

# Segmentation:

## Mean-shift & watershed

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Dr. Tushar Sandhan

# Introduction

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- Number of segments?



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# Introduction

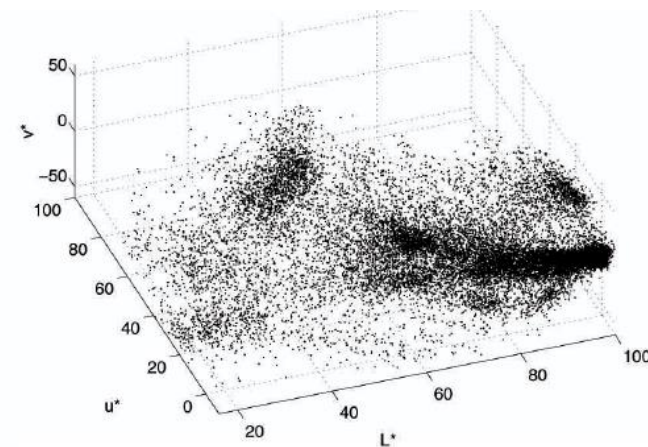
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- Number of segments?



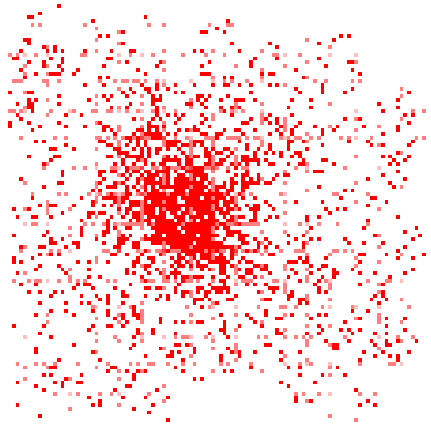
# Mean-shift

- Mean-shift clustering
  - iterative steepest ascent method
  - seeks peaks of probability density in feature space
    - finds modes or local maxima
  - it tries to find all possible cluster centres
  - no need of initial guess of K clusters



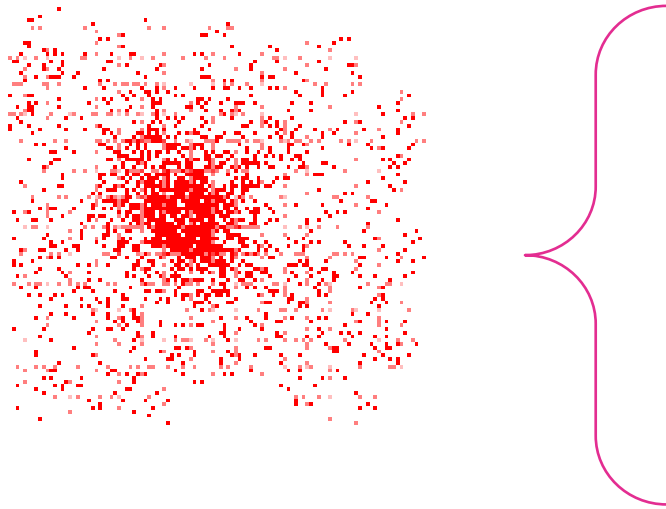
# Mean-shift

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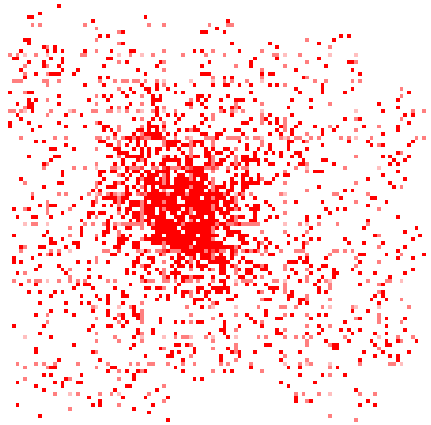
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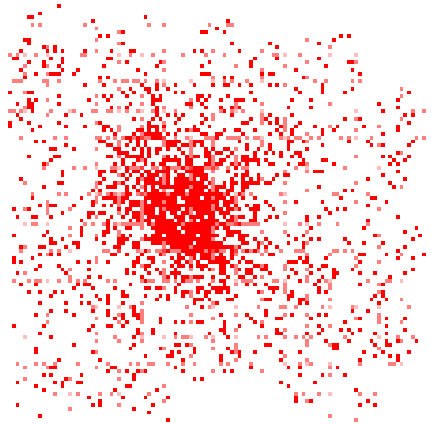


Non-parametric  
density estimation



# Mean-shift

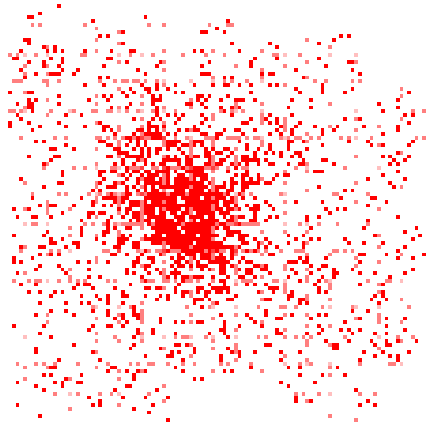
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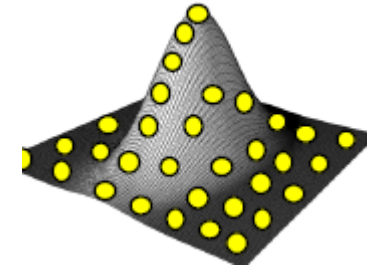
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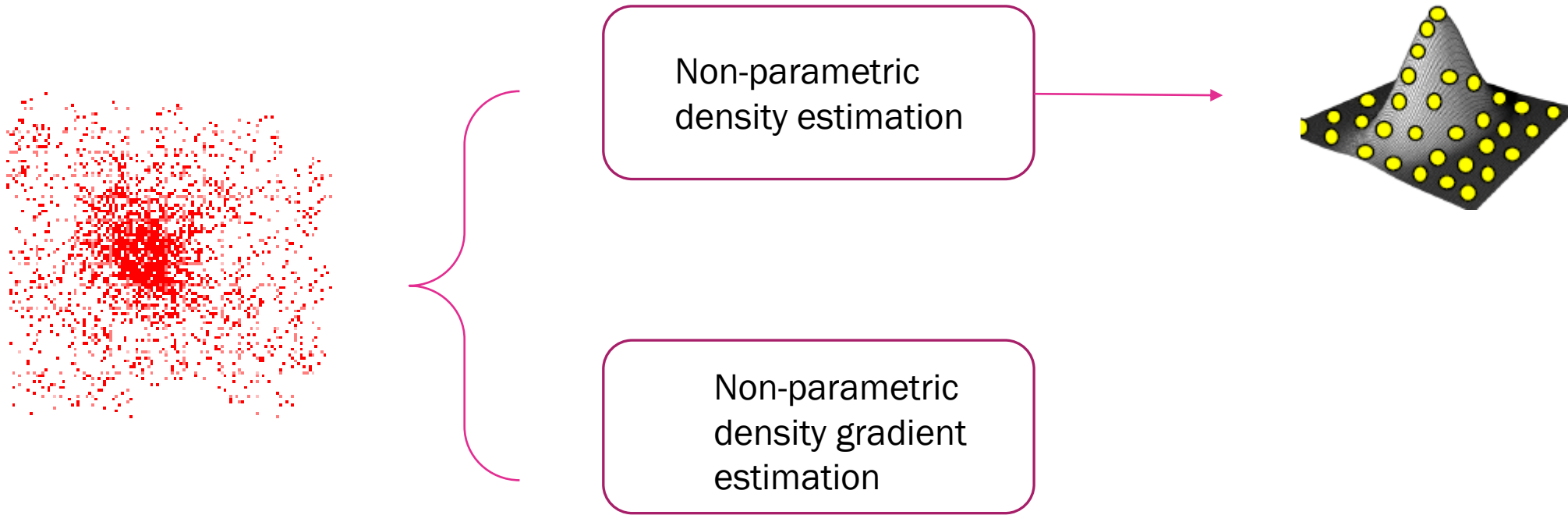


Non-parametric  
density estimation



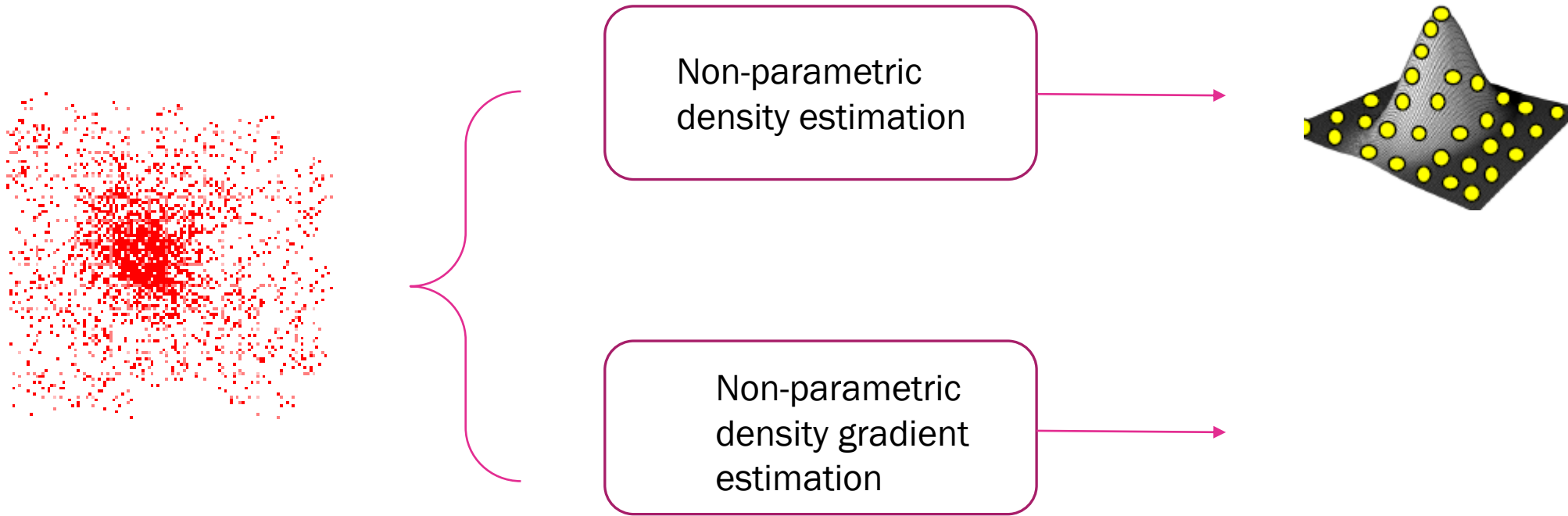
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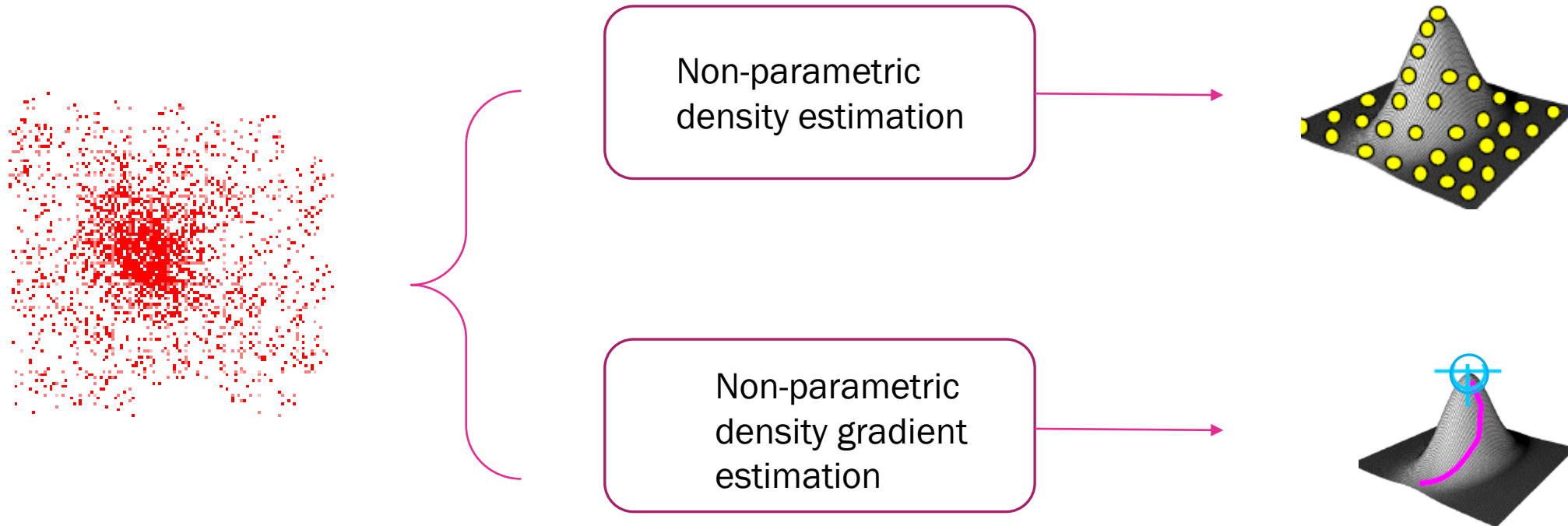


