

Sampling & Quantization

Dr. Tushar Sandhan

Introduction



Introduction

○ Input



○ Sampling



○ Quantization



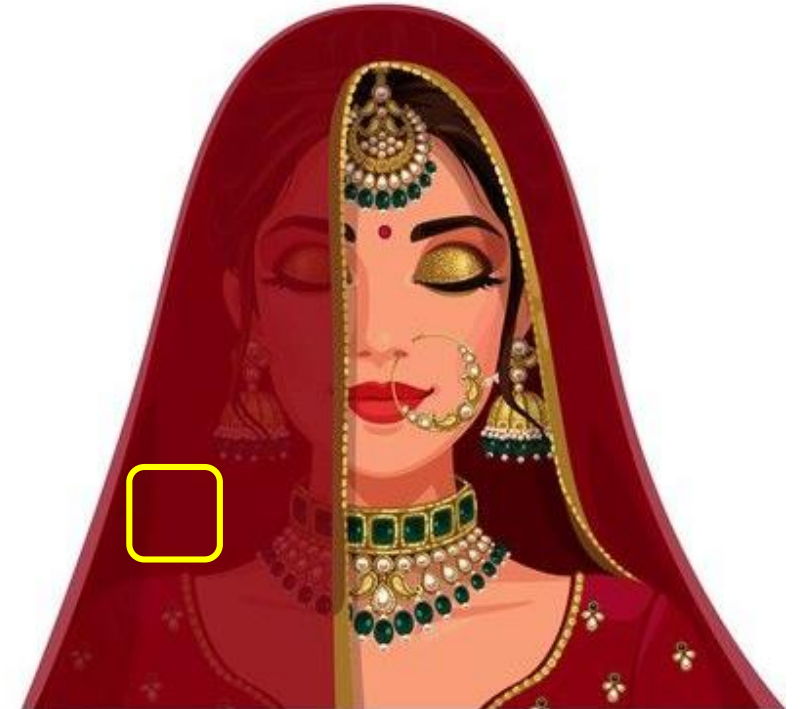
Sampling

- Sampling
 - determines spatial resolution
 - space digitization
- Image frequency
 - what are freq contents inside an image?
 - is the uniform sampling optimal?
 - is oversampling useful?
 - strive for efficient sampling
 - sampling density
 - data storage, data transmission



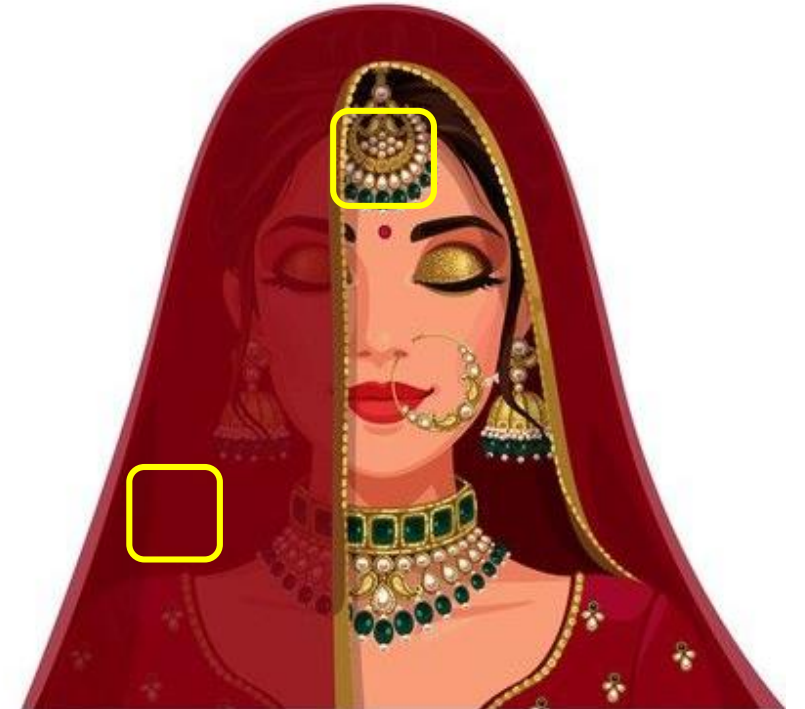
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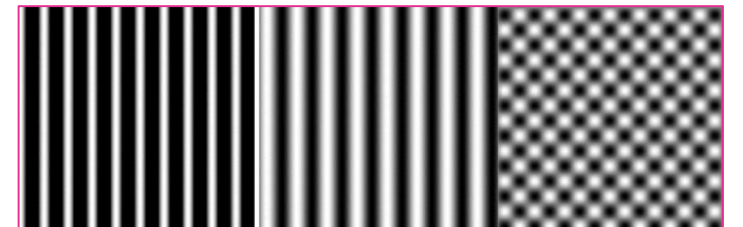
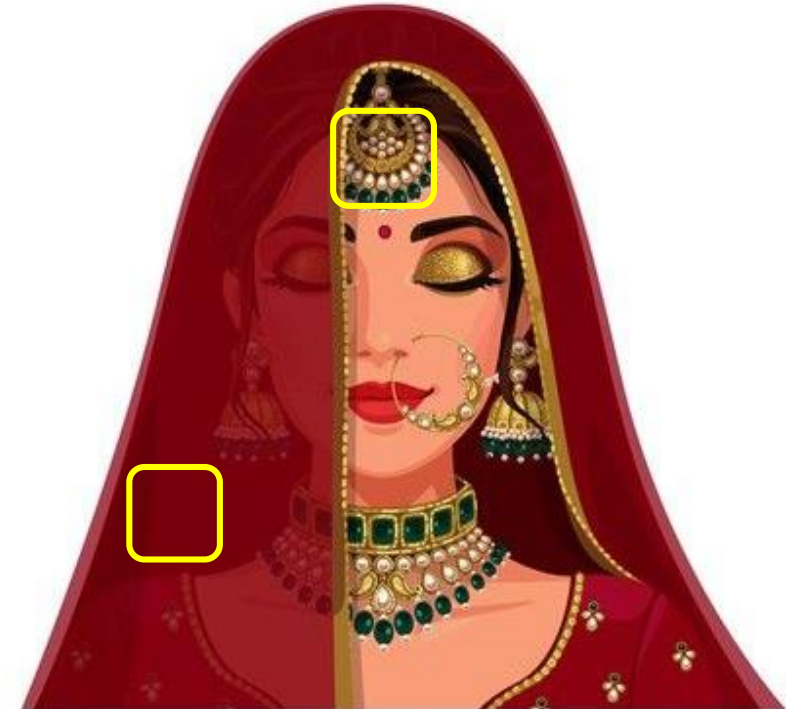
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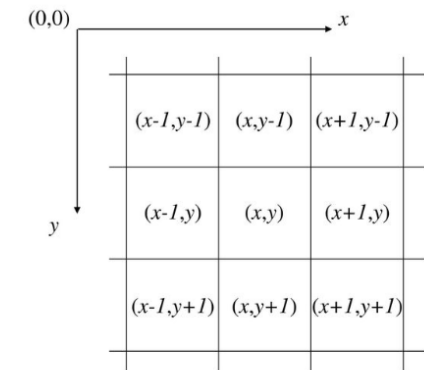
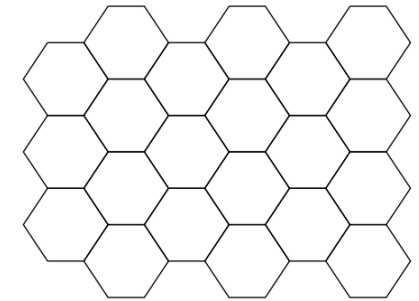
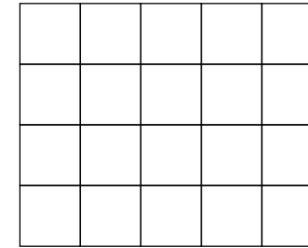
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Sampling

■ Grid

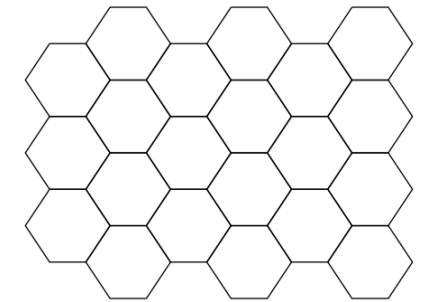
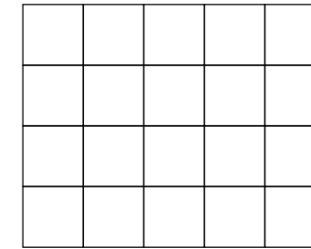
- continuous image is digitized at sampling points
- sampling points ordered in the plane
- their geometric relation – grid
- smallest grid point corresponds to – pixel (2D)
- voxel (3D)



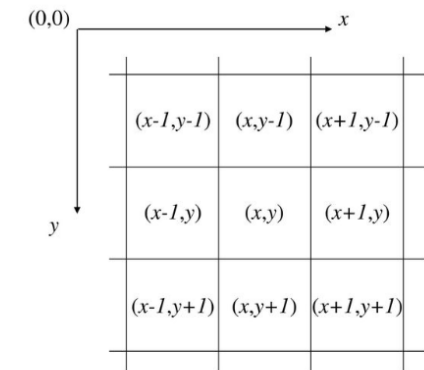
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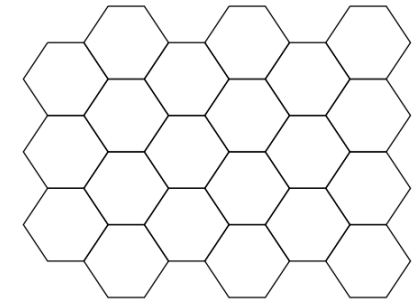
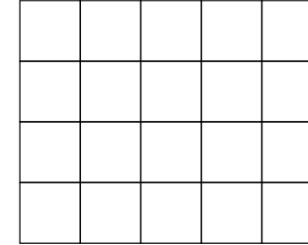
■ Neighbourhood



