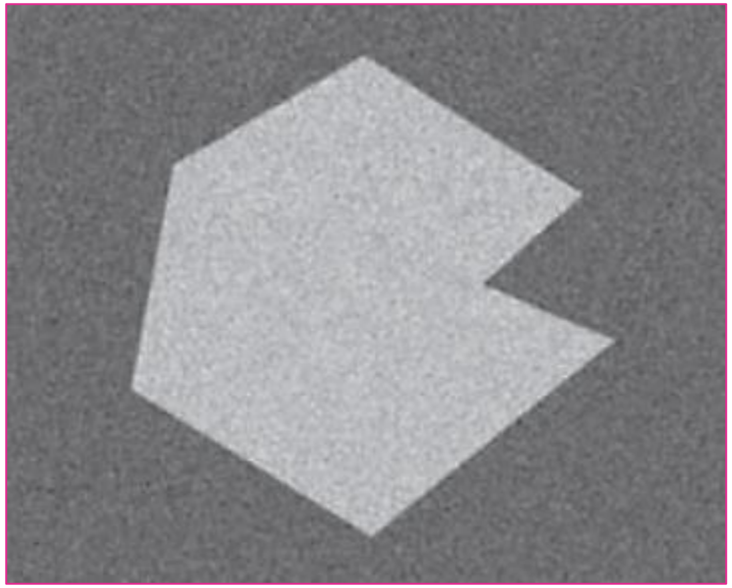
Hist



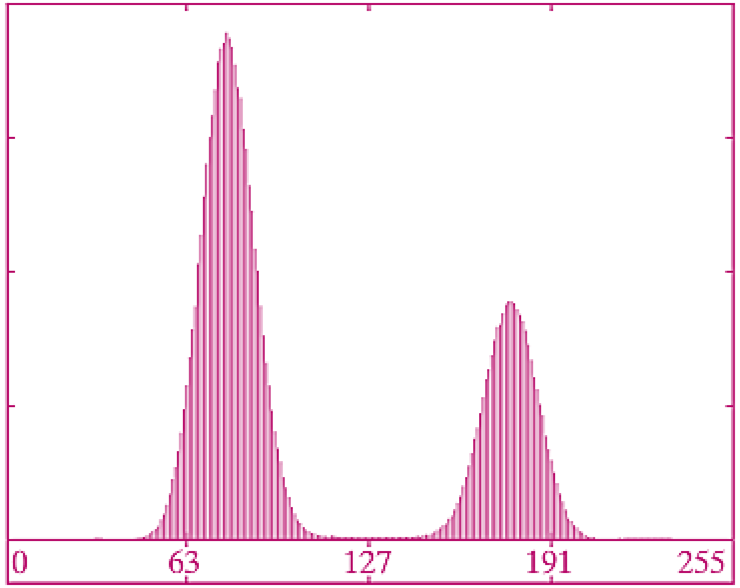
Otsu’s thresholding

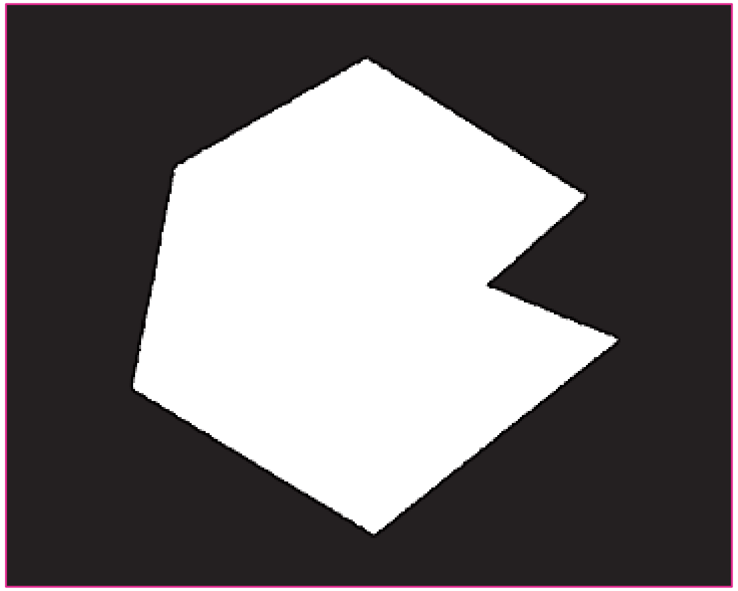
- Example
  - noisy input after minor smoothing