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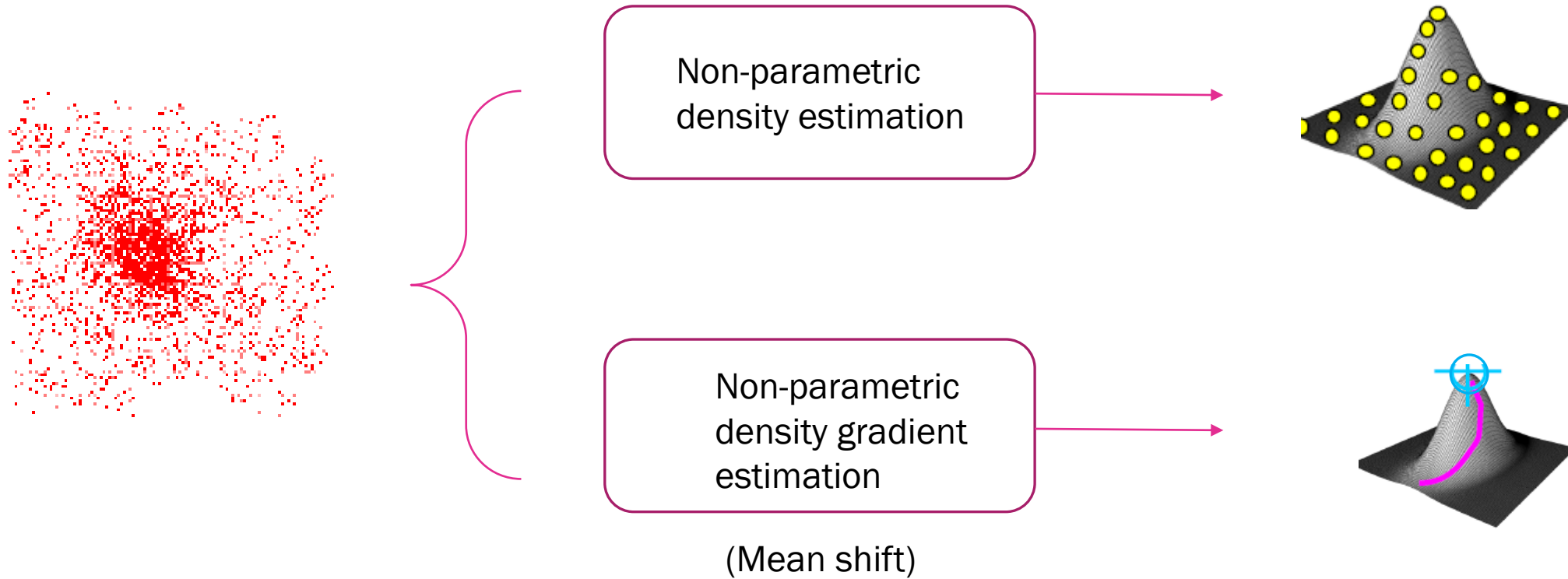
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# Mean-shift

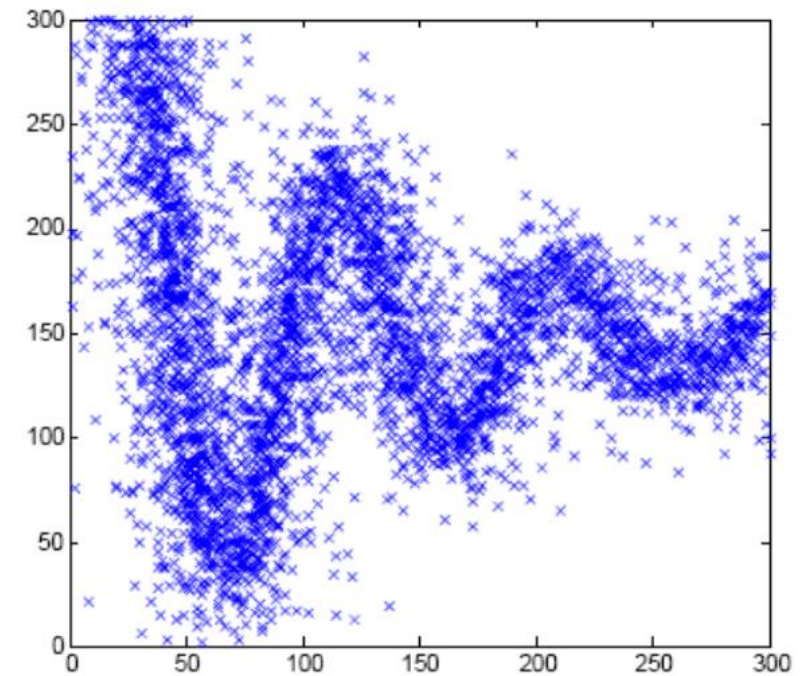
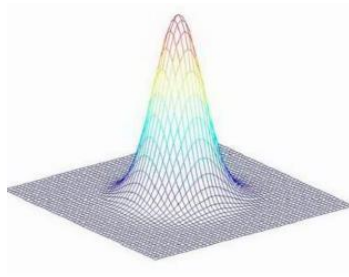


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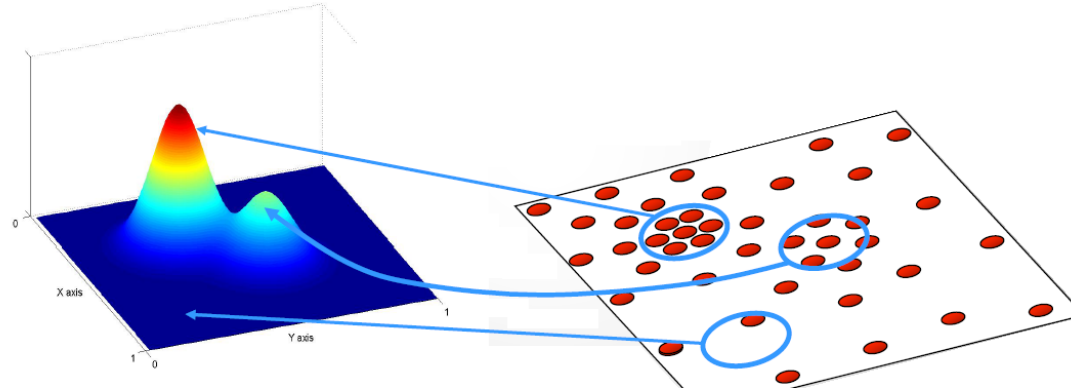


# Mean-shift

- Density estimation
  - find the underlying distribution that generates the given data
- Non-parametric density estimation
  - use data points to define distribution
  - put a small probability mass around each data-point via kernel
  - e.g. Gaussian kernel



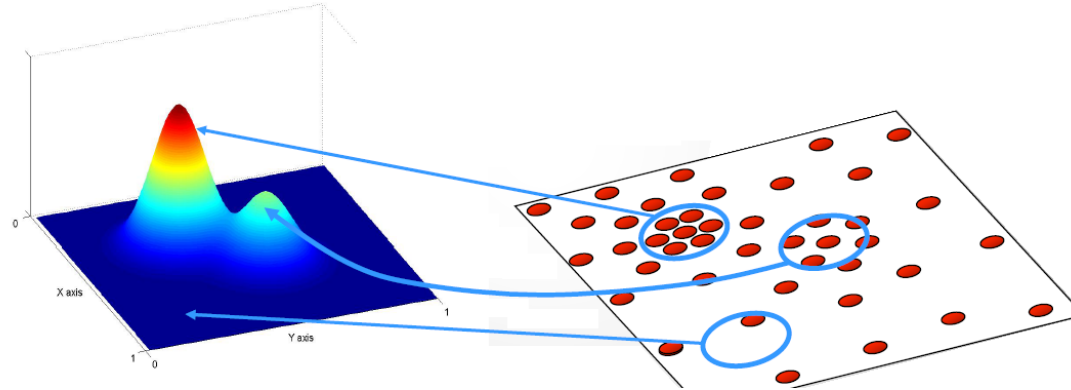
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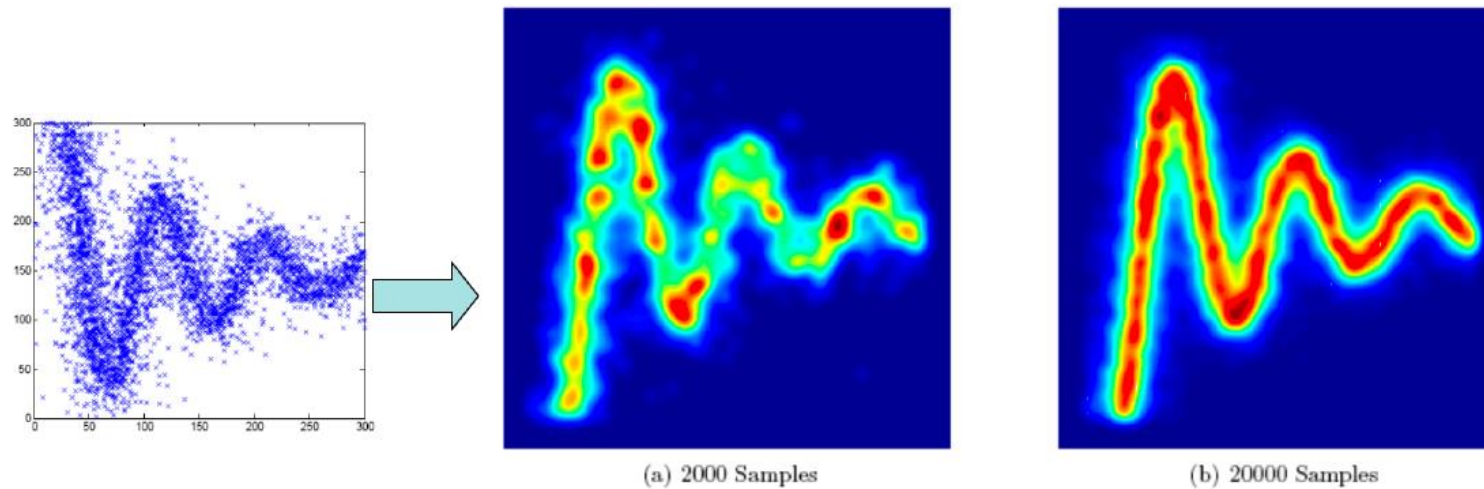
Data point density is similar to PDF value



# Mean-shift



Data point density is similar to PDF value



Courtesy: T. Tappen

# Mean-shift

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$$x_1 = x_0 + \eta \nabla f(x_0)$$

$$\hat{f}(\mathbf{x}) = \frac{1}{nh^d} \sum_{i=1}^n K\left(\frac{\mathbf{x} - \mathbf{x}_i}{h}\right)$$

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$c_{k,d}$  is volume of unit sphere  $\in R^d$

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Define a function of a kernel  $g(x) = -k'(x)$