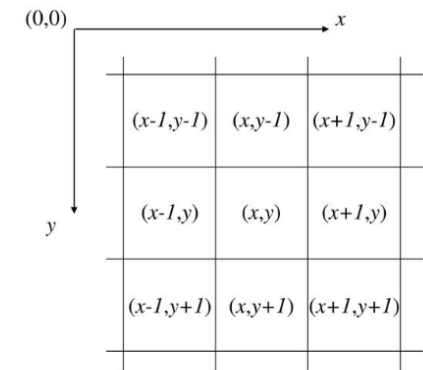
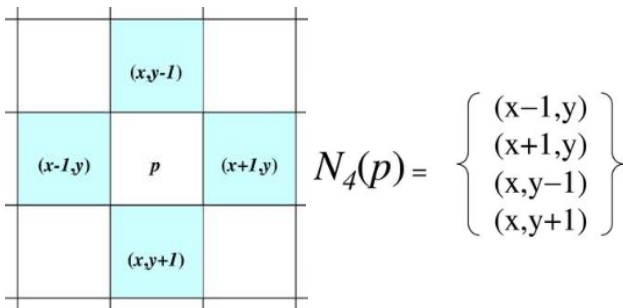
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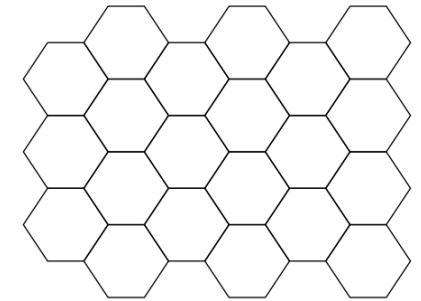
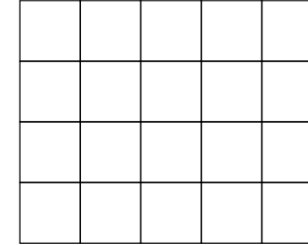
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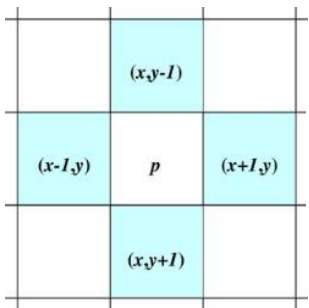
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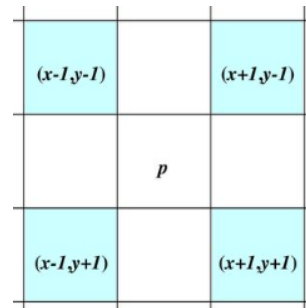
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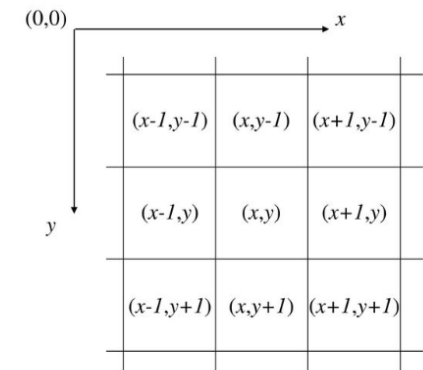
■ Neighbourhood



$$N_4(p) = \left\{ \begin{array}{l} (x-1, y) \\ (x+1, y) \\ (x, y-1) \\ (x, y+1) \end{array} \right\}$$



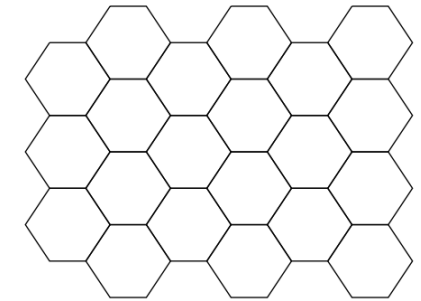
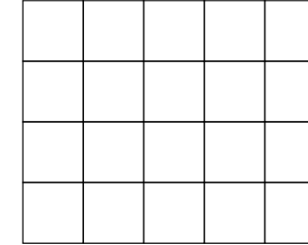
$$N_D(p)$$



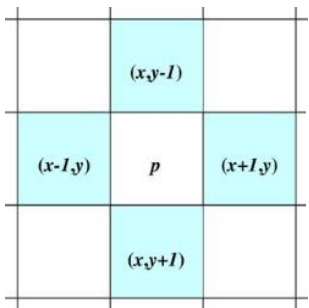
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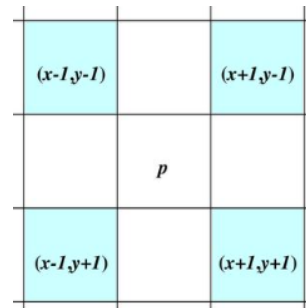
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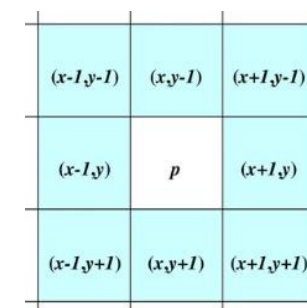
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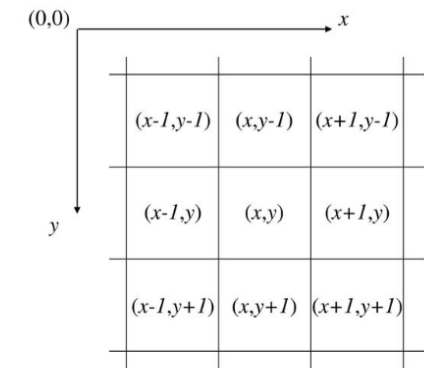
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$$N_D(p)$$

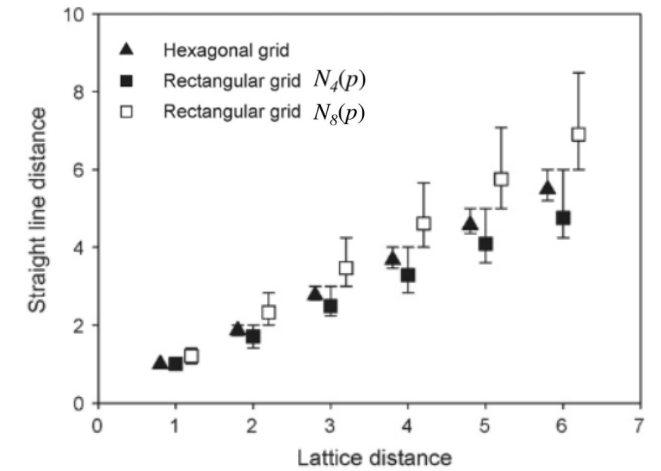
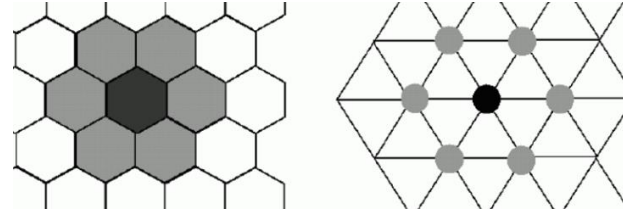


$$N_8(p)$$



Sampling

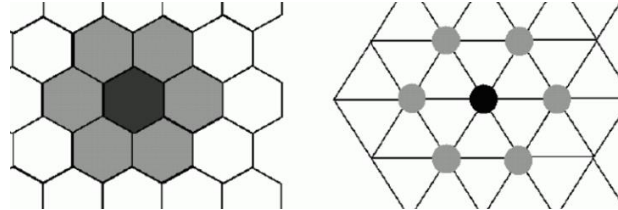
- Neighbourhood
 - hexagonal grid
 - neighbour interaction



Sampling

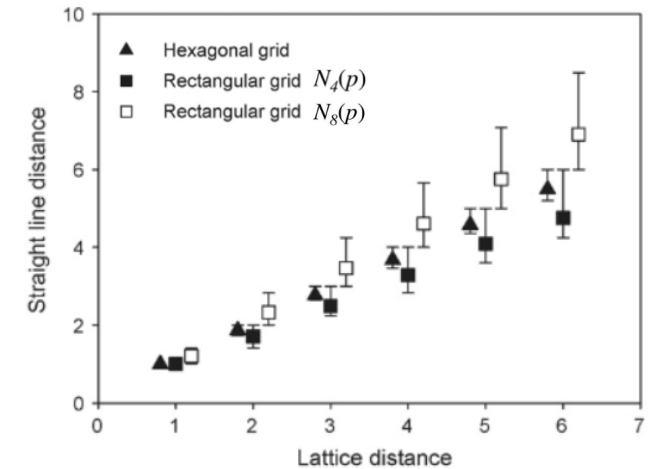
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- Neighbour interactions

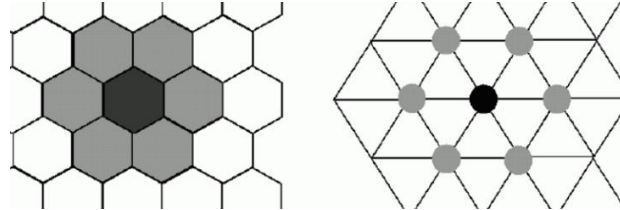
- distance, energy, edges, features
- sq. grid neighbourhood paradox
 - N_4 : broken ring encloses
 - N_8 : complete ring without enclosure



Sampling

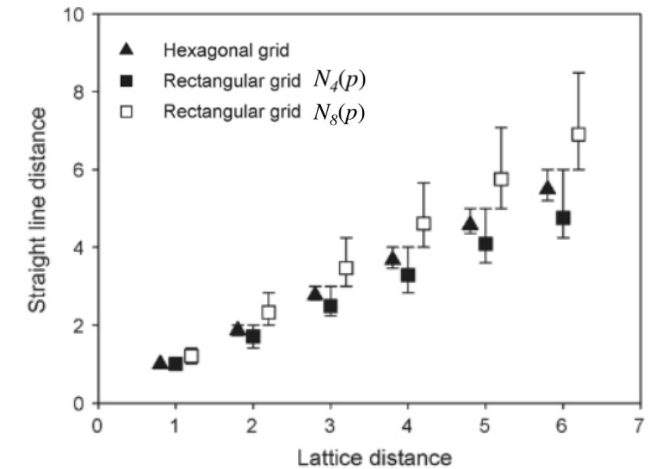
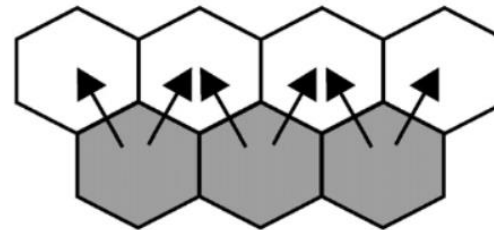
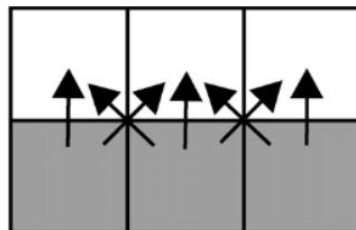
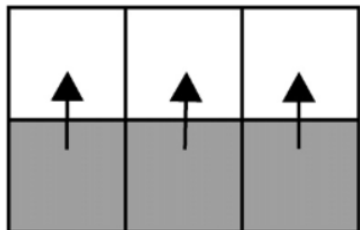
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- Neighbour interactions