Input



Hist

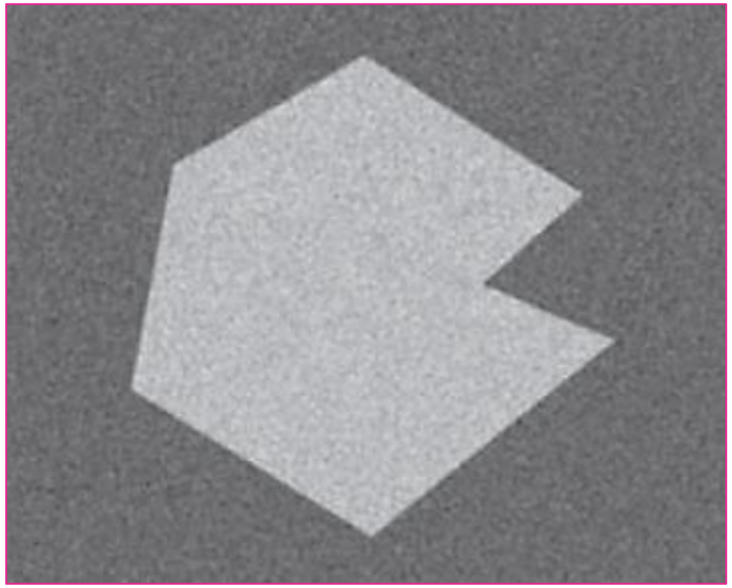




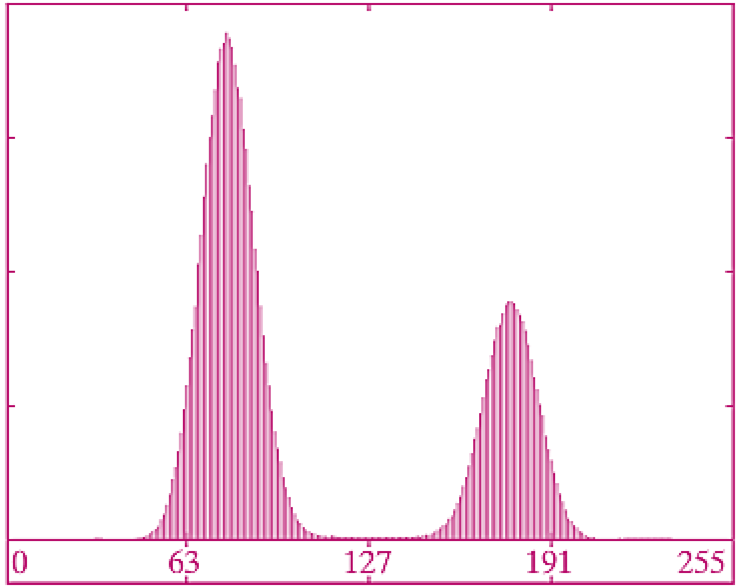
Otsu's thresholding

- Example
  - noisy input after minor smoothing

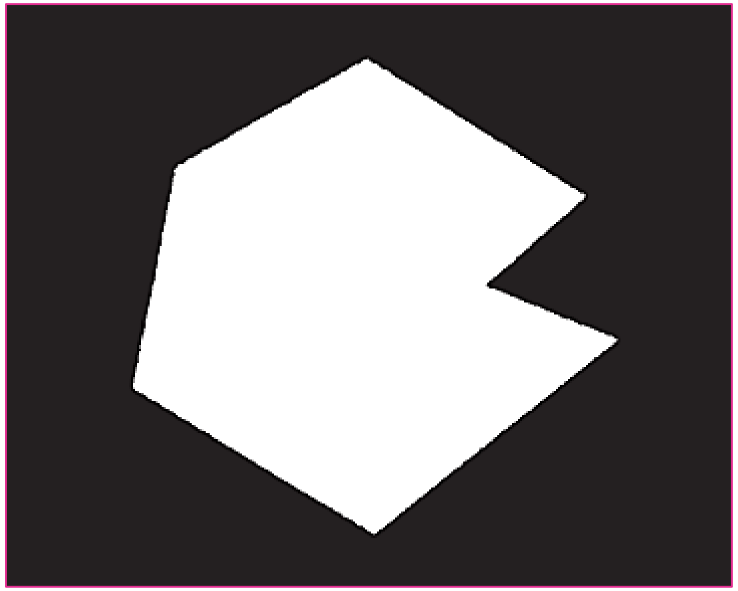
Input



Hist



Global: Otsu's TH

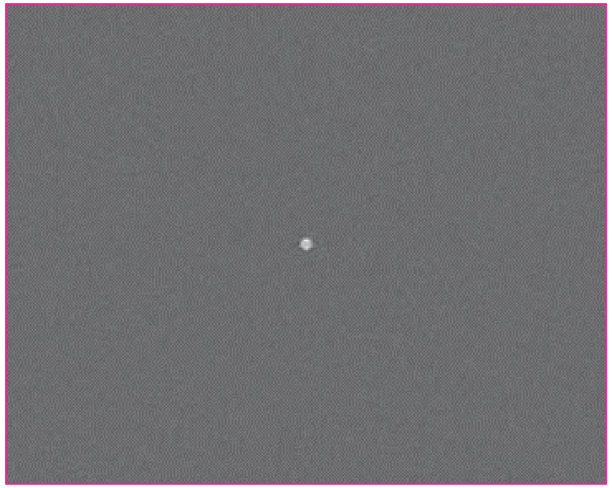


sandhan@iitk.ac.in

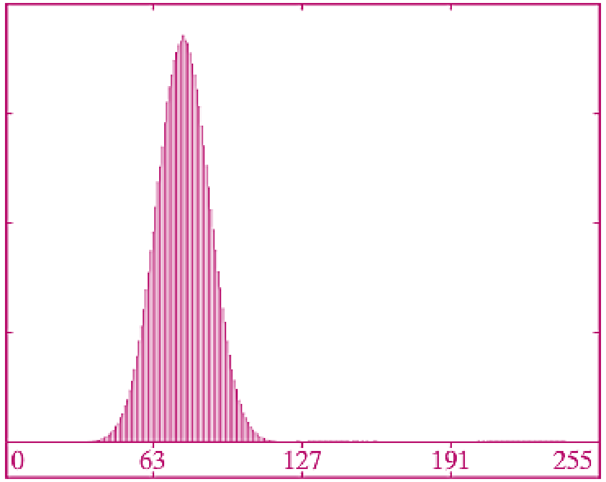