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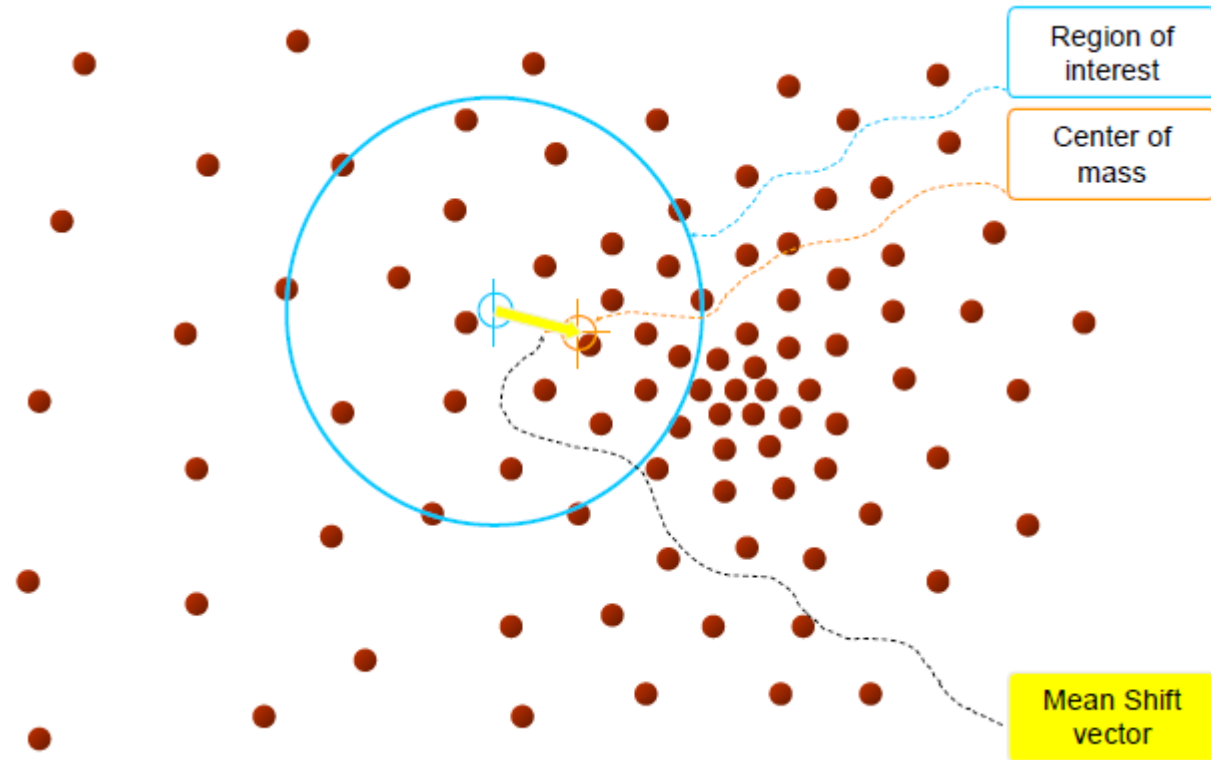
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$$\hat{\nabla} f_{h,K}(\mathbf{x}) = \hat{f}_{h,G}(\mathbf{x}) \frac{2c_{k,d}}{h^2 c_{g,d}} \mathbf{m}_{h,G}(\mathbf{x}) \quad \mathbf{m}_{h,G}(\mathbf{x}) = \frac{1}{2} h^2 c \frac{\hat{\nabla} f_{h,K}(\mathbf{x})}{\hat{f}_{h,G}(\mathbf{x})}$$