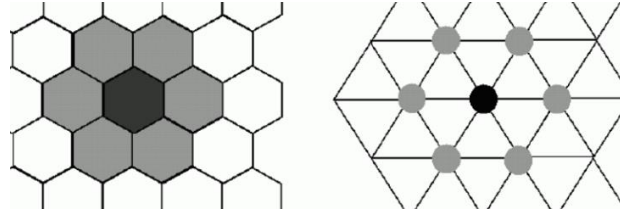
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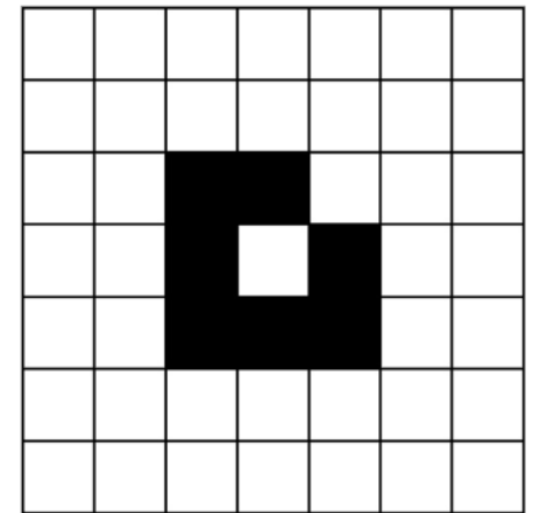
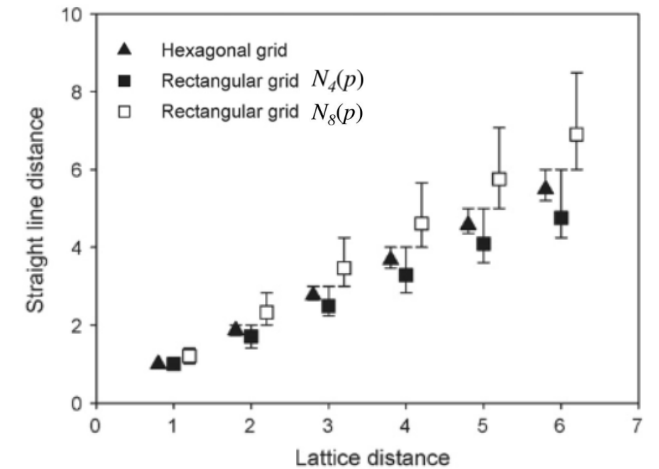
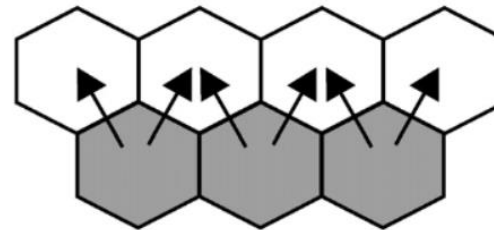
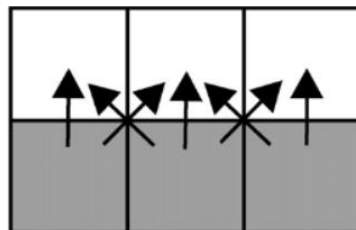
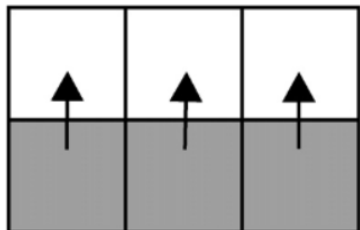
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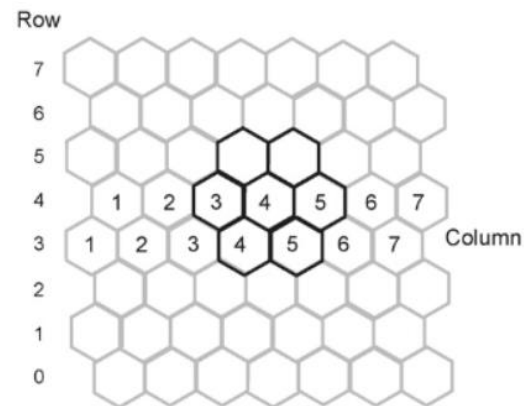
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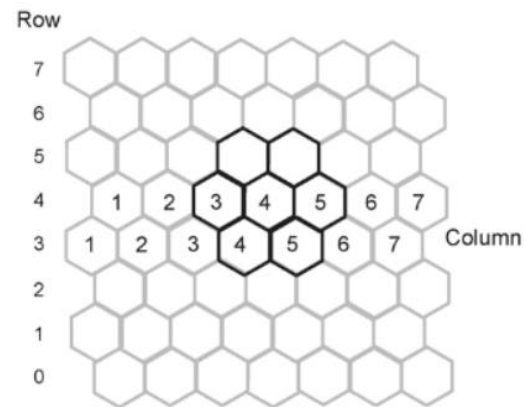
- Coordinate system
 - hexagonal grid
 - SHCS: symmetrical hexagonal coordinate system in (C)



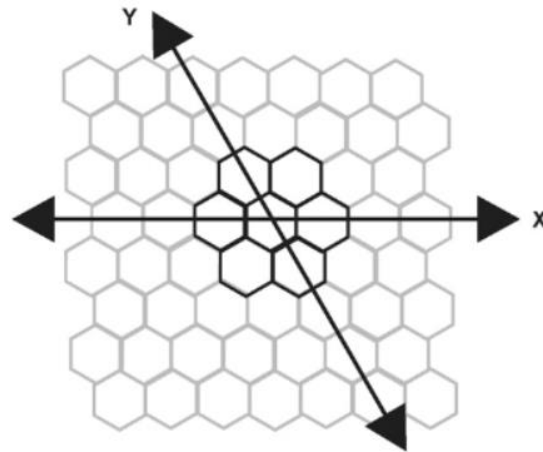
(a)

Sampling

- Coordinate system
 - hexagonal grid
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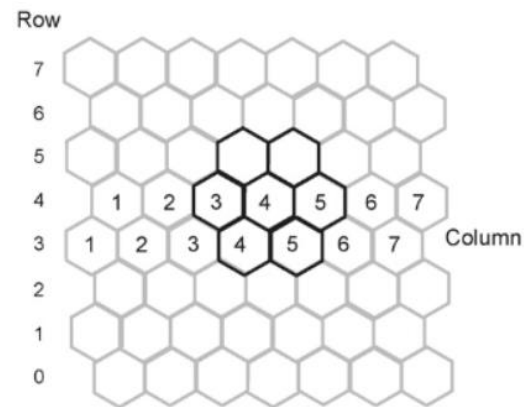
(a)



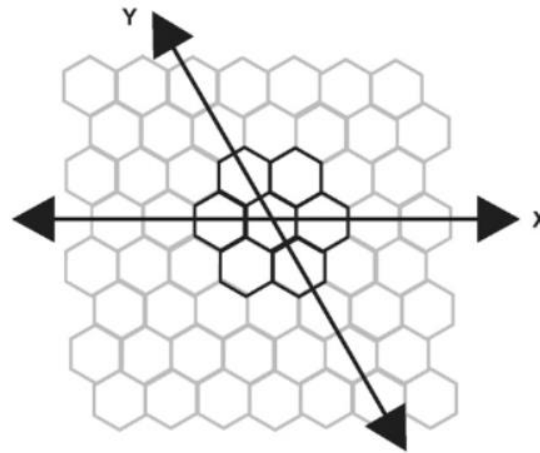
(b)

Sampling

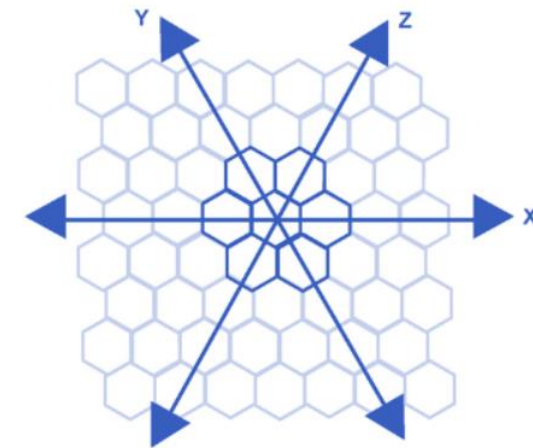
- Coordinate system
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(a)



(b)

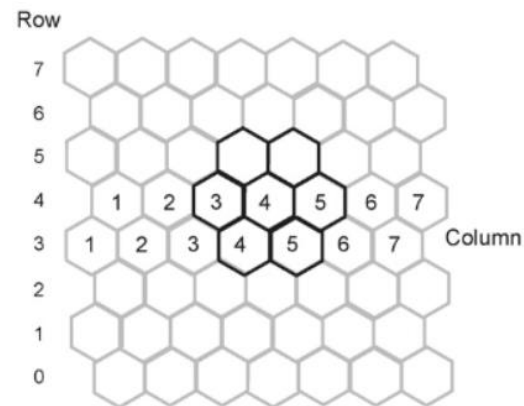


(C)

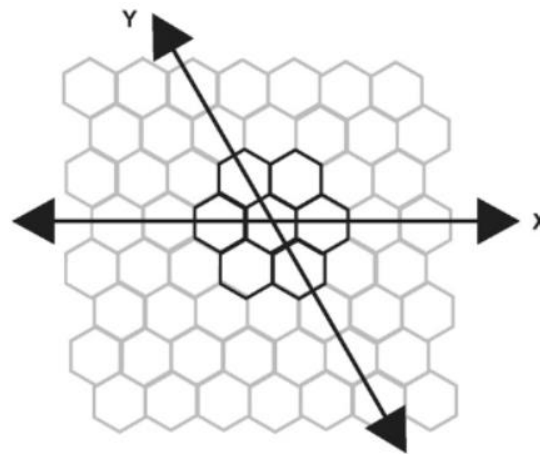
Sampling

- Coordinate system

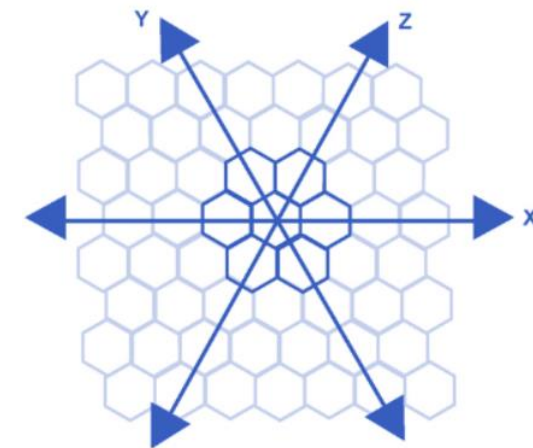
- hexagonal grid
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(a)



(b)



(C)

$$\forall (x, y, z) : x + y + z = 0$$

(x_1, y_1, z_1) and (x_2, y_2, z_2) are the co-ordinates of the two hexagons.