Otsu’s thresholding

- Example
  - small object’s noisy image

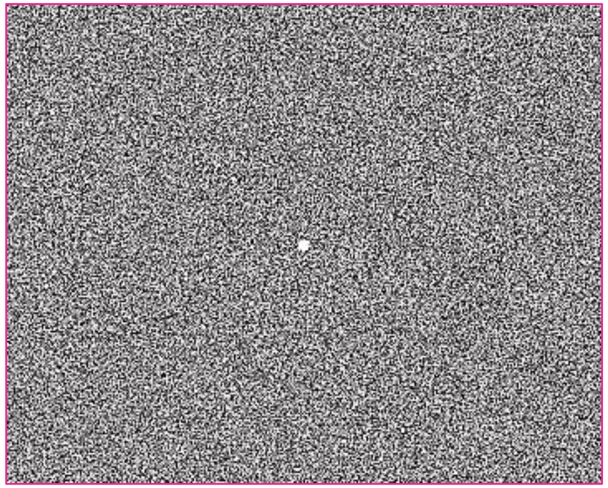
Input



Hist



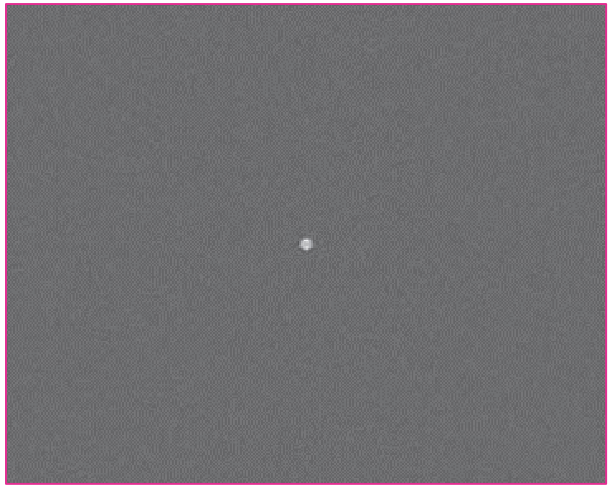
Global: Otsu’s TH



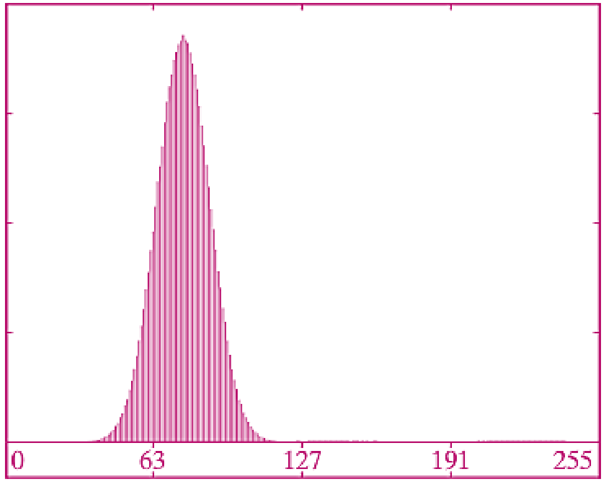
Otsu’s thresholding

- Example
  - small object’s noisy image

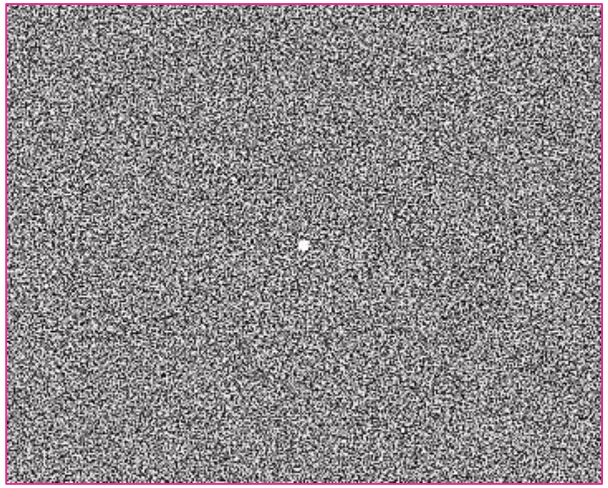
Input