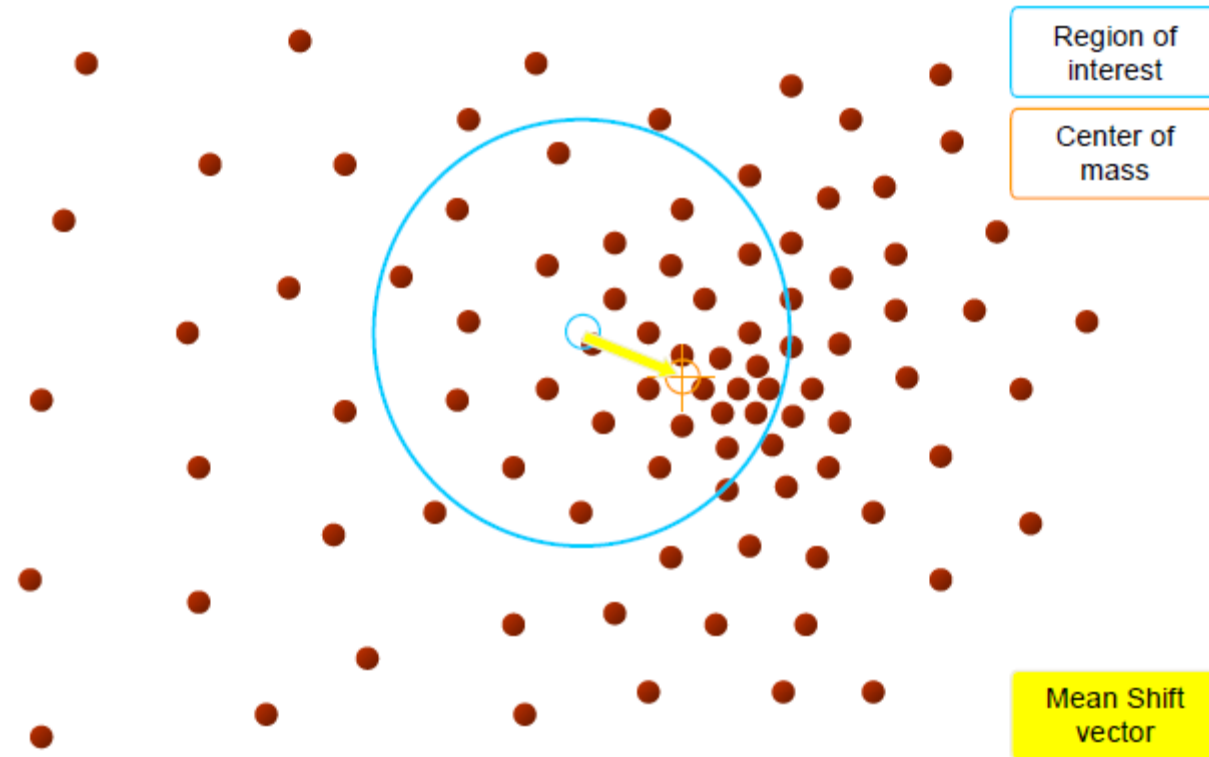
# Mean-shift

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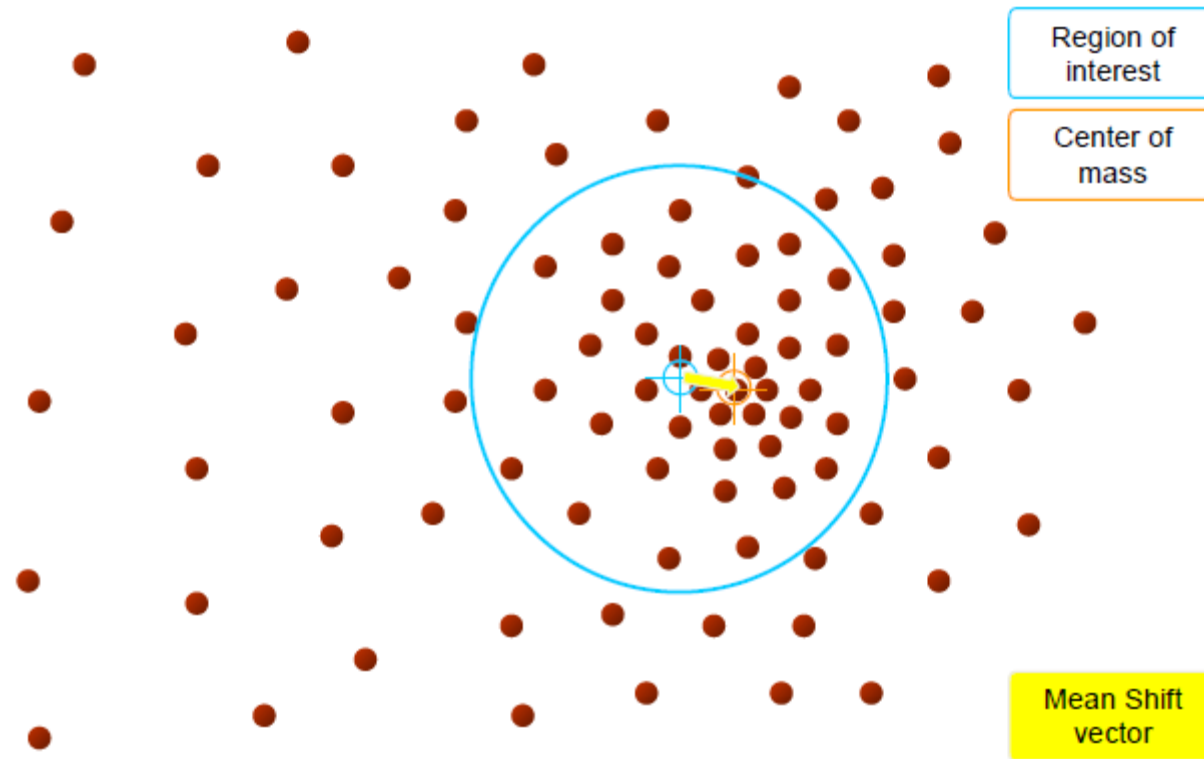
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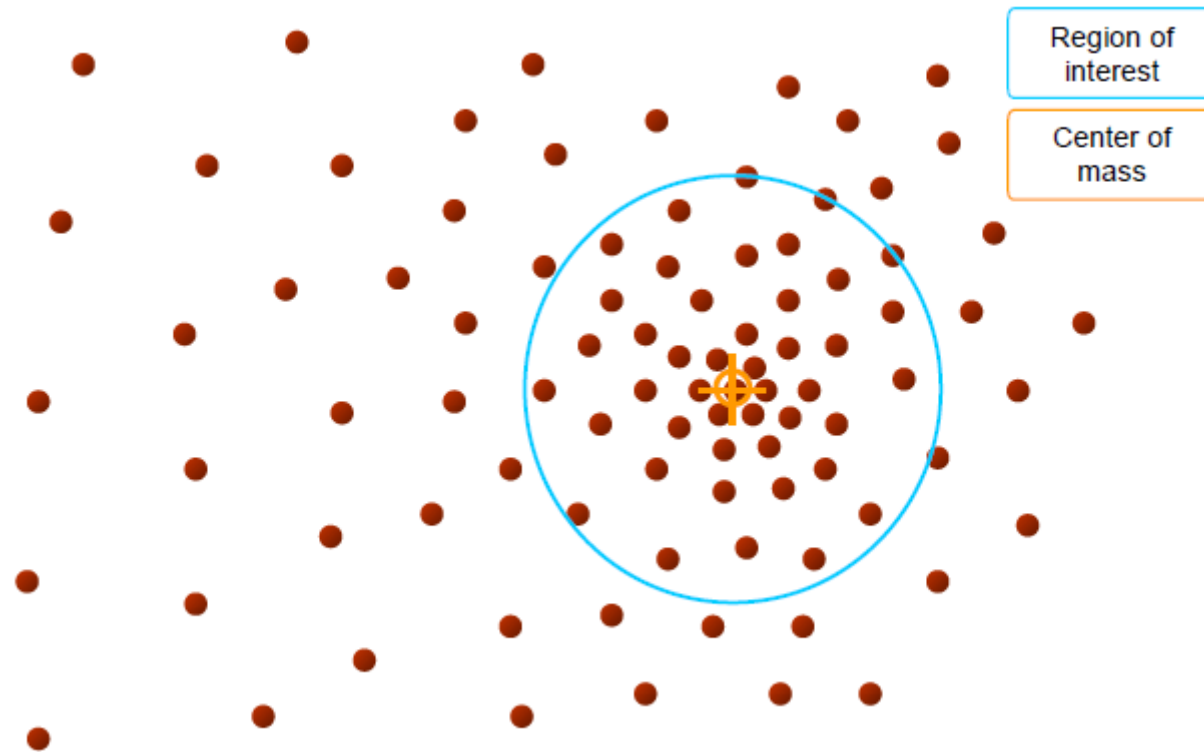
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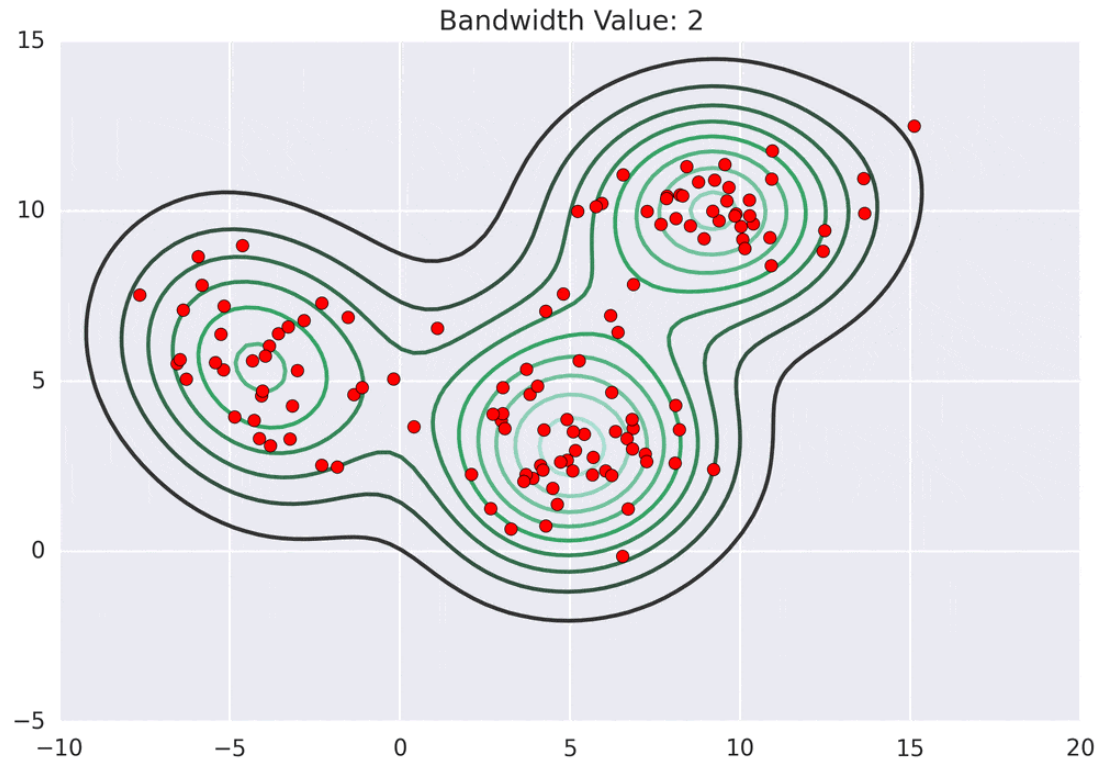
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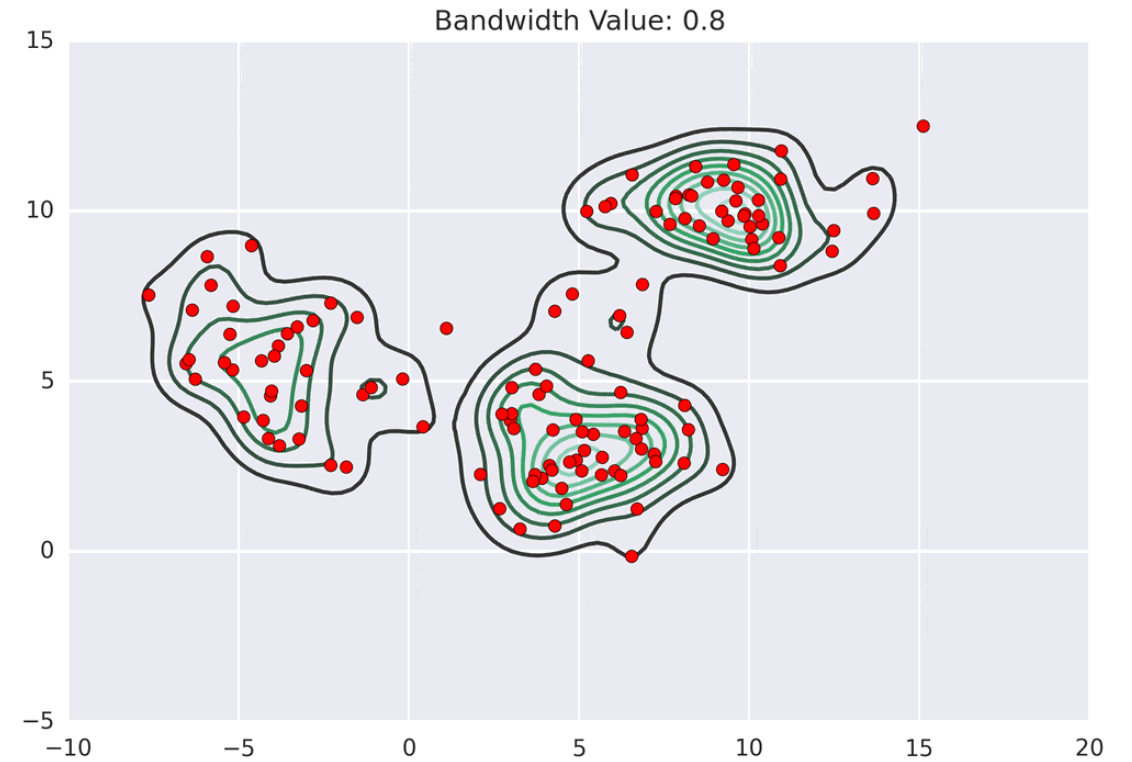
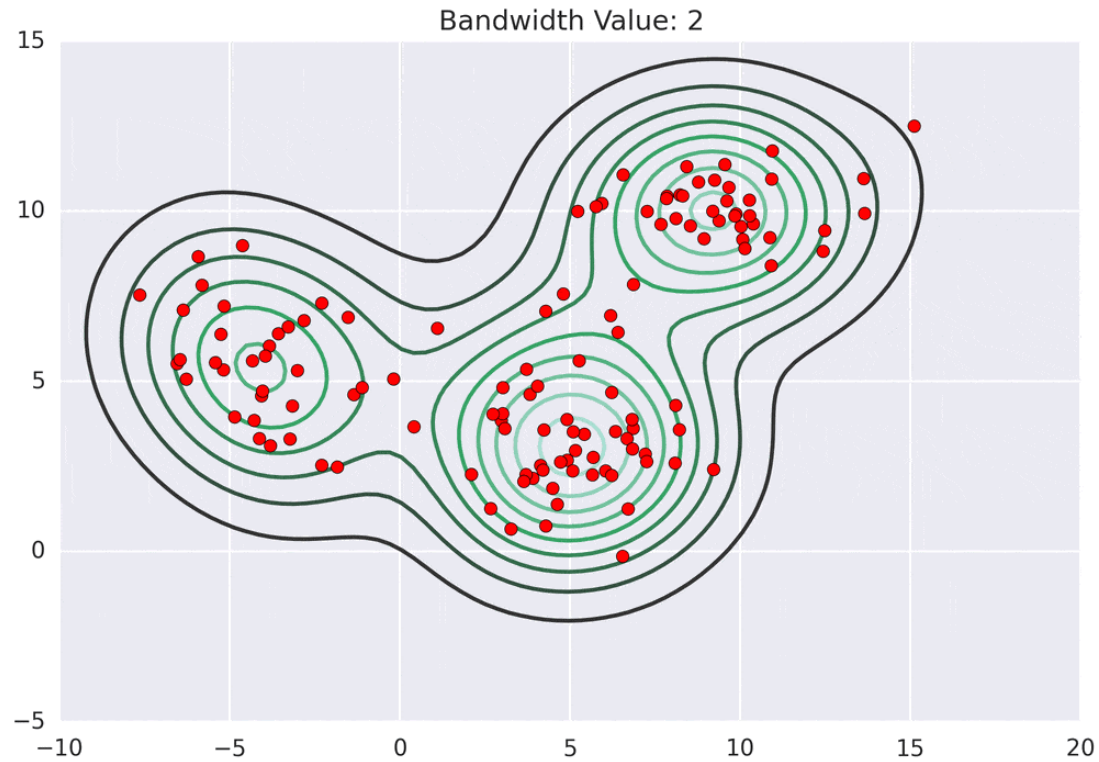


# Mean-shift

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# Mean-shift





# Mean-shift

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- Algorithm

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- transform image into feature space
  - initialize window at each feature point
  - for each window
    - compute mean shift vector  $m(x)$
    - move density estimation window by  $m(x)$
    - repeat till convergence
  - merge windows that end up near same peak
-

# Mean-shift

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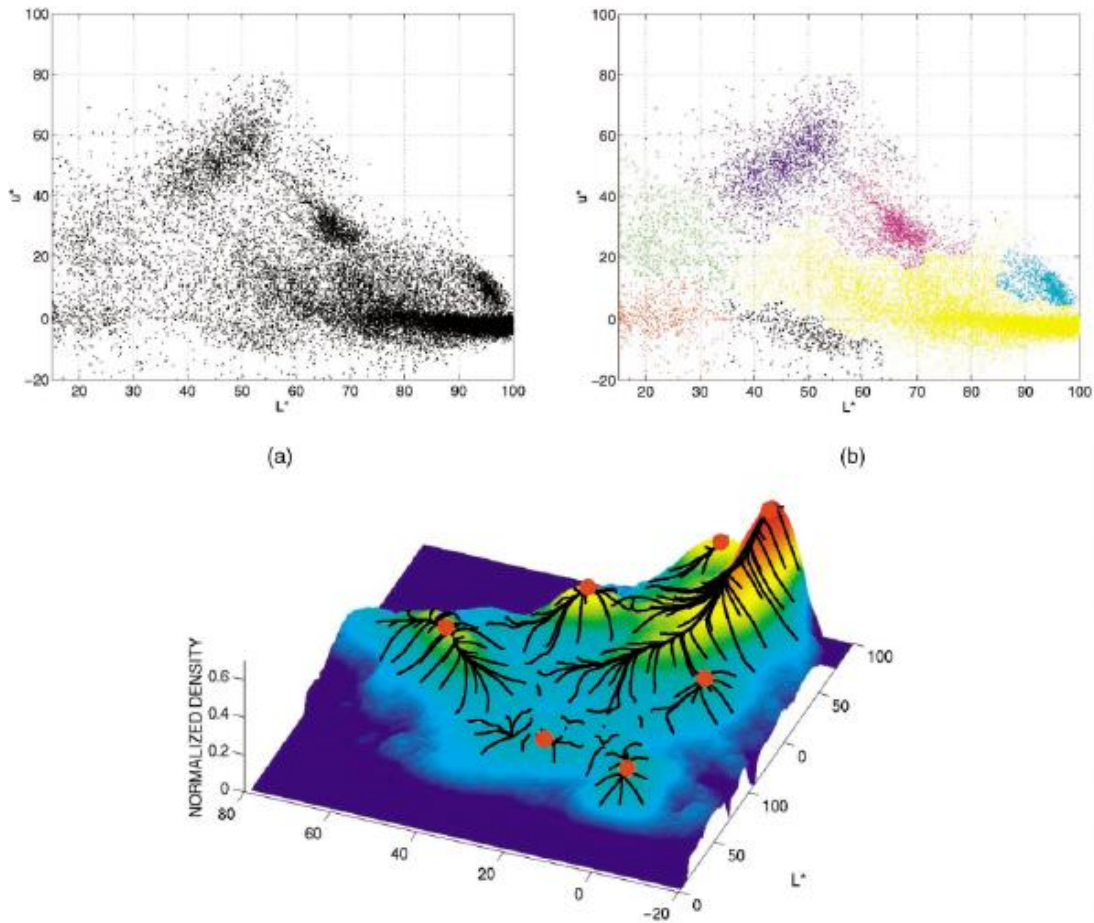
## ■ Pros

- automatically finds various number of modes
- no need of initial guessing of cluster centres
- does not assume spherical clusters
- robust to outliers
- just a single para (window size  $w$ )