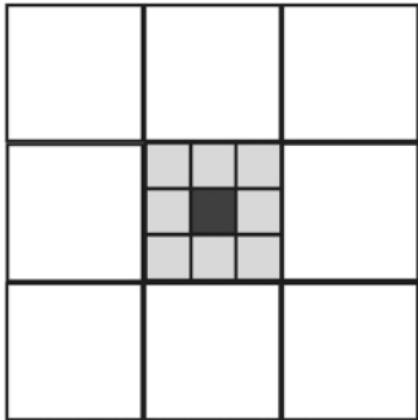
$$D_{\text{Eucl.}}[(x_1, y_1, x_1), (x_2, y_2, x_2)]$$

$$= \sqrt{\frac{1}{2}[(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2]}$$

$$D_{\text{Grid}}[(x_1, y_1, x_1), (x_2, y_2, x_2)] = \max(|x_1 - x_2|, |y_1 - y_2|, |z_1 - z_2|)$$

Sampling

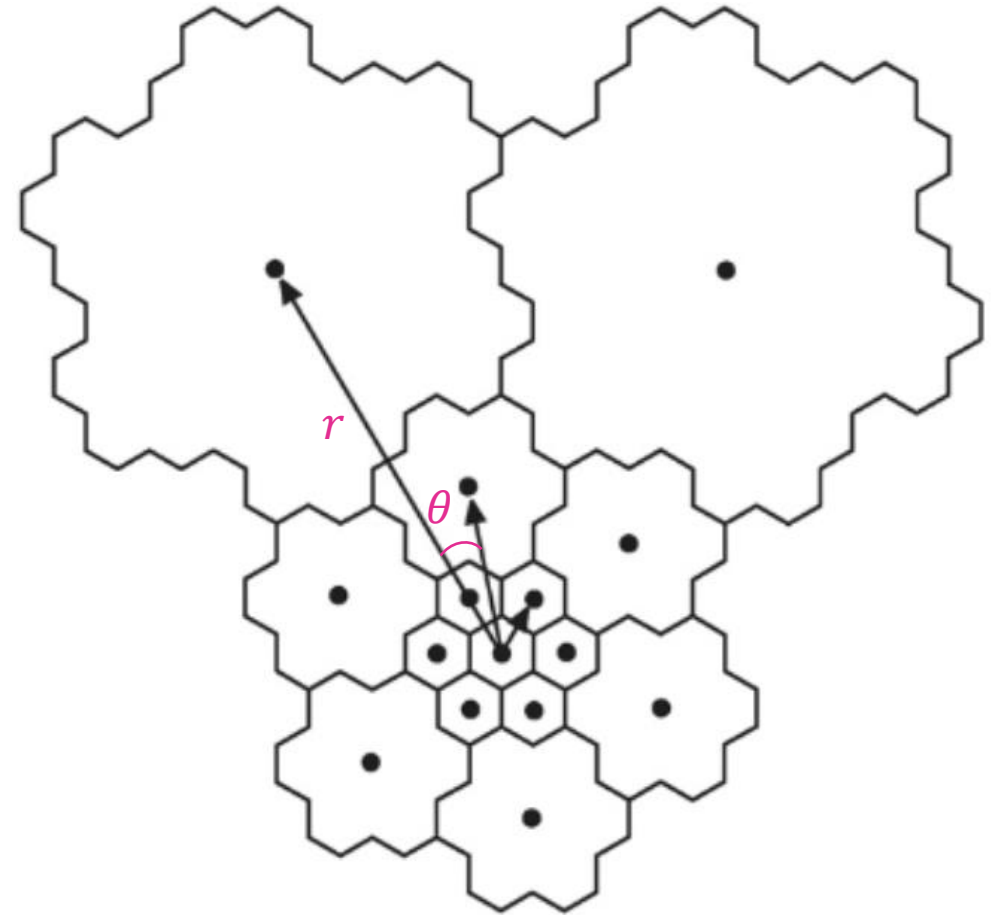
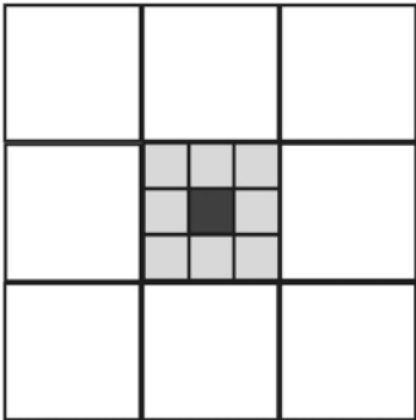
- Hierarchical grids
 - neighbours at finer scale become focal cell or centroids for coarser scale
 - smooth out or simplify some grids
 - dynamic grid resolution
 - θ, r can be used to find out current resolution scale



Sampling

- Hierarchical grids

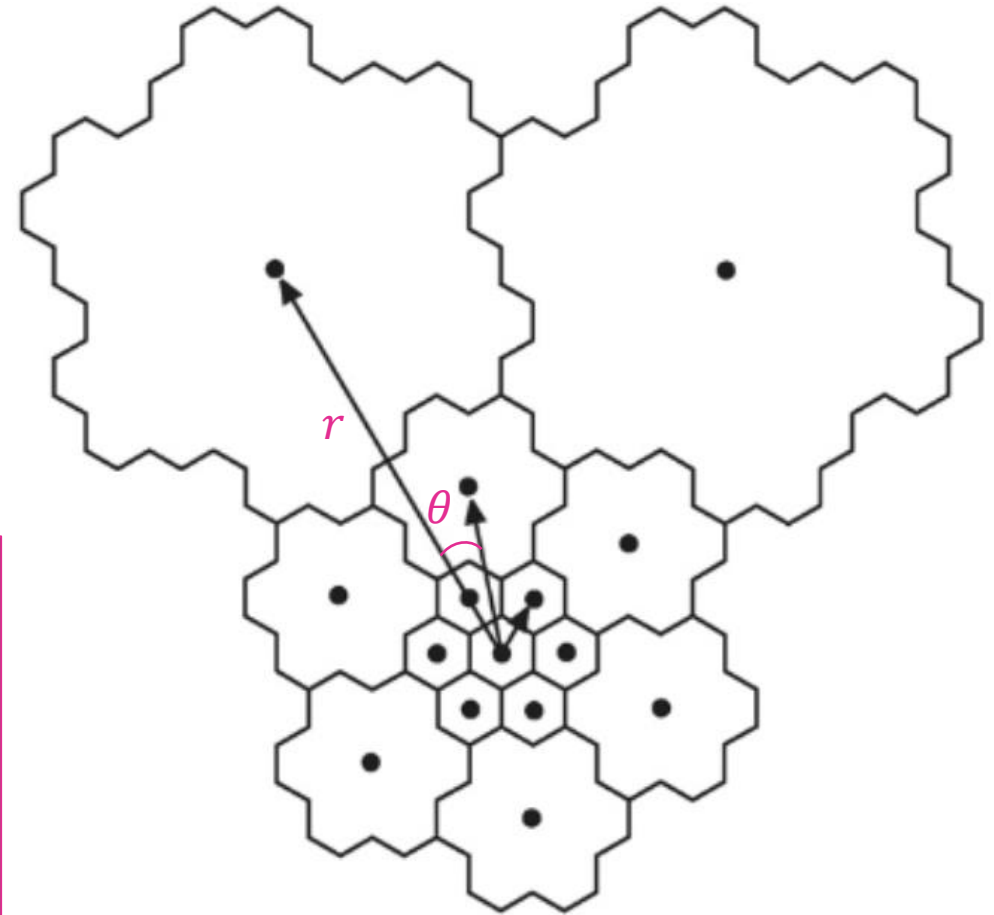
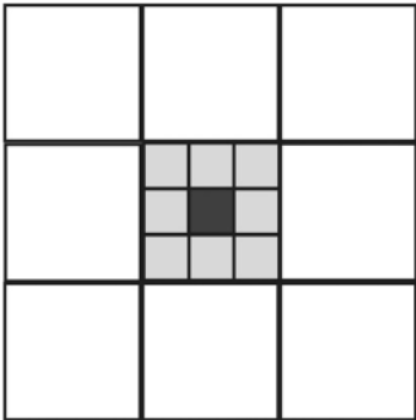
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Sampling

- Checkerboard effect
 - due to uniform non-optimal square grid sampling



Sampling

- Checkerboard effect
 - due to uniform non-optimal square grid sampling

128x128



64x64



32x32



Sampling

- Aliasing

continuous bandlimited function

$$f(x, y)$$

$$\delta(x, y) = \begin{cases} 1 & \text{if } x = y = 0 \\ 0 & \text{otherwise} \end{cases}$$

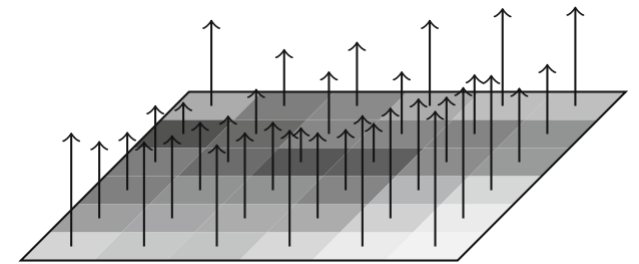
$$\sum_{x=-\infty}^{\infty} \sum_{y=-\infty}^{\infty} f(x, y) \delta(x - x_0, y - y_0) = f(x_0, y_0)$$

Sampling

- Aliasing

continuous bandlimited function

$f(x, y)$



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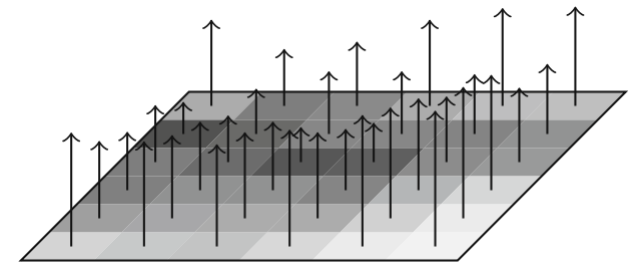
Sampling

- Aliasing

$$\text{comb}(x, y, \Delta x, \Delta y) = \sum_m \sum_n \delta(x - m\Delta x, y - n\Delta y)$$

continuous bandlimited function

$f(x, y)$



$$\delta(x, y) = \begin{cases} 1 & \text{if } x = y = 0 \\ 0 & \text{otherwise} \end{cases}$$