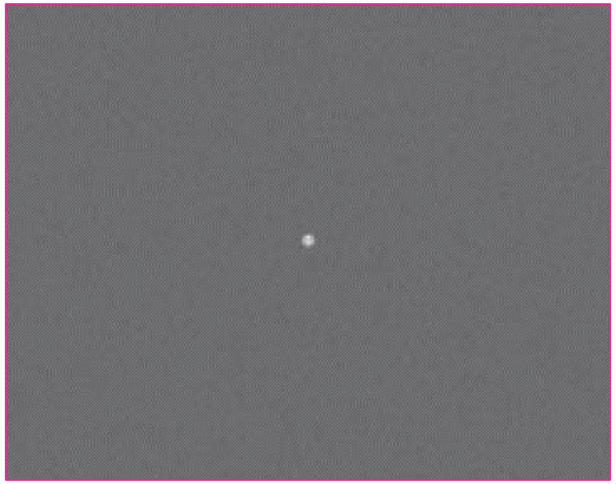


Hist



Global: Otsu’s TH

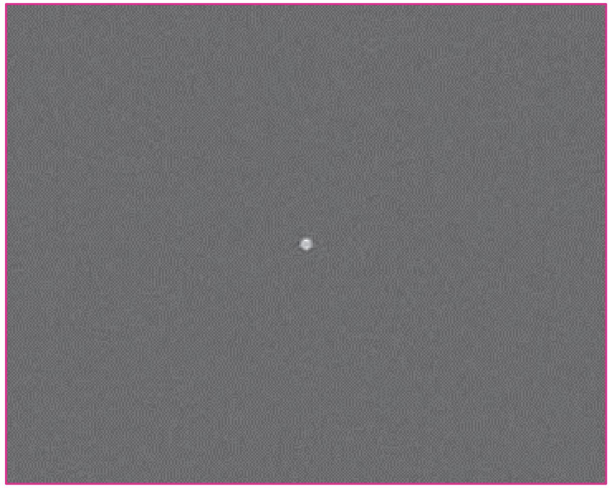




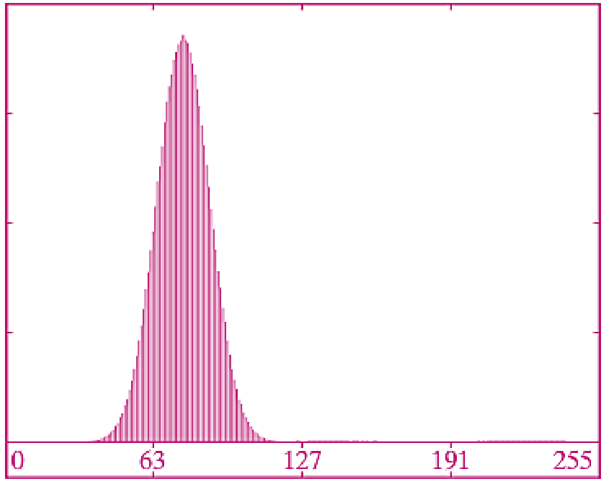
Otsu’s thresholding

- Example
  - small object’s noisy image

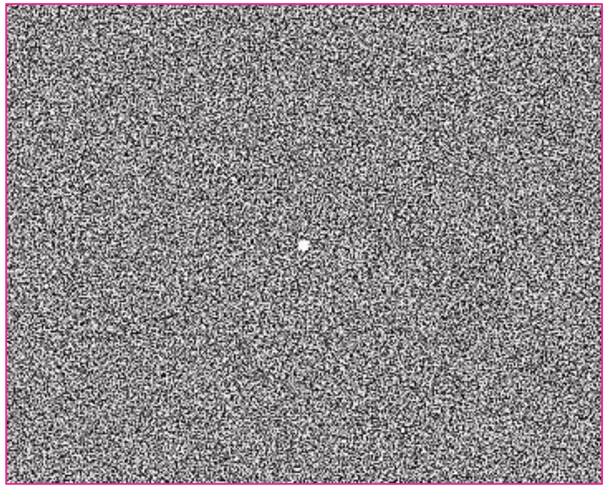
Input

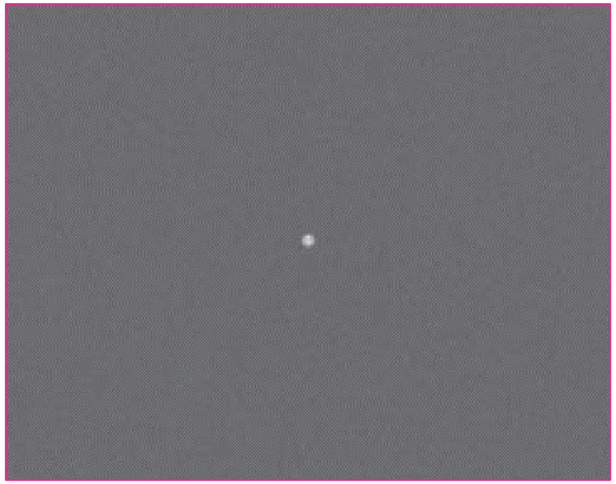


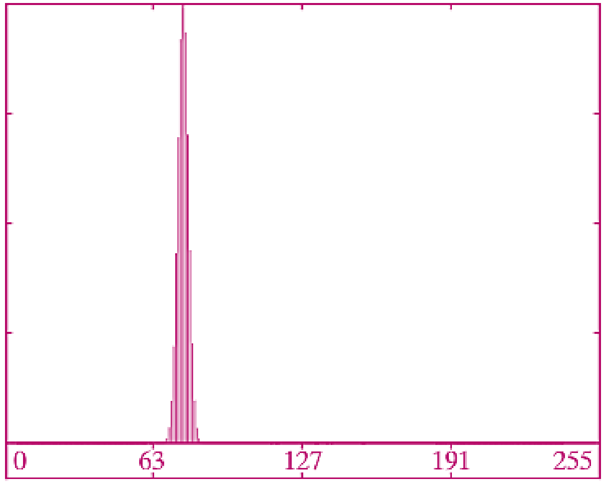
Hist