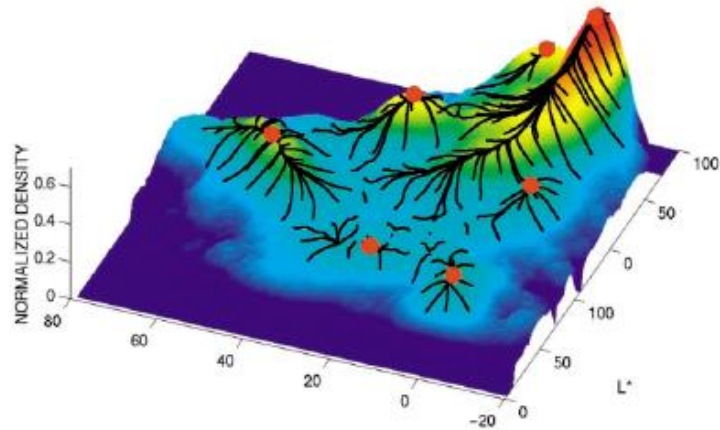
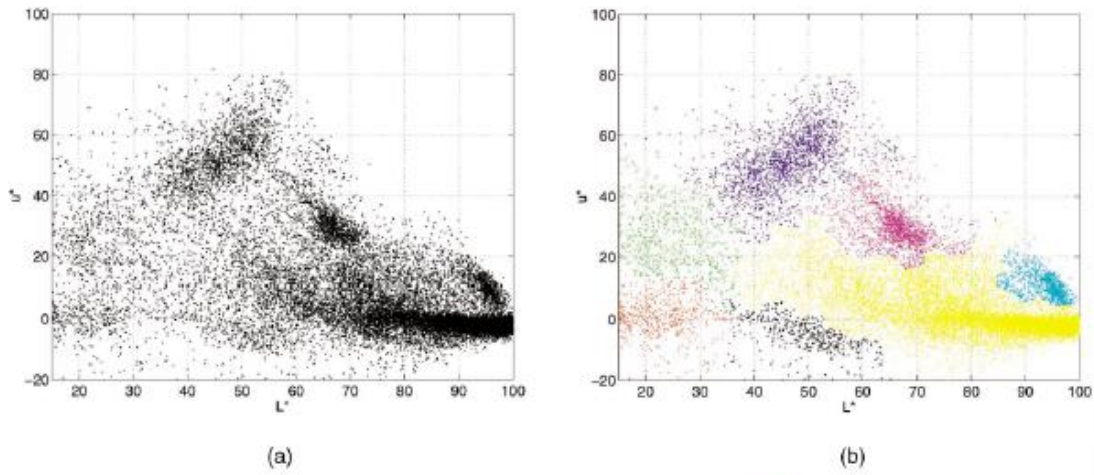
## ■ Cons

- bandwidth or windows size is an imp. Para
  - slight change in  $w$ , translates varied output
- computationally expensive
  - complexity:  $O(n^2T)$
- not scalable with dimensionality

# Mean-shift



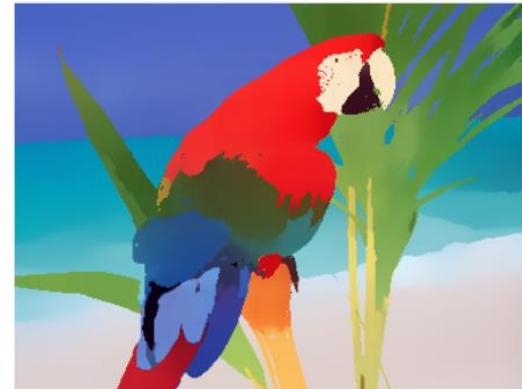
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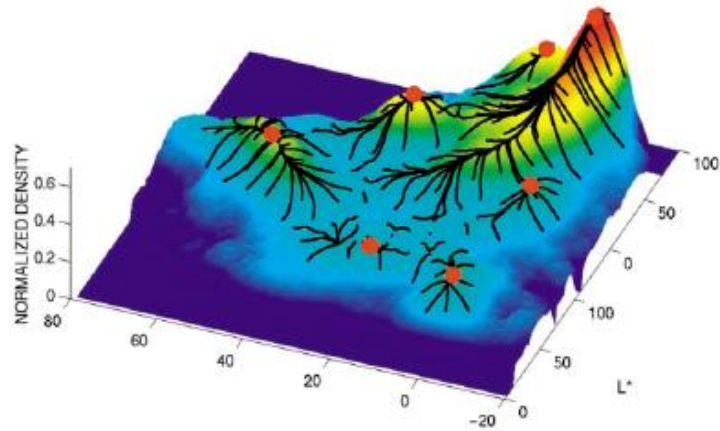
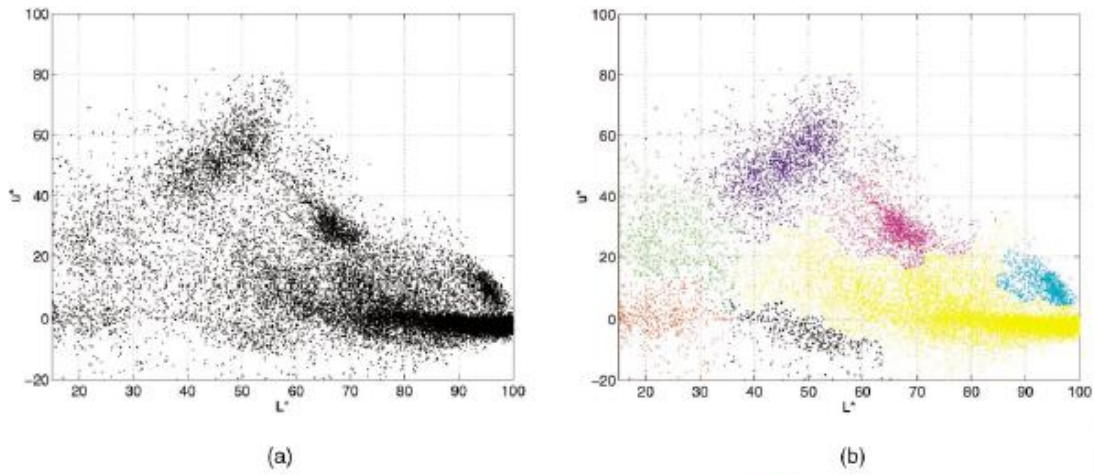
input image



output image



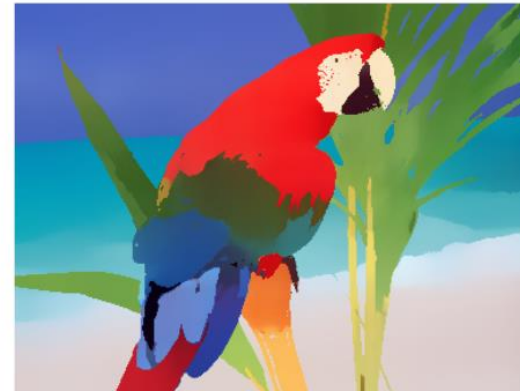
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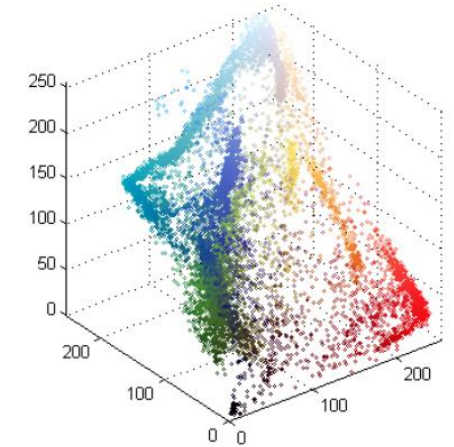
input image



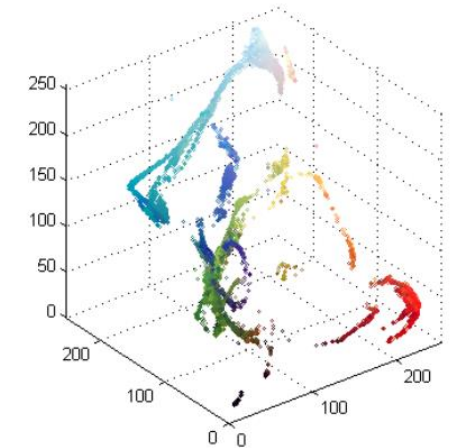
output image



Pixel Distribution Before Meanshift

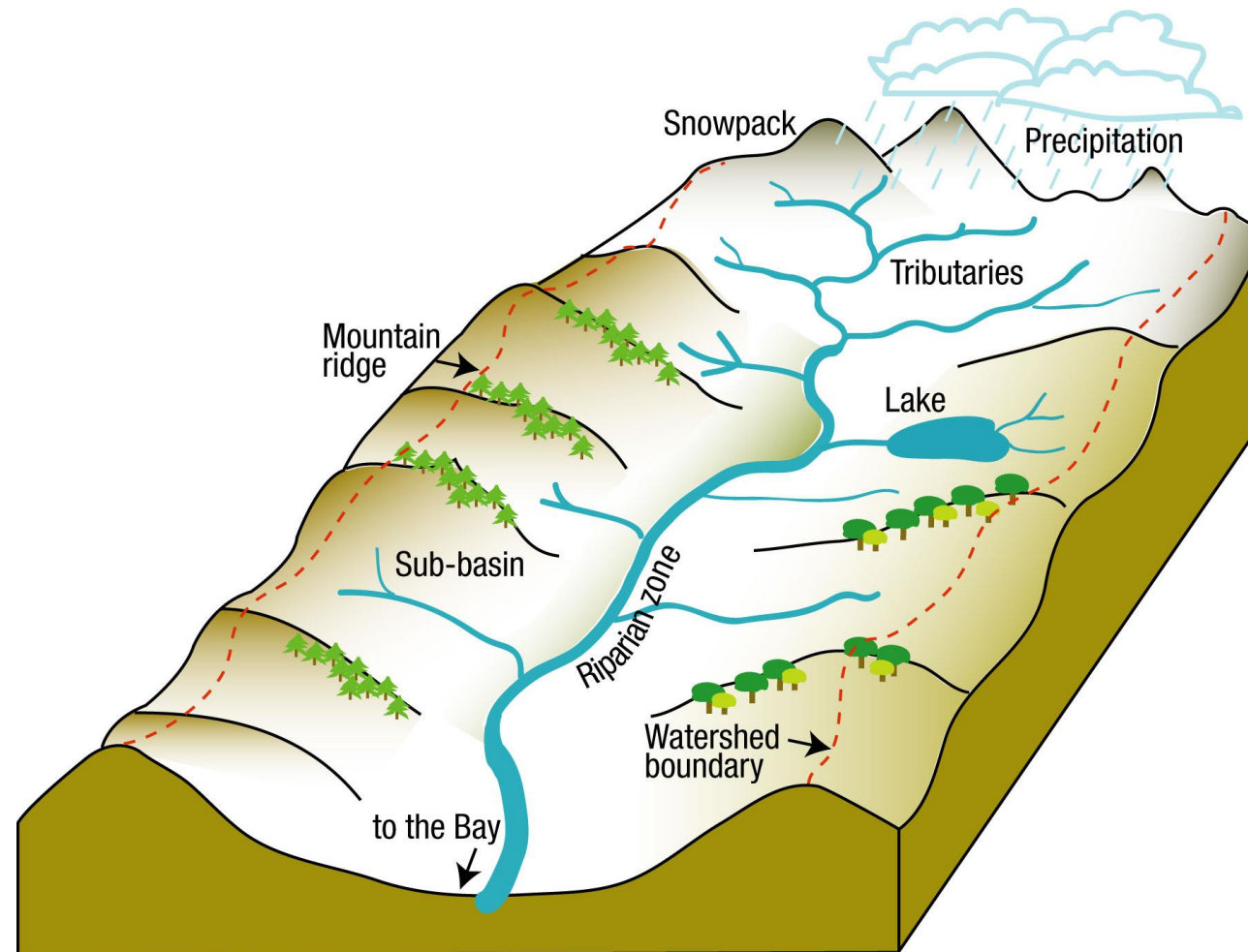


Pixel Distribution After Meanshift



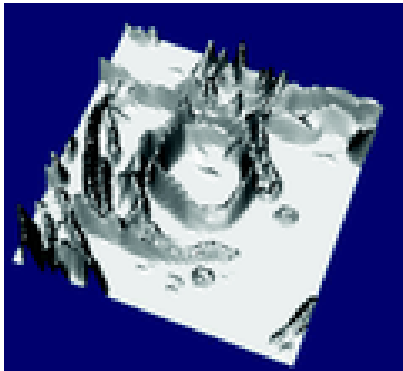


# Watershed

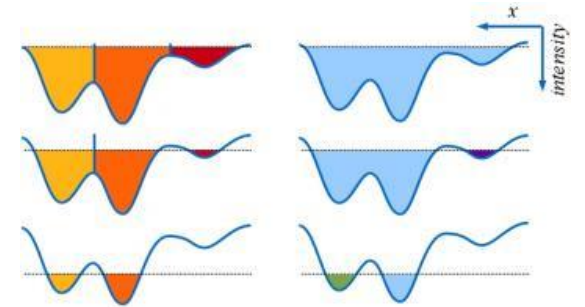


# Watershed

- Watershed segmentation
  - considers an image as a topological surface
  - pixel intensities are heights (lowest level 0, highest peak 255)
  - segmenting overlapping objects
  - consider each waterbody as a separate object