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Sampling

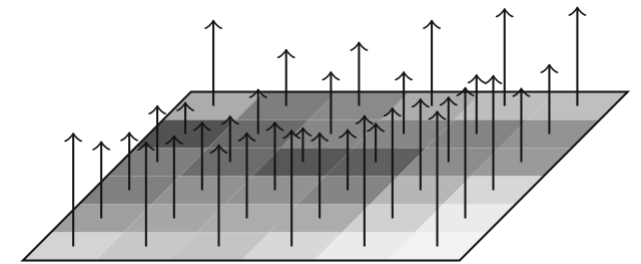
- Aliasing

$$f_s(x, y) = f(x, y) \text{comb}(x, y, \Delta x, \Delta y)$$

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continuous bandlimited function

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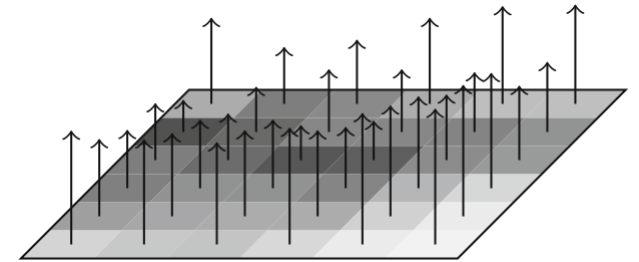
$$\text{comb}(x, y, \Delta x, \Delta y) = \sum_m \sum_n \delta(x - m\Delta x, y - n\Delta y)$$

$$\begin{aligned} F_s(\omega_x, \omega_y) &= F(\omega_x, \omega_y) * \omega_{x_s} \omega_{y_s} \sum_p \sum_q \delta(\omega_x - p\omega_{x_s}, \omega_y - q\omega_{y_s}) \\ &= \omega_{x_s} \omega_{y_s} \sum_p \sum_q F(\omega_x - p\omega_{x_s}, \omega_y - q\omega_{y_s}) \end{aligned}$$

where $\omega_{x_s} = \frac{2\pi}{\Delta x}$, and $\omega_{y_s} = \frac{2\pi}{\Delta y}$

continuous bandlimited function

$f(x, y)$



$$\delta(x, y) = \begin{cases} 1 & \text{if } x = y = 0 \\ 0 & \text{otherwise} \end{cases}$$

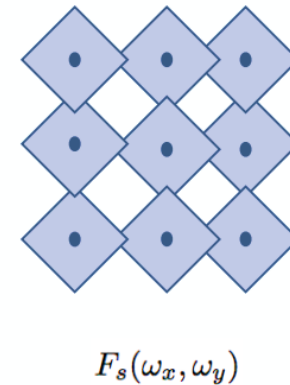
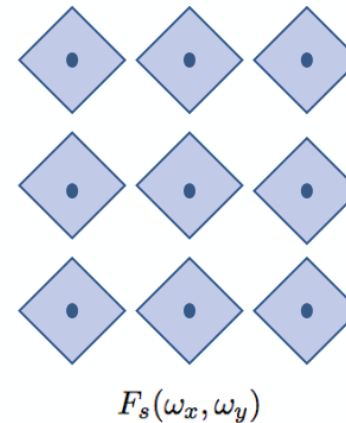
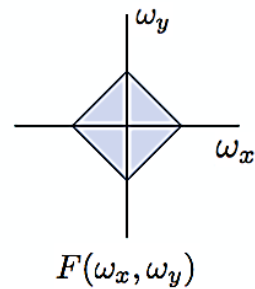
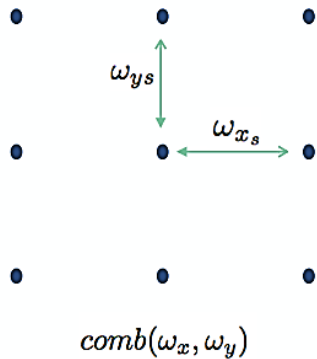
$$\sum_{x=-\infty}^{\infty} \sum_{y=-\infty}^{\infty} f(x, y) \delta(x - x_0, y - y_0) = f(x_0, y_0)$$

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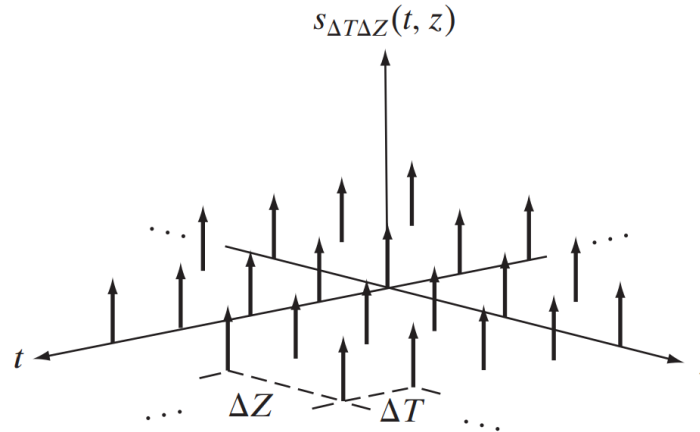
$$F_s(\omega_x, \omega_y) = \omega_{x_s} \omega_{y_s} \sum_p \sum_q F(\omega_x - p\omega_{x_s}, \omega_y - q\omega_{y_s})$$



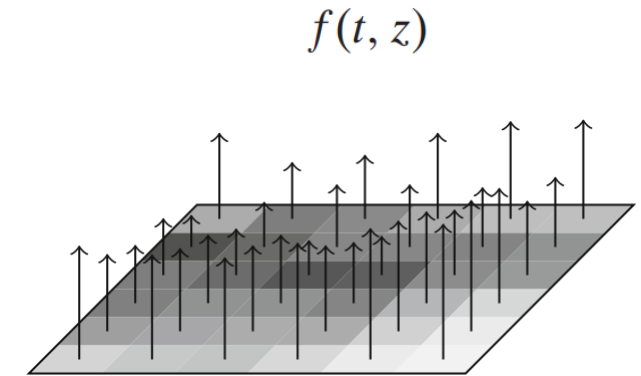
Sampling

- Sampling theorem

- $f(t, z)$ can be recovered fully with zero error from its samples
- iff grid is 'sufficiently' dense
- just change of variables to simplify notations



continuous bandlimited function



$$s_{\Delta T \Delta Z}(t, z) = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \delta(t - m\Delta T, z - n\Delta Z)$$

$$F(\mu, \nu) = 0 \quad \text{for } |\mu| \geq \mu_{\max} \text{ and } |\nu| \geq \nu_{\max}$$

... band limits

Sampling

- Sampling theorem

- $f(t, z)$ can be recovered fully with zero error from its samples
- No info is lost in the image if it is obtained via sampling at rates greater than twice the max freq content of $f(t, z)$ in both μ, ν directions.

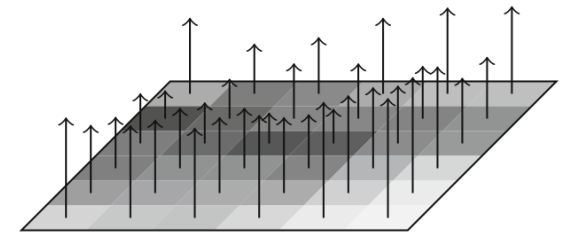
$F(\mu, \nu) = 0$ for $|\mu| \geq \mu_{\max}$ and $|\nu| \geq \nu_{\max}$... band limits

$$\frac{1}{\Delta T} > 2\mu_{\max} \quad \frac{1}{\Delta Z} > 2\nu_{\max}$$

continuous bandlimited function

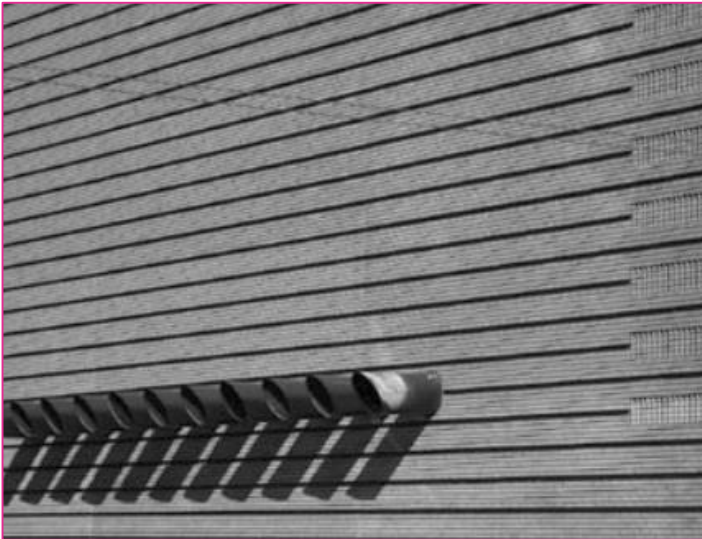
Image $\leftarrow f(t, z)$

$$s_{\Delta T \Delta Z}(t, z) = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \delta(t - m\Delta T, z - n\Delta Z)$$