Global: Otsu’s TH



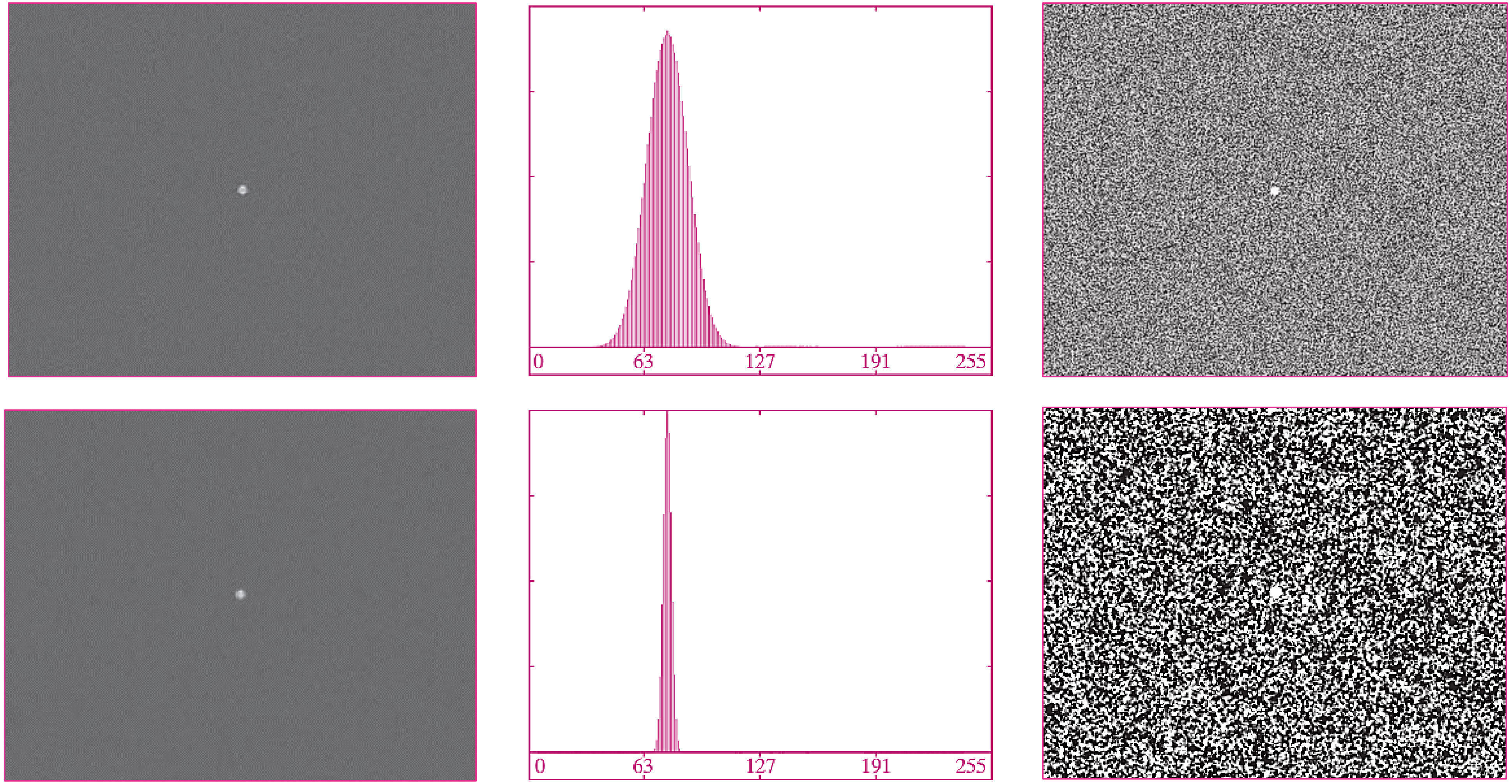




Otsu’s thresholding

- Example
  - small object’s noisy image

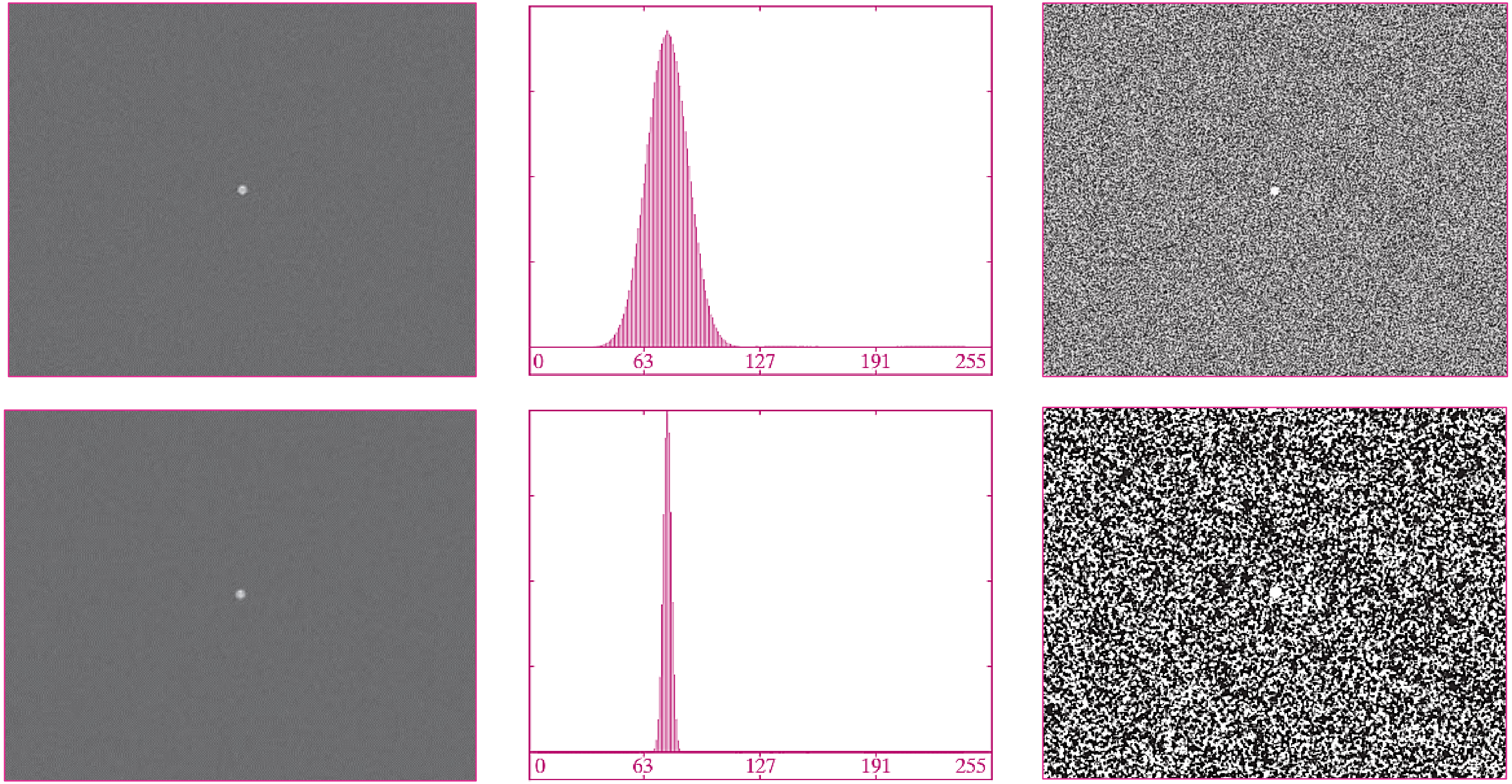
Input | Hist | Global: Otsu’s TH



Otsu's thresholding

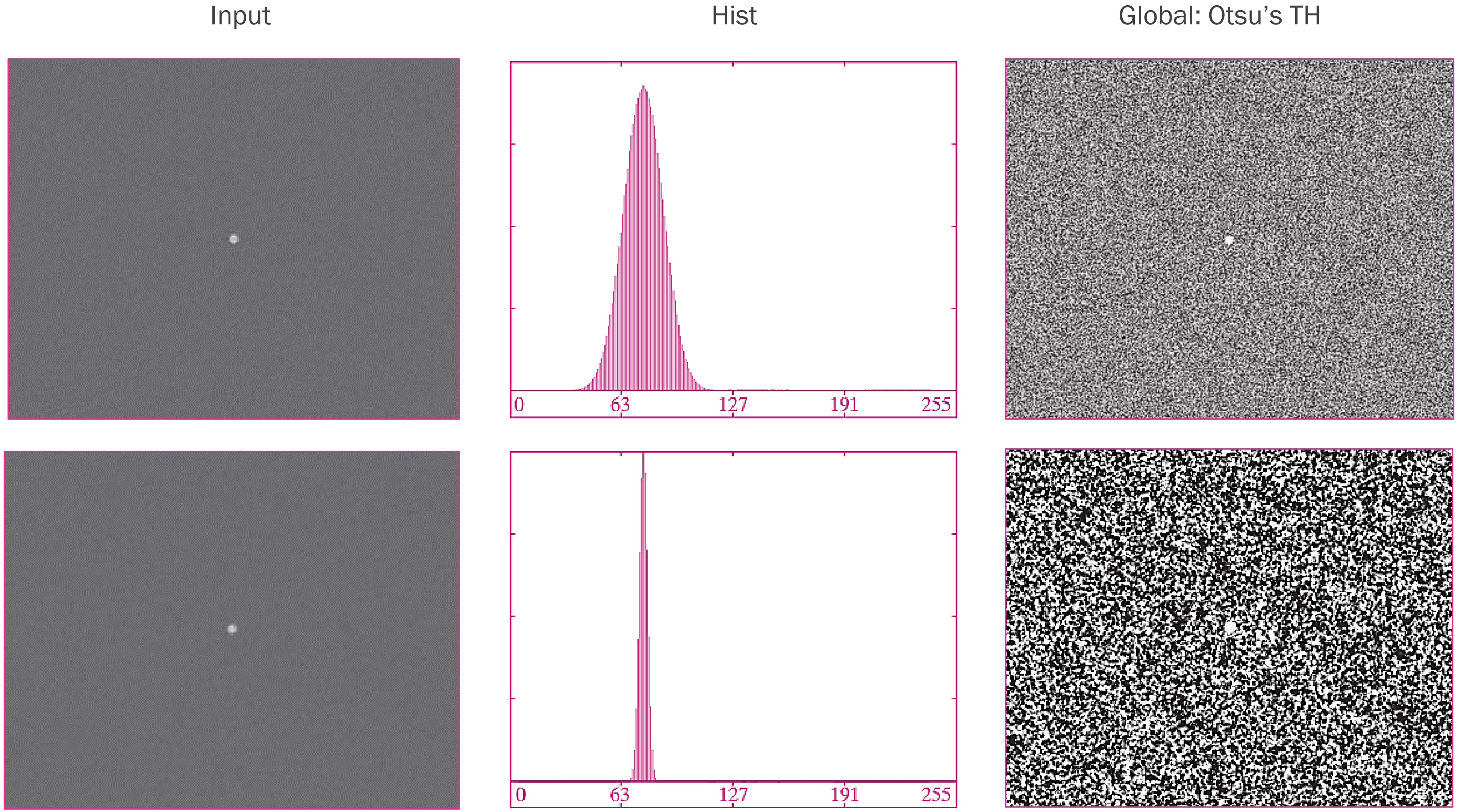
- Example
  - small object's noisy image
  - smoothing degrades the performance