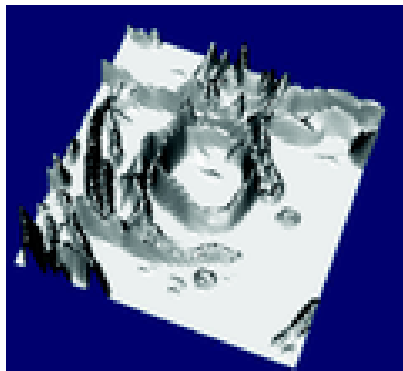
input



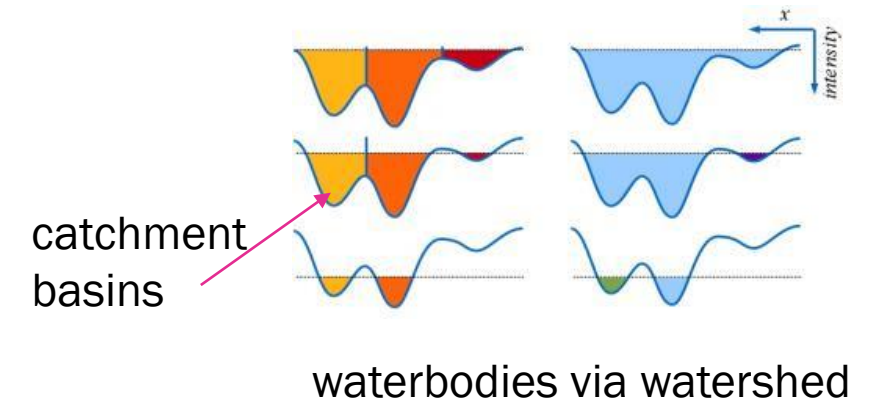
waterbodies via watershed

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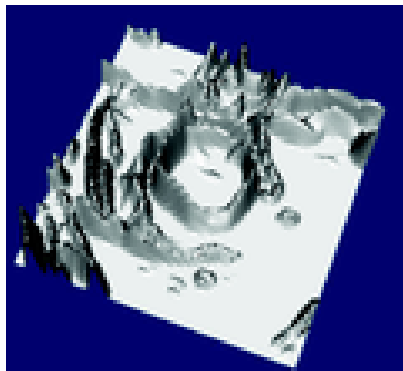
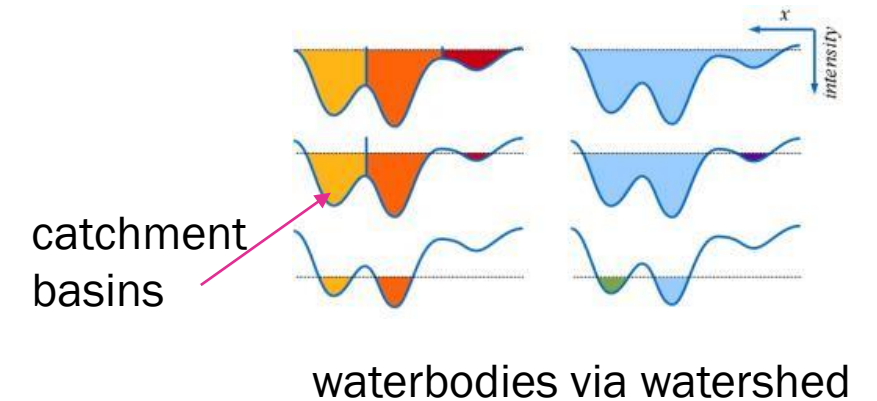
input



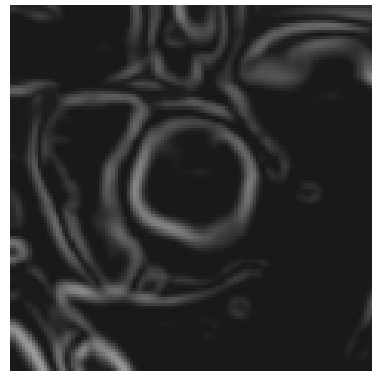


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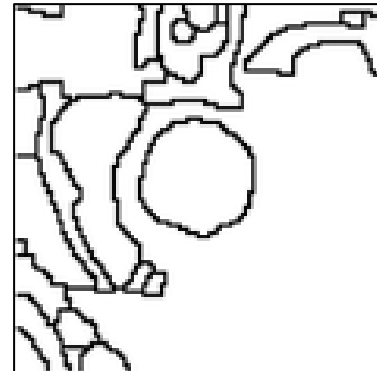
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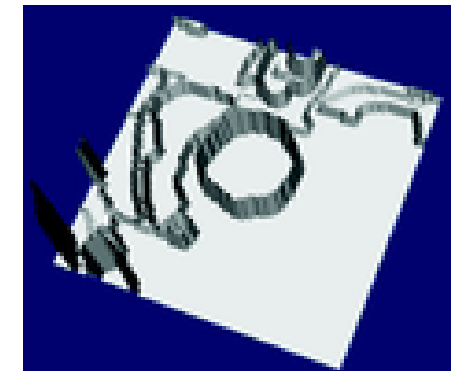
input



gradient



watershed 2D

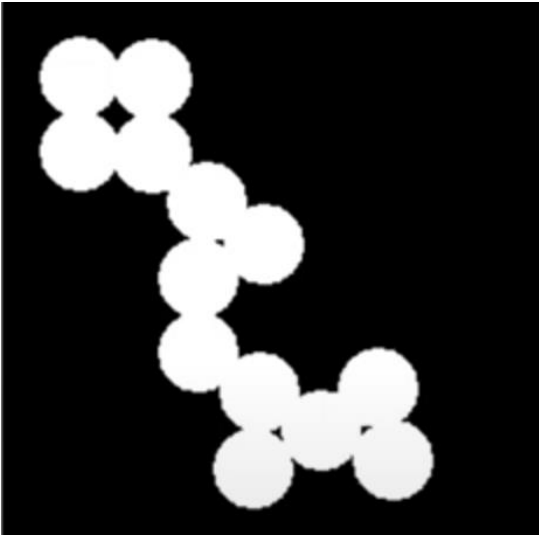


watershed 3D

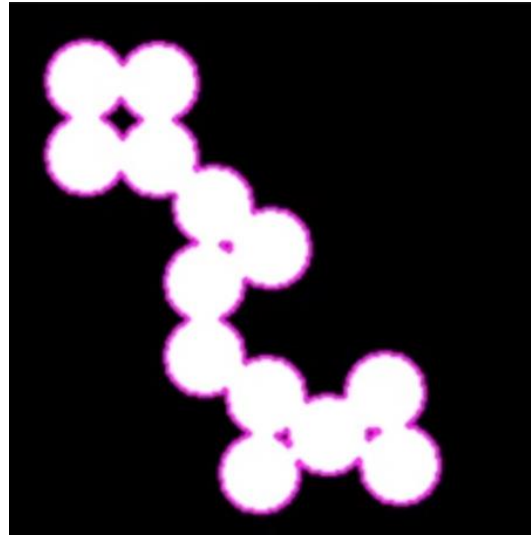
# Watershed

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- Watershed segmentation
  - overlapping objects



input

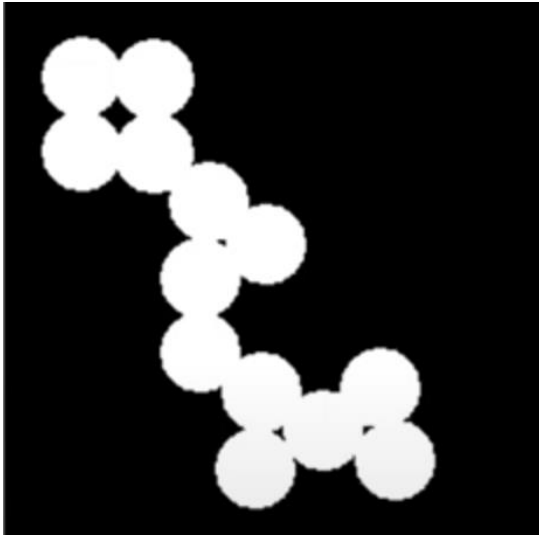


thresholding or clustering

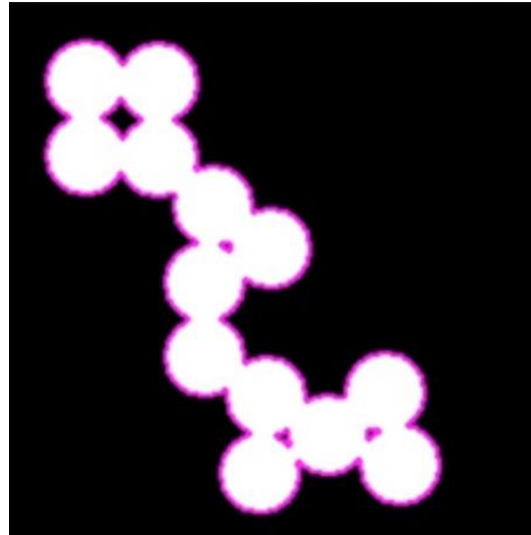
# Watershed

- Watershed segmentation

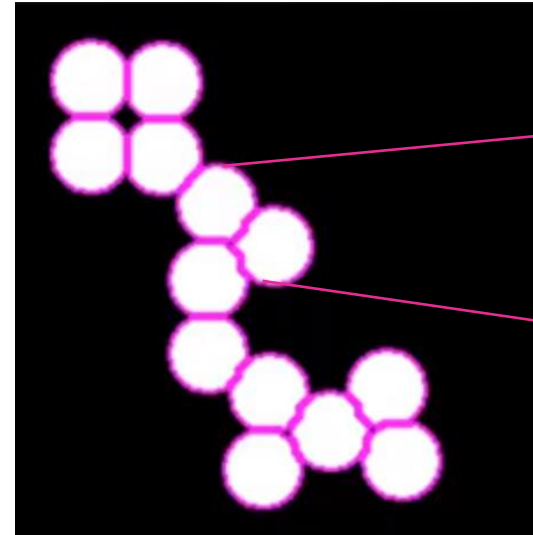
- overlapping objects



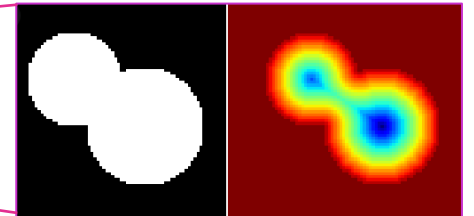
input



thresholding or clustering



watershed



# Watershed

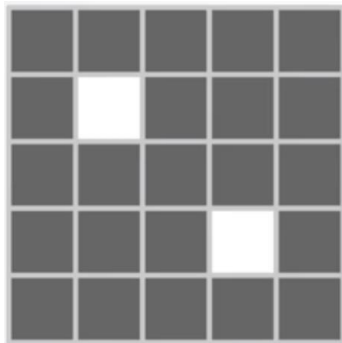
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- Distance transform
  - computes the distance between each pix and nearest nonzero pix

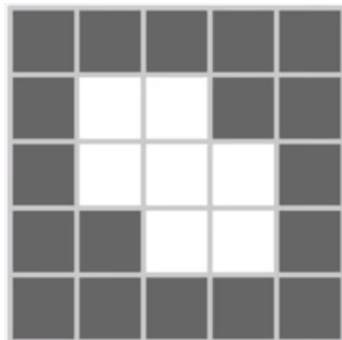


# Watershed

- Distance transform
  - computes the distance between each pix and nearest nonzero pix



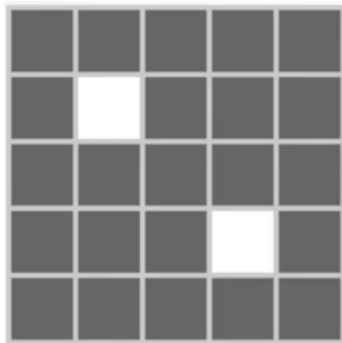
1.41	1	1.41	2.24	3.16
1	0	1	2	2.24
1.41	1	1.41	1	1.41
2.24	2	1	0	1
3.16	2.24	1.41	1	1.41



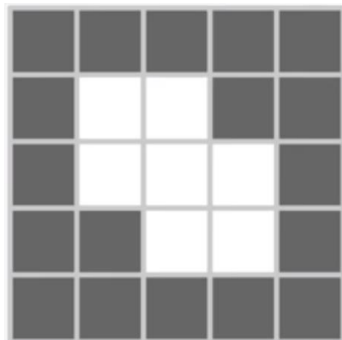
1.41	1	1	1.41	2.24
1	0	0	1	1.41
1	0	0	0	1
1.41	1	0	0	1
2.24	1.41	1	1	1.41