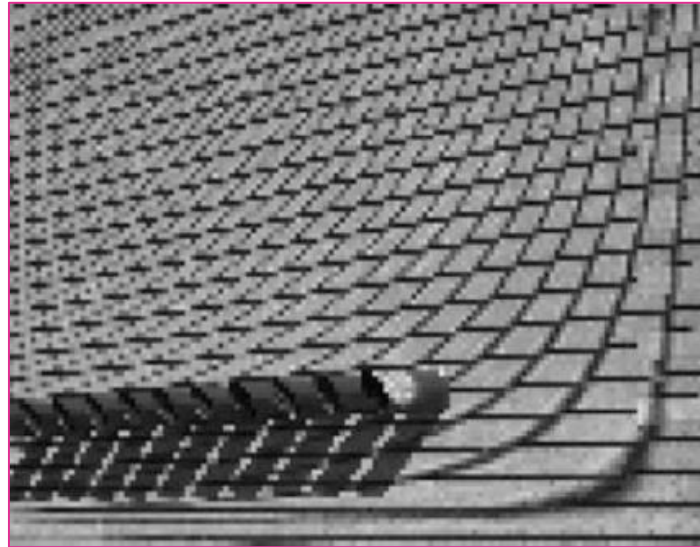
Sampling

- Erroneous effects
 - square grid sampling



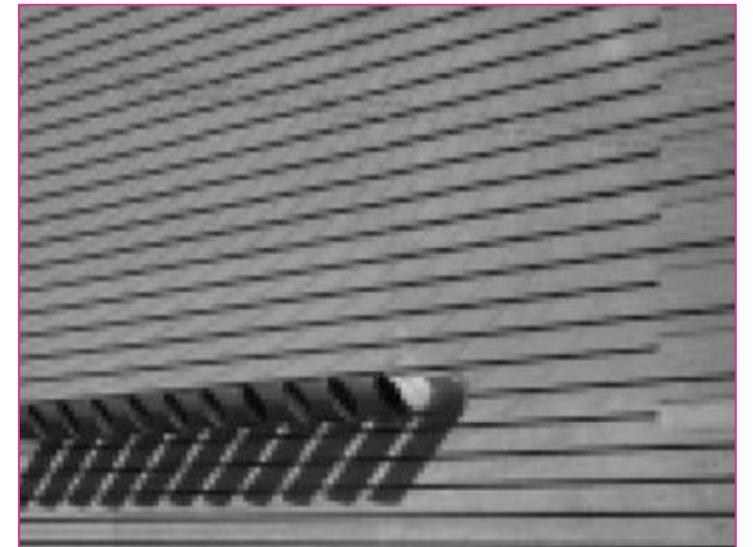
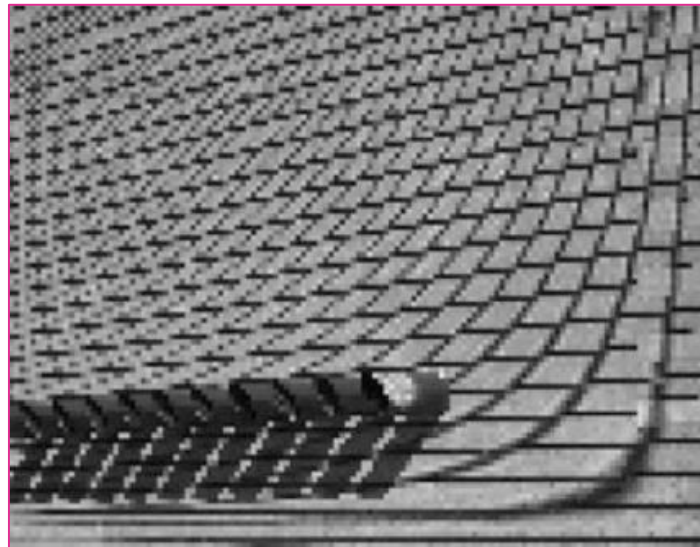
Sampling

- Erroneous effects
 - square grid sampling



Sampling

- Erroneous effects
 - square grid sampling



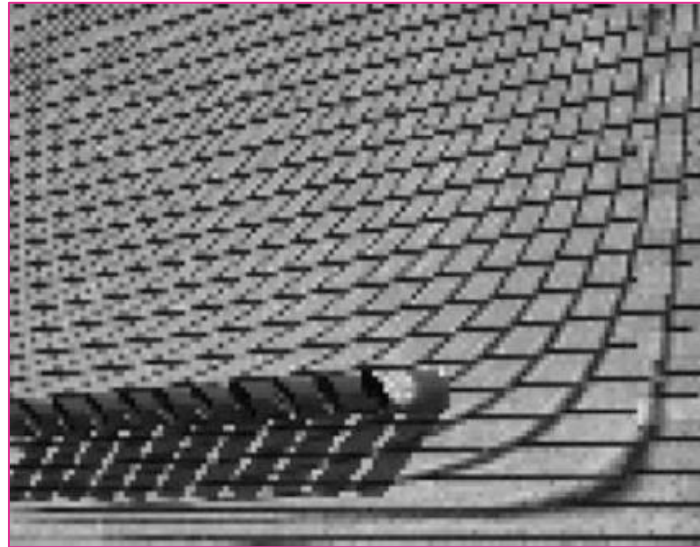
Sampling

- Erroneous effects
 - square grid sampling

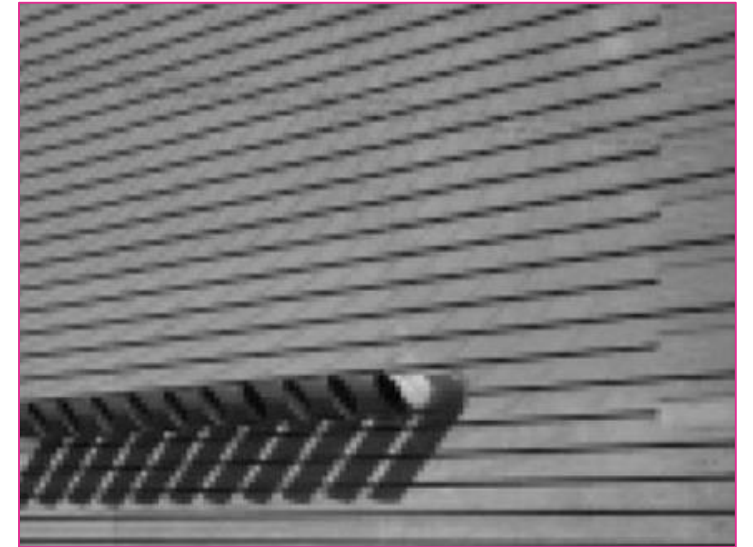
input



8x8 sq grid



8x8 mean

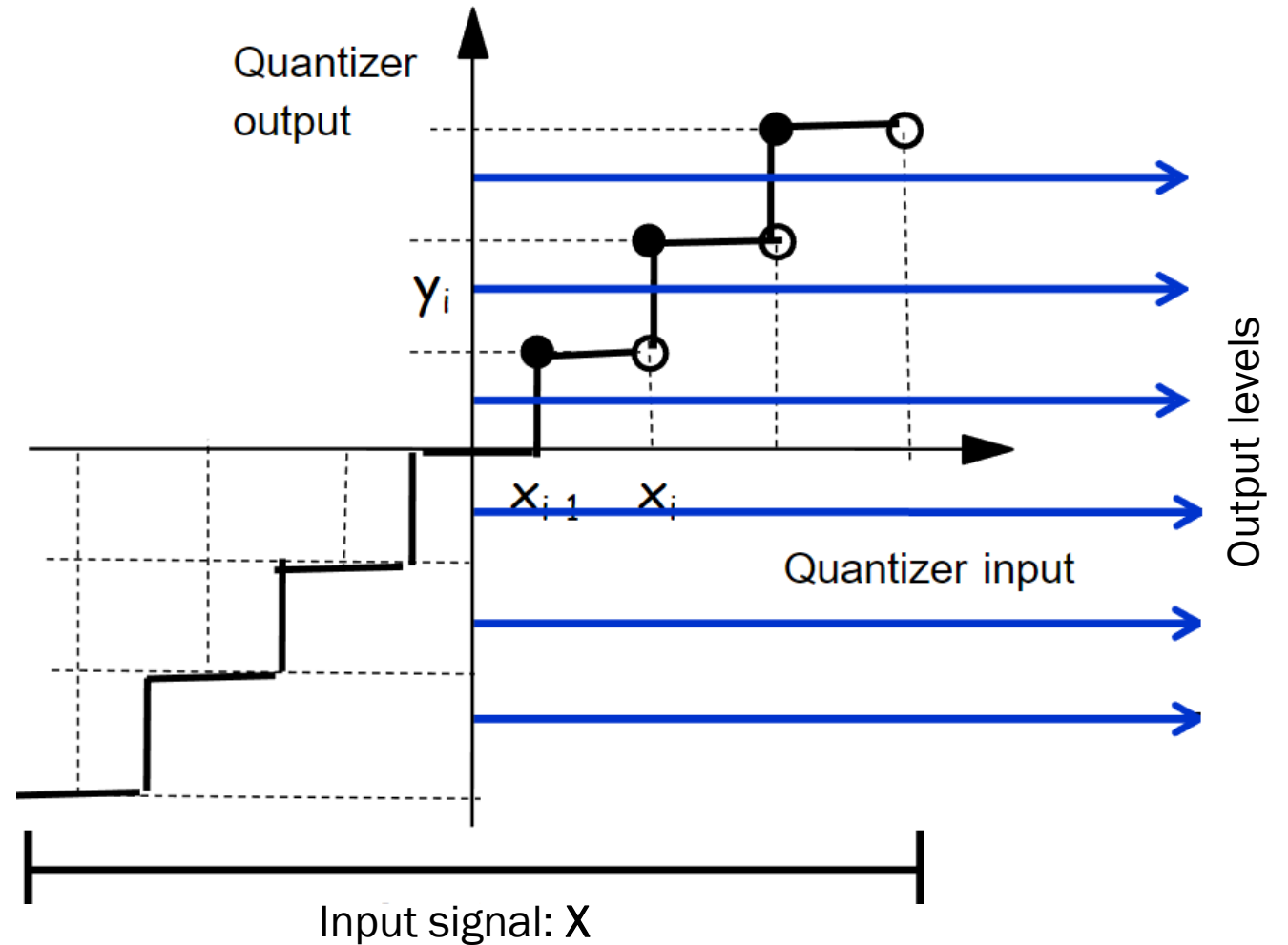


Quantization

- Quantizer

- SISO – scalar quantizer
- mappings $[x_{i-1}, x_i) \rightarrow y_i$
- what are the unknowns?

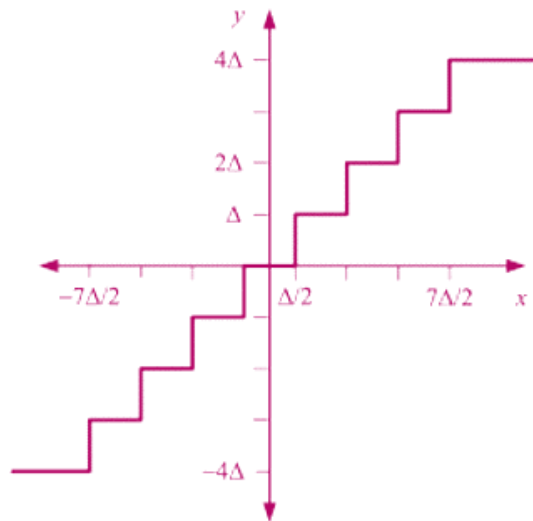
$$x \in [t_k, t_{k+1}) \Rightarrow Q(x) = r_k$$



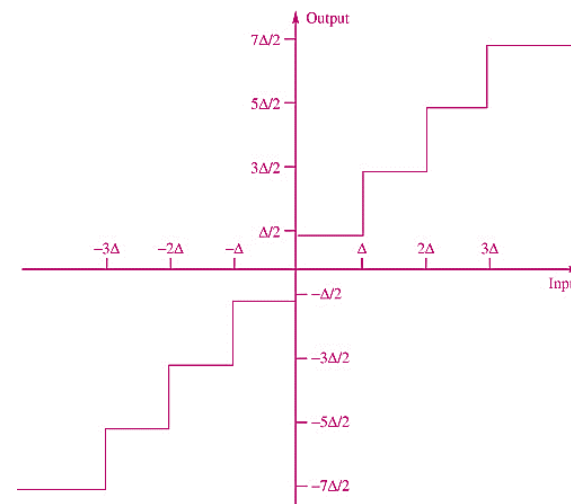
Quantization

- Uniform quantizers
 - all ranges divided equally with $\Delta = [t_k, t_{k+1})$ intervals
 - deadzone