Input | Hist | Global: Otsu's TH



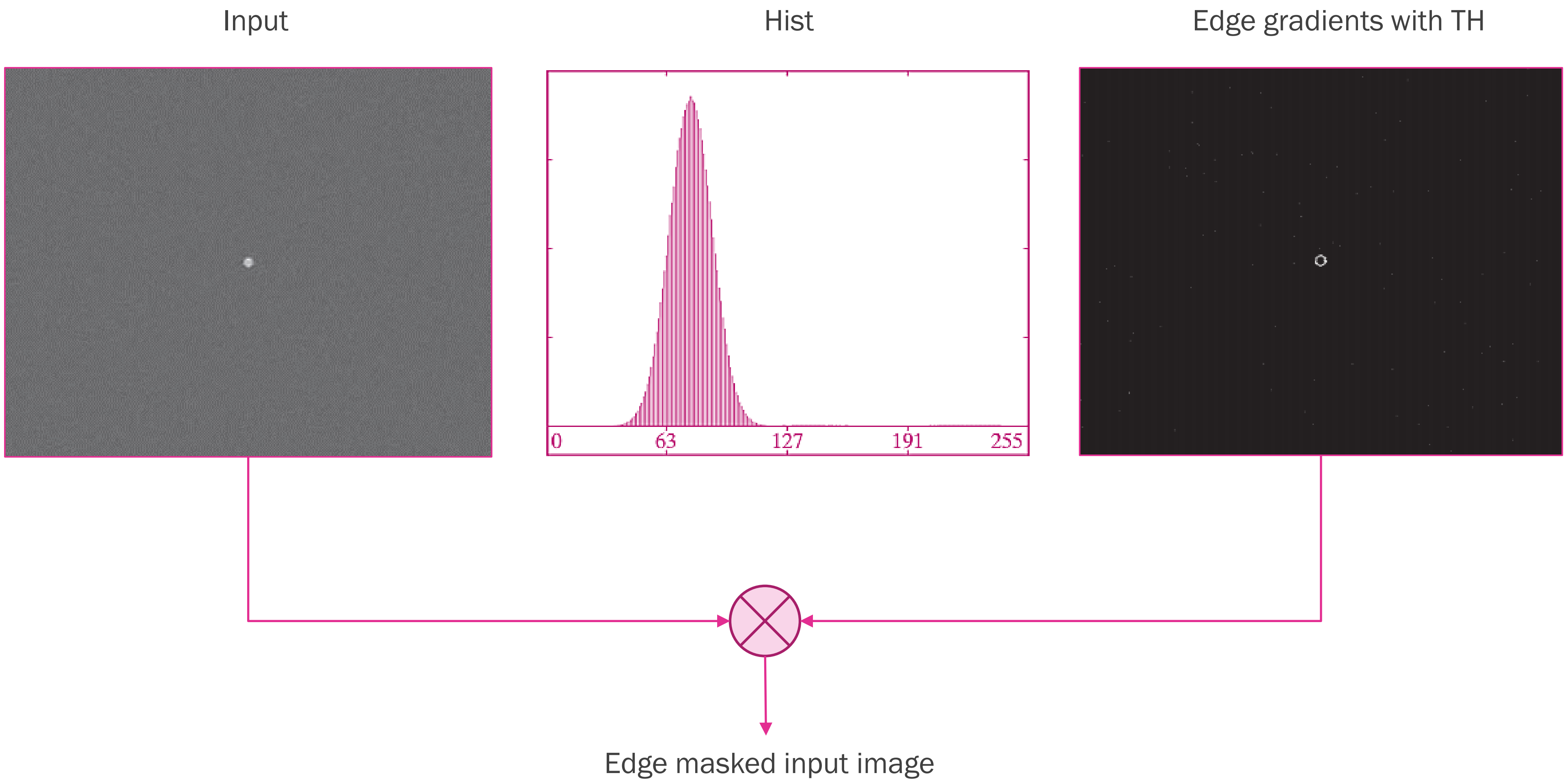
Otsu’s thresholding

- Example
  - small object’s noisy image
  - smoothing degrades the performance
  - what caused the problem?
  - how to solve the problem?



Otsu’s thresholding

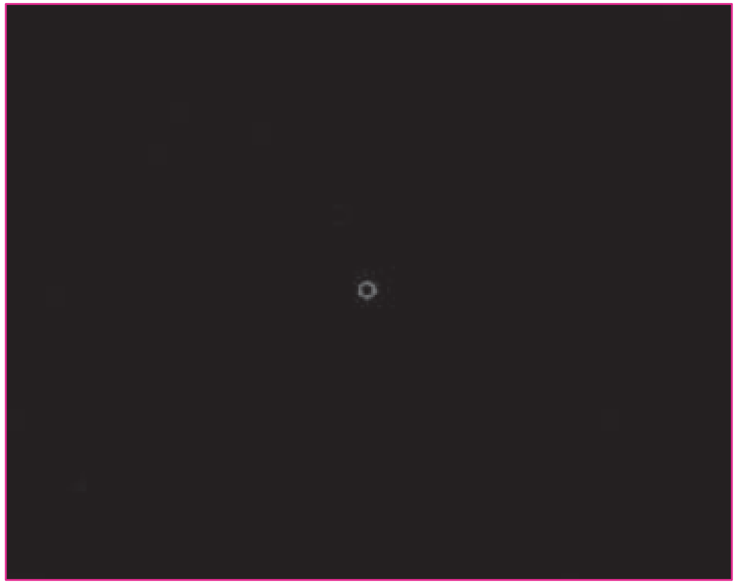
- Example
  - small object’s noisy image
  - edge masks



Otsu’s thresholding

- Example
  - small object’s noisy image
  - Otsu’s TH obtained via edge masked image but that TH is applied on the original input image

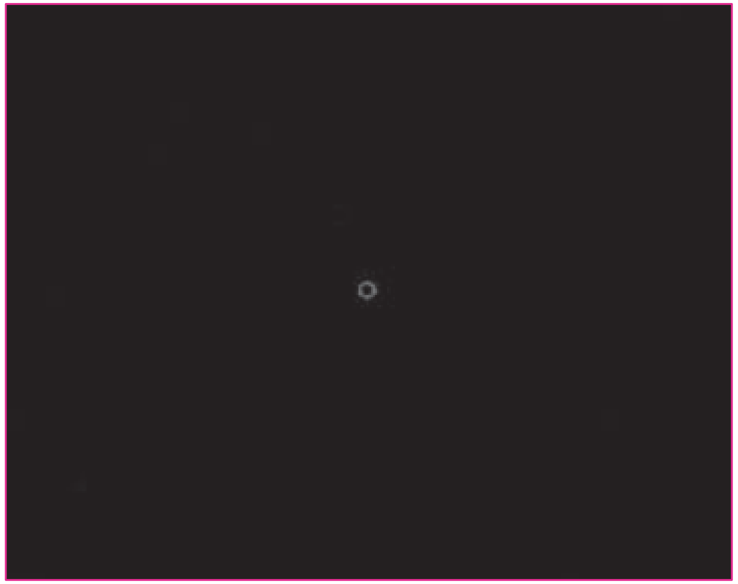
Edge masked input image

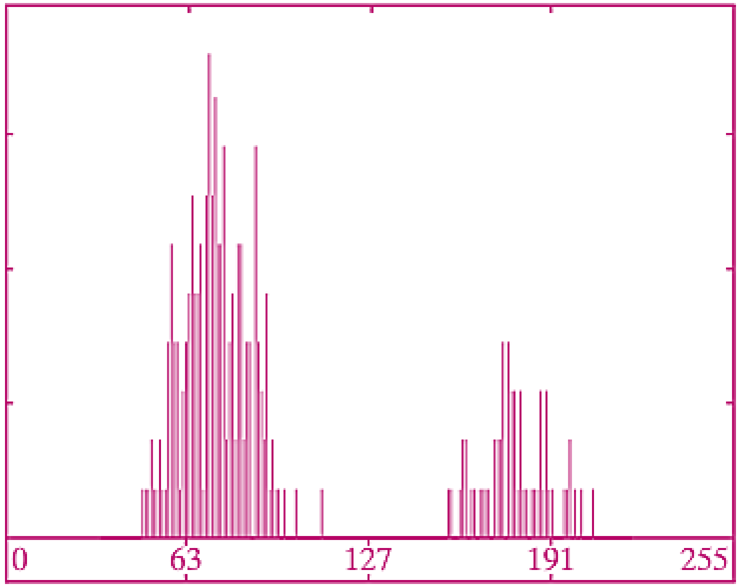


Otsu's thresholding

- Example
  - small object's noisy image
  - Otsu's TH obtained via edge masked image but that TH is applied on the original input image