# Watershed

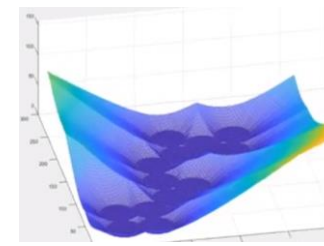
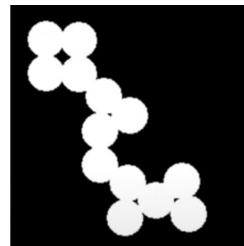
- Distance transform
  - computes the distance between each pix and nearest nonzero pix



1.41	1	1.41	2.24	3.16
1	0	1	2	2.24
1.41	1	1.41	1	1.41
2.24	2	1	0	1
3.16	2.24	1.41	1	1.41

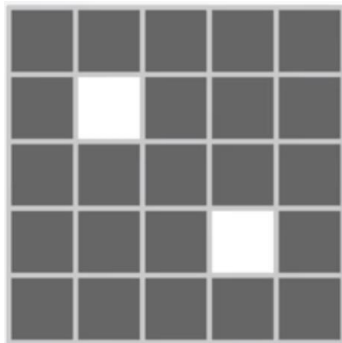


1.41	1	1	1.41	2.24
1	0	0	1	1.41
1	0	0	0	1
1.41	1	0	0	1
2.24	1.41	1	1	1.41

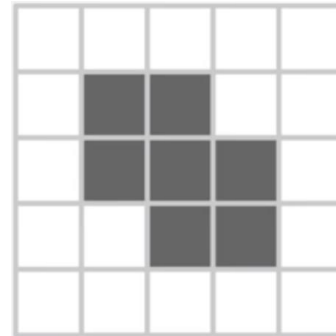


# Watershed

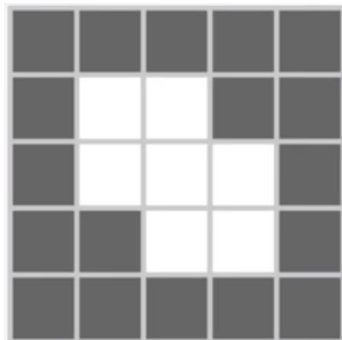
- Distance transform
  - computes the distance between each pix and nearest nonzero pix



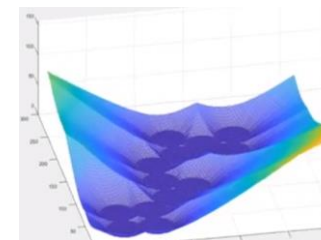
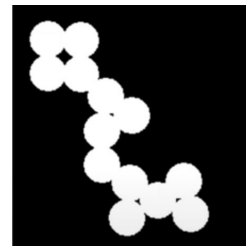
1.41	1	1.41	2.24	3.16
1	0	1	2	2.24
1.41	1	1.41	1	1.41
2.24	2	1	0	1
3.16	2.24	1.41	1	1.41



0	0	0	0	0
0	1	1	0	0
0	1	1.41	1	0
0	0	1	1	0
0	0	0	0	0



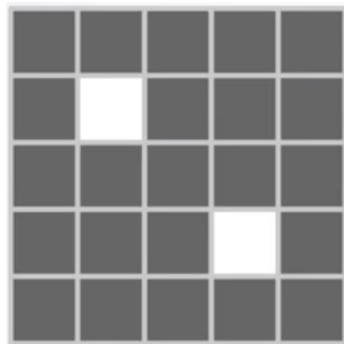
1.41	1	1	1.41	2.24
1	0	0	1	1.41
1	0	0	0	1
1.41	1	0	0	1
2.24	1.41	1	1	1.41



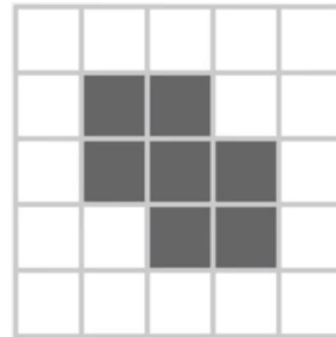
# Watershed

- Distance transform

- computes the distance between each pix and nearest nonzero pix

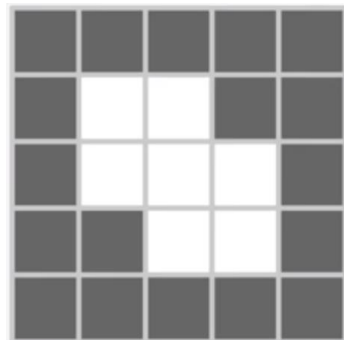


1.41	1	1.41	2.24	3.16
1	0	1	2	2.24
1.41	1	1.41	1	1.41
2.24	2	1	0	1
3.16	2.24	1.41	1	1.41

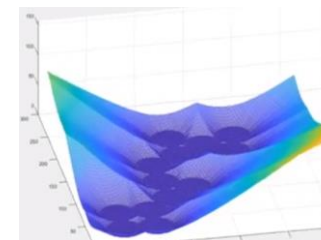
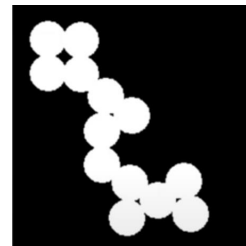


0	0	0	0	0
0	1	1	0	0
0	1	1.41	1	0
0	0	1	1	0
0	0	0	0	0

0	0	0	0	0
0	-1	-1	0	0
0	-1	-1.41	-1	0
0	0	-1	-1	0
0	0	0	0	0



1.41	1	1	1.41	2.24
1	0	0	1	1.41
1	0	0	0	1
1.41	1	0	0	1
2.24	1.41	1	1	1.41

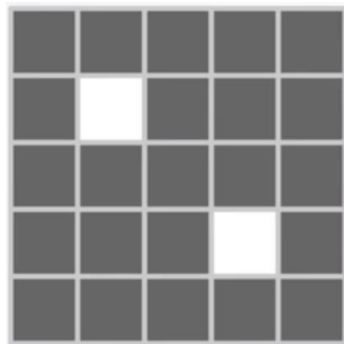




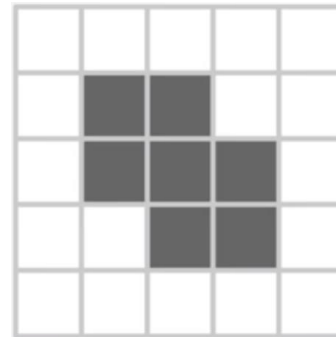
# Watershed

- Distance transform

- computes the distance between each pix and nearest nonzero pix

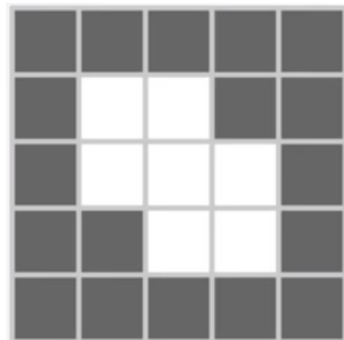


1.41	1	1.41	2.24	3.16
1	0	1	2	2.24
1.41	1	1.41	1	1.41
2.24	2	1	0	1
3.16	2.24	1.41	1	1.41

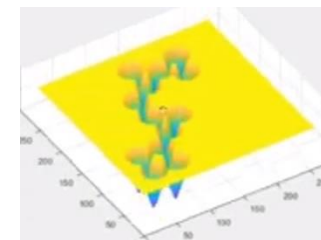
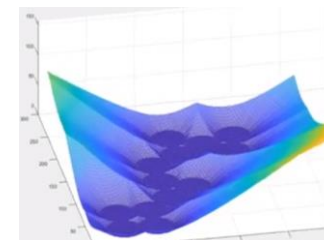
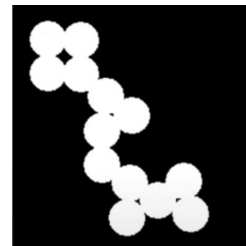


0	0	0	0	0
0	1	1	0	0
0	1	1.41	1	0
0	0	1	1	0
0	0	0	0	0

0	0	0	0	0
0	-1	-1	0	0
0	-1	-1.41	-1	0
0	0	-1	-1	0
0	0	0	0	0



1.41	1	1	1.41	2.24
1	0	0	1	1.41
1	0	0	0	1
1.41	1	0	0	1
2.24	1.41	1	1	1.41



# Watershed

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- Algorithm: outline

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- Requires:
    - each obj must be a basin, coinciding basin's bottom with approx obj center
  - Input: image  $I$
  - convert  $I$  into inverted grey:  $\hat{I} = L - \text{grey}(I)$
  - compute the negative distance transform of  $\hat{I}$
  - non-max suppression over shallow minima
  - fill basins & get watersheds
  - update each segmented mask with watersheds
-

# Watershed

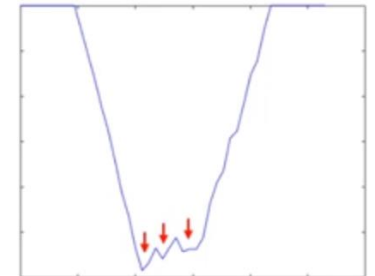
---

- Algorithm: outline

---

- Requires:
    - each obj must be a basin, coinciding basin's bottom with approx obj center
  - Input: image  $I$
  - convert  $I$  into inverted grey:  $\hat{I} = L - \text{grey}(I)$
  - compute the negative distance transform of  $\hat{I}$
  - non-max suppression over shallow minima
  - fill basins & get watersheds
  - update each segmented mask with watersheds
- 

why non-max suppression?



# Watershed

- Algorithm: getting watersheds

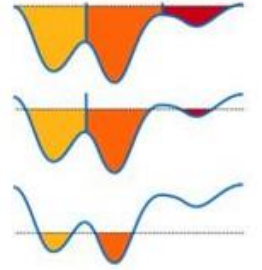
- Initialize:

- A *grey(I)* having:  $[h_{min} \ h_{max}]$
- Minima points:  $M_1, \dots, M_R$
- Thresholded set:  $T_h = \{p \in I | I(p) \leq h\}$ , where  $p$  is an pixel in  $I$  and  $h$  is some intensity level.

- Let's define Influence set

$C(M_i)$  = cluster associated with  $M_i$

$$IZ_{h+1}(M_i) = \{p \in T_{h+1} | d(p, C(M_i)) < d(p, C(M_j))\}$$
$$\forall j, i \neq j$$



# Watershed

## ■ Algorithm: getting watersheds

### ○ Initialize:

- A *grey(I)* having:  $[h_{min} \ h_{max}]$
- Minima points:  $M_1, \dots, M_R$
- Thresholded set:  $T_h = \{p \in I | I(p) \leq h\}$ , where  $p$  is an pixel in  $I$  and  $h$  is some intensity level.