Midtread quantizer



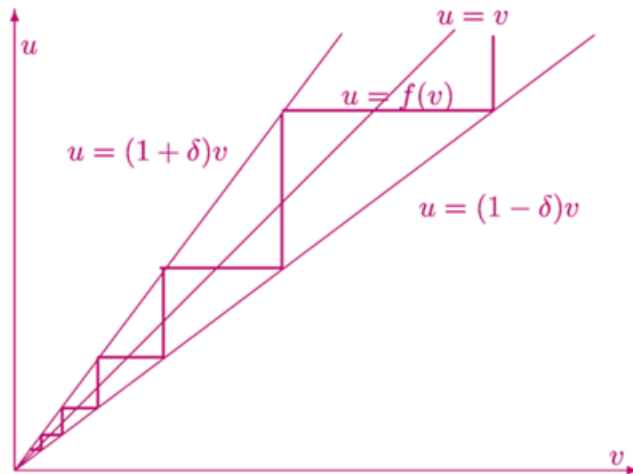
Midrise quantizer



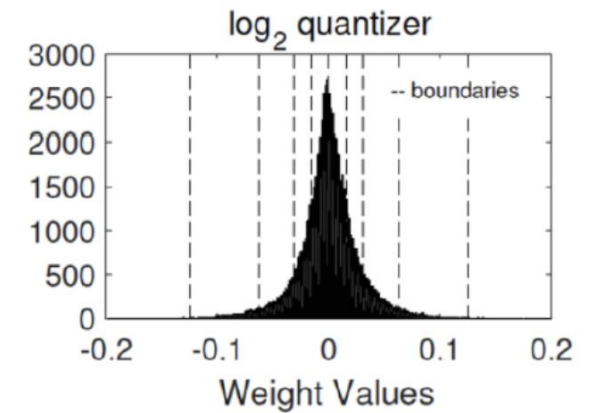
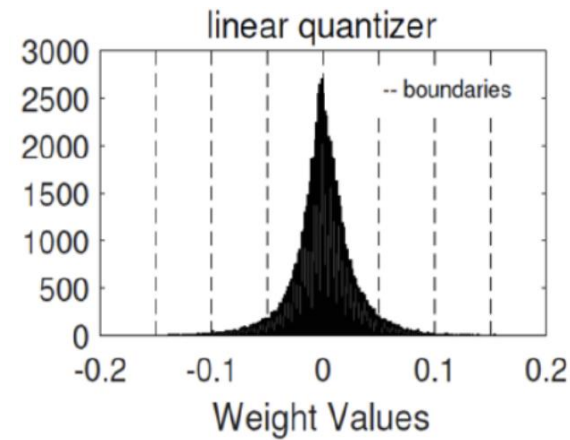
Quantization

- Non-uniform quantizers
 - ranges divided via predefined function which gives Δ intervals

Logarithmic quantizer



Logarithmic quantizer for image filter weights

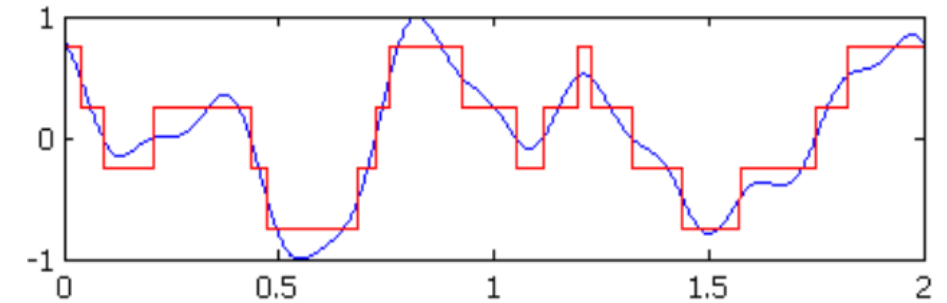


Quantization

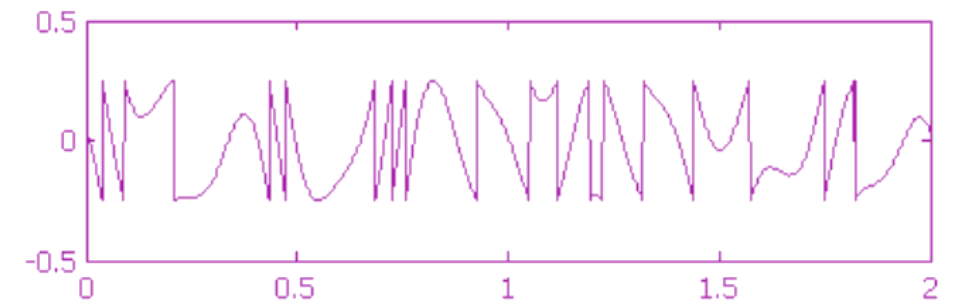
- Non-uniform quantizers

- can we design optimal quantizer?
- optimal in the sense to minimize error (which error?)
- input: $x_i \approx t_i$:thresholds
- output: $y_i \approx r_i$:reconstructions
- signal distribution is known: $p(x)$

Original and quantized signal



Quantization error

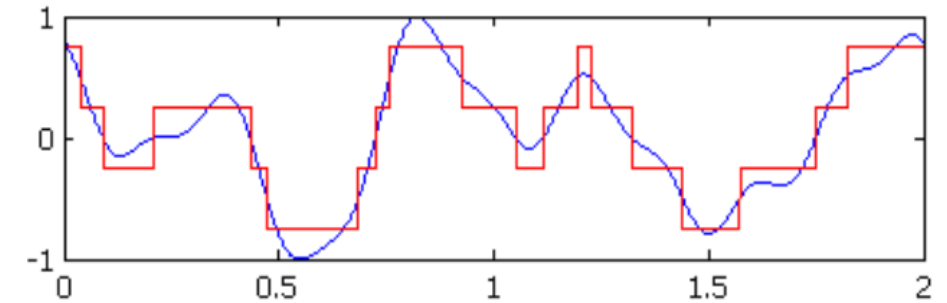


Quantization

■ Non-uniform quantizers

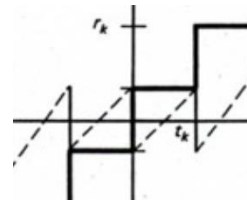
- can we design optimal quantizer?
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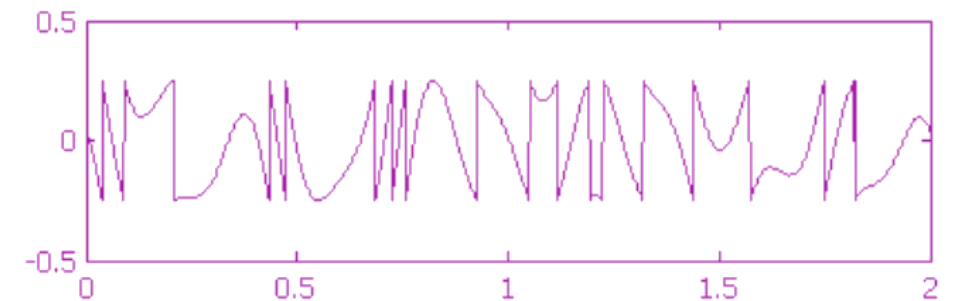


■ MSE

$$D = \int_a^b (x - Q(x))^2 p(x) dx$$



Quantization error



Quantization

$$\begin{aligned} \min_{t_k, r_k} D &= \sum_{k=1}^L \int_{t_k}^{t_{k+1}} (x - r_k)^2 p(x) dx \\ \text{sub. to} \quad &t_1 = a, t_{L+1} = b \\ &t_k \leq r_k \leq t_{k+1}, k = 0, \dots, L \end{aligned}$$

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$$\begin{aligned} \frac{\partial D}{\partial r_k} &= \frac{\partial}{\partial r_k} \sum_{j=1}^L \int_{t_j}^{t_{j+1}} (x - r_j)^2 p(x) dx \\ &= -2 \int_{t_k}^{t_{k+1}} (x - r_k) p(x) dx, \quad k = 1, \dots, L \end{aligned}$$

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Fundamental theorem of calculus:
derivative with accumulation function
(c initial const., $f(t)$ is cts in open interval)

$$\begin{aligned} A(t) &= \int_c^t f(x) dx \\ A'(t) &= f(t) \end{aligned}$$

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Assuming $p(x) > 0$ for each $x \in [a, b]$:

$$\begin{aligned} r_k &= \frac{\int_{t_k}^{t_{k+1}} x p(x) dx}{\int_{t_k}^{t_{k+1}} p(x) dx}, \quad k = 1, \dots, L \\ t_k &= \frac{r_{k-1} + r_k}{2}, \quad k = 2, \dots, L \end{aligned}$$

Fundamental theorem of calculus:
derivative with accumulation function
(c initial const., $f(t)$ is cts in open interval)

$$\begin{aligned} A(t) &= \int_c^t f(x) dx \\ A'(t) &= f(t) \end{aligned}$$

Quantization

- Lloyd-Max Quantizer
 - optimal MSE quantizer

$$r_k = \bar{x}_k = \frac{\int_{t_k}^{t_{k+1}} xp(x)dx}{\int_{t_k}^{t_{k+1}} p(x)dx}, \quad k = 1, \dots, L \quad \dots (1)$$

$$t_k = \frac{r_{k-1} + r_k}{2} = \frac{\bar{x}_{k-1} + \bar{x}_k}{2}, \quad k = 2, \dots, L \quad \dots (2)$$

Quantization

- Lloyd-Max Quantizer
 - optimal MSE quantizer

$$r_k = \bar{x}_k = \frac{\int_{t_k}^{t_{k+1}} xp(x)dx}{\int_{t_k}^{t_{k+1}} p(x)dx}, \quad k = 1, \dots, L \quad \dots (1)$$

$$t_k = \frac{r_{k-1} + r_k}{2} = \frac{\bar{x}_{k-1} + \bar{x}_k}{2}, \quad k = 2, \dots, L \quad \dots (2)$$

- Pseudo-code

- : pick initial values for t (uniform grid)
- : find r values using (1)
- : find new t values using (2)
- : repeat till both t, r converge