Edge masked input image

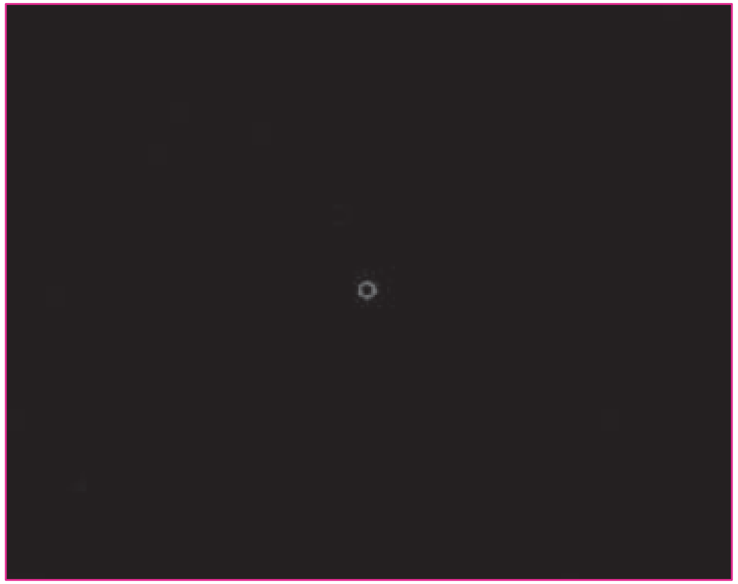




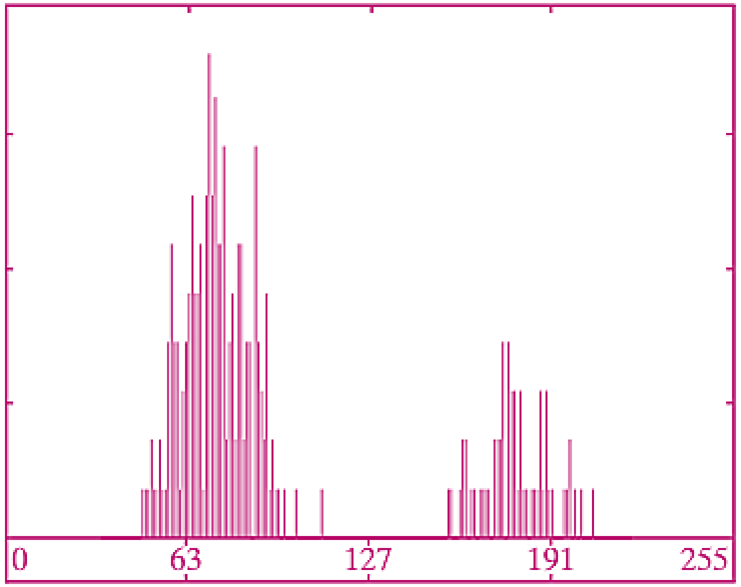
Otsu's thresholding

- Example
  - small object's noisy image
  - Otsu's TH obtained via edge masked image but that TH is applied on the original input image

Edge masked input image



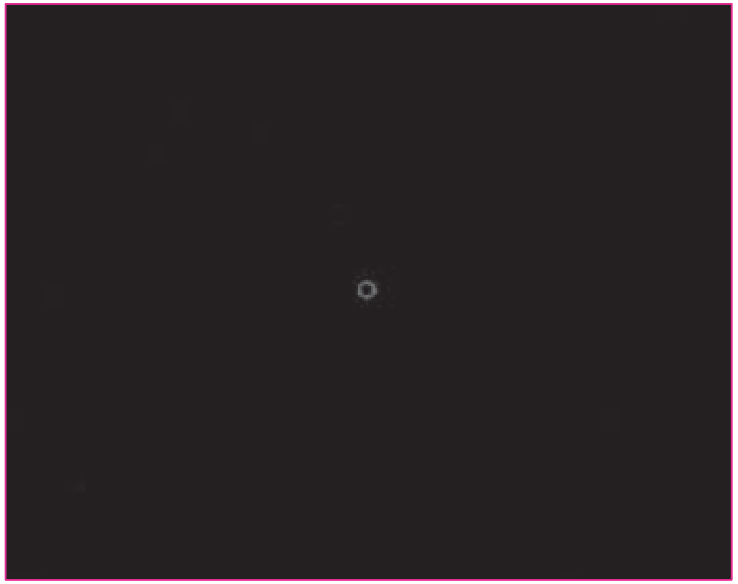
Hist



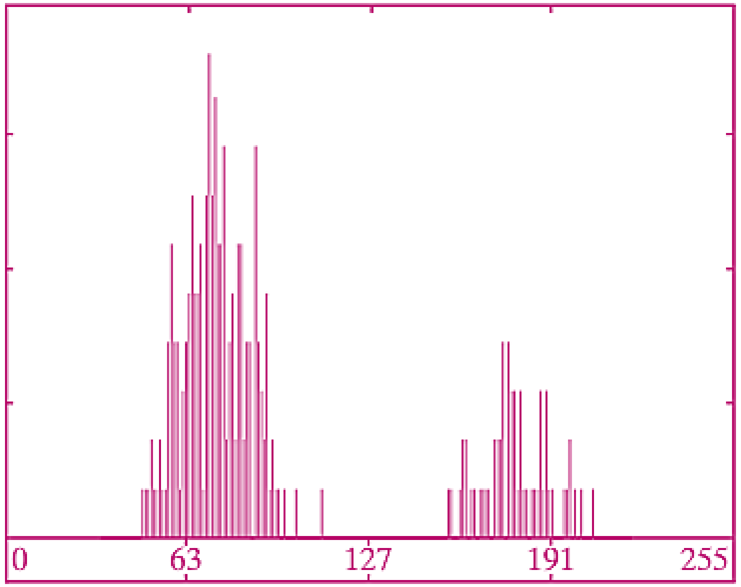
Otsu’s thresholding

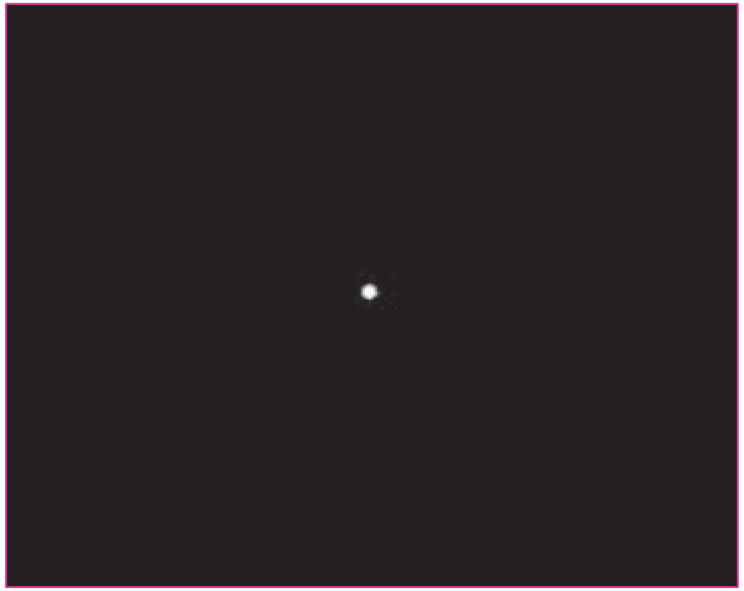
- Example
  - small object’s noisy image
  - Otsu’s TH obtained via edge masked image but that TH is applied on the original input image