### • Let's define Influence set

$C(M_i)$  = cluster associated with  $M_i$

$$IZ_{h+1}(M_i) = \{p \in T_{h+1} | d(p, C(M_i)) < d(p, C(M_j))\}$$

$$\forall j, i \neq j$$

### ○ Run:

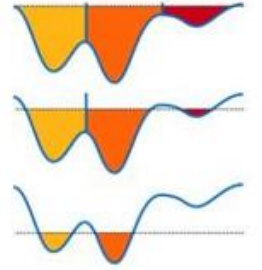
- $h = h_{min}$
- Immersed set:  $X_h = X_{h_{min}} = T_{h_{min}}$   
 $= \{p \in I | I(p) \leq h_{min}\}$

### • Loop until $h_{max}$

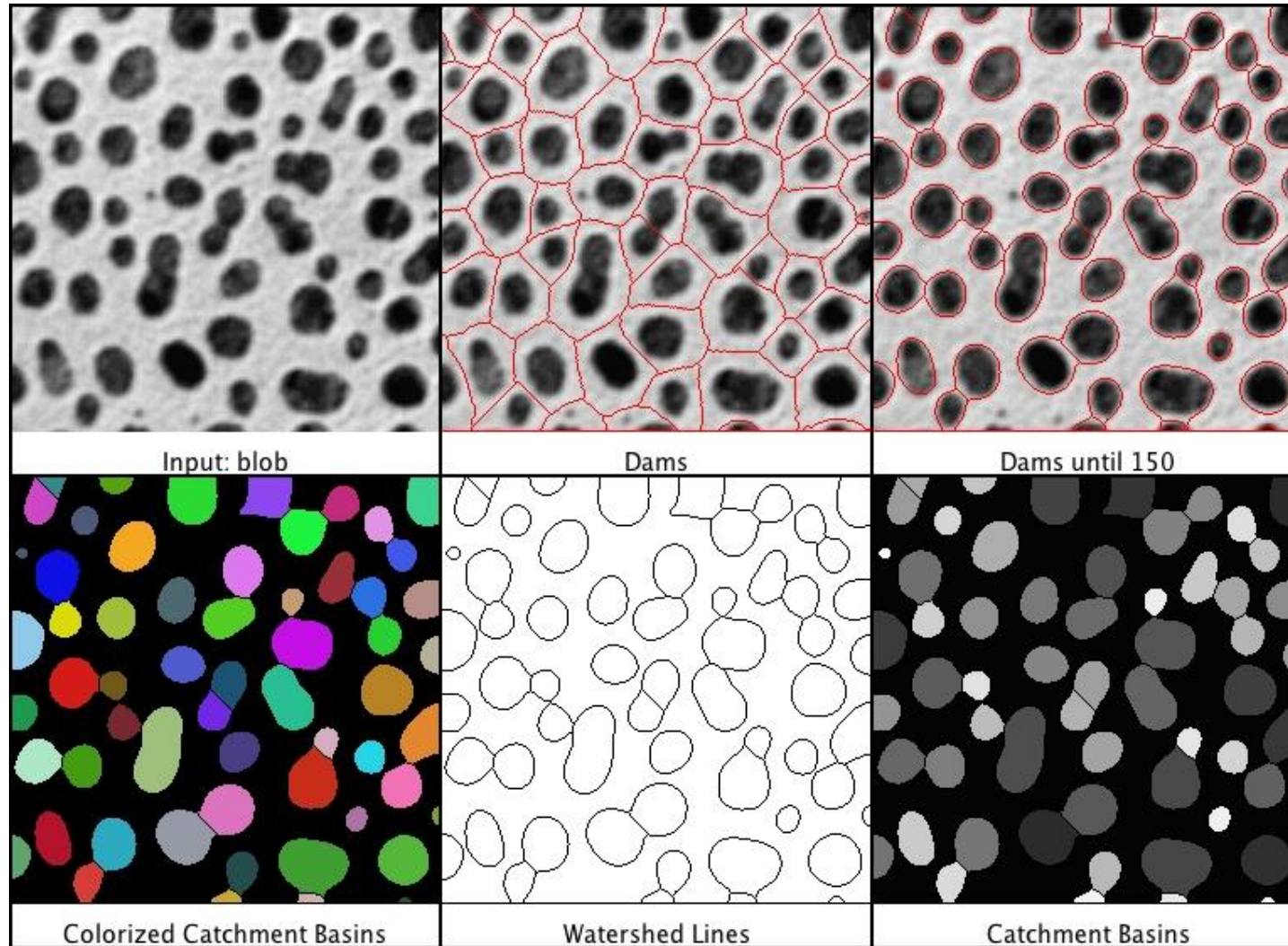
$$X_{h+1} = X_h \cup IZ_{h+1}(M_1) \dots \cup IZ_{h+1}(M_R)$$

↑  
Influence set of minima  $M_1$  at level  $h+1$

- $Watershed(I)$  = Set of all pixels in  $I \setminus X_{h_{max}}$



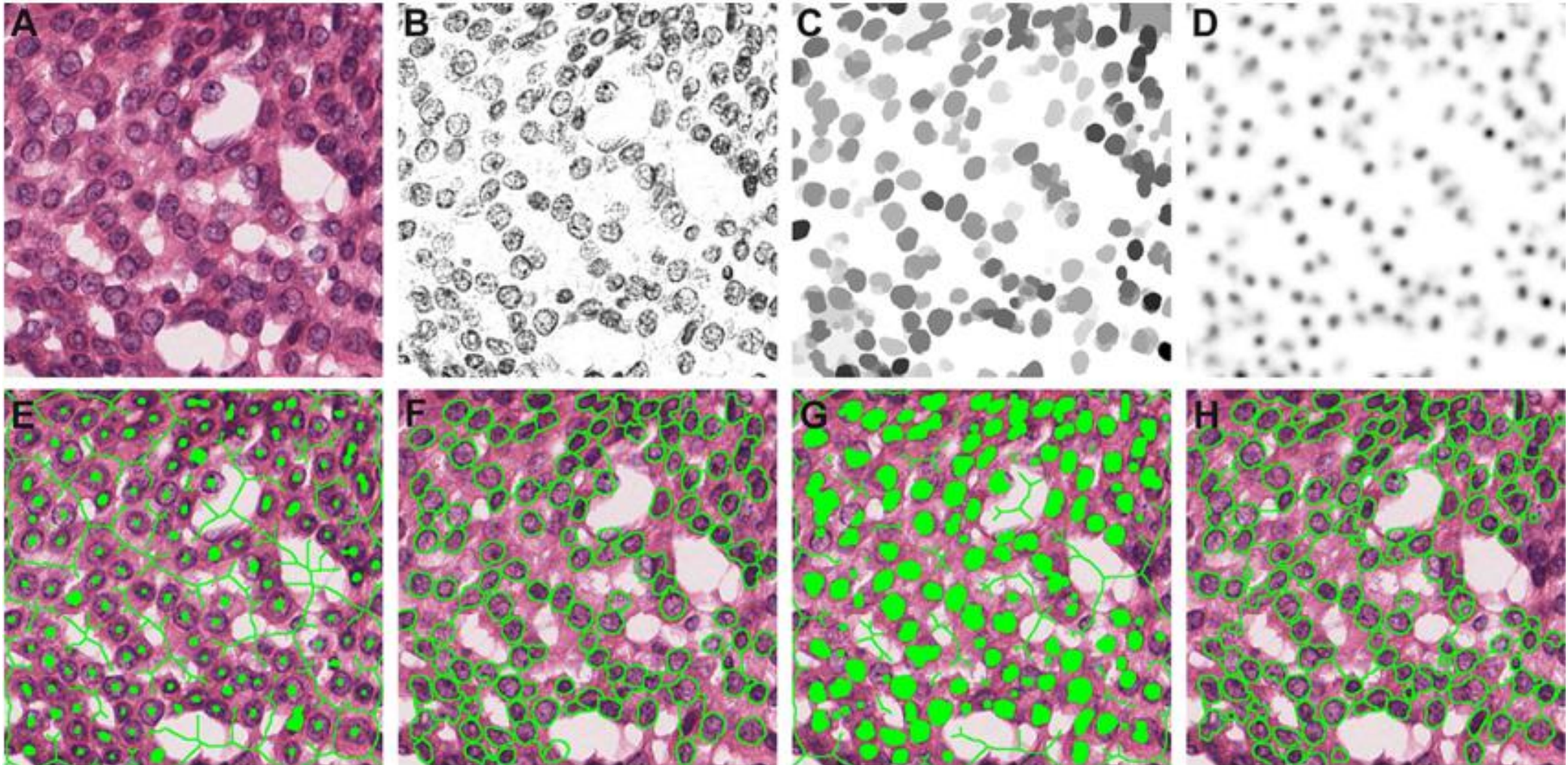
# Watershed



courtesy: EPFL



# Watershed



courtesy: M Veta

# Watershed

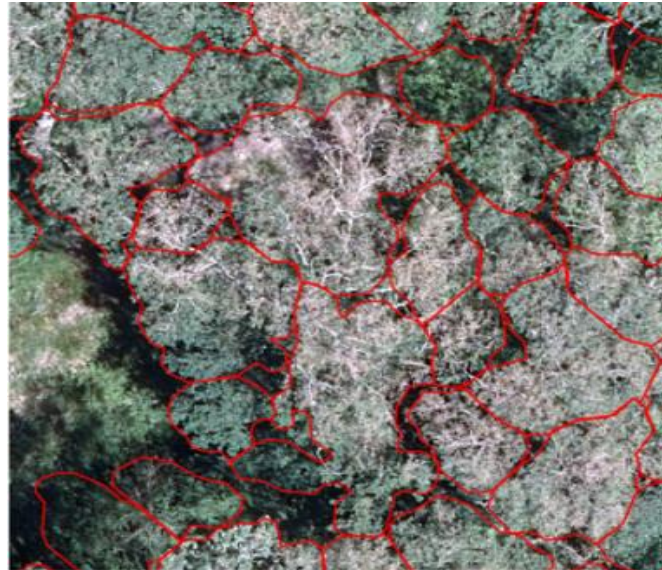
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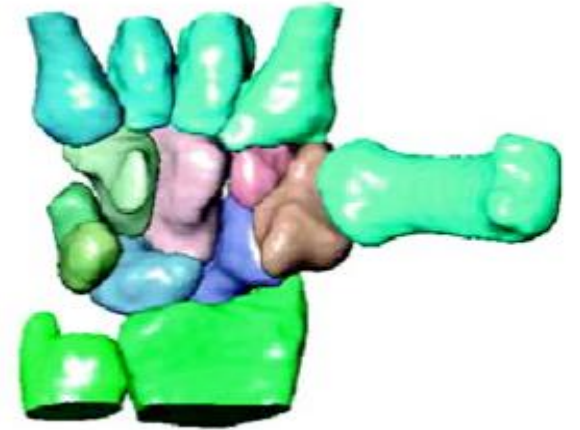
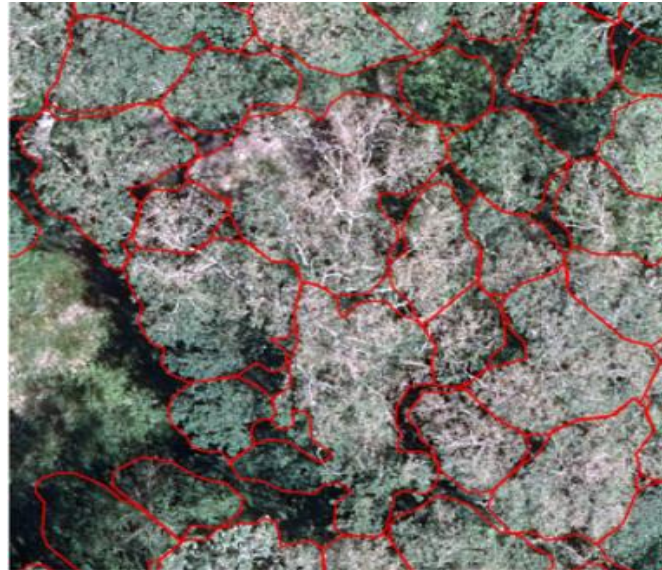
# Watershed

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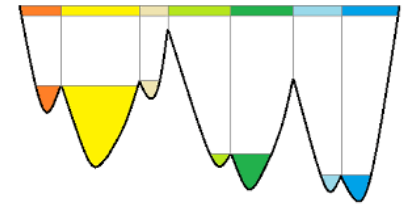
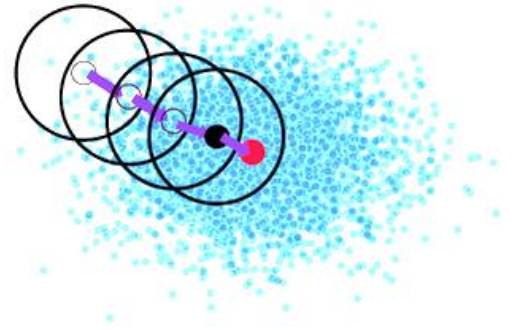
# Watershed

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# Conclusion

- Mean-shift
- Watershed

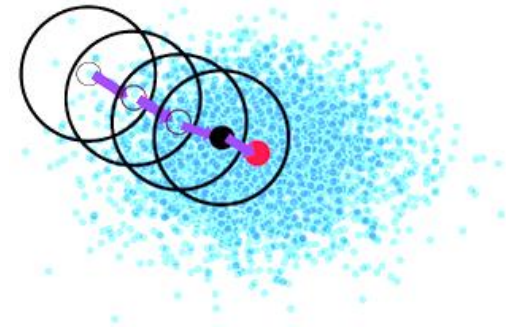


# Conclusion

- Mean-shift
- Watershed

## □ Mean-shift

- Number of cluster specification is not needed
- Mode seeking algorithm
- Computationally expensive



## □ Watershed

- Precise boundaries even for overlapping similar objects
- Images treated as topological surfaces