Quantization

- Properties of optimal quantizer

- $$\begin{aligned} E[Q(x)] &= \sum_k r_k p_k \\ &= E[x] \end{aligned}$$

$$r_k = \frac{\int_{t_k}^{t_{k+1}} x p(x) dx}{\int_{t_k}^{t_{k+1}} p(x) dx}$$

$$p_k = \int_{t_k}^{t_{k+1}} p(x) dx$$

- $$E[x - Q(x)] = 0$$

Quantization

- Properties of optimal quantizer

- $$E[(x - Q(x))Q(x)] = \sum_k \int_{t_k}^{t_{k+1}} (x - r_k)r_k p(x) dx$$

$$r_k = \frac{\int_{t_k}^{t_{k+1}} x p(x) dx}{\int_{t_k}^{t_{k+1}} p(x) dx}$$

$$p_k = \int_{t_k}^{t_{k+1}} p(x) dx$$

Quantization

- Properties of optimal quantizer

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$$= 0$$

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$$p_k = \int_{t_k}^{t_{k+1}} p(x) dx$$

Quantization

- Properties of optimal quantizer

- $$E[(x - Q(x))Q(x)] = \sum_k \int_{t_k}^{t_{k+1}} (x - r_k)r_k p(x) dx$$

$$= 0$$

$$r_k = \frac{\int_{t_k}^{t_{k+1}} x p(x) dx}{\int_{t_k}^{t_{k+1}} p(x) dx}$$

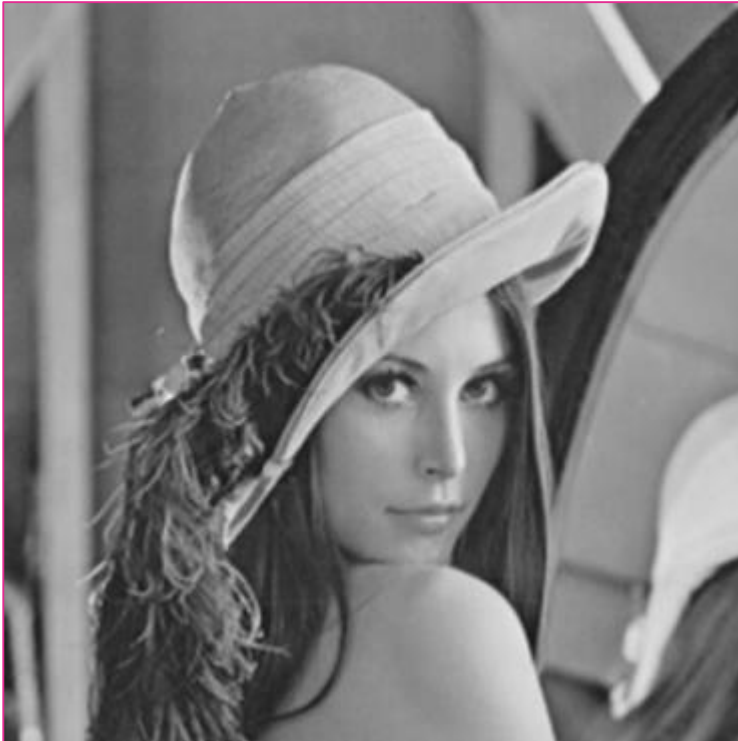
$$p_k = \int_{t_k}^{t_{k+1}} p(x) dx$$

- error is uncorrelated with the quantizer's output

Quantization

- Lloyd-Max example

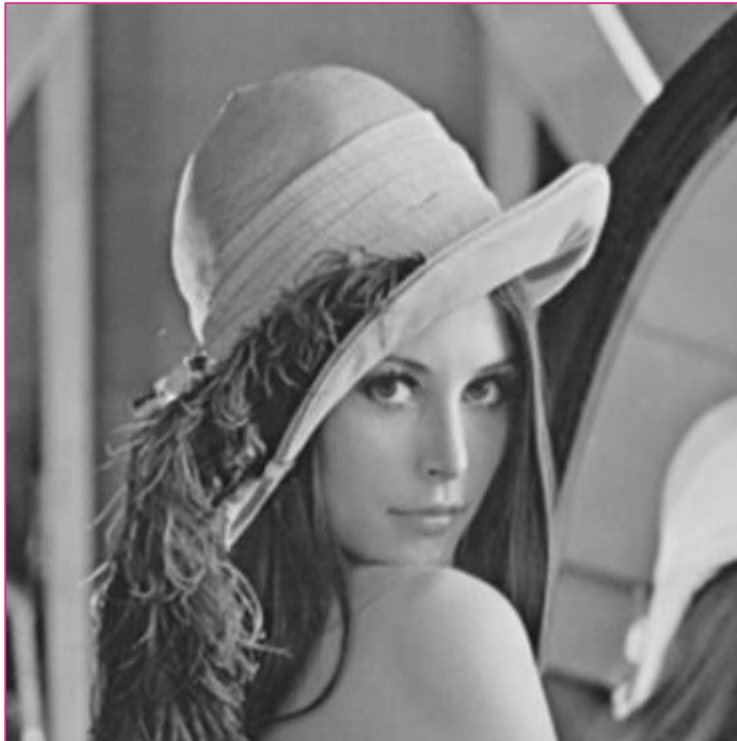
8 bpp



Quantization

- Lloyd-Max example

8 bpp



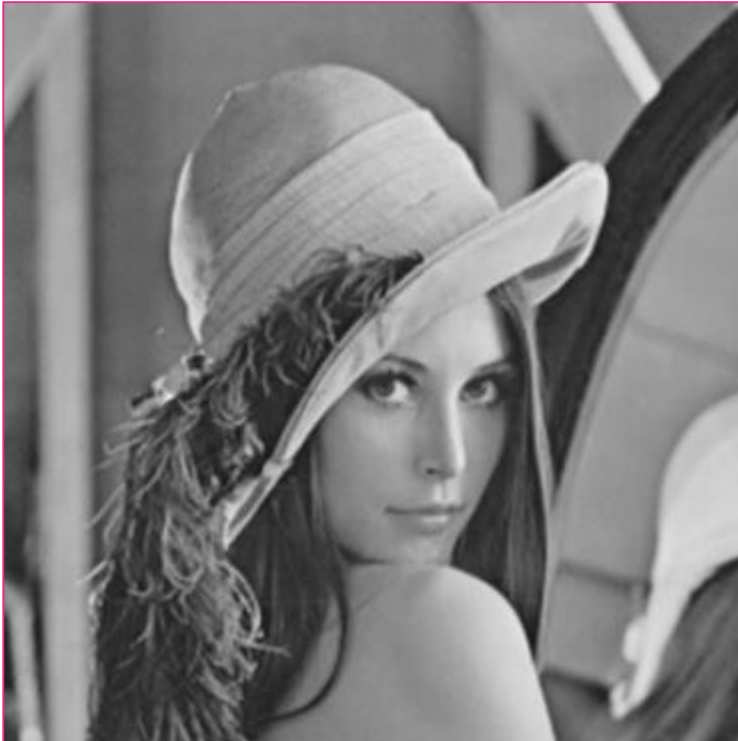
6 bpp



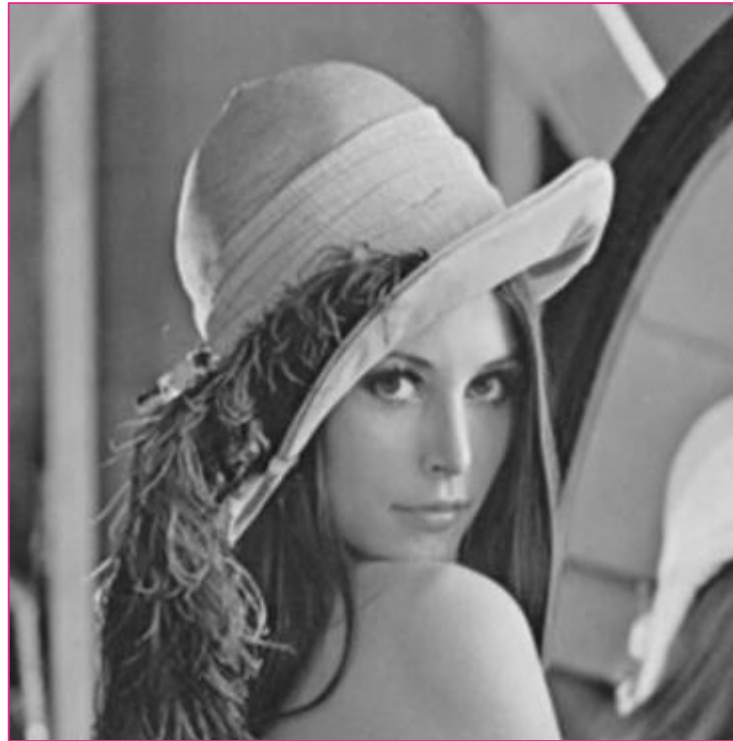
Quantization

- Lloyd-Max example

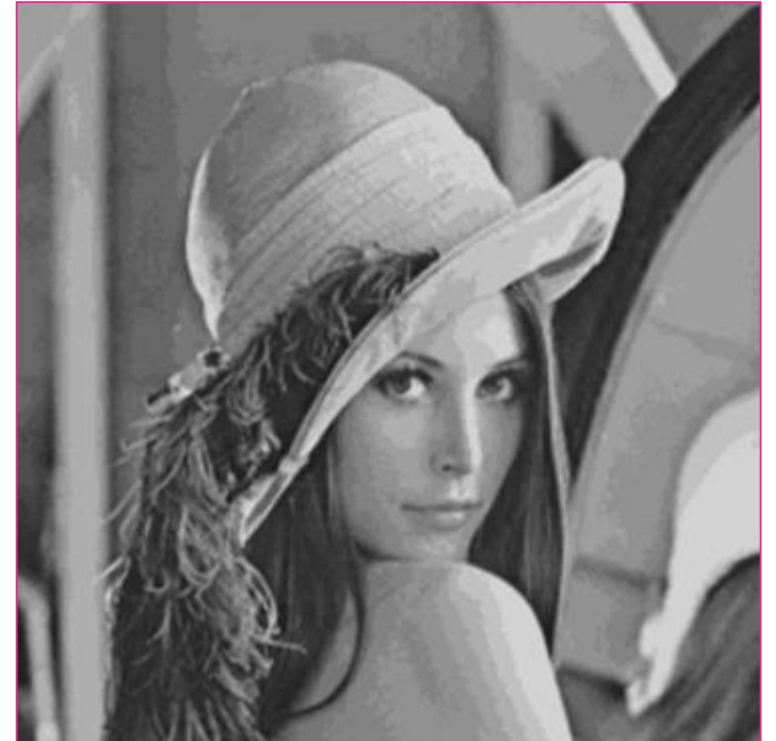
8 bpp



6 bpp



4 bpp



Conclusion

- Sampling
- Quantization

32x32



Conclusion

- Sampling
- Quantization

□ Sampling

- Squares
- Hexagonal
- Aliasing

□ Quantization

- Uniform
- Non-uniform
- Optimal

32x32