Edge masked input image



Hist

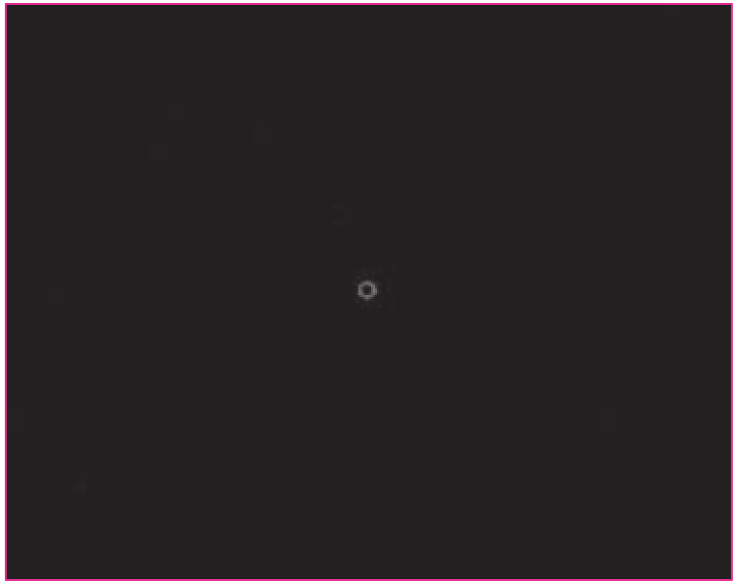




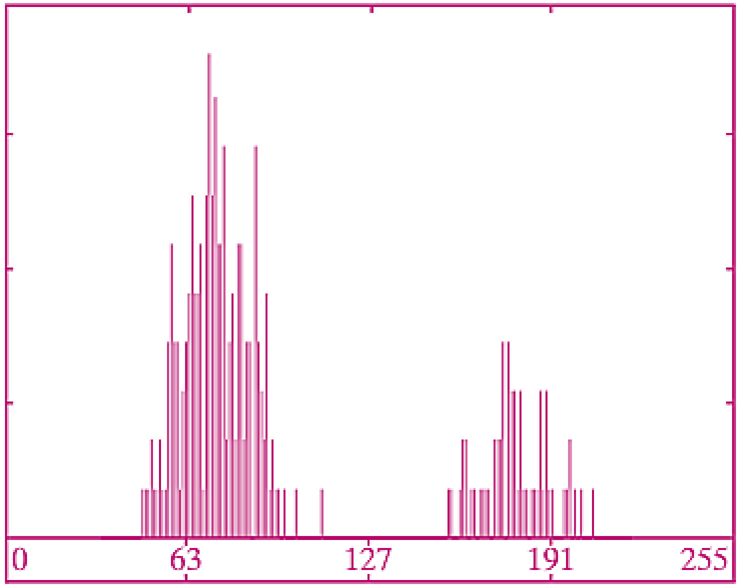
Otsu’s thresholding

- Example
  - small object’s noisy image
  - Otsu’s TH obtained via edge masked image but that TH is applied on the original input image

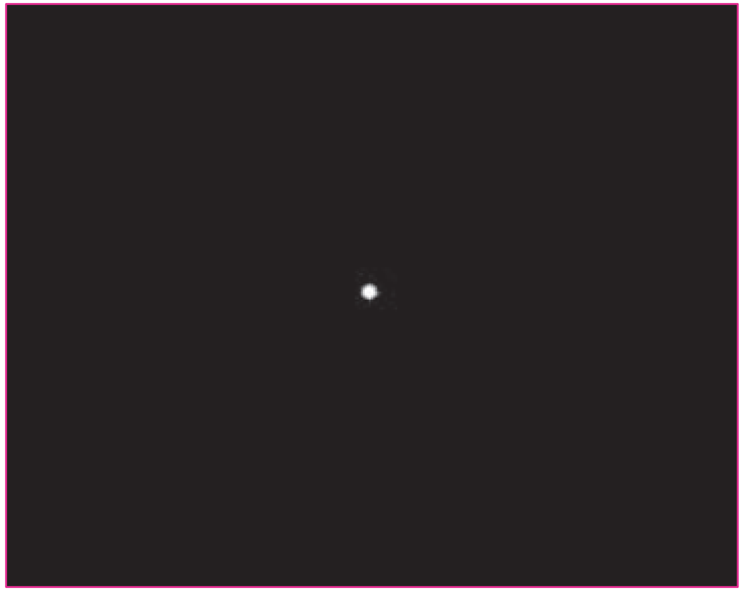
Edge masked input image



Hist



Global: Otsu’s TH



Conclusion

- Segmentation via thresholding (Otsu)

Conclusion

- Segmentation via thresholding (Otsu)

- Global optimal
- Global Otsu's method
  - Input image histogram processing
  - Noise handled via smoothing
  - Small object issues handled via edge masks

Conclusion

- Segmentation via thresholding (Otsu)

❑ Global optimal

❑ Global Otsu's method

- Input image histogram processing
- Noise handled via smoothing
- Small object issues handled via edge masks

A Threshold Selection Method from Gray-Level Histograms

NOBUYUKI OTSU

Abstract—A nonparametric and unsupervised method of automatic threshold selection for picture segmentation is presented. An optimal threshold is selected by the discriminant criterion, namely, so as to maximize the separability of the resultant classes in gray levels. The procedure is very simple, utilizing only the zeroth- and the first-order cumulative moments of the gray-level histogram. It is straightforward to extend the method to multithreshold problems. Several experimental results are also presented to support the validity of the method.

I. INTRODUCTION

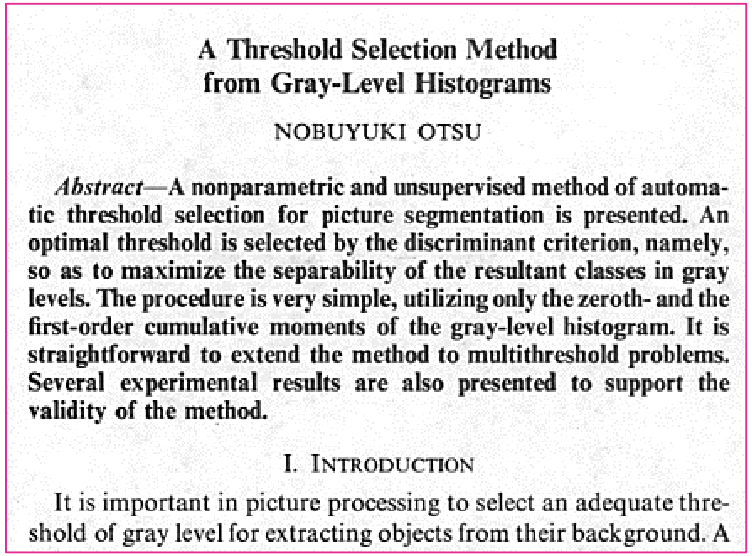
It is important in picture processing to select an adequate threshold of gray level for extracting objects from their background. A

Conclusion

- Segmentation via thresholding (Otsu)

- Global optimal
- Global Otsu's method
  - Input image histogram processing
  - Noise handled via smoothing
  - Small object issues handled via edge masks

Threshold the Otsu's paper via Otsu's method:



A Threshold Selection Method from Gray-Level Histograms

NOBUYUKI OTSU

Abstract—A nonparametric and unsupervised method of automatic threshold selection for picture segmentation is presented. An optimal threshold is selected by the discriminant criterion, namely, so as to maximize the separability of the resultant classes in gray levels. The procedure is very simple, utilizing only the zeroth- and the first-order cumulative moments of the gray-level histogram. It is straightforward to extend the method to multithreshold problems. Several experimental results are also presented to support the validity of the method.

I. Introduction

It is important in picture processing to select an adequate threshold of gray level for extracting objects from their background. A